Physics

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CHAPTER OVERVIEW

1: The Basics of Physics

Topic hierarchy

1.1: The Basics of Physics

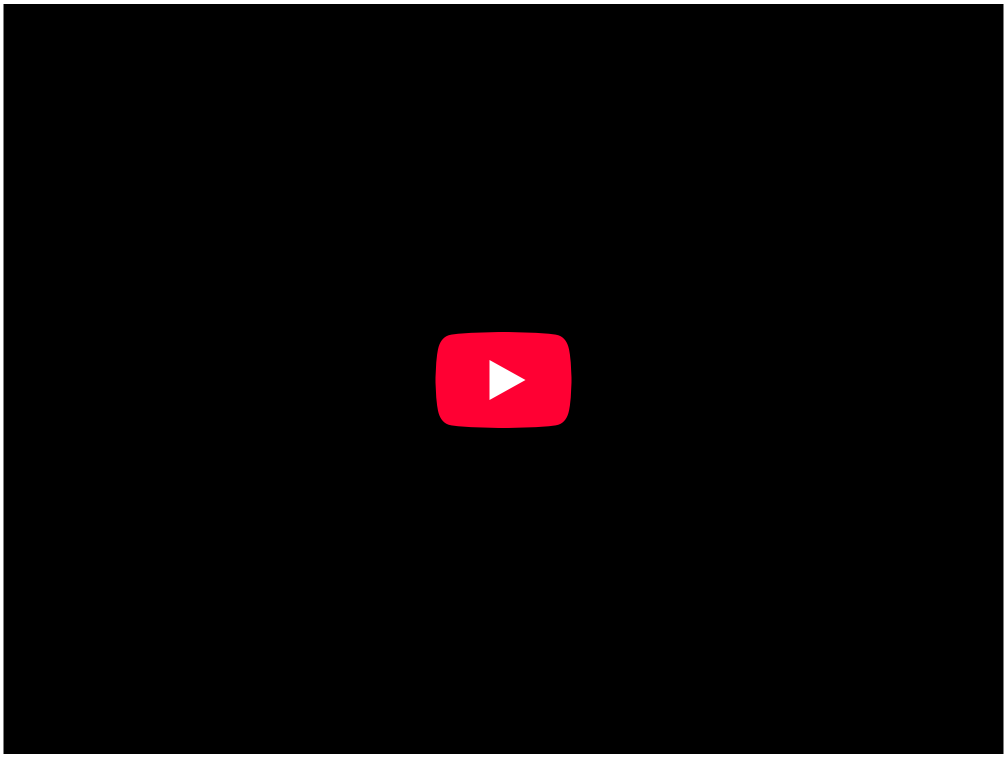
Introduction: Physics and Matter

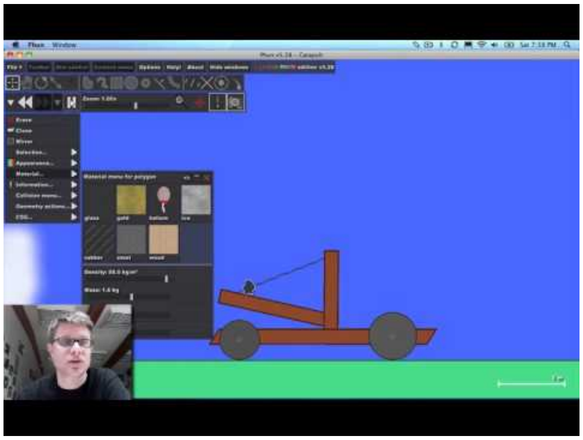
Physics is a study of how the universe behaves.

learning objectives

- Apply physics to describe the function of daily life

Physics is a natural science that involves the study of matter and its motion through space and time, along with related concepts such as energy and force. More broadly, it is the study of nature in an attempt to understand how the universe behaves.



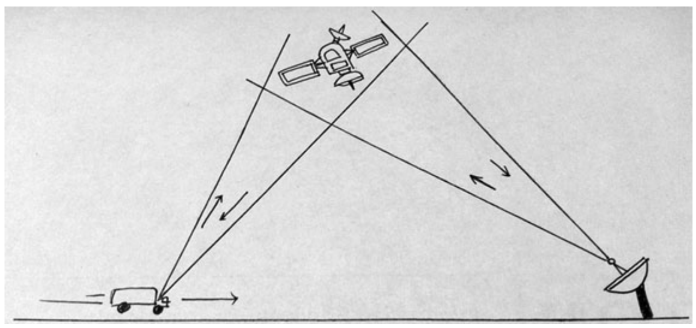


What is Physics?: Mr. Andersen explains the importance of physics as a science. History and virtual examples are used to give the discipline context.

Physics uses the scientific method to help uncover the basic principles governing light and matter, and to discover the implications of those laws. It assumes that there are rules by which the universe functions, and that those laws can be at least partially understood by humans. It is also commonly believed that those laws could be used to predict everything about the universe's future if complete information was available about the present state of all light and matter.

Matter is generally considered to be anything that has mass and volume. Many concepts integral to the study of classical physics involve theories and laws that explain matter and its motion. The law of conservation of mass, for example, states that mass cannot be created or destroyed. Further experiments and calculations in physics, therefore, take this law into account when formulating hypotheses to try to explain natural phenomena.

Physics aims to describe the function of everything around us, from the movement of tiny charged particles to the motion of people, cars, and spaceships. In fact, almost everything around you can be described quite accurately by the laws of physics. Consider a smart phone; physics describes how electricity interacts with the various circuits inside the device. This knowledge helps engineers select the appropriate materials and circuit layout when building the smart phone. Next, consider a GPS system; physics describes the relationship between the speed of an object, the distance over which it travels, and the time it takes to travel that distance. When you use a GPS device in a vehicle, it utilizes these physics equations to determine the travel time from one location to another. The study of physics is capable of making significant contributions through advances in new technologies that arise from theoretical breakthroughs.



Global Positioning System: GPS calculates the speed of an object, the distance over which it travels, and the time it takes to travel that distance using equations based on the laws of physics.

Physics and Other Fields

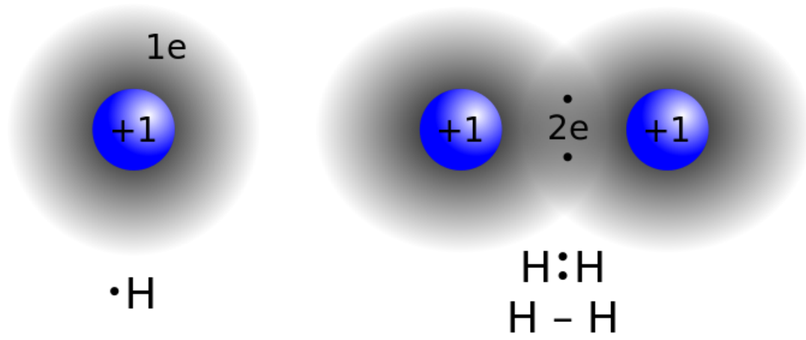
Physics is the foundation of many disciplines and contributes directly to chemistry, astronomy, engineering, and most scientific fields.

learning objectives

- Explain why the study of physics is integral to the study of other sciences

Physics and Other Disciplines

Physics is the foundation of many important disciplines and contributes directly to others. Chemistry deals with the interactions of atoms and molecules, so it is rooted in atomic and molecular physics. Most branches of engineering are applied physics. In architecture, physics is at the heart of structural stability and is involved in acoustics, heating, lighting, and the cooling of buildings. Parts of geology rely heavily on physics, such as the radioactive dating of rocks, earthquake analysis, and heat transfer in the Earth. Some disciplines, such as biophysics and geophysics, are hybrids of physics and other disciplines.



Physics in Chemistry: The study of matter and electricity in physics is fundamental towards the understanding of concepts in chemistry, such as the covalent bond.

Physics has many applications in the biological sciences. On the microscopic level, it helps describe the properties of cell walls and cell membranes. On the macroscopic level, it can explain the heat, work, and power associated with the human body. Physics is involved in medical diagnostics, such as X-rays, magnetic resonance imaging (MRI), and ultrasonic blood flow measurements. Medical therapy sometimes directly involves physics: cancer radiotherapy uses ionizing radiation, for instance. Physics can also explain sensory phenomena, such as how musical instruments make sound, how the eye detects color, and how lasers can transmit information.

The boundary between physics and the other sciences is not always clear. For instance, chemists study atoms and molecules, which are what matter is built from, and there are some scientists who would be equally willing to call themselves physical chemists or chemical physicists. It might seem that the distinction between physics and biology would be clearer, since physics seems to deal with inanimate objects. In fact, almost all physicists would agree that the basic laws of physics that apply to molecules in a test tube work equally well for the combination of molecules that constitutes a bacterium. What differentiates physics from biology is that many of the scientific theories that describe living things ultimately result from the fundamental laws of physics, but cannot be rigorously derived from physical principles.

It is not necessary to formally study all applications of physics. What is most useful is the knowledge of the basic laws of physics and skill in the analytical methods for applying them. The study of physics can also improve your problem-solving skills. Furthermore, physics has retained the most basic aspects of science, so it is used by all of the sciences. The study of physics makes other sciences easier to understand.

Models, Theories, and Laws

The terms *model*, *theory*, and *law* have exact meanings in relation to their usage in the study of physics.

learning objectives

- Define the terms model, theory, and law

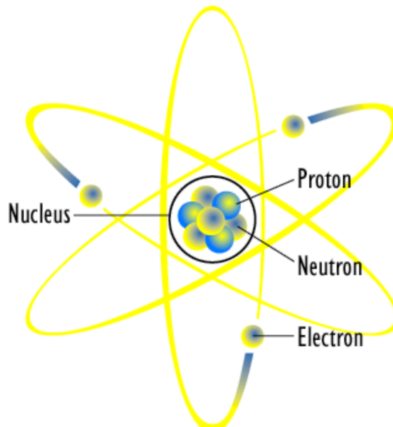
Definition of Terms: Model, Theory, Law

In colloquial usage, the terms *model*, *theory*, and *law* are often used interchangeably or have different interpretations than they do in the sciences. In relation to the study of physics, however, each term has its own specific meaning.

The *laws of nature* are concise descriptions of the universe around us. They are not explanations, but human statements of the underlying rules that all natural processes follow. They are intrinsic to the universe; humans did not create them and we cannot change them. We can only discover and understand them. The cornerstone of discovering natural laws is observation; science must describe the universe as it is, not as we may imagine it to be. Laws can never be known with absolute certainty, because it is impossible to perform experiments to establish and confirm a law in every possible scenario without exception. Physicists operate under the assumption that all scientific laws and theories are valid until a counterexample is observed. If a good-quality, verifiable experiment contradicts a well-established law, then the law must be modified or overthrown completely.

Models

A *model* is a representation of something that is often too difficult (or impossible) to display directly. While a model's design is justified using experimental information, it is only accurate under limited situations. An example is the commonly used "planetary model" of the atom, in which electrons are pictured as orbiting the nucleus, analogous to the way planets orbit the Sun. We cannot observe electron orbits directly, but the mental image helps explain the observations we can make, such as the emission of light from hot gases. Physicists use models for a variety of purposes. For example, models can help physicists analyze a scenario and perform a calculation, or they can be used to represent a situation in the form of a computer simulation.



Planetary Model of an Atom: The planetary model of the atom in which electrons are pictured as orbiting the nucleus, analogous to the way planets orbit the Sun

Theories

A *theory* is an explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers. *Some theories include models to help visualize phenomena, whereas others do not*. Newton's theory of gravity, for example, does not require a model or mental image, because we can observe the objects directly with our own senses. The kinetic theory of gases, on the other hand, makes use of a model in which a gas is viewed as being composed of atoms and molecules. Atoms and molecules are too small to be observed directly with our senses—thus, we picture them mentally to understand what our instruments tell us about the behavior of gases.

Laws

A law uses concise language to describe a generalized pattern in nature that is supported by scientific evidence and repeated experiments. Often, a law can be expressed in the form of a single mathematical equation. Laws and theories are similar in that they are both scientific statements that result from a tested hypothesis and are supported by scientific evidence. However, the designation law is reserved for a concise and very general statement that describes phenomena in nature, such as the law that energy is conserved during any process, or Newton's second law of motion, which relates force, mass, and acceleration by the simple equation $F=ma$. A theory, in contrast, is a less concise statement of observed phenomena. For example, the Theory of Evolution and the Theory of Relativity cannot be expressed concisely enough to be considered a law. The biggest difference between a law and a theory is that a law is much more complex and dynamic, and a theory is more explanatory. A law describes a single observable point of fact, whereas a theory explains an entire group of related phenomena. And, whereas a law is a postulate that forms the foundation of the scientific method, a theory is the end result of that process.

Key Points

- Physics is a natural science that involves the study of matter and its motion through space and time, along with related concepts such as energy and force.
- Matter is generally considered to be anything that has mass and volume.
- Scientific laws and theories express the general truths of nature and the body of knowledge they encompass. These laws of nature are rules that all natural processes appear to follow.
- Many scientific disciplines, such as biophysics, are hybrids of physics and other sciences.
- The study of physics encompasses all forms of matter and its motion in space and time.

- The application of physics is fundamental towards significant contributions in new technologies that arise from theoretical breakthroughs.
- Concepts in physics cannot be proven, they can only be supported or disproven through observation and experimentation.
- A model is an evidence-based representation of something that is either too difficult or impossible to display directly.
- A theory is an explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers.
- A law uses concise language, often expressed as a mathematical equation, to describe a generalized pattern in nature that is supported by scientific evidence and repeated experiments.

Key Terms

- **matter**: The basic structural component of the universe. Matter usually has mass and volume.
- **scientific method**: A method of discovering knowledge about the natural world based in making falsifiable predictions (hypotheses), testing them empirically, and developing peer-reviewed theories that best explain the known data.
- **application**: the act of putting something into operation
- **Model**: A representation of something difficult or impossible to display directly
- **Law**: A concise description, usually in the form of a mathematical equation, used to describe a pattern in nature
- **theory**: An explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers

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1.2: Units

Length

Length is a physical measurement of distance that is fundamentally measured in the SI unit of a meter.

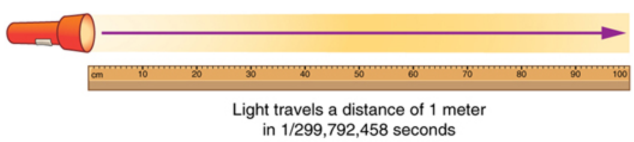
> **learning objectives**
>
> - Distinguish SI and customary units of length

Length can be defined as a measurement of the physical quantity of distance. Many qualitative observations fundamental to physics are commonly described using the measurement of length. The distance between objects, the rate at which objects are traveling, and how much force an object exerts are all dependent on length as a variable. In order to describe length in a standardized and quantitative manner, an accepted unit of measurement must be utilized.

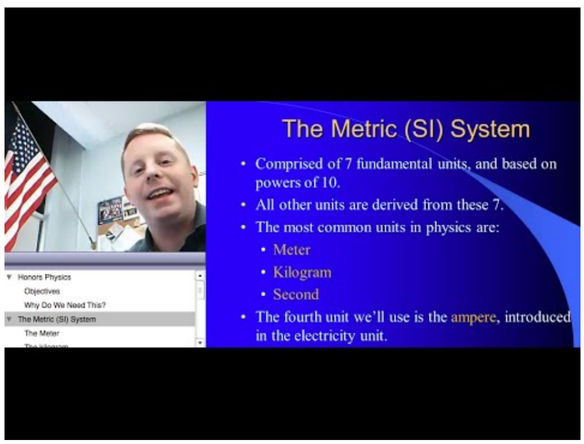
Many different units of length are used around the world. In the United States, the U.S. customary units operationally describe length in terms of the basic unit of an inch. Varying lengths are thus described in relation to the inch, such as a foot equaling 12 inches, a yard equaling three feet, and a mile equaling 1,760 yards.

Though regional use of different measurement units is not generally problematic, it can raise issues of compatibility and understanding when working abroad or collaboratively with international partners. As such, a standard unit of measurement that is internationally accepted is needed. The basic unit of length as identified by the International System of Units (SI) is the meter. The meter is expressed more specifically in terms of speed of light.

One meter is defined as the distance that light travels in a vacuum in $\frac{1}{299,792,458}$ of a second. All lengths are measured in terms related to the meter, where its multiples are devised around the convenience of the number 10. For example, a centimeter is equal to $\frac{1}{100}$ of a meter (or 10^{-2} meters), and a kilometer is equal to 1,000 meters (or 10^{3} meters).



Meter Defined by Speed of Light: The meter is defined to be the distance that light travels in $\frac{1}{299,792,458}$ of a second in a vacuum. Distance traveled is speed multiplied by time.



Metric System – Length: A brief introduction to the metric system and unit conversions.

Mass

Mass is the quantity of matter that an object contains, as measured by its resistance to acceleration.

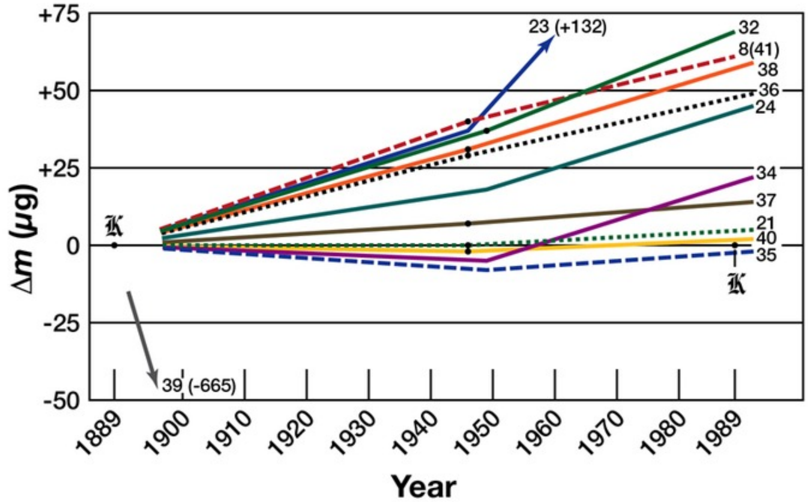
> **learning objectives**
>
> - Explain the difference between mass and weight

Mass

Mass, specifically inertial mass, is a quantitative measure of an object's resistance to acceleration. It is an intrinsic property of an object and does not change because of the environment. The SI unit of mass is the kilogram (kg).

The kilogram is defined as being equal to the mass of the International Prototype Kilogram (IPK), which is almost exactly equal to the mass of one liter of water. It is also the only SI unit that is directly defined by an artifact, rather than a fundamental physical property that can be reproduced in different laboratories. Four of the seven base units in the SI system are defined relative to the kilogram, so the stability of this measurement is crucial for accurate and consistent measurements.

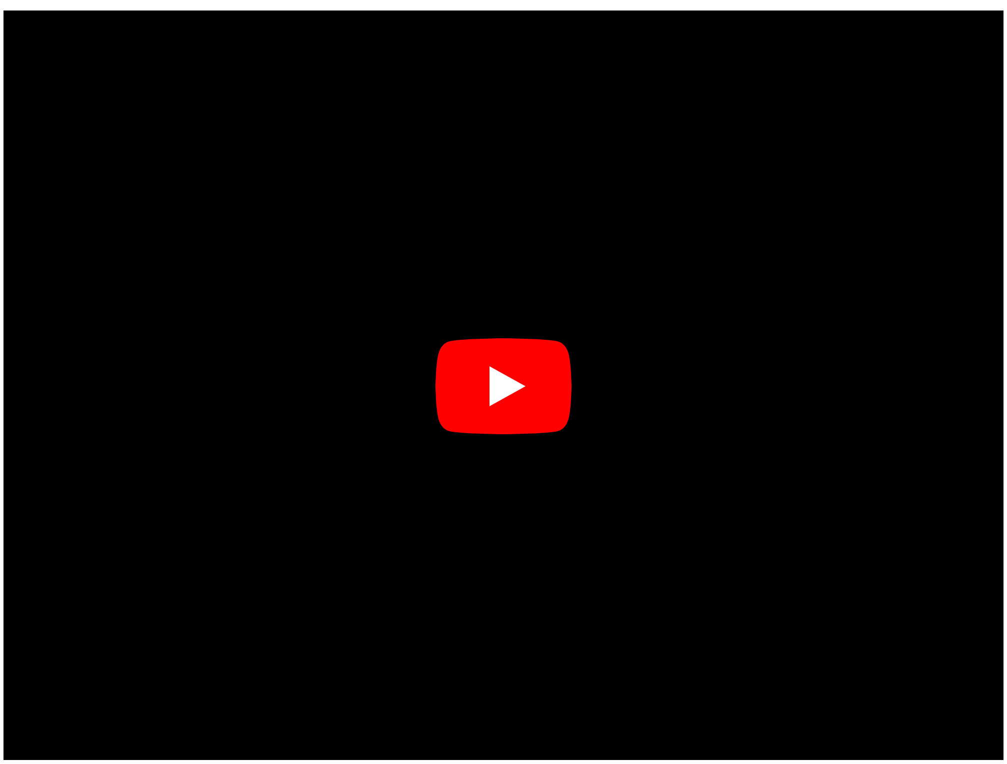
In 2005, the International Committee for Weights and Measures (CIPM) recommended that the kilogram be redefined in terms of a fundamental constant of nature, due to evidence that the International Prototype Kilogram will vary in mass over time. At its 2011 meeting, the General Conference on Weights and Measures (CGPM) agreed that the kilogram should be redefined in terms of the Planck constant. The conference deferred a final decision until its next meeting in 2014.

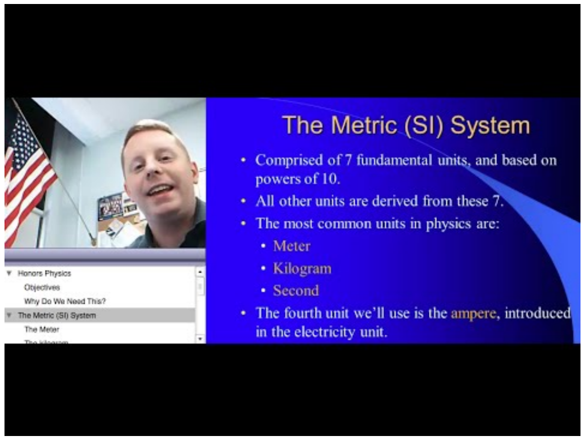


Prototype Mass Drifts: A graph of the relative change in mass of selected kilogram prototypes.

Mass and Weight

In everyday usage, the mass of an object in kilograms is often referred to as its weight. This value, though given in kilograms, is actually the non-SI unit of measure known as the kilogram- force. In scientific terms, ‘weight’ refers to the gravitational force acting on a given body. This measurement changes depending on the gravitational pull of the opposing body. For example, a person’s weight on the Earth is different than a person’s weight on the moon because of the differences in the gravitational pull of each body. In contrast, the mass of an object is an intrinsic property and remains the same regardless of gravitational fields. Accordingly, astronauts in microgravity must exert 10 times more force to accelerate a 10-kg object at the same rate as a 1-kg object, even though the differences in weight are imperceptible.





Metric System – Mass: A brief introduction to the metric system and unit conversions.

Time

Time is the fundamental physical quantity of duration and is measured by the SI Unit known as the second.

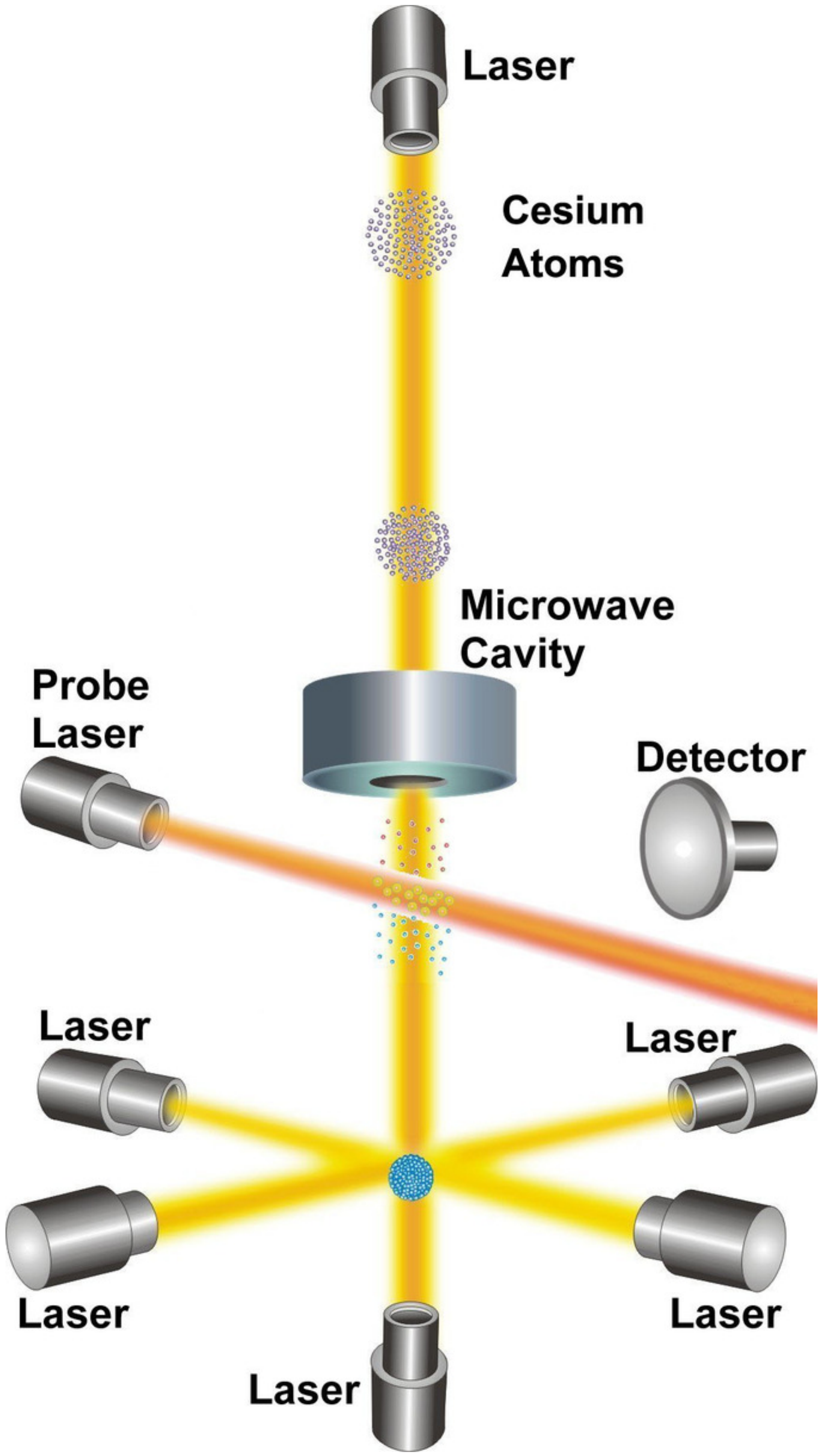
learning objectives

- Relate time with other physical quantities

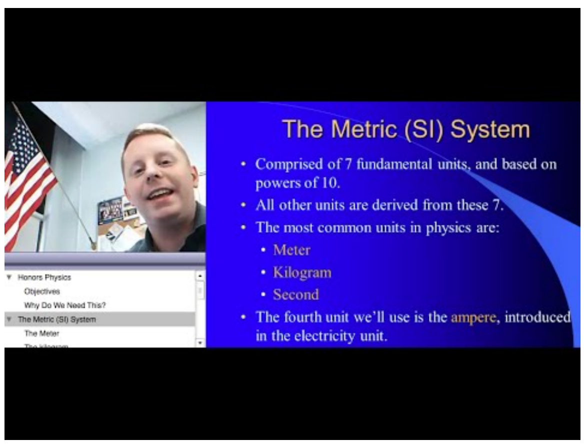
Time is one of the seven fundamental physical quantities in the International System (SI) of Units. Time is used to define other quantities, such as velocity or acceleration, and as such, it is important that it be standardized and quantified precisely. An operational definition of time is highly useful in the conduct of both advanced experiments and everyday affairs of life.

Historically, temporal measurement was a prime motivation in navigation and astronomy. Periodic events and motion have long served as standards for units of time. For example, the movement of the sun across the sky, the phases of the moon, the swing of a pendulum, and the beat of a heart have all been used as a standard for time keeping. These events and standards, however, are highly dynamic in nature and cannot reliably be utilized for accurate quantitative measures. Between 1000 and 1960 the second was defined as $\frac{1}{86,400}$ of a mean solar day. This definition changed between 1960 and 1967 and was defined in terms of the period of the Earth's orbit around the Sun in 1900. Today, the SI Unit of the second is defined in terms of radiation emitted by cesium atoms.

The second is now operationally defined as "the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom." It follows that the hyperfine splitting in the ground state of the cesium 133 atom is exactly 9,192,631,770 hertz. In other words, cesium atoms can be made to vibrate in a very steady way, and these vibrations can be readily observed and counted. The second is the time required for 9,192,631,770 of these vibrations to occur.



NIST-F1 Cesium Clock: NIST-F1 is referred to as a fountain clock because it uses a fountain-like movement of atoms to obtain its improved reckoning of time.



Metric System – Time: A brief introduction to the metric system and unit conversions.

Prefixes and Other Systems of Units

SI prefixes precede a basic unit of measure to indicate a multiple or fraction of the unit.

learning objectives

- Apply prefixes to units and distinguish between SI and customary units

Prefixes

A metric prefix, or SI prefix, is a unit prefix that precedes a basic unit of measure to indicate a multiple or fraction of the unit. Each prefix has a unique symbol that is prepended to the unit symbol. The prefix kilo-, for example, may be added to gram to indicate multiplication by one thousand; one kilogram is equal to one thousand grams (1 kg = 1000 g). The prefix centi-, likewise, may be added to meter to indicate division by one hundred; one centimeter is equal to one hundredth of a meter (1 cm = 0.01 m). Prefixes in varying multiples of 10 are a feature of all forms of the metric system, with many dating back to the system's introduction in the 1790s. Today, the prefixes are standardized for use in the International System of Units (SI) by the International Bureau of Weights and Measures.There are twenty prefixes officially specified by SI.

Metric prefixes						
Prefix	**Symbol**	**1000^m**	**10^n**	**Decimal**	**Short scale**	**Long scale**
yotta	Y	1000^8	10^{24}	1000000000000000000000000	septillion	quadrillion
zetta	Z	1000^7	10^{21}	1000000000000000000000	sextillion	trilliard
exa	E	1000^6	10^{18}	1000000000000000000	quintillion	trillion
peta	P	1000^5	10^{15}	1000000000000000	quadrillion	billiard
tera	T	1000^4	10^{12}	1000000000000	trillion	billion
giga	G	1000^3	10^9	1000000000	billion	milliard
mega	M	1000^2	10^6	1000000	million	
kilo	k	1000^1	10^3	1000	thousand	
hecto	h	$1000^{2/3}$	10^2	100	hundred	
deca	da	$1000^{1/3}$	10^1	10	ten	
		1000^0	10^0	1	one	
deci	d	$1000^{-1/3}$	10^{-1}	0.1	tenth	
centi	c	$1000^{-2/3}$	10^{-2}	0.01	hundredth	
milli	m	1000^{-1}	10^{-3}	0.001	thousandth	
micro	µ	1000^{-2}	10^{-6}	0.000001	millionth	
nano	n	1000^{-3}	10^{-9}	0.000000001	billionth	milliardth
pico	p	1000^{-4}	10^{-12}	0.000000000001	trillionth	billionth
femto	f	1000^{-5}	10^{-15}	0.000000000000001	quadrillionth	billiardth
atto	a	1000^{-6}	10^{-18}	0.000000000000000001	quintillionth	trillionth
zepto	z	1000^{-7}	10^{-21}	0.000000000000000000001	sextillionth	trilliardth
yocto	y	1000^{-8}	10^{-24}	0.000000000000000000000001	septillionth	quadrillionth

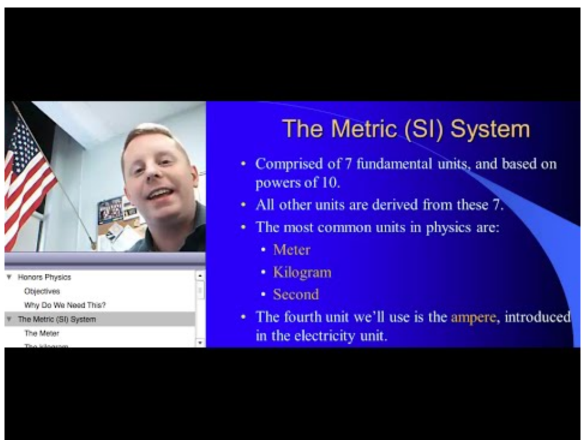
SI Unit Prefixes: The twenty prefixes officially specified by the International System of Units

It is important to note that the kilogram is the only SI unit with a prefix as part of its name and symbol. Because multiple prefixes may not be used, in the case of the kilogram the prefix names are used with the unit name "gram" and the prefix symbols are used with the unit symbol "g. " With this exception, any SI prefix may be used with any SI unit, including the degree Celsius and its symbol °C.

Other Systems of Units

The SI Unit system, or the metric system, is used by the majority of countries in the world, and is the standard system agreed upon by scientists and mathematicians. Colloquially, however, other systems of units are used in many countries. The United States, for example, teaches and uses the *United States customary units*. This system of units was developed from the English, or Imperial, unit standards of the United Kingdom.The United States customary units define measurements using different standards than those used in SI Units. The system for measuring length using the United States customary system is based on the inch, foot, yard, and mile. Likewise, units of area are measured in terms of square feet, and units of capacity and volume are measured in terms of cubic inches, cubic feet, or cubic yards. Units of mass are commonly defined in terms of ounces and pounds, rather than the SI unit of kilograms.Other commonly used units from the United States customary system include the fluid volume units of the teaspoon, tablespoon, fluid ounce, US cup, pint, quart, and gallon, as well as the degrees Fahrenheit used to measure temperature.

Some units that are widely used are not a part of the International System of Units and are considered Non-SI Units. These units, though not officially part of SI Units, are generally accepted for use in conjunction with SI units. These can include the minute, hour, and day used in temporal measurements, the liter for volumetric measurements, and the degree, minute, and second used to measure angles.



Metric System – Prefixes: A brief introduction to the metric system and unit conversions.

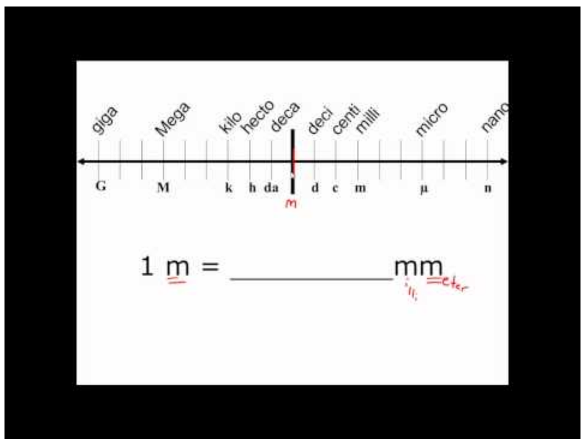
Converting Units

Converting between units can be done through the use of conversion factors or specific conversion formulas.

> **learning objectives**
>
> - Apply factor-label method for converting units

Translating Systems of Measurement

It is often necessary to convert from one type of unit to another. Conversion of units is the conversion of different units of measurement for the same quantity, typically using conversion factors. For example, if you are reading a European cookbook, some quantities may be expressed in units of liters; if you're cooking in the US in a standard kitchen with standard tools, you will need to convert those measurements to cups. Or, perhaps you are reading walking directions from one location to another and you are interested in how many miles you will be walking. In this case, you will need to convert units of feet to miles. This is a bit like translating a substitution code, using a formula that helps you understand what one measure means in terms of another system.



Unit Conversion in the Metric System: EASY Unit Conversion in the Metric System – This simple extra help video tutorial explains the metric system and how to make simple metric conversions.

Conversion Methods

There are several ways to approach doing conversions. One commonly used method is known as the Factor-label method for converting units, or the "railroad method. "

The factor-label method is the sequential application of conversion factors expressed as fractions and arranged so that any dimensional unit appearing in both the numerator and denominator of any of the fractions can be cancelled out until only the desired set of dimensional units is obtained. For example, 10 miles per hour can be converted to meters per second by using a sequence of conversion factors.

Each conversion factor is equivalent to the value of one. For example, starting with 1 mile = 1609 meters and dividing both sides of the equation by 1 mile yields $\frac{1 \text{ mile}}{1 \text{ mile}} = \frac{1609 \text{ meters}}{1 \text{ mile}}$, which when simplified yields $1 = \frac{1609 \text{ meters}}{1 \text{ mile}}$. Physically crossing out the units that cancel each other out will also help visualize what's left over.

$$1\ \cancel{\text{year}} \times \frac{365\ \cancel{\text{days}}}{1\ \cancel{\text{year}}} \times \frac{24\ \cancel{\text{hours}}}{1\ \cancel{\text{day}}} \times \frac{60\ \cancel{\text{min}}}{1\ \cancel{\text{hour}}} \times \frac{60\ \text{s}}{1\ \cancel{\text{min}}} = \\ = 3.15 \times 10^7\ \text{s}$$

Converting 1 year into seconds using the Factor-Label Method: Physically crossing out units that cancel out helps visualize the "leftover" unit(s).

So, when the units mile and hour are cancelled out and the arithmetic is done, 10 miles per hour converts to 4.47 meters per second.

A limitation of the factor-label method is that it can only convert between units that have a constant ratio that can be multiplied, or a multiplication factor. This method cannot be used between units that have a displacement, or difference factor. An example is the conversion between degrees Celsius and kelvins, or between Celsius and Fahrenheit. For these, it is best to use the specific conversion formulas.

For example, if you are planning a trip abroad in Spain and the weather forecast predicts the weather to be mostly cloudy and 16°C, you may want to convert the temperature into °F, a unit that you are more comfortable interpreting. In order to do this, you would need to know the conversion formula from Celsius to Fahrenheit. This formula is: $[^\circ\text{F}] = [^\circ\text{C}] \times \frac{9}{5} + 32$.

$$[^\circ\text{F}] = [^\circ\text{C}] \times \frac{9}{5} + 32 \tag{1.2.1}$$

$$[^\circ\text{F}] = 28.8 + 32 \tag{1.2.2}$$

$$[^\circ\text{F}] = 60.8 + 32 \tag{1.2.3}$$

So you would then know that 16°C is equivalent to 60.8°F and be able to pack the right type of clothing to be comfortable.

Key Points

- The SI unit for length is the meter.
- One meter is defined as the distance that light travels in a vacuum in $\frac{1}{299,792,458}$ of a second.
- Derivatives of measurement units related to the meter are devised around the convenience of the number 10.
- The kilogram is the only SI unit directly defined by the artifact itself.
- Mass is a property that does not depend on gravitational fields, unlike weight.
- One kilogram is defined as the mass of the International Prototype Kilogram (IPK), a platinum-iridium alloy cylinder.
- One kilogram is almost exactly equal to the mass of one liter of water.
- Time is a physical quantity of duration.
- The SI Unit for time is the second.
- The second is operationally defined in terms of radiation emitted by cesium atoms.
- The twenty standardized prefixes for use in the International System of Units are derived from multiples of 10.
- The kilogram is the only SI unit with a prefix as part of its name and symbol; as such, SI unit prefixes are prepended to the unit gram.
- The United States customary units define measurements based on the English, or Imperial, unit standards.
- Conversion of units is the conversion between different units of measurement for the same quantity, typically through multiplicative conversion factors.
- The factor-label method is the sequential application of conversion factors expressed as fractions in which units appearing in both the numerator and denominator can be cancelled out, leaving only the desired set of units.
- For conversions that have a difference factor, specific conversion formulas should be used.

Key Terms

- **Length**: How far apart objects are physically.
- **acceleration**: the rate at which the velocity of a body changes with time
- **inertia**: the tendency of an object to resist any change in its motion
- **Radiation**: the emission of energy as electromagnetic waves or as moving or oscillating subatomic particles.
- **prefix**: That which is prefixed; especially one or more letters or syllables added to the beginning of a word to modify its meaning; as, pre- in prefix, con- in conjure.
- **conversion**: a change between different units of measurement for the same quantity.

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1.3: Significant Figures and Order of Magnitude

Scientific Notation

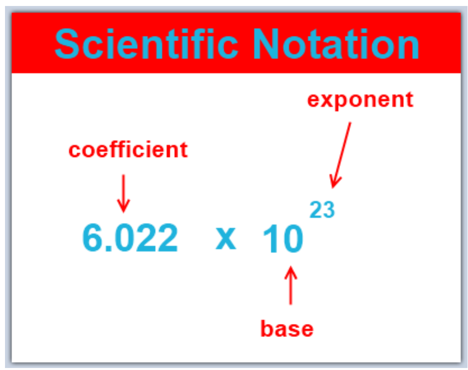
Scientific notation is a way of writing numbers that are too big or too small in a convenient and standard form.

learning objectives

- Convert properly between standard and scientific notation and identify appropriate situations to use it

Scientific Notation: A Matter of Convenience

Scientific notation is a way of writing numbers that are too big or too small in a convenient and standard form. Scientific notation has a number of useful properties and is commonly used in calculators and by scientists, mathematicians and engineers. In scientific notation all numbers are written in the form of $a \cdot 10^b$ (a multiplied by ten raised to the power of b), where the exponent b) is an integer, and the coefficient (a is any real number.



Scientific Notation: There are three parts to writing a number in scientific notation: the coefficient, the base, and the exponent.

Most of the interesting phenomena in our universe are not on the human scale. It would take about 1,000,000,000,000,000,000,000 bacteria to equal the mass of a human body. Thomas Young's discovery that light was a wave preceded the use of scientific notation, and he was obliged to write that the time required for one vibration of the wave was "$\frac{1}{500}$ of a millionth of a millionth of a second"; an inconvenient way of expressing the point. Scientific notation is a less awkward and wordy way to write very large and very small numbers such as these.

A Simple System

Scientific notation means writing a number in terms of a product of something from 1 to 10 and something else that is a power of ten.

For instance, $32 = 3.2 \cdot 10^1$

$320 = 3.2 \cdot 10^2$

$3200 = 3.2 \cdot 10^3$, and so forth…

Each number is ten times bigger than the previous one. Since 10^1 is ten times smaller than 10^2, it makes sense to use the notation 10^0 to stand for one, the number that is in turn ten times smaller than 10^1. Continuing on, we can write 10^{-1} to stand for 0.1, the number ten times smaller than 10^0. Negative exponents are used for small numbers:

$3.2 = 3.2 \cdot 10^0$

$0.32 = 3.2 \cdot 10^{-1}$

$0.032 = 3.2 \cdot 10^{-2}$

Scientific notation displayed calculators can take other shortened forms that mean the same thing. For example, $3.2 \cdot 10^6$(written notation) is the same as $3.2\mathrm{E}+6$ (notation on some calculators) and 3.2^6 (notation on some other calculators).

Round-off Error

A round-off error is the difference between the calculated approximation of a number and its exact mathematical value.

learning objectives

- Explain the impact round-off errors may have on calculations, and how to reduce this impact

Round-off Error

A round-off error, also called a rounding error, is the difference between the calculated approximation of a number and its exact mathematical value. Numerical analysis specifically tries to estimate this error when using approximation equations, algorithms, or both, especially when using finitely many digits to represent real numbers. When a sequence of calculations subject to rounding errors is made, errors may accumulate, sometimes dominating the calculation.

Calculations rarely lead to whole numbers. As such, values are expressed in the form of a decimal with infinite digits. The more digits that are used, the more accurate the calculations will be upon completion. Using a slew of digits in multiple calculations, however, is often unfeasible if calculating by hand and can lead to much more human error when keeping track of so many digits. To make calculations much easier, the results are often 'rounded off' to the nearest few decimal places.

For example, the equation for finding the area of a circle is $\mathrm{A}=\pi \mathrm{r}^{2}$. The number π(pi) has infinitely many digits, but can be truncated to a rounded representation of as 3.14159265359. However, for the convenience of performing calculations by hand, this number is typically rounded even further, to the nearest two decimal places, giving just 3.14. Though this technically decreases the accuracy of the calculations, the value derived is typically 'close enough' for most estimation purposes.

However, when doing a series of calculations, numbers are rounded off at each subsequent step. This leads to an accumulation of errors, and if profound enough, can misrepresent calculated values and lead to miscalculations and mistakes.

The following is an example of round-off error:

$$\sqrt{4.58^{2}+3.28^{2}}=\sqrt{21.0+10.8}=5.64$$

Rounding these numbers off to one decimal place or to the nearest whole number would change the answer to 5.7 and 6, respectively. The more rounding off that is done, the more errors are introduced.

Order of Magnitude Calculations

An order of magnitude is the class of scale of any amount in which each class contains values of a fixed ratio to the class preceding it.

learning objectives

- Choose when it is appropriate to perform an order-of-magnitude calculation

Orders of Magnitude

An order of magnitude is the class of scale of any amount in which each class contains values of a fixed ratio to the class preceding it. In its most common usage, the amount scaled is 10, and the scale is the exponent applied to this amount (therefore, to be an order of magnitude greater is to be 10 times, or 10 to the power of 1, greater). Such differences in order of magnitude can be measured on the logarithmic scale in "decades," or factors of ten. It is common among scientists and technologists to say that a parameter whose value is not accurately known or is known only within a range is "on the order of" some value. The order of magnitude of a physical quantity is its magnitude in powers of ten when the physical quantity is expressed in powers of ten with one digit to the left of the decimal.

Orders of magnitude are generally used to make very approximate comparisons and reflect very large differences. If two numbers differ by one order of magnitude, one is about ten times larger than the other. If they differ by two orders of magnitude, they differ by a factor of about 100. Two numbers of the same order of magnitude have roughly the same scale — the larger value is less than ten times the smaller value.

It is important in the field of science that estimates be at least in the right ballpark. In many situations, it is often sufficient for an estimate to be within an order of magnitude of the value in question. Although making order-of-magnitude estimates seems simple and natural to experienced scientists, it may be completely unfamiliar to the less experienced.

Example 1.3.1:

Some of the mental steps of estimating in orders of magnitude are illustrated in answering the following example question: Roughly what percentage of the price of a tomato comes from the cost of transporting it in a truck?

Guessing the Number of Jelly Beans: Can you guess how many jelly beans are in the jar? If you try to guess directly, you will almost certainly underestimate. The right way to do it is to estimate the linear dimensions and then estimate the volume indirectly.

Incorrect solution: Let's say the trucker needs to make a profit on the trip. Taking into account her benifits, the cost of gas, and maintenance and payments on the truck, let's say the total cost is more like 2000. You might guess about 5000 tomatoes would fit in the back of the truck, so the extra cost per tomato is 40 cents. That means the cost of transporting one tomato is comparable to the cost of the tomato itself.

The problem here is that the human brain is not very good at estimating area or volume — it turns out the estimate of 5000 tomatoes fitting in the truck is way off. (This is why people have a hard time in volume-estimation contests, such as the one shown below.) When estimating area or volume, you are much better off estimating linear dimensions and computing the volume from there.

So, here's a better solution: As before, let's say the cost of the trip is \$2000. The dimensions of the bin are probably 4m by 2m by 1m, for a volume of $8 \mathrm{~m}^3$. Since our goal is just an order-of-magnitude estimate, let's round that volume off to the nearest power of ten: $10 \mathrm{~m}^3$. The shape of a tomato doesn't follow linear dimensions, but since this is just an estimate, let's pretend that a tomato is an 0.1m by 0.1m by 0.1m cube, with a volume of $1 \cdot 10^{-3} \mathrm{~m}^3$. We can find the total number of tomatoes by dividing the volume of the bin by the volume of one tomato: $\frac{10^3 \mathrm{~m}^3}{10^{-3} \mathrm{~m}^3} = 10^6$ tomatoes. The transportation cost per tomato is $\frac{\$2000}{10^6 \text{ tomatoes}} = \$0.002$ per tomato. That means that transportation really doesn't contribute very much to the cost of a tomato. Approximating the shape of a tomato as a cube is an example of another general strategy for making order-of-magnitude estimates.

Key Points

- Scientific notation means writing a number in terms of a product of something from 1 to 10 and something else that is a power of 10.
- In scientific notation all numbers are written in the form of $a \cdot 10^b$ (a times ten raised to the power of b).
- Each consecutive exponent number is ten times bigger than the previous one; negative exponents are used for small numbers.
- When a sequence of calculations subject to rounding error is made, these errors can accumulate and lead to the misrepresentation of calculated values.
- Increasing the number of digits allowed in a representation reduces the magnitude of possible round-off errors, but may not always be feasible, especially when doing manual calculations.
- The degree to which numbers are rounded off is relative to the purpose of calculations and the actual value.
- Orders of magnitude are generally used to make very approximate comparisons and reflect very large differences.
- In the field of science, it is often sufficient for an estimate to be within an order of magnitude of the value in question.

- When estimating area or volume, you are much better off estimating linear dimensions and computing volume from those linear dimensions.

Key Terms

- **exponent**: The power to which a number, symbol or expression is to be raised. For example, the 3 in x^3.
- **Scientific notation**: A method of writing, or of displaying real numbers as a decimal number between 1 and 10 followed by an integer power of 10
- **approximation**: An imprecise solution or result that is adequate for a defined purpose.
- **Order of Magnitude**: The class of scale or magnitude of any amount, where each class contains values of a fixed ratio (most often 10) to the class preceding it. For example, something that is 2 orders of magnitude larger is 100 times larger; something that is 3 orders of magnitude larger is 1000 times larger; and something that is 6 orders of magnitude larger is one million times larger, because $10^2 = 100$, $10^3 = 1000$, and $10^6 =$ one million

1.4: Solving Physics Problems

Dimensional Analysis

Any physical quantity can be expressed as a product of a combination of the basic physical dimensions.

learning objectives

- Calculate the conversion from one kind of dimension to another

Dimensions

The dimension of a physical quantity indicates how it relates to one of the seven basic quantities. These fundamental quantities are:

- [M] Mass
- [L] Length
- [T] Time
- [A] Current
- [K] Temperature
- [mol] Amount of a Substance
- [cd] Luminous Intensity

As you can see, the symbol is enclosed in a pair of square brackets. This is often used to represent the dimension of individual basic quantity. An example of the use of basic dimensions is speed, which has a dimension of 1 in length and -1 in time; $\frac{[\mathrm{L}]}{[\mathrm{T}]} = [\mathrm{LT}^{-1}]$. Any physical quantity can be expressed as a product of a combination of the basic physical dimensions.

Dimensional Analysis

Dimensional analysis is the practice of checking relations between physical quantities by identifying their dimensions. The dimension of any physical quantity is the combination of the basic physical dimensions that compose it. Dimensional analysis is based on the fact that physical law must be independent of the units used to measure the physical variables. It can be used to check the plausibility of derived equations, computations and hypotheses.

Derived Dimensions

The dimensions of derived quantities may include few or all dimensions in individual basic quantities. In order to understand the technique to write dimensions of a derived quantity, we consider the case of force. Force is defined as:

$$\mathrm{F} = \mathrm{m} \cdot \mathrm{a} \tag{1.4.1}$$

$$\mathrm{F} = [\mathrm{M}][\mathrm{a}] \tag{1.4.2}$$

The dimension of acceleration, represented as [a], is itself a derived quantity being the ratio of velocity and time. In turn, velocity is also a derived quantity, being ratio of length and time.

$$\mathrm{F} = [\mathrm{M}][\mathrm{a}] = [\mathrm{M}][\mathrm{vT}^{-1}] \tag{1.4.3}$$

$$\mathrm{F} = [\mathrm{M}][\mathrm{LT}^{-1}\mathrm{T}^{-1}] = [\mathrm{MLT}^{-2}] \tag{1.4.4}$$

Dimensional Conversion

In practice, one might need to convert from one kind of dimension to another. For common conversions, you might already know how to convert off the top of your head. But for less common ones, it is helpful to know how to find the conversion factor:

$$\mathrm{Q} = \mathrm{n}_1\mathrm{u}_1 = \mathrm{n}_2\mathrm{u}_2 \tag{1.4.5}$$

where n represents the amount per u dimensions. You can then use ratios to figure out the conversion:

$$\mathrm{n}_2 = \frac{\mathrm{u}_2}{\mathrm{u}_1} \cdot \mathrm{n}_1 \tag{1.4.6}$$

Trigonometry

Trigonometry is central to the use of free body diagrams, which help visually represent difficult physics problems.

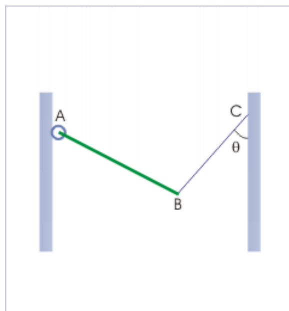
learning objectives

- Explain why trigonometry is useful in determining horizontal and vertical components of forces

Trigonometry and Solving Physics Problems

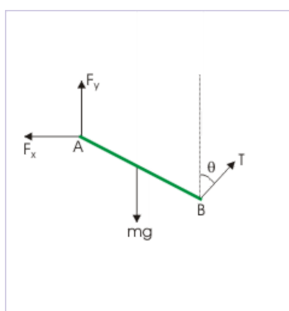
In physics, most problems are solved much more easily when a free body diagram is used. Free body diagrams use geometry and vectors to visually represent the problem. Trigonometry is also used in determining the horizontal and vertical components of forces and objects. Free body diagrams are very helpful in visually identifying which components are unknown and where the moments are applied. They can help analyze a problem, whether it is static or dynamic.

When people draw free body diagrams, often not everything is perfectly parallel and perpendicular. Sometimes people need to analyze the horizontal and vertical components of forces and object orientation. When the force or object is not acting parallel to the x or y axis, people can employ basic trigonometry to use the simplest components of the action to analyze it. Basically, everything should be considered in terms of x and y, which sometimes takes some manipulation.



Free Body Diagram: The rod is hinged from a wall and is held with the help of a string.

A rod 'AB' is hinged at 'A' from a wall and is held still with the help of a string, as shown in. This exercise involves drawing the free body diagram. To make the problem easier, the force F will be expressed in terms of its horizontal and vertical components. Removing all other elements from the image helps produce the finished free body diagram.



Free Body Diagram: The free body diagram as a finished product

Given the finished free body diagram, people can use their knowledge of trigonometry and the laws of sine and cosine to mathematically and numerical represent the horizontal and vertical components:

General Problem-Solving Tricks

Free body diagrams use geometry and vectors to visually represent the problem.

learning objectives

- Construct a free-body diagram for a physical scenario

In physics, most problems are solved much more easily when a free body diagram is used. This uses geometry and vectors to visually represent to problem, and trigonometry is also used in determining horizontal and vertical components of forces and objects.

Purpose: Free body diagrams are very helpful in visually identifying which components are unknown, where the moments are applied, and help analyze a problem, whether static or dynamic.

How to Make A Free Body Diagram

To draw a free body diagram, do not worry about drawing it to scale, this will just be what you use to help yourself identify the problems. First you want to model the body, in one of three ways:

- As a particle. This model may be used when any turning effects are zero or have zero interest even though the body itself may be extended. The body may be represented by a small symbolic blob and the diagram reduces to a set of concurrent arrows. A force on a particle is a *bound* vector.
- *rigid extended*. Stresses and strains are of no interest but turning effects are. A force arrow should lie along the line of force, but where along the line is irrelevant. A force on an extended rigid body is a*sliding* vector.
- *non-rigid extended*. The *point of application* of a force becomes crucial and has to be indicated on the diagram. A force on a non-rigid body is a *bound* vector. Some engineers use the tail of the arrow to indicate the point of application. Others use the tip.

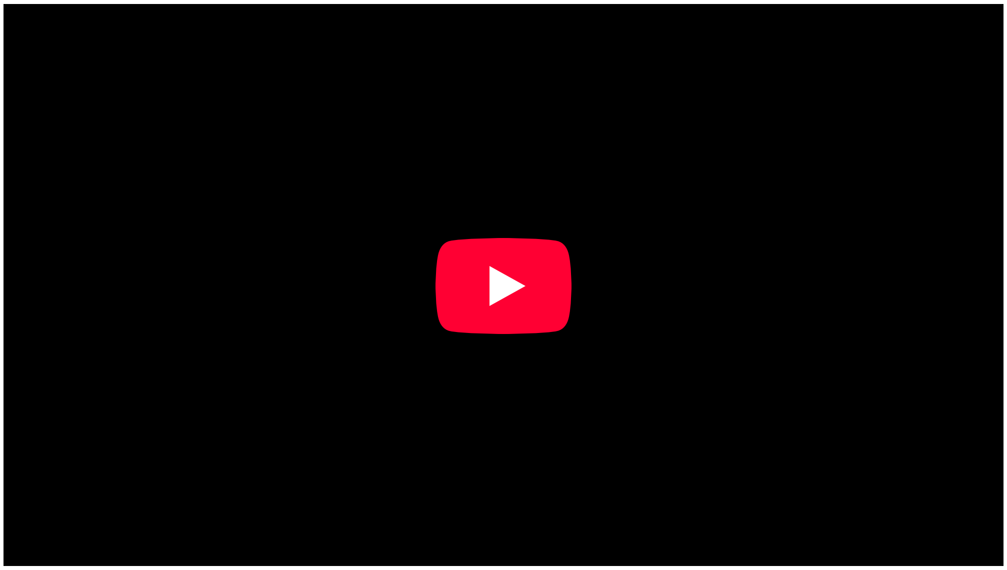
Do's and Don'ts

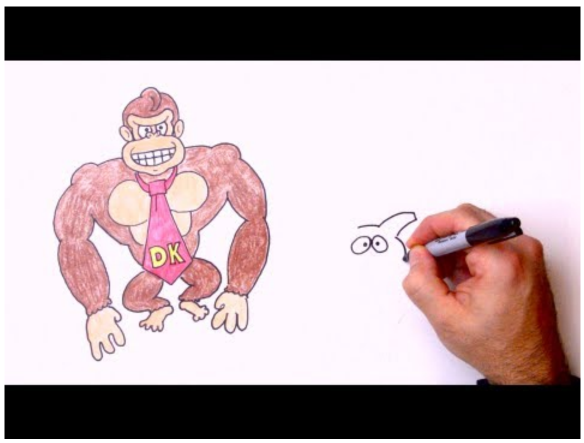
What to include: Since a free body diagram represents the body itself and the external forces on it. So you will want to include the following things in the diagram:

- The body: This is usually sketched in a schematic way depending on the body – particle/extended, rigid/non-rigid – and on what questions are to be answered. Thus if rotation of the body and torque is in consideration, an indication of size and shape of the body is needed.
- The external forces: These are indicated by labelled arrows. In a fully solved problem, a force arrow is capable of indicating the direction, the magnitude the point of application. These forces can be friction, gravity, normal force, drag, tension, etc…

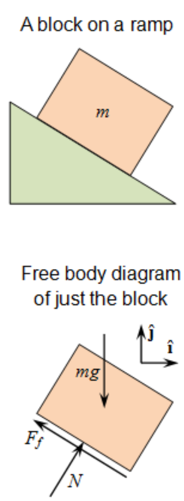
Do not include:

- Do not show bodies other than the body of interest.
- Do not show forces exerted by the body.
- Internal forces acting on various parts of the body by other parts of the body.
- Any velocity or acceleration is left out.





How To Solve Any Physics Problem: Learn five simple steps in five minutes! In this episode we cover the most effective problem-solving method I've encountered and call upon some fuzzy friends to help us remember the steps.



Free Body Diagram: Use this figure to work through the example problem.

Key Points

- Dimensional analysis is the practice of checking relations amount physical quantities by identifying their dimensions.
- It is common to be faced with a problem that uses different dimensions to express the same basic quantity. The following equation can be used to find the conversion factor between the two derived dimensions: $n_2 = \frac{u_2}{u_1} \times n_1$.
- Dimensional analysis can also be used as a simple check to computations, theories and hypotheses.
- It is important to identify the problem and the unknowns and draw them in a free body diagram.
- The laws of cosine and sine can be used to determine the vertical and horizontal components of the different elements of the diagram.
- Free body diagrams use geometry and vectors to visually represent physics problems.
- A free body diagram lets you visually isolate the problem you are trying to solve, and simplify it into simple geometry and trigonometry.
- When drawing these diagrams, it is helpful to only draw the body it self, and the forces acting on it.
- Drawing other objects and internal forces can condense the diagram and cause it to be less helpful.

Key Terms

- **dimension**: A measure of spatial extent in a particular direction, such as height, width or breadth, or depth.
- **trigonometry**: The branch of mathematics that deals with the relationships between the sides and the angles of triangles and the calculations based on them, particularly the trigonometric functions.
- **static**: Fixed in place; having no motion.
- **dynamic**: Changing; active; in motion.

- How To Solve Any Physics Problem. **Located at**: **http://www.youtube.com/watch?v=YocWuzi4JhY**. **License**: ***Public Domain: No Known Copyright***. **License Terms**: Standard YouTube license

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CHAPTER OVERVIEW

2: Kinematics

Topic hierarchy

2.1: Basics of Kinematics

Defining Kinematics

Kinematics is the study of the motion of points, objects, and groups of objects without considering the causes of its motion.

learning objectives

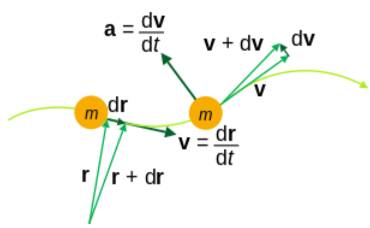
- Define kinematics

Kinematics is the branch of classical mechanics that describes the motion of points, objects and systems of groups of objects, without reference to the causes of motion (i.e., forces). The study of kinematics is often referred to as the “geometry of motion.”

Objects are in motion all around us. Everything from a tennis match to a space-probe flyby of the planet Neptune involves motion. When you are resting, your heart moves blood through your veins. Even in inanimate objects there is continuous motion in the vibrations of atoms and molecules. Interesting questions about motion can arise: how long will it take for a space probe to travel to Mars? Where will a football land if thrown at a certain angle? An understanding of motion, however, is also key to understanding other concepts in physics. An understanding of acceleration, for example, is crucial to the study of force.

To describe motion, kinematics studies the trajectories of points, lines and other geometric objects, as well as their differential properties (such as velocity and acceleration). Kinematics is used in astrophysics to describe the motion of celestial bodies and systems; and in mechanical engineering, robotics and biomechanics to describe the motion of systems composed of joined parts (such as an engine, a robotic arm, or the skeleton of the human body).

A formal study of physics begins with kinematics. The word “kinematics” comes from a Greek word “kinesis” meaning motion, and is related to other English words such as “cinema” (movies) and “kinesiology” (the study of human motion). Kinematic analysis is the process of measuring the kinematic quantities used to describe motion. The study of kinematics can be abstracted into purely mathematical expressions, which can be used to calculate various aspects of motion such as velocity, acceleration, displacement, time, and trajectory.



Kinematics of a particle trajectory: Kinematic equations can be used to calculate the trajectory of particles or objects. The physical quantities relevant to the motion of a particle include: mass m, position r, velocity v, acceleration a.

Reference Frames and Displacement

In order to describe an object’s motion, you need to specify its position relative to a convenient reference frame.

learning objectives

- Evaluate displacement within a frame of reference.

In order to describe the motion of an object, you must first describe its position — where it is at any particular time. More precisely, you need to specify its position relative to a convenient reference frame. Earth is often used as a reference frame, and we often describe the position of objects related to its position to or from Earth. Mathematically, the position of an object is generally represented by the variable x.

Frames of Reference

There are two choices you have to make in order to define a position variable x. You have to decide where to put $x = 0$ and which direction will be positive. This is referred to as choosing a coordinate system, or choosing a frame of reference. As long as you are consistent, any frame is equally valid. But you don’t want to change coordinate systems in the middle of a calculation. Imagine sitting in a train in a station when suddenly you notice that the station is moving backward. Most people would say that they just failed to notice that the train was moving — it only *seemed* like the station was moving. But this shows that there is a *third* arbitrary choice that goes into choosing a coordinate system: valid frames of reference can differ from each other by moving relative to one another. It might seem strange to use a coordinate system moving relative to the earth — but, for instance, the frame of reference moving along with a train might be far more convenient for describing things happening inside the train. Frames of reference are particularly important when describing an object's displacement.

FRAMES OF REFERENCE by Professor Hume and Professor Donald Ivey of the University of Toronto

In this classic film, Professors Hume and Ivey cleverly illustrate reference frames and distinguish between fixed and moving frames of reference.

Frames of Reference (1960) Educational Film: Frames of Reference is a 1960 educational film by Physical Sciences Study Committee. The film was made to be shown in high school physics courses. In the film University of Toronto physics professors Patterson Hume and Donald Ivey explain the distinction between inertial and nonintertial frames of reference, while demonstrating these concepts through humorous camera tricks. For example, the film opens with Dr. Hume, who appears to be upside down, accusing Dr. Ivey of being upside down. Only when the pair flip a coin does it become obvious that Dr. Ivey — and the camera — are indeed inverted. The film's humor serves both to hold students' interest and to demonstrate the concepts being discussed. This PSSC film utilizes a fascinating set consisting of a rotating table and furniture occupying surprisingly unpredictable spots within the viewing area. The fine cinematography by Abraham Morochnik, and funny narration by University of Toronto professors Donald Ivey and Patterson Hume is a wonderful example of the fun a creative team of filmmakers can have with a subject that other, less imaginative types might find pedestrian. Producer: Richard Leacock Production Company: Educational Development Corp. Sponsor: Eric Prestamon

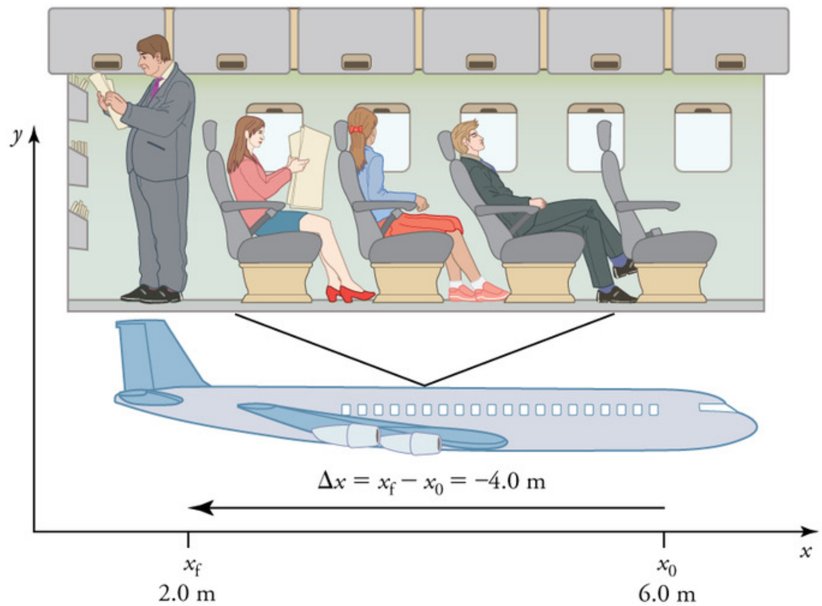
Displacement

Displacement is the change in position of an object relative to its reference frame. For example, if a car moves from a house to a grocery store, its displacement is the relative distance of the grocery store to the reference frame, or the house. The word "displacement" implies that an object has moved or has been displaced. Displacement is the change in position of an object and can be represented mathematically as follows:

$$\Delta \mathrm{x} = \mathrm{x_f} - \mathrm{x_0} \tag{2.1.1}$$

where Δx is displacement, x_f is the final position, and x_0 is the initial position.

shows the importance of using a frame of reference when describing the displacement of a passenger on an airplane.



Displacement in Terms of Frame of Reference: A passenger moves from his seat to the back of the plane. His location relative to the airplane is given by x. The -4.0m displacement of the passenger relative to the plane is represented by an arrow toward the rear of the plane. Notice that the arrow representing his displacement is twice as long as the arrow representing the displacement of the professor (he moves twice as far).

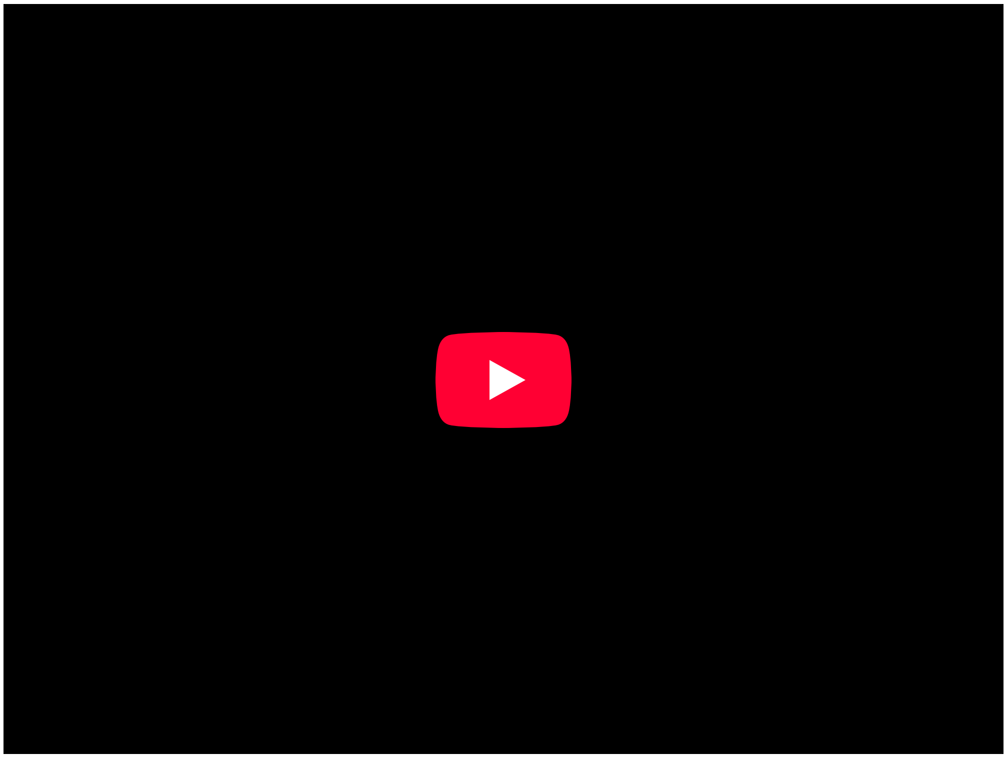
Introduction to Scalars and Vectors

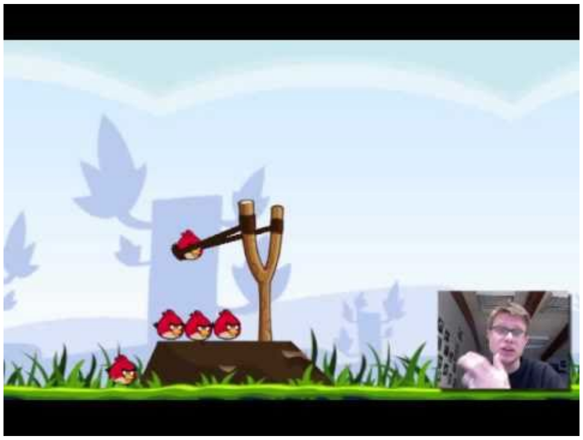
A vector is any quantity that has both magnitude and direction, whereas a scalar has only magnitude.

learning objectives

- Distinguish the difference between scalars and vectors

What is the difference between distance and displacement? Whereas displacement is defined by both direction and magnitude, distance is defined by magnitude alone. Displacement is an example of a vector quantity. Distance is an example of a scalar quantity. A vector is any quantity with both magnitude and direction. Other examples of vectors include a velocity of 90 km/h east and a force of 500 newtons straight down.





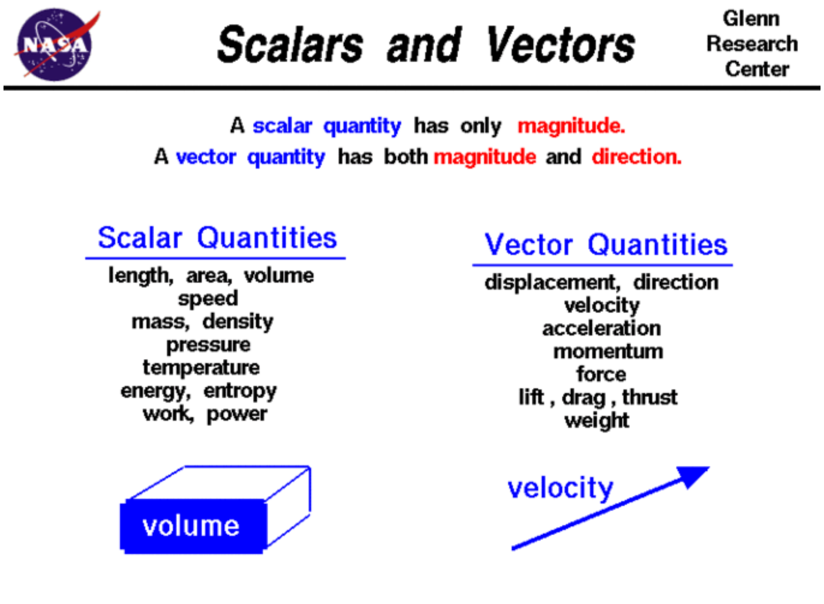
Scalars and Vectors: Mr. Andersen explains the differences between scalar and vectors quantities. He also uses a demonstration to show the importance of vectors and vector addition.

In mathematics, physics, and engineering, a vector is a geometric object that has a magnitude (or length) and direction and can be added to other vectors according to vector algebra. The direction of a vector in one-dimensional motion is given simply by a plus (+) or minus (−) sign. A vector is frequently represented by a line segment with a definite direction, or graphically as an arrow, connecting an initial point A with a terminal point B, as shown in.

$$\overrightarrow{AB}.$$

Vector representation: A vector is frequently represented by a line segment with a definite direction, or graphically as an arrow, connecting an initial point A with a terminal point B.

Some physical quantities, like distance, either have no direction or no specified direction. In physics, a scalar is a simple physical quantity that is not changed by coordinate system rotations or translations. It is any quantity that can be expressed by a single number and has a magnitude, but no direction. For example, a 20ºC temperature, the 250 kilocalories (250 Calories) of energy in a candy bar, a 90 km/h speed limit, a person's 1.8 m height, and a distance of 2.0 m are all scalars, or quantities with no specified direction. Note, however, that a scalar can be negative, such as a −20ºC temperature. In this case, the minus sign indicates a point on a scale rather than a direction. Scalars are never represented by arrows. (A comparison of scalars vs. vectors is shown in.)



Scalars vs. Vectors: A brief list of quantities that are either scalars or vectors.

Key Points

- To describe motion, kinematics studies the trajectories of points, lines and other geometric objects.
- The study of kinematics can be abstracted into purely mathematical expressions.
- Kinematic equations can be used to calculate various aspects of motion such as velocity, acceleration, displacement, and time.
- Choosing a frame of reference requires deciding where the object's initial position is and which direction will be considered positive.
- Valid frames of reference can differ from each other by moving relative to one another.
- Frames of reference are particularly important when describing an object's displacement.
- Displacement is the change in position of an object relative to its reference frame.
- A vector is any quantity that has magnitude and direction.
- A scalar is any quantity that has magnitude but no direction.
- Displacement and velocity are vectors, whereas distance and speed are scalars.

Key Terms

- **kinematics**: The branch of mechanics concerned with objects in motion, but not with the forces involved.
- **displacement**: A vector quantity that denotes distance with a directional component.
- **frame of reference**: A coordinate system or set of axes within which to measure the position, orientation, and other properties of objects in it.
- **scalar**: A quantity that has magnitude but not direction; compare vector.
- **vector**: A directed quantity, one with both magnitude and direction; the between two points.

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2.2: Speed and Velocity

Average Velocity: A Graphical Interpretation

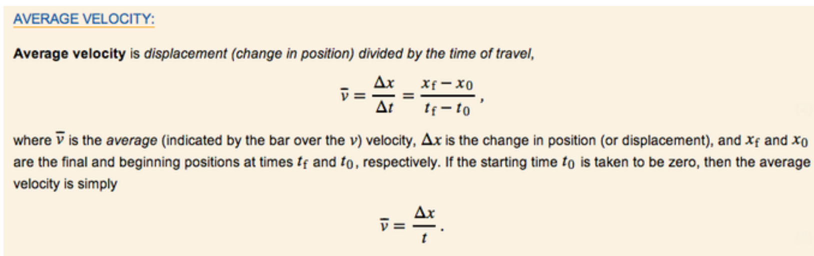
Average velocity is defined as the change in position (or displacement) over the time of travel.

learning objectives

- Contrast speed and velocity in physics

In everyday usage, the terms “speed” and “velocity” are used interchangeably. In physics, however, they are distinct quantities. Speed is a scalar quantity and has only magnitude. Velocity, on the other hand, is a vector quantity and so has both magnitude and direction. This distinction becomes more apparent when we calculate average speed and velocity.

Average speed is calculated as the distance traveled over the total time of travel. In contrast, average velocity is defined as the change in *position* (or displacement) over the total time of travel.



AVERAGE VELOCITY:

Average velocity is *displacement (change in position) divided by the time of travel,*

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_0}{t_f - t_0},$$

where $\bar{v}$ is the *average* (indicated by the bar over the v) velocity, Δx is the change in position (or displacement), and x_f and x_0 are the final and beginning positions at times t_f and t_0, respectively. If the starting time t_0 is taken to be zero, then the average velocity is simply

$$\bar{v} = \frac{\Delta x}{t}.$$

Average Velocity: The kinematic formula for calculating average velocity is the change in position over the time of travel.

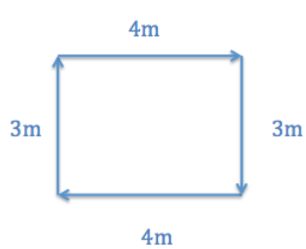
The SI unit for velocity is meters per second, or m/s, but many other units (such as km/h, mph, and cm/s) are commonly used. Suppose, for example, an airplane passenger took five seconds to move -4 m (the negative sign indicates that displacement is toward the back of the plane). His average velocity would be:

$$\mathrm{v} = \frac{\Delta \mathrm{x}}{\mathrm{t}} = \frac{-4\ \mathrm{m}}{5\ \mathrm{s}} = -0.8\,\frac{\mathrm{m}}{\mathrm{s}} \tag{2.2.1}$$

The minus sign indicates that the average velocity is also toward the rear of the plane.

The average velocity of an object does not tell us anything about what happens to it between the starting point and ending point, however. For example, we cannot tell from average velocity whether the airplane passenger stops momentarily or backs up before he gets to the back of the plane. To get more details, we must consider smaller segments of the trip over smaller time intervals.

To illustrate the difference between average speed and average velocity, consider the following additional example. Imagine you are walking in a small rectangle. You walk three meters north, four meters east, three meters south, and another four meters west. The entire walk takes you 30 seconds. If you are calculating average speed, you would calculate the entire distance (3 + 4 + 3 + 4 = 14 meters) over the total time, 30 seconds. From this, you would get an average speed of 14/30 = 0.47 m/s. When calculating average velocity, however, you are looking at the displacement over time. Because you walked in a full rectangle and ended up exactly where you started, your displacement is 0 meters. Therefore, your average velocity, or displacement over time, would be 0 m/s.



Average Speed vs. Average Velocity: If you started walking from one corner and went all the way around the rectangle in 30 seconds, your average speed would be 0.47 m/s, but your average velocity would be 0 m/s.

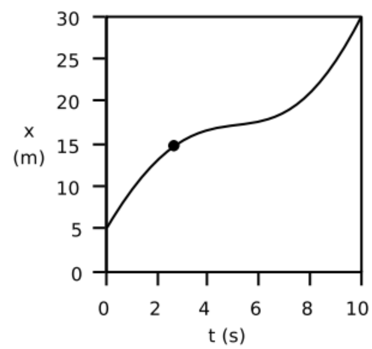
Instananeous Velocity: A Graphical Interpretation

Instantaneous velocity is the velocity of an object at a single point in time and space as calculated by the slope of the tangent line.

learning objectives

- Differentiate instantaneous velocity from other ways of determining velocity

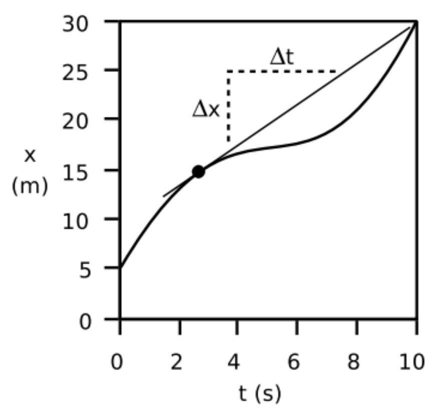
Typically, motion is not with constant velocity nor speed. While driving in a car, for example, we continuously speed up and slow down. A graphical representation of our motion in terms of distance vs. time, therefore, would be more variable or "curvy" rather than a straight line, indicating motion with a constant velocity as shown below. (We limit our discussion to one dimensional motion. It should be straightforward to generalize to three dimensional cases.)



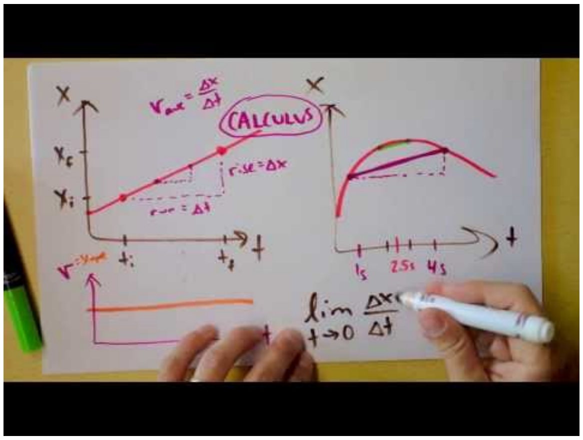
Motion with Changing Velocity: Motion is often observed with changing velocity. This would result in a curvy line when graphed with distance over time.

To calculate the speed of an object from a graph representing constant velocity, all that is needed is to find the slope of the line; this would indicate the change in distance over the change in time. However, changing velocity it is not as straightforward.

Since our velocity is constantly changing, we can estimate velocity in different ways. One way is to look at our instantaneous velocity, represented by one point on our curvy line of motion graphed with distance vs. time. In order to determine our velocity at any given moment, we must determine the slope at that point. To do this, we find a line that represents our velocity in that moment, shown graphically in. That line would be the line tangent to the curve at that point. If we extend this line, we can easily calculate the displacement of distance over time and determine our velocity at that given point. The velocity of an object at any given moment is the slope of the tangent line through the relevant point on its x vs. t graph.



Determining instantaneous velocity: The velocity at any given moment is defined as the slope of the tangent line through the relevant point on the graph



Instantaneous Velocity, Acceleration, Jerk, Slopes, Graphs vs. Time: This is how kinematics begins.

In calculus, finding the slope of curve $f(x)$ at $x = x_0$ $x=x_0$ is equivalent to finding the first derivative:

$$\frac{df(x)}{dx}|_{x=x_0}. \tag{2.2.2}$$

One interpretation of this definition is that the velocity shows how many meters the object would travel in one second if it continues moving at the same speed for at least one second.

Key Points

- Average velocity can be calculated by determining the total displacement divided by the total time of travel.
- The average velocity of an object does not tell us anything about what happens to it between the starting point and ending point.
- Average velocity is different from average speed in that it considers the direction of travel and the overall change in position.
- When velocity is constantly changing, we can estimate our velocity by looking at instantaneous velocity.
- Instantaneous velocity is calculated by determining the slope of the line tangent to the curve at the point of interest.
- Instantaneous velocity is similar to determining how many meters the object would travel in one second at a specific moment.

Key Terms

- **velocity**: A vector quantity that denotes the rate of change of position with respect to time, or a speed with a directional component.
- **instantaneous**: (As in velocity)—occurring, arising, or functioning without any delay; happening within an imperceptibly brief period of time.

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2.3: Acceleration

Graphical Interpretation

The graphical representation of acceleration over time can be derived through the graph of an object's position over time.

learning objectives

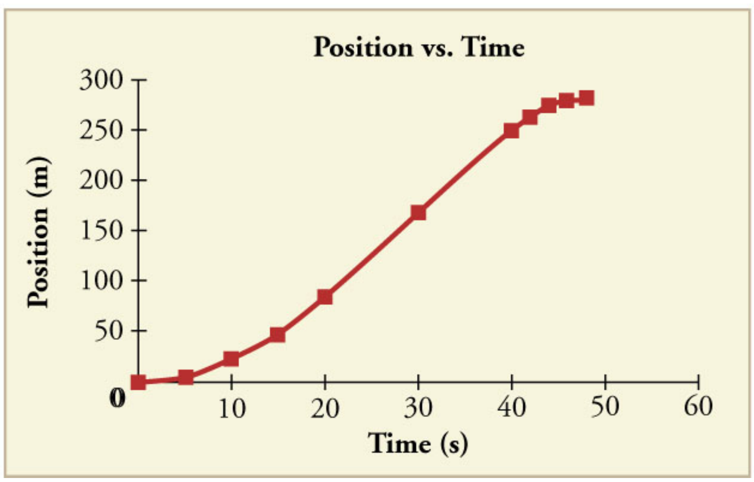
- Distinguish the difference between how to plot a velocity graph and how to plot an acceleration graph

In physics, acceleration is the rate at which the velocity of a body changes with time. It is a vector quantity with both magnitude and direction. Acceleration is accompanied by a force, as described by Newton's Second Law; the force, as a vector, is the product of the mass of the object being accelerated and the acceleration (vector), or $\mathrm{F} = \mathrm{ma}$. The SI unit of acceleration is the meter per second squared: $\frac{\mathrm{m}}{\mathrm{s}^2}$

Acceleration is a vector that points in the same direction as the change in velocity, though it may not always be in the direction of motion. For example, when an object slows down, or decelerating, its acceleration is in the opposite direction of its motion.

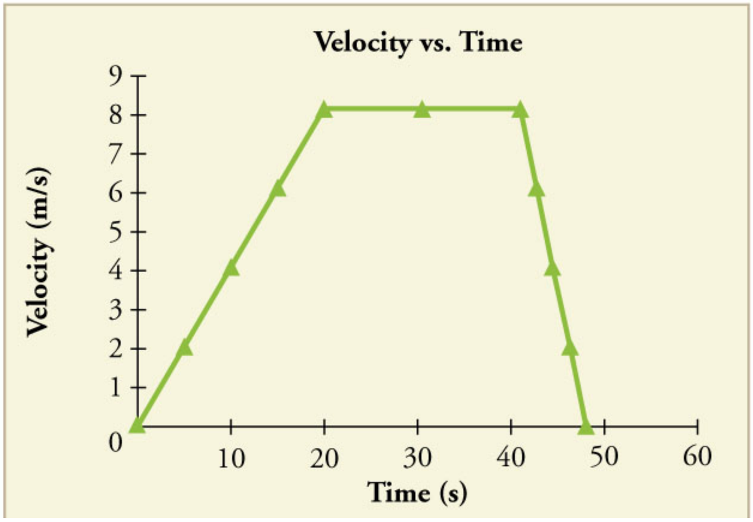
The motion of an object can be depicted graphically by plotting the position of an object over time. This distance-time graph can be used to create another graph that shows changes in velocity over time. Because acceleration is velocity in $\frac{\mathrm{m}}{\mathrm{s}}$ divided by time in s, we can further derive a graph of acceleration from a graph of an object's speed or position.

is a graph of an object's position over time. This graph is similar to the motion of a car. In the beginning, the object's position changes slowly as it gains speed. In the middle, the speed is constant and the position changes at a constant rate. As it slows down toward the end, the position changes more slowly. From this graph, we can derive a velocity vs time graph.



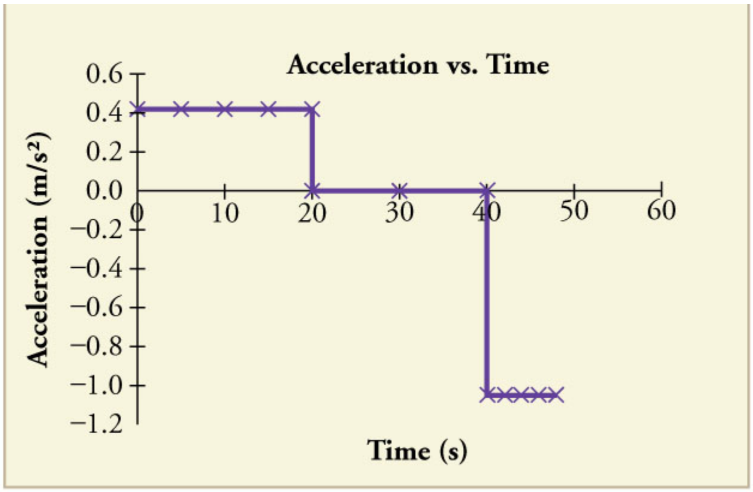
Position vs Time Graph: Notice that the object's position changes slowly at the beginning of the journey, then more and more quickly as it picks up speed. Its position then changes more slowly as it slows down at the end of the journey. In the middle of the journey, while the velocity remains constant, the position changes at a constant rate.

This shows the velocity of the object over time. The object's velocity increases in the beginning as it accelerates at the beginning, then remains constant in the middle before it slows down toward the end. Notice that this graph is a representation of the slope of the previous position vs time graph. From this graph, we can further derive an acceleration vs time graph.

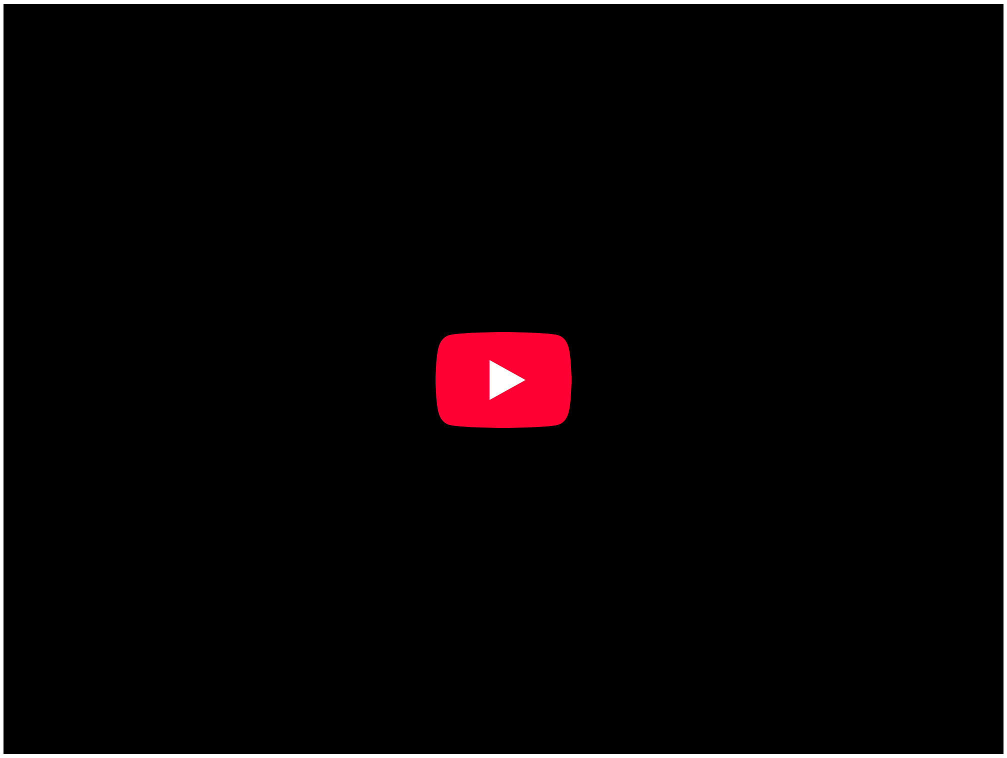


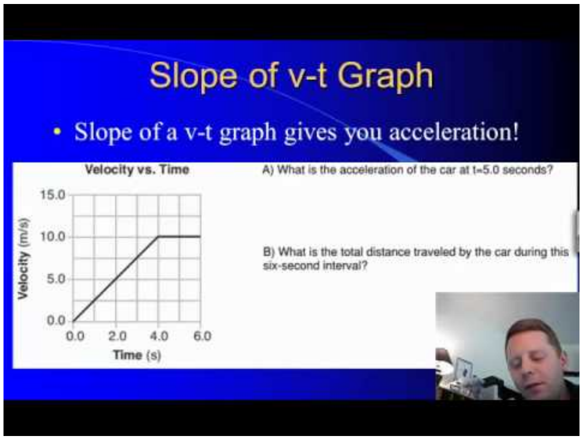
Velocity vs Time: The object's velocity increases as it accelerates at the beginning of the journey. It remains the same in the middle of the journey (where there is no acceleration). It decreases as the object decelerates at the end of the journey.

To do this, we would also plot the slope of the velocity vs time graph. In this graph, the acceleration is constant in the three different stages of motion. As we noted earlier, the object is increasing speed and changing positions slowly in the beginning. The acceleration graph shows that the object was increasing at a positive constant acceleration during this time. In the middle, when the object was changing position at a constant velocity, the acceleration was 0. This is because the object is no longer changing its velocity and is moving at a constant rate. Towards the end of the motion, the object slows down. This is depicted as a negative value on the acceleration graph. Note that in this example, the motion of the object is still forward (positive), but since it is decelerating, the acceleration is negative.



Acceleration vs Time Graph: The object has positive acceleration as it speeds up at the beginning of the journey. It has no acceleration as it travels at constant velocity in the middle of the journey. Its acceleration is negative as it slows down at the end of the journey.





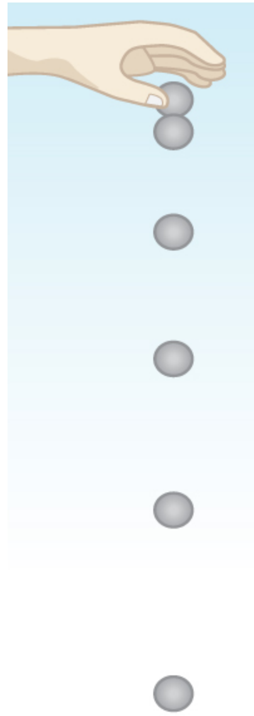
Graphing Motion: A brief introduction to particle diagrams and motion graphs.

Motion with Constant Acceleration

Constant acceleration occurs when an object’s velocity changes by an equal amount in every equal time period.

learning objectives

- Describe how constant acceleration affects the motion of an object



One-Dimensional Motion: When you drop an object, it falls vertically toward the center of the earth due to the constant acceleration of gravity.

An object experiencing constant acceleration has a velocity that increases or decreases by an equal amount for any constant period of time. Acceleration can be derived easily from basic kinematic principles. It is defined as the first time derivative of velocity (so the second derivative of position with respect to time):

$$a=\frac{\partial v}{\partial t}=\frac{\partial^2 x}{\partial t^2} \tag{2.3.1}$$

Assuming acceleration to be constant does not seriously limit the situations we can study and does not degrade the accuracy of our treatment, because in a great number of situations, acceleration *is* constant. When it is not, we can either consider it in separate parts of constant acceleration or use an average acceleration over a period of time.

The motion of falling objects is a simple, one-dimensional type of projectile motion in which there is no horizontal movement. For example, if you held a rock out and dropped it, the rock would fall only vertically downward toward the earth. If you were to throw the rock instead of just dropping it, it would follow a more projectile-like pattern, similar to the one a kicked ball follows.

Projectile motion is the motion of an object thrown or projected into the air and is subject only to the acceleration of gravity. The object thrown is called a projectile, and the object's path is called its trajectory. In two-dimensional projectile motion, there is both a vertical and a horizontal component.

Due to the algebraic properties of constant acceleration, there are kinematic equations that relate displacement, initial velocity, final velocity, acceleration, and time. A summary of these equations is given below.

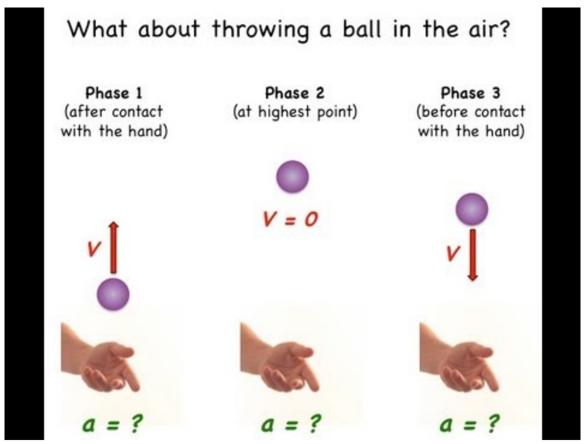
$$x=x_0+\bar{v}t \tag{2.3.2}$$

$$\bar{v}=\frac{v_0+v}{2} \tag{2.3.3}$$

$$v=v_0+at \tag{2.3.4}$$

$$x=x_0+v_0t+\frac{1}{2}at^2 \tag{2.3.5}$$

$$v^2=v_0^2+2a(x-x_0) \tag{2.3.6}$$



Constant Acceleration Explained with Vectors and Algebra: This video answers the question “what is acceleration? ".

Key Points

- Acceleration is the rate at which the velocity of a body changes with time.
- Acceleration is a vector that points in the same direction as the change in velocity, though it may not always be in the direction of motion.
- Because acceleration is velocity in m/s divided by time in s, we can derive a graph of acceleration from a graph of an object’s speed or position.
- Assuming acceleration to be constant does not seriously limit the situations we can study and does not degrade the accuracy of our treatment.
- Due to the algebraic properties of constant acceleration, there are kinematic equations that can be used to calculate displacement, velocity, acceleration, and time.
- Calculations with constant acceleration can be done in relation to one-dimensional motion as well as two-dimensional motion.

Key Terms

- **acceleration**: The amount by which a speed or velocity increases (and so a scalar quantity or a vector quantity).
- **velocity**: A vector quantity that denotes the rate of change of position with respect to time, or a speed with a directional component.
- **position**: A place or location.
- **kinematic**: of or relating to motion or kinematics

2.4: Problem-Solving for Basic Kinematics

Applications

There are four kinematic equations that describe the motion of objects without consideration of its causes.

> learning objectives
>
> - Choose which kinematics equation to use in problems in which the initial starting position is equal to zero

Kinematics is the branch of classical mechanics that describes the motion of points, bodies (objects), and systems of bodies (groups of objects) without consideration of the causes of motion. There are four kinematic equations when the initial starting position is the origin, and the acceleration is constant:

1. $v = v_0 + at$
2. $d = \frac{1}{2}(v_0 + v)t$ or alternatively $v_{average} = \frac{d}{t}$
3. $d = v_0 t + (\frac{at^2}{2})$
4. $v^2 = v_0^2 + 2ad$

Notice that the four kinematic equations involve five kinematic variables: d, v, v_0, a and t. Each of these equations contains only four of the five variables and has a different one missing. This tells us that we need the values of three variables to obtain the value of the fourth and we need to choose the equation that contains the three known variables and one unknown variable for each specific situation.

Here the basic problem solving steps to use these equations:

Step one – Identify exactly what needs to be determined in the problem (identify the unknowns).

Step two – Find an equation or set of equations that can help you solve the problem.

Step three – Substitute the knowns along with their units into the appropriate equation, and obtain numerical solutions complete with units.

Step four – Check the answer to see if it is reasonable: Does it make sense?

Problem-solving skills are obviously essential to success in a quantitative course in physics. More importantly, the ability to apply broad physical principles, usually represented by equations, to specific situations is a very powerful form of knowledge. It is much more powerful than memorizing a list of facts. Analytical skills and problem-solving abilities can be applied to new situations, whereas a list of facts cannot be made long enough to contain every possible circumstance. Such analytical skills are useful both for solving problems in a physics class and for applying physics in everyday and professional life.

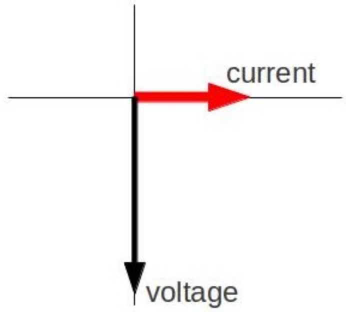
Motion Diagrams

A motion diagram is a pictorial description of an object's motion and represents the position of an object at equally spaced time intervals.

> learning objectives
>
> - Construct a motion diagram

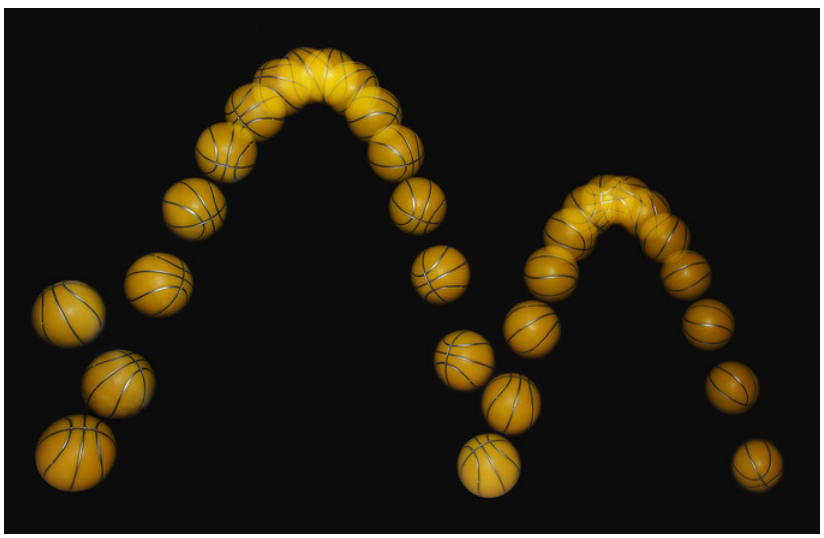
A motion diagram is a pictorial description of the motion of an object. It displays the object's location at various equally spaced times on the same diagram; shows an object's initial position and velocity; and presents several spots in the center of the diagram. These spots reveal whether or not the object has accelerated or decelerated. For simplicity, the object is represented by a simple shape, such as a filled circle, which contains information about an object's position at particular time instances. For this reason, a motion diagram is more information than a path diagram. It may also display the forces acting on the object at each time instance.

is a motion diagram of a simple trajectory. Imagine the object as a hockey puck sliding on ice. Notice that the puck covers the same distance per unit interval along the trajectory. We can conclude that the puck is moving at a constant velocity and, therefore, there is no acceleration or deceleration during the motion.



Puck Sliding on Ice: Motion diagram of a puck sliding on ice. The puck is moving at a constant velocity.

One major use of motion diagrams is the presentation of film through a series of frames taken by a camera; this is sometimes called stroboscopic technique (as seen in). Viewing an object on a motion diagram allows one to determine whether an object is speeding up or slowing down, or if it is at constant rest. As the frames are taken, we can assume that an object is at a constant rest if it occupies the same position over time. We can assume that an object is speeding up if there is a visible increase in the space between objects as time passes, and that it is slowing down if there is a visible decrease in the space between objects as time passes. The objects on the frame come very close together.



Bouncing Ball: A bouncing ball captured with a stroboscopic flash at 25 images per second.

Key Points

- The four kinematic equations involve five kinematic variables: d, v, v_0, a and t.
- Each equation contains only four of the five variables and has a different one missing.
- It is important to choose the equation that contains the three known variables and one unknown variable for each specific situation.
- Motion diagrams represent the motion of an object by displaying its location at various equally spaced times on the same diagram.
- Motion diagrams show an object's initial position and velocity and presents several spots in the center of the diagram. These spots reveal the object's state of motion.
- Motion diagrams contain information about an object's position at particular time instances and is therefore more informative than a path diagram.

Key Terms

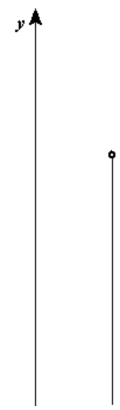
- **kinematics**: The branch of physics concerned with objects in motion.
- **stroboscopic**: Relating to an instrument used to make a cyclically moving object appear to be slow-moving, or stationary.
- **diagram**: A graph or chart.
- **motion**: A change of position with respect to time.

2.5: Free-Falling Objects

learning objectives

- Solve basic problems concerning free fall and distinguish it from other kinds of motion

The motion of falling objects is the simplest and most common example of motion with changing velocity. If a coin and a piece of paper are simultaneously dropped side by side, the paper takes much longer to hit the ground. However, if you crumple the paper into a compact ball and drop the items again, it will look like both the coin and the paper hit the floor simultaneously. This is because the amount of force acting on an object is a function of not only its mass, but also area. Free fall is the motion of a body where its weight is the only force acting on an object.



Free Fall: This clip shows an object in free fall.

Galileo also observed this phenomena and realized that it disagreed with the Aristotle principle that heavier items fall more quickly. Galileo then hypothesized that there is an upward force exerted by air in addition to the downward force of gravity. If air resistance and friction are negligible, then in a given location (because gravity changes with location), all objects fall toward the center of Earth with the *same constant acceleration, independent of their mass*, that constant acceleration is gravity. Air resistance opposes the motion of an object through the air, while friction opposes motion between objects and the medium through which they are traveling. The acceleration of free-falling objects is referred to as the acceleration due to gravity gg. As we said earlier, gravity varies depending on location and altitude on Earth (or any other planet), but the average acceleration due to gravity on Earth is 9.8 $\frac{\mathrm{m}}{\mathrm{s}^2}$. This value is also often expressed as a negative acceleration in mathematical calculations due to the downward direction of gravity.

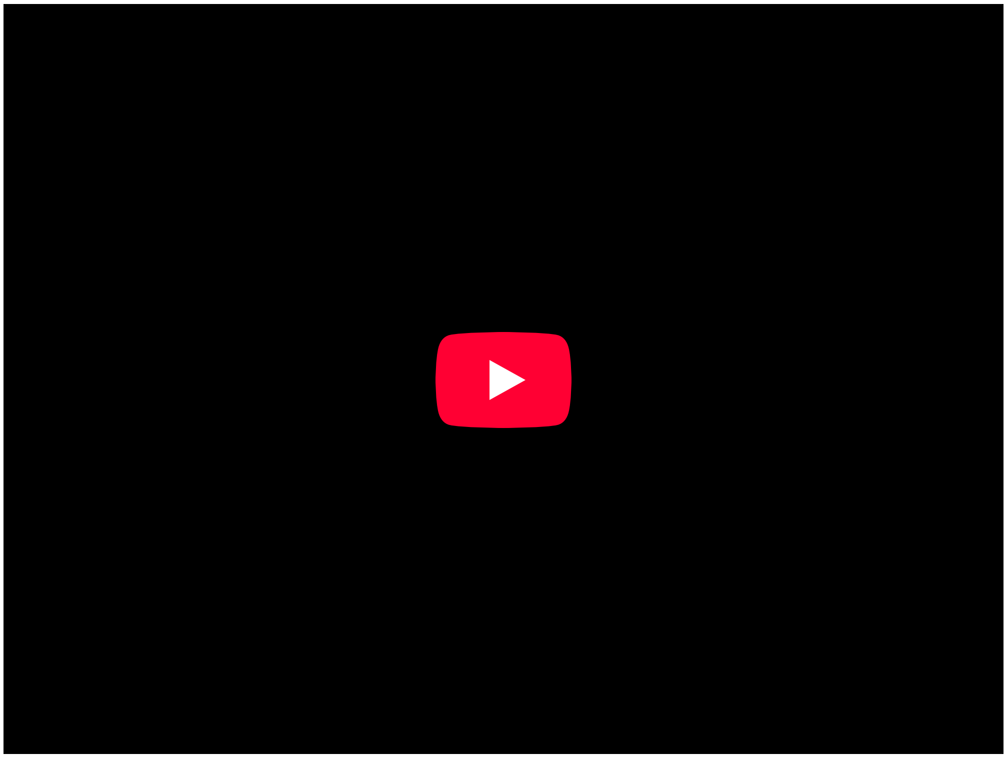
The best way to see the basic features of motion involving gravity is to start by considering straight up and down motion with no air resistance or friction. This means that if the object is dropped, we know the initial velocity is zero. Once the object is in motion, the object is in free-fall. Under these circumstances, the motion is one-dimensional and has constant acceleration, gg. The kinematic equations for objects experiencing free fall are:

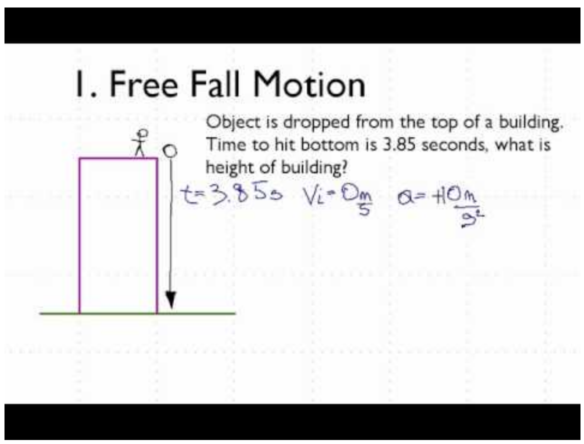
$$\mathrm{v}=\mathrm{v}_0-\mathrm{gt} \tag{2.5.1}$$

$$\mathrm{y}=\mathrm{y}_0+\mathrm{v}_0\mathrm{t}-\frac{1}{2}\mathrm{gt}^2 \tag{2.5.2}$$

$$\mathrm{v}^2=\mathrm{v}_0^2-2\mathrm{g}(\mathrm{y}-\mathrm{y}_0), \tag{2.5.3}$$

where $\mathrm{v}=\mathrm{velocity}, \mathrm{g}=\mathrm{gravity}, \mathrm{t}=\mathrm{time},$ and $\mathrm{y}=\mathrm{vertical\ displacement}.$





Video 2.5.1*: Free Fall Motion - Describes how to calculate the time for an object to fall if given the height and the height that an object fell if given the time to fall.*

Example 2.5.1:

Some examples of objects that are in free fall include:

- A spacecraft in continuous orbit. The free fall would end once the propulsion devices turned on.
- An stone dropped down an empty well.
- An object, in projectile motion, on its descent.

Key Points

- The acceleration of free-falling objects is called the acceleration due to gravity, since objects are pulled towards the center of the earth.
- The acceleration due to gravity is constant on the surface of the Earth and has the value of 9.80 $\frac{m}{s^2}$.

Glossary

Acceleration

The amount by which a speed or velocity changes within a certain period of time (and so a scalar quantity or a vector quantity).

CHAPTER OVERVIEW

3: Two-Dimensional Kinematics

3.1: Motion in Two Dimensions

Constant Velocity

An object moving with constant velocity must have a constant speed in a constant direction.

learning objectives

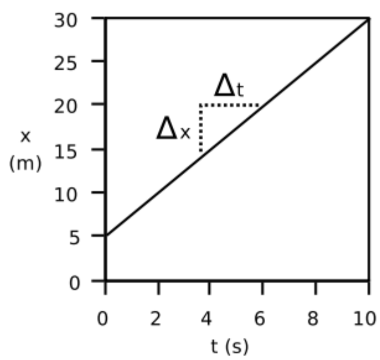
- Examine the terms for constant velocity and how they apply to acceleration

Motion with constant velocity is one of the simplest forms of motion. This type of motion occurs when an an object is moving (or sliding) in the presence of little or negligible friction, similar to that of a hockey puck sliding across the ice. To have a constant velocity, an object must have a constant speed in a constant direction. Constant direction constrains the object to motion to a straight path.

Newton's second law ($F = ma$) suggests that when a force is applied to an object, the object would experience acceleration. If the acceleration is 0, the object shouldn't have any external forces applied on it. Mathematically, this can be shown as the following:

$$a = \frac{dv}{dt} = 0 \Rightarrow v = const. \tag{3.1.1}$$

If an object is moving at constant velocity, the graph of distance vs. time (x vs. t) shows the same change in position over each interval of time. Therefore the motion of an object at constant velocity is represented by a straight line: $x = x_0 + vt$, where x_0 is the displacement when $t = 0$ (or at the y-axis intercept).



Motion with Constant Velocity: When an object is moving with constant velocity, it does not change direction nor speed and therefore is represented as a straight line when graphed as distance over time.

You can also obtain an object's velocity if you know its trace over time. Given a graph as in, we can calculate the velocity from the change in distance over the change in time. In graphical terms, the velocity can be interpreted as the slope of the line. The velocity can be positive or negative, and is indicated by the sign of our slope. This tells us in which direction the object moves.

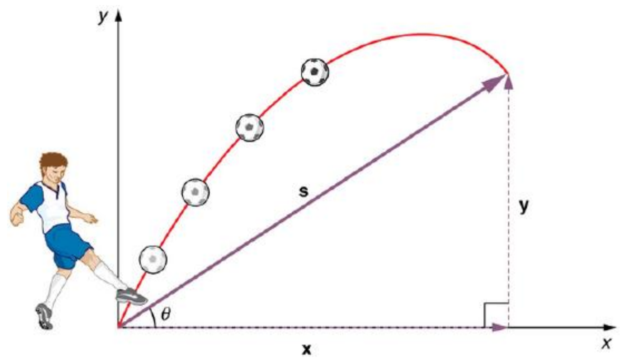
Constant Acceleration

Analyzing two-dimensional projectile motion is done by breaking it into two motions: along the horizontal and vertical axes.

learning objectives

- Analyze a two-dimensional projectile motion along horizontal and vertical axes

Projectile motion is the motion of an object thrown, or projected, into the air, subject only to the force of gravity. The object is called a projectile, and its path is called its trajectory. The motion of falling objects is a simple one-dimensional type of projectile motion in which there is no horizontal movement. In two-dimensional projectile motion, such as that of a football or other thrown object, there is both a vertical and a horizontal component to the motion.



Projectile Motion: Throwing a rock or kicking a ball generally produces a projectile pattern of motion that has both a vertical and a horizontal component.

The most important fact to remember is that motion along perpendicular axes are independent and thus can be analyzed separately. The key to analyzing two-dimensional projectile motion is to break it into two motions, one along the horizontal axis and the other along the vertical. To describe motion we must deal with velocity and acceleration, as well as with displacement.

We will assume all forces except for gravity (such as air resistance and friction, for example) are negligible. The components of acceleration are then very simple: $a_y = -g = -9.81 \frac{m}{s^2}$ (we assume that the motion occurs at small enough heights near the surface of the earth so that the acceleration due to gravity is constant). Because the acceleration due to gravity is along the vertical direction *only*, $a_x = 0$. Thus, the kinematic equations describing the motion along the x and y directions respectively, can be used:

$$x = x_0 + v_x t \tag{3.1.2}$$

$$y = v_{0y} + a_y t \tag{3.1.3}$$

$$y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2 \tag{3.1.4}$$

$$v_y^2 = v_{0y}^2 + 2a_y (y - y_0) \tag{3.1.5}$$

We analyze two-dimensional projectile motion by breaking it into two independent one-dimensional motions along the vertical and horizontal axes. The horizontal motion is simple, because $a_x = 0$ and v_x is thus constant. The velocity in the vertical direction begins to decrease as an object rises; at its highest point, the vertical velocity is zero. As an object falls towards the Earth again, the vertical velocity increases again in magnitude but points in the opposite direction to the initial vertical velocity. The xx and yy motions can be recombined to give the total velocity at any given point on the trajectory.

Key Points

- Constant velocity means that the object in motion is moving in a straight line at a constant speed.
- This line can be represented algebraically as: $x = x_0 + vt$, where x_0 represents the position of the object at $t = 0$, and the slope of the line indicates the object's speed.
- The velocity can be positive or negative, and is indicated by the sign of our slope. This tells us in which direction the object moves.
- Constant acceleration in motion in two dimensions generally follows a projectile pattern.
- Projectile motion is the motion of an object thrown or projected into the air, subject to only the (vertical) acceleration due to gravity.
- We analyze two-dimensional projectile motion by breaking it into two independent one-dimensional motions along the vertical and horizontal axes.

Key Terms

- **constant velocity**: Motion that does not change in speed nor direction.
- **kinematic**: of or relating to motion or kinematics

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3.2: Vectors

Components of a Vector

Vectors are geometric representations of magnitude and direction and can be expressed as arrows in two or three dimensions.

learning objectives

- Contrast two-dimensional and three-dimensional vectors

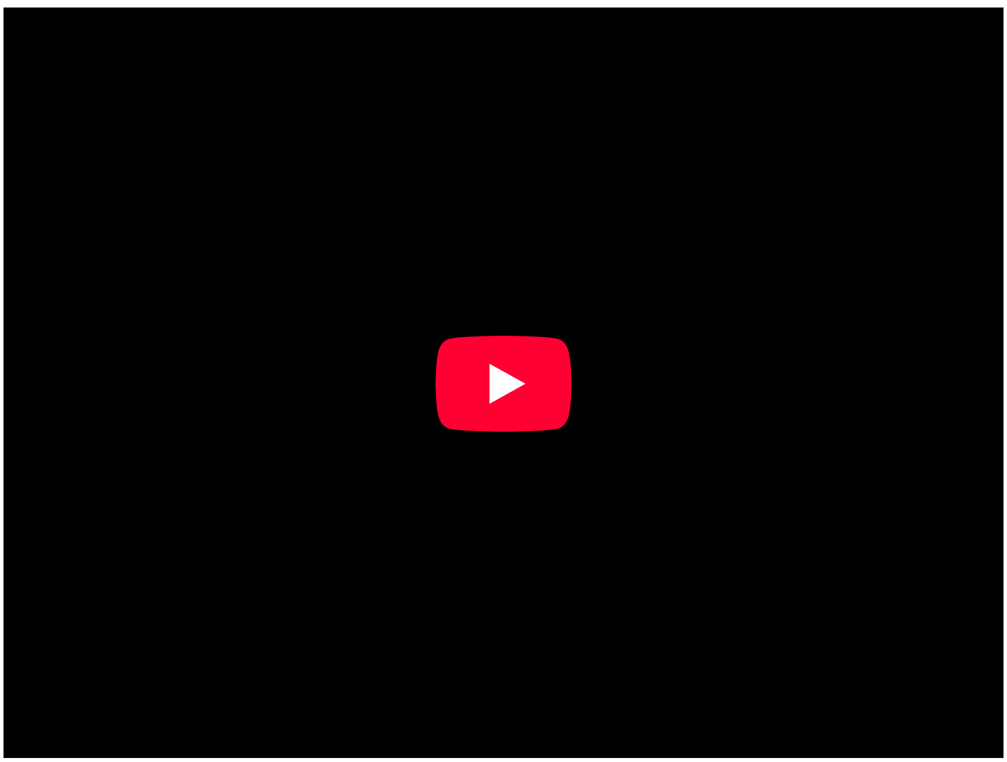
Vectors are geometric representations of magnitude and direction which are often represented by straight arrows, starting at one point on a coordinate axis and ending at a different point. All vectors have a length, called the magnitude, which represents some quality of interest so that the vector may be compared to another vector. Vectors, being arrows, also have a direction. This differentiates them from scalars, which are mere numbers without a direction.

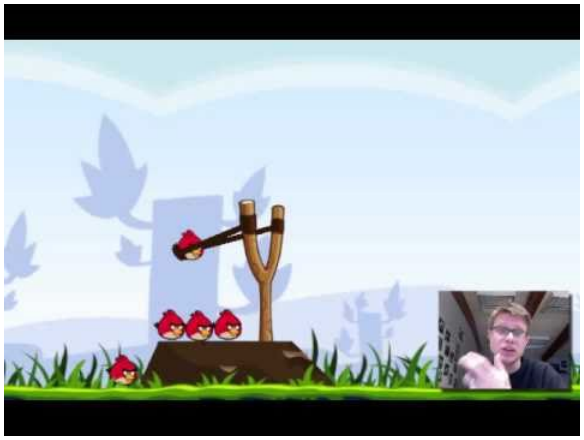
A vector is defined by its magnitude and its orientation with respect to a set of coordinates. It is often useful in analyzing vectors to break them into their component parts. For two-dimensional vectors, these components are horizontal and vertical. For three dimensional vectors, the magnitude component is the same, but the direction component is expressed in terms of xx, yy and zz.

Decomposing a Vector

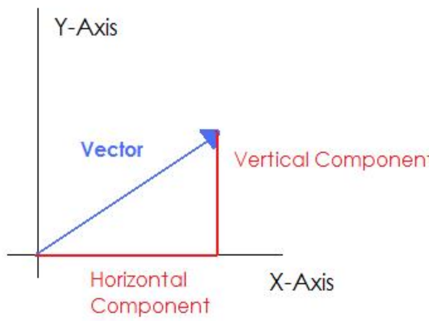
To visualize the process of decomposing a vector into its components, begin by drawing the vector from the origin of a set of coordinates. Next, draw a straight line from the origin along the x-axis until the line is even with the tip of the original vector. This is the horizontal component of the vector. To find the vertical component, draw a line straight up from the end of the horizontal vector until you reach the tip of the original vector. You should find you have a right triangle such that the original vector is the hypotenuse.

Decomposing a vector into horizontal and vertical components is a very useful technique in understanding physics problems. Whenever you see motion at an angle, you should think of it as moving horizontally and vertically at the same time. Simplifying vectors in this way can speed calculations and help to keep track of the motion of objects.





Scalars and Vectors: Mr. Andersen explains the differences between scalar and vectors quantities. He also uses a demonstration to show the importance of vectors and vector addition.



Components of a Vector: The original vector, defined relative to a set of axes. The horizontal component stretches from the start of the vector to its furthest x-coordinate. The vertical component stretches from the x-axis to the most vertical point on the vector. Together, the two components and the vector form a right triangle.

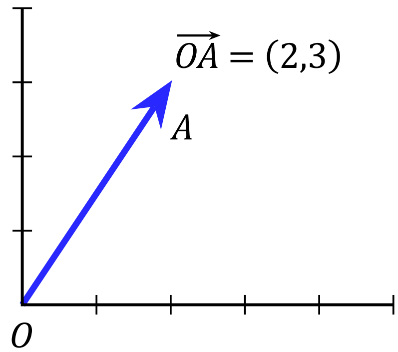
Scalars vs. Vectors

Scalars are physical quantities represented by a single number, and vectors are represented by both a number and a direction.

learning objectives

- Distinguish the difference between the quantities scalars and vectors represent

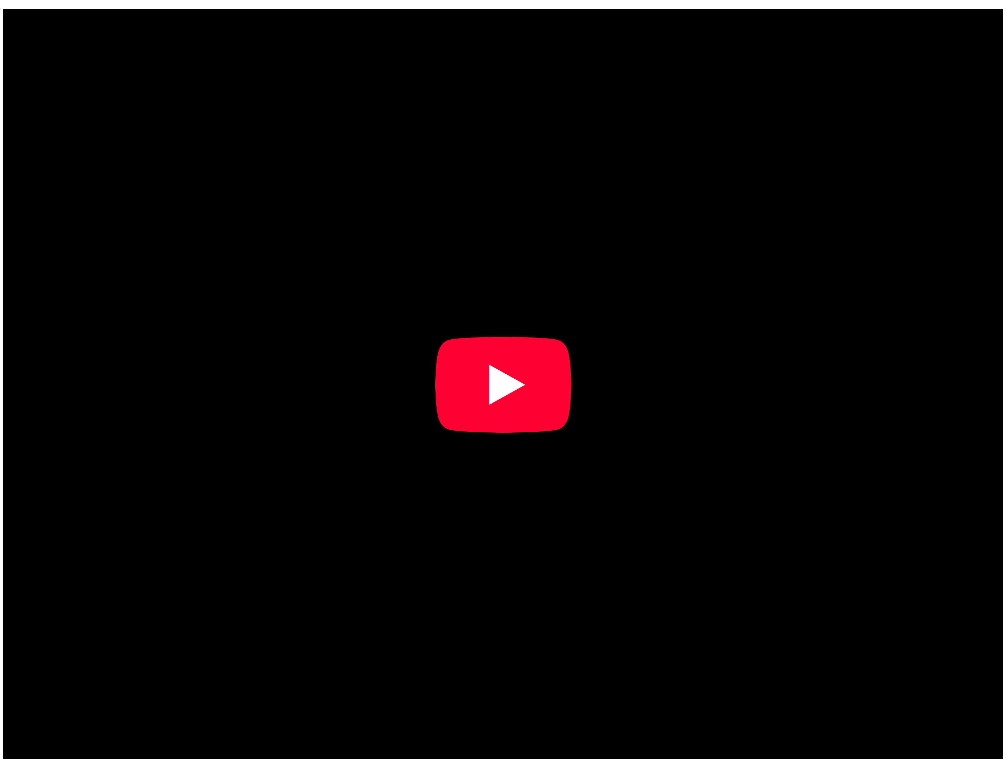
Physical quantities can usually be placed into two categories, vectors and scalars. These two categories are typified by what information they require. Vectors require two pieces of information: the magnitude and direction. In contrast, scalars require only the magnitude. Scalars can be thought of as numbers, whereas vectors must be thought of more like arrows pointing in a specific direction.

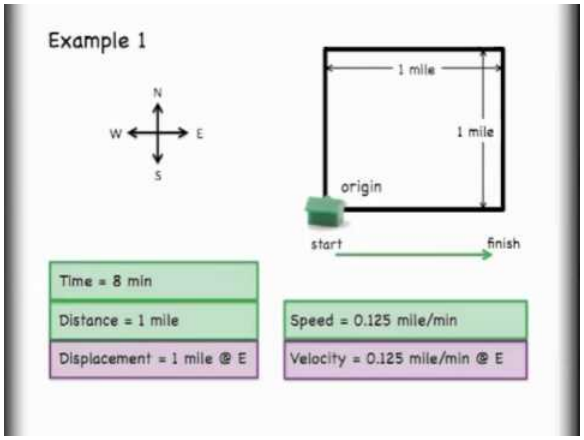


A Vector: An example of a vector. Vectors are usually represented by arrows with their length representing the magnitude and their direction represented by the direction the arrow points.

Vectors require both a magnitude and a direction. The magnitude of a vector is a number for comparing one vector to another. In the geometric interpretation of a vector the vector is represented by an arrow. The arrow has two parts that define it. The two parts are its length which represents the magnitude and its direction with respect to some set of coordinate axes. The greater the magnitude, the longer the arrow. Physical concepts such as displacement, velocity, and acceleration are all examples of quantities that can be represented by vectors. Each of these quantities has both a magnitude (how far or how fast) and a direction. In order to specify a direction, there must be something to which the direction is relative. Typically this reference point is a set of coordinate axes like the x-y plane.

Scalars differ from vectors in that they do not have a direction. Scalars are used primarily to represent physical quantities for which a direction does not make sense. Some examples of these are: mass, height, length, volume, and area. Talking about the direction of these quantities has no meaning and so they cannot be expressed as vectors.





The difference between Vectors and Scalars, Introduction and Basics: This video introduces the difference between scalars and vectors. Ideas about magnitude and direction are introduced and examples of both vectors and scalars are given.

Adding and Subtracting Vectors Graphically

Vectors may be added or subtracted graphically by laying them end to end on a set of axes.

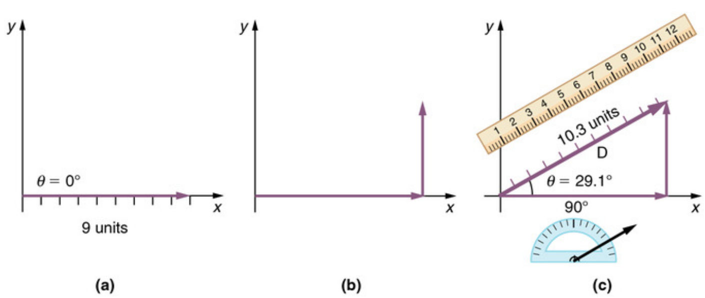
learning objectives

- Distinguish the difference between the quantities scalars and vectors represent

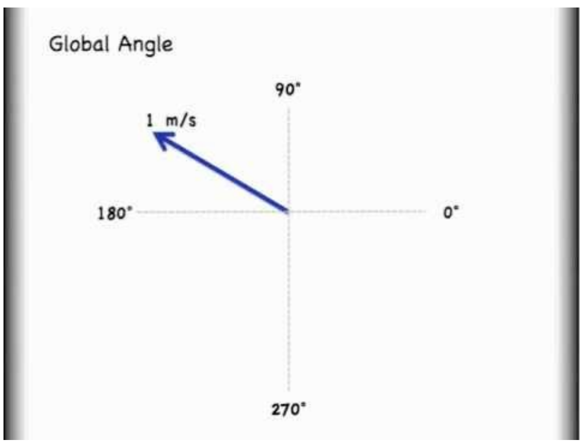
Adding and Subtracting Vectors

One of the ways in which representing physical quantities as vectors makes analysis easier is the ease with which vectors may be added to one another. Since vectors are graphical visualizations, addition and subtraction of vectors can be done graphically.

The graphical method of vector addition is also known as the head-to-tail method. To start, draw a set of coordinate axes. Next, draw out the first vector with its tail (base) at the origin of the coordinate axes. For vector addition it does not matter which vector you draw first since addition is commutative, but for subtraction ensure that the vector you draw first is the one you are subtracting *from*. The next step is to take the next vector and draw it such that its tail starts at the previous vector's head (the arrow side). Continue to place each vector at the head of the preceding one until all the vectors you wish to add are joined together. Finally, draw a straight line from the origin to the head of the final vector in the chain. This new line is the vector result of adding those vectors together.



Graphical Addition of Vectors: The head-to-tail method of vector addition requires that you lay out the first vector along a set of coordinate axes. Next, place the tail of the next vector on the head of the first one. Draw a new vector from the origin to the head of the last vector. This new vector is the sum of the original two.



Vector Addition Lesson 1 of 2: Head to Tail Addition Method: This video gets viewers started with vector addition and subtraction. The first lesson shows graphical addition while the second video takes a more mathematical approach and shows vector addition by components.

To subtract vectors the method is similar. Make sure that the first vector you draw is the one to be subtracted from. Then, to subtract a vector, proceed as if adding the *opposite* of that vector. In other words, flip the vector to be subtracted across the axes and then join it tail to head as if adding. To flip the vector, simply put its head where its tail was and its tail where its head was.

Adding and Subtracting Vectors Using Components

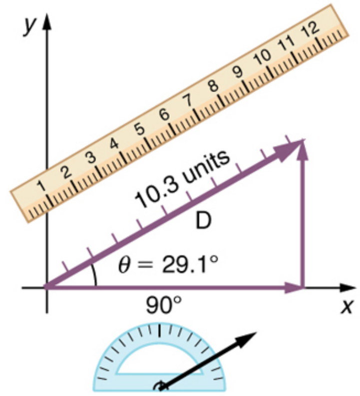
It is often simpler to add or subtract vectors by using their components.

learning objectives

- Demonstrate how to add and subtract vectors by components

Using Components to Add and Subtract Vectors

Another way of adding vectors is to add the components. Previously, we saw that vectors can be expressed in terms of their horizontal and vertical components. To add vectors, merely express both of them in terms of their horizontal and vertical components and then add the components together.

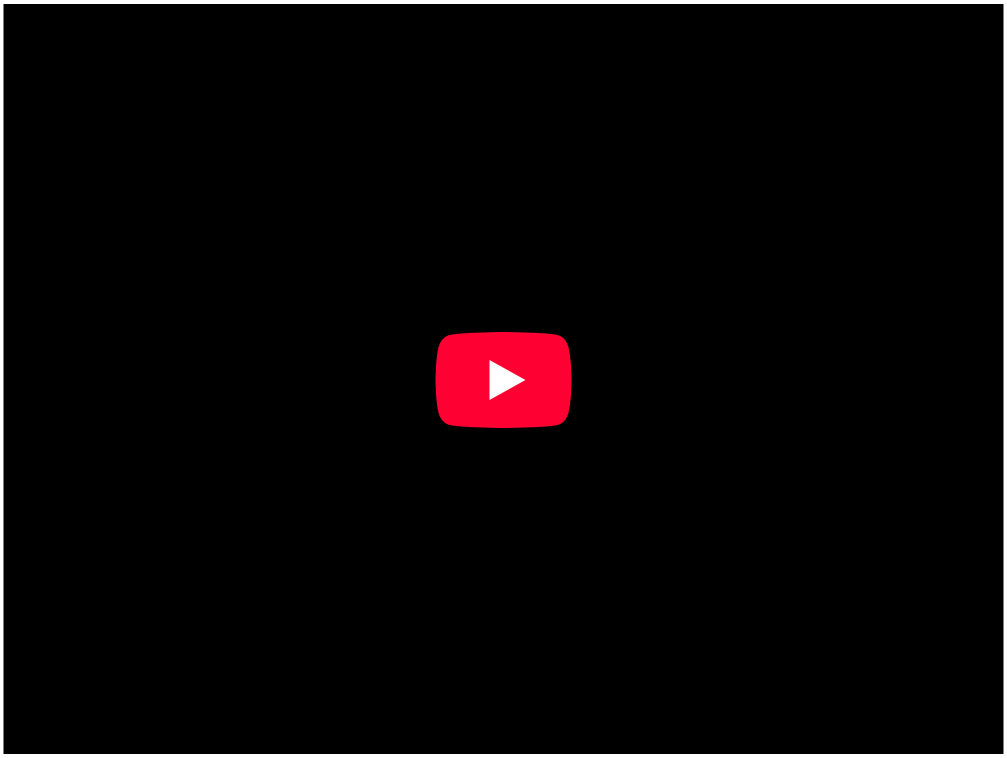


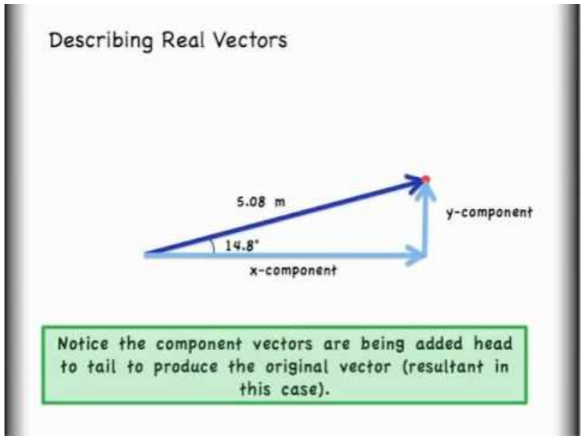
(c)

Vector with Horizontal and Vertical Components: The vector in this image has a magnitude of 10.3 units and a direction of 29.1 degrees above the x-axis. It can be decomposed into a horizontal part and a vertical part as shown.

For example, a vector with a length of 5 at a 36.9 degree angle to the horizontal axis will have a horizontal component of 4 units and a vertical component of 3 units. If we were to add this to another vector of the same magnitude and direction, we would get a vector twice as long at the same angle. This can be seen by adding the horizontal components of the two vectors $(4+4)$ and the two vertical components $(3+3)$. These additions give a new vector with a horizontal component of $8(4+4)$ and a vertical component of $6(3+3)$. To find the resultant vector, simply place the tail of the vertical component at the head (arrow side) of the horizontal component and then draw a line from the origin to the head of the vertical component. This new line is the resultant vector. It should be twice as long as the original, since both of its components are twice as large as they were previously.

To subtract vectors by components, simply subtract the two horizontal components from each other and do the same for the vertical components. Then draw the resultant vector as you did in the previous part.





Vector Addition Lesson 2 of 2: How to Add Vectors by Components: This video gets viewers started with vector addition using a mathematical approach and shows vector addition by components.

Multiplying Vectors by a Scalar

Multiplying a vector by a scalar changes the magnitude of the vector but not the direction.

learning objectives

- Summarize the interaction between vectors and scalars

Overview

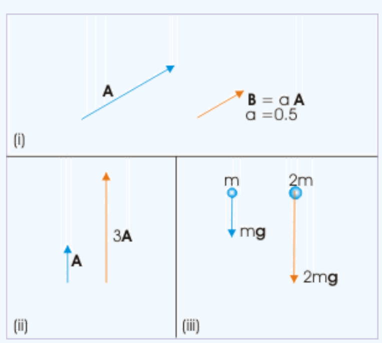
Although vectors and scalars represent different types of physical quantities, it is sometimes necessary for them to interact. While adding a scalar to a vector is impossible because of their different dimensions in space, it is possible to multiply a vector by a scalar. A scalar, however, cannot be multiplied by a vector.

To multiply a vector by a scalar, simply multiply the similar components, that is, the vector's magnitude by the scalar's magnitude. This will result in a new vector with the same direction but the product of the two magnitudes.

Example 3.2.1:

For example, if you have a vector A with a certain magnitude and direction, multiplying it by a scalar a with magnitude 0.5 will give a new vector with a magnitude of half the original. Similarly if you take the number 3 which is a pure and unit-less scalar and multiply it to a vector, you get a version of the original vector which is 3 times as long. As a more physical example take the gravitational force on an object. The force is a vector with its magnitude depending on the scalar known as mass and its direction being down. If the mass of the object is doubled, the force of gravity is doubled as well.

Multiplying vectors by scalars is very useful in physics. Most of the units used in vector quantities are intrinsically scalars multiplied by the vector. For example, the unit of meters per second used in velocity, which is a vector, is made up of two scalars, which are magnitudes: the scalar of length in meters and the scalar of time in seconds. In order to make this conversion from magnitudes to velocity, one must multiply the unit vector in a particular direction by these scalars.



Scalar Multiplication: (i) Multiplying the vector $\mathbf{A}$ by the scalar $a = 0.5$ yields the vector $\mathbf{B}$ which is half as long. (ii) Multiplying the vector $\mathbf{A}$ by 3 triples its length. (iii) Doubling the mass (scalar) doubles the force (vector) of gravity.

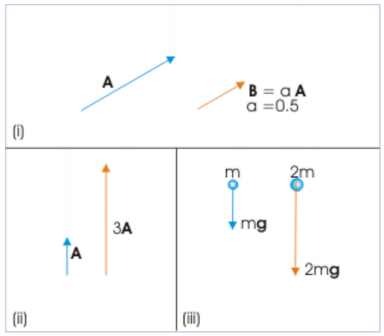
Unit Vectors and Multiplication by a Scalar

Multiplying a vector by a scalar is the same as multiplying its magnitude by a number.

learning objectives

- Predict the influence of multiplying a vector by a scalar

In addition to adding vectors, vectors can also be multiplied by constants known as scalars. Scalars are distinct from vectors in that they are represented by a magnitude but no direction. Examples of scalars include an object's mass, height, or volume.



Scalar Multiplication: (i) Multiplying the vector $\mathbf{A}$ by the scalar $a = 0.5$ yields the vector $\mathbf{B}$ which is half as long. (ii) Multiplying the vector $\mathbf{A}$ by 3 triples its length. (iii) Doubling the mass (scalar) doubles the force (vector) of gravity.

When multiplying a vector by a scalar, the direction of the vector is unchanged and the magnitude is multiplied by the magnitude of the scalar. This results in a new vector arrow pointing in the same direction as the old one but with a longer or shorter length. You can also accomplish scalar multiplication through the use of a vector's components. Once you have the vector's components, multiply each of the components by the scalar to get the new components and thus the new vector.

A useful concept in the study of vectors and geometry is the concept of a unit vector. A unit vector is a vector with a length or magnitude of one. The unit vectors are different for different coordinates. In Cartesian coordinates the directions are x and y usually denoted $\hat{x}$ and $\hat{y}$. With the triangle above the letters referred to as a "hat". The unit vectors in Cartesian coordinates describe a circle known as the "unit circle" which has radius one. This can be seen by taking all the possible vectors of length one at all the possible angles in this coordinate system and placing them on the coordinates. If you were to draw a line around connecting all the heads of all the vectors together, you would get a circle of radius one.

Position, Displacement, Velocity, and Acceleration as Vectors

Position, displacement, velocity, and acceleration can all be shown vectors since they are defined in terms of a magnitude and a direction.

learning objectives

- Examine the applications of vectors in analyzing physical quantities

Use of Vectors

Vectors can be used to represent physical quantities. Most commonly in physics, vectors are used to represent displacement, velocity, and acceleration. Vectors are a combination of magnitude and direction, and are drawn as arrows. The length represents the magnitude and the direction of that quantity is the direction in which the vector is pointing. Because vectors are constructed this way, it is helpful to analyze physical quantities (with both size and direction) as vectors.

Applications

In physics, vectors are useful because they can visually represent position, displacement, velocity and acceleration. When drawing vectors, you often do not have enough space to draw them to the scale they are representing, so it is important to denote somewhere what scale they are being drawn at. For example, when drawing a vector that represents a magnitude of 100, one may draw a line that is 5 units long at a scale of $\frac{1}{20}$. When the inverse of the scale is multiplied by the drawn magnitude, it should equal the actual magnitude.

Position and Displacement

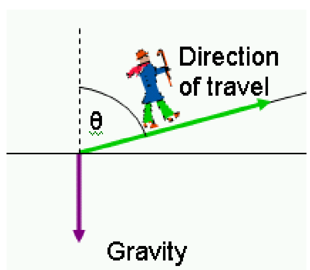
Displacement is defined as the distance, in any direction, of an object relative to the position of another object. Physicists use the concept of a position vector as a graphical tool to visualize displacements. A position vector expresses the position of an object from the origin of a coordinate system. A position vector can also be used to show the position of an object in relation to a reference point, secondary object or initial position (if analyzing how far the object has moved from its original location). The position vector is a straight line drawn from the arbitrary origin to the object. Once drawn, the vector has a length and a direction relative to the coordinate system used.

Velocity

Velocity is also defined in terms of a magnitude and direction. To say that something is gaining or losing velocity one must also say how much and in what direction. For example, an airplane flying at 200 $\frac{\mathrm{km}}{\mathrm{h}}$ to the northeast can be represented by an vector pointing in the northeast direction with a magnitude of 200 $\frac{\mathrm{km}}{\mathrm{h}}$. In drawing the vector, the magnitude is only important as a way to compare two vectors of the same units. So, if there were another airplane flying 100 $\frac{\mathrm{km}}{\mathrm{h}}$ to the southwest, the vector arrow should be half as long and pointing in the direction of southwest.

Acceleration

Acceleration, being the time rate of change of velocity, is composed of a magnitude and a direction, and is drawn with the same concept as a velocity vector. A value for acceleration would not be helpful in physics if the magnitude and direction of this acceleration was unknown, which is why these vectors are important. In a free body diagram, for example, of an object falling, it would be helpful to use an acceleration vector near the object to denote its acceleration towards the ground. If gravity is the only force acting on the object, this vector would be pointing downward with a magnitude of 9.81 $\frac{\mathrm{m}}{\mathrm{s}^2}$ of 32.2 $\frac{\mathrm{ft}}{\mathrm{s}^2}$.



Vector Diagram: Here is a man walking up a hill. His direction of travel is defined by the angle theta relative to the vertical axis and by the length of the arrow going up the hill. He is also being accelerated downward by gravity.

Key Points

- Vectors can be broken down into two components: magnitude and direction.
- By taking the vector to be analyzed as the hypotenuse, the horizontal and vertical components can be found by completing a right triangle. The bottom edge of the triangle is the horizontal component and the side opposite the angle is the vertical component.
- The angle that the vector makes with the horizontal can be used to calculate the length of the two components.
- Scalars are physical quantities represented by a single number and no direction.
- Vectors are physical quantities that require both magnitude and direction.
- Examples of scalars include height, mass, area, and volume. Examples of vectors include displacement, velocity, and acceleration.
- To add vectors, lay the first one on a set of axes with its tail at the origin. Place the next vector with its tail at the previous vector’s head. When there are no more vectors, draw a straight line from the origin to the head of the last vector. This line is the sum of the vectors.
- To subtract vectors, proceed as if adding the two vectors, but flip the vector to be subtracted across the axes and then join it tail to head as if adding.

- Adding or subtracting any number of vectors yields a resultant vector.
- Vectors can be decomposed into horizontal and vertical components.
- Once the vectors are decomposed into components, the components can be added.
- Adding the respective components of two vectors yields a vector which is the sum of the two vectors.
- A vector is a quantity with both magnitude and direction.
- A scalar is a quantity with only magnitude.
- Multiplying a vector by a scalar is equivalent to multiplying the vector's magnitude by the scalar. The vector lengthens or shrinks but does not change direction.
- A unit vector is a vector of magnitude (length) 1.
- A scalar is a physical quantity that can be represented by a single number. Unlike vectors, scalars do not have direction.
- Multiplying a vector by a scalar is the same as multiplying the vector's magnitude by the number represented by the scalar.
- Vectors are arrows consisting of a magnitude and a direction. They are used in physics to represent physical quantities that also have both magnitude and direction.
- Displacement is a physics term meaning the distance of an object from a reference point. Since the displacement contains two pieces of information: the distance from the reference point and the direction away from the point, it is well represented by a vector.
- Velocity is defined as the rate of change in time of the displacement. To know the velocity of an object one must know both how fast the displacement is changing and in what direction. Therefore it is also well represented by a vector.
- Acceleration, being the rate of change of velocity also requires both a magnitude and a direction relative to some coordinates.
- When drawing vectors, you often do not have enough space to draw them to the scale they are representing, so it is important to denote somewhere what scale they are being drawn at.

Key Terms

- **coordinates**: Numbers indicating a position with respect to some axis. Ex: x and y coordinates indicate position relative to xx and yy axes.
- **axis**: An imaginary line around which an object spins or is symmetrically arranged.
- **magnitude**: A number assigned to a vector indicating its length.
- **Coordinate axes**: A set of perpendicular lines which define coordinates relative to an origin. Example: x and y coordinate axes define horizontal and vertical position.
- **origin**: The center of a coordinate axis, defined as being the coordinate 0 in all axes.
- **Component**: A part of a vector. For example, horizontal and vertical components.
- **vector**: A directed quantity, one with both magnitude and direction; the between two points.
- **magnitude**: A number assigned to a vector indicating its length.
- **scalar**: A quantity that has magnitude but not direction; compare vector.
- **unit vector**: A vector of magnitude 1.
- **velocity**: The rate of change of displacement with respect to change in time.
- **displacement**: The length and direction of a straight line between two objects.
- **acceleration**: the rate at which the velocity of a body changes with time

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3.3: Projectile Motion

Basic Equations and Parabolic Path

Projectile motion is a form of motion where an object moves in parabolic path; the path that the object follows is called its trajectory.

learning objectives

- Assess the effect of angle and velocity on the trajectory of the projectile; derive maximum height using displacement

Projectile Motion

Projectile motion is a form of motion where an object moves in a bilaterally symmetrical, parabolic path. The path that the object follows is called its trajectory. Projectile motion only occurs when there is one force applied at the beginning on the trajectory, after which the only interference is from gravity. In a previous atom we discussed what the various components of an object in projectile motion are. In this atom we will discuss the basic equations that go along with them in the special case in which the projectile initial positions are null (i.e. $x_0 = 0$ and $y_0 = 0$).

Initial Velocity

The initial velocity can be expressed as x components and y components:

$$u_x = u \cdot \cos\theta \tag{3.3.1}$$

$$u_y = u \cdot \sin\theta \tag{3.3.2}$$

In this equation, u stands for initial velocity magnitude and θ refers to projectile angle.

Time of Flight

The time of flight of a projectile motion is the time from when the object is projected to the time it reaches the surface. As we discussed previously, T depends on the initial velocity magnitude and the angle of the projectile:

$$T = \frac{2 \cdot u_y}{g} \tag{3.3.3}$$

$$T = \frac{2 \cdot u \cdot \sin\theta}{g} \tag{3.3.4}$$

Acceleration

In projectile motion, there is no acceleration in the horizontal direction. The acceleration, a, in the vertical direction is just due to gravity, also known as free fall:

$$a_x = 0 \tag{3.3.5}$$

$$a_y = -g \tag{3.3.6}$$

Velocity

The horizontal velocity remains constant, but the vertical velocity varies linearly, because the acceleration is constant. At any time, t, the velocity is:

$$u_x = u \cdot \cos\theta \tag{3.3.7}$$

$$u_y = u \cdot \sin\theta - g \cdot t \tag{3.3.8}$$

You can also use the Pythagorean Theorem to find velocity:

$$u = \sqrt{u_x^2 + u_y^2} \tag{3.3.9}$$

Displacement

At time, t, the displacement components are:

$$x = u \cdot t \cdot \cos\theta \tag{3.3.10}$$

$$y = u \cdot t \cdot \sin\theta - \frac{1}{2} g t^2 \tag{3.3.11}$$

The equation for the magnitude of the displacement is $\Delta r = \sqrt{x^2 + y^2}$.

Parabolic Trajectory

We can use the displacement equations in the x and y direction to obtain an equation for the parabolic form of a projectile motion:

$$y = \tan\theta \cdot x - \frac{g}{2 \cdot u^2 \cdot \cos^2\theta} \cdot x^2 \tag{3.3.12}$$

Maximum Height

The maximum height is reached when $v_y = 0$. Using this we can rearrange the velocity equation to find the time it will take for the object to reach maximum height

$$t_h = \frac{u \cdot \sin\theta}{g} \tag{3.3.13}$$

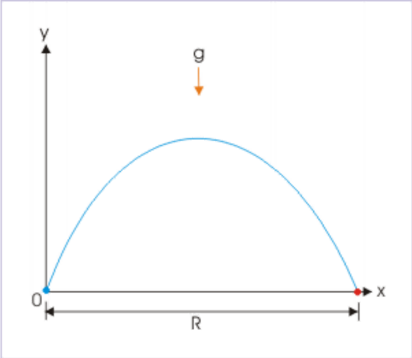
where t_h stands for the time it takes to reach maximum height. From the displacement equation we can find the maximum height

$$h = \frac{u^2 \cdot \sin^2\theta}{2 \cdot g} \tag{3.3.14}$$

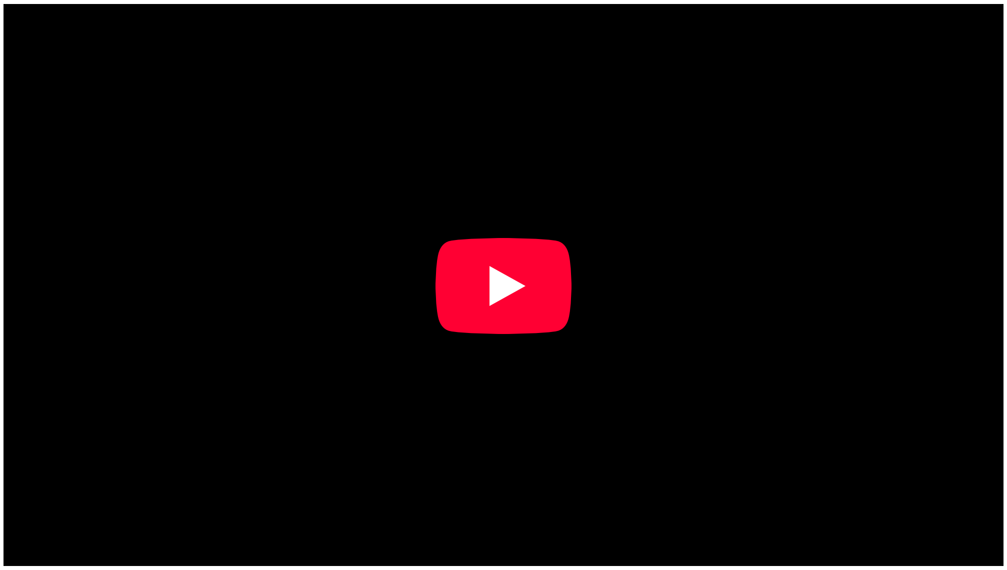
Range

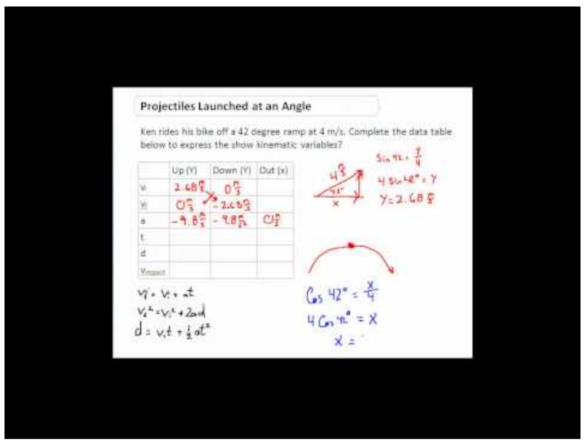
The range of the motion is fixed by the condition $y = 0$. Using this we can rearrange the parabolic motion equation to find the range of the motion:

$$R = \frac{u^2 \cdot \sin 2\theta}{g}. \tag{3.3.15}$$



Range of Trajectory: The range of a trajectory is shown in this figure.





Projectiles at an Angle: This video gives a clear and simple explanation of how to solve a problem on Projectiles Launched at an Angle. I try to go step by step through this difficult problem to layout how to solve it in a super clear way. 2D kinematic problems take time to solve, take notes on the order of how I solved it. Best wishes. Tune into my other videos for more help. Peace.

Solving Problems

In projectile motion, an object moves in parabolic path; the path the object follows is called its trajectory.

> **learning objectives**
>
> - Identify which components are essential in determining projectile motion of an object

We have previously discussed projectile motion and its key components and basic equations. Using that information, we can solve many problems involving projectile motion. Before we do this, let’s review some of the key factors that will go into this problem-solving.

What is Projectile Motion?

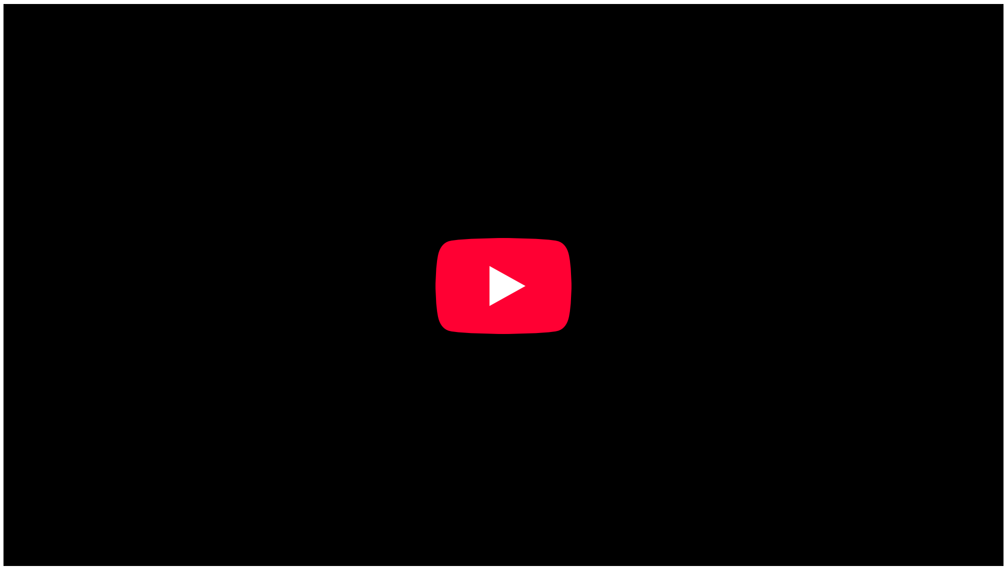
Projectile motion is when an object moves in a bilaterally symmetrical, parabolic path. The path that the object follows is called its trajectory. Projectile motion only occurs when there is one force applied at the beginning, after which the only influence on the trajectory is that of gravity.

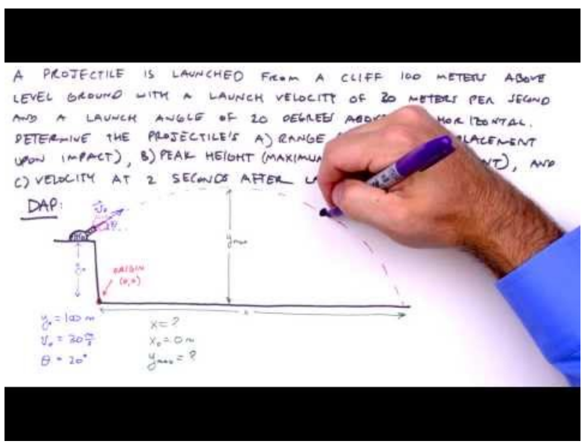
What are the Key Components of Projectile Motion?

The key components that we need to remember in order to solve projectile motion problems are:

- Initial launch angle, θ

- Initial velocity, u
- Time of flight, T
- Acceleration, a
- Horizontal velocity, v_x
- Vertical velocity, v_y
- Displacement, d
- Maximum height, H
- Range, R





How To Solve Any Projectile Motion Problem (The Toolbox Method): Introducing the “Toolbox” method of solving projectile motion problems! Here we use kinematic equations and modify with initial conditions to generate a “toolbox” of equations with which to solve a classic three-part projectile motion problem.

Now, let’s look at two examples of problems involving projectile motion.

Example 3.3.1:

Example 1

Let’s say you are given an object that needs to clear two posts of equal height separated by a specific distance. Refer to for this example. The projectile is thrown at $25\sqrt{2}$ m/s at an angle of 45°. If the object is to clear both posts, each with a height of 30m, find the minimum: (a) position of the launch on the ground in relation to the posts and (b) the separation between the posts. For simplicity’s sake, use a gravity constant of 10. Problems of any type in physics are much easier to solve if you list the things that you know (the “givens”).

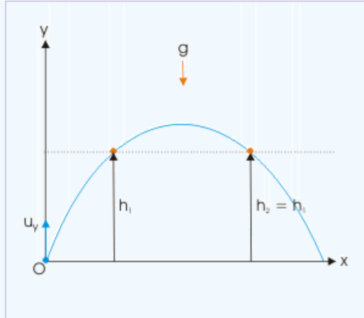


Diagram for Example 1: Use this figure as a reference to solve example 1. The problem is to make sure the object is able to clear both posts.

Solution: The first thing we need to do is figure out at what time tt the object reaches the specified height. Since the motion is in a parabolic shape, this will occur twice: once when traveling upward, and again when the object is traveling downward. For this we can use the equation of displacement in the vertical direction, $y - y_0$:

$$y - y_0 = (v_y \cdot t) - (\frac{1}{2} \cdot g \cdot t^2) \tag{3.3.16}$$

We substitute in the appropriate variables:

$$v_y = u \cdot \sin\theta = 25\sqrt{2}\frac{m}{s} \cdot \sin 45^\circ = 25\frac{m}{s} \tag{3.3.17}$$

Therefore:

$$30m = 25 \cdot t - \frac{1}{2} \cdot 10 \cdot t^2 \tag{3.3.18}$$

We can use the quadratic equation to find that the roots of this equation are 2s and 3s. This means that the projectile will reach 30m after 2s, on its way up, *and* after 3s, on its way down.

Example 2

An object is launched from the base of an incline, which is at an angle of 30°. If the launch angle is 60° *from the horizontal* and the launch speed is 10 m/s, what is the total flight time? The following information is given: $u = 10\frac{m}{s}; \theta = 60^\circ; g = 10\frac{m}{s^2}$.

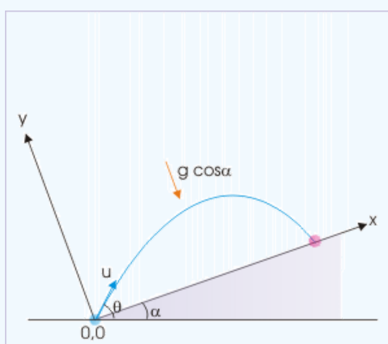


Diagram for Example 2: When dealing with an object in projectile motion on an incline, we first need to use the given information to reorient the coordinate system in order to have the object launch and fall on the same surface.

Solution: In order to account for the incline angle, we have to reorient the coordinate system so that the points of projection and return are on the same level. The angle of projection with respect to the x direction is $\theta - \alpha$, and the acceleration in the y direction is $g \cdot \cos\alpha$. We replace θ with $\theta - \alpha$ and g with $g \cdot \cos\alpha$:

$$T = \frac{2 \cdot u \cdot \sin(\theta)}{g} = \frac{2 \cdot u \cdot \sin(\theta - \alpha)}{g \cdot \cos(\alpha)} = \frac{2 \cdot 10 \cdot \sin(60 - 30)}{10 \cdot \cos(30)} = \frac{20 \cdot \sin(30)}{10 \cdot \cos(30)} \tag{3.3.19}$$

$$T = \frac{2}{\sqrt{3}} s \tag{3.3.20}$$

Zero Launch Angle

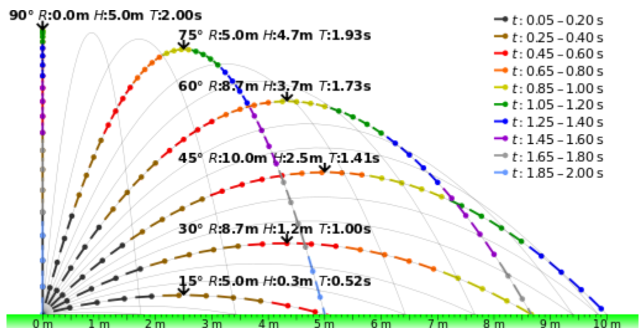
An object launched horizontally at a height H travels a range $\mathrm{v}_0\sqrt{\frac{2\mathrm{H}}{\mathrm{g}}}$ during a time of flight $\mathrm{T}=\sqrt{\frac{2\mathrm{H}}{\mathrm{g}}}$.

learning objectives

- Explain the relationship between the range and the time of flight

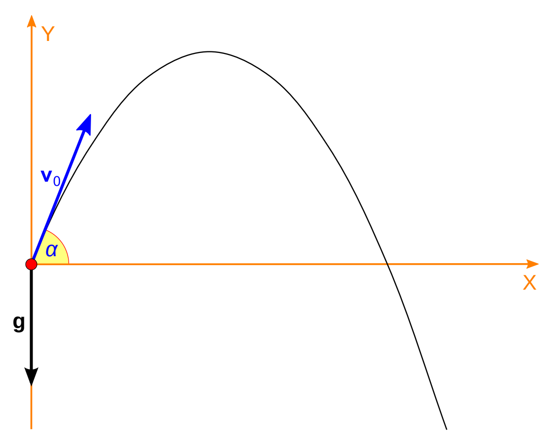
Projectile motion is a form of motion where an object moves in a parabolic path. The path followed by the object is called its trajectory. Projectile motion occurs when a force is applied at the beginning of the trajectory for the launch (after this the projectile is subject only to the gravity).

One of the key components of the projectile motion, and the trajectory it follows, is the initial launch angle. The angle at which the object is launched dictates the range, height, and time of flight the object will experience while in projectile motion. shows different paths for the same object being launched at the same initial velocity and different launch angles. As illustrated by the figure, the larger the initial launch angle and maximum height, the longer the flight time of the object.



Projectile Trajectories: The launch angle determines the range and maximum height that an object will experience after being launched.This image shows that path of the same object being launched at the same speed but different angles.

We have previously discussed the effects of different launch angles on range, height, and time of flight. However, what happens if there is no angle, and the object is just launched horizontally? It makes sense that the object should be launched at a certain height (H), otherwise it wouldn't travel very far before hitting the ground. Let's examine how an object launched horizontally at a height H travels. In our case is when α is 0.



Projectile motion: Projectile moving following a parabola.Initial launch angle is αα, and the velocity is v_0.

Duration of Flight

There is no vertical component in the initial velocity (v_0) because the object is launched horizontally. Since the object travels distance H in the vertical direction before it hits the ground, we can use the kinematic equation for the vertical motion:

$$(y - y_0) = -H = 0 \cdot T - \frac{1}{2} g T^2 \tag{3.3.21}$$

Here, T is the duration of the flight before the object its the ground. Therefore:

$$T = \sqrt{\frac{2H}{g}} \tag{3.3.22}$$

Range

In the horizontal direction, the object travels at a constant speed v_0 during the flight. Therefore, the range R (in the horizontal direction) is given as:

$$R = v_0 \cdot T = v_0 \sqrt{\frac{2H}{g}} \tag{3.3.23}$$

General Launch Angle

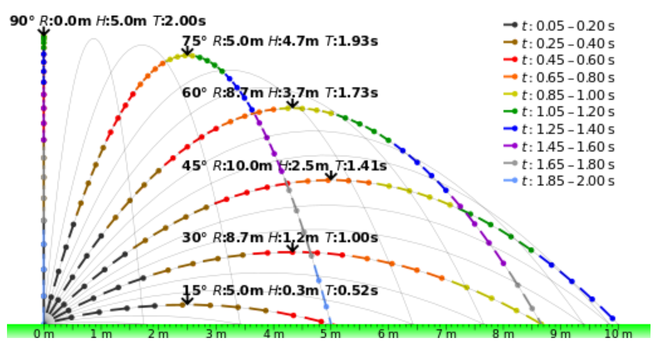
The initial launch angle (0-90 degrees) of an object in projectile motion dictates the range, height, and time of flight of that object.

learning objectives

- Choose the appropriate equation to find range, maximum height, and time of flight

Projectile motion is a form of motion where an object moves in a bilaterally symmetrical, parabolic path. The path that the object follows is called its trajectory. Projectile motion only occurs when there is one force applied at the beginning of the trajectory, after which the only interference is from gravity.

One of the key components of projectile motion and the trajectory that it follows is the initial launch angle. This angle can be anywhere from 0 to 90 degrees. The angle at which the object is launched dictates the range, height, and time of flight it will experience while in projectile motion. shows different paths for the same object launched at the same initial velocity at different launch angles. As you can see from the figure, the larger the initial launch angle, the closer the object comes to maximum height and the longer the flight time. The largest range will be experienced at a launch angle up to 45 degrees.



Launch Angle: The launch angle determines the range and maximum height that an object will experience after being launched. This image shows that path of the same object being launched at the same velocity but different angles.

The range, maximum height, and time of flight can be found if you know the initial launch angle and velocity, using the following equations:

$$R = \frac{v_i^2 \sin 2\theta_i}{g} \tag{3.3.24}$$

$$h = \frac{v_i^2 \sin^2 \theta_i}{2g} \tag{3.3.25}$$

$$T = \frac{2 v_i \sin \theta}{g} \tag{3.3.26}$$

Where R – Range, h – maximum height, T – time of flight, v_i – initial velocity, θ_i – initial launch angle, g – gravity.

Now that we understand how the launch angle plays a major role in many other components of the trajectory of an object in projectile motion, we can apply that knowledge to making an object land where we want it. If there is a certain distance, d, that you want your object to go and you know the initial velocity at which it will be launched, the initial launch angle required to get it that distance is called the angle of reach. It can be found using the following equation:

$$\theta=\frac{1}{2}\sin^{-1}(\frac{\mathrm{gd}}{\mathrm{v}^2}) \tag{3.3.27}$$

Key Points: Range, Symmetry, Maximum Height

Projectile motion is a form of motion where an object moves in parabolic path. The path that the object follows is called its trajectory.

learning objectives

- Construct a model of projectile motion by including time of flight, maximum height, and range

What is Projectile Motion ?

Projectile motion is a form of motion where an object moves in a bilaterally symmetrical, parabolic path. The path that the object follows is called its trajectory. Projectile motion only occurs when there is one force applied at the beginning on the trajectory, after which the only interference is from gravity. In this atom we are going to discuss what the various components of an object in projectile motion are, we will discuss the basic equations that go along with them in another atom, “Basic Equations and Parabolic Path”

Key Components of Projectile Motion:

Time of Flight, T:

The time of flight of a projectile motion is exactly what it sounds like. It is the time from when the object is projected to the time it reaches the surface. The time of flight depends on the initial velocity of the object and the angle of the projection, θθ. When the point of projection and point of return are on the same horizontal plane, the net vertical displacement of the object is zero.

Symmetry:

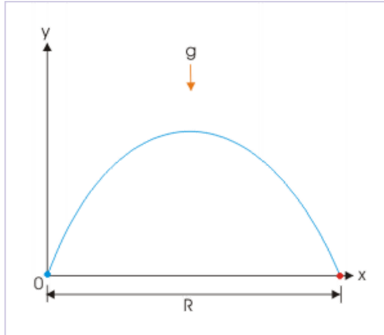
All projectile motion happens in a bilaterally symmetrical path, as long as the point of projection and return occur along the same horizontal surface. Bilateral symmetry means that the motion is symmetrical in the vertical plane. If you were to draw a straight vertical line from the maximum height of the trajectory, it would mirror itself along this line.

Maximum Height, H:

The maximum height of a object in a projectile trajectory occurs when the vertical component of velocity, vyvy, equals zero. As the projectile moves upwards it goes against gravity, and therefore the velocity begins to decelerate. Eventually the vertical velocity will reach zero, and the projectile is accelerated downward under gravity immediately. Once the projectile reaches its maximum height, it begins to accelerate downward. This is also the point where you would draw a vertical line of symmetry.

Range of the Projectile, R:

The range of the projectile is the displacement in the horizontal direction. There is no acceleration in this direction since gravity only acts vertically. shows the line of range. Like time of flight and maximum height, the range of the projectile is a function of initial speed.



Range: The range of a projectile motion, as seen in this image, is independent of the forces of gravity.

Key Points

- Objects that are projected from, and land on the same horizontal surface will have a vertically symmetrical path.
- The time it takes from an object to be projected and land is called the time of flight. This depends on the initial velocity of the projectile and the angle of projection.
- When the projectile reaches a vertical velocity of zero, this is the maximum height of the projectile and then gravity will take over and accelerate the object downward.
- The horizontal displacement of the projectile is called the range of the projectile, and depends on the initial velocity of the object.
- When solving problems involving projectile motion, we must remember all the key components of the motion and the basic equations that go along with them.
- Using that information, we can solve many different types of problems as long as we can analyze the information we are given and use the basic equations to figure it out.
- To clear two posts of equal height, and to figure out what the distance between these posts is, we need to remember that the trajectory is a parabolic shape and that there are two different times at which the object will reach the height of the posts.
- When dealing with an object in projectile motion on an incline, we first need to use the given information to reorientate the coordinate system in order to have the object launch and fall on the same surface.
- For the zero launch angle, there is no vertical component in the initial velocity.
- The duration of the flight before the object hits the ground is given as $T = \sqrt{\frac{2H}{g}}$.
- In the horizontal direction, the object travels at a constant speed v_0 during the flight. The range R (in the horizontal direction) is given as: $R = v_0 \cdot T = v_0 \sqrt{\frac{2H}{g}}$.
- If the same object is launched at the same initial velocity, the height and time of flight will increase proportionally to the initial launch angle.
- An object launched into projectile motion will have an initial launch angle anywhere from 0 to 90 degrees.
- The range of an object, given the initial launch angle and initial velocity is found with: $R = \frac{v_i^2 \sin 2\theta_i}{g}$.
- The maximum height of an object, given the initial launch angle and initial velocity is found with: $h = \frac{v_i^2 \sin^2 \theta_i}{2g}$.
- The time of flight of an object, given the initial launch angle and initial velocity is found with: $T = \frac{2v_i \sin\theta}{g}$.
- The angle of reach is the angle the object must be launched at in order to achieve a specific distance: $\theta = \frac{1}{2}\sin^{-1}(\frac{gd}{v^2})$.
- Objects that are projected from and land on the same horizontal surface will have a path symmetric about a vertical line through a point at the maximum height of the projectile.
- The time it takes from an object to be projected and land is called the time of flight. It depends on the initial velocity of the projectile and the angle of projection.
- The maximum height of the projectile is when the projectile reaches zero vertical velocity. From this point the vertical component of the velocity vector will point downwards.

- The horizontal displacement of the projectile is called the range of the projectile and depends on the initial velocity of the object.
- If an object is projected at the same initial speed, but two complementary angles of projection, the range of the projectile will be the same.

Key Terms

- **trajectory**: The path of a body as it travels through space.
- **symmetrical**: Exhibiting symmetry; having harmonious or proportionate arrangement of parts; having corresponding parts or relations.
- **reorientate**: to orientate anew; to cause to face a different direction
- **gravity**: Resultant force on Earth's surface, of the attraction by the Earth's masses, and the centrifugal pseudo-force caused by the Earth's rotation.
- **bilateral symmetry**: the property of being symmetrical about a vertical plane

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3.4: Multiple Velocities

Addition of Velocities

Relative velocities can be found by adding the velocity of the observed object to the velocity of the frame of reference it was measured in.

As learned in a previous atom, relative velocity is the velocity of an object as observed from a certain frame of reference.

demonstrates the concept of relative velocity. The girl is riding in a sled at 1.0 m/s, relative to an observer. When she throws the snowball forward at a speed of 1.5 m/s, relative to the sled, the velocity of the snowball to the observer is the sum of the velocity of the sled and the velocity of the snowball relative to the sled:

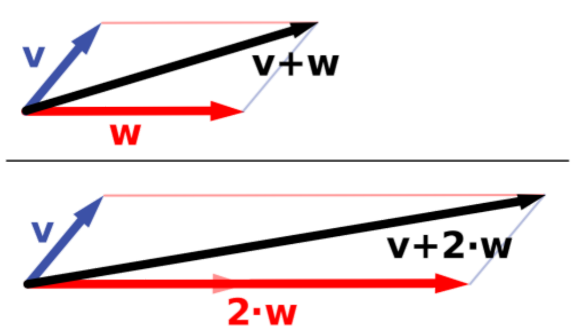
$$1.0\mathrm{m/s}+1.5\mathrm{m/s}=2.5\mathrm{m/s} \tag{3.4.1}$$

If the girl were to throw the snowball behind her at the same speed, the velocity of the ball relative to the observer would be:

$$1.0\mathrm{m/s}-1.5\mathrm{m/s}=-0.5\mathrm{m/s} \tag{3.4.2}$$

The concept of relative velocity can also be demonstrated using the example of a boat in a river with a current. The boat is only trying to move forward, but since the river is in motion, it carries the boat sideways while it moves forward. The person on the boat is only observing the forward motion, while an observer on the shore will notice that the boat is moving sideways. In order to calculate the velocity that the object is moving relative to earth, it is helpful to remember that velocity is a vector. In order to analytically add these vectors, you need to remember the relationship between the magnitude and direction of the vector and its components on the x and y axis of the coordinate system:

- Magnitude: $\mathrm{v}=\sqrt{\mathrm{v_x^2}+\mathrm{v_y^2}}$
- Direction: $\theta=\tan^{-1}(\frac{\mathrm{v_y}}{\mathrm{v_x}})$
- x-component: $\mathrm{v_x}=\mathrm{v}\cos\theta$
- y-component: $\mathrm{v_y}=\mathrm{v}\sin\theta$



Vector Addition: Addition of velocities is simply the addition of vectors.

These components are shown above. The first two equations are for when the magnitude and direction are known, but you are looking for the components. The last two equations are for when the components are known, and you are looking for the magnitude and direction. The magnitude of the observed velocity from the shore is the square root sum of the squared velocity of the boat and the squared velocity of the river.

Relative Velocity

Relative velocity is the velocity of an object B measured with respect to the velocity of another object A, denoted as $\mathrm{v_{BA}}$.

Relative velocity is the velocity of an object B, in the rest frame of another object A. This is denoted as $\mathrm{v_{AB}}$, where A is the velocity in the rest frame of B.

Galileo observed the concept of relative velocity by using an example of a fly and a boat. He observed that while you are aboard the boat, if you see a fly, you can measure its velocity, u. You can then go back on land and measure the velocity of the boat, v. Is the velocity of the fly, u, the actual velocity of the fly? No, because what you measured was the velocity of the fly relative to the velocity of the boat. To obtain the velocity of the fly relative to the shore, s, you can use the vector sum as shown: $\mathrm{s}=\mathrm{u}+\mathrm{v}$

Examples of Relative Velocity

This concept is best explained using examples. Pretend you are sitting in a passenger train that is moving east. If you were to look out the window and see a man walking in the same direction, it would appear the the man is moving much slower than he actually is. Now imagine you are standing outside and observe the same man walking next to the train. It will appear the the man is walking much faster than it seemed when you were inside the train.

Now, imagine you are on a boat, and you see a man walking from one end of the deck to the other. The velocity that you observe the man walking in will be the same velocity that he would be walking in if you both were on land. Now imagine that you are on land and see the man on the moving boat, walking from one end of the deck to another. It will now appear that the man is walking much faster than it appeared when you were on the boat with him.

Why is this? The concept of relative velocity has to do with your frame of reference. When you were on the train, your frame of reference was moving in the same direction that the man was walking, so it appeared that he was walking slower. But once you were off the train, you were in a stationary frame of reference, so you were able to observe him moving at his actual speed. When you were on the boat, you were in a moving frame of reference, but so was the object you were observing, so you were able to observe the man walking at his actual velocity. Once you were back on land, you were in a stationary frame of reference, but the man was not, so the velocity you saw was his relative velocity.

Key Points

- In order to find the velocity of an object B that is moving on object A that is observed by an observer that is not moving, add the velocity of B and A together.
- Velocity is a vector quantity, so the relationships between the magnitude, direction, x- axis component and y-axis component are important.
- These vector components can be added analytically or graphically.
- In order to calculate the magnitude and direction, you must know the values of the axis components (either x and y, or x, y, and z) and to calculate the component values, you must know the magnitude and direction.
- Relative velocity is the velocity of an object in motion being observed from a frame of reference that is either also in motion or stationary.
- If the frame of reference is moving in the same direction as the object being observed, it will appear as though the object is moving slower than it actually is.
- If the object being observed is on a moving surface, the velocity observed from that surface will be less than the velocity observed from a stationary surface looking onto the moving surface.
- In Galileo's example, the observed velocity of the fly, uu, measured in reference to the velocity of the boat, v. In order to find the velocity of the fly with respect to the shore, S, he had to add the velocity of the boat to the observed velocity of the fly: $\mathrm{s}=\mathrm{u}+\mathrm{v}$.

Key Terms

- **relative**: Expressed in relation to another item, rather than in complete form.

- Relative velocity. **Provided by**: Wikipedia. **Located at**: **en.Wikipedia.org/wiki/Relative_velocity**. **License**: ***CC BY-SA: Attribution-ShareAlike***
- Sunil Kumar Singh, Relative Velocity in Two Dimensions. September 17, 2013. **Provided by**: OpenStax CNX. **Located at**: **http://cnx.org/content/m14030/latest/**. **License**: ***CC BY: Attribution***
- relative. **Provided by**: Wiktionary. **Located at**: **http://en.wiktionary.org/wiki/relative**. **License**: ***CC BY-SA: Attribution-ShareAlike***
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CHAPTER OVERVIEW

4: The Laws of Motion

Topic hierarchy

4.1: Introduction

Newton and His Laws

There are three laws of motion that describe the relationship between forces, mass, and acceleration.

> **learning objectives**
>
> - Apply three Newton's laws of motion to relate forces, mass, and acceleration

Newton's laws of motion describe the relationship between the forces acting on a body and its motion due to those forces. For example, if your car breaks down and you need to push it, you must exert a force with your hands on the car in order for it to move. The laws of motion will tell you how quickly the car will move from your pushing. There are three laws of motion:

First law: If an object experiences no net force, then its velocity is constant: the object is either at rest (if its velocity is zero), or it moves in a straight line with constant speed (if its velocity is nonzero). For example, if you don't push the car (no force), then it doesn't move.

Second law: The acceleration aa of a body is parallel and directly proportional to the net force F acting on the body, is in the direction of the net force, and is inversely proportional to the mass mm of the body:

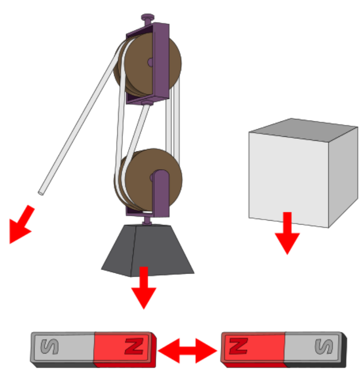
$$\mathrm{F}=\mathrm{m}\cdot\mathrm{a}\text{ or }\mathrm{a}=\frac{\mathrm{F}}{\mathrm{m}} \tag{4.1.1}$$

For example, if you push the car with a greater force it will accelerate more. But, if the car is more massive (mm is larger) then it won't accelerate as much from the same size force as a lighter car.

Third law: When a first body exerts a force F_1 on a second body, the second body simultaneously exerts a force $\mathrm{F}_2=-\mathrm{F}_1$ on the first body. This means that F_1 and F_2 are equal in magnitude and opposite in direction. For example, when you push a car, if it is exerting the same force on you that you are exerting on it, you might wonder why you don't move backwards? The answer is there are also forces from the ground on your feet pushing you forward. So, in fact, the car is pushing a force back on you that is of the same magnitude that you are using to push it forward.

In the figure below there are some practical examples illustrating the concept of force:

- Strain: by using a machine known as pulley you can easily raise or lower a massive body
- Gravitational Force: a massive body is attracted downward by the gravitational force practiced by the Earth
- Magnetic Force: two magnets repel each other when the same poles get closer



Examples of Force: Some situations in which forces are at play.

Key Points

- Acceleration of an object is proportional to the force on it.
- Force causes an object to move.
- Objects with more mass require more force to move.

Key Terms

- **force**: Any influence that causes an object to undergo a certain change, either concerning its movement, direction or geometrical construction.

4.2: Force and Mass

Force

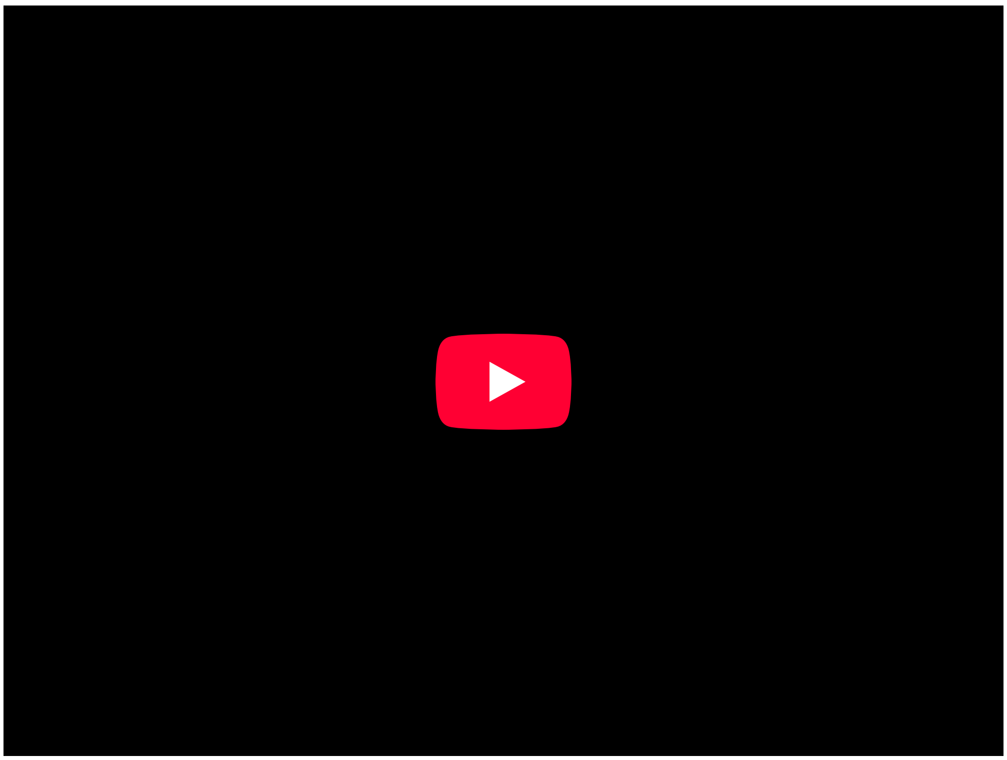
Force is any influence that causes an object to change, either concerning its movement, direction, or geometrical construction.

learning objectives

- Develop the relationship between mass and acceleration in determining force

Overview of Forces

In physics, a force is any influence that causes an object to undergo a certain change, either concerning its movement, direction, or geometrical construction. It is measured with the SI unit of Newtons. A force is that which can cause an object with mass to change its velocity, i.e., to accelerate, or which can cause a flexible object to deform. Force can also be described by intuitive concepts such as a push or pull. A force has both magnitude and direction, making it a vector quantity.



What is a force?: Describes what forces are and what they do.

Qualities of Force

The original form of Newton's second law states that the net force acting upon an object is equal to the rate at which its momentum changes. This law is further given to mean that the acceleration of an object is directly proportional to the net force acting on the object, is in the direction of the net force, and is inversely proportional to the mass of the object.

As we mentioned, force is a vector quantity. A vector is a one dimensional array with elements of both magnitude and direction. In a force vector, the mass, m, is the magnitude component and the acceleration, a, is the directional component. The equation for force is written:

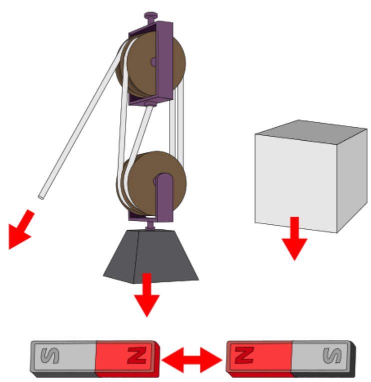
$$\mathrm{F}=\mathrm{m} \cdot \mathrm{a} \tag{4.2.1}$$

Related concepts to force include thrust, which increases the velocity of an object; drag, which decreases the velocity of an object; and torque which produces changes in rotational speed of an object. Forces which do not act uniformly on all parts of a body will also cause mechanical stresses, a technical term for influences which cause deformation of matter. While mechanical stress can remain embedded in a solid object, gradually deforming it, mechanical stress in a fluid determines changes in its pressure and volume.

Dynamics

Dynamics is the study of the forces that cause objects and systems to move. To understand this, we need a working definition of force. Our intuitive definition of force — that is, a push or a pull —is a good place to start. We know that a push or pull has both magnitude and direction (therefore, it is a vector quantity) and can vary considerably in each regard.

shows a few examples of the "push-pull" nature of force. The top left example is that of a pulley system. The force that someone would have to pull down on the cable would have to equal and exceed the force made by the mass the object and the effects of gravity on those object in order for the system to move up. The top right example shows that any object resting on a surface will still exert force on that surface. The bottom example is that of two magnets being attracted to each other due to magnetic force.



Examples of Force: Some situations in which forces are at play.

Mass

Mass is a physical property of matter that depends on size and shape of matter, and is expressed as kilograms by the SI system.

learning objectives

- Justify the significance of understanding mass in physics

What is Mass?

All elements have physical properties whose values can help describe an elements physical state. Changes to these properties can describe elemental transformations. Physical properties do not change the chemical nature of matter. The physical property we are covering in this atom is called mass.

Mass is defined as a quantitative measure of an object's resistance to acceleration. The terms mass and weight are often interchanged, however it is incorrect to do so. Weight is a different property of matter that, while related to mass, is not mass, but rather the amount of gravitational force acting on a given body of matter. Mass is an intrinsic property that never changes.

Units of Mass

In order to measure something, a standard value must be established to use in relation to the object of measurement. This relation is called a unit. The International System of Units (SI) measures mass in kilograms, or kg. There are other units of mass, including the following (only the first two are accepted by the SI system):

- t – Tonne; 1t = 1000kg
- u – atomic mass unit; 1u ~= 1.66×10^{-27}kg
- sl – slug
- lb – pound

Concepts Using Mass

- Weight – see
- Newtons Second Law – mass has a central role in determining the behavior of bodies. Newtons Second Law relates force f, exerted in a body of mass m, to the body's acceleration a: $\mathrm{F} = \mathrm{ma}$
- Momentum – mass relates a body's momentum, p, to its linear velocity, v: $\mathrm{p} = \mathrm{mv}$
- Kinetic Energy – mass relates kinetic energy, K to velocity, v: $\mathrm{K} = \frac{1}{2}\mathrm{m}|\mathrm{v}^2|$

Key Points

- Force is stated as a vector quantity, meaning it has elements of both magnitude and direction. Mass and acceleration respectively.
- In layman's terms, force is a push or pull that can be defined in terms of various standards.
- Dynamics is the study of the force that causes objects and systems to move or deform.
- External forces are any outside forces that act on a body, and internal forces are any force acting within a body.
- Mass is defined as a quantitative measure of an object's resistance to acceleration.
- According to Newton's second law of motion, if a body of fixed mass m is subjected to a single force F, its acceleration a is given by F/m.
- Mass is central in many concepts of physics, including:weight, momentum, acceleration, and kinetic energy.
- According to Newton's second law of motion, if a body of fixed mass m is subjected to a single force F, its acceleration a is given by F/m.

Key Terms

- **force**: A force is any influence that causes an object to undergo a certain change, either concerning its movement, direction or geometrical construction.
- **velocity**: A vector quantity that denotes the rate of change of position with respect to time, or a speed with a directional component.
- **vector**: A directed quantity, one with both magnitude and direction; the between two points.
- **mass**: The quantity of matter which a body contains, irrespective of its bulk or volume. It is one of four fundamental properties of matter. It is measured in kilograms in the SI system of measurement.

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- What is a force?. **Located at**: **http://www.youtube.com/watch?v=HIZpG7h5b54**. **License**: ***Public Domain: No Known Copyright***. **License Terms**: Standard YouTube license

4.3: Newton's Laws

The First Law: Inertia

Newton's first law of motion describes inertia. According to this law, a body at rest tends to stay at rest, and a body in motion tends to stay in motion, unless acted on by a net external force.

learning objectives

- Define the First Law of Motion

History

Sir Isaac Newton was an English scientist who was interested in the motion of objects under various conditions. In 1687, he published a work called *Philosophiae Naturalis Principla Mathematica*, which described his three laws of motion. Newton used these laws to explain and explore the motion of physical objects and systems. These laws form the basis for mechanics. The laws describe the relationship between forces acting on a body and the motions experienced due to these forces. The three laws are as follows:

1. If an object experiences no net force, its velocity will remain constant. The object is either at rest and the velocity is zero or it moves in a straight line with a constant speed.
2. The acceleration of an object is parallel and directly proportional to the net force acting on the object, is in the direction of the net force, and is inversely proportional to the mass of the object.
3. When a first object exerts a force on a second object, the second object simultaneously exerts a force on the first object, meaning that the force of the first object and the force of the second object are equal in magnitude and opposite in direction.

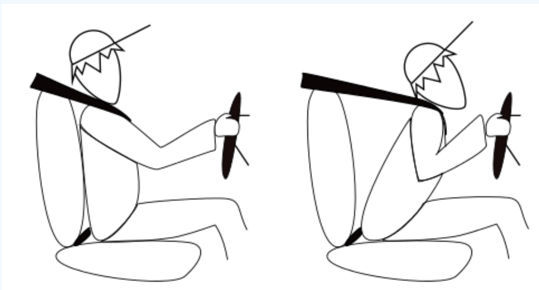
The First Law of Motion

You have most likely heard Newton's first law of motion before. If you haven't heard it in the form written above, you have probably heard that "a body in motion stays in motion, and a body at rest stays at rest." This means that an object that is in motion will not change its velocity unless an unbalanced force acts upon it. This is called uniform motion. It is easier to explain this concept through examples.

Example $4.3.1$:

If you are ice skating, and you push yourself away from the side of the rink, according to Newton's first law you will continue all the way to the other side of the rink. But, this won't actually happen. Newton says that a body in motion will stay in motion until an outside force acts upon it. In this and most other real world cases, this outside force is friction. The friction between your ice skates and the ice is what causes you to slow down and eventually stop.

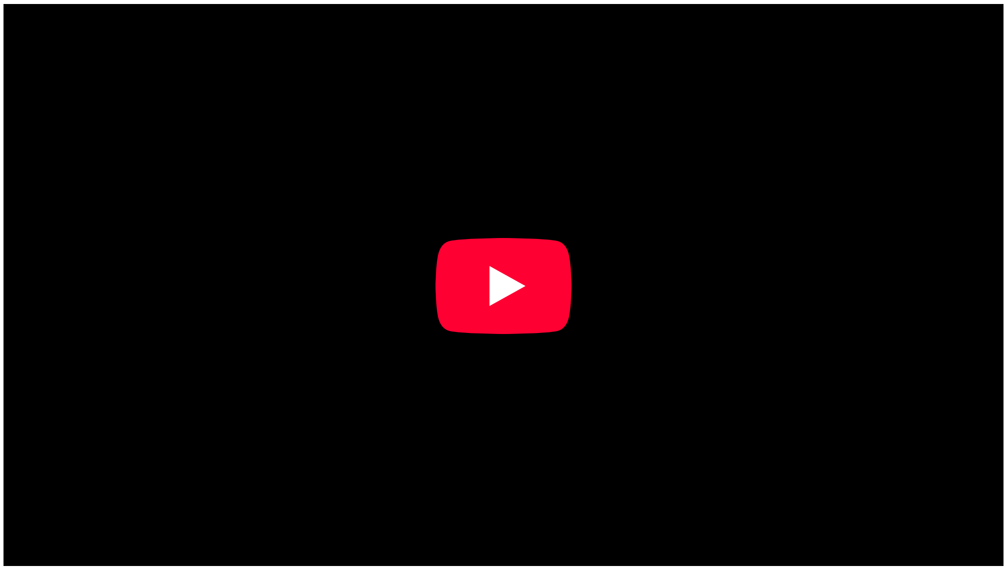
Let's look at another situation. Refer to for this example. Why do we wear seat belts? Obviously, they're there to protect us from injury in case of a car accident. If a car is traveling at 60 mph, the driver is also traveling at 60 mph. When the car suddenly stops, an external force is applied to the car that causes it to slow down. But there is no force acting on the driver, so the driver continues to travel at 60 mph. The seat belt is there to counteract this and act as that external force to slow the driver down along with the car, preventing them from being harmed.

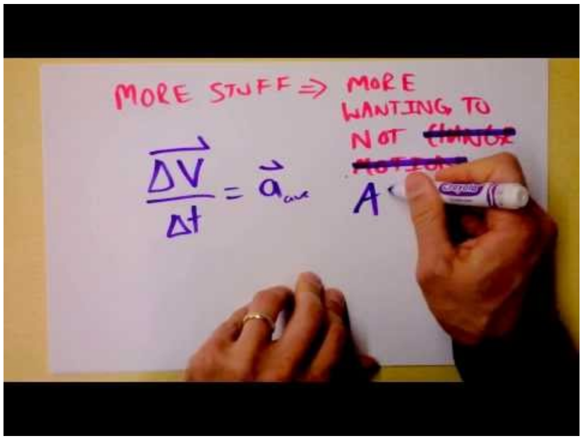


Newton's First Law: Newton's first law in effect on the driver of a car

Inertia

Sometimes this first law of motion is referred to as the law of inertia. Inertia is the property of a body to remain at rest or to remain in motion with constant velocity. Some objects have more inertia than others because the inertia of an object is equivalent to its mass. This is why it is more difficult to change the direction of a boulder than a baseball.





Doc Physics – Newton: Newton's first law is hugely counterintuitive. You may have learned it in gradeschool, though. Let's see it for the mind-blowing conclusion it really is.

The Second Law: Force and Acceleration

The second law states that the net force on an object is equal to the rate of change, or derivative, of its linear momentum.

learning objectives

- Define the Second Law of Motion

English scientist Sir Isaac Newton examined the motion of physical objects and systems under various conditions. In 1687, he published his three laws of motion in *Philosophiae Naturalis Principla Mathematica*. The laws form the basis for mechanics—they describe the relationship between forces acting on a body, and the motion experienced due to these forces. These three laws state:

1. If an object experiences no net force, its velocity will remain constant. The object is either at rest and the velocity is zero, or it moves in a straight line with a constant speed.
2. The acceleration of an object is parallel and directly proportional to the net force acting on the object, is in the direction of the net force and is inversely proportional to the mass of the object.

3. When a first object exerts a force on a second object, the second object simultaneously exerts a force on the first object, meaning that the force of the first object and the force of the second object are equal in magnitude and opposite in direction.

The first law of motion defines only the natural state of the motion of the body (i.e., when the net force is zero). It does not allow us to quantify the force and acceleration of a body. The acceleration is the rate of change in velocity; it is caused only by an external force acting on it. The second law of motion states that the net force on an object is equal to the rate of change of its linear momentum.

Linear Momentum

Linear momentum of an object is a vector quantity that has both magnitude and direction. It is the product of mass and velocity of a particle at a given time:

$$\mathrm{p=mv} \tag{4.3.1}$$

where, $\mathrm{p=momentum, m=mass,\ and\ v=velocity}$. From this equation, we see that objects with more mass will have more momentum.

The Second Law of Motion

Picture two balls of different mass, traveling in the same direction at the same velocity. If they both collide with a wall at the same time, the heavier ball will exert a larger force on the wall. This concept, illustrated below, explains Newton’s second law, which emphasizes the importance of force and motion, over velocity alone. It states: the net force on an object is equal to the rate of change of its linear momentum. From calculus we know that the rate of change is the same as a derivative. When we the linear momentum of an object we get:

Force and Mass: This animation demonstrates the connection between force and mass.

$$\mathrm{F=\frac{dp}{dt}} \tag{4.3.2}$$

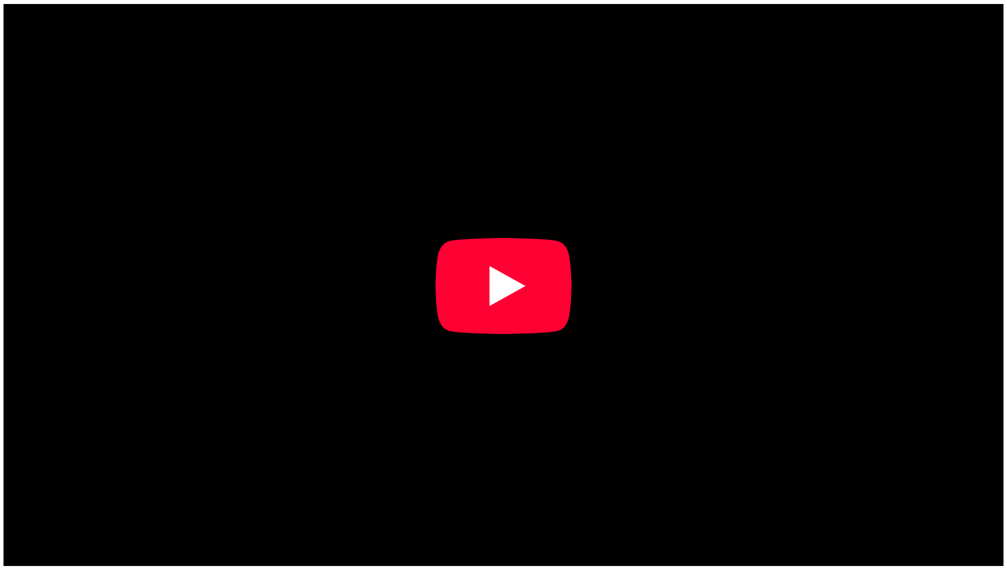
$$\mathrm{F=\frac{d(m \cdot v)}{dt}} \tag{4.3.3}$$

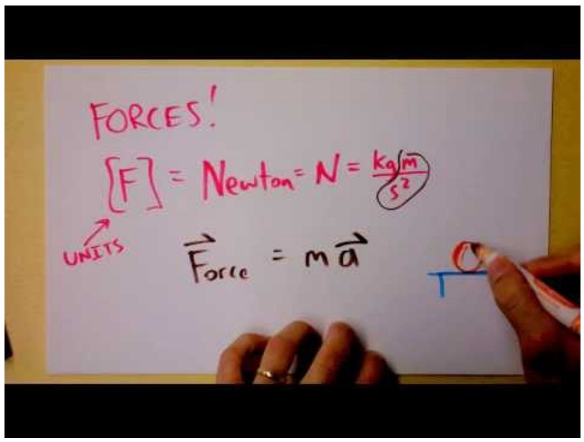
where, $\mathrm{F=Force\ and\ t=time}$. From this we can further simplify the equation:

$$\mathrm{F=m\frac{d(v)}{dt}} \tag{4.3.4}$$

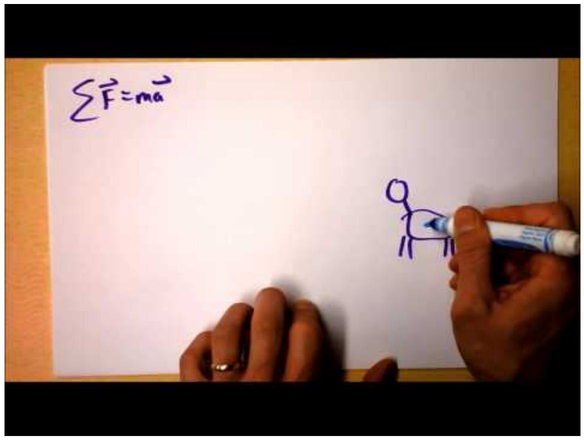
$$\mathrm{F=m \cdot a} \tag{4.3.5}$$

where, a=accelerationa=acceleration. As we stated earlier, acceleration is the rate of change of velocity, or velocity divided by time.





Newton’s Three Laws of Mechanics – Second Law – Part 1: Here we’ll see how many people can confuse your understanding of Newton’s 2nd law of motion through oversight, sloppy language, or cruel intentions.



