16.11: Energy in Waves- Intensity

Learning Objectives

By the end of this section, you will be able to:

- Calculate the intensity and the power of rays and waves.

All waves carry energy. The energy of some waves can be directly observed. Earthquakes can shake whole cities to the ground, performing the work of thousands of wrecking balls. Loud sounds pulverize nerve cells in the inner ear, causing permanent hearing loss. Ultrasound is used for deep-heat treatment of muscle strains. A laser beam can burn away a malignancy. Water waves chew up beaches.

Figure 16.11.1: The destructive effect of an earthquake is palpable evidence of the energy carried in these waves. The Richter scale rating of earthquakes is related to both their amplitude and the energy they carry. (credit: Petty Officer 2nd Class Candice Villarreal, U.S. Navy)

The amount of energy in a wave is related to its amplitude. Large-amplitude earthquakes produce large ground displacements. Loud sounds have higher pressure amplitudes and come from larger-amplitude source vibrations than soft sounds. Large ocean breakers churn up the shore more than small ones. More quantitatively, a wave is a displacement that is resisted by a restoring force. The larger the displacement x the larger the force $F = kx$ needed to create it. Because work W is related to force multiplied by distance (F_x) and energy is put into the wave by the work done to create it, the energy in a wave is related to amplitude. In fact, a wave's energy is directly proportional to its amplitude squared because

$$W \propto F_x = kx^2. \tag{16.11.1}$$

The energy effects of a wave depend on time as well as amplitude. For example, the longer deep-heat ultrasound is applied, the more energy it transfers. Waves can also be concentrated or spread out. Sunlight, for example, can be focused to burn wood. Earthquakes spread out, so they do less damage the farther they get from the source. In both cases, changing the area the waves cover has important effects. All these pertinent factors are included in the definition of **intensity** I as power per unit area:

$$I = \frac{P}{A} \tag{16.11.2}$$

where P is the power carried by the wave through area A. The definition of intensity is valid for any energy in transit, including that carried by waves. The SI unit for intensity is watts per square meter (W/m^2). For example, infrared and visible energy from the Sun impinge on Earth at an intensity of $1300\, W/m^2$ just above the atmosphere. There are other intensity-related units in use, too. The most common is the decibel. For example, a 90 decibel sound level corresponds to an intensity of $10^{-3}\, W/m^2$ (This quantity is not much power per unit area considering that 90 decibels is a relatively high sound level. Decibels will be discussed in some detail in a later chapter.

Example 16.11.1: Calculating intensity and power: How much energy is in a ray of sunlight?

The average intensity of sunlight on Earth's surface is about $700\, W/m^2$.

a. Calculate the amount of energy that falls on a solar collector having an area of $0.500\, m^2$ in $4.00\, h$.
b. What intensity would such sunlight have if concentrated by a magnifying glass onto an area 200 times smaller than its own?

Strategy a

Because power is energy per unit time or $P = \frac{E}{t}$, the definition of intensity can be written as $I = \frac{P}{A} = \frac{E/t}{A}$, and this equation can be solved for E with the given information.

Solution a

1. Begin with the equation that states the definition of intensity:

$$I = \frac{P}{A}.$$

2. Replace P with its equivalent E/t:

$$I = \frac{E/t}{A}.$$

3. Solve for E:

$$E = IAt.$$

4. Substitute known values into the equation:

$$E = (700\,W/m^2)(0.500\,m^2)[(4.00\,h)(3600\,s/h)].$$

5. Calculate to find E and convert units:

$$5.04 \times 10^6\,J.$$

Discussion a

The energy falling on the solar collector in 4 h in part is enough to be useful—for example, for heating a significant amount of water.

Strategy b

Taking a ratio of new intensity to old intensity and using primes for the new quantities, we will find that it depends on the ratio of the areas. All other quantities will cancel.

Solution b

1. Take the ratio of intensities, which yields:

$$\frac{I'}{I} = \frac{P'A'}{P/A} = \frac{A}{A'}$$

The powers cancel because $P' = P$.

2. Identify the knowns:

$$A = 200A', \tag{16.11.3}$$

$$\frac{I'}{I} = 200.$$

3. Substitute known quantities:

$$I' = 200I = 200(700\,W/m^2).$$

4. Calculate to find I':

$$I' = 1.40 \times 10^5\,W/m^2.$$

Discussion b

Decreasing the area increases the intensity considerably. The intensity of the concentrated sunlight could even start a fire.

Example 16.11.2: Determine the combined intensity of two waves: Perfect constructive interference

If two identical waves, each having an intensity of $1.00\, W/m^2$,

interfere perfectly constructively, what is the intensity of the resulting wave?

Strategy

We know from Superposition and Interference that when two identical waves, which have equal amplitudes X interfere perfectly constructively, the resulting wave has an amplitude of $2X$. Because a wave's intensity is proportional to amplitude squared, the intensity of the resulting wave is four times as great as in the individual waves.

Solution

1. Recall that intensity is proportional to amplitude squared.
2. Calculate the new amplitude:

$$I' \propto (X')^2 = (2X)^2 = 4X^2. \tag{16.11.4}$$

3. Recall that the intensity of the old amplitude was:

$$I \propto X^2. \tag{16.11.5}$$

4. Take the ratio of new intensity to the old intensity. This gives:

$$\frac{I'}{I} = 4. \tag{16.11.6}$$

5. Calculate to find I':

$$I' = 4I = 4.00\, W/m^2. \tag{16.11.7}$$

Discussion

The intensity goes up by a factor of 4 when the amplitude doubles. This answer is a little disquieting. The two individual waves each have intensities of $1.00\, W/m^2$, yet their sum has an intensity of $4.00\, W/m^2$, which may appear to violate conservation of energy. This violation, of course, cannot happen. What does happen is intriguing. The area over which the intensity is $4.00\, W/m^2$ is much less than the area covered by the two waves before they interfered. There are other areas where the intensity is zero. The addition of waves is not as simple as our first look in Superposition and Interference suggested. We actually get a pattern of both constructive interference and destructive interference whenever two waves are added. For example, if we have two stereo speakers putting out $1.00\, W/m^2$ each, there will be places in the room where the intensity is $4.00\, W/m^2$, other places where the intensity is zero, and others in between. Figure $16.11.2$shows what this interference might look like. We will pursue interference patterns elsewhere in this text.

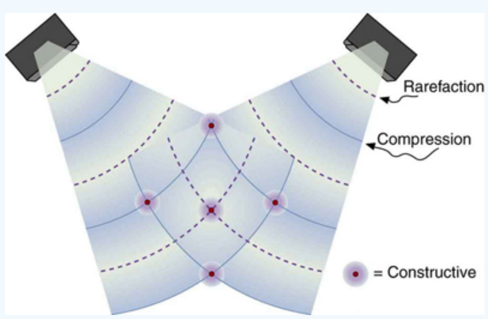

Figure 16.11.2: These stereo speakers produce both constructive interference and destructive interference in the room, a property common to the superposition of all types of waves. The shading is proportional to intensity.

Exercise 16.11.1

Which measurement of a wave is most important when determining the wave's intensity?

Answer

Amplitude, because a wave's energy is directly proportional to its amplitude squared.

Summary

- Intensity is defined to be the power per unit area: $I = \frac{P}{A}$ and has units of W/m^2.

Glossary

intensity

power per unit area