

17.E: Physics of Hearing (Exercises)

Conceptual Questions

17.2: Speed of Sound, Frequency, and Wavelength

1. How do sound vibrations of atoms differ from thermal motion?
2. When sound passes from one medium to another where its propagation speed is different, does its frequency or wavelength change? Explain your answer briefly.

17.3: Sound Intensity and Sound Level

3. Six members of a synchronized swim team wear earplugs to protect themselves against water pressure at depths, but they can still hear the music and perform the combinations in the water perfectly. One day, they were asked to leave the pool so the dive team could practice a few dives, and they tried to practice on a mat, but seemed to have a lot more difficulty. Why might this be?
4. A community is concerned about a plan to bring train service to their downtown from the town's outskirts. The current sound intensity level, even though the rail yard is blocks away, is 70 dB downtown. The mayor assures the public that there will be a difference of only 30 dB in sound in the downtown area. Should the townspeople be concerned? Why?

17.4: Doppler Effect and Sonic Booms

5. Is the Doppler shift real or just a sensory illusion?
6. Due to efficiency considerations related to its bow wake, the supersonic transport aircraft must maintain a cruising speed that is a constant ratio to the speed of sound (a constant Mach number). If the aircraft flies from warm air into colder air, should it increase or decrease its speed? Explain your answer.
7. When you hear a sonic boom, you often cannot see the plane that made it. Why is that?

17.5: Sound Interference and Resonance: Standing Waves in Air Columns

8. How does an unamplified guitar produce sounds so much more intense than those of a plucked string held taut by a simple stick?
9. You are given two wind instruments of identical length. One is open at both ends, whereas the other is closed at one end. Which is able to produce the lowest frequency?
10. What is the difference between an overtone and a harmonic? Are all harmonics overtones? Are all overtones harmonics?

17.6: Hearing

11. Why can a hearing test show that your threshold of hearing is 0 dB at 250 Hz, when Figure implies that no one can hear such a frequency at less than 20 dB?

17.7: Ultrasound

12. If audible sound follows a rule of thumb similar to that for ultrasound, in terms of its absorption, would you expect the high or low frequencies from your neighbor's stereo to penetrate into your house? How does this expectation compare with your experience?
13. Elephants and whales are known to use infrasound to communicate over very large distances. What are the advantages of infrasound for long distance communication?
14. It is more difficult to obtain a high-resolution ultrasound image in the abdominal region of someone who is overweight than for someone who has a slight build. Explain why this statement is accurate.
15. Suppose you read that 210-dB ultrasound is being used to pulverize cancerous tumors. You calculate the intensity in watts per centimeter squared and find it is unreasonably high (10^5 W/cm^2). What is a possible explanation?

Problems & Exercises

17.2: Speed of Sound, Frequency, and Wavelength

16. When poked by a spear, an operatic soprano lets out a 1200-Hz shriek. What is its wavelength if the speed of sound is 345 m/s?

Solution

0.288 m

17. What frequency sound has a 0.10-m wavelength when the speed of sound is 340 m/s?
18. Calculate the speed of sound on a day when a 1500 Hz frequency has a wavelength of 0.221 m.

Solution

332 m/s

19. (a) What is the speed of sound in a medium where a 100-kHz frequency produces a 5.96-cm wavelength?
- (b) Which substance in Table is this likely to be?
20. Show that the speed of sound in 20.0°C air is 343 m/s, as claimed in the text.

Solution

$$v_w = (331 \text{ m/s}) \sqrt{\frac{T}{273 \text{ K}}} = (331 \text{ m/s}) \sqrt{\frac{293 \text{ K}}{273 \text{ K}}} = 343 \text{ m/s}$$

21. Air temperature in the Sahara Desert can reach 56.0°C (about 134°F). What is the speed of sound in air at that temperature?
22. Dolphins make sounds in air and water. What is the ratio of the wavelength of a sound in air to its wavelength in seawater? Assume air temperature is 20.0°C.

Solution

0.223

23. A sonar echo returns to a submarine 1.20 s after being emitted. What is the distance to the object creating the echo? (Assume that the submarine is in the ocean, not in fresh water.)
24. (a) If a submarine's sonar can measure echo times with a precision of 0.0100 s, what is the smallest difference in distances it can detect? (Assume that the submarine is in the ocean, not in fresh water.)
- (b) Discuss the limits this time resolution imposes on the ability of the sonar system to detect the size and shape of the object creating the echo.