Newton's Three Laws of Mechanics – Second Law – Part Two: Equilibrium is investigated and Newton's 1st law is seen as a special case of Newton's 2nd law!

The Third Law: Symmetry in Forces

The third law of motion states that for every action, there is an equal and opposite reaction.

learning objectives

- Define the Third Law of Motion

Sir Isaac Newton was a scientist from England who was interested in the motion of objects under various conditions. In 1687, he published a work called *Philosophiae Naturalis Principla Mathematica*, which contained his three laws of motion. Newton used these laws to explain and explore the motion of physical objects and systems. These laws form the bases for mechanics. The laws describe the relationship between forces acting on a body, and the motion is an experience due to these forces. Newton's three laws are:

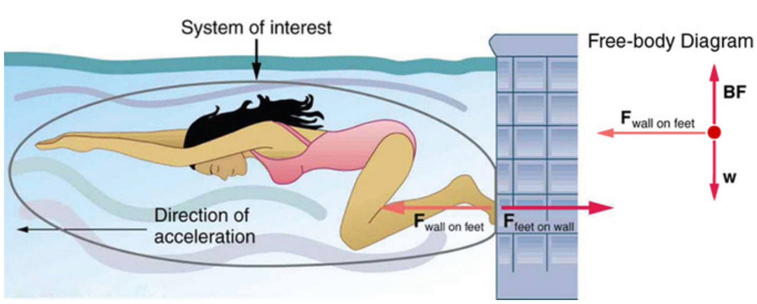
1. If an object experiences no net force, its velocity will remain constant. The object is either at rest and the velocity is zero or it moves in a straight line with a constant speed.
2. The acceleration of an object is parallel and directly proportional to the net force acting on the object, is in the direction of the net force and is inversely proportional to the mass of the object.
3. When a first object exerts a force on a second object, the second object simultaneously exerts a force on the first object, meaning that the force of the first object and the force of the second object are equal in magnitude and opposite in direction.

Newton's Third Law of Motion

Newton's third law basically states that for every action, there is an equal and opposite reaction. If object A exerts a force on object B, because of the law of symmetry, object B will exert a force on object A that is equal to the force acted on it:

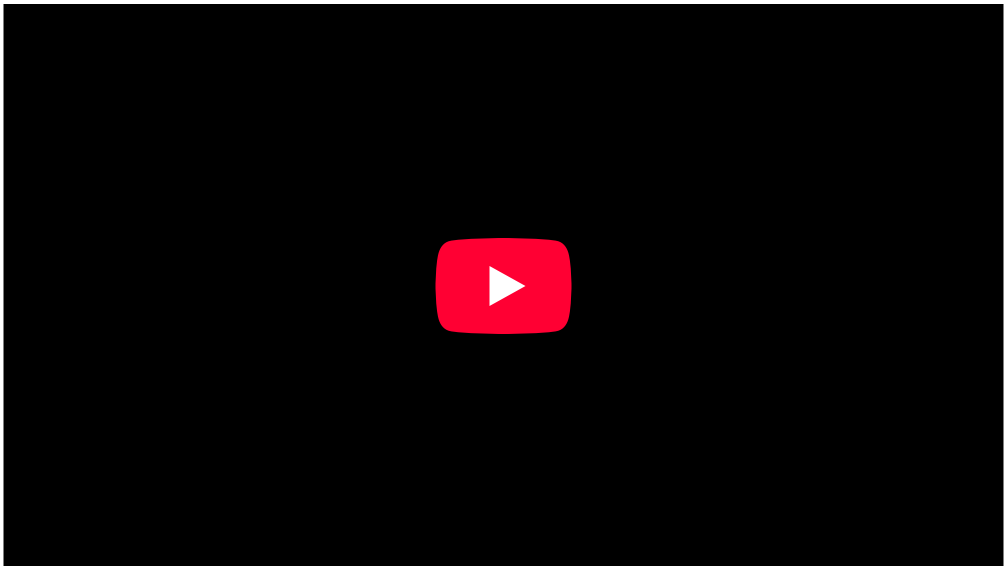
$$\mathrm{FA} = -\mathrm{FB} \tag{4.3.6}$$

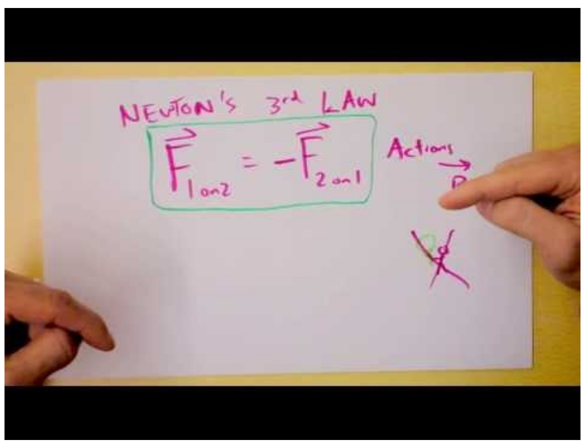
In this example, F_A is the action and F_B is the reaction. You have undoubtedly witnessed this law of motion. For example, take a swimmer who uses her feet to push off the wall in order to gain speed. The more force she exerts on the wall, the harder she pushes off. This is because the wall exerts the same force on her that she forces on it. She pushes the wall in the direction behind her, therefore the wall will exert a force on her that is in the direction in front of her and propel her forward.



Newton's Third Law of Motion: When a swimmer pushes off the wall, the swimmer is using the third law of motion.

Take as another example, the concept of thrust. When a rocket launches into outer space, it expels gas backward at a high velocity. The rocket exerts a large backward force on the gas, and the gas exerts and equal and opposite reaction force forward on the rocket, causing it to launch. This force is called thrust. Thrust is used in cars and planes as well.





Newton's Third Law: The most fundamental statement of basic physical reality is also the most often misunderstood. As your mom if she's clear on Newton's Third. Then ask her why things can move if every force has a paired opposite force all the time, forever.

Key Points

- Newton's three laws of physics are the basis for mechanics.
- The first law states that a body at rest will stay at rest until a net external force acts upon it and that a body in motion will remain in motion at a constant velocity until acted on by a net external force.
- Net external force is the sum of all of the forcing acting on an object.
- Just because there are forces acting on an object doesn't necessarily mean that there is a net external force; forces that are equal in magnitude but acting in opposite directions can cancel one another out.
- Friction is the force between an object in motion and the surface on which it moves. Friction is the external force that acts on objects and causes them to slow down when no other external force acts upon them.
- Inertia is the tendency of a body in motion to remain in motion. Inertia is dependent on mass, which is why it is harder to change the direction of a heavy body in motion than it is to change the direction of a lighter object in motion.
- Newton's three laws of motion explain the relationship between forces acting on an object and the motion they experience due to these forces. These laws act as the basis for mechanics.

- The second law explains the relationship between force and motion, as opposed to velocity and motion. It uses the concept of linear momentum to do this.
- Linear momentum p, is the product of mass m, and velocity v : $p = mv$.
- The second law states that the net force is equal to the derivative, or rate of change of its linear momentum.
- By simplifying this relationship and remembering that acceleration is the rate of change of velocity, we can see that the second law of motion is where the relationship between force and acceleration comes from.
- If an object A exerts a force on object B, object B exerts an equal and opposite force on object A.
- Newton's third law can be seen in many everyday circumstances. When you walk, the force you use to push off the ground backwards makes you move forward.
- Thrust is an application of the third law of motion. A helicopter uses thrust to push the air under the propeller down, and therefore lift off the ground.

Key Terms

- **inertia**: The property of a body that resists any change to its uniform motion; equivalent to its mass.
- **friction**: A force that resists the relative motion or tendency to such motion of two bodies in contact.
- **uniform motion**: Motion at a constant velocity (with zero acceleration). Note that an object in motion will not change its velocity unless an unbalanced force acts upon it.
- **net force**: The combination of all the forces that act on an object.
- **momentum**: (of a body in motion) the product of its mass and velocity.
- **acceleration**: The amount by which a speed or velocity increases (and so a scalar quantity or a vector quantity).
- **symmetry**: Exact correspondence on either side of a dividing line, plane, center or axis.
- **thrust**: The force generated by propulsion, as in a jet engine.

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4.4: Other Examples of Forces

Weight

Weight is taken as the force on an object due to gravity, and is different than the mass of an object.

learning objectives

- Infer what factors other than gravity will contribute to the apparent weight of an object

In physics, it is important to differentiate the weight of an object from its mass. The mass of an object is an intrinsic quantity, independent of the location of the object. On the other hand, the weight of an object is an extrinsic quantity. It is considered as the force on an object due to gravity. Since gravitational acceleration changes depending on the location in the universe, weight does as well.

Mathematically, the weight of an object (*W*) can be found by multiplying its mass (*m*) by the acceleration due to gravity (*g*): $\mathrm{W} = \mathrm{M} \cdot \mathrm{g}$. The strength of gravity varies very little over the surface of the Earth. In fact, the greatest percent difference in the value of the acceleration due to gravity on Earth is 0.5%.

For most calculations involving the weight of an object on Earth, it is sufficient to assume that $\mathrm{g} = 9.8 \frac{\mathrm{m}}{\mathrm{s}^2}$.

The weight of an object has the same SI unit as force—the Newton ($1\mathrm{N} = 1\mathrm{kg} \cdot \frac{\mathrm{m}}{\mathrm{s}^2}$).

In US customary units, the weight of an object can be expressed in pounds. Keep in mind that in US units the pound is either a unit of force or of mass. If one must find the weight (as opposed to the mass) of an object in US units, it can be calculated in terms of pounds of force.

It is important to note that the apparent weight of an object (i.e., the weight of an object determined by a scale) will vary if forces other than gravity are acting upon the object. For example, if you weigh a given mass underwater you will find a different result than if you weigh that mass in air. In this case, the weight of the object varies due to the force of buoyancy. While the mass is in the water it displaces fluid, resulting in an upward force upon it. This upward force affects the net force that the mass exerts on the scale, and thus alters its “apparent” weight.

Spring Scale: A spring scale measures weight by finding the extent to which a spring is compressed.This is proportional to the force that a mass exerts on the scale due to its weight.

Normal Forces

The normal force comes about when an object contacts a surface; the resulting force is always perpendicular to the surface of contact.

learning objectives

- Evaluate Newton's Second and Third Laws in determining the normal force on an object

Overview

The normal force, F_N, comes about when an object contacts a surface. According to Newton's third law, when one object exerts a force on a second object, the second object always exerts a force that is equal in magnitude and opposite in direction on the first object. This is the reason that the normal force exists.

A common situation in which a normal force exists is when a person stands on the ground. Because of Newton's third law, the ground exerts a force on the person that is equal in magnitude to the person's weight. In this simple case, the weight of the person and the opposing normal force are the only two forces considered on the person. The person remains still because the forces due to weight and the normal force create a net force of zero on the person.

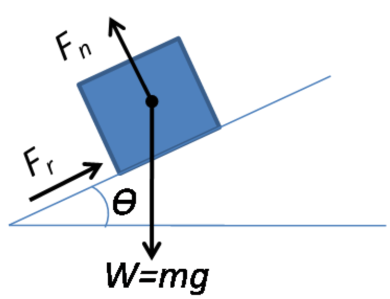
Forces on Inclined Planes

A more complex example of a situation in which a normal force exists is when a mass rests on an inclined plane. In this case, the normal force is not in the exact opposite direction as the force due to the weight of the mass. This is because the mass contacts the surface at an angle. By taking this angle into account, the magnitude of the normal force (F_N) can be found from:

$$F_N = mg\cos(\theta), \tag{4.4.1}$$

where:

- m is the mass under consideration,
- g is the acceleration due to gravity,
- and θ is the angle between the inclined surface and the horizontal.



Inclined Plane: A mass rests on an inclined plane that is at an angle θ to the horizontal. The following forces act on the mass: the weight of the mass $(m \cdot g)$,the force due to friction (Fr),and the normal force (Fn).

Another interesting example involving normal forces is when a person stands in an elevator. When the elevator goes up, the normal force is actually greater than the force due to gravity. In this situation there are only two forces acting on the person. The first is the force of gravity on the person, which does not change. The second is the normal force. By summing the forces and setting them equal to m⋅am⋅a (utilizing Newton's second law), we find:

$$F_N - m \cdot g = m \cdot a \tag{4.4.2}$$

where:

- F_N is the normal force,
- $m \cdot g$ is the force due to gravity,
- m is the mass of the person,
- and a is the acceleration.

Since acceleration is positive, the normal force must actually be greater than the force due to gravity (the weight of the person).

Key Points

- Weight is taken to be the force on an object due to gravity.
- Weight and mass are not the same thing!
- The weight of a given mass will be different when the acceleration due to gravity is different.
- Apparent weight can change because of the effect of buoyancy.
- The strength of gravity is almost the same everywhere on the surface of the Earth.
- The normal force, F_N, comes about when an object contacts a surface.
- The normal force exists because for every force, there is always an equal and opposite force.
- The normal force is always perpendicular to the plane that the object contacts or rests on.

Key Terms

- **Gravitational acceleration**: Gravitational acceleration is the acceleration that an object undergoes due solely to gravity
- **perpendicular**: at or forming a right angle (to).
- **normal**: A line or vector that is perpendicular to another line, surface, or plane.

4.5: Problem-Solving

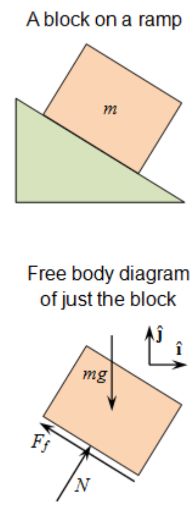
A General Approach

Basic problem-solving techniques can aid in the solution of problems involving motion (i.e., the laws of motion).

learning objectives

- Assess the laws of motion through practiced problem solving techniques

When dealing with the laws of motion, although knowledge of concepts and equations is important, understanding basic problem solving techniques can simplify the process of solving problems that may appear difficult. Your approach to problem solving can involve several key steps.



Free body diagram: An example of a drawing to help identify forces and directions.

First, gather all relevant information from the problem. Identify all quantities that are given (the *knowns*), then do the same for all quantities needed (the *unknowns*). Also, identify the physical principles involved (e.g., force, gravity, friction, etc.).

Next, a drawing may be helpful. Sometimes a drawing can even help determine the known and unknown quantities. It need not be a work of art, but it should be clear enough to illustrate proper dimension, (meaning one, two, or three dimensions). You can then use this drawing to determine which direction is positive and which is negative (making note of this on the drawing).

A next step is to use what is known to find the appropriate equation to find what is unknown. While it is easiest to find an equation that leaves only one unknown, sometimes this is not possible. In these situations, you can solve multiple equations to find the right answer. Remember that equations represent physical principles and relationships, so use the equations and drawings in tandem.

You may then substitute the knowns into the appropriate equations and find a numerical solution.

Check the answer to see if it is reasonable and makes sense. Your judgment will improve and fine tune as you solve more problems of this nature. This "judgement" step helps intuit the problem in terms of its conceptual meaning. If you can judge whether the answer is reasonable, you have a deeper understanding of physics than simply the mechanics of problem solving.

When solving problems, we tend to perform these steps in different order, as well as do several steps simultaneously. There is no rigid procedure that will work every time. Creativity and insight grow with experience. In time, the basics of problem solving can become relatively automatic.

Key Points

- Gathering all relevant information and identifying knowns and unknowns is an important first step.
- Always make a drawing to help identify directions of forces and to establish x, y, and z axes.
- Choose the correct equations, solve the problem, and check that the answer fits expectations numerically.

Key Terms

- **equation**: An assertion that two expressions are equal, expressed by writing the two expressions separated by an equal sign; from which one is to determine a particular quantity.

4.6: Vector Nature of Forces

Forces in Two Dimensions

Forces act in a particular direction and have sizes dependent upon how strong the push or pull is.

learning objectives

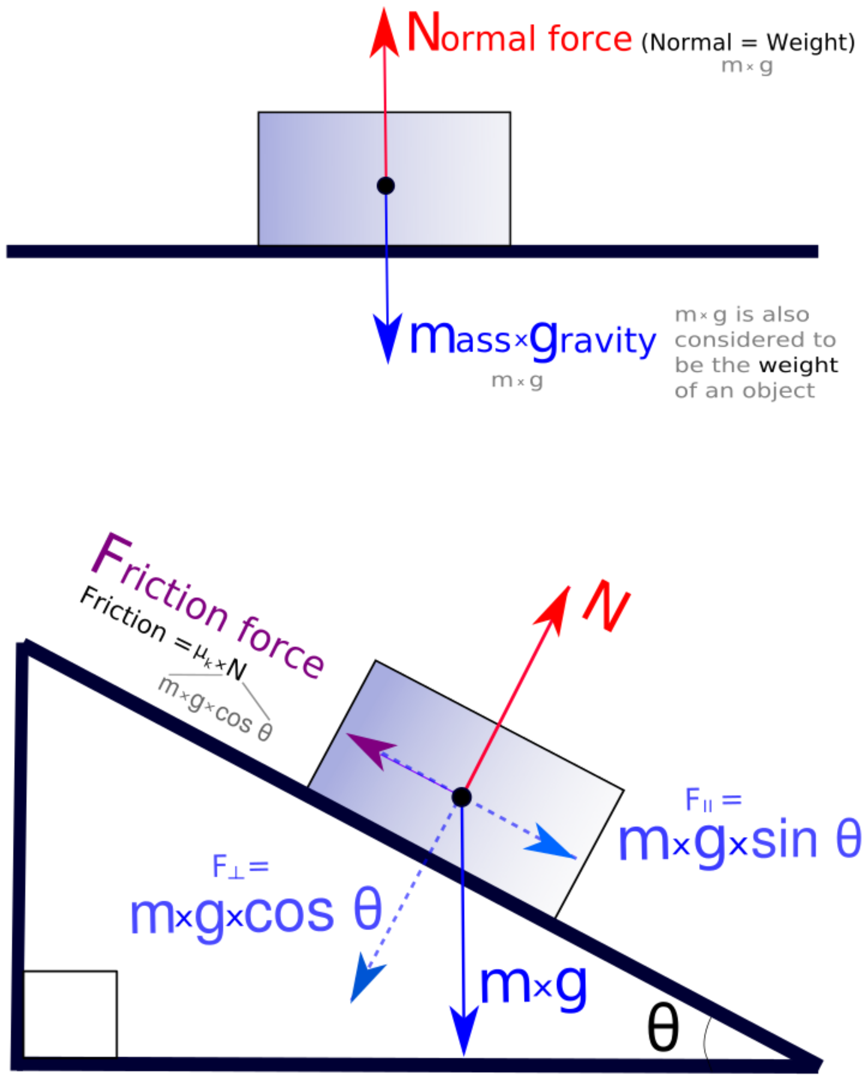
- Explain why forces are classified as "vector quantities"

Forces act in a particular direction and have sizes dependent upon how strong the push or pull is. Because of these characteristics, forces are classified as "vector quantities. " This means that forces follow a different set of mathematical rules than physical quantities that do not have direction (denoted scalar quantities).

For example, when determining what happens when two forces act on the same object, it is necessary to know both the magnitude and the direction of both forces to calculate the result. If both of these pieces of information are not known for each force, the situation is ambiguous. For example, if you know that two people are pulling on the same rope with known magnitudes of force but you do not know which direction either person is pulling, it is impossible to determine what the acceleration of the rope will be. The two people could be pulling against each other as in tug of war or the two people could be pulling in the same direction. In this simple one-dimensional example, without knowing the direction of the forces it is impossible to decide whether the net force is the result of adding the two force magnitudes or subtracting one from the other. Associating forces with vectors avoids such problems.

When two forces act on a point particle, the resulting force or the resultant (also called the net force) can be determined by following the parallelogram rule of vector addition: the addition of two vectors represented by sides of a parallelogram gives an equivalent resultant vector which is equal in magnitude and direction to the transversal of the parallelogram. The magnitude of the resultant varies from the difference of the magnitudes of the two forces to their sum, depending on the angle between their lines of action.

Free-body diagrams can be used as a convenient way to keep track of forces acting on a system. Ideally, these diagrams are drawn with the angles and relative magnitudes of the force vectors preserved so that graphical vector addition can be done to determine the net force.



Forces as Vectors: Free-body diagrams of an object on a flat surface and an inclined plane. Forces are resolved and added together to determine their magnitudes and the net force.

Key Points

- When determining what happens when two forces act on the same object, it is necessary to know both the magnitude and the direction of both forces to calculate the result.
- When two forces act on a point particle, the resulting force or the resultant (also called the net force), can be determined by following the parallelogram rule of vector addition.
- Free-body diagrams can be used as a convenient way to keep track of forces acting on an object.

Key Terms

- **vector**: A directed quantity, one with both magnitude and direction; the between two points.
- **free-body diagram**: A free body diagram, also called a force diagram, is a pictorial representation often used by physicists and engineers to analyze the forces acting on a body of interest.
- **resultant**: A vector that is the vector sum of multiple vectors

4.7: Further Applications of Newton's Laws

Applications of Newton's Laws

Net force affects the motion, postion and/or shape of objects (some important and commonly used forces are friction, drag and deformation).

learning objectives

- Explain the effect of forces on an object's motion and shape

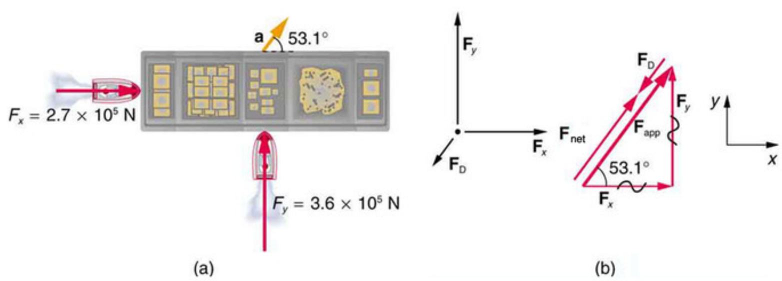
We know that a net force affects the motion, position and shape of an object. It is useful at this point to look at some particularly interesting and common forces that will provide further applications of Newton's laws of motion. Specifically, we will discuss the forces of friction, air or liquid drag, and deformation.

Friction

Friction is a force that resists movement between two surfaces sliding against each other. When surfaces in contact move relative to each other, the friction between the two surfaces converts kinetic energy into heat. This property can have a dramatic effect, as seen in the use of friction created by rubbing pieces of wood together to start a fire. Friction is not itself a fundamental force, but arises from fundamental electromagnetic forces between the charged particles constituting the two contacting surfaces.

Drag

Another interesting force in everyday life is the force of drag on an object when it is moving in a fluid (either gas or liquid). You feel this drag force when you move your hand through water, or through the wind. Like friction, the force of drag is a force that resists motion. As we will discuss in later units, the drag force is proportional to the velocity of the object moving through it. We see an illustrated example of drag force in.



Drag Force on a Barge: (a) A view from above of two tugboats pushing on a barge. (b) The free-body diagram for the ship contains only forces acting in the plane of the water. It omits the two vertical forces—the weight of the barge and the buoyant force of the water supporting it cancel and are not shown. Since the applied forces are perpendicular, the x– and y-axes are in the same direction as $\mathrm{F_x}$ and $\mathrm{F_y}$. The problem quickly becomes a one-dimensional problem along the direction of $\mathrm{F_{app}}$, since friction is in the direction opposite to $\mathrm{F_{app}}$.

Deformation

We now move from consideration of forces that affect the motion of an object (such as friction and drag) to those that affect an object's shape. If a bulldozer pushes a car into a wall, the car will not move but it will noticeably change shape. The change in shape of an object due to the application of a force is a deformation. Even very small forces are known to cause some deformation. For small deformations, two important characteristics are observed. First, the object returns to its original shape when the force is removed (that is, the deformation is elastic for small deformations). Second, the size of the deformation is proportional to the force.

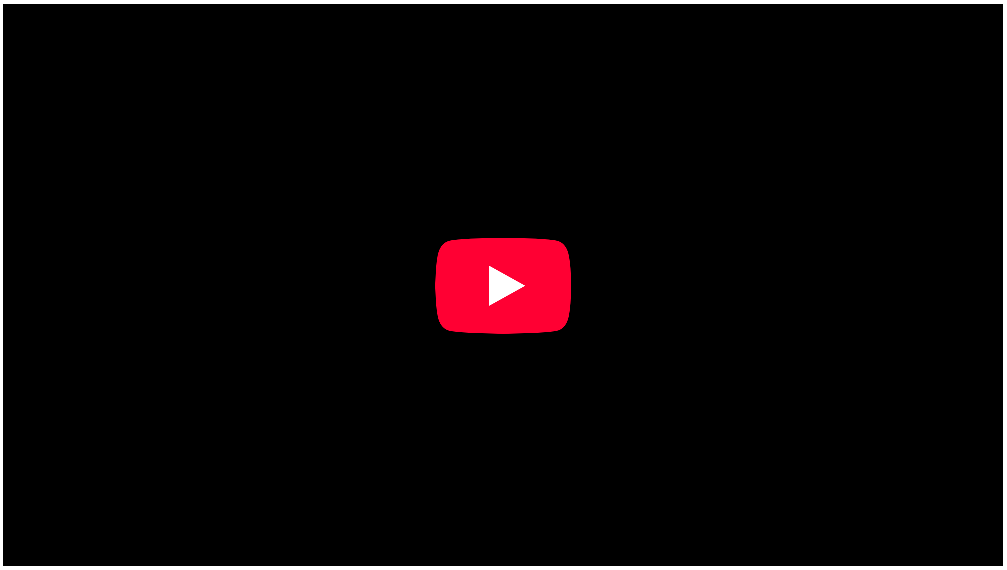
Friction: Kinetic

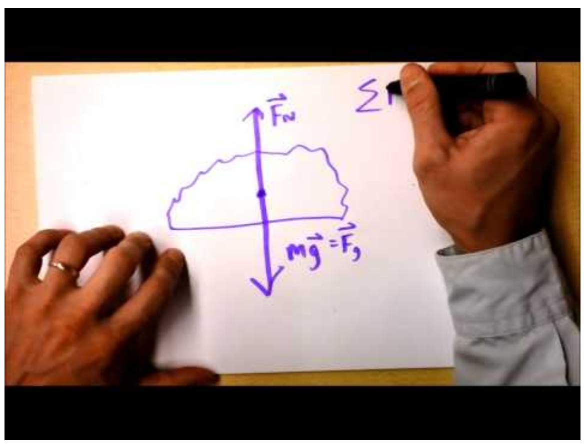
If two systems are in contact and moving relative to one another, then the friction between them is called kinetic friction.

learning objectives

- Explain the dynamics of energy for friction between two surfaces

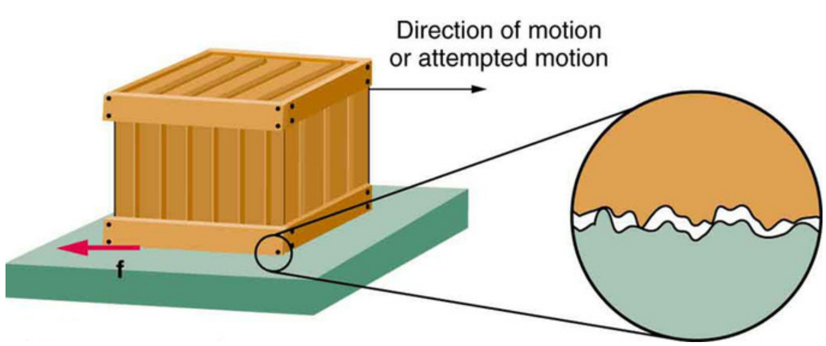
When surfaces in contact move relative to each other, the friction between the two surfaces converts kinetic energy into heat. This property can have dramatic consequences, as illustrated by the use of friction created by rubbing pieces of wood together to start a fire. Kinetic energy is converted to heat whenever motion with friction occurs, for example when a viscous fluid is stirred.





Kinetic Friction Introduction: Here, I'll explain the microscopic justification of friction and what we can know about it. The coefficient of friction, too!

Kinetic (or dynamic) friction occurs when two objects are moving relative to each other and rub together; a sled on the ground would be a good example of kinetic friction.



Friction: Frictional forces always oppose motion or attempted motion between objects in contact. Friction arises in part because of the roughness of the surfaces in contact, as seen in the expanded view. In order for the object to move, it must rise to where the peaks can skip along the bottom surface. Thus, a force is required just to set the object in motion. Some of the peaks will be broken off, also requiring a force to maintain motion. Much of the friction is actually due to attractive forces between molecules making up the two objects, so that even perfectly smooth surfaces are not friction-free. Such adhesive forces also depend on the substances the surfaces are made of, explaining, for example, why rubber-soled shoes slip less than those with leather soles.

The force of friction is what slows an object sliding over a surface. This force is what makes the brakes on cars work or causes resistance when you slide your hand across a surface. The force of friction can be represented by an equation: $\mathrm{F_{friction}} = \mu \mathrm{F_n}$. In this equation μ is something called the coefficient of friction. This is a unitless number that represents the strength of the friction of the object. A very "grippy" surface like rubber might have a high coefficient of friction, whereas a slippery surface like ice has a much lower coefficient. $\mathrm{F_n}$ is called the normal force and is the force of the surface pushing up on the object. In most cases on level ground, the normal force will be the equal and opposite of the object's weight. In other words, it is the force that the surface must exert to keep the object from falling through.

The coefficient of kinetic friction is typically represented as μ_k and is usually less than the coefficient of static friction for the same materials.

Friction: Static

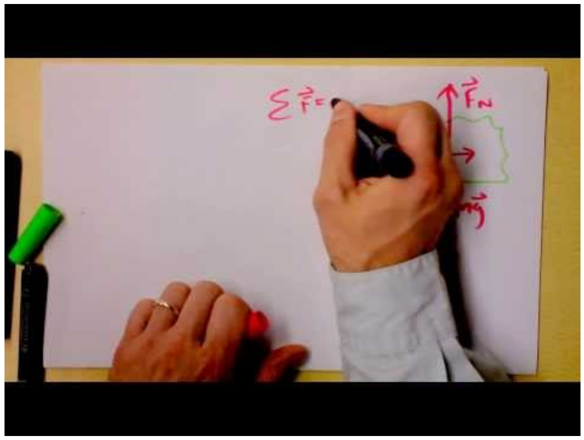
Static friction is a type of friction that occurs to resist motion when two objects are at rest against each other.

learning objectives

- Demonstrate the relationship of maximum force of static friction

Static Friction

Another type of frictional force is static friction, otherwise known as stiction. Like all friction, it acts to resist the motion of an object moving over a surface. Unlike kinetic friction, however, static friction acts to resist the start of motion.



Static Friction and some friction challenges: Here, I talk about sneaky ol’ static friction.

Static friction is friction between two objects that are not moving relative to each other. This frictional force is what prevents a parked car from sliding down a hill, for example. Before an object at rest on a surface can move, it must overcome the force of static friction.

Static friction originates from multiple sources. For any given material on another material of the same composition, friction will be greater as the material surfaces become rougher (consider sandpaper) on the macroscopic level. Additionally, intermolecular forces can greatly influence friction when two materials are put into contact. When surface area is below the micrometer range, Van der Waals’ forces, electrostatic interactions and hydrogen bonding can cause two materials to adhere to one another. A force is required to overcome these interactions and cause the surfaces to move across one another.

Like kinetic friction, the force of static friction is given by a coefficient multiplied by the normal force. The normal force is the force of the surface pushing up on the object, which is usually equal to the object’s weight. The coefficient of static friction is usually greater than the coefficient of kinetic friction and is usually represented by μ_{s}.

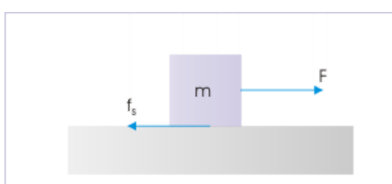
Putting these elements together gives the maximum force of static friction as:

$$\mathrm{F_s} = \mu_{\mathrm{s}} \mathrm{F_n} \tag{4.7.1}$$

In general, the force of static friction can be represented as:

$$\mathrm{F_s} \leq \mu_{\mathrm{s}} \mathrm{F_n} \tag{4.7.2}$$

As with all frictional forces, the force of friction can never exceed the force applied. Thus the force of static friction will vary between 0 and ${}_{\mathrm{s}}\mathrm{F_n}$ depending on the strength of the applied force. Any force smaller than ${}_{\mathrm{s}}\mathrm{F_n}$ attempting to slide one surface over the other is opposed by a frictional force of equal magnitude and opposite direction. Any force larger than that overcomes the force of static friction and causes sliding to occur. The instant sliding occurs, static friction is no longer applicable—the friction between the two surfaces is then called kinetic friction.



Static Friction: To move a block at rest on a surface, a force must be applied which is great enough to overcome the force of static friction.

Problem-Solving With Friction and Inclines

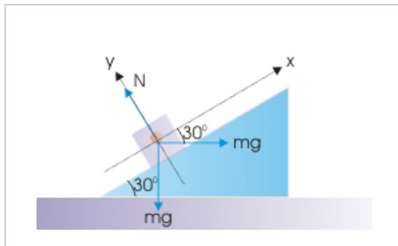
Combining motion on inclines with friction uses such concepts as equilibrium and contact force on an incline.

learning objectives

- Calculate the force of friction on an incline

Contact Force on an Incline

The incline plane has two contact or interface surfaces. One is the incline surface, where the block is placed and the other is the base of the incline, which is in contact with the surface underneath. The motion of the block, therefore, may depend on the motion of the incline itself.



Block and incline system: Forces on the block

When on an incline, calculating the force of friction is different than when the object is on a level surface. Recall that the force of friction depends on both the coefficient of friction and the normal force. $F_f = \mu F_n$ When on an incline with an angle θ, the normal force becomes $F_n = mg\cos(\theta)$

As always, the frictional force resists motion. If the block is being pushed up the incline the friction force points down the incline. If the block is being pulled down the incline, the friction force will hold the block up.

Equilibrium of Forces on an Incline

When not acted on by any other forces, only by gravity and friction, the frictional force will resist the tendency of the block to slide down the incline. If the frictional force is equal to the gravitational force the block will not slide down the incline. The block is said to be in equilibrium since the sum of the forces on it is 0.

Gravitational force down an incline is given by $mg\sin(\theta)$.

Where θ is the angle the incline makes with the horizontal. For the block to be in equilibrium, the maximum force of friction $F_f = \mu mg\cos(\theta)$ must be greater than or equal to $F_G = mg\sin(\theta)$ FG=mgsin(θ). If the maximum frictional force is greater than the force of gravity, the sum of the forces is still 0. The force of friction can never exceed the other forces acting on it. The frictional forces only act to counter motion.

Drag

The drag force is the resistive force felt by objects moving through fluids and is proportional to the square of the object's speed.

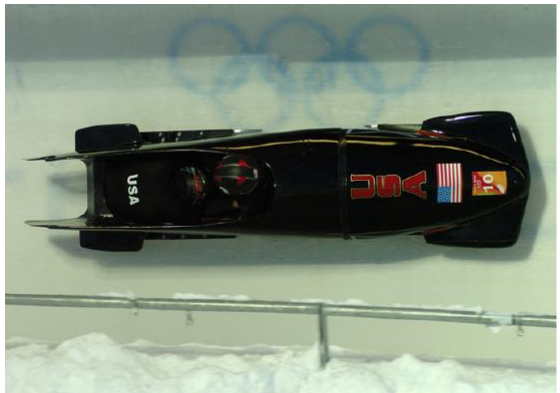
learning objectives

- Relate the magnitude of drag force to the speed of an object

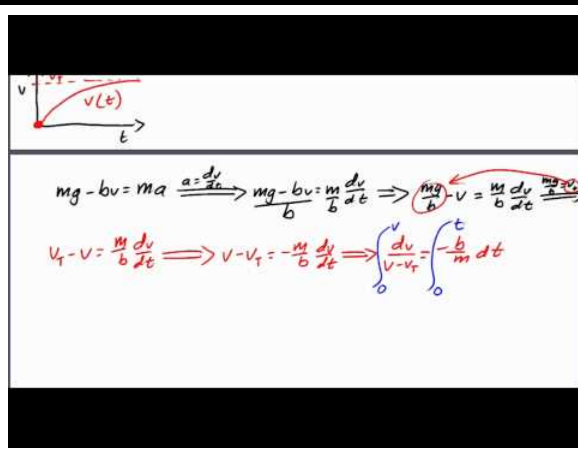
Another interesting force in everyday life is the force of drag on an object when it is moving in a fluid (either a gas or a liquid). You feel the drag force when you move your hand through water. You might also feel it if you move your hand during a strong wind. The faster you move your hand, the harder it is to move. You feel a smaller drag force when you tilt your hand so only the side goes through the air—you have decreased the area of your hand that faces the direction of motion.

Like friction, the drag force always opposes the motion of an object. Unlike simple friction, the drag force is proportional to some function of the velocity of the object in that fluid. This functionality is complicated and depends upon the shape of the object, its size, its velocity, and the fluid it is in. Aerodynamic objects tend to have small surface areas and be designed to have low drag coefficients.

For most large objects such as bicyclists, cars, and baseballs not moving too slowly, the magnitude of the drag force F_D is found to be proportional to the square of the speed of the object. We can write this relationship mathematically as $FD \propto v^2$. When taking into account other factors, this relationship becomes $F_D = \frac{\frac{1}{2C}}{rhoAv^2}$, where C is known as the drag coefficient, a unit-less number that represents the aerodynamic properties of the object, A is the cross-sectional area of the object which is facing the direction of motion, and ρ is the density of the fluid the object is moving through.



Aerodynamic Shape: From racing cars to bobsled racers, aerodynamic shaping is crucial to achieving top speeds. Bobsleds are designed for speed. They are shaped like a bullet with tapered fins. (credit: U.S. Army, via Wikimedia Commons)



Retarding and Drag Forces: A brief look at retarding (drag) forces in physics, for students in introductory physics classes that use calculus. This video walks through a single scenario of an object experiencing a drag force where the drag force is proportional to the object’s velocity.

Stress and Strain

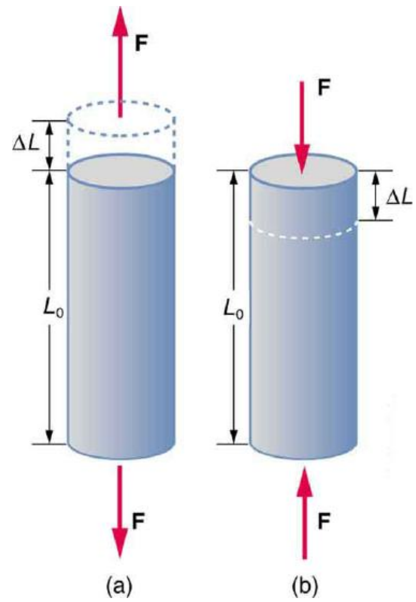
The ratio of force to area $\frac{F}{A}$ is called stress and the ratio of change in length to length $\frac{\Delta L}{L}$ is called the strain.

learning objectives

- Explain how forces affects the shape of an object

We now move from consideration of forces that affect the motion of an object (such as friction and drag) to those that affect an object's shape. If a bulldozer pushes a car into a wall, the car will not move past the wall but it will noticeably change shape. A change in shape due to the application of a force is a deformation. Even very small forces are known to cause some deformation. For small deformations, two important characteristics are observed. First, the object returns to its original shape when the force is removed—that is, the deformation is elastic for small deformations. Second, the size of the deformation is proportional to the force —that is, for small deformations, Hooke's law is obeyed. In equation form, Hooke's law is given by $F = k \cdot \Delta L$ where ΔL is the change in length and kk is a constant which depends on the material properties of the object.

Deformations come in several types: changes in length (tension and compression), sideways shear (stress), and changes in volume.



Tension/Compression: Tension: The rod is stretched a length ΔL when a force is applied parallel to its length. (b) Compression: The same rod is compressed by forces with the same magnitude in the opposite direction. For very small deformations and uniform materials, ΔL is approximately the same for the same magnitude of tension or compression. For larger deformations, the cross-sectional area changes as the rod is compressed or stretched.

The ratio of force to area $\frac{F}{A}$ is called stress and the ratio of change in length to length $\frac{\Delta L}{L}$ is called the strain.

Stress and strain are related to each other by a constant called Young's Modulus or the elastic modulus which varies depending on the material. Using Young's Modulus the relation between stress and strain is given by: $\text{stress} = Y \cdot \text{strain}$.

A material with a high elastic modulus is said to have high tensile strength. Such materials are very resistant to being stretched and require a large amount of force to deform a small amount.

Translational Equilibrium

An object is said to be in equilibrium when there is no external net force acting on it.

learning objectives

- Assess the role each type of equilibrium plays in mechanical devices

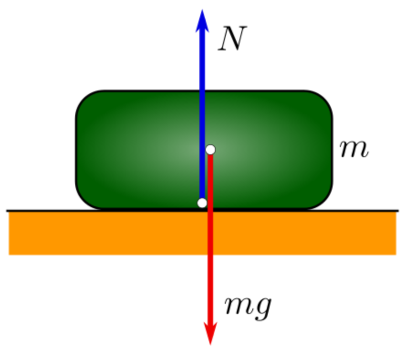
We are surrounded by great engineering architectures and mechanical devices, which are at rest in the frame of reference of Earth. A large part of engineering creations are static objects. Yet we also seek equilibrium of moving objects like that of floating ship, airplane cruising at high speed, and such other moving mechanical devices. In both cases – static or dynamic – net external forces and torques are zero.

A body is said to be in mechanical equilibrium when net external force is equal to zero and net external torque is also zero. Mathematically,

$$\Sigma \overrightarrow{\mathrm{F}}\,\mathrm{ext} = 0 \text{ and } \Sigma \overrightarrow{\tau}\,\mathrm{ext} = 0 \tag{4.7.3}$$

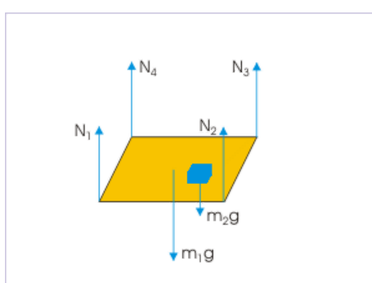
Since there is no net force on the object, the object does not accelerate. This implies two types of possible equilibrium. The first type, where all particles in the system are at rest and do have not velocity, is known as static equilibrium. In the second type, the object has a velocity, but since there are no net forces acting on it, the velocity remains constant. In the second case, the particle is said to be in dynamic equilibrium. Static or dynamic, these kinds of equilibrium can be categorized as translational equilibrium.

Examples of translational equilibrium are all around us. A book resting on a table is pushing down on the table with the force of its weight. The table, in turn, is pushing back on the book, keeping the book from falling through the table. Since neither the table nor the book are moving, this is an example of static equilibrium. The force of gravity on the book is perfectly counteracted by the force of the table pushing on it.



Forces Acting on an Object at Rest: A force diagram showing the forces acting on an object at rest on a surface. Notice that the amount of force that the table is pushing upward on the object (the N vector) is equal to the downward force of the object's weight (shown here as mg, as weight is equal to the object's mass multiplied by the acceleration due to gravity): because these forces are equal, the object is in a state of equilibrium (all the forces acting on it balance to zero).

An example of dynamic (or mechanical) equilibrium is an object sliding down a wedge. The force of gravity pulls the object down the wedge, but it is counteracted by the force of friction between the wedge and the object. If the force of friction is equal to the force of gravity, the object will proceed at a constant velocity.



Forces on a Table: These six forces are in equilibrium. The four forces of the table leg counteract the force of the table and the object pushing on them.

Connected Objects

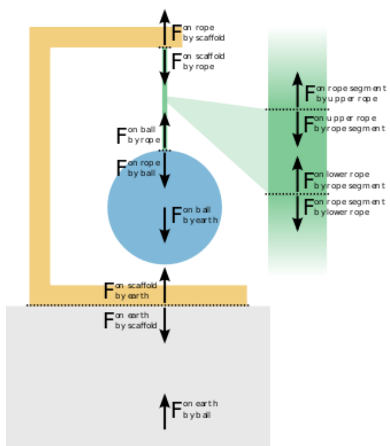
Forces can be transferred from one object to another through connections.

> **learning objectives**
>
> - Analyze the affect a rigid connection has on the movement of objects

The physics of connected objects is very similar to physics of simple objects. There are a variety of ways objects can be connected to each other, and a corresponding variety of mathematical ways to model such connections.

The simplest form of connection is a perfectly rigid connection. If two objects are connected by a perfectly rigid connector then they may be thought of as the same object. Perfectly rigid connectors cannot stretch nor deform, and transfer forces instantaneously from one side of the connection to the other. For example, given two blocks (both of mass 1 kg) connected by a perfectly rigid bar, if the first block is pulled with a force of 1 Newton, then both blocks will accelerate at the same time and the same acceleration. In this case the acceleration is $\frac{1}{2}\mathrm{m/s^2}$ —the same as if a force of mass 2 kg is exerted on one object. Thus it can be said that a perfectly rigid connection makes two objects into one large object. Of course, perfectly rigid connections do not exist in nature. Some deformation will always exist in any object as force travels along it. However, many materials are sufficiently rigid, so that using the perfectly rigid approximation is useful for simplicity's sake.

One can think of the force transferring through the connection by means of the "tension" force. Tension is the pulling force exerted by a string, chain, or similar connector on another object. If two objects are connected by a string, a force exerted on one is balanced by a tension force in the string which pulls on the other. Of course, if the tension force is greater than the rope can withstand, the rope will break.



Tension Forces: The forces involved in supporting a ball by a rope. Tension is the force of the rope on the scaffold, the force of the rope on the ball, and the balanced forces acting on and produced by segments of the rope.

Circular Motion

An object in circular motion undergoes acceleration due to centripetal force in the direction of the center of rotation.

> **learning objectives**
>
> - Develop an understanding of uniform circular motion as an indicator for net external force

Uniform circular motion describes the motion of an object along a circle or a circular arc at constant speed. It is the basic form of rotational motion in the same way that uniform linear motion is the basic form of translational motion. However, the two types of motion are different with respect to the force required to maintain the motion.

Let us consider Newton's first law of motion. It states that an object will maintain a constant velocity unless a net external force is applied. Therefore, uniform linear motion indicates the absence of a net external force. On the other hand, uniform circular motion requires that the velocity vector of an object constantly change direction. Since the velocity vector of the object is changing, an acceleration is occurring. Therefore, uniform *circular* motion indicates the *presence* of a net external force.

In uniform circular motion, the force is always perpendicular to the direction of the velocity. Since the direction of the velocity is continuously changing, the direction of the force must be as well.

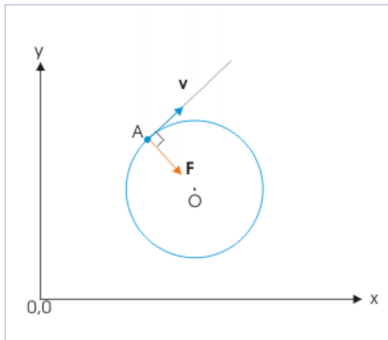
The direction of the velocity along the circular trajectory is tangential. The perpendicular direction to the circular trajectory is, therefore, the radial direction. Therefore, the force (and therefore the acceleration) in uniform direction motion is in the radial direction. For this reason, acceleration in uniform circular motion is recognized to "seek the center" — i.e., centripetal force.

The equation for the acceleration aa required to sustain uniform circular motion is:

$$a = \frac{v^2}{r} \tag{4.7.4}$$

where mm is the mass of the object, v is the velocity of the object, and r is the radius of the circle. Consequently, the net external force F_{net} required to sustain circular motion is:

$$F_{net} = \frac{m \cdot v^2}{r} \tag{4.7.5}$$



Uniform Circular Motion: In uniform circular motion, the centripetal force is perpendicular to the velocity. The centripetal force points toward the center of the circle, keeping the object on the circular track.

Key Points

- Friction is the force that resists relative motion between two surfaces sliding across each other. Friction converts kinetic energy into heat.
- Drag force is the force that resists motion of an object traveling through a fluid such as air or water. Drag force is proportional to the velocity of the object traveling.
- Deformation forces are forces caused by stretching or compressing a material. Some examples would be springs or elastics.
- Kinetic (or dynamic) friction occurs when two objects are moving relative to each other and rub together (like a sled on the ground).
- The force of friction can be represented by an equation $F_{friction} = \mu F_n$ where μ is the coefficient of friction and is a unitless number that represents the strength of the friction of the surface.
- Kinetic friction and static (stationary) friction use two different coefficients for the same material.
- Static friction is a force that acts to resist the start of motion. It is borne of macroscopic inconsistencies in the surfaces of materials in contact as well as intermolecular interactions between the materials, such as hydrogen bonding, Van der Waal's interactions and electrostatic interactions.
- Static friction uses a different, usually higher, coefficient than kinetic friction does.
- The force of static friction is $F_{fs} = \mu_s F_n$. Where μ_s is the coefficient of static friction which varies by material and FnFn is the normal force.
- Motion on an incline is resisted by friction.
- The frictional force on an incline is dependent on the angle of the incline. $F_f = \mu mg \cos(\theta)$ is the maximum friction force on an incline.
- If the friction force is greater than or equal to the forces in the direction of motion, then the net force is 0 and the object is in equilibrium.
- Objects moving through a fluid feel a force which resists their motion. This force is known as the drag force.
- The drag force is proportional to the square of the velocity of the object relative to the fluid.
- The equation for drag is $F_D = \frac{1}{2} C_\rho A v^2$. C is a constant called the drag coefficient. ρρ is the density of the fluid. A is the surface area in the direction of motion.

- The ratio of force to area $\frac{F}{A}$ is called stress and the ratio of change in length to length $\frac{\Delta L}{L}$ is called the strain.
- Stress and strain are related to each other by a constant called Young's Modulus or the elastic modulus which varies depending on the material. Using Young's Modulus the relation between stress and strain is given by: $\text{stress} = Y \cdot \text{strain}$.
- A material with a high elastic modulus is said to have high tensile strength. Such materials are very resistant to being stretched and require a large amount of force to deform a small amount.
- When there is no external net force on an object, the object is said to be in equilibrium.
- When an object is in equilibrium, it does not accelerate. If it had a velocity, the velocity remains constant; if it was at rest, it remains at rest.
- An equilibrium in motion is known as dynamic equilibrium; an equilibrium at rest is a static equilibrium.
- If two objects are connected, a force on one has an effect on the other.
- Connections can often be approximated as completely rigid. In completely rigid cases, the connection does not deform and the force is transferred instantaneously.
- Tension is the force of a rope or cable or other connector on the object it is connected to. It is one way force is transferred between objects.
- An object that is undergoing circular motion has a velocity vector that is constantly changing direction.
- The force that is needed to maintain circular motion points toward the center of the circular path. It is therefore known as the centripetal force.
- The velocity of an object in circular motion is always tangent to the circle, and the centripetal force is always perpendicular to the velocity.

Key Terms

- **kinetic energy**: The energy possessed by an object because of its motion, equal to one half the mass of the body times the square of its velocity.
- **static**: Fixed in place; having no motion.
- **kinetic**: Of or relating to motion
- **friction**: A force that resists the relative motion or tendency to such motion of two bodies in contact.
- **incline**: A slope.
- **equilibrium**: The state of a body at rest or in uniform motion, the resultant of all forces on which is zero.
- **fluid**: Any substance which can flow with relative ease, tends to assume the shape of its container, and obeys Bernoulli's principle; a liquid, gas or plasma.
- **strain**: The amount by which a material deforms under stress or force, given as a ratio of the deformation to the initial dimension of the material and typically symbolized by $\epsilon\epsilon$ is termed the engineering strain. The true strain is defined as the natural logarithm of the ratio of the final dimension to the initial dimension.
- **stress**: The internal distribution of force per unit area (pressure) within a body reacting to applied forces which causes strain or deformation and is typically symbolized by σ.
- **dynamic**: Changing; active; in motion.
- **torque**: A rotational or twisting effect of a force; (SI unit newton-meter or Nm; imperial unit foot-pound or ft-lb)
- **rigid**: Stiff, rather than flexible.
- **tangent**: a straight line touching a curve at a single point without crossing it at that point
- **perpendicular**: at or forming a right angle (to).

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CHAPTER OVERVIEW

5: Uniform Circular Motion and Gravitation

5.1: Introduction to UCM and Gravitation

Kinematics of UCM

Uniform circular motion is a motion in a circular path at constant speed.

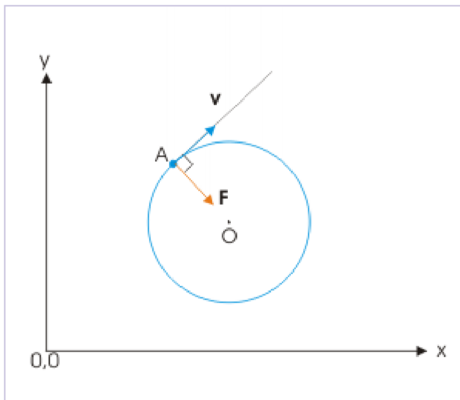
learning objectives

- Relate centripetal force and centripetal acceleration to uniform circular motion

Angular Quantities

Under uniform circular motion, angular and linear quantities have simple relations. When objects rotate about some axis, each point in the object follows a circular arc. The rotation angle is the amount of rotation and is analogous to linear distance. We define the rotation angle $\Delta\theta$ to be the ratio of the arc length to the radius of curvature:

$$\Delta\theta = \frac{\Delta \mathrm{s}}{\mathrm{r}} \tag{5.1.1}$$



Angle θθ and Arc Length ss: The radius of a circle is rotated through an angle $\Delta\theta$. The arc length $\Delta \mathrm{s}$ is described on the circumference.

We define angular velocity ωω as the rate of change of an angle. In symbols, this is $\omega = \frac{\Delta\theta}{\Delta \mathrm{t}}$, where an angular rotation $\Delta\theta$ takes place in a time $\Delta \mathrm{t}$. From the relation of s and ($\Delta \mathrm{s} = \mathrm{r}\Delta\theta$), we see:

$$\mathrm{v} = \frac{\Delta \mathrm{s}}{\Delta \mathrm{t}} = \mathrm{r}\frac{\Delta\theta}{\Delta \mathrm{t}} = \mathrm{r}\omega \tag{5.1.2}$$

Under uniform circular motion, the angular velocity is constant. The acceleration can be written as:

$$\mathrm{ac} = \frac{\mathrm{dv}}{\mathrm{dt}} = \omega\frac{\mathrm{dr}}{\mathrm{dt}} = \omega\mathrm{v} = \mathrm{r}\omega^2 = \frac{\mathrm{v}^2}{\mathrm{r}} \tag{5.1.3}$$

This acceleration, responsible for the uniform circular motion, is called centripetal acceleration.

Centripetal Force

Any force or combination of forces can cause a centripetal or radial acceleration. Just a few examples are the tension in the rope on a tether ball, the force of Earth's gravity on the Moon, friction between roller skates and a rink floor, a banked roadway's force on a car, and forces on the tube of a spinning centrifuge.

Any net force causing uniform circular motion is called a centripetal force. The direction of a centripetal force is toward the center of curvature, the same as the direction of centripetal acceleration. According to Newton's second law of motion, net force is mass times acceleration. For uniform circular motion, the acceleration is the centripetal acceleration: $\mathrm{a} = \mathrm{a_c}$. Thus, the magnitude of centripetal force $\mathrm{F_c}$ is:

$$\mathrm{F_c} = \mathrm{ma_c} = \mathrm{m}\frac{\mathrm{v}^2}{\mathrm{r}} = \mathrm{mr}\omega^2 \tag{5.1.4}$$

Dynamics of UCM

Newton's universal law of gravitation states that every particle attracts every other particle with a force along a line joining them.

learning objectives

- Relate Kepler's laws to Newton's universal law of gravitation

Newton's Universal Law of Gravitation

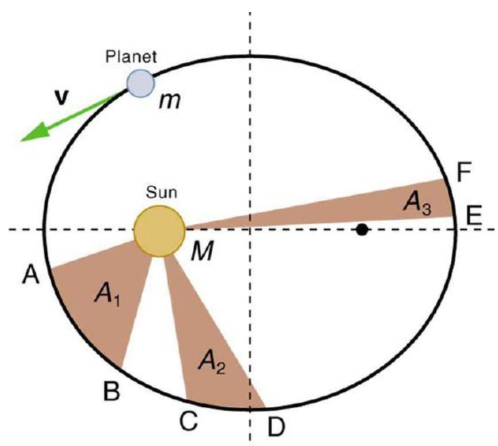
Newton's universal law of gravitation states that every particle in the universe attracts every other particle with a force along a line joining them. The force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. For two bodies having masses mm and MM with a distance rr between their centers of mass, the equation for Newton's universal law of gravitation is:

$$F = G\frac{mM}{r^2} \tag{5.1.5}$$

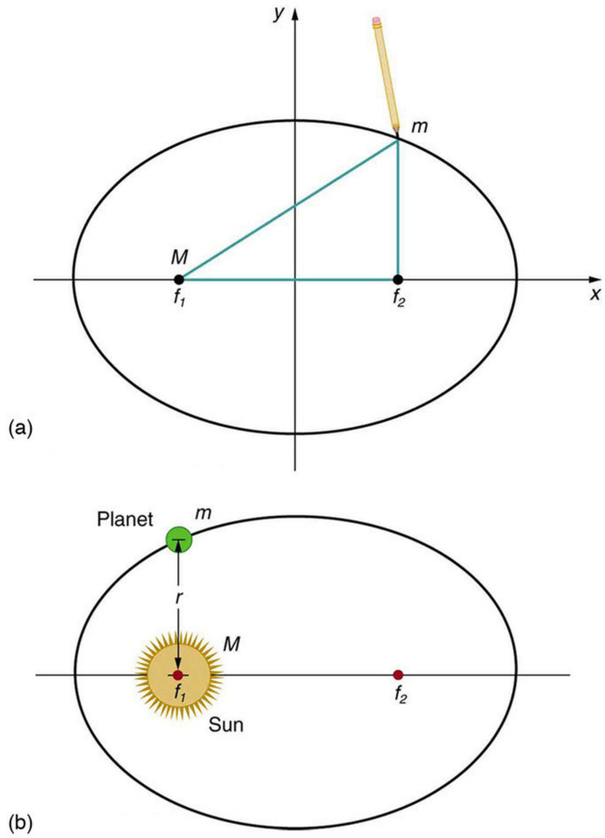
The gravitational force is responsible for artificial satellites orbiting the Earth. The Moon's orbit about Earth, the orbits of planets, asteroids, meteors, and comets about the Sun are other examples of gravitational orbits. Historically, Kepler discovered his 3 laws (called Kepler's law of planetary motion) long before the days of Newton. Kepler devised his laws after careful study (over some 20 years) of a large amount of meticulously recorded observations of planetary motion done by Tycho Brahe (1546–1601).

Kepler's Laws

- The orbit of each planet about the Sun is an ellipse with the Sun at one focus.
- Each planet moves so that an imaginary line drawn from the Sun to the planet sweeps out equal areas in equal times.
- The ratio of the squares of the periods of any two planets about the Sun is equal to the ratio of the cubes of their average distances from the Sun.



Kepler's Second Law: The shaded regions have equal areas. It takes equal times for mm to go from A to B, from C to D, and from E to F. The mass mm moves fastest when it is closest to M. Kepler's second law was originally devised for planets orbiting the Sun, but it has broader validity.



Ellipses and Kepler's First Law: (a) An ellipse is a closed curve such that the sum of the distances from a point on the curve to the two foci (f_1 and f_2) is a constant. You can draw an ellipse as shown by putting a pin at each focus, and then placing a string around a pencil and the pins and tracing a line on paper. A circle is a special case of an ellipse in which the two foci coincide (thus any point on the circle is the same distance from the center). (b) For any closed gravitational orbit, m follows an elliptical path with M at one focus. Kepler's first law states this fact for planets orbiting the Sun.

Derivation of Kepler's Third Law For Circular Orbits

Kepler's 3rd law is equivalent to:

$$\frac{T_1^2}{T_2^2} = \frac{r_3^1}{r_3^2} \tag{5.1.6}$$

T is the period (time for one orbit) and r is the average radius. We shall derive Kepler's third law, starting with Newton's laws of motion and his universal law of gravitation. We will assume a circular path (not an elliptical one) for simplicity.

Let us consider a circular orbit of a small mass mm around a large mass M, satisfying the two conditions stated at the beginning of this section. Gravity supplies the centripetal force to mass m. Therefore, for a uniform circular motion:

$$G\frac{mM}{r^2} = ma_c = m\frac{v^2}{r} \tag{5.1.7}$$

The mass m cancels, yielding:

$$G\frac{mM}{r^2} = v^2 \tag{5.1.8}$$

Now, to get at Kepler's third law, we must get the period T into the equation. By definition, period T is the time for one complete orbit. Now the average speed v is the circumference divided by the period:

$$v = \frac{2\pi r}{T} \tag{5.1.9}$$

Substituting this into the previous equation gives:

$$G\frac{M}{r}=\frac{4\pi^2 r^2}{T^2} \tag{5.1.10}$$

Solving for T^2 yields:

$$T^2=\frac{4\pi^2}{GM}r^3 \tag{5.1.11}$$

Since T^2 is proportional to r^3, their ratio is constant. This is Kepler's 3rd law.

Banked and Unbacked Highway Curves

In an "ideally banked curve," the angle θ is chosen such that one can negotiate the curve at a certain speed without the aid of friction.

learning objectives

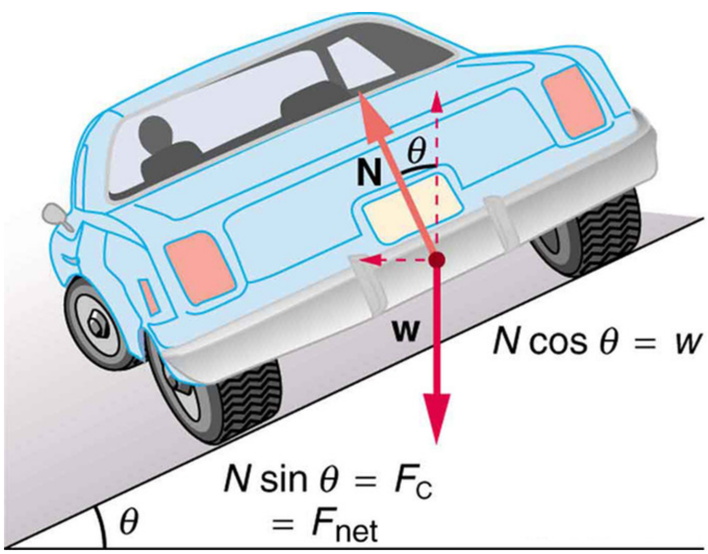
- Derive θ for an ideally banked curve for speed

Overview

As an example of a uniform circular motion and its application, let us now consider banked curves, where the slope of the road helps you negotiate the curve. The greater the angle θ, the faster you can take the curve. Race tracks for bikes as well as cars, for example, often have steeply banked curves. In an "ideally banked curve," the angle θ is such that you can negotiate the curve at a certain speed without the aid of friction between the tires and the road. We will derive an expression for θ for an ideally banked curve for speed $v$and consider an example related to it.

Uniform Circular Motion and Determining Ideal Banking Conditions

For ideal banking, the net external force equals the horizontal centripetal force in the absence of friction. The components of the normal force N in the horizontal and vertical directions must equal the centripetal force and the weight of the car, respectively. In cases in which forces are not parallel, it is most convenient to consider components along perpendicular axes—in this case, the vertical and horizontal directions.



Car on a Banked Curve: The car on this banked curve is moving away and turning to the left.

Above is a free body diagram for a car on a frictionless banked curve. The only two external forces acting on the car are its weight ww and the normal force of the road N. (A frictionless surface can only exert a force perpendicular to the surface—that is, a normal force.) These two forces must add to give a net external force that is horizontal toward the center of curvature and has magnitude $\frac{mv^2}{r}$. Only the normal force has a horizontal component, and so this must equal the centripetal force—that is:

$$N \sin\theta = \frac{mv^2}{r} \quad (5.1.12)$$

Because the car does not leave the surface of the road, the net vertical force must be zero, meaning that the vertical components of the two external forces must be equal in magnitude and opposite in direction. From the figure, we see that the vertical component of the normal force is $N \cos\theta$, and the only other vertical force is the car's weight. These must be equal in magnitude, thus:

$$N \cos\theta = mg \quad (5.1.13)$$

Dividing the above equations yields:

$$\tan\theta = \frac{v^2}{rg} \quad (5.1.14)$$

Taking the inverse tangent gives:

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) \quad (5.1.15)$$

for an ideally banked curve with no friction.

This expression can be understood by considering how θ depends on v and r. A large θ will be obtained for a large $v$and a small r. That is, roads must be steeply banked for high speeds and sharp curves. Friction helps, because it allows you to take the curve at greater or lower speed than if the curve is frictionless. Note that θ does not depend on the mass of the vehicle.

Key Points

- Under uniform circular motion, angular and linear quantities have simple relations. The length of an arc is proportional to the rotation angle and the radius. Also, $v = r\omega$.
- The acceleration responsible for the uniform circular motion is called centripetal acceleration. It is given as $a_c = r\omega^2 = \frac{v^2}{r}$.
- Any net force causing uniform circular motion is called a centripetal force. The direction of a centripetal force is toward the center of curvature and its magnitude is $m\frac{v^2}{r} = mr\omega^2$.
- The gravitational force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.
- Kepler discovered laws describing planetary motion long before the days of Newton, purely based on the observations of Tycho Brahe.
- Kepler's laws can be derived from the Newton's universal law of gravitation and his equation of motion.
- For ideal banking, the net external force equals the horizontal centripetal force in the absence of friction.
- For ideal banking, the components of the normal force NN in the horizontal and vertical directions must equal the centripetal force and the weight of the car, respectively.
- The ideal banking condition is given as $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$.

Key Terms

- **centripetal**: Directed or moving towards a center.
- **asteroid**: A naturally occurring solid object, which is smaller than a planet and is not a comet, that orbits a star.
- **planet**: A large body which directly orbits any star (or star cluster) but which has not attained nuclear fusion.
- **normal force**: Any force acting normal, to a surface, or perpendicular to the tangent plane.

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5.2: Non-Uniform Circular Motion

Overview of Non-Uniform Circular Motion

Non-uniform circular motion denotes a change in the speed of a particle moving along a circular path.

learning objectives

- Explain when a particle undergoes non-uniform circular motion

What do we mean by non-uniform circular motion? The answer lies in the definition of uniform circular motion, which is a circular motion with constant speed. It follows then that non-uniform circular motion denotes a change in the speed of the particle moving along the circular path. Note especially the change in the velocity vector sizes, denoting change in the magnitude of velocity.

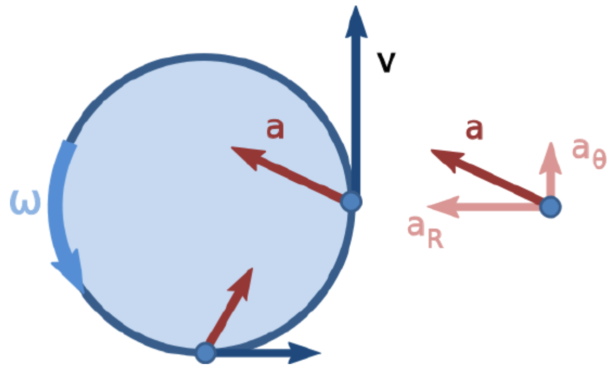

Diagram of non-uniform circular motion: In non-uniform circular motion, the magnitude of the angular velocity changes over time.

The change in direction is accounted by radial acceleration (centripetal acceleration), which is given by following relation: $a_r = \frac{v^2}{r}$. The change in speed has implications for radial (centripetal) acceleration. There are two possibilities:

1: The radius of circle is constant (like in the motion along a circular rail or motor track). A change in v will change the magnitude of radial acceleration. This means that the centripetal acceleration is not constant, as is the case with uniform circular motion. The greater the speed, the greater the radial acceleration. A particle moving at higher speed will need a greater radial force to change direction and vice-versa when the radius of the circular path is constant.

2: The radial (centripetal) force is constant (like a satellite rotating about the earth under the influence of a constant force of gravity). The circular motion adjusts its radius in response to changes in speed. This means that the radius of the circular path is variable, unlike the case of uniform circular motion. In any eventuality, the equation of centripetal acceleration in terms of “speed” and “radius” must be satisfied. The important thing to note here is that, although change in speed of the particle affects radial acceleration, the change in speed is not affected by radial or centripetal force. We need a tangential force to affect the change in the magnitude of a tangential velocity. The corresponding acceleration is called tangential acceleration.

In either case, the angular velocity in non-uniform circular motion is not constant as $\omega = \frac{v}{r}$ and v is varying.

Key Points

- In non- uniform circular motion, the size of the velocity vector (speed) changes, denoting change in the magnitude of velocity.
- The change in speed has implications for radial (centripetal) acceleration. There are two possibilities: 1) the radius of the circle is constant; or 2) the radial (centripetal) force is constant.
- In either case, the angular velocity in non-uniform circular motion is not constant, as $\omega = \frac{v}{r}$, and v varies.

Key Terms

- **radial**: Moving along a radius.
- **centripetal**: Directed or moving towards a center.

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5.3: Velocity, Acceleration, and Force

Rotational Angle and Angular Velocity

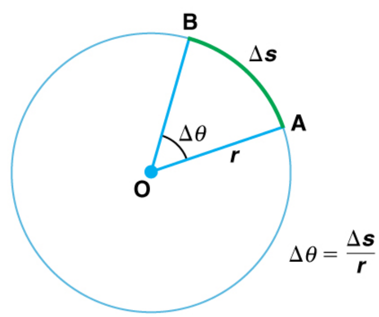
The rotational angle is a measure of how far an object rotates, and angular velocity measures how fast it rotates.

learning objectives

- Express the relationship between the rotational angle and the distance

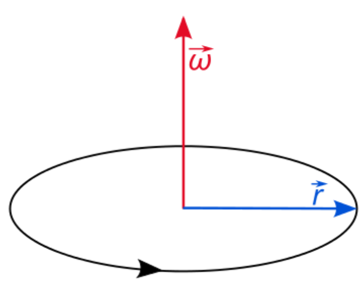
Rotational Angle and Angular Velocity

When an object rotates about an axis, as with a tire on a car or a record on a turntable, the motion can be described in two ways. A point on the edge of the rotating object will have some velocity and will be carried through an arc by riding the spinning object. The point will travel through a distance of $\Delta \mathrm{S}$, but it is often more convenient to talk about the extent the object has rotated. The amount the object rotates is called the rotational angle and may be measured in either degrees or radians. Since the rotational angle is related to the distance $\Delta \mathrm{S}$ and to the radius r by the equation $\Delta\theta = \frac{\Delta \mathrm{S}}{\mathrm{R}}$, it is usually more convenient to use radians.



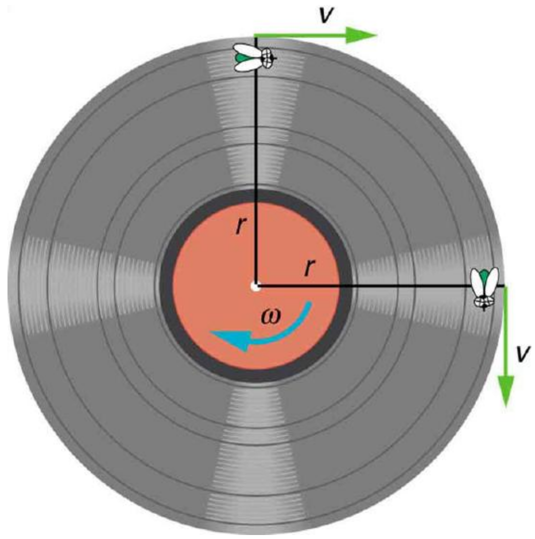
Angle θ and Arc Length s: The radius of a circle is rotated through an angle $\Delta\theta$. The arc length Δs is described on the circumference.

The speed at which the object rotates is given by the angular velocity, which is the rate of change of the rotational angle with respect to time. Although the angle itself is not a vector quantity, the angular velocity is a vector. The direction of the angular velocity vector is perpendicular to the plane of rotation, in a direction which is usually specified by the right-hand rule. Angular acceleration gives the rate of change of angular velocity. The angle, angular velocity, and angular acceleration are very useful in describing the rotational motion of an object.



The Direction of Angular Velocity: The angular velocity describes the speed of rotation and the orientation of the instantaneous axis about which the rotation occurs. The direction of the angular velocity will be along the axis of rotation. In this case (counter-clockwise rotation), the vector points upwards.

When the axis of rotation is perpendicular to the position vector, the angular velocity may be calculated by taking the linear velocity v of a point on the edge of the rotating object and dividing by the radius. This will give the angular velocity, usually denoted by ω, in terms of radians per second.



Angular Velocity: A fly on the edge of a rotating object records a constant velocity v. The object is rotating with an angular velocity equal to $\frac{v}{r}$.

Centripetial Acceleration

Centripetal acceleration is the constant change in velocity necessary for an object to maintain a circular path.

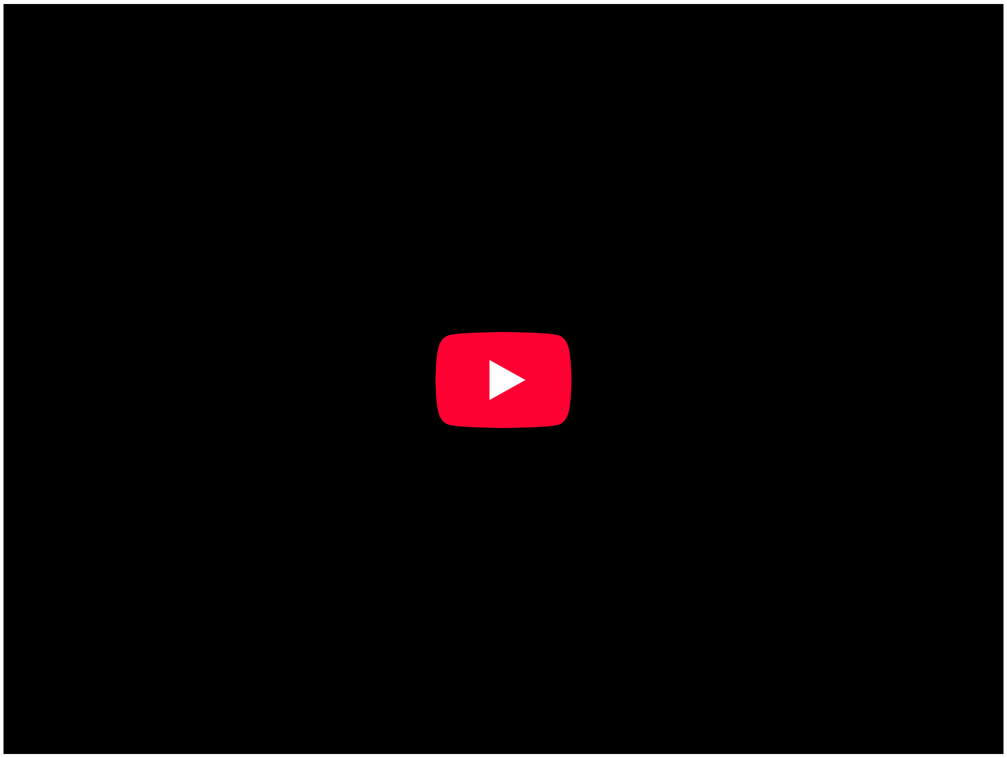
learning objectives

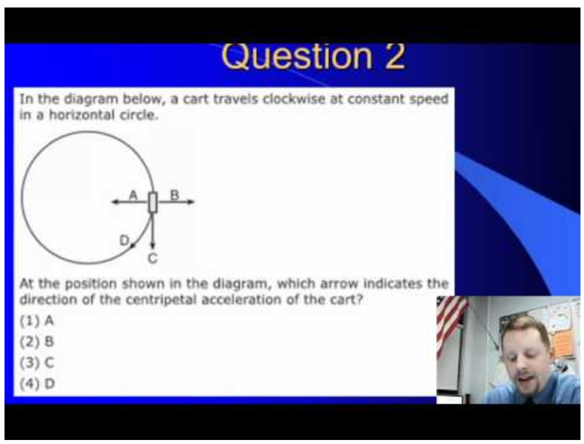
- Express the centripetal acceleration in terms of rotational velocity

Overview

As mentioned in previous sections on kinematics, any change in velocity is given by an acceleration. Often the changes in velocity are changes in magnitude. When an object speeds up or slows down this is a change in the objects velocity. Changes in the magnitude of the velocity match our intuitive and every day usage of the term accelerate. However, because velocity is a vector, it also has a direction. Therefore, any change in the direction of travel of an object must also be met with an acceleration.

Uniform circular motion involves an object traveling a circular path at constant speed. Since the speed is constant, one would not usually think that the object is accelerating. However, the direction is constantly changing as the object traverses the circle. Thus, it is said to be accelerating. One can feel this acceleration when one is on a roller coaster. Even if the speed is constant, a quick turn will provoke a feeling of force on the rider. This feeling is an acceleration.





Centripetal Acceleration: A brief overview of centripetal acceleration for high school physics students.

Calculating Centripetal Acceleration

To calculate the centripetal acceleration of an object undergoing uniform circular motion, it is necessary to have the speed at which the object is traveling and the radius of the circle about which the motion is taking place. The simple equation is:

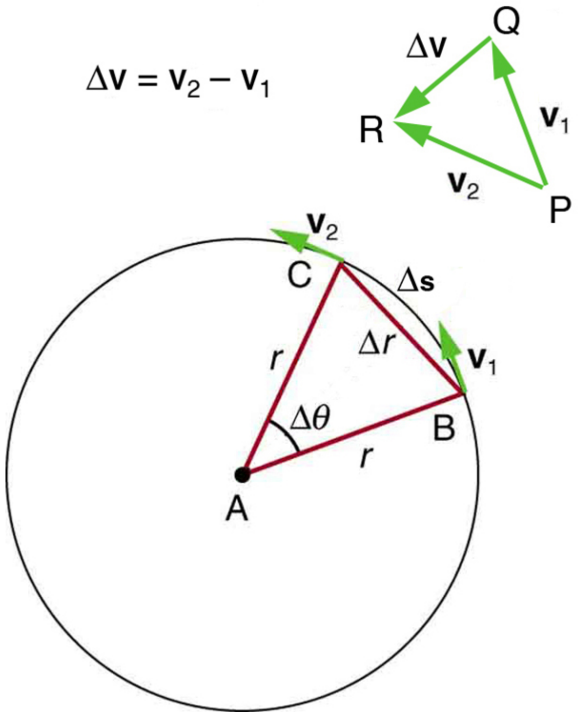
$$\mathrm{a_c} = \frac{\mathrm{v}^2}{\mathrm{r}} \tag{5.3.1}$$

where v is the linear velocity of the object and r is the radius of the circle.

The centripetal acceleration may also be expressed in terms of rotational velocity as follows:

$$\mathrm{a_c} = \omega^2 \mathrm{r} \tag{5.3.2}$$

with omega being the rotational velocity given by $\frac{\mathrm{v}}{\mathrm{r}}$.



Centripetal Acceleration: As an object moves around a circle, the direction of the velocity vector constantly changes.

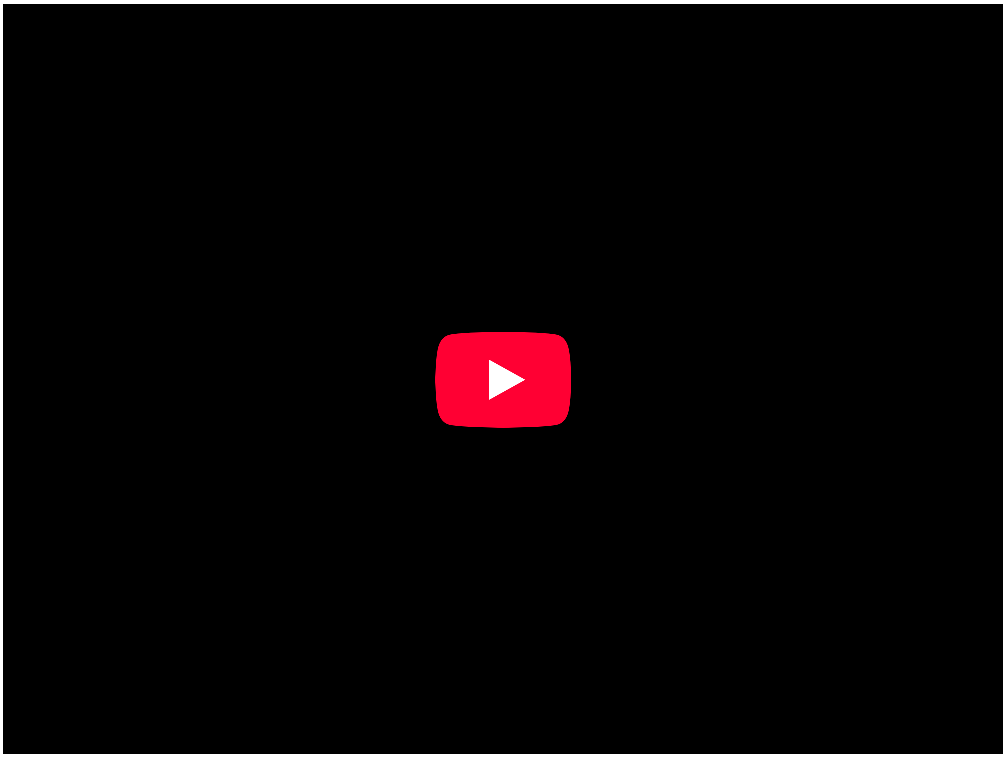
Centripetal Force

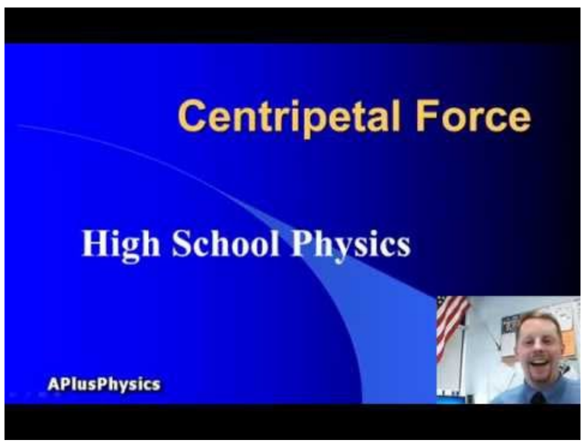
A force which causes motion in a curved path is called a centripetal force (uniform circular motion is an example of centripetal force).

learning objectives

- Express the equations for the centripetal force and acceleration

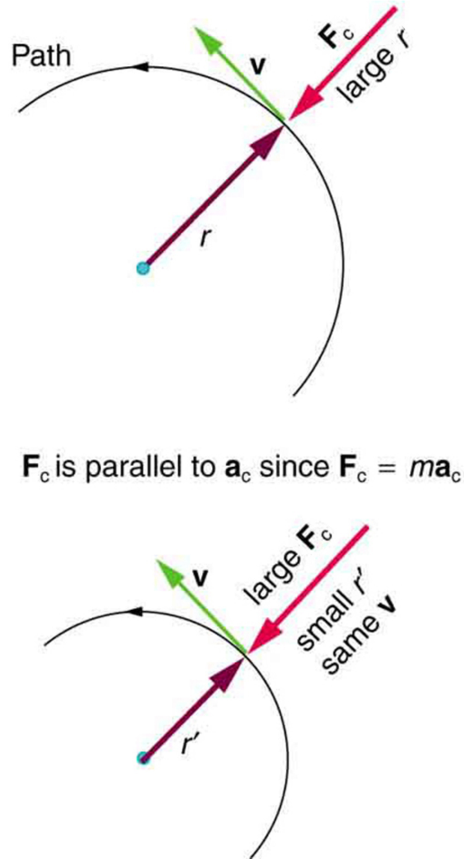
A force that causes motion in a curved path is called a centripetal force. Uniform circular motion is an example of centripetal force in action. It can be seen in the orbit of satellites around the earth, the tension in a rope in a game of tether ball, a roller coaster loop de loop, or in a bucket swung around the body.





Overview of centripetal force: A brief overview of centripetal force.

Previously, we learned that any change in a velocity is an acceleration. As the object moves through the circular path it is constantly changing direction, and therefore accelerating—causing constant force to be acting on the object. This centripetal force acts toward the center of curvature, toward the axis of rotation. Because the object is moving perpendicular to the force, the path followed by the object is a circular one. It is this force that keeps a ball from falling out of a bucket if you swing it in circular continuously.



Centripetal force: As an object travels around a circular path at a constant speed, it experiences a centripetal force accelerating it toward the center.

The equation for centripetal force is as follows:

$$\mathrm{Fc} = \frac{\mathrm{mv}^2}{\mathrm{r}} \tag{5.3.3}$$

where: $\mathrm{F_c}$ is centripetal force, m is mass, v is velocity, and r is the radius of the path of motion.

From Newton's second law $\mathrm{F} = \mathrm{m} \cdot \mathrm{a}$, we can see that centripetal acceleration is:

$$\mathrm{a_c} = \frac{\mathrm{v}^2}{\mathrm{r}} \tag{5.3.4}$$

Centripetal force can also be expressed in terms of angular velocity. Angular velocity is the measure of how fast an object is traversing the circular path. As the object travels its path, it sweeps out an arc that can be measured in degrees or radians. The equation for centripetal force using angular velocity is:

$$\mathrm{F_c} = \mathrm{mr}\omega^2 \tag{5.3.5}$$

Key Points

- When an object rotates about an axis, the points on the edge of the object travel in arcs.
- The angle these arcs sweep out is called the rotational angle, and it is usually represented by the symbol *theta*.
- A measure of how quickly the object is rotating, with respect to time, is called the angular velocity. It is usually represented by a Greek *omega* symbol. Like its counterpart linear velocity, it is a vector.
- For an object to maintain circular motion it must constantly change direction.
- Since velocity is a vector, changes in direction constitute changes in velocity.
- A change in velocity is known as an acceleration. The change in velocity due to circular motion is known as centripetal acceleration.

- Centripetal acceleration can be calculated by taking the linear velocity squared divided by the radius of the circle the object is traveling along.
- When an object is in uniform circular motion, it is constantly changing direction, and therefore accelerating. This is angular acceleration.
- A force acting on the object in uniform circular motion (called centripetal force) is acting on the object from the center of the circle.

Key Terms

- **radians**: The angle subtended at the centre of a circle by an arc of the circle of the same length as the circle's radius.
- **acceleration**: The amount by which a speed or velocity increases (and so a scalar quantity or a vector quantity).
- **circular motion**: Motion in such a way that the path taken is that of a circle.
- **velocity**: A vector quantity that denotes the rate of change of position with respect to time, or a speed with a directional component.
- **centripetal**: Directed or moving towards a center.
- **angular velocity**: A vector quantity describing an object in circular motion; its magnitude is equal to the speed of the particle and the direction is perpendicular to the plane of its circular motion.

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5.4: Types of Forces in Nature

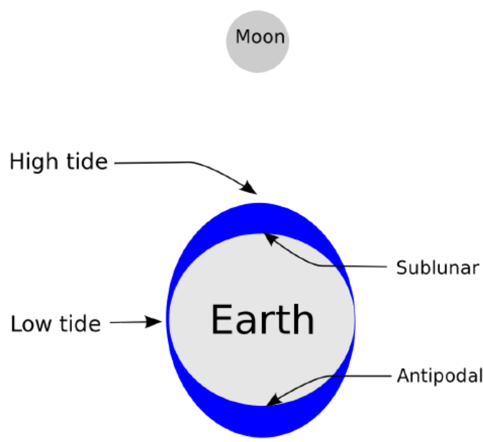
Tides

Tides are the rise and fall of sea levels due to the effects of the gravity exerted by the moon and the sun, and the rotation of the Earth.

learning objectives

- Explain factors that influence the times and amplitude of the tides at a locale

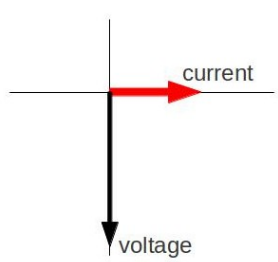
Tides are the rise and fall of sea levels due to the effects of gravitational forces exerted by the moon and the sun when combined with the rotation of the Earth. Tides occur to varying degrees and frequency, depending on location. Shorelines where two almost equally high tides and two low tides occur each day experience a semi- diurnal tide. The occurrence of only one high and one low tide each day is known as diurnal tide. A mixed tide referes to the daily occurrence of two uneven tides, or perhaps one high and one low tide. The times and amplitude of tides at various locales are influenced by the alignment of the sun and moon, the pattern of tides in the deep ocean, the shape of the coastline, and other forces.



Earth's tides.: Schematic of the lunar portion of earth's tides showing (exaggerated) high tides at the sublunar and antipodal points for the hypothetical case of an ocean of constant depth with no land. There would also be smaller, superimposed bulges on the sides facing toward and away from the sun.

Tidal Force

If we want to know the acceleration "felt" by an observer living on Earth due to the moon, a tricky part is that the Earth is not an inertial frame of reference because it is in "free fall" with respect to the moon. Given this, in order to figure out the force observed, we must subtract the acceleration of the (Earth) frame itself. The tidal force produced by the moon on a small particle located on Earth is the vector difference between the gravitational force exerted by the moon on the particle, and the gravitational force that would be exerted if it were located at the Earth's center of mass.



Moon's Gravity on the Earth: Top picture shows the gravity force due to the Moon at different locations F_r on Earth. Bottom picture shows the differential force $F_r - F_{center}$. This is the acceleration "felt" by an observer living on Earth.

As diagramed below, this is equivalent to subtracting the "red" vector from the "black" vectors on the surface of the Earth in the top picture, leading to the "differential" force represented by the bottom picture. Thus, the tidal force depends not on the strength of the lunar gravitational field, but on its gradient (which falls off approximately as the inverse cube of the distance to the originating gravitational body).

On average, the solar gravitational force on the Earth is 179 times stronger than the lunar, but because the sun is on average 389 times farther from the Earth its field gradient is weaker. The solar tidal force is 46% as large as the lunar. More precisely, the lunar tidal acceleration (along the moon-Earth axis, at the Earth's surface) is about $1.1 \cdot 10^{-7}$ g, while the solar tidal acceleration (along the sun-Earth axis, at the Earth's surface) is about $0.52 \cdot 10^{-7}$ g, where g is the gravitational acceleration at the Earth's surface. Venus has the largest effect of the other planets, at 0.000113 times the solar effect.

Tidal Energy

Energy of tides can be extracted by two means: inserting a water turbine into a tidal current, or building ponds that release/admit water through a turbine. In the first case, the energy amount is entirely determined by the timing and tidal current magnitude, but the best currents may be unavailable because the turbines would obstruct ships. In the second case, impoundment dams are expensive to construct, natural water cycles are completely disrupted, as is ship navigation. However, with multiple ponds, power can be generated at chosen times. Presently, there are few installed systems for tidal power generation (most famously, La Rance by Saint Malo, France), as many difficulties are involved. Aside from environmental issues, simply withstanding corrosion and biological fouling pose engineering challenges.

Tidal Energy Generator: Tidal energy generator that works like a wind turbine, but with the ocean currents providing the energy. The circle in the middle is the turbine. The contraption travels up and down the two legs just like a lift and sits on the sea floor when in use.

Unlike with wind power systems, tidal power proponents point out that generation levels can be reliably predicted (save for weather effects). While some generation is possible for most of the tidal cycle, in practice, turbines lose efficiency at lower operating rates. Since the power available from a flow is proportional to the cube of the flow speed, the times during which high power generation is possible are brief.

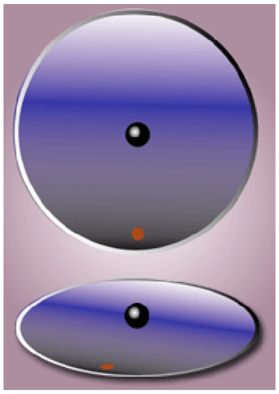
The Coriolois Force

The Coriolis effect is a deflection of moving objects when they are viewed in a rotating reference frame.

learning objectives

- Formulate relationship between the Coriolis force, mass of an object, and speed in the rotating frame

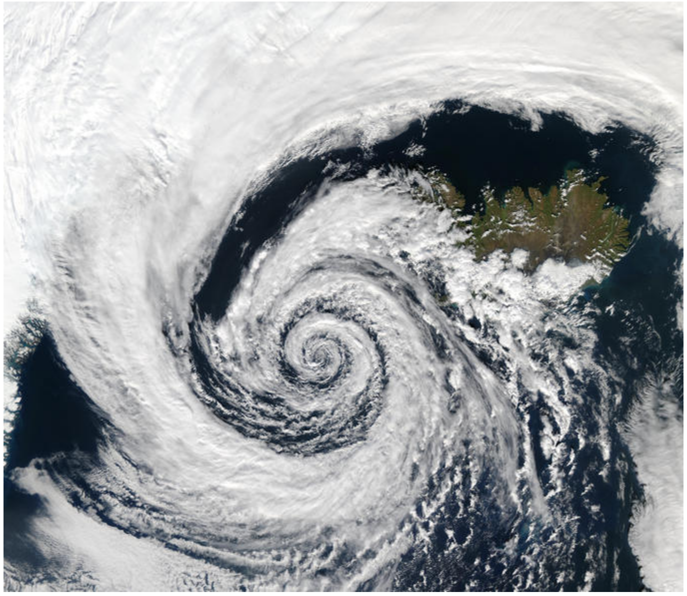
The Coriolis effect is a deflection of moving objects when they are viewed in a rotating reference frame. In a reference frame with clockwise rotation, the deflection is to the left of the motion of the object; in one with counter-clockwise rotation, the deflection is to the right. Although recognized previously by others, the mathematical expression for the Coriolis force appeared in an 1835 paper by French scientist Gaspard-Gustave Coriolis, in connection with the theory of water wheels. Early in the 20th century, the term "Coriolis force" began to be used in connection with meteorology.



Frames of Reference: In the inertial frame of reference (upper part of the picture), the black object moves in a straight line. However, the observer (red dot) who is standing in the rotating/non-inertial frame of reference (lower part of the picture) sees the object as following a curved path due to the Coriolis and centrifugal forces present in this frame.

Newton's laws of motion govern the motion of an object in a (non-accelerating) inertial frame of reference. When Newton's laws are transformed to a uniformly rotating frame of reference, the Coriolis and centrifugal forces appear. Both forces are proportional to the mass of the object. The Coriolis force is proportional to the rotation rate, and the centrifugal force is proportional to its square. The Coriolis force acts in a direction perpendicular to the rotation axis and to the velocity of the body in the rotating frame. It is proportional to the object's speed in the rotating frame. These additional forces are termed inertial forces, fictitious forces, or pseudo-forces. They allow the application of Newton's laws to a rotating system. They are correction factors that do not exist in a non-accelerating or inertial reference frame.

Perhaps the most commonly encountered rotating reference frame is the Earth. The Coriolis effect is caused by the rotation of the Earth and the inertia of the mass experiencing the effect. Because the Earth completes only one rotation per day, the Coriolis force is quite small. Its effects generally become noticeable only for motions occurring over large distances and long periods of time, such as large-scale movements of air in the atmosphere or water in the ocean. Such motions are constrained by the surface of the earth, so generally only the horizontal component of the Coriolis force is important. This force causes moving objects on the surface of the Earth to be deflected in a clockwise sense (with respect to the direction of travel) in the northern hemisphere and in a counter-clockwise sense in the southern hemisphere. Rather than flowing directly from areas of high pressure to low pressure, as they would in a non-rotating system, winds and currents tend to flow to the right of this direction north of the equator and to the left of this direction south of it. This effect is responsible for the rotation of large cyclones.



Coriolis Force: This low-pressure system over Iceland spins counter-clockwise due to balance between the Coriolis force and the pressure gradient force.

Other Geophysical Applications

Tidal and Coriolis forces may not be obvious over a small time-space scale, but they are important in meteorology, navigation, and fishing.

learning objectives

- Identify fields that have to take into account the tidal and Coriolis forces

We have studied tidal and Coriolis forces previously. To review, the tidal force is responsible for the tides — it is a "differential force," due to a secondary effect of the force of gravity. The Coriolis force is a fictitious force, representing a deflection of moving objects when they are viewed in a rotating reference frame of the Earth. Although their effects may not be obvious over a small time-space scale, these forces are important in such contexts as meteorology, navigation, fishing, and others.

The Tides

Tidal flows are important for marine navigation, and significant errors in position occur if they are not accounted for. Tidal heights are also important; for example, many rivers and harbors have a shallow "bar" at the entrance to prevent boats with significant draft from entering at low tide. Until the advent of automated navigation, competence in calculating tidal effects was important to naval officers. The certificate of examination for lieutenants in the Royal Navy once declared that the prospective officer was able to "shift his tides. "

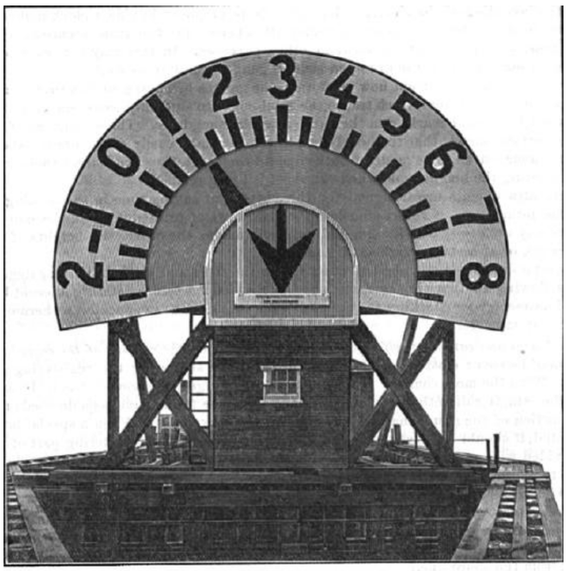
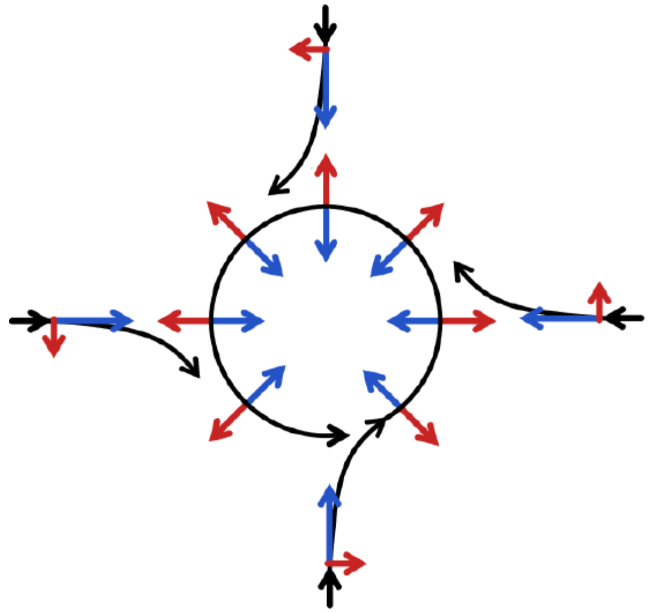


FIG. 5.—Tidal indicator, Delaware River, Delaware.

Tidal Indicator: Tidal Indicator, Delaware River, Delaware c. 1897. In the moment pictured, the tide is 1.25 feet above mean low water and is still falling, as indicated by the pointing of the arrow. The indicator is powered by a system of pulleys, cables, and a float

The Coriolis Force

The Coriolis force is quite small, and its effects generally become noticeable only when we are dealing with motions occurring over large distances and long periods of time, such as large-scale movements of air in the atmosphere or water in the ocean. The Coriolis effects also became important in ballistics calculations — for example, calculating the trajectories of very long-range artillery shells. The most famous historical example is the Paris gun, used by the Germans during World War I to bombard Paris from a range of about 120 km.



Flow Representation: A schematic representation of flow around a low-pressure area in the Northern Hemisphere. The pressure-gradient force is represented by blue arrows and the Coriolis acceleration (always perpendicular to the velocity) by red arrows

Key Points

- The tidal force depends not on the strength of the lunar gravitational field itself, but on its gradient, which falls off approximately as the inverse cube of the distance to the originating gravitational body. This is because the tidal force felt by an observer on Earth is a differential force.
- The times and amplitude of the tides at a locale are influenced by several factors, such as the alignment of the sun and moon, the pattern of tides in the deep ocean, the shape of the coastline, and others forces.
- Energy of tides can be extracted by two means: inserting a water turbine into a tidal current, or building ponds that release/admit water through a turbine.
- When Newton's laws are transformed to a uniformly rotating frame of reference, the Coriolis and centrifugal forces appear.
- The Coriolis force acts in a direction perpendicular to the rotation axis and to the velocity of the body in the rotating frame; it is proportional to the object's mass and speed in the rotating frame.
- The Coriolis effect is caused by the rotation of the Earth and the inertia of the mass experiencing the effect.
- Tidal flows are important for marine navigation, and significant errors in position occur if they are not accounted for.
- The Coriolis force is quite small, and its effects generally become noticeable only when we are dealing with motions occurring over large distances and long periods of time, such as large-scale movements of air in the atmosphere or water in the ocean.
- The tidal force is responsible for the tides. It is a "differential force," due to a secondary effect of the force of gravity. The Coriolis force is a fictitious force, representing a deflection of moving objects when they are viewed in a rotating reference frame of the Earth.

Key Terms

- **inertial frame**: A frame of reference that describes time and space homogeneously, isotropically, and in a time-independent manner.
- **diurnal**: Having a daily cycle that is completed every 24 hours, usually referring to tasks, processes, tides, or sunrise to sunset.
- **gradient**: The rate at which a physical quantity increases or decreases relative to change in a given variable, especially distance.
- **fictitious force**: an apparent force that acts on all masses in a non-inertial frame of reference, such as a rotating reference frame
- **centrifugal force**: the apparent outward force that draws a rotating body away from the center of rotation
- **ballistics**: the science of mechanics that deals with the flight, behavior, and effects of projectiles, especially bullets, gravity bombs, rockets, or the like
- **meteorology**: the interdisciplinary scientific study of the atmosphere

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5.5: Newton’s Law of Universal Gravitation

The Law of Universal Gravitation

Objects with mass feel an attractive force that is proportional to their masses and inversely proportional to the square of the distance.

learning objectives

- Express the Law of Universal Gravitation in mathematical form

While an apple might not have struck Sir Isaac Newton’s head as myth suggests, the falling of one did inspire Newton to one of the great discoveries in mechanics: *The Law of Universal Gravitation*. Pondering why the apple never drops sideways or upwards or any other direction except perpendicular to the ground, Newton realized that the Earth itself must be responsible for the apple’s downward motion.

Theorizing that this force must be proportional to the masses of the two objects involved, and using previous intuition about the inverse-square relationship of the force between the earth and the moon, Newton was able to formulate a general physical law by induction.

The Law of Universal Gravitation states that every point mass attracts every other point mass in the universe by a force pointing in a straight line between the centers-of-mass of both points, and this force is proportional to the masses of the objects and inversely proportional to their separation This attractive force always points inward, from one point to the other. The Law applies to all objects with masses, big or small. Two big objects can be considered as point-like masses, if the distance between them is very large compared to their sizes or if they are spherically symmetric. For these cases the mass of each object can be represented as a point mass located at its center-of-mass.

While Newton was able to articulate his Law of Universal Gravitation and verify it experimentally, he could only calculate the relative gravitational force in comparison to another force. It wasn’t until Henry Cavendish’s verification of the gravitational constant that the Law of Universal Gravitation received its final algebraic form:

$$\mathrm{F} = \mathrm{G}\frac{\mathrm{Mm}}{\mathrm{r}^2} \tag{5.5.1}$$

where F represents the force in Newtons, M and m represent the two masses in kilograms, and r represents the separation in meters. G represents the gravitational constant, which has a value of $6.674 \cdot 10^{-11} \mathrm{N}(\mathrm{m}/\mathrm{kg})^2$. Because of the magnitude of G, gravitational force is very small unless large masses are involved.

Forces on two masses: All masses are attracted to each other. The force is proportional to the masses and inversely proportional to the square of the distance.

Gravitational Attraction of Spherical Bodies: A Uniform Sphere

The Shell Theorem states that a spherically symmetric object affects other objects as if all of its mass were concentrated at its center.

learning objectives

- Formulate the Shell Theorem for spherically symmetric objects

Universal Gravitation for Spherically Symmetric Bodies

The Law of Universal Gravitation states that the gravitational force between two points of mass is proportional to the magnitudes of their masses and the inverse-square of their separation, d:

$$\mathrm{F} = \frac{\mathrm{GmM}}{\mathrm{d}^2} \tag{5.5.2}$$

However, most objects are not point particles. Finding the gravitational force between three-dimensional objects requires treating them as points in space. For highly symmetric shapes such as spheres or spherical shells, finding this point is simple.

The Shell Theorem

Isaac Newton proved the Shell Theorem, which states that:

1. A spherically symmetric object affects other objects gravitationally as if all of its mass were concentrated at its center,
2. If the object is a spherically symmetric shell (i.e., a hollow ball) then the net gravitational force on a body *inside* of it is zero.

Since force is a vector quantity, the vector summation of all parts of the shell/sphere contribute to the net force, and this net force is the equivalent of one force measurement taken from the sphere's midpoint, or center of mass (COM). So when finding the force of gravity exerted on a ball of 10 kg, the distance measured from the ball is taken from the ball's center of mass to the earth's center of mass.

Given that a sphere can be thought of as a collection of infinitesimally thin, concentric, spherical shells (like the layers of an onion), then it can be shown that a corollary of the Shell Theorem is that the force exerted in an object inside of a solid sphere is only dependent on the mass of the sphere inside of the radius at which the object is. That is because shells at a greater radius than the one at which the object is, do *not* contribute a force to an object inside of them (Statement 2 of theorem).

When considering the gravitational force exerted on an object at a point *inside* or *outside* a uniform spherically symmetric object of radius RR, there are two simple and distinct situations that must be examined: the case of a hollow spherical shell, and that of a solid sphere with uniformly distributed mass.

Case 1: A hollow spherical shell

The gravitational force acting by a spherically symmetric shell upon a point mass *inside* it, is the vector sum of gravitational forces acted by each part of the shell, and this vector sum is equal to zero. That is, a mass mm *within* a spherically symmetric shell of mass M, will feel no net force (Statement 2 of Shell Theorem).

The net gravitational force that a spherical shell of mass M exerts on a body *outside* of it, is the vector sum of the gravitational forces acted by each part of the shell on the outside object, which add up to a net force acting as if mass M is concentrated on a point at the center of the sphere (Statement 1 of Shell Theorem).

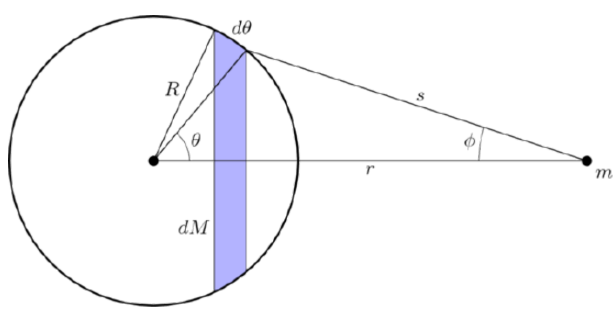


Diagram used in the proof of the Shell Theorem: This diagram outlines the geometry considered when proving The Shell Theorem. In particular, in this case a spherical shell of mass M (left side of figure) exerts a force on mass m (right side of the figure) outside of it. The surface area of a thin slice of the sphere is shown in color. (Note: The proof of the theorem is not presented here. Interested readers can explore further using the sources listed at the bottom of this article.)

Case 2: A solid, uniform sphere

The second situation we will examine is for a solid, uniform sphere of mass M and radius R, exerting a force on a body of mass m at a radius d *inside* of it (that is, $d < R$). We can use the results and corollaries of the Shell Theorem to analyze this case. The contribution of all shells of the sphere at a radius (or distance) greater than dd from the sphere's center-of-mass can be ignored (see above corollary of the Shell Theorem). Only the mass of the sphere within the desired radius $M < d$ (that is the mass of the sphere inside dd) is relevant, and can be considered as a point mass at the center of the sphere. So, the gravitational force acting upon point mass mm is:

$$F = \frac{GmM_{<d}}{d^2} \tag{5.5.3}$$

where it can be shown that $M_{<d} = \frac{4}{3}\pi d^3 \rho$

(ρ is the mass density of the sphere and we are assuming that it does not depend on the radius. That is, the sphere's mass is uniformly distributed.)

Therefore, combining the above two equations we get:

$$F = \frac{4}{3}\pi G m \rho d \tag{5.5.4}$$

which shows that mass mm feels a force that is linearly proportional to its distance, dd, from the sphere's center of mass.

As in the case of hollow spherical shells, the net gravitational force that a solid sphere of uniformly distributed mass M exerts on a body *outside* of it, is the vector sum of the gravitational forces acted by each shell of the sphere on the outside object. The resulting net gravitational force acts as if mass M is concentrated on a point at the center of the sphere, which is the center of mass, or COM (Statement 1 of Shell Theorem). More generally, this result is true even if the mass M is *not* uniformly distributed, but its density varies radially (as is the case for planets).

Weight of the Earth

When the bodies have spatial extent, gravitational force is calculated by summing the contributions of point masses which constitute them.

learning objectives

- Describe how gravitational force is calculated for the bodies with spatial extent

Newton's law of universal gravitation states that every point mass in the universe attracts every other point mass with a force that is directly proportional to the product of their masses, and inversely proportional to the square of the distance between them.

In modern language, the law states the following: *Every point mass attracts every single other point mass by a force pointing along the line intersecting both points*. The force is proportional to the product of the two masses and inversely proportional to the square of the distance between them:

$$F = G\frac{m_1 m_2}{r^2} \tag{5.5.5}$$

where F is the force between the masses, G is the gravitational constant, m_1 is the first mass, m_2 is the second mass and r is the distance between the centers of the masses.

If the bodies in question have spatial extent (rather than being theoretical point masses), then the gravitational force between them is calculated by summing the contributions of the notional point masses which constitute the bodies. In the limit, as the component point masses become "infinitely small", this entails integrating the force (in vector form, see below) over the extents of the two bodies.

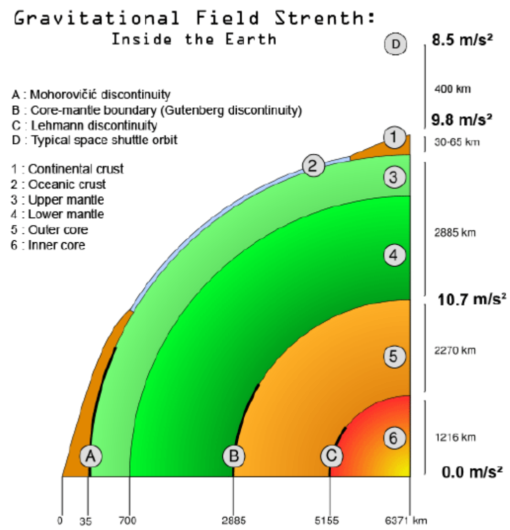
In this way it can be shown that an object with a spherically-symmetric distribution of mass exerts the same gravitational attraction on external bodies as if all the object's mass were concentrated at a point at its center.

For points inside a spherically-symmetric distribution of matter, Newton's Shell theorem can be used to find the gravitational force. The theorem tells us how different parts of the mass distribution affect the gravitational force measured at a point located a distance r_0 from the center of the mass distribution:

1. The portion of the mass that is located at radii $r < r_0$ causes the same force at r_0 as if all of the mass enclosed within a sphere of radius r_0 was concentrated at the center of the mass distribution (as noted above).
2. The portion of the mass that is located at radii $r > r_0$ exerts no net gravitational force at the distance r_0 from the center. That is, the individual gravitational forces exerted by the elements of the sphere out there, on the point at r_0 , cancel each other out.

As a consequence, for example, within a shell of uniform thickness and density there is no net gravitational acceleration anywhere within the hollow sphere. Furthermore, inside a uniform sphere the gravity increases linearly with the distance from the center; the increase due to the additional mass is 1.5 times the decrease due to the larger distance from the center. Thus, if a spherically symmetric body has a uniform core and a uniform mantle with a density that is less than $\frac{2}{3}$ of that of the core, then the gravity initially decreases outwardly beyond the boundary, and if the sphere is large enough, further outward the gravity increases again, and eventually it exceeds the gravity at the core/mantle boundary.

The gravity of the Earth may be highest at the core/mantle boundary, as shown in Figure 1:



Gravitational Field of Earth: Diagram of the gravitational field strength within the Earth.

Key Points

- Sir Isaac Newton's inspiration for the Law of Universal Gravitation was from the dropping of an apple from a tree.
- Newton's insight on the inverse-square property of gravitational force was from intuition about the motion of the earth and the moon.
- The mathematical formula for gravitational force is $\mathrm{F}=\mathrm{G}\frac{\mathrm{Mm}}{\mathrm{r}^2}$ where G is the gravitational constant.
- Since force is a vector quantity, the vector summation of all parts of the shell contribute to the net force, and this net force is the equivalent of one force measurement taken from the sphere's midpoint, or center of mass (COM).
- The gravitational force on an object within a hollow spherical shell is zero.
- The gravitational force on an object within a uniform spherical mass is linearly proportional to its distance from the sphere's center of mass (COM).
- Newton's law of universal gravitation states that every point mass in the universe attracts every other point mass with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.
- The second step in calculating earth's mass came with the development of Newton's law of universal gravitation.
- By equating Newton's second law with his law of universal gravitation, and inputting for the acceleration a the experimentally verified value of 9.8 $\frac{\mathrm{m}}{\mathrm{s}^2}$, the mass of earth is calculated to be $5.96\times10^{24}\mathrm{kg}$, making the earth's weight calculable given any gravitational field.
- The gravity of the Earth may be highest at the core/mantle boundary

Key Terms

- **induction**: Use inductive reasoning to generalize and interpret results from applying Newton's Law of Gravitation.
- **inverse**: Opposite in effect or nature or order.
- **center of mass**: The center of mass (COM) is the unique point at the center of a distribution of mass in space that has the property that the weighted position vectors relative to this point sum to zero.
- **point mass**: A theoretical point with mass assigned to it.
- **weight**: The force on an object due to the gravitational attraction between it and the Earth (or whatever astronomical object it is primarily influenced by).
- **gravitational force**: A very long-range, but relatively weak fundamental force of attraction that acts between all particles that have mass; believed to be mediated by gravitons.

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5.6: Kepler’s Laws

Kepler’s First Law

Kepler’s first law is: *The orbit of every planet is an ellipse with the Sun at one of the two foci*.

learning objectives

- Apply Kepler’s first law to describe planetary motion

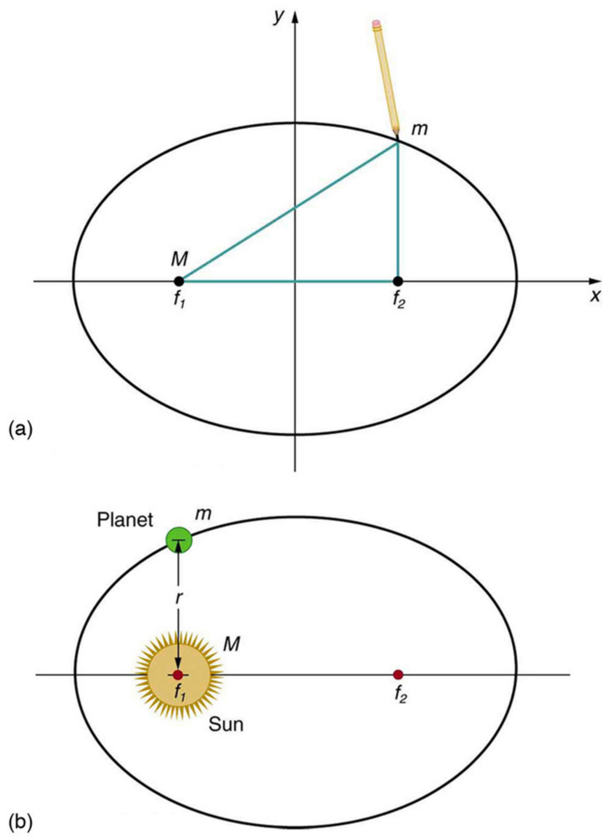
Kepler’s First Law

Kepler’s first law states that

Definition

The orbit of every planet is an ellipse with the Sun at one of the two foci.

An ellipse is a closed plane curve that resembles a stretched out circle. Note that the Sun is not at the center of the ellipse, but at one of its foci. The other focal point, f_2, has no physical significance for the orbit. The center of an ellipse is the midpoint of the line segment joining its focal points. A circle is a special case of an ellipse where both focal points coincide.



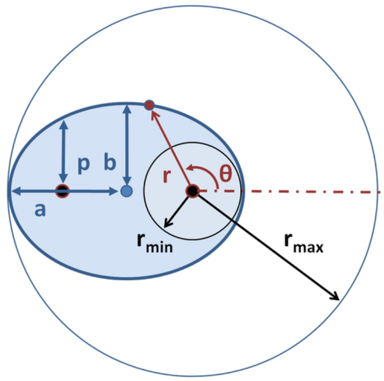
Ellipses and Kepler’s First Law: (a) An ellipse is a closed curve such that the sum of the distances from a point on the curve to the two foci (f_1 and f_2) is a constant. You can draw an ellipse as shown by putting a pin at each focus, and then placing a string around a pencil and the pins and tracing a line on paper. A circle is a special case of an ellipse in which the two foci coincide (thus any point on the circle is the same distance from the center). (b) For any closed gravitational orbit, m follows an elliptical path with M at one focus. Kepler’s first law states this fact for planets orbiting the Sun.

How stretched out an ellipse is from a perfect circle is known as its eccentricity: a parameter that can take any value greater than or equal to 0 (a circle) and less than 1 (as the eccentricity tends to 1, the ellipse tends to a parabola). The eccentricities of the planets known to Kepler varied from 0.007 (Venus) to 0.2 (Mercury). Minor bodies such as comets an asteroids (discovered after Kepler's time) can have very large eccentricities. The dwarf planet Pluto, discovered in 1929, has an eccentricity of 0.25.

Symbolically, an ellipse can be represented in polar coordinates as:

$$r = \frac{p}{1+\epsilon\cos\theta} \tag{5.6.1}$$

where (r, θ) are the polar coordinates (from the focus) for the ellipse, p is the semi-latus rectum, and ϵ is the eccentricity of the ellipse. For a planet orbiting the Sun, r is the distance from the Sun to the planet and θ is the angle between the planet's current position and its closest approach, with the Sun as the vertex.



Orbit As Ellipse: Heliocentric coordinate system (r, θ) for ellipse. Also shown are: semi-major axis a, semi-minor axis b and semi-latus rectum p; center of ellipse and its two foci marked by large dots. For $\theta = 0^\circ$, $r = r_{min}$ and for $\theta = 180^\circ$, $r = r_{max}$.

At $\theta = 0^\circ$, perihelion, the distance is minimum

$$r_{min} = \frac{p}{1+\epsilon}. \tag{5.6.2}$$

At $\theta = 90^\circ$ and at $\theta = 270^\circ$, the distance is p.

At $\theta = 180^\circ$, aphelion, the distance is maximum

$$r_{max} = \frac{p}{1-\epsilon}. \tag{5.6.3}$$

The semi-major axis a is the arithmetic mean between r_{min} and r_{max}:

$$r_{max} - a = a - r_{min} \tag{5.6.4}$$

$$a = \frac{p}{1-\epsilon^2}. \tag{5.6.5}$$

The semi-minor axis b is the geometric mean between r_{min} and r_{max}:

$$\frac{r_{max}}{b} = \frac{b}{r_{min}} \tag{5.6.6}$$

$$b = \frac{p}{\sqrt{1-\epsilon^2}} \tag{5.6.7}$$

The semi-latus rectum p is the harmonic mean between r_{min} and r_{max}:

$$\frac{1}{r_{min}} - \frac{1}{p} = \frac{1}{p} - \frac{1}{r_{max}} \tag{5.6.8}$$

$$pa = r_{max} \cdot r_{min} = b^2. \tag{5.6.9}$$

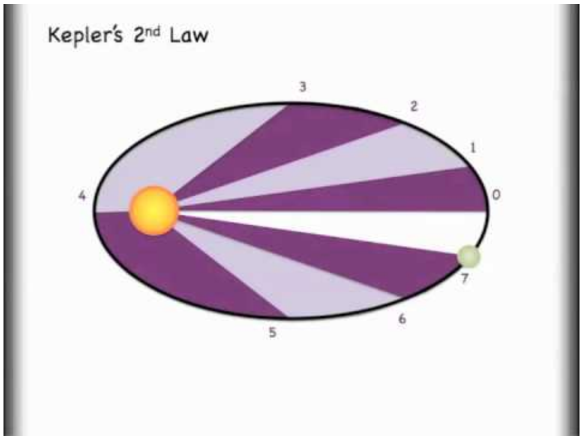
The eccentricity ϵ is the coefficient of variation between r_{min} and r_{max} :

$$\epsilon = \frac{r_{max} - r_{min}}{r_{max} + r_{min}} \tag{5.6.10}$$

The area of the ellipse is

$$A = \pi ab \tag{5.6.11}$$

The special case of a circle is $\epsilon = 0$, resulting in $r = p = r_{min} = r_{max} = a = b$ and $A = \pi r^2$. The orbits of planets with very small eccentricities can be approximated as circles.



Understanding Kepler's 3 Laws and Orbits: In this video you will be introduced to Kepler's 3 laws and see how they are relevant to orbiting objects.

Kepler's Second Law

Kepler's second law states: A line joining a planet and the Sun sweeps out equal areas during equal intervals of time.

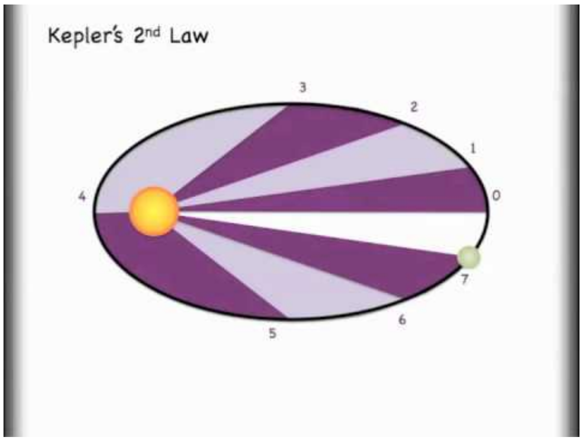
learning objectives

- Apply Kepler's second law to describe planetary motion

Kepler's second law states:

Definition

A line joining a planet and the Sun sweeps out equal areas during equal intervals of time.



Understanding Kepler's 3 Laws and Orbits: In this video you will be introduced to Kepler's 3 laws and see how they are relevant to orbiting objects.

In a small time the planet sweeps out a small triangle having base line and height. The area of this triangle is given by:

$$\mathrm{dA}=\frac{1}{2}\cdot \mathrm{r}\cdot \mathrm{r}d\theta \tag{5.6.12}$$

and so the constant areal velocity is:

$$\frac{\mathrm{dA}}{\mathrm{dt}}=\frac{1}{2}\mathrm{r}^2\frac{\mathrm{d}\theta}{\mathrm{dt}} \tag{5.6.13}$$

Now as the first law states that the planet follows an ellipse, the planet is at different distances from the Sun at different parts in its orbit. So the planet has to move faster when it is closer to the Sun so that it sweeps equal areas in equal times.

The total area enclosed by the elliptical orbit is:

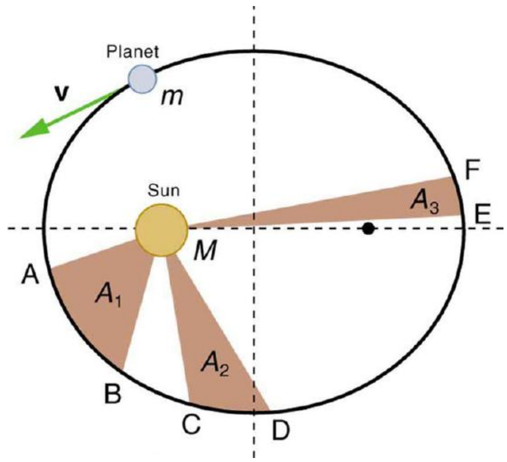
$$A = \pi ab \tag{5.6.14}$$

Therefore the period P satisfies:

$$\pi ab = P \cdot \frac{1}{2} r^2 \dot{\theta} \text{ or } r^2 \dot{\theta} = nab \tag{5.6.15}$$

Where $\dot{\theta} = \frac{d\theta}{dt}$ is the angular velocity, (using Newton notation for differentiation), and $n = \frac{2\pi}{P}$ is the mean motion of the planet around the Sun.

See below for an illustration of this effect. The planet traverses the distance between A and B, C and D, and E and F in equal times. When the planet is close to the Sun it has a larger velocity, making the base of the triangle larger, but the height of the triangle smaller, than when the planet is far from the Sun. One can see that the planet will travel fastest at perihelion and slowest at aphelion.



Kepler's Second Law: The shaded regions have equal areas. It takes equal times for m to go from A to B, from C to D, and from E to F. The mass m moves fastest when it is closest to M. Kepler's second law was originally devised for planets orbiting the Sun, but it has broader validity.

Kepler's Third Law

Kepler's third law states that *the square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit*.

learning objectives

- Apply Kepler's third law to describe planetary motion

Kepler's Third Law

Kepler's third law states:

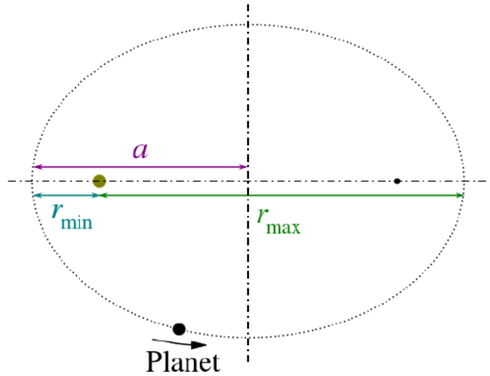
Definition

The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit.

The third law, published by Kepler in 1619, captures the relationship between the distance of planets from the Sun, and their orbital periods. Symbolically, the law can be expressed as

$$P^2 \propto a^3, \tag{5.6.16}$$

where P is the orbital period of the planet and a is the semi-major axis of the orbit (see).



Kepler's Third Law: Kepler's third law states that the square of the period of the orbit of a planet about the Sun is proportional to the cube of the semi-major axis of the orbit.

The constant of proportionality is

$$\frac{P^2_{planet}}{a^3_{planet}} = \frac{P^2_{earth}}{a^3_{earth}} = 1\,\frac{yr^2}{AU^3} \tag{5.6.17}$$

for a sidereal year (yr), and astronomical unit (AU).

Kepler enunciated this third law in a laborious attempt to determine what he viewed as the "music of the spheres" according to precise laws, and express it in terms of musical notation. Therefore, it used to be known as the harmonic law.

Derivation of Kepler's Third Law

We can derive Kepler's third law by starting with Newton's laws of motion and the universal law of gravitation. We can therefore demonstrate that the force of gravity is the cause of Kepler's laws.

Consider a circular orbit of a small mass m around a large mass M. Gravity supplies the centripetal force to mass m. Starting with Newton's second law applied to circular motion,

$$F_{net} = ma_c = m\frac{v^2}{r}. \tag{5.6.18}$$

The net external force on mass m is gravity, and so we substitute the force of gravity for F_{net}:

$$G\frac{mM}{r^2} = m\frac{v^2}{r} \tag{5.6.19}$$

The mass m cancels, as well as an r, yielding

$$G\frac{M}{r} = v^2 \tag{5.6.20}$$

The fact that m cancels out is another aspect of the oft-noted fact that at a given location all masses fall with the same acceleration. Here we see that at a given orbital radius r, all masses orbit at the same speed. This was implied by the result of the preceding worked example. Now, to get at Kepler's third law, we must get the period P into the equation. By definition, period P is the time for one complete orbit. Now the average speed v is the circumference divided by the period—that is,

$$v = \frac{2\pi r}{P}. \tag{5.6.21}$$

Substituting this into the previous equation gives

$$G\frac{M}{r} = \frac{4\pi^2 r^2}{P^2} \tag{5.6.22}$$

Solving for P^2 yields

$$P^2 = \frac{4\pi^2 r^3}{GM}. \tag{5.6.23}$$

Using subscripts 1 and 2 to denote two different satellites, and taking the ratio of the last equation for satellite 1 to satellite 2 yields

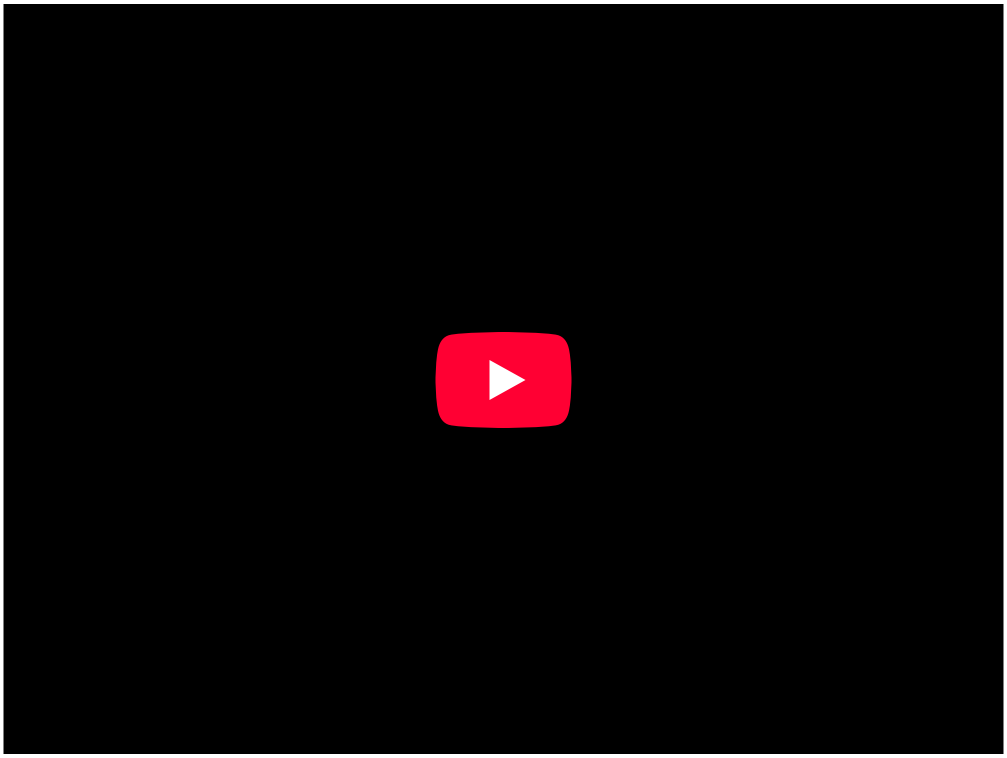
$$\frac{P_1^2}{P_2^2} = \frac{r_1^3}{r_2^3} \tag{5.6.24}$$

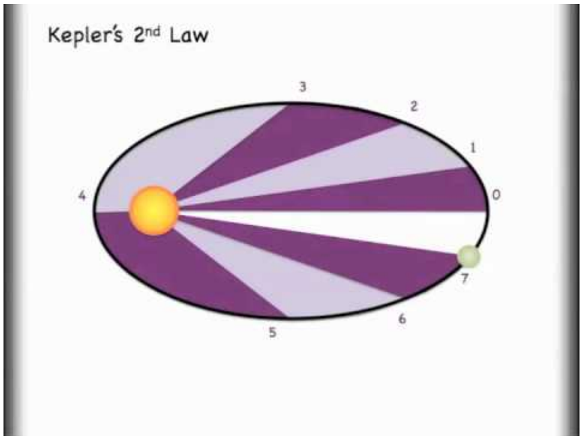
This is Kepler's third law. Note that Kepler's third law is valid only for comparing satellites of the same parent body, because only then does the mass of the parent body M cancel.

Now consider what one would get when solving $P^2 = \frac{4\pi^2 GM}{r^3}$ for the ratio $\frac{r^3}{P^2}$. We obtain a relationship that can be used to determine the mass M of a parent body from the orbits of its satellites:

$$M = \frac{4\pi^2 r^3}{GP^2} \tag{5.6.25}$$

If r and P are known for a satellite, then the mass M of the parent can be calculated. This principle has been used extensively to find the masses of heavenly bodies that have satellites. Furthermore, the ratio $\frac{r^3}{T^2}$ should be a constant for all satellites of the same parent body (because $\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$).





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Orbital Maneuvers

An orbital maneuver is the use of propulsion systems to change the orbit of a spacecraft (the rest of the flight is called "coasting").

learning objectives

- Explain purpose of an orbital maneuver

Orbital Maneuvers

In spaceflight, an orbital maneuver is the use of propulsion systems to change the orbit of a spacecraft. The rest of the flight, especially in a transfer orbit, is called coasting.

Rocket Equation

The Tsiolkovsky rocket equation or *ideal rocket equation* is an equation useful for considering vehicles that follow the basic principle of a rocket: a device that can apply acceleration to itself (a thrust) by expelling part of its mass with high speed and moving due to the conservation of momentum. Specifically, it is a mathematical equation relating the delta-v with the effective exhaust velocity and both the initial and final mass of a rocket (or other reaction engine).

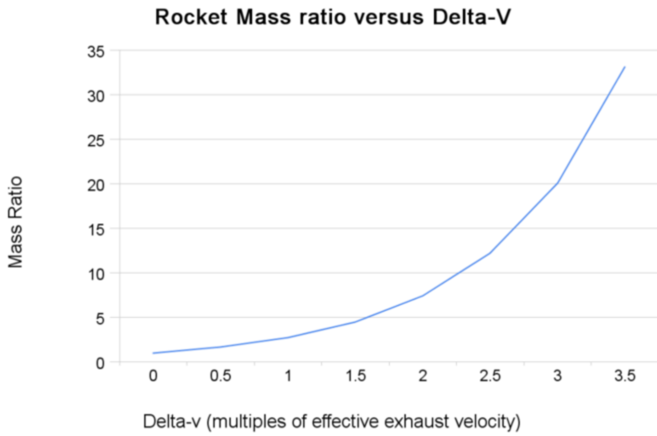
For any such maneuver (or journey involving a number of such maneuvers):

$$\Delta v = v_e \ln(\frac{m_0}{m_1}), \tag{5.6.26}$$

where:

- m_0 is the initial total mass, including propellant;
- m_1 is the final total mass;
- v_e is the effective exhaust velocity ($v_e = I_{sp} \cdot g_0$ where I_{sp} is the specific impulse expressed as a time period and g_0 is the gravitational constant); and
- Δv is delta-v the maximum change of speed of the vehicle (with no external forces acting).

See for an illustration plotting the relationship between final velocity and rocket mass ratios (according to the rocket equation).



Rocket Equation: Rocket mass ratios versus final velocity calculated from the rocket equation

Delta-v Budget:

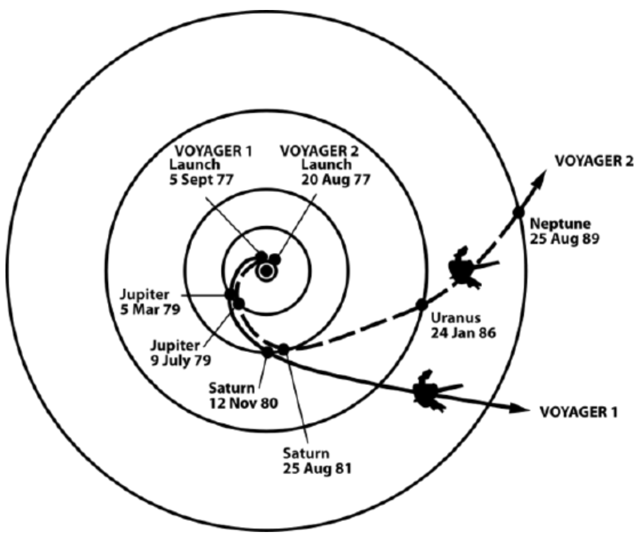
The total delta-v for each maneuver estimated for a mission is called a *delta-v budget*. With a good approximation of the delta-v budget, designers can estimate the fuel to payload requirements of the spacecraft using the rocket equation.

Oberth Effect and Gravitational Assist

In astronautics, the Oberth effect occurs when the use of a rocket engine travelling at high speed generates much more useful energy than one at low speed. This effect is the result of propellant having more usable energy (due to its kinetic energy on top of its chemical potential energy). The vehicle is able to employ this kinetic energy to generate more mechanical power.

Oberth effect is used in a powered flyby or Oberth maneuver in which the application of an impulse (typically from the use of a rocket engine) close to a gravitational body (where the gravity potential is low and the speed is high) allows for more change in kinetic energy and final speed (i.e. higher specific energy) than the same impulse applied further from the body for the same initial orbit.

In orbital mechanics, a gravitational slingshot (or gravity assist maneuver) is the use of the relative movement and gravity of a planet or other celestial body to alter the path and speed of a spacecraft, typically in an effort to save propellant, time, and expense. Gravity assistance can be used to accelerate, decelerate and/or re-direct the path of a spacecraft. This technique was used by the Voyager probes in their fly-bys of Jupiter and Saturn (see).

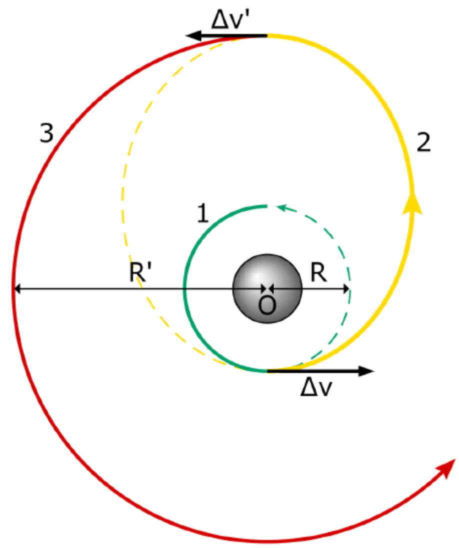


Voyager Path Using Gravity Assists: The trajectories that enabled NASA’s twin Voyager spacecraft to tour the four gas giant planets and achieve velocity to escape our solar system

Transfer Orbits

Orbit insertion is a general term used for a maneuver when it is more than a small correction. It may be used in a maneuver to change a transfer orbit or an ascent orbit into a stable one, but also to change a stable orbit into a descent (i.e., descent orbit insertion). Also, the term *orbit injection* is used, especially for changing a stable orbit into a transfer orbit—e.g., trans-lunar injection (TLI), trans-Mars injection (TMI) and trans-Earth injection (TEI).

The Hohmann transfer orbit is an elliptical orbit used to transfer between two circular orbits of different altitudes in the same plane. The orbital maneuver to perform the Hohmann transfer uses two engine impulses that move aspacecraft onto and off the transfer orbit, as diagramed in. Hohmann transfer orbits are the most efficient with fuel. Other non-Hohmann types of transfer orbits that are less efficient with fuel exist, but these may be more efficient with other resources (such as time).



Hohmann Transfer Orbit: A diagram of the Hohmann Transfer Orbit.

Orbital inclination change is an orbital maneuver aimed at changing an orbiting body's orbit inclination (this maneuver is also known as an orbital plane change as the plane of the orbit is tipped). The maneuver requires a change in the orbital velocity vector (delta-v) at the orbital nodes (i.e., the point at which the initial and desired orbits intersect: the line of orbital nodes is defined by the intersection of the two orbital planes).

Satellites

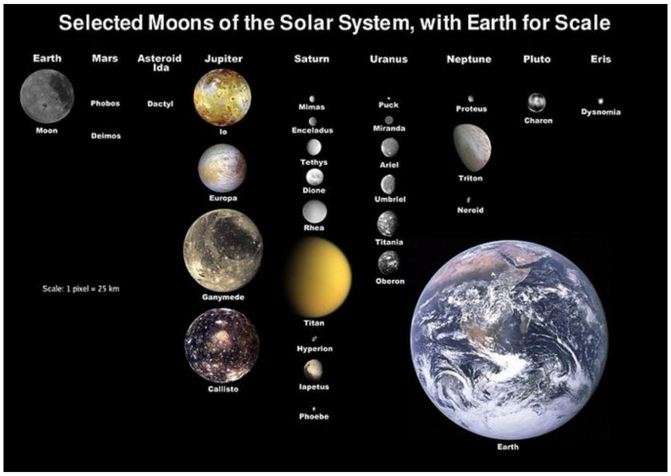
Natural satellites are celestial objects that orbit a larger body; artificial satellites are manmade objects put in the orbit of the Earth.

learning objectives

- Define the concept of a satellite, in the broadest possible terms

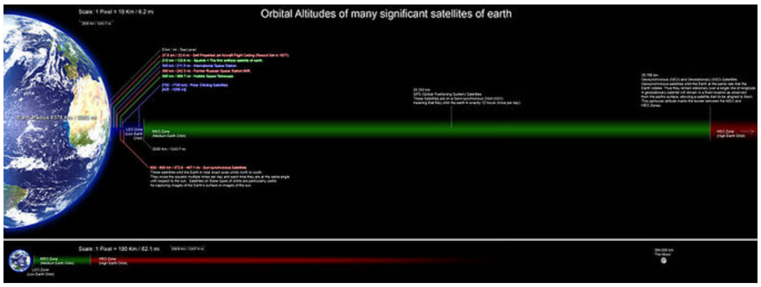
Satellites

The word "satellite" has a somewhat ambiguous definition. The broadest possible definition of a satellite is an object that orbits a larger one due to the force of gravity. Natural satellites, often called moons (see), are celestial bodies that orbit a larger body call a *primary* (often planet, though there are binary asteroids, too). It is technically correct to refer to a planet as a "satellite" of its parent star, though this is not common.



Moons of the Solar System: Nineteen natural satellites are large enough to be round, and one, Saturn's Titan, has a substantial atmosphere.

Artificial satellites (see) are man made objects put in orbit about the Earth or another planet in the Solar System. All satellites follow the laws of orbital mechanics, which can almost always be approximated with Newtonian physics.



Orbital Altitudes: Orbital Altitudes of several significant satellites of earth.

Natural satellites are often classified in terms of their size and composition, while artificial satellites are categorized in terms of their orbital parameters.

Natural Satellites

Formally classified natural satellites, or moons, include 176 planetary satellites orbiting six of the eight planets, and eight orbiting three of the five IAU-listed dwarf planets. As of January 2012, over 200 minor-planet moons have been discovered. There are 76 objects in the asteroid belt with satellites (five with two satellites each), four Jupiter trojans, 39 near-Earth objects, and 14 Mars-crossers. There are also 84 known natural satellites of trans-Neptunian objects. Planets around other stars are likely to have natural satellites as well, although none have yet been observed.

Of the inner planets, Mercury and Venus have no natural satellites; Earth has one large natural satellite, known as the Moon; and Mars has two tiny natural satellites, Phobos and Deimos. The large gas giants have extensive systems of natural satellites, including half a dozen comparable in size to Earth's Moon: the four Galilean moons, Saturn's Titan, and Neptune's Triton. Saturn has an additional six mid-sized natural satellites massive enough to have achieved hydrostatic equilibrium, and Uranus has five. It has been suggested that some satellites may potentially harbor life, though there is currently no direct evidence.

The Earth–Moon system is unique in that the ratio of the mass of the Moon to the mass of the Earth is much greater than that of any other natural satellite to planet ratio in the Solar System. Additionally the Moon's orbit with respect to the Sun is always concave.

The seven largest natural satellites in the Solar System (those bigger than 2,500 km across) are Jupiter's Galilean moons (Ganymede, Callisto, Io, and Europa), Saturn's moon Titan, Earth's moon, and Neptune's captured natural satellite Triton.

Artificial Satellites

The first satellite, Sputnik 1, was put into orbit around Earth and was therefore in geocentric orbit. By far this is the most common type of orbit with about 2,500 artificial satellites orbiting the Earth. Geocentric orbits may be further classified by their altitude, inclination and eccentricity.

The commonly used altitude classifications are Low Earth orbit (LEO), Medium Earth orbit (MEO) and High Earth orbit (HEO). Low Earth orbit is any orbit below 2000 km, and Medium Earth orbit is any orbit higher than that but still below the altitude for geosynchronous orbit at 35,786 km. High Earth orbit is any orbit higher than the altitude for geosynchronous orbit.

Altitude classifications

- Low Earth orbit (LEO): Geocentric orbits ranging in altitude from 0–2000 km (0–1240 miles)
- Medium Earth orbit (MEO): Geocentric orbits ranging in altitude from 2,000 km (1,200 mi) to just below geosynchronous orbit at 35,786 km (22,236 mi). Also known as an intermediate circular orbit.
- High Earth orbit (HEO): Geocentric orbits above the altitude of geosynchronous orbit 35,786 km (22,236 mi).

Inclination Classifications

- Inclined orbit: An orbit whose inclination in reference to the equatorial plane is not zero degrees.
- Polar orbit: An orbit that passes above or nearly above both poles of the planet on each revolution. Therefore it has an inclination of (or very close to) 90 degrees.
- Polar sun synchronous orbit: A nearly polar orbit that passes the equator at the same local time on every pass. Useful for image taking satellites because shadows will be nearly the same on every pass.

Eccentricity Classifications

- Circular orbit: An orbit that has an eccentricity of 0 and whose path traces a circle.
- Hohmann transfer orbit: An orbital maneuver that moves a spacecraft from one circular orbit to another using two engine impulses.
- Elliptic orbit: An orbit with an eccentricity greater than 0 and less than 1 whose orbit traces the path of an ellipse.
- Geosynchronous transfer orbit: An elliptic orbit where the perigee is at the altitude of a Low Earth orbit (LEO) and the apogee at the altitude of a geosynchronous orbit.
- Geostationary transfer orbit: An elliptic orbit where the perigee is at the altitude of a Low Earth orbit (LEO) and the apogee at the altitude of a geostationary orbit.

Key Points

- An ellipse is a closed plane curve that resembles a stretched out circle (The Sun is at one focus while the other focus has no physical significance. A circle is a special case of an ellipse where both focal points coincide.
- How stretched out an ellipse is from a perfect circle is known as its eccentricity: a parameter that can take any value greater than or equal to 0 (a circle) and less than 1 (as the eccentricity tends to 1, the ellipse tends to a parabola).
- Symbolically, an ellipse can be represented in polar coordinates as: $r = \frac{p}{1+\epsilon\cos\theta}$, where (r,θ) are the polar coordinates (from the focus) for the ellipse, p is the semi-latus rectum, and ϵ is the eccentricity of the ellipse.
- Perihelion is minimum distance from the Sun a planet achieves in its orbit and is given by$r_{max} = \frac{p}{1-\epsilon}$. Aphelion is the largest distance from the Sun a planet reaches in his orbit and is given by $r_{max} = \dfrac{p}{1-\epsilon}$.
- In a small time the planet sweeps out a small triangle having base line and height. The area of this triangle is given by $dA = \frac{1}{2}\cdot r\cdot rd\theta$ and so the constant areal velocity is: $\frac{dA}{dt} = \dfrac{1}{2}r^2\dfrac{d\theta}{dt}$
- The period P satisfies: $\pi ab = P\cdot\frac{1}{2}r^2\dot{\theta}$. One can see that the product of r^2 and must be constant, so that when the planet is further from the Sun it travels at a slower rate and vise versa.
- A planet travels fastest at perihelion and slowest at aphelion.
- Kepler’s third law can be represented symbolically as $P^2 \propto a^3$, where P is the orbital period of the planet and a is the semi-major axis of the orbit (see.
- The constant of proportionality is $\dfrac{P^2_{planet}}{a^3_{planet}} = \frac{P^2_{earth}}{a^3_{earth}} = 1\frac{yr^2}{AU^3}$ for a sidereal year (yr), and astronomical unit (AU).
- Kepler’s third law can be derived from Newton’s laws of motion and the universal law of gravitation. Set the force of gravity equal to the centripetal force. After substituting an expression for the velocity of the planet, one can obtain: $G\frac{M}{r} = \frac{4\pi^2}{P^2}$ which can also be written $P^2 = \frac{4\pi^2 a^3}{GM}$.
- Using the expression above we can obtain the mass of the parent body from the orbits of its satellites: $M = \frac{4\pi^2 r^3}{GP^2}$

- The ideal rocket equation related the maximum change in velocity attainable by a rocket (delta-v or Δv) as a function of the exhaust velocity (v_e) and the ratio between the mass of the rocket with and without the propellant (m_0/m_1). The equation is given by $\Delta v = v_e \ln(\frac{m_0}{m_1})$.
- The Oberth effect: where the use of a rocket engine travelling at high speed generates more useful energy than one at low speed. Thus it is more efficient to apply thrust when the spacecraft is nearest to the planet (periastron).
- A gravity assist maneuver is the use of the relative movement and gravity of a planet (or other celestial body) to alter the velocity of a spacecraft—typically in order to save propellant, time, and expense. This technique was employed by the Voyager probes (see.
- The Hohmann transfer orbit is an elliptical orbit used to transfer between two circular orbits of different altitudes, in the same plane. The orbital maneuver to perform the Hohmann transfer uses two engine impulses which move aspacecraft onto and off the transfer orbit. See.
- The broadest possible definition of a satellite is an object that orbits a larger one due to the force of gravity.
- All satellites follow the laws of orbital mechanics, which can almost always be approximated with Newtonian physics.
- Natural satellites are often classified in terms of their size and composition, while artificial satellites are categorized in terms of their orbital parameters.
- Artificial Earth-orbiting satellites have orbits categorized by their altitudes, inclinations, and eccentricities.

Key Terms

- **eccentricity**: The coefficient of variation between r_{min} and r_{max}: $\epsilon = \frac{r_{max} - r_{min}}{r_{max} + r_{min}}$ The further appart the foci are, the stronger the eccentricity.
- **perihelion**: The point in the elliptical orbit of a planet or comet etc. where it is nearest to the Sun. The point farthest from the Sun is called aphelion.
- **semi-latus rectum**: The latus rectum is a chord perpendicular to the major axis and passing through the focus. The semi-latus rectum is half the latus rectrum. See distance p in.
- **angular velocity**: A vector quantity describing an object in circular motion; its magnitude is equal to the speed of the particle and the direction is perpendicular to the plane of its circular motion.
- **mean motion**: An angle of 2π (radians) divided by the orbital period (of a celestial body in an elliptic orbit).
- **astronomical unit**: The mean distance from the Earth to the Sun (the semi-major axis of Earth's orbit), approximately 149,600,000 kilometres (symbol AU), used to measure distances in the solar system.
- **sidereal year**: The orbital period of the Earth; a measure of the time it takes for the Sun to return to the same position with respect to the stars of the celestial sphere. A sidereal year is about 20.4 minutes longer than the tropical year due to precession of the equinoxes.
- **Hohmann transfer orbit**: The Hohmann transfer orbit is an elliptical orbit used to transfer between two circular orbits of different altitudes, in the same plane. The orbital maneuver to perform the Hohmann transfer uses two engine impulses, one to move a spacecraft onto the transfer orbit and a second to move off it.
- **delta-v**: The maximum change in the scalar speed of a rocket if the rocket were operated in a vacuum away from external forces (i.e., if no other external forces act).
- **natural satellite**: A natural satellite, moon, or secondary planet is a celestial body that orbits a planet or smaller body, which is called its primary.
- **artificial satellite**: In the context of spaceflight, a satellite is an object which has been placed into orbit by human endeavour.

5.7: Gravitational Potential Energy

Defining Graviational Potential Energy

Gravitational energy is the potential energy associated with gravitational force, such as elevating objects against the Earth's gravity.

learning objectives

- Express gravitational potential energy for two masses

Gravitational energy is the potential energy associated with gravitational force, as work is required to elevate objects against Earth's gravity. The potential energy due to elevated positions is called gravitational potential energy, and is evidenced by water in an elevated reservoir or kept behind a dam. If an object falls from one point to another point inside a gravitational field, the force of gravity will do positive work on the object, and the gravitational potential energy will decrease by the same amount.

Consider a book placed on top of a table. As the book is raised from the floor to the table, some external force works against the gravitational force. If the book falls back to the floor, the "falling" energy the book receives is provided by the gravitational force. Thus, if the book falls off the table, this potential energy goes to accelerate the mass of the book and is converted into kinetic energy. When the book hits the floor, this kinetic energy is converted into heat and sound by the impact.

The factors that affect an object's gravitational potential energy are its height relative to some reference point, its mass, and the strength of the gravitational field it is in. Thus, a book lying on a table has less gravitational potential energy than the same book on top of a taller cupboard, and less gravitational potential energy than a heavier book lying on the same table. An object at a certain height above the Moon's surface has less gravitational potential energy than at the same height above the Earth's surface because the Moon's gravity is weaker. Note that "height" in the common sense of the term cannot be used for gravitational potential energy calculations when gravity is not assumed to be a constant. The following sections provide more detail.

Local Approximation

The strength of a gravitational field varies with location. However, when the change of distance is small in relation to the distances from the center of the source of the gravitational field, this variation in field strength is negligible and we can assume that the force of gravity on a particular object is constant. Near the surface of the Earth, for example, we assume that the acceleration due to gravity is a constant $g = 9.8 m/s^2$ ("standard gravity"). In this case, a simple expression for gravitational potential energy can be derived using the $W = Fd$ equation for work. The upward force required while moving at a constant velocity is equal to the weight, mg, of an object, so the work done in lifting it through a height h is the product mgh. Thus, when accounting only for mass, gravity, and altitude, the equation is:

$$U = mgh \tag{5.7.1}$$

where U is the potential energy of the object relative to its being on the Earth's surface, m is the mass of the object, g is the acceleration due to gravity, and h is the altitude of the object. If m is expressed in kilograms, g in m/s^2 and h in meters then U will be calculated in joules. In most situations, the change in potential energy is the relevant quantity:

$$\Delta U = mg\Delta h \tag{5.7.2}$$

General Formula

However, over large variations in distance, the approximation that gg is constant is no longer valid, and we have to use calculus and the general mathematical definition of work to determine gravitational potential energy. For the computation of the potential energy, we can integrate the gravitational force, whose magnitude is given by Newton's law of gravitation, with respect to the distance r between the two bodies. Using that definition, the gravitational potential energy of a system of masses m_1 and M_2 at a distance r using gravitational constant G is

$$U = -G\frac{m_1 M_2}{r} + K \tag{5.7.3}$$

where K is the constant of integration. Choosing the convention that $K = 0$ makes calculations simpler, albeit at the cost of making U negative. Note that in this case the potential energy becomes zero when r is infinite, and approaches negative infinity as r goes to zero.

Trebuchet: A trebuchet uses the gravitational potential energy of the counterweight to throw projectiles over long distances.

Key Points

- If an object falls from one point to another point inside a gravitational field, the force of gravity will do positive work on the object, and the gravitational potential energy will decrease by the same amount.
- Near the surface of the Earth, the work done in lifting an object through a height h is the product mgh, so $U = mgh$.
- The gravitational potential energy, U, of a system of masses m_1 and M_2 at a distance r using gravitational constant G is $U = -G\frac{m_1 M_2}{r} + K$.

Key Terms

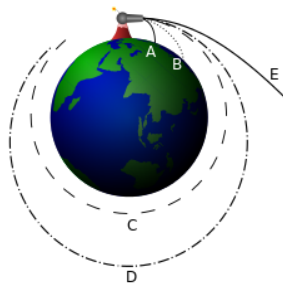
- **Newton's law of gravitation**: This law states that every point mass in the universe attracts every other point mass with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.
- **potential energy**: The energy an object has because of its position (in a gravitational or electric field) or its condition (as a stretched or compressed spring, as a chemical reactant, or by having rest mass)
- **gravity**: Resultant force on Earth's surface, of the attraction by the Earth's masses, and the centrifugal pseudo-force caused by the Earth's rotation.

5.8: Energy Conservation

learning objectives

- Calculate the escape speed of an object given its kinetic energy and the gravitational potential energy

Escape speed is the required starting speed required by an object to go from a starting point in a gravitational potential field to an ending point that is infinitely far away. It is assumed that the velocity of the object at the ending point will be zero.



Isaac Newton's Analysis of Escape Speed: In this figure, Objects A and B don't have the required escape speed and so they fall back to Earth after launch. Objects C and D don't either, they achieve a circular and an elliptical orbit respectively. Object E is launched with sufficient escape velocity and escapes the Earth.

Imagine a situation in which a spaceship that does not have a propulsion system is launched straight away from a planet. (It is moot to discuss escape speed for objects with propulsion systems.) Let us assume that the only significant force that is acting on the spaceship is the force of gravity from the planet. The escape speed of the spaceship can calculated through a simple analysis of conservation of energy. The gravitational potential energy of the spaceship is:

$$U = -\frac{GMm}{r} \tag{5.8.1}$$

Where G is the universal gravitational constant ($G = 6.67 \cdot 10^{-11} m^3 kg^{-1} s^{-2}$), M is the mass of the planet, m is the mass of the spaceship, and r is the distance of the spaceship from the planet's center of gravity.

At the ending point of the spaceship, r goes to infinity. As r goes to infinity, the value of the gravitational potential energy expression goes to 0.

The kinetic energy of the spaceship can be found from:

$$\frac{1}{2} mv^2 \tag{5.8.2}$$

Where mm is the mass of the spaceship and v is the velocity of the spaceship.

At the starting point of the spaceship, the velocity must have a magnitude equal to the escape speed (s_e). The velocity of the spaceship is 0 at its ending point, and so consequently its kinetic energy is 0 in the end as well.

Summarizing the kinetic energy (K) and potential energy (U) of the spaceship at it's initial (i) and final (f) states:

$$(K+U)_i = \frac{1}{2} m s_e^2 + \frac{-GMm}{r} \tag{5.8.3}$$

$$(K+U)_f = 0+0 \tag{5.8.4}$$

Due to conservation of energy, the initial energy must equal the final energy and so we can solve for s_e:

$$s_e = \sqrt{\frac{2GM}{r}} \tag{5.8.5}$$

Interestingly, if the spaceship were to fall to the planet from a point infinitely far away it would obtain a final speed of s_e at the planet.

It should be noted that if an object is launched from a rotating body, such as the Earth, the speed at which the body rotates will affect the required velocity that an object must have relative to the surface of the body. If a rocket is launched tangentially from the Earth's equator in the same direction that the Earth is turning, it will require a lower velocity relative to the Earth than if it were launched in the opposite direction to meet escape speed requirements.

Additionally, it is a misconception that powered vehicles (such as rockets) require escape speed to leave orbit and travel through outer-space. If the vehicle has a propulsion system to provide it with energy once it has left the surface of the planet, it is not necessary to initially meet escape speed requirements.

Key Points

- It is assumed that the velocity of the object at the ending point will be zero.
- The requisite escape speed (se)(se) of an object to escape a spherically symmetric body is given by: $\mathrm{s_e} = \sqrt{\frac{2\mathrm{GM}}{\mathrm{r}}}$, where G is the universal gravitational constant, M is the mass of the body, and r is the distance of the object from the body's center of gravity.
- Escape speed is the required speed that an object has to have to go from a starting point in a gravitational potential field to an ending point that is infinitely far away.
- The speed at which a body rotates will affect the required velocity that an object must have relative to the surface of the body.
- Objects that have propulsion systems do not need to reach escape velocity.

Key Terms

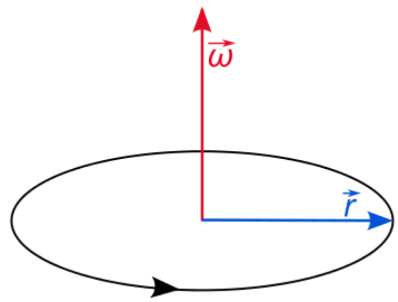
- **propulsion**: Force causing movement.
- **potential energy**: The energy an object has because of its position (in a gravitational or electric field) or its condition (as a stretched or compressed spring, as a chemical reactant, or by having rest mass)
- **kinetic energy**: The energy possessed by an object because of its motion, equal to one half the mass of the body times the square of its velocity.

5.9: Angular vs. Linear Quantities

learning objectives

- Describe properties characteristic to angular velocity and angular momentum

Linear motion is motion in a straight line. This type of motion has several familiar vector quantities associated with it, including linear velocity and momentum. These vector quantities each have a magnitude (a scalar, or number) and direction associated with them. Similarly, circular motion is motion in a circle. It has the same set of vector quantities associated with it, including angular velocity and angular momentum.



***Angular velocity diagram**: A vector diagram illustrating circular motion. The blue vector connects the origin (center) of the motion to the position of the particle. The red vector is the angular velocity vector, pointing perpendicular to the plane of motion and with magnitude equal to the instantaneous velocity. File:Angular velocity.svg - Wikipedia, the free encyclopedia. **Provided by**: Wikipedia. **Located at**: **en.Wikipedia.org/w/index.php?title=File:Angular_velocity.svg&page=1**. **License**: CC BY-SA: Attribution-ShareAlike*

Imagine a particle moving in a circle around a point at a constant speed. We will call that point the origin. At any instant in time, the particle is moving in a particular straight-line direction with that speed. In the next instant, the particle has the same speed, but the direction of its velocity has changed.

We recall from our study of linear velocity that a change in the direction of the velocity vector, is a change in velocity and a change in velocity is acceleration. However, we can define an angular momentum vector which is constant throughout this motion. The angular velocity has a direction perpendicular to the plane of circular motion, just like a bike axle points perpendicularly to the rotating wheel. This direction never changes as the object moves in its circle. The magnitude of the angular momentum is equal to the rate at which the angle of the particle advances:

$$\omega=\frac{\mathrm{d}\phi}{\mathrm{dt}} \tag{5.9.1}$$

Note that there are two vectors that are perpendicular to any plane. For example, imagine a vector pointing into your table and the opposite one pointing out of it. To remove this ambiguity, the convention in physics is to use the right hand rule: curl the fingers of your right hand in the direction of the circular motion, and your thumb will point toward the direction of the angular velocity and momentum vectors.

Right hand rule**: When determining the direction of an angular vector, use the right hand rule: curl the fingers of your right hand in the direction of the circular motion and your thumb points in the vector direction. File:Right-hand grip rule.svg - Wikipedia, the* *free encyclopedia.* ***Provided by*: Wikipedia.* ***Located at****:* ***en.Wikipedia.org/w/index.php?title=File:Right-hand_grip_rule.svg&page=1****.* ***License****: CC BY-SA: Attribution-ShareAlike*

The units of angular velocity are radians per second. Radian describes the plane angle subtended by a circular arc as the length of the arc divided by the radius of the arc. One radian is the angle subtended at the center of a circle by an arc that is equal in length to the radius of the circle. More generally, the magnitude in radians of such a subtended angle is equal to the ratio of the arc length to the radius of the circle; that is, $\theta = \frac{\mathrm{s}}{\mathrm{r}}$, where θ is the subtended angle in radians, s is arc length, and r is radius.

Thus, while the object moves in a circle at constant speed, it undergoes constant linear acceleration to keep it moving in a circle. However, it's angular velocity is constant since it continually sweeps out a constant arc length per unit time. Constant angular velocity in a circle is known as uniform circular motion.

Just as there is an angular version of velocity, so too is there an angular version of acceleration. When the object is going around a circle but its speed is changing, the object is undergoing angular acceleration. Just like with linear acceleration, angular acceleration is a change in the angular velocity vector. This change could be a change in the speed of the object or in the direction. Angular velocity can be clockwise or counterclockwise.

Key Points

- The direction of angular quantity vectors points perpendicular to the plane of the motion. You can determine this direction using the right hand rule.
- The direction of linear quantities such as velocity and momentum change as an object moves in a circle. We can instead define angular versions of these quantities which are constant throughout the circular motion.
- The units of angular quantities are per radian, a measurement of angle, rather than per linear distance (e.g. meter).

Key Terms

- **vector**: A directed quantity, one with both magnitude and direction; the between two points.
- **angular momentum**: A vector quantity describing an object in circular motion; its magnitude is equal to the momentum of the particle, and the direction is perpendicular to the plane of its circular motion.
- **angular velocity**: A vector quantity describing an object in circular motion; its magnitude is equal to the speed of the particle and the direction is perpendicular to the plane of its circular motion.

CHAPTER OVERVIEW

6: Work and Energy

6.1: Introduction

Introduction to Work and Energy

Work is the energy associated with the action of a force.

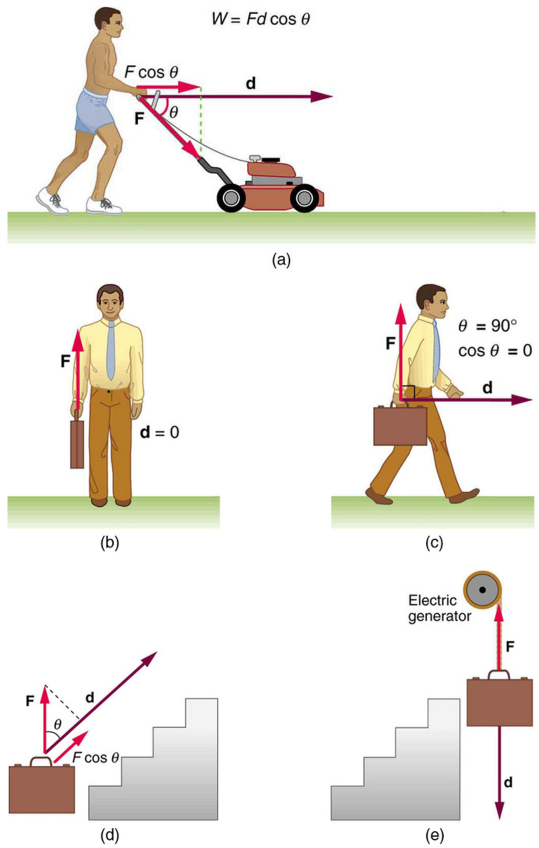
learning objectives

- Describe relationship between work, energy, and force

The work done on a system by a constant force is the product of the component of the force in the direction of motion times the distance through which the force acts. For one-way motion in one dimension, this is expressed in equation form as $\mathrm{W = Fd}\cos\theta$, where W is work, F is the magnitude of the force on the system, d is the magnitude of the displacement of the system, and θ is the angle between the force vector F and the displacement vector d.

Work, Power, and Energy: Biology is useful.

Take this example of work in action from: (A) The work done by the force F on this lawn mower is $\mathrm{W = Fd}\cos\theta$. Note that $\mathrm{W = Fd}\cos\theta$ is the component of the force in the direction of motion. (B) A person holding a briefcase does no work on it, because there is no motion. No energy is transferred to or from the briefcase. (C) The person moving the briefcase horizontally at a constant speed does no work on it, and transfers no energy to it. (D) Work is done on the briefcase by carrying it up stairs at constant speed, because there is necessarily a component of force F in the direction of the motion. Energy is transferred to the briefcase and could, in turn, be used to do work. (E) When the briefcase is lowered, energy is transferred out of the briefcase and into an electric generator. Here the work done on the briefcase by the generator is negative, removing energy from the briefcase, because F and d are in opposite directions.



Examples of Work: This is how work in progress and energy co-exist and operate. Work is the energy associated with the action of a force.

In physics, a force is said to do work when it acts on a body so that there is a displacement of the point of application, however small, in the direction of the force. Thus a force does work when there is movement under the action of the force. The work done by a constant force of magnitude F on a point that moves a distance d in the direction of the force is the product:

$$\mathrm{W} = \mathrm{Fd} \tag{6.1.1}$$

For example, if a force of 10 newton (F = 10 N) acts along point that travels two meters (d = 2 m), then it does the work W = (10 N)(2 m) = 20 N m = 20 J. This is approximately the work done lifting a 1 kg weight from the ground to over a person's head against the force of gravity. Notice that the work is doubled either by lifting twice the weight in the same distance or by lifting the same weight twice the distance.

Work is closely related to energy. The conservation of energy states that the change in total internal energy of a system equals the added heat minus the work performed by the system (see the first law of thermodynamics, and):

Baseball Pitcher: A baseball pitcher does work on a baseball by throwing the ball at some force, F, over some distance d, which for the average baseball field, is about 60 feet.

$$\delta E = \delta Q - \delta W \tag{6.1.2}$$

Also, from Newton's second law for rigid bodies, it can be shown that work on an object is equal to the change in kinetic energy of that object:

$$W = \Delta KE \tag{6.1.3}$$

The work of forces generated by a potential function is known as potential energy and the forces are said to be conservative. Therefore work on an object moving in a conservative force field is equal to minus the change of potential energy of the object:

$$W = -\Delta PE \tag{6.1.4}$$

This shows that work is the energy associated with the action of a force, and so has the physical dimensions and units of energy.

Key Points

- Work is the transfer of energy by a force acting on an object as it is displaced.
- The work done by a force is zero if the displacement is either zero or perpendicular to the force.
- The work done is positive if the force and displacement have the same direction, and the work done is negative if they have opposite direction.

Key Terms

- **energy**: A quantity that denotes the ability to do work and is measured in a unit dimensioned in mass × distance^2/time^2 (ML^2/T^2) or the equivalent.

6.2: Work Done by a Constant Force

Force in the Direction of Displacement

The work done by a constant force is proportional to the force applied times the displacement of the object.

> learning objectives
>
> - Contrast displacement and distance in constant force situations

Work Done by a Constant Force

When a force acts on an object over a distance, it is said to have done work on the object. Physically, the work done on an object is the change in kinetic energy that that object experiences. We will rigorously prove both of these claims.

The term work was introduced in 1826 by the French mathematician Gaspard-Gustave Coriolis as “weight lifted through a height,” which is based on the use of early steam engines to lift buckets of water out of flooded ore mines. The SI unit of work is the newton-meter or joule (J).

Units

One way to validate if an expression is correct is to perform dimensional analysis. We have claimed that work is the change in kinetic energy of an object and that it is also equal to the force times the distance. The units of these two should agree. Kinetic energy – and all forms of energy – have units of joules (J). Likewise, force has units of newtons (N) and distance has units of meters (m). If the two statements are equivalent they should be equivalent to one another.

$$\mathrm{N \cdot m = kg\frac{m}{s^2} \cdot m = kg\frac{m^2}{s^2} = J} \tag{6.2.1}$$

Displacement versus Distance

Often times we will be asked to calculate the work done by a force on an object. As we have shown, this is proportional to the force and the distance which the object is displaced, not moved. We will investigate two examples of a box being moved to illustrate this.

Example Problems

Here are a few example problems:

(1.a) Consider a constant force of two newtons (F = 2 N) acting on a box of mass three kilograms (M = 3 kg). Calculate the work done on the box if the box is displaced 5 meters.

(1.b) Since the box is displaced 5 meters and the force is 2 N, we multiply the two quantities together. The object’s mass will dictate how fast it is accelerating under the force, and thus the time it takes to move the object from point a to point b. Regardless of how long it takes, the object will have the same displacement and thus the same work done on it.

(2.a) Consider the same box (M = 3 kg) being pushed by a constant force of four newtons (F = 4 N). It begins at rest and is pushed for five meters (d = 5m). Assuming a frictionless surface, calculate the velocity of the box at 5 meters.

(2.b) We now understand that the work is proportional to the change in kinetic energy, from this we can calculate the final velocity. What do we know so far? We know that the block begins at rest, so the initial kinetic energy must be zero. From this we algebraically isolate and solve for the final velocity.

$$\mathrm{Fd = \Delta KE = KE_f - 0 = \frac{1}{2} m v_f^2} \tag{6.2.2}$$

$$\mathrm{v_f = \sqrt{2\frac{Fd}{m}} = \sqrt{2\frac{4N \cdot 5m}{2kg}} = \sqrt{10} m/s} \tag{6.2.3}$$

We see that the final velocity of the block is approximately 3.15 m/s.

Force at an Angle to Displacement

A force does not have to, and rarely does, act on an object parallel to the direction of motion.

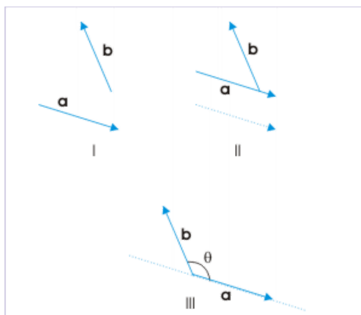
learning objectives

- Infer how to adjust one-dimensional motion for our three-dimensional world

The Fundamentals

Up until now, we have assumed that any force acting on an object has been parallel to the direction of motion. We have considered our motion to be one dimensional, only acting along the x or y axis. To best examine and understand how nature operators in our three-dimensional world, we will first discuss work in two dimensions in order to build our intuition.

A force does not have to, and rarely does, act on an object parallel to the direction of motion. In the past, we derived that $\mathrm{W}=\mathrm{Fd}$; such that the work done on an object is the force acting on the object multiplied by the displacement. But this is not the whole story. This expression contains an assumed cosine term, which we do not consider for forces parallel to the direction of motion. “Why would we do such a thing? ” you may ask. We do this because the two are equivalent. If the angle of the force along the direction of motion is zero, such that the force is parallel to the direction of motion, then the cosine term equals one and does not change the expression. As we increase the force’s angle with respect to the direction of motion, less and less work is done along the direction that we are considering; and more and more work is being done in another, perpendicular, direction of motion. This process continues until we are perpendicular to our original direction of motion, such that the angle is 90, and the cosine term would equal zero; resulting in zero work being done along our original direction. Instead, we are doing work in another direction!



Angle: Recall that both the force and direction of motion are vectors. When the angle is 90 degrees, the cosine term goes to zero. When along the same direction, they equal one.

Let’s show this explicitly and then look at this phenomena in terms of a box moving along the x and y directions.

We have discussed that work is the integral of the force and the dot product respect to x. But in fact, dot product of force and a very small distance is equal to the two terms times cosine of the angle between the two. $\mathrm{F}\times\mathrm{dx}=\mathrm{Fd}\cos(\theta)$. Explicitly,

$$\int_{\mathrm{t}_2}^{\mathrm{t}_1}\mathrm{F}\cdot\mathrm{dx}=\int_{\mathrm{t}_2}^{\mathrm{t}_1}\mathrm{Fd}\cos\theta\mathrm{dx}=\mathrm{Fd}\cos\theta \tag{6.2.4}$$

A Box Being Pushed

Consider a coordinate system such that we have x as the abscissa and y as the ordinate. More so, consider a box being pushed along the x direction. What happens in the following three scenarios?

- The box is being pushed parallel to the x direction?
- The box is being pushed at an angle of 45 degrees to the x direction?
- The box is being pushed at an angle of 60 degrees to the x direction?
- The box is being pushed at an angle of 90 degrees to the x direction?

In the first scenario, we know that all of the force is acting on the box along the x-direction, which means that work will only be done along the x-direction. More so, a vertical perspective the box is not moving – it is unchanged in the y direction. Since the force is acting parallel to the direction of motion, the angle is equal to zero and our total work is simply the force times the displacement in the x-direction.

In the second scenario, the box is being pushed at an angle of 45 degrees to the x-direction; and thus also a 45 degree angle to the y-direction. When evaluated, the cosine of 45 degrees is equal to $\frac{1}{\sqrt{2}}$, or approximately 0.71. This means is that 71% of the force is contributing to the work along the x-direction. The other 29% is acting along the y-direction.

In the third scenario, we know that the force is acting at a 60 degree angle to the x-direction; and thus also a 30 degree angle to the y-direction. When evaluated, cosine of 60 degrees is equal to 1/2. This means that the force is equally acting in the x and y-direction! The work done is linear with respect to both x and y.

In the last scenario, the box is being pushed at an angle perpendicular to the x direction. In other words, we are pushing the box in the y-direction! Thus, the box's position will be unchanged and experience no displacement along the x-axis. The work done in the x direction will be zero.

Key Points

- Understanding work is quintessential to understanding systems in terms of their energy, which is necessary for higher level physics.
- Work is equivalent to the change in kinetic energy of a system.
- Distance is not the same as displacement. If a box is moved 3 meters forward and then 4 meters to the left, the total displacement is 5 meters, not 7 meters.
- Work done on an object along a given direction of motion is equal to the force times the displacement times the cosine of the angle.
- No work is done along a direction of motion if the force is perpendicular.
- When considering force parallel to the direction of motion, we omit the cosine term because it equals 1 which does not change the expression.

Key Terms

- **work**: A measure of energy expended in moving an object; most commonly, force times displacement. No work is done if the object does not move.
- **dot product**: A scalar product.

6.3: Work Done by a Variable Force

Work Done by a Variable Force

Integration is used to calculate the work done by a variable force.

learning objectives

- Describe approaches used to calculate work done by a variable force

Using Integration to Calculate the Work Done by Variable Forces

A force is said to do work when it acts on a body so that there is a displacement of the point of application in the direction of the force. Thus, a force does work when it results in movement.

The work done by a constant force of magnitude F on a point that moves a displacement Δx in the direction of the force is simply the product

$$W = F \cdot \Delta x \tag{6.3.1}$$

In the case of a variable force, integration is necessary to calculate the work done. For example, let's consider work done by a spring. According to the Hooke's law the restoring force (or spring force) of a perfectly elastic spring is proportional to its extension (or compression), but opposite to the direction of extension (or compression). So the spring force acting upon an object attached to a horizontal spring is given by:

$$Fs = -kx \tag{6.3.2}$$

that is proportional to its displacement (extension or compression) in the x direction from the spring's equilibrium position, but its direction is opposite to the x direction. For a variable force, one must add all the infinitesimally small contributions to the work done during infinitesimally small time intervals dt (or equivalently, in infinitely small length intervals dx=$v_x$dt). In other words, an integral must be evaluated:

$$W_s = \int_0^t Fs \cdot vdt = \int_0^t -kxv_x dt = \int_{x_o}^x -kxdx = -\frac{1}{2}k\Delta x^2 \tag{6.3.3}$$

This is the work done by a spring exerting a variable force on a mass moving from position x_o to x (from time 0 to time t). The work done is positive if the applied force is in the same direction as the direction of motion; so the work done by the object on spring from time 0 to time t, is:

$$W_a = \int_0^t Fa \cdot vdt = \int_0^t -F_s \cdot vdt = \frac{1}{2}k\Delta x^2 \tag{6.3.4}$$

in this relation F_a is the force acted upon spring by the object. F_a and F_s are in fact action- reaction pairs; and W_a is equal to the elastic potential energy stored in spring.

Using Integration to Calculate the Work Done by Constant Forces

The same integration approach can be also applied to the work done by a constant force. This suggests that *integrating* the product of force and distance is the general way of determining the work done by a force on a moving body.

Consider the situation of a gas sealed in a piston, the study of which is important in Thermodynamics. In this case, the Pressure (Pressure =Force/Area) is constant and can be taken out of the integral:

$$W = \int_a^b PdV = P\int_a^b dV = P\Delta V \tag{6.3.5}$$

Another example is the work done by gravity (a constant force) on a free-falling object (we assign the y-axis to vertical motion, in this case):

$$W = \int_{t_1}^{t_2} F \cdot vdt = \int_{t_1}^{t_2} mgv_y dt = mg\int_{y_1}^{y_2} dy = mg\Delta y \tag{6.3.6}$$

Notice that the result is *the same* as we would have obtained by simply evaluating the product of force and distance.

Units Used for Work

The SI unit of work is the joule (J), which is defined as the work done by a force of one newton moving an object through a distance of one meter.

Non-SI units of work include the erg, the foot-pound, the foot-pound, the kilowatt hour, the liter-atmosphere, and the horsepower-hour.

Key Points

- The work done by a constant force of magnitude F on a point that moves a displacement d in the direction of the force is the product: $W = Fd$.
- Integration approach can be used both to calculate work done by a variable force and work done by a constant force.
- The SI unit of work is the joule; non- SI units of work include the erg, the foot-pound, the foot-poundal, the kilowatt hour, the litre-atmosphere, and the horsepower-hour.

Key Terms

- **work**: A measure of energy expended in moving an object; most commonly, force times displacement. No work is done if the object does not move.
- **force**: A physical quantity that denotes ability to push, pull, twist or accelerate a body, which is measured in a unit dimensioned in mass × distance/time^2 (ML/T^2): SI: newton (N); CGS: dyne (dyn)

6.4: Work-Energy Theorem

Kinetic Energy and Work-Energy Theorem

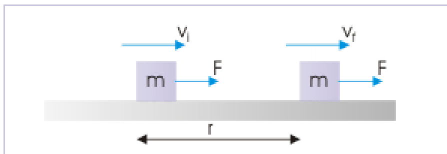
The work-energy theorem states that the work done by all forces acting on a particle equals the change in the particle's kinetic energy.

learning objectives

- Outline the derivation of the work-energy theorem

The Work-Energy Theorem

The principle of work and kinetic energy (also known as the work-energy theorem) states that the work done by the sum of all forces acting on a particle equals the change in the kinetic energy of the particle. This definition can be extended to rigid bodies by defining the work of the torque and rotational kinetic energy.



Kinetic Energy: A force does work on the block. The kinetic energy of the block increases as a result by the amount of work. This relationship is generalized in the work-energy theorem.

The work *W* done by the net force on a particle equals the change in the particle's kinetic energy K*E*:

$$\mathrm{W = \Delta KE = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2} \tag{6.4.1}$$

where v_i and v_f are the speeds of the particle before and after the application of force, and *m* is the particle's mass.

Derivation

For the sake of simplicity, we will consider the case in which the resultant force *F* is constant in both magnitude and direction and is parallel to the velocity of the particle. The particle is moving with constant acceleration *a* along a straight line. The relationship between the net force and the acceleration is given by the equation *F* = *ma* (Newton's second law), and the particle's displacement *d*, can be determined from the equation:

$$\mathrm{v_f^2 = v_i^2 + 2ad} \tag{6.4.2}$$

obtaining,

$$\mathrm{d = \frac{v_f^2 - v_i^2}{2a}} \tag{6.4.3}$$

The work of the net force is calculated as the product of its magnitude (F=ma) and the particle's displacement. Substituting the above equations yields:

$$\mathrm{W = Fd = ma\frac{v_f^2 - v_i^2}{2a} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = KE_f - KE_i = \Delta KE} \tag{6.4.4}$$

Key Points

- The work *W* done by the net force on a particle equals the change in the particle's kinetic energy K*E*: $\mathrm{W = \Delta KE = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2}$.
- The work-energy theorem can be derived from Newton's second law.
- Work transfers energy from one place to another or one form to another. In more general systems than the particle system mentioned here, work can change the potential energy of a mechanical device, the heat energy in a thermal system, or the electrical energy in an electrical device.

Key Terms

- **torque**: A rotational or twisting effect of a force; (SI unit newton-meter or Nm; imperial unit foot-pound or ft-lb)

6.5: Potential Energy and Conservation of Energy

Conservative and Nonconservative Forces

Conservative force—a force with the property that the work done in moving a particle between two points is independent of the path it takes.

learning objectives

- Describe properties of conservative and nonconservative forces

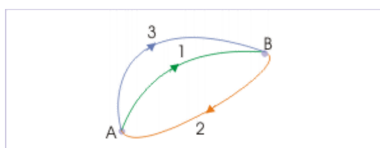
A conservative force is a force with the property that the work done in moving a particle between two points is independent of the path taken. Equivalently, if a particle travels in a closed loop, the net work done (the sum of the force acting along the path multiplied by the distance travelled) by a conservative force is zero.

A conservative force is dependent only on the position of the object. If a force is conservative, it is possible to assign a numerical value for the potential at any point. When an object moves from one location to another, the force changes the potential energy of the object by an amount that does not depend on the path taken. Gravity and spring forces are examples of conservative forces.

If a force is *not conservative*, then defining a scalar potential is not possible, because taking different paths would lead to conflicting potential differences between the start and end points. Nonconservative forces transfer energy *from* the object in motion (just like conservative force), but they do not transfer this energy back *to* the potential energy of the system to regain it during reverse motion. Instead, they transfer the energy from the system in an energy form which can not be used by the force to transfer it back to the object in motion. Friction is one such nonconservative force.

Path Independence of Conservative Force

Work done by the gravity in a closed path motion is zero. We can extend this observation to other conservative force systems as well. We imagine a closed path motion. We imagine this closed path motion be divided in two motions between points A and B as diagramed in Fig 1. Starting from point A to point B and then ending at point A via two work paths named 1 and 2 in the figure. The total work by the conservative force for the round trip is zero:



Motion Along Different Paths: Motion along different paths. For a conservative force, work done via different path is the same.

$$\mathrm{W}=\mathrm{W}_{\mathrm{AB1}}+\mathrm{W}_{\mathrm{BA2}}=0. \tag{6.5.1}$$

Let us now change the path for motion from A to B by another path, shown as path 3. Again, the total work by the conservative force for the round trip via new route is zero: $\mathrm{W}=\mathrm{W}_{\mathrm{AB3}}+\mathrm{W}_{\mathrm{BA2}}=0.$.

Comparing two equations, $\mathrm{W}_{\mathrm{AB1}}=\mathrm{W}_{\mathrm{AB3}}$. This is true for an arbitrary path. Therefore, work done for motion from A to B by conservative force along any paths are equal.

What is Potential Energy?

Potential energy is the energy difference between the energy of an object in a given position and its energy at a reference position.

learning objectives

- Relate the potential energy and the work

Potential energy is often associated with restoring forces such as a spring or the force of gravity. The action of stretching the spring or lifting the mass of an object is performed by an external force that works against the force field of the potential. This work is stored in the force field as potential energy. If the external force is removed the force field acts on the body to perform the work as it moves the body back to its initial position, reducing the stretch of the spring or causing the body to fall. The more formal definition is that potential energy is the energy difference between the energy of an object in a given position and its energy at a reference position.

Potential Energy in a Bow and Arrow: In the case of a bow and arrow, the energy is converted from the potential energy in the archer's arm to the potential energy in the bent limbs of the bow when the string is drawn back. When the string is released, the potential energy in the bow limbs is transferred back through the string to become kinetic energy in the arrow as it takes flight.

If the work for an applied force is independent of the path, then the work done by the force is evaluated at the start and end of the trajectory of the point of application. This means that there is a function $U(x)$, called a "potential," that can be evaluated at the two points $x(t = t_1)$ and $x(t_2)$ to obtain the work over any trajectory between these two points. It is tradition to define this function with a negative sign so that positive work is represented as a reduction in the potential:

$$W = \int_C F \cdot dx = \int_{x(t_1)}^{x(t_2)} F \cdot dx \tag{6.5.2}$$

$$= U(x(t_1)) - U(x(t_2)) = -\Delta U. \tag{6.5.3}$$

Examples of Potential Energy

There are various types of potential energy, each associated with a particular type of force. More specifically, every conservative force gives rise to potential energy. For example, the work of an elastic force is called elastic potential energy; work done by the gravitational force is called gravitational potential energy; and work done by the Coulomb force is called electric potential energy.

Gravity

Gravitational energy is the potential energy associated with gravitational force, as work is required to move objects against gravity.

learning objectives

- Generate an equation that can be used to express the gravitational potential energy near the earth

Gravitational energy is the potential energy associated with gravitational force (a conservative force), as work is required to elevate objects against Earth's gravity. The potential energy due to elevated positions is called gravitational potential energy, evidenced, for example, by water held in an elevated reservoir or behind a dam (as an example, shows Hoover Dam). If an object falls from one point to another point inside a gravitational field, the force of gravity will do positive work on the object, and the gravitational potential energy will decrease by the same amount.

Hoover Dam: Hoover dam uses the stored gravitational potential energy to generate electricity.

Potential Near Earth

Gravitational potential energy near the Earth can be expressed with respect to the height from the surface of the Earth. (The surface will be the zero point of the potential energy.) We can express the potential energy (gravitational potential energy) as:

$$PE = mgh, \tag{6.5.4}$$

where PE = potential energy measured in joules (J), m = mass of the object (measured in kg), and h = perpendicular height from the reference point (measured in m); g = gravitational acceleration ($9.8 m/s^2$). Near the surface of the Earth, g can be considered constant.

General Formula

However, over large variations in distance, the approximation that g is constant is no longer valid. Instead, we must use calculus and the general mathematical definition of work to determine gravitational potential energy. For the computation of the potential energy we can integrate the gravitational force, whose magnitude is given by Newton's law of gravitation (with respect to the distance r between the two bodies). Using that definition, the gravitational potential energy of a system of masses m and M at a distance r using gravitational constant G is:

$$U(r) = \int_r (G\frac{mM}{r'^2})dr' = -G\frac{mM}{r} + K, \tag{6.5.5}$$

where *K* is the constant of integration. Choosing the convention that *K*=0 makes calculations simpler, albeit at the cost of making *U* negative. For this choice, the potential at infinity is defined as 0.

Springs

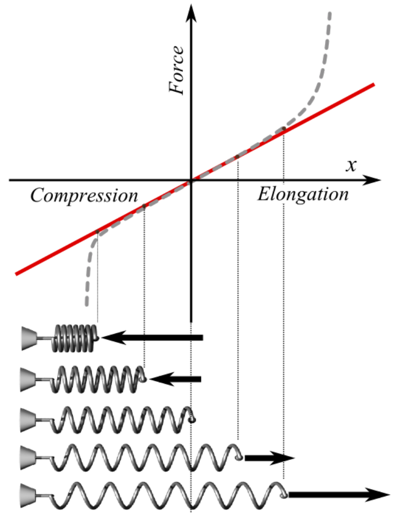
When a spring is stretched/compressed from its equilibrium position by x, its potential energy is give as $U = \frac{1}{2}kx^2$.

learning objectives

- Explain how potential energy is stored in springs

Spring force is conservative force, given by the Hooke's law: $F = -kx$, where k is spring constant, measured experimentally for a particular spring and x is the displacement. We would like to obtain an expression for the work done to the spring. From the conservation of mechanical energy (Check our Atom on "Conservation of Mechanical Energy), the work should be equal to the potential energy stored in spring. The displacement x is usually measured from the position of "neutral length" or "relaxed length"

– the length of spring corresponding to situation when spring is neither stretched nor compressed. We shall identify this position as the origin of coordinate reference (x=0).



Hooke's Law: Plot of applied force F vs. elongation X for a helical spring according to Hooke's law (solid line) and what the actual plot might look like (dashed line). Red is used extension, blue for compression. At bottom, schematic pictures of spring states corresponding to some points of the plot; the middle one is in the relaxed state (no force applied).

Let $\mathrm{x}=0$ and $\mathrm{x}=\mathrm{x}_{\mathrm{f}}(>0)$ be the initial and final positions of the block attached to the string. As the block slowly moves, we do work W on the spring: $\mathrm{W}=\int_{0}^{\mathrm{x}_{\mathrm{f}}}(\mathrm{kx})\mathrm{dx}=\frac{1}{2}\mathrm{kx}_{\mathrm{f}}^{2}$. When we stretch the spring. We have to apply force in the same direction as the displacement. (Technically, work is given as the inner product of the two vectors: force and displacement. $\mathrm{W}=\mathbf{F}\cdot\Delta\mathrm{x}$). Therefore, the overall sign in the integral is +, not -.

If the block is gently released from the stretched position (\mathrm{x = x_f}\), the stored potential energy in the spring will start to be converted to the kinetic energy of the block, and vice versa. Neglecting frictional forces, Mechanical energy conservation demands that, at any point during its motion,

$$\text{Total Energy} =\frac{1}{2}\mathrm{mv}^{2}+\frac{1}{2}\mathrm{kx}^{2} \tag{6.5.6}$$

$$=\frac{1}{2}\mathrm{kx}_{\mathrm{f}}^{2}=\text{constant.} \tag{6.5.7}$$

From the energy conservation, we can estimate that, by the time the block reaches x=0 position, its speed will be $\mathrm{v}(\mathrm{x}=0)=\sqrt{\frac{\mathrm{k}}{\mathrm{m}}}\mathrm{x}_{\mathrm{f}}$. The block will keep oscillating between x = -x_f and x_f.

Conservation of Mechanical Energy

Conservation of mechanical energy states that the mechanical energy of an isolated system remains constant without friction.

learning objectives

- Formulate the principle of the conservation of the mechanical energy

Conservation of mechanical energy states that the mechanical energy of an isolated system remains constant in time, as long as the system is free of all frictional forces. In any real situation, frictional forces and other non-conservative forces are always present, but in many cases their effects on the system are so small that the principle of conservation of mechanical energy can be used as a fair approximation. An example of a such a system is shown in. Though energy cannot be created nor destroyed in an isolated system, it can be internally converted to any other form of energy.

A Mechanical System: An example of a mechanical system: A satellite is orbiting the Earth only influenced by the conservative gravitational force and the mechanical energy is therefore conserved. This acceleration is represented by a green acceleration vector and the velocity is represented by a red velocity vector.

Derivation

Let us consider what form the work -energy theorem takes when only conservative forces are involved (leading us to the conservation of energy principle). The work-energy theorem states that the net work done by all forces acting on a system equals its change in kinetic energy (KE). In equation form, this is:

$$\mathrm{W_{net}} = \frac{1}{2}\mathrm{mv}^2 - \frac{1}{2}\mathrm{mv}_0^2 = \Delta\mathrm{KE}. \tag{6.5.8}$$

If only conservative forces act, then $\mathrm{W_{net}} = \mathrm{W_c}$, where $\mathrm{W_c}$ is the total work done by all conservative forces. Thus, $\mathrm{W_c} = \Delta\mathrm{KE}$.

Now, if the conservative force, such as the gravitational force or a spring force, does work, the system loses potential energy (PE). That is, $\mathrm{W_c} = -\mathrm{PE}$. Therefore,

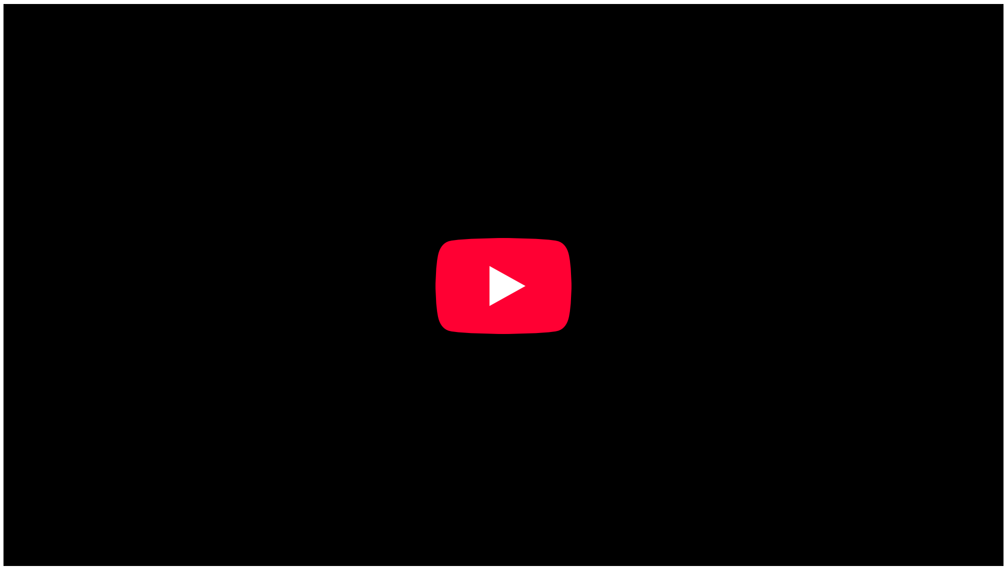
$$-\Delta\mathrm{PE} = \Delta\mathrm{KE} \tag{6.5.9}$$

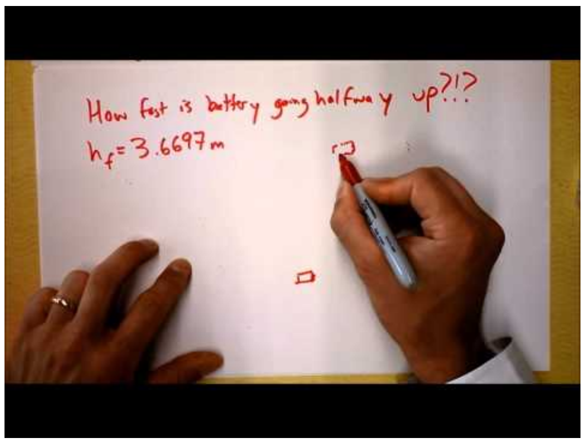
This equation means that the total kinetic and potential energy is constant for any process involving only conservative forces. That is,

$$\mathrm{KE} + \mathrm{PE} = \text{const or } \mathrm{KE_i} + \mathrm{PE_i} = \mathrm{KE_f} + \mathrm{PE_f}, \tag{6.5.10}$$

where i and f denote initial and final values. This equation is a form of the work-energy theorem for conservative forces; it is known as the conservation of mechanical energy principle.

Remember that the law applies to the extent that all the forces are conservative, so that friction is negligible. The total kinetic plus potential energy of a system is defined to be its mechanical energy $(\mathrm{KE} + \mathrm{PE})$. In a system that experiences only conservative forces, there is a potential energy associated with each force, and the energy only changes form between KE and various types of PE (with the total energy remaining constant).





Conservation of Mechanical Energy: Worked example.

Problem Solving With the Conservation of Energy

To solve a conservation of energy problem determine the system of interest, apply law of conservation of energy, and solve for the unknown.

learning objectives

- Identify steps necessary to solve a conservation of energy problem

Problem-solving Strategy

You should follow a series of steps whenever you are problem solving:

Step One

Determine the system of interest and identify what information is given and what quantity is to be calculated. For example, let's assume you have the problem with car on a roller coaster. You know that the cars of a roller coaster reach their maximum kinetic energy (KEKE) when at the bottom of their path. When they start rising, the kinetic energy begins to be converted to gravitational potential energy (PEgPEg). The sum of kinetic and potential energy in the system should remain constant, if losses to friction are ignored.

Determining Energy: The cars of a roller coaster reach their maximum kinetic energy when at the bottom of their path. When they start rising, the kinetic energy begins to be converted to gravitational potential energy. The sum of kinetic and potential energy in the system remains constant, ignoring losses to friction.

Step Two

Examine all the forces involved and determine whether you know or are given the potential energy from the work done by the forces. Then use step three or step four.

Step Three

If you know the potential energies (PE) for the forces that enter into the problem, then forces are all conservative, and you can apply conservation of mechanical energy simply in terms of potential and kinetic energy. The equation expressing conservation of energy is:

$$\mathrm{KE_i + PE_i = KE_f + PE_f}. \tag{6.5.11}$$

Step Four

If you know the potential energy for only some of the forces, then the conservation of energy law in its most general form must be used:

$$\mathrm{KE_i + PE_i + W_{nc} + OE_i = KE_f + PE_f + OE_f} \tag{6.5.12}$$

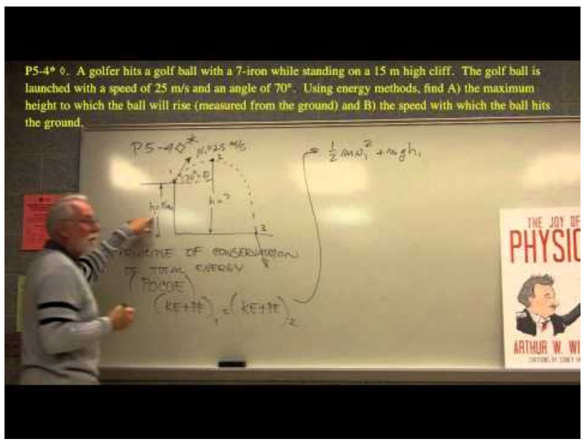
where $\mathrm{O_E}$ stand for all other energies, and $\mathrm{W_{nc}}$ stands for work done by non-conservative forces. In most problems, one or more of the terms is zero, simplifying its solution. Do *not* calculate $\mathrm{W_c}$, the work done by conservative forces; it is already incorporated in the PE terms.

Step Five

You have already identified the types of work and energy involved (in step two). Before solving for the unknown, eliminate terms wherever possible to simplify the algebra. For example, choose height $\mathrm{h=0}$ at either the initial or final point—this will allow to set PEg at zero. Then solve for the unknown in the customary manner.

Step Six

Check the answer to see if it is reasonable. Once you have solved a problem, reexamine the forms of work and energy to see if you have set up the conservation of energy equation correctly. For example, work done against friction should be negative, potential energy at the bottom of a hill should be less than that at the top, and so on.



Energy conservation: Part of a series of videos on physics problem-solving. The problems are taken from “The Joy of Physics. ” This one deals with energy conservation. The viewer is urged to pause the video at the problem statement and work the problem before watching the rest of the video.

