

## 20.3: Resistance and Resistivity

### Learning Objectives

By the end of this section, you will be able to:

- Explain the concept of resistivity.
- Use resistivity to calculate the resistance of specified configurations of material.
- Use the thermal coefficient of resistivity to calculate the change of resistance with temperature.

### Material and Shape Dependence of Resistance

The resistance of an object depends on its shape and the material of which it is composed. The cylindrical resistor in Figure 1 is easy to analyze, and, by so doing, we can gain insight into the resistance of more complicated shapes. As you might expect, the cylinder's electric resistance  $R$  is directly proportional to its length  $L$ , similar to the resistance of a pipe to fluid flow. The longer the cylinder, the more collisions charges will make with its atoms. The greater the diameter of the cylinder, the more current it can carry (again similar to the flow of fluid through a pipe). In fact,  $R$  is inversely proportional to the cylinder's cross-sectional area  $A$ .

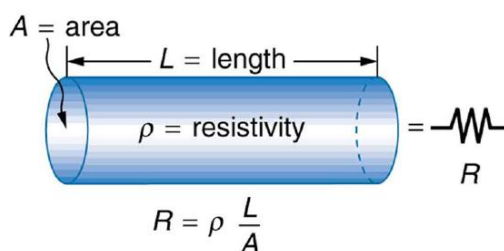


Figure 20.3.1: A uniform cylinder of length  $L$  and cross-sectional area  $A$ . Its resistance to the flow of current is similar to the resistance posed by a pipe to fluid flow. The longer the cylinder, the greater its resistance. The larger its cross-sectional area  $A$ , the smaller its resistance.

For a given shape, the resistance depends on the material of which the object is composed. Different materials offer different resistance to the flow of charge. We define the **resistivity**  $\rho$  of a substance so that the **resistance**  $R$  of an object is directly proportional to  $\rho$ . Resistivity  $\rho$  is an *intrinsic* property of a material, independent of its shape or size. The resistance  $R$  of a uniform cylinder of length  $L$ , of cross-sectional area  $A$ , and made of a material with resistivity  $\rho$ , is

$$R = \frac{\rho L}{A}. \quad (20.3.1)$$

The table below gives representative values of  $\rho$ . The materials listed in the table are separated into categories of conductors, semiconductors, and insulators, based on broad groupings of resistivities. Conductors have the smallest resistivities, and insulators have the largest; semiconductors have intermediate resistivities. Conductors have varying but large free charge densities, whereas most charges in insulators are bound to atoms and are not free to move. Semiconductors are intermediate, having far fewer free charges than conductors, but having properties that make the number of free charges depend strongly on the type and amount of impurities in the semiconductor. These unique properties of semiconductors are put to use in modern electronics, as will be explored in later chapters.

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Table 20.3.1: Resistivities  $\rho$  of Various materials at 20°C

Material	Resistivity $\rho$ ( $\Omega \cdot \text{m}$ )
Conductors	

Material	Resistivity $\rho$ ( $\Omega \cdot \text{m}$ )
Silver	$1.59 \times 10^{-8}$
Copper	$1.72 \times 10^{-8}$
Gold	$2.44 \times 10^{-8}$
Aluminum	$2.65 \times 10^{-8}$
Tungsten	$5.6 \times 10^{-8}$
Iron	$9.71 \times 10^{-8}$
Platinum	$10.6 \times 10^{-8}$
Steel	$20 \times 10^{-8}$
Lead	$22 \times 10^{-8}$
Manganin (Cu, Mn, Ni alloy)	$44 \times 10^{-8}$
Constantan (Cu, Ni alloy)	$49 \times 10^{-8}$
Mercury	$96 \times 10^{-8}$
Nichrome (Ni, Fe, Cr alloy)	$100 \times 10^{-8}$
<i>Semiconductors</i>	
Carbon (pure)	$3.5 \times 10^{-5}$
Carbon	$(3.5 - 6.0) \times 10^{-5}$
Germanium (pure)	$600 \times 10^{-3}$
Germanium	$(1 - 600) \times 10^{-3}$
Silicon (pure)	2300
Silicon	0.1–2300
<i>Insulators</i>	
Amber	$5 \times 10^{14}$
Glass	$10^9 - 10^{14}$
Lucite	$> 10^{13}$
Mica	$10^{11} - 10^{15}$
Quartz (fused)	$75 \times 10^{16}$
Rubber (hard)	$10^{13} - 10^{16}$
Sulfur	$10^{15}$
Teflon	$> 10^{13}$
Wood	$10^8 - 10^{11}$

### Example 20.3.1: Calculating Resistor Diameter: A Headlight Filament

A car headlight filament is made of tungsten and has a cold resistance of **0.350  $\Omega$** . If the filament is a cylinder 4.00 cm long (it may be coiled to save space), what is its diameter?