Solution

(a) 7.70 m

(b) This means that sonar is good for spotting and locating large objects, but it isn't able to resolve smaller objects, or detect the detailed shapes of objects. Objects like ships or large pieces of airplanes can be found by sonar, while smaller pieces must be found by other means.

25. A physicist at a fireworks display times the lag between seeing an explosion and hearing its sound, and finds it to be 0.400 s.
- (a) How far away is the explosion if air temperature is 24.0°C and if you neglect the time taken for light to reach the physicist?
- (b) Calculate the distance to the explosion taking the speed of light into account. Note that this distance is negligibly greater.
26. Suppose a bat uses sound echoes to locate its insect prey, 3.00 m away. (See Figure.)
- (a) Calculate the echo times for temperatures of 5.00°C and 35.0°C.
- (b) What percent uncertainty does this cause for the bat in locating the insect?

(c) Discuss the significance of this uncertainty and whether it could cause difficulties for the bat. (In practice, the bat continues to use sound as it closes in, eliminating most of any difficulties imposed by this and other effects, such as motion of the prey.)

Solution

(a) 18.0 ms, 17.1 ms

(b) 5.00%

(c) This uncertainty could definitely cause difficulties for the bat, if it didn't continue to use sound as it closed in on its prey. A 5% uncertainty could be the difference between catching the prey around the neck or around the chest, which means that it could miss grabbing its prey.

17.3: Sound Intensity and Sound Level

27. What is the intensity in watts per meter squared of 85.0-dB sound?

Solution

$$3.16 \times 10^{-4} \text{ W/m}^2$$

28. The warning tag on a lawn mower states that it produces noise at a level of 91.0 dB. What is this in watts per meter squared?

29. A sound wave traveling in 20°C air has a pressure amplitude of 0.5 Pa. What is the intensity of the wave?

Solution

$$3.04 \times 10^{-4} \text{ W/m}^2$$

30. What intensity level does the sound in the preceding problem correspond to?

31. What sound intensity level in dB is produced by earphones that create an intensity of $4.00 \times 10^{-2} \text{ W/m}^2$?

Solution

106 dB

32. Show that an intensity of 10^{-12} W/m^2 is the same as 10^{-16} W/cm^2 .

33. (a) What is the decibel level of a sound that is twice as intense as a 90.0-dB sound?

(b) What is the decibel level of a sound that is one-fifth as intense as a 90.0-dB sound?

Solution

(a) 93 dB

(b) 83 dB

34. (a) What is the intensity of a sound that has a level 7.00 dB lower than a $4.00 \times 10^{-9} \text{ W/m}^2$ sound?

(b) What is the intensity of a sound that is 3.00 dB higher than a $4.00 \times 10^{-9} \text{ W/m}^2$ sound?

35. (a) How much more intense is a sound that has a level 17.0 dB higher than another?

(b) If one sound has a level 23.0 dB less than another, what is the ratio of their intensities?

Solution

(a) 50.1

(b) 5.01×10^{-3} or $\frac{1}{200}$

36. People with good hearing can perceive sounds as low in level as -8.00 dB at a frequency of 3000 Hz. What is the intensity of this sound in watts per meter squared?

37. If a large housefly 3.0 m away from you makes a noise of 40.0 dB, what is the noise level of 1000 flies at that distance, assuming interference has a negligible effect?

Solution

70.0 dB

38. Ten cars in a circle at a boom box competition produce a 120-dB sound intensity level at the center of the circle. What is the average sound intensity level produced there by each stereo, assuming interference effects can be neglected?

39. The amplitude of a sound wave is measured in terms of its maximum gauge pressure. By what factor does the amplitude of a sound wave increase if the sound intensity level goes up by 40.0 dB?

Solution

100

40. If a sound intensity level of 0 dB at 1000 Hz corresponds to a maximum gauge pressure (sound amplitude) of 10^{-9} atm, what is the maximum gauge pressure in a 60-dB sound? What is the maximum gauge pressure in a 120-dB sound?