Problem Solving with Dissipative Forces

In the presence of dissipative forces, total mechanical energy changes by exactly the amount of work done by nonconservative forces (W_c).

learning objectives

- Express the energy conservation relationship that can be applied to solve problems with dissipative forces

INTRODUCTION

We have seen a problem-solving strategy with the conservation of energy in the previous section. Here we will adopt the strategy for problems with dissipative forces. Since the work done by nonconservative (or dissipative) forces will irreversibly alter the energy of the system, the total mechanical energy (KE + PE) changes by exactly the amount of work done by nonconservative forces (W_c). Therefore, we obtain $\mathrm{KE_i + PE_i + W_{nc} = KE_f + PE_f}$, where KE and PE represent kinetic and potential energies respectively. Therefore, using the new energy conservation relationship, we can apply the same problem-solving strategy as with the case of conservative forces.

EXAMPLE

Consider the situation shown in, where a baseball player slides to a stop on level ground. Using energy considerations, calculate the distance the 65.0-kg baseball player slides, given that his initial speed is 6.00 m/s and the force of friction against him is a *constant 450 N.*

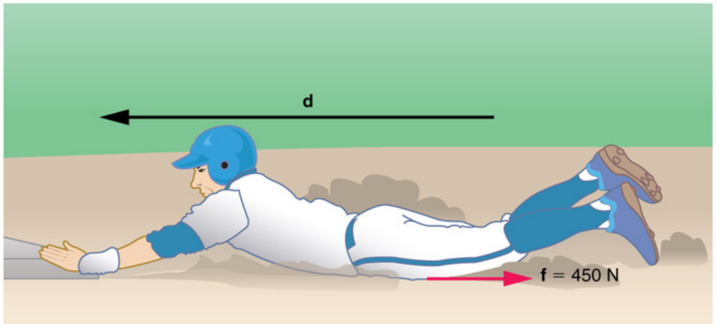


Fig 1: The baseball player slides to a stop in a distance d. In the process, friction removes the player's kinetic energy by doing an amount of work fd equal to the initial kinetic energy.

Strategy: Friction stops the player by converting his kinetic energy into other forms, including thermal energy. In terms of the work-energy theorem, the work done by friction (f), which is negative, is added to the initial kinetic energy to reduce it to zero. The work done by friction is negative, because f is in the opposite direction of the motion (that is, $\theta = 180^{\circ}$, and so $\cos\theta = -1$). Thus $W_{nc} = -fd$. The equation simplifies to $\frac{1}{2}mv_i^2 - fd = 0$.

Solution: Solving the previous equation for d and substituting known values yields, we get d = 2.60 m. The most important point of this example is that the amount of nonconservative work equals the change in mechanical energy.

Key Points

- If a particle travels in a closed loop, the net work done (the sum of the force acting along the path multiplied by the distance travelled) by a conservative force is zero.
- Conservative force is dependent only on the position of the object. If a force is conservative, it is possible to assign a numerical value for the potential at any point.
- Nonconservative force transfer the energy from the system in an energy form which can not be used by the force to transfer back to the object in motion.
- If the work for an applied force is independent of the path, then the work done by the force is evaluated at the start and end of the trajectory of the point of application. This means that there is a function U(x), called a " potential ".
- It is tradition to define the potential function with a negative sign so that positive work is represented as a reduction in the potential.
- Every conservative force gives rise to potential energy. Examples are elastic potential energy, gravitational potential energy, and electric potential energy.
- Gravitational potential energy near the earth can be expressed with respect to the height from the surface of the Earth as PE = mgh. g = gravitational acceleration (9.8m/s^2). Near the surface of the Earth, g can be considered constant.
- Over large variations in distance, the approximation that g is constant is no longer valid and a general formula should be used for the potential. It is given as: $U(r) = \int_r (G\frac{mM}{r'^2})dr' = -G\frac{mM}{r} + K$.
- Choosing the convention that the constant of integration K=0 assumes that the potential at infinity is defined to be 0.
- The displacement of spring x is usually measured from the position of "neutral length " or "relaxed length". Often, it is most convenient to identify this position as the origin of coordinate reference (x=0).
- If the block is gently released from the stretched position ($x = x_f$), energy conservation tells us that $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kx_f^2 = \text{constant}$.
- If the block is released from the stretched position ($x = x_f$), by the time the block reaches x=0 position, its speed will be $v(x=0) = \sqrt{\frac{k}{m}}x_f$. The block will keep oscillating between $x = -x_f$ and x_f.
- The conservation of mechanical energy can be written as "$KE + PE = \text{const}$ ".
- Though energy cannot be created nor destroyed in an isolated system, it can be internally converted to any other form of energy.
- In a system that experiences only conservative forces, there is a potential energy associated with each force, and the energy only changes form between KE and various types of PE, with the total energy remaining constant.
- If you know the potential energies for the forces that enter into the problem, then forces are all conservative, and you can apply conservation of mechanical energy simply in terms of potential and kinetic energy. The equation expressing conservation of

energy is: $\mathrm{KE_i + PE_i = KE_f + PE_f}$. .

- If you know the potential energy for only some of the forces, then the conservation of energy law in its most general form must be used: $\mathrm{KE_i + PE_i + W_{nc} + OE_i = KE_f + PE_f + OE_f}$, where OE stands for all other energies.
- Once you have solved a problem, always check the answer to see if it is reasonable.
- Using the new energy conservation relationship

$$\mathrm{KE_i + PE_i + W_{nc} = KE_f + PE_f} \tag{6.5.13}$$

, we can apply the same problem-solving strategy as with the case of conservative forces.

- The most important point is that the amount of nonconservative work equals the change in mechanical energy.
- The work done by nonconservative (or dissipative) forces will irreversibly dissipated in the system.

Key Terms

- **potential**: A curve describing the situation where the difference in the potential energies of an object in two different positions depends only on those positions.
- **Coulomb force**: the electrostatic force between two charges, as described by Coulomb's law
- **potential**: A curve describing the situation where the difference in the potential energies of an object in two different positions depends only on those positions.
- **conservative force**: A force with the property that the work done in moving a particle between two points is independent of the path taken.
- **Hooke's law**: the principle that the stress applied to a solid is directly proportional to the strain produced. This law describes the behavior of springs and solids stressed within their elastic limit.
- **conservation**: A particular measurable property of an isolated physical system does not change as the system evolves.
- **isolated system**: A system that does not interact with its surroundings, that is, its total energy and mass stay constant.
- **frictional force**: Frictional force is the force resisting the relative motion of solid surfaces, fluid layers, and material elements sliding against each other.
- **kinetic energy**: The energy possessed by an object because of its motion, equal to one half the mass of the body times the square of its velocity.
- **potential energy**: The energy an object has because of its position (in a gravitational or electric field) or its condition (as a stretched or compressed spring, as a chemical reactant, or by having rest mass)
- **conservative force**: A force with the property that the work done in moving a particle between two points is independent of the path taken.
- **dissipative force**: A force resulting in dissipation, a process in which energy (internal, bulk flow kinetic, or system potential) is transformed from some initial form to some irreversible final form.

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6.6: Power

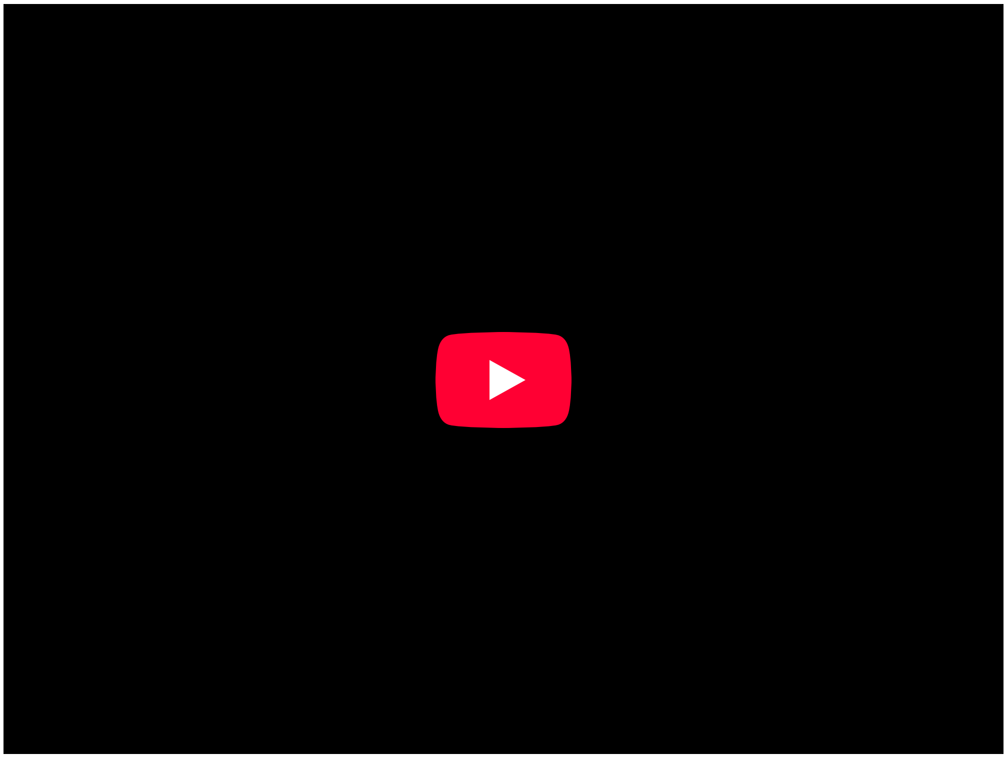
What is Power?

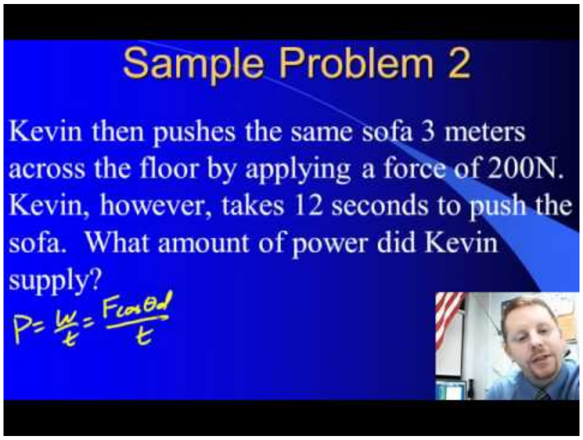
In physics, power is the rate of doing work—the amount of energy consumed per unit time.

learning objectives

- Relate power to the transfer, use, and transformation of different types of energy

In physics, power is the rate of doing work. It is the amount of energy consumed per unit time. The unit of power is the joule per second (J/s), known as the watt (in honor of James Watt, the eighteenth-century developer of the steam engine). For example, the rate at which a lightbulb transforms electrical energy into heat and light is measured in watts (W)—the more wattage, the more power, or equivalently the more electrical energy is used per unit time.





Power: A brief overview of power in an algebra-based physics course.

Energy transfer can be used to do work, so power is also the rate at which this work is performed. The same amount of work is done when carrying a load up a flight of stairs whether the person carrying it walks or runs, but more power is expended during the running because the work is done in a shorter amount of time. The output power of an electric motor is the product of the torque the motor generates and the angular velocity of its output shaft. The power expended to move a vehicle is the product of the traction force of the wheels and the velocity of the vehicle.

Examples of power are limited only by the imagination, because there are as many types as there are forms of work and energy. Sunlight reaching Earth's surface carries a maximum power of about 1.3 kilowatts per square meter (kW/m^2). A tiny fraction of this is retained by Earth over the long term. Our consumption rate of fossil fuels is far greater than the rate at which they are stored, so it is inevitable that they will be depleted. Power implies that energy is transferred, perhaps changing form. It is never possible to change one form completely into another without losing some of it as thermal energy. For example, a 60-W incandescent bulb converts only 5 W of electrical power to light, with 55 W dissipating into thermal energy. Furthermore, the typical electric power plant converts only 35 to 40 percent of its fuel into electricity. The remainder becomes a huge amount of thermal energy that must be dispersed as heat transfer, as rapidly as it is created. A coal-fired power plant may produce 1,000 megawatts; 1 megawatt (MW) is 106 W of electric power. But the power plant consumes chemical energy at a rate of about 2,500 MW, creating heat transfer to the surroundings at a rate of 1,500 MW.

Coal-fired Power Plant: Tremendous amounts of electric power are generated by coal-fired power plants such as this one in China, but an even larger amount of power goes into heat transfer to the surroundings. The large cooling towers here are needed to transfer heat as rapidly as it is produced. The transfer of heat is not unique to coal plants but is an unavoidable consequence of generating electric power from any fuel—nuclear, coal, oil, natural gas, or the like.

Humans: Work, Energy, and Power

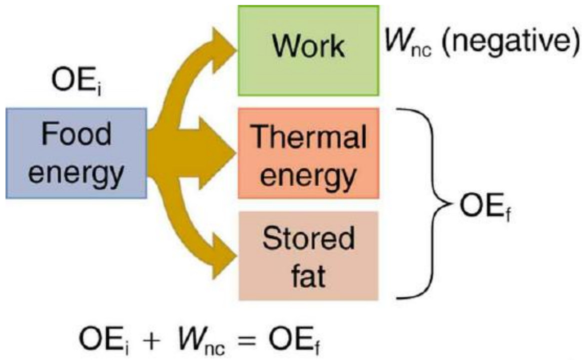
The human body converts energy stored in food into work, thermal energy, and/or chemical energy that is stored in fatty tissue.

learning objectives

- Identify what factors play a role in basal metabolic rate (BMR)

Humans: Work, Energy, and Power

Our own bodies, like all living organisms, are energy conversion machines. Conservation of energy implies that the chemical energy stored in food is converted into work, thermal energy, or stored as chemical energy in fatty tissue, as shown in. Energy consumed by humans is converted to work, thermal energy, and stored fat. By far the largest fraction goes to thermal energy, although the fraction varies depending on the type of physical activity. The fraction going into each form depends both on how much we eat and on our level of physical activity. If we eat more than is needed to do work and stay warm, the remainder goes into body fat.



Energy Conversion in Humans: Energy consumed by humans is converted to work, thermal energy, and stored fat. By far the largest fraction goes to thermal energy, although the fraction varies depending on the type of physical activity.

Functions that Require Energy

All bodily functions, from thinking to lifting weights, require energy. The many small muscle actions accompanying all quiet activity, from sleeping to head scratching, ultimately become thermal energy, as do less visible muscle actions by the heart, lungs, and digestive tract. Shivering, in fact, is an involuntary response to low body temperature that pits muscles against one another to produce thermal energy in the body (and do no work). The kidneys and liver consume a surprising amount of energy, but the biggest surprise of all it that a full 25% of all energy consumed by the body is used to maintain electrical potentials in all living cells. (Nerve cells use this electrical potential in nerve impulses.) This bioelectrical energy ultimately becomes mostly thermal energy, but some is utilized to power chemical processes such as in the kidneys and liver, and in fat production.

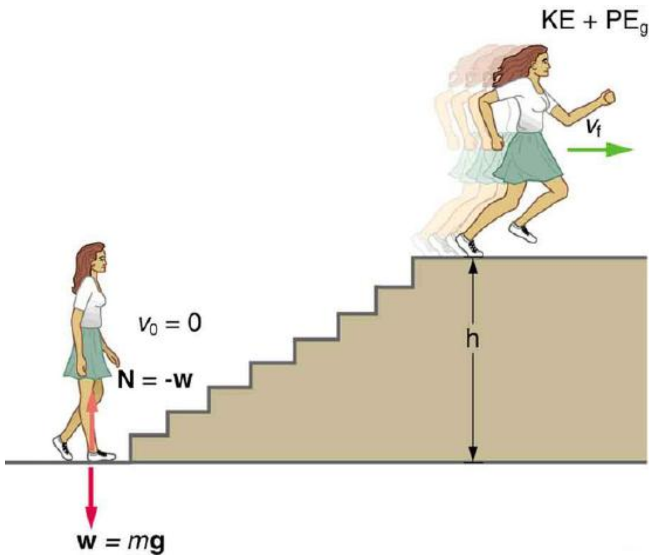
Basal Metabolic Rate

The rate at which the body uses food energy to sustain life and to do different activities is called the metabolic rate. The total energy conversion rate of a person at rest is called the basal metabolic rate (BMR) and is divided among various systems in the body. The largest fraction goes to the liver and spleen, with the brain coming next. Of course, during vigorous exercise, the energy consumption of the skeletal muscles and heart increase markedly. About 75% of the calories burned in a day go into these basic functions. The BMR is a function of age, gender, total body weight, and amount of muscle mass (which burns more calories than body fat). Athletes have a greater BMR due to this last factor.

Useful Work

Work done by a person is sometimes called useful work, which is work done on the outside world, such as lifting weights. Useful work requires a force exerted through a distance on the outside world, and so it excludes internal work, such as that done by the heart when pumping blood. Useful work does include that done in climbing stairs or accelerating to a full run, because these are accomplished by exerting forces on the outside world. Forces exerted by the body are nonconservative, so that they can change the mechanical energy (KE+PE) of the system worked upon, and this is often the goal.

For example, what is the power output for a 60.0-kg woman who runs up a 3.00 m high flight of stairs in 3.50 s, starting from rest but having a final speed of 2.00 m/s?.



Woman Running Up Stairs: When this woman runs upstairs starting from rest, she converts the chemical energy originally from food into kinetic energy and gravitational potential energy. Her power output depends on how fast she does this.

Her power output depends on how fast she does this. The work going into mechanical energy is W= KE + PE. At the bottom of the stairs, we take both KE and PE_g as initially zero; thus,

$$W = KE_f + PE_g = \frac{1}{2}mv_f^2 + mgh \tag{6.6.1}$$

where h is the vertical height of the stairs. Because all terms are given, we can calculate W and then divide it by time to get power. Substituting the expression for W into the definition of power given in the previous equation, $P = \frac{W}{t}$ yields

$$P = \frac{W}{t} = \frac{\frac{1}{2}mv_f^2 + mgh}{t} \tag{6.6.2}$$

Entering known values yields

$$P = \frac{0.5(60.0kg)(2.00m/s)^2 + (60.0kg)(9.80m/s^2)(3.00m)}{(3.50s)} = \frac{120J + 1764J}{3.50s} = 538W \tag{6.6.3}$$

The woman does 1764 J of work to move up the stairs compared with only 120 J to increase her kinetic energy; thus, most of her power output is required for climbing rather than accelerating.

Energy consumption is directly proportional to oxygen consumption because the digestive process is basically one of oxidizing food. We can measure the energy people use during various activities by measuring their oxygen use. Approximately 20 kJ of energy are produced for each liter of oxygen consumed, independent of the type of food.

Key Points

- Power implies that energy is transferred, perhaps changing form.
- Energy transfer can be used to do work, so power is also the rate at which this work is performed.
- The unit of power is the joule per second (J/s), known as the watt.
- The rate at which the body uses food energy to sustain life and to do different activities is called the metabolic rate, and the corresponding rate when at rest is called the basal metabolic rate (BMR).
- The energy included in the basal metabolic rate is divided among various systems in the body, with the largest fraction going to the liver and spleen, and the brain coming next.
- About 75% of food calories are used to sustain basic body functions included in the basal metabolic rate.
- Work done by a person is sometimes called useful work, which is work done on the outside world, such as lifting weights.
- The energy consumption of people during various activities can be determined by measuring their oxygen use, because the digestive process is basically one of oxidizing food.

Key Terms

- **power**: A measure of the rate of doing work or transferring energy.
- **watt**: In the International System of Units, the derived unit of power; the power of a system in which one joule of energy is transferred per second.
- **basal metabolic rate**: The amount of energy expended while at rest in a neutrally temperate environment, in the post-absorptive state.

6.7: CASE STUDY: World Energy Use

World Energy Use

The most prominent sources of energy used in the world are non-renewable (i.e., unsustainable).

learning objectives

- Explain why renewable energy sources must be found and utilized

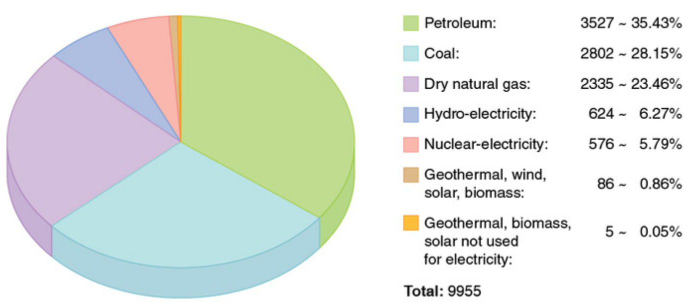
Energy Use

World energy consumption is the total amount of energy used by all humans on the planet (measured on a per-year basis). This measurement is the sum of all energy sources (and purposes) in use. Who measures this? Several organizations publish this data, including the International Energy Agency (IEA), the US Energy Information Administration (EIA), and the European Environment Agency. This data is useful because evaluating this information to discover trends might yield energy issues not currently being addressed, thereby encouraging the search for solutions. The IEA established a goal of limiting global warming to 2 degrees Celsius, but this goal is becoming more difficult to reach each year that the necessary action is not taken. In global energy use, fossil fuels make up a substantial portion. In 2011 they received over $500 billion in subsidies—six times more than that received by renewable energy sources.

Implementing new practices that will utilize different, renewable energy sources is important because having access to energy is important—it maintains our quality of life. Fossil fuels, however, are not sustainable at the rate they are currently used. About 40% of the world's energy comes from oil, but oil prices are dependent on uncertain factors (such as availability, politics, and world events). The United States alone uses 24% of the world's oil per year, yet it makes up only 4.5% of the world's population! In 2008, total worldwide energy consumption was 474 exajoules (474×10^{18} J=132,000 TWh)—equivalent to an average power usage of 15 terawatts (1.504×10^{13} W). Potential renewable energy sources include: solar energy at 1600 EJ (444,000 TWh), wind power at 600 EJ (167,000 TWh), geothermal energy at 500 EJ (139,000 TWh), biomass at 250 EJ (70,000 TWh), hydropower at 50 EJ (14,000 TWh) and ocean energy at 1 EJ (280 TWh).

Types of Energy

shows a pie chart of world energy usage by category—both renewable and nonrenewable sources. Renewable energy comes from sources with an unlimited supply. This includes energy from water, wind, the sun, and biomass. In the US, only 10% of energy comes from renewable sources (mostly hydroelectric energy). Nonrenewable sources makes up 85% of worldwide energy usage—from sources that eventually will be depleted, such as oil, natural gases and coal.



World Energy Use: This chart shows that the primary worldwide energy sources nonrenewable. If new practices are not put in place now, this model will not be sustainable.

Energy Needs

In the last 50 years, the global energy demand has tripled due to the number of developing countries and innovations in technology. It is projected to triple again over the next 30 years. In Europe, many in such developing areas recognize that the need for renewable energy sources, as the present course of energy usage cannot be sustained indefinitely. While renewable energy development makes up a only small percentage of the field, strides are being made in natural energy, particularly wind energy.

For example, by the year 2020 Germany plans to meet 10% of their total energy usage and 20% of its electricity usage with renewable resources. While some countries are making improvements in this field, coal usage is still a huge problem. In China, two thirds of the energy used each year is from commercial coal energy. India imports 50% of its oil, and 70% of its electricity is produced from coal, which is highly polluting.

Key Points

- The energy consumption increases with the increasing number of developing areas. In order for this development to continue, while maintaining quality of life, new and renewable energy sources must be found and utilized.
- Renewable energy comes from sources that will never deplete, no matter how much is used. An example of this is wind energy, which had been growing in popularity in countries like India and Germany.
- Nonrenewable energy makes up 85% the energy used on earth—the most popular form of energy being oil.

Key Terms

- **fossil fuel**: Any fuel derived from hydrocarbon deposits such as coal, petroleum, natural gas and, to some extent, peat; these fuels are irreplaceable, and their burning generates the greenhouse gas carbon dioxide.
- **renewable energy**: Energy that can be replenished at the same rate as it is used.

6.8: Further Topics

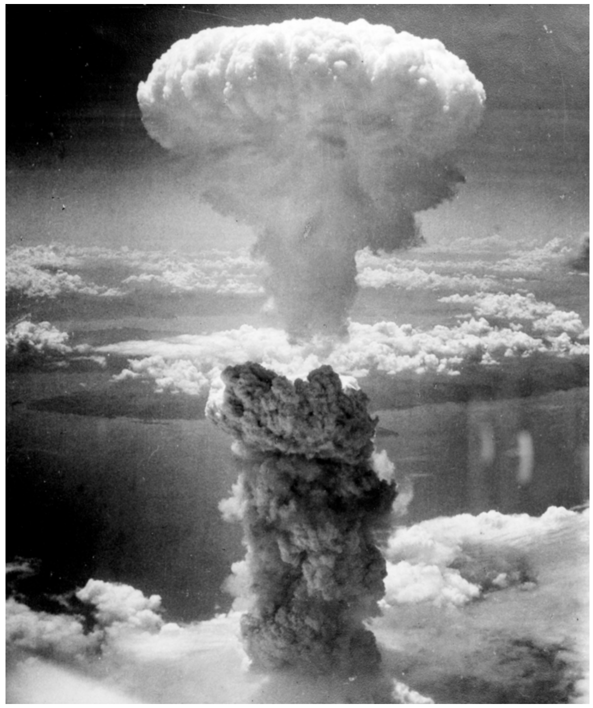
learning objectives

- Compare the different forms of energy interrelate to one another

Other Forms of Energy

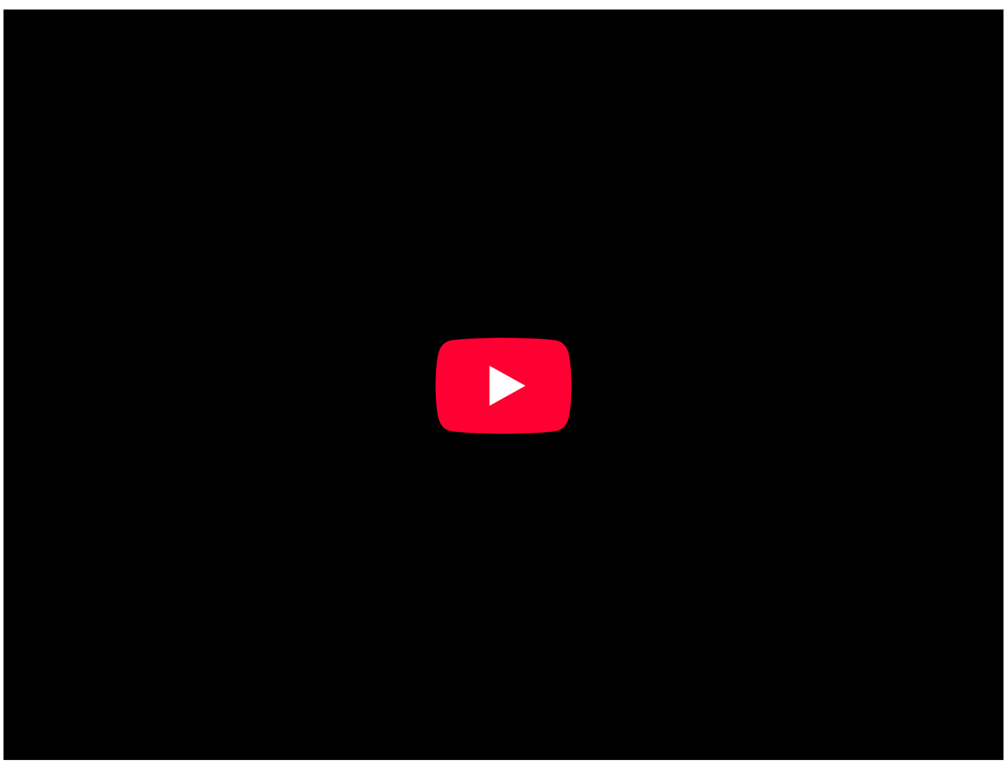
Thermal, chemical, electric, radiant, nuclear, magnetic, elastic, sound, mechanical, luminous, and mass are forms that energy can exist in. Energy can come in a variety of forms. These forms include:

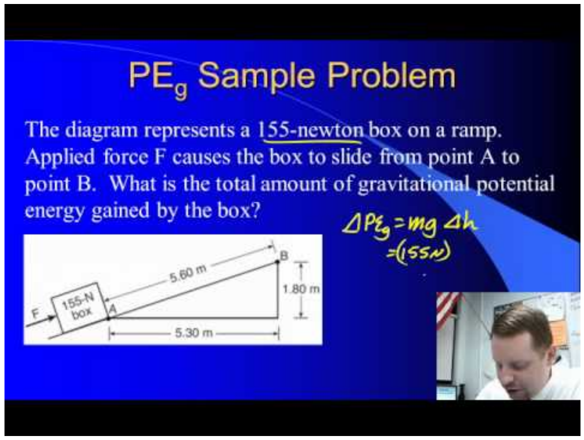
- Thermal Energy: This is energy associated with the microscopic random motion of particles in the media under consideration. An example of something that stores thermal energy is warm bath water.
- Chemical Energy: This is energy due to the way that atoms are arranged in molecules and various other collections of matter. An example of something that stores chemical energy is food. When your body digests and metabolizes food it utilizes its chemical energy.
- Electric Energy: This is energy that is from electrical potential energy, a result of Coulombic forces. Electrical potential energy is associated with the way that point charges in a system are arranged. An example of something that stores electric energy is a capacitor. A capacitor collects positive charge on one plate and negative charge on the other plate. Energy is thus stored in the resulting electrostatic field.
- Radiant Energy: This is any kind of electromagnetic radiation (see key term). An example of an electromagnetic wave is light.
- Nuclear Energy: This type of energy is liberated during the nuclear reactions of fusion and fission. Examples of things that utilize nuclear energy include nuclear power plants and nuclear weapons.
- Magnetic Energy: Technically magnetic energy is electric energy; the two are related by Maxwell's equations. An example of something that stores magnetic energy is a superconducting magnet used in an MRI.
- Elastic Energy: This is potential mechanical energy that is stored in the configuration of a material or physical system as work is performed to distort its volume or shape. An example of something that stores elastic energy is a stretched rubber band.
- Sound Energy: This is energy that is associated with the vibration or disturbance of matter. An example of something that creates sound energy is your voice box (larynx).
- Mechanical Energy: This is energy that is associated with the motion and position of an object. It is the sum of all of the kinetic and potential energy that the object has. An example of something that utilizes mechanical energy is a pendulum.
- Luminous Energy: This is energy that can be seen because it is visible light. An example of luminous energy is light from a flashlight.
- Mass: Can be converted to energy via: $\mathrm{E} = \mathrm{mc}^2$. For example, mass is converted into energy when a nuclear bomb explodes.



Atomic bomb explosion: The mushroom cloud of the atomic bombing of Nagasaki, Japan

In each of the aforementioned forms, energy exists as either kinetic energy, potential energy, or a combination of both. It is important to note that the above list is not necessarily complete as we may discover new forms of energy in the future such as “dark energy. ” Also, each of the forms that energy can take on (as listed above) are not necessarily mutually exclusive. For example, luminous energy is radiant energy.





Types of Energy: A brief overview of energy, kinetic energy, gravitational potential energy, and the work-energy theorem for algebra-based physics students.

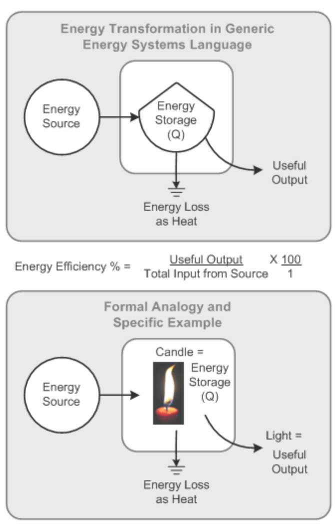
Energy Transformations

Energy transformation occurs when energy is changed from one form to another, and is a consequence of the first law of thermodynamics.

learning objectives

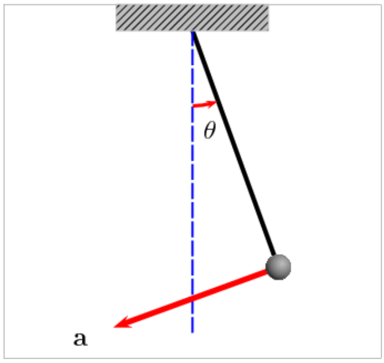
- Summarize the consequence of the first law of thermodynamics on the total energy of a system

Energy transformation occurs when energy is changed from one form to another. It is a consequence of the first law of thermodynamics that the total energy of a given system can only be changed when energy is added or subtracted from the system. Often it appears that energy has been lost from a system when it simply has been transformed. For example, an internal combustion engine converts the potential chemical energy in gasoline and oxygen into heat energy. This heat energy is then converted to kinetic energy, which is then used to propel the vehicle that is utilizing the engine. The technical term for a device that converts energy from one form to another is a *transducer*.



Energy Transformation: These figures illustrate the concepts of energy loss and useful energy output.

When analyzing energy transformations, it is important to consider the efficiency of conversion. The efficiency of conversions describes the ratio between the useful output and input of an energy conversion machine. Some energy transformations can occur with an efficiency of essentially 100%. For example, imagine a pendulum in a vacuum. As illustrated in, when the pendulum's mass reaches its maximum height, all if its energy exists in the form of potential energy. However, when the pendulum is at its lowest point, all of its energy exists in the form of kinetic energy.



Pendulum: This animation shows the velocity and acceleration vectors for a pendulum. One may note that at the maximum height of the pendulum's mass, the velocity is zero. This corresponds to zero kinetic energy and thus all of the energy of the pendulum is in the form of potential energy. When the pendulum's mass is at its lowest point, all of its energy is in the form of kinetic energy and we see its velocity vector has a maximum magnitude here.

Other energy transformations occur with a much lower efficiency of conversion. For example, the theoretical limit of the energy efficiency of a wind turbine (converting the kinetic energy of the wind to mechanical energy) is 59%. The process of photosynthesis is able to transform the light energy of the sun into chemical energy that can be used by a plant with an efficiency of conversion of a mere 6%.

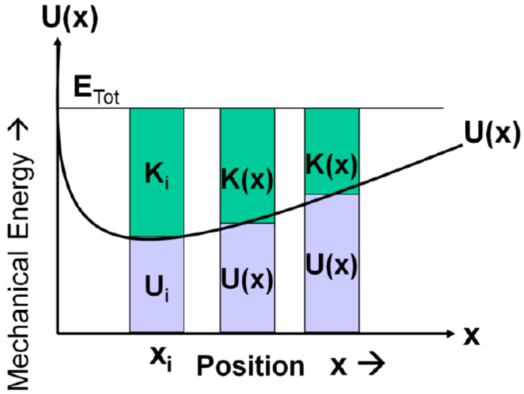
Potential Energy Curves and Equipotentials

A potential energy curve plots potential energy as a function of position; equipotential lines trace lines of equal potential energy.

learning objectives

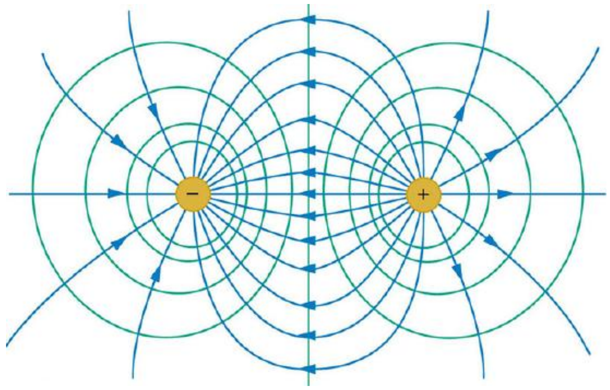
- Derive the potential of a point charge

A potential energy curve plots the potential energy of an object as a function of that object's position. For example, see. The system under consideration is a closed system, so the total energy of the system remains constant. This means that the kinetic and potential energy always have to sum to be the same value. We observe that the potential energy increases as the kinetic energy decreases and vice versa. The utility of a potential energy curve is that we can quickly determine the potential energy of the object in question at a given position.



Potential Energy Curve: This figure illustrates the potential energy of a particle as a function of position. The kinetic energy is also shown and is abbreviated K.

Equipotential lines trace lines of equal potential energy. In, if you were to draw a straight horizontal line through the center, that would be an equipotential line. In and, if you travel along an equipotential line, the electric potential will be constant.



Equipotential Lines for Two Equal and Opposite Point Charges: Electric field (blue) and equipotential lines (green) for two equal and opposite charges

Let us examine the physical explanation for the equipotentials lines in. The equation for the potential of a point charge is $\mathrm{V}=\frac{\mathrm{kQ}}{\mathrm{r}}$, where *V* is the potential, *k* is a constant with a value of $8.99 \cdot 10^9$ N m^2/C^2, *Q* is the magnitude of the point charge, and *r*is the distance from the charge. So, every point that is the same distance from the point charge will have the same electric potential energy. Therefore, if we draw a circle around the point charge, every point on the circle will have the same potential energy.

Work (*W*) is a measure of the change in potential energy $(\Delta \mathrm{PE}): \mathrm{W}=-$\DeltaPE. Since the potential energy does not change along an equipotential line, you do not need to do any work to move along one. However, you *do* need to do work to move from one equipotential line to another. Recall that work is zero if force is perpendicular to motion; in the figures shown above, the forces resulting from the electric field are in the same direction as the electric field itself. So we note that each of the equipotential lines must be perpendicular to the electric field at every point.

Key Points

- Thermal, chemical, electric, radiant, nuclear, magnetic, elastic, sound, mechanical, luminous, and mass are forms that energy can exist in.
- Energy exists as either kinetic energy, potential energy, or a combination of both.
- We may discover new forms of energy (like “dark energy”) in the future.
- The total energy of a given system can only be changed when energy is added or subtracted from the system.
- Often it appears that energy has been lost from a system when it simply has been transformed.
- The efficiency of conversions describes the ratio between the useful output of an energy conversion machine and the input.
- A potential energy curve plots the potential energy of an object as a function of its position.
- Equipotential lines trace lines of equal potential energy.
- You do not need to do any work to move along an equipotential line.

Key Terms

- **fusion**: A nuclear reaction in which nuclei combine to form more massive nuclei with the concomitant release of energy.
- **electromagnetic radiation**: radiation (quantized as photons) consisting of oscillating electric and magnetic fields oriented perpendicularly to each other, moving through space
- **fission**: The process of splitting the nucleus of an atom into smaller particles; nuclear fission.
- **pendulum**: A body suspended from a fixed support so that it swings freely back and forth under the influence of gravity; it is commonly used to regulate various devices such as clocks.
- **first law of thermodynamics**: A version of the law of conservation of energy, specialized for thermodynamical systems. It is usually formulated by stating that the change in the internal energy of a closed system is equal to the amount of heat supplied to the system, minus the amount of work done by the system on its surroundings.
- **potential energy**: The energy an object has because of its position (in a gravitational or electric field) or its condition (as a stretched or compressed spring, as a chemical reactant, or by having rest mass)
- **kinetic energy**: The energy possessed by an object because of its motion, equal to one half the mass of the body times the square of its velocity.

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CHAPTER OVERVIEW

7: Linear Momentum and Collisions

Thumbnail: A pool break-off shot. (CC-SA-BY; No-w-ay).

7.1: Introduction

Linear Momentum

Linear momentum is the product of the mass and velocity of an object, it is conserved in elastic and inelastic collisions.

learning objectives

- Calculate the momentum of two colliding objects

In classical mechanics, linear momentum, or simply momentum (SI unit kg m/s, or equivalently N s), is the product of the mass and velocity of an object. Mathematically it is stated as:

$$\mathrm{p} = \mathrm{mv} \tag{7.1.1}$$

(Note here that p and v are vectors.) Like velocity, linear momentum is a vector quantity, possessing a direction as well as a magnitude. Linear momentum is particularly important because it is a conserved quantity, meaning that in a closed system (without any external forces) its total linear momentum cannot change.

Because momentum has a direction, it can be used to predict the resulting direction of objects after they collide, as well as their speeds. Momentum is conserved in both inelastic and elastic collisions. (Kinetic energy is not conserved in inelastic collisions but is conserved in elastic collisions.) It important to note that if the collision takes place on a surface with friction, or if there is air resistance, we would need to account for the momentum of the bodies that would be transferred to the surface and/or air.

Let's take a look at a simple, one-dimensional example: The momentum of a system of two particles is the sum of their momenta. If two particles have masses m_1 and m_2, and velocities v_1 and v_2, the total momentum is:

$$\mathrm{p} = \mathrm{p}_1 + \mathrm{p}_2 = \mathrm{m}_1\mathrm{v}_1 + \mathrm{m}_2\mathrm{v}_2. \tag{7.1.2}$$

Keep in mind that momentum and velocity are vectors. Therefore, in 1D, if two particles are moving in the same direction, v1 and v2 have the same sign. If the particles are moving in opposite directions they will have opposite signs.

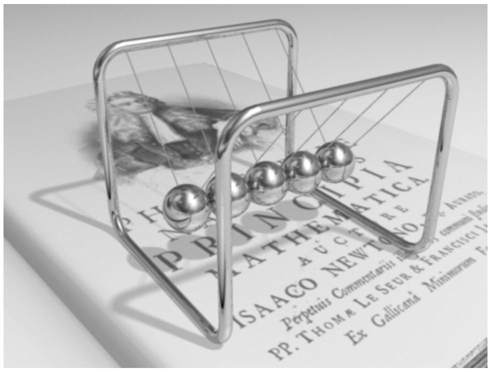
If two particles were moving on a plane we would choose our xy-plane to be on the plane of motion. We can then write the x and y component of the total momentum as:

$$\mathrm{p_x} = \mathrm{p_{1x}} + \mathrm{p_{2x}} = \mathrm{m_1 v_{1x}} + \mathrm{m_2 v_{2x}} \tag{7.1.3}$$

$$\mathrm{p_y} = \mathrm{p_{1y}} + \mathrm{p_{2y}} = \mathrm{m_1 v_{1y}} + \mathrm{m_2 v_{2y}}. \tag{7.1.4}$$

If the 2D momentum vector is decomposed into two components, the equations for each component are reduced to its 1D equivalents.

Momentum, like energy, is important because it is conserved. "Newton's cradle" shown in is an example of conservation of momentum. As we will discuss in the next concept (on Momentum, Force, and Newton's Second Law), in classical mechanics, conservation of linear momentum is implied by Newton's laws. Only a few physical quantities are conserved in nature. Studying these quantities yields fundamental insight into how nature works.



Newton's Cradle: Total momentum of the system (or Cradle) is conserved. (neglecting frictional loss in the system.)

Momentum, Force, and Newton's Second Law

In the most general form, Newton's 2nd law can be written as $\mathrm{F}=\frac{\mathrm{dp}}{\mathrm{dt}}$.

learning objectives

- Relate Newton's Second Law to momentum and force

In a closed system (one that does not exchange any matter with the outside and is not acted on by outside forces), the total momentum is constant. This fact, known as the law of conservation of momentum, is implied by Newton's laws of motion. Suppose, for example, that two particles interact. Because of the third law, the forces between them are equal and opposite. If the particles are numbered 1 and 2, the second law states that

$$\frac{\mathrm{dp}_1}{\mathrm{dt}}=-\frac{\mathrm{dp}_2}{\mathrm{dt}} \tag{7.1.5}$$

or

$$\frac{\mathrm{d}}{\mathrm{dt}}(\mathrm{p}_1+\mathrm{p}_2)=0 \tag{7.1.6}$$

Therefore, total momentum (p_1+p_2) is constant. If the velocities of the particles are u_1 and u_2 before the interaction, and afterwards they are v_1 and v_2, then

$$\mathrm{m}_1\mathrm{u}_1+\mathrm{m}_2\mathrm{u}_2=\mathrm{m}_1\mathrm{v}_1+\mathrm{m}_2\mathrm{v}_2 \tag{7.1.7}$$

This law holds regardless of the nature of the interparticle (or internal) force, no matter how complicated the force is between particles. Similarly, if there are several particles, the momentum exchanged between each pair of particles adds up to zero, so the total change in momentum is zero.

Newton's Second Law

Newton actually stated his second law of motion in terms of momentum: The net external force equals the change in momentum of a system divided by the time over which it changes. Using symbols, this law is

$$\mathrm{F}_{\mathrm{net}}=\frac{\Delta\mathrm{p}}{\Delta\mathrm{t}}, \tag{7.1.8}$$

where $\mathrm{F}_{\mathrm{net}}$ is the net external force, $\Delta\mathrm{p}$ is the change in momentum, and $\Delta\mathrm{t}$ is the change in time.

This statement of Newton's second law of motion includes the more familiar $\mathrm{F}_{\mathrm{net}}=\mathrm{ma}\color{red}{\backslash(}asaspecialcase. We canderive this form as follows. First, note that the change in momentum\color{red}{\backslash(}\Delta\mathrm{p}$ is given by $\Delta\mathrm{p}=\Delta(\mathrm{mv})$. If the mass of the system is constant, then $\Delta(\mathrm{mv})=\mathrm{m}\Delta\mathrm{v}$. So for constant mass, Newton's second law of motion becomes

$$\mathrm{F}_{\mathrm{net}}=\frac{\Delta\mathrm{p}}{\Delta\mathrm{t}}=\frac{\mathrm{m}\Delta\mathrm{v}}{\Delta\mathrm{t}}. \tag{7.1.9}$$

Because $\frac{\Delta\mathrm{v}}{\Delta\mathrm{t}}=\mathrm{a}$, we get the familiar equation $\mathrm{F}_{\mathrm{net}}=\mathrm{ma}$ when the mass of the system is constant. Newton's second law of motion stated in terms of momentum is more generally applicable because it can be applied to systems where the mass is changing, such as rockets, as well as to systems of constant mass.

Momentum in a Closed System: In a game of pool, the system of entire balls can be considered a closed system. Therefore, the total momentum of the balls is conserved.

Impulse

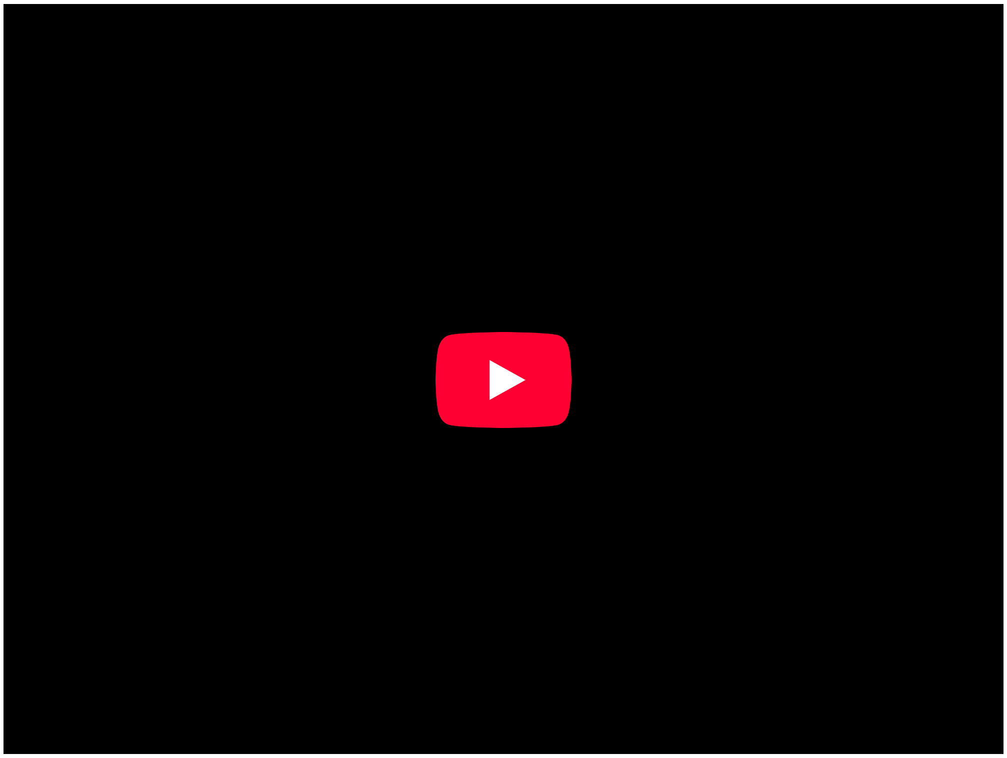
Impulse, or change in momentum, equals the average net external force multiplied by the time this force acts.

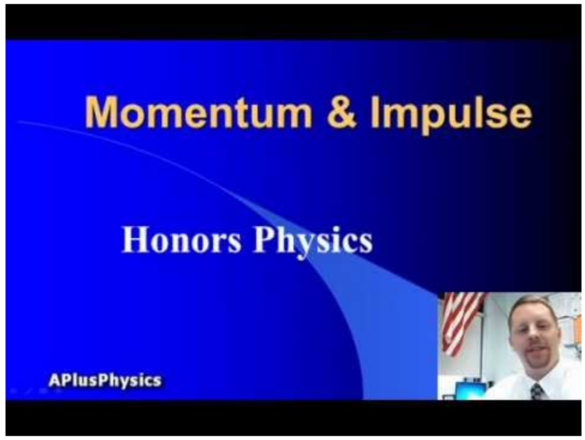
learning objectives

- Explain the relationship between change in momentum and the amount of time a force acts

Impulse

Forces produce either acceleration or deceleration on moving bodies, and the greater the force acting on an object, the greater its change in velocity and, hence, the greater its change in momentum. However, changing momentum is also related to how long a time the force acts. If a brief force is applied to a stalled automobile, a change in its momentum is produced. The same force applied over an extended period of time produces a greater change in the automobile's momentum. The quantity of impulse is *force* × *time interval*, or in shorthand notation:





Momentum & Impulse: A brief overview of momentum and impulse for high school physics students.

$$\mathrm{Impulse} = \mathrm{F}\Delta \mathrm{t}, \tag{7.1.10}$$

where F is the net force on the system, and Δt is the duration of the force.

From Newton's 2nd law:

$$\mathrm{F} = \frac{\Delta \mathrm{p}}{\Delta \mathrm{t}} \ (\Delta \mathrm{p} : \text{change in momentum}), \tag{7.1.11}$$

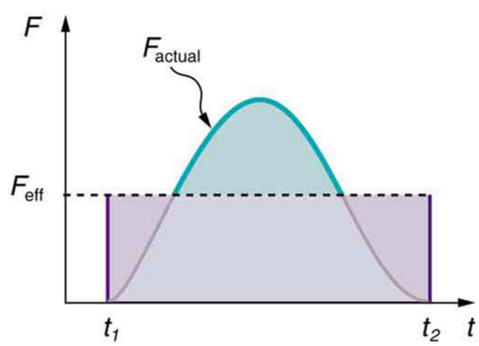
change in momentum equals the average net external force multiplied by the time this force acts.

$$\Delta \mathrm{p} = \mathrm{F}\Delta \mathrm{t}. \tag{7.1.12}$$

Therefore, impulse as defined in the previous paragraph is simply equivalent to p.

A force sustained over a long time produces more change in momentum than does the same force applied briefly. A small force applied for a long time can produce the same momentum change as a large force applied briefly because it is the product of the force and the time for which it is applied that is important. Impulse is always equal to change in momentum and is measured in Ns (Newton seconds), as both force and the time interval are important in changing momentum.

Our definition of impulse includes an assumption that the force is constant over the time interval $\Delta \mathrm{t}$. Forces are usually not constant. Forces vary considerably even during the brief time intervals considered. It is, however, possible to find an average effective force $\mathrm{F_{eff}}$ that produces the same result as the corresponding time-varying force. shows a graph of what an actual force looks like as a function of time for a ball bouncing off the floor. The area under the curve has units of momentum and is equal to the impulse or change in momentum between times $\mathrm{t_1}$ and $\mathrm{t_2}$. That area is equal to the area inside the rectangle bounded by $\mathrm{F_{eff}}, \mathrm{t_1}$, and $\mathrm{t_2}$. Thus, the impulses and their effects are the same for both the actual and effective forces. Equivalently, we can simple find the area under the curve F(t) between $\mathrm{t_1}$ and $\mathrm{t_2}$ to compute the impulse in mathematical form:



Force vs. Time: A graph of force versus time with time along the x-axis and force along the y-axis for an actual force and an equivalent effective force. The areas under the two curves are equal.

$$\mathrm{Impulse} = \int_{\mathrm{t_1}}^{\mathrm{t_2}} \mathrm{F(t)dt}. \tag{7.1.13}$$

Key Points

- Like velocity, linear momentum is a vector quantity, possessing a direction as well as a magnitude.
- Momentum, like energy, is important because it is a conserved quantity.
- The momentum of a system of particles is the sum of their momenta. If two particles have masses m_1 and m_2, and velocities v_1 and v_2, the total momentum is $p = p_1 + p_2 = m_1 v_1 + m_2 v_2.$.
- In a closed system, without any external forces, the total momentum is constant.
- The familiar equation $F = ma$ is a special case of the more general form of the 2nd law when the mass of the system is constant.
- Momentum conservation holds (in the absence of external force) regardless of the nature of the interparticle (or internal) force, no matter how complicated the force is between particles.
- A small force applied for a long time can produce the same momentum change as a large force applied briefly, because it is the product of the force and the time for which it is applied that is important.
- A force produces an acceleration, and the greater the force acting on an object, the greater its change in velocity and, hence, the greater its change in momentum. However, changing momentum is also related to how long a time the force acts.
- In case of a time-varying force, impulse can be calculated by integrating the force over the time duration. $\text{Impulse} = \int_{t_1}^{t_2} F(t)dt.$.

Key Terms

- **inelastic**: (As referring to an inelastic collision, in contrast to an elastic collision.) A collision in which kinetic energy is not conserved.
- **elastic collision**: An encounter between two bodies in which the total kinetic energy of the two bodies after the encounter is equal to their total kinetic energy before the encounter. Elastic collisions occur only if there is no net conversion of kinetic energy into other forms.
- **conservation**: A particular measurable property of an isolated physical system does not change as the system evolves.
- **closed system**: A physical system that doesn't exchange any matter with its surroundings and isn't subject to any force whose source is external to the system.
- **momentum**: (of a body in motion) the product of its mass and velocity.
- **impulse**: The integral of force over time.

7.2: Conservation of Momentum

Internal vs. External Forces

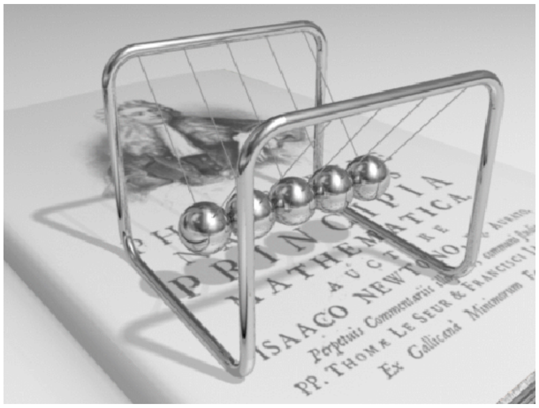
Net external forces (that are nonzero) change the total momentum of the system, while internal forces do not.

learning objectives

- Contrast the effects of external and internal forces on linear momentum and collisions

Linear Momentum and Collisions

Newton's 2nd law, applied to an isolated system composed of particles, $\mathbf{F}_{\mathrm{tot}} = \frac{\mathrm{d}\mathrm{p}_{\mathrm{tot}}}{\mathrm{d}\mathrm{t}} = 0$ indicates that the total momentum of the entire system $\mathrm{p}_{\mathrm{tot}}$ should be constant in the absence of net external forces. Forces external to the system may change the total momentum when their sum is not 0, but internal forces, regardless of the nature of the forces, will not contribute to the change in the total momentum. To analyze a mechanical system, it is important to recognize which forces are internal and which are external. Once a mechanical system is clearly defined, it's not hard to understand what part should be considered external.



Newton's Cradle: Total momentum of the system (or Cradle) is conserved. (neglecting frictional loss in the system.)

- External forces: forces caused by external agent outside of the system.
- Internal forces: forces exchanged by the particles in the system.

To give you a better idea, let's consider a simple example. We have two hockey pucks sliding across a frictionless surface, and we neglect air resistance for simplicity. They collide with each other at t=0.

Let's first list all the forces present in the system. There are mainly three kinds of forces: Gravity, normal force (between ice & pucks), and frictional forces during the collision between the pucks

How should we define our system? In most cases, we would be interested in the motion of the pucks (and nothing else). Therefore, our system consists of two pucks (and nothing else). All the rest of the universe becomes external. With this in mind, we can see that gravity and normal forces are external, while the frictional forces between pucks are internal. Since all the external forces cancel out with each other, there are no net external forces. (Gravity and normal force on each puck have the same magnitude, but are in the opposite directions) Therefore, we conclude that the total momentum of the two pucks should be a conserved quantity.

- In the previous example, it is worthwhile to note that we didn't assume anything about the nature of the collision between the two pucks. Without knowing anything about the internal forces (frictional forces during contact), we learned that the total momentum of the system is a conserved quantity (p1 and p2 are momentum vectors of the pucks.) In fact, this relation holds true both in elastic or inelastic collisions. Whether the total kinetic energy of the pucks is conserved or not, total momentum is conserved.
- Also note that, in the previous example, if we include the rest of the Earth in our system, the gravity and normal forces themselves become internal.

Key Points

- External forces are forces caused by external agent outside of the system.
- Internal forces are forces exchanged by the objects in the system.
- To determine what part should be considered external and internal, mechanical system should be clearly defined.

Key Terms

- **inelastic**: (As referring to an inelastic collision, in contrast to an elastic collision.) A collision in which kinetic energy is not conserved.
- **elastic**: referring to elastic collision, in contrast to inelastic collision. A collision in which kinetic energy is conserved

7.3: Collisions

Conservation of Energy and Momentum

In an inelastic collision the total kinetic energy after the collision is not equal to the total kinetic energy before the collision.

learning objectives

- Assess the conservation of total momentum in an inelastic collision

At this point we will expand our discussion of inelastic collisions in one dimension to inelastic collisions in multiple dimensions. It is still true that the total kinetic energy after the collision is not equal to the total kinetic energy before the collision. While inelastic collisions may not conserve total kinetic energy, they do conserve total momentum.

We will consider an example problem in which one mass ($\mathrm{m_1}$) slides over a frictionless surface into another initially stationary mass ($\mathrm{m_2}$). Air resistance will be neglected. The following things are known:

$$\mathrm{m_1 = 0.250 kg,} \tag{7.3.1}$$

$$\mathrm{m_2 = 0.400 kg,} \tag{7.3.2}$$

$$\mathrm{v_1 = 2.00 m/s,} \tag{7.3.3}$$

$$\mathrm{v_1\,' = 1.50 m/s,} \tag{7.3.4}$$

$$\mathrm{v_2 = 0 m/s,} \tag{7.3.5}$$

$$\theta_1\,' = 45.0\circ, \tag{7.3.6}$$

where $\mathrm{v_1}$ is the initial velocity of the first mass, $\mathrm{v_1\,'}$ is the final velocity of the first mass, $\mathrm{v_2}$ is the initial velocity of the second mass, and $\theta_1\,'$ is the angle between the velocity vector of the first mass and the x-axis.

The object is to calculate the magnitude and direction of the velocity of the second mass. After this, we will calculate whether this collision was inelastic or not.

Since there are no net forces at work (frictionless surface and negligible air resistance), there must be conservation of total momentum for the two masses. Momentum is equal to the product of mass and velocity. The initially stationary mass contributes no initial momentum. The components of velocities along the x-axis have the form $\mathrm{v \cdot \cos \theta}$, where θ is the angle between the velocity vector of the mass of interest and the x-axis.

Expressing these things mathematically:

$$\mathrm{m_1 v_1 = m_1 v_1\,' \cdot \cos(\theta_1) + m_2 v`_2 \cdot \cos(\theta_2). \; (Eq.\ 2)} \tag{7.3.7}$$

The components of velocities along the y-axis have the form $\mathrm{v \cdot \sin \theta}$, where θ is the angle between the velocity vector of the mass of interest and the x-axis. By applying conservation of momentum in the y-direction we find:

$$\mathrm{0 = m_1 v_1\,' \cdot \sin(\theta_1) + m_2 v`_2 \cdot \sin(\theta_2). \; (Eq.\ 3)} \tag{7.3.8}$$

If we divide Eq. 3 by Eq. 2, we will find:

$$\tan \theta_2 = \frac{\mathrm{v_1\,' \cdot \sin \theta_1}}{\mathrm{v_1\,' \cos \theta \theta_1 - v_1}} \mathrm{(Eq.\ 4)} \tag{7.3.9}$$

Eq. 4 can then be solved to find $\theta_2$approx. 312º.

Now let' use Eq. 3 to solve for $\mathrm{v\,'_2}$. Re-arranging Eq. 3, we find:

$$\mathrm{v_2\,' = \frac{-m_1 v_1\,' \cdot \sin \theta_1}{m_2 \cdot \sin \theta_2.}} \tag{7.3.10}$$

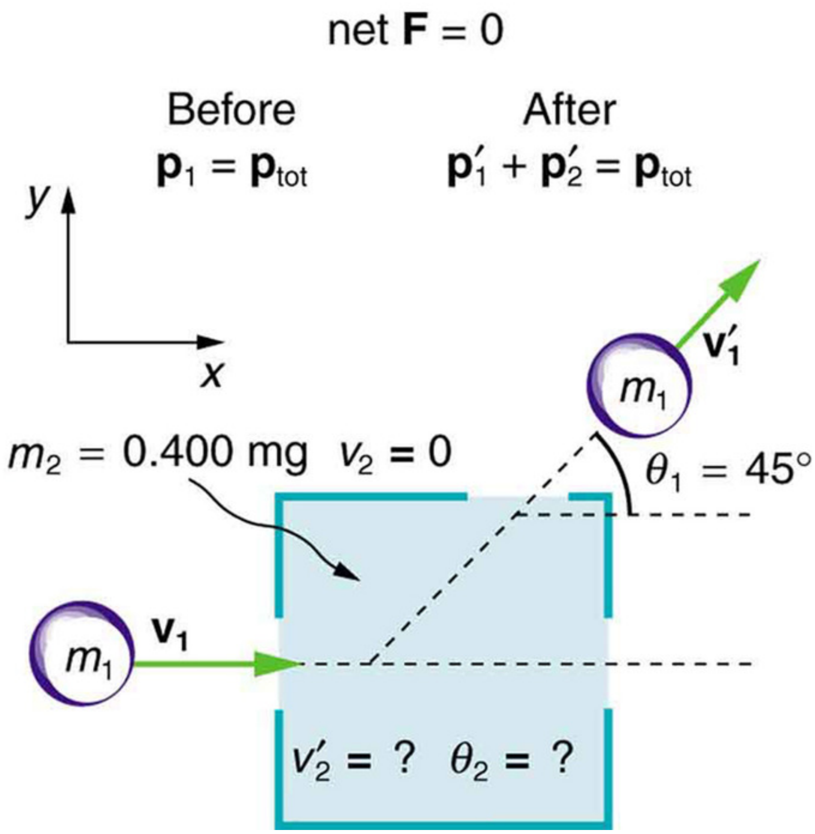
After plugging in our known values, we find that $\mathrm{v_2\,' = 0.886 m/s}$.

We can now calculate the initial and final kinetic energy of the system to see if it the same.

$$\text{Initial Kinetic Energy} = \frac{1}{2}\mathrm{m}_1 \cdot \mathrm{v}_1^2 + \frac{1}{2}\mathrm{m}_2 \cdot \mathrm{v}_2^2 = 0.5\mathrm{J}. \tag{7.3.11}$$

$$\text{Final Kinetic Energy} = \frac{1}{2}\mathrm{m}_1 \cdot \mathrm{v}_1{}'^2 + \frac{1}{2}\mathrm{m}_2 \cdot \mathrm{v}_2{}'^2 \approx 0.43\mathrm{J}. \tag{7.3.12}$$

Since these values are not the same we know that it was an inelastic collision.



Collision Example: This illustrates the example problem in which one mass collides into another mass that is initially stationary.

Glancing Collisions

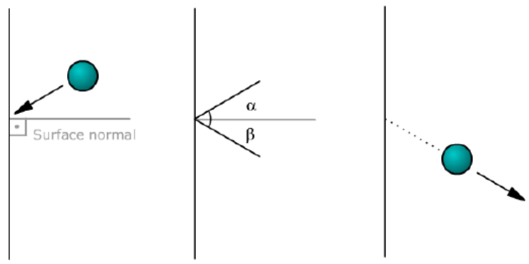
Glancing collision is a collision that takes place under a small angle, with the incident body being nearly parallel to the surface.

learning objectives

- Identify necessary conditions for a “glancing collision”

A collision is short duration interaction between two bodies or more than two bodies simultaneously causing change in motion of bodies involved due to internal forces acted between them during this. Collisions involve forces (there is a change in velocity). The magnitude of the velocity difference at impact is called the closing speed. All collisions conserve momentum. What distinguishes different types of collisions is whether they also conserve kinetic energy. Line of impact – It is the line which is common normal for surfaces are closest or in contact during impact. This is the line along which internal force of collision acts during impact and Newton’s coefficient of restitution is defined only along this line.

When dealing with an incident body that is nearly parallel to a surface, it is sometimes more useful to refer to the angle between the body and the surface, rather than that between the body and the surface normal (see), in other words 90° minus the angle of incidence. This small angle is called a glancing angle. Collision at glancing angle is called “glancing collision”.



Collision: Object is deflected after the collision withthe surface. The angles between the body and the surface normal areindicated as α and β. The angles between the body and the surface are $90-\alpha$ and $90-\beta$.

Collisions can either be elastic, meaning they conserve both momentum and kinetic energy, or inelastic, meaning they conserve momentum but not kinetic energy. An inelastic collision is sometimes also called a plastic collision.

A “perfectly-inelastic” collision (also called a “perfectly-plastic” collision) is a limiting case of inelastic collision in which the two bodies stick together after impact.

The degree to which a collision is elastic or inelastic is quantified by the coefficient of restitution, a value that generally ranges between zero and one. A perfectly elastic collision has a coefficient of restitution of one; a perfectly-inelastic collision has a coefficient of restitution of zero.

Elastic Collisions in One Dimension

An elastic collision is a collision between two or more bodies in which kinetic energy is conserved.

learning objectives

- Assess the relationship among the collision equations to derive elasticity

An elastic collision is a collision between two or more bodies in which the total kinetic energy of the bodies before the collision is equal to the total kinetic energy of the bodies after the collision. An elastic collision will not occur if kinetic energy is converted into other forms of energy. It important to understand how elastic collisions work, because atoms often undergo essentially elastic collisions when they collide. On the other hand, molecules do not undergo elastic collisions when they collide. In this atom we will review case of collision between two bodies.

The mathematics of an elastic collision is best demonstrated through an example. Consider a first particle with mass m_1 and velocity v_{1i} and a second particle with mass m_2 and velocity v_{2i}. If these two particles collide, there must be conservation of momentum before and after the collision. If we know that this is an elastic collision, there must be conservation of kinetic energy by definition. Therefore, the velocities of particles 1 and 2 after the collision (v_{1f} and v_{2f} respectively) will be related to the initial velocities by:

$\frac{1}{2}m_1 \cdot v_{1i}^2 + \frac{1}{2}m_2 \cdot v_{2i}^2 = \frac{1}{2}m_1 \cdot v_{1f}^2 + \frac{1}{2}m_2 \cdot v_{2f}^2$ (due to conservation of kinetic energy)

and

$m_1 \cdot v_{1i} + m_2 \cdot v_{2i} = m_1 \cdot v_{1f} + m_2 \cdot v_{2f}$ (due to conservation of momentum).

Since we have two equations, we are able to solve for any two unknown variables. In our case, we will solve for the final velocities of the two particles.

By grouping like terms and canceling out the ½ terms, we can rewrite our conservation of kinetic energy equation as:

$$m_1 \cdot (v_{1i}^2 - v_{1f}^2) = m_2 \cdot (v_{2f}^2 - v_{2i}^2). \text{ (Eq.1)} \tag{7.3.13}$$

By grouping like terms from our conservation of momentum equation we can find:

$$m_1 \cdot (v_{1i} - v_{1f}) = m_2 \cdot (v_{2f} - v_{2i}). \text{ (Eq. 2)} \tag{7.3.14}$$

If we then divide Eq. 1 by Eq. 2 and perform some cancelations we will find:

$$v_{1i} + v_{1f} = v_{2f} + v_{2i}. \text{ (Eq. 3)} \tag{7.3.15}$$

We can solve for v_{1f} as:

$$v_{1f} = v_{2f} + v_{2i} - v_{1i}. \text{(Eq. 4)} \tag{7.3.16}$$

At this point we see that v_{2f} is still an unknown variable. So we can fix this by plugging Eq. 4 into our initial conservation of momentum equation. Our conservation of momentum equation with Eq. 4 substituted in looks like:

$$m_1 \cdot v_{1i} + m_2 \cdot v_{2i} = m_1 \cdot (v_{2f} + v_{2i} - v_{1i}) + m_2 \cdot v_{2f}. \text{(Eq.5)} \tag{7.3.17}$$

After doing a little bit of algebra on Eq. 5 we find:

$$v_{2f} = \frac{2 \cdot m_1}{(m_2 + m_1)} v_{1i} + \frac{(m_2 - m_1)}{(m_2 + m_1)} v_{2i}. \text{(Eq.6)} \tag{7.3.18}$$

At this point we have successfully solved for the final velocity of the second particle. We still need to solve for the velocity of the first particle, so let us do that by plugging Eq. 6 into Eq. 4.

$$v_{1f} = [\frac{2 \cdot m_1}{(m_2 + m_1)} v_{1i} + \frac{(m_2 - m_1)}{(m_2 + m_1)} v_{2i}] + v_{2i} - v_{1i}. \text{(Eq. 7)} \tag{7.3.19}$$

After performing some algebraic manipulation of Eq. 7, we finally find:

$$v_{1f} = \frac{(m_1 - m_2)}{(m_2 + m_1)} v_{1i} + \frac{2 \cdot m_2}{(m_2 + m_1)} v_{2i}. \text{(Eq. 8)} \tag{7.3.20}$$

Elastic Collision of Two Unequal Masses: In this animation, two unequal masses collide and recoil.

Elastic Collisions in Multiple Dimensions

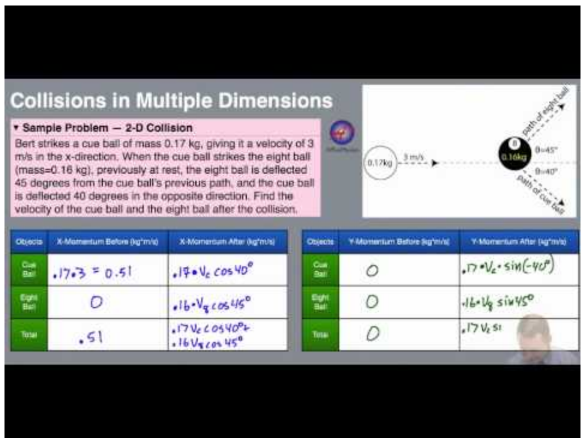
To solve a two dimensional elastic collision problem, decompose the velocity components of the masses along perpendicular axes.

learning objectives

- Construct an equation for elastic collision

Overview

As stated previously, there is conservation of total kinetic energy before and after an elastic collision. If an elastic collision occurs in two dimensions, the colliding masses can travel side to side after the collision (not just along the same line as in a one dimensional collision). The general approach to solving a two dimensional elastic collision problem is to choose a coordinate system in which the velocity components of the masses can be decomposed along perpendicular axes.



Collisions in Multiple Dimensions: A brief introduction to problem solving of collisions in two dimensions using the law of conservation of momentum.

Example 7.3.1:

In this example, we consider only point masses. These are structure-less particles that cannot spin or rotate. We will consider a case in which no outside forces are acting on the system, meaning that momentum is conserved. We will consider a situation in which one particle is initially at rest. This situation is illustrated in.

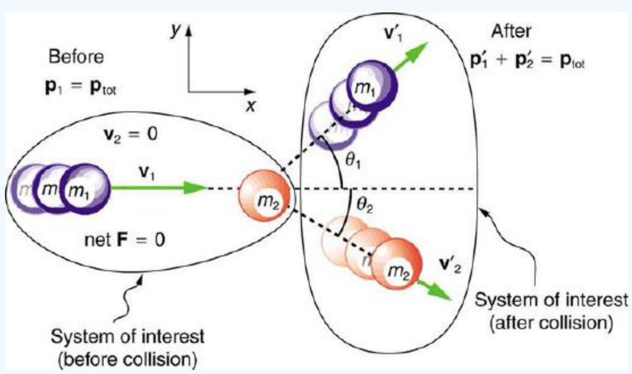


Illustration of Elastic Collision in Two Dimensions: In this illustration, we see the initial and final configurations of two masses that undergo an elastic collision in two dimensions.

By defining the x-axis to be along the direction of the incoming particle, we save ourselves time in breaking that velocity vector into its x- and y- components. Now let us consider conservation of momentum in the x direction:

$$p_{1x} + p_{2x} = p_{1x}{}' + p_{2x}{}'(\mathrm{Eq.}\ 1) \tag{7.3.21}$$

In Eq. 1, the initial momentum of the incoming particle is represented by $p_{1x,}$ the initial momentum of the stationary particle is represented by $p_{2x,}$ the final momentum of the incoming particle is represented by $p'_{1x.}$ and the final momentum of the initially stationary particle is represented by p'_{2x}.

We can expand Eq. 1 by taking into account that momentum is equal to the product of mass and velocity. Also, we know that $p_{2x} = 0$ because the initial velocity of the stationary particle is 0.

The components of velocities along the x-axis have the form $v \cdot \cos\theta$, where θ is the angle between the velocity vector of the particle of interest and the x-axis.

Therefore:

$$m_1 v_1 = m_1 v_1{}' \cdot \cos(\theta_1) + m_2 v'_2 \cdot \cos(\theta_2)(\mathrm{Eq.}\ 2) \tag{7.3.22}$$

The components of velocities along the y-axis have the form $v \cdot \sin\theta$, where θ is the angle between the velocity vector of the particle of interest (denoted in the following equations by subscript 1 or 2) and the x-axis. We can apply conservation of momentum in the y-direction in a similar way to yield:

$$0 = m_1 v'_1 \cdot \sin(\theta_1) + m_2 v'_2 \cdot \sin(\theta_2)(\mathrm{Eq.}\ 3) \tag{7.3.23}$$

In finding Eq. 3, it was taken into consideration that the incoming particle had no component of velocity along the y-axis.

Solving for Two Unknowns

Now we have gotten to a point where we have two equations, this means that we can solve for any two unknowns that we want. We also know that because the collision is elastic that there must be conservation of kinetic energy before and after the collision. This means that we may also write Eq. 4, which gives us three equations to solve for three unknowns:

$$\frac{1}{2} m_1 \cdot v_1^2 + \frac{1}{2} m_2 \cdot v_2^2 = \frac{1}{2} m_1 \cdot v_1{}'^2 + \frac{1}{2} m_2 \cdot v_2{}'^2 \tag{7.3.24}$$

The general approach to finding the defining equations for an n-dimensional elastic collision problem is to apply conservation of momentum in each of the n- dimensions. You can generate an additional equation by utilizing conservation of kinetic energy.

Inelastic Collisions in One Dimension

Collisions may be classified as either inelastic or elastic collisions based on how energy is conserved in the collision.

learning objectives

- Distinguish examples of inelastic collision from elastic collisions

Overview

In an inelastic collision the total kinetic energy after the collision is not equal to the total kinetic energy before the collision. This is in contrast to an elastic collision in which conservation of total kinetic energy applies. While inelastic collisions may not conserve total kinetic energy, they do conserve total momentum.

Collisions

If two objects collide, there are many ways that kinetic energy can be transformed into other forms of energy. For example, in the collision of macroscopic bodies, some kinetic energy is turned into vibrational energy of the constituent atoms. This causes a heating effect and results in deformation of the bodies. Another example in which kinetic energy is transformed into another form of energy is when the molecules of a gas or liquid collide. When this happens, kinetic energy is often exchanged between the molecules' translational motion and their internal degrees of freedom.

A perfectly inelastic collision happens when the maximum amount of kinetic energy in a system is lost. In such a collision, the colliding particles stick together. The kinetic energy is used on the bonding energy of the two bodies.

Sliding Block Example

Let us consider an example of a two-body sliding block system. The first block slides into the second (initially stationary block). In this perfectly inelastic collision, the first block bonds completely to the second block as shown. We assume that the surface over which the blocks slide has no friction. We also assume that there is no air resistance. If the surface had friction or if there was air resistance, one would have to account for the bodies' momentum that would be transferred to the surface and/or air.

Inelastic Collision: In this animation, one mass collides into another initially stationary mass in a perfectly inelastic collision.

Writing about the equation for conservation of momentum, one finds:

$$\mathrm{m_a u_a + m_b u_b = (m_a + m_b) v} \tag{7.3.25}$$

where $m_a$is the mass of the incoming block, u_a is the velocity of the incoming block, $m_b$is the mass of the initially stationary block, $u_b$is the velocity of initially stationary block (0 m/s), and v is the final velocity the two body system. Solving for the final velocity,

$$\mathrm{v = \frac{m_a u_a + m_b u_b}{m_a + m_b}}. \tag{7.3.26}$$

Taking into account that the blocks have the same mass and that the one of the blocks is initially stationary, the expression for the final velocity of the system may be defined as:

$$\mathrm{v = \frac{u_a}{2}}. \tag{7.3.27}$$

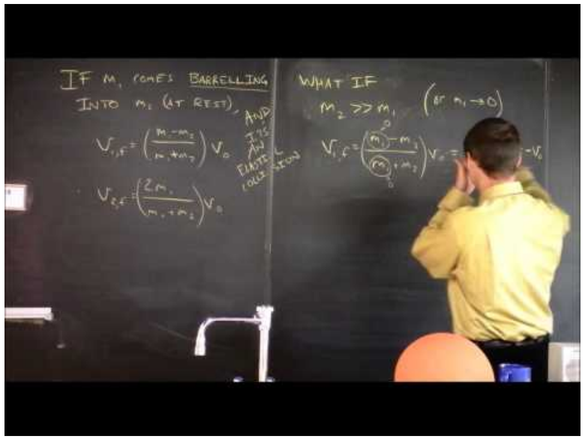
Inelastic Collisions in Multiple Dimensions

While inelastic collisions may not conserve total kinetic energy, they do conserve total momentum.

learning objectives

- Relate inelastic collision multiple dimension equations to the one dimension collisions you learned earlier

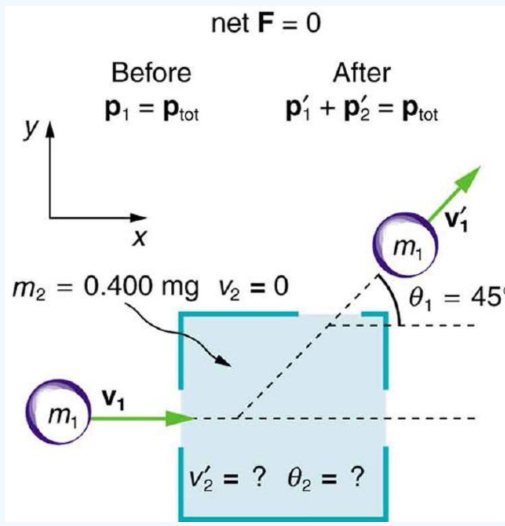
At this point we will expand our discussion of inelastic collisions in one dimension to inelastic collisions in multiple dimensions. It is still true that the total kinetic energy after the collision is not equal to the total kinetic energy before the collision. While inelastic collisions may not conserve total kinetic energy, they do conserve total momentum.



Example 7.3.2:

Examples of Collisions

We will consider an example problem, illustrated in, in which one mass (m1m1) slides over a frictionless surface into another initially stationary mass (m2m2). Air resistance will be neglected. The following quantities are known:



Collision Example: This illustrates the example problem in which one mass collides into another mass that is initially stationary.

$$m_1 = 0.250 kg, \tag{7.3.28}$$
$$m_2 = 0.400 kg, \tag{7.3.29}$$
$$v_1 = 2.00 m/s, \tag{7.3.30}$$
$$v_1{}' = 1.50 m/s, \tag{7.3.31}$$
$$v_2 = 0 m/s, \tag{7.3.32}$$
$$\theta_1{}' = 45.0^\circ, \tag{7.3.33}$$

where v1v1 is the initial velocity of the first mass, v′1v1′ is the final velocity of the first mass, v2v2is the initial velocity of the second mass, and θ′1θ1′ is the angle between the velocity vector of the first mass and the x-axis.

The object is to calculate the magnitude and direction of the velocity of the second mass. After this, we will calculate whether this collision was inelastic or not.

Since there are no net forces at work (frictionless surface and negligible air resistance), there must be conservation of total momentum for the two masses. Momentum is equal to the product of mass and velocity. The (initially) stationary mass contributes no initial momentum. The components of velocities along the x-axis have the form v⋅cosθv⋅cosθ, where θ is the angle between the velocity vector of the mass of interest and the x-axis.

Expressing these things mathematically:

$$\mathrm{m_1 v_1 = m_1 v_1{}' \cdot \cos(\theta_1) + m_2 v`_2 \cdot \cos(\theta_2). (Eq. 2)} \tag{7.3.34}$$

The components of velocities along the y-axis have the form $\mathrm{v \cdot \sin\theta}$, where θ is the angle between the velocity vector of the mass of interest and the x-axis. By applying conservation of momentum in the y-direction we find:

$$\mathrm{0 = m_1 v_1{}' \cdot \sin(\theta_1) + m_2 v`_2 \cdot \sin(\theta_2). (Eq. 3)} \tag{7.3.35}$$

If we divide Eq. 3 by Eq. 2, we will find:

$$\tan\theta_2 = \frac{\mathrm{v_1{}' \cdot \sin\theta_1}}{\mathrm{v_1{}'\cos\theta\theta_1 - v_1}} \mathrm{(Eq. 4)} \tag{7.3.36}$$

Eq. 4 can then be solved to find $\theta_2$approx. 312°.

Now let’ use Eq. 3 to solve for $\mathrm{v'_2}$. Re-arranging Eq. 3, we find:

$$\mathrm{v_2{}' = \frac{-m_1 v_1{}' \cdot \sin\theta_1}{m_2 \cdot \sin\theta_2.}} \tag{7.3.37}$$

After plugging in our known values, we find that $\mathrm{v_2{}' = 0.886 m/s}$.

We can now calculate the initial and final kinetic energy of the system to see if it the same.

$$\text{Initial Kinetic Energy} = \frac{1}{2}\mathrm{m_1 \cdot v_1^2} + \frac{1}{2}\mathrm{m_2 \cdot v_2^2 = 0.5 J.} \tag{7.3.38}$$

$$\text{Final Kinetic Energy} = \frac{1}{2}\mathrm{m_1 \cdot v_1{}'^2} + \frac{1}{2}\mathrm{m_2 \cdot v_2{}'^2 \approx 0.43 J.} \tag{7.3.39}$$

As these values are not the same, we know this was an inelastic collision.

Key Points

- In an inelastic collision the total kinetic energy after the collision is not equal to the total kinetic energy before the collision.
- If there are no net forces at work (collision takes place on a frictionless surface and there is negligible air resistance), there must be conservation of total momentum for the two masses.
- The variable θ is the angle between the velocity vector of the mass of interest and the x-axis in traditional Cartesian coordinate systems.
- Collision is short duration interaction between two bodies or more than two bodies simultaneously causing change in motion of bodies involved due to internal forces acted between them during this.
- Collisions can either be elastic, meaning they conserve both momentum and kinetic energy, or inelastic, meaning they conserve momentum but not kinetic energy.
- When dealing with an incident body that is nearly parallel to a surface, it is sometimes more useful to refer to the angle between the body and the surface, rather than that between the body and the surface normal.
- An elastic collision will not occur if kinetic energy is converted into other forms of energy.
- While molecules do not undergo elastic collisions, atoms often undergo elastic collisions when they collide.
- If two particles are involved in an elastic collision, the velocity of the first particle after collision can be expressed as: $\mathrm{v_{1f} = \frac{(m_1 - m_2)}{(m_2 + m_1)} v_{1i} + \frac{2 \cdot m_2}{(m_2 + m_1)} v_{2i}}$.
- If two particles are involved in an elastic collision, the velocity of the second particle after collision can be expressed as: $\mathrm{v_{2f} = \frac{2 \cdot m_1}{(m_2 + m_1)} v_{1i} + \frac{(m_2 - m_1)}{(m_2 + m_1)} v_{2i}}$.
- If an elastic collision occurs in two dimensions, the colliding masses can travel side to side after the collision.
- By defining the x- axis to be along the direction of the incoming particle, we can simplify the defining equations.
- The general approach to finding the defining equations for an n-dimensional elastic collision problem is to apply conservation of momentum in each of the n- dimensions. You can generate an additional equation by utilizing conservation of kinetic energy.
- In an inelastic collision, the total kinetic energy after the collision is not equal to the total kinetic energy before the collision.
- While inelastic collisions may not conserve total kinetic energy, they do conserve total momentum.

- A perfectly inelastic collision happens when the maximum amount of kinetic energy in a system is lost.
- In an inelastic collision the total kinetic energy after the collision is not equal to the total kinetic energy before the collision.
- If there are no net forces at work (i.e., collision takes place on a frictionless surface and there is negligible air resistance), there must be conservation of total momentum for the two masses.
- The variable θ is the angle between the velocity vector of the mass of interest and the x-axis in traditional Cartesian coordinate systems.

Key Terms

- **kinetic energy**: The energy possessed by an object because of its motion, equal to one half the mass of the body times the square of its velocity.
- **momentum**: (of a body in motion) the product of its mass and velocity.
- **force**: A physical quantity that denotes ability to push, pull, twist or accelerate a body which is measured in a unit dimensioned in mass × distance/time^2 (ML/T^2): SI: newton (N); CGS: dyne (dyn)
- **elastic collision**: An encounter between two bodies in which the total kinetic energy of the two bodies after the encounter is equal to their total kinetic energy before the encounter. Elastic collisions occur only if there is no net conversion of kinetic energy into other forms.
- **dimension**: A measure of spatial extent in a particular direction, such as height, width or breadth, or depth.
- **degrees of freedom**: A degree of freedom is an independent physical parameter, often called a dimension, in the formal description of the state of a physical system. The set of all dimensions of a system is known as a phase space.
- **friction**: A force that resists the relative motion or tendency to such motion of two bodies in contact.

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7.4: Rocket Propulsion

Rocket Propulsion, Changing Mass, and Momentum

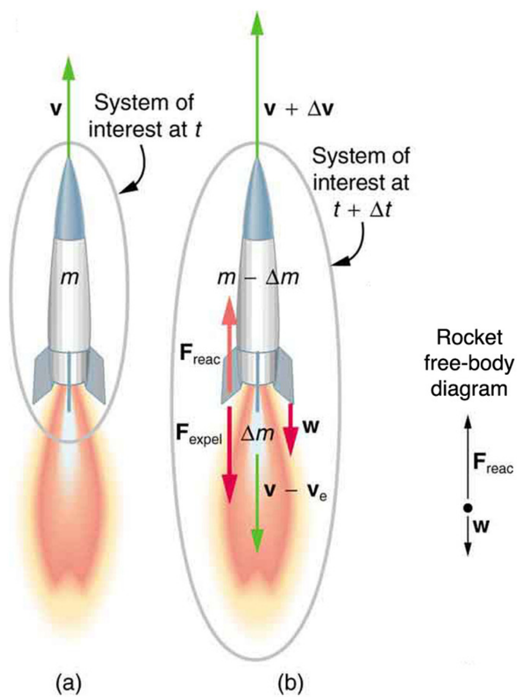
In rocket propulsion, matter is forcefully ejected from a system, producing an equal and opposite reaction on what remains.

> **learning objectives**
>
> - Identify physical principles of rocket propulsion

Rocket Propulsion, Changing Mass, and Momentum

Rockets range in size from fireworks so small that ordinary people use them to immense Saturn Vs that once propelled massive payloads toward the Moon. The propulsion of all rockets, jet engines, deflating balloons, and even squids and octopuses is explained by the same physical principle: Newton's third law of motion. Matter is forcefully ejected from a system, producing an equal and opposite reaction on what remains. Another common example is the recoil of a gun. The gun exerts a force on a bullet to accelerate it and consequently experiences an equal and opposite force, causing the gun's recoil or kick.

shows a rocket accelerating straight up. In part (a), the rocket has a mass m and a velocity v relative to Earth, and hence a momentum mv. In part (b), a time Δt has elapsed in which the rocket has ejected a mass Δm of hot gas at a velocity v_e relative to the rocket. The remainder of the mass $(m - m)$ now has a greater velocity $(v + \Delta v)$. The momentum of the entire system (rocket plus expelled gas) has actually decreased because the force of gravity has acted for a time Δt, producing a negative impulse $\Delta p = -mg\Delta t$. (Remember that impulse is the net external force on a system multiplied by the time it acts, and it equals the change in momentum of the system.) So the center of mass of the system is in free fall but, by rapidly expelling mass, part of the system can accelerate upward. It is a commonly held misconception that the rocket exhaust pushes on the ground. If we consider thrust; that is, the force exerted on the rocket by the exhaust gases, then a rocket's thrust is greater in outer space than in the atmosphere or on the launch pad. In fact, gases are easier to expel into a vacuum.



Free-body diagram of rocket propulsion: (a) This rocket has a mass m and an upward velocity v. The net external force on the system is −mg, if air resistance is neglected. (b) A time Δt later the system has two main parts, the ejected gas and the remainder of the rocket. The reaction force on the rocket is what overcomes the gravitational force and accelerates it upward.

By calculating the change in momentum for the entire system over Δt, and equating this change to the impulse, the following expression can be shown to be a good approximation for the acceleration of the rocket.

$$a = \frac{v_e}{m}\frac{\Delta m}{\Delta t} - g \tag{7.4.1}$$

where a is the acceleration of the rocket, v_e is the escape velocity, m is the mass of the rocket, Δm is the mass of the ejected gas, and Δt is the time in which the gas is ejected.

Factors of Acceleration

A rocket's acceleration depends on three major factors, consistent with the equation for acceleration of a rocket. First, the greater the exhaust velocity of the gases relative to the rocket, v_e, the greater the acceleration is. The practical limit for v_e is about 2.5×10^3 m/s for conventional (non-nuclear) hot-gas propulsion systems. The second factor is the rate at which mass is ejected from the rocket. This is the factor $\frac{\Delta m}{\Delta t}$ in the equation. The quantity $(\frac{\Delta m}{\Delta t})v_e$, with units of newtons, is called "thrust." The faster the rocket burns its fuel, the greater its thrust, and the greater its acceleration. The third factor is the mass m of the rocket. The smaller the mass is (all other factors being the same), the greater the acceleration. The rocket mass m decreases dramatically during flight because most of the rocket is fuel to begin with, so that acceleration increases continuously, reaching a maximum just before the fuel is exhausted.

To achieve the high speeds needed to hop continents, obtain orbit, or escape Earth's gravity altogether, the mass of the rocket other than fuel must be as small as possible. It can be shown that, in the absence of air resistance and neglecting gravity, the final velocity of a one-stage rocket initially at rest is

$$v = v_e \ln \frac{m_0}{m_r} \tag{7.4.2}$$

where $\ln(m_0/m_r)$ is the natural logarithm of the ratio of the initial mass of the rocket (m_0) to what is left (m_r) after all of the fuel is exhausted. (Note that v is actually the change in velocity, so the equation can be used for any segment of the flight. If we start from rest, the change in velocity equals the final velocity.)

Key Points

- The propulsion of all rockets is explained by the same physical principle: Newton's third law of motion.
- A rocket's acceleration depends on three major factors: the exhaust velocity, the rate the exhaust is ejected, and the mass of the rocket.
- To achieve the high speeds needed to hop continents, obtain orbit, or escape Earth's gravity altogether, the mass of the rocket other than fuel must be as small as possible.

Key Terms

- **Newton's third law of motion**: states that all forces exist in pairs: if one object A exerts a force FA on a second object B, then B simultaneously exerts a force FB on A, and the two forces are equal and opposite: $FA = -FB$.

7.5: Center of Mass

learning objectives

- Identify the center of mass for an object with continuous mass distribution

In the previous modules on “Center of Mass and Translational Motion,” we learned why the concept of center of mass (COM) helps solving mechanics problems involving a rigid body. Here, we will study the rigorous definition of COM and how to determine the location of it. The position of COM is mass weighted average of the positions of particles.

Definition: center of mass

The *center of mass* is a statement of spatial arrangement of mass (i.e. distribution of mass within the system). The position of COM is given a mathematical formulation which involves distribution of mass in space:

$$\mathrm{r_{COM}} = \frac{\sum_i \mathrm{m_i r_i}}{\mathrm{M}}, \tag{7.5.1}$$

where $\mathrm{r_{COM}}$ and $\mathrm{r_i}$ are vectors representing the position of COM and i-th particle respectively, and M and mi are the total mass and mass of the i-th particle, respectively. This mean means that position of COM is mass weighted average of the positions of particles.

Object with Continuous Mass Distribution

If the mass distribution is continuous with the density ρ(r) within a volume V, the position of COM is given as

$$\mathrm{r_{COM}} = \frac{1}{\mathrm{M}} \int_{\mathrm{V}} \rho(\mathrm{r}) \mathrm{r dV}, \tag{7.5.2}$$

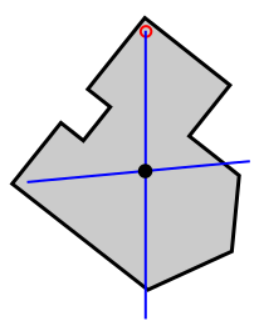
where M is the total mass in the volume. If a continuous mass distribution has uniform density, which means ρ is constant, then the center of mass is the same as the center of the volume.

Locating the Center of Mass

The experimental determination of the center of mass of a body uses gravity forces on the body and relies on the fact that in the parallel gravity field near the surface of Earth the center of mass is the same as the center of gravity.

The center of mass of a body with an axis of symmetry and constant density must lie on this axis. Thus, the center of mass of a circular cylinder of constant density has its center of mass on the axis of the cylinder. In the same way, the center of mass of a spherically symmetric body of constant density is at the center of the sphere. In general, for any symmetry of a body, its center of mass will be a fixed point of that symmetry.

In two dimensions: An experimental method for locating the center of mass is to suspend the object from two locations and to drop plumb lines from the suspension points. The intersection of the two lines is the center of mass.



Plumb Line Method for Center of Mass: Suspend the object from two locations and to drop plumb lines from the suspension points. The intersection of the two lines is the center of mass.

In three dimensions: By supporting an object at three points and measuring the forces that resist the weight of the object, COM of the three-dimensional coordinates of the center of mass can be determined.

Motion of the Center of Mass

We can describe the translational motion of a rigid body as if it is a point particle with the total mass located at the COM—center of mass.

learning objectives

- Derive the center of mass for the translational motion of a rigid body

We can describe the translational motion of a rigid body as if it is a point particle with the total mass located at the center of mass (COM). In this Atom. we will prove that the total mass (M) times the acceleration of the COM (a_{COM}), indeed, equals the sum of external forces. That is,

$$M \cdot a_{COM} = \sum F_{ext}. \tag{7.5.3}$$

You can see that the Newton's 2nd law applies as if we are describing the motion of a point particle (with mass M) under the influence of the external force.

Derivation

From the definition of the center of mass,

$$r_{COM} = \frac{\sum_i m_i r_i}{M}, \tag{7.5.4}$$

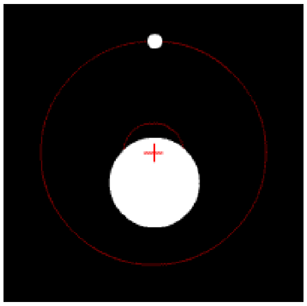
we get $M \cdot a_{COM} = \sum m_i a_i$ by taking time derivative twice on each side.

Note that $\sum m_i a_i = \sum F_i$.

In a system of particles, each particle may feel both external and internal forces. Here, external forces are forces from external sources, while internal forces are forces between particles in the system. Since the sum of all internal forces will be 0 due to the Newton's 3rd law,

$\sum F_i = \sum F_{i,ext}$. Therefore, we get $M \cdot a_{COM} = \sum F_{ext}$.

For example, when we confine our system to the Earth and the Moon, the gravitational force due to the Sun would be external, while the gravitational force on the Earth due to the Moon (and vice versa) would be internal. Since the gravitational forces between the Earth and the Moon are equal in magnitude and opposite in direction, they will cancel out each other in the sum (see).



COM of the Earth and Moon: Earth and Moon orbiting a COM inside the Earth. The red cross represents the COM of the two-body system. The COM will orbit around the Sun as if it is a point particle.

Corollary

When there is no external force, the COM momentum is conserved.

Proof: Since there is no external force, $M \cdot a_{COM} = 0$. Therefore,

$M \cdot v_{COM} = \text{constant}$.

Proof

Since there is no external force,

$$M \cdot a_{COM} = 0. \tag{7.5.5}$$

Therefore,

$M \cdot v_{COM} = constant$.

□

Center of Mass of the Human Body

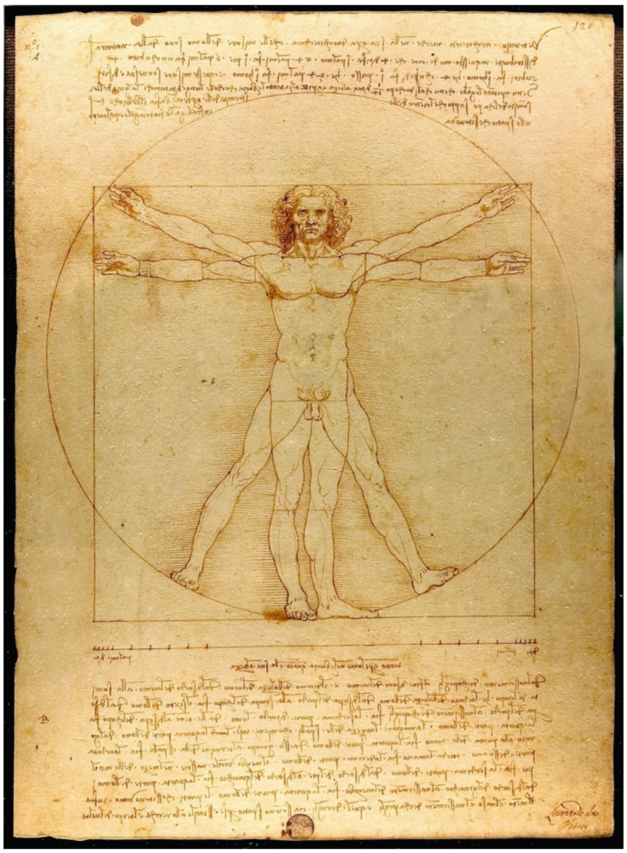
The center of mass (COM) is an important physical concept—it is the point about which objects rotate.

learning objectives

- Estimate the COM of a given object

The center of mass (COM) is an important physical concept. It is the point on an object at which the weighted relative position of the distributed mass sums to zero—the point about which objects rotate.

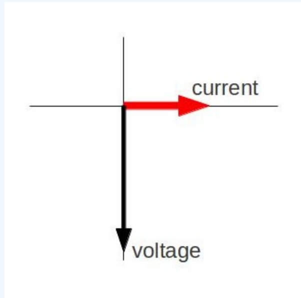
Human proportions have been important in art, measurement, and medicine (a well known drawing of the human body is seen in). Although the human body has complicated features, the location of the center of mass (COM) could be a good indicator of body proportions. The center of mass of the human body depends on the gender and the position of the limbs. In a standing posture, it is typically about 10 cm lower than the navel, near the top of the hip bones. In this Atom, we will learn how to measure the COM of a human body.



Leonardo da Vinci's "The Vitruvian Man": Vitruvian Man: A drawing created by Leonardo da Vinci. The drawing is based on the correlations of ideal human proportions with geometry described[4] by the ancient Roman architect Vitruvius in Book III of his treatise De Architectura.

Example 7.5.1:

First, let’s take two scales and a wooden beam (H meter long), long enough to contain the entire body of the subject. Put the scales H meters apart, and place the beam across the scales, as illustrated in. Now, let the subject lie on the beam. Make sure that his/her heels are aligned with one end of the beam. Measure the readings (F_1, F_2) on the scale.



The COM of a Human Body: This figure demonstrates measuring the COM of a human body.

The system (person+beam) has three external forces: gravity on the subject (F_{CM}), and normal forces from the scales F_1 and F_2. The equation of motion for force (F=ma) will give us:

$$\mathrm{F1+F2=Mg,} \tag{7.5.6}$$

where M is mass of the subject. (We assume that the wooden beam has no mass.) This equation doesn’t provide all the information to locate the COM. However, the equation of motion for torque $(\tau=\mathrm{I}\alpha)$ helps.

Since the net torque of the system is zero (hence no rotational acceleration),

$$\mathrm{hF_2-(H-h)F_1=0.(h:COM\ height)} \tag{7.5.7}$$

The COM is chosen as the origin for the torque. Therefore, gravity contributes nothing as a torque. Solving for h and using the equation of motion for force, we get

$$\mathrm{h=\frac{HF_1}{Mg}.} \tag{7.5.8}$$

Center of Mass and Translational Motion

The COM (center of mass) of a system of particles is a geometric point that assumes all the mass and external force(s) during motion.

learning objectives

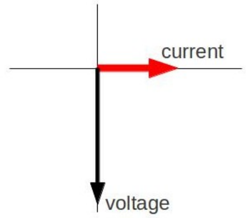
- Support the presence of COM in three dimensional bodies in motion

Introduction: COM, Linear Momentum, and Collisions

Our study of motion has been limited up to this point. We have referred to particle, object and body in the same way. We considered that actual three dimensional rigid bodies move such that all constituent particles had the same motion (i.e., same trajectory, velocity and acceleration). By doing this, we have essentially considered a rigid body as a point particle.

Center of Mass (COM)

An actual body, however, can move differently than this simplified paradigm. Consider a ball rolling down an incline plane or a stick thrown into air. Different parts of a body have different motions. While translating in the air, the stick rotates about a moving axis, as shown in. This means that such bodies may not behave like a point particle, as earlier suggested.



Forces on the COM: Left: The force appears to operate on the COM is “mgsinθ. Right: The force appears to operate on the COM is “mg”.

Describing motions of parts or particles that have different motions would be quite complicated to do in an integrated manner. However, such three dimensional bodies in motion have one surprising, simplifying characteristic—a geometric point that behaves like a particle. This point is known as center of mass, abbreviated COM (the mathematical definition of COM will be introduced in the next Atom on “Locating the Center of Mass”). It has the following two characterizing aspects:

- The center of mass appears to carry the whole mass of the body.
- At the center of mass, all external forces appear to apply.

Significantly, the center of a ball (the COM of a rolling ball) follows a straight linear path; whereas the COM of a stick follows a parabolic path (as shown in the figure above). Secondly, the forces appear to operate on the COMs in two cases (“mgsinθ and “mg”) as if they were indeed particle-like objects. This concept of COM, therefore, eliminate the complexities otherwise present in attempting to describe motions of rigid bodies.

Describing Motion in a Rigid Body

We can describe general motion of an object (with mass m) as follows:

- We describe the *translational motion* of a rigid body as if it is a point particle with mass m located at COM.
- *Rotation* of the particle, with respect to the COM, is described independently.

We “separate” the translational part of the motion from the rotational part. By introducing the concept of COM, the translational motion becomes that of a point particle with mass m. This simplifies significantly the mathematical complexity of the problem.

Key Points

- The center of mass (COM) is a statement of spatial arrangement of mass (i.e. distribution of mass within the system).
- The experimental determination of the center of mass of a body uses gravity forces on the body and relies on the fact that in the parallel gravity field near the surface of the earth the center of mass is the same as the center of gravity.
- For a 2D object, an experimental method for locating the center of mass is to suspend the object from two locations and to drop plumb lines from the suspension points. The intersection of the two lines is the center of mass.
- The total mass times the acceleration of the center of mass equals the sum of external forces.
- For the translational motion of a rigid body with mass M, Newton’s 2nd law applies as if we are describing the motion of a point particle (with mass M) under the influence of the external force.
- When there is no external force, the center of mass momentum is conserved.
- Although a human body has complicated features, the location of the center of mass (COM) could be a good indicator of the body proportions.
- We can measure the location of COM with two scales and a wooden beam. The linear and rotational equations of motion gives us the location.
- The center of mass of the human body depends on the gender and the position of the limbs. In a standing posture, it is typically about 10 cm lower than the navel, near the top of the hip bones.
- In a motion of a rigid body, different parts of the body have different motions.This means that these bodies may not behave like a point particle.
- There is a characteristic geometric point of the three dimensional body in motion. This point behaves as a particle, and is known as center of mass, abbreviated COM. COM appears to carry the whole mass of the body. All external forces appear to apply at COM.

- To describe the motion of a rigid body (with possibly a complicated geometry), we separate the translational part of the motion from the rotational part.

Key Terms

- **plumb line**: A cord with a weight attached, used to produce a vertical line.
- **rigid body**: An idealized solid whose size and shape are fixed and remain unaltered when forces are applied; used in Newtonian mechanics to model real objects.
- **center of mass**: The center of mass (COM) is the unique point at the center of a distribution of mass in space that has the property that the weighted position vectors relative to this point sum to zero.
- **torque**: A rotational or twisting effect of a force; (SI unit newton-meter or Nm; imperial unit foot-pound or ft-lb)
- **point particle**: An idealization of particles heavily used in physics. Its defining feature is that it lacks spatial extension, meaning that geometrically the particle is equivalent to a point.

- point particle. **Provided by**: Wikipedia. **Located at**: **en.Wikipedia.org/wiki/point%20particle**. **License**: ***CC BY-SA: Attribution-ShareAlike***
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CHAPTER OVERVIEW

8: Static Equilibrium, Elasticity, and Torque

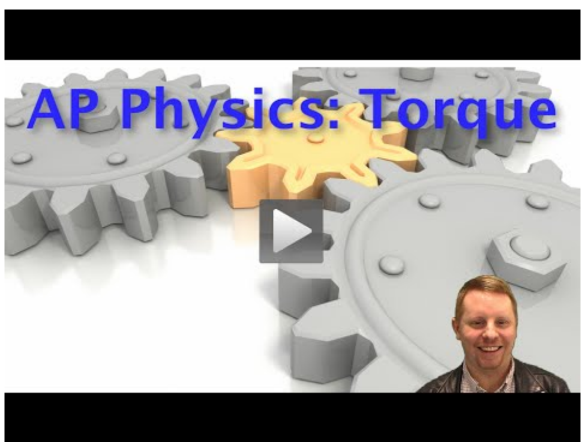
Topic hierarchy

8.1: Introduction

learning objectives

- Describe the effect of the torque on an object

Torque about a point is a concept that denotes the tendency of force to turn or rotate an object in motion. This tendency is measured in general about a point, and is termed as *moment of force*. The torque in angular motion corresponds to force in translation. It is the “cause” whose effect is either angular acceleration or angular deceleration of a particle in general motion. Quantitatively, it is defined as a vector given by:



Torque: A brief introduction to torque for students studying rotational motion in algebra-based physics courses such as AP Physics 1 and Honors Physics.

$$mathrmT = r \times F \tag{8.1.1}$$

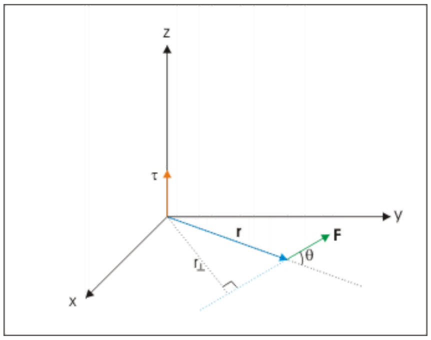
Rotation is a special case of angular motion. In the case of rotation, torque is defined with respect to an axis such that vector “r” is constrained as perpendicular to the axis of rotation. In other words, the plane of motion is perpendicular to the axis of rotation. Clearly, the torque in rotation corresponds to force in translation.

Torque is the cross product of force cross length of the moment arm; it is involved whenever there is a rotating object. Torque can also be expressed in terms of the angular acceleration of the object.

The determination of torque’s direction is relatively easier than that of angular velocity. The reason for this is simple: the torque itself is equal to vector product of two vectors, unlike angular velocity which is one of the two operands of the vector product.

Clearly, if we know the directions of two operands here, the direction of torque can easily be interpreted.

Since torque depends on both the force and the distance from the axis of rotation, the SI units of torque are newton-meters.



Torque: Torque in terms of moment arm.

Key Points

- Torque is found by multiplying the applied force by the distance to the axis of rotation, called the moment arm.
- Torque is to rotation as force is to motion.
- The unit of torque is the newton-meter.

Key Terms

- **vector**: A directed quantity, one with both magnitude and direction; the between two points.
- **angular velocity**: A vector quantity describing an object in circular motion; its magnitude is equal to the speed of the particle and the direction is perpendicular to the plane of its circular motion.
- **angular motion**: The motion of a body about a fixed point or fixed axis (as of a planet or pendulum). It is equal to the angle passed over at the point or axis by a line drawn to the body.

8.2: Conditions for Equilibrium

learning objectives

- Identify the first condition of equilibrium

First Condition of Equilibrium

For an object to be in equilibrium, it must be experiencing no acceleration. This means that both the net force and the net torque on the object must be zero. Here we will discuss the first condition, that of zero net force.

In the form of an equation, this first condition is:

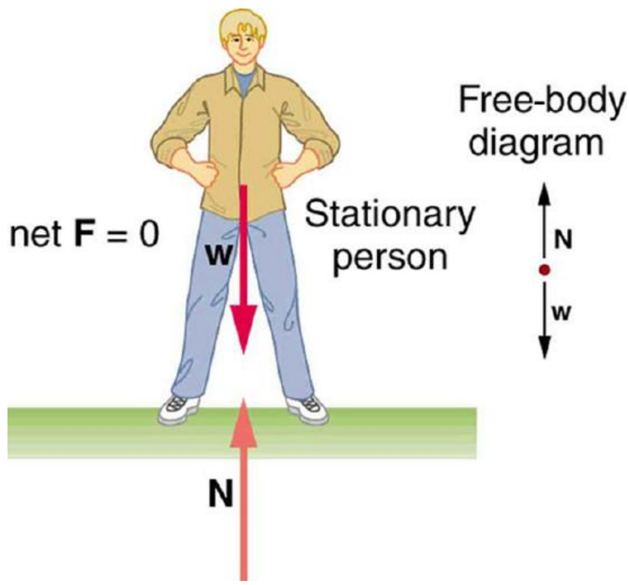
$$\mathrm{F_{net}} = 0.$$

In order to achieve this conditon, the forces acting along *each* axis of motion must sum to zero. For example, the net external forces along the typical *x*– and *y*-axes are zero. This is written as

$$\text{net } \mathrm{F_x} = 0 \text{ and net } \mathrm{F_y} = 0\,.$$

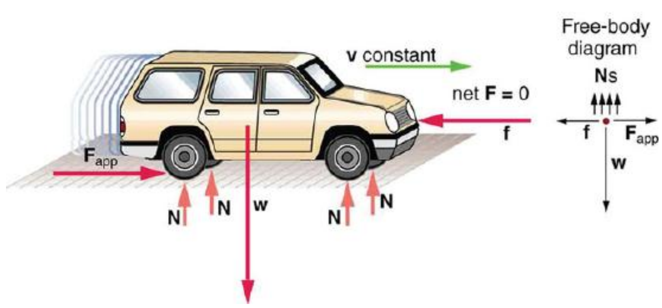
The condition $\mathrm{F_{net}} = 0$ must be true for both static equilibrium, where the object's velocity is zero, and dynamic equilibrium, where the object is moving at a constant velocity.

Below, the motionless person is in static equilibrium. The forces acting on him add up to zero. Both forces are vertical in this case.



Person in Static Equilibrium: This motionless person is in static equilibrium.

Below, the car is in dynamic equilibrium because it is moving at constant velocity. There are horizontal and vertical forces, but the net external force in any direction is zero. The applied force between the tires and the road is balanced by air friction, and the weight of the car is supported by the normal forces, here shown to be equal for all four tires.



A Car in Dynamic Equilibrium: This car is in dynamic equilibrium because it is moving at constant velocity. The forces in all directions are balanced.

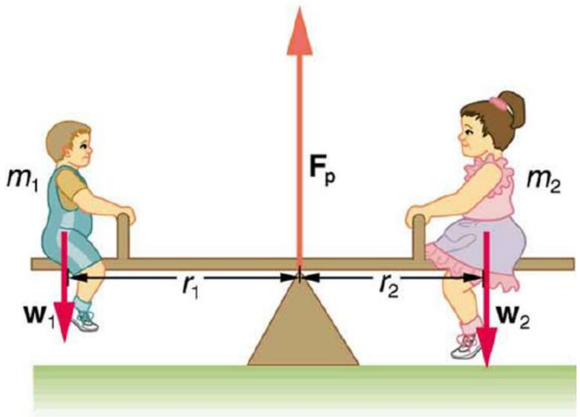
Second Condition

The second condition of static equilibrium says that the net torque acting on the object must be zero.

learning objectives

- Identify the second condition of static equilibrium

A child's seesaw, shown in, is an example of static equilibrium. An object in static equilibrium is one that has no acceleration in any direction. While there might be motion, such motion is constant.



Two children on a seesaw: The system is in static equilibrium, showing no acceleration in any direction.

If a given object is in static equilibrium, both the net force and the net torque on the object must be zero. Let's break this down:

Net Force Must Be Zero

The net force acting on the object must be zero. Therefore all forces balance in each direction. For example, a car moving along a highway at a constant speed is in equilibrium, as it is not accelerating in any forward or vertical direction. Mathematically, this is stated as $\mathrm{F_{net}} = \mathrm{ma} = 0$.

Net Torque Must Be Zero

The second condition necessary to achieve equilibrium involves avoiding accelerated rotation (maintaining a constant angular velocity). A rotating body or system can be in equilibrium if its rate of rotation is constant and remains unchanged by the forces acting on it.

To understand what factors affect rotation, let us think about what happens when you open an ordinary door by rotating it on its hinges.The magnitude, direction, and point of application of the force are incorporated into the definition of the physical quantity called torque—the rotational equivalent of a force. It is a measure of the effectiveness of a force in changing or accelerating a rotation (changing the angular velocity over a period of time).

In equation form, the magnitude of torque is defined to be $\tau = \mathrm{rF}\sin\theta$ where τ (the Greek letter tau) is the symbol for torque, r is the distance from the pivot point to the point where the force is applied, F is the magnitude of the force, and θ is the angle between the force and the vector directed from the point of application to the pivot point.

Two-Component Forces

In equilibrium, the net force and torque in any particular direction equal zero.

learning objectives

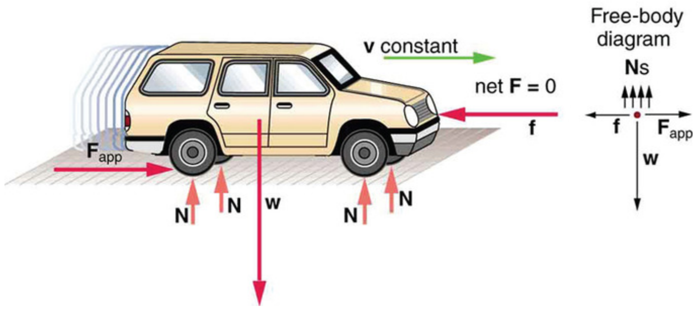
- Calculate the net force and the net torque for an object in equilibrium

An object with constant velocity has zero acceleration. A motionless object still has constant (zero) velocity, so motionless objects also have zero acceleration. Newton's second law states that:

$$\sum \mathrm{F} = \mathrm{ma} \tag{8.2.1}$$

so objects with constant velocity also have zero net external force. This means that all the forces acting on the object are balanced — that is to say, they are in equilibrium.

This rule also applies to motion in a specific direction. Consider an object moving along the *x*-axis. If no net force is applied to the object along the *x*-axis, it will continue to move along the *x*-axis at a constant velocity, with no acceleration.



Car Moving at Constant Velocity: A moving car for which the net x and y force components are zero

We can easily extend this rule to the *y*-axis. In any system, unless the applied forces cancel each other out (i.e., the resultant force is zero), there will be acceleration in the direction of the resultant force. In static systems, in which motion does not occur, the sum of the forces in all directions always equals zero. This concept can be represented mathematically with the following equations:

$$\sum \mathrm{F_x} = \mathrm{ma_x} = 0 \tag{8.2.2}$$

$$\sum \mathrm{F_y} = \mathrm{ma_y} = 0 \tag{8.2.3}$$

This rule also applies to rotational motion. If the resultant moment about a particular axis is zero, the object will have no rotational acceleration about the axis. If the object is not spinning, it will not start to spin. If the object is spinning, it will continue to spin at the same constant angular velocity. Again, we can extend this to moments about the *y*-axis as well. We can represent this rule mathematically with the following equations:

$$\sum \tau_\mathrm{x} = \mathrm{I}\alpha_\mathrm{x} = 0 \tag{8.2.4}$$

$$\sum \tau_\mathrm{y} = \mathrm{I}\alpha_\mathrm{y} = 0 \tag{8.2.5}$$

Key Points

- There are two conditions that must be met for an object to be in equilibrium.
- The first condition is that the net force on the object must be zero for the object to be in equilibrium.
- If net force is zero, then net force along any direction is zero.
- The second condition necessary to achieve equilibrium involves avoiding accelerated rotation.
- A rotating body or system can be in equilibrium if its rate of rotation is constant and remains unchanged by the forces acting on it.
- The magnitude of torque about a axis of rotation is defined to be $\tau = \mathrm{rF}\sin\theta$.
- In equilibrium, the net force in all directions is zero.
- If the net moment of inertia about an axis is zero, the object will have no rotational acceleration about the axis.
- In each direction, the net force takes the form: $\sum \mathrm{F} = \mathrm{ma} = 0$ and the net torque take the form: $\sum \tau = \mathrm{I}\alpha = 0$ where the sum represents the vector sum of all forces and torques acting.

Key Terms

- **force**: A physical quantity that denotes ability to push, pull, twist or accelerate a body which is measured in a unit dimensioned in mass × distance/time² (ML/T^2): SI: newton (N); CGS: dyne (dyn)
- **torque**: A rotational or twisting effect of a force; (SI unit newton-meter or Nm; imperial unit foot-pound or ft-lb)
- **translation**: Motion of a body on a linear path, without deformation or rotation, i.e. such that every part of the body moves at the same speed and in the same direction; also (in physics), the linear motion of a body considered independently of its rotation.
- **equilibrium**: The state of a body at rest or in uniform motion, the resultant of all forces on which is zero.

8.3: Stability

learning objectives

- Explain the relationship between how center of mass is defined and static equilibrium

For an object to be in static equilibrium, we expect it to stay in the same state indefinitely. If it starts accelerating away from its current position, it would hardly be in equilibrium. To quantify equilibrium for a single object, there are two conditions:

1. The net external force on the object is zero: $\sum_i F_i = F_{net} = 0$
2. The net external torque, regardless of choice of origin, is also zero: $\sum_i r_i \times F_i = \sum_i \tau_i = \tau_{net} = 0$

Those two conditions hold regardless of whether the object we are talking about is a single point particle, a rigid body, or a collection of discrete particles. Being in equilibrium means that we expect no changes to the linear momentum or the angular momentum. Note that this does not mean that the system is not moving or rotating; instead it simply means that its movement will not change as time goes on.

In a special case when the external forces are governed by some potential (e.g. gravitational potential) we can gain insight into the nature of the equilibrium. From the definition of a potential we know that $F_{ext} = -\frac{dU(x)}{dx}|_{x_0}$. When the first derivative is zero, we can take the second derivative to find whether the equilibrium is stable or unstable. Explicitly, if the potential is concave-up at x_0, $\frac{d^2U(x)}{dx^2}|_{x_0} > 0$, then the system is stable; conversely, if the potential is concave-down, then the equilibrium is unstable. If the second derivative is zero or does not exist, then the equilibrium is neutral—neither stable nor unstable.

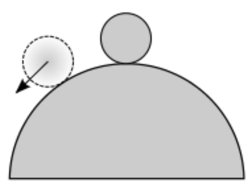
Mathematically, we can view this as a Taylor series expansion on the force slightly away from equilibrium,

$$F(x_0 + \delta x) = F(x_0) + \frac{dF(x)}{dx}|_{x_0}\ \delta x F(x0 + \delta x) = -\frac{dU(x)}{dx}|_{x_0} + (-\frac{d^2U(x)}{dx^2}|_{x_0})\delta x \quad ,$$

and when it is initially at equilibrium,

$$F(x_0) = 0 F(x_0) = 0 F(x_0 + \delta x) = -\frac{dU(x)}{dx}|_{x_0} + (-\frac{d^2U(x)}{dx^2}|_{x_0})\delta x U(y) = mgy \quad .$$

If the ball is at the top of the hill (where the potential is concave-down) it is possible for it to be perfectly balanced, and therefore at equilibrium. But if it gets pushed just slightly to the side, then it will roll down the hill with increasing speed, and the equilibrium is unstable.



Unstable Equilibrium: A ball on top of a hill can initially be balanced, but if it moves slightly left or right, it gets pushed further and further away from the initial equilibrium position. This is an example of unstable equilibrium.

Our notion of "balance" comes directly from the formulation of equilibrium. For something to be "balanced" means that the net external forces are zero. For example, a coin could balance standing up on a table. Initially the coin will feel no net external force or torque; it is in equilibrium. But if pushed slightly to the side, it will become "off-balance," experiencing both a force and a torque causing it to fall to the table. It might have been initially "balanced" and at equilibrium, but it was an unstable equilibrium, prone to being disturbed. But why all this talk of external forces, with no mention of internal forces? The reason is that all the internal forces must sum to zero. This follows directly from Newton's Third Law, $F_{12} = -F_{21}$. Every time we consider a force from particle 1 on particle 2 inside of a system, we know that it will later be cancelled out by the corresponding force from particle 2 on particle 1. We could include those forces in the sum, but it is unnecessary and internal forces are often more complicated than internal forces.

This differentiation between internal and external forces is a powerful one. It also implies that you can trace the motion of the system as a whole (ignoring motion inside the system) through the net external force acting on a center of mass. A center of mass acts as if it has the entire mass of the system, located at one point, and only feels external forces. Its position is defined as the weighted average of all the particles in the system: $\frac{R=\sum_i m_i r_i}{\sum_i m_i}$ or if we have a continuous density of mass, $\rho(r)$, then we can integrate: $R = \frac{\int V\rho(r)rdV}{\int V\rho(r)dV}$. The power of the center of mass is that it hides all the details of what is happening internally. We do not always want to lose the information of what is happening internally, but it is a useful tool to remember, when dealing with a number of complicated interactions.

Key Points

- Equilibrium is defined by no net forces or torques.
- Stability of an equilibrium can be determined by the second derivative of the potential.
- Defining a center of mass allows a simple way to study the behavior of a system or object as a whole.
- Stable equilibrium requires a restoring force. This restoring force can be derived by a Taylor expansion of the force, F(x).

Key Terms

- **stable equilibrium**: The response [of a system in static equilibrium] to a small perturbation is forces that tend to restore the equilibrium.
- **center of mass**: The center of mass (COM) is the unique point at the center of a distribution of mass in space that has the property that the weighted position vectors relative to this point sum to zero.
- **static equilibrium**: the physical state in which all components of a system are at rest and the net force is equal to zero throughout the system

8.4: Solving Statics Problems

learning objectives

- Formulate and apply six steps to solve static problems

Statics is the study of forces in equilibrium. Recall that Newton's second law states:

$$\sum \mathrm{F} = \mathrm{ma} \tag{8.4.1}$$

Therefore, for all objects moving at constant velocity (including a velocity of 0 — stationary objects), the net external force is zero. There are forces acting, but they are balanced — that is to say, they are "in equilibrium. "

When solving equilibrium problems, it might help to use the following steps:

1. First, ensure that the problem you're solving is in fact a static problem—i.e., that no acceleration (including angular acceleration) is involved Remember:$\sum \mathrm{F} = \mathrm{ma} = 0$ for these situations. If rotational motion is involved, the condition $\sum \tau = \mathrm{I}\alpha = 0$ must also be satisfied, where is torque, is the moment of inertia, and is the angular acceleration.
2. Choose a pivot point. Often this is obvious because the problem involves a hinge or a fixed point. If the choice is not obvious, pick the pivot point as the location at which you have the most unknowns. This simplifies things because forces at the pivot point create no torque because of the cross product:$\tau = \mathrm{rF}$
3. Write an equation for the sum of torques, and then write equations for the sums of forces in the *x* and *y* directions. Set these sums equal to 0. Be careful with your signs.
4. Solve for your unknowns.
5. Insert numbers to find the final answer.
6. Check if the solution is reasonable by examining the magnitude, direction, and units of the answer. The importance of this last step cannot be overstated, although in unfamiliar applications, it can be more difficult to judge reasonableness. However, these judgments become progressively easier with experience.

Key Points

- First, ensure that the problem you're solving is in fact a static problem—i.e., that no acceleration (including angular acceleration) is involved.
- Choose a pivot point — use the location at which you have the most unknowns.
- Write equations for the sums of torques and forces in the *x* and *y* directions.
- Solve the equations for your unknowns algebraically, and insert numbers to find final answers.

Key Terms

- **torque**: A rotational or twisting effect of a force; (SI unit newton-meter or Nm; imperial unit foot-pound or ft-lb)
- **moment of inertia**: A measure of a body's resistance to a change in its angular rotation velocity

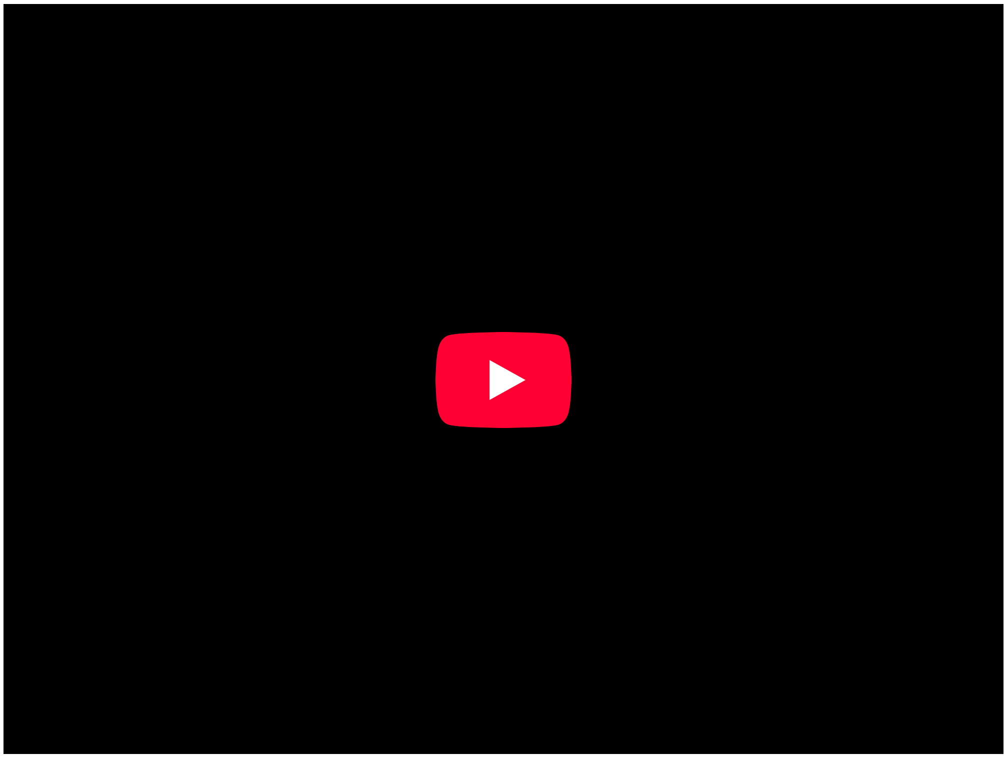
8.5: Applications of Statics

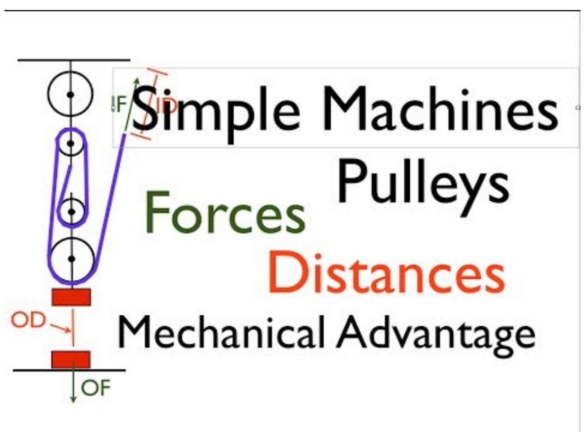
learning objectives

- Develop an understanding of how a machine applies force to work against a load force

Simple Machines

A simple machine is a device that changes the direction or magnitude of a force. They can be described as the simplest mechanisms that use mechanical advantage (or leverage) to multiply force. Usually, the term “simple machine” is referring to one of the six classical simple machines, defined by Renaissance scientists.





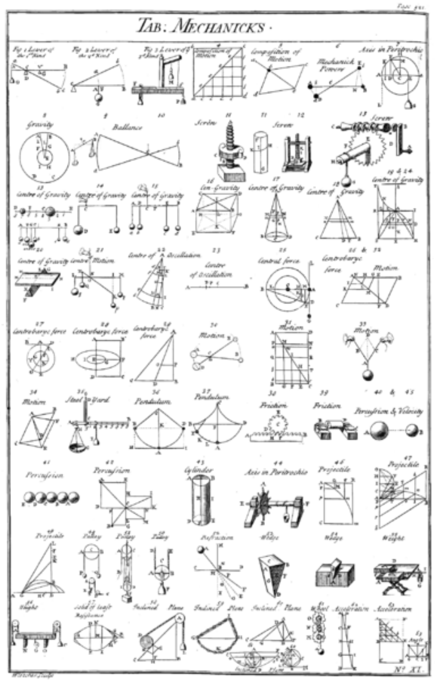
Simple Machines, Pulleys; Forces, Distances and MA: Describes the following terms as they relate to simple machine; input force, output force, input distance, output distance, mechanical advantage.

Simple machines are devices used to multiply or augment a force that we apply—often at the expense of a distance through which we apply the force. Some common examples include:

- Lever

- Wheel and Axle
- Pulley
- Inclined Plane
- Wedge
- Screw

When a device with a specific movement, called a mechanism, is joined with others to form a machine, these machines can be broken down into elementary movements. For example, a bicycle is a mechanism made up of wheels, levers, and pulleys.

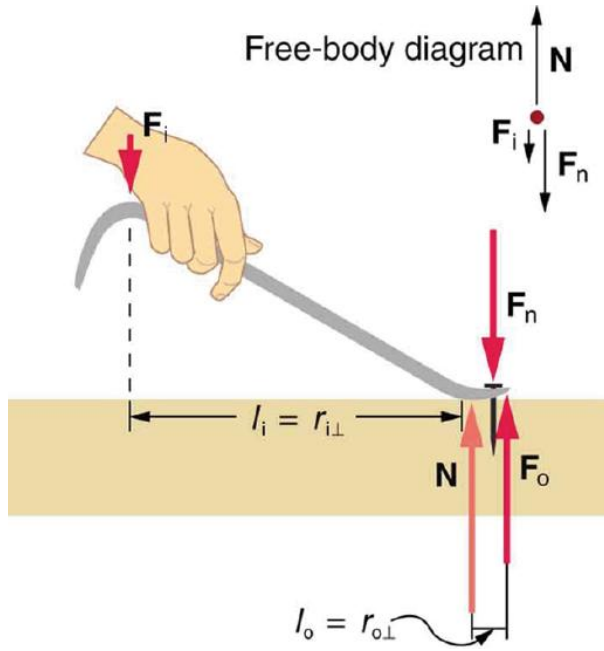


Simple Machines: Table of simple mechanisms, from Chambers' Cyclopedia, 1728. [1] Simple machines provide a "vocabulary" for understanding more complex machines.

Mechanics

A simple machine has an applied force that works against a load force. If there are no frictional losses, the work done on the load is equal to the work done by the applied force. This allows an increase in the output force at the cost of a proportional decrease in distance moved by the load. The ratio of the output force to the input force is the mechanical advantage of the machine. If the machine does not absorb energy, its mechanical advantage can be calculated from the machine's geometry. For instance, the mechanical advantage of a lever is equal to the ratio of its lever arms.

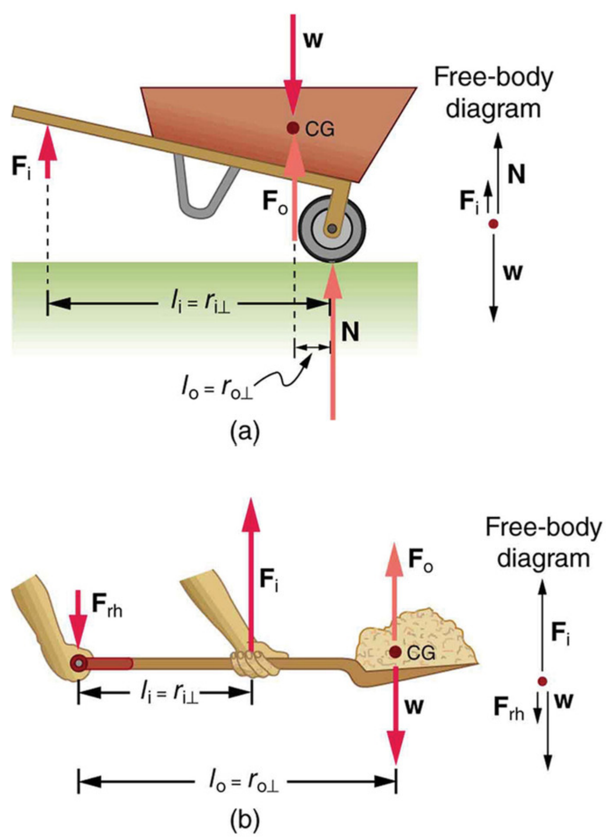
Simple machines which do not experience frictional losses are called ideal machines. For these ideal machines, the power in (rate of energy input) in equal to the power out (rate of energy output): $Pin=PoutPin=Pout$.



Lever: The amount of force produced by a machine can not be greater than the amount of force put into it.

Further Examples

Wheelbarrows and shovels are also examples of simple machines (these utilize levers). They use only three forces: the input force, output force, and force on the pivot. In the case of wheelbarrows, the output force is between the pivot (wheel's axle) and the input force. In the shovel, the input force is between the pivot and the load.



Examples of Simple Machines: Both of these machines use the concept of levers.

Arches and Domes

Arches and domes are structures that exhibit structural strength and can span large areas with no intermediate supports.

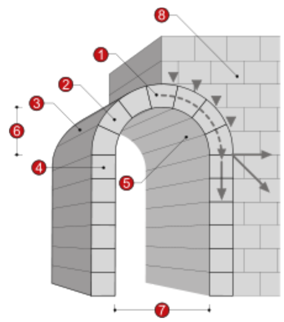
learning objectives

- Explain how an arch exhibits structural strength and how a dome can span a large area without intermediate supports

Arches and domes are structures that exhibit structural strength and can span large areas with no intermediate supports. In this atom, we will discuss the history and physics behind arches and domes.

Arches

An arch is a structure that spans a space, and supports structure and weight above it. Arches have been being built from as long ago as the second millennium, but were not used for a variety of structures until the Romans took advantage of their capabilities. Arches are a pure compression form. They span large areas by resolving forces into compressive stresses and eliminating tensile stresses (referred to as arch action). As the forces in an arch are carried toward the ground, the arch will push outward at the base (called thrust). As the height of the arch decreases, the outward thrust increases. To prevent the arch from collapsing, the thrust needs to be restrained, either with internal ties or external bracing. This external bracing is often called an abutment, as shown in.



Arches: A masonry arch1. Keystone 2. Voussoir 3. Extrados 4. Impost 5. Intrados 6. Rise 7. Clear span 8. Abutment

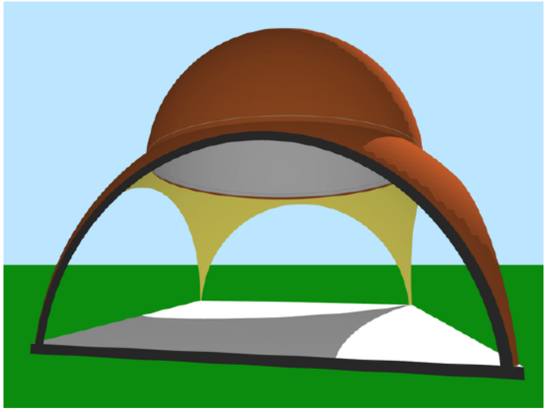
The most common true arch configurations are the fixed arch, the two-hinged arch and the three-hinged arch. The fixed arch is most often used in reinforced concrete bridge and tunnel construction, where the spans are short. Because it is subject to additional internal stress caused by thermal expansion and contraction, this type of arch is considered to be statically indeterminate. The two-hinged arch is most often used to bridge long spans. This type of arch has pinned connections at the base. Unlike the fixed arch, the pinned base is able to rotate, allowing the structure to move freely and compensate for the thermal expansion and contraction caused by changes in outdoor temperature. Because the structure is pinned between the two base connections, which can result in additional stresses, the two-hinged arch is also statically indeterminate, although not to the degree of the fixed arch.

Domes

A dome is an element of architecture that resembles the hollow upper half of a sphere. Dome structures made of various materials (from mud to stone, wood, brick, concrete, metal, glass and plastic) and have a long architectural lineage extending into prehistory.

A dome is basically an arch that has been rotated around its central vertical axis. Domes have the same properties and capabilities of arches, they can span large areas without intermediate supports and have a great deal of structural strength. When the base of a dome is not the same shape as its supporting walls, for example when a circular dome is on a square structure, techniques are employed to transition between the two. Pendentives are triangular sections of a sphere used to transition from the flat surfaces of supporting walls to the round base of a dome.

Domes can be divided into two kinds, simple and compound. Simple domes use pendentives that are part of the same sphere as the dome itself. Compound domes are part of the structure of a large sphere below that of the dome itself, forming a circular base, as shown in.



Compound Dome: A compound dome (red) with pendentives (yellow) from a sphere of greater radius than the dome.

Muscles and Joints

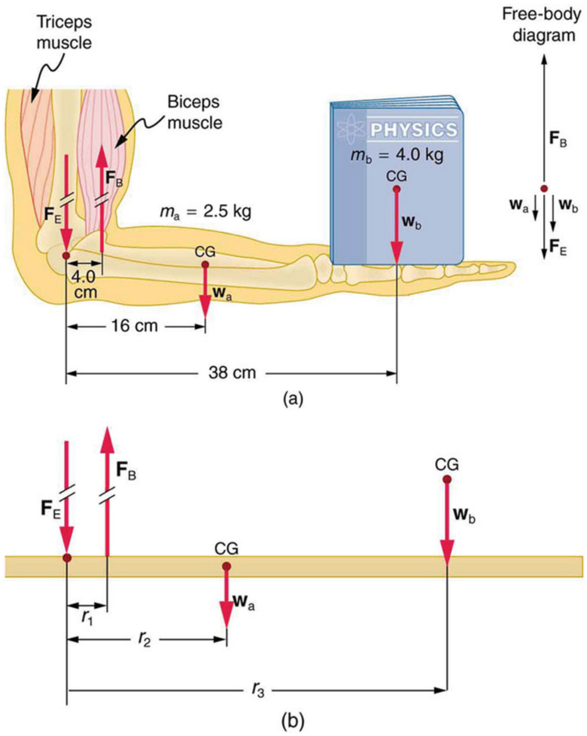
Most skeletal muscles and joints exert much larger forces within the body than the limbs will apply to the outside world.

learning objectives

- Explain the forces exerted by muscles

Muscles and Joints

Muscles and joints involve very interesting applications of statics. Muscles can only contract, so they occur in pairs. In the arm, the biceps muscle is a flexor: it closes the limb. The triceps muscle is an extensor that opens the limb. This configuration is typical of skeletal muscles, bones, and joints in humans and other vertebrates. Most skeletal muscles exert much larger forces within the body than the limbs will apply to the outside world. The reason is clear, since most muscles are attached to bones via tendons close to joints, causing these systems to have mechanical advantages much less than one. Viewing them as simple machines, the input force is much greater than the output force, as seen in.



The Forearm of a Person Holding a Book: (a.) The biceps exert a force FB to support the weight of the forearm and the book. The triceps are assumed to be relaxed. (b.) An approximately equivalent mechanical system with the pivot at the elbow joint

Very large forces are also created in the joints. Because muscles can contract but not expand beyond their resting length, joints and muscles often exert forces that act in opposite directions, and thus subtract. Forces in muscles and joints are largest when their load is far from the joint. For example, in racquet sports like tennis, the constant extension of the arm during game play creates large forces. The mass times the lever arm of a tennis racquet is an important factor, and many players use the heaviest racquet they can handle. It is no wonder that joint deterioration and damage to the tendons in the elbow, such as 'tennis elbow,' can result from repetitive motion, undue torques, and possible poor racquet selection in such sports.

Various tried techniques for holding and using a racquet, bat, or stick can not only increase sporting prowess but can minimize fatigue and long-term damage to the body. Training coaches and physical therapists use the knowledge of the relationships between forces and torques in the treatment of muscles and joints. In physical therapy, an exercise routine can apply a particular force and torque, which can revive muscles and joints in time. Some exercises should be performed under water, thus requiring the exertion of more force and further strengthening muscles.

Key Points

- The six classifications of simple machines were established by renaissance scientists; they are as follows: lever, wheel and axle, pulley, inclined plane, wedge and screw.
- Simple machines can be joined with other devices to create a more complicated machine. These building blocks are used to explain how machines work.
- The force output by a simple machine can exceed the force that was put into the machine.
- Arches span large areas by resolving forces into compressive stresses and eliminating tensile stresses.
- The most common true arch configurations are the fixed arch, the two-hinged arch, and the three-hinged arch.
- A dome is basically an arch that has been rotated around its central vertical axis.
- Domes are basically arches that have been rotated on their vertical axis, and have the same capabilities and properties of arches.
- Domes can be divided into two kinds, simple and compound.
- It is helpful to view muscles as a simple machines and draw them as free body diagrams.
- In muscles, the input force is often much greater than the output force.
- Very large forces are also created in the joints. Because muscles can contract but not expand beyond their resting length, joints and muscles often exert forces that act in opposite directions, and thus subtract.

Key Terms

- **machine**: A mechanical or electrical device that performs or assists in the performance of human tasks, whether physical or computational, laborious or for entertainment.
- **leverage**: A force amplified by means of a lever rotating around a pivot.
- **mechanical advantage**: In a simple machine, the ratio of the output force to the input force.
- **compressive stress**: Stress on materials that leads to a smaller volume.
- **tensile stress**: Stress state leading to expansion; that is, the length of a material tends to increase in the tensile direction while the volume remains constant.
- **pendentive**: The concave triangular sections of vaulting that provide the transition between a dome and the square base on which it is set and transfer the weight of the dome.
- **muscle**: A contractile form of tissue which animals use to effect movement.
- **joint**: Any part of the body where two bones join, in most cases allowing that part of the body to be bent or straightened.

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8.6: Elasticity, Stress, Strain, and Fracture

learning objectives

- Identify properties of elastic objects

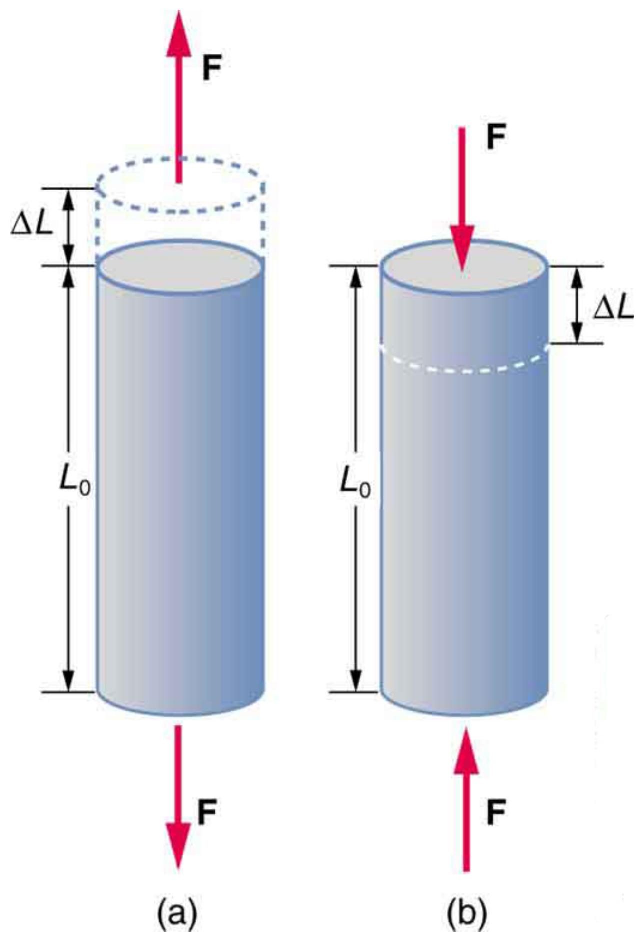
We now move from consideration of forces that affect the motion of an object (such as friction and drag) to those that affect an object's shape. If a bulldozer pushes a car into a wall, the car will not move once it hits the wall, but it will noticeably change shape. A change in shape due to the application of a force is a deformation. Even very small forces are known to cause some deformation. For small deformations, two important characteristics are observed. First, the object returns to its original shape when the force is removed—that is, the deformation is elastic for small deformations. Second, the size of the deformation is proportional to the force—that is, for small deformations, Hooke's law is obeyed. In equation form, Hooke's law is given by $\mathrm{F}=\mathrm{k} \Delta \mathrm{L}$, where $\Delta \mathrm{L}$ is the change in length.

Elasticity is a measure of how difficult it is to stretch an object. In other words it is a measure of how small kk is. Very elastic materials like rubber have small kk and thus will stretch a lot with only a small force.

Stress is a measure of the force put on the object over the area.

Strain is the change in length divided by the original length of the object.

Experiments have shown that the change in length (ΔL) depends on only a few variables. As already noted, ΔL is proportional to the force F and depends on the substance from which the object is made. Additionally, the change in length is proportional to the original length L0 and inversely proportional to the cross-sectional area of the wire or rod. For example, a long guitar string will stretch more than a short one, and a thick string will stretch less than a thin one.



Tension/Compression: Tension: The rod is stretched a length ΔL when a force is applied parallel to its length. (b) Compression: The same rod is compressed by forces with the same magnitude in the opposite direction. For very small deformations and uniform materials, ΔL is approximately the same for the same magnitude of tension or compression. For larger deformations, the cross-sectional area changes as the rod is compressed or stretched.

Fracture

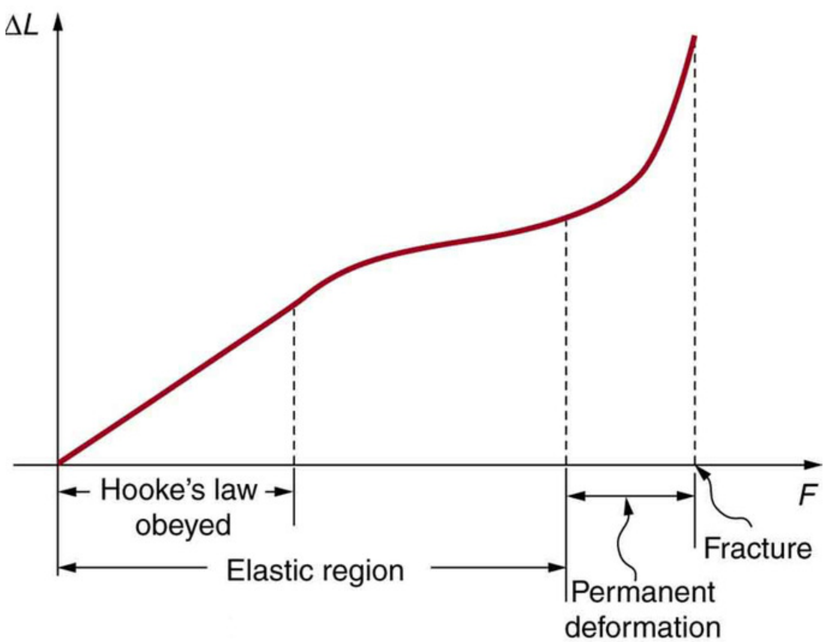
Fracture is caused by a strain placed on an object such that it deforms beyond its elastic limit and breaks.

learning objectives

- Relate fracture with the elastic limit of a material

Materials cannot stretch forever. When a strain is applied to a material it deforms elastically proportional to the force applied. However, after it has deformed a certain amount, the object can no longer take the strain and will break or fracture. The zone in which it bends under strain is called the elastic region. In that region the object will bend and then return to its original shape when the force is abated. Past that point, if more strain is added, the object may permanently deform and eventually fracture.

Fracture strength, also known as breaking strength, is the stress at which a specimen fails via fracture. This is usually determined for a given specimen by a tensile test, which charts the stress-strain curve. The final recorded point is the fracture strength.



Fracture: This is a graph of deformation ΔL versus applied force F. The straight segment is the linear region where Hooke's law is obeyed. The slope of the straight region is 1k. For larger forces, the graph is curved but the deformation is still elastic—L will return to zero if the force is removed. Still greater forces permanently deform the object until it finally fractures. The shape of the curve near fracture depends on several factors, including how the force F is applied. Note that in this graph the slope increases just before fracture, indicating that a small increase in F is producing a large increase in L near the fracture.

Bones, on the whole, do not fracture due to tension or compression. Rather they generally fracture due to sideways impact or bending, resulting in the bone shearing or snapping. The behavior of bones under tension and compression is important because it determines the load the bones can carry. Bones are classified as weight-bearing structures such as columns in buildings and trees. Weight-bearing structures have special features; columns in building have steel-reinforcing rods while trees and bones are fibrous. The bones in different parts of the body serve different structural functions and are prone to different stresses. Thus, the bone in the top of the femur is arranged in thin sheets separated by marrow while, in other places, the bones can be cylindrical and filled with marrow or just solid. Overweight people have a tendency toward bone damage due to sustained compressions in bone joints and tendons.

Key Points

- Elasticity is a measure of the deformation of an object when a force is applied. Objects that are very elastic like rubber have high elasticity and stretch easily.
- Stress is force over area.
- Strain is change in length over original length.
- Most objects behave elastically for small strains and return to their original shape after being bent.
- If the strain on an object is greater than the elastic limit of the object, it will permanently deform or eventually fracture.
- Fracture strength is a measure of the force needed to break an object.

Key Items

- **deformation**: A transformation; change of shape.
- **strain**: The amount by which a material deforms under stress or force, given as a ratio of the deformation to the initial dimension of the material and typically symbolized by ε is termed the engineering strain. The true strain is defined as the natural logarithm of the ratio of the final dimension to the initial dimension.
- **elastic**: Capable of stretching; particularly, capable of stretching so as to return to an original shape or size when force is released.

8.7: The Center of Gravity

learning objectives

- Describe how the center of mass of an oddly shaped object is found

Center of Gravity

When people think of objects, they think of them as singular particles of matter. In fact, every object is made up of millions of particles, all of which behave differently when moved. When people observe a stick being thrown in the air, it seems as though the entire object is moving at the same trajectory and velocity, but each particle is being subjected to a different motion in space and acceleration, depending on its place. The different parts of the body have different motions. shows the motion of a stick in the air: it seems to rotate around a single point. Three-dimensional bodies have a property called the center of mass, or center of gravity. This center of mass's main characteristic is that it appears to carry the whole mass of the body.

Center of Gravity: Although the center of mass is in the midpoint of the stick, all of the particles are moving as well.

The center of mass does not actually carry all the mass, despite appearances. Given a hollow sphere, the center is the center of mass, even though it does not actually have anything in it. As seen in, it looks as if the external forces of gravity appear to be working only on the center of mass, but each particle is being pushed or pulled by gravity. The center of mass is much easier to use when discussing bodies, because no one has to analyze each individual particle.

Mathematical Expression: The mathematical relation of center of gravity is read as: 'the position of the center of mass and weighted average of the position of the particles. '

Specifically: 'the total mass x the position of the center of mass= ∑ the mass of the individual particle x the position of the particle. ' The center of mass is a geometric point in three-dimensional volume. When using the definition above, it yields the following equation for center of mass:

$$\mathrm{r_{COM}} = \frac{\Sigma \mathrm{m_i r_i}}{\mathrm{M}} \tag{8.7.1}$$

where r is the reference axis x, y, or z; *m* is individual mass;$\mathrm{r_i}$ is the individual position; and M *M* is the total mass.

When taking the center of mass of an oddly shaped object, it is helpful to break it down into smaller sections whose mass and properties are easier to evaluate, and then add the products of the individual masses and positions and divide by the total mass.

Center of Mass: This child's toy uses the principles of 'center of mass' to stay balanced on a finger.

Key Points

- The center of mass 's main characteristic is that it appears to carry the whole mass of the body.
- The total mass x the position of the center of mass= ∑ mass of the individual particle x the position of the particle.
- The center of mass is a geometric point in three-dimensional volume. By using the definition above, the following equation for center of mass can be derived: $r_{COM} = \Sigma \frac{m_i r_i}{M}$.

Key Terms

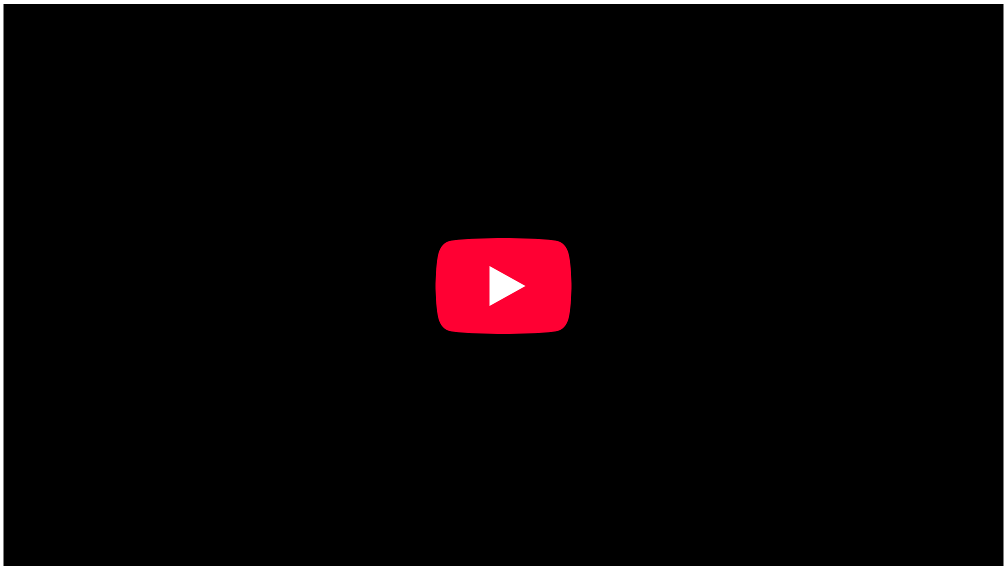
- **center of mass**: The center of mass (COM) is the unique point at the center of a distribution of mass in space that has the property that the weighted position vectors relative to this point sum to zero.

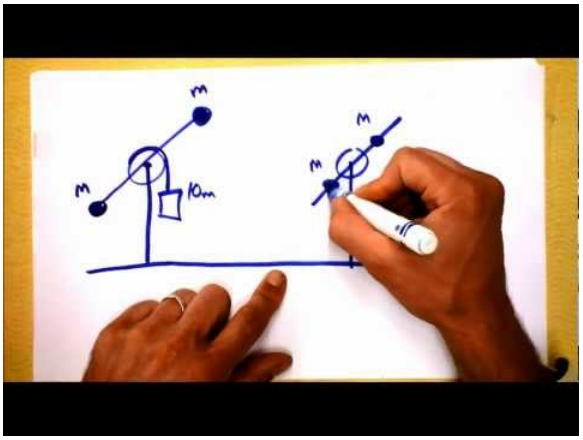
8.8: Torque and Angular Acceleration

learning objectives

- Express the relationship between the torque and the angular acceleration in a form of equation

Torque and angular acceleration are related by the following formula where is the objects moment of inertia and αα is the angular acceleration.





Torque, Angular Acceleration, and the Role of the Church in the French Revolution: Why do things change their angular velocity? Soon, you'll know.

Just like Newton's Second Law, which is force is equal to the mass times the acceleration, torque obeys a similar law. If you replace torque with force and rotational inertia with mass and angular acceleration with linear acceleration, you get Newton's Second Law back out. In fact, this equation is Newton's second law applied to a system of particles in rotation about a given axis. It makes no assumptions about constant rotational velocity.

The net torque about an axis of rotation is equal to the product of the rotational inertia about that axis and the angular acceleration, as shown in Figure 1.

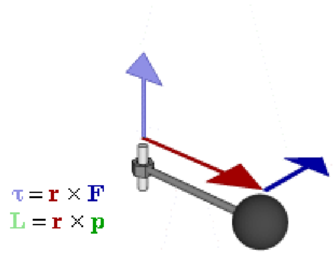


Figure 1: Relationship between force (F), torque (τ), momentum (p), and angular momentum (L) vectors in a rotating system

Similar to Newton’s Second Law, angular motion also obeys Newton’s First Law. If no outside forces act on an object, an object in motion remains in motion and an object at rest remains at rest. With rotating objects, we can say that unless an outside torque is applied, a rotating object will stay rotating and an object at rest will not begin rotating.

If a turntable were spinning counter clockwise (when viewed from the top), and you applied your fingers to opposite sides the turntable would begin to slow its spinning. From a translational viewpoint, at least, there would be no net force applied to the turntable. The force that points to one side would be cancelled by the force that points to the other. The forces of the two fingers would cancel. Therefore, the turntable would be in translational equilibrium. Despite that, the rotational velocity would be decreased meaning that the acceleration would no longer be zero. From this we might conclude that just because a rotating object is in translational equilibrium, it is not necessarily in rotational equilibrium.

Key Points

- When a torque is applied to an object it begins to rotate with an acceleration inversely proportional to its moment of inertia.
- This relation can be thought of as Newton’s Second Law for rotation. The moment of inertia is the rotational mass and the torque is rotational force.
- Angular motion obeys Newton’s First Law. If no outside forces act on an object, an object in motion remains in motion and an object at rest remains at rest.

Key Terms

- **angular acceleration**: The rate of change of angular velocity, often represented by α.
- **torque**: A rotational or twisting effect of a force; (SI unit newton-meter or Nm; imperial unit foot-pound or ft-lb)
- **rotational inertia**: The tendency of a rotating object to remain rotating unless a torque is applied to it.

CHAPTER OVERVIEW

9: Rotational Kinematics, Angular Momentum, and Energy

9.10: Conservation of Energy

learning objectives

- Conclude the interchangeability of force and radius with torque and angle of rotation in determining force

In this atom we will discuss work and energy associated with rotational motion. shows a worker using an electric grindstone propelled by a motor. Sparks are flying, and noise and vibration are created as layers of steel are pared from the pole. The stone continues to turn even after the motor is turned off, but it is eventually brought to a stop by friction. Clearly, the motor had to work to get the stone spinning. This work went into heat, light, sound, vibration, and considerable rotational kinetic energy.

Grindstone: The motor works in spinning the grindstone, giving it rotational kinetic energy. That energy is then converted to heat, light, sound, and vibration. (Credit: U.S. Navy photo by Mass Communication Specialist Seaman Zachary David Bell.)

Work must be done to rotate objects such as grindstones or merry-go-rounds. The simplest rotational situation is one in which the net force is exerted perpendicular to the radius of a disc and remains perpendicular as the disc starts to rotate. The force is parallel to the displacement, and so the net work (W) done is the product of the force (F) and the radius (r) of the disc (this is otherwise known as torque(τ)) times the angle (θ) of rotation:

$$\mathrm{W}=\mathrm{Fr}\theta=\tau\theta. \tag{9.10.1}$$

Work and energy in rotational motion are completely analogous to work and energy in translational motion and completely transferrable. Just as in translational motion (where kinetic energy equals $1/2mv^2$ where m is mass and v is velocity), energy is conserved in rotational motion. Kinetic energy (K.E.) in rotational motion is related to moment of rotational inertia (I) and angular velocity (ω):

$$\mathrm{KE}=\frac{1}{2}\mathrm{I}\omega^2. \tag{9.10.2}$$

The final rotational kinetic energy equals the work done by the torque:

$$\mathrm{W}=\tau\theta=\frac{1}{2}\mathrm{I}\omega^2=\mathrm{KE}. \tag{9.10.3}$$

This confirms that the work done went into rotational kinetic energy. To return to the grindstone example, work was done to give the grindstone rotational energy, and work is done by friction so that it loses kinetic energy. However, the energy is never destroyed; it merely changes form from rotation of the grindstone to heat when friction is applied.

Key Points

- Rotating objects have rotational kinetic energy.
- Rotational kinetic energy can change form if work is done on the object.
- Energy is never destroyed, if rotational energy is gained or lost, something must have done work on it to change the form of the energy.

Key Terms

- **work**: A measure of energy expended in moving an object; most commonly, force times displacement. No work is done if the object does not move.
- **angular velocity**: A vector quantity describing an object in circular motion; its magnitude is equal to the speed of the particle and the direction is perpendicular to the plane of its circular motion.
- **rotational inertia**: The tendency of a rotating object to remain rotating unless a torque is applied to it.

9.1: Quantities of Rotational Kinematics

learning objectives

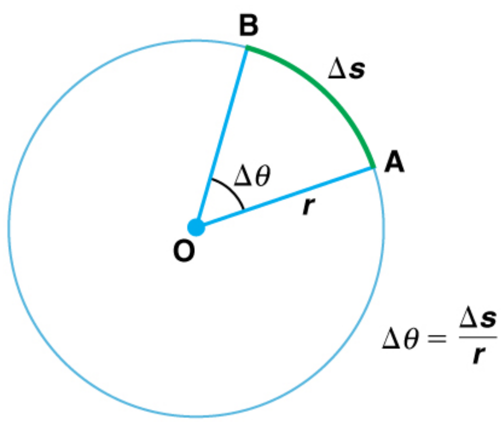
- Assess the relationship between radians the the revolution of a CD

When objects rotate about some axis—for example, when the CD (compact disc) rotates about its center—each point in the object follows a circular arc. Consider a line from the center of the CD to its edge. Each pit used to record sound along this line moves through the same angle in the same amount of time. The rotation angle is the amount of rotation, and is analogous to linear distance. We define the rotation angle $\Delta\theta$ to be the ratio of the arc length to the radius of curvature:

$\Delta\theta = \frac{\Delta s}{r}$ (illustrated in).

Rotation Angle: All points on a CD travel in circular arcs. The pits along a line from the center to the edge all move through the same angle Δ in a time Δt.

In mathematics, the angle of rotation (or angular position) is a measurement of the amount (i.e., the angle) that a figure is rotated about a fixed point (often the center of a circle, as shown in).



Angle θ and Arc Length s: The radius of a circle is rotated through an angle Δ. The arc length Δs is described on the circumference.

The arc length Δs is the distance traveled along a circular path. r is the radius of curvature of the circular path. We know that for one complete revolution, the arc length is the circumference of a circle of radius r. The circumference of a circle is 2πr. Thus, for one complete revolution the rotation angle is:

$$\Delta\theta = \frac{(2\pi r)}{r} = 2\pi. \tag{9.1.1}$$

This result is the basis for defining the units used to measure rotation angles to be radians (rad), defined so that:

$$2\pi \text{ rad} = 1 \text{ revolution.} \tag{9.1.2}$$

If $\Delta\theta = 2\pi$ rad, then the CD has made one complete revolution, and every point on the CD is back at its original position. Because there are 360º in a circle or one revolution, the relationship between radians and degrees is thus 2π rad $= 360^{\circ}$, so that:

$$1 \text{ rad} = \frac{360^{\circ}}{2\pi} = 57.3^{\circ}. \tag{9.1.3}$$

Angular Velocity, Omega

Angular velocity ω is the rate of change of an angle, mathematically defined as $\omega = \frac{\Delta\theta}{\Delta t}$.

learning objectives

- Examine how fast an object is rotating based on angular velocity

To examine how fast an object is rotating, we define angular velocity ω as the rate of change of an angle. In symbols, this is

$$\omega = \frac{\Delta\theta}{\Delta t}, \tag{9.1.4}$$

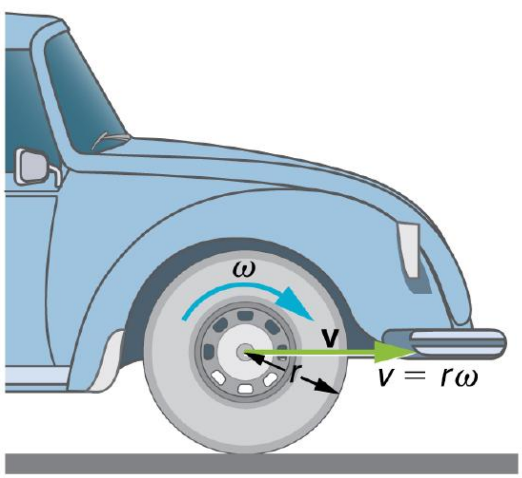
where an angular rotation Δ takes place in a time Δt. The greater the rotation angle in a given amount of time, the greater the angular velocity. The units for angular velocity are radians per second (rad/s).

Angular velocity ω is analogous to linear velocity v. To find the precise relationship between angular and linear velocity, we again consider a pit on the rotating CD. This pit moves an arc length Δs in a time Δt, and so it has a linear velocity $v = \frac{\Delta s}{\Delta t}$.

From $\Delta\theta = \frac{(\Delta s)}{r}$ we see that $\Delta s = r \cdot \Delta\theta$. Substituting this into the expression for v gives $v = \frac{(r \cdot \Delta\theta)}{(\Delta t)} = r(\frac{\Delta\theta}{\Delta t}) = r\omega$.

We can write this relationship in two different ways: $v = r\omega$ or $\omega = \frac{v}{r}$.

The first relationship states that the linear velocity v is proportional to the distance from the center of rotation, thus it is largest for a point on the rim (largest r), as you might expect. We can also call this linear speed v of a point on the rim the tangential speed. The second relationship can be illustrated by considering the tire of a moving car, as shown in the picture below. Note that the speed of the point at the center of the tire is the same as the speed v of the car. The faster the car moves, the faster the tire spins—large v means a large ω, because v=rω. Similarly, a larger-radius tire rotating at the same angular velocity (ω) will produce a greater linear speed (v) for the car.



Angular Velocity: A car moving at a velocity v to the right has a tire rotating with an angular velocity ω. The speed of the tread of the tire relative to the axle is v, the same as if the car were jacked up. Thus the car moves forward at linear velocity v=rω, where r is the tire radius. A larger angular velocity for the tire means a greater velocity for the car.

Angular Acceleration, Alpha

Angular acceleration is the rate of change of angular velocity, expressed mathematically as $\alpha = \frac{\Delta\omega}{\Delta t}$.

learning objectives

- Explain the relationship between angular acceleration and angular velocity

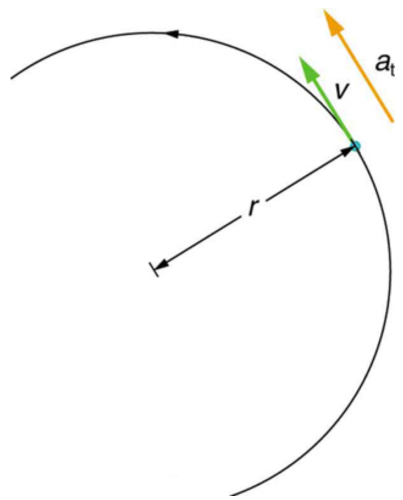
Angular acceleration is the rate of change of angular velocity. In SI units, it is measured in radians per second squared (rad/s^2), and is usually denoted by the Greek letter alpha (α).

Consider the following situations in which angular velocity is not constant: when a skater pulls in her arms, when a child starts up a merry-go-round from rest, or when a computer's hard disk slows to a halt when switched off. In all these cases, there is an angular acceleration in which ωω changes. The faster the change occurs, the greater the angular acceleration. Angular acceleration is defined as the rate of change of angular velocity. In equation form, angular acceleration is expressed as follows:

$$\alpha = \frac{\Delta\omega}{\Delta t} \tag{9.1.5}$$

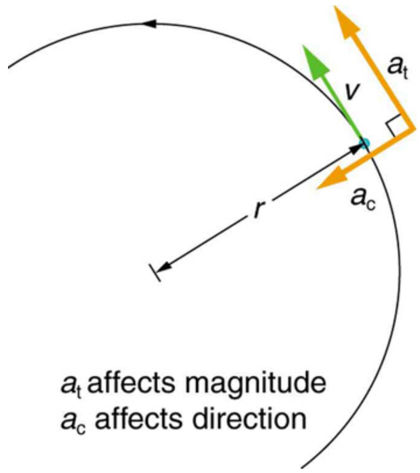
where $\Delta\omega$ is the change in angular velocity and Δt is the change in time. The units of angular acceleration are (rad/s)/s, or rad/s^2. If ωω increases, then αα is positive. If ωω decreases, then αα is negative.

It is useful to know how linear and angular acceleration are related. In circular motion, there is acceleration that is *tangent* to the circle at the point of interest (as seen in the diagram below). This acceleration is called *tangential acceleration*, a_t.



Tangential acceleration: In circular motion, acceleration can occur as the magnitude of the velocity changes: a is tangent to the motion. This acceleration is called tangential acceleration.

Tangential acceleration refers to changes in the magnitude of velocity but not its direction. In circular motion, centripetal acceleration, a_c, refers to changes in the direction of the velocity but not its magnitude. An object undergoing circular motion experiences centripetal acceleration (as seen in the diagram below.) Thus, a_t and a_c are perpendicular and independent of one another. Tangential acceleration a_t is directly related to the angular acceleration and is linked to an increase or decrease in the velocity (but not its direction).



Centripetal Acceleration: Centripetal acceleration occurs as the direction of velocity changes; it is perpendicular to the circular motion. Centripetal and tangential acceleration are thus perpendicular to each other.

Key Points

- The arc length Δs is the distance traveled along a circular path. r is the radius of curvature of the circular path.
- The rotation angle is the amount of rotation and is analogous to linear distance. We define the rotation angle $\Delta\theta$ to be the ratio of the arc length to the radius of curvature: $\Delta\theta = \frac{\Delta s}{r}$.
- For one complete revolution the rotation angle is 2π.
- The greater the rotation angle in a given amount of time, the greater the angular velocity.
- Angular velocity ω is analogous to linear velocity v.
- We can write the relationship between linear velocity and angular velocity in two different ways: $v = r\omega$ or $\omega = \frac{v}{r}$.
- The faster the change in angular velocity occurs, the greater the angular acceleration.
- In circular motion, linear acceleration is tangent to the circle at the point of interest, and is called tangential acceleration.
- In circular motion, centripetal acceleration refers to changes in the direction of the velocity but not its magnitude. An object undergoing circular motion experiences centripetal acceleration.

Key Terms

- **Angular position**: The angle in radians (degrees, revolutions) through which a point or line has been rotated in a specified sense about a specified axis.
- **angular velocity**: A vector quantity describing an object in circular motion; its magnitude is equal to the speed of the particle and the direction is perpendicular to the plane of its circular motion.
- **angular acceleration**: The rate of change of angular velocity, often represented by α.
- **tangential acceleration**: The acceleration in a direction tangent to the circle at the point of interest in circular motion.

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9.2: Angular Acceleration

learning objectives

- Relate angle of rotation, angular velocity, and angular acceleration to their equivalents in linear kinematics

Simply by using our intuition, we can begin to see the interrelatedness of rotational quantities like θ (angle of rotation), ω(angular velocity) and α (angular acceleration). For example, if a motorcycle wheel has a large angular acceleration for a fairly long time, it ends up spinning rapidly and rotating through many revolutions. The wheel's rotational motion is analogous to the fact that the motorcycle's large translational acceleration produces a large final velocity, and the distance traveled will also be large.

Kinematic Equations

Kinematics is the description of motion. We have already studied kinematic equations governing linear motion under constant acceleration:

$$\mathrm{v} = \mathrm{v}_0 + \mathrm{at} \tag{9.2.1}$$

$$\mathrm{x} = \mathrm{v}_0\mathrm{t} + \frac{1}{2}\mathrm{at}^2 \tag{9.2.2}$$

$$\mathrm{v}^2 = \mathrm{v}_0^2 + 2\mathrm{ax} \tag{9.2.3}$$

Similarly, the kinematics of rotational motion describes the relationships among rotation angle, angular velocity, angular acceleration, and time. Let us start by finding an equation relating ω, α, and t. To determine this equation, we use the corresponding equation for linear motion:

$$\mathrm{v} = \mathrm{v}_0 + \mathrm{at}. \tag{9.2.4}$$

As in linear kinematics where we assumed a is constant, here we assume that angular acceleration α is a constant, and can use the relation: $\mathrm{a} = \mathrm{r}\alpha$ Where r – radius of curve.Similarly, we have the following relationships between linear and angular values:

$$\mathrm{v} = \mathrm{r}\omega \tag{9.2.5}$$

$$\mathrm{x} = \mathrm{r}\theta \tag{9.2.6}$$

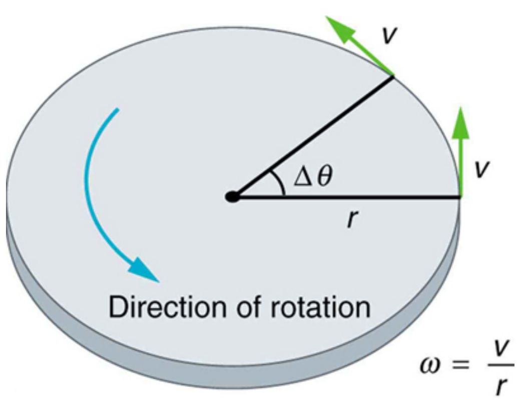
By using the relationships $\mathrm{a} = \mathrm{r}\alpha, \mathrm{v} = \mathrm{r}\omega,$ and $\mathrm{x} = \mathrm{r}\theta$, we derive all the other kinematic equations for rotational motion under constant acceleration:

$$\omega = \omega_0 + \alpha\mathrm{t} \tag{9.2.7}$$

$$\theta = \omega_0\mathrm{t} + \frac{1}{2}\alpha\mathrm{t}^2 \tag{9.2.8}$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta \tag{9.2.9}$$

The equations given above can be used to solve any rotational or translational kinematics problem in which a and α are constant. shows the relationship between some of the quantities discussed in this atom.



Linear and Angular: This figure shows uniform circular motion and some of its defined quantities.

Key Points

- The kinematic equations for rotational and/or linear motion given here can be used to solve any rotational or translational kinematics problem in which a and α are constant.
- By using the relationships between velocity and angular velocity, distance and angle of rotation, and acceleration and angular acceleration, rotational kinematic equations can be derived from their linear motion counterparts.
- To derive rotational equations from the linear counterparts, we used the relationships $a = r\alpha, v = r\omega,$ and $x = r\theta$.

Key Terms

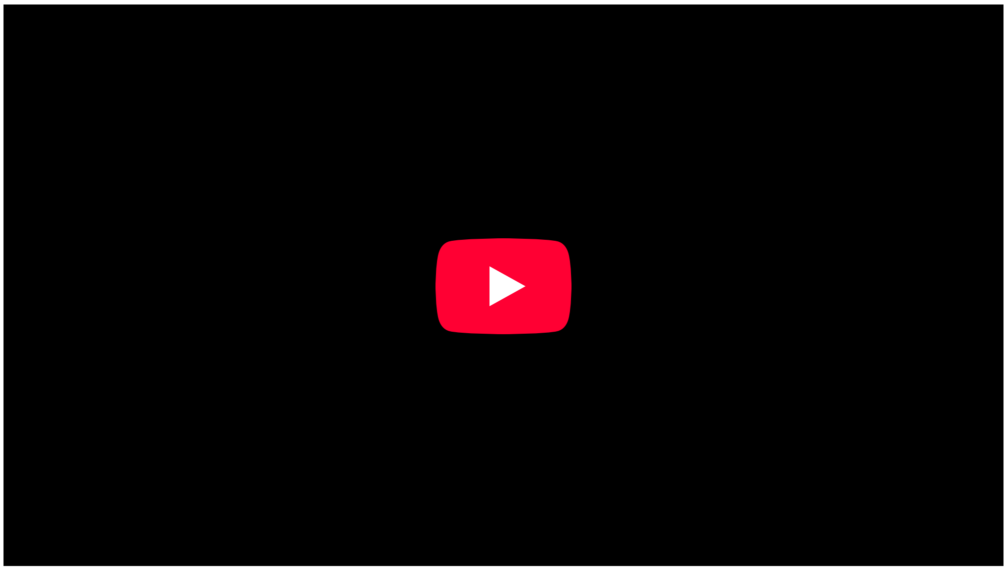
- **kinematics**: The branch of mechanics concerned with objects in motion, but not with the forces involved.
- **angular**: Relating to an angle or angles; having an angle or angles; forming an angle or corner; sharp-cornered; pointed; as in, an angular figure.

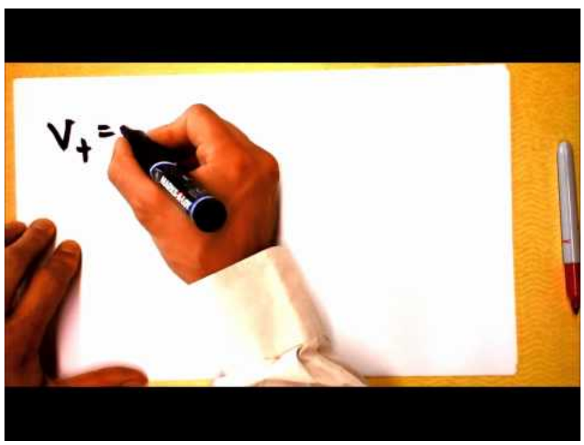
9.3: Rotational Kinematics

learning objectives

- Distinguish the two different motions in which rolling without slipping is broken down

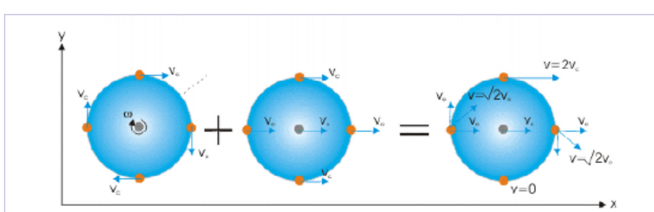
Rolling without slipping generally occurs when an object rolls without skidding. To relate this to a real world phenomenon, we can consider the example of a wheel rolling on a flat, horizontal surface.





Connecting Linear and Rotational Motion! Rolling without Slipping!: How fast does the axle of a bike wheel move? How fast does the BOTTOM of a wheel move?

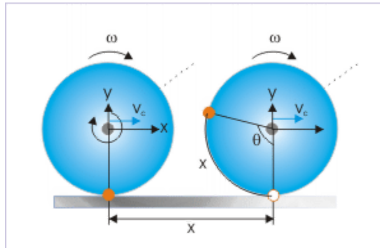
Rolling without slipping can be better understood by breaking it down into two different motions: 1) Motion of the center of mass, with linear velocity *v* (translational motion); and 2) rotational motion around its center, with angular velocity *w*.



Rolling Motion: Rolling motion is a combination of rotational and translational motion.

When an object is rolling on a plane without slipping, the point of contact of the object with the plane does not move. If we imagine a wheel moving forward by rolling on a plane at speed v, it must also be rotating about its axis at an angular speed ωω since it is rolling.

The object's angular velocity ωω is directly proportional to its velocity, because as we know, the faster we are driving in a car, the faster the wheels have to turn. So, to determine the exact relationship between linear velocity and angular velocity, we can imagine the scenario in which the wheel travels a distance of x while rotating through an angle θ.



Rolling Without Slipping: A body rolling a distance of x on a plane without slipping.

In mathematical terms, the length of the arc is equal to the angle of the segment multiplied by the object's radius, R. Therefore, we can say that the length of the arc of the wheel that has rotated an angle θ, is equal to Rθ. Furthermore, since the wheel is in constant contact with the ground, the length of the arc correlating to the angle is also equal to x. Therefore,

$$\mathrm{x}=\mathrm{R}\theta \tag{9.3.1}$$

Since xx and θθ depend on time, we can take the derivative of both sides to obtain:

$$\frac{\mathrm{dx}}{\mathrm{dt}}=\mathrm{R}\frac{\mathrm{d}\theta}{\mathrm{dt}} \tag{9.3.2}$$

where $\frac{\mathrm{dx}}{\mathrm{dt}}$ is equal to the linear velocity v, and $\frac{\mathrm{d}\theta}{\mathrm{dt}}$ is equal to the angular velocity ωω. We can then simplify this equation to:

$$\mathrm{v}=\omega\mathrm{R} \tag{9.3.3}$$

Key Points

- Rolling without slipping can be better understood by breaking it down into translational motion and rotational motion.
- When an object is rolling on a plane without slipping, the point of contact of the object with the plane does not move.
- A rolling object's velocity v is directly related to its angular velocity ω, and is mathematically expressed as $\mathrm{v}=\omega\mathrm{R}$, where R is the object's radius and v is its linear velocity.

Key Terms

- **angular velocity**: A vector quantity describing the motion of an object in circular motion; its magnitude is equal to the angular speed (ωω) of the particle, and the direction is perpendicular to the plane of its circular motion.
- **linear velocity**: A vector quantity that denotes the rate of change of position with respect to time of the object's center of mass.

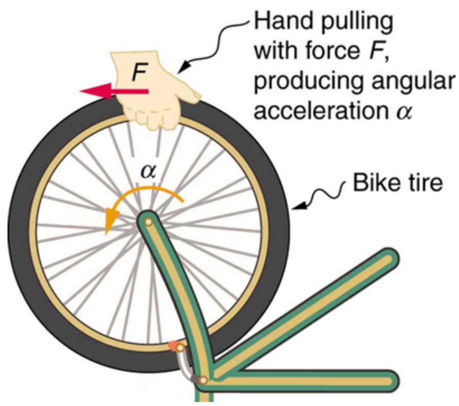
9.4: Dynamics

learning objectives

- Explain the relationship between the force, mass, radius, and angular acceleration

If you have ever spun a bike wheel or pushed a merry-go-round, you have experienced the force needed to change angular velocity. Our intuition is reliable in predicting many of the factors that are involved. For example, we know that a door opens slowly if we push too close to its hinges. Furthermore, we know that the more massive the door, the more slowly it opens. The first example implies that the farther the force is applied from the pivot, the greater the angular acceleration; another implication is that angular acceleration is inversely proportional to mass. These relationships should seem very similar to the familiar relationships among force, mass, and acceleration embodied in Newton's second law of motion. There are, in fact, precise rotational analogs to both force and mass.

Rotational inertia, as illustrated in, is the resistance of objects to changes in their rotation. In other words, a rotating object will stay rotating and a non-rotating object will stay non-rotating unless acted on by a torque. This should remind you of Newton's First Law.



Rotational Inertia: Force is required to spin the bike wheel. The greater the force, the greater the angular acceleration produced. The more massive the wheel, the smaller the angular acceleration. If you push on a spoke closer to the axle, the angular acceleration will be smaller.

To develop the precise relationship among force, mass, radius, and angular acceleration, consider what happens if we exert a force F on a point mass m that is at a distance r from a pivot point. Because the force is perpendicular to r, an acceleration $\mathrm{a}=\frac{\mathrm{F}}{\mathrm{m}}$ is obtained in the direction of F. We can rearrange this equation such that F=ma and then look for ways to relate this expression to expressions for rotational quantities. We note that a=rα, and we substitute this expression into F=ma, yielding:

$$\mathrm{F}=\mathrm{mr}\alpha. \tag{9.4.1}$$

Recall that torque is the turning effectiveness of a force. In this case, because F is perpendicular to r, torque is simply $\tau=\mathrm{Fr}$. So, if we multiply both sides of the equation above by r, we get torque on the left-hand side. That is,

$$\mathrm{rF}=\mathrm{mr}^2\alpha \tag{9.4.2}$$

, or

$$\tau=\mathrm{mr}^2\alpha. \tag{9.4.3}$$

This equation is the rotational analog of Newton's second law ($\mathrm{F}=\mathrm{ma}$), where torque is analogous to force, angular acceleration is analogous to translational acceleration, and mr^2 is analogous to mass (or inertia). The quantity mr^2 is called the rotational inertia or moment of inertia of a point mass m a distance r from the center of rotation.

Different shapes of objects have different rotational inertia which depend on the distribution of their mass.

Key Points

- The farther the force is applied from the pivot, the greater the angular acceleration.
- Angular acceleration is inversely proportional to mass.
- The equation $\tau = \mathrm{mr}^2 \alpha.$ is the rotational analog of Newton's second law ($\mathrm{F} = \mathrm{ma}$), where torque is analogous to force, angular acceleration is analogous to translational acceleration, and mr^2 is analogous to mass (or inertia).

Key Terms

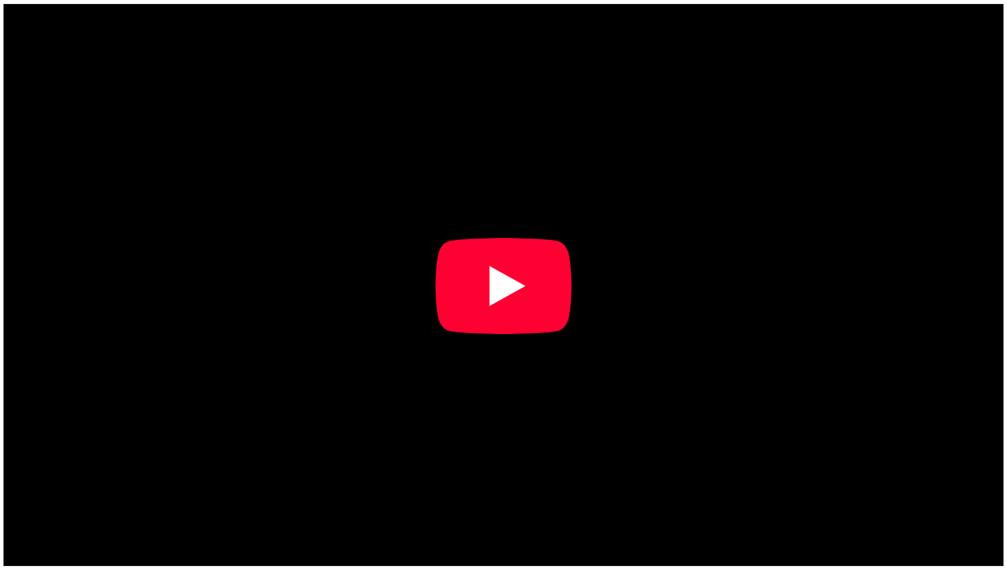
- **rotational inertia**: The tendency of a rotating object to remain rotating unless a torque is applied to it.
- **torque**: A rotational or twisting effect of a force; (SI unit newton-meter or Nm; imperial unit foot-pound or ft-lb)

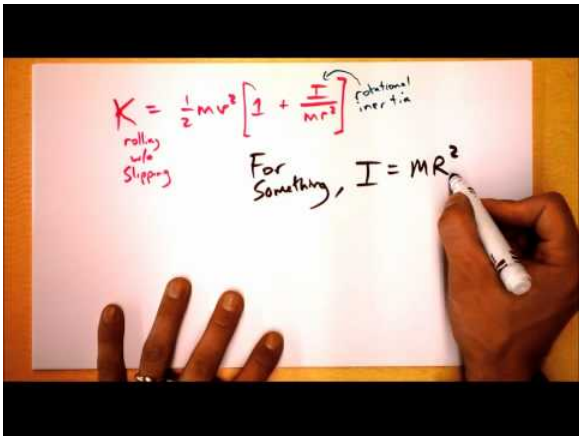
9.5: Rotational Kinetic Energy

learning objectives

- Express the rotational kinetic energy as a function of the angular velocity and the moment of inertia, and relate it to the total kinetic energy

Rotational kinetic energy is the kinetic energy due to the rotation of an object and is part of its total kinetic energy. Looking at rotational energy separately around an object's axis of rotation yields the following dependence on the object's moment of inertia:





Kinetic Energy of Rotation: Things that roll without slipping have some fraction of their energy as translational kinetic and the remainder as rotational kinetic. The ratio depends on the moment of inertia of the object that's rolling.

$$\mathrm{E_{rotational}} = \frac{1}{2}\mathrm{I}\omega^2, \tag{9.5.1}$$

where ω is the angular velocity and I is the moment of inertia around the axis of rotation.

The mechanical work applied during rotation is the torque (τ) times the rotation angle (θ) : $\mathrm{W} = \tau\theta$.

The instantaneous power of an angularly accelerating body is the torque times the angular velocity: $\mathrm{P} = \tau\omega$.

Note the close relationship between the result for rotational energy and the energy held by linear (or translational) motion:

$$\mathrm{E_{translational}} = \frac{1}{2}\mathrm{mv}^2. \tag{9.5.2}$$

In the rotating system, the moment of inertia takes the role of the mass and the angular velocity takes the role of the linear velocity.

As an example, let us calculate the rotational kinetic energy of the Earth (animated in Figure 1). As the Earth has a period of about 23.93 hours, it has an angular velocity of 7.29×10^{-5} rad/s. The Earth has a moment of inertia, $I = 8.04\times10^{37}$ kg·m2. Therefore, it has a rotational kinetic energy of 2.138×10^{29} J.

The Rotating Earth: The earth's rotation is a prominent example of rotational kinetic energy.

This can be partially tapped using tidal power. Additional friction of the two global tidal waves creates energy in a physical manner, infinitesimally slowing down Earth's angular velocity. Due to conservation of angular momentum this process transfers angular momentum to the Moon's orbital motion, increasing its distance from Earth and its orbital period.

Moment of Inertia

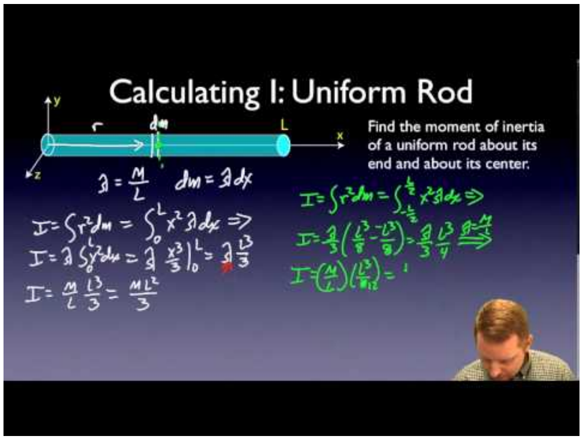
The moment of inertia is a property of a mass that measures its resistance to rotational acceleration about one or more axes.

learning objectives

- Identify a property of a mass described by the moment of inertia

The Moment of Inertia

The moment of inertia is a property of the distribution of mass in space that measures mass's resistance to rotational acceleration about one or more axes. Newton's first law, which describes the inertia of a body in linear motion, can be extended to the inertia of a body rotating about an axis using the moment of inertia. That is, an object that is rotating at constant angular velocity will remain rotating unless it is acted upon by an external torque. In this way, the moment of inertia plays the same role in rotational dynamics as mass does in linear dynamics: it describes the relationship between angular momentum and angular velocity as well as torque and angular acceleration.



Moment of Inertia: A brief introduction to moment of inertia (rotational inertia) for calculus-based physics students.

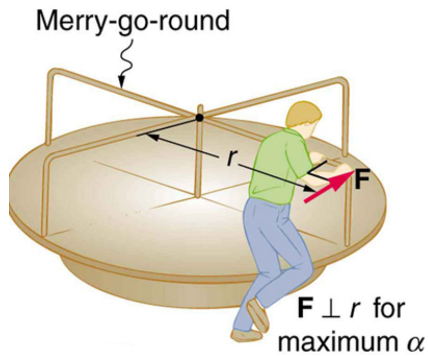
The moment of inertia *I* of an object can be defined as the sum of mr^2 for all the point masses of which it is composed, where m is the mass and r is the distance of the mass from the center of mass. It can be expressed mathematically as: $\mathrm{I}=\sum \mathrm{mr}^2$. Here, I is analogous to m in translational motion.

As an example, consider a hoop of radius r. Assuming that the hoop material is uniform, the hoop's moment of inertia can be found by summing up all the mass of the hoop and multiplying by the distance of that mass from the center of mass. Since the hoop is a circle and the mass is uniform around the circle, the moment of inertia is mr^2. All of the mass m is at a distance r from the center.

Moment of inertia also depends on the axis about which you rotate an object. Objects will usually rotate about their center of mass, but can be made to rotate about any axis. The moment of inertia in the case of rotation about a different axis other than the center of mass is given by the parallel axis theorem. The theorem states that the moment of inertia for an object rotated about a different axis parallel to the axis passing through the center of mass is $\mathrm{I_{cm}}+\mathrm{mr}^2$ where r is now the distance between the two axes and IcmIcmis the moment of inertia when rotated about the center of mass which you learned how to calculate in the previous paragraph.

A general relationship among the torque, moment of inertia, and angular acceleration is: net $\tau=\mathrm{I}\alpha$, or $\alpha=\frac{(\text{net } \tau)}{\mathrm{I}}$. Net τ is the total torque from all forces relative to a chosen axis. Such torques are either positive or negative and add like ordinary numbers. The relationship in $\tau=\mathrm{I}\alpha$ is the rotational analog to Newton's second law and is very applicable. This equation is actually valid for any torque, applied to any object, and relative to any axis.

As can be expected, the larger the torque, the larger the angular acceleration. For example, the harder a child pushes on a merry-go-round, the slower it accelerates for the same torque. The basic relationship between the moment of inertia and the angular acceleration is that the larger the moment of inertia, the smaller the angular acceleration. The moment of inertia depends not only on the mass of an object, but also on its distribution of mass relative to the axis around which it rotates. For example, it wil be much easier to accelerate a merry-go-round full of children if they stand close to its axis than if they all stand at the outer edge.



Moment of Inertia on a Merry-Go-Round: A father pushes a playground merry-go-round at its edge and perpendicular to its radius to achieve maximum torque.

Key Points

- Rotational kinetic energy can be expressed as: $\mathrm{E_{rotational}} = \frac{1}{2}\mathrm{I}\omega^2$ where ω is the angular velocity and I is the moment of inertia around the axis of rotation.
- The mechanical work applied during rotation is the torque times the rotation angle: (θ) : $\mathrm{W} = \tau\theta$.
- The instantaneous power of an angularly accelerating body is the torque times the angular velocity: $\mathrm{P} = \tau\omega$.
- There is a close relationship between the result for rotational energy and the energy held by linear (or translational) motion.
- Newton's first law, which describes the inertia of a body in linear motion, can be extended to the inertia of a body rotating about an axis using the moment of inertia.
- An object that is rotating at constant angular velocity will remain rotating unless it is acted upon by an external torque.
- The larger the torque, the larger the angular acceleration.

Key Items

- **torque**: A rotational or twisting effect of a force; (SI unit newton-meter or Nm; imperial unit foot-pound or ft-lb)
- **inertia**: The property of a body that resists any change to its uniform motion; equivalent to its mass.
- **angular velocity**: A vector quantity describing an object in circular motion; its magnitude is equal to the speed of the particle and the direction is perpendicular to the plane of its circular motion.

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9.6: Conservation of Angular Momentum

learning objectives

- Evaluate the implications of net torque on conservation of energy

Let us consider some examples of momentum: the Earth continues to spin at the same rate it has for billions of years; a high-diver who is "rotating" when jumping off the board does not need to make any physical effort to continue rotating, and indeed would be unable to stop rotating before hitting the water. These examples have the hallmarks of a *conservation law*. Following are further observations to consider:

1. *A closed system is involved*. Nothing is making an effort to twist the Earth or the high-diver. They are isolated from rotation changing influences (hence the term "closed system").
2. *Something remains unchanged*. There appears to be a numerical quantity for measuring rotational motion such that the total amount of that quantity remains constant in a closed system.
3. *Something can be transferred back and forth without changing the total amount*. A diver rotates faster with arms and legs pulled toward the chest from a fully stretched posture.

Angular Momentum

The conserved quantity we are investigating is called angular momentum. The symbol for angular momentum is the letter L. Just as linear momentum is conserved when there is no net external forces, angular momentum is constant or conserved when the net torque is zero. We can see this by considering Newton's 2nd law for rotational motion:

$\vec{\tau} = \frac{d\vec{L}}{dt}$, where τ is the torque. For the situation in which the net torque is zero, $\frac{d\vec{L}}{dt} = 0$.

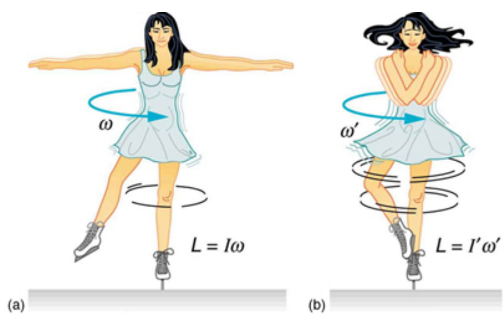
If the change in angular momentum ΔL is zero, then the angular momentum is constant; therefore,

$\vec{L} = \text{constant}$ (when net τ=0).

This is an expression for the law of conservation of angular momentum.

Example and Implications

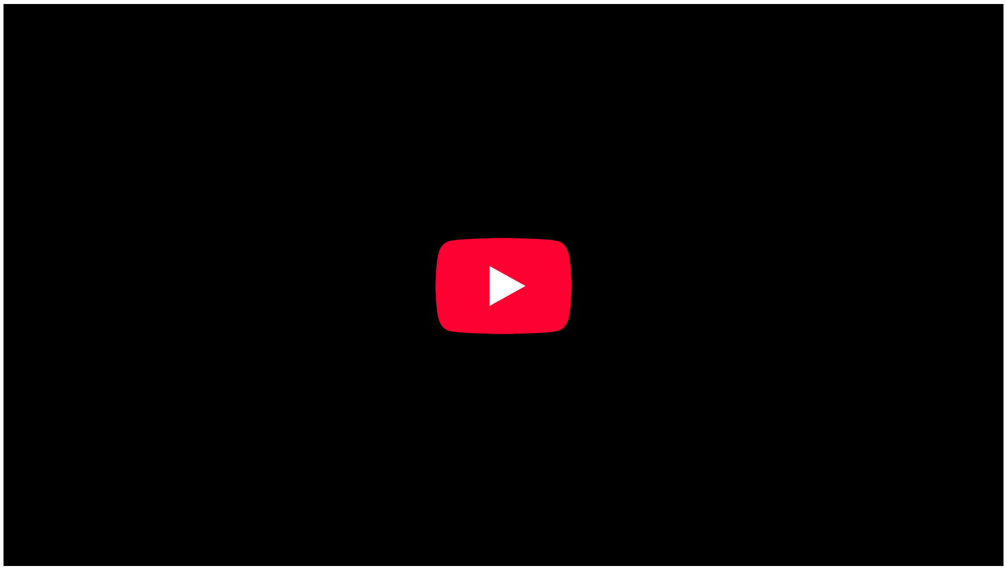
An example of conservation of angular momentum is seen in an ice skater executing a spin, as shown in. The net torque on her is very close to zero, because 1) there is relatively little friction between her skates and the ice, and 2) the friction is exerted very close to the pivot point.

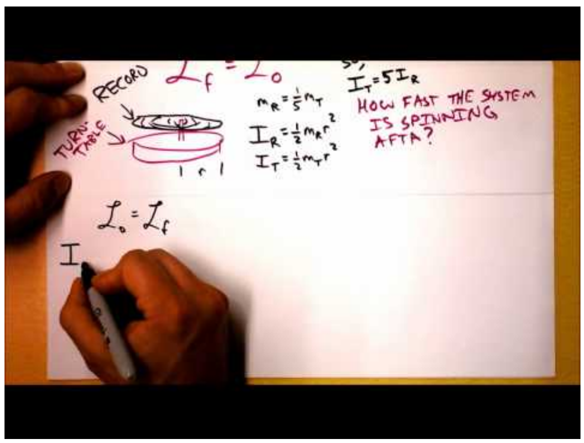


Conservation of Angular Momentum: An ice skater is spinning on the tip of her skate with her arms extended. Her angular momentum is conserved because the net torque on her is negligibly small. In the next image, her rate of spin increases greatly when she pulls in her arms, decreasing her moment of inertia. The work she does to pull in her arms results in an increase in rotational kinetic energy.

(Both F and r are small, and so $\vec{\tau} = \vec{r} \times \vec{F}$ is negligibly small.) Consequently, she can spin for quite some time. She can also increase her rate of spin by pulling in her arms and legs. When she does this, the rotational inertia decreases and the rotation rate increases in order to keep the angular momentum $L = I\omega$ constant. (I: rotational inertia, ω: angular velocity)

Conservation of angular momentum is one of the key conservation laws in physics, along with the conservation laws for energy and (linear) momentum. These laws are applicable even in microscopic domains where quantum mechanics governs; they exist due to inherent symmetries present in nature.