#### Strategy

We can rearrange the equation  $R = \frac{\rho L}{A}$  to find the cross-sectional area  $A$  of the filament from the given information. Then its diameter can be found by assuming it has a circular cross-section.

### Solution

The cross-sectional area, found by rearranging the expression for the resistance of a cylinder given in  $R = \frac{\rho L}{A}$ , is

$$A = \frac{\rho L}{R}.$$

Substituting the given values, and taking  $\rho$  from Table 20.3.1, yields

$$A = \frac{(5.6 \times 10^{-8} \Omega \cdot m)(4.00 \times 10^{-2} m)}{0.350 \Omega} = 6.40 \times 10^{-9} m^2.$$

The area of a circle is related to its diameter  $D$  by

$$A = \frac{\pi D^2}{4}.$$

Solving for the diameter  $D$ , and substituting the value found for  $A$ , gives

$$D = 2\left(\frac{A}{\pi}\right)^{\frac{1}{2}} = 2\left(\frac{6.40 \times 10^{-9} m^2}{3.14}\right)^{\frac{1}{2}} = 9.0 \times 10^{-5} m.$$

### Discussion

The diameter is just under a tenth of a millimeter. It is quoted to only two digits, because  $\rho$  is known to only two digits.

## Temperature Variation of Resistance

The resistivity of all materials depends on temperature. Some even become superconductors (zero resistivity) at very low temperatures. (See Figure 2.) Conversely, the resistivity of conductors increases with increasing temperature. Since the atoms vibrate more rapidly and over larger distances at higher temperatures, the electrons moving through a metal make more collisions, effectively making the resistivity higher. Over relatively small temperature changes (about  $100^\circ C$  or less), resistivity  $\rho$  varies with temperature change  $\Delta T$  as expressed in the following equation

$$\rho = \rho_0 (1 + \alpha \Delta T), \quad (20.3.2)$$

where  $\rho_0$  is the original resistivity and  $\alpha$  is the **temperature coefficient of resistivity**. (See the values of  $\alpha$  in the table below.) For larger temperature changes,  $\alpha$  may vary or a nonlinear equation may be needed to find  $\rho$ . Note that  $\alpha$  is positive for metals, meaning their resistivity increases with temperature. Some alloys have been developed specifically to have a small temperature dependence. Manganin (which is made of copper, manganese and nickel), for example, has  $\alpha$  close to zero (to three digits on the scale in the table), and so its resistivity varies only slightly with temperature. This is useful for making a temperature-independent resistance standard, for example.

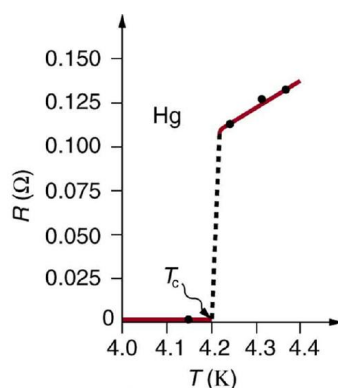


Figure 20.3.2: The resistance of a sample of mercury is zero at very low temperatures—it is a superconductor up to about 4.2 K. Above that critical temperature, its resistance makes a sudden jump and then increases nearly linearly with temperature.

Table 20.3.2: Temperature Coefficients of Resistivity  $\alpha$

Material	Coefficient $\alpha(1/^\circ C)$
Conductors	

Material	Coefficient $\alpha(1/^{\circ}\text{C})$
Silver	$3.8 \times 10^{-3}$
Copper	$3.9 \times 10^{-3}$
Gold	$3.4 \times 10^{-3}$
Aluminum	$3.9 \times 10^{-3}$
Tungsten	$4.5 \times 10^{-3}$
Iron	$5.0 \times 10^{-3}$
Platinum	$3.93 \times 10^{-3}$
Lead	$3.9 \times 10^{-3}$
Manganin (Cu, Mn, Ni alloy)	$0.000 \times 10^{-3}$
Constantan (Cu, Ni, alloy)	$0.002 \times 10^{-3}$
Mercury	$0.89 \times 10^{-3}$
Nichrome (Ni, Fe, Cr alloy)	$0.4 \times 10^{-3}$
<i>Semiconductors</i>	
Carbon (pure)	$-0.5 \times 10^{-3}$
Germanium (pure)	$-50 \times 10^{-3}$
Silicon (pure)	$-70 \times 10^{-3}$

Note also that  $\alpha$  is negative for the semiconductors listed in the table, meaning that their resistivity decreases with increasing temperature. They become better conductors at higher temperature, because increased thermal agitation increases the number of free charges available to carry current. This property of decreasing  $\rho$  with temperature is also related to the type and amount of impurities present in the semiconductors.