41. An 8-hour exposure to a sound intensity level of 90.0 dB may cause hearing damage. What energy in joules falls on a 0.800-cm-diameter eardrum so exposed?

Solution

$1.45 \times 10^{-3} J$

42. (a) Ear trumpets were never very common, but they did aid people with hearing losses by gathering sound over a large area and concentrating it on the smaller area of the eardrum. What decibel increase does an ear trumpet produce if its sound gathering area is 900cm^2 and the area of the eardrum is 0.500cm^2 , but the trumpet only has an efficiency of 5.00% in transmitting the sound to the eardrum?

(b) Comment on the usefulness of the decibel increase found in part (a).

43. Sound is more effectively transmitted into a stethoscope by direct contact than through the air, and it is further intensified by being concentrated on the smaller area of the eardrum. It is reasonable to assume that sound is transmitted into a stethoscope 100 times as effectively compared with transmission through the air. What, then, is the gain in decibels produced by a stethoscope that has a sound gathering area of 15.0cm^2 , and concentrates the sound onto two eardrums with a total area of 0.900cm^2 with an efficiency of 40.0%?

Solution

28.2 dB

44. Loudspeakers can produce intense sounds with surprisingly small energy input in spite of their low efficiencies. Calculate the power input needed to produce a 90.0-dB sound intensity level for a 12.0-cm-diameter speaker that has an efficiency of 1.00%. (This value is the sound intensity level right at the speaker.)

17.4: Doppler Effect and Sonic Booms

45. (a) What frequency is received by a person watching an oncoming ambulance moving at 110 km/h and emitting a steady 800-Hz sound from its siren? The speed of sound on this day is 345 m/s.

(b) What frequency does she receive after the ambulance has passed?

Solution

(a) 878 Hz

(b) 735 Hz

46. (a) At an air show a jet flies directly toward the stands at a speed of 1200 km/h, emitting a frequency of 3500 Hz, on a day when the speed of sound is 342 m/s. What frequency is received by the observers?

(b) What frequency do they receive as the plane flies directly away from them?

47. What frequency is received by a mouse just before being dispatched by a hawk flying at it at 25.0 m/s and emitting a screech of frequency 3500 Hz? Take the speed of sound to be 331 m/s.

Solution

$3.79 \times 10^3 \text{ Hz}$

48. A spectator at a parade receives an 888-Hz tone from an oncoming trumpeter who is playing an 880-Hz note. At what speed is the musician approaching if the speed of sound is 338 m/s?

49. A commuter train blows its 200-Hz horn as it approaches a crossing. The speed of sound is 335 m/s.

(a) An observer waiting at the crossing receives a frequency of 208 Hz. What is the speed of the train?

(b) What frequency does the observer receive as the train moves away?

Solution

- (a) 12.9 m/s
- (b) 193 Hz

50. Can you perceive the shift in frequency produced when you pull a tuning fork toward you at 10.0 m/s on a day when the speed of sound is 344 m/s? To answer this question, calculate the factor by which the frequency shifts and see if it is greater than 0.300%.

51. Two eagles fly directly toward one another, the first at 15.0 m/s and the second at 20.0 m/s. Both screech, the first one emitting a frequency of 3200 Hz and the second one emitting a frequency of 3800 Hz. What frequencies do they receive if the speed of sound is 330 m/s?

Solution

First eagle hears $4.23 \times 10^3 \text{ Hz}$
Second eagle hears $3.56 \times 10^3 \text{ Hz}$

52. What is the minimum speed at which a source must travel toward you for you to be able to hear that its frequency is Doppler shifted? That is, what speed produces a shift of 0.300% on a day when the speed of sound is 331 m/s?

17.5: Sound Interference and Resonance: Standing Waves in Air Columns

53. A “showy” custom-built car has two brass horns that are supposed to produce the same frequency but actually emit 263.8 and 264.5 Hz. What beat frequency is produced?

Solution

0.7 Hz

54. What beat frequencies will be present:

- (a) If the musical notes A and C are played together (frequencies of 220 and 264 Hz)?
- (b) If D and F are played together (frequencies of 297 and 352 Hz)?
- (c) If all four are played together?