Conservation of Angular Momentum Theory: What it do?

Rotational Collisions

In a closed system, angular momentum is conserved in a similar fashion as linear momentum.

learning objectives

- Evaluate the difference in equation variables in rotational versus angular momentum

During a collision of objects in a closed system, momentum is always conserved. This fact is readily seen in linear motion. When an object of mass m and velocity v collides with another object of mass m_2 and velocity v_2, the net momentum after the collision, $mv_{1f} + mv_{2f}$, is the same as the momentum before the collision, $mv_{1i} + mv_{2i}$.

What if an rotational component of motion is introduced? Is momentum still conserved ?

Bowling ball and pi: When a bowling ball collides with a pin, linear and angular momentum is conserved

Yes. For objects with a rotational component, there exists angular momentum. Angular momentum is defined, mathematically, as $L = I\omega$, or $L = r \times p$. This equation is an analog to the definition of linear momentum as $p = mv$. Units for linear momentum are kg⋅m/s while units for angular momentum are kg⋅m^2/s. As we would expect, an object that has a large moment of inertia *I*, such as Earth, has a very large angular momentum. An object that has a large angular velocity ω, such as a centrifuge, also has a rather large angular momentum.

So rotating objects that collide in a closed system conserve not only linear momentum p in all directions, but also angular momentum L in all directions.

For example, take the case of an archer who decides to shoot an arrow of mass m_1 at a stationary cylinder of mass m_2 and radius r, lying on its side. If the archer releases the arrow with a velocity v_{1i} and the arrow hits the cylinder at its radial edge, what's the final momentum ?

Arrow hitting cyclinde: The arrow hits the edge of the cylinder causing it to roll.

Initially, the cylinder is stationary, so it has no momentum linearly or radially. Once the arrow is released, it has a linear momentum $p = m_{1i}$ and an angular component relative to the cylinders rotating axis, $L = rp = rm_1v_{1i}$. After the collision, the arrow sticks to the rolling cylinder and the system has a net angular momentum equal to the original angular momentum of the arrow before the collision.

Key Points

- When an object is spinning in a closed system and no external torques are applied to it, it will have no change in angular momentum.
- The conservation of angular momentum explains the angular acceleration of an ice skater as she brings her arms and legs close to the vertical axis of rotation.
- If the net torque is zero, then angular momentum is constant or conserved.
- Angular momentum is defined, mathematically, as $L = I\omega$, or $L = r \times p$. Which is the moment of inertia times the angular velocity, or the radius of the object crossed with the linear momentum.
- In a closed system, angular momentum is conserved in all directions after a collision.
- Since momentum is conserved, part of the momentum in a collision may become angular momentum as an object starts to spin after a collision.

Key Items

- **quantum mechanics**: The branch of physics that studies matter and energy at the level of atoms and other elementary particles; it substitutes probabilistic mechanisms for classical Newtonian ones.

- **torque**: A rotational or twisting effect of a force; (SI unit newton-meter or Nm; imperial unit foot-pound or ft-lb)
- **angular momentum**: A vector quantity describing an object in circular motion; its magnitude is equal to the momentum of the particle, and the direction is perpendicular to the plane of its circular motion.
- **momentum**: (of a body in motion) the product of its mass and velocity.
- **rotation**: The act of turning around a centre or an axis.

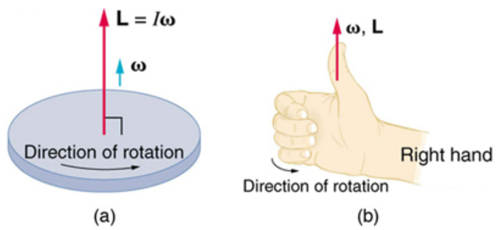
9.7: Vector Nature of Rotational Kinematics

learning objectives

- Identify the direction of a vector using the Right Hand Rule

Angular momentum and angular velocity have both magnitude and direction and, therefore, are vector quantities. The direction of these quantities is inherently difficult to track—a point on a rotating wheel is constantly rotating and changing direction. The axis of rotation of a rotating wheel is the only place that has a fixed direction. The direction of angular momentum and velocity can be determined along this axis.

Imagine the axis of rotation as a pole through the center of a wheel. The pole protrudes on both sides of the wheel and, depending on which side you're looking at, the wheel is turning either clockwise or counterclockwise. This dependency on perspective makes determining the angle of rotation slightly more difficult. As with all physical quantities, there is a standard for measurement that makes these types of quantities consistent. For angular quantities, the direction of the vector is determined using the Right Hand Rule, illustrated in.



The Right Hand Rule: Figure (a) shows a disk is rotating counterclockwise when viewed from above. Figure (b) shows the right-hand rule. The direction of angular velocity ω size and angular momentum L are defined to be the direction in which the thumb of your right hand points when you curl your fingers in the direction of the disk's rotation as shown.

The right hand rule can be used to find the direction of both the angular momentum and the angular velocity. From a spinning disc, for example, let's again imagine a pole through the center of the disc, at the axis of rotation. Using the right hand rule, your right hand would be grasping the pole so that your four fingers (index, middle, ring, and pinky) are following the direction of rotation. That is, an imaginary arrow from your wrist to your fingertips points in the same direction as the disc is rotating. In addition, your thumb is pointing straight out in the axis, perpendicular to your other fingers (or parallel to the 'pole' at the axis of rotation). Using this right hand rule, the direction of angular velocity ω and angular momentum L are defined as the direction in which the thumb of your right hand points when you curl your fingers in the direction of the disc's rotation.

Gyroscopes

A gyroscope is a spinning wheel or disk in which the axle is free to assume any orientation.

learning objectives

- Compare the concept of a rotating wheel with a gyroscope

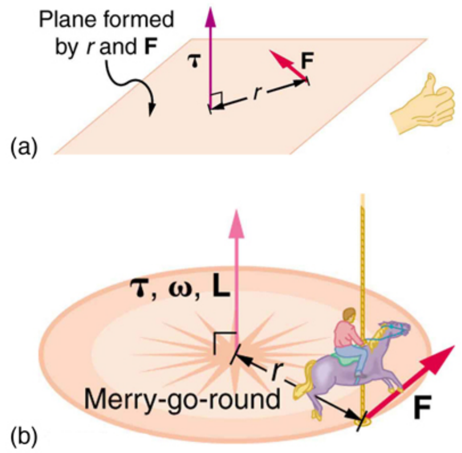
A gyroscope is a device for measuring or maintaining orientation based on the principles of angular momentum. Mechanically, a gyroscope is a spinning wheel or disk in which the axle is free to assume any orientation. Although this orientation does not remain fixed, it changes in response to an external torque much less and in a different direction than it would without the large angular momentum associated with the disk's high rate of spin and moment of inertia. The device's orientation remains nearly fixed, regardless of the mounting platform's motion, because mounting the device in a gimbal minimizes external torque.

How It Works: Examples

Torque: Torque changes angular momentum as expressed by the equation,

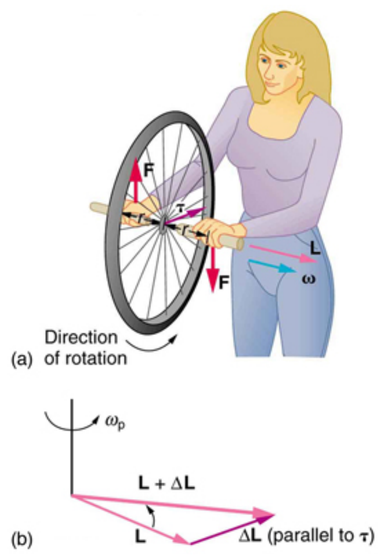
$$\tau = \frac{\Delta \mathrm{L}}{\Delta \mathrm{t}}. \tag{9.7.1}$$

This equation means that the direction of ΔL is the same as the direction of the torque that creates it, as illustrated in. This direction can be determined using the right hand rule, which says that the fingers on your hand curl towards the direction of rotation or force exerted, and your thumb points towards the direction of angular momentum, torque, and angular velocity.



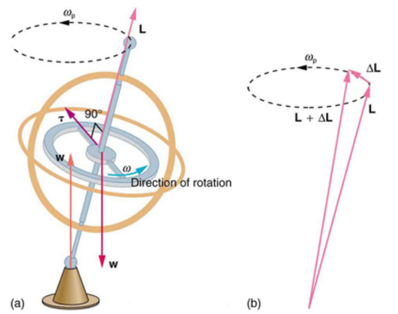
Direction of Torque and Angular Momentum: In figure (a), the torque is perpendicular to the plane formed by r and F and is the direction your right thumb would point to if you curled your fingers in the direction of F. Figure (b) shows that the direction of the torque is the same as that of the angular momentum it produces.

Rotating wheel: Consider a bicycle wheel with handles attached to it, as in. With the wheel rotating as shown, its angular momentum is to the woman's left. Suppose the person holding the wheel tries to rotate it as in the figure. Her natural expectation is that the wheel will rotate in the direction she pushes it, however, what happens is quite different. The forces exerted create a torque that is horizontal toward the person, and this torque creates a change in angular momentum L in the same direction, perpendicular to the original angular momentum L, thus changing the direction of L but not the magnitude of L. ΔL and L add, giving a new angular momentum with direction that is inclined more toward the person than before. The axis of the wheel has thus moved perpendicular to the forces exerted on it, instead of in the expected direction.



Gyroscopic Effect: In figure (a), a person holding the spinning bike wheel lifts it with her right hand and pushes down with her left hand in an attempt to rotate the wheel. This action creates a torque directly toward her. This torque causes a change in angular momentum ΔL in exactly the same direction. Figure (b) shows a vector diagram depicting how ΔL and L add, producing a new angular momentum pointing more toward the person. The wheel moves toward the person, perpendicular to the forces she exerts on it.

Gyroscope: This same logic explains the behavior of gyroscopes (see). There are two forces acting on a spinning gyroscope. The torque produced is perpendicular to the angular momentum, thus the direction of the angular momentum is changed, but not its magnitude. The gyroscope precesses around a vertical axis, since the torque is always horizontal and perpendicular to L. If the gyroscope is not spinning, it acquires angular momentum in the direction of the torque (L=ΔL), and it rotates around a horizontal axis, falling over just as we would expect.



Gyroscopes: As seen in figure (a), the forces on a spinning gyroscope are its weight and the supporting force from the stand. These forces create a horizontal torque on the gyroscope, which create a change in angular momentum ΔL that is also horizontal. In figure (b), ΔL and L add to produce a new angular momentum with the same magnitude, but different direction, so that the gyroscope precesses in the direction shown instead of falling over.

Applications

Gyroscopes serve as rotational sensors. For this reason, applications of gyroscopes include inertial navigation systems where magnetic compasses would not work (as in the Hubble telescope) or would not be precise enough (as in ICBMs). Another application is the stabilization of flying vehicles, such as radio-controlled helicopters or unmanned aerial vehicles.

Key Points

- Angular velocity and angular momentum are vector quantities and have both magnitude and direction.
- The direction of angular velocity and angular momentum are perpendicular to the plane of rotation.
- Using the right hand rule, the direction of both angular velocity and angular momentum is defined as the direction in which the thumb of your right hand points when you curl your fingers in the direction of rotation.
- Torque is perpendicular to the plane formed by r and F and is the direction your right thumb would point if you curled the fingers of your right hand in the direction of F.
- The direction of the torque is thus the same as that of the angular momentum it produces.
- The gyroscope precesses around a vertical axis, since the torque is always horizontal and perpendicular to L. If the gyroscope is not spinning, it acquires angular momentum in the direction of the torque, and it rotates about a horizontal axis, falling over just as we would expect.

Key Items

- **angular momentum**: A vector quantity describing an object in circular motion; its magnitude is equal to the momentum of the particle, and the direction is perpendicular to the plane of its circular motion.
- **right hand rule**: Direction of angular velocity ω and angular momentum L in which the thumb of your right hand points when you curl your fingers in the direction of rotation.
- **angular velocity**: A vector quantity describing an object in circular motion; its magnitude is equal to the speed of the particle and the direction is perpendicular to the plane of its circular motion.
- **gimbal**: A device for suspending something, such as a ship's compass, so that it will remain level when its support is tipped.
- **torque**: A rotational or twisting effect of a force; (SI unit newton-meter or Nm; imperial unit foot-pound or ft-lb)

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9.8: Problem Solving

learning objectives

- Develop and apply a strong problem-solving strategy for rotational kinematics

Problem-Solving Strategy For Rotational Kinematics

When solving problems on rotational kinematics:

- Examine the situation to determine that rotational kinematics (rotational motion) is involved. Rotation must be involved, but without the need to consider forces or masses that affect the motion.
- Identify exactly what needs to be determined in the problem (identify the unknowns). A sketch of the situation is useful.
- Make a list of what is given or can be inferred from the problem as stated (identify the knowns).
- Solve the appropriate equation or equations for the quantity to be determined (the unknown). It can be useful to think in terms of a translational analog because by now you are familiar with such motion.
- Substitute the known values along with their units into the appropriate equation, and obtain numerical solutions complete with units. Be sure to use units of radians for angles.
- Check your answer to see if it is reasonable: Does your answer make sense?

Example 9.8.1:

Suppose a large freight train accelerates from rest, giving its 0.350 m radius wheels an angular acceleration of 0.250 rad/s^2. After the wheels have made 200 revolutions (assume no slippage): (a) How far has the train moved down the track? (b) What are the final angular velocity of the wheels and the linear velocity of the train?

In part (a), we are asked to find x, and in (b) we are asked to find ω and v. We are given the number of revolutions θ, the radius of the wheels r, and the angular acceleration α.

The distance x is very easily found from the relationship between distance and rotation angle: $\theta=\frac{x}{r}$.

Solving this equation for x yields $x=r\theta$.

Before using this equation, we must convert the number of revolutions into radians, because we are dealing with a relationship between linear and rotational quantities:

$$\theta=(200\text{rev})\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)=1257\text{ rad}. \tag{9.8.1}$$

Substitute the known values into $x=r\theta$ to find the distance the train moved down the track:

$$x=r\theta=(0.350\text{ m})(1257\text{ rad})=440\text{ m}. \tag{9.8.2}$$

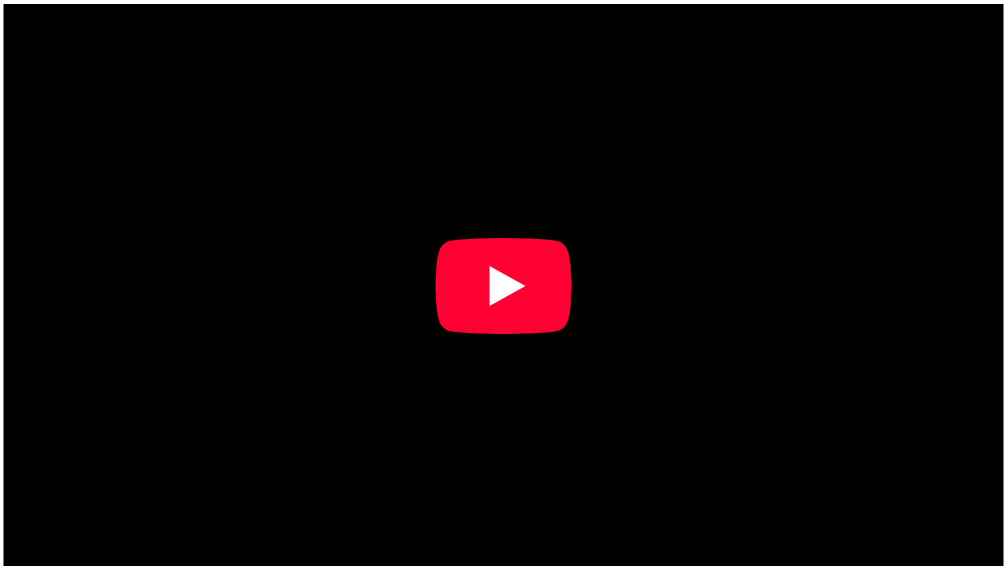
We cannot use any equation that incorporates t to find ω, because the equation would have at least two unknown values. The equation $\omega^2=\omega_0^2+2\alpha\theta$ will work, because we know the values for all variables except ω. Taking the square root of this equation and entering the known values gives

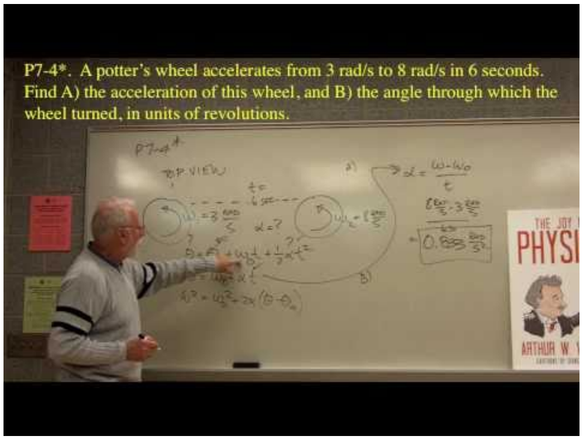
$$\omega=\sqrt{0+2(0.250\text{ rad/s}^2)(1257\text{ rad})} \tag{9.8.3}$$

$$=25.1\text{ rad/s} \tag{9.8.4}$$

One may find the linear velocity of the train, v, through its relationship to ω:

$$v=r\omega=(0.350\text{ m})(25.1\text{ rad/s})=8.77\text{ m/s} \tag{9.8.5}$$





Rotational motion: Part of a series of videos on physics problem-solving. The problems are taken from "The Joy of Physics. " This one deals with angular motion. The viewer is urged to pause the video at the problem statement and work the problem before watching the rest of the video.

Rotational	Translational	
$\theta = \overline{\omega} t$	$x = \overline{v} t$	
$\omega = \omega_0 + \alpha t$	$v = v_0 + at$	(constant α, a)
$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	$x = v_0 t + \frac{1}{2} a t^2$	(constant α, a)
$\omega^2 = {\omega_0}^2 + 2\alpha\theta$	$v^2 = {v_0}^2 + 2ax$	(constant α, a)

Table 1: **Rotational Kinematic Equations**

Equation list: Rotational and translational kinematic equations.

Key Points

- Examine the situation to determine that rotational kinematics (rotational motion) is involved, and identify exactly what needs to be determined.
- Make a list of what is given or can be inferred from the problem as stated and solve the appropriate equations.

- Substitute the known values along with their units into the appropriate equation, and obtain numerical solutions complete with units.

Key Terms

- **kinematics**: The branch of mechanics concerned with objects in motion, but not with the forces involved.

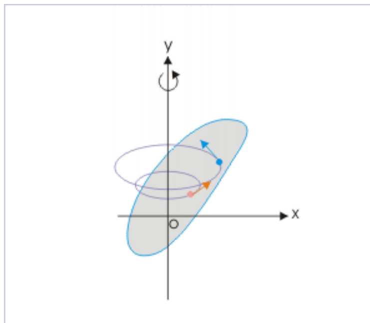
9.9: Linear and Rotational Quantities

learning objectives

- Derive uniform circular motion from linear equations

Defining Circular Motion

The description of circular motion is described better in terms of angular quantity than its linear counter part. The reasons are easy to understand. For example, consider the case of uniform circular motion. Here, the velocity of particle is changing – though the motion is “uniform”. The two concepts do not go together. The general connotation of the term “uniform” indicates “constant”, but the velocity is actually changing all the time.



A Rotating Body: Each particle constituting the body executes a uniform circular motion about the fixed axis. For the description of the motion, angular quantities are the better choice.

When we describe the uniform circular motion in terms of angular velocity, there is no contradiction. The velocity (i.e. angular velocity) is indeed constant. This is the first advantage of describing uniform circular motion in terms of angular velocity.

Second advantage is that angular velocity conveys the physical sense of the rotation of the particle as against linear velocity, which indicates translational motion. Alternatively, angular description emphasizes the distinction between two types of motion (translational and rotational).

Relationship Between Linear and Angular Speed

For simplicity, let’s consider a uniform circular motion. For the length of the arc subtending angle ” at the origin and “r” is the radius of the circle containing the position of the particle, we have $s = r\theta$.

Differentiating with respect to time, we have

$$\frac{ds}{dt} = \frac{dr}{dt}\theta + r\frac{d\theta}{dt}. \tag{9.9.1}$$

Because $\frac{dr}{dt} = 0$ for a uniform circular motion, we get $v = \omega r$. Similarly, we also get $a = \alpha r$ where a stands for linear acceleration, while α refers to angular acceleration (In a more general case, the relationship between angular and linear quantities are given as $v = \omega \times r, a = \alpha \times r + \omega \times v$

Rotational Kinematic Equations

With the relationship of the linear and angular speed/acceleration, we can derive the following four rotational kinematic equations for constant aa and αα:

$$\omega = \omega_0 + \alpha t : v = v_0 + at \tag{9.9.2}$$

$$\theta = \omega_0 t + (\frac{1}{2})\alpha t^2 : x = v_0 t + (\frac{1}{2})at^2 \tag{9.9.3}$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta : v^2 = v_0^2 + 2ax \tag{9.9.4}$$

Mass, Momentum, Energy, and Newton’s Second Law

As we use mass, linear momentum, translational kinetic energy, and Newton's 2nd law to describe linear motion, we can describe a general rotational motion using corresponding scalar/vector/tensor quantities:

- Mass/ Rotational inertia:
- Linenar/angular momentum:
- Force/ Torque:
- Kinetic energy:

For example, just as we use the equation of motion F=maF=ma to describe a linear motion, we can use its counterpart $\tau=\frac{\mathrm{dL}}{\mathrm{dt}}=\mathrm{r}\times\mathrm{F}$ to describe an angular motion. The descriptions are equivalent, and the choice can be made purely for the convenience of use.

Key Points

- As we use mass, linear momentum, translational kinetic energy, and Newton's 2nd law to describe linear motion, we can describe a general rotational motion using corresponding scalar/vector/tensor quantities.
- Angular and linear velocity have the following relationship: $\mathrm{v}=\omega\times\mathrm{r}$.
- As we use the equation of motion $\mathrm{F}=\mathrm{ma}$ to describe a linear motion, we can use its counterpart $\tau=\frac{\mathrm{dL}}{\mathrm{dt}}=\mathrm{r}\times\mathrm{F}$, to describe angular motion. The descriptions are equivalent, and the choice can be made purely for the convenience of use.

Key Items

- **uniform circular motion**: Movement around a circular path with constant speed.
- **torque**: A rotational or twisting effect of a force; (SI unit newton-meter or Nm; imperial unit foot-pound or ft-lb)
- **rotational inertia**: The tendency of a rotating object to remain rotating unless a torque is applied to it.

CHAPTER OVERVIEW

10: Fluids

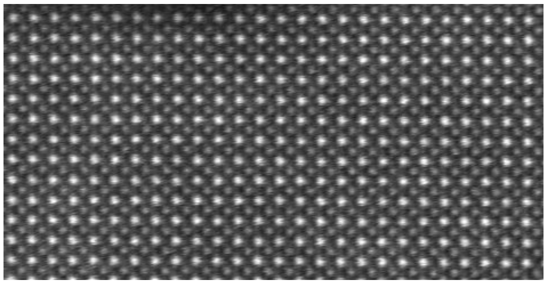
10.1: Introduction

learning objectives

- Assess the distinguishing characteristics of the four states of matter

There are a number of properties that can be observed in a material that identify what state the matter is in — solid, liquid, gas, or plasma.

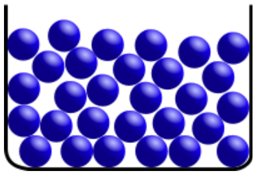
Solids



Solid: Solids are in a state of matter that maintains a fixed volume and shape.

A solid is in a state of matter that maintains a fixed volume and shape. A solid's particles fit closely together. The forces between the particles are so strong that the particles can not move freely; they can only vibrate. This causes a solid to be a stable, non-compressible shape with definite volume.

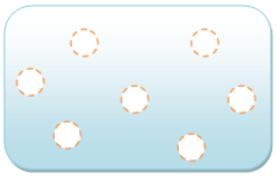
Liquids



Liquid: Liquids maintain a fixed volume, but their shape will mold to the shape of the container they are being held in.

A liquid maintains a fixed volume, but its shape will mold to the shape of the container it is being held in. In, you can see that even though the liquid's shape is determined by the container, it has a free surface that is not controlled by the container. The particles are close together but not as close as in solids; they are still able to move around, which causes the liquid to flow. Liquids usually have a higher volume than their solid counterparts, except for certain molecules, such as H_2O (water).

Gases

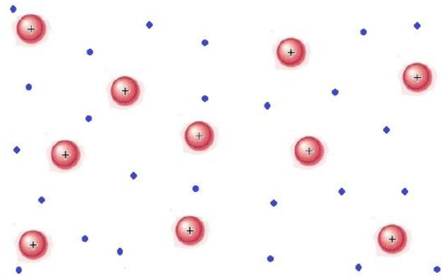


Gas: The particles are much farther from each other, usually a farther length than the size of the particles, and move a lot

shows a gas, whose particles move around a lot and are much farther apart from each other, usually farther apart than the diameter of the particles themselves. The gas behaves like a liquid; the particles are moving but are still attracted to each other, so they still flow. Unlike a solid or a liquid, the gas will try to fill whatever container it is in, adapting its volume accordingly.

Plasma

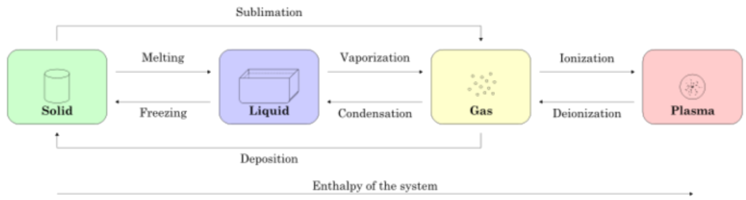
Plasma is a gas that has been ionized. That is to say, sufficient energy has been supplied to the gas such that the electrons have enough energy to escape their atoms or molecules. Plasma contains ions and electrons that can move around freely. Matter in the plasma state has variable volume and shape. Plasma is the most common form of visible matter in the universe. Lightning, sparks, neon lights, and the sun are all examples of matter in the plasma state.



Plasma: Matter in the plasma state has variable volume and shape, but as well as neutral atoms, it contains a significant number of ions and electrons, both of which can move around freely.

Phase Transitions

shows the different states of matter and how they can change from one to another as a function of enthalpy and pressure and temperature changes. A solid can change to a liquid with an enthalpy increase. The process of a liquid going to a solid is known as melting. A liquid can change into a gas when it hits its boiling point or can even enter a plasma state if the enthalpy is increased enough. When the enthalpy is lowered, a liquid can transform into a solid through freezing. Sometimes, a solid can skip freezing and go directly to a gaseous state; this process is called sublimation.



States of Matter: This figure illustrates the relationship between the enthalpy of a system and the state of matter that the system is in.

What is a Fluid?

A fluid is a substance that continually deforms (flows) under an applied shear stress.

learning objectives

- Explain the properties of substances under an applied shear stress

A fluid is a substance that continually deforms (flows) under an applied shear stress. Fluids are a subset of the states of matter and include three of the four states—liquids, gases, and plasma (shown in).

Four Fundamental States of Matter: Four fundamental states of matter: 1) top left corner corresponds to solid; 2) top right corner corresponds to liquid; 3) bottom left corner corresponds to gas; 4) bottom right corner corresponds to plasma.

Liquids form a free surface (i.e., a surface not created by the container) while gases do not. The distinction between solids and fluid is not entirely obvious. The distinction is made by evaluating the viscosity of the substance. Silly Putty can be considered to behave like a solid or a fluid, depending on the time period over which it is observed. It is best described as a viscoelastic fluid.

Fluids display properties such as:

- a) not resisting deformation or resisting it only lightly (viscosity), and
- b) the ability to flow (also described as the ability to take on the shape of the container).

This also means that all fluids have the property of fluidity. These properties are typically a function of their inability to support a shear stress in static equilibrium.

Solids can be subjected to shear stresses, and normal stresses—both compressive and tensile. In contrast, ideal fluids can only be subjected to normal, compressive stress (called pressure). Real fluids display viscosity and so are capable of being subjected to low levels of shear stress.

Although the term *fluid* includes both the liquid and gas phases, it is also commonly used as a synonym for *liquid*, with no implication that gas could also be present. For example, "brake fluid" is hydraulic oil and will not perform its required function if there is gas in it. This colloquial usage of the term is also common in the fields of medicine and nutrition (e.g., "take plenty of fluids").

Key Points

- Solids are non-compressible and have constant volume and constant shape.
- Liquids are non-compressible and have constant volume but can change shape. A liquid's shape is dictated by the shape of the container it is in.
- Gases do not have a constant volume or shape; they not only take the shape of the container they are in, they try to fill the entire container.
- Matter in the plasma state has variable volume and shape. Plasma contains ions and electrons, both of which can move around freely.
- Fluids are a subset of the states of matter and include liquids, gases, and plasma.
- Fluids display properties such as: not resisting deformation or resisting it only lightly (viscosity), and the ability to flow (also described as the ability to take on the shape of the container).
- Ideal fluids can only be subjected to normal, compressive stress which is called pressure. Real fluids display viscosity and thus are capable of being subjected to low levels of shear stress.

Key Items

- **plasma**: a state of matter consisting of partially ionized gas
- **enthalpy**: the total amount of energy in a system, including both the internal energy and the energy needed to displace its environment
- **sublimation**: the transition of a substance from the solid phase directly to the vapor state such that it does not pass through the intermediate, liquid phase
- **fluidity**: A measure of the extent to which something is fluid. The reciprocal of its viscosity.
- **viscosity**: A quantity expressing the magnitude of internal friction in a fluid, as measured by the force per unit area resisting uniform flow.
- **shear stress**: The component of stress that causes parallel layers of a material to move relative to each other in their own planes.

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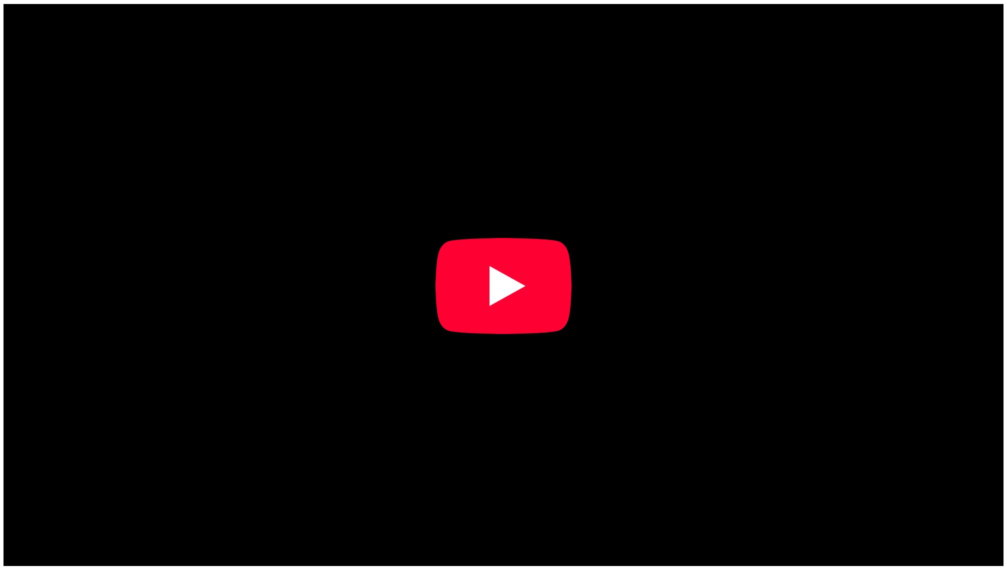
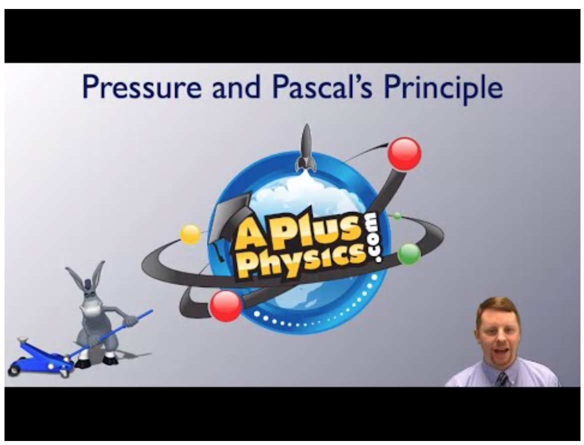
10.2: Density and Pressure

learning objectives

- Identify factors that determine the pressure exerted by the gas

Pressure is an important physical quantity—it plays an essential role in topics ranging from thermodynamics to solid and fluid mechanics. As a scalar physical quantity (having magnitude but no direction), pressure is defined as the force per unit area applied perpendicular to the surface to which it is applied. Pressure can be expressed in a number of units depending on the context of use.





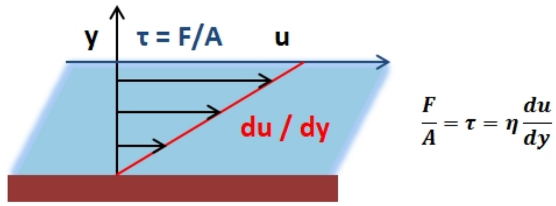
Pressure and Pascal's Principle: A brief introduction to pressure and Pascal's Principle, including hydraulics.

Units, Equations and Representations

In SI units, the unit of pressure is the Pascal (Pa), which is equal to a Newton / meter^2 (N/m^2). Other important units of pressure include the pound per square inch (psi) and the standard atmosphere (atm). The elementary mathematical expression for pressure is given by:

$$\text{pressure} = \frac{\text{Force}}{\text{Area}} = \frac{\mathrm{F}}{\mathrm{A}} \tag{10.2.1}$$

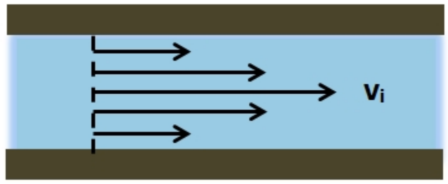
where p is pressure, F is the force acting perpendicular to the surface to which this force is applied, and A is the area of the surface. Any object that possesses weight, whether at rest or not, exerts a pressure upon the surface with which it is in contact. The magnitude of the pressure exerted by an object on a given surface is equal to its weight acting in the direction perpendicular to that surface, divided by the total surface area of contact between the object and the surface. shows the graphical representations and corresponding mathematical expressions for the case in which a force acts perpendicular to the surface of contact, as well as the case in which a force acts at angle θ relative to the surface.



Representation of Pressure: This image shows the graphical representations and corresponding mathematical expressions for the case in which a force acts perpendicular to the surface of contact, as well as the case in which a force acts at angle θ relative to the surface.

Pressure as a Function of Surface Area

Since pressure depends only on the force acting perpendicular to the surface upon which it is applied, only the force component perpendicular to the surface contributes to the pressure exerted by that force on that surface. Pressure can be increased by either increasing the force or by decreasing the area or can oppositely be decreased by either decreasing the force or increasing the area. illustrates this concept. A rectangular block weighing 1000 N is first placed horizontally. It has an area of contact (with the surface upon which it is resting) of 0.1 m^2, thus exerting a pressure of 1,000 Pa on that surface. That same block in a different configuration (also in Figure 2), in which the block is placed vertically, has an area of contact with the surface upon which it is resting of 0.01 m^2, thus exerting a pressure of 10,000 Pa—10 times larger than the first configuration due to a decrease in the surface area by a factor of 10.



Pressure as a Function of Surface Area: Pressure can be increased by either increasing the force or by decreasing the area or can oppositely be decreased by either decreasing the force or increasing the area.

A good illustration of this is the reason a sharp knife is far more effective for cutting than a blunt knife. The same force applied by a sharp knife with a smaller area of contact will exert a much greater pressure than a blunt knife having a considerably larger area of contact. Similarly, a person standing on one leg on a trampoline causes a greater displacement of the trampoline than that same person standing on the same trampoline using two legs—not because the individual exerts a larger force when standing on one leg, but because the area upon which this force is exerted is decreased, thus increasing the pressure on the trampoline. Alternatively, an object having a weight larger than another object of the same dimensionality and area of contact with a given surface will exert a greater pressure on that surface due to an increase in force. Finally, when considering a given force of constant magnitude acting on a constant area of a given surface, the pressure exerted by that force on that surface will be greater the larger the angle of that force as it acts upon the surface, reaching a maximum when that force acts perpendicular to the surface.

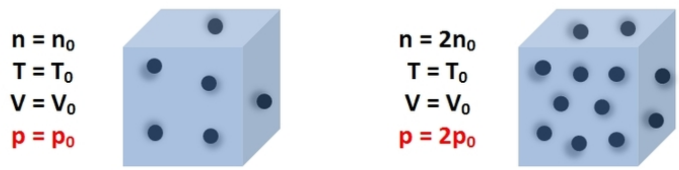
Liquids and Gases: Fluids

Just as a solid exerts a pressure on a surface upon which it is in contact, liquids and gases likewise exert pressures on surfaces and objects upon which they are in contact with. The pressure exerted by an ideal gas on a closed container in which it is confined is best analyzed on a molecular level. Gas molecules in a gas container move in a random manner throughout the volume of the container, exerting a force on the container walls upon collision. Taking the overall average force of all the collisions of the gas molecules confined within the container over a unit time allows for a proper measurement of the effective force of the gas molecules on the container walls. Given that the container acts as a confining surface for this net force, the gas molecules exert a pressure on the container. For such an ideal gas confined within a rigid container, the pressure exerted by the gas molecules can be calculated using the ideal gas law:

$$\mathrm{p} = \frac{\mathrm{nRT}}{\mathrm{V}} \tag{10.2.2}$$

where n is the number of gas molecules, R is the ideal gas constant (R = 8.314 J mol^{-1} K^{-1}), T is the temperature of the gas, and V is the volume of the container.

The pressure exerted by the gas can be increased by: increasing the number of collisions of gas molecules per unit time by increasing the number of gas molecules; increasing the kinetic energy of the gas by increasing the temperature; or decreasing the volume of the container. offers a representation of the ideal gas law, as well as the effect of varying the equation parameters on the gas pressure. Another common type of pressure is that exerted by a static liquid or hydrostatic pressure. Hydrostatic pressure is most easily addressed by treating the liquid as a continuous distribution of matter, and may be considered a measure of energy per unit volume or energy density. We will further discuss hydrostatic pressure in other sections.



$$p = \frac{nRT}{V}$$

n	n_0	$2n_0$	n_0	n_0
T	T_0	T_0	T_0	T_0
V	V_0	V_0	V_0	$2V_0$
p	p_0	$2p_0$	$2p_0$	$½p_0$

Pressure of an Ideal Gas: This image is a representation of the ideal gas law, as well as the effect of varying the equation parameters on the gas pressure.

Variation of Pressure With Depth

Pressure within static fluids depends on the properties of the fluid, the acceleration due to gravity, and the depth within the fluid.

learning obectives

- Identify factors that determine the pressure exerted by static liquids and gases

Pressure is defined in simplest terms as force per unit area. However, when dealing with pressures exerted by gases and liquids, it is most convenient to approach pressure as a measure of energy per unit volume by means of the definition of work (W = F·d). The derivation of pressure as a measure of energy per unit volume from its definition as force per unit area is given in. Since, for gases and liquids, the force acting on a system contributing to pressure does not act on a specific point or particular surface, but rather as a distribution of force, analyzing pressure as a measure of energy per unit volume is more appropriate. For liquids and gases at rest, the pressure of the liquid or gas at any point within the medium is called the hydrostatic pressure. At any such point within a medium, the pressure is the same in all directions, as if the pressure was not the same in all directions, the fluid, whether it is a gas or liquid, would not be static. Note that the following discussion and expressions pertain only to incompressible fluids at static equilibrium.

$$\Delta p = \frac{8\eta Q \Delta x}{\pi r^4}$$

Energy per Unit Volume: This equation is the derivation of pressure as a measure of energy per unit volume from its definition as force per unit area.

The pressure exerted by a static liquid depends only on the depth, density of the liquid, and the acceleration due to gravity. gives the expression for pressure as a function of depth within an incompressible, static liquid as well as the derivation of this equation from the definition of pressure as a measure of energy per unit volume (ρ is the density of the gas, g is the acceleration due to gravity, and h is the depth within the liquid). For any given liquid with constant density throughout, pressure increases with increasing depth. For example, a person under water at a depth of h_1 will experience half the pressure as a person under water at a depth of $h_2 = 2h_1$. For many liquids, the density can be assumed to be nearly constant throughout the volume of the liquid and, for virtually all practical applications, so can the acceleration due to gravity (g = 9.81 m/s^2). As a result, pressure within a liquid is therefore a function of depth only, with the pressure increasing at a linear rate with respect to increasing depth. In practical applications involving calculation of pressure as a function of depth, an important distinction must be made as to whether the absolute or relative pressure within a liquid is desired. Equation 2 by itself gives the pressure exerted by a liquid relative to atmospheric pressure, yet if the absolute pressure is desired, the atmospheric pressure must then be added to the pressure exerted by the liquid alone.

$$R_e = \frac{V}{I} \quad \rightarrow \quad \begin{cases} R_h \rightarrow R_e \\ \Delta p \rightarrow V \\ I \rightarrow Q \end{cases} \quad \rightarrow \quad \Delta p = \frac{8\eta Q \Delta x}{\pi r^4} \quad \rightarrow \quad R_h = \frac{\Delta p}{Q} = \frac{8\eta \Delta x}{\pi r^4}$$

Pressure as Energy per Unit Volume: This equation gives the expression for pressure as a function of depth within an incompressible, static liquid as well as the derivation of this equation from the definition of pressure as a measure of energy per unit volume (ρ is the density of the gas, g is the acceleration due to gravity, and h is the depth within the liquid).

When analyzing pressure within gases, a slightly different approach must be taken as, by the nature of gases, the force contributing to pressure arises from the average number of gas molecules occupying a certain point within the gas per unit time. Thus the force contributing to the pressure of a gas within the medium is not a continuous distribution as for liquids and the barometric equation given in must be utilized to determine the pressure exerted by the gas at a certain depth (or height) within the gas (p_0 is the pressure at h = 0, M is the mass of a single molecule of gas, g is the acceleration due to gravity, k is the Boltzmann constant, T is the temperature of the gas, and h is the height or depth within the gas). Equation 3 assumes that the gas is incompressible and that the pressure is hydrostatic.

$$p = p_0 e^{-\frac{Mgh}{kT}}$$

Pressure within a gas: The force contributing to the pressure of a gas within the medium is not a continuous distribution as for liquids and the barometric equation given in this figure must be utilized to determine the pressure exerted by the gas at a certain depth (or height) within the gas (p0 is the pressure at h = 0, M is the mass of a single molecule of gas, g is the acceleration due to gravity, k is the Boltzmann constant, T is the temperature of the gas, and h is the height or depth within the gas)

Static Equilibrium

Any region or point, or any static object within a static fluid is in static equilibrium where all forces and torques are equal to zero.

learning obectives

- Identify required conditions for a fluid to be in rest

Static equilibrium is a particular state of a physical system. It is qualitatively described by an object at rest and by the sum of all forces, with the sum of all torques acting on that object being equal to zero. Static objects are in static equilibrium, with the net force and net torque acting on that object being equal to zero; otherwise there would be a driving mechanism for that object to undergo movement in space. The analysis and study of objects in static equilibrium and the forces and torques acting on them is called statics—a subtopic of mechanics. Statics is particularly important in the design of static and load bearing structures. As it pertains to fluidics, static equilibrium concerns the forces acting on a static object within a fluid medium.

Fluids

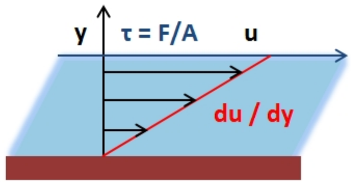
For a fluid at rest, the conditions for static equilibrium must be met at any point within the fluid medium. Therefore, the sum of the forces and torques at any point within the static liquid or gas must be zero. Similarly, the sum of the forces and torques of an object at rest within a static fluid medium must also be zero. In considering a stationary object within a liquid medium at rest, the forces acting at any point in time and at any point in space within the medium must be analyzed. For a stationary object within a static liquid, there are no torques acting on the object so the sum of the torques for such a system is immediately zero; it need not concern analysis since the torque condition for equilibrium is fulfilled.

Density

At any point in space within a static fluid, the sum of the acting forces must be zero; otherwise the condition for static equilibrium would not be met. In analyzing such a simple system, consider a rectangular region within the fluid medium with density ρ_L (same density as the fluid medium), width w, length l, and height h, as shown in. Next, the forces acting on this region within the medium are taken into account. First, the region has a force of gravity acting downwards (its weight) equal to its density object, times its volume of the object, times the acceleration due to gravity. The downward force acting on this region due to the fluid above the region is equal to the pressure times the area of contact. Similarly, there is an upward force acting on this region due to the fluid below the region equal to the pressure times the area of contact. For static equilibrium to be achieved, the sum of these forces must be zero, as shown in. Thus for any region within a fluid, in order to achieve static equilibrium, the pressure from the fluid below the region must be greater than the pressure from the fluid above by the weight of the region. This force which counteracts the weight of a region or object within a static fluid is called the buoyant force (or buoyancy).

$$\Delta p = \frac{8\eta Q\Delta x}{\pi r^4}$$

Static Equilibrium of a Region Within a Fluid: This figure shows the equations for static equilibrium of a region within a fluid.



$$\frac{F}{A} = \tau = \eta\frac{du}{dy}$$

Region Within a Static Fluid: This figure is a free body diagram of a region within a static fluid.

In the case on an object at stationary equilibrium within a static fluid, the sum of the forces acting on that object must be zero. As previously discussed, there are two downward acting forces, one being the weight of the object and the other being the force exerted by the pressure from the fluid above the object. At the same time, there is an upwards force exerted by the pressure from the fluid below the object, which includes the buoyant force. shows how the calculation of the forces acting on a stationary object within a static fluid would change from those presented in if an object having a density ρ_S different from that of the fluid medium is surrounded by the fluid. The appearance of a buoyant force in static fluids is due to the fact that pressure within the fluid changes as depth changes. The analysis presented above can furthermore be extended to much more complicated systems involving complex objects and diverse materials.

Pascal's Principle

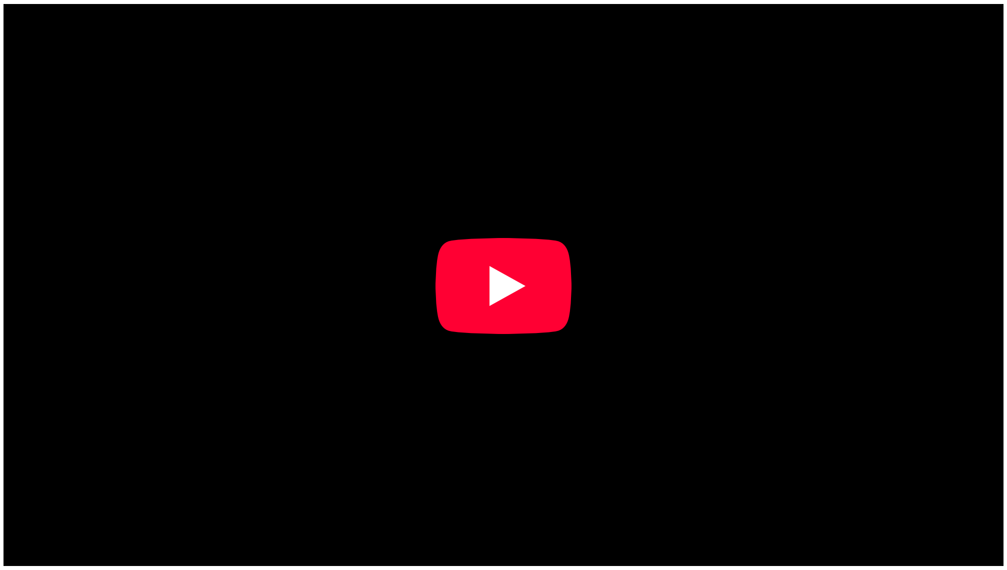
Pascal's Principle states that pressure is transmitted and undiminished in a closed static fluid.

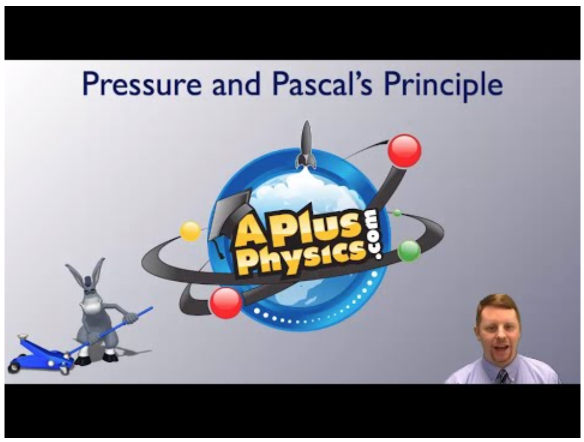
learning obectives

- Apply Pascal's Principle to describe pressure behavior in static fluids

Pascal's Principle

Pascal's Principle (or Pascal's Law) applies to static fluids and takes advantage of the height dependency of pressure in static fluids. Named after French mathematician Blaise Pascal, who established this important relationship, Pascal's Principle can be used to exploit pressure of a static liquid as a measure of energy per unit volume to perform work in applications such as hydraulic presses. Qualitatively, Pascal's Principle states that pressure is transmitted undiminished in an enclosed static liquid. Quantitatively, Pascal's Law is derived from the expression for determining the pressure at a given height (or depth) within a fluid and is defined by Pascal's Principle:





Pressure and Pascal's Principle: A brief introduction to pressure and Pascal's Principle, including hydraulics.

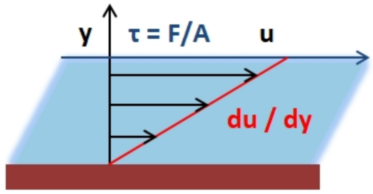
$$p_2 = p_1 + \Delta p, \Delta p = \rho g \Delta h \tag{10.2.3}$$

where p_1 is the external applied pressure, ρ is the density of the fluid, Δh is the difference in height of the static liquid, and g is the acceleration due to gravity. Pascal's Law explicitly determines the pressure difference between two different heights (or depths) within a static liquid. As, by Pascal's Law, a change in pressure is linearly proportional to a change in height within an incompressible, static liquid of constant density, doubling the height between the two points of reference will double the change of pressure, while halving the height between the two points will half the change in pressure.

Enclosed Static Liquids

While Pascal's Principle applies to any static fluid, it is most useful in terms of applications when considering systems involving rigid wall closed column configurations containing homogeneous fluids of constant density. By exploiting the fact that pressure is transmitted undiminished in an enclosed static liquid, such as in this type of system, static liquids can be used to transform small amounts of force into large amounts of force for many applications such as hydraulic presses.

As an example, referring to, a downwards force of 10 N is applied to a bottle filled with a static liquid of constant density ρ at the spout of cross-sectional area of 5 cm^2, yielding an applied pressure of 2 N/cm^2. The cross-sectional area of the bottle changes with height so that at the bottom of the bottle the cross-sectional area is 500 cm^2. As a result of Pascal's Law, the pressure change (pressure applied to the static liquid) is transmitted undiminished in the static liquid so that the applied pressure is 2 N/m^2 at the bottom of the bottle as well. Furthermore, the hydrostatic pressure due to the difference in height of the liquid is given by Equation 1 and yields the total pressure at the bottom surface of the bottle. Since the cross-sectional area at the bottom of the bottle is 100 times larger than at the top, the force contributing to the pressure at the bottom of the bottle is 1000 N plus the force from the weight of the static fluid in the bottle. This example shows how, through Pascal's Principle, the force exerted by a static fluid in a closed system can be multiplied by changing the height and the surface area of contact.

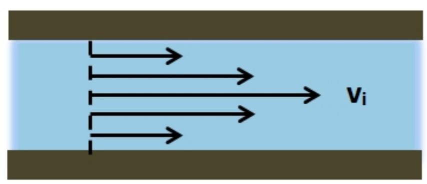


$$\frac{F}{A} = \tau = \eta \frac{du}{dy}$$

Pressure Applied to a Hydrostatic Fluid: A downwards force of 10 N is applied to a bottle filled with a static liquid of constant density ρ at the spout of cross-sectional area of 5 cm2, yielding an applied pressure of 2 N/cm2.

Pressure Transmitted Throughout an Entire Fluid

As stated by Pascal's Principle, the pressure applied to a static fluid in a closed container is transmitted throughout the entire fluid. Taking advantage of this phenomenon, hydraulic presses are able to exert a large amount of force requiring a much smaller amount of input force. This gives two different types of hydraulic press configurations, the first in which there is no difference in height of the static liquid and the second in which there is a difference in height Δh of the static liquid. In the first configuration, a force F_1 is applied to a static liquid of density ρ across a surface area of contact A_1, yielding an input pressure of P_2. On the other side of the press configuration, the fluid exerts an output pressure P_1 across a surface area of contact A_2, where $A_2 > A_1$. By Pascal's Principle, $P_1 = P_2$, yielding a force exerted by the static fluid of F_2, where $F_2 > F_1$. Depending on the applied pressure and geometry of the hydraulic press, the magnitude of F_2 can be changed. In the second configuration, the geometry of the system is the same, except that the height of the fluid on the output end is a height Δh less than the height of the fluid at the input end. The difference in height of the fluid between the input and the output ends contributes to the total force exerted by the fluid. For a hydraulic press, the force multiplication factor is the ratio of the output to the input contact areas.



Hydraulic Press Diagrams: Two different types of hydraulic press configurations, the first in which there is no difference in height of the static liquid and the second in which there is a difference in height Δh of the static liquid.

Gauge Pressure and Atmospheric Pressure

Pressure is often measured as gauge pressure, which is defined as the absolute pressure minus the atmospheric pressure.

learning obectives

- Explain the relationship among absolute pressure, gauge pressure, and atmospheric pressure

Atmospheric Pressure

An important distinction must be made as to the type of pressure quantity being used when dealing with pressure measurements and calculations. Atmospheric pressure is the magnitude of pressure in a system due to the atmosphere, such as the pressure exerted by air molecules (a static fluid) on the surface of the earth at a given elevation. In most measurements and calculations, the atmospheric pressure is considered to be constant at 1 atm or 101,325 Pa, which is the atmospheric pressure under standard conditions at sea level.

Atmospheric pressure is due to the force of the molecules in the atmosphere and is a case of hydrostatic pressure. Depending on the altitude relative to sea level, the actual atmospheric pressure will be less at higher altitudes and more at lower altitudes as the weight of air molecules in the immediate atmosphere changes, thus changing the effective atmospheric pressure. Atmospheric pressure is a measure of absolute pressure and can be affected by the temperature and air composition of the atmosphere but can generally be accurately approximated to be around standard atmospheric pressure of 101,325 Pa. Within the majority of earth's atmosphere, pressure varies with height according to. In this equation p_0 is the pressure at sea level (101,325 Pa), g is the acceleration due to gravity, M is the mass of a single molecule of air, R is the universal gas constant, T_0 is the standard temperature at sea level, and h is the height relative to sea level.

$$\Delta p = \frac{8\eta Q\Delta x}{\pi r^4}$$

Pressure and Height: Atmospheric pressure depends on altitude or height.

Gauge Pressure

For most applications, particularly those involving pressure measurements, it is more practical to use gauge pressure than absolute pressure as a unit of measurement. Gauge pressure is a relative pressure measurement which measures pressure relative to atmospheric pressure and is defined as the absolute pressure minus the atmospheric pressure. Most pressure measuring equipment give the pressure of a system in terms of gauge pressure as opposed to absolute pressure. For example, tire pressure and blood pressure are gauge pressures by convention, while atmospheric pressures, deep vacuum pressures, and altimeter pressures must be absolute.

For most working fluids where a fluid exists in a closed system, gauge pressure measurement prevails. Pressure instruments connected to the system will indicate pressures relative to the current atmospheric pressure. The situation changes when extreme vacuum pressures are measured; absolute pressures are typically used instead.

To find the absolute pressure of a system, the atmospheric pressure must then be added to the gauge pressure. While gauge pressure is very useful in practical pressure measurements, most calculations involving pressure, such as the ideal gas law, require pressure values in terms of absolute pressures and thus require gauge pressures to be converted to absolute pressures.

Measurements: Gauge Pressure and the Barometer

Barometers are devices used for measuring atmospheric and gauge pressure indirectly through the use of hydrostatic fluids.

learning obectives

- Compare design and operation of aneroid and hydrostatic based barometers

Gauge Pressure

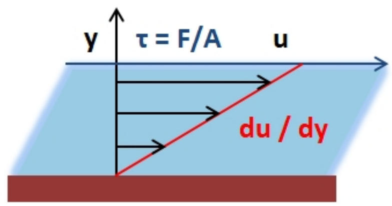
In practice, pressure is most often measured in terms of gauge pressure. Gauge pressure is the pressure of a system above atmospheric pressure. Since atmospheric pressure is mostly constant with little variation near sea level, where most practical pressure measurements are taken, it is assumed to be approximately 101,325 Pa. Modern pressure measuring devices sometimes have incorporated mechanisms to account for changes in atmospheric pressure due to elevation changes. Gauge pressure is much more convenient than absolute pressure for practical measurements and is widely used as an established measure of pressure. However, it is important to determine whether it is necessary to use absolute (gauge plus atmospheric) pressure for calculations, as is often the case for most calculations, such as those involving the ideal gas law. Pressure measurements have been accurately taken since the mid-1600s with the invention of the traditional barometer. Barometers are devices used to measure pressure and were initially used to measure atmospheric pressure.

Hydrostatic Based Barometers

Early barometers were used to measure atmospheric pressure through the use of hydrostatic fluids. Hydrostatic based barometers consist of columnar devices usually made from glass and filled with a static liquid of consistent density. The columnar section is sealed, holds a vacuum, and is partially filled with the liquid while the base section is open to the atmosphere and makes an interface with the surrounding environment. As the atmospheric pressure changes, the pressure exerted by the atmosphere on the fluid reservoir exposed to the atmosphere at the base changes, increasing as the atmospheric pressure increases and decreasing as the atmospheric pressure decreases. This change in pressure causes the height of the fluid in the columnar structure to change, increasing in height as the atmosphere exerts greater pressure on the liquid in the reservoir base and decreasing as the atmosphere exerts lower pressure on the liquid in the reservoir base. The height of the liquid within the glass column then gives a measure of the atmospheric pressure. Pressure, as determined by hydrostatic barometers, is often measured by determining the height of the liquid in the barometer column, thus the torr as a unit of pressure, but can be used to determine pressure in SI units. Hydrostatic based barometers most commonly use water or mercury as the static liquid. While the use of water is much less hazardous than mercury, mercury is often a better choice for fabricating accurate hydrostatic barometers. The density of mercury is much higher than that of water, thus allowing for higher accuracy of measurements and the ability to fabricate more compact hydrostatic barometers. In theory, a hydrostatic barometer can be placed in a closed system to measure the absolute pressure and the gauge pressure of the system by subtracting the atmospheric pressure.

Aneroid Barometer

Another type of barometer is the aneroid barometer, which consists of a small, flexible sealed metal box called an aneroid cell. The aneroid cell is made from beryllium-copper alloy and is partially evacuated. A stiff spring prevents the aneroid cell from collapsing. Small changes in external air pressure cause the cell to expand or contract. This expansion and contraction is amplified by mechanical mechanisms to give a pressure reading. Such pressure measuring devices are more practical than hydrostatic barometers for measuring system pressures. Many modern pressure measuring devices are pre-engineered to output gauge pressure measurements. While the aneroid barometer is the underlying mechanism behind many modern pressure measuring devices, pressure can also be measured using more advanced measuring mechanisms.



$$\frac{F}{A} = \tau = \eta \frac{du}{dy}$$

Hydrostatic Column Barometer: The concept of determining pressure using the fluid height in a hydrostatic column barometer

$$\Delta p = \frac{8\eta Q \Delta x}{\pi r^4}$$

Variation of Pressure with Height: The density of the liquid is p, g is the acceleration due to gravity, and h is the height of the fluid in the barometer column.

Pressure in the Body

Pressure plays an essential role in a number of critical bodily functions including respiration and blood circulation.

learning obectives

- Explain role played by pressure in the circulatory and respiratory systems

The Role of Pressure in the Circulatory System

Pressure plays an essential role in various critical bodily systems that are necessary for survival. One such critical bodily system which relies on pressure for functionality is the circulatory system, which is an example of a closed fluid system under pressure. The circulatory system is responsible for transporting oxygen and essential nutrients to all organs within the body as well as removing waste materials from these organs. Blood can be regarded as a viscous liquid contained within the circulatory system that travels throughout this closed system as a result of pressure and pressure differences within the circulatory system.

As the volume of blood within the circulatory system is confined to the veins, arteries, and capillaries there is a pressure within this closed system. Furthermore, through a complicated system of veins, arteries, and capillaries of varying diameter as well as valves and the heart acting as a continuous pump, pressure differences arise within the circulatory system that result in the potential for blood to circulate throughout the circulatory system, thus carrying out essential bodily functions for survival.

Pressure within the circulatory system is referred to as blood pressure, and is a primary and crucial vital sign which can be used to diagnose or indicate a number of medical conditions. Blood pressure varies throughout the body as well as from one individual to another and depends on a number of factors such as heart rate, blood volume, resistance of the circulatory system (veins, arteries, and capillaries), and the viscosity of blood. Any medical conditions affecting any of these factors will have an effect on blood pressure and the overall health of the circulatory system.

$$R_e = \frac{V}{I} \quad \rightarrow \quad \begin{cases} R_h \rightarrow R_e \\ \Delta p \rightarrow V \\ I \rightarrow Q \end{cases} \quad \rightarrow \quad \Delta p = \frac{8\eta Q \Delta x}{\pi r^4} \quad \rightarrow \quad R_h = \frac{\Delta p}{Q} = \frac{8\eta \Delta x}{\pi r^4}$$

Approximation for Mean Arterial Pressure: In practice, the mean arterial pressure (MAP) can be approximated from easily obtainable blood pressure measurements.

The mean arterial pressure (MAP) is the average pressure over a cardiac cycle and is determined by, where CO is the cardiac outputs, SVR is the systemic vascular resistance, and CVP is the central venous pressure (CVP). In practice, the mean arterial pressure (MAP) can be approximated from easily obtainable blood pressure measurements in, where P_{sys} is the measured systolic pressure and P_{dias} is the measured diastolic pressure. One particularly common and dangerous circulatory system condition is partial blockage of blood vessels due to a number of factors, such as plaque build-up from high cholesterol, which results in a reduction of the effective blood vessel cross-sectional diameter and a corresponding reduction in blood flow rate and thus an increase in blood pressure to restore normal blood flow according to Poiseuille's Law.

$$\Delta p = \frac{8\eta Q \Delta x}{\pi r^4}$$

Equation for Mean Arterial Pressure: The mean arterial pressure (MAP) is the average pressure over a cardiac cycle and is determined this equation, where CO is the cardiac outputs, SVR is the systemic vascular resistance, and CVP is the central venous pressure (CVP).

The Role of Pressure in the Respiratory System

Pressure also plays an essential role in the respiratory system, as it is responsible for the breathing mechanism. Pressure differences between the lungs and the atmosphere create a potential for air to enter the lungs, resulting in inhalation. The mechanism resulting in inhalation is due to lowering of the diaphragm, which increases the volume of the thoracic cavity surrounding the lungs, thus lowering its pressure as determined by the ideal gas law. The reduction in pressure of the thoracic cavity, which normally has a negative gauge pressure, thus keeping the lungs inflated, pulls air into the lungs, inflating the alveoli and resulting in oxygen transport needed for respiration. As the diaphragm restores and moves upwards, pressure within the thoracic cavity increases, resulting in exhalation. The cycle repeats itself, resulting in the respiration which as discussed is mechanically due to pressure changes. Without pressure in the body, and the corresponding potential that it has for dynamic bodily processes, essential functions such as blood circulation and respiration would not be possible.

Key Points

- Pressure is a scalar quantity defined as force per unit area. Pressure only concerns the force component perpendicular to the surface upon which it acts, thus if the force acts at an angle, the force component along the direction perpendicular to the surface must be used to calculate pressure.
- The pressure exerted on a surface by an object increases as the weight of the object increases or the surface area of contact decreases. Alternatively the pressure exerted decreases as the weight of the object decreases or the surface area of contact increases.
- Pressure exerted by ideal gases in confined containers is due to the average number of collisions of gas molecules with the container walls per unit time. As such, pressure depends on the amount of gas (in number of molecules), its temperature, and the volume of the container.
- Hydrostatic pressure refers to the pressure exerted by a fluid (gas or liquid) at any point in space within that fluid, assuming that the fluid is incompressible and at rest.
- Pressure within a liquid depends only on the density of the liquid, the acceleration due to gravity, and the depth within the liquid. The pressure exerted by such a static liquid increases linearly with increasing depth.
- Pressure within a gas depends on the temperature of the gas, the mass of a single molecule of the gas, the acceleration due to gravity, and the height (or depth) within the gas.
- Hydrostatic balance is the term used for a region or stationary object within a static fluid which is at static equilibrium, and for which the sum of all forces and sum of all torques is equal to zero.
- A region or static object within a stationary fluid experiences downward forces due to the weight of the region or object, and the pressure exerted from the fluid above the region or object, as well as an upward force due to the pressure exerted from the fluid

below the region or object.
- For a region or static object within a static fluid, the downward force due to the weight of the region or object is counteracted by the upward buoyant force, which is equal to the weight of the fluid displaced by the region or object.
- Pascal's Principle is used to quantitatively relate the pressure at two points in an incompressible, static fluid. It states that pressure is transmitted, undiminished, in a closed static fluid.
- The total pressure at any point within an incompressible, static fluid is equal to the sum of the applied pressure at any point in that fluid and the hydrostatic pressure change due to a difference in height within that fluid.
- Through the application of Pascal's Principle, a static liquid can be utilized to generate a large output force using a much smaller input force, yielding important devices such as hydraulic presses.
- Atmospheric pressure is a measure of absolute pressure and is due to the weight of the air molecules above a certain height relative to sea level, increasing with decreasing altitude and decreasing with increasing altitude.
- Gauge pressure is the additional pressure in a system relative to atmospheric pressure. It is a convenient pressure measurement for most practical applications.
- While gauge pressure is more convenient for practical measurements, absolute pressure is necessary for most pressure calculations, thus the atmospheric pressure must be added to the gauge pressure for calculations.
- Gauge pressure is the pressure of a system above atmospheric pressure, which must be converted to absolute pressure for most calculations.
- The barometer is a device which uses hydrostatic fluids to directly determine atmospheric pressure and may be used to indirectly measure the gauge pressure of systems.
- The hydrostatic column barometer uses a liquid like water or mercury for functionality, while the aneroid barometer uses an evacuated flexible metal cell.
- Pressure, along with the potential for work arising from differences in pressure, plays an essential role in the functionality of several critical bodily functions and systems necessary for survival.
- The circulatory system relies on pressure differences for circulating blood, along with oxygen, necessary nutrients, and waste products throughout the body.
- Respiration is made possible as a result of pressure differences between the thoracic cavity, the lungs, and the environment and is largely regulated by movement of the diaphragm.

Key Terms

- **ideal gas**: Theoretical gas characterized by random motion whose individual molecules do not interact with one another and are chemically inert.
- **kinetic energy**: The energy associated with a moving particle or object having a certain mass.
- **incompressible**: Unable to be compressed or condensed.
- **static equilibrium**: the physical state in which all components of a system are at rest and the net force is equal to zero throughout the system
- **Buoyancy**: The power of supporting a body so that it floats; upward pressure exerted by the fluid in which a body is immersed.
- **torque**: Something that produces or tends to produce torsion or rotation; the moment of a force or system of forces tending to cause rotation.
- **equilibrium**: A state of rest or balance due to the equal action of opposing forces.
- **hydraulic press**: Device that uses a hydraulic cylinder (closed static fluid) to generate a compressive force.
- **Gauge Pressure**: The pressure of a system above atmospheric pressure.
- **Torr**: A unit of pressure equal to one millimeter of mercury (760 torr = 101,325 Pa).
- **Aneroid Barometer**: A device for measuring pressure, often specially calibrated for use as an altimeter, consisting of a box or chamber partially exhausted of air, having an elastic top and a pointer to indicate the degree of compression of the top caused by the external air.
- **Thoracic Cavity**: A hollow place or space, or a potential space, within the body or one of its organs.
- **Poiseuille's Law**: The law that the velocity of a liquid flowing through a capillary is directly proportional to the pressure of the liquid and the fourth power of the radius of the capillary and is inversely proportional to the viscosity of the liquid and the length of the capillary.
- **Alveoli**: Small air sacs or cavities in the lung that give the tissue a honeycomb appearance and expand its surface area for the exchange of oxygen and carbon dioxide.

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10.3: Archimedes' Principle

learning objectives

- Calculate the direction of the buoyancy force

When you rise from soaking in a warm bath, your arms may feel strangely heavy. This effect is due to the loss of the buoyant support of the water. What creates this buoyant force ? Why is it that some things float and others do not? Do objects that sink get any support at all from the fluid? Is your body buoyed by the atmosphere, or are only helium balloons affected?

Buoyant Force: Cause and Calculation

We find the answers to the above questions in the fact that in any given fluid, pressure increases with depth. When an object is immersed in a fluid, the upward force on the bottom of an object is greater than the downward force on the top of the object. The result is a net upward force (a buoyant force) on any object in any fluid. If the buoyant force is greater than the object's weight, the object will rise to the surface and float. If the buoyant force is less than the object's weight, the object will sink. If the buoyant force equals the object's weight, the object will remain suspended at that depth. The buoyant force is always present in a fluid, whether an object floats, sinks or remains suspended.

The buoyant force is a result of pressure exerted by the fluid. The fluid pushes on all sides of an immersed object, but as pressure increases with depth, the push is stronger on the bottom surface of the object than in the top (as seen in).

You can calculate the buoyant force on an object by adding up the forces exerted on all of an object's sides. For example, consider the object shown in.

The top surface has area A and is at depth h_1; the pressure at that depth is:

$$\mathrm{P}_1 = \mathrm{h}_1 \rho \mathrm{g}, \tag{10.3.1}$$

where ρρ is the density of the fluid and $\mathrm{g} \approx 9.81 \frac{\mathrm{m}}{\mathrm{s}^2}$ is the gravitational acceleration. The magnitude of the force on the top surface is:

$$\mathrm{F}_1 = \mathrm{P}_1 \mathrm{A} = \mathrm{h}_1 \rho \mathrm{g} \mathrm{A}. \tag{10.3.2}$$

This force points downwards. Similarly, the force on the bottom surface is:

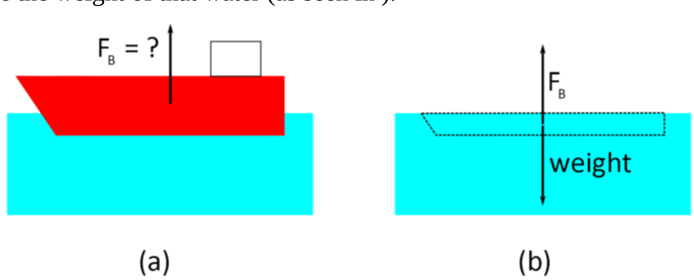
$$\mathrm{F}_2 = \mathrm{P}_2 \mathrm{A} = \mathrm{h}_2 \rho \mathrm{g} \mathrm{A} \tag{10.3.3}$$

and points upwards. Because it is cylindrical, the net force on the object's sides is zero—the forces on different parts of the surface oppose each other and cancel exactly. Thus, the net upward force on the cylinder due to the fluid is:

$$\mathrm{F}_\mathrm{B} = \mathrm{F}_2 - \mathrm{F}_1 = \rho \mathrm{g} \mathrm{A} (\mathrm{h}_2 - \mathrm{h}_1) \tag{10.3.4}$$

The Archimedes Principle

Although calculating the buoyant force in this way is always possible it is often very difficult. A simpler method follows from the Archimedes principle, which states that the buoyant force exerted on a body immersed in a fluid is equal to the weight of the fluid the body displaces. In other words, to calculate the buoyant force on an object we assume that the submersed part of the object is made of water and then calculate the weight of that water (as seen in).



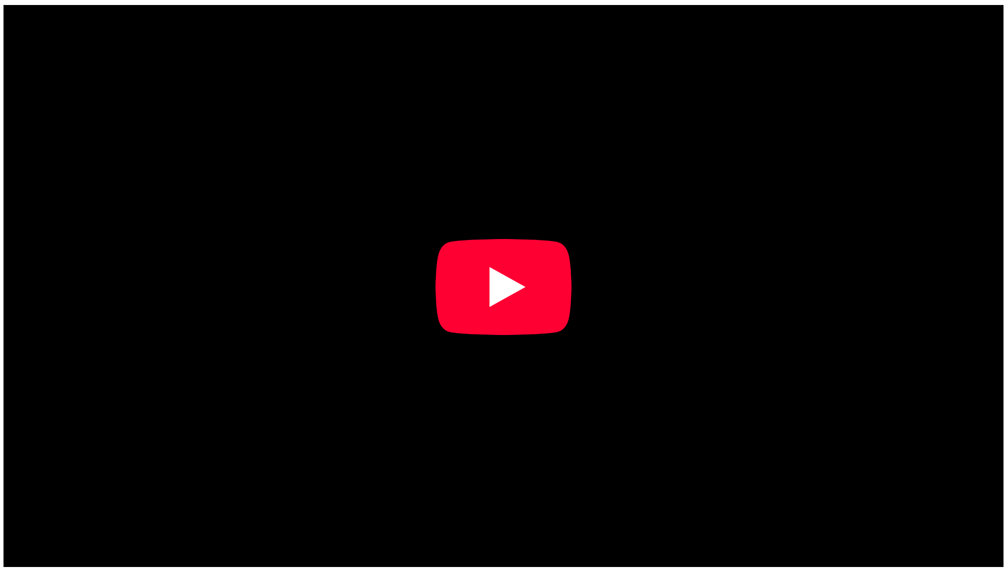
Archimedes principle: The buoyant force on the ship (a) is equal to the weight of the water displaced by the ship—shown as the dashed region in (b).

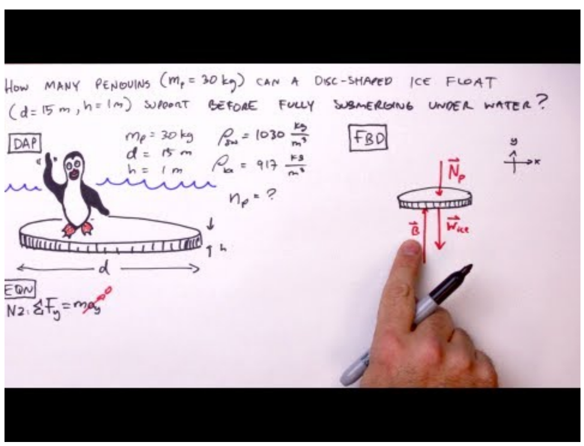
The principle can be stated as a formula:

$$F_B = w_{fl} \tag{10.3.5}$$

The reasoning behind the Archimedes principle is that the buoyancy force on an object depends on the pressure exerted by the fluid on its submerged surface. Imagine that we replace the submerged part of the object with the fluid in which it is contained, as in (b). The buoyancy force on this amount of fluid must be the same as on the original object (the ship). However, we also know that the buoyancy force on the fluid must be equal to its weight, as the fluid does not sink in itself. Therefore, the buoyancy force on the original object is equal to the weight of the "displaced fluid" (in this case, the water inside the dashed region (b)).

The Archimedes principle is valid for any fluid—not only liquids (such as water) but also gases (such as air). We will explore this further as we discuss applications of the principle in subsequent sections.





Archimedes' Principle – Simple Example: We use Archimedes' Principle to determine the number of penguins an ice float can dryly support.

Complete Submersion

The buoyancy force on a completely submerged object of volume is $F_B = V\rho g$.

learning objectivies

- Identify factors determining the buoyancy force on a completely submerged object

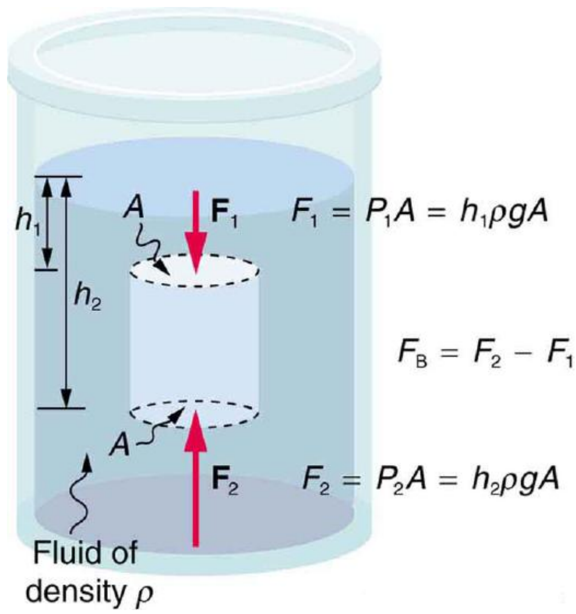
The Archimedes principle is easiest to understand and apply in the case of entirely submersed objects. In this section we discuss a few relevant examples. In general, the buoyancy force on a completely submerged object is given by the formula:

$$F_B = V\rho g, \tag{10.3.6}$$

where V is the volume of the object, ρ is the density of the fluid, and g is gravitational acceleration. This follows immediately from the Archimedes' principle, and the fact that the object is completely submerged (and so the volume of the fluid displaced is just the volume of the object).

Cylinder

In the previous section, we calculated the buoyancy force on a cylinder (shown in) by considering the force exerted on each of the cylinder's sides. Now, we'll calculate this force using Archimedes' principle. The buoyancy force on the cylinder is equal to the weight of the displaced fluid. This weight is equal to the mass of the displaced fluid multiplied by the gravitational acceleration:



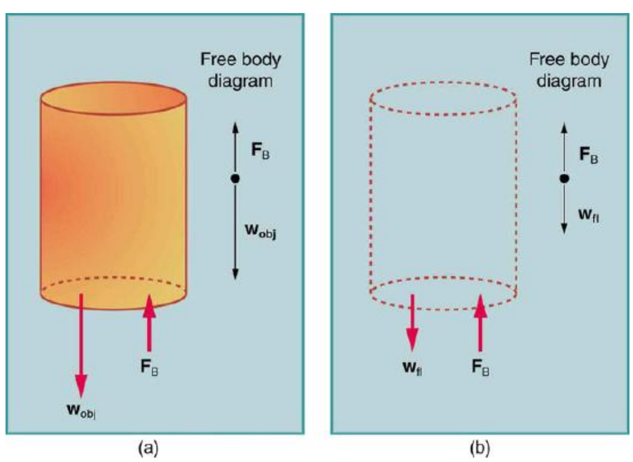
Buoyant force: The fluid pushes on all sides of a submerged object. However, because pressure increases with depth, the upward push on the bottom surface (F2) is greater than the downward push on the top surface (F1). Therefore, the net buoyant force is always upwards.

$$F_B = w_{fl} = m_{fl}g \tag{10.3.7}$$

The mass of the displaced fluid is equal to its volume multiplied by its density:

$$m_{fl} = V_{fl}\rho \tag{10.3.8}$$

However (*and this is the crucial point*), the cylinder is entirely submerged, so the volume of the displaced fluid is just the volume of the cylinder (see), and:



Archimedes principle: The volume of the fluid displaced (b) is the same as the volume of the original cylinder (a).

$$m_{fl} = V_{fl}\rho = V_{cylinder}\rho. \tag{10.3.9}$$

The volume of a cylinder is the area of its base multiplied by its height, or in our case:

$$V_{cylinder} = A(h_2 - h_1). \tag{10.3.10}$$

Therefore, the buoyancy force on the cylinder is:

$$F_B = m_{fl}g = V_{cylinder}\rho g = (h_1 - h_2)\rho g A. \tag{10.3.11}$$

This is the same result obtained in the previous section by considering the force due to the pressure exerted by the fluid.

Helium Airship

Consider the USS Macon, a helium-filled airship (shown in). Its envelope (the "balloon") contained 184,059.5 cubic meters of helium. Ignoring the small volume of the gondola, what was the buoyancy force on this airship? If the airship weighed 108,000 kg, how much cargo could it carry? Assume the density of air is 1.225 kg per meter cubed. The buoyancy force on an airship is due to the air in which it is immersed. Although we don't know the exact shape of the airship, we know its volume and the density of the air, and thus we can calculate the buoyancy force:

Helium airship: The USS Macon, a 1930s helium-filled airship.

$$F_B = V_{\rho g} = 184,059.5 \text{ kg} \times 1.225 \frac{\text{kg}}{\text{m}^3} \times 9.81 \frac{\text{m}}{\text{s}^2} \approx 2.212 \times 10^6 \text{ N} \tag{10.3.12}$$

To find the cargo capacity of the airship, we subtract the weight of the airship from the buoyancy force:

$$F_{cargo} = F_B - mg = 2.21 \times 10^6 \text{ N} - 1.08 \times 10^5 \text{ kg} \times 9.81 \frac{\text{m}}{\text{s}^2} = 1.15 \times 10^6 \text{ N} \tag{10.3.13}$$

The mass the airship can carry is:

$$m_{cargo} = \frac{F_{cargo}}{g} = 1.2 \times 10^5 \text{ kg} = 120 \text{ tons}. \tag{10.3.14}$$

Flotation

An object floats if the buoyancy force exerted on it by the fluid balances its weight.

learning objectives

- Express the relationship between the buoyancy force and the weight for a floating object

Why do some objects float, but others don't? If you put a metal coin into a glass of water it will sink. But most ships are built of metal, and they float. So how is this possible?

Condition for Flotation

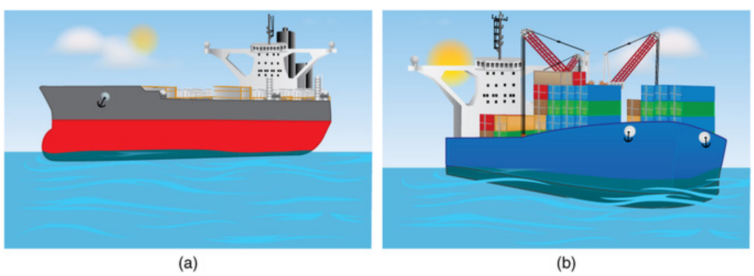
An object will float if the buoyancy force exerted on it by the fluid balances its weight, i.e. if FB=mgFB=mg.

But the Archimedes principle states that the buoyant force is the weight of the fluid displaced. So, for a floating object on a liquid, the weight of the displaced liquid is the weight of the object. Thus, only in the special case of floating does the buoyant force acting on an object equal the object's weight. Consider a one-ton block of solid iron. As iron is nearly eight times denser than water, it displaces only 1/8 ton of water when submerged, which is not enough to keep it afloat. Suppose the same iron block is reshaped into a bowl. It still weighs one ton, but when it is put in water, it displaces a greater volume of water than when it was a block. The deeper the iron bowl is immersed, the more water it displaces, and the greater the buoyant force acting on it. When the buoyant force equals one ton, it will sink no further.

When any boat displaces a weight of water equal to its own weight, it floats. This is often called the "principle of flotation" where a floating object displaces a weight of fluid equal to its own weight. Every ship, submarine, and dirigible must be designed to displace a weight of fluid equal to its own weight. A 10,000-ton ship must be built wide enough to displace 10,000 tons of water before it sinks too deep in the water. The same is true for vessels in air (as air is a fluid): A dirigible that weighs 100 tons displaces at least 100 tons of air; if it displaces more, it rises; if it displaces less, it falls. If the dirigible displaces exactly its weight, it hovers at a constant altitude.

Flotation and Density

Density plays a crucial role in Archimedes' principle. The average density of an object is what ultimately determines whether it floats. If its average density is less than that of the surrounding fluid, it will float. This is because the fluid, having a higher density, contains more mass and thus more weight in the same volume. The buoyant force, which equals the weight of the fluid displaced, is thus greater than the weight of the object. Likewise, an object denser than the fluid will sink. The extent to which a floating object is submerged depends on how the object's density is related to that of the fluid. For example, an unloaded ship has a lower density, and less of it is submerged compared with the same ship loaded with cargo. We can derive a quantitative expression for the fraction submerged by considering density. The fraction submerged is the ratio of the volume submerged to the volume of the object, or



Density and Submersion: An unloaded ship (a) floats higher in the water than a loaded ship (b).

$$\text{fraction submerged} = \frac{V_{sub}}{V_{obj}} = \frac{V_{fl}}{V_{obj}} \tag{10.3.15}$$

The volume submerged equals the volume of fluid displaced, which we call V_{fl}. Now we can obtain the relationship between the densities by substituting $\rho = mV$ into the expression. This gives

$$\text{fraction submerged} = \frac{m_{fl}/\rho_{fl}}{m_{obj}/\bar{\rho}_{obj}} \tag{10.3.16}$$

where $\bar{\rho}_{obj}$ is the average density of the object and ρflρflis the density of the fluid. Since the object floats, its mass and that of the displaced fluid are equal, and so they cancel from the equation, leaving

$$\text{fraction submerged} = \frac{\bar{\rho}_{obj}}{\rho_{fl}.} \tag{10.3.17}$$

There are a couple things to note about this expression:

1. Note that it mentions the average density of the object. This can be much less than the density of the material the object is made of. For instance, a steel ship is actually mostly filled with air (think of the corridors, cargo holds, etc.), so its average density is

between that of air and steel. To be more precise, the average density is defined as the total mass of an object divided by its total volume: $\bar{\rho} = \frac{m}{V}$.
2. This formula makes sense only if the density of the object is smaller than the density of the fluid. Otherwise, the fraction submerged becomes greater than one—a sign that the object does not float at all, but it sinks!

Key Points

- The buoyancy force is caused by the pressure exerted by the fluid in which an object is immersed.
- The buoyancy force always points upwards because the pressure of a fluid increases with depth.
- You can calculate the buoyancy force either directly by computing the force exerted on each of the object's surfaces, or indirectly by finding the weight of the displaced fluid.
- If an object is completely submerged, the volume of the fluid displaced is equal to the volume of the object.
- The buoyancy force on hot-air balloons, dirigibles and other objects can be calculated by assuming that they are entirely submerged in air.
- The buoyancy force does not depend on the shape of the object, only on its volume.
- The buoyancy force experienced by an object depends on its shape.
- The fraction of an object's volume that's submerged is given by the ratio of its average density to that of the fluid: $\frac{\bar{\rho}_{obj}}{\rho_{fl}}$.
- An object floats if the buoyancy force exerted on it by the fluid balances its weight.

Key Terms

- **buoyant force**: An upward force exerted by a fluid that opposes the weight of an immersed object.
- **Archimedes principle**: The buoyant force exerted on a body immersed in a fluid is equal to the weight of the fluid the body displaces.

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10.4: Cohesion and Adhesion

learning objectives

- Explain the phenomena of surface tension and capillary action

Attractive forces between molecules of the same type are called cohesive forces. Liquids can, for example, be kept in open containers because cohesive forces hold the molecules together. Attractive forces between molecules of different types are called adhesive forces. Such forces cause liquid drops to cling to window panes, for example. In this section we examine effects of cohesive and adhesive forces in liquids.

Surface Tension

Surface tension is a contractive tendency of the surface of a liquid that allows it to resist an external force. It is shown, for example, in the floating of some objects on the surface of water, even though they are denser than water, and in the ability of some insects (e.g., water striders) to run on water's surface. This property is caused by cohesion of similar molecules and is responsible for many of the behaviors of liquids.

The cohesive forces among liquid molecules are responsible for the phenomenon of surface tension, as shown in. In the bulk of the liquid, each molecule is pulled equally in every direction by neighboring liquid molecules, resulting in a net force of zero. The molecules at the surface do not have other molecules on all sides of them and therefore are pulled inwards. This creates some internal pressure and forces liquid surfaces to contract to the minimal area.

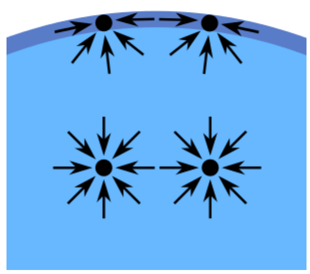


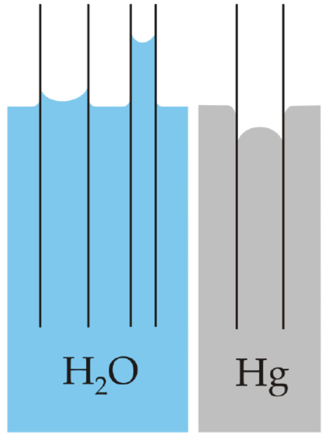
Diagram of Surface-Tension Forces: Diagram of the forces on molecules of a liquid

Surface tension has the unit of force per unit length, or of energy per unit area. The two units are equivalent. However, when we refer to energy per unit of area, we use the term surface energy, which is more general in that it applies to solids as well as liquids.

Capillary Action

Capillary action, or capillarity, is the ability of a liquid to flow in narrow spaces without the assistance of, and in opposition to, external forces like gravity. The effect can be seen in the drawing-up of liquids between the hairs of a paint-brush, in a thin tube, in porous materials such as paper, in some non-porous materials such as liquified carbon fiber, and in a cell. It occurs because of intermolecular attractive forces between the liquid and solid surrounding surfaces. If the diameter of the tube is sufficiently small, then the combination of surface tension (which is caused by cohesion within the liquid) and adhesive forces between the liquid and the container act to lift the liquid.

With some pairs of materials, such as mercury and glass (see), the intermolecular forces within the liquid exceed those between the solid and the liquid, so a convex meniscus forms, and capillary action works in reverse.



Capillarity: Capillary action of water compared to mercury, in each case with respect to glass

Key Points

- Attractive forces between molecules of the same type are called cohesive forces.
- Attractive forces between molecules of different types are called adhesive forces.
- Surface tension is a contractive tendency of the surface of a liquid that allows it to resist an external force.
- Capillary action, or capillarity, is the ability of a liquid to flow in narrow spaces without the assistance of, and in opposition to, external forces such as gravity.

Key Term

- **Pressure**: the amount of force that is applied over a given area divided by the size of that area
- **intermolecular**: from one molecule to another; between molecules

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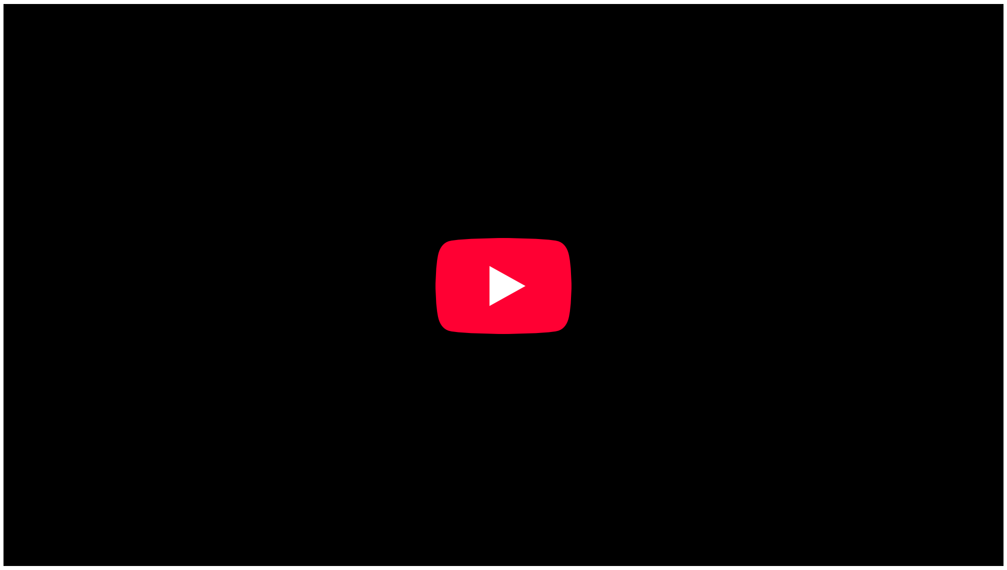
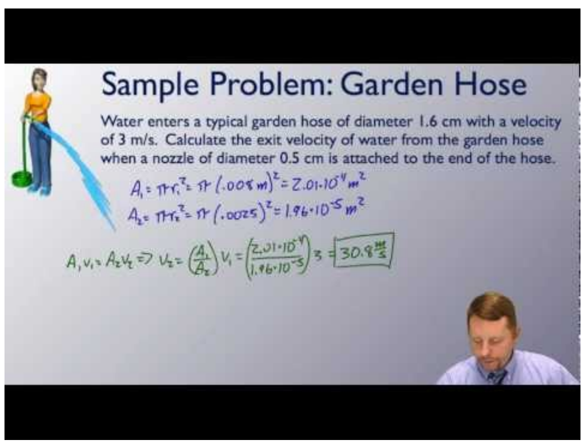
10.5: Fluids in Motion

learning objectives

- Determine the flow rate based on velocity and area or elapsed time and justify the use of continuity in expressing properties of a fluid and its motion

The flow rate of a fluid is the volume of fluid which passes through a surface in a given unit of time. It is usually represented by the symbol Q.





Continuity Equation for Fluids: A brief introduction to the Continuity Equation for Fluids.

Flow Rate

Volumetric flow rate is defined as

$$Q = v \times a, \tag{10.5.1}$$

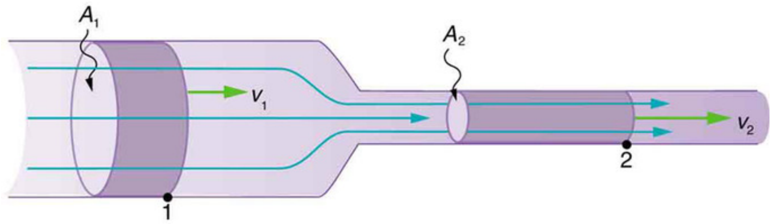
where Q is the flow rate, v is the velocity of the fluid, and a is the area of the cross section of the space the fluid is moving through. Volumetric flow rate can also be found with

$$Q = \frac{V}{t} \tag{10.5.2}$$

where Q is the flow rate, V is the Volume of fluid, and t is elapsed time.

Continuity

The equation of continuity works under the assumption that the flow in will equal the flow out. This can be useful to solve for many properties of the fluid and its motion:



Flow in = Flow out: Using the known properties of a fluid in one condition, we can use the continuity equation to solve for the properties of the same fluid under other conditions.

$$Q_1 = Q_2 \tag{10.5.3}$$

This can be expressed in many ways, for example: $A_1 v_1 = A_2 v_2$. The equation of continuity applies to any incompressible fluid. Since the fluid cannot be compressed, the amount of fluid which flows into a surface must equal the amount flowing out of the surface.

Applying the Continuity Equation

You can observe the continuity equation's effect in a garden hose. The water flows through the hose and when it reaches the narrower nozzle, the velocity of the water increases. Speed increases when cross-sectional area decreases, and speed decreases when cross-sectional area increases. This is a consequence of the continuity equation. If the flow Q is held constant, when the area A decreases, the velocity v must increase proportionally. For example, if the nozzle of the hose is half the area of the hose, the velocity must double to maintain the continuous flow.

Key Points

- Flow rate can be expressed in either terms of cross sectional area and velocity, or volume and time.
- Because liquids are incompressible, the rate of flow into an area must equal the rate of flow out of an area. This is known as the equation of continuity.
- The equation of continuity can show how much the speed of a liquid increases if it is forced to flow through a smaller area. For example, if the area of a pipe is halved, the velocity of the fluid will double.
- Although gases often behave as fluids, they are not incompressible the way liquids are and so the continuity equation does not apply.

Key Terms

- **incompressible**: Unable to be compressed or condensed.
- **continuity**: Lack of interruption or disconnection; the quality of being continuous in space or time.

- incompressible. **Provided by**: Wiktionary. **Located at**: **en.wiktionary.org/wiki/incompressible**. **License**: ***CC BY-SA: Attribution-ShareAlike***
- continuity. **Provided by**: Wiktionary. **Located at**: **en.wiktionary.org/wiki/continuity**. **License**: ***CC BY-SA: Attribution-ShareAlike***
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- OpenStax College, Flow Rate and Its Relation to Velocity. February 16, 2013. **Provided by**: OpenStax CNX. **Located at**: **http://cnx.org/content/m42205/latest/**. **License**: ***CC BY: Attribution***

10.6: Deformation of Solids

> **Learning objectives**
>
> - Explain how length of an object is determined

Length

In geometric measurements, length is the longest dimension of an object. In other contexts "length" is the measured dimension of an object. For example: it is possible to cut a length of a wire which is shorter than wire thickness. Length may be distinguished from height, which is vertical extent, and width or breadth, which are the distance from side to side, measuring across the object at right angles to the length.

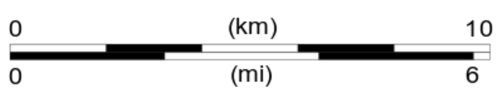
Length is a measure of one dimension, whereas area is a measure of two dimensions (length squared) and volume is a measure of three dimensions (length cubed). In most systems of measurement, the unit of length is a fundamental unit, from which other units are defined.

After Albert Einstein's Special Relativity Theory, length can no longer be thought of being constant in all reference frames. Thus, a ruler that is one meter long in one frame of reference will not be one meter long in a reference frame that is travelling at a velocity relative to the first frame. This means that the length of an object is variable depending on the observer.

Units

One of the oldest units of length measurement used in the ancient world was the 'cubit,' which was the length of the arm from the tip of the finger to the elbow. This could then be subdivided into shorter units like the foot, hand (which at 4 inches is still used today for expressing the height of horses) or finger, or added together to make longer units like the stride. The cubit could vary considerably due to the different sizes of people.

In the physical sciences and engineering, when one speaks of "units of length", the word "length" is synonymous with "distance". There are several units that are used to measure length. Units of length may be based on lengths of human body parts, the distance traveled in a number of paces, the distance between landmarks or places on the Earth, or arbitrarily on the length of some fixed object.In the International System of Units (SI), the basic unit of length is the meter and is now defined in terms of the speed of light. The centimeter and the kilometer, derived from the meter, are also commonly used units. In U.S. customary units, English or Imperial system of units, commonly used units of length are the inch, the foot, the yard, and the mile. Units used to denote distances in the vastness of space, as in astronomy, are much longer than those typically used on Earth and include the astronomical unit, the light-year, and the parsec.



Length: The metric length of one kilometre is equivalent to the imperial measurement of 0.62137 miles.

Shape

The shape of an object is a description of space that the object takes up; the shape can change if the object is deformed.

> **learning objectives**
>
> - Describe effects of deformations, rotations, and magnifications

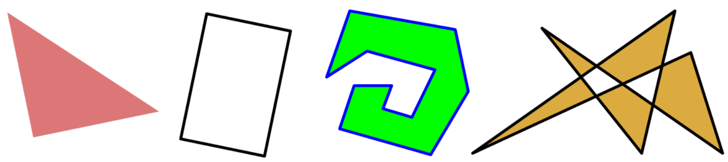
> **Shape**
>
> The shape of an object located in some space is a geometrical description of the part of that space occupied by the object, as determined by its external boundary – abstracting from location and orientation in space, size, and other properties such as color, content, and material composition

Simple and Complex Shapes

Simple shapes can be described by basic geometry objects such as a set of two or more points, a line, a curve, a plane, a plane figure (e.g. square or circle), or a solid figure (e.g. cube or sphere). Most shapes occurring in the physical world are complex. Some, such as plant structures and coastlines, may be so arbitrary as to defy traditional mathematical description – in which case they may be analyzed by differential geometry, or as fractals.

In geometry, two subsets of a Euclidean space have the same shape if one can be transformed to the other by a combination of translations, rotations (together also called rigid transformations), and uniform scalings. In other words, the shape of a set of points is all the geometrical information that is invariant to translations, rotations, and size changes. Having the same shape is an equivalence relation, and accordingly a precise mathematical definition of the notion of shape can be given as being an equivalence class of subsets of a Euclidean space having the same shape.

Shapes of physical objects are equal if the subsets of space these objects occupy satisfy the definition above. In particular, the shape does not depend on the size and placement in space of the object.



Shapes: Examples of shapes.

Volume

Volume is a measure of the three-dimensional space an object occupies, *usually* taken in terms of length, width and height.

learning objectives

- Explain how is volume measured geometrically

Volume is the quantity of three-dimensional space contained by a closed boundary; it is the space that a substance (solid, liquid, gas or plasma) or shape occupies or contains. Volume is often quantified numerically using an SI derived unit, the cubic meter. However, for liquids the unit of volume used is known as the liter (equivalent to 0.001 cubic meters).

Measuring Volume: A measuring cup can be used to measure volumes of liquids. This cup measures volume in units of cups, fluid ounces and millilitres.

Volume is measured geometrically by multiplying an object's three dimensions—usually taken as length, width and height. Some common volumes are taken as follows:

- The volume of a cube: length times width times height.
- The volume of a cylinder: the cross-sectional area times the height of the cylinder.
- The volume of a sphere: 4/3 times the radius cubed times pi.

The volume of a solid can be determined by the volume of liquid it displaces when submerged.

The volume of a container is generally understood as the capacity of the container, meaning the amount of fluid (gas or liquid) the container can hold, rather than the amount of space the container itself displaces. Measuring cups, as seen in, work by taking a known cross sectional area of a cup and multiplying that by a variable height. Since liquid will always cover the cross section (if there is enough liquid), adding more liquid will increase the height inside the container.

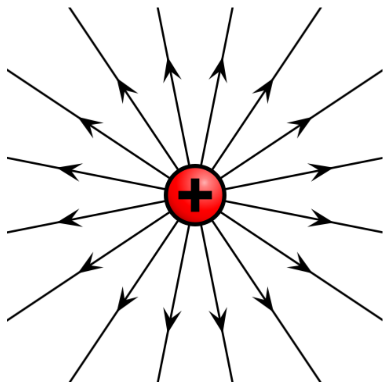
Liquids take the shape of their container, filling up the minimum height needed. Gases, on the other hand, take up the maximum amount of volume possible. Thus a measuring cup can accurately measure the volume of a liquid, whereas a gas will always fill the entire container, more or less uniformly, no matter how little gas there is.

Stress and Strain

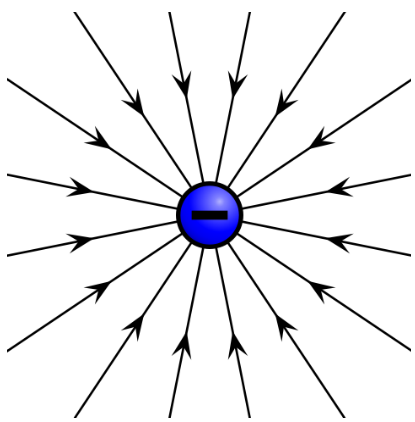
A point charge creates an electric field that can be calculated using Coulomb's Law.

The electric field of a point charge is, like any electric field, a vector field that represents the effect that the point charge has on other charges around it. The effect is felt as a force and when charged particles are not in motion this force is known as the electrostatic force. The electrostatic force is, much like gravity, a force that acts at a distance. Therefore, we rationalize this action at a distance by saying that charges create fields around them that have effects on other charges.

Given a point charge, or a particle of infinitesimal size that contains a certain charge, electric field lines emanate radially in all directions. If the charge is positive, field lines point radially away from it; if the charge is negative, field lines point radially towards it.



Electric field of positive point charge: The electric field of a positively charged particle points radially away from the charge.

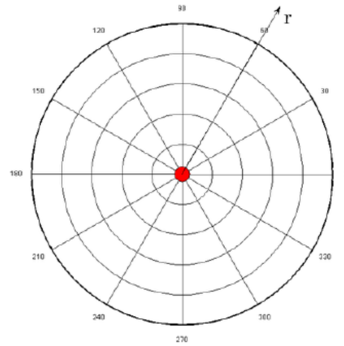


Electric field of negative point charge: The electric field of a negatively charged particle points radially toward the particle.

The reason for these directions can be seen in the derivation of the electric field of a point charge. Let's first take a look at the definition of electric field of a point particle:

$$\vec{E} = \frac{1}{4\pi\epsilon_o}\frac{q}{r^2}\hat{r} = k\frac{q}{r^2}\hat{r}. \tag{10.6.1}$$

In the above equation, q represents the charge of the particle creating the electric field and the constant k is a result of simply lumping the constants together. This charge is either positive or negative. If the charge is positive, as shown above, the electric field will be pointing in a positive radial direction from the charge q (away from the charge) and the following text explains why. The above equation is defined in radial coordinates which can be seen in.



Radial Coordinate System: The electric field of a point charge is defined in radial coordinates. The positive r direction points away from the origin, and the negative r direction points toward the origin. The electric field of a point charge is symmetric with respect to the θ direction.

If we now place another positive charge, Q (called the test charge), at some radial distance, R, away from the original particle, the test charge will feel a force given by

$$\overrightarrow{\mathrm{F}}=\mathrm{Q}\overrightarrow{\mathrm{E}}=\mathrm{Q}\frac{1}{4\pi\epsilon_{\mathrm{o}}}\frac{\mathrm{q}}{\mathrm{R}^{2}}\hat{\mathrm{r}} \tag{10.6.2}$$

The thing to keep in mind is that the force above is acting on the test charge Q, in the positive radial direction as defined by the original charge q. This means that because the charges are both positive and will repel one another, the force on the test charge points away from the original charge.

If the test charge were negative, the force felt on that charge would be

$$\overrightarrow{\mathrm{F}}=\mathrm{Q}\overrightarrow{\mathrm{E}}=-\mathrm{Q}\frac{1}{4\pi\epsilon_{\mathrm{o}}}\frac{\mathrm{q}}{\mathrm{R}^{2}}\hat{\mathrm{r}} \tag{10.6.3}$$

Notice that this points in the negative $\hat{\mathrm{r}}$ direction, which is toward the original charge. This makes sense because opposite charges attract and the force on the test charge will tend to push it toward the original positive charge creating the field. The above mathematical description of the electric field of a point charge is known as Coulomb ‘s Law.

Key Points

- Length is typically a measure of the longest dimension of an object.
- The deformation of an object is typically a change in length.
- The SI unit of length is the meter.
- The shape of an object is a representation of the space taken up by the object.
- Deformations can change the shape of an object.
- Objects that have the same shape can be transformed into each other by rotation or magnification.
- Volume is often quantified numerically using an SI derived unit, the cubic meter. However, for liquids the unit of volume used is known as the liter (equivalent to 0.001 cubic meters).
- This can also be understood as the amount of fluid a submerged object displaces.
- Volume can be measured for geometrically regular objects by simple formulas. However, more complicated objects are easier to measure with fluid displacement.
- The electric field is a vector field around a charged particle that represents the force that other charged particles would feel when placed near the particle creating the electric field.
- Given a point charge, or a particle of infinitesimal size, that contains a certain charge, electric field lines emanate from equally in all radial directions.
- If the point charge is positive, field lines point away from it; if the charge is negative, field lines point towards it.

Key Terms

- **dimension**: A measure of spatial extent in a particular direction, such as height, width or breadth, or depth.
- **special relativity**: A theory that (neglecting the effects of gravity) reconciles the principle of relativity with the observation that the speed of light is constant in all frames of reference.
- **plane**: A level or flat surface.
- **Euclidean**: Adhering to the principles of traditional geometry, in which parallel lines are equidistant.
- **cross section**: A section formed by a plane cutting through an object, usually at right angles to an axis.
- **dimension**: A measure of spatial extent in a particular direction, such as height, width or breadth, or depth.
- **coulomb's law**: the mathematical equation calculating the electrostatic force vector between two charged particles
- **vector field**: a construction in which each point in a Euclidean space is associated with a vector; a function whose range is a vector space

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CHAPTER OVERVIEW

11: Fluid Dynamics and Its Applications

11.1: Overview

learning objectives

- Interpret the circulatory system in terms of your knowledge of fluid dynamics

We have discussed many situations in which fluids are static, though there are many situations where fluids flow. For example, a column of smoke rises from a camp fire, water streams from a fire hose, blood courses through your veins. Why does rising smoke curl and twist? How does a nozzle increase the speed of water emerging from a hose? How does the body regulate blood flow? Fluid dynamics, the physics of fluids in motion, allows us to answer these and many other questions.

Application in the Circulatory System

For example, consider the circulatory system—a connected series of tubes with fluid flowing through them. The heart is the driver of the circulatory system, generating cardiac output (CO) by rhythmically contracting and relaxing. This creates changes in regional pressures and (combined with a complex valvular system in the heart and the veins) ensures that the blood moves around the circulatory system in one direction. The "beating" of the heart generates pulsatile blood flow, conducted into the arteries across the micro-circulation and then back via the venous system to the heart.

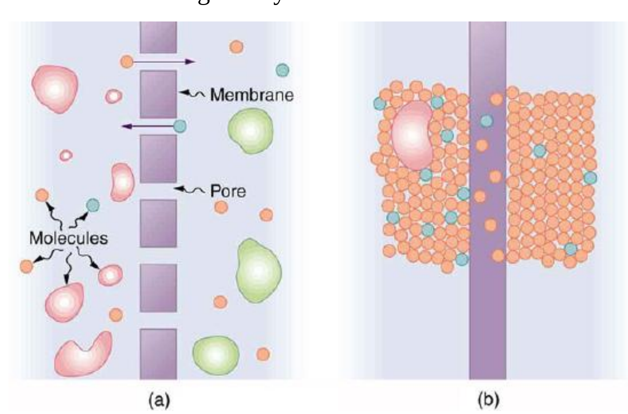
The aorta, the main artery, leaves the left side of the heart and proceeds to divide into smaller and smaller arteries that first become arterioles and eventually become capillaries, through which oxygen transfer occurs. The capillaries connect to venules, into which the deoxygenated blood passes from the cells back into the blood. The blood then travels back through the network of veins to the right heart. The micro-circulation (arterioles, capillaries and venules) constitutes most of the area of the vascular system and is the site of the transfer of O_2 into the cells.

The venous system returns the de-oxygenated blood to the right heart where it is pumped into the lungs to become oxygenated. This is also where CO_2 and other gaseous wastes are exchanged and expelled during breathing. Blood then returns to the left side of the heart where it begins the process again. The heart, vessels and lungs are all actively involved in maintaining healthy cells and organs, and all influence the fluid dynamics of the blood.

Fluids and Diffusion

Now consider how nutrients are transported through a human body. Diffusion is the movement of substances due to random thermal molecular motion. Fluids can even diffuse through solids (such as fumes or odors entering ice cubes). Diffusion is the dominant mechanism by which the exchange of nutrients and waste products occurs between the blood and tissue, and between air and blood in the lungs. In the evolutionary process, as organisms became larger they needed quicker methods of transportation than net diffusion, due to the larger distances involved in the transport. This factor lead to the development of circulatory systems. Less sophisticated, single-celled organisms still rely totally on diffusion for the removal of waste products and the uptake of nutrients.

Another important form of fluid movement is osmosis—the transport of water through a semipermeable membrane (shown in) from a region of high concentration to a region of low concentration. It is driven by the imbalance in water concentration. Similarly, dialysis is the transport of any other molecule through a semipermeable membrane due to its concentration difference. Both osmosis and dialysis are used by the kidneys to cleanse the blood, and the medical application of dialysis through machinery is important in the treatment of individuals with failing kidney function.



A Semipermeable Membrane: A semipermeable membrane with small pores that allow only small molecules to pass through.

Flow Rate and Velocity

Flow velocity and volumetric flow rates are important quantities in fluid dynamics used to quantify motion of a fluid and are interrelated.

learning objectives

- Assess the significance of studying volumetric flow in addition to flow velocity

Fluid dynamics is the study of fluids in motion and corresponding phenomena. A fluid in motion has a velocity, just as a solid object in motion has a velocity. Like the velocity of a solid, the velocity of a fluid is the rate of change of position per unit of time. In mathematical terms, the velocity of a fluid is the derivative of the position vector of the fluid with respect to time, and is therefore itself a vector quantity. The flow velocity vector is a function of position, and if the velocity of the fluid is not constant then it is also a function of time. Equation 1 shows the mathematical expression for the velocity of a fluid in motion. As a vector quantity, fluid velocity must have at least one non-zero directional component and may have up to three non-zero directional components. The velocity vector has non-zero components in any orthogonal direction along which motion of the fluid occurs.

$$\Delta p = \frac{8\eta Q \Delta x}{\pi r^4}$$

Flow Velocity: Mathematical Expression for Flow Velocity

Turbulent Flow vs. Laminar Flow

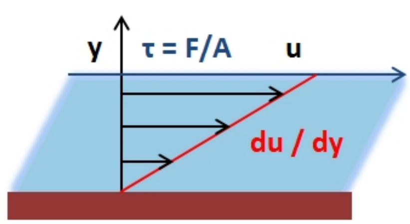
Fluid velocity can be affected by the pressure of the fluid, the viscosity of the fluid, and the cross-sectional area of the container in which the fluid is travelling. These factors affect fluid velocity depending on the nature of the fluid flow—particularly whether the flow is turbulent or laminar in nature. In the case of turbulent flow, the flow velocity is complex in nature and thus hard to predict; it must be analyzed on a system per system basis. In the case of Laminar flow, however, fluid flow is much simpler and flow velocity can be accurately calculated using Poiseuille’s Law. In SI units, fluid flow velocity is expressed in terms of meters per seconds. The magnitude of the fluid flow velocity is the fluid flow speed. Fluid flow velocity effectively describes everything about the motion of a fluid.

Volumetric Flow

In addition to flow velocity, volumetric flow rate is an important quantity in fluid dynamics analysis. Volumetric flow is defined as the volume of fluid that passes through a given surface per unit time. Qualitatively, Figure 1 shows the notion of volumetric flow rate regarding a cross-sectional surface of area A. Mathematically, volumetric flow rate is the derivative of the volume of fluid that passes through a given surface with respect to time; in SI units this is expressed as meterscubed per second. Volumetric flow rate is related to the flow velocity vector as the surface integral with respect to the surface in question. If the surface area in question is a flat, plane cross-section, the surface integral reduces as shown in Equation 2, where A is the surface area of the surface in question and v is the flow velocity of the fluid.

$$R_e = \frac{V}{I} \quad \rightarrow \quad \begin{cases} R_h \rightarrow R_e \\ \Delta p \rightarrow V \\ I \rightarrow Q \end{cases} \quad \rightarrow \quad \Delta p = \frac{8\eta Q \Delta x}{\pi r^4} \quad \rightarrow \quad R_h = \frac{\Delta p}{Q} = \frac{8\eta \Delta x}{\pi r^4}$$

Volumetric Flow Rate: Volumetric Flow Rate Surface Integral and its simplification



$$\frac{F}{A} = \tau = \eta \frac{du}{dy}$$

Flow Velocity – Volumetric Flow Rate Relation: This figure shows the relation between flow velocity and volumetric flow rate.

Moreover, only the flow velocity component parallel to the surface normal of the surface in question, or alternatively the flow velocity component perpendicular to the surface in question contributes to the volumetric flow rate. Figure 1 and Equation 2 illustrate decomposition of the flow velocity vector, making an angle θ with respect to the normal of the surface plane in order to calculate volumetric flow rate through that surface. Thus, volumetric flow rate for a given fluid velocity and cross-sectional surface area increases as θ decreases, and is maximized when $\theta = 0$. Volumetric flow rate is an important scalar quantity in fluid dynamics and is used widely in fluid flow measurements. Volumetric flow rate can be converted to mass flow rate if the density of the fluid is known. Flow of fluids through a closed system is often analyzed as a hydraulic circuit analogous to electron flow in an electronic circuit where: 1) the volumetric fluid flow is analogous to the electric current, 2) pressure is analogous to the voltage, and 3) fluid velocity is analogous to current density.

Key Points

- There are many fluids in biology and understanding their behavior in motion is crucial to effective medicine.
- The heart pumps a fluid, blood, throughout a series of tubes in the body.
- Circulation may be understood through a study of fluid dynamics.
- Diffusion is the dominant mechanism by which the exchange of nutrients and waste products occur between the blood and tissue, and between air and blood in the lungs.
- Both osmosis and dialysis are used by the kidneys to cleanse the blood, and the medical application of dialysis through machinery is important in the treatment of individuals with failing kidney function.
- Flow velocity is a vector quantity used to describe the motion of a fluid. It can be easily determined for laminar flow but complex to determine for turbulent flow.
- Volumetric flow rate is the volume of a liquid that passes through a given surface per unit time. It is found from the flow velocity and the surface area of the surface through which the fluid passes.
- Fluid flow through a closed hydraulic system is analyzed much like electron flow through an electronic circuit —where volumetric flow rate is analogous to current, flow velocity is analogous to current density, and pressure is analogous to voltage (electrical potential).

Key Terms

- **vascular**: Of, pertaining to, or containing vessels that conduct or circulate fluids (such as blood, lymph, or sap) through the body of an animal or plant.
- **osmosis**: The net movement of solvent molecules from a region of high solvent potential to a region of lower solvent potential through a partially permeable membrane.
- **dialysis**: A method of separating molecules or particles of different sizes by differential diffusion through a semipermeable membrane.
- **Laminar Flow**: Non-turbulent motion of a fluid in which parallel layers have different velocities relative to each other.
- **Turbulent Flow**: The motion of a fluid having local velocities and pressures that fluctuate randomly.

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11.2: Flow in Tubes

learning objectives

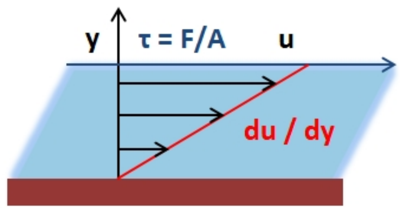
- Contrast turbulent and laminar flow in constant velocity

Virtually all moving fluids exhibit viscosity, which is a measure of the resistance of a fluid to flow. Viscosity is a basic property necessary for the analysis of fluid flow.

Measure of Fluid Friction

It describes a fluid's internal resistance to movement and can be thought of as a measure of fluid friction. The greater the viscosity, the 'thicker' the fluid and the more the fluid will resist movement.

Mathematically, viscosity is a proportionality constant relating an applied shear stress to the resulting shear velocity and is given, along with a representative diagram, (see). As shown, when a force is applied to a fluid, creating a shear stress, the fluid will undergo a certain displacement. The viscosity of the fluid is then its inherent resistance to undergo this displacement.



$$\frac{F}{A} = \tau = \eta \frac{du}{dy}$$

Representation of Viscosity: A proportionality constant relating an applied shear stress to the resulting shear velocity.

Different fluids exhibit different viscous behavior yet, in this analysis, only Newtonian fluids (fluids with constant velocity independent of applied shear stress) will be considered. Viscosity in fluids generally decreases with increasing temperature. The study of the viscous nature of fluids is called *rheology*.

In analyzing the properties of moving fluids, it is necessary to determine the nature of flow of the fluid. This is generally split into two categories, laminar and turbulent flow.

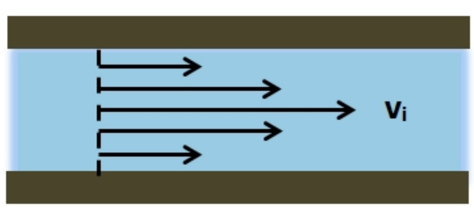
Turbulent Flow

Turbulent flow is characterized by irregular flow of a fluid in which there are both inconsistent flow patterns and velocity variations throughout the volume of the fluid in motion. Analysis of turbulent flow can be very complex and often requires advanced mathematical analysis to simulate flow in systems on a near case-by-case basis.

It occurs when the Reynolds number is above a certain critical threshold while mixed turbulent–laminar flow occurs within a range of Reynolds number below this threshold value. At the lower limit of this mixed turbulent–laminar flow Reynolds number region there is another critical threshold value, below which only laminar flow is possible.

Laminar Flow

Laminar flow consists of a regular-flow pattern with constant-flow velocity throughout the fluid volume and is much easier to analyze than turbulent flow.



Relative Magnitudes of Velocity Vectors: Laminar fluid flow in a circular pipe at the same direction.

Laminar flow is often encountered in common hydraulic systems, such as where fluid flow is through an enclosed, rigid pipe; the fluid is incompressible, has constant viscosity, and the Reynolds number is below this lower critical threshold value. It is characterized by the flow of a fluid in parallel layers, in which there is no disruption or interaction between the different layers, and in which each layer flows at a different velocity along the same direction. The variation in velocity between adjacent parallel layers is due to the viscosity of the fluid and resulting shear forces.

This figure (see) gives a representation of the relative magnitudes of the velocity vectors of each of these layers for laminar fluid flow through a circular pipe, in a direction parallel to the pipe axis.

$$\Delta p = \frac{8\eta Q \Delta x}{\pi r^4}$$

Poiseuille's Equation: Can be used to determine the pressure drop of a constant viscosity fluid exhibiting laminar flow through a rigid pipe.

Considering laminar flow of a constant density, incompressible fluid such as for a Newtonian fluid traveling in a pipe, with a Reynolds number below the upper limit level for fully laminar flow, the pressure difference between two points along the pipe can be found from the volumetric flow rate, or vice versa. For such a system with a pipe radius of r, fluid viscosity η, distance between the two points along the pipe $\Delta x = x_2 - x_1$, and the volumetric flow rate Q, of the fluid, the pressure difference between the two points along the pipe Δp is given by Poiseuille's equation (see).

This equation is valid for laminar flow of incompressible fluids only, and may be used to determine a number of properties in the hydraulic system, if the others are known or can be measured. In practice, Poiseuille's equation holds for most systems involving laminar flow of a fluid, except at regions where features disrupting laminar flow, such as at the ends of a pipe, are present.

Poiseuille's equation as given in this example (see) is analogous to Ohm 's equation for determining the resistance in an electronic circuit and is of great practical use in hydraulic-circuit analysis.

$$R_e = \frac{V}{I} \quad \rightarrow \quad \begin{cases} R_h \rightarrow R_e \\ \Delta p \rightarrow V \\ I \rightarrow Q \end{cases} \quad \rightarrow \quad \Delta p = \frac{8\eta Q \Delta x}{\pi r^4} \quad \rightarrow \quad R_h = \frac{\Delta p}{Q} = \frac{8\eta \Delta x}{\pi r^4}$$

Poiseuille's Equation: Analogous to Ohm's Law Analogy

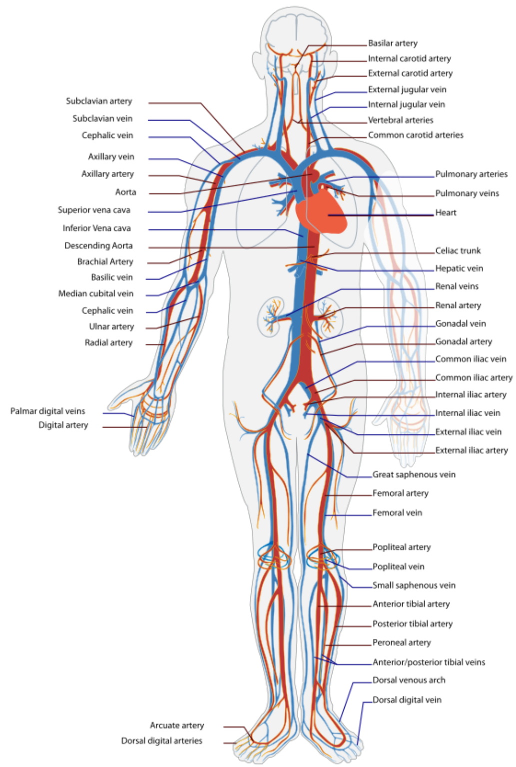
Blood Flow

Blood flow is the continuous running of blood through the cardiovascular system, which consists of the vessels and the heart.

learning objectives

- Outline how normal plasma behaves in a mammalian cardiovascular system

Blood flow is the continuous running of blood through vessels in the cardiovascular system (the mammalian cardiovascular system is shown in). Blood is the viscous fluid composed of plasma and cells. The composition of the blood includes plasma, red blood cells, white blood cells and platelets. In microcirculation, the properties of the blood cells have an important influence on flow.



An illustrative overview of the mammalian cardiovascular system: Keep in mind that both circular paths are working simultaneously and not in a sequential manner as the numbering in the illustration might suggest. Both the ventricles are working together in harmony; as tiny amounts of blood are moving in the pulmonary circuit, the remainder of the blood moves through the systemic circuit.

The cardiovascular system, which consists of blood vessels and the heart, helps to distribute nutrients, O_2, and other products of metabolism. The blood moves in the blood vessels, while the heart serves as the pump for the blood. The vessel walls of the heart are elastic and movable, therefore causing the blood and the wall to exert forces on each other and in turn influencing their respective motion.

The major quantity of interest in describing the motion of blood particles is velocity—the rate of change of the position of an object with time:

$$\mathrm{v}=\frac{\Delta \mathrm{x}}{\Delta \mathrm{t}} \tag{11.2.1}$$

Blood velocities in arteries are higher during systole than during diastole. One parameter to quantify this difference is pulsatility index (PI), which is equal to the difference between the peak systolic velocity and the minimum diastolic velocity divided by the mean velocity during the cardiac cycle.

Another important parameter is the acceleration—the rate of change of velocity: $\mathrm{a}=\frac{\Delta \mathrm{v}}{\Delta \mathrm{t}}$

Normal plasma behaves like a Newtonian fluid at rates of shear. Typical values for the viscosity of normal human plasma at 37°C is $1.2Nsm^{-2}$. The viscosity of normal plasma varies with temperature in the same way as does that of its solvent, water. (a 5°C increase of temperature in the physiological range reduces plasma viscosity by about 10%).

The osmotic pressure of the plasma affects the mechanics of the circulation in several ways. An alteration of the osmotic pressure difference across the membrane of a blood cell causes a shift of water and a change in cell volume. The change, both in shape and flexibility, affects the mechanical properties of whole blood. Therefore, a change in plasma osmotic pressure alters the hematocrit (the volume concentration of red cells in the whole blood) by redistributing water between the intravascular and extravascular spaces. This in turn affects the mechanics of the whole blood.

Key Points

- Viscosity is the resistance of a fluid to flow. Virtually all fluids have viscosity which generally changes as a function of temperature; although different types of fluids exhibit different types of fluid–shear velocity dependencies.
- Laminar flow of a fluid is characterized by its flow in parallel layers in which there is no disruption or interaction between the different layers, and in which each layer flows at a different velocity along the same direction.
- Poiseuille's equation pertains to moving incompressible fluids exhibiting laminar flow. It relates the difference in pressure at different spatial points to volumetric flow rate for fluids in motion in certain cases, such as in the flow of fluid through a rigid pipe.
- The major quantity of interest in describing the motion of blood particles is the velocity – the rate of change of the position of an object with time: $\mathrm{v}=\frac{\Delta \mathrm{x}}{\Delta \mathrm{t}}$.
- Blood velocities in arteries are higher during systole than during diastole.
- The mechanics of the circulation depends on osmotic pressure of plasma.

Key Terms

- **viscosity**: The property of a fluid that resists the force which tends to cause it to flow.
- **shear stress**: The external force acting on an object or surface parallel to the slope or plane in which it lies; the stress tending to produce shear.
- **Reynolds Number**: A dimensionless number, $\frac{\mathrm{v}\rho \mathrm{l}}{\eta}$, where v is the fluid velocity, ρ the density, η the viscosity and l a dimension of the system. The value of the number indicates the type of fluid flow.
- **systole**: The rhythmic contraction of the heart, by which blood is driven through the arteries.
- **vessel**: A tube or canal that carries fluid in an animal or plant.
- **diastole**: The phase or process of relaxation and dilation of the heart chambers, between contractions, during which they fill with blood; an instance of the process.

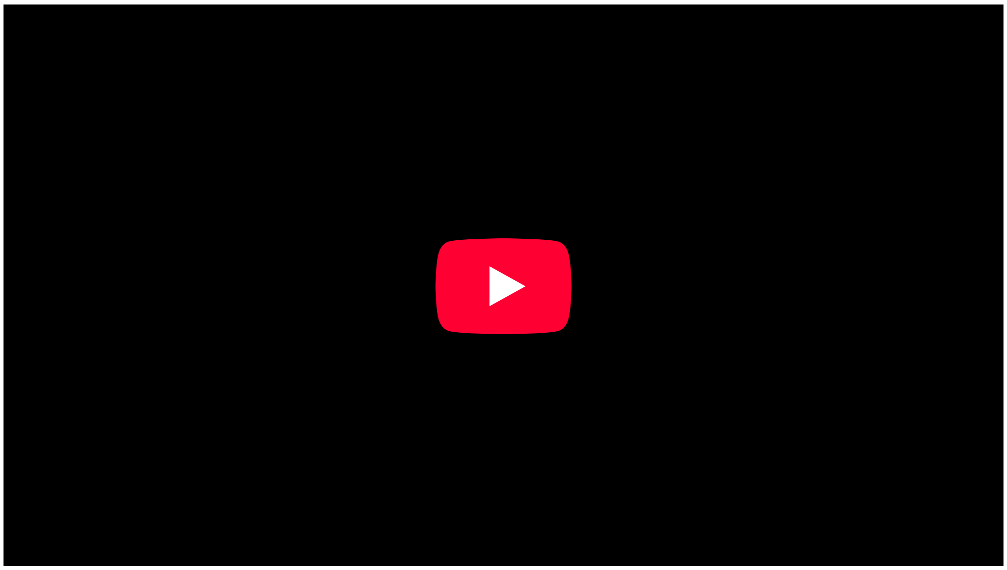
11.3: Bernoulli’s Equation

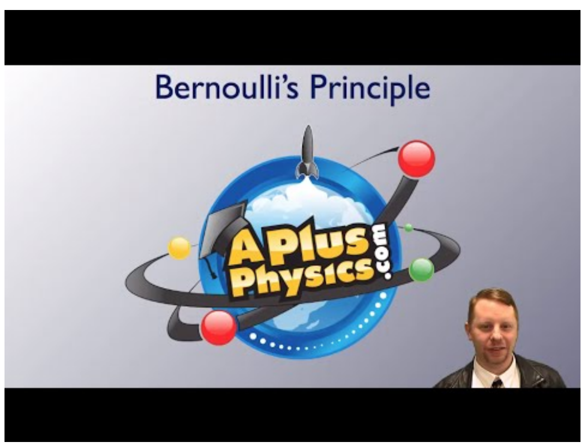
learning objectives

- Adapt Bernoulli’s equation for flows that are either unsteady or compressible

Application of Bernoulli’s Equation

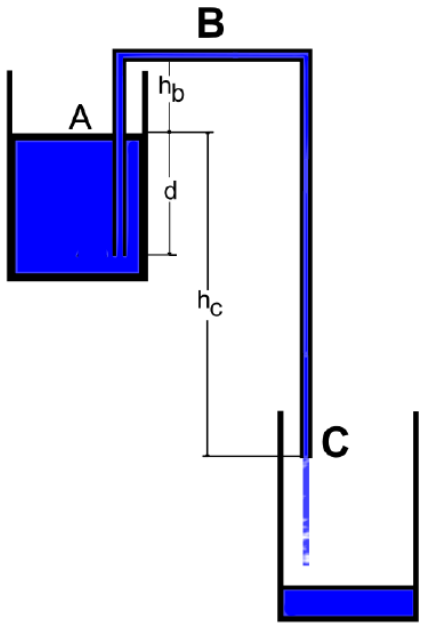
The relationship between pressure and velocity in ideal fluids is described quantitatively by Bernoulli’s equation, named after its discoverer, the Swiss scientist Daniel Bernoulli (1700–1782). Bernoulli’s equation states that for an incompressible and inviscid fluid, the total mechanical energy of the fluid is constant. (An inviscid fluid is assumed to be an ideal fluid with no viscosity.)





Bernoulli’s Principle*: A brief introduction to Bernoulli’s Principle for students studying fluids.*

The total mechanical energy of a fluid exists in two forms: potential and kinetic. The kinetic energy of the fluid is stored in static pressure, psps, and dynamic pressure, 12ρV212ρV2, where \rho is the fluid density in (SI unit: kg/m^3) and V is the fluid velocity (SI unit: m/s). The SI unit of static pressure and dynamic pressure is the pascal.



Syphoning*: Syphoning fluid between two reservoirs. The flow rate out can be determined by drawing a streamline from point (A) to point (C).*

Static pressure is simply the pressure at a given point in the fluid, dynamic pressure is the kinetic energy per unit volume of a fluid particle. Thus, a fluid will not have dynamic pressure unless it is moving. Therefore, if there is no change in potential energy along a streamline, Bernoulli's equation implies that the total energy along that streamline is constant and is a balance between static and dynamic pressure. Mathematically, the previous statement implies:

$$\mathrm{p_s} + \frac{1}{2}\rho \mathrm{V}^2 = \text{constant} \tag{11.3.1}$$

along a streamline. If changes there are significant changes in height or if the fluid density is high, the change in potential energy should not be ignored and can be accounted for with,

$$\Delta \mathrm{PE} = \rho \mathrm{g} \Delta \mathrm{h}. \tag{11.3.2}$$

This simply adds another term to the above version of the Bernoulli equation and results in

$$\mathrm{p_s} + \frac{1}{2}\rho \mathrm{V}^2 + \rho \mathrm{g} \Delta \mathrm{h} = \text{constant}. \tag{11.3.3}$$

Deriving Bernoulli's Equation

The Bernoulli equation can be derived by integrating Newton's 2nd law along a streamline with gravitational and pressure forces as the only forces acting on a fluid element. Given that any energy exchanges result from conservative forces, the total energy along a streamline is constant and is simply swapped between potential and kinetic.

Applying Bernoulli's Equation

Bernoulli's equation can be applied when syphoning fluid between two reservoirs. Another useful application of the Bernoulli equation is in the derivation of Torricelli's law for flow out of a sharp edged hole in a reservoir. A streamline can be drawn from the top of the reservoir, where the total energy is known, to the exit point where the static pressure and potential energy are known but the dynamic pressure (flow velocity out) is not.

Adapting Bernoulli's Equation

The Bernoulli equation can be adapted to flows that are both unsteady and compressible. However, the assumption of inviscid flow remains in both the unsteady and compressible versions of the equation. Compressibility effects depend on the speed of the flow relative to the speed of sound in the fluid. This is determined by the dimensionless quantity known as the Mach number. The Mach number represents the ratio of the speed of an object moving through a medium to the speed of sound in the medium.

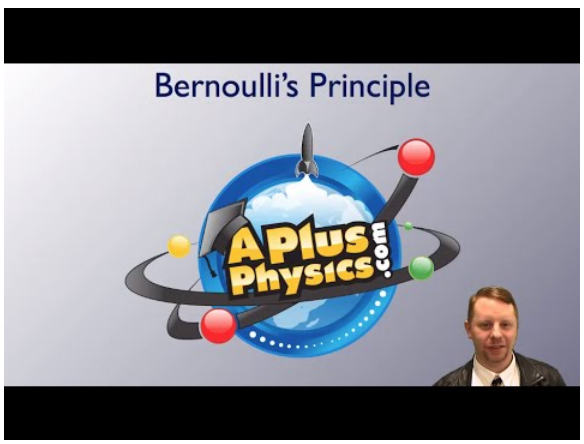
Torricelli’s Law

Torricelli’s law is theorem about the relation between the exit velocity of a fluid from a hole in a reservoir to the height of fluid above the hole.

learning objectives

- Infer the exit velocity through examining the Bernoulli equation

Torricelli’s law is theorem in fluid dynamics about the relation between the exit velocity of a fluid from a sharp-edged hole in a reservoir to the height of the fluid above that exit hole. This relationship applies for an “ideal” fluid (inviscid and incompressible) and results from an exchange of potential energy,



Torricelli’s Principle: A brief introduction to Torricelli’s Principle for students studying fluids.

mghmgh, for kinetic energy,

$\frac{1}{2}\rho v^2$, at the exit.

This relationship can be derived by applying the Bernoulli equation between the top of the reservoir and the exit hole. Applying Bernoulli between the top of a reservoir and an exit hole at a height h below the top of the reservoir results in,

$$PE \leftrightarrow KE$$

Exchange of Energy*: Potential energy at the top of the reservoir becomes kinetic energy at the exit.*

$$p_t + \frac{1}{2}\rho v_t^2 + \rho g h_t = p_e + \frac{1}{2}\rho v_e^2 + \rho g h_e \tag{11.3.4}$$

where subscript t implies evaluation at the top of the reservoir and subscript e implies evaluation at the exit. If we assume both the top of reservoir and the exit are open to the atmosphere, the zero for potential energy is at the exit hole, and the fluid velocity at the top of the reservoir is essentially zero (large reservoir, small hole), we arrive at

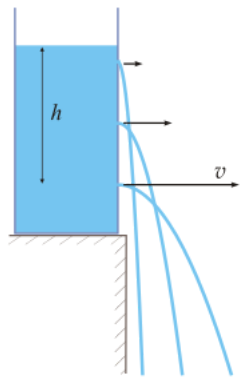
$$\rho g h_t = \frac{1}{2}\rho v_e^2 \tag{11.3.5}$$

This can be solved for the exit velocity, resulting in,

$$v_e = \sqrt{2 g h_t} \tag{11.3.6}$$

where again h_t is the height difference between the top of the reservoir and the exit hole. Due to the assumption of an ideal fluid, all forces acting on the fluid are conservative and thus there is an exchange between potential and kinetic energy. The result is that the velocity acquired by the fluid is the same that a body would acquire when simply dropped from the height h_t.

A simple experiment to test Torricelli's law involves filling a soda bottle with water and puncturing the bottom with a small hole (about 1 cm in diameter). As the height in the reservoir decreases, the exit velocity will decrease as well. The exit velocity can be increased by capping the top of the reservoir and pressurizing it.



Toricelli's Law: The exit velocity depends on the height of the fluid above the exit hole.

$$\mu = 0$$

$$\nabla \cdot \vec{u} = 0$$

Ideal Fluid: Applies to an ideal fluid (inviscid, incompressible)

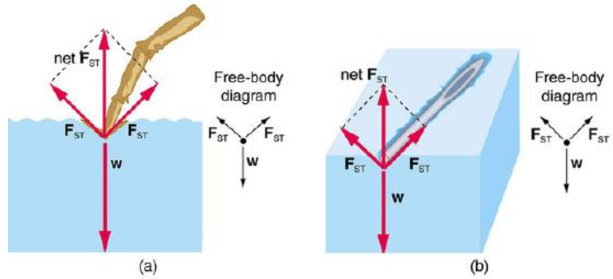
Surface Tension

The tendency of the surface of a liquid to resist a force and behave like a membrane and is a result of cohesion between liquid molecules.

learning objectives

- Summarize the cause for different surface tensions at a liquid's surface

Surface tension is the tendency of a liquid surface to resist forces applied to it. This effect is a result of cohesion of the molecules of the liquid causing the surface of the liquid to contract to the smallest area possible. This effect is visible in nature with water strider insects that are able to walk on water. Also, a paper clip or pin can be supported by the surface tension at a water air interface.

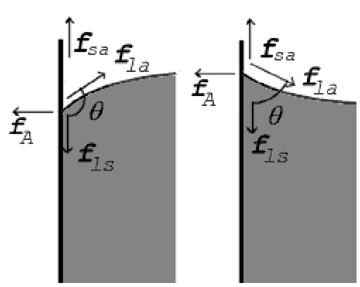


***Surface Tension FBD**: Force diagrams showing the direction of forces for water supporting a water strider (insect) foot and a pin. In both cases, the vertical component of the surface tension is enough to support the weight of the object.*

In the bulk of the liquid, the molecules are pulled equally in all directions. The molecules at the surface feel a greater attractive force toward the bulk material than the interface material.

The surface of a liquid is an interface between another fluid, a solid body, or both. Therefore, the surface tension will be a property of the interface rather than simply the liquid. Adhesion describes the attractive force between molecules of different types. The surface of a liquid in a container is an interface between the liquid, the air, and the container. Where the surfaces meet, forces must be in equilibrium. This results in a contact angle at the interface. The contact angle is measured in the liquid and depends on the relative strength of cohesive forces in the liquid and adhesive forces between the liquid and interface materials. If liquid molecules are strongly attracted to the molecules of the solid surface (adhesive forces > cohesive forces), the drop will tend to spread out and the contact angle will be close to zero degrees. If the cohesive forces are greater than the adhesive forces, the resulting contact angles will be large and will form a more circular drop.

***Water Droplet on Leaf**: When a water droplet forms on a leaf, the cohesive forces between the water molecules are greater than the adhesive forces between the water and leaf surface. The leaf is a hydrophobic surface.*



***Contact Angle**: The contact angle is the angle, measured in the fluid, that results when a liquid-gas interface, meets a solid surface.*

When the liquid is water, a surface where the contact angle is small is said to be hydrophilic. Large contact angles are present on hydrophobic surfaces. The contact angle determines the wettability of the surface.

Key Points

- The simplest form of Bernoulli's equation (steady and incompressible flow) states that the sum of mechanical energy, potential energy and kinetic energy, along a streamline is constant. Therefore, any increase in one form results in a decrease in the other.
- Bernoulli's equation considers only pressure and gravitational forces acting on the fluid particles. Therefore, if there is no change in height along a streamline, Bernoulli's equation becomes a balance between static pressure and velocity.
- The steady-state, incompressible Bernoulli equation, can be derived by integrating Newton's 2nd law along a streamline.
- Torricelli's law applies to an inviscid, incompressible fluid ("ideal" fluid).
- You can ascertain results from applying the Bernoulli equation between the top of the reservoir and the exit hole.
- The relationship arises from an exchange of potential energy at the top of the reservoir to kinetic energy at the exit.
- The final kinetic energy is equivalent to what a solid body would acquire when falling from height h.
- Surface tension is a result of cohesion between the molecules of the liquid. The molecules at the surface of the liquid feel an attractive force pulling them toward the bulk of the liquid more than the solid or fluid at the interface.
- When a liquid-solid-gas interface is encountered, the contact angle represents a measure of the relative strength of adhesive and cohesive forces.
- The contact angle determines the wettability of a surface.

Key Terms

- **viscosity**: A quantity expressing the magnitude of internal friction in a fluid, as measured by the force per unit area resisting uniform flow.
- **Ideal Fluid**: An inviscid and incompressible fluid
- **incompressible**: Unable to be compressed or condensed.
- **inviscid**: A fluid with zero viscosity (internal friction). In reality viscosity is always present. However, it is often very small compared with other forces (e.g. gravity, pressure) and for common fluids (water and air) the fluid can be approximated as having zero viscosity.
- **cohesion**: Various intermolecular forces that hold solids and liquids together.
- **wettability**: The ability of a solid surface to reduce the surface tension of a liquid in contact with it such that it spreads over the surface and wets it.
- **adhesion**: The ability of a substance to stick to an unlike substance.

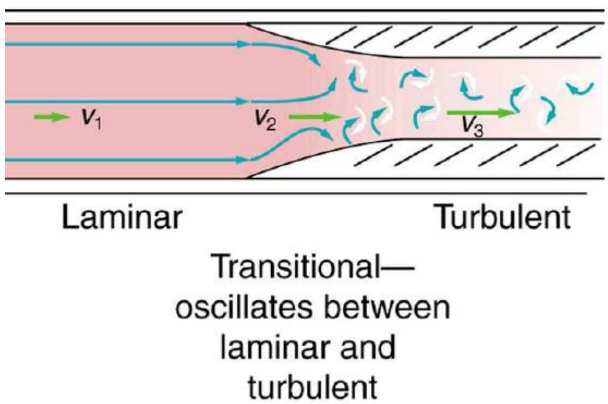
11.4: Other Applications

learning objectives

- Predict if flow will be laminar or turbulent

It is possible to predict if flow will be laminar or turbulent. At low velocity, flow in a very smooth tube or around a smooth, streamlined object will be laminar. At high velocity, even the flow in a smooth tube or around a smooth object will experience turbulence. However, between low and high velocity, flow is more difficult to predict. In fact, at intermediate velocities, flow may oscillate back and forth indefinitely between laminar and turbulent.

An occlusion (narrowing) of an artery, such as shown in, is likely to cause turbulence because of the irregularity of the blockage, as well as the complexity of blood as a fluid. Turbulence in the circulatory system (such as aneurysms, or ballooning of arteries) is noisy and can sometimes be detected with a stethoscope (such as when measuring diastolic pressure in the upper arm's partially collapsed brachial artery). These turbulent sounds, at the onset of blood flow when the cuff pressure becomes sufficiently small, are called *Korotkoff* sounds. Heart murmurs, consistent with their name, are sounds produced by turbulent flow around damaged and insufficiently closed heart valves. Another method of detecting this type of turbulence is ultrasound, used as a medical indicator in a process analogous to Doppler-shift radar (used to detect storms).



Turbulent Flow in an Artery: Flow is laminar in the large part of this blood vessel and turbulent in the part narrowed by plaque, where velocity is high. In the transition region, the flow can oscillate chaotically between laminar and turbulent flow.

Turbulence manifests in other areas, with varying causes. During an airplane flight, for example, the turbulence experienced is due to the mixing of warm and cold air in the atmosphere, causing the airplane to shake. The mixing currents in oceans creates a similar effect.

The phenomenon of turbulent air flow must be accounted for in many applications. For example, race cars are unable to follow each other around fast corners because the leading car creates turbulent air flow in its wake (this can lead to under-steering).

Industrial equipment, such as pipes, ducts, and heat exchangers are often designed to induce the flow regime of interest (laminar or turbulent). When flow is turbulent, particles exhibit additional transverse motion. This enhances the rate of energy and momentum exchange between them, increasing the heat transfer. Turbulent flow is thus desirable in applications where a relatively cool fluid is mixed with a warmer fluid to reduce the temperature of the warmer fluid.

It is imperative to take into account turbulent flow when designing certain structures, such as a bridge support, as shown in. In the late summer and fall, when river flow is slow, water flows smoothly around the support legs. In the spring, when the flow is faster, the flow may start off laminar but it is quickly separated from the leg and becomes turbulent. The bridge supports must be designed so that they can withstand the turbulent flow of the water in the spring.

Longtown Bridge: Turbulent flow is visible around the bridge supports of the Longtown bridge.

Motionof an Object in a Viscous Field

Objects moving in a viscous fluid feel a resistive force proportional to the viscosity of the fluid.

learning objectives

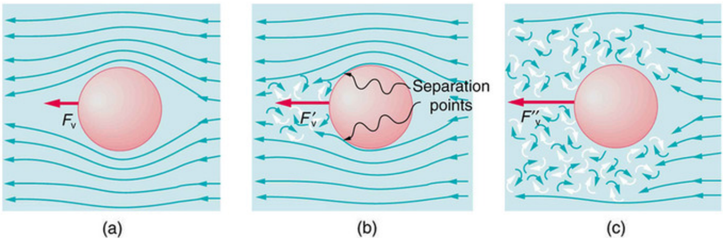
- Assess the relationship of the parameters to one another in determining the inertia of an object moving in fluid

Overview

A moving object in a viscous fluid is equivalent to a stationary object in a flowing fluid stream. (For example, when you ride a bicycle at 10 m/s in still air, you feel the air in your face exactly as if you were stationary in a 10-m/s wind.) Flow of the stationary fluid around a moving object may be laminar, turbulent, or a combination of the two. Just as with flow in tubes, it is possible to predict when a moving object creates turbulence. We use another form of the Reynolds number N′R, defined for an object moving in a fluid to be

$$\mathrm{N'R} = \frac{\rho \mathrm{vL}}{\eta} \tag{11.4.1}$$

where L is a characteristic length of the object (a sphere's diameter, for example), the fluid density, its viscosity, and v the object's speed in the fluid. If N′R is less than about 1, flow around the object can be laminar, particularly if the object has a smooth shape. The transition to turbulent flow occurs for N′R between 1 and about 10, depending on surface roughness and so on. Depending on the surface, there can be a *turbulent wake* behind the object with some laminar flow over its surface. For an N′R between 10 and 10^6, the flow may be either laminar or turbulent and may oscillate between the two. For N′R greater than about 10^6, the flow is entirely turbulent, even at the surface of the object. (See.) Laminar flow occurs mostly when the objects in the fluid are small, such as raindrops, pollen, and blood cells in plasma.



Motion of an object in a viscous fluid.: (a) Motion of this sphere to the right is equivalent to fluid flow to the left. Here the flow is laminar with N′R less than 1. There is a force, called viscous drag FV, to the left on the ball due to the fluid's viscosity. (b) At a higher speed, the flow becomes partially turbulent, creating a wake starting where the flow lines separate from the surface. Pressure in the wake is less than in front of the sphere, because fluid speed is less, creating a net force to the left F′V that is significantly greater than for laminar flow. Here N′R is greater than 10. (c) At much higher speeds, where N′R is greater than 10^6, flow becomes turbulent everywhere on the surface and behind the sphere. Drag increases dramatically.

Viscous Drag

One of the consequences of viscosity is a resistance force called viscous drag FVFV that is exerted on a moving object. This force typically depends on the object's speed (in contrast with simple friction). Experiments have shown that for laminar flow (N′R less than about one) viscous drag is proportional to speed, whereas for N′R between about 10 and 106, viscous drag is proportional to speed squared. (This relationship is a strong dependence and is pertinent to bicycle racing, where even a small headwind causes significantly increased drag on the racer. Cyclists take turns being the leader in the pack for this reason.) For N′R greater than 106, drag increases dramatically and behaves with greater complexity. For laminar flow around a sphere, FVFV is proportional to fluid viscosity, the object's characteristic size L, and its speed v. All of which makes sense—the more viscous the fluid and the larger the object, the more drag we expect. Recall Stoke's law FS=6πrηvFS=6πrηv. For the special case of a small sphere of radius R, moving slowly in a fluid of viscosity, the drag force FSFS is given by

FS=6πRηvFS=6πRηv.

Molecular Transport Phenomena

Molecular transport phenomena are ways in which molecules are transported from one region to another. These include diffusion and osmosis.

learning objectives

- Predict the role diffusion plays in blood transport throughout the body

Diffusion

Atoms and molecules are in constant motion at any temperature. In fluids they move about randomly even in the absence of macroscopic flow.

Diffusion is the movement of substances due to random thermal molecular motion. Fluids, like fish fumes or odors entering ice cubes, can even diffuse through solids. Diffusion is a slow process over macroscopic distances. The densities of common materials are great enough that molecules cannot travel very far before having a collision that can scatter them in any direction, including straight backward. More massive molecules diffuse more slowly.

Another interesting point is that the diffusion rate for oxygen in air is much greater than for oxygen in water. In water, an oxygen molecule makes many more collisions in its random walk and is slowed considerably. In water, an oxygen molecule moves only about 40μm in 1 s. (Each molecule actually collides about 1010 times per second!). Finally, note that diffusion constants increase with temperature, because average molecular speed increases with temperature. This is because the average kinetic energy of molecules, $1/2mv^2$, is proportional to absolute temperature. Because diffusion is typically very slow, its most important effects occur over small distances. For example, the cornea of the eye gets most of its oxygen by diffusion through the thin tear layer covering it.

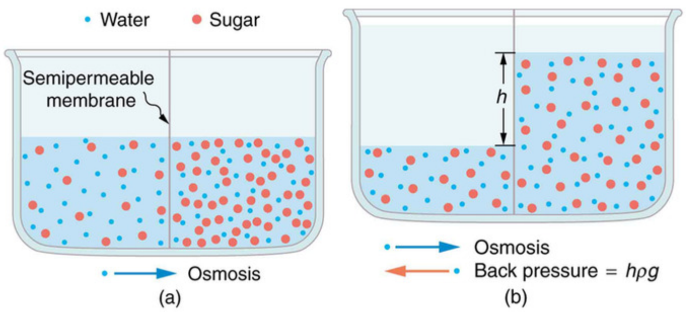
If you very carefully place a drop of food coloring in a still glass of water, it will slowly diffuse into the colorless surroundings until its concentration is the same everywhere. This type of diffusion is called free diffusion, because there are no barriers inhibiting it. Let us examine its direction and rate. Molecular motion is random in direction, and so simple chance dictates that more molecules will move out of a region of high concentration than into it. The net rate of diffusion is higher initially than after the process is partially completed. The rate of diffusion is proportional to the concentration difference. Many more molecules will leave a region of high concentration than will enter it from a region of low concentration. In fact, if the concentrations were the same, there would be no net movement. The rate of diffusion is also proportional to the diffusion constant D, which is determined experimentally. Many of the factors that affect the rate are hidden in the diffusion constant D. For example, temperature and cohesive and adhesive forces all affect values of D. Diffusion is the dominant mechanism by which the exchange of nutrients and waste products occur between the blood and tissue, and between air and blood in the lungs. In the evolutionary process, as organisms became larger, they needed quicker methods of transportation than net diffusion, because of the larger distances involved in the transport, leading to the development of circulatory systems. Less sophisticated, single-celled organisms still rely totally on diffusion for the removal of waste products and the uptake of nutrients.

Food Coloring: Food coloring spreading on a thin water film.

Osmosis and Dialysis – Diffusion Across Different Membranes

Some of the most interesting examples of diffusion occur through barriers that affect the rates of diffusion. For example, when you soak a swollen ankle in Epsom salt, water diffuses through your skin. Many substances regularly move through cell membranes; oxygen moves in, carbon dioxide moves out, nutrients go in, and wastes go out, for example. Because membranes are thin structures (typically 6.5×10^{-9} to 10×10^{-9} m across) diffusion rates through them can be high.

Diffusion through membranes is an important method of transport. Membranes are generally selectively permeable, or semipermeable. In other types of membranes, the molecules may actually dissolve in the membrane or react with molecules in the membrane while moving across. Membrane function, in fact, is the subject of much current research, involving not only physiology but also chemistry and physics. Osmosis is driven by the imbalance in water concentration. For example, water is more concentrated in your body than in Epsom salt. When you soak a swollen ankle in Epsom salt, the water moves out of your body into the lower-concentration region in the salt. Similarly, dialysis is the transport of any other molecule through a semipermeable membrane due to its concentration difference. Both osmosis and dialysis are used by the kidneys to cleanse the blood.



Diffusion: (a) Two sugar-water solutions of different concentrations, separated by a semipermeable membrane that passes water but not sugar. Osmosis will be to the right, since water is less concentrated there. (b) The fluid level rises until the back pressure ρgh equals the relative osmotic pressure; then, the net transfer of water is zero.

Pumps and the Heart

The heart pumps blood through the body by contracting and relaxing, increasing and decreasing the pressure.

learning objectives

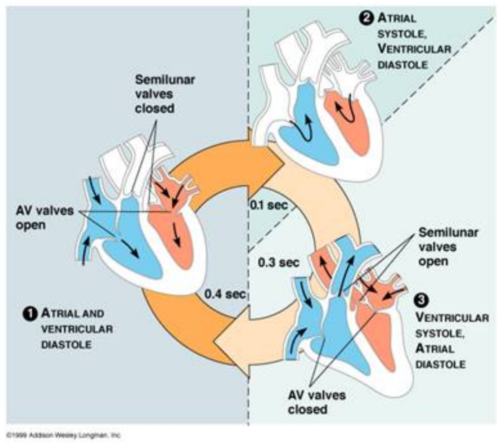
- Contrast systole and diastole in cardiovascular circulation

The Heart and its Parts

The heart is made up of four chambers. Two atria at the top of the heart receive blood and two ventricles at the bottom of the heart pump blood out of the heart. The septum divides the left and right side of the heart, while the valves of the heart ensure that blood only flows in one direction.They include the tricuspid valve-found between the right atrium and the right ventricle-and the mitral valve-found between the left atrium and the left ventricle. The list of heart valves also includes the semi-lunar valves, which are located at the bottom of the aorta and pulmonary artery. Strong tendinous chords attached to valves prevent them from turning inside out when they close.

The human heart will undergo over 3 billion contraction cycles during a normal lifetime. A complete cardiac cycle is one round of the heart pumping blood and consists of two parts: systole (contraction of the heart muscle) and diastole (relaxation of the heart muscle). During the cycle, the top half of the heart works as one unit, while the bottom half of the heart works as one unit.

The heart beat can be heard as a sound that the valves make when they close. The 'lub' sound is made when the atrio ventricular valves close and the 'dub' sound is made when the semi lunar valves close. Blood pressure is produced by the left ventricle contractions.The rhythm of ventricle diastole, often just referred to as diastole, causes the pulse, which can be felt by holding two fingers to the side of the throat.



Cardiac Cycle: The heart pumps blood through the body.

Key Points

- For low velocity, flow in a smooth tube will be laminar.
- At higher velocities or if there are obstructions, the flow turns turbulent.
- Turbulent flow is very chaotic, with rapid variations in velocity and pressure.
- Viscous fluids exert a resitive force on objects attempting to move through them.
- This resistive force is called viscous drag and is proportional to the viscosity of the fluid and the motion of the object.
- An object moving in a fluid can be thought of as a stationary object in a moving fluid.
- Diffusion is the movement of molecules due to random thermal motion.
- Osmosis is the movement of molecules due to different concentrations. Molecules will move from regions of high concentrations to lower concentrations.
- These transport phenomena can take place through membranes if the pressure is great enough.

Key Terms

- **turbulent**: Being in, or causing, disturbance or unrest.
- **streamlined**: Designed to offer little resistance to the flow of fluid, especially by having sleek, graceful lines.
- **laminar**: Of fluid motion, smooth and regular, flowing as though in different layers.
- **viscosity**: A quantity expressing the magnitude of internal friction in a fluid, as measured by the force per unit area resisting uniform flow.
- **turbulence**: Disturbance in a gas or fluid, characterized by evidence of internal motion or unrest.
- **diffusion**: the intermingling of the molecules of a fluid due to random thermal agitation
- **ventricle**: One of two lower chambers of the heart.
- **contraction**: A reversible reduction in size.
- **atrium**: An upper chamber of the heart that receives blood from the veins and forces it into a ventricle. In higher vertebrates, the right atrium receives blood from the superior vena cava and inferior vena cava, and the left atrium receives blood from the left and right pulmonary veins.

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CHAPTER OVERVIEW

12: Temperature and Kinetic Theory

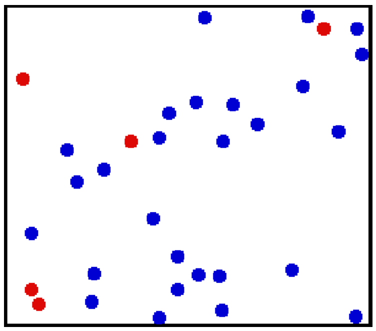
Topic hierarchy

12.1: Introduction

> **learning objectives**
>
> - Describe gas using the kinetic theory of gases

Introduction to Temperature and Kinetic Theory

The kinetic theory of gases describes a gas as a large number of small particles (atoms or molecules), all of which are in constant, random motion. The rapidly moving particles constantly collide with each other, and with the walls of the container. Kinetic theory explains macroscopic properties of gases (such as pressure, temperature, and volume) by considering their molecular composition and motion. Essentially, the theory posits that pressure is due not to static repulsion between molecules (as was Isaac Newton's conjecture) but rather due to collisions between molecules moving at different velocities through Brownian motion. Also, the temperature of an ideal monatomic gas is a measure of the average kinetic energy of its atoms, as illustrated in.



Translational Motion of Helium: Real gases do not always behave according to the ideal model under certain conditions, such as high pressure. Here, the size of helium atoms relative to their spacing is shown to scale under 1950 atmospheres of pressure.

The kinetic theory of gases uses the model of the ideal gas to relate temperature to the average translational kinetic energy of the molecules in a container of gas in thermodynamic equilibrium. Classical mechanics defines the translational kinetic energy of a gas molecule as follows:

$$\mathrm{E_k} = \frac{1}{2}\mathrm{mv}^2, \tag{12.1.1}$$

where m is the particle mass and v its speed (the magnitude of its velocity). The distribution of the speeds (which determine the translational kinetic energies) of the particles in a classical ideal gas is called the Maxwell-Boltzmann distribution. In kinetic theory, the temperature of a classical ideal gas is related to its average kinetic energy per degree of freedom $\mathrm{E_k}$ via the equation:

$$\bar{\mathrm{E}}_\mathrm{k} = \frac{1}{2}\mathrm{kT}, \tag{12.1.2}$$

(k: Boltzmann's constant). We will derive this relationship in the following atoms. We will also derive the ideal gas law:

$$\mathrm{pV} = \mathrm{nRT}, \tag{12.1.3}$$

(R: ideal gas constant, n: number of moles of gas) from a microscopic theory.

Atomic Theory of Matter

Atomic theory is a scientific theory of the nature of matter which states that matter is composed of discrete units called atoms.

> **learning objectives**
>
> - Formulate five postulates of John Dalton's atomic theory

Atomic theory is a scientific theory of the nature of matter which states that matter is composed of discrete units called atoms, as opposed to the obsolete notion that matter could be divided into any arbitrarily small quantity. Although physicists discovered that the so-called "indivisible atom" was actually a conglomerate of various subatomic particles, the concept of atoms is still important because they are building blocks of matter and form the basis of chemistry.

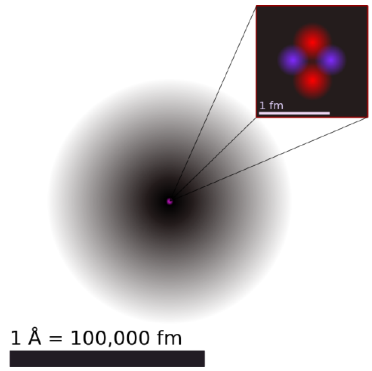


Illustration of the Helium Atom: This is an illustration of the helium atom, depicting the nucleus (pink) and the electron cloud distribution (black). The nucleus (upper right) in helium-4 is in reality spherically symmetric and closely resembles the electron cloud, although for more complicated nuclei this is not always the case. The black bar is one angstrom (10-10 m, or 100 pm).

Dalton's Atomic Hypothesis

Philosophical proposals regarding atoms have been suggested since the years of the ancient Greeks, but John Dalton was the first to propose a scientific theory of atoms. He based his study on two laws about chemical reactions that emerged (without referring to the notion of an atomic theory) in the late 18^{th}century. The first was the law of conservation of mass, formulated by Antoine Lavoisier in 1789, which states that the total mass in a chemical reaction remains constant (that is, the reactants have the same mass as the products). The second was the law of definite proportions, first proven by the French chemist Joseph Louis Proust.

Dalton proposed that each chemical element is composed of atoms of a single, unique type, and though they cannot be altered or destroyed by chemical means, they can combine to form more complex structures (chemical compounds). This marked the first truly scientific theory of the atom, since Dalton reached his conclusions by experimentation and examination of the results in an empirical fashion. For this reason, Dalton is considered the originator of modern atomic theory.

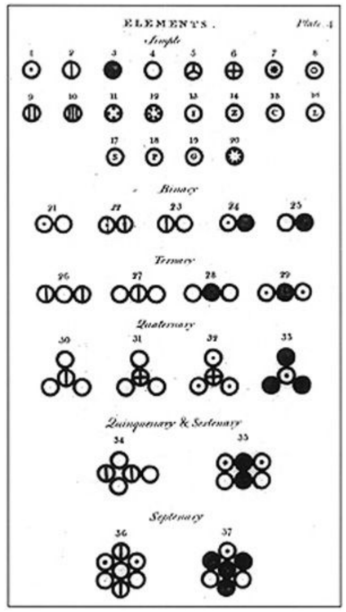
5 Main Points

Dalton's atomic theory had 5 main points:

1. Elements are made of extremely small particles called atoms.
2. Atoms of a given element are identical in size, mass, and other properties; atoms of different elements differ in size, mass, and other properties.
3. Atoms cannot be subdivided, created, or destroyed.
4. Atoms of different elements combine in simple whole-number ratios to form chemical compounds.
5. In chemical reactions, atoms are combined, separated, or rearranged.

Of these five, only three are still considered valid today. 1, 4, and 5 are valid, while 2 and 3 have turned out not to be the case. Atoms can be broken down into smaller pieces, and atoms of a given element can vary in mass and other properties (see isotopes and ions).

Knowing that a gas is composed of small atomic and molecular particles, it is natural to try to explain properties of the gas from a microscopic point of view. This effort led to the development of the kinetic theory of gases, where macroscopic properties of gases, such as pressure, temperature, and volume, are explained by considering their molecular composition and motion.



John Dalton's A New System of Chemical Philosophy: Various atoms and molecules as depicted in John Dalton's A New System of Chemical Philosophy (1808).

Key Points

- The kinetic theory posits that pressure is due to collisions between molecules moving at different velocities through Brownian motion.
- The temperature of an ideal monatomic gas is a measure of the average kinetic energy of its atoms. In kinetic theory, it is related to its average kinetic energy per degree of freedom E_k via the equation: $\bar{E}_k = \frac{1}{2} kT$.
- The kinetic theory of gases uses the model of the ideal gas to relate temperature to the average translational kinetic energy of the molecules in a container of gas in thermodynamic equilibrium.
- John Dalton was the first to propose a scientific theory of atoms. He based his study on two laws: the law of conservation of mass and the law of definite proportions.
- Dalton proposed that each chemical element is composed of atoms of a single, unique type, and though they cannot be altered or destroyed by chemical means, they can combine to form more complex structures.
- Kinetic theory of gases explain macroscopic properties of gases, such as pressure, temperature, and volume, by considering their molecular composition and motion.
- While Dalton's idea of matter being composed of various atoms was correct, he was wrong about some of their properties. Atoms can be broken down into smaller parts. Atoms of the same element can have slightly different masses and behave differently. See isotopes and ions for examples.

Key Terms

- **ideal gas**: A hypothetical gas whose molecules exhibit no interaction and undergo elastic collision with each other and with the walls of the container.
- **degree of freedom**: Any of the coordinates, a minimum number of which are needed to specify the motion of a mechanical system.
- **Brownian motion**: Random motion of particles suspended in a fluid, arising from those particles being struck by individual molecules of the fluid.
- **atom**: The smallest possible amount of matter which still retains its identity as a chemical element, now known to consist of a nucleus surrounded by electrons.
- **kinetic theory of gases**: The kinetic theory of gases describes a gas as a large number of small particles (atoms or molecules), all of which are in constant, random motion.
- **chemical reaction**: A process, involving the breaking or making of interatomic bonds, in which one or more substances are changed into others.

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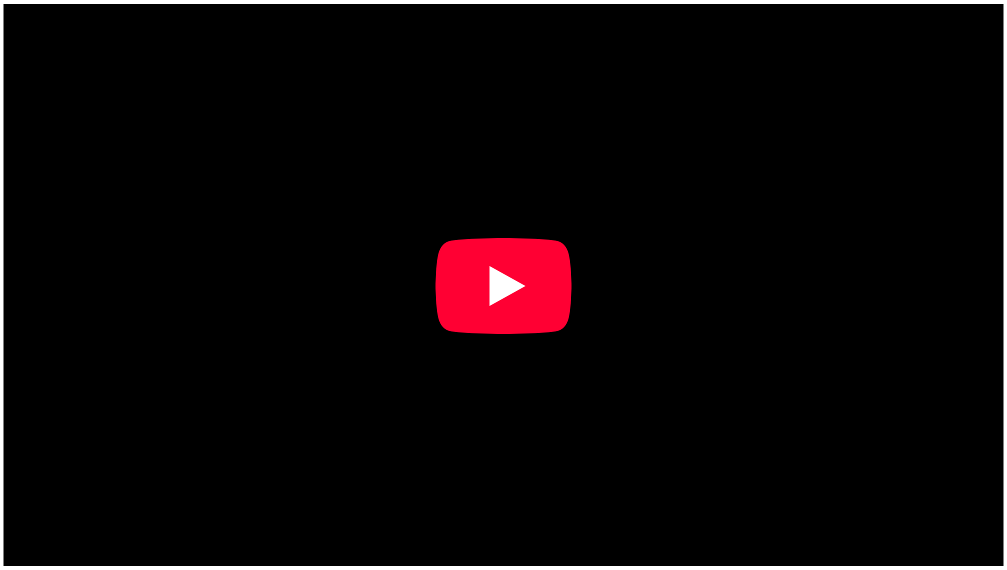
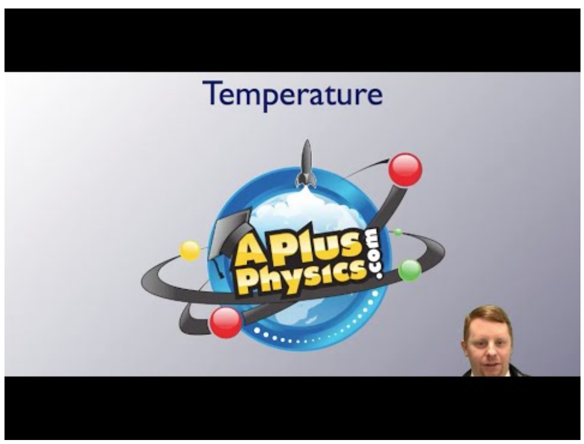
12.2: Temperature and Temperature Scales

learning objectives

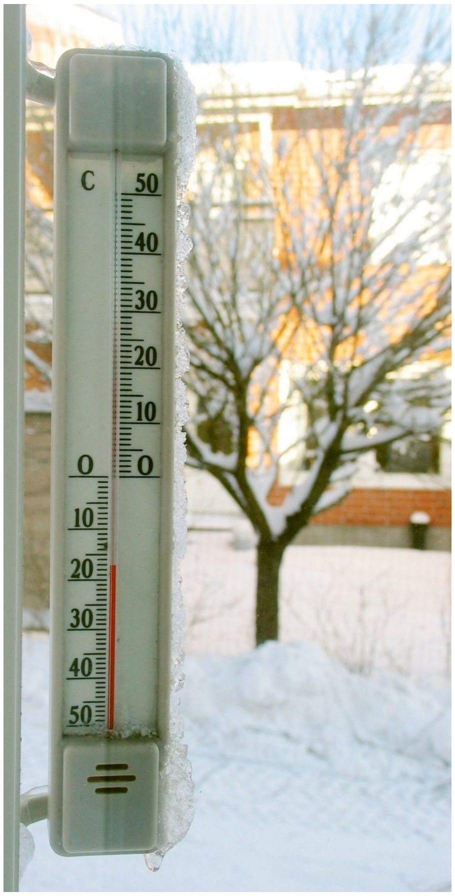
- Explain how the Celsius scale is defined

Celsius, also known as centigrade, is a scale to measure temperature. The unit of measurement is the degree Celsius (°C). It is one of the most commonly used temperature units in the world. The unit system is named after the Swedish astronomer Anders Celsius (1701-1744), who developed a similar temperature scale.



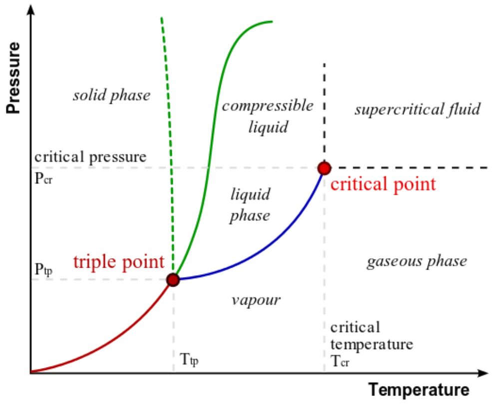


Temperature Scales: A brief introduction to temperature and temperature scales for students studying thermal physics or thermodynamics.



Thermometer: A thermometer calibrated in degrees Celsius

From 1743 until 1954, 0°C was defined as the freezing point of water, and 100°C was defined as the boiling point of water, both at a pressure of one standard atmosphere, with mercury as the working material. Although these defining correlations are commonly taught in schools today, by international agreement the unit “degree Celsius” and the Celsius scale are currently defined by two different temperatures: absolute zero and the triple point of Vienna Standard Mean Ocean Water (VSMOW; specially purified water). This definition also precisely relates the Celsius scale to the Kelvin scale, which defines the SI base unit of thermodynamic temperature and which uses the symbol K. Absolute zero, the lowest temperature possible (the temperature at which matter reaches minimum entropy), is defined as being precisely 0K and -273.15°C. The temperature of the triple point of water is defined as precisely 273.16K and 0.01°C. Based on this, the relationship between degree Celsius and Kelvin is as follows:



Phase Diagram of Water: In this typical phase diagram of water, the green lines mark the freezing point, and the blue line marks the boiling point, showing how they vary with pressure. The dotted line illustrates the anomalous behavior of water. Note that water changes states based on the pressure and temperature.

[Math Processing Error]

Besides expressing specific temperatures along its scale (e.g., “Gallium melts at 29.7646°C” and “The temperature outside is 23 degrees Celsius”), the degree Celsius is also suitable for expressing temperature intervals — differences between temperatures, or their uncertainties (e.g. “The output of the heat exchanger is hotter by 40 degrees Celsius” and “Our standard uncertainty is ±3°C”). Because of this dual usage, one must not rely upon the unit name or its symbol to denote that a quantity is a temperature interval; it must be clear through context or explicit statement that the quantity is an interval.

Farenheit Scale

In the Fahrenheit scale, the freezing of water is defined at 32 degrees, while the boiling point of water is defined to be 212 degrees.

learning objectives

- Explain how the Fahrenheit scale is defined and convert between it and Celsius

The Fahrenheit scale measures temperature. It is based on a scale proposed in 1724 by physicist Daniel Gabriel Fahrenheit (1686-1736). The unit of this scale is the degree Fahrenheit (°F). On this scale, water’s freezing point is defined to be 32 degrees, while water’s boiling point is defined to be 212 degrees.

Historically, the zero point of the Fahrenheit scale was determined by evaluating a thermometer placed in brine. Fahrenheit himself used a mixture of ice, water, and ammonium chloride (a salt) at a 1:1:1 ratio. This is a frigorific mixture, which stabilizes its temperature automatically; the stable temperature of this mixture was defined as 0 °F (-17.78 °C). The second determining point, 32 degrees, was a mixture of just ice and water at a 1:1 ratio. The third determining point, 96 degrees, was approximately the temperature of the human body, then called “blood-heat. ”

The Fahrenheit system puts the boiling and freezing points of water exactly 180 degrees apart. Therefore, a degree on the Fahrenheit scale is 1/180 of the interval between the freezing point and the boiling point. On the Celsius scale, the freezing and boiling points of water are 100 degrees apart. A temperature interval of 1 °F is equal to an interval of 5/9 degrees Celsius (°C). To convert °F to °C, you can use the following formula:

[Math Processing Error]

The Fahrenheit and Celsius scales intersect at -40° (-40 °F and -40 °C represent the same temperature). Absolute zero (-273.15 °C, or 0K) is defined as -459.67 °F.

The Fahrenheit scale was replaced by the Celsius scale in most countries in the mid- to late-20th century, though Canada retains it as a supplementary scale that can be used alongside the Celsius scale. The Fahrenheit scale remains the official scale of the United States, the Cayman Islands, Palau, the Bahamas, and Belize.

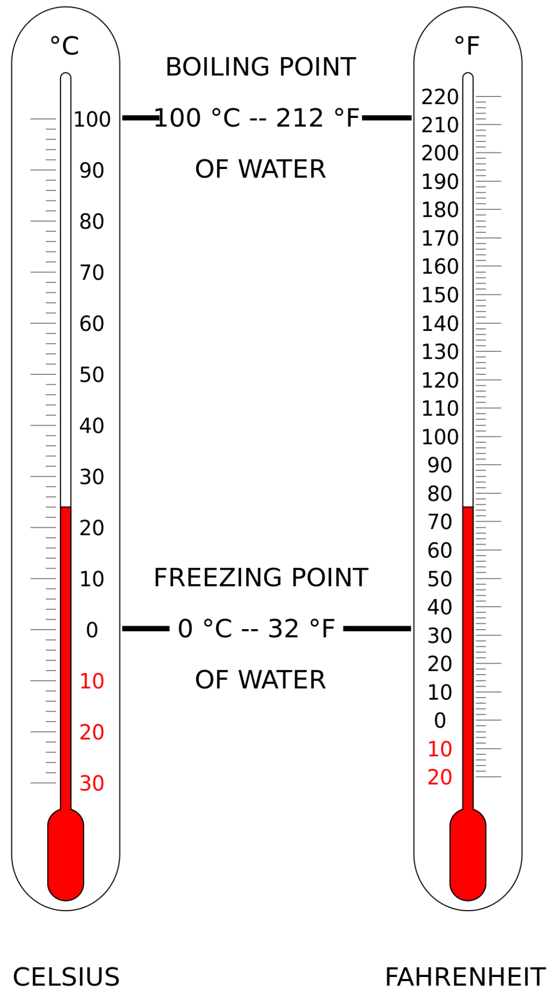
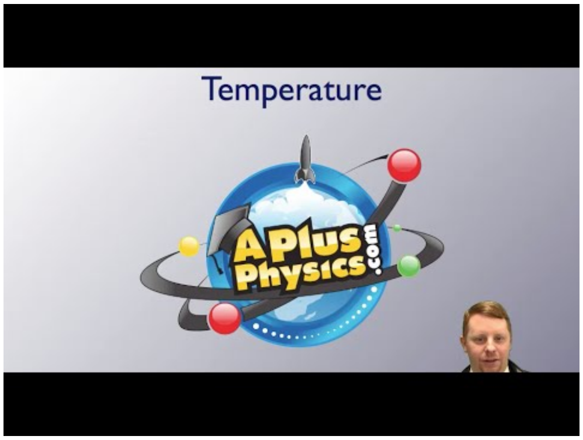


Fig 2: Comparison of Celsius vs Fahrenheit scales.



Temperature Scales: A brief introduction to temperature and temperature scales for students studying thermal physics or thermodynamics.

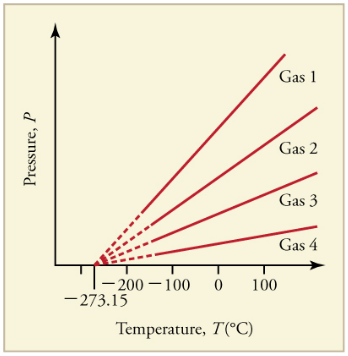
Absolute Zero

Absolute zero is the coldest possible temperature; formally, it is the temperature at which entropy reaches its minimum value.

learning objectives

- Explain why absolute zero is a natural choice as the null point for a temperature unit system

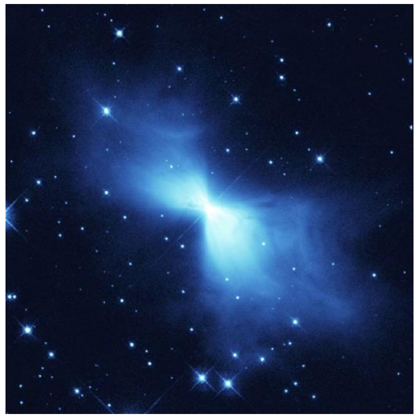
Absolute zerois the coldest possible temperature. Formally, it is the temperature at which entropy reaches its minimum value. More simply put, absolute zero refers to a state in which all the energy of a system is extracted (by definition, the lowest energy state the system can have). Absolute zero is universal in the sense that all matteris in ground state at this temperature. Therefore, it is a natural choice as the null point for a temperature unit system.



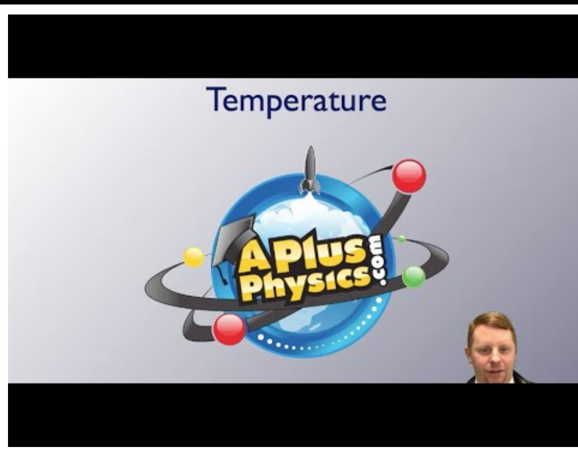
Graph of Pressure Versus Temperature: Graph of pressure versus temperature for various gases kept at a constant volume. Note that all of the graphs extrapolate to zero pressure at the same temperature

To be precise, a system at absolute zero still possesses quantum mechanical zero-point energy, the energy of its ground state. The uncertainty principle states that the position of a particle cannot be determined with absolute precision; therefore a particle is in motion even if it is at absolute zero, and a ground state still carries a minimal amount of kinetic energy. However, in the interpretation of classical thermodynamics, kinetic energy can be zero, and the thermal energy of matter vanishes.

The zero point of a thermodynamic temperature scale, such as the Kelvin scale, is set at absolute zero. By international agreement, absolute zero is defined as 0K on the Kelvin scale and as -273.15° on the Celsius scale (equivalent to -459.67° on the Fahrenheit scale). Scientists have brought systems to temperatures very close to absolute zero, at which point matter exhibits quantum effects such as superconductivity and superfluidity. The lowest temperature that has been achieved in the laboratory is in the 100 pK range, where pK (pico-Kelvin) is equivalent to 10^{-12} K. The lowest natural temperature ever recorded is approximately 1K, observed in the rapid expansion of gases leaving the Boomerang Nebula, shown below.



Boomerang Nebula: The rapid expansion of gases resulting in the Boomerang Nebula causes the lowest observed temperature outside a laboratory.



Temperature Scales: A brief introduction to temperature and temperature scales for students studying thermal physics or thermodynamics.

Kelvin Scale

The kelvin is a unit of measurement for temperature; the null point of the Kelvin scale is absolute zero, the lowest possible temperature.

learning objectives

- Explain how the Kelvin scale is defined

The kelvin is a unit of measurement for temperature. It is one of the seven base units in the International System of Units (SI) and is assigned the unit symbol K. The Kelvin scale is an absolute, thermodynamic temperature scale using absolute zero as its null point. In the classical description of thermodynamics, absolute zero is the temperature at which all thermal motion ceases.

The choice of absolute zero as null point for the Kelvin scale is logical. Different types of matter boil or freeze at different temperatures, but at 0K (absolute zero), *all* thermal motions of *any* matter are maximally suppressed. The Kelvin scale is used extensively in scientific work because a number of physical quantities, such as the volume of an ideal gas, are directly related to absolute temperature.

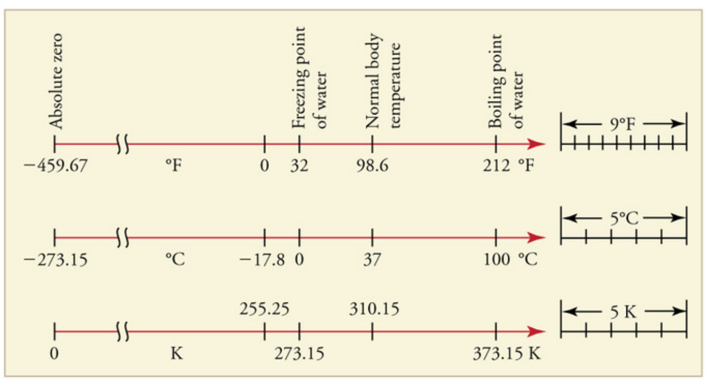
The Kelvin scale is named after Glasgow University engineer and physicist William Thomson, 1st Baron Kelvin (1824-1907), who wrote of the need for an “absolute thermometric scale. ” Unlike the degree Fahrenheit and the degree Celsius, the kelvin is not referred to or typeset as a degree. The kelvin is the primary unit of measurement in the physical sciences, but it is often used in conjunction with the degree Celsius, which has the same magnitude. The kelvin is defined as the fraction 1/273.16 of the thermodynamic temperature of the triple point of water (exactly 0.01°C, or 32.018°F). To convert kelvin to degrees Celsius, we use the following formula:

[Math Processing Error]

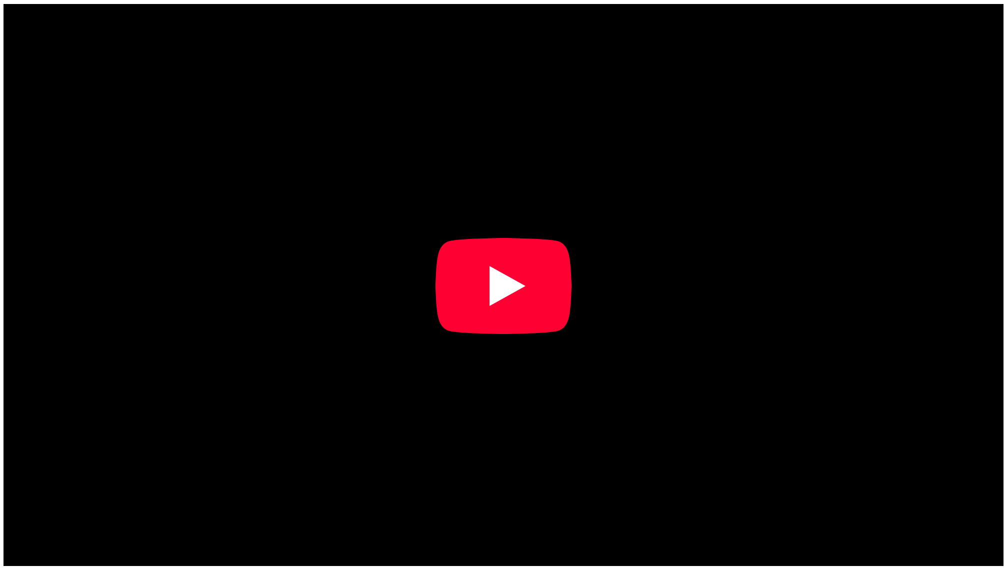
Subtracting 273.16K from the temperature of the triple point of water, 0.01°C, makes absolute zero (0K) equivalent to -273.15°C and -460°F.

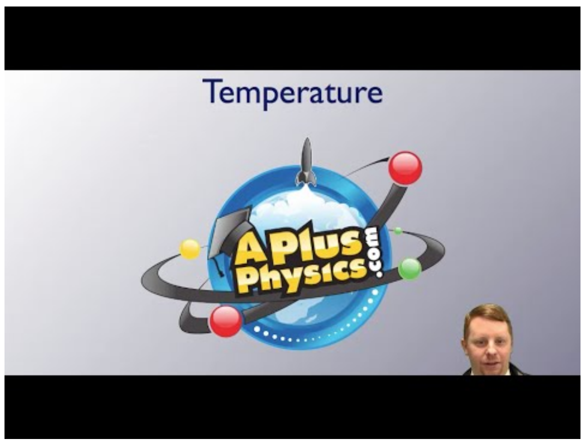
where n_1 = # of scores in group 1
n_2 = # of scores in group 2
R_1 = sum of ranks for group 1
R_2 = sum of ranks for group 2

Calculating U



Relationships Between the Temperature Scales: Relationships between the Fahrenheit, Celsius, and Kelvin temperature scales, rounded to the nearest degree. The relative sizes of the scales are also shown





Temperature Scales: A brief introduction to temperature and temperature scales for students studying thermal physics or thermodynamics.

Key Points

- The degree Celsius (°C) can refer to a specific temperature on the Celsius scale as well as a unit to indicate a temperature interval, a difference between two temperatures or an uncertainty.
- The Celsius scale is currently defined by two different temperatures: absolute zero and the triple point of Vienna Standard Mean Ocean Water (VSMOW; specially purified water).
- Based on this, the relationship between Celsius and Kelvin is as follows: *[Math Processing Error]*.
- The Fahrenheit system puts the boiling and freezing points of water exactly 180 degrees apart. Therefore, a degree on the Fahrenheit scale is 1⁄180 of the interval between the freezing point and the boiling point.
- To convert °F to °C, you can use the following formula: *[Math Processing Error]*. The Fahrenheit and Celsius scales intersect at −40°.
- The Fahrenheit scale was replaced by the Celsius scale in most countries during the mid to late 20th century. Fahrenheit remains the official scale of the United States, Cayman Islands, Palau, Bahamas and Belize.
- Absolute zero is universal in the sense that all matter is in ground state at this temperature. Therefore, it is a natural choice as the null point for a temperature unit system.
- K system at absolute zero still possesses quantum mechanical zero-point energy, the energy of its ground state. However, in the interpretation of classical thermodynamics, kinetic energy can be zero, and the thermal energy of matter vanishes.
- The lowest temperature that has been achieved in the laboratory is in the 100 pK range, where pK (pico- Kelvin) is equivalent to 10-12 K. The lowest natural temperature ever recorded is approximately 1K, observed in the rapid expansion of gases leaving the Boomerang Nebula.

- 0K (absolute zero) is universal because all thermal motions of all matter are maximally suppressed at this temperature. Absolute zero is therefore the natural choice as the null point of the Kelvin scale.
- The Kelvin scale is used extensively in scientific work because a number of physical quantities, such as the volume of an ideal gas, are directly related to absolute temperature.
- To convert kelvin to degree Celsius, we use the following formula: *[Math Processing Error]*.

Key Terms

- **kelvin**: in the International System of Units, the base unit of thermodynamic temperature; 1/273.16 of the thermodynamic temperature of the triple point of water; symbolized as K
- **absolute zero**: The coldest possible temperature: zero on the Kelvin scale and approximately -273.15°C and -459.67°F. The total absence of heat; the temperature at which motion of all molecules would cease.
- **standard atmosphere**: an international reference pressure defined as 101.325 kPa and formerly used as a unit of pressure
- **brine**: a solution of salt (usually sodium chloride) in water
- **frigorific mixture**: A mixture of two or more chemicals that reaches an equilibrium temperature independent of the temperature of any of its constituent chemicals. The temperature is also relatively independent of the quantities of mixtures as long as a significant amount of each original chemical is present in its pure form
- **entropy**: A measure of how evenly energy (or some analogous property) is distributed in a system.
- **thermodynamics**: a branch of natural science concerned with heat and its relation to energy and work
- **absolute zero**: The coldest possible temperature: zero on the Kelvin scale and approximately -273.15°C and -459.67°F. The total absence of heat; the temperature at which motion of all molecules would cease.
- **Triple point**: The unique temperature and pressure at which the solid, liquid and gas phases of a substance are all in equilibrium.
- **ideal gas**: A hypothetical gas whose molecules exhibit no interaction and undergo elastic collision with each other and with the walls of the container.

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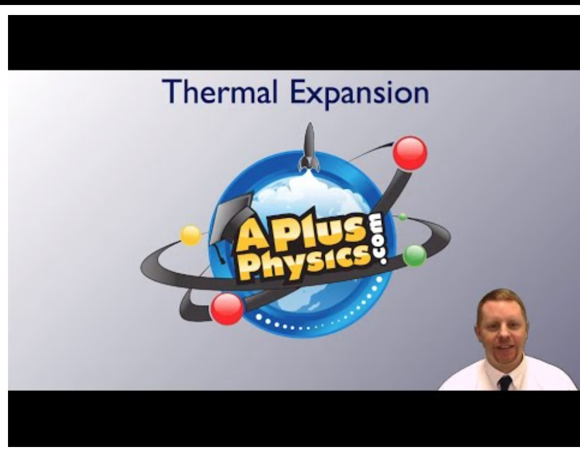
12.3: Thermal Expansion

learning objectives

- Describe volume changes that take place in response to a temperature change

Thermal expansion is the tendency of matter to change in volume in response to a change in temperature. (An example of this is the buckling of railroad track, as seen in.) Atoms and molecules in a solid, for instance, constantly oscillate around its equilibrium point. This kind of excitation is called thermal motion. When a substance is heated, its constituent particles begin moving more, thus maintaining a greater average separation with their neighboring particles. The degree of expansion divided by the change in temperature is called the material’s coefficient of thermal expansion; it generally varies with temperature.

Fig 1: Thermal expansion of long continuous sections of rail tracks is the driving force for rail buckling. This phenomenon resulted in 190 train derailments during 1998–2002 in the US alone.



Thermal Expansion: A brief introduction to thermal expansion for students.

Expansion, Not Contraction

Why does matter usually expand when heated? The answer can be found in the shape of the typical particle-particle potential in matter. Particles in solids and liquids constantly feel the presence of other neighboring particles. This interaction can be represented mathematically as a potential curve. Fig 2 illustrates how this inter-particle potential usually takes an asymmetric form rather than a symmetric form, as a function of particle-particle distance. Note that the potential curve is steeper for shorter distance. In the diagram, (b) shows that as the substance is heated, the equilibrium (or average) particle-particle distance increases. Materials which contract or maintain their shape with increasing temperature are rare. This effect is limited in size, and only occurs within limited temperature ranges.

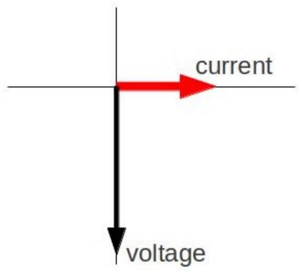


Fig 2: Typical inter-particle potential in condensed matter (such as solid or liquid).

Linear Expansion

To a first approximation, the change in length measurements of an object (*linear*dimension as opposed to, for example, volumetric dimension) due to thermal expansion is related to temperature change by a *linear expansion coefficient*. It is the fractional change in length per degree of temperature change. Assuming negligible effect of pressure, we may write:

$$\alpha_{L} = \frac{1}{L}\frac{dL}{dT}, \tag{12.3.1}$$

where L is a particular length measurement and $\frac{dL}{dT}$ is the rate of change of that linear dimension per unit change in temperature. From the definition of the expansion coefficient, the change in the linear dimension ΔL over a temperature range ΔT can be estimated to be:

$$\frac{\Delta L}{L} = \alpha_{L}\Delta T. \tag{12.3.2}$$

This equation works well as long as the linear-expansion coefficient does not change much over the change in temperature. If it does, the equation must be integrated.

Area Expansion

Objects expand in all dimensions. That is, their areas and volumes, as well as their lengths, increase with temperature.

learning objectives

- Express the area thermal expansion coefficient in the form of an equation

We learned about the linear expansion (in one dimension) in the previous Atom. Objects expand in all dimensions, and we can extend the thermal expansion for 1D to two (or three) dimensions. That is, their areas and volumes, as well as their lengths, increase with temperature.

Quiz

Before we look into details, here is an interesting question. Imagine that we have a rectangular sheet of metal with a circular hole in the middle. If the metal is heated, we can guess that the the piece, in general, will get larger due to thermal expansion. Now, what is going to happen with the circular hole in the middle? Is the hole going to be larger or smaller? Answer: Imagine that we have a similar metal sheet but without a hole. Draw an imaginary circular line representing the circular hole in our quiz. How does this imaginary circle change as the metal is heated? Yes. It will get bigger. Therefore, you can guess that the hole in our quiz will get larger.

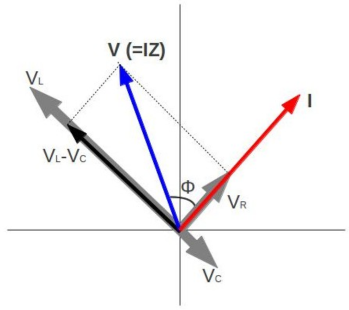


Fig 1: In general, objects expand in all directions as temperature increases. In these drawings, the original boundaries of the objects are shown with solid lines, and the expanded boundaries with dashed lines. (a) Area increases because both length and width increase. The area of a circular plug also increases. (b) If the plug is removed, the hole it leaves becomes larger with increasing temperature, just as if the expanding plug were still in place.

Area thermal expansion coefficient

The area thermal expansion coefficient relates the change in a material's area dimensions to a change in temperature. It is the fractional change in area per degree of temperature change. Ignoring pressure, we may write: $\alpha_A = \frac{1}{A}\frac{dA}{dT}$, where is some area of interest on the object, and $\frac{dA}{dT}$ is the rate of change of that area per unit change in temperature. The change in the linear dimension can be estimated as: $\frac{\Delta A}{A} = \alpha_A \Delta T$. This equation works well as long as the linear expansion coefficient does not change much over the change in temperature ΔT. If it does, the equation must be integrated.

Relationship to linear thermal expansion coefficient

For isotropic materials, and for small expansions, the linear thermal expansion coefficient is one half of the area coefficient. To derive the relationship, let's take a square of steel that has sides of length *L*. The original area will be $A = L^2$,and the new area, after a temperature increase, will be

$$A + \Delta A = (L + \Delta L)^2 \tag{12.3.3}$$
$$= L^2 + 2L\Delta L + (\Delta L)^2 \tag{12.3.4}$$
$$\approx L^2 + 2L\Delta L \tag{12.3.5}$$
$$= A + 2A\frac{\Delta L}{L} \tag{12.3.6}$$

The approximation holds for a sufficiently small ΔL campared to L. Since $\frac{\Delta A}{A} = 2\frac{\Delta L}{L}$ from the equation above (and from the definitions of the thermal coefficients), we get $\alpha_A = 2_{\alpha L}$.

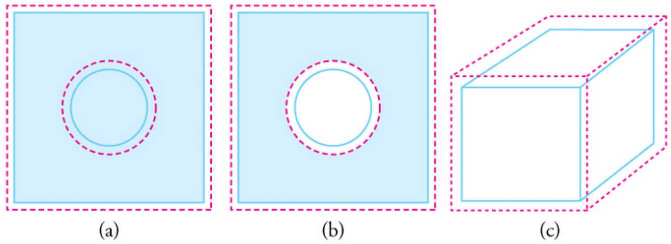
Volume Expansion

Substances expand or contract when their temperature changes, with expansion or contraction occurring in all directions.

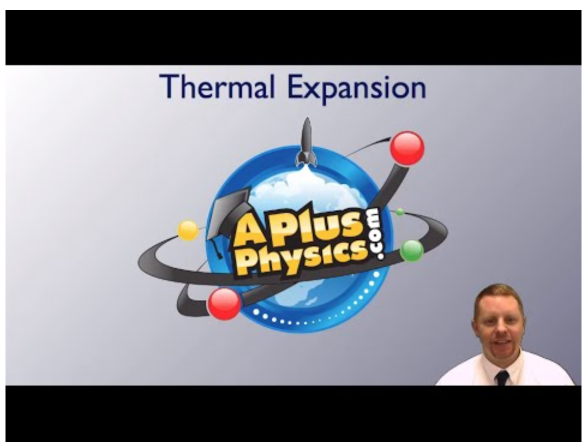
learning objectives

- Compare the effects of the pressure on the expansion of gaseous and solid materials

The volumetric thermal expansion coefficient is the most basic thermal expansion coefficient. illustrates that, in general, substances expand or contract when their temperature changes, with expansion or contraction occurring in all directions. Such substances that expand in all directions are called isotropic. For isotropic materials, the area and linear coefficients may be calculated from the volumetric coefficient (discussed below).



Volumetric Expansion: In general, objects expand in all directions as temperature increases. In these drawings, the original boundaries of the objects are shown with solid lines, and the expanded boundaries with dashed lines. (a) Area increases because both length and width increase. The area of a circular plug also increases. (b) If the plug is removed, the hole it leaves becomes larger with increasing temperature, just as if the expanding plug were still in place. (c) Volume also increases, because all three dimensions increase.



Thermal Expansion – Volume Expansion: A brief introduction to thermal expansion for students.

Mathematical definitions of these coefficients are defined below for solids, liquids, and gasses:

$$\alpha_{\mathrm{V}} = \frac{1}{\mathrm{V}}\left(\frac{\partial \mathrm{V}}{\partial \mathrm{T}}\right)\mathrm{p}. \tag{12.3.7}$$

The subscript *p* indicates that the pressure is held constant during the expansion. In the case of a gas, the fact that the pressure is held constant is important, as the volume of a gas will vary appreciably with pressure as well as with temperature.

For a solid, we can ignore the effects of pressure on the material, thus the volumetric thermal expansion coefficient can be written:

$$\alpha_V = \frac{1}{V}\frac{dV}{dT}, \tag{12.3.8}$$

where V is the volume of the material, and is dV/dT the rate of change of that volume with temperature. This means that the volume of a material changes by some fixed fractional amount. For example, a steel block with a volume of 1 cubic meter might expand to 1.002 cubic meters when the temperature is raised by 50 °C. This is an expansion of 0.2%. The volumetric expansion coefficient would be 0.2% for 50 °C, or 0.004% per degree C.