The resistance of an object also depends on temperature, since  $R_0$  is directly proportional to  $\rho$ . For a cylinder we know  $R = \rho L/A$ , and so, if  $L$  and  $A$  do not change greatly with temperature,  $R$  will have the same temperature dependence as  $\rho$ . (Examination of the coefficients of linear expansion shows them to be about two orders of magnitude less than typical temperature coefficients of resistivity, and so the effect of temperature on  $L$  and  $A$  is about two orders of magnitude less than on  $\rho$ .) Thus,

$$R = R_0 (1 + \alpha \Delta T) \quad (20.3.3)$$

is the temperature dependence of the resistance of an object, where  $R_0$  is the original resistance and  $R$  is the resistance after a temperature change  $\Delta T$ . Numerous thermometers are based on the effect of temperature on resistance. (See Figure 3.) One of the most common is the thermistor, a semiconductor crystal with a strong temperature dependence, the resistance of which is measured to obtain its temperature. The device is small, so that it quickly comes into thermal equilibrium with the part of a person it touches.



Figure 20.3.3: These familiar thermometers are based on the automated measurement of a thermistor's temperature-dependent resistance. (credit: Biol, Wikimedia Commons)

#### Example 20.3.2: Calculating Resistance: Hot-Filament Resistance:

Although caution must be used in applying  $\rho = \rho_0 (1 + \alpha \Delta T)$  and  $R = R_0 (1 + \alpha \Delta T)$  for temperature changes greater than  $100^{\circ}\text{C}$ , for tungsten the equations work reasonably well for very large temperature changes. What, then, is the resistance of the

tungsten filament in the previous example if its temperature is increased from room temperature ( $20^{\circ}\text{C}$ ) to a typical operating temperature of  $2850^{\circ}\text{C}$ ?

### Strategy

This is a straightforward application of  $R = R_0 (1 + \alpha \Delta T)$ , since the original resistance of the filament was given to be  $R_0 = 0.350\Omega$ , and the temperature change is  $\Delta T = 2830^{\circ}\text{C}$ .

### Solution

The hot resistance  $R$  is obtained by entering known values into the above equation:

$$\begin{aligned} R &= R_0 (1 + \alpha \Delta T) \\ &= (0.350\Omega) [1 + (4.5 \times 10^{-3}/^{\circ}\text{C})] \\ &= 4.8\Omega \end{aligned}$$

### Discussion

This value is consistent with the headlight resistance example in 20.3.

## PHET EXPLORATIONS: RESISTANCE IN A WIRE

Learn about the physics of resistance in a wire. Change its resistivity, length, and area to see how they affect the wire's resistance. The sizes of the symbols in the equation change along with the diagram of a wire.



## PhET Interactive Simulation

Figure 20.3.4: [Resistance in a Wire](#)

## Summary

- The resistance  $R$  of a cylinder of length  $L$  and cross-sectional area  $A$  is  $R = \frac{\rho L}{A}$ , where  $\rho$  is the resistivity of the material.
- Values of  $\rho$  in the Table show that materials fall into three groups—conductors, semiconductors, and insulators.
- Temperature affects resistivity; for relatively small temperature changes  $\Delta T$ , resistivity is  $\rho = \rho_0 (1 + \alpha \Delta T)$ , where  $\rho_0$  is the original resistivity and  $\alpha$  is the temperature coefficient of resistivity.
- The Table gives values for  $\alpha$ , the temperature coefficient of resistivity.
- The resistance  $R$  of an object also varies with temperature:  $R = R_0 (1 + \alpha \Delta T)$ , where  $R_0$  is the original resistance, and  $R$  is the resistance after the temperature change.

## Footnotes

1 Values depend strongly on amounts and types of impurities

2 Values at  $20^{\circ}\text{C}$ .

## Glossary

### resistivity

an intrinsic property of a material, independent of its shape or size, directly proportional to the resistance, denoted by  $\rho$

### temperature coefficient of resistivity

an empirical quantity, denoted by  $\alpha$ , which describes the change in resistance or resistivity of a material with temperature

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