55. What beat frequencies result if a piano hammer hits three strings that emit frequencies of 127.8, 128.1, and 128.3 Hz?

Solution

0.3 Hz, 0.2 Hz, 0.5 Hz

56. A piano tuner hears a beat every 2.00 s when listening to a 264.0-Hz tuning fork and a single piano string. What are the two possible frequencies of the string?

57. (a) What is the fundamental frequency of a 0.672-m-long tube, open at both ends, on a day when the speed of sound is 344 m/s?

- (b) What is the frequency of its second harmonic?

Solution

- (a) 256 Hz
- (b) 512 Hz

58. If a wind instrument, such as a tuba, has a fundamental frequency of 32.0 Hz, what are its first three overtones? It is closed at one end. (The overtones of a real tuba are more complex than this example, because it is a tapered tube.)

59. What are the first three overtones of a bassoon that has a fundamental frequency of 90.0 Hz? It is open at both ends. (The overtones of a real bassoon are more complex than this example, because its double reed makes it act more like a tube closed at one end.)

Solution

180 Hz, 270 Hz, 360 Hz

60. How long must a flute be in order to have a fundamental frequency of 262 Hz (this frequency corresponds to middle C on the evenly tempered chromatic scale) on a day when air temperature is **20.0°C**? It is open at both ends.

61. What length should an oboe have to produce a fundamental frequency of 110 Hz on a day when the speed of sound is 343 m/s? It is open at both ends.

Solution

1.56 m

62. What is the length of a tube that has a fundamental frequency of 176 Hz and a first overtone of 352 Hz if the speed of sound is 343 m/s?

63. (a) Find the length of an organ pipe closed at one end that produces a fundamental frequency of 256 Hz when air temperature is **18.0°C**.

(b) What is its fundamental frequency at **25.0°C**?

Solution

(a) 0.334 m

(b) 259 Hz

64. By what fraction will the frequencies produced by a wind instrument change when air temperature goes from **10.0°C** to **30.0°C**? That is, find the ratio of the frequencies at those temperatures.

65. The ear canal resonates like a tube closed at one end. (See [link].) If ear canals range in length from 1.80 to 2.60 cm in an average population, what is the range of fundamental resonant frequencies? Take air temperature to be **37.0°C**, which is the same as body temperature. How does this result correlate with the intensity versus frequency graph ([link] of the human ear?

Solution

3.39 to 4.90 kHz

66. Calculate the first overtone in an ear canal, which resonates like a 2.40-cm-long tube closed at one end, by taking air temperature to be **37.0°C**. Is the ear particularly sensitive to such a frequency? (The resonances of the ear canal are complicated by its nonuniform shape, which we shall ignore.)

67. A crude approximation of voice production is to consider the breathing passages and mouth to be a resonating tube closed at one end. (See Figure.)

(a) What is the fundamental frequency if the tube is 0.240-m long, by taking air temperature to be **37.0°C**?

(b) What would this frequency become if the person replaced the air with helium? Assume the same temperature dependence for helium as for air.

Solution

(a) 367 Hz

(b) 1.07 kHz

68. (a) Students in a physics lab are asked to find the length of an air column in a tube closed at one end that has a fundamental frequency of 256 Hz. They hold the tube vertically and fill it with water to the top, then lower the water while a 256-Hz tuning fork is rung and listen for the first resonance. What is the air temperature if the resonance occurs for a length of 0.336 m?

(b) At what length will they observe the second resonance (first overtone)?

69. What frequencies will a 1.80-m-long tube produce in the audible range at 20.0°C if:

(a) The tube is closed at one end?

(b) It is open at both ends?

Solution

(a) $f_n = n(47.6\text{ Hz})$, $n = 1, 3, 5, \dots, 419$

(b) $f_n = n(95.3\text{ Hz})$, $n = 1, 2, 3, \dots, 210$

17.6: Hearing

70. The factor of 10^{-12} in the range of intensities to which the ear can respond, from threshold to that causing damage after brief exposure, is truly remarkable. If you could measure distances over the same range with a single instrument and the

smallest distance you could measure was 1 mm, what would the largest be?