Relationship to Linear Thermal Expansion Coefficient

For isotropic material, and for small expansions, the linear thermal expansion coefficient is one third the volumetric coefficient. To derive the relationship, let's take a cube of steel that has sides of length *L*. The original volume will be $V = L^3$,and the new volume, after a temperature increase, will be:

$$V + \Delta V = (L + \Delta L)^3 \tag{12.3.9}$$
$$= L^3 + 3L^2\Delta L + 3L(\Delta L)^2 + (\Delta L)^3 \tag{12.3.10}$$
$$\approx L^3 + 3L^2\Delta L \tag{12.3.11}$$
$$= V + 3V\frac{\Delta L}{L}. \tag{12.3.12}$$

The approximation holds for a sufficiently small ΔL compared to L. Since:

$$\frac{\Delta V}{V} = 3\frac{\Delta L}{L} \tag{12.3.13}$$

(and from the definitions of the thermal coefficients), we arrive at:

$$\alpha_V = 3\alpha_L \tag{12.3.14}$$

Special Properties of Water

Objects will expand with increasing temperature, but water is the most important exception to the general rule.

learning objectives

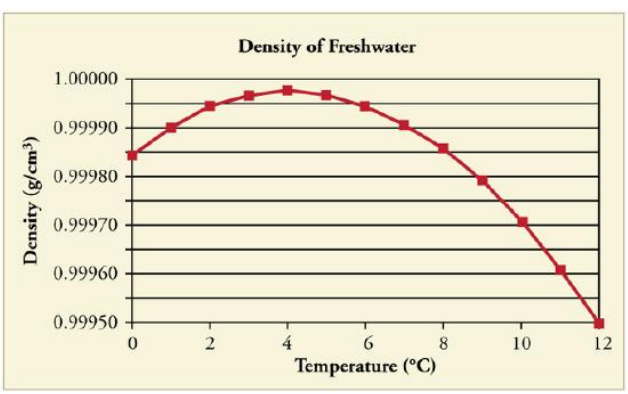
- Describe thermal expansion properties of water

Special Properties of Water

In general, objects will expand with increasing temperature. However, a number of materials contract on heating within certain temperature ranges; this is usually called negative thermal expansion, rather than "thermal contraction. " Water is the most important exception to the general rule. Water has this unique characteristic because of the particular nature of the hydrogen bond in H_2O.

Density of Water as Temperature Changes

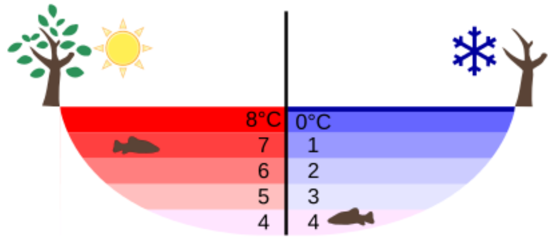
At temperatures greater than 4ºC (40ºF) water expands with increasing temperature (its density decreases). However, it expands with decreasing temperature when it is between +4ºC and 0ºC (40ºF to 32ºF). Water is densest at +4ºC.



Water Density vs. Temperature: The density of water as a function of temperature. Note that the thermal expansion is actually very small. The maximum density at +4ºC is only 0.0075% greater than the density at 2ºC, and 0.012% greater than that at 0ºC.

Perhaps the most striking effect of this phenomenon is the freezing of water in a pond. When water near the surface cools down to 4ºC it is denser than the remaining water and thus will sink to the bottom. This "turnover" results in a layer of warmer water near the surface, which is then cooled. Eventually the pond has a uniform temperature of 4ºC. If the temperature in the surface layer drops below 4ºC, the water is less dense than the water below, and thus stays near the top.

As a result, the pond surface can completely freeze over, while the bottom may remain at 4ºC. The ice on top of liquid water provides an insulating layer from winter's harsh exterior air temperatures. Fish and other aquatic life can survive in 4ºC water beneath ice, due to this unusual characteristic of water. It also produces circulation of water in the pond that is necessary for a healthy ecosystem of the body of water.



Temperature in a Lake: Temperature distribution in a lake on warm and cold days in winter

Ice Versus Water

The solid form of most substances is denser than the liquid phase; thus, a block of most solids will sink in the liquid. However, a block of ice floats in liquid water because ice is less dense. Upon freezing, the density of water decreases by about 9%.

Key Points

- Inter-particle potential usually takes an asymmetric form, rather than a symmetric form as a function of particle-particle distance. This is why matters expands and contracts as temperature changes.
- The change in length measurements of an object due to thermal expansion is related to temperature change by a "linear expansion coefficient", which is given as $\alpha_{\mathrm{L}}=\frac{1}{\mathrm{L}} \frac{\mathrm{dL}}{\mathrm{dT}}$.
- The linear expansion coefficient is as an approximation over a narrow temperature interval only.
- The area thermal expansion coefficient relates the change in a material's area dimensions to a change in temperature. It is defined as $\alpha_{\mathrm{A}}=\frac{1}{\mathrm{A}} \frac{\mathrm{dA}}{\mathrm{dT}}$.
- The relationship between the area and linear thermal expansion coefficient is given as the following: $\alpha_{\mathrm{A}}=2 \alpha_{\mathrm{L}}$.
- Just like the linear expansion coefficient, the area thermal expansion coefficient works as an approximation over a narrow temperature interval only.
- Substances that expand at the same rate in every direction are called isotropic.
- In the case of a gas, expansion depends on how the pressure changed in the process because the volume of a gas will vary appreciably with pressure as well as temperature.
- For a solid, we can ignore the effects of pressure on the material, and the volumetric thermal expansion coefficient can be written as $\alpha_{\mathrm{V}}=\frac{1}{\mathrm{V}} \frac{\mathrm{dV}}{\mathrm{dT}}$. For isotropic materials, $\alpha_{\mathrm{V}}=3 \alpha_{\mathrm{L}}$.
- Water expands with increasing temperature (its density decreases) when it is at temperatures greater than 4ºC (40ºF). However, it expands with decreasing temperature when it is between +4ºC and 0ºC (40ºF to 32ºF). Water is densest at +4ºC.
- Due to the peculiar thermal expansion property of water, a pond surface can completely freeze over, while the bottom may remain at 4ºC. Fish and other aquatic life can survive in 4ºC water beneath ice, due to this unusual characteristic of water.
- The solid form of most substances is denser than the liquid phase; thus, a block of most solids will sink in the liquid. However, a block of ice floats in liquid water because ice is less dense.

Key Terms

- **potential**: A curve describing the situation where the difference in the potential energies of an object in two different positions depends only on those positions.
- **linear thermal expansion coefficient**: The fractional change in length per degree of temperature change.
- **isotropic**: Having properties that are identical in all directions; exhibiting isotropy.
- **hydrogen bond**: A weak bond in which a hydrogen atom in one molecule is attracted to an electronegative atom (usually nitrogen or oxygen) in the same or different molecule.

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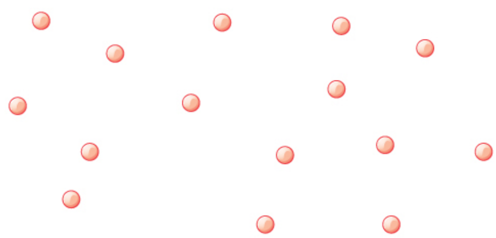
12.4: Ideal Gas Law

learning objectives

- Describe how ideal gas law was derived.

The ideal gas law is the equation of state of a hypothetical ideal gas (an illustration is offered in). In an ideal gas, there is no molecule-molecule interaction, and only elastic collisions are allowed. It is a good approximation to the behavior of many gases under many conditions, although it has several limitations. It was first stated by Émile Clapeyron in 1834 as a combination of Boyle's law and Charles' law.



Atoms and Modules in a Gas: Atoms and molecules in a gas are typically widely separated, as shown. Because the forces between them are quite weak at these distances, they are often described by the ideal gas law.

Empirical Derivation

Boyle's law states that pressure P and volume V of a given mass of confined gas are inversely proportional:

$$P \propto \frac{1}{V}, \tag{12.4.1}$$

while Charles' law states that volume of a gas is proportional to the absolute temperature T of the gas at constant pressure

$$V \propto T. \tag{12.4.2}$$

By combining the two laws, we get

$$\frac{PV}{T} = C, \tag{12.4.3}$$

where C is a constant which is directly proportional to the amount of gas, n (representing the number of moles).

The proportionality factor is the universal gas constant, R, i.e. $C = nR$.

Hence the ideal gas law

$$PV = nRT \tag{12.4.4}$$

Equivalently, it can be written as $PV = NkT$,

where k is Boltzmann's constant and N is the number of molecules.

(Since N = nN_A, you can see that $R = N_{Ak}$, where N_A is Avogadro's number.)

Note that the empirical derivation does not consider microscopic details. However, the equation can be derived from first principles in the classical thermodynamics (which goes beyond the scope of this Atom).

Microscopic version

We have seen in the Atom on "Origin of Pressure" that

$$P = \frac{Nm\overline{v^2}}{3V}, \tag{12.4.5}$$

where P is the pressure, N is the number of molecules, m is the mass of the molecule, v is the speed of molecules, and V is the volume of the gas. Therefore, we derive a microscopic version of the ideal gas law

$$PV = \frac{1}{3}Nm\bar{v}^2 \tag{12.4.6}$$

Isotherms

An isothermal process is a change of a system in which the temperature remains constant: $\Delta T = 0$.

learning objectives

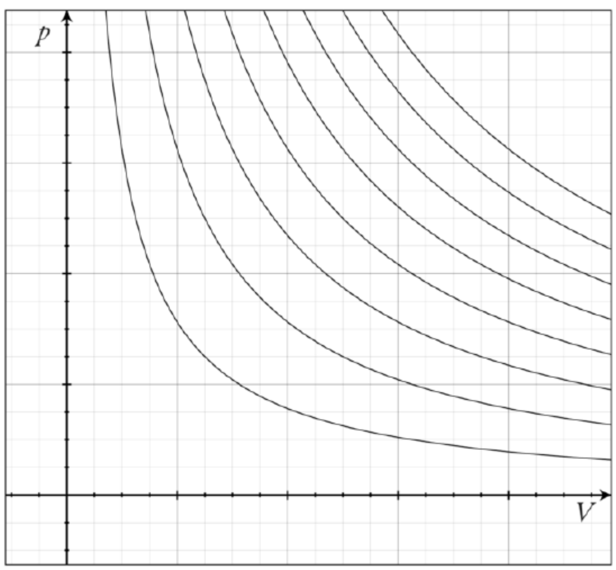
- Identify conditions at which isothermal processes can occur.

An isothermal process is a change of a system in which the temperature remains constant: ΔT = 0. Typically this occurs when a system is in contact with an outside thermal reservoir (heat bath), and the change occurs slowly enough to allow the system to adjust continually to the temperature of the reservoir through heat exchange. In contrast, an adiabatic process occurs when a system exchanges no heat with its surroundings (Q = 0). In other words, in an isothermal process, the value ΔT = 0 but Q ≠ 0, while in an adiabatic process, ΔT ≠ 0 but Q = 0.

For an ideal gas, the product PV (P: pressure, V: volume) is a constant if the gas is kept at isothermal conditions (Boyle's law). According to the ideal gas law, the value of the constant is NkT, where N is the number of molecules of gas and k is Boltzmann's constant.

This means that $p = \frac{NkT}{V} = \frac{Constant}{V}$ holds.

The family of curves generated by this equation is shown in the graph presented in. Each curve is called an isotherm. Such graphs are termed indicator diagrams—first used by James Watt and others to monitor the efficiency of engines. The temperature corresponding to each curve in the figure increases from the lower left to the upper right.



Isotherms of an Ideal Gas: Several isotherms of an ideal gas on a PV diagram.

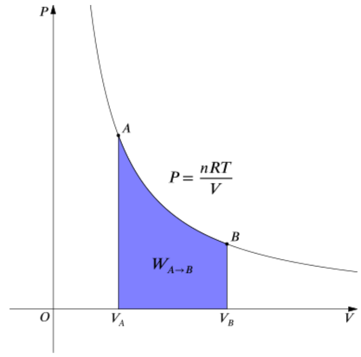
Calculation of Work

In thermodynamics, the work involved when a gas changes from state A to state B is simply:

$$W_{A\rightarrow B} = \int_{V_A}^{V_B} PdV. \tag{12.4.7}$$

(This equation is derived in our Atom on "Constant Pressure" under kinetic theory. Note that $P = \frac{F}{A}$. This definition is consistent with our definition of work being force times distance.)

For an isothermal, reversible process, this integral equals the area under the relevant pressure-volume isotherm, and is indicated in blue in for an ideal gas. Again, $P = \frac{nRT}{V}$ applies and with T being constant (as this is an isothermal process), we have:



Work Done by Gas During Expansion: The blue area represents "work" done by the gas during expansion for this isothermal change.

$$W_{A\to B} = \int_{V_A}^{V_B} pdV = \int_{V_A}^{V_B} \frac{NkT}{V} dV \tag{12.4.8}$$

$$= NkT \ln \frac{V_B}{V_A}. \tag{12.4.9}$$

By convention, work is defined as the work the system does on its environment. If, for example, the system expands by a piston moving in the direction of force applied by the internal pressure of a gas, then the work is counted as positive. As this work is done by using internal energy of the system, the result is that the internal energy decreases. Conversely, if the environment does work on the system so that its internal energy increases, the work is counted as negative (for details on internal energy, check our Atom on "Internal Energy of an Ideal Gas").

Constant Pressure

Isobaric processis a thermodynamic process in which the pressure stays constant (at constant pressure, work done by a gas is $P\Delta V$).

learning objectives

- Describe behavior of monatomic gas during isobaric processes.

Under a certain constraint (e.g., pressure), gases can expand or contract; depending on the type of constraint, the final state of the gas may change. For example, an ideal gas that expands while its temperature is kept constant (called isothermal process) will exist in a different state than a gas that expands while pressure stays constant (called isobaric process). This Atom addresses isobaric process and correlated terms. We will discuss isothermal process in a subsequent Atom.

Isobaric Process

An isobaric process is a thermodynamic process in which pressure stays constant: $\Delta P = 0$. For an ideal gas, this means the volume of a gas is proportional to its temperature (historically, this is called Charles' law). Let's consider a case in which a gas does work on a piston at constant pressure P, referring to Fig 1 as illustration. Since the pressure is constant, the force exerted is constant and the work done is given as W=Fd, where F (=PA) is the force on the piston applied by the pressure and d is the displacement of the piston. Therefore, the work done by the gas (W) is:

$$W = PAd \tag{12.4.10}$$

Because the change in volume of a cylinder is its cross-sectional area A times the displacement d, we see that Ad=ΔV, the change in volume. Thus,

$$W = P\Delta V \tag{12.4.11}$$

(as seen in Fig 2—isobaric process). Note: if ΔV is positive, then W is positive, meaning that work is done by the gas on the outside world. Using the ideal gas law $PV = NkT(P = \text{const})$,

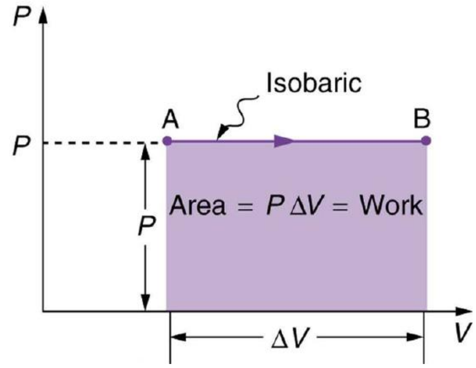


Fig 2: A graph of pressure versus volume for a constant-pressure, or isobaric process. The area under the curve equals the work done by the gas, since W=PΔV.

$$W = Nk\Delta T \tag{12.4.12}$$

(Eq. 1) for an ideal gas undergoing an isobaric process.

Monatomic Gas

According to the first law of thermodynamics,

$$Q = \Delta U + W \tag{12.4.13}$$

(Eq. 2), where W is work done by the system, U is internal energy, and Q is heat. The law says that the heat transferred to the system does work but also changes the internal energy of the system. Since,

$U = \frac{3}{2}NkT$ for a monatomic gas, we get $\Delta U = \frac{3}{2}Nk\Delta T$

(Eq. 3; for the details on internal energy, see our Atom on "Internal Energy of an Ideal Gas"). By using the Equations 1 and 3, Eq. 2 can be written as:

$Q = \frac{5}{2}Nk\Delta T$ for monatomic gas in an isobaric process.

Specific Heat

Specific heat at constant pressure is defined by the following equation:

$Q = ncP\Delta T$

Here n is the amount of particles in a gas represented in moles. By noting that $N = N_A n$ and $R = kN_A$ (N_A: Avogadro's number, R: universal gas constant), we derive:

$c_P = \frac{5}{2}kN_A = \frac{5}{2}R$ for a monatomic gas.

Problem Solving

With the ideal gas law we can figure pressure, volume or temperature, and the number of moles of gases under ideal thermodynamic conditions.

learning objectives

- Identify steps used to solve the ideal gas equation.

The Ideal Gas Law is the equation of state of a hypothetical ideal gas. It is a good approximation to the behavior of many gases under many conditions, although it has several limitations. It is most accurate for monatomic gases at high temperatures and low pressures.

The ideal gas law has the form:

$$PV = nRT, \tag{12.4.14}$$

where R is the universal gas constant, and with it we can find values of the pressure P, volume V, temperature T, or number of moles n under a certain ideal thermodynamic condition. Typically, you are given enough parameters to calculate the unknown. Variations of the ideal gas equation may help solving the problem easily. Here are some general tips.

The ideal gas law can also come in the form:

$$\mathrm{PV=NkT}, \tag{12.4.15}$$

where N is the number of particles in the gas and k is the Boltzmann constant.

To solve the ideal gas equation:

1. Write down all the information that you know about the gas.
2. If necessary, convert the known values to SI units.
3. Choose a relevant gas law equation that will allow you to calculate the unknown variable.
4. Substitute the known values into the equation. Calculate the unknown variable.

Remember that the general gas equation only applies if the molar quantity of the gas is fixed. For example, if a gas is mixed with another gas, you may have to apply the equation separately for individual gases.

Example

Let's imagine that at the beginning of a journey a truck tire has a volume of 30,000 cm^3 and an internal pressure of 170 kPa. The temperature of the tire is 16˚C. By the end of the trip, the volume of the tire has increased to 32,000 cm^3 and the temperature of the air inside the tire is 40˚C. What is the tire pressure at the end of the journey?

Tire Pressure: Tire pressure may change significantly during the operation of the vehicle. This is mostly due to the temperature change of the air in tires.

Solution:

Step 1. Write down all the information that you know about the gas: P_1= 170 kPa and P_2 is unknown. V_1 = 30,000 cm^3 and V_2 = 32,000 cm^3. T_1 = 16˚C and T_2 = 40˚C.

Step 2. Convert the known values to SI units if necessary: Here, temperature must be converted into Kelvin. Therefore, T_1 = 16 + 273 = 289 K, T_2 = 40 + 273 = 313 K

Step 3. Choose a relevant gas law equation that will allow you to calculate the unknown variable: We can use the general gas equation to solve this problem: $\mathrm{\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}}$.

Therefore, $\mathrm{P_2=\frac{P_1\times V_1\times T_2}{T_1\times V_2}}$.

Step 4. Substitute the known values into the equation. Calculate the unknown variable:

$$P_2 = \frac{170 \times 30,000 \times 313}{289 \times 32,000} = 173 \text{ kPa} \tag{12.4.16}$$

The pressure of the tire at the end of the journey is 173 kPa.

Note that in Step 2 we did not bother to convert the volume values to m^3. In Step 4, pressure appears both in the numerator and denominator. In this case the conversion was not necessary.

Avogador's Number

The number of molecules in a mole is called Avogadro's number (N_A)—defined as 6.02x 10^{23} mol^{-1}.

learning objectives

- Explain relationship between Avogadro's number and mole.

When measuring the amount of substance, it is sometimes easier to work with a unit other than the number of molecules. A mole (abbreviated mol) is a base unit in the International System of Units (SI). It is defined as any substance containing as many atoms or molecules as there are in exactly 12 grams (0.012 kg) of carbon-12. The actual number of atoms or molecules in one mole is called *Avogadro's constant (N_A)*, in recognition of Italian scientist Amedeo Avogadro.

Amadeo Avogadro: Amedeo Avogadro (1776–1856). He established the relationship between the masses of the same volume of different gases (at the same temperature and pressure) corresponds to the relationship between their respective molecular weights.

Avogadro's number (N) refers to the number of molecules in one gram-molecule of oxygen. This indicates an amount of substance as opposed to an independent dimension of measurement. In 1811 Amedeo Avogadro first proposed that the volume of a gas (at a given pressure and temperature) is proportional to the number of atoms or molecules, regardless of the nature of the gas (i.e., this number is universal and independent of the type of gas). In 1926, Jean Perrin won the Nobel Prize in Physics, largely for his work in determining the Avogadro constant (by several different methods). The value of Avogadro's constant, N_A , has been found to equal $6.02\times10^{23} mol^{-1}$.

Role in Science

Avogadro's constant is a scaling factor between macroscopic and microscopic (atomic scale) observations of nature. As such, it provides the relation between other physical constants and properties. For example, it establishes a relationship between the gas constant R and the Boltzmann constant k,

$$R = kN_A = 8.314472(15)\mathrm{J\ mol^{-1}K^{-1}}; \tag{12.4.17}$$

and the Faraday constant F and the elementary charge e,

$$F = N_A e = 96485.3383(83)\mathrm{C\ mol^{-1}}. \tag{12.4.18}$$

Measuring N_A

The determination of N_A is crucial to the calculation of an atom's mass, since the latter is obtained by dividing the mass of a mole of the gas by Avogadro's constant. In his study on Brownian motion in 1905, Albert Einstein proposed that this constant could be determined based on the quantities observable in Brownian motion. Subsequently, Einstein's idea was verified, leading to the first determination of N_A in 1908 through the experimental work of Jean Baptiste Perrin.

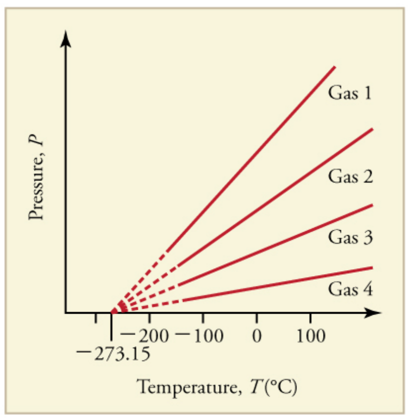
Absolute Temperature

Absolute temperature is the most commoly used thermodyanmic temperature unit and is the standard unit of temperature.

learning objectives

- Describe relationship between absolute temperature and kinetic energy.

Thermodynamic temperature is the absolute measure of temperature. It is one of the principal parameters of thermodynamics and kinetic theory of gases. Thermodynamic temperature is an "absolute" scale because it is the measure of the fundamental property underlying temperature: its null or zero point ("absolute zero") is the temperature at which the particle constituents of matter have minimal motion and cannot become any colder. That is, they have minimal motion, retaining only quantum mechanical motion, as diagramed in.



Graph of Pressure Versus Temperature: Graph of pressure versus temperature for various gases kept at a constant volume. Note that all of the graphs extrapolate to zero pressure at the same temperature

At its simplest, "temperature" arises from the kinetic energy of the random motions of matter's particle constituents such as molecules or atoms, as seen in. Therefore, it is reasonable to choose absolute zero, where all classical motion ceases, as the reference point (T=0) of our temperature system. By using the absolute temperature scale (Kelvin system), which is the most commonly used thermodynamic temperature, we have shown that the average translational kinetic energy (KE) of a particle in a gas has a simple relationship to the temperature:

Translational Motion of Helium: Real gases do not always behave according to the ideal model under certain conditions, such as high pressure. Here, the size of helium atoms relative to their spacing is shown to scale under 1950 atmospheres of pressure.

$$\bar{KE} = \frac{3}{2}kT. \tag{12.4.19}$$

Note that this equation would not look this elegant if the Fahrenheit scale were used instead.

The Kelvin scale

The kelvin (or "absolute temperature") is the standard thermodyanmic temperature unit. It is one of the seven base units in the International System of Units (SI) and is assigned the unit symbol K. By international agreement, the unit kelvin and its scale are defined by two points: absolute zero and the triple point of Vienna Standard Mean Ocean Water (water with a specified blend of hydrogen and oxygen isotopes). Absolute zero, the lowest possible temperature, is defined precisely as 0 K and −273.15 °C. The triple point of water is defined precisely as 273.16 K and 0.01 °C.

Key Points

- Ideal gas law was derived empirically by combining Boyle's law and Charles' law.
- Although the empirical derivation of the equation does not consider microscopic details, the ideal gas law can be derived from first principles in the classical thermodynamics.
- Pressure and volume of a gas can be related to the average velocity of molecues: $PV = \frac{1}{3}Nm\bar{v}^2$.
- Isothermal processes typically occur when a system is in contact with an outside thermal reservoir (heat bath), and the change occurs slowly enough to allow the system to adjust continually to the temperature of the reservoir through heat exchange.
- For an ideal gas, from the ideal gas law $PV = NkT, PV$ remains constant through an isothermal process. A curve in a P-V diagram generated by the equation $PV = \text{const}$ is called an isotherm.
- For an isothermal, reversible process, the work done by the gas is equal to the area under the relevant pressure -volume isotherm. It is given as $W_A \rightarrow B = NkT \ln \frac{V_B}{V_A}$.
- Gases can expand or contract under a certain constraint. Depending on the constraint, the final state of the gas may change.
- The heat transferred to the system does work but also changes the internal energy of the system. In an isobaric process for a monatomic gas, heat and the temperature change satisfy the following equation: $Q = \frac{5}{2}Nk\Delta T$.
- For a monatomic ideal gas, specific heat at constant pressure is $\frac{5}{2}R$.
- Write down all the information that you know about the gas and convert the known values to SI units if necessary.
- Choose a relevant gas law equation that will allow you to calculate the unknown variable, and substitute the known values into the equation. Then calculate the unknown variable.
- The general gas equation only applies if the molar quantity of the gas is fixed.
- Avogadro hypothesized that equal volumes of gas, at the same pressure and temperature, contain equal numbers of molecules, regardless of the type of gas.
- Avogadro's constant is a scaling factor between macroscopic and microscopic (atomic scale) observations of nature. It provides the relation between other physical constants and properties.
- Albert Einstein proposed that Avogadro's number could be determined based on the quantities observable in Brownian motion. NA was measured for the first time by Jean Baptiste Perrin in 1908.
- Temperature arises from the kinetic energy of the random motions of matter 's particle constituents such as molecules or atoms. Therefore, it is reasonable to choose absolute zero, where all classical motion ceases, as the reference point.

- By international agreement, the unit kelvin and its scale are defined by two points: absolute zero and the triple point of the standardized water.
- At absolute zero, the particle constituents of matter have minimal motion and cannot become any colder. They retain minimal, quantum mechanical motion.

Key Terms

- **mole**: In the International System of Units, the base unit of amount of substance; the amount of substance of a system which contains as many elementary entities as there are atoms in 12 g of carbon-12. Symbol: mol.
- **ideal gas**: A hypothetical gas whose molecules exhibit no interaction and undergo elastic collision with each other and with the walls of the container.
- **Avogadro's number**: the number of constituent particles (usually atoms or molecules) in one mole of a given substance. It has dimensions of reciprocal mol and its value is equal to 6.02214129·1023 mol-1
- **adiabatic**: Occurring without gain or loss of heat.
- **internal energy**: The sum of all energy present in the system, including kinetic and potential energy; equivalently, the energy needed to create a system, excluding the energy necessary to displace its surroundings.
- **the first law of thermodynamics**: A version of the law of energy conservation: the change in the internal energy of a closed system is equal to the amount of heat supplied to the system, minus the amount of work done by the system on its surroundings.
- **specific heat**: The ratio of the amount of heat needed to raise the temperature of a unit mass of substance by a unit degree to the amount of heat needed to raise that of the same mass of water by the same amount.
- **SI units**: International System of Units (abbreviated SI from French: Le Système international d'unités). It is the modern form of the metric system.
- **gas constant**: A universal constant, R, that appears in the ideal gas law, (PV = nRT), derived from two fundamental constants, the Boltzman constant and Avogadro's number, (R = NAk).
- **Faraday constant**: The magnitude of electric charge per mole of electrons.
- **Brownian motion**: Random motion of particles suspended in a fluid, arising from those particles being struck by individual molecules of the fluid.
- **absolute zero**: The coldest possible temperature: zero on the Kelvin scale and approximately -273.15°C and -459.67°F. The total absence of heat; the temperature at which motion of all molecules would cease.
- **International System of Units**: (SI): The standard set of basic units of measurement used in scientific literature worldwide.
- **Vienna Standard Mean Ocean Water**: A standard defining a standardized isotopic composition of water.

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12.5: Kinetic Theory

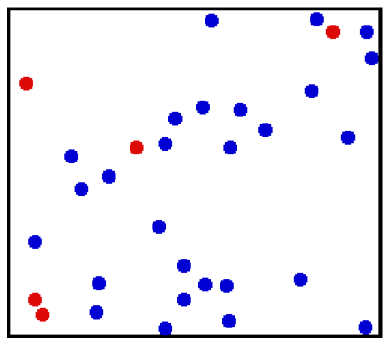
learning objectives

- Express the relationship between the pressure and the average kinetic energy of gas molecules in the form of equation

In Newtonian mechanics, if pressure is the force divided by the area on which the force is exerted, then what is the origin of pressure in a gas? What forces create the pressure? We can gain a better understanding of pressure (and temperature as well) from the kinetic theory of gases, which assumes that atoms and molecules are in continuous random motion.

Microscopic Origin of Pressure

Pressure is explained by kinetic theory as arising from the force exerted by molecules or atoms impacting on the walls of a container, as illustrated in the figure below. Consider a gas of N molecules, each of mass m, enclosed in a cubical container of volume $V=L^3$. When a gas molecule collides with the wall of the container perpendicular to the x coordinate axis and bounces off in the opposite direction with the same speed (an elastic collision), then the momentum lost by the particle and gained by the wall (Δp) is:



Translational Motion of Helium: Real gases do not always behave according to the ideal model under certain conditions, such as high pressure. Here, the size of helium atoms relative to their spacing is shown to scale under 1950 atmospheres of pressure.

$$\Delta p = p_{i,x} - p_{f,x} = p_{i,x} - (-p_{i,x}) \tag{12.5.1}$$

$$= 2p_{i,x} = 2mv_x \tag{12.5.2}$$

where v_x is the x-component of the initial velocity of the particle.

The particle impacts one specific side wall once every $\Delta t = \frac{2L}{v_x}$,

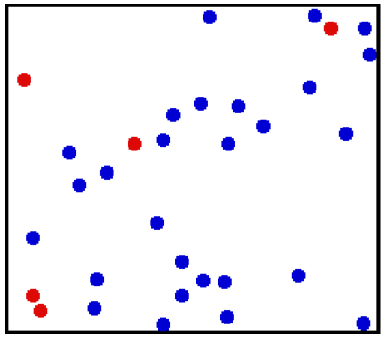
(where L is the distance between opposite walls). The force due to this particle is:

$$F = \frac{\Delta p}{\Delta t} = \frac{mv_x^2}{L}. \tag{12.5.3}$$

The total force on the wall, therefore, is:

$$F = \frac{Nm\bar{v_x^2}}{L} \tag{12.5.4}$$

where the bar denotes an average over the *N* particles. Since the assumption is that the particles move in random directions, if we divide the velocity vectors of all particles in three mutually perpendicular directions, the average value of the squared velocity along each direction must be same. (This does not mean that each particle always travel in 45 degrees to the coordinate axes.)



Pressure: Pressure arises from the force exerted by molecules or atoms impacting on the walls of a container.

This gives $\bar{v_x^2} = \frac{\bar{v^2}}{3}$. We can rewrite the force as $F = \frac{Nm\bar{v^2}}{3L}$.

This force is exerted on an area L^2. Therefore the pressure of the gas is:

$$P = \frac{F}{L^2} = \frac{Nm\bar{v^2}}{3V} = \frac{nm\bar{v^2}}{3}, \tag{12.5.5}$$

where $V=L^3$ is the volume of the box. The fraction n=N/V is the number density of the gas. This is a first non-trivial result of the kinetic theory because it relates pressure (a macroscopic property) to the average (translational) kinetic energy per molecule which is a microscopic property.

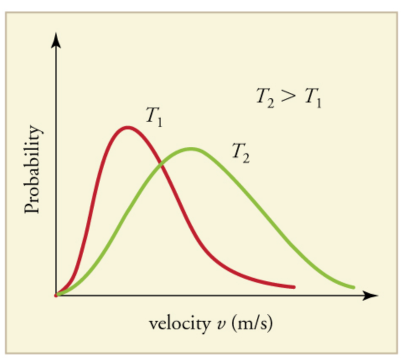
Speed Distribution of Molecules

A gas of many molecules has a predictable distribution of molecular speeds, known as the Maxwell-Boltzmann distribution.

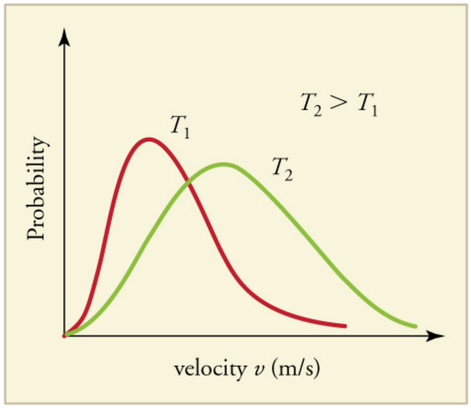
learning objectives

- Describe the shape and temperature dependence of the Maxwell-Boltzmann distribution curve

The motion of molecules in a gas is random in magnitude and direction for individual molecules, but a gas of many molecules has a predictable distribution of molecular speeds, known as the Maxwell-Boltzmann distribution (illustrated in). The distribution has a long tail because some molecules may go several times the rms speed. The most probable speed v_p (at the peak of the curve) is less than the rms speed v_{rms}. As shown in, the curve is shifted to higher speeds at higher temperatures, with a broader range of speeds.



Maxwell-Boltzmann Distribution at Higher Temperatures: The Maxwell-Boltzmann distribution is shifted to higher speeds and is broadened at higher temperatures.



Maxwell-Boltzmann Distribution: The Maxwell-Boltzmann distribution of molecular speeds in an ideal gas. The most likely speed v_p is less than the rms speed v_rms. Although very high speeds are possible, only a tiny fraction of the molecules have speeds that are an order of magnitude greater than v_rms.

Maxwell-Boltzmann Distribution

Maxwell-Boltzmann distribution is a probability distribution. It applies to ideal gases close to thermodynamic equilibrium, and is given as the following equation:

$$\mathrm{f_v(v_x, v_y, v_z) = (\frac{m}{2\pi kT})^{3/2} exp[-\frac{m(v_x^2+v_y^2+v_z^2)}{2kT}],} \tag{12.5.6}$$

where fv is the velocity probability density function. (Derivation of the formula goes beyond the scope of introductory physics.) The formula calculates the probability of finding a particle with velocity in the infinitesimal element [dv_x, dv_y, dv_z] about velocity v = [v_x, v_y, v_z] is:

$$\mathrm{f_v(v_x, v_y, v_z) dv_x dv_y dv_z.} \tag{12.5.7}$$

It can also be shown that the Maxwell–Boltzmann velocity distribution for the vector velocity [v_x, v_y, v_z] is the product of the distributions for each of the three directions:

$$\mathrm{f_v(v_x, v_y, v_z) = f_v(v_x) f_v(v_y) f_v(v_z)} \tag{12.5.8}$$

where the distribution for a single direction is,

$$\mathrm{f_v(v_i) = \sqrt{\frac{m}{2\pi kT}} \, exp[\frac{-mv_i^2}{2kT}].} \tag{12.5.9}$$

This makes sense because particles are moving randomly, meaning that each component of the velocity should be independent.

Distribution for the Speed

Usually, we are more interested in the speeds of molecules rather than their component velocities. The Maxwell–Boltzmann distribution for the speed follows immediately from the distribution of the velocity vector, above. Note that the speed is:

$$\mathrm{v = \sqrt{v_x^2 + v_y^2 + v_z^2}} \tag{12.5.10}$$

and the increment of volume is:

$$\mathrm{dv_x \, dv_y \, dv_z = v^2 \sin\phi \, dv \, d\theta \, d\phi,} \tag{12.5.11}$$

where θ and ϕ are the "course" (azimuth of the velocity vector) and "path angle" (elevation angle of the velocity vector). Integration of the normal probability density function of the velocity, above, over the course (from 0 to 2π) and path angle (from 0 to π), with substitution of the speed for the sum of the squares of the vector components, yields the following probability density function (known simply as the Maxwell distribution):

$\mathrm{f(v) = \sqrt{(\frac{m}{2\pi kT})^3} 4\pi v^2 \, exp(\frac{-mv^2}{2kT})}$ for speed v.

Temperature

Temperature is directly proportional to the average translational kinetic energy of molecules in an ideal gas.

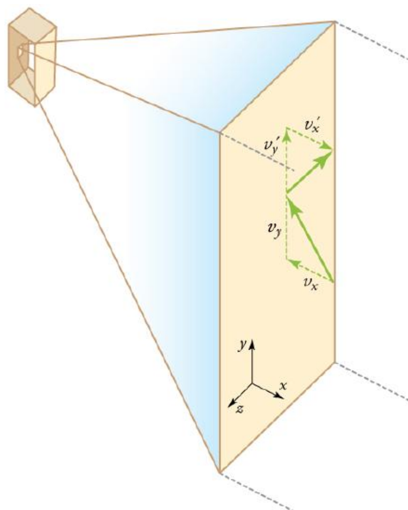
learning objectives

- Describe relationship between temperature and energy of molecules in an ideal gas

Intuitively, hotter air suggests faster movement of air molecules. In this atom, we will derive an equation relating the temperature of a gas (a macroscopic quantity) to the average kinetic energy of individual molecules (a microscopic quantity). This is a basic and extremely important relationship in the kinetic theory of gases.

Microscopic View

We assume that a molecule is small compared with the separation of molecules in the gas (confined in a three dimensional container), and that its interaction with other molecules can be ignored. Also, we assume elastic collisions when molecules hit the wall of the container, as illustrated in.



Elastic Collisions When Molecules Hit the Wall of the Container: Gas in a box exerts an outward pressure on its walls. A molecule colliding with a rigid wall has the direction of its velocity and momentum in the x-direction reversed. This direction is perpendicular to the wall. The components of its velocity momentum in the y- and z-directions are not changed, which means there is no force parallel to the wall.

We have seen in the Atom on “Origin of Pressure ” that, for an ideal gas under our assumptions:

$$\mathrm{P}=\frac{\mathrm{Nm}\bar{\mathrm{v}}^2}{3\mathrm{V}}, \tag{12.5.12}$$

where P is the pressure, N is the number of molecules, m is the mass of the molecule, v is the speed of molecules, and V is the volume of the gas. From the equation, we get:

$$\mathrm{PV}=\frac{1}{3}\mathrm{Nm}\bar{\mathrm{v}}^2 \text{ (Eq. 1)}. \tag{12.5.13}$$

What can we learn from this atomic and molecular version of the ideal gas law ? We can derive a relationship between temperature and the average translational kinetic energy of molecules in a gas. Recall the macroscopic expression of the ideal gas law:

$$\mathrm{PV}=\mathrm{NkT} \text{ (Eq. 2)}, \tag{12.5.14}$$

where N is the number of molecules, T is the temperature of the gas, and k is the Boltzmann constant.

Equating the right hand sides of the macroscopic and microscopic versions of the ideal gas law (Eq. 1 & 2) gives:

$$\frac{1}{3}\mathrm{m}\bar{\mathrm{v}}^2=\mathrm{kT}. \tag{12.5.15}$$

Thermal Energy

Note that the average kinetic energy (KE) of a molecule in the gas is:

$$\frac{1}{2}m\bar{v^2}. \tag{12.5.16}$$

Therefore, we derive the relation between average KE and temperature as follows:

$$\bar{KE} = \frac{1}{2}m\bar{v^2} = \frac{3}{2}kT, \text{ (Eq. 3)}. \tag{12.5.17}$$

The average translational kinetic energy of a molecule is called thermal energy.

RMS Speed

Eq. 3 is a molecular interpretation of temperature. It has been found to be valid for gases and reasonably accurate in liquids and solids. It is another definition of temperature based on an expression of the molecular energy. It is sometimes useful to know the average speed of molecules in a gas in terms of temperature:

$$\bar{v^2} = v_{rms} = \sqrt{\frac{3kT}{m}}, \tag{12.5.18}$$

where v_{rms} stands for root-mean-square (rms) speed.

Internal Energy of an Ideal Gas

Internal energy is the total energy contained by a thermodynamic system, and has two major components: kinetic energy and potential energy.

learning objectives

- Determine the number of degrees of freedom and calculate the internal energy for an ideal gas molecule

In thermodynamics, internal energy is the total energy contained by a thermodynamic system. Internal energy has two major components: kinetic energy and potential energy. The kinetic energy is due to the motion of the system's particles (e.g., translations, rotations, vibrations). In ideal gases, there is no inter-particle interaction. Therefore, we will disregard potential energy and only focus on the kinetic energy contribution to the internal energy.

Monatomic Gases

A monatomic gas is one in which atoms are not bound to each other. Noble gases (He, Ne, etc.) are typical examples. A helium balloon is shown in the following figure. In this case, the kinetic energy consists only of the translational energy of the individual atoms. Monoatomic particles do not vibrate, and their rotational energy can be neglected because atomic moment of inertia is so small. Also, they are not electronically excited to higher energies except at very high temperatures. Therefore, practical internal energy changes in an ideal gas may be described solely by changes in its translational kinetic energy.

Helium Blimp: Helium, like other noble gases, is a monatomic gas, which often can be described by the ideal gas law. It is the gas of choice to fill airships such as the Goodyear blimp.

The average kinetic energy (KE) of a particle in an ideal gas is given as:

$$\bar{KE} = \frac{1}{2}m\bar{v^2} = \frac{3}{2}kT, \tag{12.5.19}$$

where k is the Boltzmann's constant. (See the Atom on "Temperature" in kinetic theory.) With N atoms in the gas, its total internal energy U is given as:

$$U = \frac{3}{2}NkT \tag{12.5.20}$$

where N is the number of atoms in the gas. Note that there are three degrees of freedom in monatomic gases: translation in x, y and z directions.

Since atomic motion is random (and therefore isotropic), each degrees of freedom contribute $\frac{1}{2}kT$ per atom to the internal energy.

Diatomic gases

A diatomic molecule (H_2, O_2, N_2, etc.) has 5 degrees of freedom (3 for translation in x, y and z directions, and 2 for rotation). Therefore, the internal energy for diatomic gases is $U = \frac{5}{2}NkT$.

Key Points

- We can gain a better understanding of pressure (and temperature as well) from the kinetic theory of gases, which assumes that atoms and molecules are in continuous random motion.
- Pressure, a macroscopic property, can be related to the average (translational) kinetic energy per molecule which is a microscopic property by $P = \frac{nm\bar{v^2}}{3}$.
- Since the assumption is that the particles move in random directions, the average value of velocity squared along each direction must be same. This gives: $\bar{v_x^2} = \bar{v_y^2} = \bar{v_z^2} = \bar{v^2}/3$.
- The Maxwell-Boltzmann distribution has a long tail, and the most probable speed v_p is less than the rms speed v_{rms}. The distribution curve is shifted to higher speeds at higher temperatures, with a broader range of speeds.
- Maxwell-Boltzmann distribution is given as follows: $f_v(v_x, v_y, v_z) = (\frac{m}{2\pi kT})^{3/2} \exp[-\frac{m(v_x^2+v_y^2+v_z^2)}{2kT}]$. It is a product of three independent 1D Maxwell-Boltzmann distributions.
- Molecular speed distribution is given as $f(v) = \sqrt{(\frac{m}{2\pi kT})^3} 4\pi v^2 \exp(\frac{-mv^2}{2kT})$. This is simply called Maxwell distribution.
- The average translational kinetic energy of a molecule is equivalent to $\frac{3}{2}kT$ and is called thermal energy.
- In kinematic theory of gases, macroscopic quantities (such as press and temperature) are explained by considering microscopic (random) motion of molecules.
- The rms speed of molecules in a gas is given as $\sqrt{\frac{3kT}{m}}$.
- In ideal gases, there is no inter-particle interaction. Therefore, only the kinetic energy contribute to the internal energy.
- Each degrees of freedom contribute $\frac{1}{2}kT$ per atom to the internal energy.
- For monatomic ideal gases with N atoms, its total internal energy U is given as $U = \frac{3}{2}NkT$. For diatomic gases, $U = \frac{5}{2}NkT$.

Key Terms

- **kinetic theory of gases**: The kinetic theory of gases describes a gas as a large number of small particles (atoms or molecules), all of which are in constant, random motion.
- **Newtonian mechanics**: Early classical mechanics as propounded by Isaac Newton, especially that based on his laws of motion and theory of gravity.
- **rms**: Root mean square: a statistical measure of the magnitude of a varying quantity.
- **ideal gas**: A hypothetical gas whose molecules exhibit no interaction and undergo elastic collision with each other and with the walls of the container.
- **Boltzmann's constant**: The physical constant relating energy at the particle level with temperature observed at the bulk level. It is the gas constant R divided by Avogadro's number, NA.
- **moment of inertia**: A measure of a body's resistance to a change in its angular rotation velocity
- **noble gas**: Any of the elements of group 18 of the periodic table, being monatomic and (with very limited exceptions) inert.

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12.6: Phase Changes

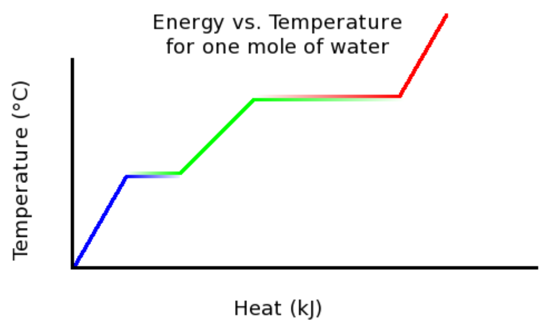
learning objectives

- Describe behavior of the medium during a phase transition

A phase of a thermodynamic system and the states of matter have uniform physical properties. During a phase transition of a given medium certain properties of the medium change, often discontinuously, as a result of some external condition, such as temperature or pressure. For example, a liquid may become gas upon heating to the boiling point, resulting in an abrupt change in volume. The measurement of the external conditions at which the transformation occurs is termed the phase transition. The term is most commonly used to describe transitions between solid, liquid and gaseous states of matter and, in rare cases, plasma.

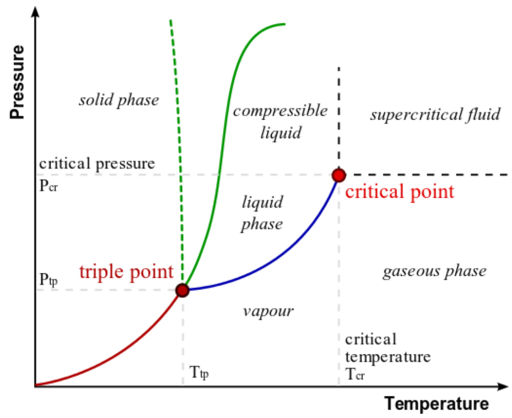
As an example, if you boil water, it never goes above 100 degrees Celsius. Only after it has completely evaporated will it get any hotter. This is because once water reaches the boiling point, extra energy is used to change the state of matter and increase the potential energy instead of the kinetic energy. The opposite happens when water freezes. To boil or melt one mole of a substance, a certain amount of energy is required. These amounts of energy are the molar heat of vaporization and molar heat of fusion. If that amount of energy is added to a mole of that substance at boiling or freezing point, all of it will melt or boil, but the temperature won't change.

Temperature increases linearly with heat, until the melting point. But the heat added does not change the temperature; that heat energy is instead used to break intermolecular bonds and convert ice into water. At this point, there is a mixture of both ice and water. Once all ice has been melted, the temperature again rises linearly with heat added. At the boiling point, temperature no longer rises with heat added because the energy is once again being used to break intermolecular bonds. Once all water has been boiled to steam, the temperature will continue to rise linearly as heat is added.



Temperature vs. Heat: This graph shows the temperature of ice as heat is added.

The plots of pressure versus temperatures provide considerable insight into thermal properties of substances. There are well-defined regions on these graphs that correspond to various phases of matter, so PT graphs are called phase diagrams. Using the graph, if you know the pressure and temperature you can determine the phase of water. The solid lines—boundaries between phases—indicate temperatures and pressures at which the phases coexist (that is, they exist together in ratios, depending on pressure and temperature). For example, the boiling point of water is 100º C at 1.00 atm. As the pressure increases, the boiling temperature rises steadily to 374º C at a pressure of 218 atm. A pressure cooker (or even a covered pot) will cook food faster because the water can exist as a liquid at temperatures greater than 100º C without all boiling away. The curve ends at a point called the critical point, because at higher temperatures the liquid phase does not exist at any pressure. The critical temperature for oxygen is -118ºC, so oxygen cannot be liquefied above this temperature.



Phase Diagram of Water: In this typical phase diagram of water, the green lines mark the freezing point, and the blue line marks the boiling point, showing how they vary with pressure. The dotted line illustrates the anomalous behavior of water. Note that water changes states based on the pressure and temperature.

Humidity, Evaporation, and Boiling

The amount of water vapor in air is a result of evaporation or boiling, until an equilibrium is reached.

learning objectives

- Explain why water boils at 100 °C

Overview

The term relative humidity refers to how much water vapor is in the air compared with the maximum possible. At its maximum, denoted as saturation, the relative humidity is 100%, and evaporation is inhibited. The amount of water vapor the air can hold depends on its temperature. For example, relative humidity rises in the evening, as air temperature declines, sometimes reaching the dew point. At the dew point temperature, relative humidity is 100%, and fog may result from the condensation of water droplets if they are small enough to stay in suspension. Conversely, if one wished to dry something, it is more effective to blow hot air over it rather than cold air, because, among other things, hot air can hold more water vapor.

Evaporation

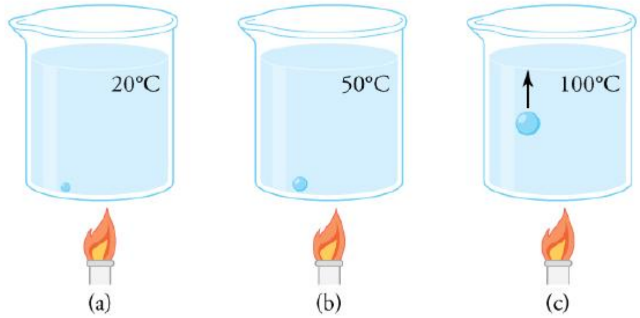
The capacity of air to hold water vapor is based on vapor pressure of water. The liquid and solid phases are continuously giving off vapor because some of the molecules have high enough speeds to enter the gas phase, a process called evaporation; see (a). For the molecules to evaporate, they must be located near the surface, be moving in the proper direction, and have sufficient kinetic energy to overcome liquid-phase intermolecular forces. When only a small proportion of the molecules meet these criteria, the rate of evaporation is low. Since the kinetic energy of a molecule is proportional to its temperature, evaporation proceeds more quickly at higher temperatures.

If a lid is placed over the container, as in (b), evaporation continues, increasing the pressure, until sufficient vapor has built up for condensation to balance evaporation. Then equilibrium has been achieved, and the vapor pressure is equal to the partial pressure of water in the container. Vapor pressure increases with temperature because molecular speeds are higher as temperature increases.

As the faster-moving molecules escape, the remaining molecules have lower average kinetic energy, and the temperature of the liquid decreases. This phenomenon is also called evaporative cooling. This is why evaporating sweat cools the human body. Evaporation also tends to proceed more quickly with higher flow rates between the gaseous and liquid phase and in liquids with higher vapor pressure. For example, laundry on a clothes line will dry (by evaporation) more rapidly on a windy day than on a still day.

Application for Boiling Water

Why does water boil at 100ºC? The vapor pressure of water at 100ºC is 1.01×10^5 Pa, or 1.00 atm. Thus, it can evaporate without limit at this temperature and pressure. But why does it form bubbles when it boils? This is because water ordinarily contains significant amounts of dissolved air and other impurities, which are observed as small bubbles of air in a glass of water. If a bubble starts out at the bottom of the container at 20ºC, it contains water vapor (about 2.30%). The pressure inside the bubble is fixed at 1.00 atm (we ignore the slight pressure exerted by the water around it). As the temperature rises, the amount of air in the bubble stays the same, but the water vapor increases; the bubble expands to keep the pressure at 1.00 atm. At 100ºC, water vapor enters the bubble continuously since the partial pressure of water is equal to 1.00 atm in equilibrium. It cannot reach this pressure, however, since the bubble also contains air and total pressure is 1.00 atm. The bubble grows in size and thereby increases the buoyant force. The bubble breaks away and rises rapidly to the surface, resulting in boiling. (See.)



Close-up of the Boiling Process: (a) An air bubble in water starts out saturated with water vapor at 20ºC. (b) As the temperature rises, water vapor enters the bubble because its vapor pressure increases. The bubble expands to keep its pressure at 1.00 atm. (c) At 100ºC, water vapor enters the bubble continuously because water's vapor pressure exceeds its partial pressure in the bubble, which must be less than 1.00 atm. The bubble grows and rises to the surface.

Key Points

- The term is most commonly used to describe transitions between solid, liquid and gaseous states of matter and, in rare cases, plasma.
- Once water reaches the boiling point, extra energy is used to change the state of matter and increase the potential energy instead of the kinetic energy.
- Plots of pressure versus temperatures, an example of a phase diagram, provide considerable insight into thermal properties of substances.
- Relative humidity is the fraction of water vapor in a gas compared to the saturation value.
- Since the kinetic energy of a molecule is proportional to its temperature, evaporation proceeds more quickly at higher temperatures.
- Vapor pressure increases with temperature because molecular speeds are higher as temperature increases.
- Water boils at 100 °C because the vapor pressure exceeds atmospheric pressure at this temperature.

Key Terms

- **intermolecular**: from one molecule to another; between molecules
- **plasma**: a state of matter consisting of partially ionized gas
- **thermodynamic**: Relating to the conversion of heat into other forms of energy.
- **equilibrium**: The state of a body at rest or in uniform motion, the resultant of all forces on which is zero.
- **vapor pressure**: The pressure that a vapor exerts, or the partial pressure if it is mixed with other gases.
- **humidity**: The amount of water vapor in the air.

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12.7: The Zeroth Law of Thermodynamics

learning objectives

- Identify major implications of the Zeroth Law of Thermodynamics

The Zeroth Law of Thermodynamics

There are a few ways to state the Zeroth Law of Thermodynamics, but the simplest is as follows: systems that are in thermal equilibrium exist at the same temperature.

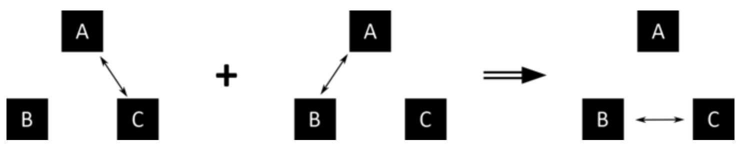
Systems are in thermal equilibrium if they do not transfer heat, even though they are in a position to do so, based on other factors. For example, food that's been in the refrigerator overnight is in thermal equilibrium with the air in the refrigerator: heat no longer flows from one source (the food) to the other source (the air) or back.

What the Zeroth Law of Thermodynamics means is that temperature is something worth measuring, because it indicates whether heat will move between objects. This will be true regardless of how the objects interact. Even if two objects don't touch, heat may still flow between them, such as by radiation (as from a heat lamp). However, according to the Zeroth Law of Thermodynamics, if the systems are in thermal equilibrium, no heat flow will take place.

There are more formal ways to state the Zeroth Law of Thermodynamics, which is commonly stated in the following manner:

Let A, B, and C be three systems. If A and C are in thermal equilibrium, and A and B are in thermal equilibrium, then B and C are in thermal equilibrium.

This statement is represented symbolically in. Temperature is not mentioned explicitly, but it's implied that temperature exists. Temperature is the quantity that is always the same for all systems in thermal equilibrium with one another.



Zeroth Law of Thermodynamics: The double arrow represents thermal equilibrium between systems. If systems A and C are in equilibrium, and systems A and B are in equilibrium, then systems B and C are in equilibrium. The systems A, B, and C are at the same temperature.

Key Points

- Assuming A, B, and C are three systems, if A and C are in thermal equilibrium, and A and B are in thermal equilibrium, then B and C are in thermal equilibrium.
- Two systems are in thermal equilibrium if they could transfer heat between each other, but do not.
- The Zeroth Law of Thermodynamics implies that temperature is a quantity worth measuring.

Key Terms

- **thermal equilibrium**: Two systems are in thermal equilibrium if they could transfer heat between each other, but don't.
- **zeroth law of thermodynamics**: Let A, B and C be three systems. If A and C are in thermal equilibrium, and A and B are in thermal equilibrium, then B and C are in thermal equilibrium.

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12.8: Thermal Stresses

learning objectives

- Formulate relationship between thermal stress and thermal expansion

Thermal Expansion

Thermal expansion is the change in size or volume of a given mass with temperature. The expansion of alcohol in a thermometer is one of many commonly encountered examples of this. Hot air rises because its volume increases, which causes the hot air's density to be smaller than the density of surrounding air, causing a buoyant (upward) force on the hot air. The same happens in all liquids and gases, driving natural heat transfer upward in homes, oceans, and weather systems. Solids also undergo thermal expansion. Railroad tracks and bridges, for example, have expansion joints to allow them to freely expand and contract with temperature changes.

Thermal Expansion Joints: Thermal expansion joints like these in the Auckland Harbour Bridge in New Zealand allow bridges to change length without buckling. (credit: Ingolfson, Wikimedia Commons)

What are the basic properties of thermal expansion? First, thermal expansion is clearly related to temperature change. The greater the temperature change, the more a bimetallic strip will bend. Second, it depends on the material. In a thermometer, for example, the expansion of alcohol is much greater than the expansion of the glass containing it.

What is the underlying cause of thermal expansion? An increase in temperature implies an increase in the kinetic energy of the individual atoms. In a solid, unlike in a gas, the atoms or molecules are closely packed together, but their kinetic energy (in the form of small, rapid vibrations) pushes neighboring atoms or molecules apart from each other. This neighbor-to-neighbor pushing results in a slightly greater distance, on average, between neighbors, and adds up to a larger size for the whole body. For most substances under ordinary conditions, there is no preferred direction, and an increase in temperature will increase the solid's size by a certain fraction in each dimension.

To be more quantitative, the change in length ΔL is proportional to length L. The dependence of thermal expansion on temperature, substance, and length is summarized in the equation

$$\Delta \mathrm{L} = \alpha \mathrm{L} \Delta \mathrm{T} \tag{12.8.1}$$

where ΔL is the change in length L, ΔT is the change in temperature, and α is the coefficient of linear expansion, which varies slightly with temperature.

Thermal Stress

Thermal stress is created by thermal expansion or contraction. Thermal stress can be destructive, such as when expanding gasoline ruptures a tank. It can also be useful, for example, when two parts are joined together by heating one in manufacturing, then slipping it over the other and allowing the combination to cool. Thermal stress can explain many phenomena, such as the weathering of rocks and pavement by the expansion of ice when it freezes.

Forces and pressures created by thermal stress can be quite large. Railroad tracks and roadways can buckle on hot days if they lack sufficient expansion joints. Power lines sag more in the summer than in the winter, and will snap in cold weather if there is insufficient slack. Cracks open and close in plaster walls as a house warms and cools. Glass cooking pans will crack if cooled rapidly or unevenly, because of differential contraction and the stresses it creates. (Pyrex® is less susceptible because of its small coefficient of thermal expansion.) Nuclear reactor pressure vessels are threatened by overly rapid cooling, and although none have failed, several have been cooled faster than considered desirable. Biological cells are ruptured when foods are frozen, detracting from their taste. Repeated thawing and freezing accentuates the damage. Even the oceans can be affected. A significant portion of the rise in sea level that is resulting from global warming is due to the thermal expansion of sea water.

Metal is regularly used in the human body for hip and knee implants. Most implants need to be replaced over time because, among other things, metal does not bond with bone. Researchers are trying to find better metal coatings that would allow metal-to-bone bonding. One challenge is to find a coating that has an expansion coefficient similar to that of metal. If the expansion coefficients are too different, the thermal stresses during the manufacturing process lead to cracks at the coating-metal interface.

Another example of thermal stress is found in the mouth. Dental fillings can expand differently from tooth enamel. It can give pain when eating ice cream or having a hot drink. Cracks might occur in the filling. Metal fillings (gold, silver, etc.) are being replaced by composite fillings (porcelain), which have smaller coefficients of expansion, and are closer to those of teeth.

Key Points

- Thermal expansion is the change in size or volume of a given mass with changing temperature.
- An increase in temperature implies an increase in the kinetic energy of the individual atoms, which will increase a solid's size by a certain fraction in each dimension.
- Thermal stress is created when thermal expansion is constrained.

Key Terms

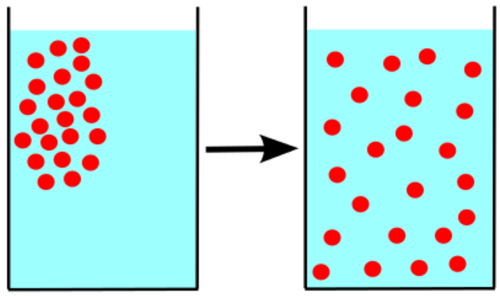
- **stress**: The internal distribution of force per unit area (pressure) within a body reacting to applied forces which causes strain or deformation and is typically symbolized by σ.
- **differential**: A qualitative or quantitative difference between similar or comparable things.

12.9: Diffusion

learning objectives

- Discuss the process and results of diffusion, identifying factors that affect its rate

Diffusion is the movement of particles move from an area of high concentration to an area of low concentration until equilibrium is reached. A distinguishing feature of diffusion is that it results in mixing or mass transport without requiring bulk motion. Thus, diffusion should not be confused with convection or advection, which are other transport mechanisms that use bulk motion to move particles from one place to another.



Diffusion: Particles moving from areas of high concentration to areas of low concentration.

Molecular diffusion, often called simply diffusion, is the thermal motion of all (liquid or gas) particles at temperatures above absolute zero. The rate of this movement is a function of temperature, viscosity of the fluid and the size (mass) of the particles. Diffusion explains the net flux of molecules from a region of higher concentration to one of lower concentration. However, diffusion can still occur in the absence of a concentration gradient.

The result of diffusion is a gradual mixing of material. In a phase with uniform temperature, absent external net forces acting on the particles, the diffusion process will eventually result in complete mixing.

Key Points

- Molecular diffusion, often called simply diffusion, is the thermal motion of all (liquid or gas) particles at temperatures above absolute zero.
- The result of diffusion is a gradual mixing of material. In a phase with uniform temperature, absent external net forces acting on the particles, the diffusion process will eventually result in complete mixing.
- Diffusion can also occur in the absence of a concentration gradient — equilibrium particles are still moving around their container.

Key Terms

- **equilibrium**: The state of a body at rest or in uniform motion, the resultant of all forces on which is zero.
- **diffusion**: Diffusion is the movement of particles from regions of high concentration toward regions of lower concentration.
- **concentration**: The proportion of a substance in a mixture.

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CHAPTER OVERVIEW

13: Heat and Heat Transfer

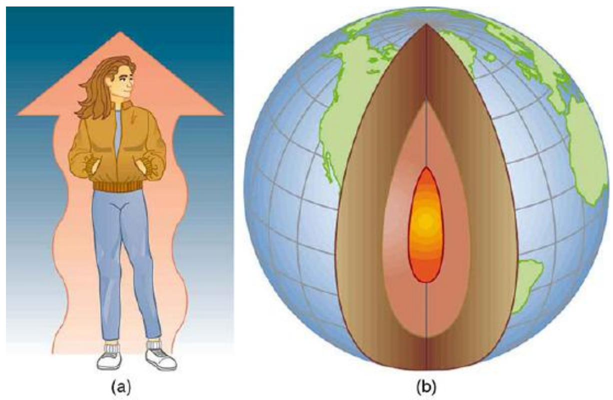
13.1: Introduction

learning objectives

- Distinguish three modes of heat transfer

Introduction to Heat and Heat Transfer

Energy can exist in many forms and heat is one of the most intriguing. Heat is often hidden, as it only exists when in transit, and is transferred by a number of distinctly different methods. Heat transfer touches every aspect of our lives and helps us understand how the universe functions. It explains the chill we feel on a clear breezy night, or why Earth's core has yet to cool. This module defines and explores heat transfer, its effects, and the methods by which heat is transferred. These topics are fundamental, as well as practical, and will often be referred to in the modules ahead.



Examples of Heat Transfer: (a) The chilling effect of a clear breezy night is produced by the wind and by radiative heat transfer to cold outer space. (b) There was once great controversy about the Earth's age, but it is now generally accepted to be about 4.5 billion years old. Much of the debate is centered on the Earth's molten interior. According to our understanding of heat transfer, if the Earth is really that old, its center should have cooled off long ago. The discovery of radioactivity in rocks revealed the source of energy that keeps the Earth's interior molten, despite heat transfer to the surface, and from there to cold outer space.

Definitions

Scottish physicist James Clerk Maxwell, in his 1871 classic *Theory of Heat*, was one of many who began to build on the already established idea that heat was something to do with matter in motion. This was the same idea put forwards by Sir Benjamin Thompson in 1798, who said he was only following on from the work of many others. One of Maxwell's recommended books was *Heat as a Mode of Motion* by John Tyndall. Maxwell outlined four stipulations for the definition of heat:

- It is something which may be transferred from one body to another.
- It is a measurable quantity, and thus treated mathematically.
- It cannot be treated as a substance, because it may be transformed into something that is not a substance, such as mechanical work.
- Heat is one of the forms of energy.

In the following sections, we will define heat more rigorously, paying particular attention to how it can be measured and quantified.

Estimation of Quantity of Heat

The quantity of heat transferred by some process can either be directly measured, or determined indirectly through calculations based on other quantities. Direct measurement is by calorimetry and is the primary empirical basis of the idea of quantity of heat transferred in a process. The transferred heat is measured by changes in a body of known properties, for example, temperature rise, change in volume or length, or phase change, such as melting of ice. Indirect estimations of quantity of heat transferred rely on the law of conservation of energy, and, in particular cases, on the first law of thermodynamics (explored in the following sections). Indirect estimation is the primary approach of many theoretical studies of quantity of heat transferred.

Heat Transfer Methods

After defining and quantifying heat transfer and its effects on physical systems, we will discuss the methods by which heat is transferred. So many processes involve heat transfer, so that it is hard to imagine a situation where no heat transfer occurs. Yet every process involving heat transfer takes place by only three methods:

1. Conduction is heat transfer through stationary matter by physical contact. (The matter is stationary on a macroscopic scale—we know there is thermal motion of the atoms and molecules at any temperature above absolute zero.) Heat transferred between the electric burner of a stove and the bottom of a pan is transferred by conduction.
2. Convection is the heat transfer by the macroscopic movement of a fluid. This type of transfer takes place in a forced-air furnace and in weather systems, for example.
3. Heat transfer by radiation occurs when microwaves, infrared radiation, visible light, or another form of electromagnetic radiation is emitted or absorbed. An obvious example is the warming of the Earth by the Sun. A less obvious example is thermal radiation from the human body.

Heat as Energy Transfer

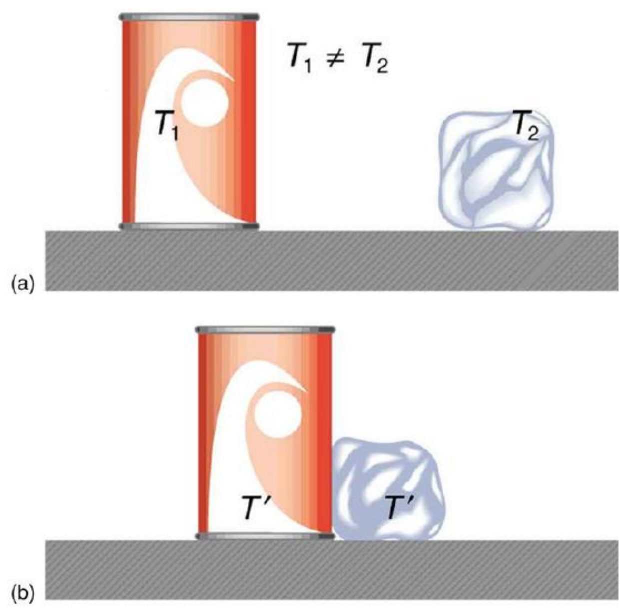
Heat is the spontaneous transfer of energy due to a temperature difference.

learning objectives

- Identify SI and common units of heat

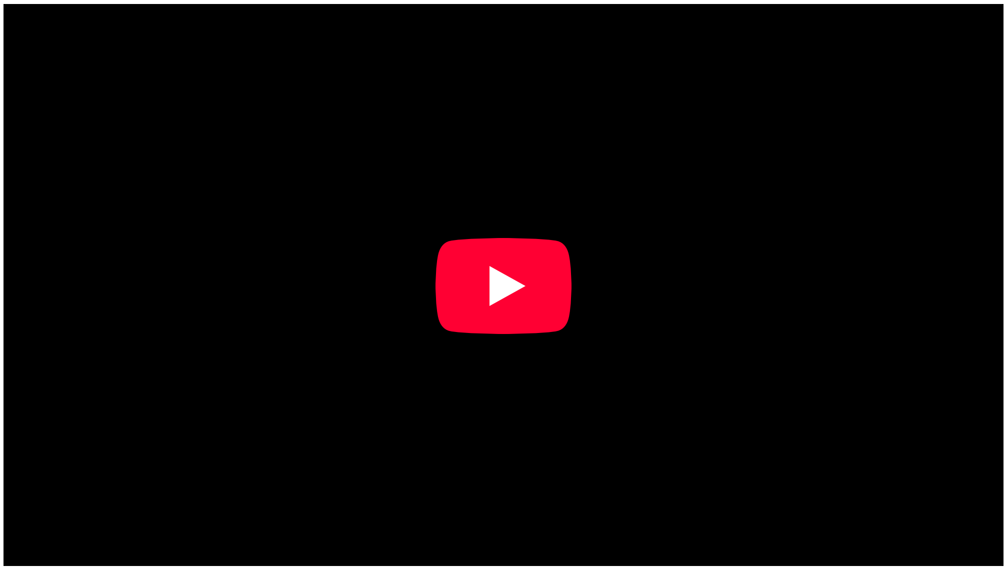
Heat as Energy Transfer

Consider two objects at different temperatures that are brought together. Energy is transferred from the hotter object to the cooler one, until both objects reach thermal equilibrium (i.e., both become the same temperature). How is this energy transferred? No work is done by either object, because no force acts through a distance. The transfer of energy is caused by the temperature difference, and ceases once the temperatures are equal. This observation leads to the following definition of heat: Heat is the spontaneous transfer of energy due to a temperature difference.



Heat Transfer and Equilibrium: (a) The soft drink and the ice have different temperatures, T1 and T2, and are not in thermal equilibrium. (b) When the soft drink and ice are allowed to interact, energy is transferred until they reach the same temperature T, achieving equilibrium. Heat transfer occurs due to the difference in temperatures. In fact, since the soft drink and ice are both in contact with the surrounding air and bench, the equilibrium temperature will be the same for both.

Where Is the Most Heat Lost?: Use movable thermometers to discover where a house has poor insulation.



Heat Transfer: A brief introduction to heat transfer for students.

Heat is often confused with temperature. For example, we may say the heat was unbearable, when we actually mean that the temperature was high. Heat is a form of energy, whereas temperature is not. The misconception arises because we are sensitive to the flow of heat, rather than the temperature.

Units

Owing to the fact that heat is a form of energy, it has the SI unit of *joule* (J). The *calorie* (cal) is a common unit of energy, defined as the energy needed to change the temperature of 1.00 g of water by 1.00ºC —specifically, between 14.5ºC and 15.5ºC, since there is a slight temperature dependence. Another common unit of heat is the kilocalorie (kcal), which is the energy needed to change the temperature of 1.00 kg of water by 1.00ºC. Since mass is often specified in kilograms, kilocalorie is commonly used. Food calories (given the notation Cal, and sometimes called “big calorie”) are actually kilocalories (1kilocalorie=1000 calories), a fact not easily determined from package labeling in the United States, but more common in Europe and elsewhere. In some engineering fields, the British Thermal Unit (BTU), equal to about 1.055 kilo-joules, is widely used.

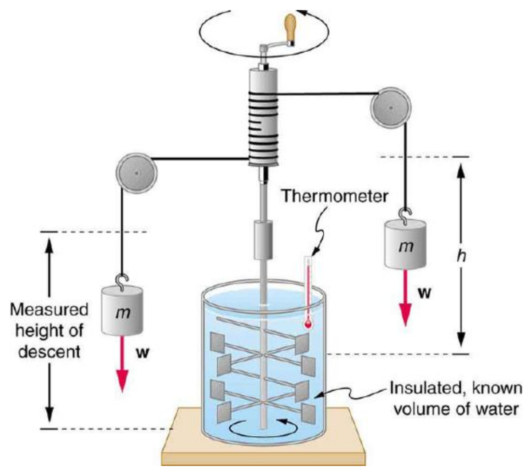


Figure 1 Equivalence of Heat and Work: Schematic depiction of Joule's experiment that established the equivalence of heat and work

The total amount of energy transferred as heat is conventionally written as Q for algebraic purposes. Heat released by a system into its surroundings is by convention a negative quantity ($Q < 0$); when a system absorbs heat from its surroundings, it is positive ($Q > 0$).

Mechanical Equivalent of Heat

It is also possible to change the temperature of a substance by doing work. Work can transfer energy into or out of a system. This realization helped establish the fact that heat is a form of energy. James Prescott Joule (1818–1889) performed many experiments to establish the mechanical equivalent of heat—*the work needed to produce the same effects as heat transfer*. In terms of the units used for these two terms, the best modern value for this equivalence is 1.000 kcal = 4186 J. We consider this equation as the conversion between two different units of energy.

Figure 1 shows one of Joule's most famous experimental setups for demonstrating the mechanical equivalent of heat. It demonstrated that work and heat can produce the same effects, and helped establish the principle of conservation of energy. Gravitational potential energy (PE) (work done by the gravitational force) is converted into kinetic energy (KE), and then randomized by viscosity and turbulence into increased average kinetic energy of atoms and molecules in the system, producing a temperature increase. His contributions to the field of thermodynamics were so significant that the SI unit of energy was named after him.

Heat added or removed from a system changes its internal energy (a concept we will discuss in the following section) and thus its temperature. Such a temperature increase is observed while cooking. However, adding heat does not necessarily increase the temperature. An example is melting of ice; that is, when a substance changes from one phase to another. Work done on the system or by the system can also change the internal energy of the system. Joule demonstrated that the temperature of a system can be increased by stirring. If an ice cube is rubbed against a rough surface, work is done by the frictional force. A system has a well-defined internal energy, but we cannot say that it has a certain "heat content" or "work content." We use the phrase "heat transfer" to emphasize its nature.

Internal Energy

The internal energy of a system is the sum of all kinetic and potential energy in a system.

learning objectives

- Express the internal energy in terms of kinetic and potential energy

Internal Energy

James Joule showed that both heat and work can produce the same change in the internal energy of a substance, establishing the principle of the mechanical equivalence of heat. Heat is emphatically a quantity that solely describes energy being transferred. It makes no sense to speak of the total 'heat' an object or system contains. However, a system does contain a quantifiable amount of energy called the internal energy of a system. The internal energy of a system is the quantity that changes with the addition or subtraction of work or heat. It is closely related to temperature.

Definition

The internal energy is the energy required to create a system, excluding the energy necessary to displace its surroundings. Internal energy has two components: kinetic energy and potential energy. The kinetic energy consists of all the energy involving the motions of the particles constituting the system, including translation, vibration, and rotation. The potential energy is associated with the static constituents of matter, static electric energy of atoms within molecules or crystals, and the energy from chemical bonds. The equation describing the total internal energy of a system is then:

$$U = U_{kinetic} + U_{potential}. \tag{13.1.1}$$

We can also think of the internal energy as the sum of all the energy states of each component in the system:

$$U = \sum_i E_i. \tag{13.1.2}$$

At any finite temperature, kinetic and potential energies are constantly converted into each other, but the total energy remains constant in an isolated system. The kinetic energy portion of internal energy gives rise to the temperature of the system. We can use statistical mechanics to relate the (somewhat) random motions of particles in a system to the mean kinetic energy of the ensemble of particles, and thus the empirically measurable quantity expressed as temperature.

We can see that internal energy is an extensive property: it depends on the size of the system or on the amount of substance it contains.

In most cases, we are not concerned with the *total* amount of internal energy in the system, as it is rarely convenient or necessary to consider all energies belonging to the system. Rather, we are far more interested in the *change* in internal energy, given some transfer of work or heat. This can be expressed as:

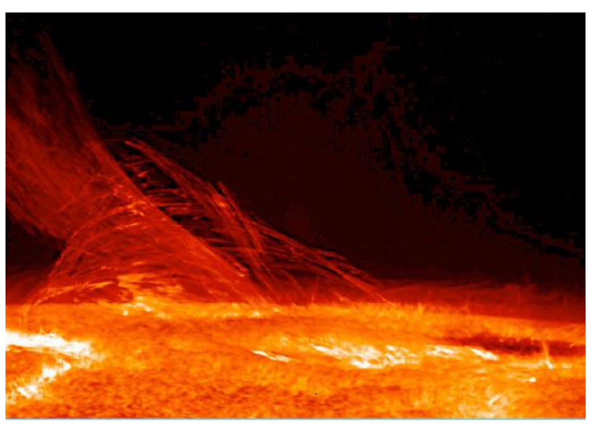
$$\Delta U = Q + W_{mech} + W_{other}. \tag{13.1.3}$$

Q is heat added to a system and W_{mech} is the mechanical work performed by the surroundings due to pressure or volume changes in the system. All other perturbations and energies added by other processes, such as an electric current introduced into an electronic circuit, is summarized as the term W_{extra}.

We can calculate a small change in internal energy of the system by considering the infinitesimal amount of heat δQ added to the system minus the infinitesimal amount of work δW done by the system:

$$dU = \delta Q - \delta W. \tag{13.1.4}$$

This expression is the first law of thermodynamics.



The Sun and Internal Energy: Nuclear fusion in the sun converts nuclear potential energy into available internal energy and keeps the temperature of the Sun very high.

Key Points

- Heat is a crucial concept that touches every aspect of our lives. James Clerk Maxwell set down important principles that couple into the definition of heat.
- The quantity of heat transfer can be directly measured or estimated indirectly through the science of calorimetry.
- There are three modes of heat transfer: conduction, convection, and radiation.
- If two objects at different temperature are brought together, energy will transfer from the hotter object to the cooler one until both are at the same temperature. This transfer of energy is known as heat.
- Heat should not be confused with temperature. Temperature describes the internal state of an object, while heat refers to the energy transferred to or from the object.
- Since heat is a form of energy, its SI unit is the joule. Other common units of heat energy include the calorie and kilocalorie, equal to 4.186 and 4,186 joules, respectively.
- Because heat and work both involve the transfer of energy, they can each produce the same effects. The concept of the mechanical equivalent of heat was instrumental in establishing the principle of conservation of energy.
- While a system does not contain ' heat,' it does contain a total amount of energy called internal energy.
- The internal energy is the energy necessary to create a system, minus the energy necessary to displace its surroundings.
- Most of the time, we are interested in the change in internal energy rather than the total internal energy.
- The first law of thermodynamics, $dU = \Delta Q - \Delta W$, describes small changes in internal energy.

Key Terms

- **heat transfer**: The transmission of thermal energy via conduction, convection, or radiation.
- **calorimetry**: The science of measuring the heat absorbed or evolved during the course of a chemical reaction or change of state.
- **kilocalorie**: A non-SI unit of energy equal to 1,000 calories or 4,186 joules; equal to the "calorie" or "Calorie" used in nutritional labeling. Symbol: kcal.
- **thermal equilibrium**: Two systems are in thermal equilibrium if they could transfer heat between each other, but don't.
- **mechanical equivalent of heat**: The work needed to produce the same effects as heat transfer.
- **internal energy**: The sum of all energy present in the system, including kinetic and potential energy; equivalently, the energy needed to create a system, excluding the energy necessary to displace its surroundings.
- **isolated system**: A system that does not interact with its surroundings, that is, its total energy and mass stay constant.

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13.2: Specific Heat

learning objectives

- Explain the enthalpy in a system with constant volume and pressure

Heat capacity (usually denoted by a capital C, often with subscripts), or thermal capacity, is the measurable physical quantity that characterizes the amount of heat required to change a substance's temperature by a given amount. In SI units, heat capacity is expressed in units of joules per kelvin (J/K).

An object's heat capacity (symbol *C*) is defined as the ratio of the amount of heat energy transferred to an object to the resulting increase in temperature of the object.

$$\mathrm{C}=\frac{\mathrm{Q}}{\Delta \mathrm{T}}. \tag{13.2.1}$$

Heat capacity is an extensive property, so it scales with the size of the system. A sample containing twice the amount of substance as another sample requires the transfer of twice as much heat (Q) to achieve the same change in temperature (ΔT). For example, if it takes 1,000 J to heat a block of iron, it would take 2,000 J to heat a second block of iron with twice the mass as the first.

The Measurement of Heat Capacity

The heat capacity of most systems is not a constant. Rather, it depends on the state variables of the thermodynamic system under study. In particular, it is dependent on temperature itself, as well as on the pressure and the volume of the system, and the ways in which pressures and volumes have been allowed to change while the system has passed from one temperature to another. The reason for this is that pressure-volume work done to the system raises its temperature by a mechanism other than heating, while pressure-volume work done by the system absorbs heat without raising the system's temperature. (The temperature dependence is why the definition a calorie is formally the energy needed to heat 1 g of water from 14.5 to 15.5 °C instead of generally by 1 °C.)

Different measurements of heat capacity can therefore be performed, most commonly at constant pressure and constant volume. The values thus measured are usually subscripted (by p and V, respectively) to indicate the definition. Gases and liquids are typically also measured at constant volume. Measurements under constant pressure produce larger values than those at constant volume because the constant pressure values also include heat energy that is used to do work to expand the substance against the constant pressure as its temperature increases. This difference is particularly notable in gases where values under constant pressure are typically 30% to 66.7% greater than those at constant volume.

Thermodynamic Relations and Definition of Heat Capacity

The internal energy of a closed system changes either by adding heat to the system or by the system performing work. Recalling the first law of thermodynamics,

$$\mathrm{dU}=\delta \mathrm{Q}-\delta \mathrm{W}. \tag{13.2.2}$$

For work as a result of an increase of the system volume we may write,

$$\mathrm{dU}=\delta \mathrm{Q}-\mathrm{PdV}. \tag{13.2.3}$$

If the heat is added at constant volume, then the second term of this relation vanishes and one readily obtains

$$\left(\frac{\partial \mathrm{U}}{\partial \mathrm{T}}\right)_{\mathrm{V}}=\left(\frac{\partial \mathrm{Q}}{\partial \mathrm{T}}\right)_{\mathrm{V}}=\mathrm{C}_{\mathrm{V}}. \tag{13.2.4}$$

This defines the *heat capacity at constant volume*, C_V. Another useful quantity is the *heat capacity at constant pressure*, C_P. With the *enthalpy* of the system given by

$$\mathrm{H}=\mathrm{U}+\mathrm{PV}, \tag{13.2.5}$$

our equation for d*U* changes to

$$\mathrm{dH}=\delta \mathrm{Q}+\mathrm{VdP}, \tag{13.2.6}$$

and therefore, at constant pressure, we have

$$(\frac{\partial H}{\partial T})_P = (\frac{\partial Q}{\partial T})_P = C_P. \tag{13.2.7}$$

Specific Heat

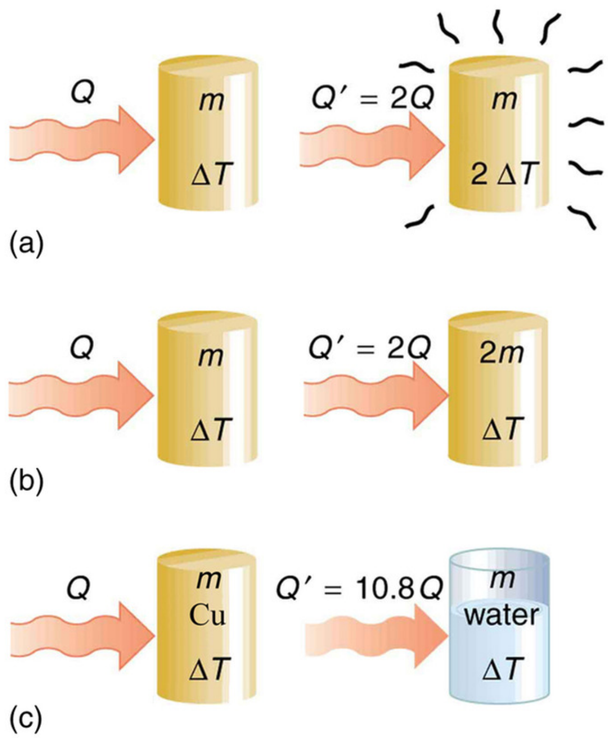
The specific heat is an intensive property that describes how much heat must be added to a particular substance to raise its temperature.

learning objectives

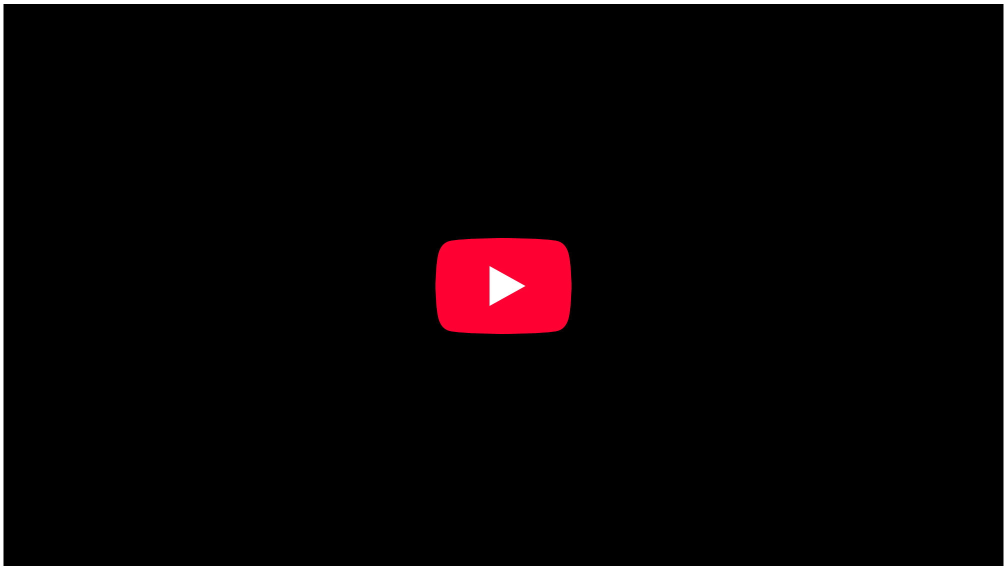
- Summarize the quantitative relationship between heat transfer and temperature change

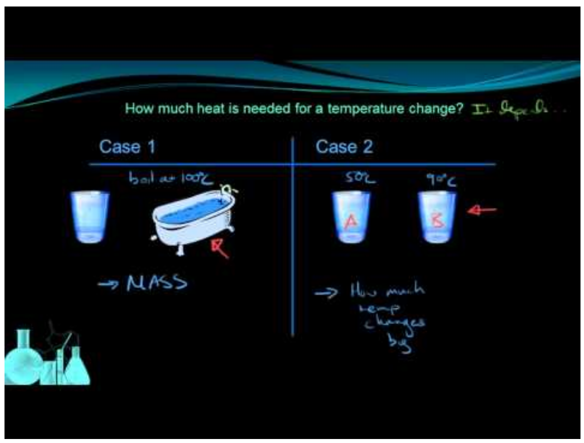
Specific Heat

The heat capacity is an extensive property that describes how much heat energy it takes to raise the temperature of a given system. However, it would be pretty inconvenient to measure the heat capacity of every unit of matter. What we want is an intensive property that depends only on the type and phase of a substance and can be applied to systems of arbitrary size. This quantity is known as the specific heat capacity (or simply, the specific heat), which is the heat capacity per unit mass of a material. Experiments show that the transferred heat depends on three factors: (1) The change in temperature, (2) the mass of the system, and (3) the substance and phase of the substance. The last two factors are encapsulated in the value of the specific heat.



Heat Transfer and Specific Heat Capacity: The heat Q transferred to cause a temperature change depends on the magnitude of the temperature change, the mass of the system, and the substance and phase involved. (a) The amount of heat transferred is directly proportional to the temperature change. To double the temperature change of a mass m, you need to add twice the heat. (b) The amount of heat transferred is also directly proportional to the mass. To cause an equivalent temperature change in a doubled mass, you need to add twice the heat. (c) The amount of heat transferred depends on the substance and its phase. If it takes an amount Q of heat to cause a temperature change ΔT in a given mass of copper, it will take 10.8 times that amount of heat to cause the equivalent temperature change in the same mass of water assuming no phase change in either substance.





Specific Heat Capacity: This lesson relates heat to a change in temperature. We discuss how the amount of heat needed for a temperature change is dependent on mass and the substance involved, and that relationship is represented by the specific heat capactiy of the substance, C.

The dependence on temperature change and mass are easily understood. Because the (average) kinetic energy of an atom or molecule is proportional to the absolute temperature, the internal energy of a system is proportional to the absolute temperature and the number of atoms or molecules. Since the transferred heat is equal to the change in the internal energy, the heat is proportional to the mass of the substance and the temperature change. The transferred heat also depends on the substance so that, for example, the heat necessary to raise the temperature is less for alcohol than for water. For the same substance, the transferred heat also depends on the phase (gas, liquid, or solid).

The quantitative relationship between heat transfer and temperature change contains all three factors:

$$\mathrm{Q=mc\Delta T,} \tag{13.2.8}$$

where Q is the symbol for heat transfer, m is the mass of the substance, and ΔT is the change in temperature. The symbol c stands for specific heat and depends on the material and phase.

The specific heat is the amount of heat necessary to change the temperature of 1.00 kg of mass by 1.00ºC. The specific heat c is a property of the substance; its SI unit is J/(kg⋅K) or J/(kg⋅C). Recall that the temperature change (ΔT) is the same in units of kelvin and degrees Celsius. Note that the total heat capacity C is simply the product of the specific heat capacity c and the mass of the substance m, i.e.,

$$\mathrm{C=mc \text{ or } c=\frac{C}{m}=\frac{C}{\rho V},} \tag{13.2.9}$$

where ϱ is the density of the substance and V is its volume.

Values of specific heat must generally be looked up in tables, because there is no simple way to calculate them. Instead, they are measured empirically. In general, the specific heat also depends on the temperature. The table below lists representative values of specific heat for various substances. Except for gases, the temperature and volume dependence of the specific heat of most substances is weak. The specific heat of water is five times that of glass and ten times that of iron, which means that it takes five times as much heat to raise the temperature of water the same amount as for glass and ten times as much heat to raise the temperature of water as for iron. In fact, water has one of the largest specific heats of any material, which is important for sustaining life on Earth.

Substances	**Specific heat (*c*)**	
Solids	J/kg·°C	kcal/kg·°C[2]
Aluminum	900	0.215
Asbestos	800	0.19
Concrete, granite (average)	840	0.20
Copper	387	0.0924
Glass	840	0.20
Gold	129	0.0308
Human body (average at 37 °C)	3500	0.83
Ice (average, -50°C to 0°C)	2090	0.50
Iron, steel	452	0.108
Lead	128	0.0305
Silver	235	0.0562
Wood	1700	0.4
Liquids		
Benzene	1740	0.415
Ethanol	2450	0.586
Glycerin	2410	0.576
Mercury	139	0.0333
Water (15.0 °C)	4186	1.000
Gases [3]		
Air (dry)	721 (1015)	0.172 (0.242)
Ammonia	1670 (2190)	0.399 (0.523)
Carbon dioxide	638 (833)	0.152 (0.199)
Nitrogen	739 (1040)	0.177 (0.248)
Oxygen	651 (913)	0.156 (0.218)
Steam (100°C)	1520 (2020)	0.363 (0.482)

Specific Heats: Listed are the specific heats of various substances. These values are identical in units of cal/(g·C).3. cv at constant volume and at 20.0ºC, except as noted, and at 1.00 atm average pressure. Values in parentheses are cp at a constant pressure of 1.00 atm.

Calorimetry

Calorimetry is the measurement of the heat of chemical reactions or physical changes.

learning objectives

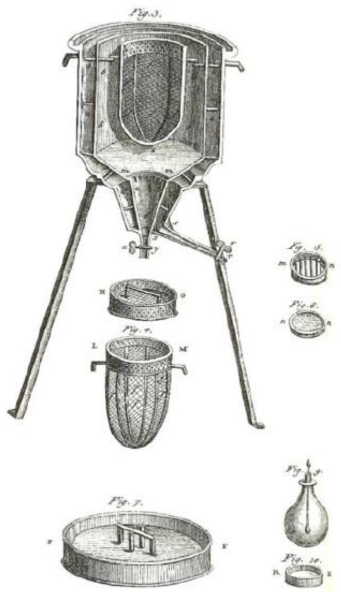
- Analyze the relationship between the gas constant for an ideal gas yield and volume

Calorimetry

Overview

Calorimetry is the science of measuring the heat of chemical reactions or physical changes. Calorimetry is performed with a calorimeter. A simple calorimeter just consists of a thermometer attached to a metal container full of water suspended above a combustion chamber. The word calorimetry is derived from the Latin word *calor*, meaning heat. Scottish physician and scientist Joseph Black, who was the first to recognize the distinction between heat and temperature, is said to be the founder of calorimetry.

Calorimetry requires that the material being heated have known thermal properties, i.e. specific heat capacities. The classical rule, recognized by Clausius and by Kelvin, is that the pressure exerted by the calorimetric material is fully and rapidly determined solely by its temperature and volume; this rule is for changes that do not involve phase change, such as melting of ice. There are many materials that do not comply with this rule, and for them, more complex equations are required than those below.



Ice Calorimeter: The world's first ice-calorimeter, used in the winter of 1782-83, by Antoine Lavoisier and Pierre-Simon Laplace, to determine the heat evolved in variouschemical changes; calculations which were based on Joseph Black's prior discovery of latent heat. These experiments mark the foundation of thermochemistry.

Basic Calorimetry at Constant Value

Constant-volume calorimetry is calorimetry performed at a constant volume. This involves the use of a constant-volume calorimeter (one type is called a Bomb calorimeter). For constant-volume calorimetry:

$$\delta Q = CV\Delta T = mcV\Delta T \tag{13.2.10}$$

where δQ is the increment of heat gained by the sample, C_V is the heat capacity at constant volume, c_v is the specific heat at constant volume, and ΔT is the change in temperature.

Measuring Enthalpy Change

To find the enthalpy change per mass (or per mole) of a substance A in a reaction between two substances A and B, the substances are added to a calorimeter and the initial and final temperatures (before the reaction started and after it has finished) are noted. Multiplying the temperature change by the mass and specific heat capacities of the substances gives a value for the energy given off or absorbed during the reaction:

$$\delta Q = \Delta T(m_A c_A + m_B c_B) \tag{13.2.11}$$

Dividing the energy change by how many grams (or moles) of A were present gives its enthalpy change of reaction. This method is used primarily in academic teaching as it describes the theory of calorimetry. It does not account for the heat loss through the container or the heat capacity of the thermometer and container itself. In addition, the object placed inside the calorimeter shows that the objects transferred their heat to the calorimeter and into the liquid, and the heat absorbed by the calorimeter and the liquid is equal to the heat given off by the metals.

Constant-Pressure Calorimetry

A constant-pressure calorimeter measures the change in enthalpy of a reaction occurring in solution during which the atmospheric pressure remains constant. An example is a coffee-cup calorimeter, which is constructed from two nested Styrofoam cups and a lid with two holes, allowing insertion of a thermometer and a stirring rod. The inner cup holds a known amount of a solute, usually water, that absorbs the heat from the reaction. When the reaction occurs, the outer cup provides insulation. Then

$$C_P = \frac{W\Delta H}{M\Delta T} \tag{13.2.12}$$

where C_p is the specific heat at constant pressure, ΔH is the enthalpy of the solution, ΔT is the change in temperature, W is the mass of the solute, and M is the molecular mass of the solute. The measurement of heat using a simple calorimeter, like the coffee cup calorimeter, is an example of constant-pressure calorimetry, since the pressure (atmospheric pressure) remains constant during the process. Constant-pressure calorimetry is used in determining the changes in enthalpy occurring in solution. Under these conditions the change in enthalpy equals the heat (Q=ΔH).

Specific Heat for an Ideal Gas at Constant Pressure and Volume

An ideal gas has different specific heat capacities under constant volume or constant pressure conditions.

learning objectives

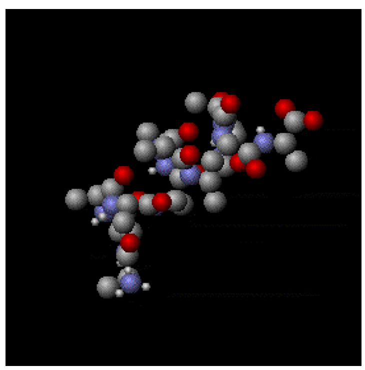
- Explain how to derive the adiabatic index

Specific Heat for an Ideal Gas at Constant Pressure and Volume

The heat capacity at constant volume of nR = 1 $J\cdot K^{-1}$ of any gas, including an ideal gas is:

$$(\frac{\partial U}{\partial T})_V = c_v \tag{13.2.13}$$

This represents the dimensionless heat capacity at constant volume; it is generally a function of temperature due to intermolecular forces. For moderate temperatures, the constant for a monoatomic gas is c_v=3/2 while for a diatomic gas it is c_v=5/2 (see). Macroscopic measurements on heat capacity provide information on the microscopic structure of the molecules.



Molecular internal vibrations: When a gas is heated, translational kientic energy of molecules in the gas will increase. In addition, molecules in the gas may pick up many characteristic internal vibrations. Potential energy stored in these internal degrees of freedom contributes to specific heat of the gas.

The heat capacity at constant pressure of 1 $J\cdot K^{-1}$ ideal gas is:

$$(\frac{\partial H}{\partial T})_V = c_p = c_v + R \tag{13.2.14}$$

where H=U+pV is the enthalpy of the gas.

Measuring the heat capacity at constant volume can be prohibitively difficult for liquids and solids. That is, small temperature changes typically require large pressures to maintain a liquid or solid at constant volume (this implies the containing vessel must be nearly rigid or at least very strong). It is easier to measure the heat capacity at constant pressure (allowing the material to expand or contract freely) and solve for the heat capacity at constant volume using mathematical relationships derived from the basic thermodynamic laws.

Utilizing the Fundamental Thermodynamic Relation we can show:

$$C_p - C_V = T(\frac{\partial P}{\partial T})_V, N(\frac{\partial V}{\partial T})_{p,N} \tag{13.2.15}$$

where the partial derivatives are taken at: constant volume and constant number of particles, and at constant pressure and constant number of particles, respectively.

The heat capacity ratio or adiabatic index is the ratio of the heat capacity at constant pressure to heat capacity at constant volume. It is sometimes also known as the isentropic expansion factor:

$$\gamma = \frac{C_P}{C_V} = \frac{c_p}{c_v} \tag{13.2.16}$$

For an ideal gas, evaluating the partial derivatives above according to the equation of state, where R is the gas constant for an ideal gas yields:

$$pV = RT \tag{13.2.17}$$

$$C_p - C_V = T(\frac{\partial P}{\partial T})_V(\frac{\partial V}{\partial T})_p \tag{13.2.18}$$

$$C_p - C_V = -T(\frac{\partial P}{\partial V})_V(\frac{\partial V}{\partial T})_p^2 \tag{13.2.19}$$

$$P = \frac{RT}{V}n \rightarrow (\frac{\partial P}{\partial V})_T = \frac{-RT}{V^2} = \frac{-P}{V} \tag{13.2.20}$$

$$V = \frac{RT}{P}n \rightarrow (\frac{\partial V}{\partial T})_p^2 = \frac{R^2}{P^2} \tag{13.2.21}$$

substituting:

$$-T(\frac{\partial P}{\partial V})_V(\frac{\partial V}{\partial T})_p^2 = -T\frac{-P}{V}\frac{R^2}{P^2} = R \tag{13.2.22}$$

This equation reduces simply to what is known as Mayer's relation:

Julius Robert Mayer: Julius Robert von Mayer (November 25, 1814 – March 20, 1878), a German physician and physicist, was one of the founders of thermodynamics. He is best known for his 1841 enunciation of one of the original statements of the conservation of energy (or what is now known as one of the first versions of the first law of thermodynamics): “Energy can be neither created nor destroyed. ” In 1842, Mayer described the vital chemical process now referred to as oxidation as the primary source of energy for any living creature. His achievements were overlooked and credit for the discovery of the mechanical equivalent of heat was attributed to James Joule in the following year. von Mayer also proposed that plants convert light into chemical energy.

$$C_P - C_V = R. \tag{13.2.23}$$

It is a simple equation relating the heat capacities under constant temperature and under constant pressure.

Solving Problems with Calorimetry

Calorimetry is used to measure the amount of heat produced or consumed in a chemical reaction.

learning objectives

- Explain a bomb calorimeter is used to measure heat evolved in a combustion reaction

Calorimeters are designed to minimize energy exchange between the system being studied and its surroundings. They range from simple coffee cup calorimeters used by introductory chemistry students to sophisticated bomb calorimeters used to determine the energy content of food.

Calorimetry is used to measure amounts of heat transferred to or from a substance. To do so, the heat is exchanged with a calibrated object (calorimeter). The change in temperature of the measuring part of the calorimeter is converted into the amount of heat (since the previous calibration was used to establish its heat capacity). The measurement of heat transfer using this approach requires the definition of a system (the substance or substances undergoing the chemical or physical change) and its surroundings (the other components of the measurement apparatus that serve to either provide heat to the system or absorb heat from the system). Knowledge of the heat capacity of the surroundings, and careful measurements of the masses of the system and surroundings and their temperatures before and after the process allows one to calculate the heat transferred as described in this section.

A calorimeter is a device used to measure the amount of heat involved in a chemical or physical process. For example, when an exothermic reaction occurs in solution in a calorimeter, the heat produced by the reaction is absorbed by the solution, which increases its temperature. When an endothermic reaction occurs, the heat required is absorbed from the thermal energy of the solution, which decreases its temperature. The temperature change, along with the specific heat and mass of the solution, can then be used to calculate the amount of heat involved in either case.

Coffee-Cup Calorimeters

General chemistry students often use simple calorimeters constructed from polystyrene cups. These easy-to-use “coffee cup” calorimeters allow more heat exchange with their surroundings, and therefore produce less accurate energy values.

Structure of the Constant Volume (or “Bomb”) Calorimeter

Bomb Calorimeter: This is the picture of a typical setup of bomb calorimeter.

A different type of calorimeter that operates at constant volume, colloquially known as a bomb calorimeter, is used to measure the energy produced by reactions that yield large amounts of heat and gaseous products, such as combustion reactions. (The term “bomb” comes from the observation that these reactions can be vigorous enough to resemble explosions that would damage other calorimeters.) This type of calorimeter consists of a robust steel container (the “bomb”) that contains the reactants and is itself submerged in water. The sample is placed in the bomb, which is then filled with oxygen at high pressure. A small electrical spark is used to ignite the sample. The energy produced by the reaction is trapped in the steel bomb and the surrounding water. The temperature increase is measured and, along with the known heat capacity of the calorimeter, is used to calculate the energy produced by the reaction. Bomb calorimeters require calibration to determine the heat capacity of the calorimeter and ensure accurate results. The calibration is accomplished using a reaction with a known q, such as a measured quantity of benzoic acid ignited by a spark from a nickel fuse wire that is weighed before and after the reaction. The temperature change produced by the known reaction is used to determine the heat capacity of the calorimeter. The calibration is generally performed each time before the calorimeter is used to gather research data.

Example 13.2.1: Identifying a Metal by Measuring Specific Heat

A 59.7 g piece of metal that had been submerged in boiling water was quickly transferred into 60.0 mL of water initially at 22.0 °C. The final temperature is 28.5 °C. Use these data to determine the specific heat of the metal. Use this result to identify the metal.

Solution

Assuming perfect heat transfer, the heat given off by metal is the negative of the heat taken in by water, or:

$$q_{metal} = -q_{water} \tag{13.2.24}$$

In expanded form, this is:

$$c_{metal} \times m_{metal} \times (T_{f,metal} - T_{i,metal}) = c_{water} \times m_{water} \times (T_{f,water} - T_{i,water}) \tag{13.2.25}$$

Noting that since the metal was submerged in boiling water, its initial temperature was 100.0 °C; and that for water, 60.0 mL = 60.0 g; we have:

$$(c_{metal})(59.7 g)(28.5^oC - 100.0^oC) = (4.18 J/g^oC)(60.0 g)(28.5oC - 22.0oC) \tag{13.2.26}$$

Solving this:

$$c_{metal} = \frac{-(4.184 J/g^oC)(60.0 g)(6.5^oC)}{(59.7 g)(-71.5^oC)} = 0.38 J/g^oC \tag{13.2.27}$$

Our experimental specific heat is closest to the value for copper (0.39 J/g °C), so we identify the metal as copper.

Key Points

- Heat capacity is the measurable physical quantity that characterizes the amount of heat required to change a substance’s temperature by a given amount. It is measured in joules per Kelvin and given by.
- The heat capacity is an extensive property, scaling with the size of the system.
- The heat capacity of most systems is not constant (though it can often be treated as such). It depends on the temperature, pressure, and volume of the system under consideration.
- Unlike the total heat capacity, the specific heat capacity is independent of mass or volume. It describes how much heat must be added to a unit of mass of a given substance to raise its temperature by one degree Celsius. The units of specific heat capacity are J/(kg °C) or equivalently J/(kg K).
- The heat capacity and the specific heat are related by $C = cm$ or $c = \frac{C}{m}$.
- The mass m, specific heat c, change in temperature ΔT, and heat added (or subtracted) Q are related by the equation: $Q = mc\Delta T$.
- Values of specific heat are dependent on the properties and phase of a given substance. Since they cannot be calculated easily, they are empirically measured and available for reference in tables.
- A calorimeter is used to measure the heat generated (or absorbed) by a physical change or chemical reaction. The science of measuring these changes is known as calorimetry.
- In order to do calorimetry, it is crucial to know the specific heats of the substances being measured.

- Calorimetry can be performed under constant volume or constant pressure. The type of calculation done depends on the conditions of the experiment.
- The specific heat at constant volume for a gas is given as $\left(\frac{\partial U}{\partial T}\right)_V = c_v$.
- The specific heat at constant pressure for an ideal gas is given as $\left(\frac{\partial H}{\partial T}\right)_V = c_p = c_v + R$.
- The heat capacity ratio (or adiabatic index) is the ratio of the heat capacity at constant pressure to heat capacity at constant volume.
- Calorimetry is used to measure amounts of heat transferred to or from a substance.
- A calorimeter is a device used to measure the amount of heat involved in a chemical or physical process.
- This means that the amount of heat produced or consumed in the reaction equals the amount of heat absorbed or lost by the solution.

Key Terms

- **heat capacity**: The amount of heat energy needed to raise the temperature of an object or unit of matter by one degree Celsius; in units of joules per kelvin (J/K).
- **enthalpy**: the total amount of energy in a system, including both the internal energy and the energy needed to displace its environment
- **specific heat capacity**: The amount of heat that must be added (or removed) from a unit mass of a substance to change its temperature by one degree Celsius. It is an intensive property.
- **constant-pressure calorimeter**: An instrument used to measure the heat generated during changes that do not involve changes in pressure.
- **calorimeter**: An apparatus for measuring the heat generated or absorbed by either a chemical reaction, change of phase or some other physical change.
- **constant-volume calorimeter**: An instrument used to measure the heat generated during changes that do not involve changes in volume.
- **Fundamental Thermodynamic Relation**: In thermodynamics, the fundamental thermodynamic relation expresses an infinitesimal change in internal energy in terms of infinitesimal changes in entropy, and volume for a closed system in thermal equilibrium in the following way: dU=TdS-PdV. Here, U is internal energy, T is absolute temperature, S is entropy, P is pressure and V is volume.
- **adiabatic index**: The ratio of the heat capacity at constant pressure to heat capacity at constant volume.
- **specific heat**: The ratio of the amount of heat needed to raise the temperature of a unit mass of substance by a unit degree to the amount of heat needed to raise that of the same mass of water by the same amount.
- **heat of reaction**: The enthalpy change in a chemical reaction; the amount of heat that a systems gives up to its surroundings so it can return to its initial temperature.
- **combustion**: A process where two chemicals are combined to produce heat.

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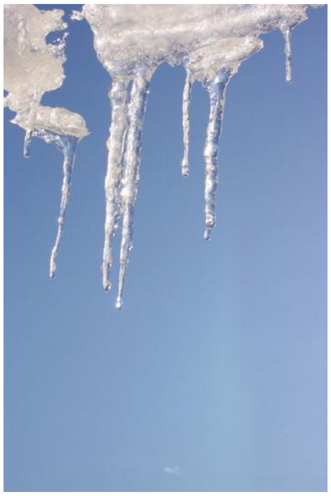
13.3: Phase Change and Latent Heat

learning objectives

- Describe the latent heat as a form of energy

Latent Heat

Previously, we have discussed temperature change due to heat transfer. No temperature change occurs from heat transfer if ice melts and becomes liquid water (i.e., during a phase change). For example, consider water dripping from icicles melting on a roof warmed by the Sun. Conversely, water freezes in an ice tray cooled by lower-temperature surroundings.



Melting Icicle: Heat from the air transfers to the ice causing it to melt.

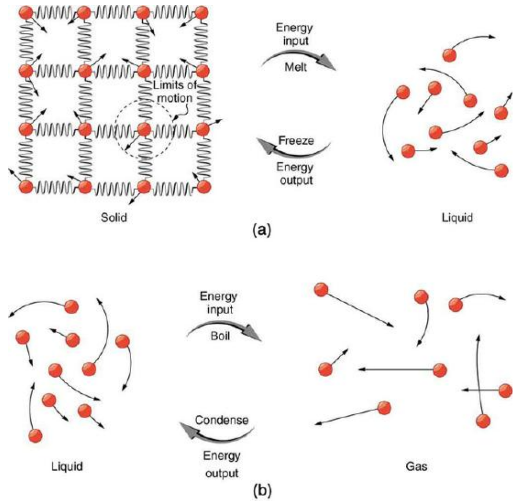
Energy is required to melt a solid because the cohesive bonds between the molecules in the solid must be broken apart so that the molecules can move around at comparable kinetic energies; thus, there is no rise in temperature. Similarly, energy is needed to vaporize a liquid, because molecules in a liquid interact with each other via attractive forces. There is no temperature change until a phase change is complete. The temperature of a glass of lemonade initially at 0 ºC stays at 0 ºC until all the ice has melted. Conversely, energy is released during freezing and condensation, usually in the form of thermal energy. Work is done by cohesive forces when molecules are brought together. The corresponding energy must be given off (dissipated) to allow them to stay together.

The energy involved in a phase change depends on two major factors: the number and strength of bonds or force pairs. The number of bonds is proportional to the number of molecules and thus to the mass of the sample. The strength of forces depends on the type of molecules. The heat Q required to change the phase of a sample of mass m is given by

$\mathrm{Q = mL_f}$ (melting or freezing)

$\mathrm{Q = mL_v}$ (evaporating or condensing)

where the latent heat of fusion, L_f, and latent heat of vaporization, L_v, are material constants that are determined experimentally.



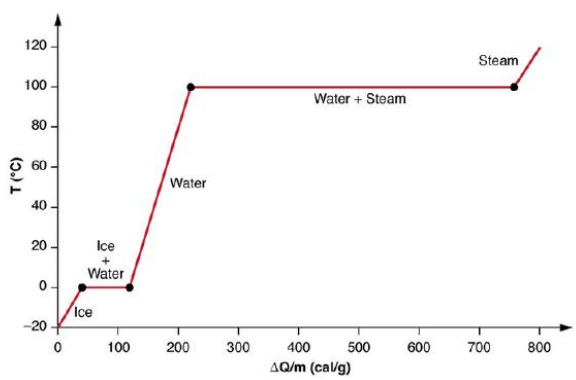
Phase Transitions: (a) Energy is required to partially overcome the attractive forces between molecules in a solid to form a liquid. That same energy must be removed for freezing to take place. (b) Molecules are separated by large distances when going from liquid to vapor, requiring significant energy to overcome molecular attraction. The same energy must be removed for condensation to take place. There is no temperature change until a phase change is complete.

Latent heat is an intensive property measured in units of J/kg. Both L_f and L_v depend on the substance, particularly on the strength of its molecular forces as noted earlier. L_f and L_v are collectively called latent heat coefficients. They are *latent*, or hidden, because in phase changes, energy enters or leaves a system without causing a temperature change in the system; so, in effect, the energy is hidden. Note that melting and vaporization are endothermic processes in that they absorb or require energy, while freezing and condensation are exothermic process as they release energy.

Heating Up Ice: Andrew Vanden Heuvel explores latent heat while trying to cool down his soda.

Significant amounts of energy are involved in phase changes. Let us look, for example, at how much energy is needed to melt a kilogram of ice at 0º C to produce a kilogram of water at 0°C. Using the equation for a change in temperature and the value for water (334 kJ/kg), we find that Q=mLf=(1.0kg)(334kJ/kg)=334kJ is the energy to melt a kilogram of ice. This is a lot of energy as it represents the same amount of energy needed to raise the temperature of 1 kg of liquid water from 0ºC to 79.8ºC. Even more energy is required to vaporize water; it would take 2256 kJ to change 1 kg of liquid water at the normal boiling point (100ºC at atmospheric pressure) to steam (water vapor). This example shows that the energy for a phase change is enormous compared to energy associated with temperature changes without a phase change.

Phase changes can have an enormous stabilizing effect (see figure below). Consider adding heat at a constant rate to a sample of ice initially at -20 ºC. Initially the temperature of the ice rises linearly, absorbing heat at a constant rate of 0.50 cal/g⋅C until it reaches 0 ºC. Once at this temperature, the ice begins to melt until all the entire sample has melted, absorbing a total of 79.8 cal/g of heat. The temperature remains constant at 0 ºC during this phase change. Once all the ice has melted, the temperature of the liquid water rises, absorbing heat at a new constant rate of 1.00 cal/g⋅C (remember that specific heats are dependent on phase). At 100ºC, the water begins to boil and the temperature again remains constant until the water absorbs 539 cal/g of heat to complete this phase change. When all the liquid has become steam, the temperature rises again, absorbing heat at a rate of 0.482 cal/g⋅C.



Heating and Phase Changes of Water: A graph of temperature versus energy added. The system is constructed so that no vapor evaporates while ice warms to become liquid water, and so that, when vaporization occurs, the vapor remains in of the system. The long stretches of constant temperature values at 0ºC and 100ºC reflect the large latent heat of melting and vaporization, respectively.

A phase change we have neglected to mention thus far is sublimation, the transition of solid directly into vapor. The opposite case, where vapor transitions directly into a solid, is called deposition. Sublimation has its own latent heat L*s* and can be used in the same way as $L_v$and L_f.

Key Points

- Energy is required to change the phase of a substance, such as the energy to break the bonds between molecules in a block of ice so it may melt.

- During a phase change energy my be added or subtracted from a system, but the temperature will not change. The temperature will change only when the phase change has completed.
- The heat Q required to change the phase of a sample of mass m is given by $Q = mL_f$ (melting or freezing) and $Q = mL_v$ (evaporating or condensing), where L_f and L_v are the latent heat of fusion and the latent heat of vaporization, respectively.

Key Terms

- **latent heat of fusion**: the energy required to transition one unit of a substance from solid to liquid; equivalently, the energy liberated when one unit of a substance transitions from liquid to solid.
- **latent heat of vaporization**: the energy required to transition one unit of a substance from liquid to vapor; equivalently, the energy liberated when one unit of a substance transitions from vapor to liquid.
- **sublimation**: the transition of a substance from the solid phase directly to the vapor state such that it does not pass through the intermediate, liquid phase

13.4: Methods of Heat Transfer

learning objectives

- Assess why particular characteristics are necessary for effective conduction

Conduction

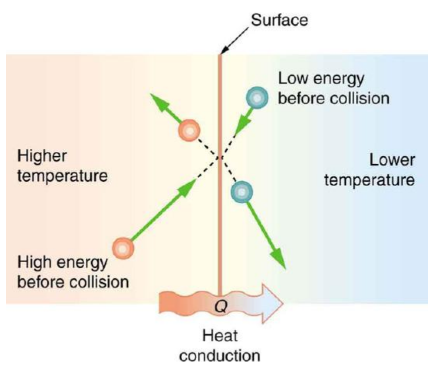
Conduction is the transfer of heat through stationary matter by physical contact. (The matter is stationary on a macroscopic scale—we know there is thermal motion of the atoms and molecules at any temperature above absolute zero.) Heat transferred from an electric stove to the bottom of a pot is an example of conduction.

Some materials conduct thermal energy faster than others. For example, the pillow in your room may the same temperature as the metal doorknob, but the doorknob feels cooler to the touch. In general, good conductors of electricity (metals like copper, aluminum, gold, and silver) are also good heat conductors, whereas insulators of electricity (wood, plastic, and rubber) are poor heat conductors.

Microscopic Description of Conduction

On a microscopic scale, conduction occurs as rapidly moving or vibrating atoms and molecules interact with neighboring particles, transferring some of their kinetic energy. Heat is transferred by conduction when adjacent atoms vibrate against one another, or as electrons move from one atom to another. Conduction is the most significant means of heat transfer within a solid or between solid objects in thermal contact. Conduction is greater in solids because the network of relatively close fixed spatial relationships between atoms helps to transfer energy between them by vibration.

Fluids and gases are less conductive than solids. This is due to the large distance between atoms in a fluid or (especially) a gas: fewer collisions between atoms means less conduction.

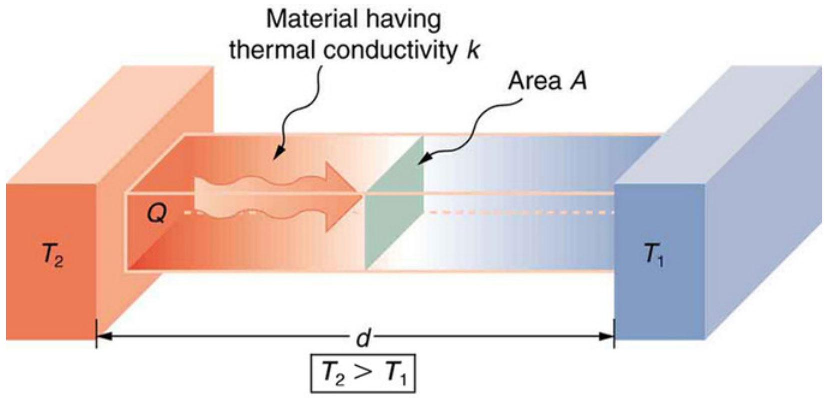


Microscopic Illustration of Conduction: The molecules in two bodies at different temperatures have different average kinetic energies. Collisions occurring at the contact surface tend to transfer energy from high-temperature regions to low-temperature regions. In this illustration, a molecule in the lower temperature region (right side) has low energy before collision, but its energy increases after colliding with the contact surface. In contrast, a molecule in the higher temperature region (left side) has high energy before collision, but its energy decreases after colliding with the contact surface.

The (average) kinetic energy of a molecule in the hot body is higher than in the colder body. If two molecules collide, an energy transfer from the hot to the cold molecule occurs (see the above figure). The cumulative effect from all collisions results in a net flux of heat from the hot body to the colder body. The heat flux thus depends on the temperature difference $\mathrm{T} = \mathrm{T}_{\text{hot}} - \mathrm{T}_{\text{cold}}$. Therefore, you will get a more severe burn from boiling water than from hot tap water. Conversely, if the temperatures are the same, the net heat transfer rate falls to zero, and equilibrium is achieved. Owing to the fact that the number of collisions increases with increasing area, heat conduction depends on the cross-sectional area. If you touch a cold wall with your palm, your hand cools faster than if you just touch it with your fingertip.

Factors Affecting the Rate of Heat Transfer Through Conduction

In addition to temperature and cross-sectional area, another factor affecting conduction is the thickness of the material through which the heat transfers. Heat transfer from the left side to the right side is accomplished by a series of molecular collisions. The thicker the material, the more time it takes to transfer the same amount of heat. If you get cold during the night, you may retrieve a thicker blanket to keep warm.



Effect of Thickness on Heat Conduction: Heat conduction occurs through any material, represented here by a rectangular bar. The temperature of the material is T_2 on the left and T_1 on the right, where T_2 is greater than T_1. The rate of heat transfer by conduction is directly proportional to the surface area A, the temperature difference $T_2 - T_1$, and the substance's conductivity kk. The rate of heat transfer is inversely proportional to the thickness d.

Lastly, the heat transfer rate depends on the material properties described by the coefficient of thermal conductivity. All four factors are included in a simple equation that was deduced from and is confirmed by experiments. The rate of conductive heat transfer through a slab of material, such as the one in the figure above is given by $\frac{Q}{t} = \frac{kA(T_2 - T_1)}{d}$ where $\frac{Q}{t}$ is the rate of heat transfer in Joules per second (Watts), k is the thermal conductivity of the material, A and d are its surface area and thickness, and $(T_2 - T_1)$ is the temperature difference across the slab.

Convection

Convection is the heat transfer by the macroscopic movement of a fluid, such as a car's engine kept cool by the water in the cooling system.

learning objectives

- Illustrate the mechanisms of convection with phase change

Example 13.4.1:

Calculating Heat Transfer by Convection: Convection of Air Through the Walls of a House.

Most houses are not airtight: air goes in and out around doors and windows, through cracks and crevices, following wiring to switches and outlets, and so on. The air in a typical house is completely replaced in less than an hour.

Suppose that a moderately-sized house has inside dimensions 12.0 m × 18.0 m × 3.00 m high, and that all air is replaced in 30.0 min. Calculate the heat transfer per unit time in watts needed to warm the incoming cold air by 10.0 ºC, thus replacing the heat transferred by convection alone.

Strategy:

Heat is used to raise the temperature of air so that $Q = mc\Delta T$. The rate of heat transfer is then $\frac{Q}{t}$, where t is the time for air turnover. We are given that ΔT is 10.0ºC, but we must still find values for the mass of air and its specific heat before we can calculate QQ. The specific heat of air is a weighted average of the specific heats of nitrogen and oxygen, which is $c = cp \cong 1000\ J/kg \cdot C$ (note that the specific heat at constant pressure must be used for this process).

Solution

(1) Determine the mass of air from its density and the given volume of the house. The density is given from the density ρ and the volume $m = \rho V = (1.29\ kg/m3)(12.0\ m \times 18.0\ m \times 3.00\ m) = 836\ kg$

(2) Calculate the heat transferred from the change in air temperature: $Q = mc\Delta T$ so that $Q = (836\ kg)(1000\ J/kg \cdot \circ C)(10 \circ C) = 8.36 \times 10^6\ J$

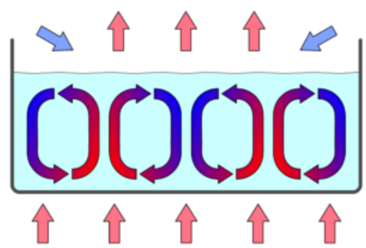
(3) Calculate the heat transfer from the heat Q and the turnover time t. Since air is turned over in $t = 0.500\ h = 1800\ s$, the heat transferred per unit time is $\frac{Q}{t} = \frac{8.36 \times 10^6\ J}{1800\ s} = 4.64\ kW$.

This rate of heat transfer is equal to the power consumed by about forty-six 100-W light bulbs.

Newly constructed homes are designed for a turnover time of 2 hours or more, rather than 30 minutes for the house of this example. Weather stripping, caulking, and improved window seals are commonly employed. More extreme measures are sometimes taken in very cold (or hot) climates to achieve a tight standard of more than 6 hours for one air turnover. Still longer turnover times are unhealthy, because a minimum amount of fresh air is necessary to supply oxygen for breathing and to dilute household pollutants. The term used for the process by which outside air leaks into the house from cracks around windows, doors, and the foundation is called “air infiltration.”

Convection

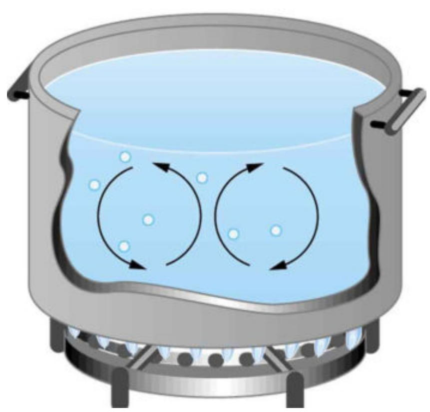
Convection (illustrated in) is the concerted, collective movement of ensembles of molecules within fluids (e.g., liquids, gases). Convection of mass cannot take place in solids, since neither bulk current flows nor significant diffusion can occur in solids. Instead heat diffusion in solids is called heat conduction, which we’ve just reviewed.



Convection Cells: Convection cells in a gravity field.

Convection is driven by large-scale flow of matter. In the case of Earth, the atmospheric circulation is caused by the flow of hot air from the tropics to the poles, and the flow of cold air from the poles toward the tropics. (Note that Earth’s rotation causes changes in the direction of airflow depending on latitude.). An example of convection is a car engine kept cool by the flow of water in the cooling system, with the water pump maintaining a flow of cool water to the pistons.

While convection is usually more complicated than conduction, we can describe convection and perform some straightforward, realistic calculations of its effects. Natural convection is driven by buoyant forces: hot air rises because density decreases as temperature increases. This principle applies equally with any fluid. For example, the pot of water on the stove in is kept warm in this manner; ocean currents and large-scale atmospheric circulation transfer energy from one part of the globe to another.



Convection in a Pot of Water: Convection plays an important role in heat transfer inside this pot of water. Once conducted to the inside, heat transfer to other parts of the pot is mostly by convection. The hotter water expands, decreases in density, and rises to transfer heat to other regions of the water, while colder water sinks to the bottom. This process keeps repeating.

Convection and Insulation

Although air can transfer heat rapidly by convection, it is a poor conductor and thus a good insulator. The amount of available space for airflow determines whether air acts as an insulator or conductor. The space between the inside and outside walls of a house, for example, is about 9 cm (3.5 in)—large enough for convection to work effectively. The addition of wall insulation prevents airflow, so heat loss (or gain) is decreased. Similarly, the gap between the two panes of a double-paned window is about 1 cm, which prevents convection and takes advantage of air's low conductivity to prevent greater loss. Fur, fiber and fiberglass also take advantage of the low conductivity of air by trapping it in spaces too small to support convection. In animals, fur and feathers are lightweight and thus ideal for their protection.

Convection and Phase Changes

Some interesting phenomena happen when convection is accompanied by a phase change. It allows us to cool off by sweating, even if the temperature of the surrounding air exceeds body temperature. Heat from the skin is required in order for sweat to evaporate from the skin, but without air flow the air becomes saturated and evaporation stops. Air flow caused by convection replaces the saturated air by dry air and thus evaporation continues.

Another important example of the combination of phase change and convection occurs when water evaporates from the ocean. Heat is removed from the ocean when water evaporates. If the water vapor condenses in liquid droplets as clouds form, heat is released in the atmosphere (this heat release is latent heat) . Thus, an overall transfer of heat from the ocean to the atmosphere occurs. This process is the driving power behind thunderheads—great cumulus clouds that rise as much as 20.0 km into the stratosphere. Water vapor carried in by convection condenses, releasing tremendous amounts of energy, and this energy allows air to become more buoyant (warmer than its surroundings) and rise. As the air continues to rise, more condensation occurs, which in turn drives the cloud even higher. Such a mechanism is called positive feedback, since the process reinforces and accelerates itself. These systems sometimes produce violent storms with lightning and hail, and constitute the mechanism that drives hurricanes.

Cumulus Clouds: Cumulus clouds are caused by water vapor that rises because of convection. The rise of clouds is driven by a positive feedback mechanism.

Radiation

Radiation is the transfer of heat through electromagnetic energy

learning objectives

- Explain how the energy of electromagnetic radiation corresponds with wavelength

Radiation

You can feel heat transfer from a fire or the Sun. Yet the space between Earth and the Sun is largely empty, without any possibility of heat transfer by convection or conduction. Similarly, you can tell that an oven is hot without touching it or looking inside—it just warms you as you walk by.

In these examples, heat is transferred by radiation. The hot body emits electromagnetic waves that are absorbed by our skin, and no medium is required for them to propagate. We use different names for electromagnetic waves of different wavelengths: radio waves, microwaves, infrared radiation, visible light, ultraviolet radiation, X-rays, and gamma rays .

Radiation from a Fire: Most of the heat transfer from this fire to the observers is through infrared radiation. The visible light, although dramatic, transfers relatively little thermal energy. Convection transfers energy away from the observers as hot air rises, while conduction is negligibly slow here. Skin is very sensitive to infrared radiation so that you can sense the presence of a fire without looking at it directly.

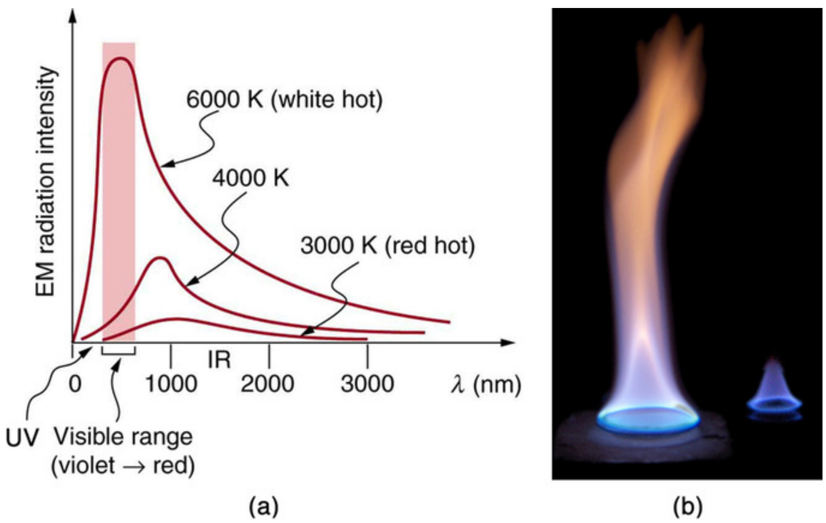
The energy of electromagnetic radiation depends on its wavelength (color) and varies over a wide range; a smaller wavelength (or higher frequency) corresponds to a higher energy. We can write this as:

$$\mathrm{E}=\mathrm{hf}=\frac{\mathrm{hc}}{\lambda} \tag{13.4.1}$$

where E is the energy, f is the frequency, λ is the wavelength, and h is a constant.

Because more heat is radiated at higher temperatures, a temperature change is accompanied by a color change. For example, an electrical element on a stove glows from red to orange, while the higher-temperature steel in a blast furnace glows from yellow to white. The radiation you feel is mostly infrared, which is lower in temperature still.

The radiated energy depends on its intensity, which is represented by the height of the distribution .



Radiation Spectrum: (a) A graph of the spectra of electromagnetic waves emitted from an ideal radiator at three different temperatures. The intensity or rate of radiation emission increases dramatically with temperature, and the spectrum shifts toward the visible and ultraviolet parts of the spectrum. The shaded portion denotes the visible part of the spectrum. It is apparent that the shift toward the ultraviolet with temperature makes the visible appearance shift from red to white to blue as temperature increases. (b) Note the variations in color corresponding to variations in flame temperature.

Heat Transfer

All objects absorb and emit electromagnetic radiation. The rate of heat transfer by radiation is largely determined by the color of the object. Black is the most effective, and white the least. People living in hot climates generally avoid wearing black clothing, for instance. Similarly, black asphalt in a parking lot will be hotter than the adjacent gray sidewalk on a summer day, because black absorbs better than gray. The reverse is also true—black radiates better than gray. Thus, on a clear summer night the asphalt will be colder than the gray sidewalk because black radiates energy more rapidly than gray.

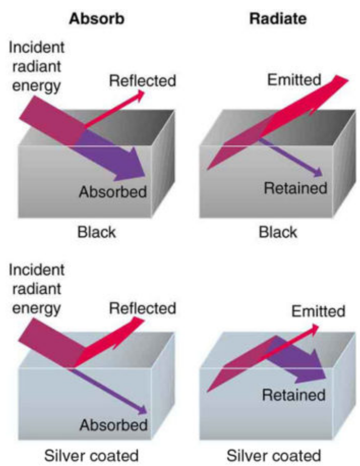
An ideal radiator, often called a blackbody, is the same color as an ideal absorber, and captures all the radiation that falls on it. In contrast, white is a poor absorber and also a poor radiator. A white object reflects all radiation, like a mirror. (A perfect, polished white surface is mirror-like in appearance, and a crushed mirror looks white.)

There is a clever relation between the temperature of an ideal radiator and the wavelength at which it emits the most radiation. It is called Wien's displacement law and is given by:

$$\mathrm{\lambda_max T = b} \tag{13.4.2}$$

where b is a constant equal to $2.9 \times 10^{-3}\ \mathrm{m \cdot K}$.

Gray objects have a uniform ability to absorb all parts of the electromagnetic spectrum. Colored objects behave in similar but more complex ways, which gives them a particular color in the visible range and may make them special in other ranges of the nonvisible spectrum. Take, for example, the strong absorption of infrared radiation by the skin, which allows us to be very sensitive to it .



Good and Poor Radiators: A black object is a good absorber and a good radiator, while a white (or silver) object is a poor absorber and a poor radiator. It is as if radiation from the inside is reflected back into the silver object, whereas radiation from the inside of the black object is "absorbed" when it hits the surface and finds itself on the outside and is strongly emitted.

The rate of heat transfer by emitted radiation is determined by the Stefan-Boltzmann law of radiation:

$$\mathrm{\frac{Q}{t} = \sigma e A T^4} \tag{13.4.3}$$

where $\sigma = 5.67 \times 10^{-8}\ \frac{\mathrm{J}}{\mathrm{s \cdot m^2 \cdot K^4}}$ is the Stefan-Boltzmann constant, A is the surface area of the object, and T is its absolute temperature in kelvin. The symbol e stands for the emissivity of the object, which is a measure of how well it radiates. An ideal jet-black (or blackbody) radiator has e=1e=1, whereas a perfect reflector has $\mathrm{e=0}$. Real objects fall between these two values. For example, tungsten light bulb filaments have an ee of about 0.5, and carbon black (a material used in printer toner), has the (greatest known) emissivity of about 0.99.

The radiation rate is directly proportional to the fourth power of the absolute temperature—a remarkably strong temperature dependence. Furthermore, the radiated heat is proportional to the surface area of the object. If you knock apart the coals of a fire, there is a noticeable increase in radiation due to an increase in radiating surface area.

Net Rate of Heat Transfer

The net rate of heat transfer by radiation (absorption minus emission) is related to both the temperature of the object and that of its surroundings. Assuming that an object with a temperature $\mathrm{T_1}$ is surrounded by an environment with uniform temperature $\mathrm{T_2}$, the net rate of heat transfer by radiation is:

\[\mathrm{\dfrac{Q_{net}}{t}=eAσ(T_2^4−T_1^4)}\)

where e is the emissivity of the object alone. In other words, it does not matter whether the surroundings are white, gray, or black; the balance of radiation into and out of the object depends on how well it emits and absorbs radiation. When $T_2 > T_1$, the quantity $\frac{Q_{net}}{t}$ is positive; that is, the net heat transfer is from hotter objects to colder objects.

Key Points

- On a microscopic scale, conduction occurs as rapidly moving or vibrating atoms and molecules interact with neighboring particles, transferring some of their kinetic energy.
- Conduction is the most significant form of heat transfer within a solid object or between solids in thermal contact.
- Conduction is most significant in solids, and less though in liquids and gases, due to the space between molecules.
- The rate of heat transfer by conduction is dependent on the temperature difference, the size of the area in contact, the thickness of the material, and the thermal properties of the material(s) in contact.
- Convection is driven by the large scale flow of matter in fluids. Solids cannot transport heat through convection.
- Natural convection is driven by buoyant forces: hot air rises because density decreases as temperature increases. This principle applies equally with any fluid.
- Convection can transport heat much more efficiently than conduction. Air is a poor conductor and a good insulator if the space is small enough to prevent convection.
- Convection often accompanies phase changes, such as when sweat evaporates from your body. This mass flow during convection allows humans to cool off even if the surrounding air’s temperature exceeds the body temperature.
- The energy of electromagnetic radiation depends on the wavelength (color) and varies over a wide range: a smaller wavelength (or higher frequency) corresponds to a higher energy.
- All objects emit and absorb electromagnetic energy. The color of an object is related emissivity, or its efficiency of radiating away energy. Black is the most effective while white is the least effective ($e = 1$ and $e = 0$, respectively).
- An ideal radiator, often called a blackbody, is the same color as an ideal absorber and captures all the radiation that falls on it.
- The rate of heat transfer by emitted radiation is determined by the Stefan-Boltzmann law of radiation: $\frac{Q}{t} = \sigma A T^4$ where $\sigma = 5.67 \times 10^{-8} \frac{J}{s \cdot m^2 \cdot K^4}$ is the Stefan-Boltzmann constant, A is the surface area of the object, and T is its absolute temperature in kelvin.
- The net rate of heat transfer is related to the temperature of the object and the temperature of its surroundings. The larger the difference, the higher the net heat flux.
- The temperature of an object is very significant, because the radiation emitted is proportional to this quantity to the fourth power.

Key Terms

- **thermal conductivity**: the measure of a material’s ability to conduct heat
- **natural convection**: A method for heat transport. A fluid surrounding a heat source receives heat, becomes less dense and rises. The surrounding, cooler fluid then moves to replace it. This cooler fluid is then heated and the process continues, forming a convection current.
- **positive feedback**: a feedback loop in which the output of a system is amplified with a net positive gain each cycle.
- **blackbody**: A theoretical body, approximated by a hole in a hollow black sphere, that absorbs all incident electromagnetic radiation and reflects none; it has a characteristic emission spectrum.
- **emissivity**: The energy-emitting propensity of a surface, usually measured at a specific wavelength.

13.5: Global Warming

learning objectives

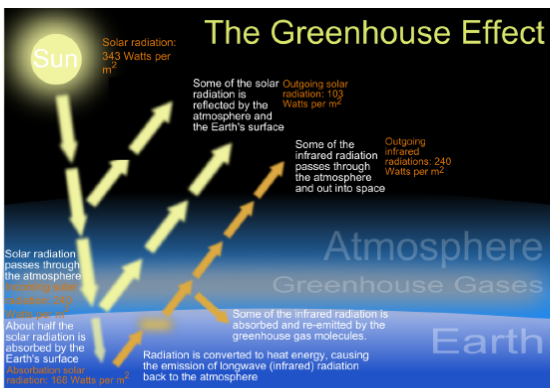
- Describe effect of greenhouse gases on the Earth's equilibrium temperature

Radiation is a natural process of heat transfer; everything is constantly radiating heat. When a system is at equilibrium, we do not notice this exchange of energy because every object absorbs and emits precisely the amount of energy it receives, but this equilibrium is dynamic–the exchange is always taking place. In general, as the temperature of an object increases, it emits energy more rapidly by emitting photons at a faster rate, and by emitting photons of a greater average energy.

If we approximate the Earth as a perfect absorber and emitter of the radiation received from the sun (called a blackbody), we would expect the Earth to be at an average temperature of 5°C, rather than the 14°C which we observe. The extra temperature comes from the Earth and it's atmosphere selectively absorbing certain wavelengths of radiation while reflecting other wavelengths. The 9°C discrepancy is due to the greenhouse effect. The gases of the atmosphere are "selective absorbers"; energy in the visible part of the electromagnetic spectrum passes through the atmosphere directly to the Earth's surface (with some reflection occurring as well). The Earth absorbs this energy and then re-emits radiation in the infrared portion of the spectrum. The gases in the atmosphere, primarily CO_2 and water vapor are highly absorbent in the infrared part of the spectrum. The atmosphere absorbs the infrared radiation from the Earth, preventing it from escaping to space. Because all objects are continually emitting radiation, the atmosphere (having absorbed the Earth's radiation) then emits radiation, some of which is then reabsorbed by the Earth's surface. Thus the greenhouse effect is a continuous cycle of absorption and emission of energy between the Earth and atmosphere. This causes the Earth's atmosphere and surface to be warmer than otherwise expected.

Radiative Transfer

The greenhouse effect is a phenomenon of radiative transfer, the process by which the energy of light waves is exchanged in matter. Radiative transfer dictates what energy is reflected, absorbed, and emitted.



The greenhouse effect: A summary of the heat transfer in the Earth's atmosphere.

Atmospheric Absorbers

The radiative transfer properties of atmospheric chemicals depend on the energy of the radiation (both for absorption and emission), and those properties are unique to each chemical. In the context of global warming, we find that it is important to consider both how chemicals and particles reflect sunlight and how they absorb energy radiated from the Earth. Atmospheric reflecters, notably sulfates and nitrates, reflect and scatter light before it ever hits the surface of the Earth, effectively reducing the power that the Earth receives. On the other hand, greenhouse gases such as carbon dioxide (CO_2), methane (CH_4), and nitrous oxide (N_2O) are characteristically strong absorbers of the energy radiated by the Earth's surface. They absorb so strongly because they typically exhibit resonant absorption behavior in the same energy range as the radiation emitted by the earth. These trap heat before it leaves the Earth, insulating the Earth and increasing the Earth's equilibrium temperature.

Entropy and Solar Radiation

The Second Law of Thermodynamics implies that in order to produce energy sources, entropy must be produced. On a planetary scale, this production of entropy is primarily accounted for by the absorption and re-emisson of radiation. In general, the earth radiates the same energy that it receives. (If greenhouse gases increase, then temperature increases, and higher temperatures cause the Earth to radiate more, compensating for the greater energy absorbed in the atmosphere.) With light radiation, we find the entropy carried by the photons decreases as temperature increases, $S \propto \frac{1}{T}$. Since the Earth is cooler than the Sun, as the Earth absorbs radiation from the Sun and re-emits radiation from the Earth's surface, there is a net production of entropy. This net production of entropy allows energy to be stored and reused, particularly in the form of chemical energy stored by cells through photosynthesis.

Key Points

- The atmosphere can both absorb and reflect radiation, either increasing or decreasing the Earth's average temperature.
- All bodies naturally emit radiation; warmer bodies emit more photons, with higher average energies.
- In absorbing sunlight and emitting infrared radiation, the Earth produces entropy.

Key Terms

- **selective absorber**: An object that will absorb radiation over a particular set of wavelengths but will not (is transparent) at other wavelengths.
- **greenhouse effect**: The process by which a planet is warmed by its atmosphere.
- **greenhouse gas**: Any gas, such as carbon dioxide or CFCs, that contributes to the greenhouse effect when released into the atmosphere.
- **radiative transfer**: The transfer of radiation (energy) leaving one object and being absorbed by another.
- **blackbody**: An object that is a perfect absorber and emitter of radiation.

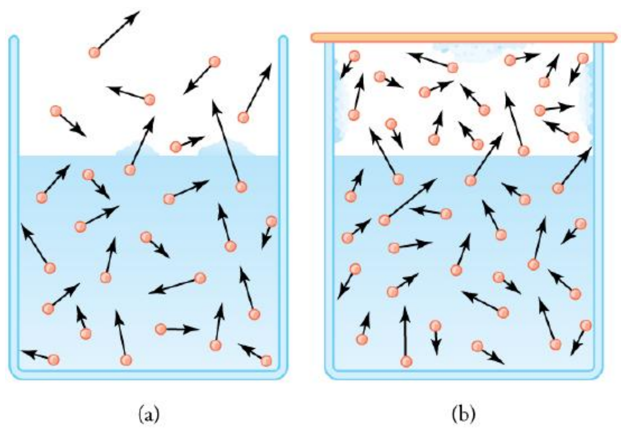
13.6: Phase Equilbrium

learning objectives

- Illustrate the causes for evaporation near the surface of a liquid

Evaporation is a type of vaporization of a liquid that only occurs on the liquid's surface. Usually, the molecules in a glass of water do not have enough heat energy to escape from the liquid. With sufficient heat, however, the liquid would quickly turn into vapor. When the molecules collide, they transfer energy to each other in varying degrees. Sometimes the transfer is so one-sided for a molecule near the surface that it achieves enough energy to escape the liquid.

Three key parts to evaporation are heat, atmospheric pressure (determines the percent humidity) and air movement. For molecules of a liquid to evaporate, they must be located near the surface, be moving in the proper direction, and have sufficient kinetic energy to overcome liquid- phase intermolecular forces. When only a small proportion of the molecules meet these criteria, the rate of evaporation is low. Since the kinetic energy of a molecule is proportional to its temperature, evaporation proceeds more quickly at higher temperatures. As the faster-moving molecules escape, the remaining molecules have lower average kinetic energy, and the temperature of the liquid decreases. This phenomenon is also called evaporative cooling. This is why evaporating sweat cools the human body. Evaporation also tends to proceed more quickly with higher flow rates between the gaseous and liquid phases and in liquids with higher vapor pressure. For example, laundry on a clothes line will dry (by evaporation) more rapidly on a windy day than on a still day.



Vapor Pressure Diagram: (a) Because of the distribution of speeds and kinetic energies, some water molecules can break away to the vapor phase even at temperatures below the ordinary boiling point. (b) If the container is sealed, evaporation will continue until there is enough vapor density for the condensation rate to equal the evaporation rate. This vapor density and the partial pressure it creates are the saturation values. They increase with temperature and are independent of the presence of other gases, such as air. They depend only on the vapor pressure of water.

Evaporation is an essential part of the water cycle. The sun (solar energy) drives evaporation of water from oceans, lakes, moisture in the soil, and other sources of water. In hydrology, evaporation and transpiration (which involves evaporation within plant stomata) are collectively termed evapotranspiration. Evaporation of water occurs when the surface of the liquid is exposed, allowing molecules to escape and form water vapor; this vapor can then rise and form clouds.

The Evaporating Atmosphere

At equilibrium, evaporation and condensation processes exactly balance and there is no net change in the volume of either phase.

learning objectives

- Explain how a substance can have multiple distinct phases in the same environment

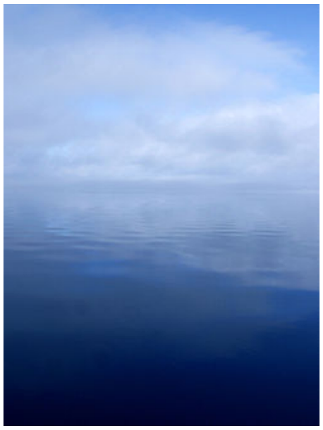
Phase Equilibrium

Left to equilibration, many compositions will form a uniform single phase, but depending on the temperature and pressure even a single substance may separate into two or more distinct phases. Within each phase, the properties are uniform but between the two phases properties differ.

Water in a closed jar with an air space over it forms a two phase system. Most of the water is in the liquid phase, where it is held by the mutual attraction of water molecules. Even at equilibrium, molecules are constantly in motion and, once in a while, a molecule in the liquid phase gains enough kinetic energy to break away from the liquid phase and enter the gas phase. Likewise, every once in a while a vapor molecule collides with the liquid surface and condenses into the liquid. At equilibrium, evaporation and condensation processes exactly balance and there is no net change in the volume of either phase.

At room temperature and pressure, the water jar reaches equilibrium when the air over the water has a humidity of about 3%. This percentage increases as the temperature goes up. At 100 °C and atmospheric pressure, equilibrium is not reached until the air is 100% water. If the liquid is heated a little over 100 °C, the transition from liquid to gas will occur not only at the surface, but throughout the liquid volume: the water boils.

The Earth's atmosphere is not unchanging. The water vapor in it changes phases. It is in a phase equilibrium. Collisions between water molecules in the atmosphere allows some to condense and some to remain in vapor. Similarly, several lighter gases can escape the gravitational field entirely.



Water Vapor in the Atmosphere: Water vapor condenses in the atmosphere

Key Points

- Evaporation turns liquids into gas.
- Evaporation can take place at temperatures below boiling point since the molecules in the liquid have different energies.
- As the molecules in a liquid collide, some achieve higher energies, allowing them to escape. This process lowers the energy of the remaining molecules and is the source of cooling in evaporating liquids.
- The atmosphere is made of gases in a phase equilibrium.
- As molecules in the atmosphere collide, they gain and lose energy.
- As water evaporates from the surface of the earth, water condenses in the atmosphere.

Key Terms

- **Vaporization**: a conversion of a solid or a liquid into a gas
- **Evaporation**: The process of a liquid converting to the gaseous state.
- **condensation**: The conversion of a gas to a liquid; the condensate so formed
- **equilibrium**: The state of a body at rest or in uniform motion, the resultant of all forces on which is zero.

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CHAPTER OVERVIEW

14: Thermodynamics

Topic hierarchy

14.1: Introduction

learning objectives

- Analyze the necessity to exclude energy transferred between system as heat from mechanical work

Work

In thermodynamics, work performed by a closed system is the energy transferred to another system that is measured by mechanical constraints on the system. Thermodynamic work encompasses mechanical work (gas expansion,) plus many other types of work, such as electrical. As such, thermodynamic work is a generalization of the concept of mechanical work in mechanics. It necessarily excludes energy transferred between systems as heat, which is modeled distinctly in thermodynamics. For closed systems, energy changes in a system other than as work transfer are as heat.

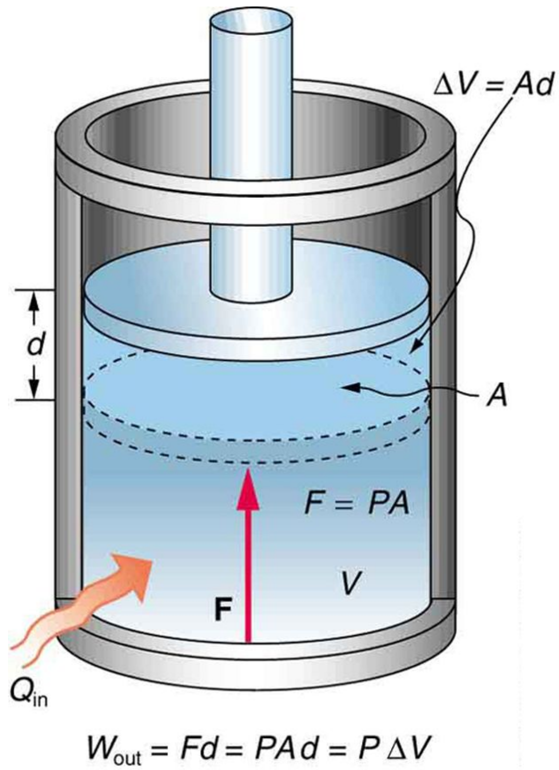


Fig 1: An isobaric expansion of a gas requires heat transfer during the expansion to keep the pressure constant. Since pressure is constant, the work done is PΔV.

Heat and Work

Heat transfer (often represented by Q) and doing work (W) are the two everyday means of bringing energy into or taking energy out of a system. The processes are quite different. Heat transfer, a less organized process, is driven by temperature differences. Work, a quite organized process(as in gas expansion), involves a macroscopic force exerted through a distance. Nevertheless, heat and work can produce identical results. Both heat and work can cause a temperature increase.

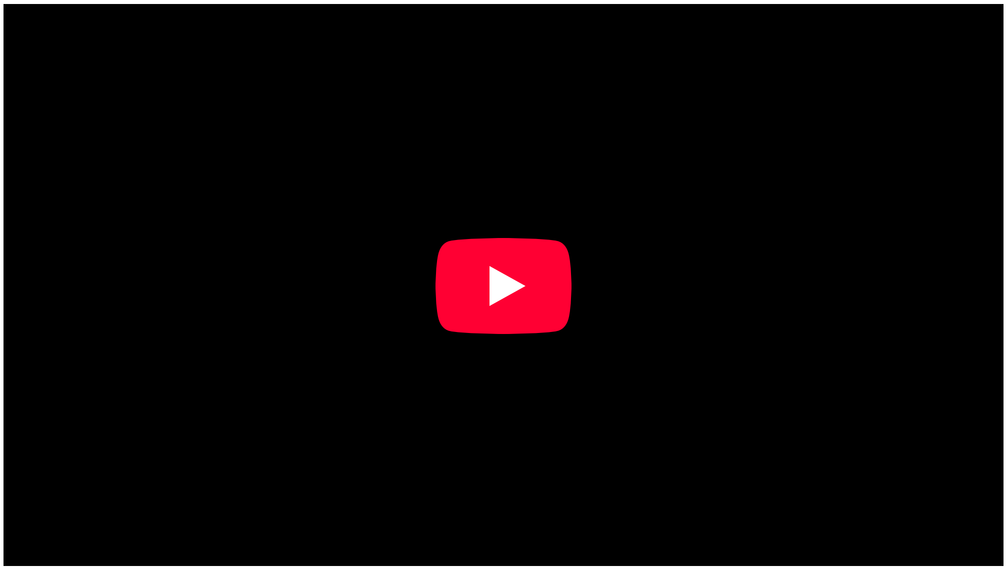
Heat transfer into a system, such as when the Sun warms the air in a bicycle tire, can increase its temperature, and so can work done on the system, as when the bicyclist pumps air into the tire. Once the temperature increase has occurred, it is impossible to tell whether it was caused by heat transfer or by doing work.

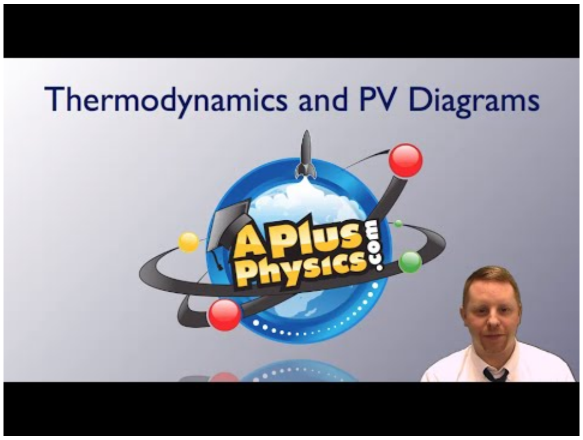
This uncertainty is an important point. Heat transfer and work are both energy in transit—neither is stored as such in a system. However, both can change the internal energy of a system. Internal energy is a form of energy completely different from either heat or work.

A Review of the Zeroth Law

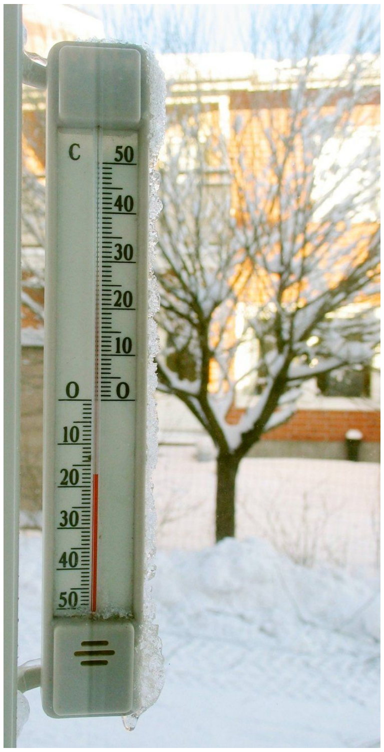
Zeroth law justifies the use of thermodynamic temperature, defined as the shared temperature of three designated systems at equilibrium.

> **learning objectives**
>
> - Discuss how the Zeroth Law of Thermodynamics justifies the use of thermodynamic temperature





Thermodynamics and PV Diagrams: A brief introduction to the zeroth and 1st laws of thermodynamics as well as PV diagrams for students.



Thermometer: A thermometer calibrated in degrees Celsius

The Zeroth Law of Thermodynamics states: *If two systems, A and B, are in thermal equilibrium with each other, and B is in thermal equilibrium with a third system, C, then A is also in thermal equilibrium with C.*

This law was postulated in the 1930s, after the first and second laws of thermodynamics had been developed and named. It is called the "zeroth" law because it comes logically before the first and second laws (discussed in Atoms on the 1st and 2nd laws).

Two systems are in thermal equilibrium if they could transfer heat between each other, but don't. Indeed, experiments have shown that if two systems, A and B, are in thermal equilibrium with each other, and B is in thermal equilibrium with a third system C, then A is also in thermal equilibrium with C. This conclusion may seem obvious, because all three have the same temperature, but zeroth law is basic to thermodynamics. Zeroth law justifies the use of thermodynamic temperature: the common "label" that the three systems in the definition above share is defined as the temperature of the systems.

Temperature

Thermometers actually take their own temperature, not the temperature of the object they are measuring. This raises the question of how we can be certain that a thermometer measures the temperature of the object with which it is in contact. The answer lies in the fact that any two systems placed in thermal contact (meaning heat transfer can occur between them) will reach the same temperature. That is, heat will flow from the hotter object to the cooler one until they reach exactly the same temperature. The objects are then in thermal equilibrium, and no further changes will occur. The systems interact and change because their temperatures differ, and the changes stop once their temperatures are the same. Thus, if enough time is allowed for this transfer of heat to run its course, the temperature a thermometer registers does represent the system with which it achieves thermal equilibrium.

Key Points

- Thermodynamic work is a generalization of the concept of mechanical work in mechanics.
- For closed systems, energy changes in a system other than as work transfer are as heat.
- Work in thermodynamics is a quite organized process (as in gas expansion), involving a macroscopic force exerted through a distance.
- The zeroth law of thermodynamics states that when systems, A and B, are in thermal equilibrium with each other, and B is in thermal equilibrium with a third system, C, then A is also in thermal equilibrium with C.
- Two systems are in thermal equilibrium if they could transfer heat between each other, but don't.

- If enough time is allowed for heat transfer to occur between a thermometer and a system, the temperature the thermometer registers does represent the system with which it achieves thermal equilibrium.

Key Terms

- **heat**: energy transferred from one body to another by thermal interactions
- **thermodynamics**: a branch of natural science concerned with heat and its relation to energy and work
- **internal energy**: The sum of all energy present in the system, including kinetic and potential energy; equivalently, the energy needed to create a system, excluding the energy necessary to displace its surroundings.
- **thermal equilibrium**: Two systems are in thermal equilibrium if they could transfer heat between each other, but don't.
- **thermodynamic temperature**: Temperature defined in terms of the laws of thermodynamics rather than the properties of a real material: expressed in kelvins

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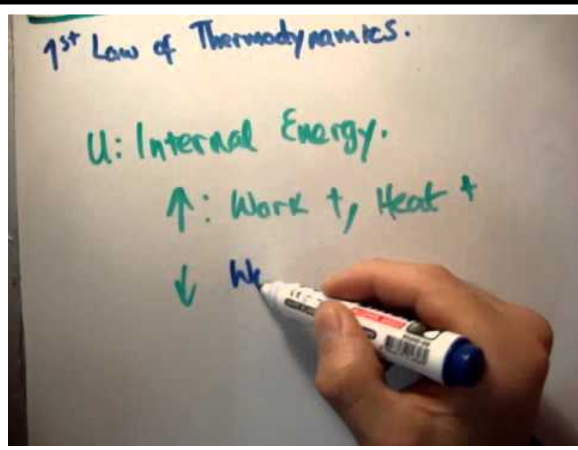
14.2: The First Law of Thermodynamics

learning objectives

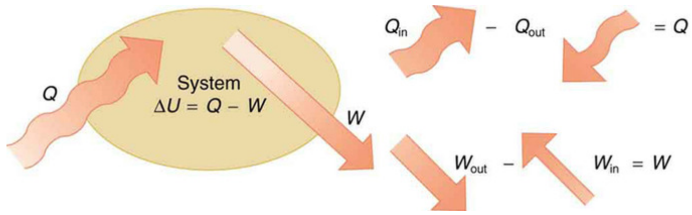
- Explain how the net heat transferred and net work done in a system relate to the first law of thermodynamics

The first law of thermodynamics is a version of the law of conservation of energy specialized for thermodynamic systems. It is usually formulated by stating that the change in the internal energy of a closed system is equal to the amount of heat supplied to the system, minus the amount of work done by the system on its surroundings. The law of conservation of energy can be stated like this: The energy of an isolated system is constant.



First Law of Thermodynamics: In this video I continue with my series of tutorial videos on Thermal Physics and Thermodynamics. It's pitched at undergraduate level and while it is mainly aimed at physics majors, it should be useful to anybody taking a first course in thermodynamics such as engineers etc..

If we are interested in how heat transfer is converted into work, then the conservation of energy principle is important. The first law of thermodynamics applies the conservation of energy principle to systems where heat transfer and doing work are the methods of transferring energy into and out of the system. In equation form, the first law of thermodynamics is



Internal Energy: The first law of thermodynamics is the conservation-of-energy principle stated for a system where heat and work are the methods of transferring energy for a system in thermal equilibrium. Q represents the net heat transfer—it is the sum of all heat transfers into and out of the system. Q is positive for net heat transfer into the system. W is the total work done on and by the system. W is positive when more work is done by the system than on it. The change in the internal energy of the system, ΔU, is related to heat and work by the first law of thermodynamics, $\Delta U = Q - W$.

$$\Delta U = Q - W. \tag{14.2.1}$$

Here ΔU is the change in internal energy U of the system, Q is the net heat transferred into the system, and W is the net work done by the system. We use the following sign conventions: if Q is positive, then there is a net heat transfer into the system; if W is positive, then there is net work done by the system. So positive Q adds energy to the system and positive W takes energy from the system. Thus $\Delta U = Q - W$. Note also that if more heat transfer into the system occurs than work done, the difference is stored as internal energy. Heat engines are a good example of this—heat transfer into them takes place so that they can do work.

Constant Pressure and Volume

Isobaric process is one in which a gas does work at constant pressure, while an isochoric process is one in which volume is kept constant.

learning objectives

- Contrast isobaric and isochoric processes

According to the first law of thermodynamics, heat transferred to a system can be either converted to internal energy or used to do work to the environment. A process in which a gas does work on its environment at constant pressure is called an isobaric process, while one in which volume is kept constant is called an isochoric process.

Isobaric Process (Constant Pressure)

An isobaric process occurs at constant pressure. Since the pressure is constant, the force exerted is constant and the work done is given as PΔV. An example would be to have a movable piston in a cylinder, so that the pressure inside the cylinder is always at atmospheric pressure, although it is isolated from the atmosphere. In other words, the system is dynamically connected, by a movable boundary, to a constant-pressure reservoir. If a gas is to expand at a constant pressure, heat should be transferred into the system at a certain rate. This process is called an isobaric expansion.

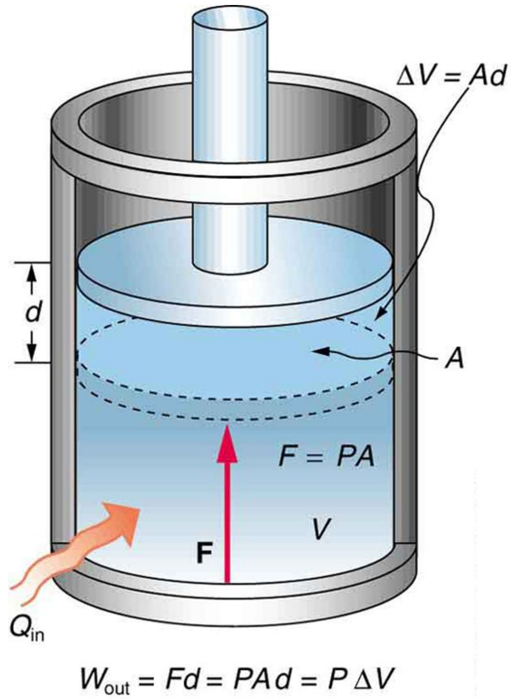


Fig 1: An isobaric expansion of a gas requires heat transfer during the expansion to keep the pressure constant. Since pressure is constant, the work done is PΔV.

Isochoric Process (Constant Volume)

An isochoric process is one in which the volume is held constant, meaning that the work done by the system will be zero. It follows that, for the simple system of two dimensions, any heat energy transferred to the system externally will be absorbed as internal energy. An isochoric process is also known as an isometric process or an isovolumetric process. An example would be to place a closed tin can containing only air into a fire. To a first approximation, the can will not expand, and the only change will be that the gas gains internal energy, as evidenced by its increase in temperature and pressure. Mathematically,

$$\Delta Q = \Delta U. \tag{14.2.2}$$

We may say that the system is dynamically insulated, by a rigid boundary, from the environment.

Isothermal Processes

An isothermal process is a change of a thermodynamic system, in which the temperature remains constant.

learning objectives

- Identify the typical systems in which an isothermal process occurs

An isothermal process is a change of a system, in which the temperature remains constant: $\Delta T = 0$. This typically occurs when a system is in contact with an outside thermal reservoir (heat bath), and the change occurs slowly enough to allow the system to continually adjust to the temperature of the reservoir through heat exchange. In contrast, an adiabatic process is where a system exchanges no heat with its surroundings ($Q = 0$). (See our atom on “Adiabatic Process. “) In other words, in an isothermal process, the value $\Delta T = 0$ but $Q \neq 0$, while in an adiabatic process, $\Delta T \neq 0$ but $Q = 0$.

Ideal Gas in an Isothermal Process

For an ideal, the product of pressure and volume (PV) is a constant if the gas is kept at isothermal conditions. (This is historically called Boyle’s law.) However, the cases where the product PV is an exponential term, does not comply. The value of the constant is nRT, where n is the number of moles of gas present and R is the ideal gas constant. In other words, the ideal gas law $PV = nRT$ applies. This means that

$$P = \frac{nRT}{V} = \frac{constant}{V} \tag{14.2.3}$$

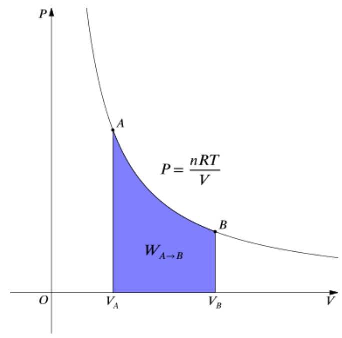
holds.

Calculation of Work

In thermodynamics, the work involved when a gas changes from state A to state B is simply

$$W_{A \to B} = \int_{V_A}^{V_B} P\, dV. \tag{14.2.4}$$

For an isothermal, reversible process, this integral equals the area under the relevant pressure-volume isotherm, and is indicated in blue in for an ideal gas. Again, $P = \frac{nRT}{V}$ applies and with T being constant (as this is an isothermal process), we have



Work Done by Gas During Expansion: The blue area represents "work" done by the gas during expansion for this isothermal change.

$$W_{A \to B} = nRT \int_{V_A}^{V_B} \frac{1}{V} dV = nRT \ln \frac{V_B}{V_A}. \tag{14.2.5}$$

It is also worth noting that, for many systems, if the temperature is held constant, the internal energy of the system also is constant, and so ΔU=0ΔU=0. From the first law of thermodynamics, it follows that Q=−WQ=−W for this same isothermal process.

Adiabatic Processes

An adiabatic process is any process occurring without gain or loss of heat within a system.

learning objectives

- Assess the environments in which isothermal processes typically occur

An isothermal process is a change of a system, in which the temperature remains constant: $\Delta T = 0$. This typically occurs when a system is in contact with an outside thermal reservoir (heat bath), and the change occurs slowly enough to allow the system to continually adjust to the temperature of the reservoir through heat exchange. In contrast, an adiabatic process is where a system exchanges no heat with its surroundings ($Q = 0$). (See our atom on "Adiabatic Process. ") In other words, in an isothermal process, the value $\Delta T = 0$ but $Q \neq 0$, while in an adiabatic process, $\Delta T \neq 0$ but $Q = 0$.

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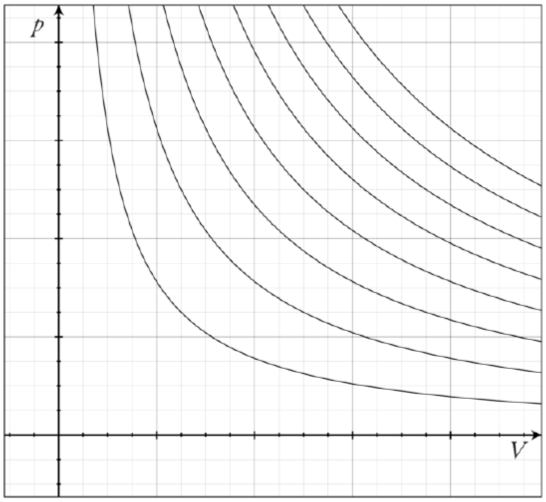
$$P = \frac{nRT}{V} = \frac{constant}{V} \tag{14.2.6}$$

holds. The family of curves generated by this equation is shown in. Each curve is called an isotherm.



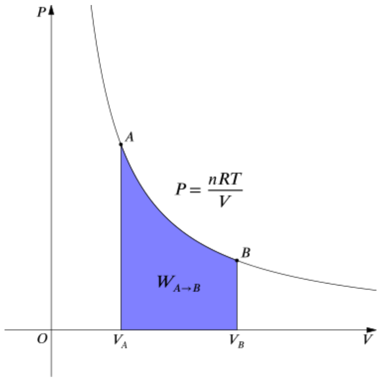
Isotherms of an Ideal Gas: Several isotherms of an ideal gas on a PV diagram.

Calculation of Work

In thermodynamics, the work involved when a gas changes from state A to state B is simply

$$W_{A\rightarrow B} = \int_{V_A}^{V_B} P\ dV. \tag{14.2.7}$$

For an isothermal, reversible process, this integral equals the area under the relevant pressure-volume isotherm, and is indicated in blue in for an ideal gas. Again, P = nRT / V applies and with T being constant (as this is an isothermal process), we have



Work Done by Gas During Expansion: The blue area represents "work" done by the gas during expansion for this isothermal change.

$$W_{A\rightarrow B} = nRT \int_{V_A}^{V_B} \frac{1}{V}\ dV = nRT \ln \frac{V_B}{V_A}. \tag{14.2.8}$$

It is also worth noting that, for many systems, if the temperature is held constant, the internal energy of the system also is constant, and so $\Delta U = 0$. From the first law of thermodynamics, it follows that $Q = -W$ for this same isothermal process.

Human Metabolism

The 1st law of thermodynamics explains human metabolism: the conversion of food into energy that is used by the body to perform activities.

learning objectives

- Contrast catabolism and anabolism in regards to energy

Metabolism in humans is the conversion of food into energy, which is then used by the body to perform activities. It is an example of the first law of thermodynamics in action. Considering the body as the system of interest, we can use the first law to examine heat transfer, doing work, and internal energy in activities ranging from sleep to heavy exercise. For example, one major factor in such activities is body temperature—normally kept constant by heat transfer to the surroundings, meaning that Q is negative (i.e., our body loses heat). Another factor is that the body usually does work on the outside world, meaning that W is positive. Thus, in such situations the body loses internal energy, since $\Delta U = Q - W$ is negative.

Eating

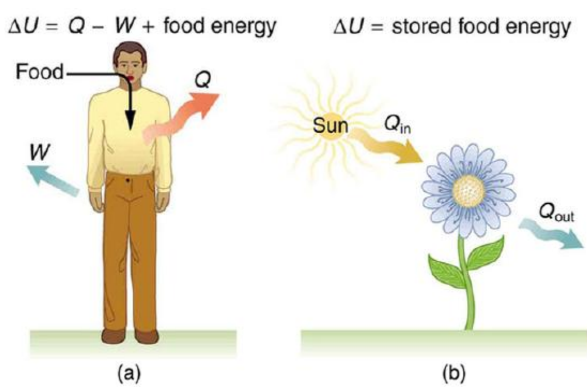
Now consider the effects of eating. The body metabolizes all the food we consume. Eating increases the internal energy of the body by adding chemical potential energy. In essence, metabolism uses an oxidation process in which the chemical potential energy of food is released. This implies that food input is in the form of work. Food energy is reported in a special unit, known as the Calorie. This energy is measured by burning food in a calorimeter, which is how the units are determined.

Catabolism and Anabolism

Catabolism is the pathway that breaks down molecules into smaller units and produces energy. Anabolism is the building up of molecules from smaller units. Anabolism uses up the energy produced by the catabolic break down of your food to create molecules more useful to your body.

Internal energy

Our body loses internal energy, and there are three places this internal energy can go—to heat transfer, to doing work, and to stored fat (a tiny fraction also goes to cell repair and growth). As shown in Fig 1 heat transfer and doing work take internal energy out of the body, and then food puts it back. If you eat just the right amount of food, then your average internal energy remains constant. Whatever you lose to heat transfer and doing work is replaced by food, so that, in the long run, $\Delta U=0$. If you overeat repeatedly, then ΔU is always positive, and your body stores this extra internal energy as fat. The reverse is true if you eat too little. If ΔU is negative for a few days, then the body metabolizes its own fat to maintain body temperature and do work that takes energy from the body. This process is how dieting produces weight loss.



Metabolism: (a) The first law of thermodynamics applied to metabolism. Heat transferred out of the body (Q) and work done by the body (W) remove internal energy, while food intake replaces it. (Food intake may be considered as work done on the body.) (b) Plants convert part of the radiant heat transfer in sunlight to stored chemical energy, a process called photosynthesis.

Metabolism

Life is not always this simple, as any dieter knows. The body stores fat or metabolizes it only if energy intake changes for a period of several days. Once you have been on a major diet, the next one is less successful because your body alters the way it responds to low energy intake. Your basal metabolic rate is the rate at which food is converted into heat transfer and work done while the body is at complete rest. The body adjusts its basal metabolic rate to compensate (partially) for over-eating or under-eating. The body will decrease the metabolic rate rather than eliminate its own fat to replace lost food intake. You will become more easily chilled and feel less energetic as a result of the lower metabolic rate, and you will not lose weight as fast as before. Exercise helps with weight loss because it produces both heat transfer from your body and work, and raises your metabolic rate even when you are at rest.

Irreversibility

The body provides us with an excellent indication that many thermodynamic processes are irreversible. An irreversible process can go in one direction but not the reverse, under a given set of conditions. For example, although body fat can be converted to do work and produce heat transfer, work done on the body and heat transfer into it cannot be converted to body fat. Otherwise, we could skip lunch by sunning ourselves or by walking down stairs. Another example of an irreversible thermodynamic process is photosynthesis. This process is the intake of one form of energy—light—by plants and its conversion to chemical potential energy. Both applications of the first law of thermodynamics are illustrated in. One great advantage of such conservation laws is that they accurately describe the beginning and ending points of complex processes (such as metabolism and photosynthesis) without regard to the complications in between.

Key Points

- The first law of thermodynamics is a version of the law of conservation of energy, specialized for thermodynamical systems.
- In equation form, the first law of thermodynamics is $\Delta U = Q - W$.
- Heat engines are a good example of the application of the 1st law; heat transfer into them takes place so that they can do work.
- An isobaric process occurs at constant pressure. Since the pressure is constant, the force exerted is constant and the work done is given as $P\Delta V$.
- An isobaric expansion of a gas requires heat transfer to keep the pressure constant.
- An isochoric process is one in which the volume is held constant, meaning that the work done by the system will be zero. The only change will be that a gas gains internal energy.
- For an ideal gas, the product of pressure and volume (PV) is a constant if the gas is kept at isothermal conditions.
- For an ideal gas, the work involved when a gas changes from state A to state B through an isothermal process is given as $W_{A \rightarrow B} = nRT \ln \frac{V_B}{V_A}$.
- For many systems, if the temperature is held constant, the internal energy of the system also is constant. It follows that $Q = -W$ in this case.
- Adiabatic processes can occur if the container of the system has thermally-insulated walls or the process happens in an extremely short time.
- For an adiabatically expanding ideal monatomic gas which does work on its environment (W is positive), internal energy of the gas should decrease.
- In a sense, isothermal process can be considered as the opposite extreme of adiabatic process. In isothermal processes, heat exchange is slow enough so that the system's temperature remains constant.
- Human metabolism is a complicated process. The 1st law of thermodynamics describes the beginning and ending points of these processes.
- Our body loses internal energy. There are three places this internal energy can go—to heat transfer, to doing work, and to stored fat.
- Our body provides a good example of irreversible processes. Although body fat can be converted to do work and produce heat transfer, work done on the body and heat transfer into it cannot be converted to body fat.

Key Terms

- **internal energy**: The sum of all energy present in the system, including kinetic and potential energy; equivalently, the energy needed to create a system, excluding the energy necessary to displace its surroundings.
- **heat**: energy transferred from one body to another by thermal interactions
- **law of conservation of energy**: The law stating that the total amount of energy in any isolated system remains constant, and cannot be created or destroyed, although it may change forms.
- **internal energy**: The sum of all energy present in the system, including kinetic and potential energy; equivalently, the energy needed to create a system, excluding the energy necessary to displace its surroundings.
- **reversible**: Capable of returning to the original state without consumption of free energy and increase of entropy.
- **ideal gas**: A hypothetical gas whose molecules exhibit no interaction and undergo elastic collision with each other and with the walls of the container.
- **Boyle's law**: The observation that the pressure of an ideal gas is inversely proportional to its volume at constant temperature.

- **ideal gas**: A hypothetical gas whose molecules exhibit no interaction and undergo elastic collision with each other and with the walls of the container.
- **metabolism**: The complete set of chemical reactions that occur in living cells.
- **oxidation**: A reaction in which the atoms of an element lose electrons and the valence of the element increases.
- **calorie**: The energy needed to increase the temperature of 1 kilogram of water by 1 kelvin. It is equivalent to 1,000 (small) calories.

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14.3: The Second Law of Thermodynamics

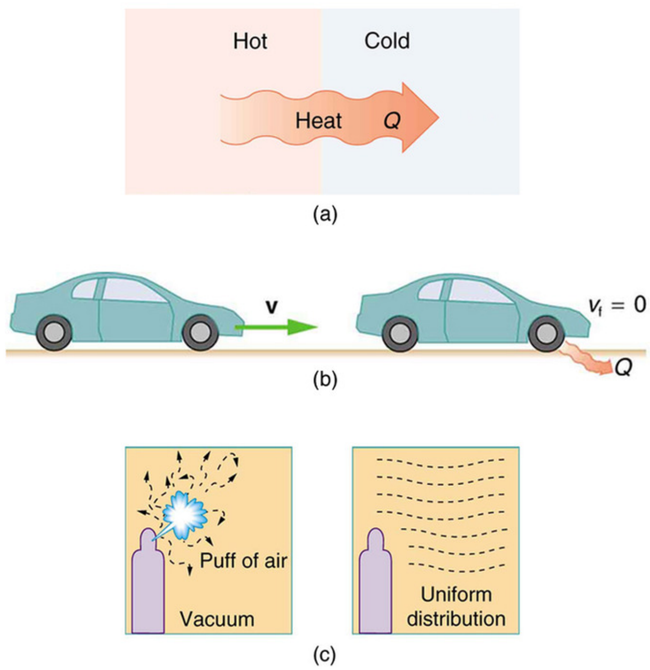
learning objectives

- Contrast the concept of irreversibility between the First and Second Laws of Thermodynamics

Irreversibility

The second law of thermodynamics deals with the direction taken by spontaneous processes. Many processes occur spontaneously in one direction only—that is, they areirreversible, under a given set of conditions. Although irreversibility is seen in day-to-day life—a broken glass does not resume its original state, for instance—complete irreversibility is a statistical statement that cannot be seen during the lifetime of the universe. More precisely, an irreversible process is one that depends on path. If the process can go in only one direction, then the reverse path differs fundamentally and the process cannot be reversible.

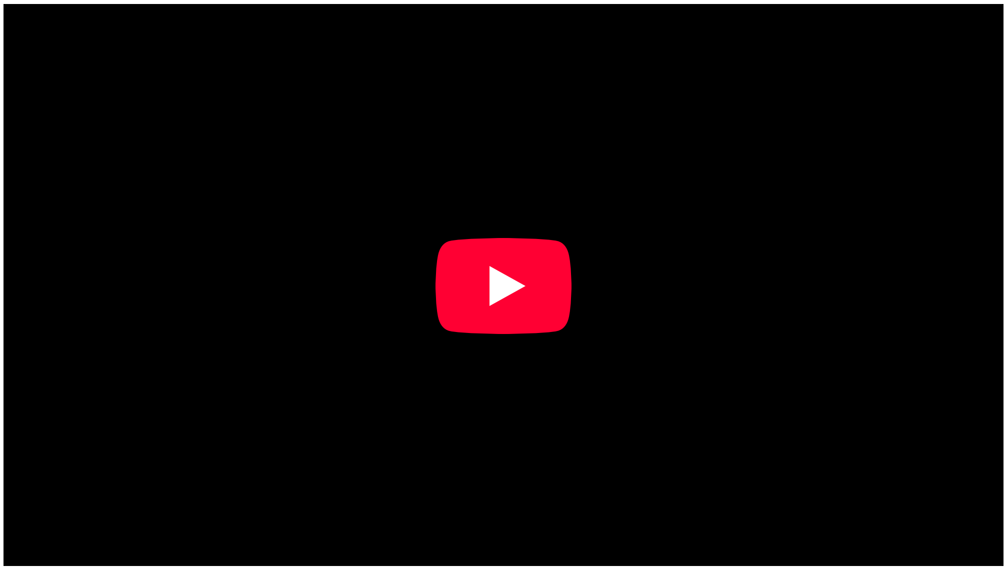
For example, heat involves the transfer of energy from higher to lower temperature. A cold object in contact with a hot one never gets colder, transferring heat to the hot object and making it hotter. Furthermore, mechanical energy, such as kinetic energy, can be completely converted to thermal energy by friction, but the reverse is impossible. A hot stationary object never spontaneously cools off and starts moving. Yet another example is the expansion of a puff of gas introduced into one corner of a vacuum chamber. The gas expands to fill the chamber, but it never regroups in the corner. The random motion of the gas molecules could take them all back to the corner, but this is never observed to happen.

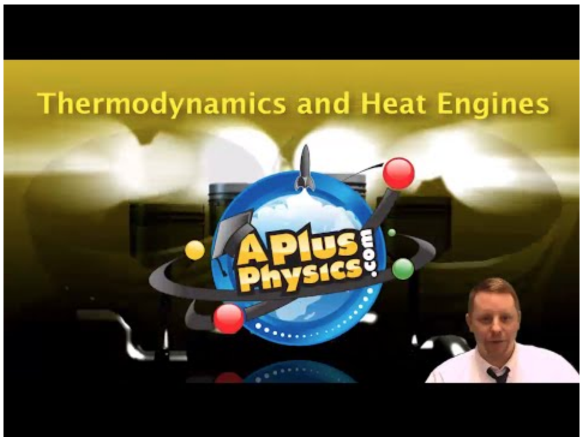


One-Way Processed in Nature: Examples of one-way processes in nature. (a) Heat transfer occurs spontaneously from hot to cold and not from cold to hot. (b) The brakes of this car convert its kinetic energy to heat transfer to the environment. The reverse process is impossible. (c) The burst of gas let into this vacuum chamber quickly expands to uniformly fill every part of the chamber. The random motions of the gas molecules will never return them to the corner.

Second Law of Thermodynamics

The fact that certain processes never occur suggests that there is a law forbidding them to occur. The first law of thermodynamics would allow them to occur—none of those processes violate conservation of energy. The law that forbids these processes is called the second law of thermodynamics. We shall see that the second law can be stated in many ways that may seem different, but these many ways are, in fact, equivalent. Like all natural laws, the second law of thermodynamics gives insights into nature, and its several statements imply that it is broadly applicable, fundamentally affecting many apparently disparate processes. The already familiar direction of heat transfer from hot to cold is the basis of our first version of the second law of thermodynamics.





Thermodynamics and Heat Engines: A brief introduction to heat engines and thermodynamic concepts such as the Carnot Engine for students.

The Second Law of Thermodynamics(first expression): *Heat transfer occurs spontaneously from higher- to lower-temperature bodies but never spontaneously in the reverse direction.*

The law states that it is impossible for any process to have as its sole result heat transfer from a cooler to a hotter object. We will express the law in other terms later on, most importantly in terms of entropy.

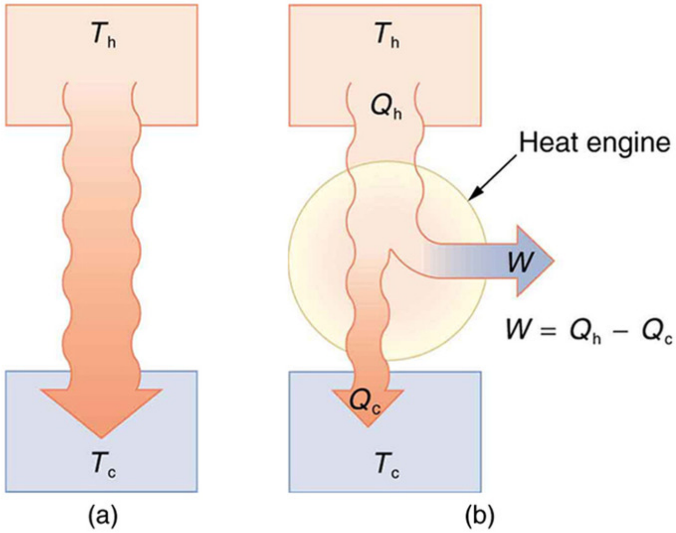
Heat Engines

In thermodynamics, a heat engine is a system that performs the conversion of heat or thermal energy to mechanical work.

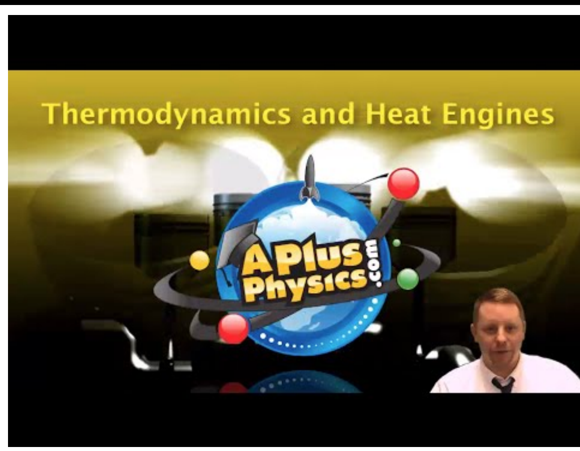
learning objectives

- Justify why efficiency is one of the most important parameters for any heat engine

In thermodynamics, a heat engine is a system that performs the conversion of heat or thermal energy to mechanical work. Gasoline and diesel engines, jet engines, and steam turbines are all heat engines that do work by using part of the heat transfer from some source. Heat transfer from the hot object (or hot reservoir) is denoted as Q_h, while heat transfer into the cold object (or cold reservoir) is Q_c, and the work done by the engine is W. The temperatures of the hot and cold reservoirs are T_h and T_c, respectively.



Heat Transfer: (a) Heat transfer occurs spontaneously from a hot object to a cold one, consistent with the second law of thermodynamics. (b) A heat engine, represented here by a circle, uses part of the heat transfer to do work. The hot and cold objects are called the hot and cold reservoirs. Qh is the heat transfer out of the hot reservoir, W is the work output, and Qc is the heat transfer into the cold reservoir.



Thermodynamics and Heat Engines: A brief introduction to heat engines and thermodynamic concepts such as the Carnot Engine for students.

Because the hot reservoir is heated externally, which is energy intensive, it is important that the work is done as efficiently as possible. In fact, we would like W to equal Q_h, and for there to be no heat transfer to the environment (Q_c=0). Unfortunately, this is impossible. The second law of thermodynamics (second expression) also states, with regard to using heat transfer to do work: *It is impossible in any system for heat transfer from a reservoir to completely convert to work in a cyclical process in which the system returns to its initial state.*

A cyclical process brings a system, such as the gas in a cylinder, back to its original state at the end of every cycle. Most heat engines, such as reciprocating piston engines and rotating turbines, use cyclical processes. The second law, in its second form, clearly states that such engines cannot have perfect conversion of heat transfer into work done.

Efficiency

A cyclical process brings the system back to its original condition at the end of every cycle. By definition, such a system's internal energy U is the same at the beginning and end of every cycle—that is, $\Delta U = 0$. The first law of thermodynamics states that $\Delta U = Q - W$, where Q is the net heat transfer during the cycle ($W = Q_h - Q_c$) and W is the net work done by the system. Since $\Delta U = 0$ for a complete cycle, we have $W = Q$. Thus the net work done by the system equals the net heat transfer into the system, or

$W = Q_h - Q_c$ (cyclical process),

just as shown schematically in (b).

Efficiency is one of the most important parameters for any heat engine. The problem is that in all processes, there is significant heat transfer Q_c lost to the environment. In the conversion of energy to work, we are always faced with the problem of getting less out than we put in. We define the efficiency of a heat engine (*Eff*) to be its net work output W divided by heat transfer to the engine Q_h:

$$E_{ff} = WQ_h. \tag{14.3.1}$$

Since $W = Q_h - Q_c$ in a cyclical process, we can also express this as

$Eff = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h}$ (for cyclical process),

making it clear that an efficiency of 1, or 100%, is possible only if there is no heat transfer to the environment (Q_c=0).

Carnot Cycles

The Carnot cycle is the most efficient cyclical process possible and uses only reversible processes through its cycle.

learning objectives

- Analyze why the Carnot engine is considered the perfect engine

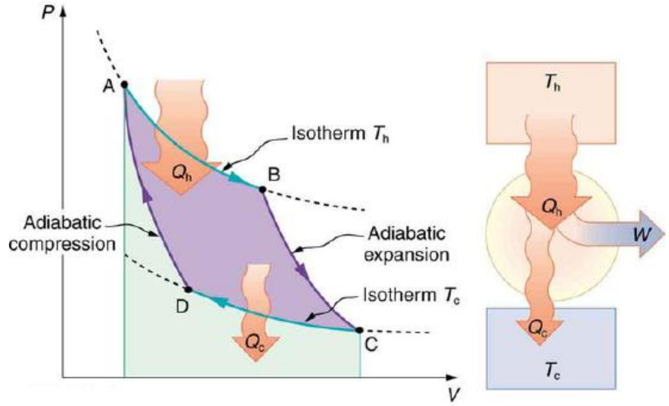
We know from the second law of thermodynamics that a heat engine cannot be 100 percent efficient, since there must always be some heat transfer Q_c to the environment. (See our atom on "Heat Engines. ") How efficient can a heat engine be then? This question was answered at a theoretical level in 1824 by a young French engineer, Sadi Carnot (1796-1832), in his study of the then-emerging heat engine technology crucial to the Industrial Revolution. He devised a theoretical cycle, now called the Carnot cycle, which is the most efficient cyclical process possible. The second law of thermodynamics can be restated in terms of the Carnot cycle, and so what Carnot actually discovered was this fundamental law. Any heat engine employing the Carnot cycle is called a Carnot engine.

What is crucial to the Carnot cycle is that only reversible processes are used. Irreversible processes involve dissipative factors, such as friction and turbulence. This increases heat transfer Q_c to the environment and reduces the efficiency of the engine. Obviously, then, reversible processes are superior.

The second law of thermodynamics (a third form): *A Carnot engine operating between two given temperatures has the greatest possible efficiency of any heat engine operating between these two temperatures. Furthermore, all engines employing only reversible processes have this same maximum efficiency when operating between the same given temperatures.*

Efficiency

The Carnot cycle comprises two isothermal and two adiabatic processes. Recall that both isothermal and adiabatic processes are, in principle, reversible.



PV Diagram for a Carnot Cycle: PV diagram for a Carnot cycle, employing only reversible isothermal and adiabatic processes. Heat transfer Qh occurs into the working substance during the isothermal path AB, which takes place at constant temperature Th. Heat transfer Qc occurs out of the working substance during the isothermal path CD, which takes place at constant temperature Tc. The net work output W equals the area inside the path ABCDA. Also shown is a schematic of a Carnot engine operating between hot and cold reservoirs at temperatures Th and Tc.

Carnot also determined the efficiency of a perfect heat engine—that is, a Carnot engine. It is always true that the efficiency of a cyclical heat engine is given by: $\mathrm{Eff} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h}$.

What Carnot found was that for a perfect heat engine, the ratio \mathrm{\frac{Q_c}{Q_h}} equals the ratio of the absolute temperatures of the heat reservoirs. That is, $\frac{Q_c}{Q_h} = \frac{T_c}{T_h}$ for a Carnot engine, so that the maximum or Carnot efficiency Eff_C is given by $\mathrm{Eff_c} = 1 - \frac{T_c}{T_h}$, where T_h and T_c are in kelvins. (Derivation of the formula is slightly beyond the scope of this atom.) No real heat engine can do as well as the Carnot efficiency—an actual efficiency of about 0.7 of this maximum is usually the best that can be accomplished.

Heat Pumps and Refrigerators

A heat pump is a device that transfers heat energy from a heat source to a heat sink against a temperature gradient.

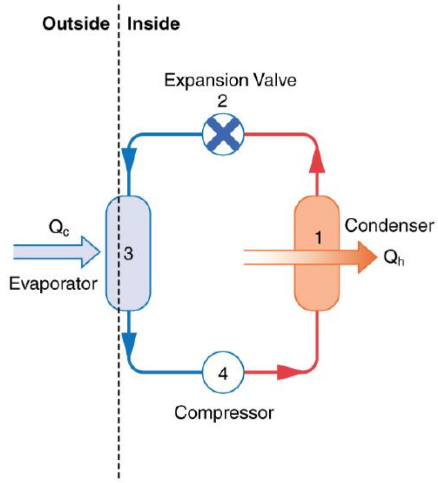
learning objectives

- Explain how the the components of a heat pump cause heat to transfer from a cold reservoir to a hot reservoir

Heat pumps, air conditioners, and refrigerators utilize heat transfer from cold to hot. Heat transfer (Q_c) occurs from a cold reservoir and into a hot one. This requires work input W, which is also converted to heat transfer. Thus the heat transfer to the hot reservoir is Q_h=Q_c+W. A heat pump's mission is for heat transfer Q_h to occur into a warm environment, such as a home in the winter. The mission of air conditioners and refrigerators is for heat transfer Q_c to occur from a cool environment, such as chilling a room or keeping food at lower temperatures than the environment. Actually, a heat pump can be used both to heat and cool a space. It is essentially an air conditioner and a heating unit all in one. In this section we will concentrate on its heating mode.

Heat Pumps

A working fluid such as a non-CFC refrigerant is used in a basic heat pump. The basic components of a heat pump in are a condenser, an expansion valve, an evaporator and a compressor. In the outdoor coils (the evaporator), heat transfer Q_c occurs to the working fluid from the cold outdoor air, turning it into a gas. The electrically driven compressor (work input W) raises the temperature and pressure of the gas and forces it into the condenser coils that are inside the heated space. Because the temperature of the gas is higher than the temperature inside the room, heat transfer to the room occurs and the gas condenses to a liquid. The liquid then flows back through a pressure-reducing valve to the outdoor evaporator coils, being cooled through expansion. (In a cooling cycle, the evaporator and condenser coils exchange roles and the flow direction of the fluid is reversed.)



Simple Heat Pump: A simple heat pump has four basic components: (1) condenser, (2) expansion valve, (3) evaporator, and (4) compressor.

Coefficient of Performance

The quality of a heat pump is judged by how much heat transfer $Q_h$occurs into the warm space compared with how much work input W is required. We define a heat pump's coefficient of performance (COP_{hp}) to be

$$\mathrm{COP_{hp}} = \frac{Q_h}{W}. \tag{14.3.2}$$

Since the efficiency of a heat engine is *Eff*=W/Q_h, we see that COP_{hp}=1/*Eff*. Since the efficiency of any heat engine is less than 1, it means that COP_{hp} is always greater than 1—that is, a heat pump always has more heat transfer Q_h than work put into it. Another interesting point is that heat pumps work best when temperature differences are small. The efficiency of a perfect engine (or Carnot engine) is

$$\mathrm{Eff_C} = 1\frac{T_c}{T_h}; \tag{14.3.3}$$

thus, the smaller the temperature difference, the smaller the efficiency and the greater the COP_{hp}.

Air Conditioners and Refrigerators

Air conditioners and refrigerators are designed to cool something down in a warm environment. As with heat pumps, work input is required for heat transfer from cold to hot. The quality of air conditioners and refrigerators is judged by how much heat transfer Q_c occurs from a cold environment compared with how much work input W is required. What is considered the benefit in a heat pump is considered waste heat in a refrigerator. We thus define the coefficient of performance (COP_{ref}) of an air conditioner or refrigerator to be

$$\mathrm{COP_{ref}} = \frac{Q_c}{W}. \tag{14.3.4}$$

Since $Q_h = Q_c + W$ and $\mathrm{COP_{hp}} = \frac{Q_h}{W}$, we derive that

$$\mathrm{COP_{ref}} = \mathrm{COP_{hp}} - 1. \tag{14.3.5}$$

Also, from Q_h>Q_c, we see that an air conditioner will have a lower coefficient of performance than a heat pump.

Key Points

- Many thermodynamic phenomena, allowed to occur by the first law of thermodynamics, never occur in nature.
- Many processes occur spontaneously in one direction only, and the second law of thermodynamics deals with the direction taken by spontaneous processes.
- According to the second law of thermodynamics, it is impossible for any process to have heat transfer from a cooler to a hotter object as its sole result.

- A cyclical process brings a system, such as the gas in a cylinder, back to its original state at the end of every cycle. Most heat engines, such as reciprocating piston engines and rotating turbines, use cyclical processes.
- The second law of thermodynamics can be expressed as the following: It is impossible in any system for heat transfer from a reservoir to completely convert to work in a cyclical process in which the system returns to its initial state.
- The efficiency of a heat engine (Eff) is defined to be the engine's net work output W divided by heat transfer to the engine: $Eff=\frac{W}{Q_h}=1-\frac{Q_c}{Q_h}$, where Q_c and Q_h denotes heat transfer to hot (engine) and cold (environment) reservoir.
- The second law of thermodynamics indicates that a Carnot engine operating between two given temperatures has the greatest possible efficiency of any heat engine operating between these two temperatures.
- Irreversible processes involve dissipative factors, which reduces the efficiency of the engine. Obviously, reversible processes are superior from the efficiency perspective.
- Carnot efficiency, the maximum achievable heat engine efficiency, is given as $Eff_c=1-\frac{T_c}{T_h}$.
- A heat pump 's mission is for heat transfer Qh to occur into a warm environment, such as a home in the winter.
- The mission of air conditioners and refrigerators is for heat transfer Qc to occur from a cool environment, such as chilling a room or keeping food at lower temperatures than the environment.
- A heat pump can be used both to heat and cool a space. It is essentially an air conditioner and a heating unit all in one. This is made possible by reversing the flow of its refrigerant, changing the direction net heat transfer.

Key Terms

- **entropy**: A measure of how evenly energy (or some analogous property) is distributed in a system.
- **the first law of thermodynamics**: A version of the law of conservation of energy, specialized for thermodynamical systems. Usually expressed as $\Delta U=Q-W$.
- **thermal energy**: The internal energy of a system in thermodynamic equilibrium due to its temperature.
- **internal energy**: The sum of all energy present in the system, including kinetic and potential energy; equivalently, the energy needed to create a system, excluding the energy necessary to displace its surroundings.
- **the second law of thermodynamics**: A law stating that states that the entropy of an isolated system never decreases, because isolated systems spontaneously evolve toward thermodynamic equilibrium—the state of maximum entropy. Equivalently, perpetual motion machines of the second kind are impossible.
- **heat engine**: Any device which converts heat energy into mechanical work.
- **CFC**: An organic compound that was commonly used as a refrigerant. Not commonly used anymore because of its ozone depletion effect.

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14.4: Entropy

learning objectives

- Calculate the total change in entropy for a system in a reversible process

In this and following Atoms, we will study entropy. By examining it, we shall see that the directions associated with the second law — heat transfer from hot to cold, for example—are related to the tendency in nature for systems to become disordered and for less energy to be available for use as work. The entropy of a system can in fact be shown to be a measure of its disorder and of the unavailability of energy to do work.

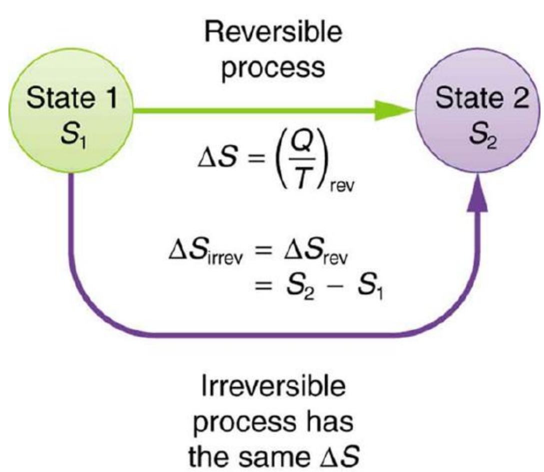
Definition of Entropy

We can see how entropy is defined by recalling our discussion of the Carnot engine. We noted that for a Carnot cycle, and hence for any reversible processes, $Q_c/Q_h=T_c/T_h$. Rearranging terms yields $\frac{Q_c}{T_c}=\frac{Q_h}{T_h}$ for any reversible process. Q_c and $Q_h$are absolute values of the heat transfer at temperatures T_c and T_h, respectively. This ratio of Q/T is defined to be the change in entropy ΔS for a reversible process,

$$\Delta S=(\frac{Q}{T})_{rev}, \tag{14.4.1}$$

where Q is the heat transfer, which is positive for heat transfer into and negative for heat transfer out of, and T is the absolute temperature at which the reversible process takes place. The SI unit for entropy is joules per kelvin (J/K). If temperature changes during the process, then it is usually a good approximation (for small changes in temperature) to take T to be the average temperature, avoiding the need to use integral calculus to find ΔS.

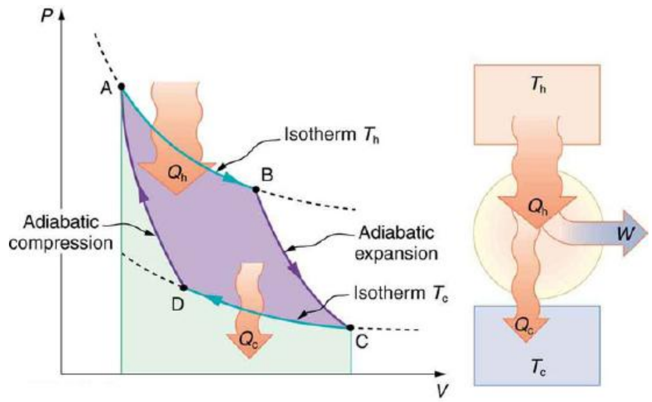
The definition of ΔS is strictly valid only for reversible processes, such as used in a Carnot engine. However, we can find ΔS precisely even for real, irreversible processes. The reason is that the entropy S of a system, like internal energy U, depends only on the state of the system and not how it reached that condition. Entropy is a property of state. Thus the change in entropy ΔS of a system between state one and state two is the same no matter how the change occurs. We just need to find or imagine a reversible process that takes us from state one to state two and calculate ΔS for that process. That will be the change in entropy for any process going from state one to state two.



Change in Entropy: When a system goes from state one to state two, its entropy changes by the same amount ΔS, whether a hypothetical reversible path is followed or a real irreversible path is taken.

Example

Now let us take a look at the change in entropy of a Carnot engine and its heat reservoirs for one full cycle. The hot reservoir has a loss of entropy $\Delta Sh=-Q_h/T_h$, because heat transfer occurs out of it (remember that when heat transfers out, then Q has a negative sign). The cold reservoir has a gain of entropy $\Delta S_c=Q_c/T_c$, because heat transfer occurs into it. (We assume the reservoirs are sufficiently large that their temperatures are constant.) So the total change in entropy is



PV Diagram for a Carnot Cycle: PV diagram for a Carnot cycle, employing only reversible isothermal and adiabatic processes. Heat transfer Qh occurs into the working substance during the isothermal path AB, which takes place at constant temperature Th. Heat transfer Qc occurs out of the working substance during the isothermal path CD, which takes place at constant temperature Tc. The net work output W equals the area inside the path ABCDA. Also shown is a schematic of a Carnot engine operating between hot and cold reservoirs at temperatures Th and Tc.

$$\Delta S_{tot} = \Delta S_h + \Delta S_c. \tag{14.4.2}$$

Thus, since we know that $Q_h/T_h = Q_c/T_c$ for a Carnot engine,

$$\Delta S_{tot} = -\frac{Q_h}{T_h} + \frac{Q_c}{T_c} = 0. \tag{14.4.3}$$

This result, which has general validity, means that the total change in entropy for a system in any reversible process is zero.

Stastical Interpretation of Entropy

According the second law of thermodynamics, disorder is vastly more likely than order.

learning objectives

- Calculate the number of microstates for simple configurations

The various ways of formulating the second law of thermodynamics tell what happens rather than why it happens. Why should heat transfer occur only from hot to cold? Why should the universe become increasingly disorderly? The answer is that it is a matter of overwhelming probability. Disorder is simply vastly more likely than order. To illustrate this fact, we will examine some random processes, starting with coin tosses.

Coin Tosses

What are the possible outcomes of tossing 5 coins? Each coin can land either heads or tails. On the large scale, we are concerned only with the total heads and tails and not with the order in which heads and tails appear. The following table shows all possibilities along with numbers of possible configurations (or microstate; a detailed description of every element of a system). For example, 4 heads and 1 tail instance may occur on 5 different configurations, with any one of the 5 coins showing tail and all the rest heads. (HHHHT, HHHTH, HHTHH, HTHHH, THHHH)

- 5 heads, 0 tails: 1 microstate
- 4 heads, 1 tail: 5 microstates
- 3 heads, 2 tails: 10 microstates
- 2 heads, 3 tails: 10 microstates
- 1 head, 4 tails: 5 microstates
- 0 head, 5 tails: 1 microstate

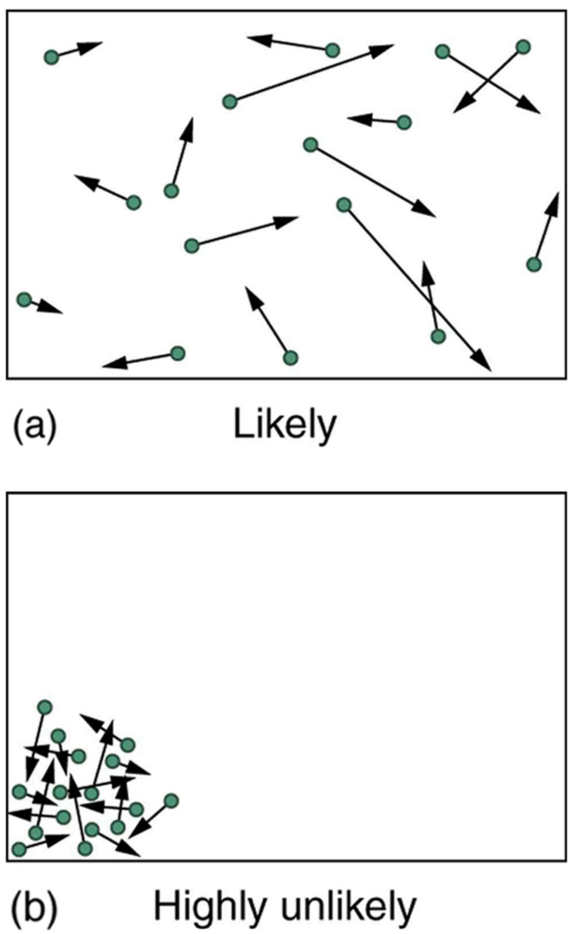
Note that all of these conclusions are based on the crucial assumption that each microstate is equally probable. Otherwise, the analysis will be erroneous.

The two most orderly possibilities are 5 heads or 5 tails. (They are more structured than the others.) They are also the least likely, only 2 out of 32 possibilities. The most disorderly possibilities are 3 heads and 2 tails and its reverse. (They are the least structured.) The most disorderly possibilities are also the most likely, with 20 out of 32 possibilities for the 3 heads and 2 tails and its reverse. If we start with an orderly array like 5 heads and toss the coins, it is very likely that we will get a less orderly array as a result, since 30 out of the 32 possibilities are less orderly. So even if you start with an orderly state, there is a strong tendency to go from order to disorder, from low entropy to high entropy. The reverse can happen, but it is unlikely.

This result becomes dramatic for larger systems. Consider what happens if you have 100 coins instead of just 5. The most orderly arrangements (most structured) are 100 heads or 100 tails. The least orderly (least structured) is that of 50 heads and 50 tails. There is only 1 way (1 microstate) to get the most orderly arrangement of 100 heads. The total number of different ways 100 coins can be tossed—is an impressively large 1.27×10^{30}. Now, if we start with an orderly macrostate like 100 heads and toss the coins, there is a virtual certainty that we will get a less orderly macrostate. If you tossed the coins once each second, you could expect to get either 100 heads or 100 tails once in 2×10^{22} years! In contrast, there is an 8% chance of getting 50 heads, a 73% chance of getting from 45 to 55 heads, and a 96% chance of getting from 40 to 60 heads. Disorder is highly likely.

Real Gas

The fantastic growth in the odds favoring disorder that we see in going from 5 to 100 coins continues as the number of entities in the system increases. In a volume of 1 m^3, roughly 10^{23} molecules (or the order of magnitude of Avogadro's number) are present in a gas. The most likely conditions (or macrostate) for the gas are those we see all the time—a random distribution of atoms in space with a Maxwell-Boltzmann distribution of speeds in random directions, as predicted by kinetic theory as shown in (a). This is the most disorderly and least structured condition we can imagine.



Kinetic Theory: (a) The ordinary state of gas in a container is a disorderly, random distribution of atoms or molecules with a Maxwell-Boltzmann distribution of speeds. It is so unlikely that these atoms or molecules would ever end up in one corner of the container that it might as well be impossible. (b) With energy transfer, the gas can be forced into one corner and its entropy greatly reduced. But left alone, it will spontaneously increase its entropy and return to the normal conditions, because they are immensely more likely.

In contrast, one type of very orderly and structured macrostate has all of the atoms in one corner of a container with identical velocities. There are very few ways to accomplish this (very few microstates corresponding to it), and so it is exceedingly unlikely ever to occur. (See (b).) Indeed, it is so unlikely that we have a law saying that it is impossible, which has never been observed to be violated—the second law of thermodynamics.

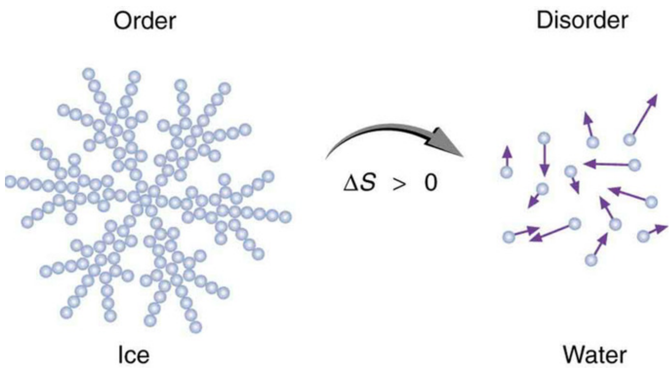
Order to Disorder

Entropy is a measure of disorder, so increased entropy means more disorder in the system.

learning objectives

- Discuss entropy and disorder within a system

Entropy is a measure of disorder. This notion was initially postulated by Ludwig Boltzmann in the 1800s. For example, melting a block of ice means taking a highly structured and orderly system of water molecules and converting it into a disorderly liquid in which molecules have no fixed positions. There is a large increase in entropy in the process.



Entropy of Ice: When ice melts, it becomes more disordered and less structured. The systematic arrangement of molecules in a crystal structure is replaced by a more random and less orderly movement of molecules without fixed locations or orientations. Its entropy increases because heat transfer occurs into it. Entropy is a measure of disorder.

Example 14.4.1:

As an example, suppose we mix equal masses of water originally at two different temperatures, say 20.0º C and 40.0º C. The result is water at an intermediate temperature of 30.0º C. Three outcomes have resulted:

- Entropy has increased.
- Some energy has become unavailable to do work.
- The system has become less orderly.

Entropy, Energy, and Disorder

Let us think about each of the results. First, entropy has increased for the same reason that it did in the example above. Mixing the two bodies of water has the same effect as heat transfer from the hot one and the same heat transfer into the cold one. The mixing decreases the entropy of the hot water but increases the entropy of the cold water by a greater amount, producing an overall increase in entropy.

Second, once the two masses of water are mixed, there is only one temperature—you cannot run a heat engine with them. The energy that could have been used to run a heat engine is now unavailable to do work.

Third, the mixture is less orderly, or to use another term, less structured. Rather than having two masses at different temperatures and with different distributions of molecular speeds, we now have a single mass with a uniform temperature.

These three results—entropy, unavailability of energy, and disorder—are not only related but are in fact essentially equivalent.

Heat Death

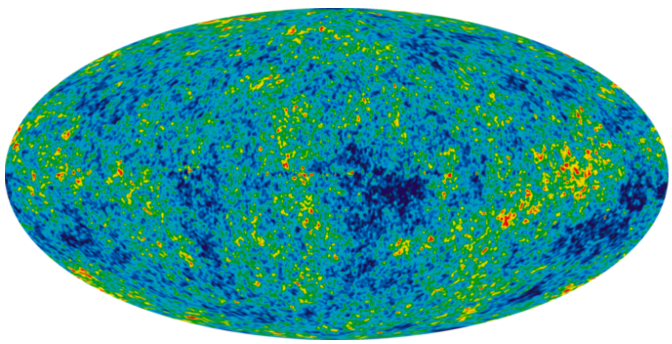
The entropy of the universe is constantly increasing and is destined for thermodynamic equilibrium, called the heat death of the universe.

learning objectives

- Describe processes that lead to the heat death of the universe

In the early, energetic universe, all matter and energy were easily interchangeable and identical in nature. Gravity played a vital role in the young universe. Although it may have seemed disorderly, there was enormous potential energy available to do work—all the future energy in the universe.

As the universe matured, temperature differences arose, which created more opportunity for work. Stars are hotter than planets, for example, which are warmer than icy asteroids, which are warmer still than the vacuum of the space between them. Most of these are cooling down from their usually violent births, at which time they were provided with energy of their own—nuclear energy in the case of stars, volcanic energy on Earth and other planets, and so on. Without additional energy input, however, their days are numbered.



Infant Universe: The image of an infant universe reveals temperature fluctuations (shown as color differences) that correspond to the seeds that grew to become the galaxies.

As entropy increases, less and less energy in the universe is available to do work. On Earth, we still have great stores of energy such as fossil and nuclear fuels; large-scale temperature differences, which can provide wind energy; geothermal energies due to differences in temperature in Earth's layers; and tidal energies owing to our abundance of liquid water. As these are used, a certain fraction of the energy they contain can never be converted into doing work. Eventually, all fuels will be exhausted, all temperatures will equalize, and it will be impossible for heat engines to function, or for work to be done.

Since the universe is a closed system, the entropy of the universe is constantly increasing, and so the availability of energy to do work is constantly decreasing. Eventually, when all stars have died, all forms of potential energy have been utilized, and all temperatures have equalized (depending on the mass of the universe, either at a very high temperature following a universal contraction, or a very low one, just before all activity ceases) there will be no possibility of doing work.

Either way, the universe is destined for thermodynamic equilibrium—maximum entropy. This is often called the heat death of the universe, and will mean the end of all activity. However, whether the universe contracts and heats up, or continues to expand and cools down, the end is not near. Calculations of black holes suggest that entropy can easily continue for at least 10^{100} years.

Living Systems and Evolution

It is possible for the entropy of one part of the universe to decrease, provided the total change in entropy of the universe increases.

learning objectives

- Formulate conditions that allow decrease of the entropy in one part of the universe

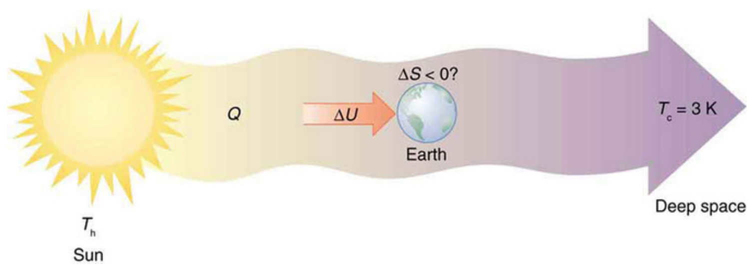
Some people misunderstand the second law of thermodynamics, stated in terms of entropy, to say that the process of the evolution of life violates this law. Over time, complex organisms evolved from much simpler ancestors, representing a large decrease in entropy of the Earth's biosphere. It is a fact that living organisms have evolved to be highly structured, and much lower in entropy than the substances from which they grow. But it is always possible for the entropy of one part of the universe to decrease, provided the total change in entropy of the universe increases. In equation form, we can write this as

$$\Delta S_{tot} = \Delta S_{sys} + \Delta S_{env} > 0. \tag{14.4.4}$$

Thus ΔS_{sys} can be negative as long as ΔS_{env} is positive and greater in magnitude.

How is it possible for a system to decrease its entropy? Energy transfer is necessary. If I gather iron ore from the ground and convert it into steel and build a bridge, my work (and used energy) has decreased the entropy of that system. Energy coming from the Sun can decrease the entropy of local systems on Earth—that is, ΔS_{sys} is negative. But the overall entropy of the rest of the universe increases by a greater amount—that is, ΔS_{env} is positive and greater in magnitude. Thus, $\Delta S_{tot}>0$, and the second law of thermodynamics is not violated.

Every time a plant stores some solar energy in the form of chemical potential energy, or an updraft of warm air lifts a soaring bird, the Earth can be viewed as a heat engine operating between a hot reservoir supplied by the Sun and a cold reservoir supplied by dark outer space—a heat engine of high complexity, causing local decreases in entropy as it uses part of the heat transfer from the Sun into deep space. However, there is a large total increase in entropy resulting from this massive heat transfer. A small part of this heat transfer is stored in structured systems on Earth, producing much smaller local decreases in entropy.



Earth's Entropy: Earth's entropy may decrease in the process of intercepting a small part of the heat transfer from the Sun into deep space. Entropy for the entire process increases greatly while Earth becomes more structured with living systems and stored energy in various forms.

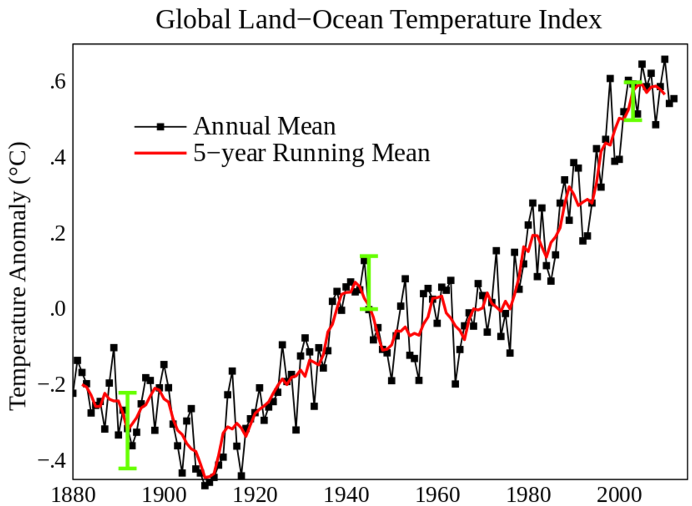
Global Warming Revisited

The Second Law of Thermodynamics may help provide explanation for the global warming over the last 250 years.

learning objectives

- Describe effect of the heat dumped into the environment on the Earth's atmospheric temperature

The Second Law of Thermodynamics may help provide explanation for why there have been increases in Earth's temperatures over the last 250 years (often called "Global Warming"), and many professionals are concerned that the entropy increase of the universe is a real threat to the environment.



Global Land-Ocean Temperature: Global mean land-ocean temperature change from 1880 – 2012, relative to the 1951 – 1980 mean. The black line is the annual mean and the red line is the five-year running mean. The green bars show uncertainty estimates.

As an engine operates, heat flows from a heat tank of greater temperature to a heat sink of lesser temperature. In between these states, the heat flow is turned into useful energy with the help of heat engines. As these engines operate, however, a great deal of heat is lost to the environment due to inefficiencies. In a Carnot engine, which is the most efficient theoretical engine based on Carnot cycle, the maximum efficiency is equal to one minus the temperature of the heat sink (T_c) divided by the temperature of the heat source (T_h).

$$(\mathrm{Eff_c} = 1 - \frac{\mathrm{T_c}}{\mathrm{T_h}}). \tag{14.4.5}$$

This ratio shows that for a greater efficiency to be achieved there needs to be the greatest difference in temperature available. This brings up two important points: optimized heat sinks are at absolute zero, and the longer engines dump heat into an isolated system the less efficient engines will become.

Unfortunately for engine efficiency, day-to-day life never operates in absolute zero. In an average car engine, only 14% to 26% of the fuel which is put in is actually used to make the car move forward. This means that 74% to 86% is lost heat or used to power accessories. According to the U.S. Department of Energy, 70% to 72% of heat produced by burning fuel is heat lost by the engine. The excess heat lost by the engine is then released into the heat sink, which in the case of many modern engines would be the Earth's atmosphere. As more heat is dumped into the environment, Earth's atmospheric (or heat sink) temperature will increase. With the entropy of the environment constantly increasing, searching for new, more efficient technologies and new non-heat engines has become a priority.

Thermal Pollution

Thermal pollution is the degradation of water quality by any process that changes ambient water temperature.

learning objectives

- Identify factors that lead to thermal pollution and its ecological effects

Thermal pollution is the degradation of water quality by any process that changes ambient water temperature. A common cause of thermal pollution is the use of water as a coolant, for example, by power plants and industrial manufacturers. When water used as a coolant is returned to the natural environment at a higher temperature, the change in temperature decreases oxygen supply, and affects ecosystem composition.

As we learned in our Atom on "Heat Engines", all heat engines require heat transfer, achieved by providing (and maintaining) temperature difference between engine's heat source and heat sink. Water, with its high heat capacity, works extremely well as a coolant. But this means that cooling water should be constantly replenished to maintain its cooling capacity.

Cooling Tower: This is a cooling tower at Gustav Knepper Power Station, Dortmund, Germany. Cooling water is circulated inside the tower.

Ecological Effects

Elevated water temperature typically decreases the level of dissolved oxygen of water. This can harm aquatic animals such as fish, amphibians, and other aquatic organisms. An increased metabolic rate may result in fewer resources; the more adapted organisms moving in may have an advantage over organisms that are not used to the warmer temperature. As a result, food chains of the old and new environments may be compromised. Some fish species will avoid stream segments or coastal areas adjacent to a thermal discharge. Biodiversity can decrease as a result. Many aquatic species will also fail to reproduce at elevated temperatures.

Some may assume that by cooling the heated water, we can possibly fix the issue of thermal pollution. However, as we noted in our previous Atom on “Heat Pumps and Refrigerators”, work required for the additional cooling leads to more heat exhaust into the environment. Therefore, it makes the situation even worse.

Key Points

- This ratio of Q/T is defined to be the change in entropy ΔS for a reversible process: $\Delta S=(QT)rev\Delta S=(QT)rev$.
- Entropy is a property of state. Therefore, the change in entropy ΔS of a system between two states is the same no matter how the change occurs.
- The total change in entropy for a system in any reversible process is zero.
- Each microstate is equally probable in the example of coin toss. However, as a macrostate, there is a strong tendency for the most disordered state to occur.
- When tossing 100 coins, if the coins are tossed once each second, you could expect to get either all 100 heads or all 100 tails once in 2×10^{22} years.
- Molecules in a gas follow the Maxwell-Boltzmann distribution of speeds in random directions, which is the most disorderly and least structured condition out of all the possibilities.
- Mixing two systems may decrease the entropy of one system, but increase the entropy of the other system by a greater amount, producing an overall increase in entropy.
- After mixing water at two different temperatures, the energy in the system that could have been used to run a heat engine is now unavailable to do work. Also, the process made the whole system more less structured.
- Entropy, unavailability of energy, and disorder are not only related but are in fact essentially equivalent.
- In the early, energetic universe, all matter and energy were easily interchangeable and identical in nature.
- As entropy increases, less and less energy in the universe is available to do work.
- The universe is destined for thermodynamic equilibrium —maximum entropy. This is often called the heat death of the universe, and will mean the end of all activity.
- Living organisms have evolved to be highly structured, and much lower in entropy than the substances from which they grow.
- It possible for a system to decrease its entropy provided the total change in entropy of the universe increases: $\Delta Stot=\Delta Ssys+\Delta Senv>0\Delta Stot=\Delta Ssys+\Delta Senv>0$.
- The Earth can be viewed as a heat engine operating between a hot reservoir supplied by the Sun and a cold reservoir supplied by dark outer space.
- As heat engines operate, a great deal of heat is lost to the environment due to inefficiencies.

- Even in a Carnot engine, which is the most efficient theoretical engine, there is a heat loss determined by the ratio of temperature of the engine and its environment.
- As more heat is dumped into the environment, Earth's atmospheric temperature will increase.
- All heat engines require heat transfer, achieved by providing (and maintaining) temperature difference between engine's heat source and heat sink. Cooling water is typically used to maintain the temperature difference.
- Elevated water temperature typically decreases the level of dissolved oxygen of water, affecting ecosystem composition.
- Cooling heated water is not a solution for thermal pollution because extra work is required for the cooling, leading to more heat exhaust into the environment.

Key Terms

- **Carnot cycle**: A theoretical thermodynamic cycle. It is the most efficient cycle for converting a given amount of thermal energy into work.
- **reversible**: Capable of returning to the original state without consumption of free energy and increase of entropy.
- **disorder**: Absence of some symmetry or correlation in a many-particle system.
- **Maxwell-Boltzmann distribution**: A distribution describing particle speeds in gases, where the particles move freely without interacting with one another, except for very brief elastic collisions in which they may exchange momentum and kinetic energy.
- **entropy**: A measure of how evenly energy (or some analogous property) is distributed in a system.
- **geothermal**: Pertaining to heat energy extracted from reservoirs in the Earth's interior.
- **asteroid**: A naturally occurring solid object, which is smaller than a planet and is not a comet, that orbits a star.
- **Carnot cycle**: A theoretical thermodynamic cycle. It is the most efficient cycle for converting a given amount of thermal energy into work.
- **absolute zero**: The coldest possible temperature: zero on the Kelvin scale and approximately -273.15°C and -459.67°F. The total absence of heat; the temperature at which motion of all molecules would cease.
- **heat engine**: Any device which converts heat energy into mechanical work.
- **heat pump**: A device that transfers heat from something at a lower temperature to something at a higher temperature by doing work.

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14.5: The Third Law of Thermodynamics

learning objectives

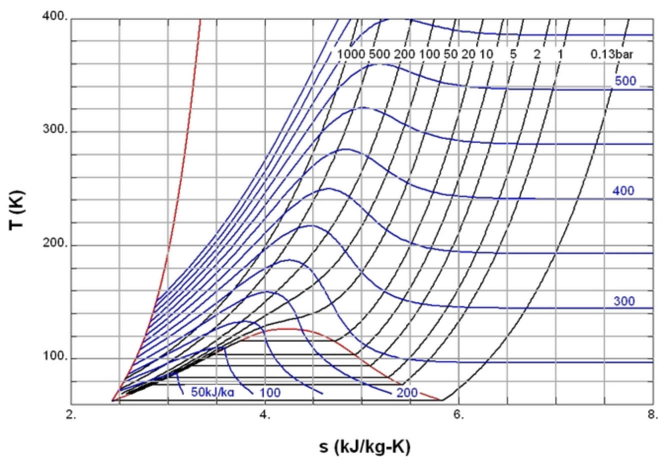
- Explain how absolute zero affects entropy

The third law of thermodynamics is sometimes stated as follows: *The entropy of a perfect crystal at absolute zero is exactly equal to zero.*

At zero kelvin the system must be in a state with the minimum possible energy, thus this statement of the third law holds true if the perfect crystal has only one minimum energy state. Entropy is related to the number of possible microstates, and with only one microstate available at zero kelvin the entropy is exactly zero.

The third law was developed by the chemist Walther Nernst during the years 1906-1912. It is often referred to as Nernst's theorem or Nernst's postulate. Nernst proposed that the entropy of a system at absolute zero would be a well-defined constant. Instead of being 0, entropy at absolute zero could be a nonzero constant, due to the fact that a system may have degeneracy (having several ground states at the same energy).

In simple terms, the third law states that the entropy of a perfect crystal approaches zero as the absolute temperature approaches zero. This law provides an absolute reference point for the determination of entropy. (diagrams the temperature entropy of nitrogen.) The entropy (S) determined relative to this point is the absolute entropy represented as follows:



Temperature Entropy of Nitrogen: Temperature–entropy diagram of nitrogen. The red curve at the left is the melting curve. Absolute value of entropy can be determined shown here, thanks to the third law of thermodynamics.

$$S = k_B \log W, \tag{14.5.1}$$

where k_B is the Bolzmann constant and W is the number of microstates. Provided that the ground state is unique (or $W = 1$), the entropy of a *perfect* crystal lattice as defined by Nernst's theorem is zero provided that its ground state is unique, because $\log(1) = 0$.

Adiabatic Processes

It is impossible to reduce the temperature of any system to zero temperature in a finite number of finite operations.

learning objectives

- Illustrate isentropic process, for example in terms of adiabtic demangetization

In our Atom on "Adiabatic Processes" (category: the First Law of Thermodynamics), we learned that an adiabatic process is any process occurring without gain or loss of heat within a system. We also learned a monatomic ideal gas expands adiabatically. In this

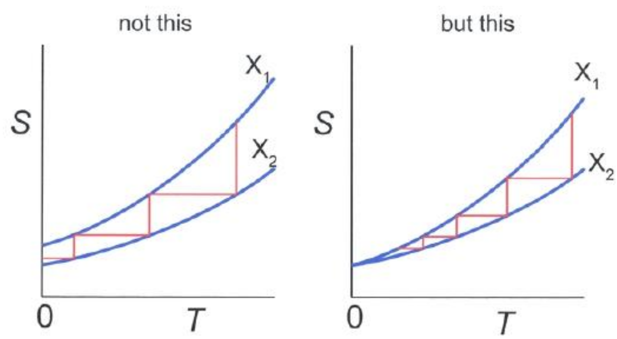
Atom, we discuss an adiabatic cooling process that can be used to cool a gas, as well as whether absolute zero can be obtained in real systems.

Absolute Zero?

Previously, we learned about the third law of thermodynamics, which states: *the entropy of a perfect crystal at absolute zero is exactly equal to zero.*

According to the third law, the reason that T=0 cannot be reached is explained as follows: Suppose the temperature of a substance can be reduced in an isentropic process by changing the parameter X from X_2 to X_1. As an example, one can think of a multistage adiabatic magnetization-demagnetization cycle setup where a magnetic field is switched on and off in a controlled way. (See below. The parameter X in this case would be the magnetization of the gas.)

Assuming an entropy difference at absolute zero, T=0 could be reached in a finite number of steps. However, going back to the third law, at T=0 there is no entropy difference, and therefore an infinite number of stepswould be needed for this process (illustrated in).



Can Absolute Zero be Reached?: Temperature-Entropy diagram. Horizontal lines represent isentropic processes, while vertical lines represent isothermal processes. Left side: Absolute zero can be reached in a finite number of steps if S(T=0,X1)≠S(T=0, X2). Right: An infinite number of steps is needed since S(0,X1)= S(0,X2).

Adiabatic Demagnetization Cooling

In simple terms, the cooling scheme mentioned above occurs by repeating the following steps:

1. A strong magnetic field is applied to adiabatically align magnetic moments of the particles in the gas.
2. The magnetic field is reduced adiabatically, and thermal energy of the gas causes “ordered” magnetic moments to become random again.

In the second step, thermal energy in the gas is used to cause disorder of magnetic moments. Therefore, the temperature of the gas is reduced (hence cooling works). This scheme is called *adiabatic demagnetization cooling*.

Key Points

- Entropy is related to the number of possible microstates, and with only one microstate available at zero kelvin, the entropy is exactly zero.
- The third law of thermodynamics provides an absolute reference point for the determination of entropy. The entropy determined relative to this point is the absolute entropy.
- Absolute entropy can be written as $S = k_B \textcolor{red}{\backslash \log W}$, where W is the number of available microstates.
- Since at $T = 0$ there is no entropy difference, an infinite number of steps in a thermodynamic cooling process is required to reach T=0.
- In adiabatic demagnetization cooling, energy transfers from thermal entropy to magnetic entropy (or disorder of the magnetic moments).
- The fact that T=0 cannot be achieved in reality is a direct consequence of the third law of thermodynamics.

Key Terms

- **microstate**: The specific detailed microscopic configuration of a system.
- **absolute zero**: The coldest possible temperature: zero on the Kelvin scale and approximately -273.15°C and -459.67°F. The total absence of heat; the temperature at which motion of all molecules would cease.
- **degeneracy**: Two or more different quantum states are said to be degenerate if they are all at the same energy level.
- **isentropic**: Having a constant entropy.
- **demagnetization**: The process of removing the magnetic field from an object.

CHAPTER OVERVIEW

15: Waves and Vibrations

Topic hierarchy

15.1: Introduction

learning objectives

- Contrast mechanical and electromagnetic waves

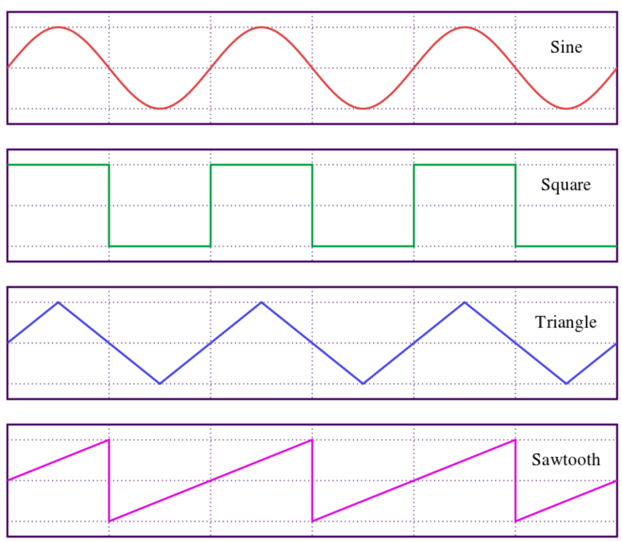
Overview

A wave is an oscillation that travels through space, accompanied by a transfer of energy. Wave motion transfers energy from one point to another, often with no permanent displacement of the particles of the medium —that is, with little or no associated mass transport. They consist, instead, of oscillations or vibrations around almost fixed locations. There are two main types of waves. Mechanical waves propagate through a medium, and the substance of this medium is deformed. The deformation reverses itself owing to restoring forces resulting from its deformation.

The second main type of wave, electromagnetic waves, do not require a medium (although they may still propagate through a medium). Instead, they consist of periodic oscillations in electrical and magnetic fields generated by charged particles, and can therefore travel through a vacuum.

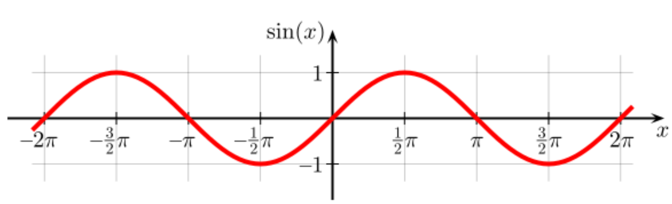
Wave Equation

The shape of a wave can take the form of any function that repeats itself over some characteristic spatial scale λλ, the wavelength (see). More generally, waveforms are scalar functions uu which satisfy the wave equation, *[Math Processing Error]*. This equation simply states that the acceleration of the waveform (Left: second derivative with respect to time) is proportional to the Laplacian (Right: second spatial derivative) of the same waveform. The constant of proportionality, *[Math Processing Error]*, is the square of the propagation speed of the wave.



Common waveforms: A sample of several common, simple waveforms. A waveform is a function that repeats in space.

Sine Wave



Plot of Sine: The sine function graphed on the Cartesian plane. In this graph, the angle *x* is given in radians (π = 180°).

Consider one of the most common waveforms, the sinusoid. A general form of a sinusoidal wave is *[Math Processing Error]*, where A is the amplitude of the wave, *[Math Processing Error]* is the wave's angular frequency, k is the wavenumber, and *[Math*

Processing Error] is the phase of the sine wave given in radians. This waveform gives the displacement position ("y") of a particle in a medium from its equilibrium as a function of both position "x" and time "t".

By taking derivatives, it is evident that the wave equation given above holds for *[Math Processing Error]*, which is also called the phase speed of the wave. To find the velocity of a particle in the medium at x and t, we take the temporal derivative of the waveform to get *[Math Processing Error]*. Likewise, to find the acceleration of the displaced particle in the medium at x and t, we take the second derivative to get *[Math Processing Error]*. Note the phase relationship among the trigonometric functions in *[Math Processing Error]*. When the particle displacement is maximum or minimum, the velocity is 0. When the displacement is 0, particle velocity is either maximum or minimum. Similarly, the particle acceleration is maximum (or minimum) when the particle displacement is minimum (or maximum), respectively.

Arbitrary Wave

We looked closely into the sinusoidal wave. But how about waves that has a general form? One important aspect of the wave equation is its linearity: the wave equation is linear in *u* and it is left unaltered by translations in space and time. Since a wave with an arbitrary shape can be represented by a sum of many sinusoidal waves (this is called Fourier analysis), we can generate a great variety of solutions of the wave equation by translating and summing sine waves that we just looked closely into.

Key Points

- The wave equation requires that the second time derivative of the waveform be proportional to its second spatial derivative.
- Waveforms describe the shape of physical waves, and can take the form of any function that repeats in space.
- One of the most common waveforms in physics is the sinusoid. Since any arbitray waveform can be generated by adding a set of sine waves, the physics governing a wave of an arbitrary shape can be described by using its sine wave components.

Key Terms

- **waveform**: The shape of a physical wave, such as sound or electromagnetic radiation. The shape can be any function that repeats in space.
- **Fourier analysis**: The study of the way general functions may be represented or approximated by sums of simpler trigonometric functions.

15.2: Hooke’s Law

learning objectives

- Generate the mathematical expression of Hooke’s law

In mechanics (physics), Hooke’s law is an approximation of the response of elastic (i.e., springlike) bodies. It states: the extension of a spring is in direct proportion with the load applied to it. For instance, the spring is pulled downwards with either no load, F_p, or twice F_p.

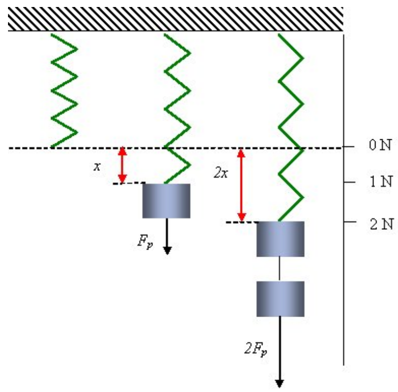
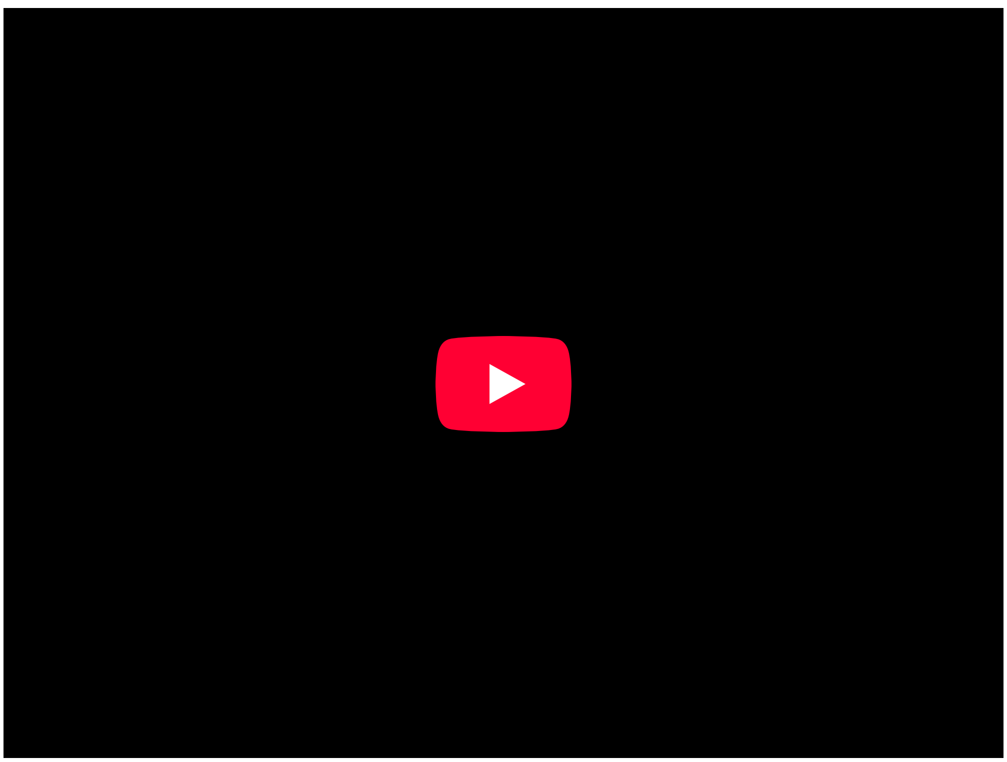
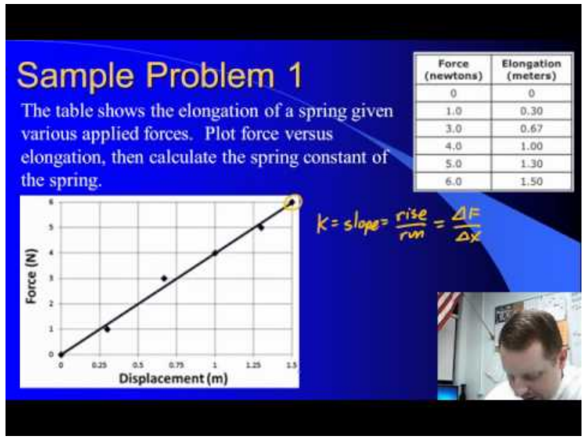


Diagram of Hooke’s Law: The extension of the spring is linearly proportional to the force.





Springs and Hooke's Law: A brief overview of springs, Hooke's Law, and elastic potential energy for algebra-based physics students.

Many materials obey this law of elasticity as long as the load does not exceed the material's elastic limit. Materials for which Hooke's law is a useful approximation are known as linear-elastic or "Hookean" materials. Hookean materials are broadly defined and include springs as well as muscular layers of the heart. In simple terms, Hooke's law says that stress is directly proportional to strain. Mathematically, Hooke's law is stated as:

$$\mathrm{F}=-\mathrm{kx} \tag{15.2.1}$$

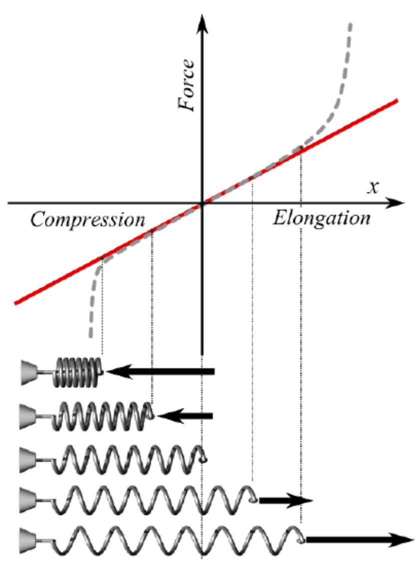
where:

- x is the displacement of the spring's end from its equilibrium position (a distance, in SI units: meters);
- F is the restoring force exerted by the spring on that end (in SI units: N or kg·m/s^2); and
- k is a constant called the rate or spring constant (in SI units: N/m or kg/s^2). When this holds, the behavior is said to be linear. If shown on a graph, the line should show a direct variation.

It's possible for multiple springs to act on the same point. In such a case, Hooke's law can still be applied. As with any other set of forces, the forces of many springs can be combined into one resultant force.

When Hooke's law holds, the behavior is linear; if shown on a graph, the line depicting force as a function of displacement should show a direct variation. There is a negative sign on the right hand side of the equation because the restoring force always acts in the opposite direction of the displacement (for example, when a spring is stretched to the left, it pulls back to the right).

Hooke's law is named after the 17th century British physicist Robert Hooke, and was first stated in 1660 as a Latin anagram, whose solution Hooke published in 1678 as *Ut tensio, sic vis*, meaning, "As the extension, so the force."



Hooke's Law: The red line in this graph illustrates how force, F, varies with position according to Hooke's law. The slope of this line corresponds to the spring constant k. The dotted line shows what the actual (experimental) plot of force might look like. The pictures of spring states at the bottom of the graph correspond to some points of the plot; the middle one is in the relaxed state (no force applied).

Elastic Potential Energy

If a force results in only deformation, with no thermal, sound, or kinetic energy, the work done is stored as elastic potential energy.

learning objectives

- Express elastic energy stored in a spring in a mathematical form

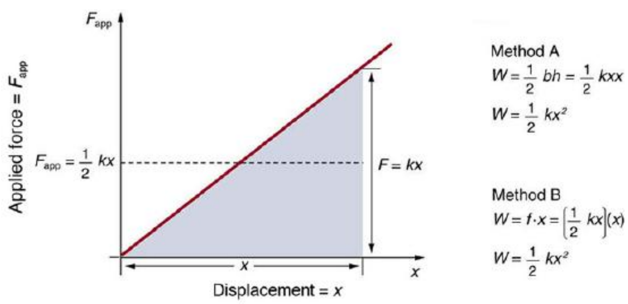
Elastic Potential Energy

In order to produce a deformation, work must be done. That is, a force must be exerted through a distance, whether you pluck a guitar string or compress a car spring. If the only result is deformation and no work goes into thermal, sound, or kinetic energy, then all the work is initially stored in the deformed object as some form of potential energy. Elastic energy is the potential mechanical energy stored in the configuration of a material or physical system when work is performed to distort its volume or shape. For example, the potential energy PE_{el} stored in a spring is

$$PE_{el} = \frac{1}{2}kx^2 \tag{15.2.2}$$

where k is the elastic constant and x is the displacement.

It is possible to calculate the work done in deforming a system in order to find the energy stored. This work is performed by an applied force F_{app}. The applied force is exactly opposite to the restoring force (action-reaction), and so $F_{app} = kx$. A graph shows the applied force versus deformation x for a system that can be described by Hooke's law. Work done on the system is force multiplied by distance, which equals the area under the curve, or $\frac{1}{2}kx^2$ (Method A in the figure). Another way to determine the work is to note that the force increases linearly from 0 to kx, so that the average force is $\frac{1}{2}kx$, the distance moved is x, and thus



Applied force versus deformation: A graph of applied force versus distance for the deformation of a system that can be described by Hooke's law is displayed. The work done on the system equals the area under the graph or the area of the triangle, which is half its base multiplied by its height, or $W = \frac{1}{2}kx^2$.

$W = F_{app}d = (\frac{1}{2}kx)(x) = \frac{1}{2}kx^2$ (Method B in the figure).

Elastic energy of or within a substance is static energy of configuration. It corresponds to energy stored principally by changing the inter-atomic distances between nuclei. Thermal energy is the randomized distribution of kinetic energy within the material, resulting in statistical fluctuations of the material about the equilibrium configuration. There is some interaction, however. For example, for some solid objects, twisting, bending, and other distortions may generate thermal energy, causing the material's temperature to rise. This energy can also produce macroscopic vibrations sufficiently lacking in randomization to lead to oscillations that are merely the exchange between (elastic) potential energy within the object and the kinetic energy of motion of the object as a whole.

Key Points

- Mathematically, Hooke's Law can be written as $F = -kx$.
- Many materials obey this law as long as the load does not exceed the material's elastic limit.

- The rate or spring constant, k, relates the force to the extension in SI units: N/m or kg/s2.
- In order to produce a deformation, work must be done.
- The potential energy stored in a spring is given by $PE_{el} = \frac{1}{2}kx^2$, where k is the spring constant and x is the displacement.
- Deformation can also be converted into thermal energy or cause an object to begin oscillating.

Key Terms

- **elasticity**: The property by virtue of which a material deformed under the load can regain its original dimensions when unloaded
- **deformation**: A transformation; change of shape.
- **kinetic energy**: The energy possessed by an object because of its motion, equal to one half the mass of the body times the square of its velocity.
- **oscillating**: Moving in a repeated back-and-forth motion.

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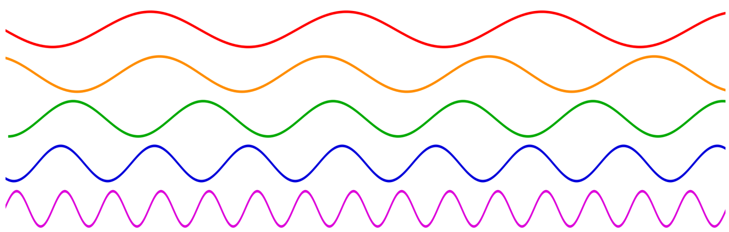
15.3: Periodic Motion

learning objectives

- Practice converting between frequency and period

Period and Frequency

The usual physics terminology for motion that repeats itself over and over is *periodic motion*, and the time required for one repetition is called the *period*, often expressed as the letter *T*. (The symbol *P* is not used because of the possible confusion with momentum.) One complete repetition of the motion is called a cycle. The frequency is defined as the number of cycles per unit time. Frequency is usually denoted by a Latin letter *f* or by a Greek letter *ν* (nu). Note that period and frequency are reciprocals of each other.



Sinusoidal Waves of Varying Frequencies: Sinusoidal waves of various frequencies; the bottom waves have higher frequencies than those above. The horizontal axis represents time.

$$f = \frac{1}{T} \tag{15.3.1}$$

For example, if a newborn baby's heart beats at a frequency of 120 times a minute, its period (the interval between beats) is half a second. If you calibrate your intuition so that you expect *large frequencies* to be paired with *short periods*, and vice versa, you may avoid some embarrassing mistakes on physics exams.

Units

Locomotive Wheels: The locomotive's wheels spin at a frequency of f cycles per second, which can also be described as ω radians per second. The mechanical linkages allow the linear vibration of the steam engine's pistons, at frequency f, to drive the wheels.

In SI units, the unit of frequency is the *hertz* (Hz), named after the German physicist Heinrich Hertz: *1 Hz* indicates that an event repeats once per second. A traditional unit of measure used with rotating mechanical devices is revolutions per minute, abbreviated *RPM*. 60 RPM equals one hertz (i.e., one revolution per second, or a period of one second). The SI unit for period is the second.

Angular Frequency

Often periodic motion is best expressed in terms of angular frequency, represented by the Greek letter ω (omega). Angular frequency refers to the angular displacement per unit time (e.g., in rotation) or the rate of change of the phase of a sinusoidal waveform (e.g., in oscillations and waves), or as the rate of change of the argument of the sine function.

$$y(t) = \sin(\theta(t)) = \sin(\omega t) = \sin(2\pi f t) \tag{15.3.2}$$

$$\omega = 2\pi f \tag{15.3.3}$$

Angular frequency is often represented in units of radians per second (recall there are 2π radians in a circle).

Period of a Mass on a Spring

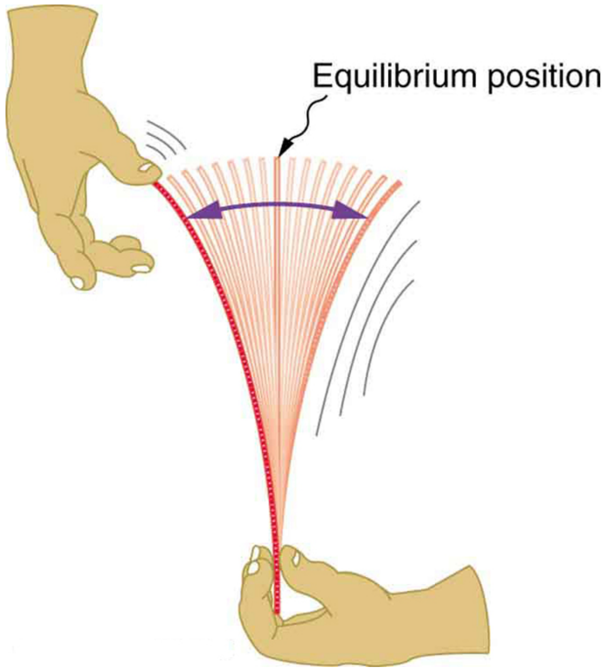
The period of a mass m on a spring of spring constant k can be calculated as $T = 2\pi\sqrt{\frac{m}{k}}$.

learning objectives

- Identify parameters necessary to calculate the period and frequency of an oscillating mass on the end of an ideal spring

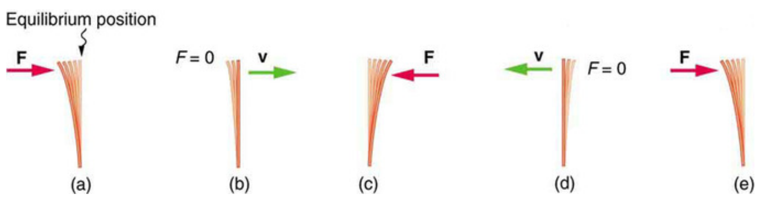
Understanding the Restoring Force

Newton's first law implies that an object oscillating back and forth is experiencing forces. Without force, the object would move in a straight line at a constant speed rather than oscillate. It is important to understand how the force on the object depends on the object's position. If an object is vibrating to the right and left, then it must have a leftward force on it when it is on the right side, and a rightward force when it is on the left side. In one dimension, we can represent the direction of the force using a positive or negative sign, and since the force changes from positive to negative there must be a point in the middle where the force is zero. This is the equilibrium point, where the object would stay at rest if it was released at rest. It is common convention to define the origin of our coordinate system so that *x* equals zero at equilibrium.



Oscillating Ruler: When displaced from its vertical equilibrium position, this plastic ruler oscillates back and forth because of the restoring force opposing displacement. When the ruler is on the left, there is a force to the right, and vice versa.

Consider, for example, plucking a plastic ruler shown in the first figure. The deformation of the ruler creates a force in the opposite direction, known as a *restoring force*. Once released, the restoring force causes the ruler to move back toward its stable equilibrium position, where the net force on it is zero. However, by the time the ruler gets there, it gains momentum and continues to move to the right, producing the opposite deformation. It is then forced to the left, back through equilibrium, and the process is repeated until dissipative forces (e.g., friction) dampen the motion. These forces remove mechanical energy from the system, gradually reducing the motion until the ruler comes to rest.



Restoring force, momentum, and equilibrium: (a) The plastic ruler has been released, and the restoring force is returning the ruler to its equilibrium position. (b) The net force is zero at the equilibrium position, but the ruler has momentum and continues to move to the right. (c) The restoring force is in the opposite direction. It stops the ruler and moves it back toward equilibrium again. (d) Now the ruler has momentum to the left. (e) In the absence of damping (caused by frictional forces), the ruler reaches its original position. From there, the motion will repeat itself.

Hooke's Law

The simplest oscillations occur when the restoring force is directly proportional to displacement. The name that was given to this relationship between force and displacement is Hooke's law:

$$\mathrm{F} = \mathrm{kx} \tag{15.3.4}$$

Here, F is the restoring force, x is the displacement from equilibrium or deformation, and k is a constant related to the difficulty in deforming the system (often called the spring constant or force constant). Remember that the minus sign indicates the restoring force is in the direction opposite to the displacement. The force constant k is related to the rigidity (or stiffness) of a system—the larger the force constant, the greater the restoring force, and the stiffer the system. The units of k are newtons per meter (N/m). For example, k is directly related to Young's modulus when we stretch a string. A typical physics laboratory exercise is to measure restoring forces created by springs, determine if they follow Hooke's law, and calculate their force constants if they do.

Mass on a Spring

A common example of an objecting oscillating back and forth according to a restoring force directly proportional to the displacement from equilibrium (i.e., following Hooke's Law) is the case of a mass on the end of an ideal spring, where "ideal" means that no messy real-world variables interfere with the imagined outcome.

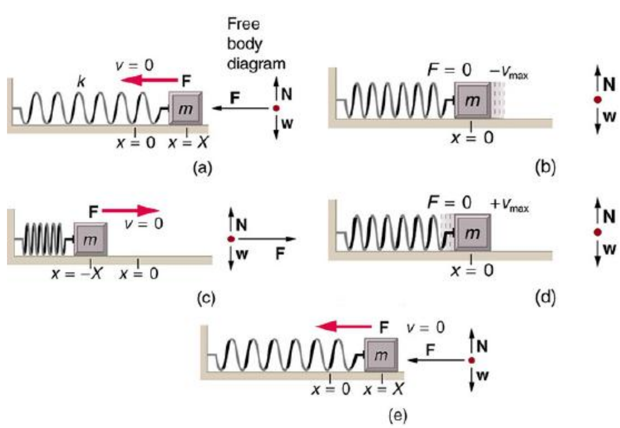
The motion of a mass on a spring can be described as Simple Harmonic Motion (SHM), the name given to oscillatory motion for a system where the net force can be described by Hooke's law. We can now determine how to calculate the period and frequency of an oscillating mass on the end of an ideal spring. The period T can be calculated knowing only the mass, m, and the force constant, k:

$$\mathrm{T} = 2\pi\sqrt{\frac{\mathrm{m}}{\mathrm{k}}} \tag{15.3.5}$$

When dealing with $\mathrm{f} = \frac{1}{\mathrm{T}}$, the frequency is given by:

$$\mathrm{f} = \frac{1}{2\pi}\sqrt{\frac{\mathrm{k}}{\mathrm{m}}} \tag{15.3.6}$$

We can understand the dependence of these equations on m and k intuitively. If one were to increase the mass on an oscillating spring system with a given k, the increased mass will provide more inertia, causing the acceleration due to the restoring force F to decrease (recall Newton's Second Law: $\mathrm{F} = \mathrm{ma}$). This will lengthen the oscillation period and decrease the frequency. In contrast, increasing the force constant k will increase the restoring force according to Hooke's Law, in turn causing the acceleration at each displacement point to also increase. This reduces the period and increases the frequency. The maximum displacement from equilibrium is known as the *amplitude X*.



Motion of a mass on an ideal spring: An object attached to a spring sliding on a frictionless surface is an uncomplicated simple harmonic oscillator. When displaced from equilibrium, the object performs simple harmonic motion that has an amplitude X and a period T. The object’s maximum speed occurs as it passes through equilibrium. The stiffer the spring is, the smaller the period T. The greater the mass of the object is, the greater the period T. (a) The mass has achieved its greatest displacement X to the right and now the restoring force to the left is at its maximum magnitude. (b) The restoring force has moved the mass back to its equilibrium point and is now equal to zero, but the leftward velocity is at its maximum. (c) The mass’s momentum has carried it to its maximum displacement to the right. The restoring force is now to the right, equal in magnitude and opposite in direction compared to (a). (d) The equilibrium point is reach again, this time with momentum to the right. (e) The cycle repeats.

Simple Harmonic Motion

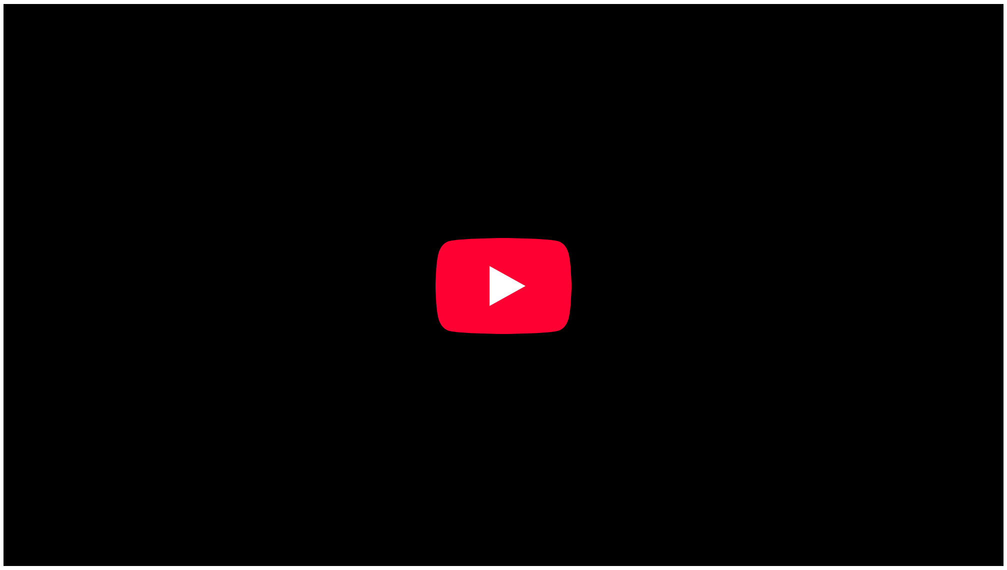
Simple harmonic motion is a type of periodic motion where the restoring force is directly proportional to the displacement.

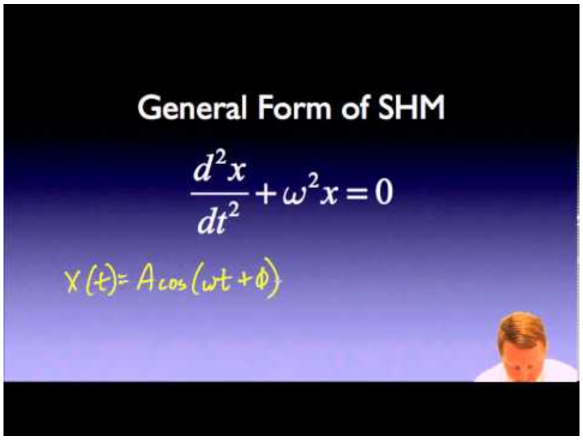
learning objectives

- Relate the restoring force and the displacement during the simple harmonic motion

Simple Harmonic Motion

Simple harmonic motion is a type of periodic motion where the restoring force is directly proportional to the displacement (i.e., it follows Hooke’s Law). It can serve as a mathematical model of a variety of motions, such as the oscillation of a spring. In addition, other phenomena can be approximated by simple harmonic motion, such as the motion of a simple pendulum, or molecular vibration.





Simple Harmonic Motion: A brief introduction to simple harmonic motion for calculus-based physics students.

Simple harmonic motion is typified by the motion of a mass on a spring when it is subject to the linear elastic restoring force given by Hooke’s Law. A system that follows simple harmonic motion is known as a *simple harmonic oscillator.*

Dynamics of Simple Harmonic Oscillation

For one-dimensional simple harmonic motion, the equation of motion (which is a second-order linear ordinary differential equation with constant coefficients) can be obtained by means of Newton's second law and Hooke's law.

$$F_{net} = m\frac{d^2x}{dt^2} = -kx, \tag{15.3.7}$$

where *m* is the mass of the oscillating body, *x* is its displacement from the equilibrium position, and *k* is the spring constant. Therefore:

$$\frac{d^2x}{dt^2} = -(\frac{k}{m})x. \tag{15.3.8}$$

Solving the differential equation above, a solution which is a sinusoidal function is obtained.

$$x(t) = c_1\cos(\omega t) + c_2\sin(\omega t) = A\cos(\omega t - \varphi), \tag{15.3.9}$$

where

$$\omega = \sqrt{\frac{k}{m}}, \tag{15.3.10}$$

$$A = \sqrt{c_1^2 + c_2^2}, \tag{15.3.11}$$

$$\tan\varphi = (\frac{c_2}{c_1}). \tag{15.3.12}$$

In the solution, c_1 and c_2 are two constants determined by the initial conditions, and the origin is set to be the equilibrium position. Each of these constants carries a physical meaning of the motion: A is the amplitude (maximum displacement from the equilibrium position), $\omega = 2\pi f$ is the angular frequency, and φ is the phase.

We can use differential calculus and find the velocity and acceleration as a function of time:

$$v(t) = \frac{dx}{dt} = -A\omega\sin(\omega t - \varphi) \tag{15.3.13}$$

$$a(t) = \frac{d^2x}{dt^2} = -A\omega^2\cos(\omega t - \varphi). \tag{15.3.14}$$

Acceleration can also be expressed as a function of displacement:

$$a(t) = -\omega^2 x. \tag{15.3.15}$$

Then since $\omega = 2\pi f$,

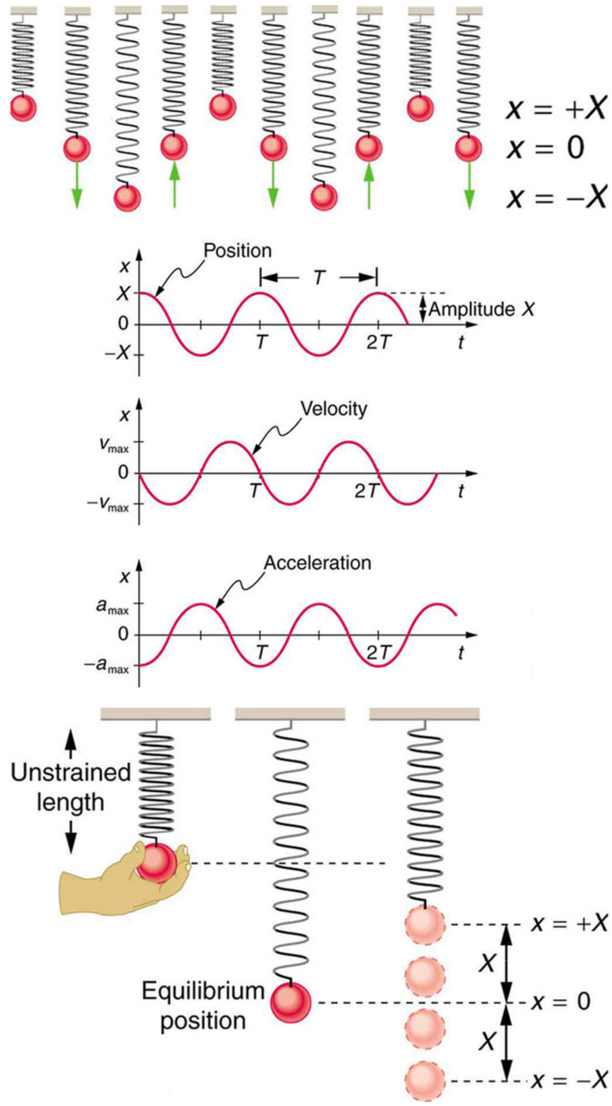
$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}. \tag{15.3.16}$$

Recalling that $T = \frac{1}{f}$,

$$T = 2\pi\sqrt{\frac{m}{k}}. \tag{15.3.17}$$

Using Newton's Second Law, Hooke's Law, and some differential Calculus, we were able to derive the period and frequency of the mass oscillating on a spring that we encountered in the last section! Note that the period and frequency are completely independent of the amplitude.

The below figure shows the simple harmonic motion of an object on a spring and presents graphs of x(t),v(t), and a(t) versus time. You should learn to create mental connections between the above equations, the different positions of the object on a spring in the cartoon, and the associated positions in the graphs of x(t), v(t), and a(t).



Visualizing Simple Harmonic Motion: Graphs of x(t),v(t), and a(t) versus t for the motion of an object on a spring. The net force on the object can be described by Hooke's law, and so the object undergoes simple harmonic motion. Note that the initial position has the vertical displacement at its maximum value X; v is initially zero and then negative as the object moves down; and the initial acceleration is negative, back toward the equilibrium position and becoming zero at that point.

Simple Harmonic Motion and Uniform Circular Motion

Simple harmonic motion is produced by the projection of uniform circular motion onto one of the axes in the x-y plane.

learning objectives

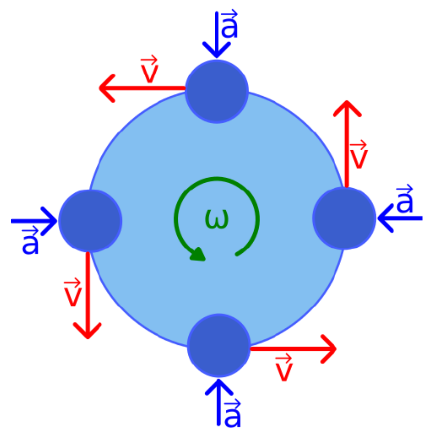
- Describe relationship between the simple harmonic motion and uniform circular motion

Uniform Circular Motion

Uniform circular motion describes the motion of a body traversing a circular path at constant speed. The distance of the body from the center of the circle remains constant at all times. Though the body's speed is constant, its velocity is not constant: velocity (a vector quantity) depends on both the body's speed and its direction of travel. Since the body is constantly changing direction as it travels around the circle, the velocity is changing also. This varying velocity indicates the presence of an acceleration called the centripetal acceleration. Centripetal acceleration is of constant magnitude and directed at all times towards the center of the circle. This acceleration is, in turn, produced by a centripetal force —a force in constant magnitude, and directed towards the center.

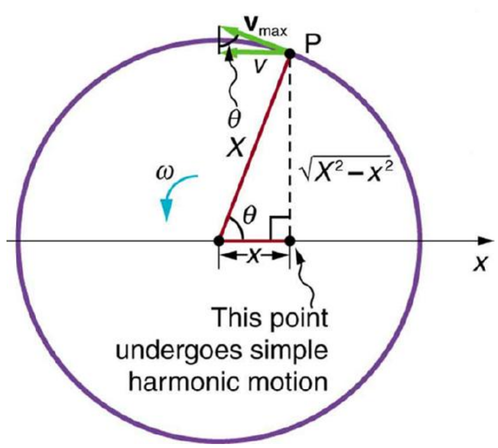
Velocity

The above figure illustrates velocity and acceleration vectors for uniform motion at four different points in the orbit. Since velocity *v* is tangent to the circular path, no two velocities point in the same direction. Although the object has a constant speed, its direction is always changing. This change in velocity is due to an acceleration, *a*, whose magnitude is (like that of the velocity) held constant, but whose direction also is always changing. The acceleration points radially inwards (centripetally) and is perpendicular to the velocity. This acceleration is known as centripetal acceleration.



Uniform Circular Motion (at Four Different Point in the Orbit): Velocity v and acceleration a in uniform circular motion at angular rate ω; the speed is constant, but the velocity is always tangent to the orbit; the acceleration has constant magnitude, but always points toward the center of rotation

Displacement around a circular path is often given in terms of an angle θ. This angle is the angle between a straight line drawn from the center of the circle to the objects starting position on the edge and a straight line drawn from the objects ending position on the edge to center of the circle. See for a visual representation of the angle where the point p started on the x- axis and moved to its present position. The angle θ describes how far it moved.



Projection of Uniform Circular Motion: A point P moving on a circular path with a constant angular velocity ω is undergoing uniform circular motion. Its projection on the x-axis undergoes simple harmonic motion. Also shown is the velocity of this point around the circle, v−max, and its projection, which is v. Note that these velocities form a similar triangle to the displacement triangle.

For a path around a circle of radius *r*, when an angle θ(measured in radians) is swept out, the distance traveled on the edge of the circle is $s = r\theta$. You can prove this yourself by remembering that the circumference of a circle is $2\pi r$, so if the object traveled around the whole circle (one circumference) it will have gone through an angle of 2π radians and traveled a distance of $2\pi r$. Therefore, the speed of travel around the orbit is:

$$v = r\frac{d\theta}{dt} = r\omega, \tag{15.3.18}$$

where the angular rate of rotation is ω. (Note that $\omega = \frac{v}{r}$.) Thus,v is a constant, and the velocity vector v also rotates with constant magnitude v, at the same angular rate ω.

Acceleration

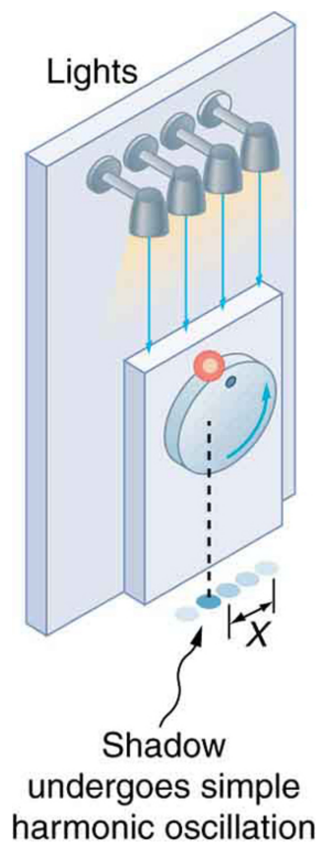
The acceleration in uniform circular motion is always directed inward and is given by:

$$a = v\frac{d\theta}{dt} = v\omega = \frac{v^2}{r}. \tag{15.3.19}$$

This acceleration acts to change the direction of v, but not the speed.

Simple Harmonic Motion from Uniform Circular Motion

There is an easy way to produce simple harmonic motion by using uniform circular motion. The figure below demonstrates one way of using this method. A ball is attached to a uniformly rotating vertical turntable, and its shadow is projected onto the floor as shown. The shadow undergoes simple harmonic motion.



Shadow of a Ball Undergoing Simple Harmonic Motion: The shadow of a ball rotating at constant angular velocity ω on a turntable goes back and forth in precise simple harmonic motion.

The next figure shows the basic relationship between uniform circular motion and simple harmonic motion. The point P travels around the circle at constant angular velocity ω. The point P is analogous to the ball on a turntable in the figure above. The projection of the position of P onto a fixed axis undergoes simple harmonic motion and is analogous to the shadow of the object. At a point in time assumed in the figure, the projection has position x and moves to the left with velocity v. The velocity of the point P around the circle equals $|v_{max}|$. The projection of $|v_{max}|$ on the x-axis is the velocity v of the simple harmonic motion along the x-axis.

To see that the projection undergoes simple harmonic motion, note that its position x is given by:

$$x = X\cos\theta, \tag{15.3.20}$$

where $\theta=\omega t$, ω is the constant angular velocity, and X is the radius of the circular path. Thus,

$$x = X\cos\omega t. \tag{15.3.21}$$

The angular velocity ω is in radians per unit time; in this case 2π radians is the time for one revolution *T*. That is, $\omega = \frac{2\pi}{T}$. Substituting this expression for ω, we see that the position *x* is given by:

$$x(t) = \cos(\frac{2\pi t}{T}) = \cos(2\pi f t). \tag{15.3.22}$$

Note: This equation should look familiar from our earlier discussion of simple harmonic motion.

The Simple Pendulum

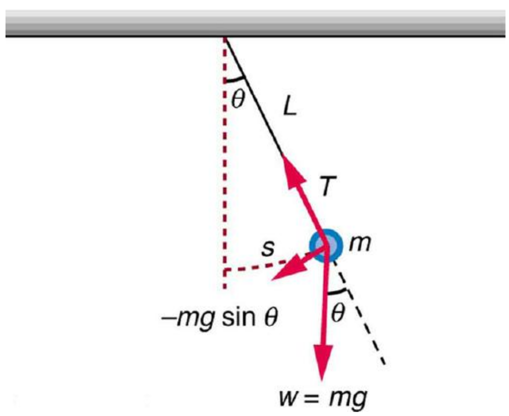
A simple pendulum acts like a harmonic oscillator with a period dependent only on L and g for sufficiently small amplitudes.

learning objectives

- Identify parameters that affect the period of a simple pendulum

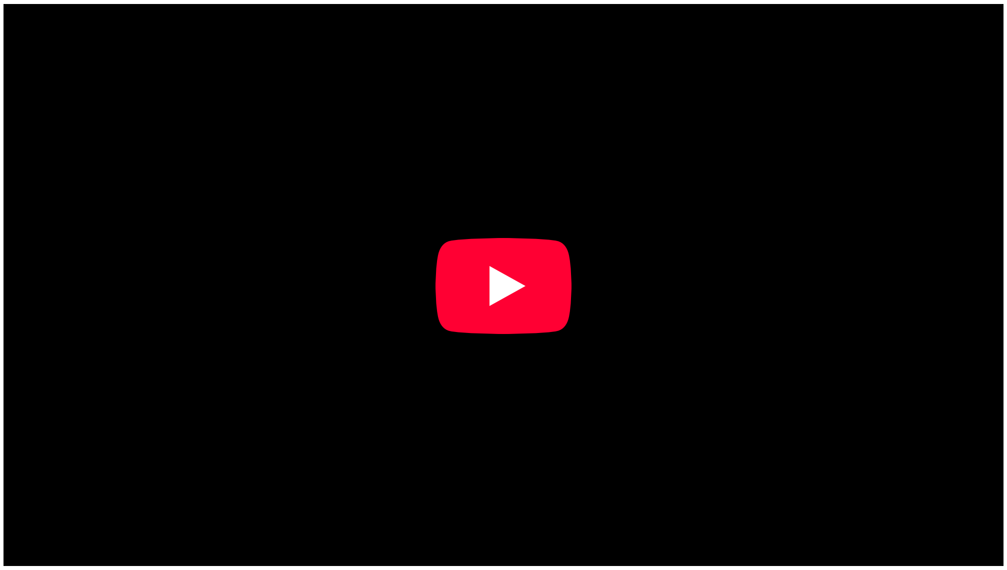
The Simple Pendulum

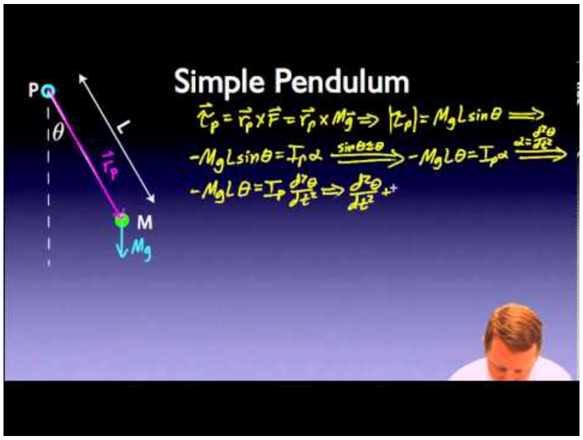
A pendulum is a weight suspended from a pivot so that it can swing freely. When a pendulum is displaced sideways from its resting equilibrium position, it is subject to a restoring force; after it reaches its highest point in its swing, gravity will accelerate it back toward the equilibrium position. When released, the restoring force combined with the pendulum's mass causes it to oscillate about the equilibrium position, swinging back and forth.



Simple Pendulum: A simple pendulum has a small-diameter bob and a string that has a very small mass but is strong enough not to stretch appreciably. The linear displacement from equilibrium is s, the length of the arc. Also shown are the forces on the bob, which result in a net force of −mgsinθ toward the equilibrium position—that is, a restoring force.

For small displacements, a pendulum is a simple harmonic oscillator. A simple pendulum is defined to have an object that has a small mass, also known as the pendulum bob, which is suspended from a wire or string of negligible mass, such as shown in the illustrating figure. Exploring the simple pendulum a bit further, we can discover the conditions under which it performs simple harmonic motion, and we can derive an interesting expression for its period.





Pendulums: A brief introduction to pendulums (both ideal and physical) for calculus-based physics students from the standpoint of simple harmonic motion.

We begin by defining the displacement to be the arc length *s*. We see from the figure that the net force on the bob is tangent to the arc and equals $-mg\sin\theta$. (The weight mg has components $mg\cos\theta$ along the string and $mg\sin\theta$ tangent to the arc.) Tension in the string exactly cancels the component $mg\cos\theta$ parallel to the string. This leaves a net restoring force drawing the pendulum back toward the equilibrium position at $\theta = 0$.

Now, if we can show that the restoring force is directly proportional to the displacement, then we have a simple harmonic oscillator. In trying to determine if we have a simple harmonic oscillator, we should note that for small angles (less than about 15º), $\sin\theta \approx \theta$ ($\sin\theta$ and θ differ by about 1% or less at smaller angles). Thus, for angles less than about 15º, the restoring force F is

$$\mathrm{F} \approx -\mathrm{mg}\theta. \tag{15.3.23}$$

The displacement s is directly proportional to θ. When θ is expressed in radians, the arc length in a circle is related to its radius (*L* in this instance) by:

s=Lθs=Lθ

so that

$$\theta = \mathrm{sL}. \tag{15.3.24}$$

For small angles, then, the expression for the restoring force is:

$$\mathrm{F} \approx \frac{\mathrm{mgL}}{\mathrm{s}}. \tag{15.3.25}$$

This expression is of the form of Hooke's Law:

$$\mathrm{F} \approx -\mathrm{kx} \tag{15.3.26}$$

where the force constant is given by $k=mg/L$ and the displacement is given by $x=s$. For angles less than about 15º, the restoring force is directly proportional to the displacement, and the simple pendulum is a simple harmonic oscillator.

Using this equation, we can find the period of a pendulum for amplitudes less than about 15º. For the simple pendulum:

$$\mathrm{T}=2\pi\sqrt{\frac{\mathrm{m}}{\mathrm{k}}}=2\pi\sqrt{\frac{\mathrm{m}}{\frac{\mathrm{mg}}{\mathrm{L}}}}. \tag{15.3.27}$$

Thus,

$$\mathrm{T}=2\pi\sqrt{\frac{\mathrm{L}}{\mathrm{g}}} \tag{15.3.28}$$

or the period of a simple pendulum. This result is interesting because of its simplicity. The only things that affect the period of a simple pendulum are its length and the acceleration due to gravity. The period is completely independent of other factors, such as mass. Even simple pendulum clocks can be finely adjusted and accurate. Note the dependence of T on g. If the length of a pendulum is precisely known, it can actually be used to measure the acceleration due to gravity. If θ is less than about 15º, the period T for a pendulum is nearly independent of amplitude, as with simple harmonic oscillators. In this case, the motion of a pendulum as a function of time can be modeled as:

$$\theta(\mathrm{t})=\theta_{\mathrm{o}}\cos(\frac{2\pi\mathrm{t}}{\mathrm{T}}) \tag{15.3.29}$$

For amplitudes larger than 15º, the period increases gradually with amplitude so it is longer than given by the simple equation for T above. For example, at an amplitude of θ_0 = 23° it is 1% larger. The period increases asymptotically (to infinity) as $\theta_0$approaches 180°, because the value θ_0 = 180° is an unstable equilibrium point for the pendulum.

The Physical Pendulum

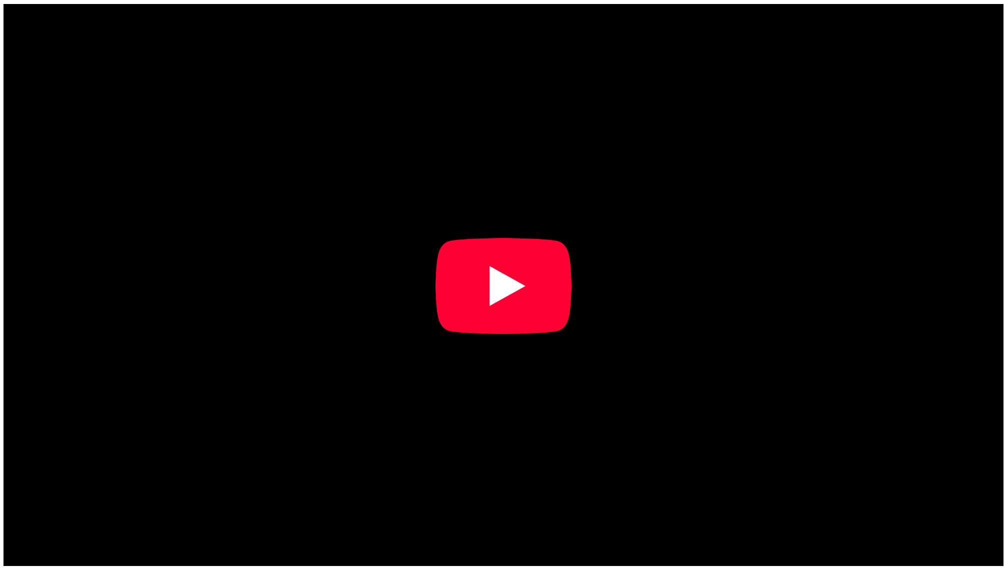
The period of a physical pendulum depends upon its moment of inertia about its pivot point and the distance from its center of mass.

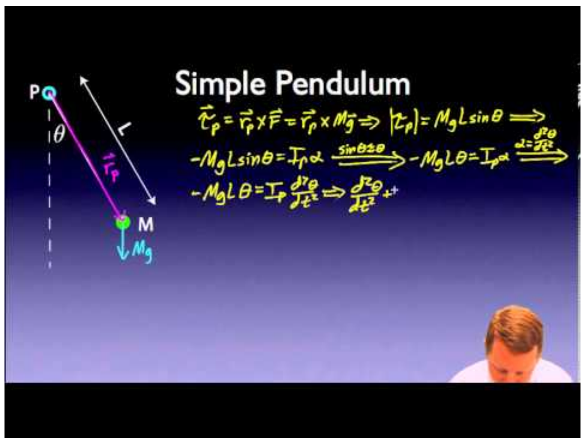
learning objectives

- Identify parameters that affect the period of a physical pendulum

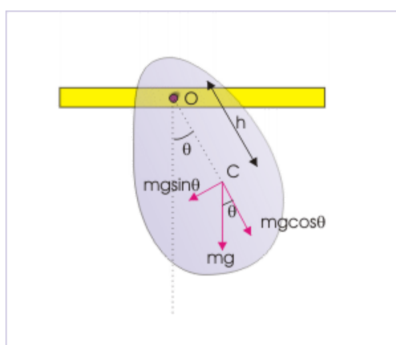
The Physical Pendulum

Recall that a simple pendulum consists of a mass suspended from a massless string or rod on a frictionless pivot. In that case, we are able to neglect any effect from the string or rod itself. In contrast, a *physical pendulum* (sometimes called a compound pendulum) may be suspended by a rod that is not massless or, more generally, may be an arbitrarily-shaped, rigid body swinging by a pivot (see). In this case, the pendulum's period depends on its moment of inertia around the pivot point.





Pendulums – Physical Pendulum: A brief introduction to pendulums (both ideal and physical) for calculus-based physics students from the standpoint of simple harmonic motion.



A Physical Pendulum: An example showing how forces act through center of mass. We can calculate the period of this pendulum by determining the moment of inertia of the object around the pivot point.

Gravity acts through the center of mass of the rigid body. Hence, the length of the pendulum used in equations is equal to the linear distance between the pivot and the center of mass (h).

The equation of torque gives:

$$\tau = \mathrm{I}\alpha, \tag{15.3.30}$$

where α is the angular acceleration, τ is the torque, and I is the moment of inertia.

The torque is generated by gravity so:

$$\tau=\mathrm{mgh}\sin\theta, \tag{15.3.31}$$

where *h* is the distance from the center of mass to the pivot point and θ is the angle from the vertical.

Hence, under the small-angle approximation sin\theta \approx \theta,

$$\alpha\approx-\frac{\mathrm{mgh}\theta}{\mathrm{I}}. \tag{15.3.32}$$

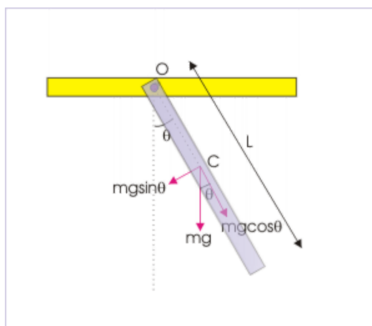
This is of the same form as the conventional simple pendulum and this gives a period of:

$$\mathrm{T}=2\pi\sqrt{\frac{\mathrm{I}}{\mathrm{mgh}}}. \tag{15.3.33}$$

And a frequency of:

\[\mathrm{f=\dfrac{1}{T}=\dfrac{1}{2π}\sqrt{\dfrac{mgh}{I}}.}\]

In case we know the moment of inertia of the rigid body, we can evaluate the above expression of the period for the physical pendulum. For illustration, let us consider a uniform rigid rod, pivoted from a frame as shown (see). Clearly, the center of mass is at a distance *L/2* from the point of suspension:



Uniform Rigid Rod: A rigid rod with uniform mass distribution hangs from a pivot point. This is another example of a physical pendulum.

$$\mathrm{h}=\frac{\mathrm{L}}{2}. \tag{15.3.34}$$

The moment of inertia of the rigid rod about its center is:

$$\mathrm{I_c}=\frac{\mathrm{mL}^2}{12}. \tag{15.3.35}$$

However, we need to evaluate the moment of inertia about the pivot point, not the center of mass, so we apply the parallel axis theorem:

$$\mathrm{I_o}=\mathrm{I_c}+\mathrm{mh}^2=\frac{\mathrm{mL}^2}{12}+\mathrm{m}(\frac{\mathrm{L}}{2})^2=\frac{\mathrm{mL}^2}{3}. \tag{15.3.36}$$

Plugging this result into the equation for period, we have:

$$\mathrm{T}=2\pi\sqrt{\frac{\mathrm{I}}{\mathrm{mgh}}}=2\pi\sqrt{\frac{2\mathrm{mL}^2}{3\mathrm{mgL}}}=2\pi\sqrt{\frac{2\mathrm{L}}{3\mathrm{g}}}. \tag{15.3.37}$$

The important thing to note about this relation is that the period is still independent of the mass of the rigid body. However, it is not independent of the *mass distribution* of the rigid body. A change in shape, size, or mass distribution will change the moment of inertia. This, in turn, will change the period.

As with a simple pendulum, a physical pendulum can be used to measure *g*.

Energy in a Simple Harmonic Oscillator

The total energy in a simple harmonic oscillator is the constant sum of the potential and kinetic energies.

learning objectives

- Explain why the total energy of the harmonic oscillator is constant

Energy in a Simple Harmonic Oscillator

To study the energy of a simple harmonic oscillator, we first consider all the forms of energy it can have. Recall that the potential energy (*PE*), stored in a spring that follows Hooke's Law is:

$$\mathrm{PE} = \frac{1}{2}\mathrm{kx}^2, \tag{15.3.38}$$

where PE is the potential energy, *k* is the spring constant, and *x* is the magnitude of the displacement or deformation. Because a simple harmonic oscillator has no *dissipative forces* , the other important form of energy is kinetic energy (*KE*). Conservation of energy for these two forms is:

$$\mathrm{KE} + \mathrm{PE} = \mathrm{constant}, \tag{15.3.39}$$

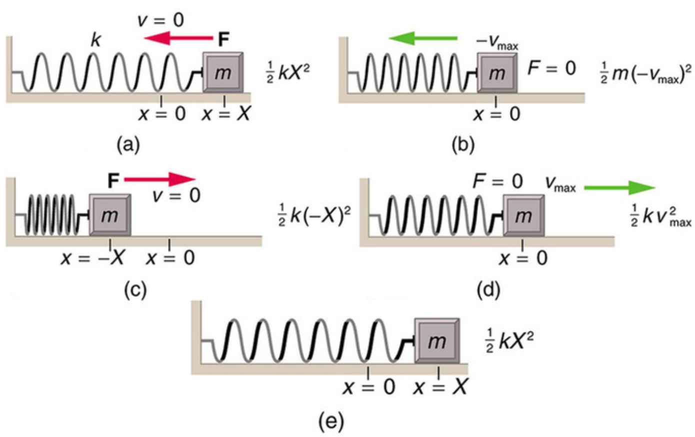
which can be written as:

$$\frac{1}{2}\mathrm{mv}^2 + \frac{1}{2}\mathrm{kx}^2 = \mathrm{constant}. \tag{15.3.40}$$

This statement of conservation of energy is valid for *all* simple harmonic oscillators, including ones where the gravitational force plays a role. For example, for a simple pendulum we replace the velocity with $v=L\omega$, the spring constant with $k=mg/L$, and the displacement term with $x=L\theta$. Thus:

$$\frac{1}{2}\mathrm{mL}^2\omega^2 + \frac{1}{2}\mathrm{mgL}\theta^2 = \mathrm{constant}. \tag{15.3.41}$$

In the case of undamped, simple harmonic motion, the energy oscillates back and forth between kinetic and potential, going completely from one to the other as the system oscillates. So for the simple example of an object on a frictionless surface attached to a spring, as shown again (see), the motion starts with all of the energy stored in the spring. As the object starts to move, the elastic potential energy is converted to kinetic energy, becoming entirely kinetic energy at the equilibrium position. It is then converted back into *elastic potential energy* by the spring, the velocity becomes zero when the kinetic energy is completely converted, and so on. This concept provides extra insight here and in later applications of simple harmonic motion, such as alternating current circuits.



Energy in a Simple Harmonic Oscillator: The transformation of energy in simple harmonic motion is illustrated for an object attached to a spring on a frictionless surface. (a) The mass has achieved maximum displacement from equilibrium. All energy is potential energy. (b) As the mass passes through the equilibrium point with maximum speed all energy in the system is in kinetic energy. (c) Once again, all energy is in the potential form, stored in the compression of the spring (in the first panel the energy was stored in the extension of the spring). (d) Passing through equilibrium again all energy is kinetic. (e) The mass has completed an entire cycle.

The conservation of energy principle can be used to derive an expression for velocity v. If we start our simple harmonic motion with zero velocity and maximum displacement (x=X), then the total energy is:

$$\mathrm{E}=\frac{1}{2}\mathrm{kX}^2. \tag{15.3.42}$$

This total energy is constant and is shifted back and forth between kinetic energy and potential energy, at most times being shared by each. The conservation of energy for this system in equation form is thus:

$$\frac{1}{2}\mathrm{mv}^2+\frac{1}{2}\mathrm{kx}^2=\frac{1}{2}\mathrm{kX}^2. \tag{15.3.43}$$

Solving this equation for v yields:

$$\mathrm{v}=\pm\sqrt{\frac{\mathrm{k}}{\mathrm{m}}(\mathrm{X}^2-\mathrm{x}^2)}. \tag{15.3.44}$$

Manipulating this expression algebraically gives:

$$\mathrm{v}=\pm\sqrt{\frac{\mathrm{k}}{\mathrm{m}}}\mathrm{X}\sqrt{1-\frac{\mathrm{x}^2}{\mathrm{X}^2}}, \tag{15.3.45}$$

and so:

$$\mathrm{v}=\pm\mathrm{v}_{\max}\sqrt{1-\frac{\mathrm{x}^2}{\mathrm{X}^2}}, \tag{15.3.46}$$

where:

$$\mathrm{v}_{\max}=\sqrt{\frac{\mathrm{k}}{\mathrm{m}}}\mathrm{X}. \tag{15.3.47}$$

From this expression, we see that the velocity is a maximum (v_{max}) at x=0. Notice that the maximum velocity depends on three factors. It is directly proportional to amplitude. As you might guess, the greater the maximum displacement, the greater the maximum velocity. It is also greater for stiffer systems because they exert greater force for the same displacement. This observation is seen in the expression for v_{max}; it is proportional to the square root of the force constant k. Finally, the maximum velocity is smaller for objects that have larger masses, because the maximum velocity is inversely proportional to the square root of m. For a given force, objects that have large masses accelerate more slowly.

A similar calculation for the simple pendulum produces a similar result, namely:

$$\omega_{\max}=\sqrt{\frac{\mathrm{g}}{\mathrm{L}}}\theta_{\max}. \tag{15.3.48}$$

Experience with a simple harmonic oscillator: A known mass is hung from a spring of known spring constant and allowed to oscillate. The time for one oscillation (period) is measured. This value is compared to a predicted value, based on the mass and spring constant.

Sinusoidal Nature of Simple Harmonic Motion

The solutions to the equations of motion of simple harmonic oscillators are always sinusoidal, i.e., sines and cosines.

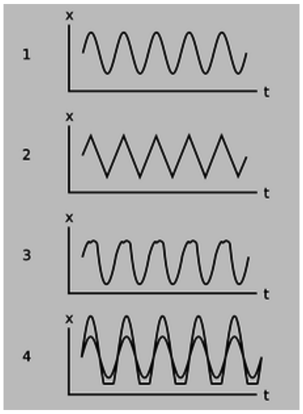
learning objectives

- Review factors responsible for the sinusoidal behavior of uniform circular motion

Sinusoidal Nature of Simple Harmonic Motion

Why are sine waves so common?

If the mass -on-a-spring system discussed in previous sections were to be constructed and its motion were measured accurately, its *x*–*t* graph would be a near-perfect sine-wave shape, as shown in. It is called a “sine wave” or “sinusoidal” even if it is a cosine, or a sine or cosine shifted by some arbitrary horizontal amount. It may not be surprising that it is a wiggle of this general sort, but why is it a specific mathematically perfect shape? Why is it not a sawtooth shape, like in (2); or some other shape, like in (3)? It is notable that a vast number of apparently unrelated vibrating systems show the same mathematical feature. A tuning fork, a sapling pulled to one side and released, a car bouncing on its shock absorbers, all these systems will exhibit sine-wave motion under one condition: the amplitude of the motion must be small.



Sinusoidal and Non-Sinusoidal Vibrations: Only the top graph is sinusoidal. The others vary with constant amplitude and period, but do no describe simple harmonic motion.

Hooke's Law and Sine Wave Generation

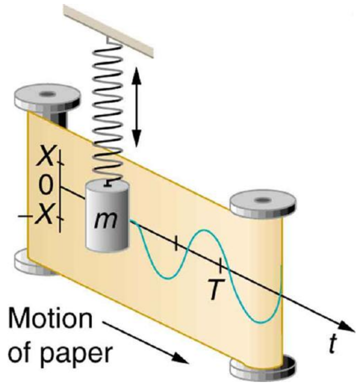
The key to understanding how an object vibrates is to know how the force on the object depends on the object's position. If a system follows Hooke's Law, the restoring force is proportional to the displacement. As touched on in previous sections, there exists a second order differential equation that relates acceleration and displacement.

$$\mathrm{F_{net}} = \mathrm{m}\frac{\mathrm{d^2x}}{\mathrm{dt^2}} = -\mathrm{kx}. \tag{15.3.49}$$

When this general equation is solved for the position, velocity and acceleration as a function of time:

- $\mathrm{x(t)} = \mathrm{A}\cos(\omega\mathrm{t} - \varphi)$
- $\mathrm{v(t)} = \frac{\mathrm{dx}}{\mathrm{dt}} = -\mathrm{A}\omega\sin(\omega\mathrm{t} - \varphi)$
- $\mathrm{a(t)} = \frac{\mathrm{d^2x}}{\mathrm{dt^2}} = -\mathrm{A}\omega^2\cos(\omega\mathrm{t} - \varphi)$

These are all sinusoidal solutions. Consider a mass on a spring that has a small pen inside running across a moving strip of paper as it bounces, recording its movements.

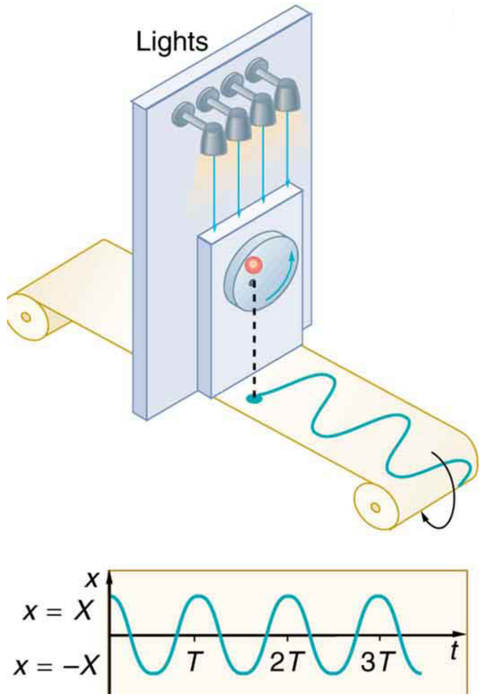


Mass on Spring Producing Sine Wave: The vertical position of an object bouncing on a spring is recorded on a strip of moving paper, leaving a sine wave.

The above equations can be rewritten in a form applicable to the variables for the mass on spring system in the figure.

- $\mathrm{x(t)} = \mathrm{X}\cos(\frac{2\pi\mathrm{t}}{\mathrm{T}})$
- $\mathrm{v(t)} = -\mathrm{v_{max}}\sin(\frac{2\pi\mathrm{t}}{\mathrm{T}})$
- $\mathrm{a(t)} = -\frac{\mathrm{kX}}{\mathrm{m}}\cos(\frac{2\pi\mathrm{t}}{\mathrm{T}})$

Recall that the projection of uniform circular motion can be described in terms of a simple harmonic oscillator. Uniform circular motion is therefore also sinusoidal, as you can see from.



Sinusoidal Nature of Uniform Circular Motion: The position of the projection of uniform circular motion performs simple harmonic motion, as this wavelike graph of x versus t indicates.

Instantaneous Energy of Simple Harmonic Motion

The equations discussed for the components of the total energy of simple harmonic oscillators may be combined with the sinusoidal solutions for *x(t)*, *v(t)*, and *a(t)* to model the changes in kinetic and potential energy in simple harmonic motion.

The kinetic energy *K* of the system at time t is:

$$\mathrm{K(t)=\frac{1}{2}mv^2(t)=\frac{1}{2}m\omega^2A^2\sin^2(\omega t-\varphi)=\frac{1}{2}kA^2\sin^2(\omega t-\varphi).} \tag{15.3.50}$$

The potential energy *U* is:

$$\mathrm{U(t)=\frac{1}{2}kx^2(t)==\frac{1}{2}kA^2\cos^2(\omega t-\varphi).} \tag{15.3.51}$$

Summing *K(t)* and *U(t)* produces the total mechanical energy seen before:

$$\mathrm{E=K+U=\frac{1}{2}kA^2.} \tag{15.3.52}$$

Key Points

- Motion that repeats itself regularly is called periodic motion. One complete repetition of the motion is called a cycle. The duration of each cycle is the period.
- The frequency refers to the number of cycles completed in an interval of time. It is the reciprocal of the period and can be calculated with the equation f=1/T.
- Some motion is best characterized by the angular frequency (ω). The angular frequency refers to the angular displacement per unit time and is calculated from the frequency with the equation $\omega=2\pi f$.
- If an object is vibrating to the right and left, then it must have a leftward force on it when it is on the right side, and a rightward force when it is on the left side.
- The restoring force causes an oscillating object to move back toward its stable equilibrium position, where the net force on it is zero.
- The simplest oscillations occur when the restoring force is directly proportional to displacement. In this case the force can be calculated as $\mathrm{F=-kx}$, where F is the restoring force, k is the force constant, and x is the displacement.

- The motion of a mass on a spring can be described as *Simple Harmonic Motion* (SHM): oscillatory motion that follows Hooke's Law.
- The period of a mass on a spring is given by the equation $T = 2\pi\sqrt{\frac{m}{k}}$
- Simple harmonic motion is often modeled with the example of a mass on a spring, where the restoring force obey's Hooke's Law and is directly proportional to the displacement of an object from its equilibrium position.
- Any system that obeys simple harmonic motion is known as a simple harmonic oscillator.
- The equation of motion that describes simple harmonic motion can be obtained by combining Newton's Second Law and Hooke's Law into a second-order linear ordinary differential equation: $F_{net} = m\frac{d^2x}{dt^2} = -kx.$
- Uniform circular motion describes the movement of an object traveling a circular path with constant speed. The one-dimensional projection of this motion can be described as simple harmonic motion.
- In uniform circular motion, the velocity vector v is always tangent to the circular path and constant in magnitude. The acceleration is constant in magnitude and points to the center of the circular path, perpendicular to the velocity vector at every instant.
- If an object moves with angular velocity ω around a circle of radius r centered at the origin of the x-y plane, then its motion along each coordinate is simple harmonic motion with amplitude r and angular frequency ω.
- A simple pendulum is defined as an object that has a small mass, also known as the pendulum bob, which is suspended from a wire or string of negligible mass.
- When displaced, a pendulum will oscillate around its equilibrium point due to momentum in balance with the restoring force of gravity.
- When the swings (amplitudes) are small, less than about 15º, the pendulum acts as a simple harmonic oscillator with period $T = 2\pi\sqrt{\frac{L}{g}}$, where L is the length of the string and g is the acceleration due to gravity.
- A physical pendulum is the generalized case of the simple pendulum. It consists of any rigid body that oscillates about a pivot point.
- For small amplitudes, the period of a physical pendulum only depends on the moment of inertia of the body around the pivot point and the distance from the pivot to the body's center of mass. It is calculated as:$T = 2\pi\sqrt{\frac{I}{mgh}}$.
- The period is still independent of the total mass of the rigid body. However, it is not independent of the mass distribution of the rigid body. A change in shape, size, or mass distribution will change the moment of inertia and thus, the period.
- The sum of the kinetic and potential energies in a simple harmonic oscillator is a constant, i.e., $KE + PE = constant$. The energy oscillates back and forth between kinetic and potential, going completely from one to the other as the system oscillates.
- In a spring system, the conservation equation is written as: $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = constant = \frac{1}{2}kX^2$, where X is the maximum displacement.
- The maximum velocity depends on three factors: amplitude, the stiffness factor, and mass: $v_{max} = \sqrt{\frac{k}{m}}X$.
- For simple harmonic oscillators, the equation of motion is always a second order differential equation that relates the acceleration and the displacement. The relevant variables are x, the displacement, and k, the spring constant.
- Solving the differential equation above always produces solutions that are sinusoidal in nature. For example, $x(t), v(t), a(t), K(t),$ and $U(t)$ all have sinusoidal solutions for simple harmonic motion.
- Uniform circular motion is also sinusoidal because the projection of this motion behaves like a simple harmonic oscillator.

Key Terms

- **period**: The duration of one cycle in a repeating event.
- **angular frequency**: The angular displacement per unit time.
- **frequency**: The quotient of the number of times n a periodic phenomenon occurs over the time t in which it occurs: f = n / t.
- **Restoring force**: A variable force that gives rise to an equilibrium in a physical system. If the system is perturbed away from the equilibrium, the restoring force will tend to bring the system back toward equilibrium. The restoring force is a function only of position of the mass or particle. It is always directed back toward the equilibrium position of the system
- **amplitude**: The maximum absolute value of some quantity that varies.
- **simple harmonic oscillator**: A device that implements Hooke's law, such as a mass that is attached to a spring, with the other end of the spring being connected to a rigid support, such as a wall.
- **oscillator**: A pattern that returns to its original state, in the same orientation and position, after a finite number of generations.

- **centripetal acceleration**: Acceleration that makes a body follow a curved path: it is always perpendicular to the velocity of a body and directed towards the center of curvature of the path.
- **uniform circular motion**: Movement around a circular path with constant speed.
- **simple pendulum**: A hypothetical pendulum consisting of a weight suspended by a weightless string.
- **physical pendulum**: A pendulum where the rod or string is not massless, and may have extended size; that is, an arbitrarily-shaped, rigid body swinging by a pivot. In this case, the pendulum's period depends on its moment of inertia around the pivot point.
- **mass distribution**: Describes the spatial distribution, and defines the center, of mass in an object.
- **elastic potential energy**: The energy stored in a deformable object, such as a spring.
- **dissipative forces**: Forces that cause energy to be lost in a system undergoing motion.
- **sinusoidal**: In the form of a wave, especially one whose amplitude varies in proportion to the sine of some variable (such as time).

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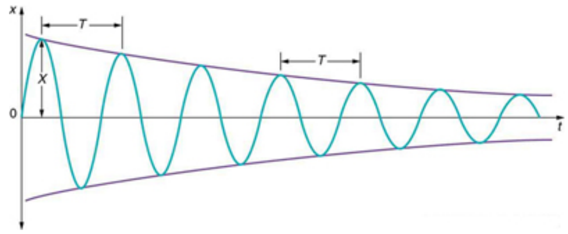
15.4: Damped and Driven Oscillations

learning objectives

- Describe the time evolution of the motion of the damped harmonic oscillator

The Physical Situation

The simple harmonic oscillator describes many physical systems throughout the world, but early studies of physics usually only consider ideal situations that do not involve friction. In the real world, however, frictional forces – such as air resistance – will slow, or dampen, the motion of an object. Sometimes, these dampening forces are strong enough to return an object to equilibrium over time.



Damped Harmonic Motion: Illustrating the position against time of our object moving in simple harmonic motion. We see that for small damping, the amplitude of our motion slowly decreases over time.

The simplest and most commonly seen case occurs when the frictional force is proportional to an object's velocity. Note that other cases exist which may lead to nonlinear equations which go beyond the scope of this example.

Consider an object of mass *m* attached to a spring of constant *k*. Let the damping force be proportional to the mass' velocity by a proportionality constant, *b,* called the *vicious damping coefficient.* We can describe this situation using Newton's second law, which leads to a second order, linear, homogeneous, ordinary differential equation. We simply add a term describing the damping force to our already familiar equation describing a simple harmonic oscillator to describe the general case of damped harmonic motion.

$$\begin{aligned} F_{net} &= m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0 && (15.4.1) \\ &= \frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0 && (15.4.2) \\ &= \frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \omega_0^2 x = 0 && (15.4.3) \end{aligned}$$

$$\omega_0^2 = \frac{k}{m}, \gamma = \frac{b}{m} \tag{15.4.4}$$

This notation uses $\frac{d^2x}{dt^2}$, the acceleration of our object, $\frac{dx}{dt}$, the velocity of our object, ω_0, undamped angular frequency of oscillation, and ɣ, which we can call the damping ratio.

Solving the Differential Equation; Interpreting Results

We solve this differential equation for our equation of motion of the system, x(t). We assume a solution in the form of an exponential, where a is a constant value which we will solve for.

$$x(t) = e^{at} \tag{15.4.5}$$

Plugging this into the differential equation we find that there are three results for a, which will dictate the motion of our system. We can solve for a by using the quadratic equation.

$$\begin{aligned} F_{net} &= a^2x + \gamma ax + \omega_0^2 x = 0 && (15.4.6) \\ &= a^2 + \gamma a + \omega_0^2 = 0 && (15.4.7) \end{aligned}$$

$$a = \frac{\gamma \pm \sqrt{\gamma^2 - 4\omega_0^2}}{2} \tag{15.4.8}$$

The physical situation has three possible results depending on the value of a, which depends on the value of what is under our radical. This expression can be positive, negative, or equal to zero which will result in overdamping, underdamping, and critical damping, respectively.

$\gamma^2 > 4\omega_0^2$ is the *Over Damped* case. In this case, the system returns to equilibrium by exponentially decaying towards zero. The system will not pass the equilibrium position more than once.

$\gamma^2 < 4\omega_0^2$ is the *Under Damped* case. In this case, the system oscillates as it slowly returns to equilibrium and the amplitude decreases over time. Figure 1 depicts an underdamped case.

$\gamma^2 = 4\omega_0^2$ is the*Critically Damped* case. In this case, the system returns to equilibrium very quickly without oscillating and without passing the equilibrium position at all.

Driven Oscillations and Resonance

Driven harmonic oscillators are damped oscillators further affected by an externally applied force.

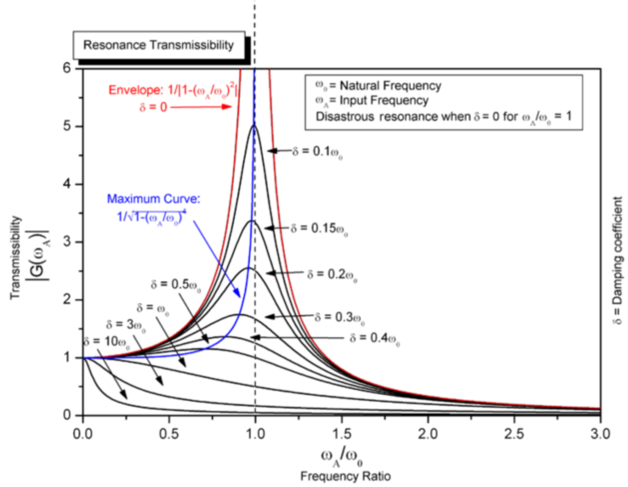
learning objectives

- Describe a driven harmonic oscillator as a type of damped oscillator

In classical mechanics, a harmonic oscillator is a system that, when displaced from its equilibrium position, experiences a restoring force, F, proportional to the displacement, $\mathrm{xF} \rightarrow = -\mathrm{k}\overrightarrow{\mathrm{x}}$ where k is a positive constant. If a frictional force (damping) proportional to the velocity is also present, the harmonic oscillator is described as a damped oscillator.

Driven harmonic oscillators are damped oscillators further affected by an externally applied force F(t). Newton's second law takes the form $\mathrm{F(t)} - \mathrm{kx} - \mathrm{c}\frac{\mathrm{dx}}{\mathrm{dt}} = \mathrm{m}\frac{\mathrm{d^2x}}{\mathrm{dt^2}}$. It is usually rewritten into the form $\frac{\mathrm{d^2x}}{\mathrm{dt^2}} + 2\zeta\omega_0\frac{\mathrm{dx}}{\mathrm{dt}} + \omega_0^2\mathrm{x} = \frac{\mathrm{F(t)}}{\mathrm{m}}$. This equation can be solved exactly for any driving force, using the solutions z(t) which satisfy the unforced equation: $\frac{\mathrm{d^2z}}{\mathrm{dt^2}} + 2\zeta\omega_0\frac{\mathrm{dz}}{\mathrm{dt}} + \omega_0^2\mathrm{z} = 0$, and which can be expressed as damped sinusoidal oscillations$\mathrm{z(t)} = \mathrm{Ae}^{-\zeta\omega_0\mathrm{t}}\sin(\sqrt{1-\zeta^2}\,\omega_0\mathrm{t}+\varphi)$ in the case where $\zeta \leq 1$. The amplitude A and phase φ determine the behavior needed to match the initial conditions. In the case $\zeta < 1$ and a unit step input with $\mathrm{x}(0) = 0$ the solution is: $\mathrm{x(t)} = 1 - \mathrm{e}^{-\zeta\omega_0\mathrm{t}}\frac{\sin(\sqrt{1-\zeta^2}\omega_0\mathrm{t}+\varphi)}{\sin(\varphi)}$ with phase φ given by $\cos\varphi = \zeta$. The time an oscillator needs to adapt to changed external conditions is of the order \(\mathrm{τ =\frac{1}{(ζ_0)}}\). In physics, the adaptation is called relaxation, and τ is called the relaxation time.

In the case of a sinusoidal driving force: $\frac{\mathrm{d^2x}}{\mathrm{dt^2}} + 2\zeta\omega_0\frac{\mathrm{dx}}{\mathrm{dt}} + \omega_0^2\mathrm{x} = \frac{1}{\mathrm{m}}\mathrm{F_0}\sin(\omega\mathrm{t})$, where $\mathrm{F_0}$ is the driving amplitude and ωω is the driving frequency for a sinusoidal driving mechanism. This type of system appears in AC driven RLC circuits (resistor-inductor-capacitor) and driven spring systems having internal mechanical resistance or external air resistance. The general solution is a sum of a transient solution that depends on initial conditions, and a steady state that is independent of initial conditions and depends only on the driving amplitude $\mathrm{F_0}$, driving frequency ωω, undamped angular frequency ω_0, and the damping ratio ζ. For a particular driving frequency called the resonance, or resonant frequency $\omega_\mathrm{r} = \omega_0\sqrt{1-2\zeta^2}$, the amplitude (for a given $\mathrm{F_0}$) is maximum. This resonance effect only occurs when $\zeta < 1/\sqrt{2}$, i.e. for significantly underdamped systems. For strongly underdamped systems the value of the amplitude can become quite large near the resonance frequency (see).



Resonance: Steady state variation of amplitude with frequency and damping of a driven simple harmonic oscillator.

Key Points

- To describe a damped harmonic oscillator, add a velocity dependent term, bx, where b is the *vicious dampingcoefficient*.
- Solve the differential equation for the equation of motion, x(t).
- Depending on the values of the damping coefficient and undamped angular frequency, the results will be one of three cases: an under damped system, an over damped system, or a critically damped system.
- Newton's second law takes the form $\mathrm{F}(\mathrm{t})-\mathrm{kx}-\mathrm{c}\frac{\mathrm{dx}}{\mathrm{dt}}=\mathrm{m}\frac{\mathrm{d}^2\mathrm{x}}{\mathrm{dt}^2}$ for driven harmonic oscillators.
- The resonance effect occurs only in the underdamped systems.
- For strongly underdamped systems the value of the amplitude can become quite large near the resonance frequency.

Key Terms

- **Under Damped**: "The condition in which damping of an oscillator causes it to return to equilibrium with the amplitude gradually decreasing to zero; system returns to equilibrium faster but overshoots and crosses the equilibrium position one or more times. "
- **Critically Damped**: "The condition in which the damping of an oscillator causes it to return as quickly as possible to its equilibrium position without oscillating back and forth about this position. "
- **Over Damped**: "The condition in which damping of an oscillator causes it to return to equilibrium without oscillating; oscillator moves more slowly toward equilibrium than in the critically damped system. "
- **oscillator**: A pattern that returns to its original state, in the same orientation and position, after a finite number of generations.
- **equilibrium**: The state of a body at rest or in uniform motion, the resultant of all forces on which is zero.
- **force**: A physical quantity that denotes ability to push, pull, twist or accelerate a body which is measured in a unit dimensioned in mass × distance/time² (ML/T^2): SI: newton (N); CGS: dyne (dyn)

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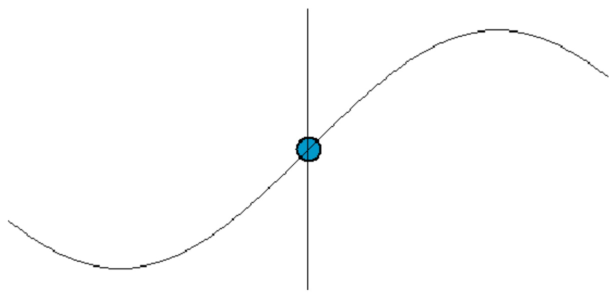
15.5: Waves

learning objectives

- Describe process of energy and mass transfer during wave motion

Vibrations and waves are extremely important phenomena in physics. In nature, oscillations are found everywhere. From the jiggling of atoms to the large oscillations of sea waves, we find examples of vibrations in almost every physical system. In physics a wave can be thought of as a disturbance or oscillation that travels through space-time, accompanied by a transfer of energy. Wave motion transfers energy from one point to another, often with no permanent displacement of the particles of the medium —that is, with little or no associated mass transport. They consist, instead, of oscillations or vibrations around almost fixed locations.

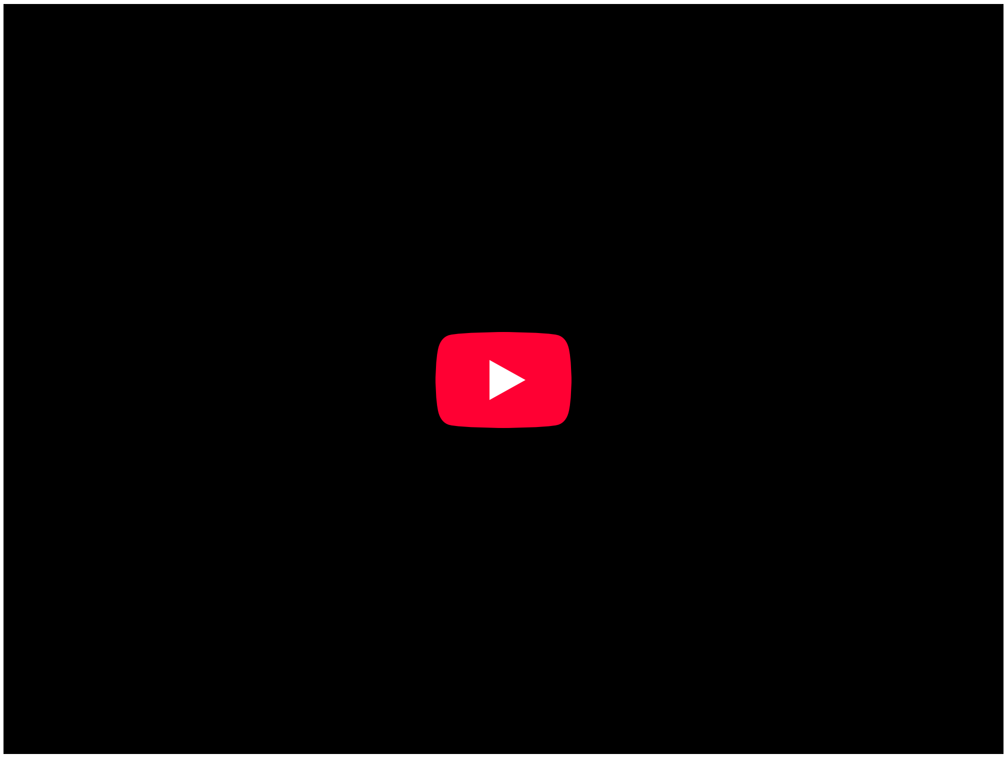
The emphasis of the last point highlights an important misconception of waves. Waves transfer energy not mass. An easy way to see this is to imagine a floating ball a few yards out to sea. As the waves propagate (i.e., travel) towards the shore, the ball will not come towards the shore. It may come to shore eventually due to the tides, current or wind, but the waves themselves will not carry the ball with them. A wave only moves mass perpendicular to the direction of propagation—in this case up and down, as illustrated in the figure below:

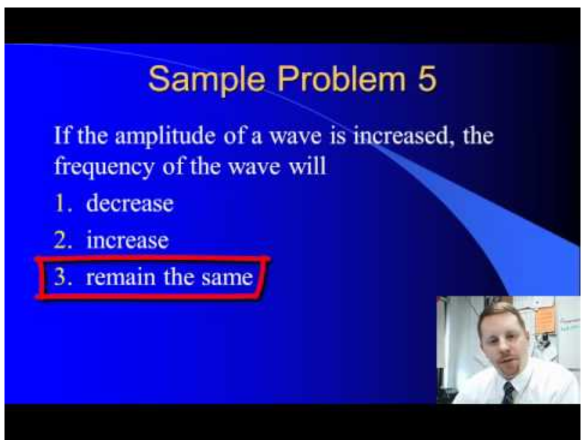


Wave motion: The point along the axis is analogous to the floating ball at sea. We notice that while it moves up and down it does not move in the direction of the wave's propagation.

A wave can be transverse or longitudinal depending on the direction of its oscillation. Transverse waves occur when a disturbance causes oscillations perpendicular (at right angles) to the propagation (the direction of energy transfer). Longitudinal waves occur when the oscillations are parallel to the direction of propagation. While mechanical waves can be both transverse and longitudinal, all electromagnetic waves are transverse. Sound, for example, is a longitudinal wave.

The description of waves is closely related to their physical origin for each specific instance of a wave process. For example, acoustics is distinguished from optics in that sound waves are related to a mechanical rather than an electromagnetic (light) wave transfer caused by vibration. Therefore, concepts such as mass, momentum, inertia or elasticity become crucial in describing acoustic (as distinct from optic) wave processes. This difference in origin introduces certain wave characteristics particular to the properties of the medium involved. In this chapter we will closely examine the difference between longitudinal and transverse waves along with some of the properties they possess. We will also learn how waves are fundamental in describing motion of many applicable physical systems.





The Wave Equation: A brief introduction to the wave equation, discussing wave velocity, frequency, wavelength, and period.

Transverse Waves

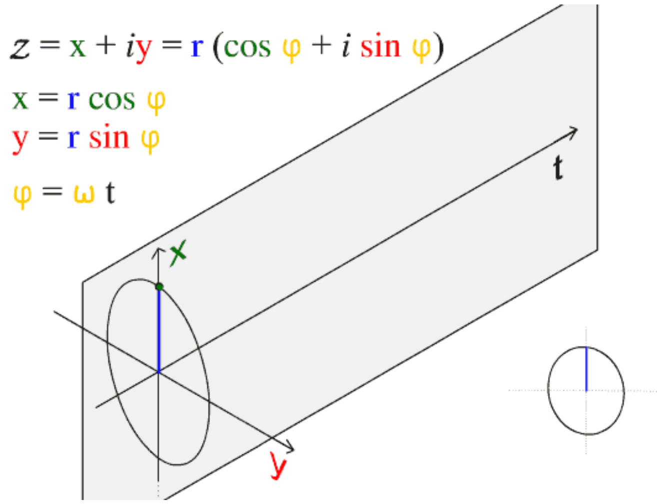
Transverse waves propagate through media with a speed $\vec{v}_w$ orthogonally to the direction of energy transfer.

learning objectives

- Describe properties of the transverse wave

A transverse wave is a moving wave that consists of oscillations occurring perpendicular (or right angled) to the direction of energy transfer. If a transverse wave is moving in the positive *x*-direction, its oscillations are in up and down directions that lie in the *y*–*z* plane. Light is an example of a transverse wave. For transverse waves in matter, the displacement of the medium is perpendicular to the direction of propagation of the wave. A ripple on a pond and a wave on a string are easily visualized transverse waves.

Transverse waves are waves that are oscillating perpendicularly to the direction of propagation. If you anchor one end of a ribbon or string and hold the other end in your hand, you can create transverse waves by moving your hand up and down. Notice though, that you can also launch waves by moving your hand side-to-side. This is an important point. There are two independent directions in which wave motion can occur. In this case, these are the *y* and *z* directions mentioned above. depicts the motion of a transverse wave. Here we observe that the wave is moving in *t* and oscillating in the *x*-*y* plane. A wave can be thought as comprising many particles (as seen in the figure) which oscillate up and down. In the figure we observe this motion to be in *x*-*y* plane (denoted by the red line in the figure). As time passes the oscillations are separated by units of time. The result of this separation is the sine curve we expect when we plot position versus time.

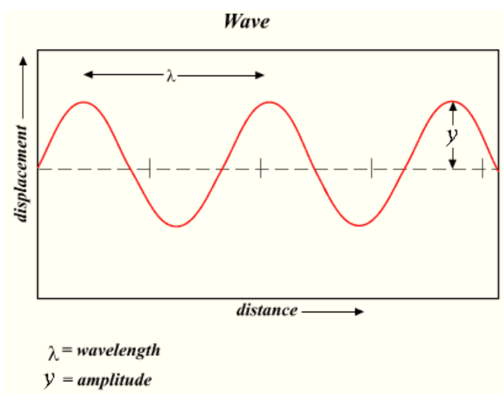


Sine Wave: The direction of propagation of this wave is along the t axis.

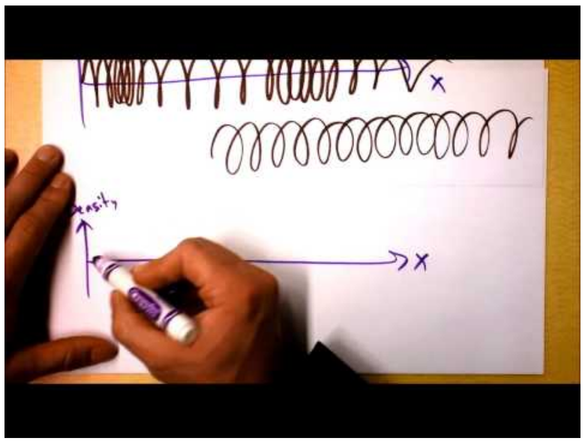
When a wave travels through a medium–i.e., air, water, etc., or the standard reference medium (vacuum)–it does so at a given speed: this is called the speed of propagation. The speed at which the wave propagates is denoted and can be found using the following formula:

$$v = f\lambda \tag{15.5.1}$$

where *v* is the speed of the wave, *f* is the frequency , and is the wavelength. The wavelength spans crest to crest while the amplitude is 1/2 the total distance from crest to trough. Transverse waves have their applications in many areas of physics. Examples of transverse waves include seismic S (secondary) waves, and the motion of the electric (E) and magnetic (M) fields in an electromagnetic plane waves, which both oscillate perpendicularly to each other as well as to the direction of energy transfer. Therefore an electromagnetic wave consists of two transverse waves, visible light being an example of an electromagnetic wave.



Wavelength and Amplitude: The wavelength is the distance between adjacent crests. The amplitude is the 1/2 the distance from crest to trough.



Two Types of Waves: Longitudinal vs. Transverse: Even ocean waves!

Longitudinal Waves

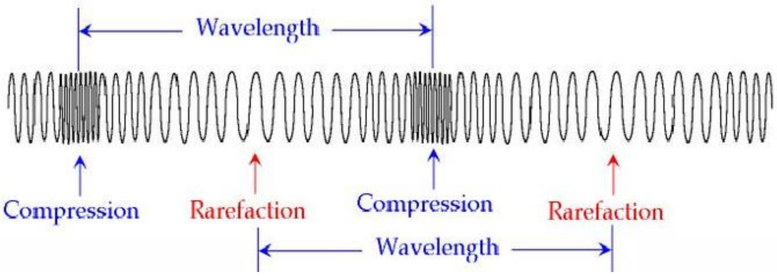
Longitudinal waves, sometimes called compression waves, oscillate in the direction of propagation.

learning objectives

- Give properties and provide examples of the longitudinal wave

Longitudinal Waves

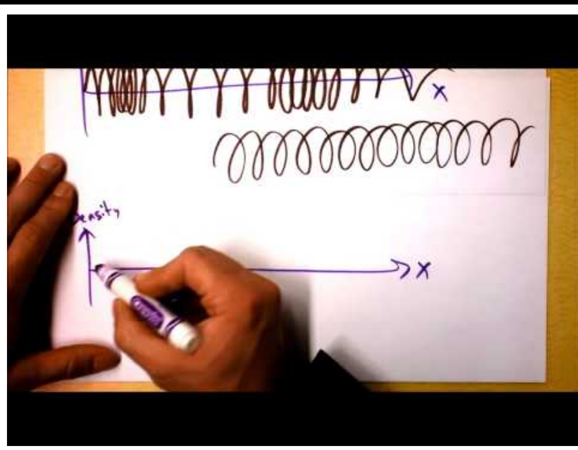
Longitudinal waves have the same direction of vibration as their direction of travel. This means that the movement of the medium is in the same direction as the motion of the wave. Some longitudinal waves are also called compressional waves or compression waves. An easy experiment for observing longitudinal waves involves taking a Slinky and holding both ends. After compressing and releasing one end of the Slinky (while still holding onto the end), a pulse of more concentrated coils will travel to the end of the Slinky.



Longitudinal Waves: A compressed Slinky is an example of a longitudinal wave. The wve propagates in the same direction of oscillation.

Like transverse waves, longitudinal waves do not displace mass. The difference is that each particle which makes up the medium through which a longitudinal wave propagates oscillates along the axis of propagation. In the example of the Slinky, each coil will oscillate at a point but will not travel the length of the Slinky. It is important to remember that energy, in this case in the form of a pulse, is being transmitted and not the displaced mass.

Longitudinal waves can sometimes also be conceptualized as pressure waves. The most common pressure wave is the sound wave. Sound waves are created by the compression of a medium, usually air. Longitudinal sound waves are waves of alternating pressure deviations from the equilibrium pressure, causing local regions of compression and rarefaction. Matter in the medium is periodically displaced by a sound wave, and thus oscillates. When people make a sound, whether it is through speaking or hitting something, they are compressing the air particles to some significant amount. By doing so, they create transverse waves. When people hear sounds, their ears are sensitive to the pressure differences and interpret the waves as different tones.



Two Types of Waves: Longitudinal vs. Transverse: Even ocean waves!

Water Waves

Water waves can be commonly observed in daily life, and comprise both transverse and longitudinal wave motion.

learning objectives

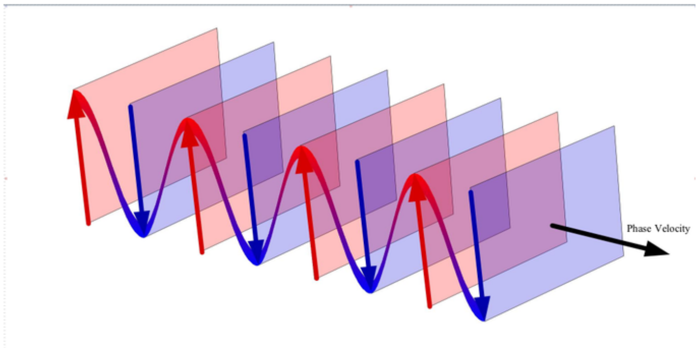
- Describe particle movement in water waves

Water waves, which can be commonly observed in our daily lives, are of specific interest to physicists. Describing detailed fluid dynamics in water waves is beyond the scope of introductory physics courses. Although we often observe water wave propagating in 2D, in this atom we will limit our discussion to 1D propagation.

Water waves: Surface waves in water

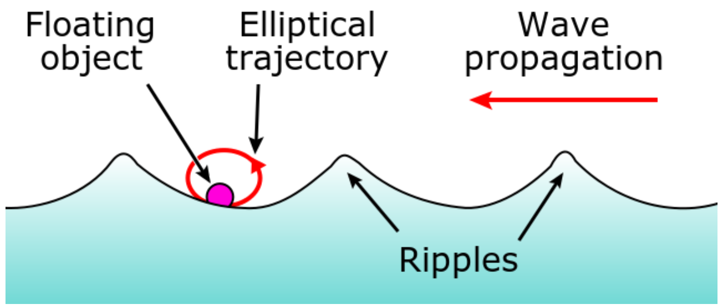
The uniqueness of water waves is found in the observation that they comprise both transverse and longitudinal wave motion. As a result, the particles composing the wave move in clockwise circular motion, as seen in. Oscillatory motion is highest at the surface and diminishes exponentially with depth. Waves are generated by wind passing over the surface of the sea. As long as the waves propagate slower than the wind speed just above the waves, there is an energy transfer from the wind to the waves. Both air pressure differences between the upwind and the lee side of a wave crest, as well as friction on the water surface by the wind (making the water to go into the shear stress), contribute to the growth of the waves.

In the case of monochromatic linear plane waves in deep water, particles near the surface move in circular paths, creating a combination of longitudinal (back and forth) and transverse (up and down) wave motions. When waves propagate in shallow water (where the depth is less than half the wavelength), the particle trajectories are compressed into ellipses. As the wave amplitude (height) increases, the particle paths no longer form closed orbits; rather, after the passage of each crest, particles are displaced slightly from their previous positions, a phenomenon known as Stokes drift.



Plane wave: We see a wave propagating in the direction of the phase velocity. The wave can be thought to be made up of planes orthogonal to the direction of the phase velocity.

Since water waves transport energy, attempts to generate power from them have been made by utilizing the physical motion of such waves. Although larger waves are more powerful, wave power is also determined by wave speed, wavelength, and water density. Deep water corresponds with a water depth larger than half the wavelength, as is a common case in the sea and ocean. In deep water, longer-period waves propagate faster and transport their energy faster. The deep-water group velocity is half the phase velocity. In shallow water for wavelengths larger than about twenty times the water depth (as often found near the coast), the group velocity is equal to the phase velocity. These methods have proven viable in some cases but do not provide a fully sustainable form of renewable energy to date.



Water waves: The motion water waves causes particles to follow clockwise circular motion. This is a result of the wave having both transverse and longitudinal properties.

Wavelength, Freqency in Relation to Speed

Waves are defined by its frequency, wavelength, and amplitude among others. They also have two kinds of velocity: phase and group velocity.

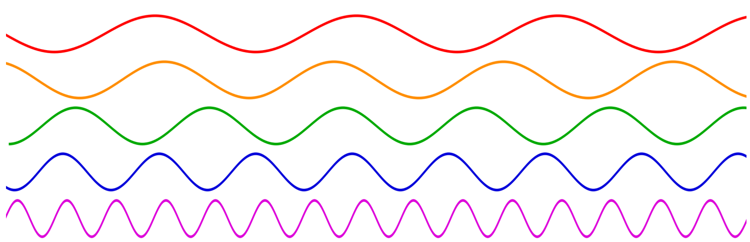
learning objectives

- Identify major characteristic properties of waves

Characteristics of Waves

Waves have certain characteristic properties which are observable at first notice. The first property to note is the amplitude. The amplitude is half of the distance measured from crest to trough. We also observe the wavelength, which is the spatial period of the wave (e.g. from crest to crest or trough to trough). We denote the wavelength by the Greek letter λλ.

The frequency of a wave is the number of cycles per unit time — one can think of it as the number of crests which pass a fixed point per unit time. Mathematically, we make the observation that,



Frequencies of different sine waves.: The red wave has a low frequency sine there is very little repetition of cycles. Conversely we say that the purple wave has a high frequency. Note that time increases along the horizontal.

$$\mathrm{f}=\frac{1}{\mathrm{T}} \tag{15.5.2}$$

where T is the period of oscillation. Frequency and wavelength can also be related-* with respects to a “speed” of a wave. In fact,

\[\mathrm{v=fλ}]

where v is called the wave speed, or more commonly,the phase velocity, the rate at which the phase of the wave propagates in space. This is the velocity at which the phase of any one frequency component of the wave travels. For such a component, any given phase of the wave (for example, the crest) will appear to travel at the phase velocity.

Finally, the group velocity of a wave is the velocity with which the overall shape of the waves’ amplitudes — known as the modulation or envelope of the wave — propagates through space. In, one may see that the overall shape (or “envelope”) propagates to the right, while the phase velocity is negative.

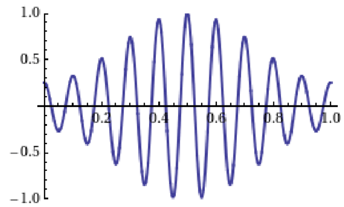


Fig 2: This shows a wave with the group velocity and phase velocity going in different directions. (The group velocity is positive and the phase velocity is negative.)

Energy Transportation

Waves transfer energy which can be used to do work.

learning objectives

- Relate direction of energy and wave transportation

Energy transportion is essential to waves. It is a common misconception that waves move mass. Waves carry energy along an axis defined to be the direction of propagation. One easy example is to imagine that you are standing in the surf and you are hit by a significantly large wave, and once you are hit you are displaced (unless you hold firmly to your ground!). In this sense the wave has done work (it applied a force over a distance). Since work is done over time, the energy carried by a wave can be used to generate power.

Water Wave: Waves that are more massive or have a greater velocity transport more energy.

Similarly we find that electromagnetic waves carry energy. Electromagnetic radiation (EMR) carries energy—sometimes called radiant energy—through space continuously away from the source (this is not true of the near-field part of the EM field). Electromagnetic waves can be imagined as a self-propagating transverse oscillating wave of electric and magnetic fields. EMR also carries both momentum and angular momentum. These properties may all be imparted to matter with which it interacts (through work). EMR is produced from other types of energy when created, and it is converted to other types of energy when it is destroyed. The photon is the quantum of the electromagnetic interaction, and is the basic “unit” or constituent of all forms of EMR. The quantum nature of light becomes more apparent at high frequencies (or high photon energy). Such photons behave more like particles than lower-frequency photons do.

Electromagnetic Wave: Electromagnetic waves can be imagined as a self-propagating transverse oscillating wave of electric and magnetic fields. This 3D diagram shows a plane linearly polarized wave propagating from left to right.

In general, there is a relation of waves which states that the velocity (v) of a wave is proportional to the frequency (f) times the wavelength (λ):

$$\mathrm{v}=\mathrm{f}\lambda \tag{15.5.3}$$

We also know that classical momentum pp is given by p=mvp=mv which relates to force via Newton's second law: $\mathrm{F}=\frac{\mathrm{dp}}{\mathrm{dt}}$

EM waves with higher frequencies carry more energy. This is a direct result of the equations above. Since $\mathrm{v} \propto \mathrm{f}$ we find that higher frequencies imply greater velocity. If velocity is increased then we have greater momentum which implies a greater force (it gets a little bit tricky when we talk about particles moving close to the speed of light, but this observation holds in the classical sense). Since energy is the ability of an object to do work, we find that for $\mathrm{W}=\mathrm{Fd}$ a greater force correlates to more energy transfer. Again, this is an easy phenomenon to experience empirically; just stand in front of a faster wave and feel the difference!

Key Points

- A wave can be thought of as a disturbance or oscillation that travels through space-time, accompanied by a transfer of energy.
- The direction a wave propagates is perpendicular to the direction it oscillates for transverse waves.
- A wave does not move mass in the direction of propagation; it transfers energy.
- Transverse waves oscillate in the z-y plane but travel along the x axis.
- A transverse wave has a speed of propagation given by the equation $\mathrm{v}=\mathrm{f}\lambda$.
- The direction of energy transfer is perpendicular to the motion of the wave.
- While longitudinal waves oscillate in the direction of propagation, they do not displace mass since the oscillations are small and involve an equilibrium position.
- The longitudinal 'waves' can be conceptualized as pulses that transfer energy along the axis of propagation.
- Longitudinal waves can be conceptualized as pressure waves characterized by compression and rarefaction.
- The particles which make up a water wave move in circular paths.
- If the waves move slower than the wind above them, energy is transfered from the wind to the waves.
- The oscillations are greatest on the surface of the wave and become weaker deeper in the fluid.
- The wavelength is the spatial period of the wave.
- The frequency of a wave refers to the number of cycles per unit time and is not to be confused with angular frequency.
- The phase velocity can be expressed as the product of wavelength and frequency.
- Waves which are more massive transfer more energy.
- Waves with greater velocities transfer more energy.
- Energy of a wave is transported in the direction of the waves transportation.

Key Terms

- **medium**: The material or empty space through which signals, waves or forces pass.
- **direction of propagation**: The axis along which the wave travels.
- **wave**: A moving disturbance in the energy level of a field.
- **wavelength**: The length of a single cycle of a wave, as measured by the distance between one peak or trough of a wave and the next; it is often designated in physics as λ, and corresponds to the velocity of the wave divided by its frequency.
- **trough**: A long, narrow depression between waves or ridges.
- **speed of propagation**: The speed at which a wave moves through a medium.
- **crest**: The ridge or top of a wave.
- **transverse wave**: Any wave in which the direction of disturbance is perpendicular to the direction of travel.
- **rarefaction**: a reduction in the density of a material, especially that of a fluid
- **Longitudinal**: Running in the direction of the long axis of a body.
- **compression**: to increase in density; the act of compressing, or the state of being compressed; compaction
- **phase velocity**: The velocity of propagation of a pure sine wave of infinite extent and infinitesimal amplitude.
- **group velocity**: The propagation velocity of the envelope of a modulated travelling wave, which is considered as the propagation velocity of information or energy contained in it.

- **plane wave**: A constant-frequency wave whose wavefronts (surfaces of constant phase) are infinite parallel planes of constant peak-to-peak amplitude normal to the phase velocity vector.
- **wave speed**: The absolute value of the velocity at which the phase of any one frequency component of the wave travels.
- **wavelength**: The length of a single cycle of a wave, as measured by the distance between one peak or trough of a wave and the next; it is often designated in physics as λ, and corresponds to the velocity of the wave divided by its frequency.
- **frequency**: The quotient of the number of times n a periodic phenomenon occurs over the time t in which it occurs: $f = \frac{n}{t}$.
- **energy**: A quantity that denotes the ability to do work and is measured in a unit dimensioned in mass × distance²/time² (ML^2/T^2) or the equivalent.
- **power**: A measure of the rate of doing work or transferring energy.
- **work**: A measure of energy expended in moving an object; most commonly, force times displacement. No work is done if the object does not move.

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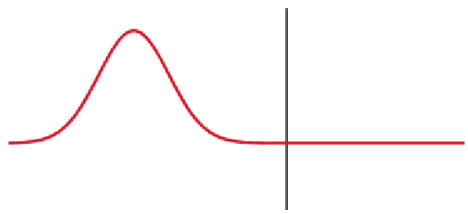
15.6: Wave Behavior and Interaction

learning objectives

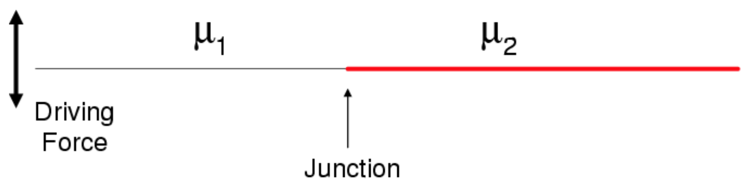
- Distinguish transmission and reflection phenomena

When the medium through which a wave travels suddenly changes, the wave often experiences partial transmission and partial refection at the interface. Reflection is a wave phenomenon that changes the direction of a wavefront at an interface between two different media so that the wavefront returns into the medium from which it originated. Transmission permits the passage of wave, with some or none of the incident wave being absorbed. Reflection and transmission often occur at the same time.



Partial Transmittance and Partial Reflectance: A wave experiences partial transmittance and partial reflectance when the medium through which it travels suddenly changes.

Consider a long string made by connecting two sub-strings with different density μ_1, μ_2. When the string is driven by an external force, partial reflection and transmission occur as in the figure above. For the incoming, reflected, and transmitted waves, we can try a solution of the following forms:



Two Strings With Different Density: Two strings with different density are connected and driven by an external driving force.

$$y_{inc} = A\cos(k_1 x - \omega t) \tag{15.6.1}$$
$$y_{ref} = B\cos(k_1 x + \omega t) \tag{15.6.2}$$
$$y_{trans} = C\cos(k_2 x - \omega t) \tag{15.6.3}$$

k_1 and k_2 are determined by the speed of the wave in each medium. We choose our coordinates such that the junction of two sub-strings is located at x=0. In choosing a trial solution for the waves, we assumed that the incident and transmitted waves travel to the right, while the reflected waves travel to the left. (This is why the '+' sign is chosen before ωtωt in the reflected wave. On the left side of the junction, we have

$$y_l = y_{inc} + y_{ref} = A\cos(k_1 x - \omega t). \tag{15.6.4}$$

On the right side, we have

$$y_r = y_{trans} = C\cos(k_2 x - \omega t) \tag{15.6.5}$$

We will impose additional restriction on the waves by applying "boundary conditions" at x=0. At the boundary x=0, the wave must be continuous and there should be no kinks in it. Thus we must have

$$y_l(x=0,t) = y_r(x=0,t) \tag{15.6.6}$$
$$\frac{\partial y_l(x,t)}{\partial x})_{x=0} = (\frac{\partial y_r(x,t)}{\partial x})_{x=0} \tag{15.6.7}$$

From the first equation, we get $A + B = C$. From the second equation, we get $A - B = (\frac{k_2}{k_1})C$.

Thus, we get the following result.

$$A = \frac{1}{2}(1 + \frac{k_2}{k_1})C \tag{15.6.8}$$

$$B = \frac{1}{2}(1 - \frac{k_2}{k_1})C \tag{15.6.9}$$

We can define the transmission (t) and reflection (r) coefficients as

$$t = \frac{C}{A} = \frac{2k_1}{k_1 + k_2}, r = \frac{B}{A} = \frac{k_1 - k_2}{k_1 + k_2}. \tag{15.6.10}$$

Superposition and Interference

A wave may have a complicated shape that can result from superposition and interference of several waves.

learning objectives

- Distinguish destructive and constructive interference and identify conditions that are required for the superposition of waves

Most waves do not look very simple. They look are often more complex than the simple water waves often considered in textbooks. Simple waves may be created by a simple harmonic oscillation, and thus have a sinusoidal shape. Complex waves are more interesting, even beautiful, but they look formidable. Most waves appear complex because they result from several simple waves adding together. Luckily, the rules for adding waves are quite simple.

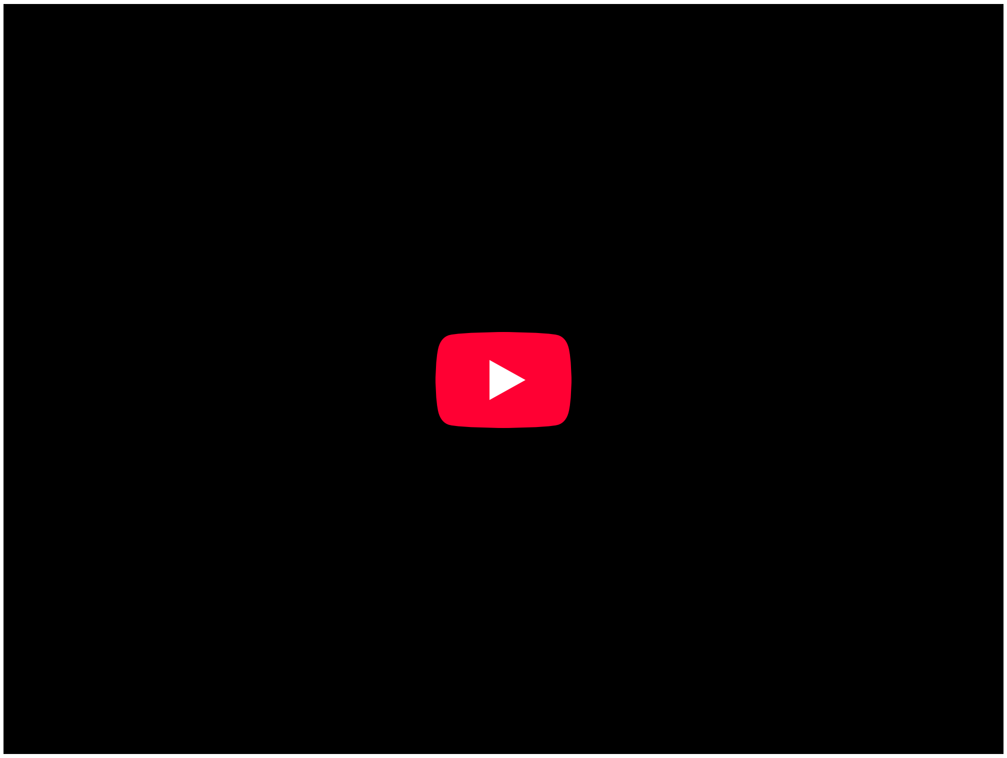
Complex Pattern of Waves: These waves result from the superposition of several waves from different sources, producing a complex pattern.

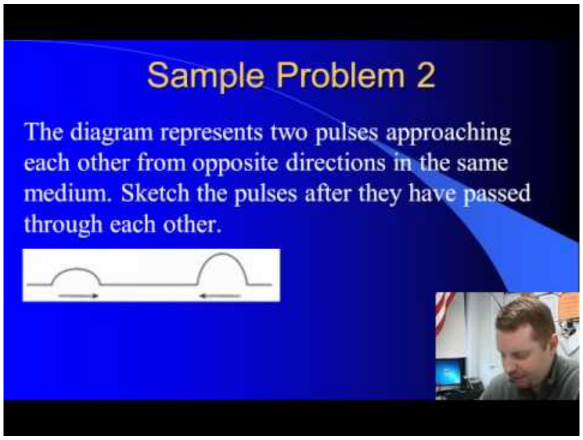
Superposition

When two or more waves arrive at the same point, they superimpose themselves on one another. More specifically, the disturbances of waves are superimposed when they come together—a phenomenon called superposition. Each disturbance corresponds to a force, and forces add. If the disturbances are along the same line, then the resulting wave is a simple addition of the disturbances of the individual waves—that is, their amplitudes add.

Interference

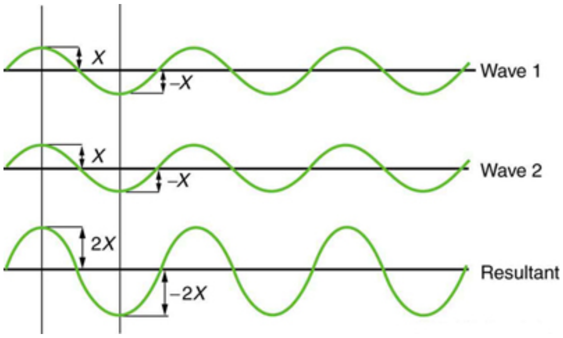
As a result of superposition of waves, interference can be observed. Interference is an effect caused by two or more waves.





Wave Interference: A brief introduction to constructive and destructive wave interference and the principle of superposition.

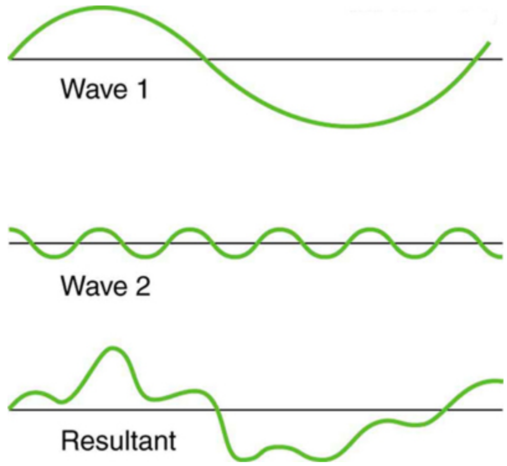
When two identical waves arrive at the same point exactly in phase the crests of the two waves are precisely aligned, as are the troughs. This superposition produces pure constructive interference. Because the disturbances add, constructive interference may produce a wave that has twice the amplitude of the individual waves, but has the same wavelength.



Constructive Interference: Pure constructive interference of two identical waves produces one with twice the amplitude, but the same wavelength.

If two identical waves that arrive exactly out of phase—that is, precisely aligned crest to trough—they may produce pure destructive interference. Because the disturbances are in the opposite direction for this superposition, the resulting amplitude may be zero for destructive interference, and the waves completely cancel.

While pure constructive and pure destructive interference do occur, they require precisely aligned identical waves. The superposition of most waves produces a combination of constructive and destructive interference and can vary from place to place and time to time. Here again, the disturbances add and subtract, producing a more complicated looking wave.



Superposition of Non-Identical Waves: Superposition of non-identical waves exhibits both constructive and destructive interference.

Standing Waves and Resonance

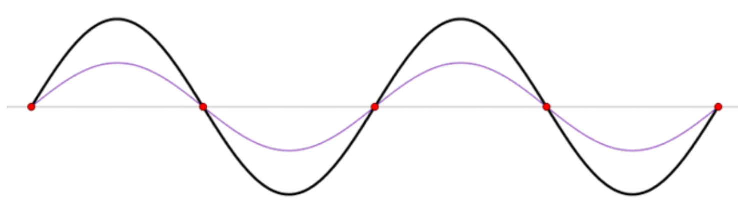
A standing wave is one in which two waves superimpose to produce a wave that varies in amplitude but does not propagate.

learning objectives

- Describe properties of a standing wave

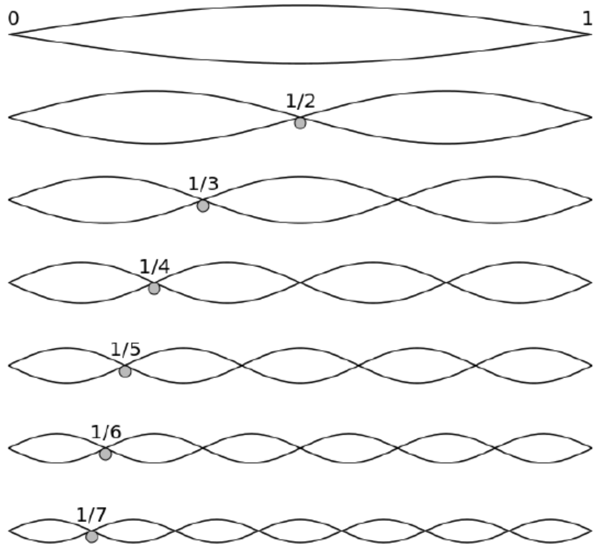
Standing Wave

Sometimes waves do not seem to move, but rather they just vibrate in place. These waves are formed by the superposition of two or more moving waves for two identical waves moving in opposite directions. The waves move through each other with their disturbances adding as they go by. If the two waves have the same amplitude and wavelength then they alternate between constructive and destructive interference. The resultant looks like a wave standing in place and, thus, is called a standing wave.



Standing Wave: A standing wave (black) depicted as the sum of two propagating waves traveling in opposite directions (red and blue).

Standing waves are found on the strings of musical instruments and are due to reflections of waves from the ends of the string. shows seven standing waves that can be created on a string that is fixed at both ends. Nodes are the points where the string does not move; more generally, nodes are where the wave disturbance is zero in a standing wave. The fixed ends of strings must be nodes, too, because the string cannot move there. The word antinode is used to denote the location of maximum amplitude in standing waves. Standing waves on strings have a frequency that is related to the propagation speed v_w of the disturbance on the string. The wavelength λ is determined by the distance between the points where the string is fixed in place.

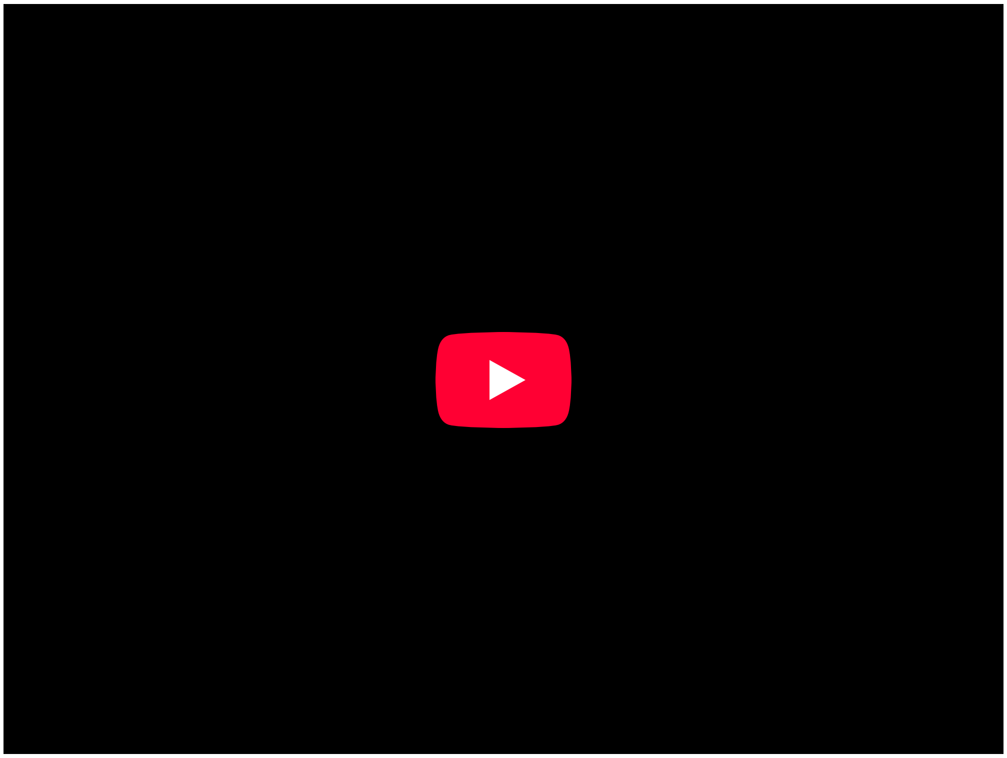


Standing Waves: Standing waves in a string, the fundamental mode and the first six overtones.

The lowest frequency, called the fundamental frequency, is thus for the longest wavelength, twice the length of the string. The overtones or harmonics are multiples of the fundamental frequency. shows the fundamental mode along with six overtones.

Resonance

A closer look at earthquakes provides evidence for conditions appropriate for resonance: standing waves, and constructive and destructive interference. A building may be vibrated for several seconds with a driving frequency matching that of the natural frequency of the vibration of the building—producing a resonance resulting in one building collapsing while neighboring buildings do not. Often buildings of a certain height are devastated while other taller buildings remain intact. The building height matches the condition for setting up a standing wave for that particular height. As the earthquake waves travel along the surface of Earth and reflect off denser rocks, constructive interference occurs at certain points. Often areas closer to the epicenter are not damaged while areas farther away are damaged.



Resonance: A brief overview of resonance, targeted toward introductory physics students.

Harmonic Wave Functions

When vibrations in the string are simple harmonic motion, waves are described by harmonic wave functions.

learning objectives

- Express relationship between the wave number and the wavelength, and frequency and period, of the harmonic wave function

In this Atom we shall consider wave motion resulting from harmonic vibrations and discuss harmonic transverse wave in the context of a string. We assume there is no loss of energy during transmission of wave along the string. This can be approximated when the string is light and taught. In such condition, if we oscillate the free end in harmonic manner, then the vibrations in the string are simple harmonic motion (SHM), perpendicular to the direction of wave motion. The amplitude of wave form remains intact through its passage along the string.

We know that a traveling wave function representing motion in x-direction has the form:

$$y(x,t) = Af(ax - bt). \tag{15.6.11}$$

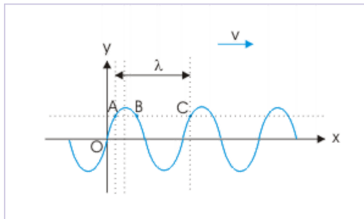
(Read our Atom on “Mathematical Representation of a Traveling Wave. “) For the case of harmonic vibration, we represent harmonic wave motion in terms of either harmonic sine or cosine function:

$$y(x,t) = A\sin(kx - \omega t). \tag{15.6.12}$$

Harmonic Oscillatory Properties

Each particle (or a small segment of string) vibrates in SHM. The particle attains the greatest speed at the mean position and reduces to zero at extreme positions. On the other hand, acceleration of the particle is greatest at extreme positions and zero at the mean position. The vibration of particle is represented by a harmonic sine or cosine function. For x=0:

$$y(x=0,t) = A\sin(\omega t) = A\sin(\omega t). \tag{15.6.13}$$



Harmonic Waves: Harmonic waves are described by sinusoidal functions. The wavelength is equal to linear distance between repetitions of transverse disturbance or phase.

Clearly, the displacement in y-direction is described by the bounded sine or cosine function. The important point here is to realize that oscillatory attributes (like time period, angular and linear frequency) of wave motion is same as that of vibration of a particle in transverse direction.

We know that time period in SHM is equal to time taken by the particle to complete one oscillation. It means that displacement of the particle from the mean position at a given position such as x=0 has same value after time period “T” for:

$\omega T = 2\pi$. Therefore, $\omega = \frac{2\pi}{T}$.

Similarly, displacement of the particle from the mean position at a given time such as t=0 has same value by change the position by “, where $k\lambda = 2\pi$. k is called wavenumber.

We can determine speed of the wave by noting that wave travels a linear distance ” in one period (T). Thus, speed of wave is given by:

$$v = \frac{\lambda}{T} = \frac{\omega}{k}. \tag{15.6.14}$$

Refraction

Refraction is a surface phenomenon that occurs as the change in direction of a wave due to a change in its medium.

learning objectives

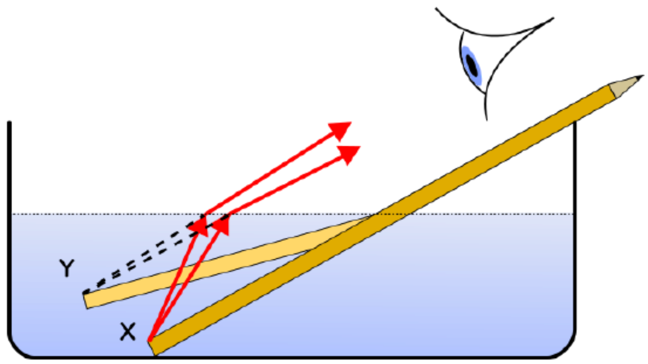
- Formulate the law of conservation of energy and momentum as it is applied to refraction

Refraction is the change in direction of a wave due to a change in its medium. Essentially, it is a surface phenomenon—mainly in governance to the law of conservation of energy and momentum. Due to change of medium, the phase velocity of the wave is changed but its frequency remains constant (most commonly observed when a wave passes from one medium to another at any angle other than 90° or 0°). Refraction of light is the most commonly observed phenomenon, but any type of wave can refract when it interacts with a medium (e.g., when sound waves pass from one medium into another or when water waves move into water of a different depth). Refraction is described by Snell’s law, which states that for a given pair of media and a wave with a single frequency, the ratio of the sines of the angle of incidence θ1 and angle of refraction θ2 is equivalent to the ratio of phase velocities (v_1/v_2) in the two media, or equivalently, to the opposite ratio of the indices of refraction (n_2/n_1):

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1}. \tag{15.6.15}$$

In optics, refraction is a phenomenon that often occurs when waves travel from a medium with a given refractive index to a medium with another at an oblique angle. For example, a light ray will refract as it enters and leaves glass, assuming there is a change in refractive index. A ray traveling along the normal (perpendicular to the boundary) will change speed, but not direction. Refraction still occurs in this case. Understanding of refraction led to the invention of lenses and the refracting telescope.

Refraction can be seen when looking into a bowl of water, as illustrated in. Air has a refractive index of about 1.0003, and water has a refractive index of about 1.33. If a person looks at a straight object, such as a pencil or straw, placed partially in the water at a slant, the object appears to bend at the water's surface. This is due to the bending of light rays as they move from the water to the air. Once the rays reach the eye, the eye traces them back as straight lines (lines of sight). The lines of sight (shown as dashed lines) intersect at a higher position than where the actual rays originated (causing the pencil to appear higher and the water to appear shallower than they actually are).



Refraction in Water: An object (in this case a pencil) partially immersed in water looks bent due to refraction: the light waves from X change direction and so seem to originate at Y. (More accurately, for any angle of view, Y should be vertically above X, and the pencil should appear shorter, not longer as shown.)

Diffraction

Diffraction refers to various phenomena such as the bending of waves around obstacles and the spreading out of waves past small openings.

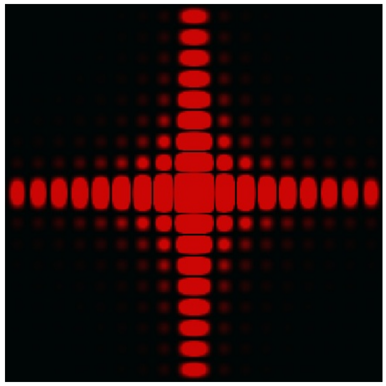
learning objectives

- Describe the phenomenon of diffraction, according to classical physics

Diffraction refers to various phenomena which occur when a wave encounters an obstacle. In classical physics, the diffraction phenomenon is described as the apparent bending of waves around small obstacles and the spreading out of waves past small openings. Similar effects occur when a light wave travels through a medium with a varying refractive index, or a sound wave travels through one with varying acoustic impedance.

Diffraction occurs with all waves, including sound waves, water waves, and electromagnetic waves such as visible light, X-rays and radio waves. As physical objects have wave-like properties (at the atomic level), diffraction also occurs with matter and can be studied according to the principles of quantum mechanics.

Diffraction effects are generally most pronounced for waves whose wavelengths are roughly similar to the dimensions of the diffracting objects. If the obstructing object provides multiple, closely-spaced openings, a complex pattern of varying intensity can result. This is due to the superposition, or interference, of different parts of a wave that travel to the observer by different paths. A good example would be diffraction gratings.



Intensity Pattern: Intensity pattern formed on a screen by diffraction from a square aperture.

The effects of diffraction are often seen in everyday life. The most striking examples of diffraction are those involving light. For example, the closely spaced tracks on a CD or DVD act as a diffraction grating to form the familiar rainbow pattern seen when looking at a disk. This principle can be extended to engineer a grating with a structure such that it will produce any diffraction pattern desired, like the hologram on a credit card. Diffraction in the atmosphere by small particles can cause a bright ring to be visible around a bright light source like the sun or the moon. A shadow of a solid object, using light from a compact source, shows small fringes near its edges. The speckle pattern which is observed when laser light falls on an optically rough surface is also a diffraction phenomenon. All these effects are a consequence of the fact that light propagates as a wave.

Mathematical Represenation of a Traveling Wave

The most general solution of the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ is given as $u(x,t) = f(x+ct) + g(x-ct)$, where f and g are arbitrary functions.

learning objectives

- Formulate solution of the wave equation for a traveling wave

In general, one dimensional waves satisfy the 1D wave equation:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}. \tag{15.6.16}$$

For example, a sinusoidal form

$$u(x,t) = A\sin(kx - \omega t) \tag{15.6.17}$$

is a solution of the wave equation for $c = \frac{\omega}{k}$. In this atom, we will obtain a general mathematical form of a traveling wave.

Solving the Wave Equation

First, we notice that any function u(x,t) satisfying

$$\frac{\partial u}{\partial t} = \pm c \frac{\partial u}{\partial x} (\text{Eq. } 1) \tag{15.6.18}$$

is a solution to the wave equation. To show this, note that

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial t}(\frac{\partial u}{\partial t}) = \pm c \frac{\partial}{\partial t}(\frac{\partial u}{\partial x}) \tag{15.6.19}$$

$$= \pm c \frac{\partial}{\partial x}(\frac{\partial u}{\partial t}) = c^2 \frac{\partial^2 u}{\partial x^2}. \tag{15.6.20}$$

In the middle, we used the equation 1 along with the fact that partial derivatives are interchangeable.

To solve the equation 1, let's introduce new variables: $\phi = x - ct, \psi = x + ct.$ From the chain rules,

$$\frac{\partial}{\partial t}=\frac{\partial \phi}{\partial t}\frac{\partial}{\partial \phi}+\frac{\partial \psi}{\partial t}\frac{\partial}{\partial \psi}=-c\frac{\partial}{\partial \phi}+c\frac{\partial}{\partial \psi}. \tag{15.6.21}$$

$$\frac{\partial}{\partial x}=\frac{\partial \phi}{\partial x}\frac{\partial}{\partial \phi}+\frac{\partial \psi}{\partial x}\frac{\partial}{\partial \psi}=\frac{\partial}{\partial \phi}+\frac{\partial}{\partial \psi}. \tag{15.6.22}$$

With the change of variables, the equation 1 becomes $\frac{\partial u_+}{\partial \phi}=0$ for the equation with the “+” sign and $\frac{\partial u_-}{\partial \psi}=0$ for the “-” sign. Therefore, we see that

$$u_+(\phi,\psi)=f(\psi), u_-(\phi,\psi)=g(\phi), \tag{15.6.23}$$

where f and g are arbitrary functions. Converting back to the original variables of x and t, we conclude that the solution of the original wave equation is

$$u(x,t)=f(x+ct)+g(x-ct). \tag{15.6.24}$$

f(x+ct) represents a left-going traveling wave, while g(x-ct) represents a right-going traveling wave. In other words, solutions of the 1D wave equation are sums of a left traveling function f and a right traveling function g. “Traveling” means that the shape of these individual arbitrary functions with respect to x stays constant, however the functions are translated left and right with time at the speed c. This solution was derived by Jean le Rond d’Alembert.

Boundary Condition

Any function that contains “x+ct” or “x-ct” can be a solution of the wave equation. The wave function is further determined by taking additional information, usually given as boundary conditions and some others. For example, in the case of a string in a guitar, we know that the wave has zero amplitude at both ends: u(x=0)=u(x=L)=0. Also, the shape of the function at an instance can be provided to determine the function.

Wave Equation in Two Dimensions: A solution of the wave equation in two dimensions with a zero-displacement boundary condition along the entire outer edge.

Energy, Intensity, Frequency, and Amplitude

The energy in a wave is proportional to its amplitude squared and the intensity of a wave is defined as power per unit area.

learning objectives

- Describe relationship between the energy and the amplitude, and energy and intensity, of a wave

All waves carry energy. This is seen in practical applications (e.g., in medicine), as well as effects in nature. Some examples of are:

- ultrasound used for deep-heat treatment of muscle strains
- a laser beam to burn away malignant tissue
- water waves that erode beaches
- earthquakes that topple cities

The amount of energy in a wave is related to its amplitude. Large-amplitude earthquakes produce large ground displacements, as seen in. Loud sounds have higher pressure amplitudes and come from larger-amplitude source vibrations than soft sounds. Large ocean breakers erode the shore more than small ones. More quantitatively, a wave is a displacement that is resisted by a restoring force. The larger the displacement x, the larger the force F=-kx needed to create it. Because work W is related to force multiplied by distance (Fx) and energy is put into the wave by the work done to create it, the energy in a wave is related to amplitude. In fact, a wave’s energy is directly proportional to its amplitude squared because:

Earthquake Destruction: The destructive effect of an earthquake is palpable evidence of the energy carried in these waves. The Richter scale rating of earthquakes is related to both their amplitude and the energy they carry.

$$W = \int F(x)dx = \frac{1}{2}kx^2. \tag{15.6.25}$$

The energy effects of a wave depend on time as well as amplitude. For example, the longer deep-heat ultrasound is applied, the more energy it transfers. Therefore, power is more appropriate than energy to describe the "intensity" of a wave. Waves can also be concentrated or spread out. Sunlight, for example, can be focused to burn wood. Earthquakes "spread out" so they do less damage the farther they spread from their source. In both cases, changing the area the waves cover has important effects. All these pertinent factors are included in the definition of intensity I as power (P) per unit area:

$$I = \frac{P}{A}, \tag{15.6.26}$$

where P is the power carried by the wave through area A.

Energy vs. Frequency

In classic wave theory, energy of a wave doesn't depend on the frequency of the wave. However, this is not the case in the microscopic world, as shown in experiments on photoelectric effects (see our Atom on "Photoelectric Effect"). As Einstein postulated to explain photoelectric effects, a quantum of light (photon) carries a specific amount of energy proportional to the frequency of light. Although you can increase the number of photons by increasing the intensity of a beam, the energy of individual photons in the beam is determined by the frequency of the beam.

Key Points

- Reflection is a wave phenomenon that changes the direction of a wavefront at an interface between two different media so that the wavefront returns into the medium from which it originated.
- At the boundary, a wave must be continuous and there should be no kinks in it.
- By imposing boundary conditions, we can solve wave equation and get the form of the waves. Reflection and transmission coefficients are defined as ratio of reflected/transmitted amplitudes and the incoming amplitude.
- The disturbances of waves are superimposed when they come together—a phenomenon called superposition.
- As a result of superposition of waves, inteference can be observed. Interference is an effect caused by two or more waves. Waves can interfere constructively or destructively.
- The superposition of most waves produces a combination of constructive and destructive interference and can vary from place to place and time to time.
- If two waves with the same amplitude and wavelength travel in opposite directions they alternate between constructive and destructive interference. The resultant looks like a wave standing in place and, thus, is called a standing wave.
- Nodes are points of no motion in standing waves. An antinode is the location of maximum amplitude of a standing wave.
- During an earthquake, buildings with a certain height may collapse more easily. This occurs when the building height matches the condition for setting up a standing wave for that particular height.
- We represent harmonic wave motion in terms of either harmonic sine or cosine function: $y(x,t) = A\sin(kx - \omega t)$.
- k and ω in the harmonic wave functinon are related to wavelength and period as follows: $k = \frac{2\pi}{\lambda}, \omega = \frac{2\pi}{T}$.
- The speed of a harmonic wave is given by ω/k.
- Refraction is mainly in governance to the law of conservation of energy and momentum. Due to change of medium, the phase velocity of the wave is changed but its frequency remains constant.

- Refraction is described by Snell's law, which states that for a given pair of media and a wave with a single frequency, $\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$.
- An object partially immersed in water looks bent due to refraction.
- Diffraction is a wave phenomenon. It occurs with all waves, including sound waves, water waves, and electromagnetic waves such as visible light, X-rays and radio waves.
- Diffraction effects are generally most pronounced for waves whose wavelengths are roughly similar to the dimensions of the diffracting objects.
- The effects of diffraction are often seen in everyday life. All these effects are a consequence of the fact that light propagates as a wave.
- Any function u(x,t) satisfying $\frac{\partial u}{\partial t} = \pm c\frac{\partial u}{\partial x}$ is a solution the wave equation. To solve this new equation, we introduced new variables $\phi = x - ct, \psi = x + ct$.
- The solutions of the 1D wave equation are sums of a left traveling function and a right traveling function.
- The wave function is further determined by taking additional information, usually given as boundary conditions and some others.
- The energy effects of a wave depend on the amplitude and duration (time) of the wave. Waves can also be concentrated or spread out. Considering all these factors, intensity is defined as power per unit area.
- In the classical wave theory, energy of a wave doesn't depend on the frequency of the wave. However, the energy of individual photons in a beam is determined by the frequency of the beam.
- Wave's energy is directly proportional to its amplitude squared.

Key Terms

- **boundary condition**: A set of restraints at the boundaries, used to solve a differential equation.
- **superposition**: The summing of two or more field contributions occupying the same space.
- **interference**: An effect caused by the superposition of two systems of waves, such as a distortion on a broadcast signal due to atmospheric or other effects.
- **constructive interference**: Occurs when waves interfere with each other crest to crest and the waves are exactly in phase with each other.
- **destructive interference**: Occurs when waves interfere with each other crest to trough (peak to valley) and are exactly out of phase with each other.
- **resonance**: The increase in the amplitude of an oscillation of a system under the influence of a periodic force whose frequency is close to that of the system's natural frequency.
- **simple harmonic motion**: (SHM) — Oscillating motion (as of a pendulum) in which the acceleration of the oscillator has an equal magnitude but opposite direction to the displacement of it from the equilibrium position.
- **Snell's law**: A formula used to describe the relationship between the angles of incidence and refraction.
- **refractive index**: The ratio of the speed of light in air or vacuum to that in another medium.
- **wave equation**: An important second-order linear partial differential equation for the description of waves such as sound waves, light waves, and water waves.
- **Restoring force**: If the system is perturbed away from the equilibrium, the restoring force will tend to bring the system back toward equilibrium. The restoring force is a function only of position of the mass or particle. It is always directed back toward the equilibrium position of the system.An example is the action of a spring. An idealized spring exerts a force that is proportional to the amount of deformation of the spring from its equilibrium length, exerted in a direction to oppose the deformation. Pulling the spring to a greater length causes it to exert a force that brings the spring back toward its equilibrium length. The amount of force can be determined by multiplying the spring constant of the spring by the amount of stretch.
- **ultrasound**: Sound with a frequency greater than the upper limit of human hearing; approximately 20 kilohertz.
- **photoelectric effects**: In photoelectric effects, electrons are emitted from matter (metals and non-metallic solids, liquids or gases) as a consequence of their absorption of energy from electromagnetic radiation.

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15.7: Waves on Strings

learning objectives

- Calculate the speed of a wave on a string

When studying waves, it is helpful to use a string to observe the physical properties of waves visually. Imagine you are holding one end of a string, and the other end is secured and the string is pulled tight. Now, if you were to flick the string either up and down. The wave that occurs due to this motion is called a transverse wave. A transverse wave is defined as a wave where the movement of the particles of the medium is perpendicular to the direction of the propagation of the wave. Figure 1 shows this in a diagram. In this case, the medium through which the waves propagate is the rope. The wave traveled from one end to the other, while the rope moved up and down.

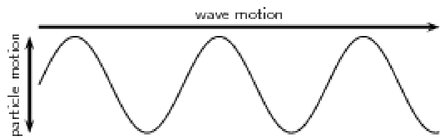


Figure 1: In transverse waves, the media the wave is traveling in moves perpendicular to the direction of the wave.

Wave Properties

Transverse waves have what are called peaks and troughs. The peak is the crest, or top point of the wave and the trough is the valley or bottom point of the wave. Refer to Figure 2 for a visual representation of these terms.The amplitude is the maximum displacement of a particle from its equilibrium position.Wavelength, usually denoted with a lambda (λ) and measured in meters, is the distance from either one peak to the next peak, or one trough to the next trough.Period, usually denoted as T and measured in seconds, is the time it takes for two successive peaks, or one wavelength, to pass through a fixed point.Frequency, *f*, is the number of wavelengths that pass through a given point in 1 second. Frequency is measured by taking the reciprocal of a period: $\mathrm{f}=\frac{1}{\mathrm{T}}$

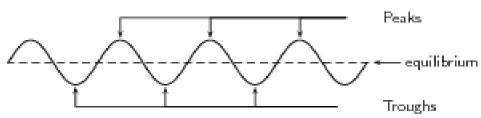


Figure 2: Peaks are the top most points of the waves and troughs are the bottom, or valleys of the waves.

Speed of a Wave on a String

Velocity is found by dividing the distance traveled by the time it took to travel that distance. In waves, this is found by dividing the wavelength by the period: v=λTv=λTWe can take the inverse proportionality to period and frequency and apply it to this situation:

$$\mathrm{v}=\frac{\lambda}{\mathrm{T}} \tag{15.7.1}$$

$$\mathrm{v}=\lambda \frac{1}{\mathrm{T}} \tag{15.7.2}$$

$$\mathrm{v}=\lambda \mathrm{f} \tag{15.7.3}$$

Speed of a Wave on a Vibrating String

Another example of waves on strings are of the waves on vibrating strings, such as in musical instruments. Pianos and guitars both use vibrating strings to produce music. In these cases, the frequency is what characterizes the pitch and therefore the note. The speed of a wave on this kind of string is proportional to the square root of the tension in the string and inversely proportional to the square root of the linear density of the string:$\mathrm{v}=\sqrt{\frac{\mathrm{T}}{\mu}}$

Reflections

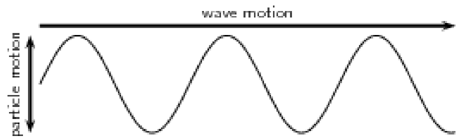
When transverse waves in strings meet one end, they are reflected, and when the incident wave meets the reflected wave, interference occurs.

learning objectives

- Explain when a standing wave occurs

Overview

Imagine you are holding one end of a string, and the other end is secured and the string is pulled tight. Now, if you were to flick the string either up and down. The wave that occurs due to this motion is called a transverse wave. A transverse wave is defined as a wave where the movement of the particles of the medium is perpendicular to the direction of the propagation of the wave. shows this in a diagram. In this case, the medium through which the waves propagate is the rope. The wave traveled from one end to the other, while the rope moved up and down.



Transverse Wave: Diagram of a transverse wave. The wave motion moves perpendicular to the medium it is traveling in.

Properties of Waves

- Transverse waves have what are called peaks and troughs. The peak is the crest, or top point of the wave and the trough is the valley or bottom point of the wave.
- The amplitude is the maximum displacement of a particle from its equilibrium position.
- Wavelength, usually denoted with a lambda (λ) and measured in meters, is the distance from either one peak to the next peak, or one trough to the next trough.
- Period, usually denoted as T and measured in seconds, is the time it takes for two successive peaks, or one wavelength, to pass through a fixed point.
- Frequency, f, is the number of wavelengths that pass through a given point in 1 second. Frequency is measured by taking the reciprocal of a period: $f = \frac{1}{T}$
- Transverse waves can occur while being fixed at the end point or while being free at the end point.

Reflections of Transverse Waves

The way in which a transverse wave reflects depends on whether or not it is fixed at both ends. First we will look at waves that are fixed at both ends:

shows an image of a transverse wave that is reflected from a fixed end. When a transverse wave meets a fixed end, the wave is reflected, but inverted. This swaps the peaks with the troughs and the troughs with the peaks.

Transverse Wave With a Fixed End Point: A transverse wave that is fixed at the end point. The reflected wave is inverted.

is an image of a transverse wave on a string that meets a free end. The wave is reflected, but unlike a transverse wave with a fixed end, it is not inverted.

Transverse Wave With a Free End: When a transverse wave meets a free end, it is reflected.

Standing Waves

When either of the two scenarios of wave reflection occurs, the incident wave meets the reflected wave. These waves move past each other in opposite directions, causing interference. When these two waves have the same frequency, the product of this is called the standing waves. Standing waves appear to be standing still, hence the name. To understand how standing waves occur, we can analyze them further: When the incident wave and reflected wave first meet, both waves have an amplitude is zero. As the waves continue to move past each other, they continue to interfere with each other either constructively of destructively.

As you may remember from previous atoms, when waves are completely in phase and interfere with each other constructively, they are amplified, and when they are completely out of phase and interfere destructively they cancel out. As the waves continue to move past each other, and are reflected from the opposite end, they continue to interfere both ways, and a standing wave is produced.

Every point in the medium containing a standing wave oscillates up and down and the amplitude of the oscillations depends on the location of the point. When we observe standing waves on strings, it looks like the wave is not moving and standing still. The principle of standing waves is the basis of resonance and how many musical instruments get their sound. The points in a standing wave that appear to remain flat and do not move are called nodes. The points which reach the maximum oscillation height are called antinodes.

Key Points

- The type of wave that occurs in a string is called a transverse wave. In a transverse wave, the wave direction is perpendicular the the direction that the string oscillates in.
- The period of a wave is indirectly proportional to the frequency of the wave: $\mathrm{T}=\frac{1}{\mathrm{f}}$.
- The speed of a wave is proportional to the wavelength and indirectly proportional to the period of the wave: $\mathrm{v}=\frac{\lambda}{\mathrm{T}}$.
- This equation can be simplified by using the relationship between frequency and period:$\mathrm{v}=\lambda \mathrm{f}$.
- When a transverse wave on a string is fixed at the end point, the reflected wave is inverted from the incident wave. When a transverse wave on a string is free at the end point, the reflected wave is not inverted from the incident wave.
- A standing wave occurs when an incident wave meets a reflected wave on a string.
- The points in a standing wave that appear to remain flat and do not move are called nodes. The points which reach the maximum oscillation height are called antinodes.
- Every point in the medium containing a standing wave oscillates up and down and the amplitude of the oscillations depends on the location of the point.
- A standing wave has some points that remain flat due to destructive interference. These are called antinodes.
- The points on a standing wave that have reached maximum oscillation do so from constructive interference, and are called nodes.

Key Terms

- **transverse wave**: Any wave in which the direction of disturbance is perpendicular to the direction of travel.
- **oscillate**: To swing back and forth, especially if with a regular rhythm.
- **amplitude**: The maximum absolute value of some quantity that varies.
- **standing wave**: A wave form which occurs in a limited, fixed medium in such a way that the reflected wave coincides with the produced wave. A common example is the vibration of the strings on a musical stringed instrument.
- **transverse wave**: Any wave in which the direction of disturbance is perpendicular to the direction of travel.

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CHAPTER OVERVIEW

16: Sound

Topic hierarchy

16.1: Introduction

learning objectives

- Explain how humans can characterize sound

Before delving too far into the physics of calculating sound waves (studying things like the Doppler Effect, for example), lets address some basics of sound. Sound is a wave—a longitudinal wave of pressure that travels through compressible medias (i.e., solid, liquid, gaseous, or made of plasma). There is no sound in a vacuum; by definition, a vacuum is a space free of any particles or matter. Thus there in a vacuum, there is no media through which sound waves can travel. Following are some characteristics of sound:

- Sound travels in longitudinal waves. When drawn these are also called sinusoidal waves, a visual example of which is shown in (we will cover this in more detail in a different section).
- Sound waves have frequency; that is, the pitch of sounds goes up or down.
- The amplitude of a sound determines its volume (loudness).
- Tone is a measure of the quality of a sound wave.
- Sound travels faster in a hot meduim, or in a solid. It also travels faster at sea level (where air pressure is higher).
- Sound intensity is the energy transmitted over a certain area. Intensity is a measure of the sound's frequency.
- Ultrasound uses sound waves with high frequencies to see things normally hard to detect, like tumors. Animals, like bats and dolphins, use ultrasound (echolocation) to navigate and locate things. Ships also use a similar technique (known as SONAR) to locate things underwater. (This point will be discussed further in a more advanced Atom.)

Sound Perception

Every sound wave has properties that define its frequency, such as wavelength, amplitude and intensity. Calculating these properties is outside the scope of this atom and will be addressed later. For now, it is important to know the basics of sound. As with light waves, sound frequencies have a range. Each living creature has a different level of sound perception. For example, consider the following examples of sound ranges (in Hz, Hertz):

- Humans 20 – 20,000 Hz
- Dogs 50 – 45,000 Hz
- Bats 20 – 120,000 Hz

By this comparison, humans have a relatively low sound perception.

Sound Speed

Breaking the Sound Barrier: This familiar image is of a plane that is moving faster than the speed of sound.

As mentioned previously, the speed at which sound travels depends on the media through which the sound is traveling. It is much faster in a solid than in a liquid or gas. The general formula for calculating the speed of sound is given as:

$$\mathrm{c}=\sqrt{\frac{\mathrm{K}}{\rho}}, \tag{16.1.1}$$

where K is the coefficient of stiffness of the material (also called the Bulk modulus) and p is the density of the material. We will examine this further in another section. Generally, the expression 'faster than the speed of sound' refers to 344 m/s. is an image demonstrating a plane moving faster than the speed of sound. This general measurement is taken at sea level—at a temperature of 21 degrees Celsius under normal atmospheric conditions.

Frequency of Sound Waves

Frequency is the number of occurrences of a repeating event per unit of time. The perception of frequency is called pitch.

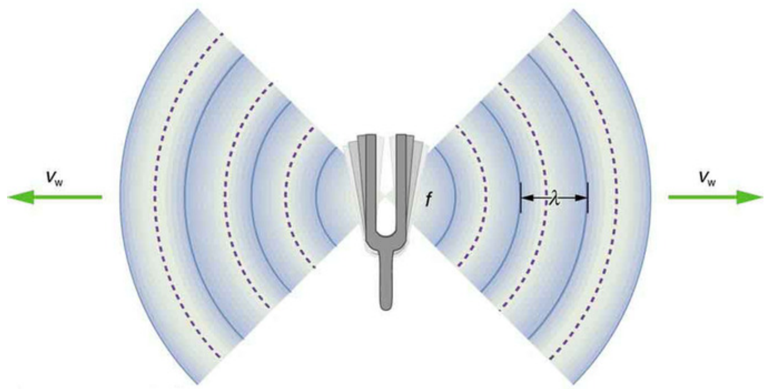
learning objectives

- Relate frequency with the wavelength and speed of sound

Sound waves, like all other waves, have a property called frequency. Frequency is the number of occurrences of a repeating event per unit of time. The perception of frequency is called pitch.

Frequency is dependent on wavelength and the speed of sound. It is calculated with the following equation: $f = \frac{v_s}{\lambda}$

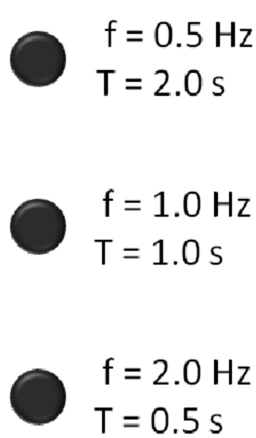
This figure,, shows how the frequency is connected to wavelength.



Frequency: A sound wave emanates from a source vibrating at a frequency f, propagates at v, and has a wavelength λ.

Alternatively, you can use the frequency and the wavelength to find the speed of sound in a specific medium. Remember that sound travels at different speeds in different mediums; sound moves fastest through a solid. The following equation is used to find the specific speed of sound, and is often easier to use than the standard speed of sound equation: *vs=f∗λv_s=f*\lambda*

A period is the duration of one cycle of a repeating event, and is the reciprocal or inverse of the frequency. The following animation shows different frequencies and their periods, from lowest to highest.



Frequency Animation: Three flashing lights, from lowest frequency (top) to highest frequency (bottom). *f* is the frequency in hertz (*Hz*); or the number of cycles per second. *T* is the period in seconds (*s*); or the number of seconds per cycle. *T* and *f* are reciprocals.

Hertz

The SI unit of frequency is called a Hertz, denoted Hz. A hertz is defined as the number of cycles per second. For example, 100 Hz signifies 100 cycles per second.

Different species can hear different frequency ranges. Humans can only hear from 20 Hz to 20,000 Hz, while dogs can hear up to 60,000 Hz. Bats can hear the highest ranges, up to 120,000 Hz. Bats use this super hearing, or ultrasound, to locate objects and prey. By bouncing sound waves off of another object and hearing how long it takes for the sound to echo back to them, they are able to approximate the distance between themselves and the object. This is called echolocation.

Sound Production: Vibrating String and Air Columns

Sound can be produced by many different devices. A vibrating string or air column can both create music and have unique properties.

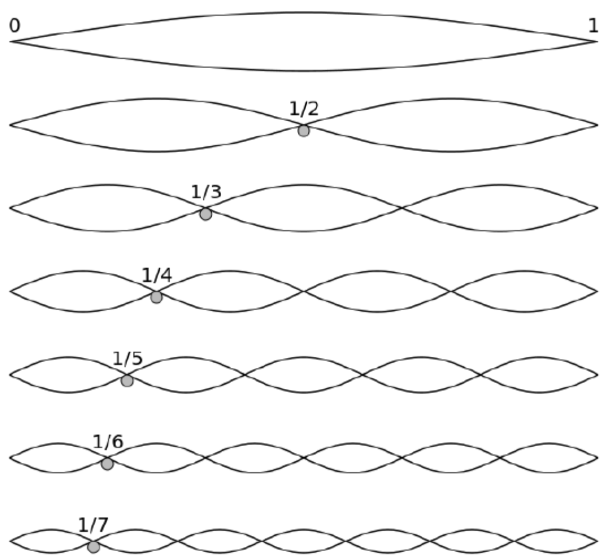
learning objectives

- Calculate the frequency of the sound wave produced by the string and a column of air

Sound can be produced in many ways, by many different instruments and devices. A vibrating string or air column can both make interesting music and it is good to understand the physics behind both.

Vibrating Strings

There are many instruments that produce sound based on strings. Guitars, cellos, pianos and many other examples. These sounds are produced by standing waves in the strings. These waves and their frequencies are constant, and therefor the sound and pitch produced by them is constant. This figure shows a visual of a standing wave in a string: The speed of the wave is proportional to the root of the string tension, and inversely proportional to the root of the string density, shown by the following equation $v=\sqrt{\frac{T}{\mu}}$ Pitch, and the way the sound is perceived depends on the frequency of the sound wave. Using the wave velocity, you can find the frequency using this equation: $f=\frac{v}{2L}$, where L is the length of the string.

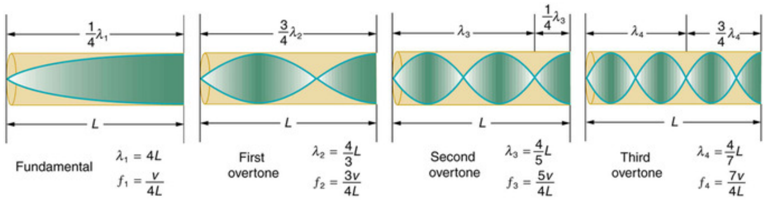


Standing Waves in a String: Vibration, standing waves in a string. The fundamental and the first 6 overtones form a harmonic series.

Air Columns

Many instruments make music by causing a vibration to a column of air in a resonator. These are usually called wind instruments. Examples of these are flutes, recorders, and saxophones. They are either open at both ends, or closed at one end and open at the other. The pitch of the sound is a function of the frequency. There are many factors that go into finding the frequency. Lets start with the tubes that are closed at one end and open at the other.

Closed Air Tubes:The maximum displacement of the air occurs at the open end of the tube, and is called the antinode. The air movement is constrained at the closed end, and there is no displacement, and this is called the node. The distance from the node to the antinode is 1/4 the length of one wavelength, and equal to the length of the tube, as shown in this equation:$\lambda = 4\mathrm{L}$This can also be seen in this figure: The frequency is equal to the speed of sound in the air divided by the wavelength, or:$\mathrm{f} = \frac{\mathrm{v_w}}{\lambda} = \frac{\mathrm{v_w}}{4\mathrm{L}}$, where $\mathrm{v_w}$ is the speed of sound in the air, which we learned how to find in a previous atom.



An Air column in a tube closed at one end: The fundamental and three lowest overtones for a tube closed at one end. All have maximum air displacements at the open end and none at the closed end.

Open Air tubes:Air tubes can also be open at both ends. They are very similar to the ones we talked about above, but there is an antinode at both ends, since they are both open, as shown in this figure: Since there is an antinode at both ends, we can see that the length of a wavelength is found by this equation: $\lambda = 2\mathrm{L}$ And the frequency can be found in the following equation:$\mathrm{f} = \frac{\mathrm{v_w}}{\lambda} = \frac{\mathrm{v_w}}{2\mathrm{L}}$

Quality of Sound

Sound quality is an assessment of accuracy or enjoyability of how a sound is perceived.

Skills to Develop

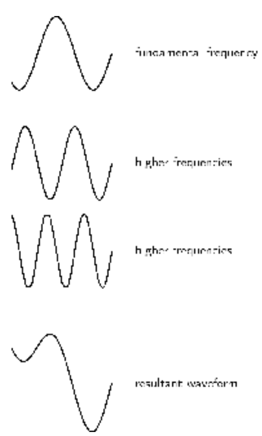
- Identify factors that influence the sound quality of a reproduction of music

Sound quality is an assessment of accuracy or enjoyability of how a sound is perceived. It can be measured objectively or subjectively. What is the difference? Objective measurement is taken when tools are used to gauge accuracy. Subjective measurement is more of an opinion. When human listeners hear a sound and compare it to another sound they have heard, and decide which one they enjoy more, this is a subjective measurement of sound quality.

Music Sound Quality

The quality of sound of live music is referred to as its tone. What makes this tone? Why does the same note being played on a piano sound different from that note played on a flute, or a guitar? When you hear a note, you mostly hear the fundamental frequency. But there are other harmonics present. You hear these, but they are much, much fainter than the fundamental, or main frequency. These are called *overtones*.

Each instrument has a different mix of these overtones, and this is why the sounds of similar notes sound different. The quality of this sound depends on these overtones to not take over what you are hearing. The overtones and fundamental frequency combine to make a unique frequency that you hear, which is illustrated in.



Sound Frequencies: The quality of a tone depends on its mixture of harmonics.

Reproduced Sound Quality

The sound quality of a reproduction of music depends on many factors. These include:

- Recording equipment
- Processing and mastering
- Reproduction equipment
- Listening environment

When you listen to a recording of music, you are hearing not only the music itself, but also any interference that may have been mistakenly recorded as well. The simplest form of digitally-stored audio is PCM.

As samples are placed closer together in time, higher frequencies can be reproduced. According to the sampling theorem, any signal with bandwidth *B* can be perfectly described by more than *2B* samples per second. Audio must be sampled at above 40kHz: 44.1kHz for CD recordings and 48kHz for DVD recordings.

The amount of space required to store PCM depends on the number of bits per sample, the number of samples per second, and the number of channels. For CD audio, this is 44,100 samples per second, 16 bits per sample, and 2 channels for stereo audio leading to 1,411,200 bits per second.

Speed of Sound

The speed of sound is is the distance traveled in a unit of time by a sound wave through an elastic medium, and is usually given as 344 m/s.

Skills to Develop

- Calculate the speed of sound from the properties of the media

Speed of Soud

The speed of sound is is the distance traveled in a unit of time by a sound wave through an elastic medium. This medium can be a solid, liquid, gas or even plasma. The speed of sound is dependent on the properties of the media the sound is travelling through. The general value given for the speed of sound is the speed of a sound wave in air, at sea level, at normal atmospheric pressure; that number is 344 m/s. However, this number is not constant. Sound travels faster in a solid than in a liquid, and faster in a liquid that in a gas.

Types of Sound Waves: Compression and Shear

There are two different kinds of sound waves: *compression waves* and *shear waves*. Compression waves can travel through any media, but shear waves can only travel through solids. The speed of a compression wave is determined by the media's compression capacity, shear modulus, and density, while the speed of the shear wave is only determined by the shear modulus and density. The shear modulus is a measurement of the elasticity or rigidity of a material. Calculating this is outside of the scope of this atom, but there are tables which tell you its value for materials.

Calculating the Speed of Sound

The speed of sound is usually denoted by cc, and a general equation can be used to calculate it. This is called the Newton-Laplace equation:c=√Kρc=KρK is the coefficient of stiffness, and p is the density of the media. From this equation, it is easy to see that the speed of sound will increase with stiffness and decrease with density. This is a very general equation, there are more specific derivations, for example:

The speed of sound in air at sea level is given by the following equation: $\mathrm{c_{air}} = 331\,\mathrm{\frac{m}{s}} \times \sqrt{\frac{\mathrm{T}}{273\mathrm{K}}}$ T is the temperature in Kelvin.

Mach Number

You may have heard the term *Mach number* in relation to speed of space craft or jets before. This is a ratio of an object's speed in relation to the speed of sound. The Mach number is given by the following, dimensionless equation: $\mathrm{M} = \frac{\mathrm{v}}{\mathrm{a}}$ – Mach numberv – Velocity of object a – Speed of sound in medium. If something is travelling at the speed of sound, that would make the equation equal to 1, and can be denoted as Mach 1. shows a jet that is travelling at the speed of sound or faster. The *vapor cone* is made just before it reaches the speed of sound and is caused by a sudden drop in air pressure.

Faster than the Speed of Sound: This is a jet that is just about to break the sound barrier.

Key Points

- Sound travels in longitudinal sinusoidal waves.
- Humans can characterize sound by frequency, amplitude and tone.
- The speed at which sound travels depends on the media through which the sound is traveling. It can be calculated using: $c = \sqrt{\frac{K}{\rho}}$, where K is the coefficient of stiffness of the material (also called the Bulk modulus) and p is the density of the material.
- Frequency is dependent on wavelength and the speed of sound. It is calculated with the following equation: $f = \frac{v_s}{\lambda}$
- A period is the duration of one cycle of a repeating event, and is the reciprocal or inverse of the frequency.
- The SI unit of frequency is called a Hertz, denoted Hz. A hertz is defined as the number of cycles per second. For example, 100 Hz signifies 100 cycles per second.
- The frequency, and therefore the pitch of a string instrument depends on the velocity of the sound wave and the length of the string. The frequency is found by: $f = \frac{\sqrt{\frac{T}{\mu}}}{2L}$.
- Sound in a tube of air that is closed at one end is found by the following equation: $f = \frac{v_w}{\lambda} = \frac{v_w}{4L}$.
- Sound in a tube of air that is open at one end is found by the following equation: $f = \frac{v_w}{\lambda} = \frac{v_w}{2L}$.
- Objective measurement is taken when tools are used to gauge accuracy. Subjective measurement is more of an opinion. When human listeners hear a sound and compare it to another sound they have heard, and decide which one they enjoy more, this is a subjective measurement of sound quality.
- The quality of sound of live music is referred to as its tone. When you hear a note, you mostly hear the fundamental frequency. But there are other harmonics present. You hear these, but they are much, much fainter than the fundamental, or main frequency. These are called overtones.
- The sound quality of a reproduction of music depends on many factors. These include: recording equipment, processing and mastering,reproduction equipment, and even listening environment.
- Sound can travel through any compressible material. These media can be solid, liquid, gas, or even plasma.
- The speed of sound is dependent on the properties of the material it travels through. It will travel faster through a solid than a liquid, and faster through a liquid than a gas.
- The general number given for the speed of sound is calculated at sea level, in air, at normal atmospheric pressure. That value is 344 m/s.