Solution

$$1 \times 10^6 \text{ km}$$

71. The frequencies to which the ear responds vary by a factor of 10^3 . Suppose the speedometer on your car measured speeds differing by the same factor of 10^3 , and the greatest speed it reads is 90.0 mi/h. What would be the slowest nonzero speed it could read?

72. What are the closest frequencies to 500 Hz that an average person can clearly distinguish as being different in frequency from 500 Hz? The sounds are not present simultaneously.

Solution

498.5 or 501.5 Hz

73. Can the average person tell that a 2002-Hz sound has a different frequency than a 1999-Hz sound without playing them simultaneously?

74. If your radio is producing an average sound intensity level of 85 dB, what is the next lowest sound intensity level that is clearly less intense?

Solution

82 dB

75. Can you tell that your roommate turned up the sound on the TV if its average sound intensity level goes from 70 to 73 dB?

76. Based on the graph in Figure, what is the threshold of hearing in decibels for frequencies of 60, 400, 1000, 4000, and 15,000 Hz? Note that many AC electrical appliances produce 60 Hz, music is commonly 400 Hz, a reference frequency is 1000 Hz, your maximum sensitivity is near 4000 Hz, and many older TVs produce a 15,750 Hz whine.

Solution

approximately 48, 9, 0, -7, and 20 dB, respectively

77. What sound intensity levels must sounds of frequencies 60, 3000, and 8000 Hz have in order to have the same loudness as a 40-dB sound of frequency 1000 Hz (that is, to have a loudness of 40 phons)?

78. What is the approximate sound intensity level in decibels of a 600-Hz tone if it has a loudness of 20 phons? If it has a loudness of 70 phons?

Solution

(a) 23 dB

(b) 70 dB

79. (a) What are the loudnesses in phons of sounds having frequencies of 200, 1000, 5000, and 10,000 Hz, if they are all at the same 60.0-dB sound intensity level?

(b) If they are all at 110 dB?

(c) If they are all at 20.0 dB?

80. Suppose a person has a 50-dB hearing loss at all frequencies. By how many factors of 10 will low-intensity sounds need to be amplified to seem normal to this person? Note that smaller amplification is appropriate for more intense sounds to avoid further hearing damage.

Solution

Five factors of 10

81. If a woman needs an amplification of 5.0×10^{12} times the threshold intensity to enable her to hear at all frequencies, what is her overall hearing loss in dB? Note that smaller amplification is appropriate for more intense sounds to avoid further damage to her hearing from levels above 90 dB.

82. (a) What is the intensity in watts per meter squared of a just barely audible 200-Hz sound?

(b) What is the intensity in watts per meter squared of a barely audible 4000-Hz sound?

Solution

- (a) $2 \times 10^{-10} \text{ W/m}^2$
(b) $2 \times 10^{-13} \text{ W/m}^2$

83. (a) Find the intensity in watts per meter squared of a 60.0-Hz sound having a loudness of 60 phons.
(b) Find the intensity in watts per meter squared of a 10,000-Hz sound having a loudness of 60 phons.
84. A person has a hearing threshold 10 dB above normal at 100 Hz and 50 dB above normal at 4000 Hz. How much more intense must a 100-Hz tone be than a 4000-Hz tone if they are both barely audible to this person?

Solution

2.5

85. A child has a hearing loss of 60 dB near 5000 Hz, due to noise exposure, and normal hearing elsewhere. How much more intense is a 5000-Hz tone than a 400-Hz tone if they are both barely audible to the child?
86. What is the ratio of intensities of two sounds of identical frequency if the first is just barely discernible as louder to a person than the second?

Solution

1.26

17.7: Ultrasound

Unless otherwise indicated, for problems in this section, assume that the speed of sound through human tissues is 1540 m/s

87. What is the sound intensity level in decibels of ultrasound of intensity 105 W/m^2 , used to pulverize tissue during surgery?

Solution

170 dB

88. Is 155-dB ultrasound in the range of intensities used for deep heating? Calculate the intensity of this ultrasound and compare this intensity with values quoted in the text.
89. Find the sound intensity level in decibels of $2.00 \times 10^{-2} \text{ W/m}^2$ ultrasound used in medical diagnostics.