Key Terms

- **frequency**: The quotient of the number of times n a periodic phenomenon occurs over the time t in which it occurs: $f = \frac{n}{t}$.
- **Hertz**: Measurement of sound frequency.
- **media**: General term for different types of materials.
- **period**: The duration of one cycle in a repeating event.
- **node**: Point on a wave where there is no displacement.
- **antinode**: A region of maximum amplitude situated between adjacent nodes of a vibrating body, such as a string
- **Subjective measurement**: Based on a comparison to a previous experience, opinion.
- **Objective measurement**: Taken by tools to gauge accuracy.
- **elasticity**: The property by virtue of which a material deformed under the load can regain its original dimensions when unloaded
- **kelvin**: in the International System of Units, the base unit of thermodynamic temperature; 1/273.16 of the thermodynamic temperature of the triple point of water; symbolized as K

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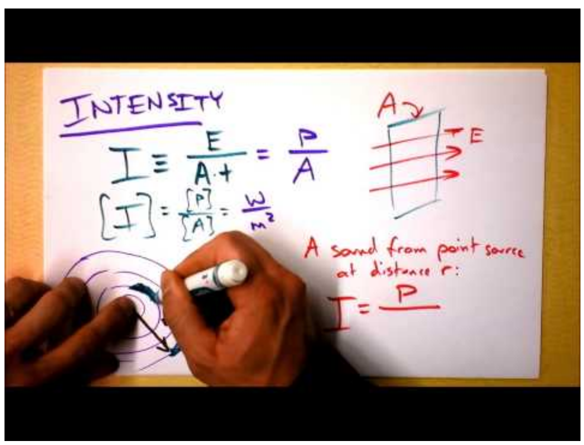
16.2: Sound Intensity and Level

learning objectives

- Calculate sound intensity from power

Overview of Intensity

Sound Intensity is the power per unit area carried by a wave. Power is the rate that energy is transferred by a wave.

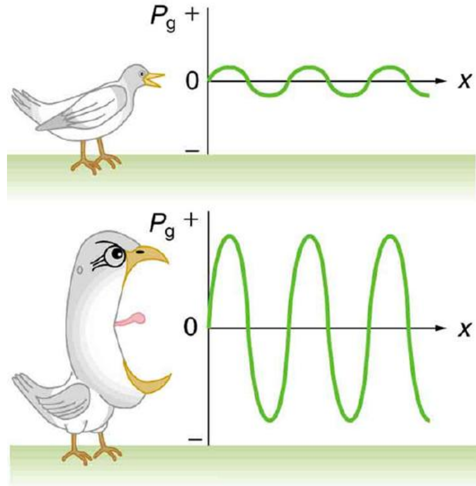


Sound Intensity and Decibels

The equation used to calculate this intensity, I, is:I*[Math Processing Error]*Where P is the power going through the area, A. The SI unit for intensity is watts per meter squared or,Wm2Wm2. This is the general intensity formula, but lets look at it from a sound perspective.

Sound Intensity

Sound intensity can be found from the following equation: *[Math Processing Error]*. Δp – change in pressure, or amplitudeρ – density of the material the sound is traveling throughv_w – speed of observed sound.Now we have a way to calculate the sound intensity, so lets talk about observed intensity. The pressure variation, amplitude, is proportional to the intensity, So it is safe to say that the larger your sound wave oscillation, the more intense your sound will be. This figure shows this concept.



Sound Intensity: Graphs of the gauge pressures in two sound waves of different intensities. The more intense sound is produced by a source that has larger-amplitude oscillations and has greater pressure maxima and minima. Because pressures are higher in the greater-intensity sound, it can exert larger forces on the objects it encounters

Although the units for sound intensity are technically watts per meter squared, it is much more common for it to be referred to as decibels, dB. A decibel is a ratio of the observed amplitude, or intensity level to a reference, which is 0 dB. The equation for this is: \(\mathrm{β=10 \log _{10} \frac{I}{I_0}β}\) – decibel levelI – Observed intensityI_0– Reference intensity.For more on decibels, please refer to the Decibel Atom.

For a reference point on intensity levels, below are a list of a few different intensities:

- 0 dB, I = 1×10^{-12} –> Threshold of human hearing
- 10 dB, I = 1×10^{-11} –> Rustle of leaves
- 60 dB, I = 1×10^{-6} –> Normal conversation
- 100 dB, I = 1×10^{-2} –> Loud siren
- 160 dB, I = 1×10^{4}–> You just burst your eardrums

Human Perception of Sound

The study of human perception of sound is called psychoacoustics.

Skills to Develop

- Explain how frequency is perceived by humans

The study of the human perception of sound is called psychoacoustics. Many factors go into hearing, including wave properties, sensory and brain processes. First, the wave has to be made, and it has a specific wavelength and frequency. Then the sound wave reaches the human ear, and is processed through many areas. Finally, the sound wave makes it through the ear and to the human brain, where even more action happens. You might think that when something makes a noise that you hear it instantaneously but, in reality, it goes through many steps first.

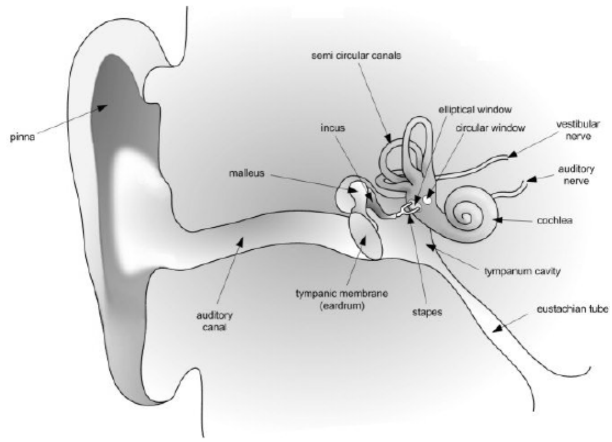
Wave Properties

We are not going to go into too much detail about the wave's physical properties, since it is out of the scope of this atom, but remember:

- Frequency is perceived by humans as pitch;
- The sound intensity is the amplitude;
- Humans can only hear a specific range of sound, usually from 20 Hz to 20,000 Hz;
- The factors that go into a sound are its intensity, frequency and overtones (which are like interference, or background noises).

The Human Ear

The human ear is made up of three main sections, as shown in:



The Human Ear: A detailed diagram of the human ear.

1. The outer ear
2. The middle ear
3. The inner ear

We are going to start where a sound wave would start and follow it on its journey from outside your ear all the way to your brain. When you look at someone's ear, you are really only seeing the pinna, the outer most portion of the ear. It collects and focuses the sound wave. The wave then goes through your ear canal to the eardrum. The sound waves cause the eardrum to vibrate. Then we are in the middle ear, which has three very, very small bones: the malleus, incus and stapes. These can also be referred to as the hammer, anvil and stirrup, respectively. These three bones transmit the signal to the elliptical window. This is the beginning of the inner ear. The sound waves are then transmitted from the elliptical window through the inner ear's semicircular canals, the cochlea, and the audio nerve, which is filled with fluid. This fluid is what allows the body to detect movements and maintain balance. Your cochlea is shaped like a snail, and is full of teeny tiny hairs. These hairs vibrate differently depending on the frequencies. These vibrations release electrical impulses to the auditory nerve and are then sent to your brain, where they are understood as sound. So while this seems to happen very quickly, sound waves have to travel a long way before you ever hear anything!

Decibels

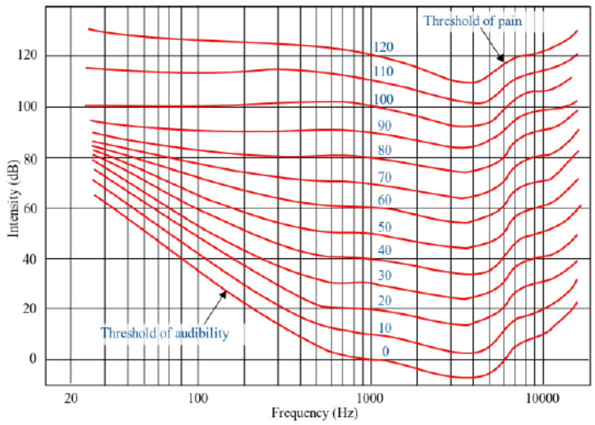
The decibel is a logarithmic unit used to quantify sound levels, by comparing a physical quantity to a reference level.

Skills to Develop

- Identify how decibel is quantified in acoustics

The decibel, dB, is commonly used to quantify sound levels, although it is not a unit of sound, but a unit of pressure. The decibel is a logarithmic unit that indicates the ratio if a physical quantity to a reference level. It is one tenth of a Bel, which was named after the inventor of the telephone, Alexander Graham Bell. The word decibell comes from the prefix, deci, that is 1/10 of the word it precedes For more information on how to convert units, refer to the unit conversion atom. Although the decibel can be used to talk about a number of different subjects, in this atom we are going to cover its use in acoustics and sound level.

In acoustics, the decibel is quantified relative to a reference which has been set at a sound pressure level of 20 micropascals, and is called a 0 dB. This reference level is a typical threshold of human hearing perception. The following equation is used to calculate the sound pressure level, or amplitude: *[Math Processing Error]*s_o is the reference pressure which is 20 micropascals or 0 dB, and s is the observed sound pressure.The human ear has a standard sound threshold of 120 dB, which expressed logarithmically is around 10^{12}. This is a standard threshold, but it also depends on frequency. Loudness is a measure of sound intensity taking frequency into account, and is called a A-weighted decibel, dB(A), or a phon. This figure shows The Fletcher Munson Chart, which demonstrates the different sound frequencies and decibels that the human ear perceives as the same.



The Fletcher Munson Chart: The Fletcher-Munson equal-loudness contours. Phons are labelled in blue

Key Points

- Sound intensity can be found from the following equation: *[Math Processing Error]*. Δ p – change in pressure, or amplitude ρ – density of the material the sound is traveling through v_w– speed of observed sound.
- The larger your sound wave oscillation, the more intense your sound will be.
- Although the units for sound intensity are technically watts per meter squared, it is much more common for it to be referred to as decibels, dB.
- Frequency is perceived by humans as pitch. The sound intensity is what humans can hear, and is generally only a specific range of sound, usually from 20 Hz to 20,000 Hz. The factors that go into a sound are its intensity, frequency and overtones (which are like interference or background noises).
- Your ear is made up of three major sections: the inner, middle and outer ear.
- Your cochlea, which is in your inner ear, not only transmits sound waves to your brain, but also contains a liquid that helps humans maintain their balance.
- In acoustics, the decibel is quantified relative to a reference which has been set at a sound pressure level of 20 micropascals, and is called a 0 dB.
- The following equation is used to calculate the sound pressure level, or amplitude: *[Math Processing Error]*. *[Math Processing Error]* is the reference pressure which is 20 micropascals or 0 dB, and s is the observed sound pressure.
- The human ear has a standard sound threshold of 120 dB, which expressed logarithmically is around 1012. This is a standard threshold, but it also depends on frequency. Loudness is a measure of sound intensity taking frequency into account, and is called a A-weighted decibel, dB(A), or a phon.

Key Terms

- **decibel**: A common measure of sound intensity that is one tenth of a bel on the logarithmic intensity scale. It is defined as *[Math Processing Error]*, where *[Math Processing Error]* and *[Math Processing Error]* are the relative powers of the sound.
- **amplitude**: The maximum absolute value of some quantity that varies.
- **eardrum**: A thin membrane that separates the outer ear from the middle ear and transmits sound from the air to the malleus.
- **cochlea**: The complex, spirally coiled, tapered cavity of the inner ear in which sound vibrations are converted into nerve impulses.
- **phon**: A unit of apparent loudness, equal in number to the intensity in decibels of a 1,000-hertz tone judged to be as loud as the sound being measured.

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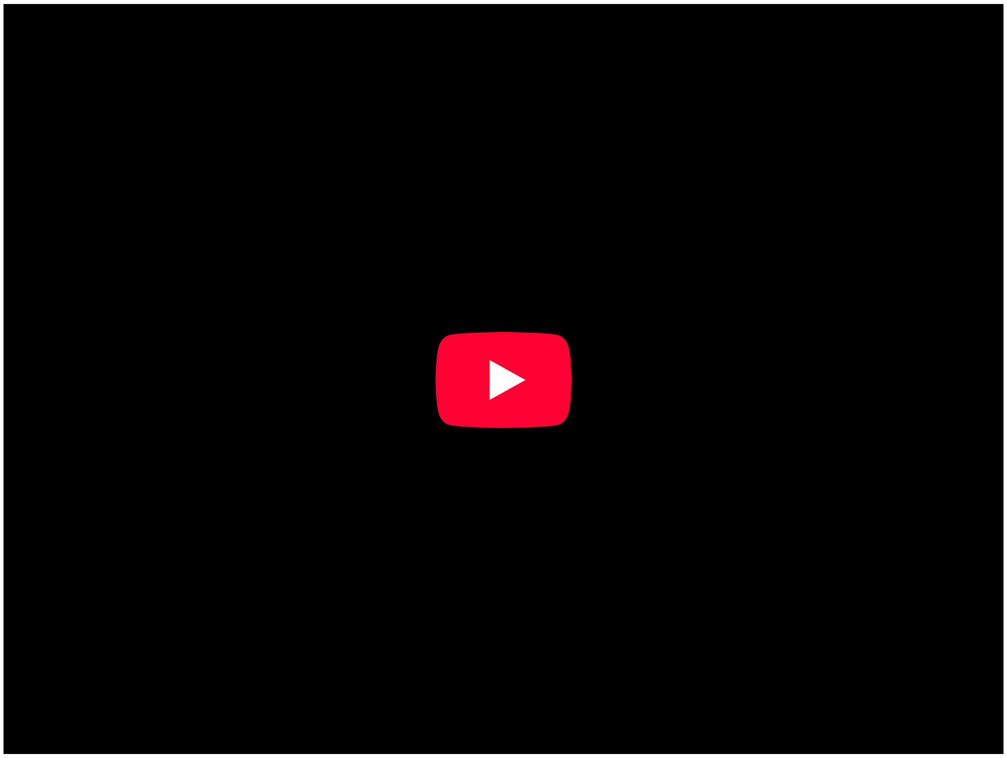
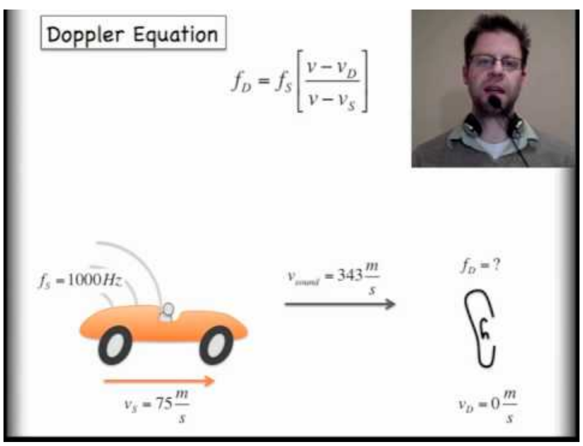
16.3: Doppler Effect and Sonic Booms

learning objectives

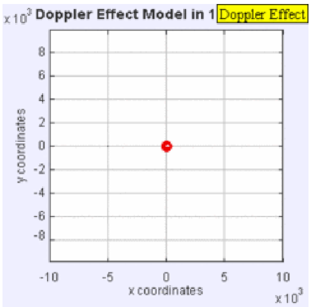
- Identify parameters required to calculate the frequency perceived by the observer moving towards the sound source

In this atom, we are going to cover the Doppler effect, but specifically when the observer is the one in motion.





Sound and the Doppler Effect: This video introduces sound waves. The first video describes the basics of sound while the second video looks at the Doppler Effect.



The Doppler Effect: The same sound source is radiating sound waves at a constant frequency in the same medium. However, now the sound source is moving to the right with a speed υs = 0.7 c (Mach 0.7). The wave-fronts are produced with the same frequency as before. However, since the source is moving, the centre of each new wavefront is now slightly displaced to the right. As a result, the wave-fronts begin to bunch up on the right side (in front of) and spread further apart on the left side (behind) of the source.

When the observer moves toward an sound source, each successive wave is encountered sooner than the previous wave. Thus, it will take just a little less time for the observer to hear the next one. Since the time between waves is reduced, the frequency is increased. Similarly if the observer is moving away from the sound source, the frequency, and therefor pitch, is decreased. While the frequency will change whether the observer or sound source is moving, it is easier to show with the sound source as the one moving. This figure demonstrated the sound source moving:

Unless the observer is moving directly towards the sound source, this angle needs to be taken into account when calculating the newly perceived frequency. Before we can start this calculation, we must know:

- The original sound wave frequency, f_0
- The velocity of the observer, v_r
- The speed of sound in the air, or medium, c
- The angle of the line of sight from the observer to the sound source, θθ

Although the sound waves are being emitted from the sound source at a uniform frequency, the observer is perceiving them differently. The equation for the perceived wave frequency is as follows: *[Math Processing Error]*And the equation for v_r, is: *[Math Processing Error]* If the observer is moving towards the sound source, you are going to use a plus sign in front of the observers velocity. If the observer is moving away from sound source, you are going to use a negative sign in front of the observers velocity.

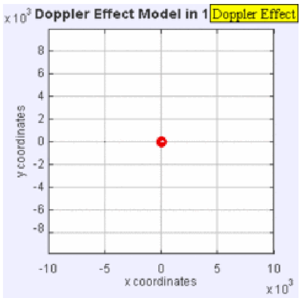
Moving Source

The Doppler effect is the apparent change in frequency of a wave when the observer and the source of the wave move relative to each other.

learning objectives

- Identify parameters required to calculate the perceived frequency of the sound source moving towards an observer

The Doppler Effect —When the Sound Source is in Motion



The Doppler Effect: The same sound source is radiating sound waves at a constant frequency in the same medium. However, now the sound source is moving to the right with a speed υs = 0.7 c (Mach 0.7). The wave-fronts are produced with the same frequency as before. However, since the source is moving, the centre of each new wavefront is now slightly displaced to the right. As a result, the wave-fronts begin to bunch up on the right side (in front of) and spread further apart on the left side (behind) of the source.

When the sound source moves toward an observer, each successive wave is emitted closer to the observer than the previous wave and takes just a little less time to reach the observer than the previous one. Since the time between waves is reduced, the frequency is increased. Similarly, if the sound source is moving away from the observer, the frequency (and therefore pitch) is decreased. While the frequency will change whether the observer or sound source is moving, the effect is more easily demonstrated by the sound source. This Doppler Effect is illustrated in.

Unless the observer is moving directly towards the sound source, this angle must be considered when calculating the newly perceived frequency. Before attempting this calculation, we must know:

- The original sound wave frequency, f_0
- The velocity of the observer, v_r
- The speed of sound in the air, or medium, c
- The angle of the line of sight from the observer to the sound source, *[Math Processing Error]*

Although the sound waves are being emitted from the sound source at a uniform frequency, the observer is perceiving them differently. The equation for the perceived wave frequency is as follows: *[Math Processing Error]* And the equation for v_s, is: *[Math Processing Error]*

If the sound source is moving towards the observer, a plus sign is used in front of the sound source's velocity. If the sound source is moving away from the observer, then a negative sign is used in front of the sound source's velocity.

General Case

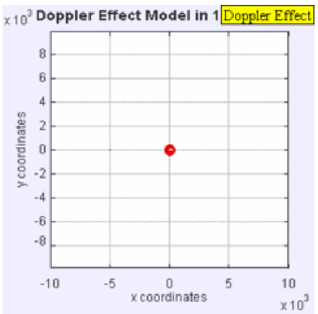
The Doppler effect is the apparent change in frequency of a wave when the observer and the source of the wave move relative to each other.

learning objectives

- Compare change in the frequency due to the doppler effect when the object in motion moves towards or away from the other object

Have you ever wondered why as a firetruck speeds by, the sound of the siren seems to change? Or where the very distinguishable sound at a race track comes from as the cars whisk by you? Well believe it or not, this is science! The name of this is called the Doppler effect, named for the scientist who discovered it. His name was Christian Doppler, and he discovered it around 1840 in Prague. He preformed the experiment in two segments. In the first experiment, he had a band play music while aboard a moving train, and would have observers listen to the music in a stationary location while the train went by. In the second experiment, he had a group of observers sit aboard a moving train, while the band played in a stationary location. In both cases, observers noticed the phenomenon. The change in sound perception can be explained through relativity.

When the sound source moves toward an observer, each successive wave is emitted closer to the observer than the previous wave. Thus, it will take just a little less time to reach the observer than the previous one. Since the time between waves is reduced, the frequency is increased. Similarly if the sound source is moving away from the observer, the frequency, and therefor pitch, is decreased. If the observer is moving away from the sound source, the frequency will be lowered, and if the observer moves closer to the sound source, the frequency is increased. While the frequency will change whether the observer or sound source is moving, it is easier to show with the sound source as the one moving. This figure demonstrated the sound source moving:



The Doppler Effect: The same sound source is radiating sound waves at a constant frequency in the same medium. However, now the sound source is moving to the right with a speed υs = 0.7 c (Mach 0.7). The wave-fronts are produced with the same frequency as before. However, since the source is moving, the centre of each new wavefront is now slightly displaced to the right. As a result, the wave-fronts begin to bunch up on the right side (in front of) and spread further apart on the left side (behind) of the source.

In classical physics, where the speeds of source and the receiver relative to the medium are lower than the velocity of waves in the medium, the relationship between observed frequency *(f)* and emitted frequency (f_o) is given by: *[Math Processing Error]* where c – velocity of the sound waves in the medium, v_r– velocity of the observer or reciever, v_s – velocity of the sound source, and f_0 – original frequency of the sound waves.

The above formula assumes that the source is either directly approaching or receding from the observer. If the source approaches the observer at an angle (but still with a constant velocity), the observed frequency that is first heard is higher than the object's emitted frequency. Thereafter, there is a monotonic decrease in the observed frequency as it gets closer to the observer, through equality when it is closest to the observer, and a continued monotonic decrease as it recedes from the observer. When the observer is very close to the path of the object, the transition from high to low frequency is very abrupt. When the observer is far from the path of the object, the transition from high to low frequency is gradual.

If the speeds and are small compared to the speed of the wave, the relationship between observed frequency and emitted frequency is approximately

Observed frequency: *[Math Processing Error]*

Change in frequency: *[Math Processing Error]*

where *[Math Processing Error]* is the velocity of the receiver relative to the source: it is positive when the source and the receiver are moving towards each other.

Sonic Booms

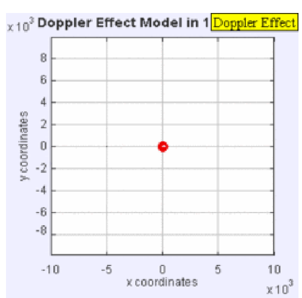
A sonic boom is the sound associated with the shock waves created by an object traveling through the air faster than the speed of sound.

learning objectives

- Identify conditions that lead to a sonic boom and discuss its properties

Sonic Booms

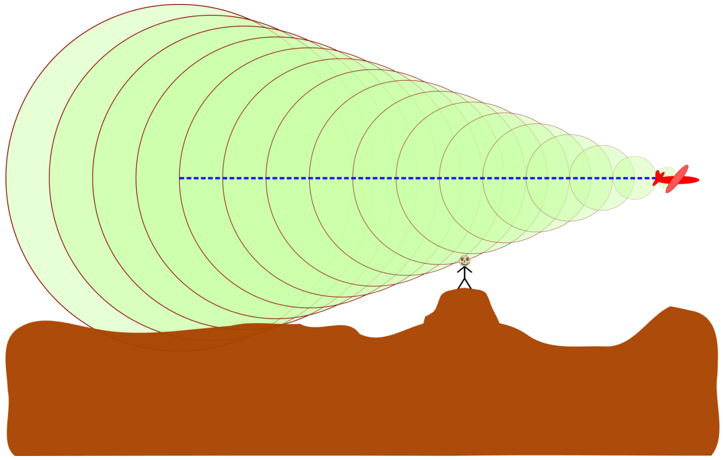
A sonic boom is a sound associated with the shock waves created by an object traveling through the air faster than the speed of sound. It can be viewed as a Doppler effect on steroids; sonic booms generate an enormous amount of energy and sound like explosions. The first man-made object to ever create this phenomenon was a bullwhip. The 'crack' of the whip is a result of this sonic boom. This version of a Doppler effect is demonstrated by.



Sonic Boom Gif: The sound source has now broken through the sound speed barrier, and is traveling at 1.4 times the speed of sound, (Mach 1.4). Since the source is moving faster (with a speed) than the sound waves it creates, it actually leads the advancing wavefront. The sound source will pass by a stationary observer (with a speed) before the observer actually hears the sound it creates.

When the sound source passes through the air, it creates a series of pressure waves. These waves are travelling at the speed of sound, and as the speed of the sound source increases, the waves, not being able to get out of each other's way, are forced together.

They eventually merge into a single shock wave traveling at the speed of sound. This is a critical speed, known as Mach. The shock waves radiate out from the sound source, and create a "Mach cone'. The half-angle, αα, can be found using the equation



Sonic Boom: A sonic boom produced by an aircraft moving at M=2.92, calculated from the cone angle of 20 degrees. An observer hears the boom when the shock wave, on the edges of the cone, crosses his or her location

[Math Processing Error]

From previous atoms, we know that *[Math Processing Error]* is the sound source's Mach number.

At the front of the sound source, there is a sudden rise in pressure, while at the end of the source there is a decreasing pressure. This 'overpressure profile' is known as an N-wave. There is a big boom when there is a sudden change in pressure, and since the pressure changes twice, this is a double boom.

Key Points

- When the object in motion moves towards the other, the frequency is increased because the time between successive sound waves is shortened. Therefore the pitch is higher.
- When the object in motion moves away from the other, the frequency is decreased because the time between successive sound waves is lengthened. Therefor the pitch is lowered.
- Unless the objects are in each other's direct path, you need to account for the angle they are at relative to each other. The following equation needs to be substituted for the 'movers' velocity. The angle used needs to be the angle from the line of sight of the observer to the sound source. vradial=v∗cosθvradial=v∗cosθ.
- When the object in motion moves towards the other, the frequency is increased because the time between successive sound waves is shortened (therefore the pitch is higher).
- When the object in motion moves away from the other, the frequency is decreased because the time between successive sound waves is lengthened (therefore the pitch is lowered).
- Unless the objects are in each other's direct path, you need to account for their angle relative to each other. The following equation must be substituted for the 'movers' velocity. The angle used must be the angle from the line of sight of the observer to the sound source.
- When the object in motion moves towards the other, the frequency is increased because the time between successive sound waves is shortened. Therefore the pitch is higher.
- When the object in motion moves away from the other, the frequency is decreased because the time between successive sound waves is lengthened. Therefor the pitch is lowered.
- If the speeds and are small compared to the speed of the wave, the relationship between observed frequency and emitted frequency is approximately: *[Math Processing Error]* where *[Math Processing Error]* is the velocity of the receiver relative to the source: it is positive when the source and the receiver are moving towards each other.
- A sonic boom happens when a sound source passes an observer at a speed either very close to, or faster than the speed of sound.
- Sonic booms generate an enormous amount of energy, and sound like explosions.
- The half angle of the cone of sound waves produced by the sonic boom can be found by taking the inverse sine of the object's Mach number.

Key Terms

- **Doppler's effect**: Apparent change in frequency of a wave when the observer and the source of the wave move relative to each other.
- **classical physics**: All aspects of physics developed before the rise of quantum mechanics.
- **frequency**: The quotient of the number of times n a periodic phenomenon occurs over the time t in which it occurs: *[Math Processing Error]*.
- **Mach number**: The ratio of the velocity of a body to that of sound in the surrounding medium.
- **sonic boom**: The audible effect of a shock wave in the air, especially one caused by an aircraft flying faster than the speed of sound

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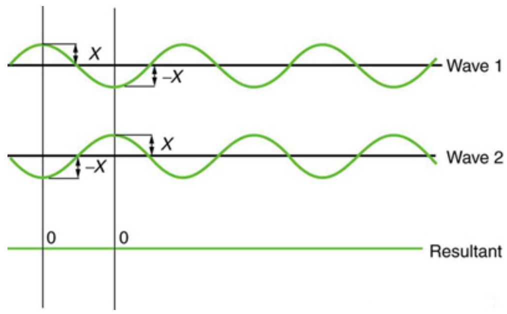
16.4: Interactions with Sound Waves

learning objectives

- Identify conditions required for the superposition of two waves

Waves are most commonly described by variations in some parameter through space and time—height in a water wave, pressure in a sound wave, or the electromagnetic field in a light wave. The value of this parameter is called the amplitude of the wave; the wave itself is a function specifying the amplitude at each point.

When two or more waves arrive at the same point, they superimpose themselves onto one another. More specifically, the disturbances of waves are superimposed when they come together (a phenomenon called superposition). Each disturbance corresponds to a force, or amplitude (and the forces add). If the disturbances are along the same line, the resulting wave is a simple addition of the disturbances of the individual waves. That is, their amplitudes add.

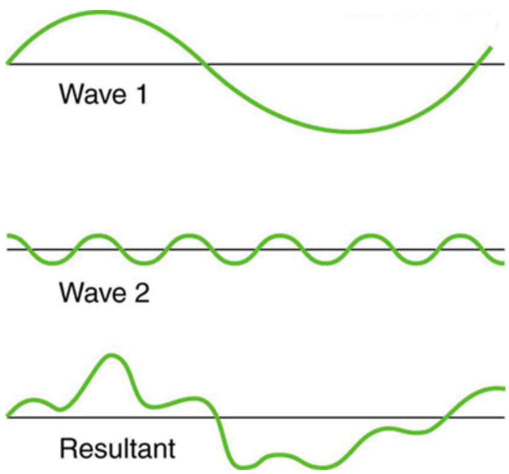


Superposition: Superposition is when two waves add together. In this figure, the two waves add together and cancel out leaving no wave. This is destructive interference.

Superposition of waves leads to what is known as interference, which manifests in two types: constructive and destructive. Constructive interference occurs when two waves add together in superposition, creating a wave with cumulatively higher amplitude, as shown in. In destructive interference, the two waves add together but cancel out (like adding a positive and negative number). Destructive interference is shown in.

While pure constructive and pure destructive interference do occur, they require precisely aligned identical waves. The superposition of most waves produces a combination of constructive and destructive interference, and can vary from place to place and time to time. Sound from a stereo, for example, can be loud in one spot but quiet in another. Varying loudness means the sound waves add partially constructively and partially destructively at different locations. A stereo has at least two speakers creating sound waves, and waves can reflect from walls. All these waves superimpose. An example of sounds that vary over time from constructive to destructive is found in the combined whine of airplane jets heard by a stationary passenger. The combined sound can fluctuate up and down in volume as the sound from the two engines varies in time from constructive to destructive.

These examples are of waves that are similar. illustrates that when non-identical waves superimpose, the outcome is a mixture of constructive and destructive interference.



Superposition of Non-Identical Waves: Superposition of non-identical waves exhibits both constructive and destructive interference.

Interference

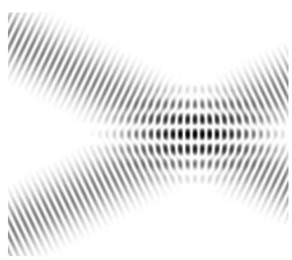
Interference occurs when multiple waves interact with each other, and is a change in amplitude caused by several waves meeting.

learning objectives

- Contrast constructive and destructive interference

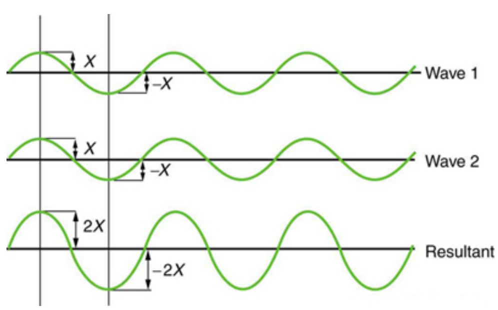
Unlike solid objects, two waves can share a point in space. In physics, interference is a phenomenon in which two waves (passing through the same point) superimpose to form a resultant wave of greater or lower amplitude. Interference usually refers to the interaction of waves that are correlated or coherent with each other (i.e, "interfere" with each other), either because they come from the same source or because they have the same or nearly the same frequency.

The effects of interference can be observed with all types of waves, for example, light, radio, acoustic and surface water waves. The idea that interference is caused by superposition means that when two waves meet their two amplitudes (their maximum absolute value) combine together.



Interference: Two overlapping waves exhibit interference.

Interference can be constructive or destructive. In constructive interference, the two amplitudes of the waves add together and result in a higher displacement than would have been the case if there were only one wave. An example of constructive interference may be seen in.



Constructive Interference: Pure constructive interference of two identical waves produces one with twice the amplitude, but the same wavelength.

Destructive interference is when two waves add together and the result is a smaller displacement than would have been the case. An example of destructive interference can be seen in. When the waves have opposite amplitudes at the point they meet they can destructively interfere, resulting in no amplitude at that point. For example, this is how noise cancelling headphones work. By playing a sound with the opposite amplitude as the incoming sound, the two sound waves destructively interfere and this cancel each other out.

Beats

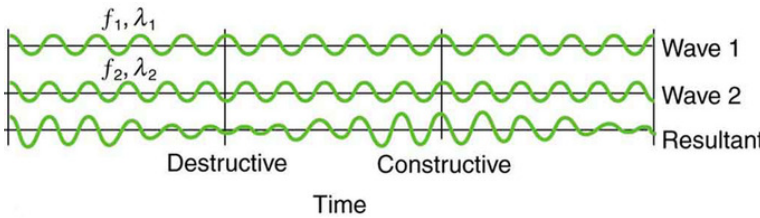
The superposition of two waves of similar but not identical frequencies produces a pulsing known as a beat.

learning objectives

- Identify superposition conditions that lead to beat

Striking two adjacent keys on a piano produces a warbling combination (usually considered unpleasant to the ear). The culprit is the superposition of two waves of similar but not identical frequencies. When two waves of similar frequency arrive at the same point and superimpose, they alternately constructively and destructively interfere. This alternating is known as a beat because it produces an unpleasant pulsing sound.

Another example is often noticeable in a taxiing jet aircraft (particularly the two-engine variety). The loudness of the combined sound of the engines increases and decreases. This varying loudness occurs because the sound waves have similar but not identical frequencies. The discordant warbling of the piano and the fluctuating loudness of the jet engine noise are both due to alternately constructive and destructive interference as the two waves go in and out of phase. illustrates this phenomenon graphically.



Beat Frequency: Beats are produced by the superposition of two waves of slightly different frequencies but identical amplitudes.The waves alternate in time between constructive interference and destructive interference, giving the resulting wave a time-varying amplitude.

The wave resulting from the superposition of two similar-frequency waves has a frequency that is the average of the two. This wave fluctuates in amplitude, or beats, with a frequency called the beat frequency. We can determine the beat frequency mathematically by adding two waves together.

One can also measure the beat frequency directly. When you hear a beat coming from two discordant sounds (say, two notes on a piano) you can count the number of beats per second. The number of beats per second, or the beat frequency, shows the difference in frequency between the two notes. Musicians often use this phenomena to ensure that two notes are in tune (if they are in tune then there are no beats).

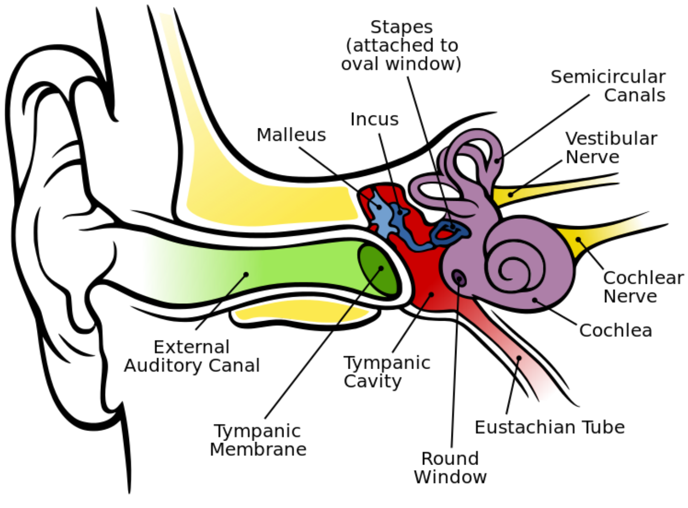
The Ear

The ear is the sensory organ that picks up sound waves from the air and turns them into nerve impulses that can be sent to the brain.

learning objectives

- Describe how sound waves are collected and transformed into nerve impulses

Sound waves are vibrations in the air. The ear is the sensory organ that picks up sound waves from the surrounding air and turns them into nerve impulses, which are then sent to the brain. The sound waves carry a lot of information — language, music, and noise — all mixed together. The task of the ear is to turn the signals in these waves of bouncing air molecules into electrical nerve signals while keeping as much of the information in the signal as possible. (It's the brain's job to then sort the signals and make sense of them.) It's not easy to turn one kind of signal into another without losing information, but the ear is well designed for the task.



Anatomy of the Human Ear: Anatomy of the human ear; the length of the auditory canal is exaggerated for viewing purposes

Air surrounds the head and fills the ear canal and middle ear. Therefore, when the outer part of the ear collects sound and the middle ear amplifies this sound pressure, these processes occur in the medium of air. However, the hollow channels of the inner ear (which is embedded in the temporal bone, the densest bone of the body) are filled with liquid. So as the sound travels into the inner ear, it passes from the medium of air into a liquid medium. These inner-ear channels contain a sensory epithelium that is studded with hair cells. The microscopic "hairs" are structural protein filaments that project out into the fluid. The hair cells release a chemical neurotransmitter when stimulated. Sound waves moving through fluid push the filaments; if the filaments bend over enough, the hair cells fire chemical signals. In this way sound waves are transformed into nerve impulses. The nerve impulses travel from the left and right ears through the eighth cranial nerve to both sides of the brain stem and up to the part of the cerebral cortex dedicated to sound (auditory cortex, located in the temporal lobe).

Applications: Ultrasound, Sonar, and Medical Imaging

Sound waves reflect off different materials differently (when the reflections are collected, they can provide information and images).

learning objectives

- Discuss application of sound waves in medicine and navigation

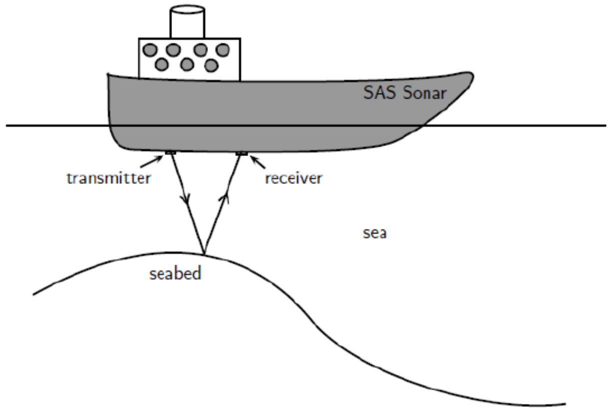
The controlled use of sound waves has many applications in science. Controlled reflection of such waves allows images to be received.

Ultrasound

Ultrasound is sound with a frequency higher than 20 kHz. This is above the human range of hearing. The most common use of ultrasound, creating images, has industrial and medical applications. The use of ultrasound to create images is based on the reflection and transmission of a wave at a boundary. When an ultrasound wave travels inside an object that is made up of different materials (such as the human body), each time it encounters a boundary (e.g., between bone and muscle, or muscle and fat), part of the wave is reflected and part of it is transmitted. The reflected rays are detected and used to construct an image of the object.

Sonar

illustrates how a ship on the ocean utilizes the reflecting properties of sound waves to determine the depth of the ocean. A sound wave is transmitted and bounces off the seabed. Because the speed of sound is known and the time lapse between sending and receiving the sound can be measured, the distance from the ship to the bottom of the ocean can be determined. This technique is called *sonar* (originally an acronym for SOund Navigation And Ranging).



Sonar: Ships on the ocean make use of the reflecting properties of sound waves to determine the depth of the ocean. A sound wave is transmitted and bounces off the seabed. Because the speed of sound is known and the time lapse between sending and receiving the sound can be measured, the distance from the ship to the bottom of the ocean can be determined.

Just as ships on the ocean, certain animals, like dolphins and bats, make use of sounds waves (sonar) to navigate or find their way. Ultrasound waves are sent out then reflected off the objects around the animal. Bats or dolphins then use the reflected sounds to form a “picture” of their surroundings (this is known as echolocation).

Key Points

- When two waves occupy the same point, superposition occurs. Superposition results in adding the two waves together.
- Constructive interference is when two waves superimpose and the resulting wave has a higher amplitude than the previous waves.
- Destructive interference is when two waves superimpose and cancel each other out, leading to a lower amplitude.
- Most wave superpositions involve a mixture of constructive and destructive interference since the waves are not perfectly identical.
- Interference is a phenomenon of wave interactions. When two waves meet at a point, they interfere with each other.
- There are two types of interference, constructive and destructive.
- In constructive interference, the amplitudes of the two waves add together resulting in a higher wave at the point they meet.
- In destructive interference, the two waves cancel out resulting in a lower amplitude at the point they meet.
- When two waves of similar frequencies interfere, the result is a beat frequency.
- A beat frequency is a pulsing sound that goes up and down in loudness.
- As the two waves go in and out of phase, the varying constructive and destructive interference makes the wave grow and shrink in amplitude. For sound waves this produces a beating sound.
- The task of the ear is to turn the signals in the waves of bouncing air molecules into electrical nerve signals while keeping as much of the information in the signals as possible.
- Sound is collected in the outer part of the ear; sound pressure is amplified through the middle part of the ear and is passed from the medium of air into a liquid medium.
- That sound pressure is amplified through the middle portion of the ear and passed from the medium of air into a liquid medium.
- Sound waves moving through the fluid in the inner ear stimulate hair cells, making them release chemical neurotransmitters. In this way sound waves are transformed into nerve impulses.
- When waves encounter a boundary between two materials, part of the wave is reflected and part is transmitted.
- By using high frequency sound waves, doctors can create images of parts of the body normally not visible.
- By transmitting sound waves and measuring the time between the transmission and receiving the reflection, ships can use sound waves to navigate. This is called sonar.

Key Terms

- **superimpose**: To place an object over another object.
- **displacement**: A vector quantity that denotes distance with a directional component.
- **amplitude**: The maximum absolute value of some quantity that varies.
- **coherent**: Of waves having the same direction, wavelength and phase, as light in a laser.

- **frequency**: The quotient of the number of times n a periodic phenomenon occurs over the time tt in which it occurs: \(\mathrm{f=n/t\).
- **interfere**: (of waves) To be correlated with each other when overlapped or superposed.
- **superposition**: The summing of two or more field contributions occupying the same space.
- **epithelium**: a membranous tissue composed of one or more layers of cells that forms the covering of most internal and external surfaces of the body and its organs (internally, the lining of vessels and other small cavities; externally, the skin)
- **nerve impulse**: the signal transmitted along a nerve fiber, either in response to a stimulus (such as touch, pain, or heat), or as an instruction (such as causing a muscle to contract)
- **neurotransmitter**: any substance, such as acetylcholine or dopamine, responsible for sending nerve signals across a synapse between two neurons

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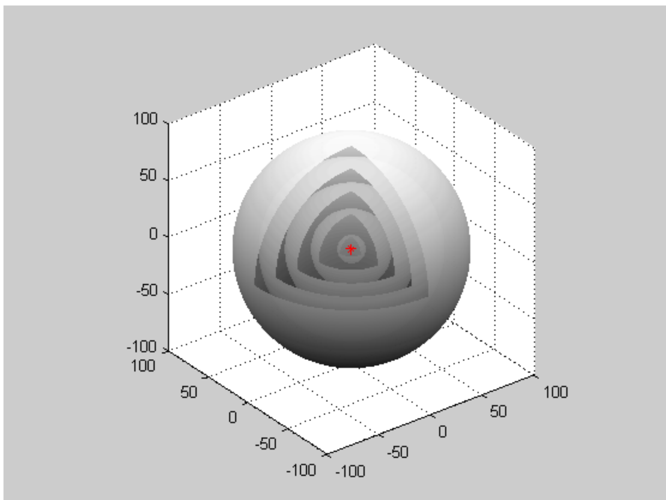
16.5: Further Topics

learning objectives

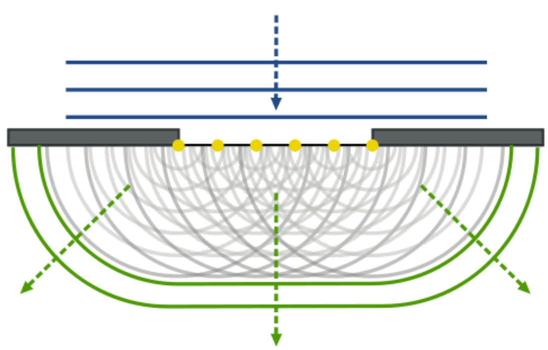
- Identify sources of spherical and plane wave patterns

Spherical Waves

Christiaan Huygens was a Dutch scientist who developed a useful technique for determining how and where waves propagate. In 1678, he proposed that every point that a luminous disturbance touches becomes itself a source of a spherical wave; the sum of these secondary waves determines the form of the wave at any subsequent time. The Huygen-Fresnel Principle shows that as the waves interact with each other, they interfere either constructively or destructively. Constructive interference occurs when waves are completely in phase with each other and amplifies the waves. Destructive interference occurs when waves are exactly out of phase with either other, and if waves are perfectly out of phase with each other, the wave will be canceled out completely. Since the waves all come from one point source, the waves happen in a spherical pattern. All the waves come from a single point source and are spherical.



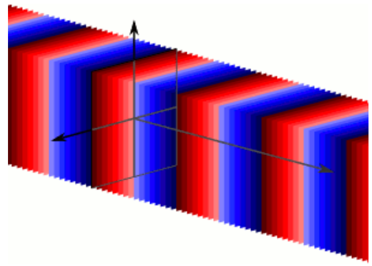
Spherical Wave*: When waves are produced from a point source, they are spherical waves.*



Huygen-Fresnel Principle: The Huygen-Fresnel Principle uses the law of refraction. Each point on the wave produces waves which interfere with each other either constructively or destructively.

Plane Waves

A plane wave is a constant-frequency wave whose wavefronts (surfaces of constant phase) are infinite parallel planes of constant peak-to-peak amplitude normal to the phase velocity vector. It is not possible in practice to have a true plane wave; only a plane wave of infinite extent will propagate as a plane wave. However, many waves are approximately plane waves in a localized region of space. For example, a localized source such as an antenna produces a field that is approximately a plane wave far from the antenna in its far-field region. Similarly, if the length scales are much longer than the wave's wavelength, as is often the case for light in the field of optics, one can treat the waves as light rays which correspond locally to plane waves.



Plane Wave: *Plane waves are an infinite number of wavefronts normal to the direction of the propogation.*

Standing Waves on a String

Standing wave occurs due to the interference when transverse waves in strings are reflected and the incident and reflected waves meet.

> **learning objectives**
>
> - Identify when a standing wave occurs

A standing wave is a wave that appears stationary, meaning it remains in a constant position. In a string, a standing wave is a type of transverse wave—where the movement of the particles of the medium is perpendicular to the direction of the propagation of the wave. A standing wave can occur when two identical waves moving in different directions along the string interfere.

There are two scenarios of waves in strings: the string is fixed at both ends, or the string is fixed at one end and free at the other. A transverse wave will move along the string until it reaches the other end. It is then reflected from that end and starts to move back towards the original direction; at this point interference occurs. shows a transverse wave that is reflected from a fixed end. When a transverse wave meets a fixed end, the wave is reflected, but inverted. This swaps the peaks with the troughs and the troughs with the peaks. diagrams a transverse wave on a string that meets a free end. The wave is reflected, but unlike a transverse wave with a fixed end, it is not inverted.

Free End Reflection: The wave is reflected, but unlike a transverse wave with a fixed end, it is not inverted.

Fixed End Reflection: When a transverse wave meets a fixed end, the wave is reflected, but inverted.

Standing Waves

When either of the two scenarios of wave reflection occurs, the incident wave meets the reflected wave. These waves move past each other in opposite directions, causing interference. When these two waves have the same frequency, the product of this is called the standing waves. Standing waves appear to be standing still, hence the name. illustrates a very slow moving standing wave. (One application of the principle of standing waves is in music with the concept of resonance—and how many musical instruments, like guitars and pianos, get their sound.) Let us now examine how standing waves occur.

Standing Wave on a String: This is what a standing wave would look like if you were to slow it down. The wave is caused by an incident wave on a string being reflected and then traveling back in the direction it came from. The two waves then meet and interfere with each other causing this phenomenon.

Constructive vs. Destructive Interference

When the incident wave and reflected wave first meet, both waves have an amplitude is zero. As the waves continue to move past each other they continue to interfere with each other, either constructively of destructively. As discussed in previous atoms, when waves are completely in phase and interfere with each other constructively they are amplified, and when they are completely out of phase and interfere destructively they cancel out. As the waves continue to move past each other, and are reflected from the opposite end, they continue to interfere both ways; a standing wave is produced. Every point in the medium containing a standing wave oscillates up and down, and the amplitude of the oscillations depends on the location of the point. When we observe a standing wave on strings, it appears the wave is not moving but standing still. In summary:

- The points which reach the maximum oscillation height are called antinodes, and are results of complete constructive interference.
- The points in a standing wave that appear to remain flat and do not move are called nodes. These are due to complete destructive interference.

Standing Waves in Air Columns

Standing waves in air columns is the physical phenomenon that gives wind instruments their resonance and, therefore, sound.

learning objectives

- Identify the type of a standing wave in an air column

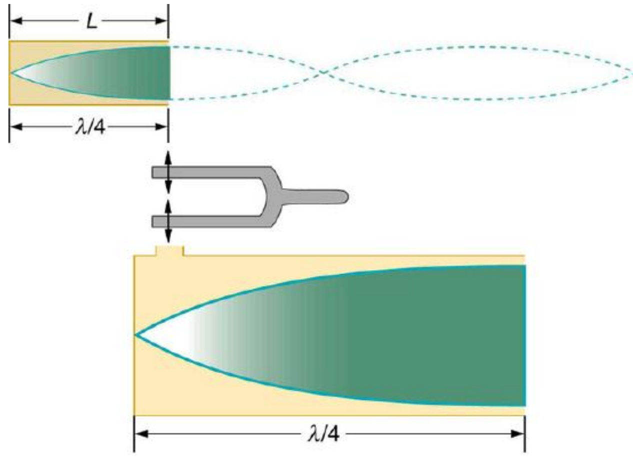
Standing Waves

A standing wave is a wave that appears to be stationary, meaning it remains in a constant position. In an air column, a standing wave can form as either a longitudinal or transverse wave. A transverse wave, you may remember, is a wave where the movement of the particles of the medium is perpendicular to the direction of the propagation of the wave. A longitudinal wave, on the other hand, is parallel to the direction of propagation. A standing wave can occur when two identical waves moving in different directions interfere.

Air Columns

When a standing wave is formed in a tube, the standing wave has a maximum air displacement at the open end called an antinode. Here, the motion is unconstrained. At the closed end, there is no displacement; this is called a node, and the air is halted. The distance from a node to antinode is 1/4 of a wavelength, and is equal to the length of the tube.

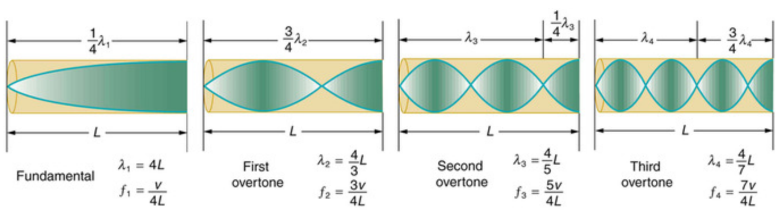
shows that this resonance can be produced by a vibration introduced at or near the closed end of the tube as well. This is considered to be a natural vibration of the air column independently of how it is induced. Given that maximum air displacements are possible at the open end and none at the closed end, there are other, shorter wavelengths that can resonate in the tube. For example, a standing wave can have three-fourths of its wavelength in the tube, or $L=(3/4)\lambda$. This can be rendered as follows:



Standing Wave in Air Column: The same standing wave is created in the tube by a vibration introduced near its closed end.

$$\lambda' = \frac{4\mathrm{L}}{3} \tag{16.5.1}$$

Continuing this process reveals a whole series of shorter-wavelength and higher-frequency sounds that resonate in the tube. We use specific terms for the resonances in any system. The lowest resonant frequency is called the fundamental, while all higher resonant frequencies are called overtones. All resonant frequencies are integral multiples of the fundamental, and they are collectively called harmonics. The fundamental is the first harmonic, the first overtone is the second harmonic, and so on. shows hows the fundamental and the first three overtones (the first four harmonics) in a tube closed at one end.



The First Four Harmonics: The fundamental and three lowest overtones for a tube closed at one end. All have maximum air displacements at the open end and none at the closed end.

Now let us look for a pattern in the resonant frequencies for a simple tube that is closed at one end. The fundamental has =4L, and frequency is related to wavelength and the speed of sound as given by the following:

$$\mathbf{v}_\mathrm{w} = \mathrm{f}\lambda \tag{16.5.2}$$

Solving for f in this equation gives a more helpful form:

$$\mathrm{f} = \frac{\mathrm{v_w}}{\lambda} = \frac{\mathbf{v}_\mathrm{w}}{4\mathrm{L}} \tag{16.5.3}$$

Here, f is frequency, v_w is speed of sound in air, λ is wavelength, and L is the length of the air column. The first overtone has = 4L/3. From this, we can deduce the following:

$$\mathrm{f}' = 3\frac{v_\mathrm{w}}{4\mathrm{L}} = 3\mathrm{f} \tag{16.5.4}$$

Because f′ = 3f, we call the first overtone the third harmonic. Continuing this process, we see a pattern that can be generalized in a single expression. The resonant frequencies of a tube closed at one end are:

$$\mathrm{f_n} = \mathrm{n}\frac{\mathrm{vw}}{4\mathrm{L}}, \mathrm{n} = 1, 3, 5 \tag{16.5.5}$$

Here, f1 is the fundamental, f3 is the first overtone, and so on. It is interesting that the resonant frequencies depend on the speed of sound and, hence, on temperature. This dependence poses a noticeable problem for organs in old unheated cathedrals, and it is also the reason why musicians commonly bring their wind instruments to room temperature before playing them.

Example 16.5.1:

The fundamental and overtones can be present simultaneously in a variety of combinations. For example, middle C on a trumpet has a sound distinctively different from middle C on a clarinet, both instruments being modified versions of a tube closed at one end. The fundamental frequency is the same (and usually the most intense), but the overtones and their mix of intensities are different and subject to shading by the musician. This mix is what gives various musical instruments (and human voices) their distinctive characteristics, whether they have air columns, strings, sounding boxes, or drumheads. In fact, much of our speech is determined by shaping the cavity formed by the throat and mouth and positioning the tongue to adjust the fundamental and combination of overtones. Simple resonant cavities can be made to resonate with the sound of the vowels, for example.

Forced Vibrations and Resonance

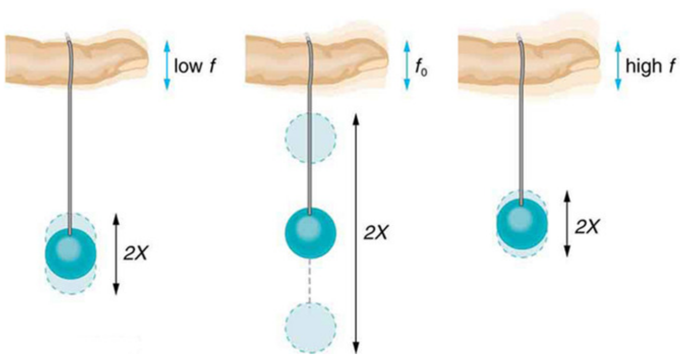
The phenomenon of driving a system with a frequency equal to its natural frequency is called resonance.

learning objectives

- Explain the relationship between the resonance curve and damping

Forced Vibration and Resonance

Many people have played with toys involving an object supported by an elastic band: something like the paddle ball suspended from a finger in. Say a person drives the paddle ball by moving his or her finger up and down at a certain frequency. In this example, he or she is causing a forced oscillation (or vibration). At first the finger is held steady, and the ball bounces up and down with a small amount of damping. If the finger is moved up and down slowly, the ball will follow along without bouncing much on its own.

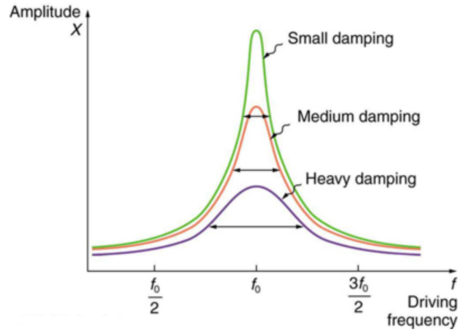


Paddle Balls and Frequencies: The paddle ball on its rubber band moves in response to the finger supporting it. If the finger moves with the natural frequency f0 of the ball on the rubber band, then a resonance is achieved, and the amplitude of the ball's oscillations increases dramatically. At higher and lower driving frequencies, energy is transferred to the ball less efficiently, and it responds with lower-amplitude oscillations.

As the frequency at which the finger is moved up and down increases, the ball will respond by oscillating with increasing amplitude. After driving the ball at its natural frequency, the ball's oscillations increase in amplitude with each oscillation for as long as it is driven. The phenomenon of driving a system with a frequency equal to its natural frequency is called resonance. As the driving frequency gets progressively higher than the resonant or natural frequency, the amplitude of the oscillations becomes smaller until the oscillations nearly disappear and the finger simply moves up and down with little effect on the ball.

Effects of Damping

In real life, most oscillators have damping present in the system. It is interesting that the widths of the resonance curves shown in depend on damping: the less the damping, the narrower the resonance. For a driven oscillator to resonate at a very specific frequency, there needs to be as little damping as possible, as is the case for piano strings and many other musical instruments. Conversely, for small-amplitude oscillations, such as in a car's suspension system, there needs to be heavy damping. Heavy damping reduces the amplitude, but the tradeoff is that the system responds at more frequencies.



Damping: The amplitude of a harmonic oscillator is a function of the frequency of the driving force. The curves represent the same oscillator with the same natural frequency but with different amounts of damping. Resonance occurs when the driving frequency equals the natural frequency, and the greatest response is for the least amount of damping. The narrowest response is also for the least amount of damping.

Example 16.5.2:

These features of driven harmonic oscillators apply to a huge variety of systems. When tuning a radio, for example, people are adjusting the resonant frequency of the radio circuit so that it only oscillates to the desired station's broadcast (or driving) frequency. The more selective the radio is in discriminating between stations, the smaller its damping. A child on a swing is driven by a parent at the swing's natural frequency to achieve maximum amplitude. In all of these cases, the efficiency of energy transfer from the driving force into the oscillator is best at resonance. On gravel roads that are corrugated, if people travel at the 'wrong' speed, the bumps are very noticeable. At other speeds, it is difficult to feel the bumps at all. shows a photograph of a famous example (the Tacoma Narrows Bridge) of the destructive effects of a driven harmonic oscillation. Unfortunately, heavy winds happened to drive the bridge at its natural frequency, leading to the collapse.

Collapse of the Tacoma Narrows Bridge: In 1940, the Tacoma Narrows Bridge in Washington state collapsed. Heavy cross winds drove the bridge into oscillations at its resonant frequency. The damping decreased when support cables broke loose and started to slip over the towers, allowing increasingly greater amplitudes until the structure failed. (credit: PRI's Studio 360, via Flickr)

Key Points

- The waves either interfere with each other constructively or destructively, which will either amplify or minimize the wave, respectively.
- Spherical waves are emitted from a single point source in a spherical shape.
- A plane wave is a constant-frequency wave whose wavefronts (surfaces of constant phase) are infinite parallel planes of constant peak-to-peak amplitude normal to the phase velocity vector.
- Although it is not possible in practice to have a true plane wave, many waves approximate plane wave behavior.
- The reflected wave is inverted from the incident wave when a transverse wave on a string is fixed at the end point. The reflected wave is not inverted from the incident wave when a transverse wave on a string is free at the end point.
- A standing wave occurs when an incident wave meets a reflected wave on a string.
- A standing wave contains nodes (points that remain flat due to the destructive interference) and antinodes (points with maximum oscillation due to the constructive interference).

- Every point in the string oscillates up and down and the amplitude of the oscillations depends on the location of the point.
- A standing wave has some points that remain flat due to destructive interference. These are called antinodes.
- The points on a standing wave that have reached maximum oscillation do so from constructive interference, and are called nodes.
- A standing wave in an air column is a transverse wave.
- A node occurs at the closed end of an air tube where there is no wave displacement.
- An antinode occurs at the open end of the air tube where the maximum displacement occurs.
- The resonant frequencies of a tube closed at one end are as follows: $\mathrm{f_n = n\frac{vw}{4L}}$, n=1,3,5.
- Any oscillator has a natural frequency. Driven at the natural frequency, oscillations increase in amplitude with each oscillation.
- The widths of the resonance curves depend on damping: less damping corresponds to a narrower resonance. With less damping, the resonance peak becomes more pronounced.
- The relationship between resonance and damping applies to a huge variety of systems. Damping is often reduced or enhanced to induce the desired response of an oscillator.

Key Terms

- **wavefront**: An imaginary surface passing through points of a medium oscillating in phase.
- **destructive interference**: Occurs when waves interfere with each other crest to trough (peak to valley) and are exactly out of phase with each other.
- **constructive interference**: Occurs when waves interfere with each other crest to crest and the waves are exactly in phase with each other.
- **transverse wave**: Any wave in which the direction of disturbance is perpendicular to the direction of travel.
- **natural frequency**: The frequency at which a system vibrates on its own. For a spring (spring constant k) with an object of mass m attached, the natural frequency is given as $\mathrm{f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}}$.
- **damping**: The reduction in the magnitude of oscillations by the dissipation of energy

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CHAPTER OVERVIEW

17: Electric Charge and Field

Topic hierarchy

17.1: Overview

learning objectives

- Identify factors that determine the atom's net charge

Overview of Atomic Electrical Charges

Atoms, the fundamental building blocks of all molecules, consist of three types of particles: protons, neutrons, and electrons. Of these three subatomic particle types, two (protons and electrons) carry a net electric charge, while neutrons are neutral and have no net charge.

Both protons and electrons have charge that is quantized. That is, the magnitude of their respective charges, which are equal each other, is 1. This standard value is equal to approximately 1.6×10^{-19} Coulombs.

Protons

Protons are found in the center of the atom; they, with neutrons, make up the nucleus. Protons have a charge of +1 and a mass of 1 atomic mass unit, which is approximately equal to 1.66×10^{-24} grams. The number of protons in an atom defines the identity of the element (an atom with 1 proton is hydrogen, for example, and an atom with two protons is helium). As such, protons are relatively stable; their number rarely changes, only in the instance of radioactive decay.

Electrons

Electrons are found in the periphery of the atom and have a charge of -1. They are much smaller than protons; their mass is 1183611836 amu. Typically in modeling atoms, protons and neutrons are regarded as stationary, while electrons move about in the space outside the nucleus like a cloud. The negatively charged electronic cloud indicates the regions of the space where electrons are likely to be found. The electrons cloud patterns are extremely complex and is of no importance to the discussion of electric charge in the atom. More important is the fact that electrons are labile; that is, they can be transferred from one atom to the next. It is through electronic transfer that atoms become charged.

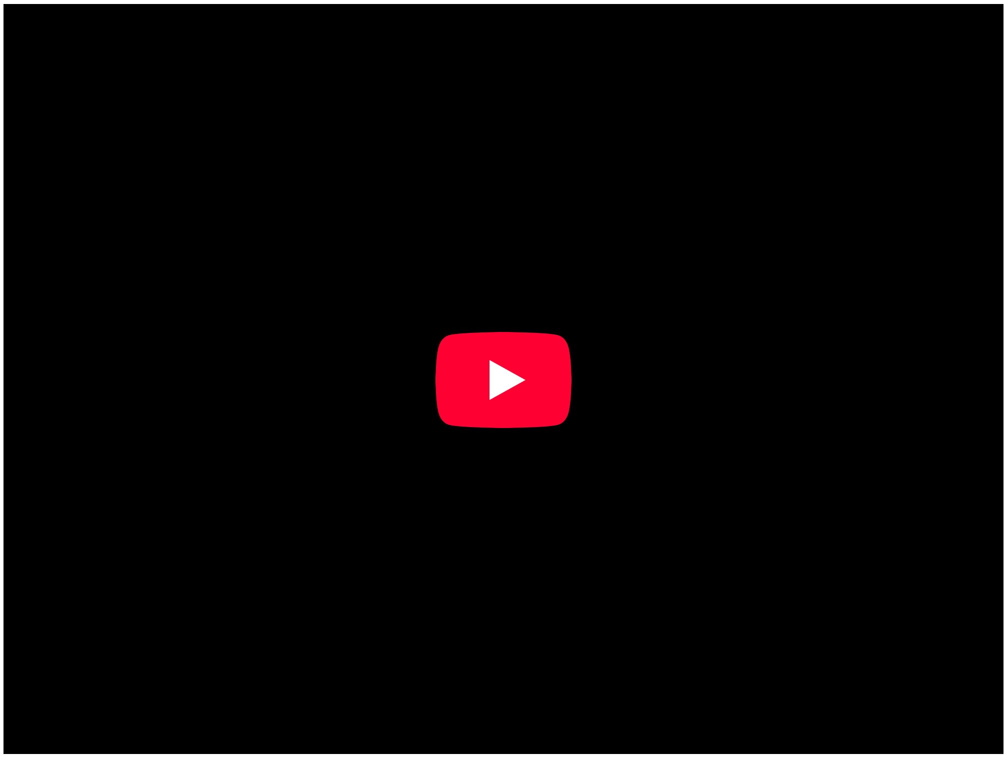
Ions

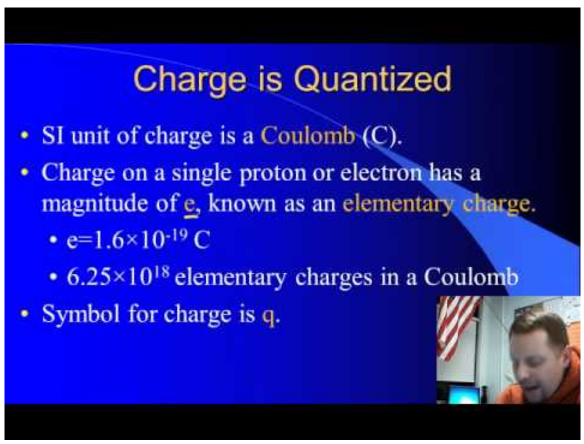
In the ground state, an atom will have an equal number of protons and electrons, and thus will have a net charge of 0. However, because electrons can be transferred from one atom to another, it is possible for atoms to become charged. Atoms in such a state are known as ions.

If a neutral atom gains an electron, it becomes negative. This kind of ion is called an anion.

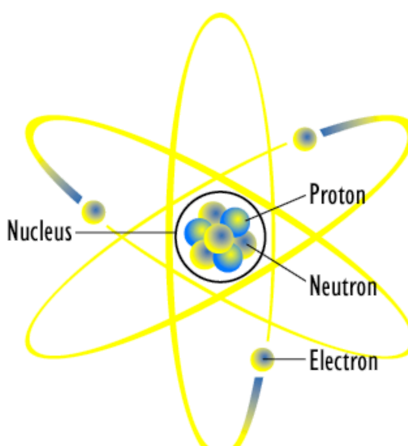
If a neutral atom loses an electron, it becomes positive. This kind of ion is called a cation.

The steady flow of electrons is called current. Current is what flows through electrical wires and powers electronics items, from light bulbs to televisions.





Electric Charge: A brief overview of atoms, ions, and electrical charge.



Planetary Model of an Atom: Small electrons orbit the large and relatively fixed nucleus of protons and neutrons.

Properties of Electric Charges

Electric charge is a fundamental physical property of matter that has many parallels to mass.

learning objectives

- Describe properties of electric charge, such as its relativistic invariance and its conservation in closed systems

Properties of Electric Charge

Electric charge, like mass and volume, is a physical property of matter. Its SI unit is known as the Coulomb (C), which represents $6.242 \times 10^{18} e$, where e is the charge of a proton. Charges can be positive or negative; a singular proton has a charge of 1.602×10^{-19} C, while an electron has a charge of -1.602×10^{-19} C.

Invariance

Like mass, electric charge in a closed system is conserved. As long as a system is impermeable, the amount of charge inside it will neither increase nor decrease; it can only be transferred. However, electric charge differs from other properties—like mass—in that it is a relativistic invariant. That is, charge is *independent of speed*. The mass of a particle will rise exponentially as its speed approaches that of light, its charge, however, will remain constant.

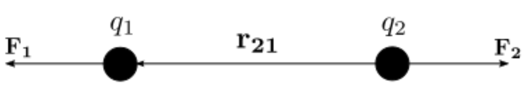
The independence of electric charge from speed was proven through an experiment in which one fast-moving helium nucleus (two protons and two neutrons bound together) was proven to have the same charge as two separate, slow-moving deuterium nuclei (one proton and one neutron bound together in each nucleus).

Attraction and Repulsion

Electric charge is a property that produces forces that can attract or repel matter. Mass is similar, although it can only attract matter, not repel it. Still, the formula describing the interactions between charges is remarkably similar to that which characterizes the interactions between masses. For electric fields, the force (F) is related to the charges (q_1, q_2) and the distance (r) between them as:

$$\mathrm{F} = \frac{1}{4\pi\epsilon_0} \frac{\mathrm{q}_1 \mathbf{q}_2}{\mathrm{r}^2} \tag{17.1.1}$$

where π and ϵ0ϵ0 are constants. This is known as Coulomb's Law.



Coulomb's Law: The forces (F_1 and F_2) sum to produce the total force, which is calculated by Coulomb's Law and is proportional to the product of the charges q_1 and q_2, and inversely proportional to the square of the distance (r_{21}) between them.

The formula for gravitational force has exactly the same form as Coulomb's Law, but relates the product of two masses (rather than the charges) and uses a different constant. Both act in a vacuum and are central (depend only on distance between the forces) and conservative (independent of path taken). However, it should be noted that when comparing similar terms, charge-based interaction is substantially greater than that based on mass. For example, the electric repulsion between two electrons is about 10^{42} times stronger than their gravitational attraction.

Charge Separation

Charge separation, often referred to as static electricity, is the building of space between particles of opposite charges.

learning objectives

- Identify factors that can create charge separation

All matter is composed of atoms made up of negatively-charged electrons and positively-charged protons. In the ground state, each atom is of neutral charge—its protons and electrons are equal in number, and it exists with no permanent dipole. Because electrons are labile (i.e., they can be transferred from atom to atom) it is possible for the phenomenon of "charge separation" (often referred to as static electricity) to occur.

Static Electricity: Due to friction between her hair and the plastic slide, the girl on the left has created charge separation, resulting in her hair being attracted to the slide.

In chemistry, this charge separation is illustrated simply by the transfer of an electron from one atom to another as an ionic bond is formed. In physics, there are many other instances of charge separation that cannot be written as formal chemical reactions. Consider, for example, rubbing a balloon on your hair. Once you pull the balloon away, your hair will stand on end and "reach" towards the balloon. This is because electrons from one have transferred to the other, causing one to be positive and the other to be negative. Thus, the opposite charges attract. A similar example can be seen in playground slides (as shown in).

Charge separation can be created not only by friction, but by pressure, heat, and other charges. Both pressure and heat increase the energy of a material and can cause electrons to break free and separate from their nuclei. Charge, meanwhile, can attract electrons to or repel them from a nucleus. For example, a nearby negative charge can "push" electrons away from the nucleus around which they typically orbit. Charge separation occurs often in the natural world. It can have an extreme effect if it reaches a critical level, whereat it becomes discharged. Lightning is a common example.

Polarization

Dielectric polarization is the phenomenon that arises when positive and negative charges in a material are separated.

learning objectives

- Identify two ways polarization can occur on the molecular level

The concept of polarity is very broad and can be applied to molecules, light, and electric fields. For the purposes of this atom, we focus on its meaning in the context of what is known as dielectric polarization—the separation of charges in materials.

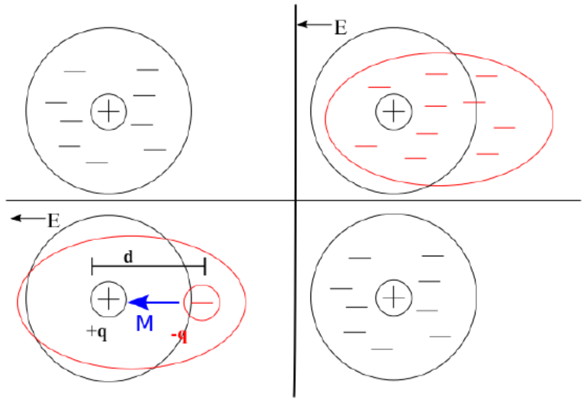
Dielectrics

A dielectric is an insulator that can be polarized by an electric field, meaning that it is a material in which charge does not flow freely, but in the presence of an electric field it can shift its charge distribution. Positive charge in a dielectric will migrate towards the applied field, while negative charges will shift away. This creates a weak local field within the material that opposes the applied field.

Different materials will react differently to an induced field, depending on their dielectric constant. This constant is the degree of their polarizability (the extent to which they become polarized).

Atomic Model

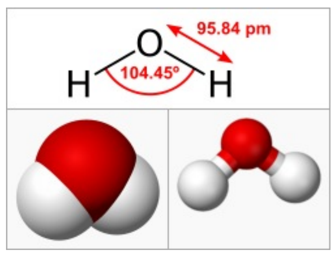
The most basic view of dielectrics involves considering their charged components: protons and electrons. If an electric field is applied to an atom, the electrons in the atom will migrate away from the applied field. The protons, however, remain relatively exposed to the field. This separation creates a dipole moment, as shown in.



Reaction of an Atom to an Applied Electric Field: When an electric field (E) is applied, electrons drift away from the field. Their average location is displaced from the average location of the protons (which hasn't moved) by a distance of d. The atom's dipole moment is represented by M.

Dipole Polarization

On the molecular level, polarization can occur with both dipoles and ions. In polar bonds, electrons are more attracted to one nucleus than to the other. One example of a dipole molecule is water, (H_2O), which has a bent shape (the H-O-H angle is 104.45°) and in which the oxygen pulls electron density away from the H atoms, leaving the H relatively positive and the O relatively negative, as shown in.



Water Molecule: Water is an example of a dipole molecule, which has a bent shape (the H-O-H angle is 104.45°) and in which the oxygen pulls electron density away from the H atoms, leaving the H relatively positive and the O relatively negative.

When a dipolar molecule is exposed to an electric field, the molecule will align itself with the field, with the positive end towards the electric field and the negative end away from it.

Ionic Polarization

Ionic compounds are those that are formed from permanently charge-separated ions. For example, table salt (NaCl) is formed from Na^+ and Cl^- ions that are not formally bound to one another through a chemical bond, but interact very strongly due to their opposite charges.

Ions are still free from one another and will naturally move at random. If they happen to move in a way that is asymmetrical, and results in a greater concentration of positive ions in one area and a greater concentration of negative ions in another, the sample of ionic compound will be polarized—a phenomenon is known as ionic polarization.

Static Electricity, Charge, and the Conservation of Charge

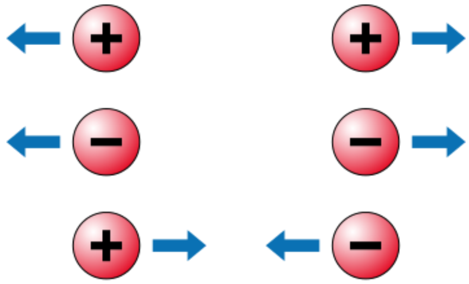
Electric charge is a physical property that is perpetually conserved in amount; it can build up in matter, which creates static electricity.

learning objectives

- Formulate rules that apply to the creation and the destruction of electric charge

Electric charge is a physical property of matter. It is created by an imbalance in a substance's number of protons and electrons. The matter is positively charged if it contains more protons than electrons, and it is negatively charged if it contains more electrons than protons. In both instances, charged particles will experience a force when in the presence of other charged matter.

Charges of like sign (positive and positive, or negative and negative) will repel each other, whereas charges of opposite sign (positive and negative) will attract each another, as shown in.



Charge Repulsion and Attraction: Charges of like sign (positive and positive, or negative and negative) will repel each other, whereas charges of opposite sign (positive and negative) will attract each other.

The SI unit for charge is the Coulomb (C), which is approximately equal to 6.24×10186.24×1018 elementary charges. (An elementary charge is the magnitude of charge of a proton or electron.)

Conservation of Charge

Charge, like matter, is essentially constant throughout the universe and over time. In physics, charge conservation is the principle that electric charge can neither be created nor destroyed. The net quantity of electric charge, the amount of positive charge minus the amount of negative charge in the universe, is always conserved.

For any finite volume, the law of conservation of charge (Q) can be written as a continuity equation:

$$Q\left(t_2\right)=Q\left(t_1\right)+Q_{\text{ in }}-Q_{\text{ out}} \tag{17.1.2}$$

where $Q(t_1)$ is the charge in the system at a given time, $Q(t_2)$ is the charge in the same system at a later time, Q_{in} is the charge that has entered the system between the two times, and Q_{out} is the amount of charge that has left the system between the two times.

This does not mean that individual positive and negative charges cannot be created or destroyed. Electric charge is carried by subatomic particles such as electrons and protons, which can be created and destroyed. For example, when particles are destroyed, equal numbers of positive and negative charges are destroyed, keeping the net amount of charge unchanged.

Static Electricity

Static electricity is when an excess of electric charge collects on an object's surface. It can be created through contact between materials, a buildup of pressure or heat, or the presence of a charge. Static electricity can also be created through friction between a balloon (or another object) and human hair (see). It can be observed in storm clouds as a result of pressure buildup; lightning (see) is the discharge that occurs after the charge exceeds a critical concentration.

Static Electricity: Due to friction between her hair and the plastic slide, the girl on the left has created charge separation, resulting in her hair being attracted to the slide.

Lightning: Lightning is a dramatic natural example of static discharge.

Conductors and Insulators

Based on the ability to conduct current, materials are divided into conductors and insulators.

learning objectives

- Identify conductors and insulators among common materials

Overview

All materials can be categorized as either insulators or conductors based on a physical property known as resistivity.

An insulator is a material in which, when exposed to an electric field, the electric charges do not flow freely—it has a high resistivity. Conversely, a conductor is a material that permits the flow of electric charges in one or more directions—its resistivity is low.

Conductors

All conductors contain electric charges that, when exposed to a potential difference, move towards one pole or the other. The positive charges in a conductor will migrate towards the negative end of the potential difference; the negative charges in the material will move towards the positive end of the potential difference. This flow of charge is electric current.

Ionic substances and solutions can conduct electricity, but the most common and effective conductors are metals. Copper is commonly used in wires due to its high conductivity and relatively inexpensive price. However, gold-plated wires are sometimes used in instances in which especially high conductivity is necessary.

Every conductor has a limit to its ampacity, or amount of current it can carry. This usually is the current at which the heat released due to resistance melts the material.

Insulators

Insulators are materials in which the internal charge cannot flow freely, and thus cannot conduct electric current to an appreciable degree when exposed to an electric field.

While there is no perfect insulator with infinite resistivity, materials like glass, paper and Teflon have very high resistivity and can effectively serve as insulators in most instances.

Just as conductors are used to carry electrical current through wires, insulators are commonly used as coating for the wires.

Insulators, like conductors, have their physical limits. When exposed to enough voltage, an insulator will experience what is known as electrical breakdown, in which current suddenly spikes through the material as it becomes a conductor.

Conductor and Insulator in a Wire: This wire consists of a core of copper (a conductor) and a coating of polyethylene (an insulator). The copper allows current to flow through the wire, while the polyethylene ensures that the current does not escape.

The Millikan Oil-Drop Experiment

In 1911, using charged droplets of oil, Robert Millikan was able to determine the charge of an electron.

learning objectives

- Explain the difference in value of a real electron's charge and the charge measured by Robert Millikan

The Oil-Drop Experiment

The Oil-Drop Experiment, otherwise known as the Millikan Oil-Drop Experiment, is one of the most influential studies in the history of physical science.

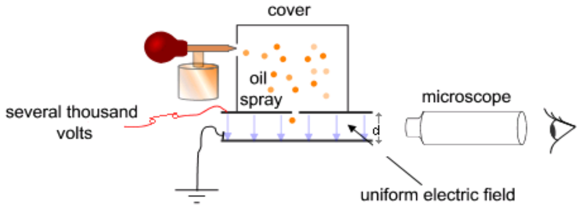
Performed by Robert Millikan and Harvey Fletcher in 1911, the experiment was designed to determine the charge of a single electron, otherwise known as the elementary electric charge.

Millikan designed his experiment to measure the force on oil droplets between two electrodes.

He used an atomizer to spray a mist of tiny oil droplets into a chamber, which included a hole. Some droplets would fall through this hole and into a chamber, where he measured their terminal velocity and calculated their mass.

Millikan then exposed the droplets to X-rays, which ionized molecules in the air and caused electrons to attach to the oil droplets, thus making them charged. The top and bottom of the chamber were attached to a battery, and the potential difference between the top and bottom produced an electric field that acted on the charged oil drops.

Adjusting the voltage perfectly, Millikan was able to balance the force of gravity (which was exerted downward) with the force of the electric field on the charged particles (which was exerted upward), causing the oil droplets to be suspended in mid-air.



Simplified scheme of Millikan's oil-drop experiment: This apparatus has a parallel pair of horizontal metal plates. A uniform electric field is created between them. The ring has three holes for illumination and one for viewing through a microscope. Special oil for vacuum apparatus is sprayed into the chamber, where drops become electrically charged. The droplets enter the space between the plates and can be controlled by changing the voltage across the plates.

Millikan then calculated the charge on particles suspended in mid-air. His assumptions were that the force of gravity, which is the product of mass (m) and gravitational acceleration (g), was equal to the force of the electric field (the product of the charge (q) and the electric field (E)):

$$\mathbf{q} \cdot \mathbf{E} = \mathbf{m} \cdot \mathbf{g} \tag{17.1.3}$$

$$\mathrm{q} = \frac{\mathrm{m} \cdot \mathrm{g}}{\mathrm{E}} \tag{17.1.4}$$

Since he already knew the mass of the oil droplets and the acceleration due to gravity (9.81 m/s^2), as well as the energy of the x-rays he was using, he was able to calculate the charge.

Although the charge of each droplet was unknown, Millikan adjusted the strength of the X-rays ionizing the air and measured many values of (q) from many different oil droplets. In each instance, the charge measured was a multiple of $1.5924(17)\times10^{-19}$ C. Thus, it was concluded that the elementary electric charge was $1.5924(17)\times10^{-19}$ C.

The results were very accurate. The calculated value from the Oil-Drop Experiment differs by less than one percent of the current accepted value of $1.602176487(40)\times10^{-19}$ C.

The Oil-Drop Experiment was tremendously influential at the time, not only for determining the charge of an electron, but for helping prove the existence of particles smaller than atoms. At the time, it was not fully accepted that protons, neutrons, and electrons existed.

Key Points

- A proton is a positively charged particle located in the nucleus of an atom. An electron has $\frac{1}{1836}$ times the mass of a proton, but an equal and opposite negative charge.
- An elementary charge — that of a proton or electron — is approximately equal to 1.6×10-19Coulombs.
- Unlike protons, electrons can move from atom to atom. If an atom has an equal number of protons and electrons, its net charge is 0. If it gains an extra electron, it becomes negatively charged and is known as an anion. If it loses an electron, it becomes positively charged and is known as a cation.
- Charge is measured in Coulombs (C), which represent 6.242×10^{18} e, where e is the charge of a proton. Charges can be positive or negative, and as such a singular proton has a charge of 1.602×10^{-19} C, while an electron has a charge of -1.602×10^{-19} C.
- Electric charge, like mass, is conserved. The force generated by two charges is of the same form as that generated by two masses and, like gravity, force from an electrical field is both conservative and central.
- Electric charge is a relativistic invariant. That is, charge (unlike mass) is independent of speed. Whereas the mass of a particle will exponentially rise as its speed approaches that of light, charge will remain constant.
- Because electrons are labile (i.e., they can be transferred from atom to atom), it is possible for “charge separation” to occur. This phenomenon is often commonly referred to as static electricity.
- Charge separation can be created by friction, pressure, heat, and other charges.
- Charge separation can reach a critical level, whereat it is discharged. Lightning is a common example.
- Dielectrics are insulators that are capable of being polarized by an electric field. That is, their charges cannot flow freely, but can still be induced to redistribute unevenly.
- Electric fields applied to atoms will push electrons away from the field. In the case of polar molecules, the negative ends thereof will align themselves away from the field while the positive ends will be towards the field.
- An instantaneous polarization occurs when ions, through natural, random vibrations, become distributed asymmetrically such that one area is more dense with one type of ion than another.
- Electric charge is a physical property of matter created by an imbalance in the number of protons and electrons in a substance.
- Charge can be created or destroyed. However, any creation or elimination of charge occurs at a ratio of 1:1 between positive and negative charges.
- Static electricity is when an excess of electric charge collects on an object’s surface.
- Resistivity, a physical property that measures the ability of a material to carry current, is the main factor in determining whether a substance is a conductor or an insulator.
- Conductors contain electric charges that, when exposed to a potential difference, move towards one pole or the other. This flow of charge is electric current.
- Insulators are materials in which the internal charge cannot flow freely, and thus cannot conduct electric current to an appreciable degree when exposed to an electric field.
- The Oil-Drop Experiment involved ionizing droplets of oil as they fell through the air, and balancing the force of gravity with the force of an electric field applied by electrodes above and below the droplet.
- Millikan could not directly count the number of electrons on each oil droplet, but found that the common denominator between all measured charges was equal to $1.5924(17)\times10^{-19}$ C, and thus concluded that this value was the charge of an electron.
- The measured value of an electron’s charge, $1.5924(17)\times10^{-19}$ C, differs from the accepted value of $1.602176487(40)\times10^{-19}$ C by less than one percent.

Key Terms

- **nucleus**: the massive, positively charged central part of an atom, made up of protons and neutrons
- **coulomb**: In the International System of Units, the derived unit of electric charge; the amount of electric charge carried by a current of 1 ampere flowing for 1 second. Symbol: C
- **gravity**: Resultant force on Earth's surface, of the attraction by the Earth's masses, and the centrifugal pseudo-force caused by the Earth's rotation.
- **electric field**: A region of space around a charged particle, or between two voltages; it exerts a force on charged objects in its vicinity.
- **discharge**: the act of releasing an accumulated charge
- **static electricity**: an electric charge that has built up on an insulated body, often due to friction
- **dipole moment**: The vector product of the charge on either pole of a dipole and the distance separating them.
- **dielectric**: An electrically insulating or nonconducting material considered for its electric susceptibility (i.e., its property of polarization when exposed to an external electric field).
- **insulator**: A substance that does not transmit heat (thermal insulator), sound (acoustic insulator) or electricity (electrical insulator).
- **electric charge**: A quantum number that determines the electromagnetic interactions of some subatomic particles; by convention, the electron has an electric charge of -1 and the proton +1, and quarks have fractional charge.
- **conductor**: A material which contains movable electric charges.
- **resistivity**: In general, the resistance to electric current of a material; in particular, the degree to which a material resists the flow of electricity.
- **voltage**: The amount of electrostatic potential between two points in space.
- **terminal velocity**: The speed at which an object in free-fall and not in a vacuum ceases to accelerate downwards because the force of gravity is equal and opposite to the drag force acting against it.

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17.2: Shelding and Charging Through Induction

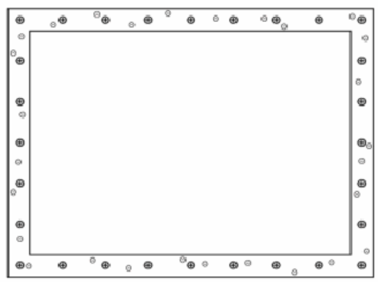
learning objectives

- Describe structure of a Faraday cage

Electrostatic shielding is the phenomenon that is observed when a Faraday cage operates to block the effects of an electric field. Such a cage can block the effects of an external field on its internal contents, or the effects of an internal field on the outside environment.

A Faraday cage is a closed chamber consisting of a conducting material or a mesh of such a material. This type of cage was first invented by Michael Faraday in 1836, and can block external static and non-static electric fields.

When an external electric field operates on a Faraday cage, the charges within the cage (which are mobile, as the cage is a conductor) rearrange themselves to directly counteract the field and thus “shield” the interior of the cage from the external field



Faraday Cage in Presence of an External Electrical Field: As the field is applied, the negative charge from the cage migrates toward the positive end of the field, canceling the effects of the field at both ends of the cage.

The action of a Faraday cage may depend on whether or not it is grounded. Consider a charge placed within a cage. If the cage is not grounded, electrons in the cage will redistribute such that the interior wall of the cage takes on a charge opposite the internal charge. This would leave an exterior wall of opposite charge to that of the interior. If it is grounded, however, excess charges on the exterior of the cage will go to the ground, leaving the exterior wall of neutral charge.

Limitations

Faraday cages are limited in their effectiveness, and cannot block static and slowly varying magnetic fields, such as that of the planet Earth. They can, however, shield the interior from external magnetic radiation provided that the mesh is smaller than the wavelength of the radiation and that the shield is sufficiently thick.

Applications

Microwave ovens contain energy within themselves, shielding the outside from harmful radiation.

Electrical linemen often wear suits made of Faraday cages so as to avoid electrocution.

Elevators can act as unintended Faraday cages, shielding cell phones and radios from signal from the outside.

Induced Charge

Electrostatic induction is the redistribution of charges within an object that occurs as a reaction to the presence of a nearby charge.

learning objectives

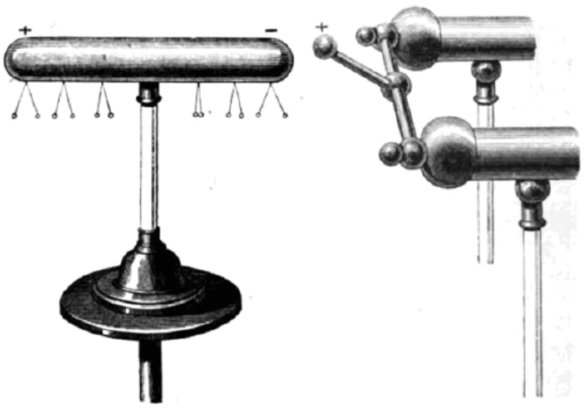
- Compare electrostatic induction processes in conductors and dielectrics

Electrostatic induction is the redistribution of charge within an object, which occurs as a reaction to a nearby charge.

Normally, a unit of matter will have equal parts positive and negative charge, distributed evenly throughout the object. As such, it has no net charge.

If a charged piece of matter is brought into close proximity with respect to an uncharged piece of matter, it can induce charge redistribution in the neutral material.

In such an event, the electrons in the neutral object move (the protons are relatively inert) according to the charge of the nearby charged object (inductor). If the inductor is positive, electrons migrate toward it, making the uncharged object more negative in that area and positive in the region opposite it. If the inductor is negative, the electrons in the neutral object are repelled, leaving a positive charge near the inductor and a negative charge opposite it.



Electric Induction Experiment: Circa 1870, the positive end of an electrostatic generator is placed near an uncharged brass cylinder, causing the cylinder to polarize as its left end becomes positive and its right end becomes negative.

If a charged object comes in touch with an uncharged object, or the two come sufficiently close to one another to cause a discharge that bridges the gap between them, the previously uncharged object will become charged. Depending on the sign of the charge of the inductor, electrons will go to or leave the previously uncharged object. Total charge is conserved, and that of the inductor decreases as it transfers charge to its subject.

Subjects that can react to inductors include conductors and dielectrics. In the case of the former, the free flow of charges makes it possible for strong polarization to occur. In the case of the latter, the force is comparatively weak.

Key Points

- Electrostatic shielding is the phenomenon that is observed when a Faraday cage operates to block the effects of an electric field. Such a cage can block the effects of an external field on its internal contents, or the effects of an internal field on the outside environment.
- A Faraday cage is a closed chamber consisting of a conducting material or a mesh of such a material. This type of cage can block external static and non-static electric fields.
- Faraday cages cannot block static and slowly varying magnetic fields, such as that of the planet Earth. They can, however, shield the interior from external magnetic radiation provided that the mesh is smaller than the wavelength of the radiation and that the shield is sufficiently thick.
- If a charged piece of matter is brought into close proximity with respect to an uncharged piece of matter, it can induce charge redistribution in the neutral material. This is one form of induction.
- If a charged object comes in touch with an uncharged object, or the two come sufficiently close to one another to cause a discharge that bridges the gap between them, the previously uncharged object will become charged. This is another form of induction.
- Subjects that can react to inductors include conductors and dielectrics. In the case of the former, the free flow of charges makes it possible for strong polarization to occur. In the case of the latter, the force is comparatively weak.

Key Terms

- **wavelength**: The length of a single cycle of a wave, as measured by the distance between one peak or trough of a wave and the next; it is often designated in physics as λ, and corresponds to the velocity of the wave divided by its frequency.
- **discharge**: the act of releasing an accumulated charge
- **inductor**: A passive device that introduces inductance into an electrical circuit.

- **dielectric**: An electrically insulating or nonconducting material considered for its electric susceptibility (i.e., its property of polarization when exposed to an external electric field).

17.3: Coulomb's Law

learning objectives

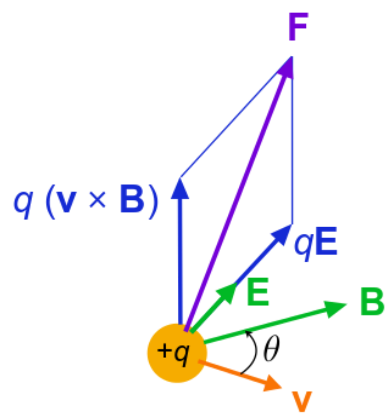
- Apply the superposition principle to determine the net response caused by two or more stimuli

The superposition principle (also known as superposition property) states that: *for all linear systems, the net response at a given place and time caused by two or more stimuli is the sum of the responses which would have been caused by each stimulus individually*. For Coulomb's law, the stimuli are forces. Therefore, the principle suggests that total force is a vector sum of individual forces.

Coulomb Force

The scalar form of Coulomb's Law relates the magnitude and sign of the electrostatic force F, acting simultaneously on two point charges q_1 and q_2:

$$|\mathrm{F}| = \frac{1}{4\pi ar\epsilon_0}\frac{|q_1 q_2|}{\mathrm{r}^2} \tag{17.3.1}$$



Lorentz Force on a Moving Particle: Lorentz force f on a charged particle (of charge q) in motion (instantaneous velocity v). The E field and B field vary in space and time.

where r is the separation distance and ε_0 is electric permittivity. If the product q_1q_2 is positive, the force between them is repulsive; if q_1q_2 is negative, the force between them is attractive. The principle of linear superposition allows the extension of Coulomb's law to include any number of point charges—in order to derive the force on any one point charge by a vector addition of these individual forces acting alone on that point charge. The resulting force vector happens to be parallel to the electric field vector at that point, with that point charge removed.

To calculate the force on a small test charge q at position rr, due to a system of N discrete charges:

$$\mathrm{F}(\mathrm{r}) = \frac{\mathrm{q}}{4\pi \mathrm{ar}\epsilon_0}\sum_{i=1}^{N} q_i \frac{\mathrm{r}-\mathrm{r_i}}{|\mathrm{r}-\mathrm{r_i}|^3} = \frac{q}{r4\pi ar\epsilon_0}\sum_{\mathrm{i}=1}^{\mathrm{N}} \mathrm{q_i}\frac{\hat{\mathrm{R}}_i}{|\mathrm{R_i}|^2} \tag{17.3.2}$$

where $\mathrm{q_i}$ and $\mathrm{r_i}$ are the magnitude and position vector of the i-th charge, respectively, and $\hat{\mathrm{R}}_\mathrm{i}$ is a unit vector in the direction of $\mathrm{R_i} = \mathrm{r} - \mathrm{r_i}$ (a vector pointing from charges $\mathrm{q_i}$ to q.)

Of course, our discussion of superposition of forces applies to any types (or combinations) of forces. For example, when a charge is moving in the presence of a magnetic field as well as an electric field, the charge will feel both electrostatic and magnetic forces. Total force, affecting the motion of the charge, will be the vector sum of the two forces. (In this particular example of the moving charge, the force due to the presence of electromagnetic field is collectively called Lorentz force (see).

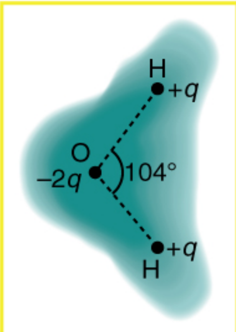
Spherical Distribution of Charge

The charge distribution around a molecule is spherical in nature, and creates a sort of electrostatic "cloud" around the molecule.

learning objectives

- Describe shape of a Coulomb force from a spherical distribution of charge

Through the work of scientists in the late 18th century, the main features of the electrostatic force —the existence of two types of charge, the observation that like charges repel, unlike charges attract, and the decrease of force with distance—were eventually refined, and expressed as a mathematical formula. The mathematical formula for the electrostatic force is called Coulomb 's law after the French physicist Charles Coulomb (1736–1806), who performed experiments and first proposed a formula to calculate it.



Charge distribution in a water molecule: Schematic representation of the outer electron cloud of a neutral water molecule. The electrons spend more time near the oxygen than the hydrogens, giving a permanent charge separation as shown. Water is thus a polar molecule. It is more easily affected by electrostatic forces than molecules with uniform charge distributions.

Modern experiments have verified Coulomb's law to great precision. For example, it has been shown that the force is inversely proportional to distance between two objects squared ($F \propto 1/r^2$) to an accuracy of 1 part in 1016. No exceptions have ever been found, even at the small distances within the atom.

Coulomb's law holds even within the atoms, correctly describing the force between the positively charged nucleus and each of the negatively charged electrons. This simple law also correctly accounts for the forces that bind atoms together to form molecules and for the forces that bind atoms and molecules together to form solids and liquids.

Generally, as the distance between ions increases, the energy of attraction approaches zero and ionic bonding is less favorable. As the magnitude of opposing charges increases, energy increases and ionic bonding is more favorable.

An electric field is a vector field which associates to each point of the space the Coulomb force that will experience a test unity charge. Given the electric field, the strength and direction of a force F on a quantity charge q in an electric field E is determined by the electric field. For a positive charge, the direction of the electric field points along lines directed radially away from the location of the point charge, while the direction is towards for a negative charge.

This distribution around a charged molecule is spherical in nature, and creates a sort of electrostatic "cloud" around the molecule. The attraction or repulsion forces within the spherical distribution of charge is stronger closer to the molecule, and becomes weaker as the distance from the molecule increases.

This image shows the outer electron cloud of a neutral water molecule. The charge distribution of the oxygen molecule is negative, and attracts the two positive hydrogen molecules. The attraction between the two opposing charges forms a neutral water molecule. It is a polar molecule because there is still a permanent charge separation because the electrons spend more time near the oxygen than the hydrogens.

Solving Problems with Vectors and Coulomb's Law

Coulomb's Law, which calculates the electric force between charged particles, can be written in vector notation as $\mathrm{F(E)} = \frac{\mathrm{kq_1q_2}}{\mathrm{r^2}}\mathrm{r}+$.

learning objectives

- Explain when the vector notation of Coulomb's Law can be used

Solving Problems with Vectors and Coulomb's Law

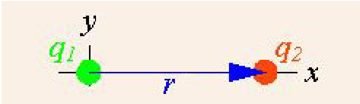
Electric Force Between Two Point Charges

To address the electrostatic forces among electrically charged particles, first consider two particles with electric charges q and Q, separated in empty space by a distance r. Suppose that we want to find the electric force vector on charge q. (The electric force vector has both a magnitude and a direction.) We can express the location of charge q as r_q, and the location of charge Q as r_Q. In this way we can know both how strong the electric force is on a charge, but also what direction that force is directed in. Coulomb's Law using vectors can be written as:

$$\mathbf{F_E} = \frac{\mathrm{kq} Q \left(\mathrm{r_q} - \mathrm{r}_Q\right)}{\left|\mathrm{r_q} - \mathrm{r}_Q\right|^3} \tag{17.3.3}$$

In this equation, k is equal to $\frac{1}{4\pi\varepsilon_0\varepsilon}$,where ε_0 is the permittivity of free space and εε is the relative permittivity of the material in which the charges are immersed. The variables $\mathbf{F_E}, \boldsymbol{\Gamma}_\mathrm{q}$ and $\mathbf{R}_Q$ are in bold because they are vectors. Thus, we need to find $\mathbf{r}_\mathrm{q} - \mathbf{r}_\mathrm{Q}$ by performing standard vector subtraction. This means that we need to subtract the corresponding components of vector \mathbf{r}_\mathrm{Q}} from vector \mathrmbf$r_\mathrm{q}$.

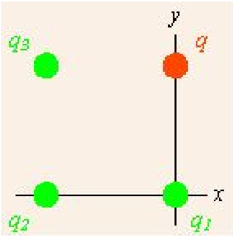
This vector notation can be used in the simple example of two point charges where only one of which is a source of charge.



Application of Coulomb's Law: In a simple example, the vector notation of Coulomb's Law can be used when there are two point charges and only one of which is a source charge.

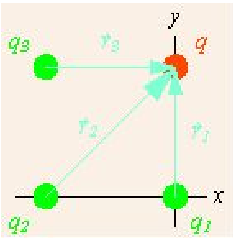
Electric Force on a Field Charge Due to Fixed Source Charges

Suppose there is more than one point source charges providing forces on a field charge. diagrams a fairly simple example with three source charges (shown in green and indexed by subscripts) and one field charge (in red, designated q). We assume that the source charges are fixed in space, and the field charge q is subject to forces from the source charges.



Multiple point charges: Coulomb's Law applied to more than one point source charges providing forces on a field charge.

Note the coordinate system that has been chosen. All of the charges lie on the corners of a square, and the origin is chosen to collocate with the lower right source charge, and aligned with the square. Since we can have only one origin of coordinates, no more than one of the source points can lie at the origin, and the displacements from different source points to the field point differ. The total force on the field charge q is due to applications of the force described in the vector notation of Coulomb's Law from each of the source charges. The total force is therefore the sum of these individual forces.



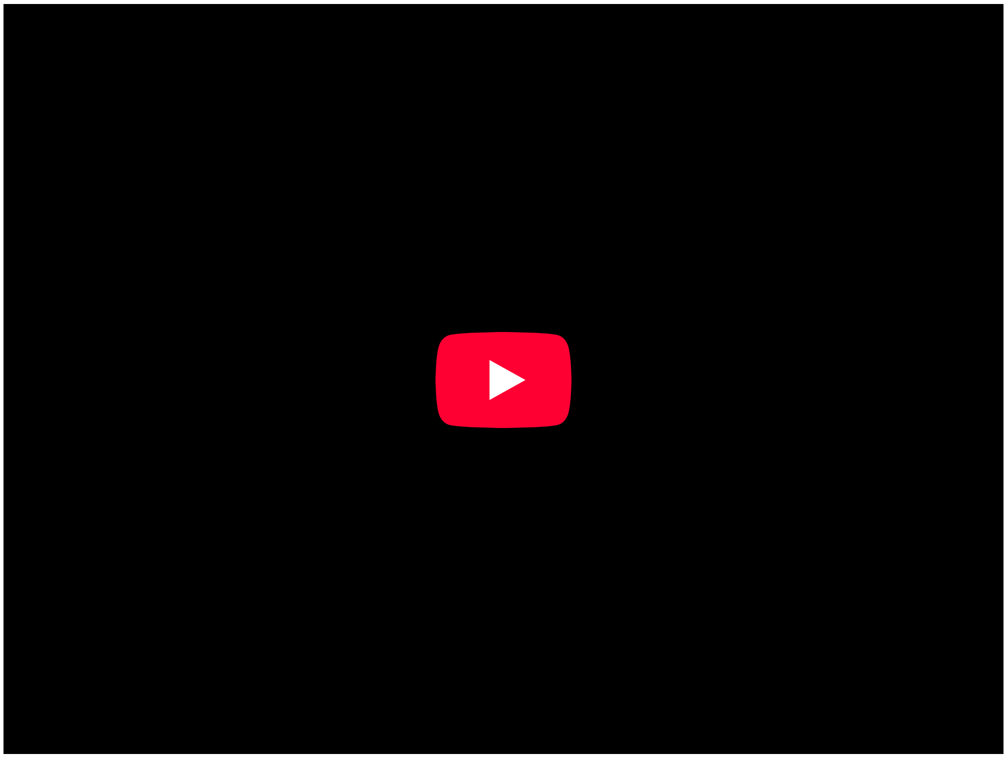
Displacements of field charge: The displacements of the field charge from each source charge are shown as light blue arrows.

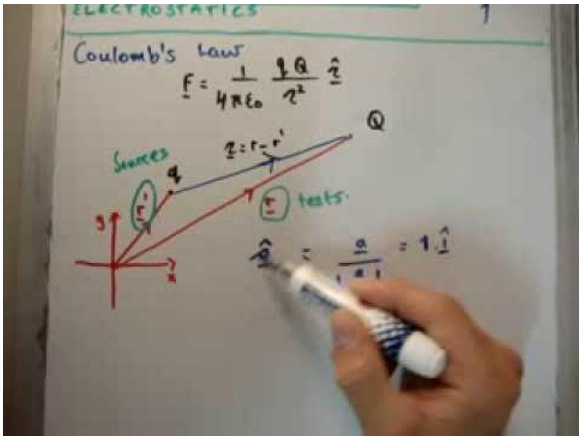
Applying Coulomb's Law three times and summing the results gives us:

$$\mathbf{F}_{\mathbf{E_q}}=\frac{\mathrm{kq}\cdot q_1\left(\mathbf{r}_q-\mathbf{r}_{q1}\right)}{\left|\mathbf{r}_{\mathrm{q}}-\mathbf{r}_{\mathrm{q}1}\right|^3}+\frac{\mathrm{kq}\cdot\mathbf{q}_2\left(\mathbf{r}_{\mathrm{q}}-\mathbf{r}_{\mathrm{q}2}\right)}{\left|\mathbf{r}_{\mathrm{q}}-\mathbf{r}_{\mathrm{q}2}\right|^3}+\frac{\mathrm{kq}\cdot\mathrm{q}_3\left(\mathbf{r}_{\mathrm{q}}-\mathbf{r}_{\mathrm{q}3}\right)}{\left|\mathbf{r}_{\mathrm{q}}-\mathbf{r}_{\mathrm{q}3}\right|^3} \tag{17.3.4}$$

This equation can further be simplified and applied to a fixed number of charge points.

$$\mathbf{F}_{\mathbf{n}}=\sum_{i\neq n}\frac{q_n q_i\left(\mathbf{r}_n-\mathbf{r}_i\right)}{4\pi\epsilon_0\left|\mathbf{r}_{\mathrm{n}}-\mathbf{r}_{\mathrm{i}}\right|^3} \tag{17.3.5}$$





Coulomb's Law: In this video I continue with my series of tutorial videos on Electrostatics. It's pitched at undergraduate level and while it is mainly aimed at physics majors, it should be useful to anybody taking a first course in electricity and magnetism such as engineers etc.

Key Points

- The superposition principle suggests that the net response at a given place and time caused by two or more stimuli is the sum of the responses which would have been caused by each stimulus individually.
- Total Coulomb force on a test charge due to a group of charges is equal to the vector sum of all the Coulomb forces between the test charge and other individual charges.
- The superposition of forces is not limited to Coulomb forces. It applies to any types (or combinations) of forces.

- The force between two objects is inversely proportional to the square of the distance between two objects.
- The attraction or repulsion forces within the spherical distribution of charge is stronger closer to the molecule and becomes weaker as the distance from the molecule increases.
- This law also accounts for the forces that bind atoms together to form molecules and for the forces that bind atoms and molecules together to form solids and liquids.
- The vector notation of Coulomb 's Law can be used in the simple example of two point charges where only one of which is a source of charge.
- The total force on the field charge for multiple point source charges is the sum of these individual forces.
- Coulomb's Law can be further simplified and applied to a fixed number of charge points.

Key Items

- **Lorentz force**: The force exerted on a charged particle in an electromagnetic field.
- **unit vector**: A vector with length 1.
- **electrostatic force**: The electrostatic interaction between electrically charged particles; the amount and direction of attraction or repulsion between two charged bodies.
- **coulomb's law**: the mathematical equation calculating the electrostatic force vector between two charged particles

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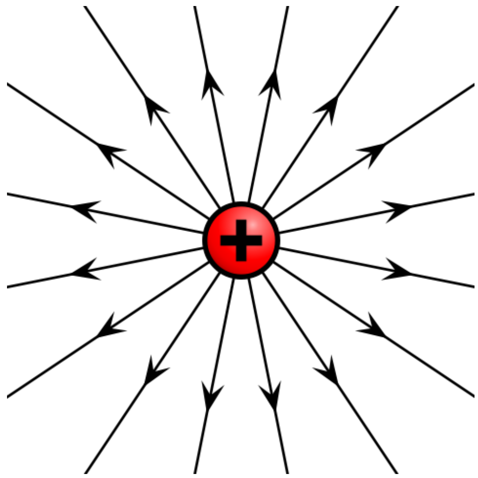
17.4: The Electric Field Revisited

learning objectives

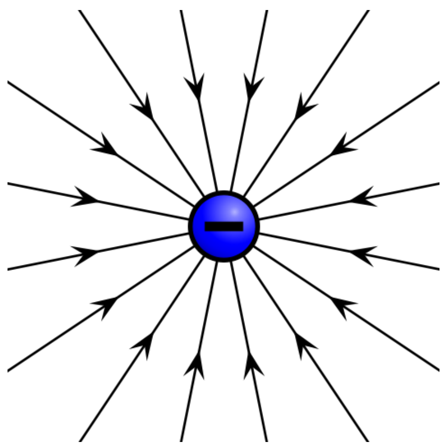
- Identify law that can be used to calculate an electric field created by a point charge

The electric field of a point charge is, like any electric field, a vector field that represents the effect that the point charge has on other charges around it. The effect is felt as a force, and when charged particles are not in motion, this force is known as the electrostatic force. The electrostatic force is, much like gravity, a force that acts at a distance. Therefore, we rationalize this action at a distance by saying that charges create fields around them that have effects on other charges.

Given a point charge, or a particle of infinitesimal size that contains a certain charge, electric field lines emanate radially in all directions. If the charge is positive, field lines point radially away from it; if the charge is negative, field lines point radially towards it.



Electric field of positive point charge: The electric field of a positively charged particle points radially away from the charge.



Electric field of negative point charge: The electric field of a negatively charged particle points radially toward the particle.

The reason for these directions can be seen in the derivation of the electric field of a point charge. Let's first take a look at the definition of the electric field of a point particle:

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The above equation is defined in radial coordinates, which can be seen in. The constant k is a result of simply combining the constants together, and q is the charge of the particle creating the electric field. This charge is either positive or negative. If the charge is positive, as shown above, the electric field will be pointing in a positive radial direction from the charge q (away from the charge).

As a demonstration of this phenomenon, if we now place another positive charge, *Q* (called the test charge), at some radial distance, *R*, away from the original particle, the test charge will feel a force given by

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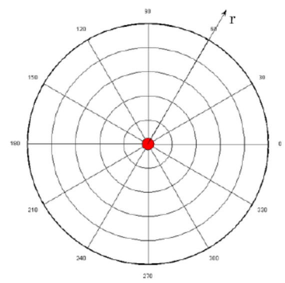

Radial Coordinate System: The electric field of a point charge is defined in radial coordinates. The positive r direction points away from the origin, and the negative r direction points toward the origin. The electric field of a point charge is symmetric with respect to the θ direction.

The thing to keep in mind is that the force above is acting on the test charge *Q*, in the positive radial direction as defined by the original charge *q*. This means that because the charges are both positive and will repel one another, the force on the test charge points away from the original charge.

If the test charge were negative, the force felt on that charge would be:

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Notice that this points in the negative ^rr^ direction, which is toward the original charge. This makes sense because opposite charges attract, and the force on the test charge will tend to push it toward the original positive charge creating the field. The above mathematical description of the electric field of a point charge is known as Coulomb ‘s law.

Superposition of Fields

The resultant of multiple electric fields acting on the same point is the sum of the strength of each field’s applied force at that point.

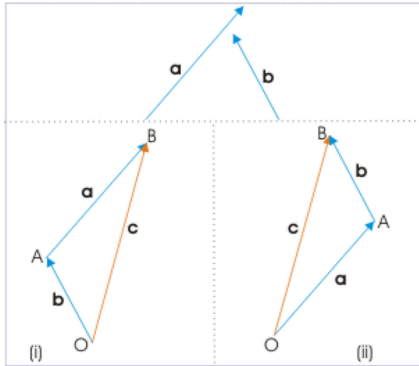
learning objectives

- Formulate the superposition principle for a linear system

As vector fields, electric fields obey the superposition principle. This principle states that for all linear systems, the net response to multiple stimuli at a given place and time is equal to the sum of the responses that would have resulted from each stimulus individually.

Possible stimuli include but are not limited to: numbers, functions, vectors, vector fields, and time-varying signals. It should be noted that the superposition principle is applicable to any linear system, including algebraic equations, linear differential equations, and systems of equations of the aforementioned forms.

For example, if forces A and B are constant and simultaneously act upon an object, illustrated as O in, the resultant force will be the sum of forces A and B. Vector addition is commutative, so whether adding A to B or B to A makes no difference on the resultant vector; this is also the case for subtraction of vectors.



Vector addition: Forces a and b act upon an object at point O. Their sum is commutative, and results in a resultant vector c.

Electric fields are continuous fields of vectors, so at a given point, one can find the forces that several fields will apply to a test charge and add them to find the resultant. To do this, first find the component vectors of force applied by each field in each of the orthogonal axes. This can be done using trigonometric functions. Then once the component vectors are found, add the components in each axis that are applied by the combined electric fields.

This is one only form of solution. An overall resultant vector can be found by using the Pythagorean theorem to find the resultant (the hypotenuse of the triangle created with applied forces as legs) and the angle with respect to a given axis by equating the inverse tangent of the angle to the ratio of the forces of the adjacent and opposite legs.

Electric Field Lines: Multiple Charges

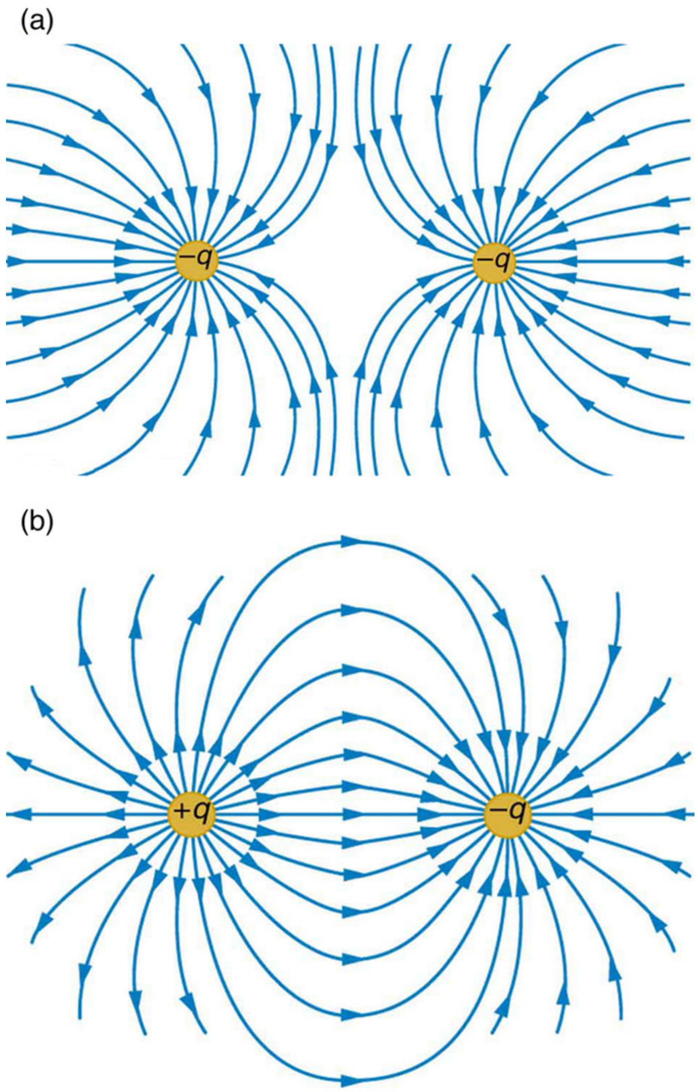
Electric fields created by multiple charges interact as do any other type of vector field; their forces can be summed.

learning objectives

- Calculate the resultant force of the multiple electric charges on a test charge

Thus far, we have looked at electric field lines pertaining to isolated point charges. But what if another charge is introduced? Each will have its own electric field, and the two fields will interact.

When modeling the electric fields of multiple charges, it’s important to take into consideration the sign and magnitude of each charge. Such models are not meant to be absolute, but must be self-consistent. For example, the number field of lines should be proportional to the value of the charge that gives rise to them. This means that if charges q_1 (with a +1 value) q_2 (+2 charge) and q_3 (+3 charge) are in the same field, one can connect 4, 8, and 12 field lines, respectively, to the charges. One could also choose to connect 3, 6, and 9 field lines, respectively, to q_1, q_2, and q_3; what matters is that the number of lines are related to the charge values by the same proportionality constant. Field lines should always point away from positive charges and towards negative charge.



Field lines between like and unlike charges: Example a shows how the electric field is weak between like charges (the concentration of field lines is low between them). Example b, by contrast, has a strong field between the charges, as exhibited by the high concentration of field lines connecting them.

If there are opposite charges in consideration, connect one to the other with field lines. If charges are the same, do not connect them in any way.

The strength of the electric field depends proportionally upon the separation of the field lines. More field lines per unit area perpendicular to the lines means a stronger field. It should also be noted that at any point, the direction of the electric field will be tangent to the field line.

Determining net force on a test charge

As vector fields, electric fields exhibit properties typical of vectors and thus can be added to one another at any point of interest. Thus, given charges q_1, q_2,… q_n, one can find their resultant force on a test charge at a certain point using vector addition: adding the component vectors in each direction and using the inverse tangent function to solve for the angle of the resultant relative to a given axis.

Parallel-Plate Capacitor

A parallel-plate capacitor is an electrical component used to store energy in an electric field between two charged, flat surfaces.

learning objectives

- Describe general structure of a capacitor

Overview

A capacitor is an electrical component used to store energy in an electric field. Capacitors can take many forms, but all involve two conductors separated by a dielectric material. For the purpose of this atom, we will focus on parallel-plate capacitors.

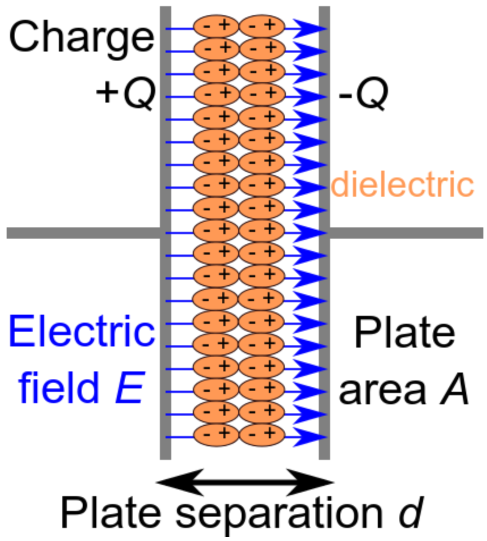


Diagram of a Parallel-Plate Capacitor: Charges in the dielectric material line up to oppose the charges of each plate of the capacitor. An electric field is created between the plates of the capacitor as charge builds on each plate.

Capacitance

All capacitors collect charge on the two, separate conductive surfaces; one side is positive and the other negative. An electric field is created as charge builds on the opposite surfaces, storing energy. The dielectric between the conductors is meant to act as an insulator, preventing charge from bridging the gap between the two plates. Such dielectrics are commonly composed of glass, air, paper, or empty space (a vacuum). In practice, dielectrics do not act as perfect insulators, and permit a small amount of leakage current to pass through them.

Capacitors are limited in their ability to prevent charge flow from one conductive surface to the other; their ability to hold charge is measured in Farads (F), which are defined as 1 ampere-second per volt, one joule per square volt and one Coulomb per volt, among other ways.

For a parallel-plate capacitor, capacitance (C) is related to dielectric permittivity (ε), surface area (A), and separation between the plates (d):

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Voltage (V) of a capacitor is related to distance between the plates, dielectric permittivity, conductor surface area, and charge (Q) on the plates:

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Depending on the dielectric strength (E_{ds}) and distance (d) between plates, a capacitor will "break" at a certain voltage (V_{bd}). This is calculated by:

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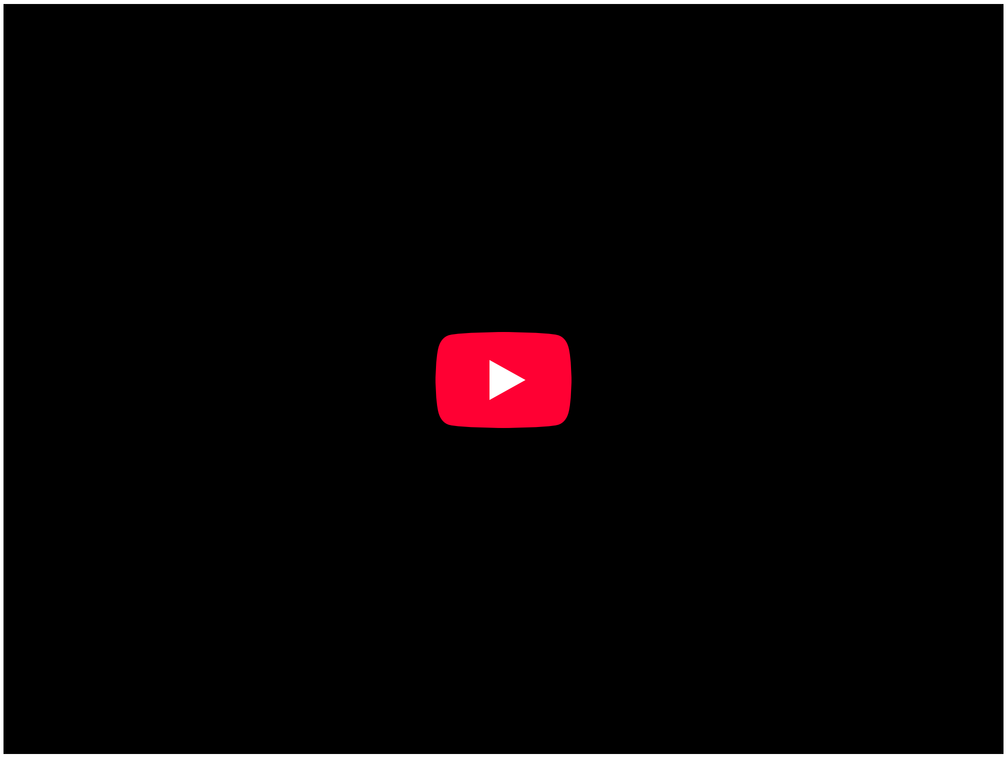
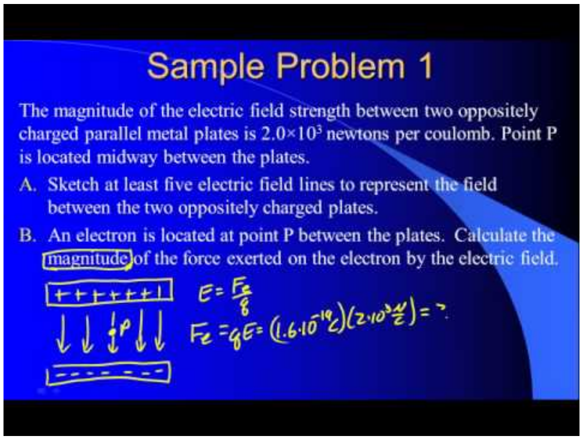




Parallel Plates and Equipotential Lines: A brief overview of parallel plates and equipotential lines from the viewpoint of electrostatics.

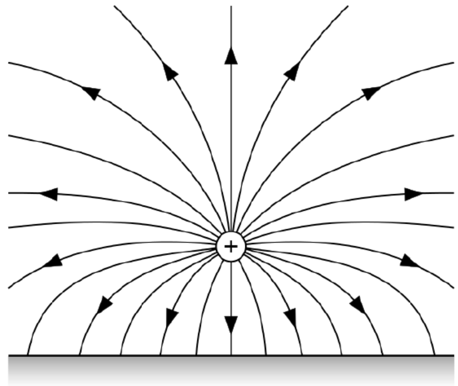
Electric Fields and Conductors

Electric fields in the presence of conductors have several unique and not necessarily intuitive properties.

learning objectives

- Describe unique properties expressed by electric fields in the presence of conductors

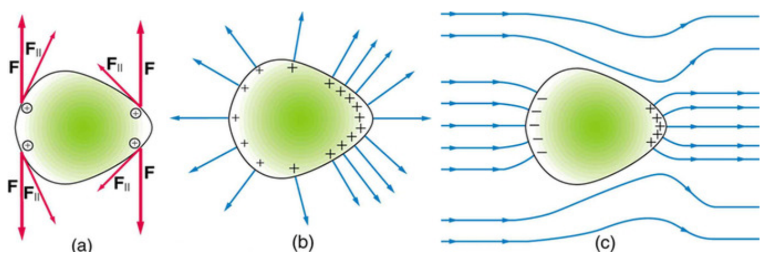
An electric field, like other fields (e.g., gravitational or magnetic), is a vector field that surrounds an object. Electric fields are found around electric charges and help determine the direction and magnitude of force the charge exerts on a nearby charged particle. It measures units of force exerted per unit of charge, and its SI units are N/C.



Field Lines Created by a Point Charge: Lines around the positive charge represent the electric field it creates.

Electrical conductors are materials in which internal charges can move freely. Therefore, they can facilitate the flow of charge, or current. When a conductor is placed in the presence of an electric field, it exhibits some interesting properties:

1. There is no electric field inside a charged conductor. A charged conductor at electrostatic equilibrium will contain charges only on its outer surface and will have no net electric field within itself. This is because all the charges in such a conductor will symmetrically oppose other charges within the conductor, causing the net result to sum to 0.
2. Charged surfaces align themselves perpendicularly relative to electric fields. Provided a conductor is at electrostatic equilibrium, the electric field upon the surface will be aligned perpendicularly with respect to that surface. If there were a nonzero parallel component of the electric field, with respect to any charge on the surface of a conductor, that charge would exert a force and would move. If the conductor is at equilibrium, such a force cannot exist, and therefore the direction of the electric field must be completely perpendicular to the surface.
3. Curvature on the surface of a conductor allows for increased charge concentration. Charge will not necessarily distribute itself evenly over the surface of a conductor. If the surface of the conductor is flat, charge will be very evenly distributed. But as the surface becomes more sharply curved, charge can be found more densely packed in areas, even if the conductor is at electrostatic equilibrium. Charges on a curved surface repel one another less strongly than they would on a smooth surface. This is because, based on how the charges are positioned, much of the repulsion the charges exert is in the direction away from the surface of the conductor, not along its surface. And it is harder for charges to be pushed off a surface than along it. Therefore, the repulsion between charges on a curved surface is weaker.



Electrical Charge at a Sharp Point of a Conductor: Repulsive forces towards the more sharply curved surface on the right aim more outward than along the surface of the conductor.

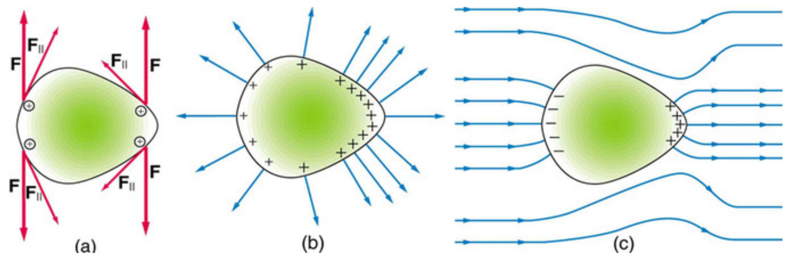
Conductors and Fields in Static Equilibrium

In the presence of charge or an electric field, the charges in a conductor will redistribute until they reach static equilibrium.

learning objectives

- Describe behavior of charges in a conductor in the presence of charge or an electric field and under static equilibrium

Conductors are materials in which charges can move freely. If conductors are exposed to charge or an electric field, their internal charges will rearrange rapidly. For example, if a neutral conductor comes into contact with a rod containing a negative charge, some of that negative charge will transfer to the conductor at the point of contact. But the charge will not stay local to the contact point — it will distribute itself evenly over the surface of the conductor. Once the charges are redistributed, the conductor is in a state of electrostatic equilibrium. It should be noted that the distribution of charges depends on the shape of the conductor and that static equilibrium may not necessarily involve an even distribution of charges, which tend to aggregate in higher concentrations around sharp points. This is explained in.



Electrical Charge at a Sharp Point of a Conductor: Forces between like charges at either end of the conductor are identical, but the components of the forces parallel to the surfaces are different. The component parallel to the surface is greatest on the flattest surface and therefore moves charges away from one another more freely. This explains the difference in concentration of charge on flat vs. pointed areas of a conductor.

Similarly, if a conductor is placed in an electric field, the charges within the conductor will move until the field is perpendicular to the surface of the conductor. Negative charges in the conductor will align themselves towards the positive end of the electric field, leaving positive charges at the negative end of the field. The conductor thus becomes polarized, with the electric field becoming stronger near the conductor but disintegrating inside it. This occurrence is similar to that observed in a Faraday cage, which is an enclosure made of a conducting material that shields the inside from an external electric charge or field or shields the outside from an internal electric charge or field.

Key Points

- The electric field is a vector field around a charged particle. It represents the force that other charged particles would feel if placed near the particle creating the electric field.
- Given a point charge, or a particle of infinitesimal size, that contains a certain charge, electric field lines emanate from equally in all radial directions.
- If the point charge is positive, field lines point away from it; if the charge is negative, field lines point toward it.
- The superposition principle states that for all linear systems, the net response to multiple stimuli at a given place and time is equal to the sum of the responses that would have been caused by each stimulus individually.
- Possible stimuli include but are not limited to numbers, functions, vectors, vector fields, and time-varying signals.
- The superposition principle is applicable to any linear system, including algebraic equations, linear differential equations, and systems of equations of the aforementioned forms.
- Electric fields are continuous fields of vectors, so at a given point, one can find the forces that several fields will apply to a test charge and add them to find the resultant.
- When multiple electric charges interact, their resultant force on a test charge can be calculated using vector addition.
- If there are opposite charges in consideration, connect one to the other with field lines. If charges are the same, do not connect them in any way.
- When modeling the electric fields of multiple charges, take into consideration the sign and magnitude of each charge. The number of field lines should be proportional to the value of the charge that gives rise to them.
- Capacitors can take many forms, but all involve two conductors separated by a dielectric material.
- All capacitors collect charge on the two, separate conductive surfaces; one side is positive and the other negative. An electric field is created as charge builds on the opposite surfaces, storing energy. The dielectric acts as an insulator, isolating the charged surfaces.
- The ability of capacitors to hold charge is measured in Farads (F). Capacitors will typically allow a small leakage of current through the dielectric, but after a certain voltage, the entire capacitor breaks down as the dielectric becomes a conductor.
- There is no electric field inside a charged conductor. This is because charges, which are located on the surface of the conductor, symmetrically oppose one another and sum to 0 in all locations.
- Charged surfaces align themselves perpendicularly relative to electric fields to achieve electrostatic equilibrium. If charges are not distributed as such, they will exert net force upon one another, which will move them. In such an instance, the charges will not be at static equilibrium.

- Curvature on the surface of a field allows for increased charge concentration. Much of the repulsion charges exert is in the direction moving away from the surface of the conductor, rather than along its surface. Charges thus push one another more weakly along the surface of a curved conductor.
- The presence of charge or an electric field will force charges in a conductor to redistribute along the conductor's surface until static equilibrium is achieved.
- At static equilibrium, charge will be more concentrated in sharp, pointy areas of conductors than elsewhere.
- At static equilibrium, the inside of a conductor will be entirely shielded from an external electric field.

Key Terms

- **coulomb's law**: the mathematical equation calculating the electrostatic force vector between two charged particles
- **vector field**: a construction in which each point in a Euclidean space is associated with a vector; a function whose range is a vector space
- **orthogonal**: Of two objects, at right angles; perpendicular to each other.
- **superposition principle**: The principle that a linear combination of two or more solutions of an equation is itself a solution; it is a feature of many physical laws.
- **vector**: A directed quantity, one with both magnitude and direction; the between two points.
- **capacitor**: An electronic component capable of storing an electric charge, especially one consisting of two conductors separated by a dielectric.
- **dielectric**: An electrically insulating or nonconducting material considered for its electric susceptibility (i.e., its property of polarization when exposed to an external electric field).
- **conductor**: A material which contains movable electric charges.
- **equilibrium**: The state of a body at rest or in uniform motion, the resultant of all forces on which is zero.
- **static equilibrium**: the physical state in which all components of a system are at rest and the net force is equal to zero throughout the system

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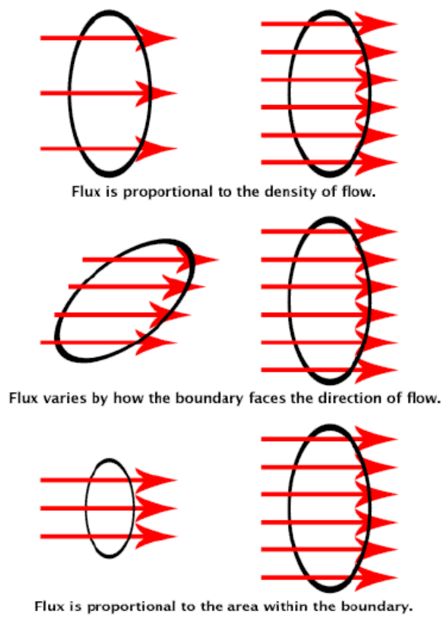
17.5: Electric Flux and Gauss's Law

learning objectives

- Express the electric flux for uniform and non-uniform electric fields

Electric flux is the rate of flow of the electric field through a given area (see). Electric flux is proportional to the number of electric field lines going through a virtual surface.



Electric Flux: Electric flux visualized. The ring shows the surface boundaries. The red arrows for the electric field lines.

If the electric field is uniform, the electric flux passing through a surface of vector area S is $\Phi_E = \mathbf{E} \cdot \mathbf{S} = ES\cos\theta$ where E is the magnitude of the electric field (having units of V/m), S is the area of the surface, and θ is the angle between the electric field lines and the normal (perpendicular) to S.

For a non-uniform electric field, the electric flux $d\Phi_E$ through a small surface area dS is given by $d\Phi_E = \mathbf{E} \cdot d\mathbf{S}$ (the electric field, E, multiplied by the component of area perpendicular to the field).

Gauss' Law describes the electric flux over a surface S as the surface integral: $\Phi_E = \iint_S \mathbf{E} \cdot d\mathbf{S}$ where E is the electric field and dS is a differential area on the closed surface S with an outward facing surface normal defining its direction.

It is important to note that while the electric flux is not affected by charges that are not within the closed surface, the net electric field, E, in the Gauss' Law equation, can be affected by charges that lie outside the closed surface. While Gauss' Law holds for all situations, it is only useful for "by hand" calculations when high degrees of symmetry exist in the electric field. Examples include spherical and cylindrical symmetry.

Electric flux has SI units of volt metres (V m), or, equivalently, newton metres squared per coulomb ($N\ m^2\ C^{-1}$). Thus, the SI base units of electric flux are $kg{\cdot}m^3{\cdot}s^{-3}{\cdot}A^{-1}$.

Gauss's Law

Gauss's law is a law relating the distribution of electric charge to the resulting electric field.

learning objectives

- Describe relationship between the Gauss's law and the Coulomb's law

Gauss's law, also known as Gauss's flux theorem, is a law relating the distribution of electric charge to the resulting electric field.

The law was formulated by Carl Friedrich Gauss (see) in 1835, but was not published until 1867. It is one of the four Maxwell's equations which form the basis of classical electrodynamics, the other three being Gauss's law for magnetism, Faraday's law of induction, and Ampère's law with Maxwell's correction.

Carl Friedrich Gauss: Carl Friedrich Gauss (1777–1855), painted by Christian Albrecht Jensen

Gauss's law can be used to derive Coulomb's law, and vice versa. Note that since Coulomb's law only applies to stationary charges, there is no reason to expect Gauss's law to hold for moving charges based on this derivation alone. In fact, Gauss's law does hold for moving charges, and in this respect Gauss's law is more general than Coulomb's law.

In words, Gauss's law states that: The net outward normal electric flux through any closed surface is proportional to the total electric charge enclosed within that closed surface.

The law can be expressed mathematically using vector calculus in integral form and differential form, both are equivalent since they are related by the divergence theorem, also called Gauss's theorem. Each of these forms in turn can also be expressed two ways: In terms of a relation between the electric field E and the total electric charge, or in terms of the electric displacement field D and the free electric charge.

Gauss's law has a close mathematical similarity with a number of laws in other areas of physics, such as Gauss's law for magnetism and Gauss's law for gravity. In fact, any "inverse-square law" can be formulated in a way similar to Gauss's law: For example, Gauss's law itself is essentially equivalent to the inverse-square Coulomb's law, and Gauss's law for gravity is essentially equivalent to the inverse-square Newton's law of gravity.

Key Points

- If the electric field is uniform, the electric flux passing through a surface of vector area S is $\Phi_{\mathrm{E}} = \mathbf{E} \cdot \mathbf{S} = \mathrm{ES}\cos\theta$.
- For a non-uniform electric field, the electric flux is.
- Electrical flux has SI units of volt metres (V m).
- Gauss's law is one of the four Maxwell's equations which form the basis of classical electrodynamics.
- Gauss's law can be used to derive Coulomb 's law, and vice versa.
- Gauss's law states that: The net outward normal electric flux through any closed surface is proportional to the total electric charge enclosed within that closed surface.

Key Terms

- **electric field**: A region of space around a charged particle, or between two voltages; it exerts a force on charged objects in its vicinity.
- **electric displacement field**: A vector field that appears in Maxwell's equations.
- **electric charge**: A quantum number that determines the electromagnetic interactions of some subatomic particles; by convention, the electron has an electric charge of -1 and the proton +1, and quarks have fractional charge.

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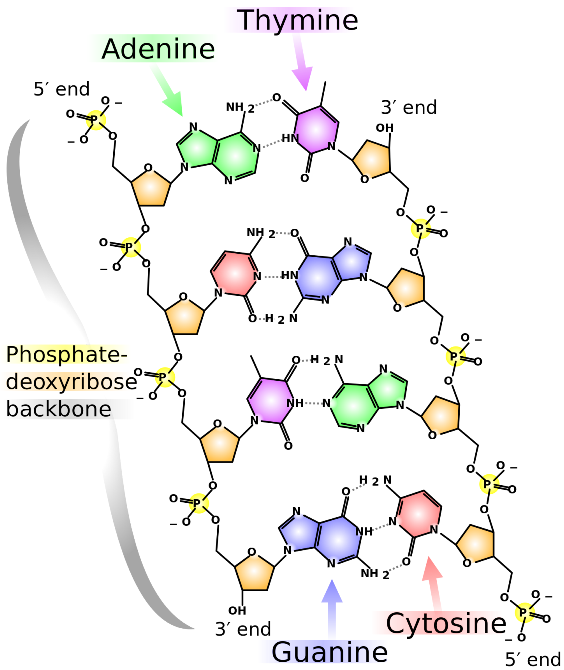
17.6: Applications of Electrostatics

learning objectives

- Describe effect of the DNA replication process on the hydrogen bonds

All living organisms contain either DNA or RNA. These molecules contain genetic information that defines (or at least influences) all of the organism’s characteristics. In both DNA and RNA, electrostatics plays a major role. For the purpose of this atom we focus specifically on DNA.

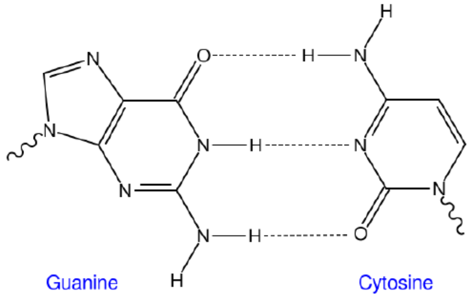
DNA is a large macromolecule that (in three-dimensional space) forms the shape of a double helix, as shown in. The “backbone” of each helix is formed from alternating deoxyribose and phosphate subunits as illustrated in. Each deoxyribose is connected to a base that extends between the two backbones. Four bases can be found in DNA: adenine, guanine, cytosine and thymine. Their ordering in sequence determines the genetic characteristics of the parent organism.



Phosphate in DNA: The complex molecules that make up our DNA are held together by a phosphate-deoxyribose backbone, as shown.

The bases of each strand of DNA are positioned close to one another but not as a result of formal chemical bonds. Electrostatic interactions, in this case otherwise known as hydrogen bonds, are what hold these bonds together.

In, (in the instance of guanine and cytosine) there are three instances in which hydrogen atoms are attracted to nearby nitrogen and oxygen atoms (denoted by dashed lines). For adenine and thymine, there are two such hydrogen bonds. These interactions can be modeled as electrostatic interactions—electrons from the oxygen and nitrogen atoms are negative, and are attracted to the exposed nuclei (which are positive) of the nearby hydrogen atoms. These hydrogen bonds are relatively weak, but their sum is strong enough to hold the two DNA strands together.



Hydrogen bonding between Guanine and Cytosine: In the instance of guanine and cytosine, there are three instances in which hydrogen atoms are attracted to nearby nitrogen and oxygen atoms (denoted by dashed lines).

When DNA replicates, as it does in cellular reproduction, it is first "unzipped" by any of a number of enzymes known as DNA Helicases. These enzymes break the electrostatic hydrogen bonds between the two strands.

Photocopy Machines and Printers

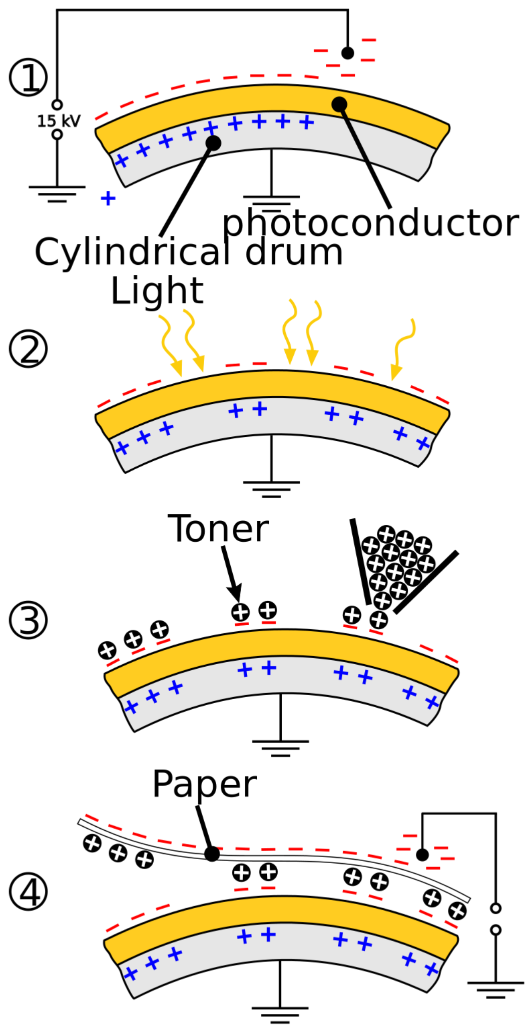
Photocopiers use xerography, a process that uses principles of electrostatics, to copy images.

learning objectives

- Describe functions of a photocopier and distinguish the three steps of xerography

A photocopier is a machine that prints copies of documents and images on paper. Currently, most photocopiers use a process called xerography, which is dry and uses heat in printing.

Different models of photocopiers vary somewhat in their methods, but the first instance of xerography involved use of a photosensor with a copy camera and a separate processing unit. To this day, xerography is conducted in a five-step process, in which electrostatics play an important role. This process is diagrammed in with explanations of each step outlined in the text below.



How a photocopier works: This image describes how a photocopier works.

Charging

In the first step of xerography, a high-voltage device (either a corona wire or charge roller) charges a cylindrical drum. This occurs by corona discharge, with output limited by a control grid or screen: a negative charge on the wire ionizes the space between the wire and conductor, so electrons are repelled and pushed onto the conductor. The conductor is put on top of a conducting surface and is kept at ground potential.

Polarity of charge can be altered depending on desired effects. A positive process is used to produce black on white copies, while a negative process is used to produce black on white from negative originals.

Exposure

A lamp illuminates the original document. The white areas of the original reflect light onto the surface of the drum, which exhibits photoconductivity (it becomes a conductor in the presence of light). The areas of the drum exposed to light then discharge to the ground; the other parts of the drum (which are not conductive, having not been exposed to light) remain negatively charged. Thus, the result is an electric image on the surface of the drum.

Developing

The toner, a powder used to form images on the paper, is positively charged. When it is placed on the drum, it is attracted to the negative (black) areas. This develops the image.

Transfer

The image with toner on the drum is transferred to a piece of paper with more negative charge than the drum.

Fusing

The toner is melted and bonded to the paper by heat and pressure rollers.

It should be noted that the explanation above is typical for modern copiers, but some older copiers use a positive drum and paper, and a negatively charged toner.

Van de Graff Generators

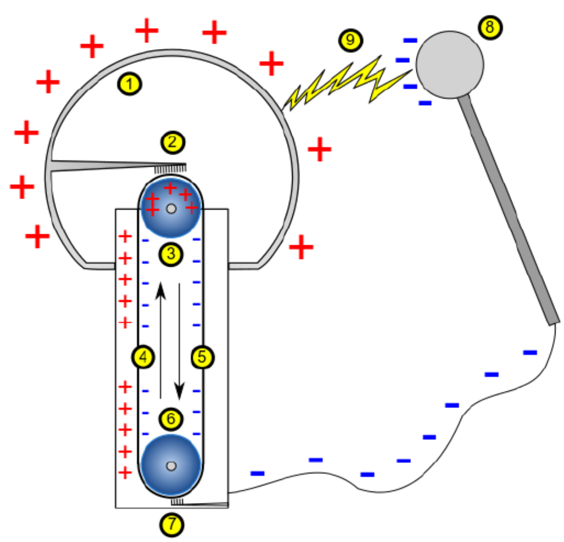
A Van de Graaff generator is a device that can be used to separate charges and create potential differences in the range of megavolts.

learning objectives

- Identify and describe functions of the major parts of a Van de Graaff generator

A Van de Graaff generator is a device used to separate electric charges. Using a moving belt, it can create extremely high potential differences. In some cases these can reach a value in excess of five megavolts.

Fundamentally, a Van de Graaff generator consists of a flexible dielectric belt (silk is commonly used) running over two metal pulleys. One of these pulleys is surrounded by a hollow metal sphere. Two electrodes are positioned near the bottom of the lower pulley and inside the sphere, over the upper pulley. One comb is connected to the sphere, and another is connected to the ground. In this figure, a high, positive DC potential is applied to the upper roller.



Schematic of a Van de Graaff Generator: Numbers in the diagram indicate: 1) hollow metal sphere; 2) upper electrode; 3) upper roller (for example an acrylic glass); 4) side of the belt with positive charges; 5) opposite side of the belt with negative charges; 6) lower roller (metal); 7) lower electrode (ground); 8) spherical device with negative charges, used to discharge the main sphere; 9) spark produced by the difference of potentials

The separation of charge in a Van de Graaff generator is a complex, multistep process. Due to the electric field around the lower pulley, the belt receives negative charge as it passes the lower comb. When the belt comes into contact with the upper roller, it leaves the roller with a negative charge as electrons are transferred. Electrons seep from the belt to the upper comb and then the terminal, leaving the belt positively charged and the terminal negatively charged. The sphere acts as a Faraday shield, shielding the upper roller and comb from the electric field produced by charges on the outside of the sphere. Eventually, discharge occurs, and the belt changes polarity. As the belt continues, a charging current travels via the belt, and the sphere takes on more and more negative charge until the rates at which it gains and loses charge (which occurs by leakage and corona discharge) are equal. Final potential is proportional to the size of the sphere and its distance from the ground.

Key Points

- In DNA, hydrogen bonds between base pairs hold two opposite strands together.
- The hydrogen bonds that hold DNA together are electrostatic interactions between the nucleus of a hydrogen atom and the electrons of an oxygen or a nitrogen atom.
- When DNA replicates, a helicase enzyme "unzips" the double helix, breaking the hydrogen bonds that hold it together in the center.
- In the first step of xerography, a high-voltage device (either a corona wire or charge roller) charges a cylindrical drum.
- In the second step, a lamp illuminates the original document. The white areas of the original reflect light onto the surface of the drum, which is photoconductive. The areas of the drum exposed to light then discharge to the ground.
- Next, the toner, a positively-charged powder used to form images on the paper, is placed on the drum. It is attracted to the negative (black) areas. This develops the image.
- The image with toner on the drum is then transferred to a piece of paper with more negative charge than the drum.
- Finally, the toner is melted and bonded to the paper by heat and pressure rollers.
- Fundamentally, a Van de Graaff generator consists of a flexible dielectric belt (silk is commonly used) running over two metal pulleys.
- The belt is used to transport charge, isolating positive charge from negative charge.
- The maximum potential difference a Van de Graaff generator can create is achieved when the rates at which the sphere (which holds charge) gains and loses charge are equal. Charge can be lost by leakage and corona discharge.

Key Terms

- **base**: A nucleotide's nucleobase in the context of a DNA or RNA biopolymer.
- **photoconductivity**: An increase in the electrical conductivity of a material as a result of incident electromagnetic radiation
- **xerography**: a photocopying process in which a negative image formed on an electrically charged plate is transferred as a positive to paper and thermally fixed
- **Faraday shield**: A Faraday cage or Faraday shield is an enclosure formed by conducting material or by a mesh of such material. Such an enclosure blocks external static and non-static electric fields.
- **polarity**: the separation, alignment or orientation of something into two opposed poles
- **corona discharge**: an electrical discharge brought on by the ionization of a fluid surrounding a conductor that is electrically energized

CHAPTER OVERVIEW

18: Electric Potential and Electric Field

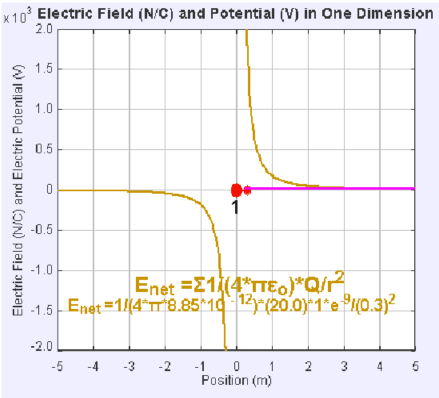
Topic hierarchy

18.1: Overview

learning objectives

- Explain the relationship between the electric potential and the electric field

The relationship between electric potential and field is similar to that between gravitational potential and field in that the potential is a property of the field describing the action of the field upon an object (see).



Electric field and potential in one dimension: The presence of an electric field around the static point charge (large red dot) creates a potential difference, causing the test charge (small red dot) to experience a force and move.

The electric field is like any other vector field—it exerts a force based on a stimulus, and has units of force times inverse stimulus. In the case of an electric field the stimulus is charge, and thus the units are NC^{-1}. In other words, the electric field is a measure of force per unit charge.

The electric potential at a point is the quotient of the potential energy of any charged particle at that location divided by the charge of that particle. Its units are JC^{-1}. Thus, the electric potential is a measure of energy per unit charge.

In terms of units, electric potential and charge are closely related. They share a common factor of inverse Coulombs (C^{-1}), while force and energy only differ by a factor of distance (energy is the product of force times distance).

Thus, for a uniform field, the relationship between electric field (E), potential difference between points A and B (Δ), and distance between points A and B (d) is:

$$\mathrm{E} = -\frac{\Delta\phi}{\mathrm{d}} \tag{18.1.1}$$

The -1 coefficient arises from repulsion of positive charges: a positive charge will be pushed away from the positively charged plate, and towards a location of higher-voltage.

The above equation is an algebraic relationship for a uniform field. In a more pure sense, without assuming field uniformity, electric field is the gradient of the electric potential in the direction of x:

$$\mathrm{E_x} = -\frac{\mathrm{dV}}{\mathrm{dx}} \tag{18.1.2}$$

This can be derived from basic principles. Given that ΔP=W (change in the energy of a charge equals work done on that charge), an application of the law of conservation of energy, we can replace ΔP and W with other terms. ΔP can be substituted for its definition as the product of charge (q) and the differential of potential (dV). We can then replace W with its definition as the product of q, electric field (E), and the differential of distance in the x direction (dx):

$$\mathrm{qdV} = -q\mathrm{E_{xdx}} \tag{18.1.3}$$

Dividing both sides of the equation by q yields the previous equation.

Electric Potential Energy and Potential Difference

Electric potential energy results from forces between charges; potential difference is the energy needed to move a charge from point A to B.

learning objectives

- Calculate the potential energy between the charges

Electric potential energy is a type of potential energy that results from Coulomb forces. It is measured in joules and depends on the positioning of charged particles relative to one another, as well as the magnitude of their respective charges.

The potential energy ($\mathrm{U_E}$) between charges q and Q can be calculated as a function of distance between the charges (r):

$$\mathrm{U_E(r)=\frac{q\mathit{Q}}{4\pi\epsilon_0 r}} \tag{18.1.4}$$

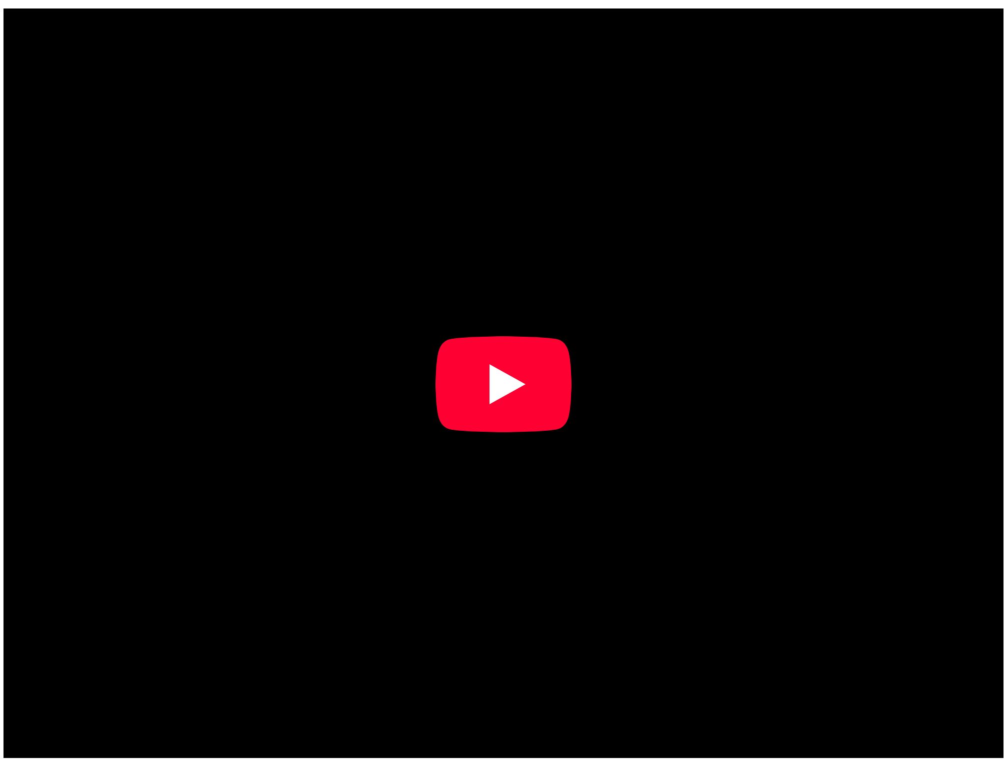
If there are three or more charges, the above formula can be modified so that the potential energies between all charges are summed. Consider, for example, the case involving charges $\mathrm{Q_1}$, $\mathrm{Q_2}$ and $\mathrm{Q_3}$:

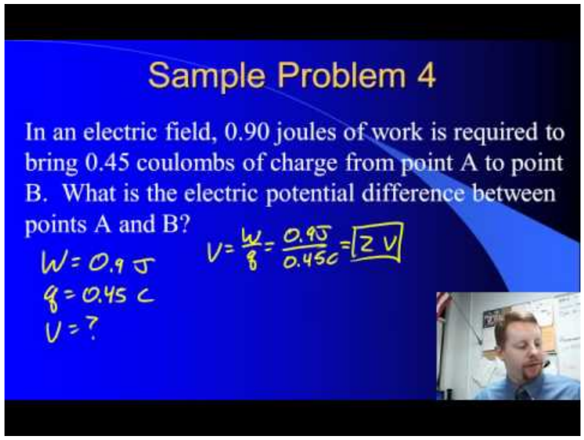
$$\mathrm{U_E=\frac{1}{4\pi\epsilon_{0r}}\left(\frac{Q_1Q_2}{r12}+\frac{Q_2Q_3}{r23}+\frac{Q_1Q_3}{r13}\right)} \tag{18.1.5}$$

In this example, $\mathrm{r_{12}}$ represents the distance between $\mathrm{Q_1}$ and $\mathrm{Q_2}$, $\mathrm{r_{23}}$ represents the distance between $\mathrm{Q_2}$ and $\mathrm{Q_3}$, and $\mathrm{r_{13}}$represents the distance between $\mathrm{Q_1}$ and $\mathrm{Q_3}$. The above formula can be modified for any number of charges.

Potential Difference

Potential difference, or voltage, is the difference in electric potential energy between two points. It is denoted by ΔV and has units of volts, or joules per Coulomb.

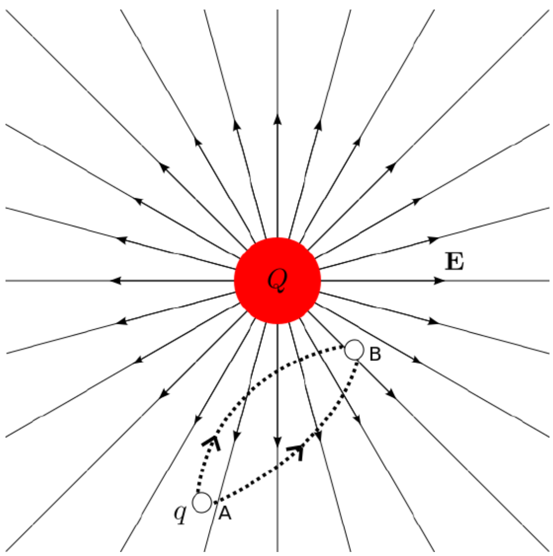




Electric Potential Difference: A brief overview of electric potential difference and electric potential energy for beginning physics students.

Voltage denotes the work per unit charge that must be done against a static electric field to move a charge from one point to another. It may represent a source of energy, or lost, stored or used energy. Voltage also is defined such that negative charges are pulled towards higher voltages, while positive charges move towards lower voltages. Thus, current in wires flows from higher to lower voltages.

Potential difference is independent of path taken from one point to another, and may be measured by any of a number of instruments. These include the voltmeter, the potentiometer, and the oscilloscope. It is most typically measured in circuits, and in such situations can be calculated using Ohm ‘s Law, which will be covered in a later atom.



Potential difference in a static field: When a charge q moves from point A to point B, the potential difference is independent of path taken.

Electric Field and Changing Electric Potential

Electric field is the gradient of potential, which depends inversely upon distance of a given point of interest from a charge.

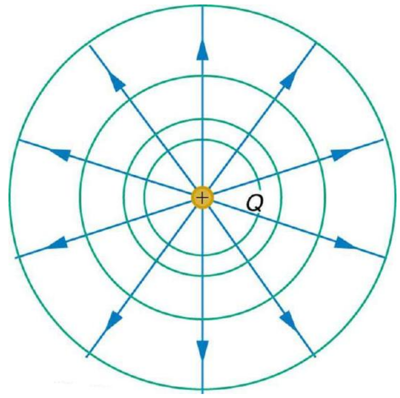
learning objectives

- Calculate the electric potential created by a charge distribution of constant value

Any charge will create a vector field around itself (known as an electric field). Electric field is the gradient of potential, which depends inversely upon distance of a given point of interest from a charge. Placing a second charge in the system (a “test charge”) results in the two charges experiencing a force (the field’s units are Newtons, a measure of force per Coulomb), causing the charges to move relative to one another. It is easiest to model interactions between two charges such that one is considered stationary while the test charge moves.

As the test charge moves, the potential between it and another charge changes, as does the electric field. The relationship between potential and field (E) is a differential: electric field is the gradient of potential (V) in the x direction. This can be represented as:

$$E_x = -\frac{dV}{dx} \tag{18.1.6}$$



Equipotential Lines: An isolated point charge Q with its electric field lines (blue) and equipotential lines (green)

Thus, as the test charge is moved in the x direction, the rate of the its change in potential is the value of the electric field.

The instant before the test charge moves, its potential energy is at a maximum, and its kinetic energy is 0. For any charge of constant value (Q), the potential at a certain distance from it (r) can be calculated from the following equation:

$$V_E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r} \tag{18.1.7}$$

where ε_0 is the electric constant, otherwise known as permittivity of free space. Moving towards and away from the charge results in change of potential; the relationship between distance and potential is inverse.

For one point charge, potential will be constant for all points a certain radial distance away. Multiple points of the same potential are known as equipotential. In the case of fields created by a single point charge, all points on any circle centered around the point charge will be equipotential, as illustrated in.

shows that when multiple charges create a field, the equipotential lines become irregularly shaped. This is due to the fact that the fields created by each charge overlap, thus potential is increased at any point relative to that which would have arisen from one or the other charge.

Potentials and Charged Conductors

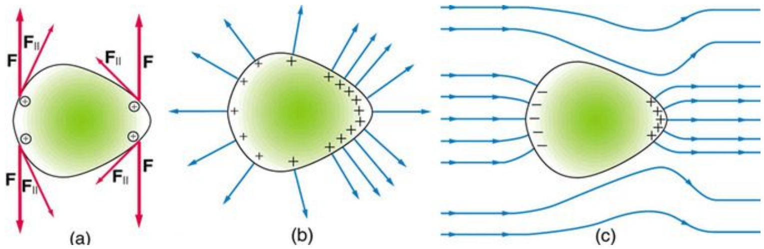
Electric potential within a charged conductor is equal to zero, but can be calculated as a nonzero value outside of a charged conductor.

learning objectives

- Determine the electric potential within and outside a charged conductor

When a conductor becomes charged, that charge distributes across its surface until electrostatic equilibrium is reached. Its surface is equipotential.

All points within a charged conductor experience an electric field of 0. This is because field lines from charges on the surface of the conductor oppose one another equally. However, having the electric field equal to zero at all points within a conductor, the electric potential within a conductor is not necessarily equal to zero for all points within that same conductor. This can be proven by relating electric field and potential.



Electrical Charge at a Sharp Point of a Conductor: Repulsive forces towards the more sharply curved surface on the right aim more outward than along the surface of the conductor.

Given that work is the difference in final and initial potential energies (ΔU), we can relate this difference to the dot product of force at every infinitesimal distance l along the path between the points within the conductor:

\Delta \mathrm { U } = - \int _ { \mathrm { i } } ^ { f } \vec { \mathrm { F } } \cdot \mathrm { d } \vec { }

This is the equation for work, with ΔU substituted in place of W. Rewriting U as the product of charge (q) and potential difference (V), and force as the product of charge and electric field (E), we can assert:

$$\Delta(\mathrm{qV}) = -\int_{\mathrm{i}}^{\mathrm{f}} (\mathrm{qE}) \cdot \mathrm{d}\vec{\mathrm{l}} \tag{18.1.8}$$

Dividing both sides by the common term of q, we simplify the equation to:

$$\Delta \mathrm{V} = -\int_{\mathrm{i}}^{\mathrm{f}} \vec{\mathrm{E}} \cdot \mathrm{d}\vec{\mathrm{l}} \tag{18.1.9}$$

Finally we derive the equation:

$$\mathrm{dV} = -\vec{\mathrm{E}} \cdot \mathrm{d}\vec{\mathrm{l}} = 0 \tag{18.1.10}$$

Thus we can conclude that, given that the electric field is constantly 0 for any location within the charged conductor, the *potential difference* in that same volume needs to be constant and equal to 0.

On the other hand, for points outside a conductor, potential is nonzero and can be defined by the very same equation, according to field and distance from the conductor.

Uniform Electric Field

An electric field that is uniform is one that reaches the unattainable consistency of being constant throughout.

learning objectives

- Describe properties and approximations of the uniform electric field

A uniform field is that in which the electric field is constant throughout. Just like the so-called "frictionless surface" in mechanics, the uniform field is an ideal but unreal situation that makes for simpler calculations. Equations involving non-uniform electric fields require use of differential calculus.

Uniformity in an electric field can be approximated by placing two conducting plates parallel to one another and creating a potential difference between them. In such a case there will be slight variations in the field near its edges, but it will be approximately constant throughout every other area.

The equation for magnitude of a uniform electric field is:

$$\mathrm{E} = \frac{-\Delta\phi}{\mathrm{d}} \tag{18.1.11}$$

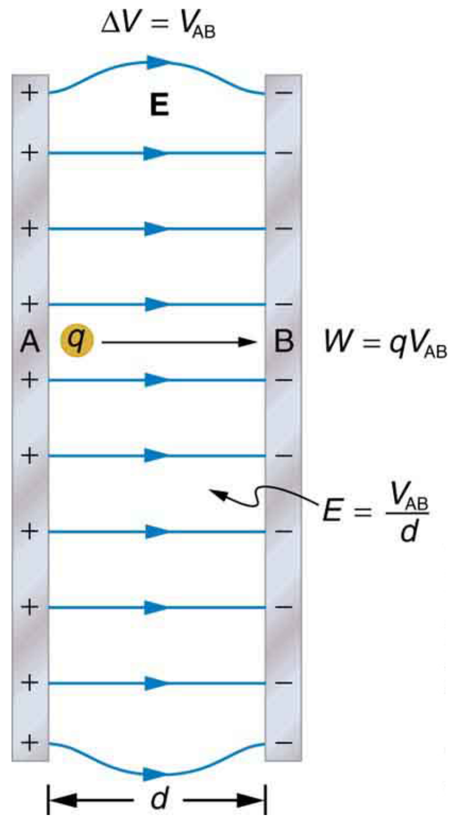
where E is the field, Δ is the potential difference between the plates, and d is the distance between the plates. The coefficient of -1 arises from the fact that positive charges repel, and thus a positive charge will be pushed away from the positive plate and in a direction opposite that of the increasing voltage.

Uniformity of an electric field allows for simple calculation of work performed when a test charge is moved across it. For the case of a positive charge q to be moved from a point A with a certain potential (V_1) to a point B with another potential (V_2), that equation is:

$$W = -q\left(V_2 - V_1\right) \tag{18.1.12}$$

The difference (V_2-V_1) can also be represented as ΔV or V_{AB}. In uniform fields it is also simple to relate ΔV to field strength and distance (d) between points A and B:

$$\mathbf{V}_{AB} = Ed \tag{18.1.13}$$



Relationships within a uniform electric field: In this image, Work (W), field strength (E), and potential difference (ΔV) are defined for points A and B within the constructs of a uniform potential field between the positive and negative plates.

Energy Conservation

Energy is conserved in the movement of a charged particle through an electric field, as it is in every other physical situation.

learning objectives

- Formulate energy conservation principle for a charged particle in an electric field

Energy is conserved in the movement of a charged particle through an electric field, as it is in every other physical situation. This phenomenon can be expressed as the equality of summed kinetic (E_{kin}) and electric potential (E_{el}) energies:

$$\left(E_{kin} + E_{el}\right)_{\text{initial}} = \left(E_{kin} + E_{el}\right)_{\text{final}} \tag{18.1.14}$$

Given a stationary test charge in a certain location, an applied electric field will cause the charge to move to one end or the other, depending on the charge (positive test charges will move in the direction of the field; negative charges will move in the opposite direction). In all cases, a charge will naturally move from an area of higher potential energy to an area of lower potential energy.

At the instant at which the field is applied, the motionless test charge has 0 kinetic energy, and its electric potential energy is at a maximum. After that moment, the charge accelerates, and its kinetic energy (from motion) increases as its potential energy decreases. Throughout this time, the sum of potential and kinetic energies remains constant.

Another way to express the previous equation is:

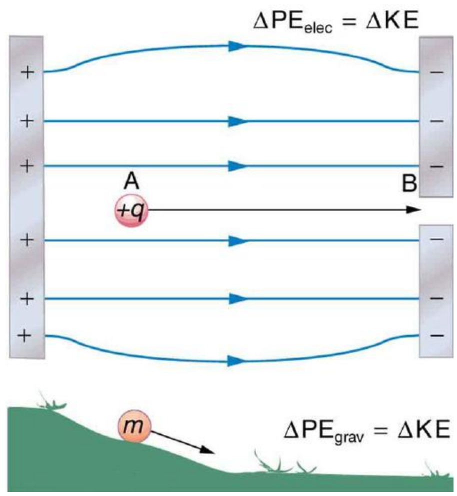
$$(\frac{1}{2}mv^2+U)_{\text{initial}}=(\frac{1}{2}mv^2+U)_{\text{final}} \tag{18.1.15}$$

where *m* and *v* are the mass and velocity of the electron, respectively, and *U* is the electric potential energy. *U* can be calculated as follows:

$$U=q_0V=k\frac{q_0q}{r} \tag{18.1.16}$$

where *V* is the potential difference, *k* is a constant, q_0 is a test charge, *q* is another charge, and *r* is the distance between the charges.

The terms involved in the formula for conservation of energy can be rewritten in many ways, but all expressions are based on the simple premise of equating the initial and final sums of kinetic and potential energy.



Similarities Between Activity of Gravitational and Electric Fields on an Object: The charge, +q, is moved down the electric field in the same way that the object, m, is moved down the hill. In both instances, the particle in motion goes from a higher to a lower potential energy state.

The Electron-Volt

The electron volt is a unit of energy useful in the physics of elementary charges and electricity.

learning objectives

- Convert between electron volts and SI units of energy

Overview

The electron volt, symbolized as eV and sometimes written as electronvolt, is a unit of energy useful in the physics of elementary charges and electricity.

The electron volt is defined as the amount of energy gained or lost by the charge of an electron moved across a one-volt electric potential difference. As such, it is equal to the product of one volt (1 J/C) and one elementary charge, giving it a value in joules approximately equal to 1.602×10^{-19} J.

Not an SI unit in itself, the electron volt became useful through experimentation. Scientists working with electrostatic particle accelerators commonly used the relationship between energy (E), charge (q), and potential difference (V) in their work:

$$E=qV \tag{18.1.17}$$

All calculations of energy from the above equation were quantized as multiples of the elementary charge, q, for a given voltage, and thus arose the common usage of the electron volt as a unit of measurement.

Momentum

Both electron volts and momentum are measures of energy, and the two are related in high-energy physics. Applying a potential difference to an electron gives it energy, which manifests itself in motion of the electron through it. Given that the electron has both mass and velocity, it has momentum. Dividing electron volts by a constant with units of velocity results in a momentum.

Mass

Given that mass is equivalent to energy, the electron volt can measure mass. In particle physics, the equation $E=mc^2$ can be rearranged to solve for mass:

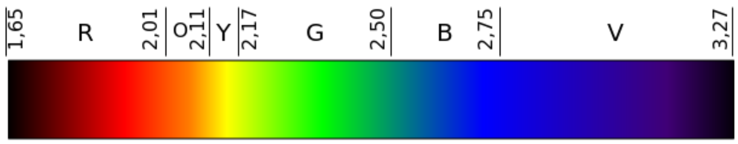
$$m=\frac{E}{c^2} \tag{18.1.18}$$

Wavelength

The energy *E*, frequency *v*, and wavelength λ of a photon are related by

$$E(eV)=hv=\frac{hc}{\lambda} \tag{18.1.19}$$

where h is the Planck constant and c is the speed of light. Thus, a photon with a wavelength of 532 nm (green light) would have an energy of approximately 2.33 eV. Similarly, 1 eV would correspond to an infrared photon of wavelength 1240 nm, and so on.



Energy of Photons in the Visible Spectrum: Relationship between wavelength and energy expressed in electron volts.

Temperature

In plasma physics, the electron volt can be used as a unit of temperature. To convert to Kelvins, simply divide the value of 1 eV (in Joules) by the Boltzmann constant ($1.3806505(24)\times10^{-23}$ J/K).

Dipole Moments

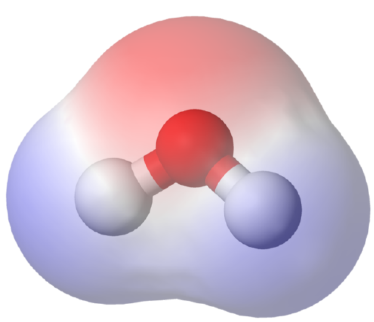
The electric dipole moment is a measure of polarity in a system.

learning objectives

- Relate the electric dipole moment to the polarity in a system

The electric dipole moment is a measure of polarity, which is the separation of positive and negative charges in a system. It is measured in units of Coulomb-meters (C m). There are many different types of dipole moments, including electric dipole moments, magnetic dipole moments, and topological dipole moments.

Among the subset of electric dipole moments are transition dipole moments, molecular dipole moments, bond dipole moments, and electron electric dipole moments. For the purposes of this atom we will focus on a broad overview of electric dipole moment in static situations.



Molecular Dipole Moment in Water: This water (H_2O) molecule has a high density of electrons (denoted by the red shading) near the red O atom. Closer to the white H atoms, there is a low density of electrons. Therefore, the molecule is is a dipole, with negativity near the O and positivity closer to the H atoms.

Definition

Fundamentally, for the case of point charges with values +q and -q, an electric dipole moment (p) can be defined as the vector product of the charges and the displacement vector d:

$$\mathrm{p} = \mathrm{qd} \tag{18.1.20}$$

The displacement vector is the vector with a magnitude equal to the distance between the charges and a direction pointing from the negative charge to the positive charge. It is essentially interchangeable with the "radius" variable in many other equations (such as those determining gravitational and electrostatic forces), except that it includes the factor of direction.

Torque

All dipoles will experience a torsional force, or torque, when they are placed in external electric fields. This torque rotates the dipole to align it with the field. It is brought on by the need to minimize potential energy. Torque (τ) can be calculated as the cross product of the electric dipole moment and the electric field (E), assuming that E is spatially uniform:

$$\tau = \mathrm{p} \times \mathrm{E} \tag{18.1.21}$$

Key Points

- The electric field is a measure of force per unit charge; the electric potential is a measure of energy per unit charge.
- For a uniform field, the relationship between electric field (E), potential difference between points A and B (Δ), and distance between points A and B (d) is: $\mathrm{E} = -\frac{\Delta\phi}{\mathrm{d}}$ If the field is not uniform, calculus is required to solve.
- Potential is a property of the field that describes the action of the field upon an object.
- Electric potential energy is a type of potential energy that results from Coulomb forces. The potential energy (UE) between charges q and Q can be calculated as a function of distance between the charges (r): $\mathrm{U_E(r)} = \frac{\mathrm{q}Q}{4\pi\epsilon_0 \mathrm{r}}$.
- The formula for potential energy can be modified for potential between many charges, so long as the interactions of each charge with every other charge in the system are considered. For example, potential between three charges can be solved using the following formula: $\mathrm{U_E} = \frac{1}{4\pi\epsilon_0 \mathrm{r}}\left(\frac{Q_1Q_2}{\mathrm{r12}} + \frac{\mathrm{Q_2Q_3}}{\mathrm{r23}} + \frac{\mathrm{Q_1}Q_3}{\mathrm{r13}}\right)$.
- Potential difference, or voltage, is the difference in electric potential energy between two points. It is denoted by ΔV and has units of volts, or joules per Coulomb.
- For any charge of constant value (Q), the potential (VE) at a certain distance from it (r) can be calculated from the equation: $\mathrm{V_E} = \frac{1}{4\pi\epsilon_0}\frac{\mathrm{Q}}{\mathrm{r}}$ Where ε_0 is the electric constant, otherwise known as permittivity of free space.
- For one point charge, potential will be constant for all points a certain radial distance away. Multiple points of the same potential are known as equipotential.
- When multiple charges create a field, the equipotential lines become irregularly shaped. This is because the fields created by each charge overlap, thus the potential is increased at any point relative to that which would have arisen from one or the other charge.
- Electric potential (ΔV) and field (E) are related according to the integral: $\Delta\mathrm{V} = -\int_\mathrm{i}^\mathrm{f} \vec{\mathrm{E}} \cdot \mathrm{d}\vec{\mathrm{l}}$ where l is the distance between two points between which the potential difference is being found.
- Given that the electric field is constantly 0 for any location within the charged conductor, it is impossible for potential difference in that same volume to have any value other than 0.
- For points outside a conductor, potential is nonzero and can be calculated according to field and distance from the conductor.
- The uniform electric field is an approximation that makes for simple calculations that don't require differential calculus. Every field will have at least some irregularity, although some can be very nearly uniform.
- The equation for magnitude of a uniform electric field is: $\mathrm{E} = \frac{-\Delta\phi}{\mathrm{d}}$ where E is the field, Δ is the potential difference between the plates, and d is the distance between the plates.
- For the case of a positive charge q to be moved from a point A with a certain potential (V_1) to a point B with another potential (V_2), that equation is: $\mathrm{W} = -\mathrm{q}(\mathrm{V_2} - \mathrm{V_1})$ The difference (V_2-V_1) can also be represented as ΔV or V_{AB}.
- In uniform fields it is also simple to calculate potential difference: $\mathrm{V_{AB}} = \mathrm{Ed}$ In this case, field strength is E, and distance between points A and B is d.

- Given a stationary test charge in a certain location, an applied electric field will cause the charge to move to one end or the other, depending on the charge.
- Positive test charges will move in the direction of the field; negative charges will move in the opposite direction.
- At the instant at which the field is applied, the motionless test charge has 0 kinetic energy, and its electric potential energy is at a maximum. Then, the charge accelerates, and its kinetic energy (from motion) increases as its potential energy decreases.The sum of energies is always constant.
- The formula illustrating conservation of energy can be written in many ways, but all expressions are based on the simple premise of equating the initial and final sums of kinetic and potential energy.
- The electron volt is defined as the amount of energy gained or lost by the charge of an electron moved across a one-volt electric potential difference. Its value is approximately equal to 1.602×10^{-19} J.
- The electron volt became useful through experimentation. Scientists working with electrostatic particle accelerators commonly used the relationship between energy (E), charge (q), and potential difference (V) in their work. This relationship is: $\mathrm{E=qV}$.
- As an energy, the electron volt can be used in many calculations, including momentum, mass, wavelength, and temperature.
- Electric dipole moments are used to measure the separation of positive and negative charges (polarity) in a system. They are measured in units of Coulomb-meters (C m).
- For point charges with values +q and -q, electric dipole moment (p) can be defined as: $\mathrm{p=qd}$ where q represents the charges and d represents the displacement vector. The displacement vector has a magnitude of the distance between the charges and a direction from the negative to the positive charge.
- All dipoles will experience a torque that rotates the dipole to align it with an electric field. This torque can be calculated as the cross product of the electric dipole moment and the electric field.

Key Terms

- **electric field**: A region of space around a charged particle, or between two voltages; it exerts a force on charged objects in its vicinity.
- **electric potential**: The potential energy per unit charge at a point in a static electric field; voltage.
- **coulomb**: In the International System of Units, the derived unit of electric charge; the amount of electric charge carried by a current of 1 ampere flowing for 1 second. Symbol: C
- **potential energy**: The energy an object has because of its position (in a gravitational or electric field) or its condition (as a stretched or compressed spring, as a chemical reactant, or by having rest mass)
- **equipotential**: A region whose every point has the same potential.
- **radial**: Moving along a radius.
- **electric field**: A region of space around a charged particle, or between two voltages; it exerts a force on charged objects in its vicinity.
- **work**: A measure of energy expended in moving an object; most commonly, force times displacement. No work is done if the object does not move.
- **potential difference**: The difference in potential energy between two points in an electric field; the difference in charge between two points in an electrical circuit; voltage.
- **kinetic energy**: The energy possessed by an object because of its motion, equal to one half the mass of the body times the square of its velocity.
- **particle accelerator**: A device that accelerates electrically charged particles to extremely high speeds, for the purpose of inducing high-energy reactions or producing high-energy radiation.
- **electron volt**: A unit for measuring the energy of subatomic particles; the energy equal to that attained by an electron moving through a potential difference of one volt. Equivalent to 1.6022 x 10-19 joules.
- **dipole moment**: The vector product of the charge on either pole of a dipole and the distance separating them.
- **vector**: A directed quantity, one with both magnitude and direction; the between two points.
- **torque**: A rotational or twisting effect of a force; (SI unit newton-meter or Nm; imperial unit foot-pound or ft-lb)

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18.2: Equipotential Surfaces and Lines

learning objectives

- Compare conductivity and resistivity of an ideal conductor, commenting on the presence of ideal conductors in nature

An ideal conductor is one that exists only in the world of theory. It is one that has certain “ideal” properties that make calculations of electric potential and field, and other properties simple to perform; it essentially eliminates the need for “corrections” to account for slight deviations from the so-called “ideal. ” These ideal properties are:

$$\sigma = \infty \tag{18.2.1}$$

Conductivity (σ) is the inverse of resistance, measured in units of current per potential difference. In an ideal conductor, the material’s conductivity is infinite, and its resistance approaches 0. This means that, ideally, it needs a minute amount of voltage (potential difference) to carry an extremely high amperage (current). The principle of near-zero resistance is akin to that of frictionless surfaces: t. Theoretically, with the slightest force (voltage), an object (current) on a frictionless surface (zero-resistance conductor) can proceed without restriction.

$$\mathrm{E_{tan} = D_{tan} = 0} \tag{18.2.2}$$

The electric field (E_{tan}) and electric flux density (D_{tan}) tangential to the surface of a conductor must be equal to 0. This is because any such field or flux that is tangential to the surface of the conductor must also exist inside the conductor, which by definition touches the tangential field or density at one point.

If an electric field exists inside the metal, there must be a drop in voltage between any two points along the surface of the metal. In a perfect conductor this drop should not exist because it implies a less-than-infinite conductivity.

$$\mathrm{D_n = Charge\ Density_{surface}} \tag{18.2.3}$$

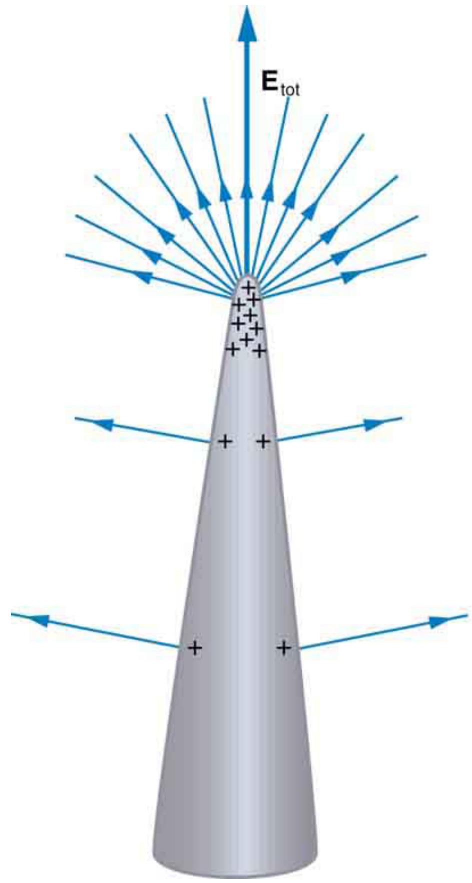
Electric flux density normal to the conductor’s surface is equal to surface charge density. Essentially this means that the conductor’s charge exists on its surface, not in its interior.

$$\overrightarrow{\mathrm{E}}_{\mathrm{int}} = 0 \tag{18.2.4}$$

This means that the electric field inside a perfect conductor is 0. The charges along the surface all act equally and opposite to one another, and their sum at any point is equal to 0.

$$\Phi_{\mathrm{surface}} = \mathrm{constant} \tag{18.2.5}$$

Charge distribution may vary depending on shape, but the potential over the surface of an ideal conductor is, at electrostatic equilibrium, constant throughout.



Charge Distribution on a Conductor with an Irregular Surface: Curvature causes electric field lines to extend such that they further distance themselves from one another with increasing distance from the conductor surface. As such, charges (and field lines) aggregate around areas of curvature.

Electric Potential in Human

Electric potentials are commonly found in the body, across cell membranes and in the firing of neurons.

learning objectives

- Give examples of the electric potentials in human body

Electric potentials are not limited in function to inorganic processes. In fact, they can be commonly seen in living organisms. In humans, they are seen in cell membranes and nerve impulses in particular.

Cell Membranes

Cell membranes are only semipermeable; water can freely travel in and out, but ions can be selectively admitted passage across them. As a result, a cell can contain a concentration of a given ion that differs from that which exists outside. Thus, a potential, called the resting potential, is created on either side of the membrane.

Typical ions used to generate resting potential include potassium, chloride, and bicarbonate.

Resting membrane potential is approximately -95 mV in skeletal muscle cells, -60 mV in smooth muscle cells, -80 to -90 mV in astroglia, and -60 to -70 mV in neurons.

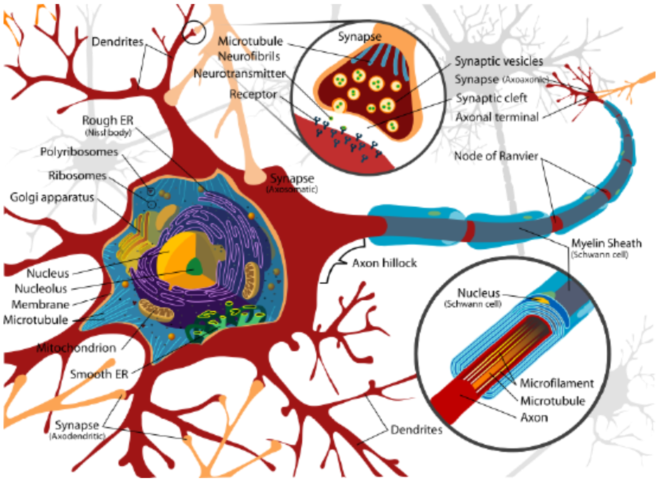
Potentials can change as ions move across the cell membrane. This can occur passively, as ions diffuse through ion channels in the membrane. No energy is required for this to occur, and therefore ions can only move from areas of higher concentration to those of lower concentration.

Active transport of ions across a cell membrane is also a possibility. This involves ion pumps using energy to push an ion from an area of lower concentration to one of higher concentration.

Nerve Impulses

When the brain decides on an action, it sends an impulse that cascades to the extremity where a muscle contracts.

Neurons receive an impulse at the dendrites. This impulse is passed through the axon, a long extension of the cell, in the form of an electrical potential created by differing concentrations of sodium and potassium ions on either side of a membrane in the axon.



The Neuron: Neurons receive an impulse at the dendrites. This impulse is passed through the axon, a long extension of the cell, in the form of an electrical potential created by differing concentrations of sodium and potassium ions on either side of a membrane in the axon.

When the signal reaches the end of the axon, neurotransmitters are released, which then are received by the dendrites of the next neuron. The next neuron repeats the process outlined above.

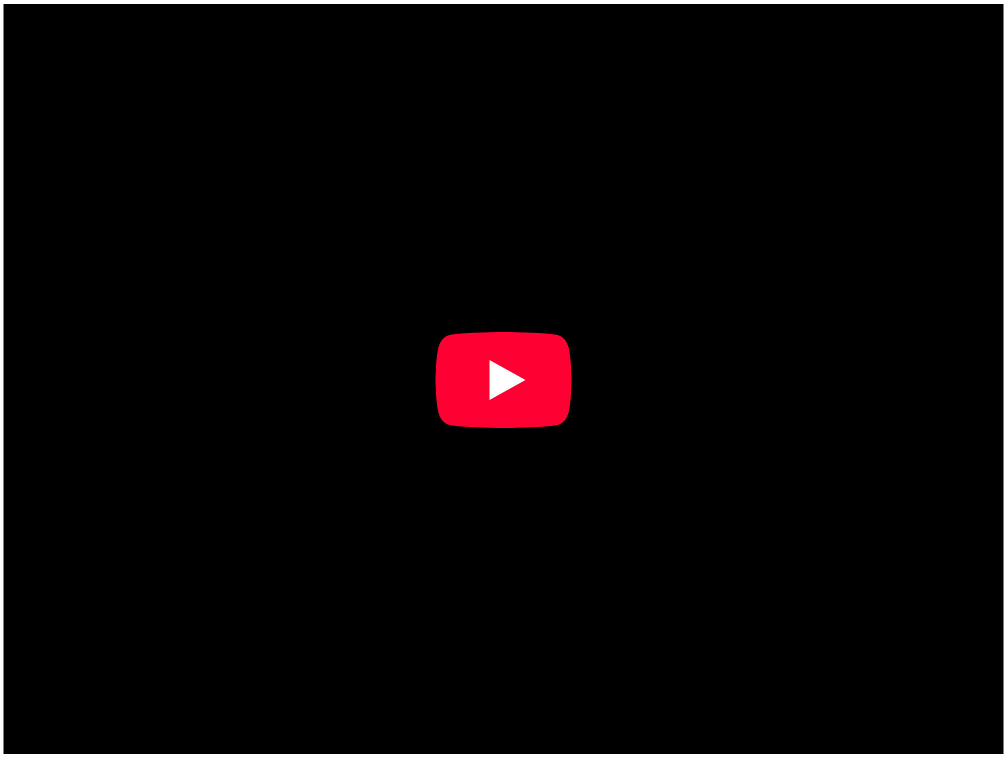
Equipotential Lines

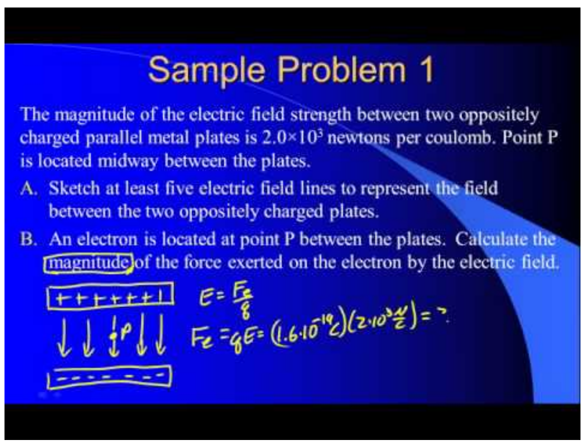
Equipotential lines depict one-dimensional regions in which the electric potential created by one or more nearby charges is constant.

learning objectives

- Describe the shape of the equipotential lines for several charge configurations

Equipotential lines depict one-dimensional regions in which the electric potential created by one or more nearby charges has a constant value. This means that if a charge is at any point on a given equipotential line, no work will be required to move it from one point to another on that same line.





Parallel Plates and Equipotential Lines: A brief overview of parallel plates and equipotential lines from the viewpoint of electrostatics.

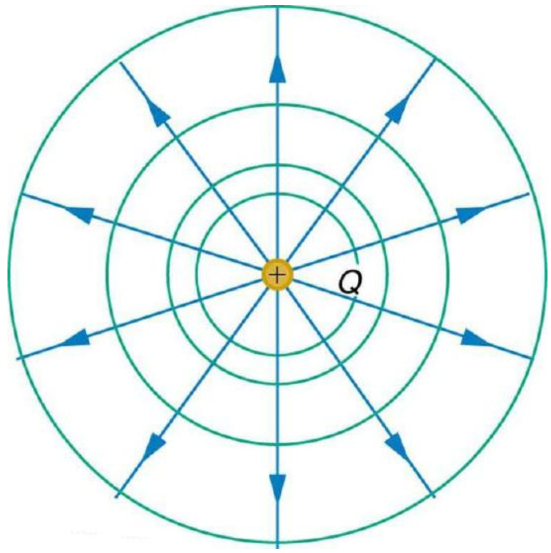
Equipotential lines may be straight, curved, or irregularly shaped, depending on the orientation of charges that give rise to them. Since they are located radially around a charged body, they are perpendicular to electric field lines, which extend radially from the center of a charged body.

A Single Point Charge

For a single, isolated point charge, the formula for potential (V) is functionally dependent upon charge (Q) and inversely dependent upon radial distance from the charge (r):

$$\mathrm{V}=\frac{\mathrm{kQ}}{\mathrm{r}} \tag{18.2.6}$$

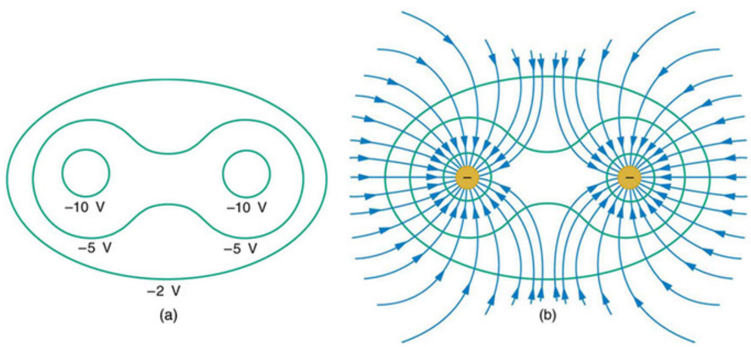
The radial dependence means that at any point a certain distance from the point charge, potential will be the same. Therefore, equipotential lines for a single point charge are circular, with the point charge at the center.



Equipotential Lines: An isolated point charge Q with its electric field lines (blue) and equipotential lines (green)

Multiple Point Charges

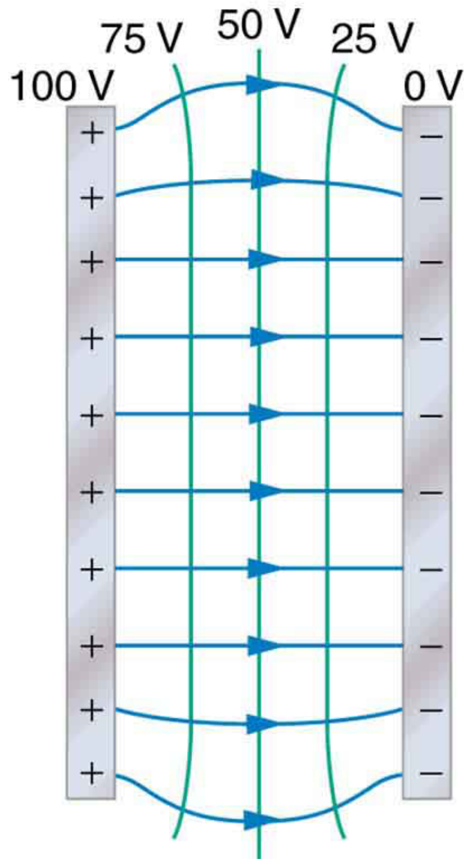
When multiple, discrete charges interact, their fields overlap. The combining of fields also results in a combining of potential, which results in the skewing of equipotential lines in areas that are close enough to both charges to "feel" the effects of both fields.



Equipotential lines with multiple charges: Equipotential lines become irregularly shaped when there are multiple charges vicinal to one another. At a point between the charges, a test charge may "feel" the effects of both charges.

Continuous Charge

If charges are distributed across two conductor plates in static equilibrium, in which charges are continuous and distributed in a straight line, the equipotential lines will be approximately straight. This is because the continuity of charges results in a continuity of action upon any point a certain distance from either plate.



Equipotential Lines Between Two Plates: When charges are lined up and continuous on conducting plates, equipotential lines are straight between them. The only exception is a curving of the lines near the edges of the conductor plates.

This continuity is broken towards the ends of the plates, however, which causes curvature in these areas. This curvature is known as “edge effects. ”

Key Points

- In an ideal conductor, conductivity is assumed to be infinite (as resistivity approaches 0).
- The electric field (Etan) and electric flux density (Dtan) tangential to the surface of a conductor must be equal to 0.
- Electric flux density normal to the conductor’s surface is equal to surface charge density. Essentially this means that the conductor’s charge exists on its surface, not in its interior.
- Electric field inside a perfect conductor is 0. The charges along the surface all act equally and opposite to one another, and their sum at any point is equal to 0.
- Charge distribution may vary depending on shape, but the potential over the surface of an ideal conductor is, at electrostatic equilibrium, constant throughout.
- The resting potential is a potential created by an imbalance of ions on either side of a cell membrane.
- Resting potential can be altered by passive diffusion (which requires no energy) or active transport of ions across a cell membrane.
- When the brain decides on an action, it sends an impulse that cascades to the extremity where a muscle contracts. Along the axon of a neuron, this impulse manifests itself in a potential created by an imbalance of sodium and potassium ions across a membrane.
- For a single, isolated point charge, potential is inversely dependent upon radial distance from the charge. Therefore, equipotential lines for a single point charge are circular, with the point charge at the center.
- When multiple, discrete charges interact, their fields overlap, meaning their potentials combine. This results in the skewing of equipotential lines in areas that are close enough to both charges to “feel” the effects of both fields.
- If charges are distributed across two conductor plates in static equilibrium, in which charges are continuous and distributed in a straight line, the equipotential lines will be approximately straight.

Key Terms

- **conductor**: A material which contains movable electric charges.
- **flux density**: A measure of rate of flow of a fluid, particles or energy per unit area.
- **neuron**: A cell of the nervous system, which conducts nerve impulses; consisting of an axon and several dendrites. Neurons are connected by synapses.
- **axon**: A nerve fiber which is a long, slender projection of a nerve cell, and which conducts nerve impulses away from the body of the cell to a synapse.
- **cell membrane**: The semipermeable membrane that surrounds the cytoplasm of a cell.
- **equipotential**: A region whose every point has the same potential.
- **static equilibrium**: the physical state in which all components of a system are at rest and the net force is equal to zero throughout the system

- Parallel Plates and Equipotential Lines. **Located at**: **http://www.youtube.com/watch?v=EJ910SiVha0**. **License**: ***Public Domain: No Known Copyright***. **License Terms**: Standard YouTube license

18.3: Point Charge

learning objectives

- Express the electric potential generated by a single point charge in a form of equation

Electric Potential Due to a Point Charge

Overview

Recall that the electric potential is defined as the electric potential energy per unit charge

$$V = \frac{PE}{q} \tag{18.3.1}$$

The electric potential tells you how much potential energy a single point charge at a given location will have. The electric potential at a point is equal to the electric potential energy (measured in joules) of any charged particle at that location divided by the charge (measured in coulombs) of the particle. Since the charge of the test particle has been divided out, the electric potential is a "property" related only to the electric field itself and not the test particle. Another way of saying this is that because PE is dependent on q, the q in the above equation will cancel out, so V is not dependent on q.

The potential difference between two points ΔV is often called the voltage and is given by

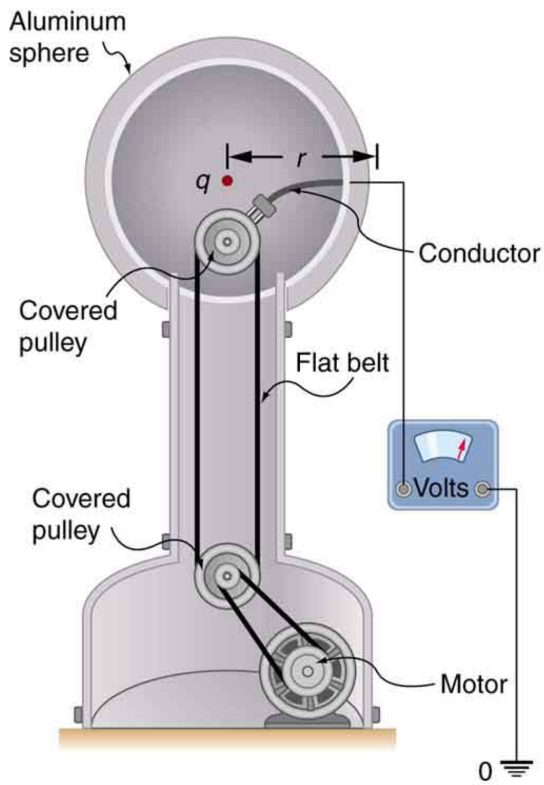
$$\Delta V = V_B - V_A = \frac{\Delta PE}{q} \tag{18.3.2}$$

Point Charges

Point charges, such as electrons, are among the fundamental building blocks of matter. Furthermore, spherical charge distributions (like on a metal sphere, see figure below) create external electric fields exactly like a point charge. The electric potential due to a point charge is, thus, a case we need to consider. Using calculus to find the work needed to move a test charge *q* from a large distance away to a distance of r from a point charge *Q*, and noting the connection between work and potential (W=–qΔV), it can be shown that the *electric potential V of a point charge* is

$V = \frac{kQ}{r}$ (point charge)

where *k* is a constant equal to $9.0\times10^9\ N\cdot m^2/C^2$.