Solution

103 dB

90. The time delay between transmission and the arrival of the reflected wave of a signal using ultrasound traveling through a piece of fat tissue was 0.13 ms. At what depth did this reflection occur?
91. In the clinical use of ultrasound, transducers are always coupled to the skin by a thin layer of gel or oil, replacing the air that would otherwise exist between the transducer and the skin.
- (a) Using the values of acoustic impedance given in Table calculate the intensity reflection coefficient between transducer material and air.
- (b) Calculate the intensity reflection coefficient between transducer material and gel (assuming for this problem that its acoustic impedance is identical to that of water).
- (c) Based on the results of your calculations, explain why the gel is used.

Solution

- (a) 1.00
(b) 0.823
(c) Gel is used to facilitate the transmission of the ultrasound between the transducer and the patient's body.
92. (a) Calculate the minimum frequency of ultrasound that will allow you to see details as small as 0.250 mm in human tissue.
(b) What is the effective depth to which this sound is effective as a diagnostic probe?
93. (a) Find the size of the smallest detail observable in human tissue with 20.0-MHz ultrasound.
(b) Is its effective penetration depth great enough to examine the entire eye (about 3.00 cm is needed)?

(c) What is the wavelength of such ultrasound in 0°C air?

Solution

(a) $77.0\ \mu\text{m}$

(b) Effective penetration depth = $3.85\ \text{cm}$, which is enough to examine the eye.

(c) $16.6\ \mu\text{m}$

94. (a) Echo times are measured by diagnostic ultrasound scanners to determine distances to reflecting surfaces in a patient. What is the difference in echo times for tissues that are 3.50 and $3.60\ \text{cm}$ beneath the surface? (This difference is the minimum resolving time for the scanner to see details as small as $0.100\ \text{cm}$, or $1.00\ \text{mm}$. Discrimination of smaller time differences is needed to see smaller details.)

(b) Discuss whether the period T of this ultrasound must be smaller than the minimum time resolution. If so, what is the minimum frequency of the ultrasound and is that out of the normal range for diagnostic ultrasound?

95. (a) How far apart are two layers of tissue that produce echoes having round-trip times (used to measure distances) that differ by $0.750\ \mu\text{s}$?

(b) What minimum frequency must the ultrasound have to see detail this small?

Solution

(a) $5.78 \times 10^{-4}\ \text{m}$

(b) $2.67 \times 10^6\ \text{Hz}$

96. (a) A bat uses ultrasound to find its way among trees. If this bat can detect echoes $1.00\ \text{ms}$ apart, what minimum distance between objects can it detect?

(b) Could this distance explain the difficulty that bats have finding an open door when they accidentally get into a house?

97. A dolphin is able to tell in the dark that the ultrasound echoes received from two sharks come from two different objects only if the sharks are separated by $3.50\ \text{m}$, one being that much farther away than the other.

(a) If the ultrasound has a frequency of $100\ \text{kHz}$, show this ability is not limited by its wavelength.

(b) If this ability is due to the dolphin's ability to detect the arrival times of echoes, what is the minimum time difference the dolphin can perceive?

Solution

$$(a) v_w = 1540\ \text{m/s} = f\lambda \Rightarrow \lambda = \frac{1540\ \text{m/s}}{100 \times 10^3\ \text{Hz}} = 0.0154\ \text{m} < 3.50\ \text{m}.$$

Because the wavelength is much shorter than the distance in question, the wavelength is not the limiting factor.

(b) $4.55\ \text{ms}$

98. A diagnostic ultrasound echo is reflected from moving blood and returns with a frequency $500\ \text{Hz}$ higher than its original $2.00\ \text{MHz}$. What is the velocity of the blood? (Assume that the frequency of $2.00\ \text{MHz}$ is accurate to seven significant figures and $500\ \text{Hz}$ is accurate to three significant figures.)

99. Ultrasound reflected from an oncoming bloodstream that is moving at $30.0\ \text{cm/s}$ is mixed with the original frequency of $2.50\ \text{MHz}$ to produce beats. What is the beat frequency? (Assume that the frequency of $2.50\ \text{MHz}$ is accurate to seven significant figures.)

Solution

$974\ \text{Hz}$

(Note: extra digits were retained in order to show the difference.)

Contributors and Attributions

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