Van de Graaff Generator: The voltage of this demonstration Van de Graaff generator is measured between the charged sphere and ground. Earth's potential is taken to be zero as a reference. The potential of the charged conducting sphere is the same as that of an equal point charge at its center.

The potential at infinity is chosen to be zero. Thus V for a point charge decreases with distance, whereas E for a point charge decreases with distance squared:

$$E = \frac{F}{q} = \frac{kQ}{r^2} \tag{18.3.3}$$

The electric potential is a scalar while the electric field is a vector. Note the symmetry between electric potential and gravitational potential – both drop off as a function of distance to the first power, while both the electric and gravitational fields drop off as a function of distance to the second power.

Superposition of Electric Potential

To find the total electric potential due to a system of point charges, one adds the individual voltages as numbers.

learning objectives

- Explain how the total electric potential due to a system of point charges is found

Superposition of Electric Potential

We've seen that the electric potential is defined as the amount of potential energy per unit charge a test particle has at a given location in an electric field, i.e.

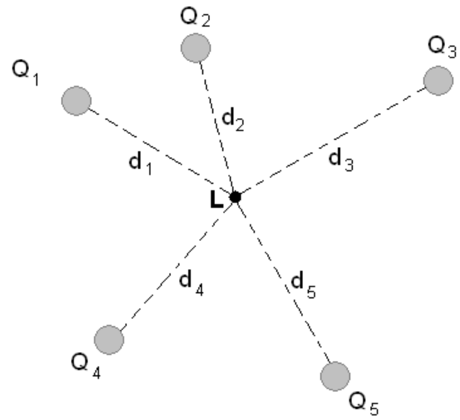
$$V = \frac{PE}{q} \tag{18.3.4}$$

We've also seen that the electric potential due to a point charge is

where *k* is a constant equal to $9.0 \times 10^9\ N \cdot m^2/C^2$. The equation for the electric potential of a point charge looks similar to the equation for the electric field generated for a point particle

$$E = \frac{F}{q} = \frac{kQ}{r^2} \tag{18.3.5}$$

with the difference that the electric field drops off with the square of the distance while the potential drops off linearly with distance. This is analogous to the relationship between the gravitational field and the gravitational potential.



Superposition of Electric Potential: The electric potential at point L is the sum of voltages from each point charge (scalars).

Recall that the electric potential *V* is a scalar and has no direction, whereas the electric field *E* is a vector. To find the voltage due to a combination of point charges, you add the individual voltages as numbers. So for example, in the figure above the electric potential at point L is the sum of the potential contributions from charges Q_1, Q_2, Q_3, Q_4, and Q_5 so that

$$V_L = k\left[\frac{Q_1}{d_1} + \frac{Q_2}{d_2} + \frac{Q_3}{d_3} + \frac{Q_4}{d_4} + \frac{Q_5}{d_5}\right] \tag{18.3.6}$$

To find the total electric field, you must add the individual fields as *vectors*, taking magnitude and direction into account. This is consistent with the fact that *V* is closely associated with energy, a scalar, whereas *E* is closely associated with force, a vector.

The summing of all voltage contributions to find the total potential field is called the superposition of electric potential. Summing voltages rather than summing the electric simplifies calculations significantly, since addition of potential scalar fields is much easier than addition of the electric vector fields. Note that there are cases where you might need to sum potential contributions from sources other than point charges; however, that is beyond the scope of this section.

Key Points

- Recall that the electric potential is defined as the potential energy per unit charge, i.e. $V = \frac{PE}{q}$.
- The potential difference between two points ΔV is often called the voltage and is given b $\Delta V = V_B - V_A = \frac{\Delta PE}{q}$. The potential at an infinite distance is often taken to be zero.
- The case of the electric potential generated by a point charge is important because it is a case that is often encountered. A spherical sphere of charge creates an external field just like a point charge, for example.
- The equation for the electric potential due to a point charge is $V = \frac{kQ}{r}$, where k is a constant equal to $9.0 \times 10^9 N \cdot m^2/C^2$.
- The electric potential *V* is a scalar and has no direction, whereas the electric field *E* is a vector.
- To find the voltage due to a combination of point charges, you add the individual voltages as numbers. So for example, in the electric potential at point L is the sum of the potential contributions from charges Q_1, Q_2, Q_3, Q_4, and Q_5 so that $V_L = k\left[\frac{Q_1}{d_1} + \frac{Q_2}{d_2} + \frac{Q_3}{d_3} + \frac{Q_4}{d_4} + \frac{Q_5}{d_5}\right]$.
- To find the total electric field, you must add the individual fields as vectors, taking magnitude and direction into account. This is consistent with the fact that V is closely associated with energy, a scalar, whereas E is closely associated with force, a vector.
- The summing of all voltage contributions to find the total potential field is called the superposition of electric potential. It is much easier to sum scalars than vectors, so often the preferred method for solving problems with electric fields involves the summing of voltages.

Key Terms

- **electric potential**: The potential energy per unit charge at a point in a static electric field; voltage.
- **voltage**: The amount of electrostatic potential between two points in space.
- **vector**: A directed quantity, one with both magnitude and direction; the between two points.
- **scalar**: A quantity that has magnitude but not direction; compare vector.
- **superposition**: The summing of two or more field contributions occupying the same space.

18.4: Capacitors and Dielectrics

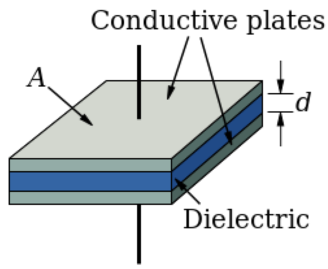
learning objectives

- Express the relationship between the capacitance, charge of an object, and potential difference in the form of equation

Capacitance is the measure of an object's ability to store electric charge. Any body capable of being charged in any way has a value of capacitance.

The unit of capacitance is known as the Farad (F), which can be adjusted into subunits (the millifarad (mF), for example) for ease of working in practical orders of magnitude. The Farad can be equated to many quotients of units, including JV^{-2}, WsV^{-2}, CV^{-1}, and C^2J^{-1}.

The most common capacitor is known as a parallel-plate capacitor which involves two separate conductor plates separated from one another by a dielectric. Capacitance (C) can be calculated as a function of charge an object can store (q) and potential difference (V) between the two plates:



Parallel-Plate Capacitor: The dielectric prevents charge flow from one plate to the other.

$$C = \frac{q}{V} \tag{18.4.1}$$

Ultimately, in such a capacitor, q depends on the surface area (A) of the conductor plates, while V depends on the distance (d) between the plates and the permittivity (ε_r) of the dielectric between them. For a parallel-plate capacitor, this equation can be used to calculate capacitance:

$$C = \epsilon_r \epsilon_0 \frac{A}{d} \tag{18.4.2}$$

Where ε_0 is the electric constant. The product of length and height of the plates can be substituted in place of A.

In storing charge, capacitors also store potential energy, which is equal to the work (W) required to charge them. For a capacitor with plates holding charges of +q and -q, this can be calculated:

$$W_{charging} = \int_0^Q \frac{q}{C} dq = \frac{CV^2}{2} = W_{stored} \tag{18.4.3}$$

Thus, either through calculus or algebraically (if C and V are known), stored energy (W_{stored}) can be calculated. In a parallel-plate capacitor, this can be simplified to:

$$W_{stored} = \frac{\epsilon_{\epsilon_0 A} V^2}{2d} \tag{18.4.4}$$

Capacitors with Dielectrics

A dielectric partially opposes a capacitor's electric field but can increase capacitance and prevent the capacitor's plates from touching.

learning objectives

- Describe the behavior of the dielectric material in a capacitor's electric field

In order for a capacitor to hold charge, there must be an interruption of a circuit between its two sides. This interruption can come in the form of a vacuum (the absence of any matter) or a dielectric (an insulator).

When a dielectric is used, the material between the parallel plates of the capacitor will polarize. The part near the positive end of the capacitor will have an excess of negative charge, and the part near the negative end of the capacitor will have an excess of positive charge. This redistribution of charge in the dielectric will thus create an electric field opposing the field created by the capacitor.

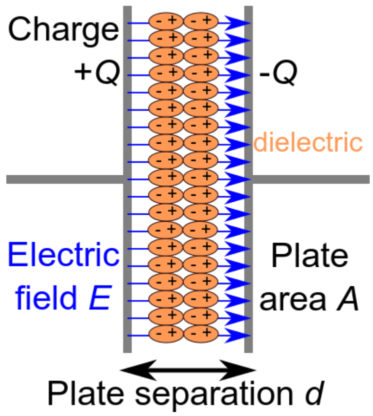


Diagram of a Parallel-Plate Capacitor: Charges in the dielectric material line up to oppose the charges of each plate of the capacitor. An electric field is created between the plates of the capacitor as charge builds on each plate.

Therefore, the net field created by the capacitor will be partially decreased, as will the potential difference across it, by the dielectric. On the other hand, the dielectric prevents the plates of the capacitor from coming into direct contact (which would render the capacitor useless). If it has a high permittivity, it also increases the capacitance for any given voltage. The capacitance for a parallel-plate capacitor is given by:

c=ϵAdc=ϵAd

where ε is the permittivity, A is the area of the capacitor plates (assuming both are the same size and shape), and d is the thickness of the dielectric.

Any insulator can be used as a dielectric, but the materials most commonly used are selected for their ability to resist ionization. The more resistant a material is to ionization, the more tolerance it has for operating at higher voltages. Eventually every material has a "dielectric breakdown point," at which the potential difference becomes too high for it to insulate, and it ionizes and permits the passage of current.

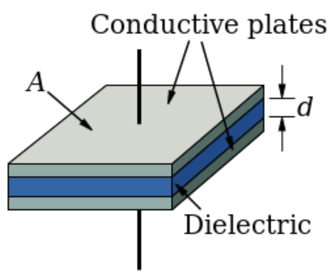
Parallel-Plate Capacitor

The parallel-plate capacitor is one that includes two conductor plates, each connected to wires, separated from one another by a thin space.

learning objectives

- Calculate the maximum storable energy in a parallel-plate capacitor

One of the most commonly used capacitors in industry and in the academic setting is the parallel-plate capacitor. This is a capacitor that includes two conductor plates, each connected to wires, separated from one another by a thin space. Between them can be a vacuum or a dielectric material, but not a conductor.



Parallel-Plate Capacitor: In a capacitor, the opposite plates take on opposite charges. The dielectric ensures that the charges are separated and do not transfer from one plate to the other.

The purpose of a capacitor is to store charge, and in a parallel-plate capacitor one plate will take on an excess of positive charge while the other becomes more negative.

Assuming the plates extend uniformly over an area of A and hold ± Q charge, their charge density is ±, where ρ=Q/A. Assuming that the dimensions of length and width for the plates are significantly greater than the distance (d) between them, the electric field (E) near the center of the plates can be calculated by:

$$\mathrm{E}=\frac{\rho}{\epsilon} \tag{18.4.5}$$

Potential (V) between the plates can be calculated from the line integral of the electric field (E):

$$\mathrm{V}=\int_0^{\mathrm{d}} \mathrm{Edz} \tag{18.4.6}$$

where z is the axis perpendicular to both plates. Through simplification and substitution, this integral can be changed to:

$$\mathrm{V}=\frac{\rho \mathrm{d}}{\epsilon}=\frac{\mathrm{Qd}}{\epsilon \mathrm{A}} \tag{18.4.7}$$

Given that capacitance is the quotient of charge and potential:

$$\mathrm{C}=\frac{\epsilon \mathrm{A}}{\mathrm{d}} \tag{18.4.8}$$

Accordingly, capacitance is greatest in devices with high permittivity, large plate area, and minimal separation between the plates.

The maximum energy (U) a capacitor can store can be calculated as a function of U_d, the dielectric strength per distance, as well as capacitor's voltage (V) at its breakdown limit (the maximum voltage before the dielectric ionizes and no longer operates as an insulator):

$$\mathrm{U}=\frac{\mathrm{CV}^2}{2}=\frac{\epsilon \mathrm{A}\left(\mathrm{U}_{\mathrm{dd}}\right)^2}{2 \mathrm{d}}=\frac{\epsilon \mathrm{AdU}_{\mathrm{d}}^2}{2} \tag{18.4.9}$$

Combinations of Capacitors: Series and Parallel

Like any other form of electrical circuitry device, capacitors can be used in series and/or in parallel within circuits.

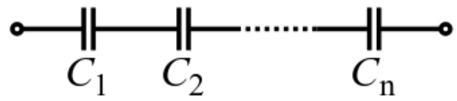
learning objectives

- Calculate the total capacitance for the capacitors connected in series and in parallel

Like any other form of electrical circuitry device, capacitors can be used in combination in circuits. These combinations can be in series (in which multiple capacitors can be found along the same path of wire) and in parallel (in which multiple capacitors can be found along different paths of wire).

Capacitors in Series

Like in the case of resistors in parallel, the reciprocal of the circuit's total capacitance is equal to the sum of the reciprocals of the capacitance of each individual capacitor:



Capacitors in Series: This image depicts capacitors C1, C2 and so on until Cn in a series.

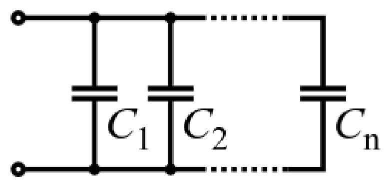
$$\frac{1}{\mathrm{C_{total}}}=\frac{1}{\mathrm{C_1}}+\frac{1}{\mathrm{C_2}}+\ldots+\frac{1}{\mathrm{C_n}} \tag{18.4.10}$$

This can also be expressed as:

$$\mathrm{C_{total}}=\frac{1}{\frac{1}{\mathrm{C_1}}+\frac{1}{\mathrm{C_2}}+\ldots+\frac{1}{\mathrm{C_n}}} \tag{18.4.11}$$

Parallel Capacitors

Total capacitance for a circuit involving several capacitors in parallel (and none in series) can be found by simply summing the individual capacitances of each individual capacitor.

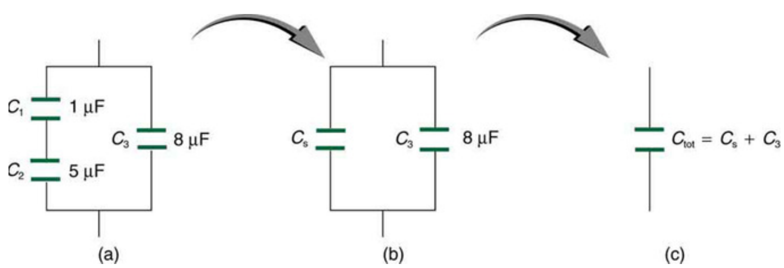


Parallel Capacitors: This image depicts capacitors C1, C2, and so on until Cn in parallel.

$$\mathrm{C_{total}}=\mathrm{C_1}+\mathrm{C_2}+\ldots+\mathrm{C_n} \tag{18.4.12}$$

Capacitors in Series and in Parallel

It is possible for a circuit to contain capacitors that are both in series and in parallel. To find total capacitance of the circuit, simply break it into segments and solve piecewise.



Capacitors in Series and in Parallel: The initial problem can be simplified by finding the capacitance of the series, then using it as part of the parallel calculation.

The circuit shown in (a) contains C_1 and C_2 in series. However, these are both in parallel with C_3. If we find the capacitance for the series including C_1 and C_2, we can treat that total as that from a single capacitor (b). This value can be calculated as approximately equal to 0.83 μF.

With effectively two capacitors left in parallel, we can add their respective capacitances (c) to find the total capacitance for the circuit. This sum is approximately 8.83 μF.

Dieletrics and their Breakdown

Dielectric breakdown is the phenomenon in which a dielectric loses its ability to insulate, and instead becomes a conductor.

learning objectives

- Identify conditions that can lead to a dielectric breakdown and its effect on materials

Dielectric breakdown (illustrated in) is the phenomenon in which a dielectric loses its ability to insulate, and instead becomes a conductor. Dielectrics are commonly used either to isolate conductors from a variable external environment (e.g., as coating for electrical wires) or to isolate conductors from one another (e.g., between plates of a parallel-plate capacitor). In all applications, they are selected for their ability to act as insulators. By definition, an insulator is unable to conduct electricity. Under certain conditions, however, a material that is an insulator can become a conductor.

Eventually, exposing any insulator to increasing voltage will result in the insulator becoming conductive. This point (the minimum voltage for the insulator to become a conductor) is known as the breakdown voltage. Breakdown is more of a rough concept than an exact science. A material's breakdown voltage cannot be precisely defined. As a failure, there is a probabilistic element and thus a dielectric may experience a breakdown at any of a range of voltages. Additionally, the nature of the voltage used to induce breakdown must be considered. Short pulses can be used in stress testing to resemble lightning strikes, as could a continuous applied voltage.

However, for the case of a gas being used as a dielectric, the following equation has been proven to be rather reliable in predicting breakdown voltage (V_b):

$$V_b = \frac{Bpd}{\ln Apd - \ln\left(\ln\left(1 + \frac{1}{\gamma_{se}}\right)\right)} \tag{18.4.13}$$

where A and B are constants that depend on the surrounding gas, p is the pressure of the surrounding gas, d is distance between the electrodes (in cm) and γ_{se} is the secondary electron emission coefficient. Gaseous dielectrics commonly experience breakdown in nature (the phenomenon of lightning is the most common example).

Dielectric breakdown of plexiglas: The treelike pattern in the plexiglas stems from the root of the breakdown. Current is dispersed in many different directions, creating different stems.

Key Points

- The unit of capacitance is known as the farad (F), which can be equated to many quotients of units, including JV^{-2}, WsV^{-2}, CV^{-1}, and C^2J^{-1}.

- Capacitance (C) can be calculated as a function of charge an object can store (q) and potential difference (V) between the two plates: $C = \frac{q}{V}$. Q depends on the surface area of the conductor plates, while V depends on the distance between the plates and the permittivity of the dielectric between them.
- In storing charge, capacitors also store potential energy, which is equal to the work (W) required to charge them. For a capacitor with plates holding charges of +q and -q, this can be calculated: $W_{stored} = \frac{CV^2}{2}$. The above can be equated with the work required to charge the capacitor.
- When a dielectric is used, the material between the plates will polarize to oppose the dielectric's field. The net field created by the capacitor will be partially decreased, as will the potential difference across it, by the dielectric.
- Capacitance for a parallel -plate capacitor is given by: $\mathbf{c} = \frac{\epsilon A}{d}$ where *ε* is the permittivity, *A* is the area of the capacitor plates (assuming both are the same size and shape), and *d* is the thickness of the dielectric.
- Any insulator can be used as a dielectric, but the materials most commonly used are selected for their ability to resist ionization. The more resistant a material is to ionization, the more tolerance it has for operating at higher voltages.
- Assuming the plates extend uniformly over an area of A and hold ± Q charge, their charge density is ±, where ρ=Q/A.
- Assuming that the dimensions of length and width for the plates are significantly greater than the distance (d) between them, $E = \frac{\rho}{\epsilon}$ can be used to calculate the electric field (E) near the center of the plates. In this equation, ε represents permittivity.
- $\mathbf{V} = \frac{\rho d}{\epsilon} = \frac{Qd}{\epsilon A}$ can be used to calculate the potential between the plates.
- $C = \frac{\epsilon A}{d}$ can be found from the previous equation, adjusting the terms to solve for capacitance (C).
- $U = \frac{CV^2}{2} = \frac{\epsilon A (U_{dd})^2}{2d} = \frac{\epsilon A d U_d^2}{2}$ solves for the maximum storable energy in a parallel-plate capacitor (U) as a function of U_d, the dielectric strength per distance as well as capacitor's voltage (V) at its breakdown limit.
- $\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n}$ Capacitors in series follow the law of reciprocals; the reciprocal of the circuit 's total capacitance is equal to the sum of the reciprocals of the capacitances of each individual capacitor.
- $C_{total} = C_1 + C_2 + \ldots + C_n$ For capacitors in parallel, summing the capacitances of individual capacitors affords the total capacitance in the circuit.
- When capacitors are found both in series and in parallel in the same circuit, it is best to simplify the circuit by solving parts of it in sequence.
- All insulators can, when exposed to enough voltage, experience dielectric breakdown and become conductors.
- Because dielectric breakdown is a failure that depends on a probability, an exact breakdown voltage is in most cases impossible to calculate with a high degree of certainty.
- Lightning is a common instance of dielectric breakdown, as air loses its ability to separate the potential difference between clouds and the point of a lightning bolt's impact.

Key Terms

- **dielectric**: An electrically insulating or nonconducting material considered for its electric susceptibility (i.e., its property of polarization when exposed to an external electric field).
- **capacitance**: The property of an electric circuit or its element that permits it to store charge, defined as the ratio of stored charge to potential over that element or circuit (Q/V); SI unit: farad (F).
- **capacitor**: An electronic component capable of storing an electric charge, especially one consisting of two conductors separated by a dielectric.
- **permittivity**: A property of a dielectric medium that determines the forces that electric charges placed in the medium exert on each other.
- **circuit**: A pathway of electric current composed of individual electronic components, such as resistors, transistors, capacitors, inductors and diodes, connected by conductive wires or traces through which electric current can flow. T
- **conductor**: A material which contains movable electric charges.
- **breakdown**: A failure, particularly mechanical; something that has failed.

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18.5: Applications

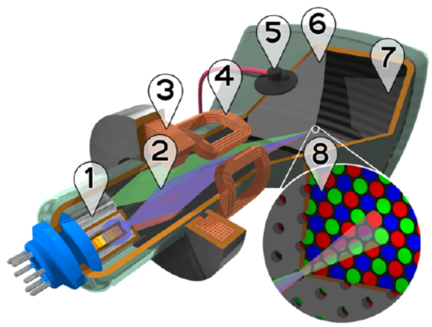
learning objectives

- Identify the primary components of a cathode ray tube and describe the use of cathode ray tubes

Cathode Ray Tube, TV and Computer Monitors, and the Oscilloscope

Introduction

The *cathode ray tube* (CRT) is a vacuum tube containing one or more electron guns (a source of directed electrons) and a fluorescent screen used to view images. It has a means to accelerate and deflect the electron beam onto the fluorescent screen to create the images. The images are generated when electrons strike fluorescent phosphors on the screen, which then emit light (the color varies depending on the phosphor used,).



Color Cathode Ray Tube: Cutaway rendering of a color CRT: 1) Three Electron guns (for red, green, and blue phosphor dots) 2) Electron beams 3) Focusing coils 4) Deflection coils 5) Anode connection 6) Mask for separating beams for red, green, and blue part of displayed image 7) Phosphor layer with red, green, and blue zones 8) Close-up of the phosphor-coated inner side of the screen

The CRT uses an evacuated glass envelope which is large, deep (i.e., long from front screen face to rear end), fairly heavy, and relatively fragile. As a matter of safety, the face is typically made of thick lead glass so as to be highly shatter-resistant and to block most X-ray emissions, particularly if the CRT is used in a consumer product.

Phosphors

The phosphors in a CRT's screen are the materials that directly produce the photons generated by the CRT. These phosphors are struck by incoming electrons from the electron gun, absorb energy, and then re-emit some or all of that energy in the form of light (this process is called phosphorescence). By varying the type of phosphor used, one may vary the wavelength of light emitted by the phosphor when excited. For example, black and white TV screens use one type of phosphor, while color TVs use three (blue, red, and green). Early computer terminal monitors used only green phosphors.

CRT Devices

Monochrome Computer CRT Monitor: Monochrome monitor – this CRT uses only one type of phosphor.

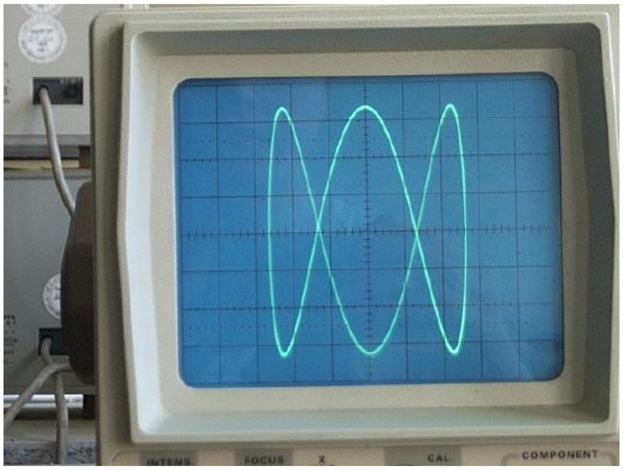
Televisions and Computer Monitors

In television sets and computer monitors, the entire front area of the tube is scanned repetitively and systematically in a fixed pattern called a raster. An image is produced by controlling the intensity of each of the three electron beams, one for each additive primary color (red, green, and blue) with a video signal as a reference. In all modern CRT monitors and televisions, the beams are bent by magnetic deflection, which is a varying magnetic field generated by coils and driven by electronic circuits around the neck of the tube.

Although a mainstay of display technology for decades, CRT-based computer monitors and televisions constitute a dead technology. The demand for CRT screens has dropped precipitously since 2000, and this falloff has been accelerating in the latter half of that decade. The rapid advances and falling prices of LCD flat panel technology, first for computer monitors and then for televisions, has been the key factor in the demise of competing display technologies such as CRT, rear-projection, and plasma display.

Oscilloscope

An oscilloscope is a device that measures and displays voltages as a time versus voltage graph. The voltage difference between the positive and negative probe leads is measured, buffered, and displayed on the screen as a continuous curve. Oscilloscopes are generally used to see if a circuit is performing as expected, but oscilloscopes are also useful for comparing different signals to each other.



Oscilloscope Display: Example of an analog oscilloscope display. Shown is a Lissajous figure, showing a harmonic relationship of one horizontal oscillation cycle to three vertical oscillation cycles.

Many oscilloscopes also use CRT displays, though LCD displays are becoming more common. In oscilloscope CRTs, electrostatic deflection is used, rather than the magnetic deflection commonly used with television and other large CRTs. The beam is deflected horizontally by applying an electric field between a pair of plates to its left and right, and vertically by applying an electric field to plates above and below.

Oscilloscopes use electrostatic rather than magnetic deflection because the inductive reactance of the magnetic coils would limit the frequency response of the instrument. The color of the oscilloscope phosphor is much less important than in the case of color televisions or computer monitors since the primary purpose is to evaluate signal voltages rather than construct complex images; however, the persistence of the phosphor may be more important. Phosphors are available with persistences ranging from less than one microsecond to several seconds. For visual observation of brief transient events, a long persistence phosphor may be desirable. For events which are fast and repetitive, or high frequency, a short-persistence phosphor is generally preferable.

Key Points

- The primary components of a cathode ray tube (CRT) consist of a vacuum tube containing an electron gun and a screen lined with phosphors. CRTs are used to produce images.
- The phosphors in a CRT's screen are the materials that directly produce the photons generated by the CRT. These phosphors are struck by incoming electrons from the electron gun, absorb energy, and then re-emit some or all of that energy in the form of

light.
- CRT technology used to be common in televisions and computer monitors. Color CRTs contain three electron guns corresponding to three types of phosphors, one for each primary color (red, blue, and green). Examples of monochromatic CRTs include black and white TVs and old computer terminals.
- Oscilloscopes, devices used to measure and display voltages, also use CRT displays. In this case, the persistence of the phosphor is more important than the color.
- The CRTs in televisions and computer monitors bend the electron beams with magnetic deflection, while oscilloscopes rely on electrostatic deflection.

Key Terms

- **phosphor**: A substance that exhibits the phenomenon of luminescence; often transition metal compounds or rare earth compounds of various types. The most common uses of phosphors are in CRT displays and fluorescent lights.
- **raster**: A scanning pattern of parallel lines that form the display of an image projected on a cathode-ray tube of a television set or display screen.
- **electron gun**: Any device that produces a stream of electrons, especially a narrow stream that is focused onto a phosphor screen.

CHAPTER OVERVIEW

19: Electric Current and Resistance

Topic hierarchy

19.1: Overview

learning objectives

- Explain difference between conductor and resistor

Introduction to Electric Current and Resistance

From ceiling lights to circuit chips, from power steering to Internet browsing, electricity provides the basis for our technology and civilization. The firing of neurons in your brain is also an example of electric current – that is, the movement of electric charge through a conductive medium. In electric circuits, this charge is often carried by moving electrons in a wire. It can also be carried by ions in an electrolyte, or by both ions and electrons such as in a plasma.

Electric Current

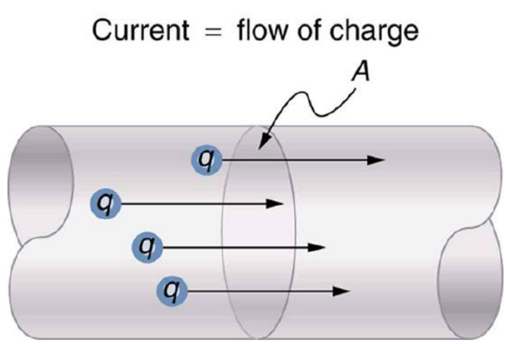
When we speak of electric current, often we are referring to a specific quantity – the *rate* at which charge flows. A large current, such as that used to start a truck engine, moves a large amount of charge in a small time, whereas a small current, such as that used to operate a hand-held calculator, moves a small amount of charge over a long period of time. In equation form, electric current I is defined to be

$$\mathrm{I}=\frac{\Delta \mathrm{Q}}{\Delta \mathrm{T}} \tag{19.1.1}$$

where Q is the amount of charge passing through a given area in time t. The SI unit for current is the ampere (A), named for the French physicist André-Marie Ampère (1775–1836). Since $I=\Delta Q/\Delta t$, we see that an ampere is one coulomb per second:

$$1 \mathrm{A}=1 \mathrm{C} / \mathrm{s} \tag{19.1.2}$$

The flow of electricity requires a medium in which charge can flow. We call an object or medium that allows charge to flow a *conductor*, while the empirical measure of a material's ability to conduct charge is called the electrical *conductance*. The SI unit for conductance is the siemens (S).



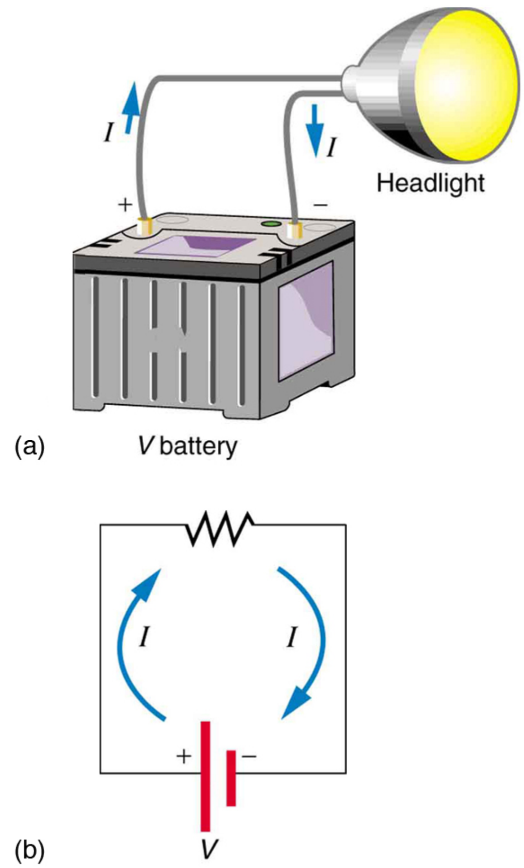
Electric Current: The rate of flow of charge is current. An ampere is the flow of one coulomb through an area in one second.

Resistance

The opposite of conductance is resistance – a quantity that describes how strongly a material opposes the flow of electric current. An object or medium that has high electrical resistance is called a *resistor.* We will see that the resistance of an object depends on its shape and the material of which it is composed. The SI unit for resistance is the *ohm* (symbol:).

Electric Circuits

A useful and practical way to learn about electric current and resistance is to study circuits. The figure above shows a simple circuit and the standard schematic representation of a battery, conducting path, and load (a resistor). Schematics are very useful in visualizing the main features of a circuit. A single schematic can represent a wide variety of situations. The schematic in (b), for example, can represent anything from a truck battery connected to a headlight lighting the street in front of the truck to a small battery connected to a penlight lighting a keyhole in a door. Such schematics are useful because the analysis is the same for a wide variety of situations. We need to understand a few schematics to apply the concepts and analysis to many more situations.



Simple Electric Circuit: (a) A simple electric circuit. A closed path for current to flow through is supplied by conducting wires connecting a load to the terminals of a battery. (b) In this schematic, the battery is represented by the two parallel red lines, conducting wires are shown as straight lines, and the zigzag represents the load. The schematic represents a wide variety of similar circuits.

Note that the direction of current flow in the figure is from positive to negative. The direction of conventional current is the direction that positive charge would flow. Depending on the situation, positive charges, negative charges, or both may move. In metal wires, for example, current is carried by electrons—that is, negative charges move. In ionic solutions, such as salt water, both positive and negative charges move.

It is important to realize that there is an electric field in conductors responsible for producing the current. Unlike static electricity, where a conductor in equilibrium cannot have an electric field in it, conductors carrying a current have an electric field and are not in static equilibrium. An electric field is needed to supply energy to move the charges.

Armed with these basics, we'll begin to tackle the harder details of this topic in the next section.

Key Points

- Electric current is the movement of electric charge through a conductive medium.
- We also use the term "current" as a quantity to describe the rate at which charge flows through a medium. The SI unit for current is the ampere (A), which is equal to a coulomb per second (C/s).
- Conductance is a quantity describing how easily charge can flow through a material, while resistance is the inverse, a measure of how strongly a material opposes electric flow.
- An object that allows charge to flow easily is called a conductor, while an object that resists the flow of charge is called a resistor.

Key Terms

- **conductive medium**: A material that can transmit electricity.

- **electrical resistance**: The opposition offered by an electrical conductor to the flow of a current through itself, resulting in a conversion of electrical energy into heat and radiation. The SI derived unit of resistance is the ohm. Symbol: R.
- **electric charge**: A quantum number that determines the electromagnetic interactions of some subatomic particles; by convention, the electron has an electric charge of -1 and the proton +1, and quarks have fractional charge.

19.2: Electric Current

learning objectives

- Describe the functions and identify the major components of a battery

A battery is a device that converts chemical energy directly to electrical energy. It consists of a number of voltaic cells connected in series by a conductive electrolyte containing anions and cations. One half-cell includes electrolyte and the anode, or negative electrode; the other half-cell includes electrolyte and the cathode, or positive electrode. In the redox (reduction-oxidation) reaction that powers the battery, cations are reduced (electrons are added) at the cathode, while anions are oxidized (electrons are removed) at the anode. The electrodes do not touch each other but are electrically connected by the electrolyte. Some cells use two half-cells with different electrolytes. A separator between half-cells allows ions to flow, but prevents mixing of the electrolytes.

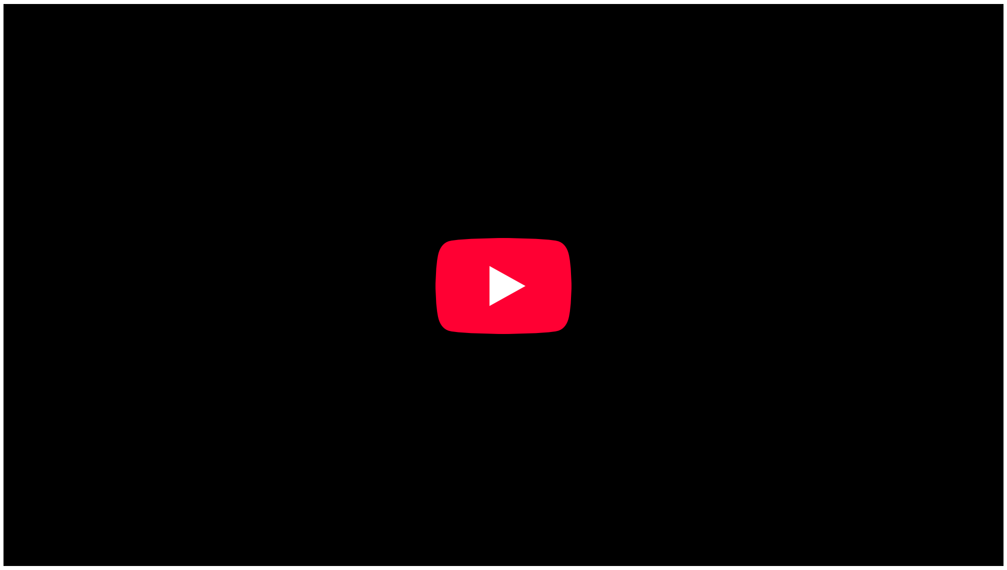
Each half-cell has an electromotive force (or emf), determined by its ability to drive electric current from the interior to the exterior of the cell. The net emf of the cell is the difference between the emfs of its half-cells, or the difference between the reduction potentials of the half-reactions.

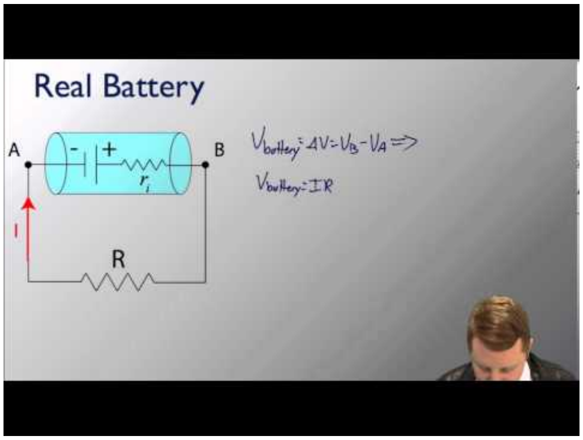
The electrical driving force across the terminals of a cell is known as the terminal voltage (difference) and is measured in volts. When a battery is connected to a circuit, the electrons from the anode travel through the circuit toward the cathode in a direct circuit. The voltage of a battery is synonymous with its electromotive force, or emf. This force is responsible for the flow of charge through the circuit, known as the electric current.

A battery stores electrical potential from the chemical reaction. When it is connected to a circuit, that electric potential is converted to kinetic energy as the electrons travel through the circuit. Electric potential is defined as the potential energy per unit charge (q). The voltage, or potential difference, between two points is defined to be the change in potential energy of a charge q moved from point 1 to point 2, divided by the charge. Rearranged, this mathematical relationship can be described as:

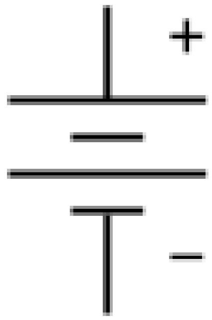
$$\Delta \mathrm{PE} = \mathrm{q}\Delta \mathrm{V} \tag{19.2.1}$$

Voltage is not the same as energy. Voltage is the energy per unit charge. Thus a motorcycle battery and a car battery can both have the same voltage (more precisely, the same potential difference between battery terminals), yet one stores much more energy than the other. The car battery can move more charge than the motorcycle battery, although both are 12V batteries.





Ideal and Real Batteries: A brief introduction to ideal and real batteries for students studying circuits.



Symbol of a Battery in a Circuit Diagram: This is the symbol for a battery in a circuit diagram. It originated as a schematic drawing of the earliest type of battery, a voltaic pile. Notice the positive cathode and negative anode. This orientation is important when drawing circuit diagrams to depict the correct flow of electrons.

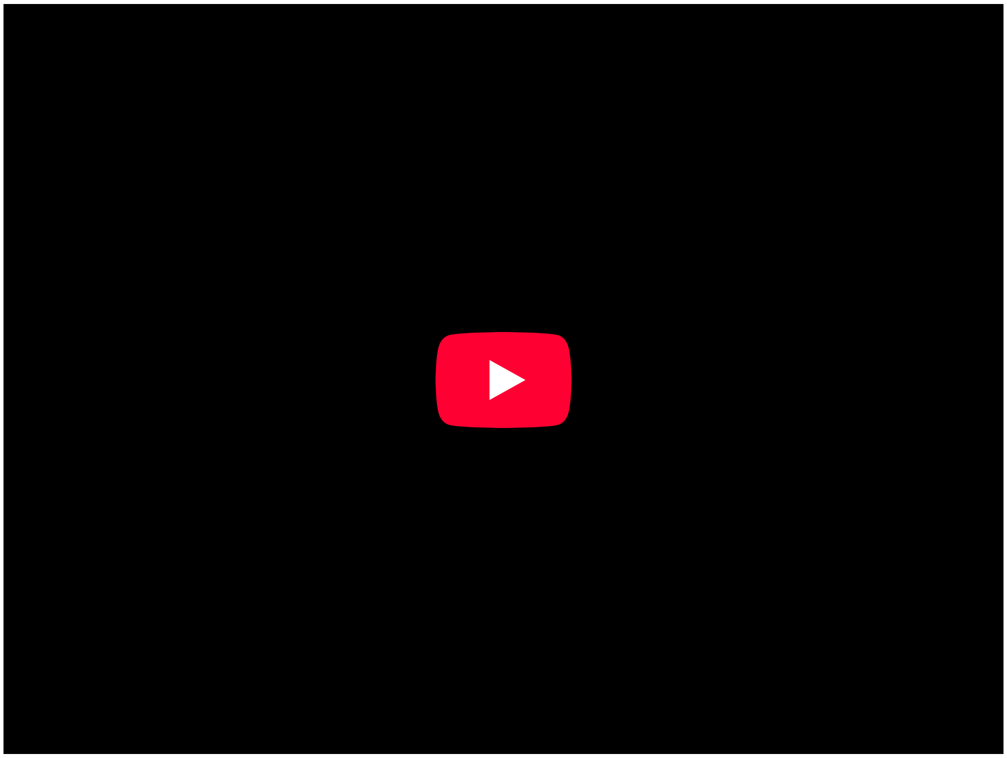
Current and Voltage Measurements in Circuits

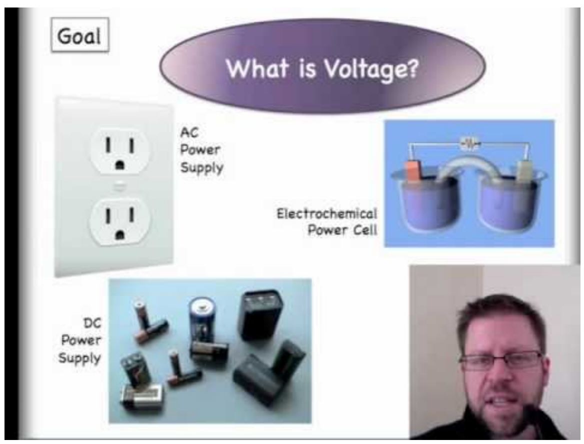
The electrical current is directly proportional to the voltage applied and inversely related to the resistance in a circuit.

learning objectives

- Describe the relationship between the electrical current, voltage, and resistance in a circuit

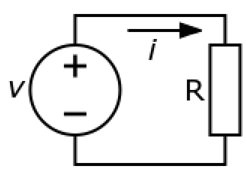
To understand how to measure current and voltage in a circuit, you must also have a general understanding of how a circuit works and how its electrical measurements are related.





What is Voltage?: This video helps with a conceptual understanding of voltage.

An electrical circuit is a type of network that has a closed loop, which provides a return path for the current. A simple circuit consists of a voltage source and a resistor and can be schematically represented as in.



A Simple Circuit: A simple electric circuit made up of a voltage source and a resistor

According to Ohm's law, The electrical current *I*, or movement of charge, that flows through most substances is directly proportional to the voltage *V* applied to it. The electric property that impedes current (crudely similar to friction and air resistance) is called resistance *R*. Collisions of moving charges with atoms and molecules in a substance transfer energy to the substance and limit current. Resistance is inversely proportional to current. Ohm's law can therefore be written as follows:

$$I = \frac{V}{R} \tag{19.2.2}$$

where I is the current through the conductor in amperes, V is the potential difference measured across the conductor in volts, and R is the resistance of the conductor in ohms (Ω). More specifically, Ohm's law states that R in this relation is constant, independent of the current. Using this equation, we can calculate the current, voltage, or resistance in a given circuit.

For example, if we had a 1.5V battery that was connected in a closed circuit to a lightbulb with a resistance of 5Ω, what is the current flowing through the circuit? To solve this problem, we would just substitute the given values into Ohm's law: I = 1.5V/5Ω; I = 0.3 amperes. If we know the current and the resistance, we can rearrange the Ohm's law equation and solve for voltage V:

$$V = IR \tag{19.2.3}$$

A Microscopic View: Drift Speed

The drift velocity is the average velocity that a particle achieves due to an electric field.

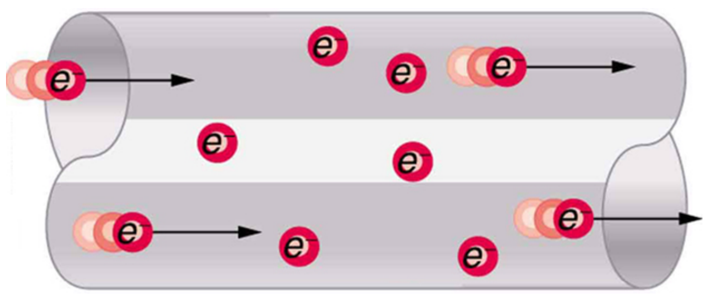
learning objectives

- Relate the drift velocity with the velocity of free charges in conductors

Drift Speed

Electrical signals are known to move very rapidly. Telephone conversations carried by currents in wires cover large distances without noticeable delays. Lights come on as soon as a switch is flicked. Most electrical signals carried by currents travel at speeds on the order of 10^8m/s, a significant fraction of the speed of light. Interestingly, the individual charges that make up the current move much more slowly on average, typically drifting at speeds on the order of 10^{-4}m/s.

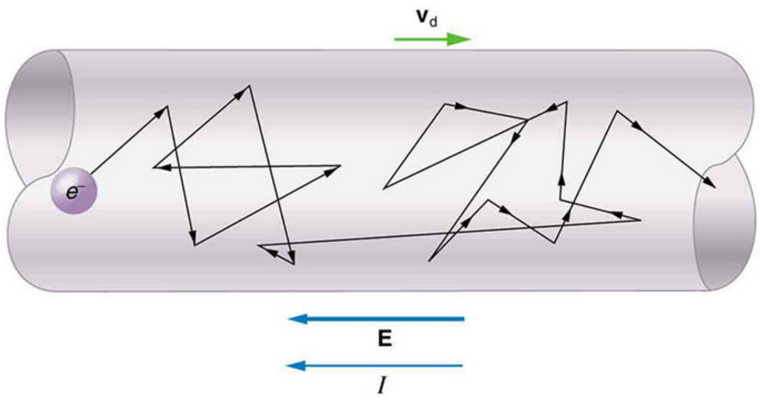
The high speed of electrical signals results from the fact that the force between charges acts rapidly at a distance. Thus, when a free charge is forced into a wire, the incoming charge pushes other charges ahead of it, which in turn push on charges farther down the line. The resulting electrical shock wave moves through the system at nearly the speed of light. To be precise, this rapidly moving signal or shock wave is a rapidly propagating change in the electric field.



Electrons Moving Through a Conductor: When charged particles are forced into this volume of a conductor, an equal number are quickly forced to leave. The repulsion between like charges makes it difficult to increase the number of charges in a volume. Thus, as one charge enters, another leaves almost immediately, carrying the signal rapidly forward.

Drift Velocity

Good conductors have large numbers of free charges in them. In metals, the free charges are free electrons. The distance that an individual electron can move between collisions with atoms or other electrons is quite small. The electron paths thus appear nearly random, like the motion of atoms in a gas. However, there is an electric field in the conductor that causes the electrons to drift in the direction shown (opposite to the field, since they are negative). The drift velocity $v_d$is the average velocity of the free charges after applying the field. The drift velocity is quite small, since there are so many free charges. Given an estimate of the density of free electrons in a conductor (the number of electrons per unit volume), it is possible to calculate the drift velocity for a given current. The larger the density, the lower the velocity required for a given current.



Drift Speed: Free electrons moving in a conductor make many collisions with other electrons and atoms. The path of one electron is shown. The average velocity of the free charges is called the drift velocity and is in the direction opposite to the electric field for electrons. The collisions normally transfer energy to the conductor, requiring a constant supply of energy to maintain a steady current.

It is possible to obtain an expression for the relationship between the current and drift velocity by considering the number of free charges in a segment of wire. *The number of free charges per unit volume* is given the symbol *n* and depends on the material. *Ax* is the volume of a segment, so that the number of free charges in it is *nAx*. The charge *ΔQ* in this segment is thus *qnAx*, where *q* is the amount of charge on each carrier. (Recall that for electrons, *q* is 1.60×10−19C.) The current is the charge moved per unit time. Thus, if all the original charges move out of this segment in time t, the current is:

$$\mathrm{I}=\frac{\Delta \mathrm{Q}}{\Delta \mathrm{t}}=\mathrm{qnA} \frac{\mathrm{x}}{\Delta \mathrm{t}} \tag{19.2.4}$$

Notably, x/Δt is the magnitude of the drift velocity v_d, since the charges move an average distance *x* in a time t. Rearranging terms gives: $I = qnAv_d$, where *I* is the current through a wire of cross-sectional area *A* made of a material with a free charge density *n*. The carriers of the current each have charges *q* and move with a drift velocity of magnitude v_d.

Current density is the electric current per unit area of cross-section. It has units of Amperes per square meter.

Key Points

- A battery stores electrical potential from the chemical reaction. When it is connected to a circuit, that electric potential is converted to kinetic energy as the electrons travel through the circuit.
- The voltage or potential difference between two points is defined to be the change in potential energy of a charge q moved from point 1 to point 2, divided by the charge.
- The voltage of a battery is synonymous with its electromotive force, or emf. This force is responsible for the flow of charge through the circuit, known as the electric current.
- A simple circuit consists of a voltage source and a resistor.
- Ohm 's law gives the relationship between current *I*, voltage *V*, and resistance *R* in a simple circuit: *I* = *V*/*R*.
- The SI unit for measuring the rate of flow of electric charge is the ampere, which is equal to a charge flowing through some surface at the rate of one coulomb per second.
- There is an electric field in conductors that causes electrons to drift in the direction opposite to the field. The drift velocity is the average velocity of these free charges.
- The expression for the relationship between the current and drift velocity can be obtained by considering the number of free charges in a segment of wire.
- *I* = *qnAv* relates the drift velocity to the current, where *I* is the current through a wire of cross-sectional area *A* made of a material with a free charge density *n*. The carriers of the current each have a charge *q* and move with a drift velocity of magnitude *v*.

Key Terms

- **battery**: A device that produces electricity by a chemical reaction between two substances.
- **current**: The time rate of flow of electric charge.

- **voltage**: The amount of electrostatic potential between two points in space.
- **electrical current**: the movement of charge through a circuit
- **ohm**: in the International System of Units, the derived unit of electrical resistance; the electrical resistance of a device across which a potential difference of one volt causes a current of one ampere; symbol: Ω
- **ampere**: A unit of electrical current; the standard base unit in the International System of Units. Abbreviation: amp. Symbol: A.
- **drift velocity**: The average velocity of the free charges in a conductor.

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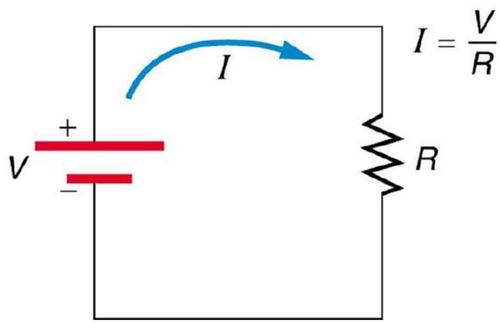
19.3: Resistance and Resistors

- Contrast shape of current-voltage plots for ohmic and non-ohmic circuits

Ohm's Law

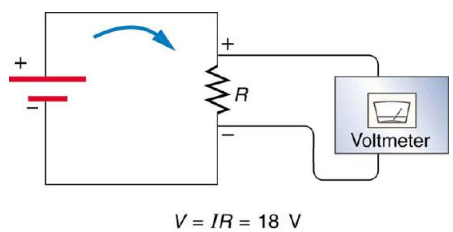
What drives current? We can think of various devices—such as batteries, generators, wall outlets, and so on—which are necessary to maintain a current. All such devices create a potential difference and are loosely referred to as voltage sources. When a voltage source is connected to a conductor, it applies a potential difference V that creates an electric field. The electric field, in turn, exerts force on charges, causing current. The current that flows through most substances is directly proportional to the voltage V applied to it. The German physicist Georg Simon Ohm (1787-1854) was the first to experimentally demonstrate that the current in a metal wire is directly proportional to the voltage applied:\ $mathrmI \propto \mathrm{V}$.

This important relationship is known as Ohm's law. It can be viewed as a cause-and-effect relationship, with voltage the cause and current the effect. This is an empirical law like that for friction—an experimentally observed phenomenon. Such a linear relationship doesn't always occur. Recall that while voltage drives current, resistance impedes it. Collisions of moving charges with atoms and molecules in a substance transfer energy to the substance and limit current. The current is therefore inversely proportional to the resistance: $\mathrm{I} \propto \frac{1}{\mathrm{R}}$.



Simple Circuit: A simple electric circuit in which a closed path for current to flow is supplied by conductors (usually metal wires) connecting a load to the terminals of a battery, represented by the red parallel lines. The zigzag symbol represents the single resistor and includes any resistance in the connections to the voltage source.

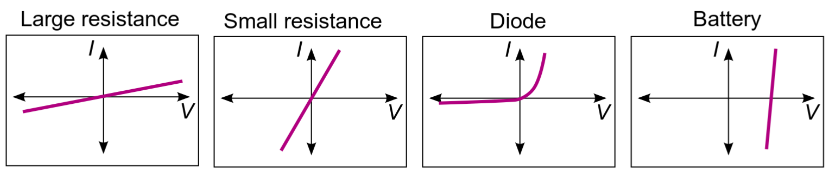
The unit for resistance is the ohm where 1Ω = 1 V/A. We can combine the two relations above to obtain I = V/R. This relationship is also called Ohm's law. In this form Ohm's law really defines resistance for certain materials. Ohm's law (like Hooke's law) is not universally valid. The many substances for which Ohm's law holds are called ohmic. These include good conductors like copper and aluminum, and some poor conductors under certain circumstances. Ohmic materials have a resistance R that is independent of voltage V and current I. An object that has simple resistance is called a resistor, even if its resistance is small.



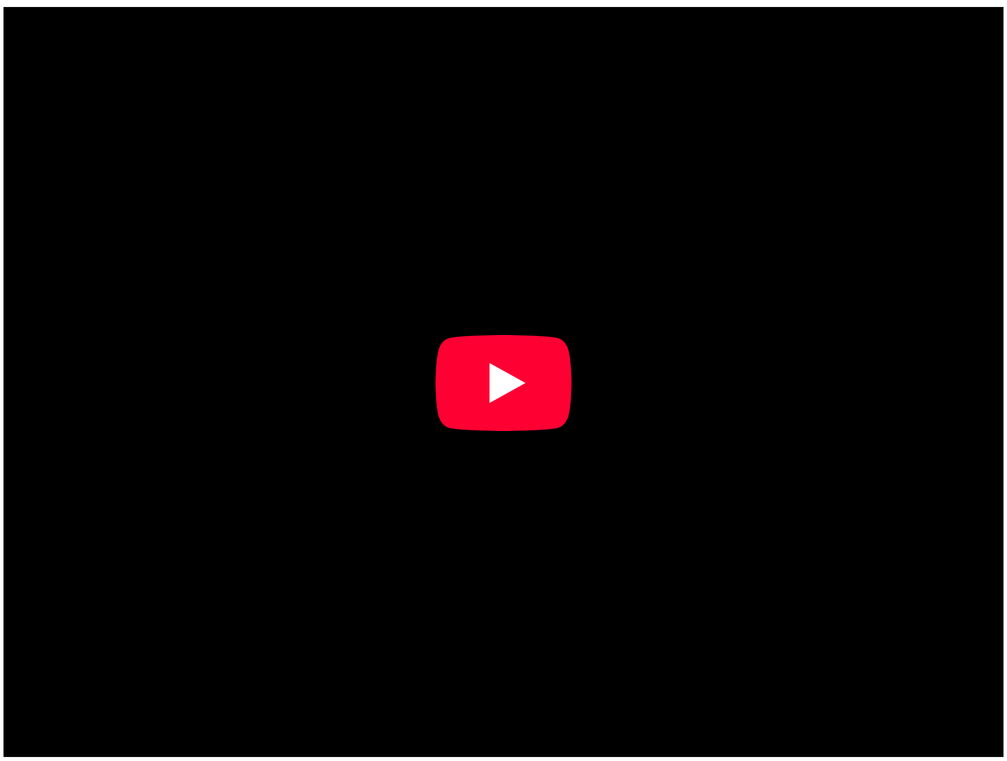
Voltage Drop: The voltage drop across a resistor in a simple circuit equals the voltage output of the battery.

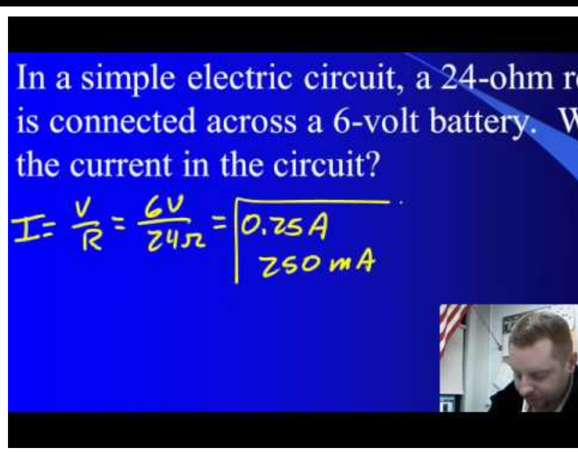
Additional insight is gained by solving I=V/R for V, yielding V=IR. This expression for V can be interpreted as the voltage drop across a resistor produced by the flow of current I. The phrase IR drop is often used for this voltage. If voltage is measured at various points in a circuit, it will be seen to increase at the voltage source and decrease at the resistor. Voltage is similar to fluid pressure. The voltage source is like a pump, creating a pressure difference, causing current—the flow of charge. The resistor is like a pipe that reduces pressure and limits flow because of its resistance. Conservation of energy has important consequences here. The voltage source supplies energy (causing an electric field and a current), and the resistor converts it to another form (such as thermal energy). In a simple circuit (one with a single simple resistor), the voltage supplied by the source equals the voltage drop across the resistor, since E=qΔV, and the same q flows through each. Thus, the energy supplied by the voltage source and the energy converted by the resistor are equal.

In a true ohmic device, the same value of resistance will be calculated from R = V/I regardless of the value of the applied voltage V. That is, the ratio of V/I is constant, and when current is plotted as a function of voltage the curve is linear (a straight line). If voltage is forced to some value V, then that voltage V divided by measured current I will equal R. Or if the current is forced to some value I, then the measured voltage V divided by that current I is also R. We visualize the plot of I versus V as a straight line. There are, however, components of electrical circuits which do not obey Ohm's law; that is, their relationship between current and voltage (their I–V curve) is nonlinear (or non-ohmic). An example is the p-n junction diode.



Current-Voltage Curves: The I–V curves of four devices: two resistors, a diode, and a battery. The two resistors follow Ohm's law: The plot is a straight line through the origin. The other two devices do not follow Ohm's law.





Ohm’s Law: A brief overview of Ohm’s Law.

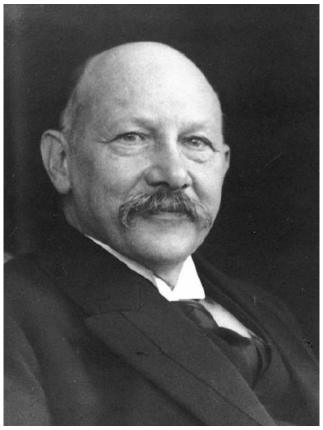
Temperature and Superconductivity

Superconductivity is a phenomenon of zero electrical resistance and expulsion of magnetic fields in certain materials below a critical temp.

learning objectives

- Describe behaviors of a superconductor below a critical temperature and in a weak external magnetic field

Superconductivity is a phenomenon of exactly zero electrical resistance and expulsion of magnetic fields occurring in certain materials when cooled below a characteristic critical temperature. It was discovered by Heike Kamerlingh Onnes (shown in) on April 8, 1911 in Leiden.

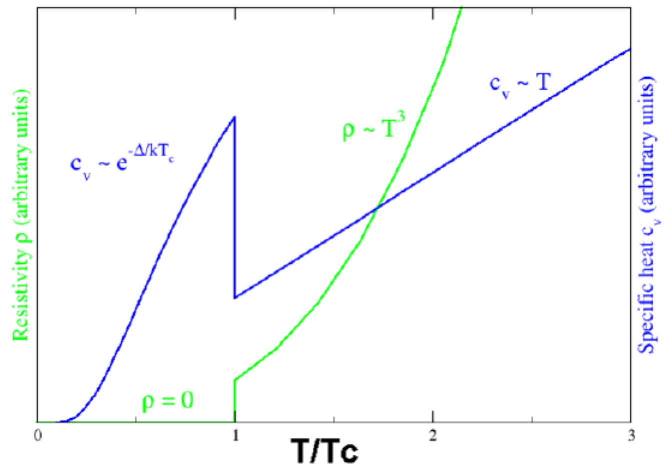


Heike Kamerlingh Onnes: Heike Kamerlingh Onnes (1853-1926).

Most of the physical properties of superconductors vary from material to material, such as the heat capacity and the critical temperature, critical field, and critical current density at which superconductivity is destroyed. On the other hand, there is a class of properties independent of the underlying material. For instance, all superconductors have exactly zero resistivity to low applied currents when there is no magnetic field present or if the applied field does not exceed a critical value. The existence of these “universal” properties implies that superconductivity is a thermodynamic phase, and thus possesses certain distinguishing properties that are largely independent of microscopic details.

In superconducting materials, the characteristics of superconductivity appear when the temperature T is lowered below a critical temperature T_c. The onset of superconductivity is accompanied by abrupt changes in various physical properties—the hallmark of a phase transition. For example, the electronic heat capacity is proportional to the temperature in the normal (non-superconducting) regime. At the superconducting transition, it suffers a discontinuous jump and thereafter ceases to be linear, as illustrated in.

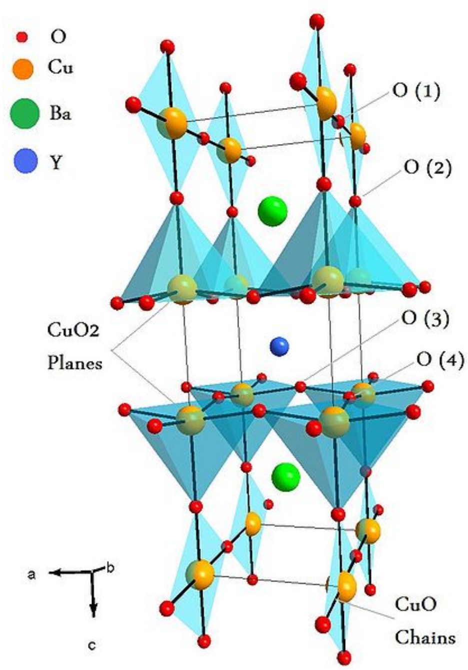
When a superconductor is placed in a weak external magnetic field H, and cooled below its transition temperature, the magnetic field is ejected. The Meissner effect does not cause the field to be completely ejected. Rather, the field penetrates the superconductor to a very small distance (characterized by a parameter λ), called the London penetration depth. It decays exponentially to zero within the bulk of the material. The Meissner effect is a defining characteristic of superconductivity. For most superconductors, the London penetration depth is on the order of 100 nm.



Superconducting phase transition: Behavior of heat capacity (cv, blue) and resistivity (ρ, green) at the superconducting phase transition.

Superconductors are also able to maintain a current with no applied voltage whatsoever—a property exploited in superconducting electromagnets such as those found in MRI machines. Experiments have demonstrated that currents in superconducting coils can persist for years without any measurable degradation. Experimental evidence points to a current lifetime of at least 100,000 years. Theoretical estimates for the lifetime of a persistent current can exceed the estimated lifetime of the universe, depending on the wire geometry and the temperature.

The value of this critical temperature varies from material to material. Usually, conventional superconductors have critical temperatures ranging from around 20 K to less than 1 K. Solid mercury, for example, has a critical temperature of 4.2 K. As of 2009, the highest critical temperature found for a conventional superconductor is 39 K for magnesium diboride (MgB_2), although this material's exotic properties cause some doubt about accurately classifying it as a "conventional" superconductor. High-temperature superconductors can have much higher critical temperatures. For example, $YBa_2Cu_3O_7$, one of the first cuprate superconductors to be discovered, has a critical temperature of 92 K; mercury-based cuprates have been found with critical temperatures in excess of 130 K. It is of note that the chemical composition and crystal structure of superconducting materials can be quite complex, as seen in.



Unit Cell of YBaCuO superconductor: Unit Cell of YBaCuO superconductor. Atoms are indicated with different colors.

Resistance and Resistivity

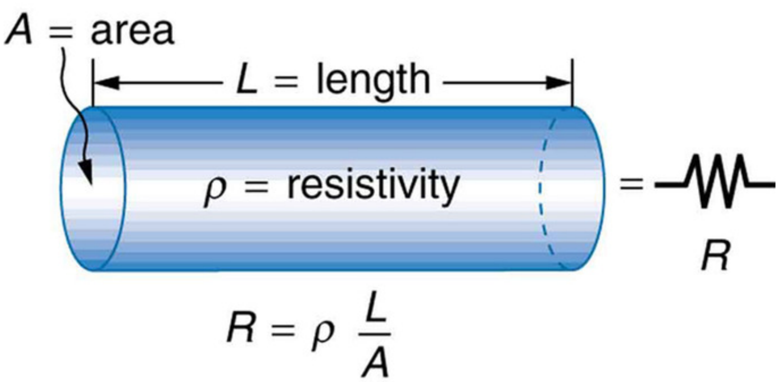
Resistance and resistivity describe the extent to which an object or material impedes the flow of electric current.

learning objectives

- Identify properties of the material that are described by the resistance and resistivity

Resistance and Resistivity

Resistance is the electric property that impedes a current. A current flowing through a wire (or resistor) is like water flowing through a pipe, and the voltage drop across the wire is like the pressure drop which pushes water through the pipe. Resistance is proportional to how much pressure is required to achieve a given flow, while conductance is proportional to how much flow occurs for a given pressure. Conductance and resistance are reciprocals. The resistance of an object depends on its shape and the material of which it is composed. The cylindrical resistor is easy to analyze, and by so doing we can gain insight into the resistance of more complicated shapes. As you might expect, the cylinder's electric resistance R is directly proportional to its length L, similar to the resistance of a pipe to fluid flow. The longer the cylinder, the more collisions charges will make with its atoms. The greater the diameter of the cylinder, the more current it can carry (again, similar to the flow of fluid through a pipe). In fact, R is inversely proportional to the cylinder's cross-sectional area A.



Cylindrical Resistor: A uniform cylinder of length L and cross-sectional area A. Its resistance to the flow of current is similar to the resistance posed by a pipe to fluid flow. The longer the cylinder, the greater its resistance. The larger its cross-sectional area A, the smaller its resistance.

As mentioned, for a given shape, the resistance depends on the material of which the object is composed. Different materials offer different resistance to the flow of charge. We define the resistivity ρ of a substance so that theresistanceR of an object is directly proportional to ρ. Resistivity ρ is an *intrinsic* property of a material, independent of its shape or size. In contrast, the resistance R is an extrinsic property that does depend on the size an shape of the resistor. (A similar intrinsic/extrinsic relation exists between heat capacity C and the specific heat c). Recall that an object whose resistance is proportional to the voltage and current is known as a resistor.

Typical Resistor: A typical axial-lead resistor.

What determines resistivity? The resistivity of different materials varies by an enormous amount. For example, the conductivity of teflon is about 1030 times lower than the conductivity of copper. Why is there such a difference? Loosely speaking, a metal has large numbers of "delocalized" electrons that are not stuck in any one place, but free to move across large distances, whereas in an insulator (like teflon), each electron is tightly bound to a single atom, and a great force is required to pull it away. Likewise, resistors range over many orders of magnitude. Some ceramic insulators, such as those used to support power lines, have resistances of $10^{12}\ \Omega$ or more. A dry person may have a hand-to-foot resistance of $10^5\ \Omega$, whereas the resistance of the human heart is about $10^3\ \Omega$. A meter-long piece of large-diameter copper wire may have a resistance of $10^{-5}\ \Omega$, and superconductors have no resistance at all (they are non-ohmic). The potential difference (voltage) seen across the network is the sum of those voltages, thus the total resistance (the series equivalent resistance) can be found as the sum of those resistances:

$$\mathrm{R_{eq}=R_1+R_2+\cdots+R_N} \tag{19.3.1}$$

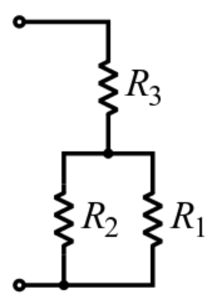
As a special case, the resistance of N resistors connected in series, each of the same resistance R, is given by NR.Resistors in a parallel configuration are each subject to the same potential difference (voltage), however the currents through them add. Thus the equivalent resistance (Req) of the network can be computed:

$$\mathrm{\frac{1}{Req}=\frac{1}{R_1}+\frac{1}{R_2}+\cdots+\frac{1}{RN}} \tag{19.3.2}$$

The parallel equivalent resistance can be represented in equations by two vertical lines "||" (as in geometry) as a simplified notation. Occasionally two slashes "//" are used instead of "||", in case the keyboard or font lacks the vertical line symbol. For the case of two resistors in parallel, this can be calculated using:

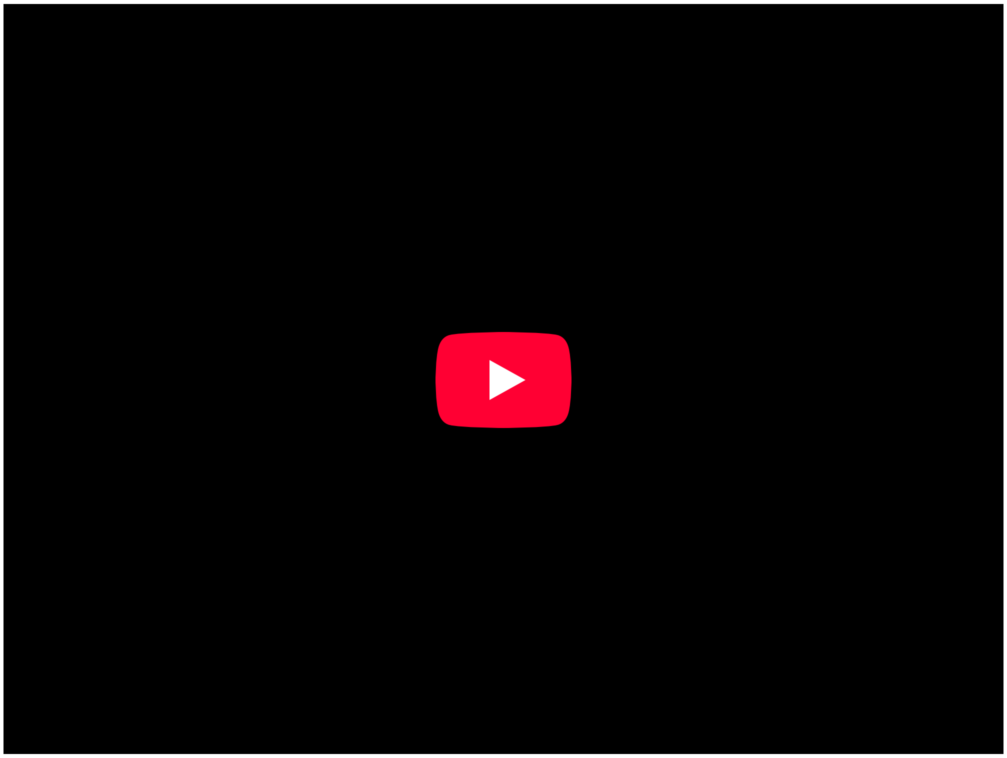
$$\mathrm{R_{eq}=R_1\|R_2=\frac{R_1R_2}{R_1+R_2}} \tag{19.3.3}$$

As a special case, the resistance of N resistors connected in parallel, each of the same resistance R, is given by R/N. A resistor network that is a combination of parallel and series connections can be broken up into smaller parts that are either one or the other, such as is shown in.



Resistor Network: In this combination circuit, the circuit can be broken up into a series component and a parallel component.

However, some complex networks of resistors cannot be resolved in this manner. These require a more sophisticated circuit analysis. One practical application of these relationships is that a non-standard value of resistance can generally be synthesized by connecting a number of standard values in series or parallel. This can also be used to obtain a resistance with a higher power rating than that of the individual resistors used. In the special case of N identical resistors all connected in series or all connected in parallel, the power rating of the individual resistors is thereby multiplied by N.



Resistance, Resistors, and Resistivity: A brief overview of resistance, resistors, and resistivity.

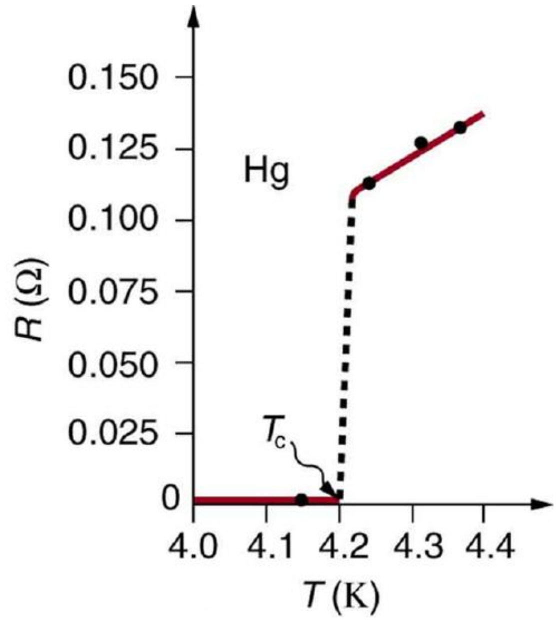
Dependence of Resistance on Temperature

Resistivity and resistance depend on temperature with the dependence being linear for small temperature changes and nonlinear for large.

learning objectives

- Compare temperature dependence of resistivity and resistance for large and small temperature changes

The resistivity of all materials depends on temperature. Some materials can become superconductors (zero resistivity) at very low temperatures (see). Conversely, the resistivity of conductors increases with increasing temperature. Since the atoms vibrate more rapidly and over larger distances at higher temperatures, the electrons moving through a metal, for example, create more collisions, effectively making the resistivity higher. Over relatively small temperature changes (about 100ºC or less), resistivity ρ varies with temperature change ΔT as expressed in the following equation:



Resistance of a sample of mercury: The resistance of a sample of mercury is zero at very low temperatures—it is a superconductor up to about 4.2 K. Above that critical temperature, its resistance makes a sudden jump and then increases nearly linearly with temperature.

$$\mathrm{p} = \mathrm{p}_0(1 + \alpha\Delta\mathrm{T}) \tag{19.3.4}$$

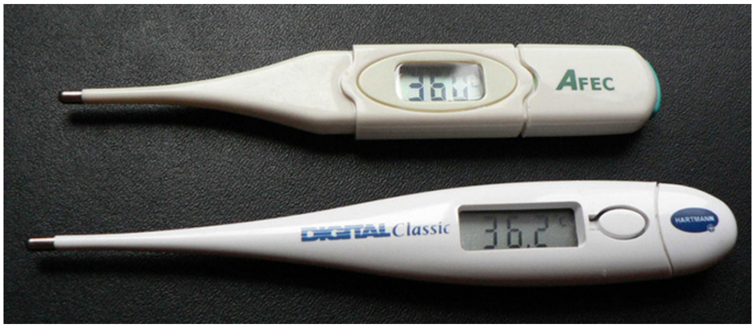
where ρ_0 is the original resistivity and α is the temperature coefficient of resistivity. For larger temperature changes, α may vary, or a nonlinear equation may be needed to find ρ. For this reason it is usual to specify a suffix for the temperature at which the substance was measured (such as α_{15}) and the relationship only holds in a range of temperatures around the reference. Note that α is positive for metals, meaning their resistivity increases with temperature. The temperature coefficient is typically $+3\times10^{-3}\ K^{-1}$ to $+6\times10^{-3}\ K^{-1}$ for metals near room temperature. Some alloys have been developed specifically to have a small temperature dependence. Manganin (made of copper, manganese and nickel), for example, has α close to zero, so its resistivity varies only slightly with temperature. This is useful for making a temperature-independent resistance standard, for example.

Note also that α is negative for semiconductors, meaning that their resistivity decreases with increasing temperature. They become better conductors at higher temperature because increased thermal agitation increases the number of free charges available to carry current. This property of decreasing ρ with temperature is also related to the type and amount of impurities present in the semiconductors.

The resistance of an object also depends on temperature, since R_0 is directly proportional to ρ. For a cylinder we know R=ρL/A, so if L and A do not change greatly with temperature, R will have the same temperature dependence as ρ. (Examination of the coefficients of linear expansion shows them to be about two orders of magnitude less than typical temperature coefficients of resistivity, and so the effect of temperature on L and A is about two orders of magnitude less than on ρ.) Thus,

$$\mathrm{R} = \mathrm{R}_0(1 + \alpha\Delta\mathrm{T}) \tag{19.3.5}$$

is the temperature dependence of the resistance of an object, where R_0 is the original resistance and R is the resistance after a temperature change T. Numerous thermometers are based on the effect of temperature on resistance (see). One of the most common is the thermistor, a semiconductor crystal with a strong temperature dependence, the resistance of which is measured to obtain its temperature. The device is small so it quickly comes into thermal equilibrium with the part of a person it touches.



Thermometers: These familiar thermometers are based on the automated measurement of a thermistor's temperature-dependent resistance.

Key Points

- Voltage drives current while resistance impedes it.
- Ohm 's Law refers to the proportion relation between voltage and current. It also applies to the specific equation V=IR, which is valid when considering circuits that contain simple resistors (whose resistance is independent of voltage and current).
- Circuits or components that obey the relation V=IR are known as ohmic and have current-voltage plots that are linear and pass through the origin.
- There are components and circuits that are non-ohmic; their I-V plots are not linear and/or don't pass through the origin.
- Superconductivity is a Superconductivity is a thermodynamic phase and possesses certain distinguishing properties which are largely independent of microscopic details.
- In superconducting materials, the characteristics of superconductivity appear when the temperature is lowered below a critical temperature. The onset of superconductivity is accompanied by abrupt changes in various physical properties.
- When a superconductor is placed in a weak external magnetic field H, and cooled below its transition temperature, the magnetic field is ejected.
- Superconductors are able to maintain a current with no applied voltage.
- The resistance of an object (i.e., a resistor) depends on its shape and the material of which it is composed.
- Resistivity ρ is an intrinsic property of a material and directly proportional to the total resistance R, an extrinsic quantity that depends on the length and cross-sectional area of a resistor.
- The resistivity of different materials varies by an enormous amount. Likewise, resistors range over many orders of magnitude.
- Resistors are arranged in series or parallel configurations. The equivalent resistance of a network of resistors in series is the sum of all the resistance. The inverse of the equivalent resistance of a network of resistors in parallel is the sum of the inverse of the resistance of each resistor.
- Over temperature changes of 100ºC or less, resistivity (ρ) varies with temperature change ΔT as: p=p0(1+αΔT)p=p0(1+αΔT) where ρ_0 is the original resistivity and α is the temperature coefficient of resistivity.
- For large temperature changes, nonlinear variation of resistivity with temperature is observed.
- The resistance of an object demonstrates similar temperature dependence as resistivity since resistance is directly proportional to resistivity.

Key Terms

- **simple circuit**: A circuit with a single voltage source and a single resistor.
- **ohmic**: That which obeys Ohm's law.
- **high-temperature superconductors**: Materials that behave as superconductors at unusually high temperatures (above about 30 K).
- **critical temperature**: In superconducting materials, the characteristics of superconductivity appears at (and continues below) this temperature.
- **superconductivity**: The property of a material whereby it has no resistance to the flow of an electric current.
- **series equivalent resistance**: The resistance of a network of resistors arranged such that the voltage across the network is the sum of the voltage across each resistor. In this case, the equivalent resistance is the sum of the resistance of all the resistors in the network.

- **parallel equivalent resistance**: the resistance of a network such that each resistor is subject to the same potential difference (voltage), so that the currents through them add. In this case the inverse of the equivalent resistance is equal to the sum of the inverse resistance of all the resistors in the network.
- **resistivity**: In general, the resistance to electric current of a material; in particular, the degree to which a material resists the flow of electricity.
- **temperature coefficient of resistivity**: An empirical quantity, denoted by α, which describes the change in resistance or resistivity of a material with temperature.
- **semiconductor**: A substance with electrical properties intermediate between a good conductor and a good insulator.

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19.4: Electric Power and Energy

learning objectives

- Formulate the relationship between the energy usage and the electric power

In many cases it is necessary to calculate the energy usage by an electric device or a collection of devices, such as in a home. For example, we (or the electric power utility) may want to calculate the amount of money owed for electricity consumed. In another case, we might need to determine the energy necessary to power a component or apparatus for a given period of time. The last distinction is crucial – the energy used by a circuit or component is *the time integral of the electric power*.

Power

Recall that power is the rate at which work is done – or the rate at which energy is consumed or produced – and is measured in watts (W). The electric power in watts produced by an electric current I consisting of a charge of Q coulombs every t seconds passing through an electric potential (voltage) difference of V is $\mathrm{P}=\frac{\mathrm{QV}}{\mathrm{t}}=\mathrm{IV}$, where Q is electric charge in coulombs, t is time in seconds, I is electric current in amperes, and V is electric potential or voltage in volts.

In resistive circuits where Ohm ‘s Law applies, the power can be expressed as $\mathrm{P}=\mathrm{I}^2\mathrm{R}=\frac{\mathrm{V}^2}{\mathrm{R}}$, where R is the electrical resistance. Power isn’t necessarily constant; it may vary over time. The general expression for electric power is then

$$\mathrm{P}(\mathrm{t})=\mathrm{I}(\mathrm{t})\mathrm{V}(\mathrm{t}) \tag{19.4.1}$$

where the current I and voltage V may be time variable.

Energy

In any given time interval, the energy consumed (or provided, depending on your perspective) is given by $\mathrm{PE}=\mathrm{qV}$, where E is the electric energy, V is the voltage, and q is the amount of charge moved in the time interval under consideration. We can relate the total energy consumed to the power by integrating over time:. Positive energy corresponds to consumed energy and negative energy corresponds to energy production. Note that a circuit element having a power profile that is both positive and negative over some time interval could consume or produce energy according to the sign of the integral of power. If the power is constant over the time interval then the energy can be expressed simply as:

$$\mathrm{E}=\mathrm{Pt} \tag{19.4.2}$$

Units of Energy Usage

We are of course very familiar with the SI unit of energy, the joule. However, typically, residential energy bills state household energy consumption in kilowatt-hours (kWh). Additionally, this unit is often seen elsewhere when the energy usage of power consuming devices, structures, or jurisdictions is under consideration. We can parse out the conversion from kilowatt-hours to joules in this way: 1 W = 1 J/s and a kilowatt is 1000 W while one hour is 3,600 seconds, so 1 kWh is (1000 J/s)(3600 s)=3,600,000 joules. This is the scale of American home energy usage, which is on the order of hundreds of kilowatt-hours per month.

Reducing Energy Usage

The electrical energy (E) used can be reduced either by reducing the time of use or by reducing the power consumption of that appliance or fixture. This will not only reduce the cost, but it will also result in a reduced impact on the environment. Improvements to lighting are some of the fastest ways to reduce the electrical energy used in a home or business. About 20% of a home’s use of energy goes to lighting, while the number for commercial establishments is closer to 40%. Fluorescent lights are about four times more efficient than incandescent lights—this is true for both the long tubes and the compact fluorescent lights (CFL). Thus, a 60-W incandescent bulb can be replaced by a 15-W CFL, which has the same brightness and color. CFLs have a bent tube inside a globe or a spiral-shaped tube, all connected to a standard screw-in base that fits standard incandescent light sockets. (Original problems with color, flicker, shape, and high initial investment for CFLs have been addressed in recent years.) The heat transfer from these CFLs is less, and they last up to 10 times longer.

Compact Fluorescent Light (CFL): CFLs are much more efficient than incandescent bulbs and so consume much less energy for the intensity light produces.

Key Points

- Recall that power is the rate work is done, or the rate at which energy is consumed or produced. In terms of current and voltage it is P=IV.
- The energy used is the amount of charge q moved through voltage V in a time interval t. It is equal to the integral of power over time.
- A common unit used to describe energy usage is the kilowatt-hour, the energy of 1000 W acting over one hour.

Key Terms

- **kilowatt-hour**: a unit of electrical energy equal to that done by one kilowatt acting for one hour; equal to 3.6 mega-joules. Symbol: kWh.

19.5: Alternating Currents

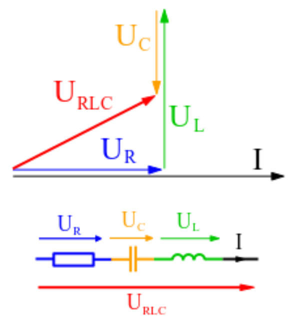
learning objectives

- Discuss applications of a phase vector

Phasors

Complex numbers play an important role in physics. Usually, complex numbers are written in terms of their real part plus the imaginary part. For example, $a + bi$ where a and b are real numbers, and ii signals the imaginary part. However, it is often practical to write complex numbers in the form of an exponential called a phasor.

In physics, a phase vector, or phasor, is a representation of a sinusoidal function whose amplitude (A), frequency (ω), and phase (θ) are time-invariant, as diagramed in. Phasors separate the dependencies on A, ω, and θ into three independent factors. This can be particularly useful because the frequency factor (which includes the time-dependence of the sinusoid) is often common to all the components of a linear combination of sinusoids. In those situations, phasors allow this common feature to be factored out, leaving just the A and θ features. The result is that trigonometry reduces to algebra, and linear differential equations become algebraic ones. The term phasor therefore often refers to just those two factors.



Phasor Diagram: *An example of series RLC circuit and respective phasor diagram for a specific ω. Electrical engineers, electronics engineers, electronic engineering technicians and aircraft engineers all use phasor diagrams to visualize complex constants and variables (phasors). Like vectors, arrows drawn on graph paper or computer displays represent phasors.*

Phasors are often used in electrical systems when considering voltages and currents that vary sinusoidally in time, such as in RLC circuits.

Definition

Sinusoids can be represented mathematically as the sum of two complex-valued functions:

$$A \cdot \cos(\omega t + \theta) = A \cdot \frac{e^{i(\omega t+\theta)} + e^{-i(\omega t+\theta)}}{2} \tag{19.5.1}$$

or as the real part of one of the functions:

$$A \cdot \cos(\omega t + \theta) = \mathrm{Re}\left\{A \cdot e^{i(\omega t+\theta)}\right\} = \mathrm{Re}\{Ae^{i\theta} \cdot e^{i\omega t}\} \tag{19.5.2}$$

As indicated above, phasor can refer to either Aeiθ·eiωtAeiθ·eiωt or just the complex constant, AeiθAeiθ. In the latter case, it is understood to be a shorthand notation, encoding the amplitude and phase of an underlying sinusoid.

Phasor Representation of Signals

There are two key ideas behind the phasor representation of a signal:

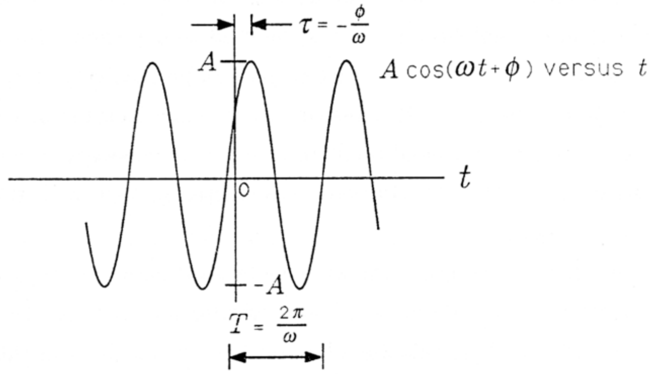
1. a real, time-varying signal may be represented by a complex, time-varying signal; and
2. a complex, time-varying signal may be represented as the product of a complex number that is independent of time and a complex signal that is dependent on time.

The signal:

$$x(t) = A\cos(\omega t + \theta) \tag{19.5.3}$$

illustrated in the figure below is a cosinusoidal signal with amplitude *A*, frequency, and phase θ. The amplitude *A* characterizes the peak-to-peak swing of *2A*, the angular frequency ω characterizes the period $T = 2\pi/\omega$ between negative- to-positive zero crossings (or positive peaks or negative peaks), and the phase θ characterizes the time $\tau = -\theta/\omega$ when the signal reaches its first peak. With so defined, the signal *x(t)* may also be written as

$$x(t) = A\cos(t - \tau) \tag{19.5.4}$$



Cosinusoidal Signal: A Cosinusoidal Signal.

When τ is positive, then τ is a "time delay" that describes the time (greater than zero) when the first peak is achieved. When τ is negative, then τ is a "time advance" that describes the time (less than zero) when the last peak was achieved. With the substitution $=2\pi/T$ we obtain a third way of writing x(t):

$$x(t) = A\cos\frac{2\pi}{T}(t - \boldsymbol{\tau}) \tag{19.5.5}$$

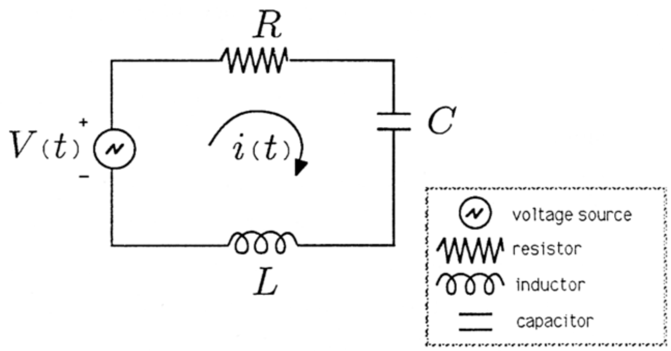
In this form the signal is easy to plot. Simply draw a cosinusoidal wave with amplitude *A* and period *T*; then strike the origin (t=0) so that the signal reaches its peak at τ. In summary, the parameters that determine a cosinusoidal signal have the following units:

- *A*, arbitrary (e.g., volts or meters/sec, depending upon the application)
- ω, in radians /sec (rad/sec)
- *T*, in seconds (sec)
- θ, in radians (rad)
- τ, in seconds (sec)

Sinusoidal Steady State and the Series RLC CircuitPhasors may be used to analyze the behavior of electrical and mechanical systems that have reached a kind of equilibrium called sinusoidal steady state.

In the sinusoidal steady state, every voltage and current (or force and velocity) in a system is sinusoidal with angular frequency ω. However, the amplitudes and phases of these sinusoidal voltages and currents are all different.

For example, the voltage across a resistor might lead the voltage across a capacitor by 90° and lag the voltage across an inductor by 90°. In order to make our application of phasors to electrical systems concrete, we consider the series *RLC* circuit illustrated in. The arrow labeled *i(t)* denotes a current that flows in response to the voltage applied.



Series RLC Circuit: Series RLC Circuit.

We will assume that the voltage source is an audio oscillator that produces the voltage:

$$V(t)=A\cos(\omega t+\theta) \tag{19.5.6}$$

We represent this voltage as the complex signal:

$$V(t)\leftrightarrow Ae^{i\theta}\cdot e^{i\omega t} \tag{19.5.7}$$

and give it the phasor representation,

$$V(t)\leftrightarrow V;V=Ae^{i\theta} \tag{19.5.8}$$

We then describe the voltage source by the phasor V and remember that we can always compute the actual voltage by multiplying by $e^{i\omega t}$and taking the real part.

Root Mean Square Values

The root mean square (RMS) voltage or current is the time-averaged voltage or current in an AC system.

learning objectives

- Relate the root mean square voltage and current in an alternating circut with the peak voltage and current and the average power

Root Mean Square Values and Alternating Current

Recall that in the case of alternating current (AC) the flow of electric charge periodically reverses direction. Unlike direct current (DC), where the currents and voltages are constant, AC currents and voltages vary over time. Recall that most residential and commercial power sources use AC. It is often the case that we wish to know the *time averaged*current, or voltage. Given the current or voltage as a function of time, we can take the root mean square over time to report the average quantities.

Definition

The root mean square (abbreviated RMS or rms), also known as the quadratic mean, is a statistical measure of the magnitude of a varying quantity. It is especially useful when the function alternates between positive and negative values, e.g., sinusoids.The RMS value of a set of values (or a continuous-time function such as a sinusoid) is the square root of the arithmetic mean of the squares of the original values (or the square of the function). In the case of a set of n values $\{x_1,x_2,\ldots,x_n\}$, the RMS value is given by this formula:

$$x_{rms}=\sqrt{\frac{1}{n}\left(x_1^2+x_2^2+\cdots+x_n^2\right)} \tag{19.5.9}$$

The corresponding formula for a continuous function *f(t)*defined over the interval $T_1\leq t\leq T_2$ is as follows:

$$f_{rms}=\sqrt{\frac{1}{T_2-T_1}\int_{T_1}^{T_2}[t(t)]^2dt} \tag{19.5.10}$$

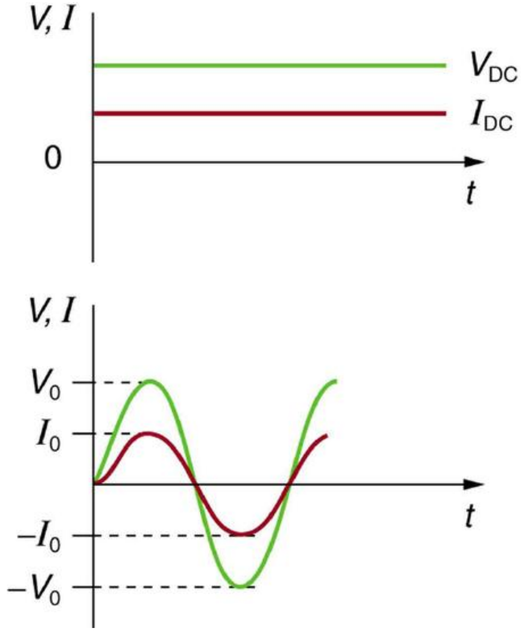
The RMS for a function over all time is below.

$$f_{rms} = \lim_{T \to \infty} \sqrt{\frac{1}{T} \int_0^T [t(t)]^2 dt} \tag{19.5.11}$$

The RMS over all time of a periodic function is equal to the RMS of one period of the function. The RMS value of a continuous function or signal can be approximated by taking the RMS of a series of equally spaced samples.

Application to Voltage and Current

Consider the case of sinusoidally varying voltage:



Sinusoidal Voltage and Current: (a) DC voltage and current are constant in time, once the current is established. (b) A graph of voltage and current versus time for 60-Hz AC power. The voltage and current are sinusoidal and are in phase for a simple resistance circuit. The frequencies and peak voltages of AC sources differ greatly.

$$V = V_0 \sin(2\pi ft) \tag{19.5.12}$$

V is the voltage at time t, V_0 is the peak voltage, and f is the frequency in hertz. For this simple resistance circuit, I=V/R, and so the AC current is as follows:

$$\mathbf{I} = \mathbf{I}_0 \sin(2\pi ft) \tag{19.5.13}$$

Here, I is the current at time t, and I_0=V_0/R is the peak current. Now using the definition above, let's calculate the rms voltage and rms current. First, we have

$$V_{rms} = \sqrt{\frac{1}{T_2 - T_1} \int_{T_1}^{T_2} [V_0 \sin(\omega t)]^2 dt} \tag{19.5.14}$$

Here, we have replaced 2πf with ω. Since V_0 is a constant, we can factor it out of the square root, and use a trig identity to replace the squared sine function.

$$V_{rms} = V_0 \sqrt{\frac{1}{T_2 - T_1} \int_{T_1}^{T_2} \frac{1 - \cos(2\omega t)}{2} dt} \tag{19.5.15}$$

Integrating the above, we have:

$$V_{rms} = V_0 \sqrt{\frac{1}{T_2 - T_1} \left[\frac{t}{2} - \frac{\sin(2\omega t)}{4\omega}\right]_{T_1}^{T_2}} \tag{19.5.16}$$

Since the interval is a whole number of complete cycles (per definition of RMS), the terms will cancel out, leaving:

$$\mathrm{V_{rm}} = \mathrm{V_0}\sqrt{\frac{1}{\mathrm{T_2 - T_1}}\left[\frac{\mathrm{t}}{2}\right]\mathrm{T_1}} = \mathrm{V_0}\sqrt{\frac{1}{\mathrm{T_2 - T_1}}\frac{\mathrm{T_2 - T_1}}{2}} \tag{19.5.17}$$

$$= \frac{V_0}{\sqrt{2}} \tag{19.5.18}$$

Similarly, you can find that the RMS current can be expressed fairly simply:

$$\mathrm{I_{rms}} = \mathrm{I_0}/\sqrt{2} \tag{19.5.19}$$

Updated Circuit Equations for AC

Many of the equations we derived for DC current apply equally to AC. If we are concerned with the time averaged result and the relevant variables are expressed as their rms values. For example, Ohm ‘s Law for AC is written as follows:

$$\mathrm{I_{rms}} = \frac{\mathrm{V_{rms}}}{\mathrm{R}} \tag{19.5.20}$$

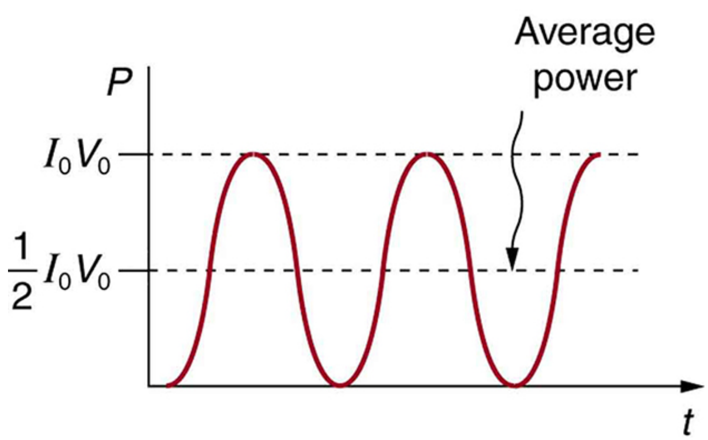
The various expressions for AC power are below:

$$\mathrm{P_{ave}} = \mathrm{I_{rms}V_{rms}} \tag{19.5.21}$$

$$\mathrm{P_{ave}} = \frac{\mathrm{V_{ms}^2}}{\mathrm{R}} \tag{19.5.22}$$

$$\mathrm{P_{ave}} = \mathrm{I_{rms}^2 R} \tag{19.5.23}$$

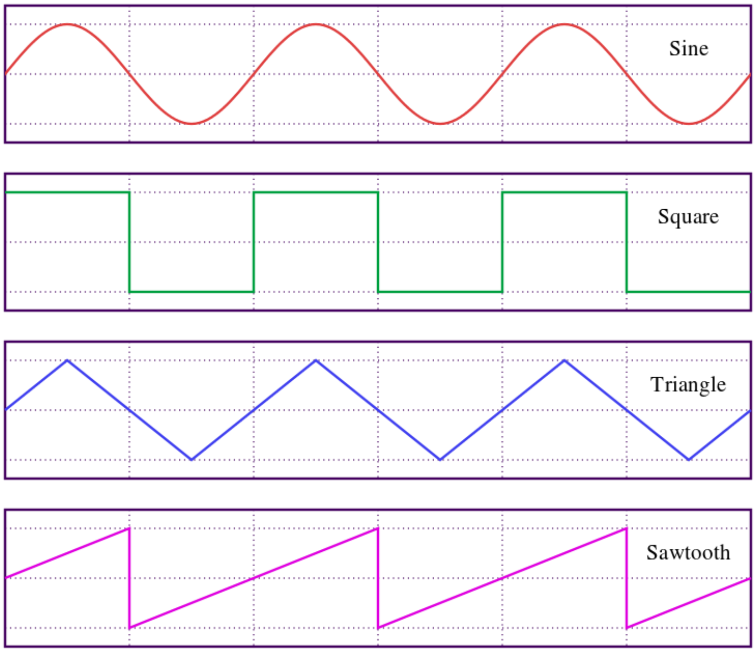
We can see from the above equations that we can express the average power as a function of the peak voltage and current (in the case of sinusoidally varying current and voltage):



Average Power: AC power as a function of time. Since the voltage and current are in phase here, their product is non-negative and fluctuates between zero and I0V0. Average power is (1/2)I0V0.

$$\mathrm{P_{ave}} = \mathrm{I_{rms}}V_{\mathrm{rms}} = \frac{\mathrm{I_0}}{\sqrt{2}}\frac{\mathrm{V_0}}{\sqrt{2}} = \frac{1}{2}\mathrm{V_0I_0} \tag{19.5.24}$$

The RMS values are also useful if the voltage varies by some waveform other than sinusoids, such as with a square, triangular or sawtooth waves.



Waveforms: Sine, square, triangle, and sawtooth waveforms

Safety Precautions in the Household

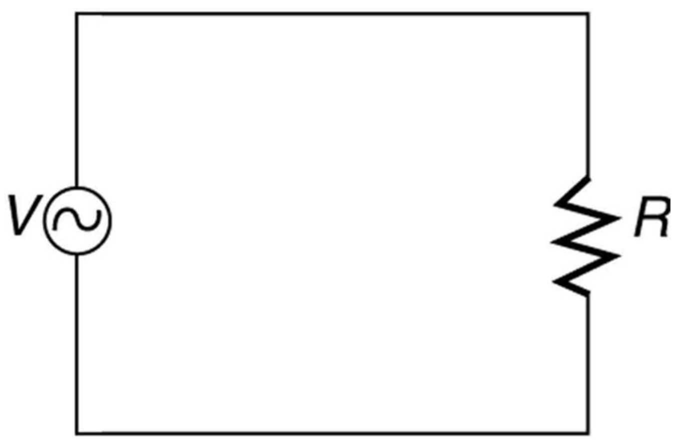
Electrical safety systems and devices are designed and widely used to reduce the risks of thermal and shock hazards.

learning objectives

- Identify major risks associated with the electrical circuits and strategies to mitigate those risks

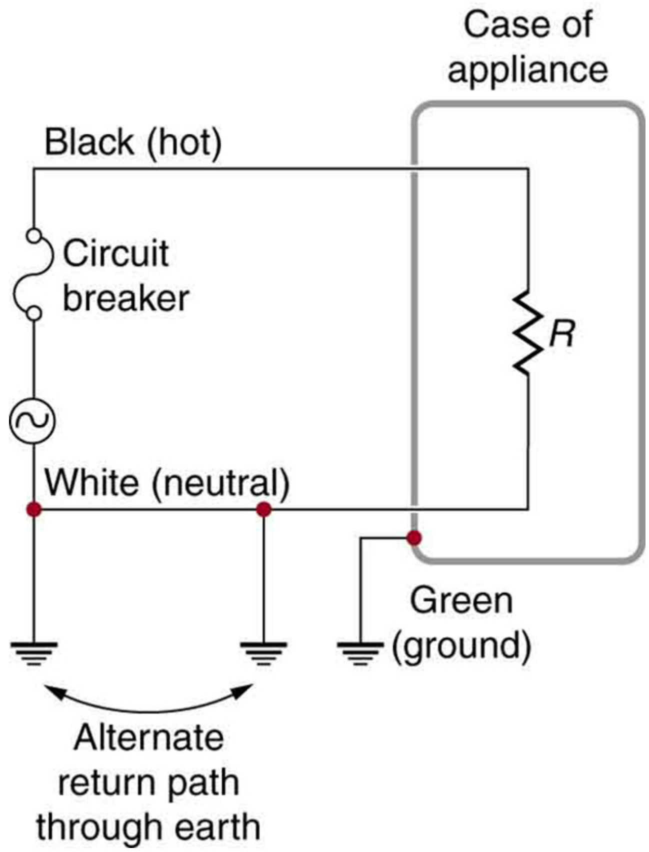
Electrical Safety and Household Appliances

Electricity has two hazards. A thermal hazard occurs in cases of electrical overheating. A shock hazard occurs when an electric current passes through a person. There are many systems and devices that prevent electrical hazards.



AC Circuit Lacking Safety Features: A schematic of a simple AC circuit with a voltage source and a single appliance represented by the resistance R. It lacks safety features.

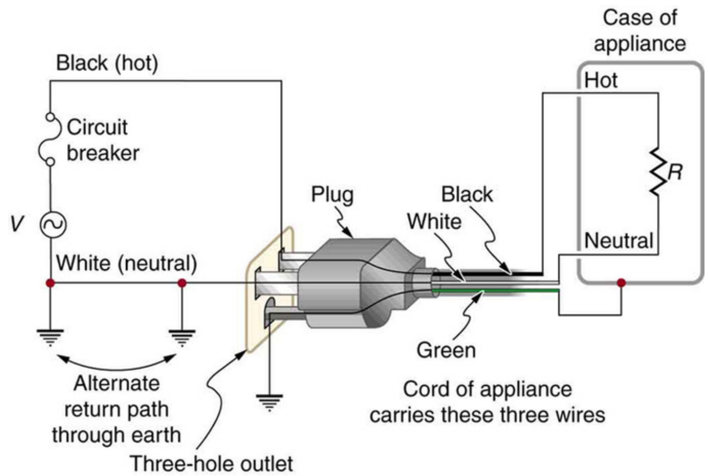
In practice, a simple AC circuit with no safety features is not how power is distributed. Modern household and industrial wiring requires the three-wire system, which has several safety features. The first safety feature is the familiar circuit breaker (or fuse) that prevents thermal overload. Secondly, there is a protective case around the appliance, as with a toaster or refrigerator. The case prevents people from touching exposed wires and coming into electrical contact with the circuit, helping prevent shocks.



Three-Wire System: The three-wire system connects the neutral wire to the earth at the voltage source and the user location. It exists at zero volts and supplies an alternative return path for the current through the earth. The case of the appliance is also grounded to zero volts. A circuit breaker or fuse prevents thermal overload and exists in series on the active (live/hot) wire. Wire insulation colors vary by region. It is essential to check locally to determine which color codes are in use, even if they were followed in one particular installation.

There are three connections to the earth or ground (earth/ground,). An earth/ground connection is a low-resistance path directly to the earth. The two earth/ground connections on the neutral wire force it to exist at zero volts relative to the earth, giving the wire its name. This wire is therefore safe to touch even if its insulation is missing. The neutral wire is the return path for the current to follow in order to complete the circuit.

The two earth/ground connections supply an alternative path through the earth to complete the circuit, since the earth is a good conductor. The earth/ground connection closest to the power source could be at the generating plant, while the other is situated at the user's location. The third earth/ground connection involves the case of the appliance, through the green earth/ground wire, forcing the case to be at zero volts. The live or hot wire (live/hot) supplies the voltage and current to operate the appliance. The three-wire system is connected to an appliance through a three-prong plug.



Three-Prong Plug: The standard three-prong plug can only be inserted one way to ensure the proper function of the three-wire system.

The Three-Prong Plug

The three-wire system replaced the older two-wire system, which lacks an earth/ground wire. Under ordinary circumstances, insulation on the live/hot and neutral wires prevents the case from being situated directly within the circuit, so that the earth/ground wire may seem like double protection. Grounding the case solves more than one problem, however. The simplest problem is worn insulation on the live/hot wire that allows it to contact the case. When lacking an earth/ground connection (some people cut the third prong off the plug because they only have outdated two-hole receptacles), a severe shock is possible. This is particularly dangerous in the kitchen, where a good earth/ground connection is available through water on the floor or a water faucet.

With the earth/ground connection intact, the circuit breaker will trip, thus requiring appliance repair. Some appliances are still sold with two-prong plugs. These appliances, including power tools with impact resistant plastic cases, have nonconducting cases and are called ‘doubly insulated. ‘ Modern two-prong plugs can be inserted into the asymmetric standard outlet in only one way, ensuring the proper connection of live/hot and neutral wires.

Color-Coding

Insulating plastic is color-coded to identify live/hot, neutral, and ground wires, but these codes vary throughout the world. Live/hot wires may be brown, red, black, blue, or grey. Neutral wires may be blue, black, or white. Since the same color may be used for live/hot or neutral wires in different parts of the world, it is essential to confirm the color code for any given local region. The only exception is the earth/ground wire, which is often green but may be yellow or ‘bare wire. ‘ Striped coatings are sometimes used for the benefit of those who are colorblind.

Induction and Leakage Current

Electromagnetic induction causes a subtler problem solved by grounding the case. The alternating current in appliances can induce an EMF on the case. If grounded, the case voltage is kept near zero, but if the case is not grounded, a shock can occur. Current that is driven by the induced case EMF is called a leakage current, although current does not necessarily pass from the resistor to the case.

Key Points

- A phasor is a representation of a sinusoidal function whose amplitude (A), frequency (ω), and phase (θ) are time-invariant. If ω is shared by all components of the system, it can be factored out, leaving just A and ω. The term phasor usually refers to the last two factors.
- Phasors greatly reduce the complexity of expressing sinusoidally varying signals.
- Phasors may be used to analyze the behavior of electrical systems, such as RLC circuits, that have reached a kind of equilibrium called sinusoidal steady state. In the sinusoidal steady state, every voltage and current in a system is sinusoidal with angular frequency ω.
- Phasors allow us to apply techniques used to solve DC circuits to solve RC circuits.

- Recall that unlike DC current and voltage, which are constant, AC current and voltage vary over time. This is called alternating current because the direction alternates.
- The root mean square (abbreviated RMS or rms) is a statistical measure of the magnitude of a varying quantity. We use the root mean square to express the average current or voltage in an AC system.
- The RMS current and voltage (for sinusoidal systems) are the peak current and voltage over the square root of two.
- The average power in an AC circuit is the product of the RMS current and RMS voltage.
- Electrical circuits carry the risks of overheating and potential electrical shocks.
- Fuses and circuit breakers are used to stop currents that exceed a set safety limit, thus preventing overheating.
- The three-wire system protects against thermal and shock hazards by using live, neutral, and ground wires, and grounding the neutral wire and the conducting cases of appliances.
- Before altering any circuitry, it is important to establish the correct color-coding scheme for your region (the color of live/hot, neutral, and ground wires).
- Alternating current has the potential to induce an EMF on the case of an appliance, which poses a shock hazard, so it is important to ground the case.

Key Terms

- **sinusoidal steady state**: Indicates every voltage and current in a system is sinusoidal with the same angular frequency ω.
- **complex numbers**: Numbers that have an imaginary part. Usually represented as i.
- **phasor**: A representation of a complex number in terms of a complex exponential.
- **root mean square**: The square root of the arithmetic mean of the squares.
- **rms current**: the root mean square of the current, $\mathrm{I_{rms}} = \frac{\mathrm{I_0}}{\sqrt{2}}$, where I0 is the peak current, in an AC system
- **rms voltage**: the root mean square of the voltage, $\mathrm{V_{rms}} = \frac{\mathrm{V_0}}{\sqrt{2}}$, where V0 is the peak voltage, in an AC system
- **thermal hazard**: an electrical hazard caused by overheating (e.g., in a resistive element)
- **shock hazard**: an electrical hazard that poses the risk of passing current through the body
- **three-wire system**: a modern wiring system with safety precautions; contains live, neutral, and ground wires

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19.6: Electricity in the World

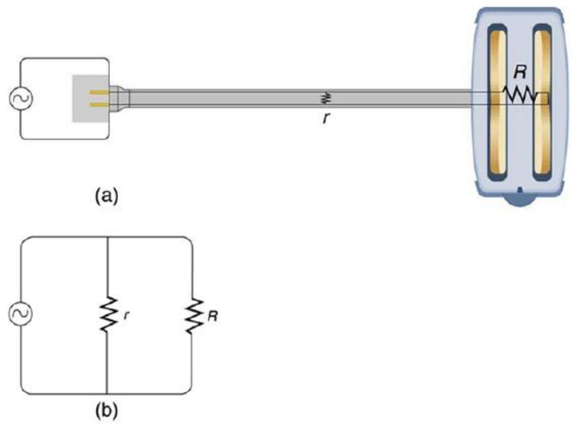
learning objectives

- Identify factors that determine the lethality of an electric shock

There are two known categories of electrical hazards: thermal hazards and shock hazards. A thermal hazard is when excessive electric power causes undesired thermal effects, such as starting a fire in the wall of a house. A shock hazard occurs when electric current passes through a person. Shocks range in severity from painful but otherwise harmless to heart-stoppingly lethal.

Thermal Hazards

Electric power causes undesired heating effects whenever electric energy is converted to thermal energy at a rate faster than it can be safely dissipated. A classic example of this is the short circuit, shown in. A short circuit is a low- resistance path between terminals of a voltage source. Insulation on the appliance's wires has worn through, allowing the two wires to come into contact. Such an undesired contact with a high voltage is called a short. Since the resistance of the short, r, is very small, the power dissipated in the short, $P = V^2/r$, is very large. For example, if V is 120 V and r is 0.100 Ω, then the power is 144 kW, much greater than that used by a typical household appliance. Thermal energy delivered at this rate will very quickly raise the temperature of surrounding materials, melting or perhaps igniting them.



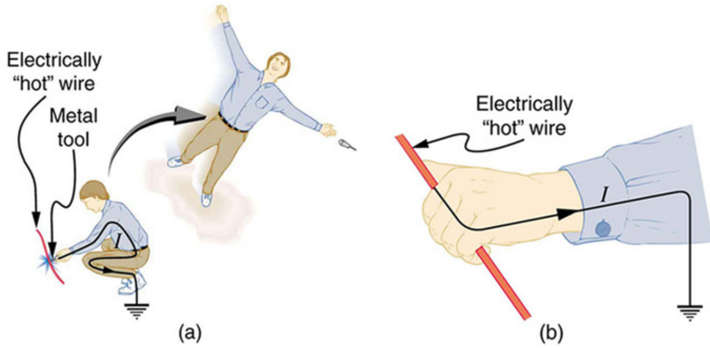
Short Circuit: A short circuit is an undesired low-resistance path across a voltage source. (a) Worn insulation on the wires of a toaster allows them to come into contact with a low resistance r. Since P = V2/r, thermal power is created so rapidly that the cord melts or burns. (b) A schematic of the short circuit.

A thermal hazard can be created even when a short circuit is not present if the wires in a circuit are overloaded with too much current. The power dissipated in the supply wires is $P = I^2R_w$, where R_w is the resistance of the wires and I the current flowing through them. If either I or R_w is too large, the wires overheat. Thermal hazards can cause moderate to severe burns to those who come in contact with the affected appliance or circuit.

Shock Hazards

Electric shock occurs upon contact of a body part with any source of electricity that causes a sufficient current through the skin, muscles, or hair. Typically, the expression is used to describe an injurious exposure to electricity.The minimum current a human can feel depends on the current type (AC or DC) and frequency. A person can feel at least 1 mA (rms) of AC current at 60 Hz and at least 5 mA of DC current. The current may, if it is high enough, cause tissue damage or fibrillation, which leads to cardiac arrest. 60 mA of AC (rms, 60 Hz) or 300-500 mA of DC can cause fibrillation.The potential severity of the shock depends on paths through the body that the currents take. If an electrical circuit is established by electrodes introduced in the body, bypassing the skin, then the potential for lethality is much higher if a circuit through the heart is established. This is known as a microshock. Currents of only 10 µA can be sufficient to cause fibrillation in this case.

A very dangerous possibility is the "can't let go" effect illustrated in. The muscles that close the fingers are stronger than those that open them, so the hand involuntarily closes around the wire shocking it. This can prolong the shock indefinitely. It can also be a danger to a person trying to rescue the victim, because the rescuer's hand may close about the victim's wrist. Usually the best way to help the victim is to give the fist a hard blow with an insulator or to throw an insulator at the fist.

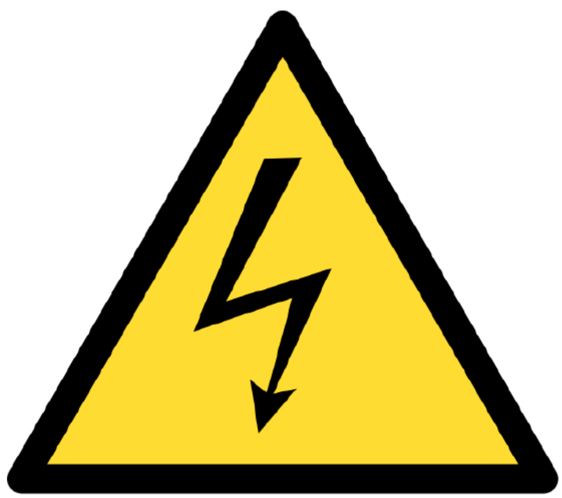


Electric Shock and Muscular Contractions: An electric current can cause muscular contractions with varying effects. (a) The victim is "thrown" backward by involuntary muscle contractions that extend the legs and torso. (b) The victim can't let go of the wire that is stimulating all the muscles in the hand. Those that close the fingers are stronger than those that open them.

Factors in the Lethality of Electric Shock

The lethality of an electric shock is dependent on several variables:

- Current: The higher the current, the more likely it is lethal. Since current is proportional to voltage when resistance is fixed (Ohm 's law), high voltage is an indirect risk for producing higher currents.
- Duration: The longer the duration, the more likely it is lethal — safety switches may limit the time of current flow.
- Pathway: If current flows through the heart muscle, it is more likely to be lethal.
- Very high voltage (over about 600 volts): This poses an additional risk beyond the simple ability of high voltage to cause high current at a fixed resistance. Very high voltage, enough to cause burns, will cause dielectric breakdown at the skin, actually lowering total body resistance and, ultimately, causing even higher current than when the voltage was first applied. Contact with voltages over 600 volts can cause enough skin burning to decrease the total resistance of a path though the body to 500 ohms or less.
- Frequency: Very high-frequency electric current causes tissue burning but does not penetrate the body far enough to cause cardiac arrest.



High Voltage Warning: International safety symbol "Caution, risk of electric shock" (ISO 3864), also known as the high-voltage symbol

Nerve Conduction and Electrocardiograms

Voltage pulses along a cell membrane, called action potentials, allow us to sense the world, control parts of our body, and think.

learning objectives

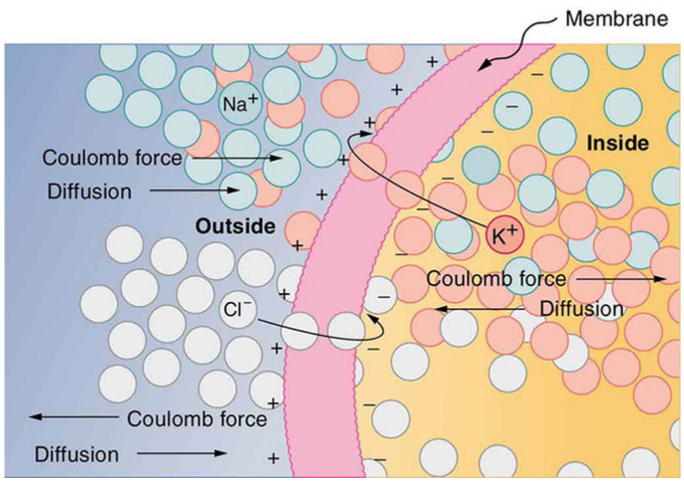
- Explain purpose of the electrocardiogram and identify functions performed by electric currents in the nerve system

Nerve Conduction and Electrocardiograms

Nerve Conduction

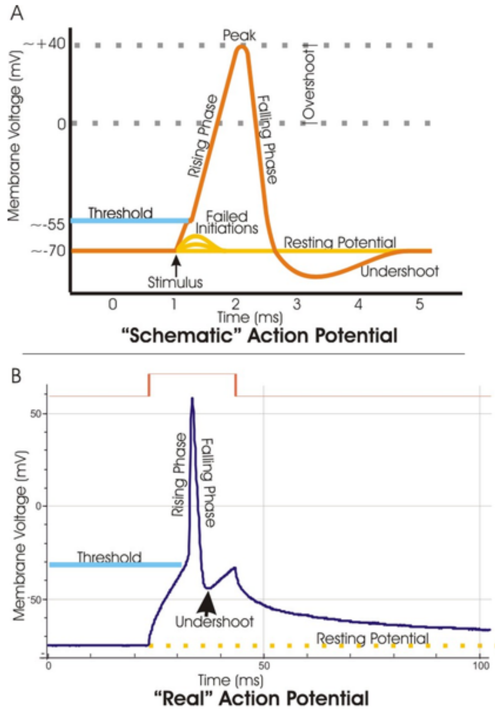
Electric currents in the complex system of nerves in our body allow us to sense the world, control parts of our body, and think. There are three major functions of nerves. First, nerves carry messages from our sensory organs to the central nervous system, consisting of the brain and spinal cord. Second, nerves carry messages from the central nervous system to muscles and other organs. Third, nerves transmit and process signals within the central nervous system.

Nerve conduction is a general term for electrical signals carried by nerve cells. A voltage is created across the cell membrane of a neuron in its resting state. This membrane separates electrically neutral fluids having differing concentrations of ions, the most important varieties being Na^+, K^+, and Cl^- (these are sodium, potassium, and chlorine ions). Free ions will diffuse from a region of high concentration to one of low concentration. The cell membrane is *semipermeable*, meaning that some ions may cross it while others cannot. In its resting state, the cell membrane is permeable to K^+ and Cl^-, and impermeable to Na^+. Diffusion of K^+ and Cl^- thus creates the layers of positive and negative charge on the outside and inside of the membrane, and the Coulomb force prevents the ions from diffusing across in their entirety.



Creating a Voltage Across a Cell Membrane: The semipermeable membrane of a cell has different concentrations of ions inside and out. Diffusion moves the K+ and Cl− ions in the direction shown, until the Coulomb force halts further transfer. This results in a layer of positive charge on the outside, a layer of negative charge on the inside, and thus a voltage across the cell membrane. The membrane is normally impermeable to Na+.

Once the charge layer has built up, the repulsion of like charges prevents more from moving across, and the attraction of unlike charges prevents more from leaving either side. The result is two layers of charge right on the membrane, with diffusion being balanced by the Coulomb force. A tiny fraction of the charges move across and the fluids remain neutral, while a separation of charge and a voltage have been created across the membrane. Electric currents along the cell membrane are created by any stimulus that changes the membrane's permeability. The membrane thus temporarily becomes permeable to Na^+, which then rushes in, driven both by diffusion and the Coulomb force. This inrush of Na+ first neutralizes the inside membrane (called depolarization), and then makes it slightly positive. The depolarization causes the membrane to again become impermeable to Na^+, and the movement of K^+ quickly returns the cell to its resting potential, referred to as repolarization. This sequence of events results in a voltage pulse, called the action potential and is shown in.



Voltage channels are critical in the generation of an action potential: Top: view of an idealized action potential shows its various phases as the action potential passes a point on a cell membrane. Bottom: Recordings of action potentials are often distorted compared to the schematic view because of variations in electrophysiological techniques used to make the recording.

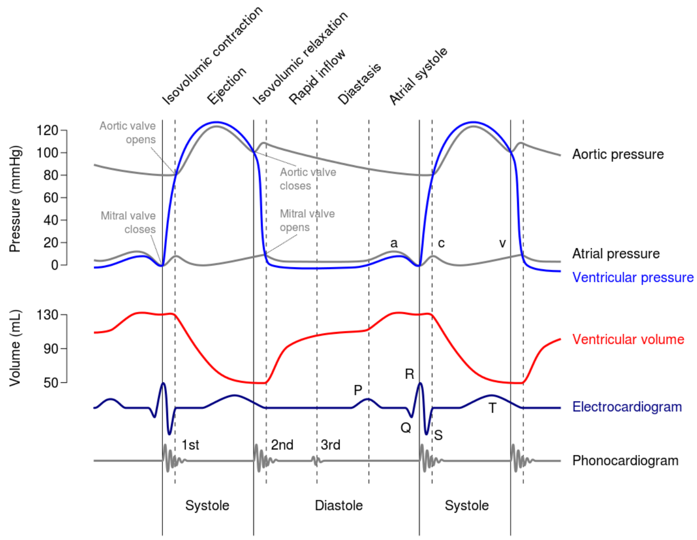
Only small fractions of the ions move, so that the cell can fire many hundreds of times without depleting the excess concentrations of Na+ and K+. This is an example of active transport, wherein cell energy is used to move ions across membranes against diffusion gradients and the Coulomb force. The action potential is a voltage pulse at one location on a cell membrane.

How does it get transmitted along the cell membrane as a nerve impulse ? The changing voltage and electric fields affect the permeability of the adjacent cell membrane, so that the same process takes place there. The adjacent membrane depolarizes, affecting the membrane farther down, and so on. Thus the action potential stimulated at one location triggers a nerve impulse that moves slowly (about 1 m/s) along the cell membrane.

Electrocardiograms

Just as nerve impulses are transmitted by depolarization and repolarization of an adjacent membrane, the depolarization that causes muscle contraction can also stimulate adjacent muscle cells to depolarize (fire) and contract. Thus, a depolarization wave can be sent across the heart, coordinating its rhythmic contractions and enabling it to perform its vital function of propelling blood through the circulatory system. An electrocardiogram (ECG) is a record of the voltages created by the wave of depolarization (and subsequent repolarization) in the heart. Historically, ECGs were performed by placing electrodes on the left and right arms and the left leg. The voltage between the right arm and the left leg is called the lead II potential and is an indicator of heart-muscle function.

shows an ECG of the lead II potential and a graph of other major events during the cardiac cycle. The major features are labeled P, Q, R, S, and T. The *P wave* is generated by the depolarization and contraction of the atria as they pump blood into the ventricles. The *QRS complex* has a characteristic shape and time span, and is created by the depolarization of the ventricles as they pump blood to the body. The lead II QRS signal also masks the repolarization of the atria. Finally, the *T wave*is generated by the repolarization of the ventricles and is followed by the P wave in the next heartbeat. Arterial blood pressure varies with each part of the heartbeat, with systolic (maximum) pressure occurring closely after the QRS complex, signaling the contraction of the ventricles.



ECG Curve: A normal ECG curve synchronized with other major events during the cardiac cycle.

Electric Activity in the Heart

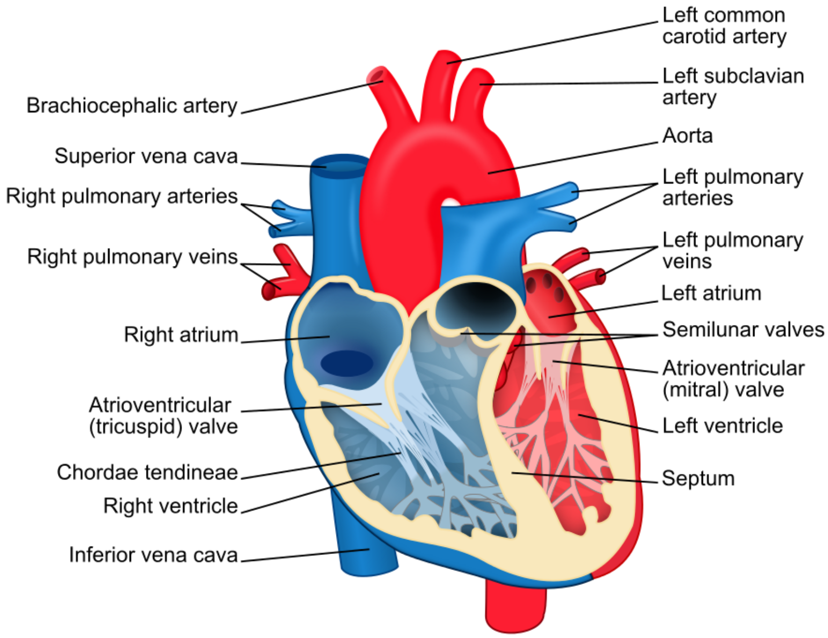
Electric energy stimulating the heart occurs in the sinoatrial node, the heart's pacemaker, and is transmitted partially by Perkinje fibers.

learning objectives

- Identify part of the heart that acts as a pacemaker

Electric Activity in the Heart

The human heart provides continuous blood circulation through the cardiac cycle and is unsurprisingly one of the most vital organs in the human body. The heart is divided into four main chambers: the two upper chambers are called the left and right atria (singular atrium) and two lower chambers are called the right and left ventricles. As can be seen in, there is a thick wall of muscle separating the right side and the left side of the heart called the septum. Normally with each beat the right ventricle pumps the same amount of blood into the lungs that the left ventricle pumps out into the body. Physicians commonly refer to the right atrium and right ventricle together as the right heart and to the left atrium and left ventricle as the left heart.



Structure of the heart: Structure diagram of a coronal section of the human heart from an anterior view. The two larger chambers are the ventricles.

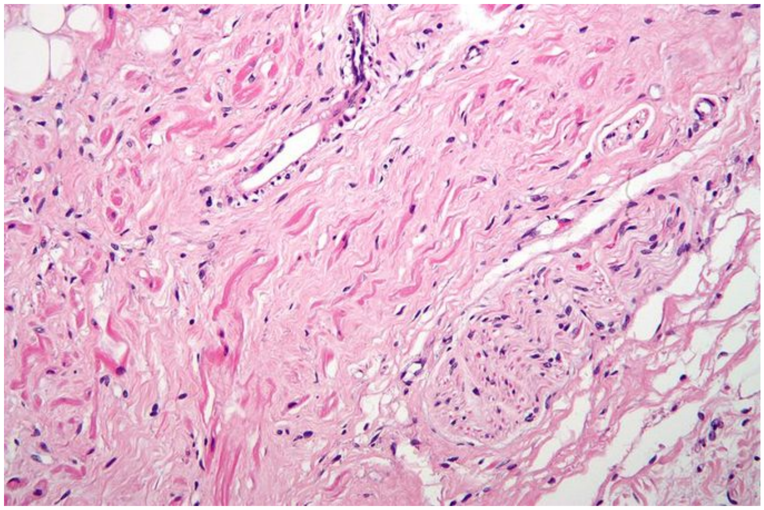
The electric energy that stimulates the heart occurs in the sinoatrial node, which produces a definite potential and then discharges, sending an impulse across the atria. In the atria the electrical signal moves from cell to cell (see section on nerve conduction and the electrocardiogram) while in the ventricles the signal is carried by specialized tissue called the Purkinje fibers which then transmit the electric charge to the myocardium. shows the isolated heart conduction system.

Conduction System of the Heart: Isolated heart conduction system, showing the sinoatrial and Purkinje fibers.

The Sinotrial Node's Role as a Pacemaker

The sinoatrial node (also commonly spelled sinuatrial node) is the impulse-generating (pacemaker) tissue located in the right atrium of the heart: i.e., generator of normal sinus rhythm. It is a group of cells positioned on the wall of the right atrium. These cells are specialized cardiomycetes (cardiac muscle cells).

The sinoatrial node (also commonly spelled sinuatrial node, abbreviated SA node) is the impulse-generating (pacemaker) tissue located in the right atrium of the heart, and thus the generator of normal sinus rhythm. It is a group of cells positioned on the wall of the right atrium. Although all of the heart's cells have the ability to generate the electrical impulses (or action potentials) that trigger cardiac contraction, the sinoatrial node normally initiates it, simply because it generates impulses slightly faster than the other areas with pacemaker potential.



Sinoatrial Node Tissue: High magnification micrograph of sinoatrial node tissue and an adjacent nerve fiber.

Cells in the SA node, located in the upper right corner of the heart, will typically discharge (create action potentials) at about 60-100 beats/minute. Because the sinoatrial node is responsible for the rest of the heart's electrical activity, it is sometimes called the primary pacemaker. If the SA node does not function, or the impulse generated in the SA node is blocked before it travels down the electrical conduction system, a group of cells further down the heart will become the heart's pacemaker. These cells form the atrioventricular node (AV node), which is an area between the atria and ventricles, within the atrial septum. If the AV node also fails, Purkinje fibers are capable of acting as the pacemaker. The reason Purkinje cells do not normally control the heart rate is that they generate action potentials at a lower frequency than the AV or SA nodes.

Purkinje Fibers

The Purkinje fibers are located in the inner ventricular walls of the heart. These fibers consist of specialized cardiomyocytes that are able to conduct cardiac action potentials more quickly and efficiently than any other cells in the heart. Purkinje fibers allow the heart's conduction system to create synchronized contractions of its ventricles, and are therefore essential for maintaining a consistent heart rhythm.

During the ventricular contraction portion of the cardiac cycle, the Purkinje fibers carry the contraction impulse from both the left and right bundle branch to the myocardium of the ventricles. This causes the muscle tissue of the ventricles to contract, thus enabling a force to eject blood out of the heart. Atrial and ventricular discharge through the Purkinje trees is assigned on a standard Electrocardiogram as the P Wave and QRS complex, respectively.

Purkinje fibers also have the ability of automatically firing at a rate of 15-40 beats per minute if left to their own devices. In contrast, the SA node outside of parasympathetic control can fire a rate of almost 100 beats per minute. In short, they generate action potentials, but at a slower rate than sinoatrial node and other atrial ectopic pacemakers. Thus they serve as the last resort when other pacemakers fail.

Key Points

- Electric power causes undesired heating effects whenever electric energy is converted to thermal energy at a rate faster than it can be safely dissipated, such as in the case of a short circuit. This is referred to as a thermal hazard.
- A shock hazard occurs when electric current passes through a person. If the current is high enough it can cause tissue damage or fibrillation, which can lead to cardiac arrest.
- The lethality of an electric shock is dependent on many variables, including current, duration, pathway, and the presence or absence of very high voltage.
- Nerves carry messages from our sensory organs to the central nervous system, from the central nervous system to muscles, and within the central nervous system itself.
- The effects of diffusion and the Coulomb force act together to allow ions like Na+, K+, and Cl- to create a voltage across cell membranes.
- An action potential is an event which alters the permeability of a cell membrane due to electric currents.

- The depolarization that causes muscle contraction can also stimulate contraction in adjacent muscle cells. A depolarization wave across the heart is responsible for rhythmic contractions. An electrocardiogram (ECG) is a record of the voltages in the heart.
- The human heart provides continuous blood circulation through the cardiac cycle and is unsurprisingly one of the most vital organs in the human body.
- The heart is divided into four main chambers: the two upper chambers are called the left and right atria (singular atrium) and two lower chambers are called the right and left ventricles. See for an illustration. There is a conduction system that transports impulses through the heart.
- The electric energy that stimulates the heart occurs in the sinoatrial node, which produces a definite potential and then discharges, sending an impulse across the atria. It is thus the generator of normal sinus rhythm and functions as the heart's pacemaker.
- In the atria the electrical signal moves from cell to cell while in the ventricles the signal is carried by specialized tissue called the Purkinje fibers.
- Purkinje fibers allow the heart's conduction system to create synchronized contractions of its ventricles, and are therefore essential for maintaining a consistent heart rhythm.

Key Terms

- **shock hazard**: an electrical hazard that poses the risk of passing current through the body
- **thermal hazard**: an electrical hazard caused by overheating (e.g., in a resistive element)
- **fibrillation**: the rapid, irregular, and unsynchronized contraction of the muscle fibers of the heart
- **diffusion**: the intermingling of the molecules of a fluid due to random thermal agitation
- **electrocardiogram**: The visual output that an electrocardiograph produces
- **action potential**: A short term change in the electrical potential that travels along a cell such as a nerve or muscle fiber.
- **sinoatrial node**: a group of specialized cardiac muscle cells (tissue) located in the right atrium of the heart, which generates the impulses that establish the normal sinus rhythm.
- **Purkinje fibers**: specialized cardiac muscle cells that are able to conduct cardiac muscle potentials quickly and efficiently; essential for maintaining consistent heart rhythm.
- **myocardium**: The middle of the three layers forming the wall of the heart.

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CHAPTER OVERVIEW

20: Circuits and Direct Currents

Topic hierarchy

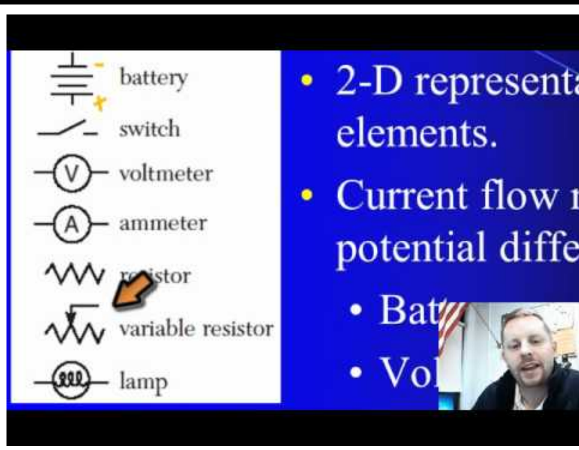
20.1: Overview

learning objectives

- Describe structure of an electrical circuit and identify elements of a direct current circuit

Overview

An electrical network is an interconnection of electrical elements such as resistors, inductors, capacitors, transmission lines, voltage sources, current sources and switches. An electrical circuit is a special type of network, one that has a closed loop giving a return path for the current. Electrical networks that consist only of sources (voltage or current), linear lumped elements (resistors, capacitors, inductors), and linear distributed elements (transmission lines) can be analyzed by algebraic and transform methods. A resistive circuit is a circuit containing only resistors and ideal current and voltage sources. Analysis of resistive circuits is less complicated than analysis of circuits containing capacitors and inductors. If the sources are constant (DC) sources, the result is a DC circuit. A network that contains active electronic components is known as an electronic circuit. Such networks are generally nonlinear and require more complex design and analysis tools.



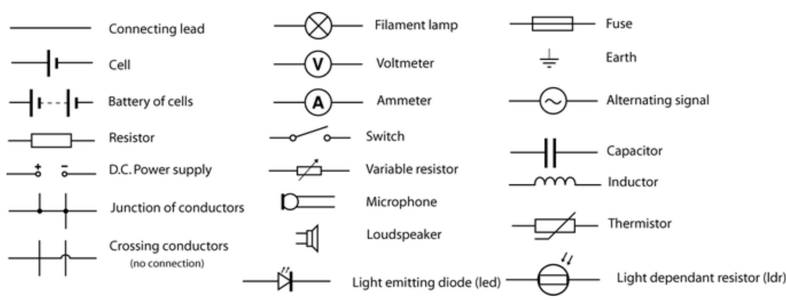
Circuits: A brief introduction to electric circuits and current flow for introductory physics students.

Direct Current Circuits

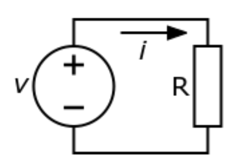
Direct current (DC) is the unidirectional flow of electric charge. Direct current is produced by sources such as batteries, thermocouples, solar cells, and commutator-type electric machines of the dynamo type. Direct current may flow in a conductor such as a wire, but can also flow through semiconductors, insulators, or even through a vacuum as in electron or ion beams. The electric charge flows in a constant direction, distinguishing it from alternating current (AC). A term formerly used for direct current was galvanic current.

A direct current circuit is an electrical circuit that consists of any combination of constant voltage sources, constant current sources, and resistors. In this case, the circuit voltages and currents are independent of time. A particular circuit voltage or current does not depend on the past value of any circuit voltage or current. This implies that the system of equations that represent a DC circuit do not involve integrals or derivatives with respect to time. If a capacitor or inductor is added to a DC circuit, the resulting circuit is not, strictly speaking, a DC circuit. However, most such circuits have a DC solution. This solution gives the circuit voltages and currents when the circuit is in DC steady state. Such a circuit is represented by a system of differential equations. The solution to these equations usually contain a time varying or transient part as well as constant or steady state part. It is this steady state part that is the DC solution. In electronics, it is common to refer to a circuit that is powered by a DC voltage source such as a battery or the output of a DC power supply as a DC circuit even though what is meant is that the circuit is DC powered.

A simple DC circuit is illustrated in. In circuit diagrams such as this, electrical elements are represented by symbols and usually labeled with appropriate characteristics, such as the resistance *r* of a resistor. The electric potential and current may also be labeled at various points of the circuit. However, please be aware that circuit diagram conventions do differ among textbooks and subject fields, leading to different symbols being used for the same circuit elements.



Example Circuit Element Symbols: A set of example circuit elements and their associated symbols commonly used in circuit diagrams.



A Simple Circuit: A simple DC circuit with a voltage source (V) and resistor (R). The current i flowing through the circuit is given by Ohm's law.

Physical Laws Related to Circuit Analysis

A number of electrical laws apply to all electrical networks. These include Ohm's law, which has been discussed in the "Resistance and Resistors" module, Kirchhoff's current and voltage laws, which are covered in the "Kirchhoff's Rules" module. The two Kirchoff laws along with the current-voltage characteristic (I-V curve) of each electrical element completely describe a circuit. With these, it is possible to calculate the electric potential and current at each point in the circuit. In addition, Norton's theorem and Thévenin's theorem are useful in analyzing complicated circuits.

Sources of EMF

Electromotive force (EMF) is the voltage voltage generated by a battery or by the magnetic force according to Faraday's Law of Induction.

learning objectives

- Give examples of the devices that can provide the electromotive force

Electromotive Force

Electromotive force, also called EMF (denoted and measured in volts) refers to voltage generated by a battery or by the magnetic force according to Faraday's Law of Induction, which states that a time varying magnetic field will induce an electric current.

Electromotive "force" is not considered a force (as force is measured in newtons) but a potential, or energy per unit of charge, measured in volts. Formally, EMF is classified as the external work expended per unit of charge to produce an electric potential difference across two open-circuited terminals. By separating positive and negative charges, electric potential difference is produced, generating an electric field. The created electrical potential difference drives current flow if a circuit is attached to the source of EMF. When current flows, however, the voltage across the terminals of the source of EMF is no longer the open-circuit value, due to voltage drops inside the device due to its internal resistance.

Devices that can provide EMF include electrochemical cells (batteries), thermoelectric devices, solar cells, electrical generators, transformers, and even Van de Graaff generators (examples shown in).

Examples of generators of electromotive force.: A variety of voltage sources (clockwise from top left): the Brazos Wind Farm in Fluvanna, Texas (credit: Leaflet, Wikimedia Commons); the Krasnoyarsk Dam in Russia (credit: Alex Polezhaev); a solar farm (credit: U.S. Department of Energy); and a group of nickel metal hydride batteries (credit: Tiaa Monto). The voltage output of each depends on its construction and load, and equals emf only if there is no load.

In the case of a battery, charge separation that gives rise to a voltage difference is accomplished by chemical reactions at the electrodes; voltaic cells can be thought of as having a "charge pump" of atomic dimensions at each electrode.

In the case of an electrical generator, a time-varying magnetic field inside the generator creates an electric field via electromagnetic induction, which in turn creates an energy difference between generator terminals. Charge separation takes place within the generator, with electrons flowing away from one terminal and toward the other, until, in the open-circuit case, sufficient electric field builds up to make further movement unfavorable. Again the EMF is countered by the electrical voltage due to charge separation. If a load is attached, this voltage can drive a current. The general principle governing the EMF in such electrical machines is Faraday's law of Induction.

In nature, EMF is generated whenever magnetic field fluctuations occur through a surface. An example of this is the varying Earth magnetic field during a geomagnetic storm, acting on anything on the surface of the planet, like an extended electrical grid.

Key Points

- Direct current (DC) is the unidirectional flow of electric charge. Direct current is produced by sources such as batteries, thermocouples, solar cells, and commutator-type electric machines of the dynamo type.
- A direct current circuit is an electrical circuit that consists of constant voltage sources, constant current sources, and resistors. It is common to refer to a circuit that is powered by a DC voltage source as a DC circuit even though what is meant is that the

circuit is DC powered.

- A number of electrical laws apply to all electrical networks: Ohm 's law, Kirchhoff's current and voltage laws, Norton's theorem, and Thévenin's theorem. With these, it is possible to calculate the electric potential and current at each point in the circuit.
- EMF is classified as the external work expended per unit of charge to produce an electric potential difference across two open-circuited terminals.
- Devices that can provide EMF include electrochemical cells, thermoelectric devices, solar cells, electrical generators, transformers, and even Van de Graaff generators.
- In nature, EMF is generated whenever magnetic field fluctuations occur through a surface.

Key Terms

- **current-voltage characteristic**: A current–voltage characteristic or I–V curve (current–voltage curve) is a relationship between the electric current through a circuit, device, or material, and the corresponding voltage, or potential difference across it.
- **electrical circuit**: An interconnection of electrical elements such as resistors, inductors, capacitors, transmission lines, voltage sources, current sources and switches that has a closed loop giving a return path for the current.
- **direct current**: An electric current in which the electrons flow in one direction, but may vary with time.
- **battery**: A device that produces electricity by a chemical reaction between two substances.
- **electromotive force**: (EMF)—The voltage generated by a battery or by the magnetic force according to Faraday's Law. It is measured in units of volts, not newtons, and thus, is not actually a force.
- **Faraday's law of induction**: Is a basic law of electromagnetism that predicts how a magnetic field will interact with an electric circuit to produce an electromotive force.

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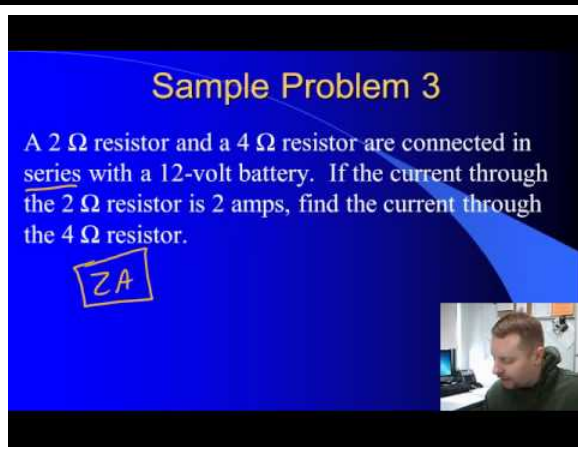
20.2: Resistors in Series and Parallel

learning objectives

- Calculate the total resistance in the circuit with resistors connected in series

Overview

Most circuits have more than one component, called a resistor, that limits the flow of charge in the circuit. A measure of this limit on charge flow is called resistance. The simplest combinations of resistors are the series and parallel connections. The total resistance of a combination of resistors depends on both their individual values and how they are connected.



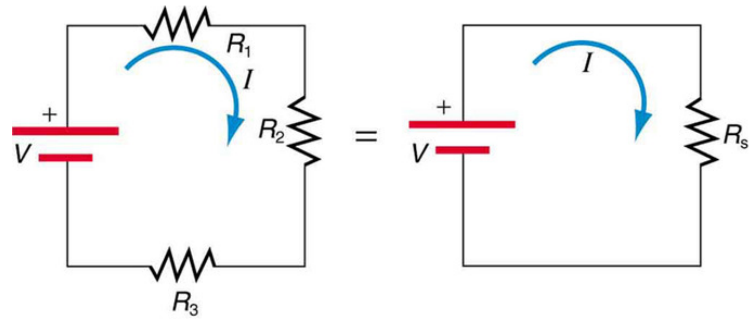
Series Circuits: A brief introduction to series circuit and series circuit analysis, including Kirchhoff's Current Law (KCL) and Kirchhoff's Voltage Law (KVL).

Resistors in Series

Resistors are in series whenever the flow of charge, or the current, must flow through components sequentially.

Resistors in Series: These four resistors are connected in series because if a current was applied at one end, it would flow through each resistor sequentially to the end.

shows resistors in series connected to a voltage source. The total resistance in the circuit is equal to the sum of the individual resistances, since the current has to pass through each resistor in sequence through the circuit.



Resistors connected in a series circuit: Three resistors connected in series to a battery (left) and the equivalent single or series resistance (right).

Using Ohm 's Law to Calculate Voltage Changes in Resistors in Series

According to Ohm's law, the voltage drop, V, across a resistor when a current flows through it is calculated by using the equation V=IR, where I is current in amps (A) and R is the resistance in ohms (Ω).

So the voltage drop across R_1 is *[Math Processing Error]*, across R_2 is *[Math Processing Error]*, and across R_3 is *[Math Processing Error]* . The sum of the voltages would equal: *[Math Processing Error]*, based on the conservation of energy and charge. If we substitute the values for individual voltages, we get:

[Math Processing Error]

or

[Math Processing Error]

This implies that the total resistance in a series is equal to the sum of the individual resistances. Therefore, for every circuit with *N* number of resistors connected in series:

[Math Processing Error]

Since all of the current must pass through each resistor, it experiences the resistance of each, and resistances in series simply add up.

Since voltage and resistance have an inverse relationship, individual resistors in series do not get the total source voltage, but divide it. This is indicated in an example of when two light bulbs are connected together in a series circuit with a battery. In a simple circuit consisting of one 1.5V battery and one light bulb, the light bulb would have a voltage drop of 1.5V across it. If two lightbulbs were connected in series with the same battery, however, they would each have 1.5V/2, or 0.75V drop across them. This would be evident in the brightness of the lights: each of the two light bulbs connected in series would be half as dim as the single light bulb. Therefore, resistors connected in series use up the same amount of energy as a single resistor, but that energy is divided up between the resistors depending on their resistances.

Resistors in Parallel

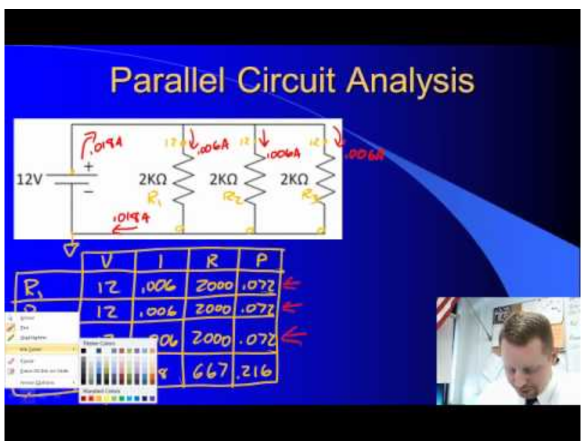
The total resistance in a parallel circuit is equal to the sum of the inverse of each individual resistances.

learning objectives

- Calculate the total resistance in the circuit with resistors connected in parallel

Overview

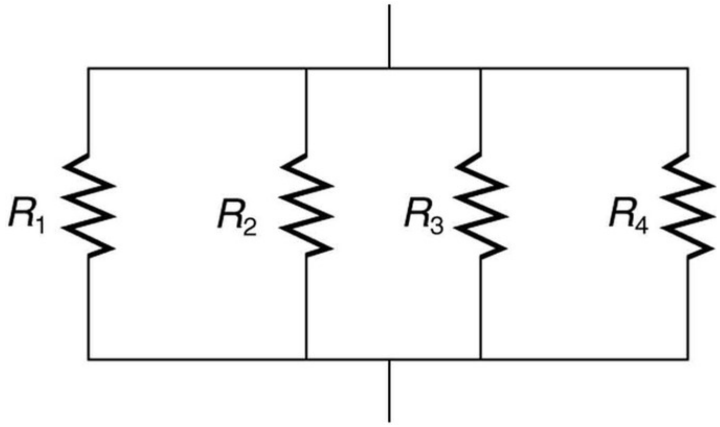
Resistors in a circuit can be connected in series or in parallel. The total resistance of a combination of resistors depends on both their individual values and how they are connected.



Parallel Circuits: A brief overview of parallel circuit analysis using VIRP tables for high school physics students.

Resistors in Parallel

Resistors are in parallel when each resistor is connected directly to the voltage source by connecting wires having negligible resistance. Each resistor thus has the full voltage of the source applied to it.

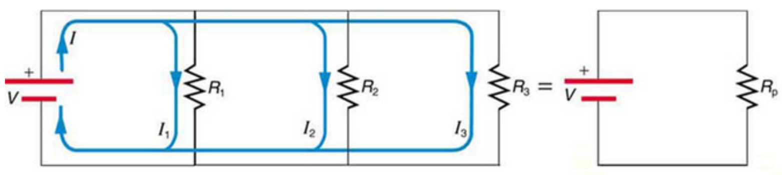


Resistors in Parallel: A parallel connection of resistors.

Each resistor draws the same current it would if it were the only resistor connected to the voltage source. This is true of the circuitry in a house or apartment. Each outlet that is connected to a appliance (the "resistor") can operate independently, and the current does not have to pass through each appliance sequentially.

Ohm 's Law and Parallel Resistors

Each resistor in the circuit has the full voltage. According to Ohm's law, the currents flowing through the individual resistors are *[Math Processing Error]* and *[Math Processing Error]*. Conservation of charge implies that the total current is the sum of these currents:



Parallel resistors: Three resistors connected in parallel to a battery and the equivalent single or parallel resistance.

[Math Processing Error]

Substituting the expressions for individual currents gives:

[Math Processing Error]

or

[Math Processing Error]

This implies that the total resistance in a parallel circuit is equal to the sum of the inverse of each individual resistances. Therefore, for every circuit with nn number or resistors connected in parallel,

[Math Processing Error]

This relationship results in a total resistance that is less than the smallest of the individual resistances. When resistors are connected in parallel, more current flows from the source than would flow for any of them individually, so the total resistance is lower.

Each resistor in parallel has the same full voltage of the source applied to it, but divide the total current amongst them. This is exemplified by connecting two light bulbs in a parallel circuit with a 1.5V battery. In a series circuit, the two light bulbs would be half as dim when connected to a single battery source. However, if the two light bulbs were connected in parallel, they would be equally as bright as if they were connected individually to the battery. Because the same full voltage is being applied to both light bulbs, the battery would also die more quickly, since it is essentially supplying full energy to both light bulbs. In a series circuit, the battery would last just as long as it would with a single light bulb, only the brightness is then divided amongst the bulbs.

Combination Circuits

A combination circuit can be broken up into similar parts that are either series or parallel.

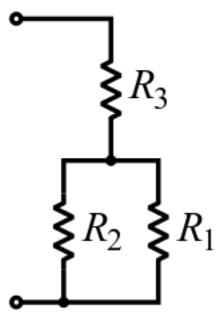
learning objectives

- Describe arrangement of resistors in a combination circuit and its practical implications

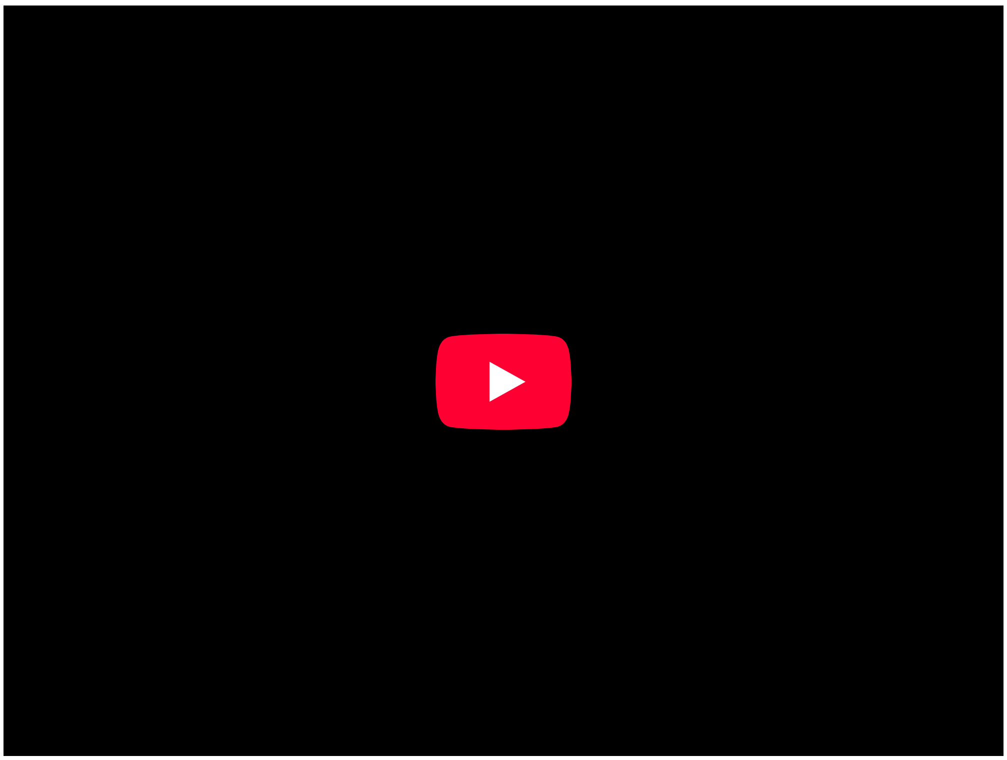
Combination Circuits

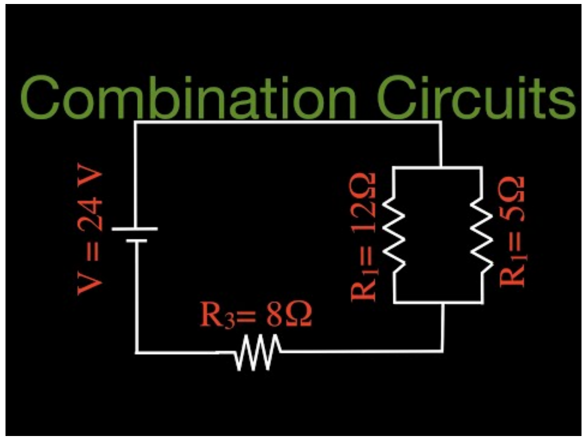
More complex connections of resistors are sometimes just combinations of series and parallel. This is commonly encountered, especially when wire resistances is considered. In that case, wire resistance is in series with other resistances that are in parallel.

A combination circuit can be broken up into similar parts that are either series or parallel, as diagrammed in. In the figure, the total resistance can be calculated by relating the three resistors to each other as in series or in parallel. R_1 and R_2 are connected in parallel in relation to each other, so we know that for that subset, the inverse of resistance would be equal to:



Resistor Network: In this combination circuit, the circuit can be broken up into a series component and a parallel component.





Combination Circuits: Two parallel resistors in series with one resistor.

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R_3 is connected in series to *both* R_1 and R_2, so the resistance would be calculated as:

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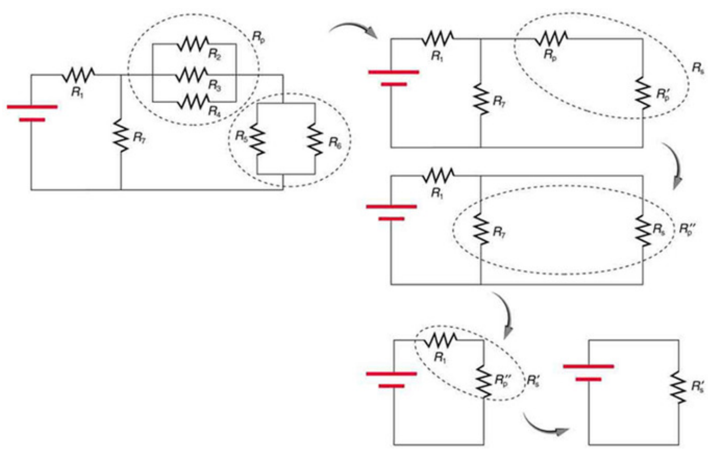
Complex Combination Circuits

For more complicated combination circuits, various parts can be identified as series or parallel, reduced to their equivalents, and then further reduced until a single resistance is left, as shown in. In this figure, the combination of seven resistors was identified by being either in series or in parallel. In the initial image, the two circled sections show resistors that are in parallel.



Reducing a combination circuit: This combination of seven resistors has both series and parallel parts. Each is identified and reduced to an equivalent resistance, and these are further reduced until a single equivalent resistance is reached.

Reducing those parallel resistors into a single R value allows us to visualize the circuit in a more simplified manner. In the top right image, we can see that the circled portion contains two resistors in series. We can further reduce that to another R value by adding them. The next step shows that the circled two resistors are in parallel. Reducing those highlights that the last two are in series, and thus can be reduced to a single resistance value for the entire circuit.

One practical implication of a combination circuit is that resistance in wires reduces the current and power delivered to a resistor. Combination circuit can be transformed into a series circuit, based on an understanding of the equivalent resistance of parallel branches to a combination circuit. A series circuit can be used to determine the total resistance of the circuit. Essentially, wire resistance is a series with the resistor. It thus increases the total resistance and decreases the current. If wire resistance is relatively large, as in a worn (or a very long) extension cord, then this loss can be significant. If a large current is drawn, the IR drop in the wires can also be significant.

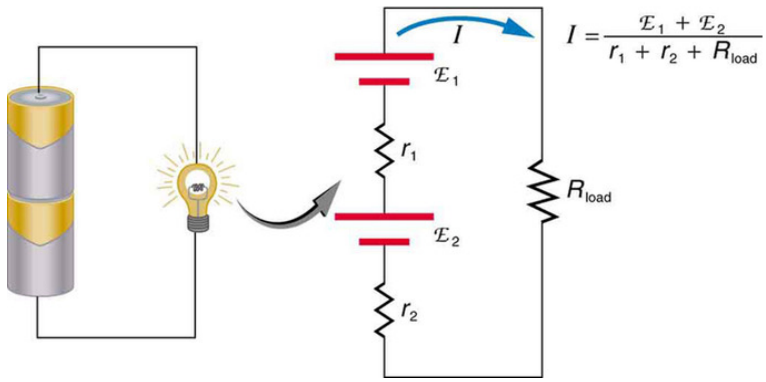
Charging a Battery: EMFs in Series and Parallel

When voltage sources are connected in series, their emfs and internal resistances are additive; in parallel, they stay the same.

learning objectives

- Compare the resistances and electromotive forces for the voltage sources connected in the same and opposite polarity, and in series and in parallel

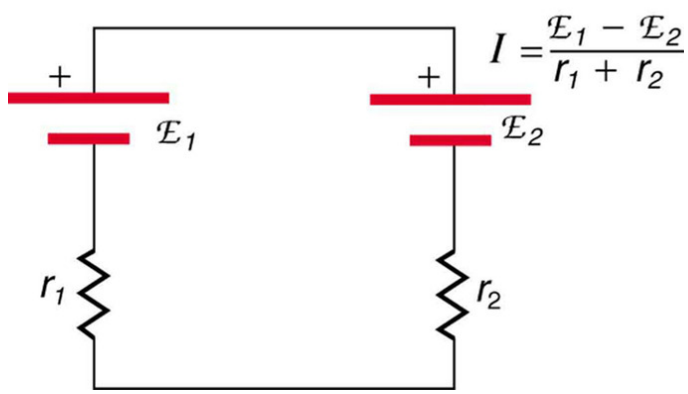
When more than one voltage source is used, they can be connected either in series or in parallel, similar to resistors in a circuit. When voltage sources are in series facing the same direction, their internal resistances add and their electromotive force, or emf, add algebraically. These types of voltage sources are common in flashlights, toys, and other appliances. Usually, the cells are in series in order to produce a larger total emf.



Flashlight and Bulb: A series connection of two voltage sources in the same direction. This schematic represents a flashlight with two cells (voltage sources) and a single bulb (load resistance) in series.

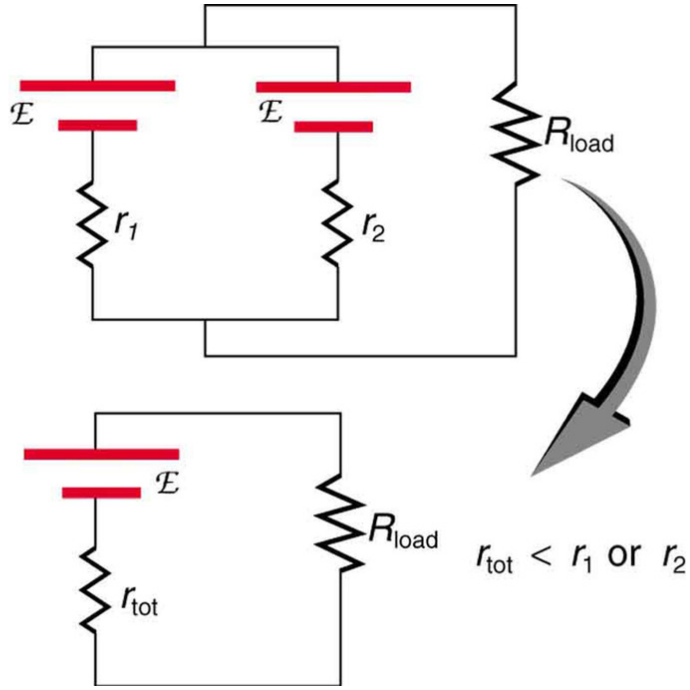
A battery is a multiple connection of voltaic cells. The disadvantage of series connections of cells in this manner, though, is that their internal resistances add. This can sometimes be problematic. For example, if you placed two 6v batteries in your car instead of the typical 12v single battery, you would be adding both the emfs and the internal resistances of each battery. You would therefore end up with the same 12v emf, though the internal resistance would then be doubled, causing issues for you when you want to start your engine.

But, if the cells oppose one another—such as when one is put into an appliance backwards—the total emf is less, since it is the algebraic sum of the individual emfs. When it is reversed, it produces an emf that opposes the other, and results in a difference between the two voltage sources.



Battery Charger: This represents two voltage sources connected in series with their emfs in opposition. Current flows in the direction of the greater emf and is limited by the sum of the internal resistances. (Note that each emf is represented by script E in the figure.) A battery charger connected to a battery is an example of such a connection. The charger must have a larger emf than the battery to reverse current through it.

When two voltage sources with identical emfs are connected in parallel and also connected to a load resistance, the total emf is the same as the individual emfs. But the total internal resistance is reduced, since the internal resistances are in parallel. Thus, the parallel connection can produce a larger current.



Two Identical EMFs: Two voltage sources with identical emfs (each labeled by script E) connected in parallel produce the same emf but have a smaller total internal resistance than the individual sources. Parallel combinations are often used to deliver more current.

EMF and Terminal Voltage

The output, or terminal voltage of a voltage source such as a battery, depends on its electromotive force and its internal resistance.

learning objectives

- Express the relationship between the electromotive force and terminal voltage in a form of equation

When you forget to turn off your car lights, they slowly dim as the battery runs down. Why don't they simply blink off when the battery's energy is gone? Their gradual dimming implies that battery output voltage decreases as the battery is depleted. The reason for the decrease in output voltage for depleted or overloaded batteries is that all voltage sources have two fundamental parts—a source of electrical energy and an internal resistance.

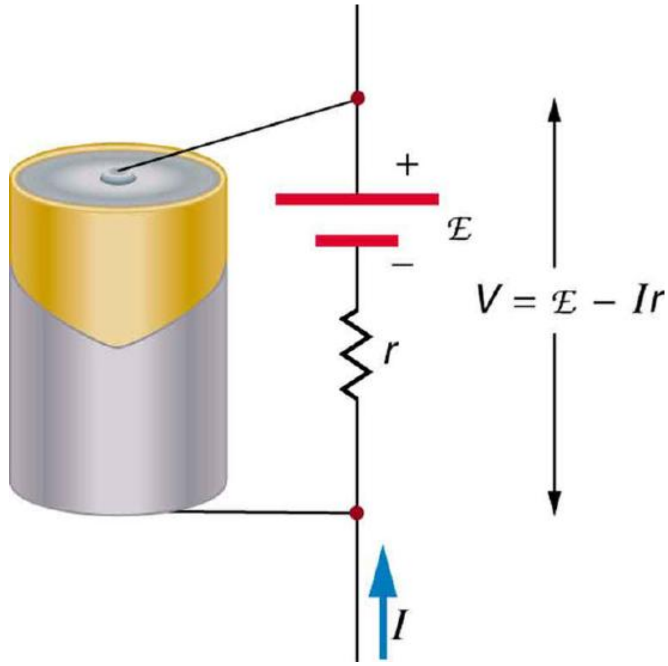
Electromotive Force

All voltage sources create a potential difference and can supply current if connected to a resistance. On a small scale, the potential difference creates an electric field that exerts force on charges, causing current. We call this potential difference the electromotive force (abbreviated emf). Emf is not a force at all; it is a special type of potential difference of a source when no current is flowing. Units of emf are volts.

Electromotive force is directly related to the source of potential difference, such as the particular combination of chemicals in a battery. However, emf differs from the voltage output of the device when current flows. The voltage across the terminals of a battery, for example, is less than the emf when the battery supplies current, and it declines further as the battery is depleted or loaded down. However, if the device's output voltage can be measured without drawing current, then output voltage will equal emf (even for a very depleted battery).

Terminal Voltage

presents a schematic representation of a voltage source. The voltage output of a device is measured across its terminals and is called its terminal voltage *V*. Terminal voltage is given by the equation:



Schematic Representation of a Voltage Source: Any voltage source (in this case, a carbon-zinc dry cell) has an emf related to its source of potential difference, and an internal resistance r related to its construction. (Note that the script E stands for emf.) Also shown are the output terminals across which the terminal voltage V is measured. Since V=emf−Ir, terminal voltage equals emf only if there is no current flowing.

[Math Processing Error]

where r is the internal resistance and I is the current flowing at the time of the measurement.

I is positive if current flows away from the positive terminal. The larger the current, the smaller the terminal voltage. Likewise, it is true that the larger the internal resistance, the smaller the terminal voltage.

Key Points

- The same current flows through each resistor in series.
- Individual resistors in series do not get the total source voltage, but divide it.
- The total resistance in a series circuit is equal to the sum of the individual resistances: *[Math Processing Error]*.
- The total resistance in a parallel circuit is less than the smallest of the individual resistances.
- Each resistor in parallel has the same voltage of the source applied to it (voltage is constant in a parallel circuit).
- Parallel resistors do not each get the total current; they divide it (current is dependent on the value of each resistor and the number of total resistors in a circuit).
- More complex connections of resistors are sometimes just combinations of series and parallel.
- Various parts of a combination circuit can be identified as series or parallel, reduced to their equivalents, and then further reduced until a single resistance is left.
- Resistance in wires reduces the current and power delivered to a resistor. If the resistance in wires is relatively large, as in a worn (or a very long) extension cord, then this loss can be significant and affect power output into appliances.
- Emfs connected in the same polarity in series are additive and result in a higher total emf.
- Two emfs connected in the opposite polarity in series have a total emf equal to the difference between them, and can be used to charge the lower voltage source.
- Two voltage sources with identical emfs connected in parallel have a net emf equivalent to one emf source, however, the net internal resistance is less, and therefore produces a higher current.

Key Terms

- **series**: A number of things that follow on one after the other or are connected one after the other.
- **resistance**: The opposition to the passage of an electric current through that element.
- **parallel**: An arrangement of electrical components such that a current flows along two or more paths.
- **combination circuit**: An electrical circuit containing multiple resistors that are connected in a combination of both series and parallel connections.
- **electromotive force**: (EMF)—The voltage generated by a battery or by the magnetic force according to Faraday's Law. It is measured in units of volts, not newtons, and thus, is not actually a force.

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20.3: Kirchhoff's Rules

learning objectives

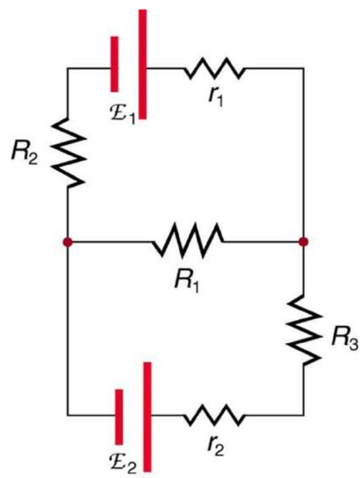
- Describe relationship between the Kirchhoff's circuit laws and the energy and charge in the electrical circuits

Introduction to Kirchhoff's Laws

Kirchhoff's circuit laws are two equations first published by Gustav Kirchhoff in 1845. Fundamentally, they address conservation of energy and charge in the context of electrical circuits.

Although Kirchhoff's Laws can be derived from the equations of James Clerk Maxwell, Maxwell did not publish his set of differential equations (which form the foundation of classical electrodynamics, optics, and electric circuits) until 1861 and 1862. Kirchhoff, rather, used Georg Ohm's work as a foundation for *Kirchhoff's current law (KCL)* and *Kirchhoff's voltage law (KVL)*.

Kirchhoff's laws are extremely important to the analysis of closed circuits. Consider, for example, the circuit illustrated in the figure below, consisting of five resistors in a combination of in series and parallel arrangements. Simplification of this circuit to a combination of series and parallel connections is impossible. However, using Kirchhoff's rules, one can analyze the circuit to determine the parameters of this circuit using the values of the resistors (R_1, R_2, R_3, r_1 and r_2). Also of importance in this example is that the values E_1 and E_2 represent sources of voltage (e.g., batteries).



Closed Circuit: To determine all variables (i.e., current and voltage drops across the different resistors) in this circuit, Kirchhoff's rules must be applied.

As a final note, Kirchhoff's laws depend on certain conditions. The voltage law is a simplification of Faraday's law of induction, and is based on the assumption that there is *no* fluctuating magnetic field within the closed loop. Thus, although this law can be applied to circuits containing resistors and capacitors (as well as other circuit elements), it can only be used as an approximation to the behavior of the circuit when a changing current and therefore magnetic field are involved.

The Junction Rule

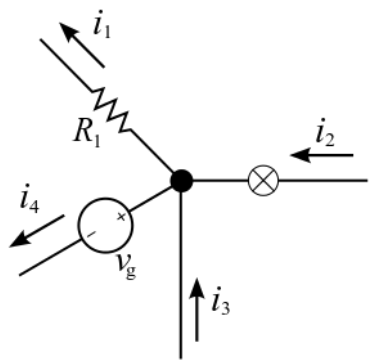
Kirchhoff's junction rule states that at any circuit junction, the sum of the currents flowing into and out of that junction are equal.

learning objectives

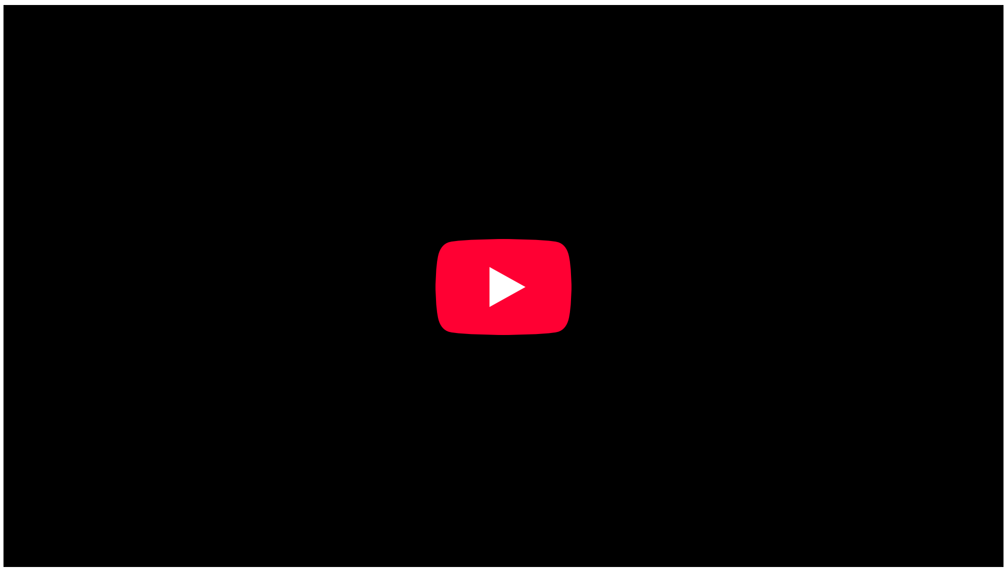
- Describe relationship between the Kirchhoff's circuit laws and the energy and charge in the electrical circuits

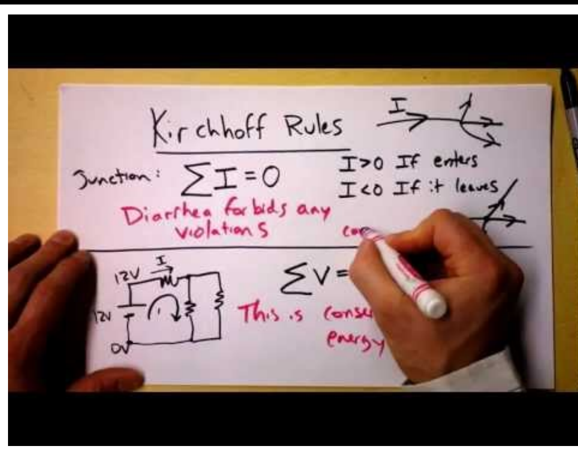
Kirchhoff's junction rule, also known as Kirchhoff's current law (KCL), Kirchoff's first law, Kirchhoff's point rule, and Kirchhoff's nodal rule, is an application of the principle of conservation of electric charge.

Kirchhoff's junction rule states that at any junction (node) in an electrical circuit, the sum of the currents flowing into that junction is equal to the sum of the currents flowing out of that junction. In other words, given that a current will be positive or negative depending on whether it is flowing towards or away from a junction, the algebraic sum of currents in a network of conductors meeting at a point is equal to zero. A visual representation can be seen in.



Kirchhoff's Junction Law: Kirchhoff's Junction Law illustrated as currents flowing into and out of a junction.





Kirchhoff's Loop and Junction Rules Theory: We justify Kirchhoff's Rules from diarrhea and conservation of energy. Some people call 'em laws, but not me!

Thus, Kirchoff's junction rule can be stated mathematically as a sum of currents (I):

$$\sum_{k=1}^{n} I_k = 0 \tag{20.3.1}$$

where n is the total number of branches carrying current towards or away from the node.

This law is founded on the conservation of charge (measured in coulombs), which is the product of current (amperes) and time (seconds).

Limitation

Kirchhoff’s junction law is limited in its applicability. It holds for all cases in which total electric charge (Q) is constant in the region in consideration. Practically, this is always true so long as the law is applied for a specific point. Over a region, however, charge density may not be constant. Because charge is conserved, the only way this is possible is if there is a flow of charge across the boundary of the region. This flow would be a current, thus violating Kirchhoff’s junction law.

The Loop Rule

Kirchhoff’s loop rule states that the sum of the emf values in any closed loop is equal to the sum of the potential drops in that loop.

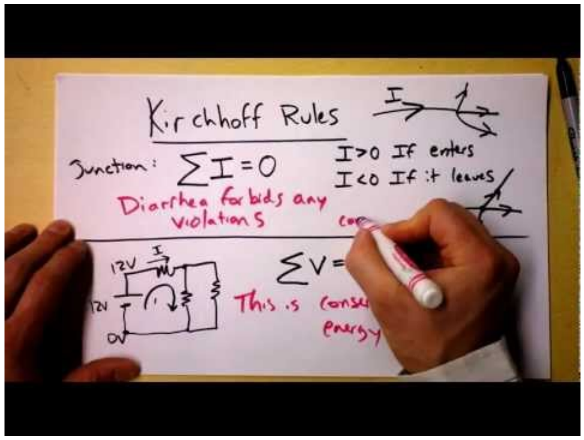
learning objectives

- Formulate the Kirchhoff’s loop rule, noting its assumptions

Kirchhoff’s loop rule (otherwise known as *Kirchhoff’s voltage law (KVL), Kirchhoff’s mesh rule, Kirchhoff’s second law,* or*Kirchhoff’s second rule*) is a rule pertaining to circuits, and is based on the principle of conservation of energy.

Conservation of energy—the principle that energy is neither created nor destroyed—is a ubiquitous principle across many studies in physics, including circuits. Applied to circuitry, it is implicit that the directed sum of the electrical potential differences (voltages) around any closed network is equal to zero. In other words, the sum of the electromotive force (emf) values in any closed loop is equal to the sum of the potential drops in that loop (which may come from resistors).

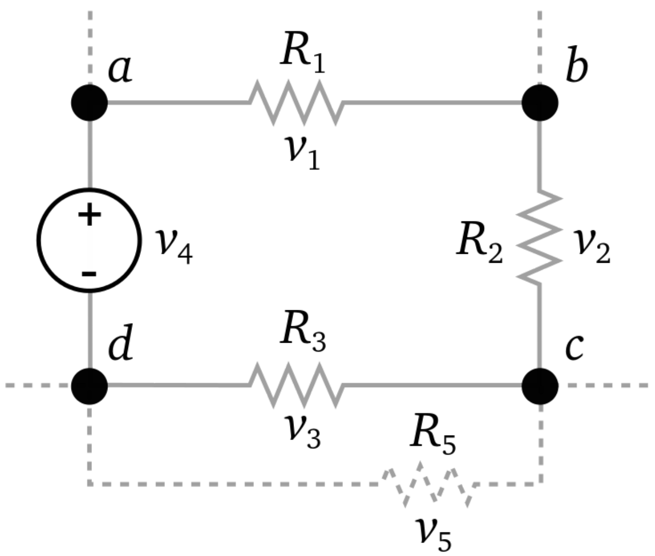
Another equivalent statement is that the algebraic sum of the products of resistances of conductors (and currents in them) in a closed loop is equal to the total electromotive force available in that loop. Mathematically, Kirchhoff’s loop rule can be represented as the sum of voltages in a circuit, which is equated with zero:



Kirchhoff's Loop and Junction Rules Theory: We justify Kirchhoff's Rules from diarrhea and conservation of energy. Some people call 'em laws, but not me!

$$\sum_{k=1}^{n} V_k = 0 \tag{20.3.2}$$

Here, V_k is the voltage across element k, and n is the total number of elements in the closed loop circuit. An illustration of such a circuit is shown in. In this example, the sum of v_1, v_2, v_3, and v_4 (and v_5 if it is included), is zero.

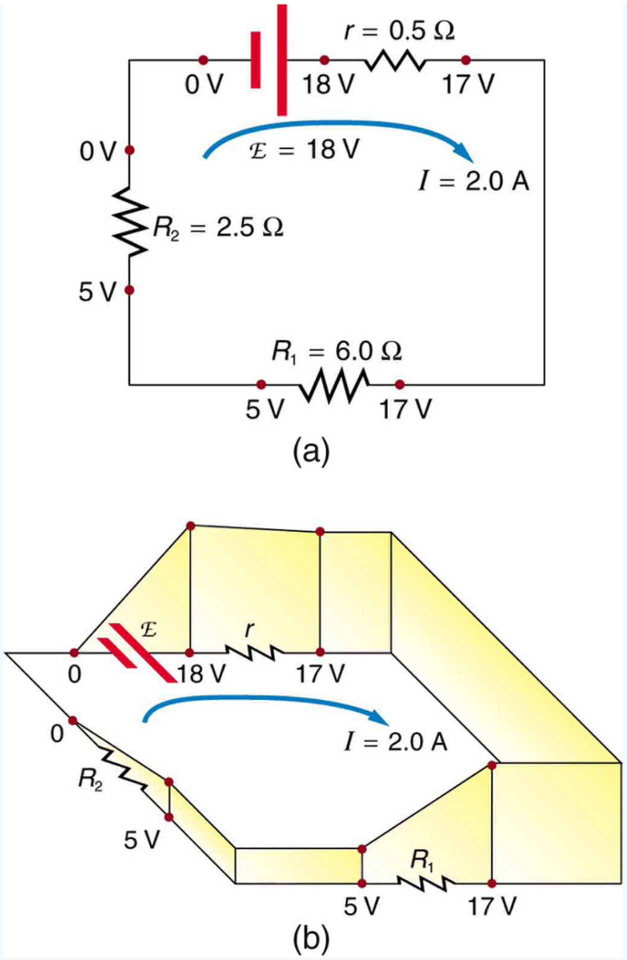


Kirchhoff's Loop Rule: Kirchhoff's loop rule states that the sum of all the voltages around the loop is equal to zero: v1 + v2 + v3 – v4 = 0.

Given that voltage is a measurement of energy per unit charge, Kirchhoff's loop rule is based on the law of conservation of energy, which states: *the total energy gained per unit charge must equal the amount of energy lost per unit of charge*.

Example 20.3.1:

illustrates the changes in potential in a simple series circuit loop. Kirchhoff's second rule requires $emf - Ir - IR_1 - IR_2 = 0$. Rearranged, this is $emf = Ir + IR_1 + IR_2$, meaning that the emf equals the sum of the IR (voltage) drops in the loop. The emf supplies 18 V, which is reduced to zero by the resistances, with 1 V across the internal resistance, and 12 V and 5 V across the two load resistances, for a total of 18 V.



The Loop Rule: An example of Kirchhoff's second rule where the sum of the changes in potential around a closed loop must be zero. (a) In this standard schematic of a simple series circuit, the emf supplies 18 V, which is reduced to zero by the resistances, with 1 V across the internal resistance, and 12 V and 5 V across the two load resistances, for a total of 18 V. (b) This perspective view represents the potential as something like a roller coaster, where charge is raised in potential by the emf and lowered by the resistances. (Note that the script E stands for emf.)

Limitation

Kirchhoff's loop rule is a simplification of Faraday's law of induction, and holds under the assumption that there is no fluctuating magnetic field linking the closed loop. In the presence of a variable magnetic field, electric fields could be induced and emf could be produced, in which case Kirchhoff's loop rule breaks down.

Applications

Kirchhoff's rules can be used to analyze any circuit and modified for those with EMFs, resistors, capacitors and more.

learning objectives

- Describe conditions when the Kirchoff's rules are useful to apply

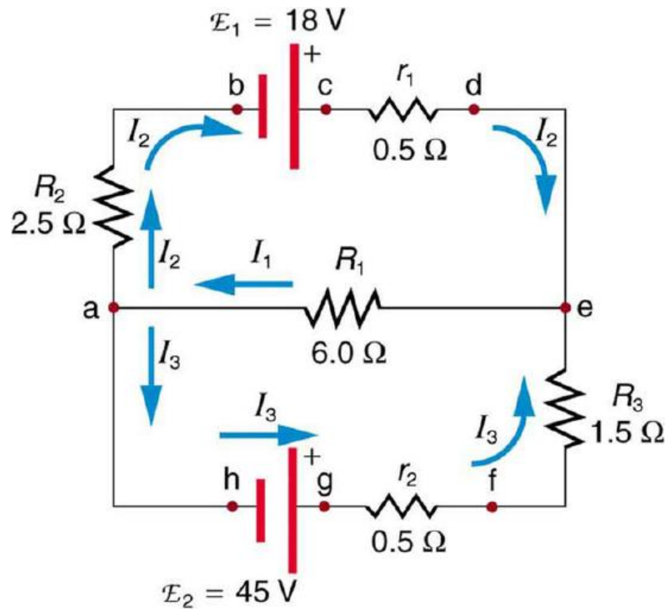
Overview

Kirchhoff's rules can be used to analyze any circuit by modifying them for those circuits with electromotive forces, resistors, capacitors and more. Practically speaking, however, the rules are only useful for characterizing those circuits that cannot be simplified by combining elements in series and parallel.

Combinations in series and parallel are typically much easier to perform than applying either of Kirchhoff's rules, but Kirchhoff's rules are more broadly applicable and should be used to solve problems involving complex circuits that cannot be simplified by combining circuit elements in series or parallel.

Example of Kirchoff's Rules

shows a very complex circuit, but Kirchhoff's loop and junction rules can be applied. To solve the circuit for currents I_1, I_2, and I_3, both rules are necessary.



Kirchhoff's Rules: sample problem: This image shows a very complicated circuit, which can be reduced and solved using Kirchoff's Rules.

Applying Kirchhoff's junction rule at point a, we find:

$$I_1 = I_2 + I_3 \tag{20.3.3}$$

because I_1 flows into point a, while I_2 and I3 flow out. The same can be found at point e. We now must solve this equation for each of the three unknown variables, which will require three different equations.

Considering loop abcdea, we can use Kirchhoff's loop rule:

$$-I_2R_2 + emf_1 - I_2r_1 - I_1R_1 = -I_2\left(R_2\right) + r_1) + emf_1 - I_1R_1 = 0 \tag{20.3.4}$$

Substituting values of resistance and emf from the figure diagram and canceling the ampere unit gives:

$$-3I_2 + 18 - 6I_1 = 0 \tag{20.3.5}$$

This is the second part of a system of three equations that we can use to find all three current values. The last can be found by applying the loop rule to loop aefgha, which gives:

$$I_1R_1 + I_3R_3 + I_3r_2 - emf_2 = I_1R_1 + I_3\left(R_3 + r_2\right) - emf_2 = 0 \tag{20.3.6}$$

Using substitution and simplifying, this becomes:

$$6I_1 + 2I_3 - 45 = 0 \tag{20.3.7}$$

In this case, the signs were reversed compared with the other loop, because elements are traversed in the opposite direction.

We now have three equations that can be used in a system. The second will be used to define I_2, and can be rearranged to:

$$I_2 = 6 - 2I_1 \tag{20.3.8}$$

The third equation can be used to define I_3, and can be rearranged to:

$$I_3 = 22.5 - 3I_1 \tag{20.3.9}$$

Substituting the new definitions of I_2 and I_3 (which are both in common terms of I_1), into the first equation ($I_1=I_2+I_3$), we get:

$$II_1 = (6 - 2I_1) + (22.5 - 3I_1) = 28.5 - 5I_1 \tag{20.3.10}$$

Simplifying, we find that $I_1=4.75$ A. Inserting this value into the other two equations, we find that $I_2=-3.50$ A and $I_3=8.25$ A.

Key Points

- Kirchhoff used Georg Ohm 's work as a foundation to create Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL) in 1845. These can be derived from Maxwell's Equations, which came 16-17 years later.
- It is impossible to analyze some closed-loop circuits by simplifying as a sum and/or series of components. In these cases, Kirchhoff's laws can be used.
- Kirchhoff's laws are special cases of conservation of energy and charge.
- Kirchhoff's junction rule is an application of the principle of conservation of electric charge: current is flow of charge per time, and if current is constant, that which flows into a point in a circuit must equal that which flows out of it.
- The mathematical representation of Kirchhoff's law is: $\sum_{k=1}^{n} I_k = 0$ where I_k is the current of k, and n is the total number of wires flowing into and out of a junction in consideration.
- Kirchhoff's junction law is limited in its applicability over regions, in which charge density may not be constant. Because charge is conserved, the only way this is possible is if there is a flow of charge across the boundary of the region. This flow would be a current, thus violating the law.
- Kirchhoff's loop rule is a rule pertaining to circuits that is based upon the principle of conservation of energy.
- Mathematically, Kirchoff's loop rule can be represented as the sum of voltages (V_k) in a circuit, which is equated with zero: $\sum_{k=1}^{n} V_k = 0$.
- Kirchhoff's loop rule is a simplification of Faraday's law of induction and holds under the assumption that there is no fluctuating magnetic field linking the closed loop.
- Kirchhoff's rules can be applied to any circuit, regardless of its composition and structure.
- Because combining elements is often easy in parallel and series, it is not always convenient to apply Kirchhoff's rules.
- To solve for current in a circuit, the loop and junction rules can be applied. Once all currents are related by the junction rule, one can use the loop rule to obtain several equations to use as a system to find each current value in terms of other currents. These can be solved as a system.

Key Terms

- **resistor**: An electric component that transmits current in direct proportion to the voltage across it.
- **electromotive force**: (EMF)—The voltage generated by a battery or by the magnetic force according to Faraday's Law. It is measured in units of volts (not newtons, N; EMF is not a force).
- **capacitor**: An electronic component consisting of two conductor plates separated by empty space (sometimes a dielectric material is instead sandwiched between the plates), and capable of storing a certain amount of charge.
- **electric charge**: A quantum number that determines the electromagnetic interactions of some subatomic particles; by convention, the electron has an electric charge of -1 and the proton +1, and quarks have fractional charge.
- **current**: The time rate of flow of electric charge.

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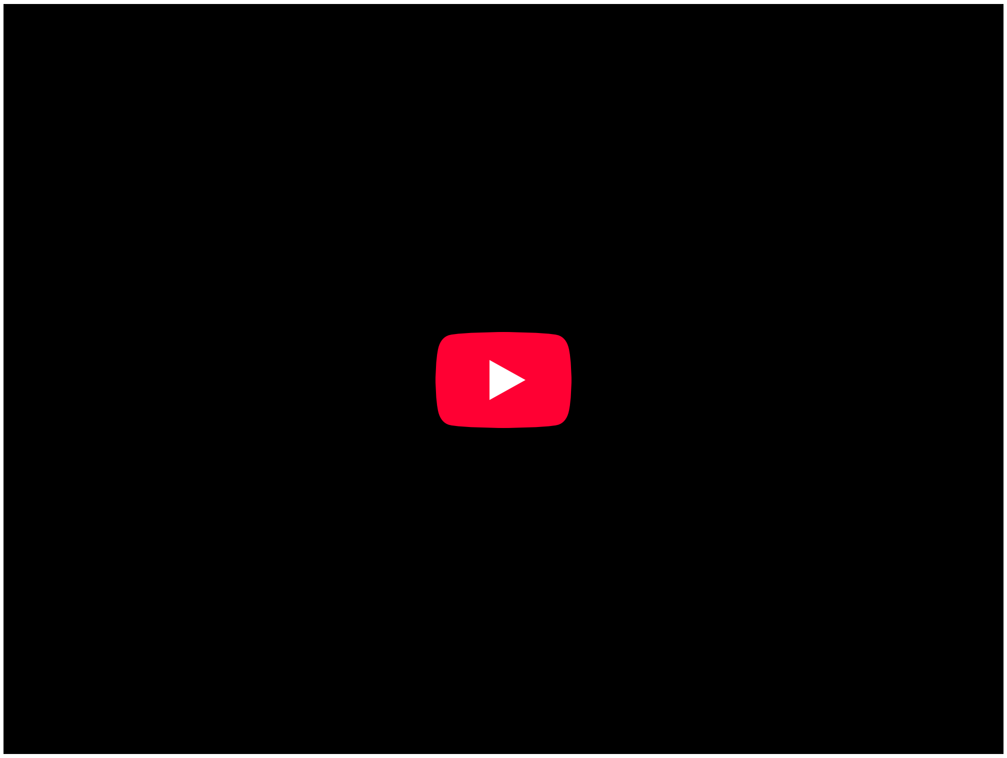
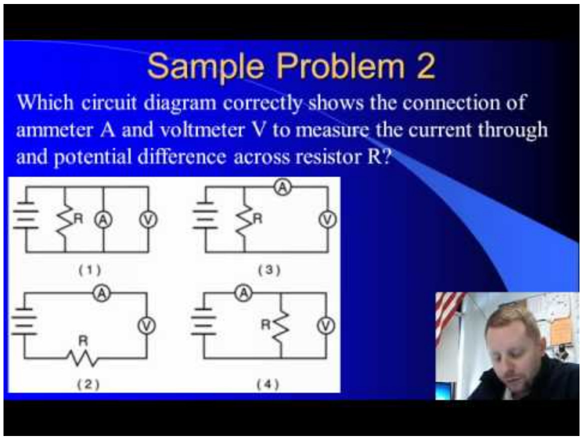
20.4: Voltmeters and Ammeters

learning objectives

- Compare circuit connection of an ammeter and a voltmeter

Voltmeters and ammeters measure the voltage and current, respectively, of a circuit. Some meters in automobile dashboards, digital cameras, cell phones, and tuner-amplifiers are voltmeters or ammeters.

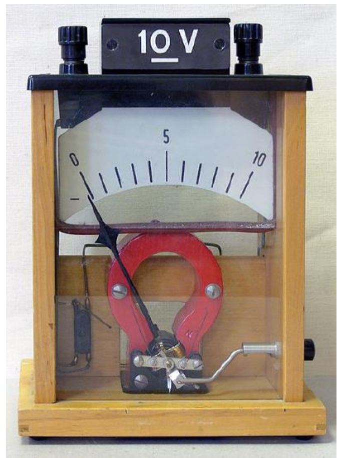




Voltmeters and Ammeters: A brief introduction to voltmeters and ammeters for introductory physics students.

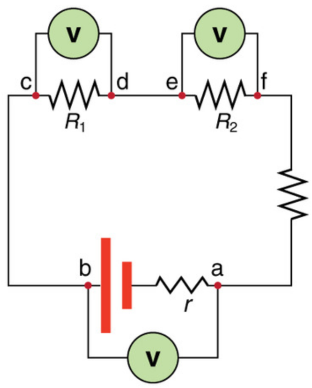
Voltmeters

A voltmeter is an instrument that measures the difference in electrical potential between two points in an electric circuit. An analog voltmeter moves a pointer across a scale in proportion to the circuit's voltage; a digital voltmeter provides a numerical display. Any measurement that can be converted to voltage can be displayed on a meter that is properly calibrated; such measurements include pressure, temperature, and flow.



Voltmeter: Demonstration voltmeter from a physics class

In order for a voltmeter to measure a device's voltage, it must be connected in parallel to that device. This is necessary because objects in parallel experience the same potential difference.



(a)

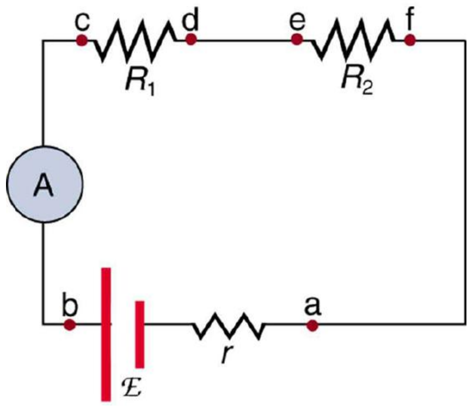
(b)

Voltmeter in Parallel: (a) To measure the potential difference in this series circuit, the voltmeter (V) is placed in parallel with the voltage source or either of the resistors. Note that terminal voltage is measured between points a and b. It is not possible to connect the voltmeter directly across the EMF without including its internal resistance, r. (b) A digital voltmeter in use

Ammeters

An ammeter measures the electric current in a circuit. The name is derived from the name for the SI unit for electric current, amperes (A).

In order for an ammeter to measure a device's current, it must be connected in series to that device. This is necessary because objects in series experience the same current. They must not be connected to a voltage source — ammeters are designed to work under a minimal burden, (which refers to the voltage drop across the ammeter, typically a small fraction of a volt).



Ammeter in Series: An ammeter (A) is placed in series to measure current. All of the current in this circuit flows through the meter. The ammeter would have the same reading if located between points d and e or between points f and a, as it does in the position shown. (Note that the script capital E stands for EMF, and r stands for the internal resistance of the source of potential difference.)

Galvanometers (Analog Meters)

Analog meters have needles that swivel to point at numbers on a scale, as opposed to digital meters, which have numerical readouts.The heart of most analog meters is a device called a galvanometer, denoted by G. Current flow through a galvanometer, I_G, produces a proportional movement, or deflection, of the needle.

The two crucial characteristics of any galvanometer are its resistance and its current sensitivity. Current sensitivity is the current that gives a full-scale deflection of the galvanometer's needle — in other words, the maximum current that the instrument can measure. For example, a galvanometer with a current sensitivity of 50 μA has a maximum deflection of its needle when 50 μA flows through it, is at the scale's halfway point when 25 μA flows through it, and so on.

If such a galvanometer has a 25-Ω resistance, then a voltage of only $\mathrm{V=IR=(50\mu A)(25\Omega)=1.25\ mV}$ produces a full-scale reading. By connecting resistors to this galvanometer in different ways, you can use it as either a voltmeter or ammeter to measure a broad range of voltages or currents.

Galvanometers as Voltmeters

A galvanometer can function as a voltmeter when it is connected in series with a large resistance R. The value of R is determined by the maximum voltage that will be measured. Suppose you want 10 V to produce a full-scale deflection of a voltmeter containing a 25-Ω galvanometer with a 50-μA sensitivity. Then 10 V applied to the meter must produce a current of 50 μA. The total resistance must be:

$$\mathrm{R_{tot}=R+r=\frac{V}{I}=\frac{10V}{50\mu A}=200k\Omega} \tag{20.4.1}$$

or:

$$\mathrm{R=R_{tot}-r=200k\Omega-25\Omega\approx 200k\Omega} \tag{20.4.2}$$

(R is so large that the galvanometer resistance, r, is nearly negligible.) Note that 5 V applied to this voltmeter produces a half-scale deflection by sending a 25-μA current through the meter, and so the voltmeter's reading is proportional to voltage, as desired. This voltmeter would not be useful for voltages less than about half a volt, because the meter deflection would be too small to read accurately. For other voltage ranges, other resistances are placed in series with the galvanometer. Many meters allow a choice of scales, which involves switching an appropriate resistance into series with the galvanometer.

Galvanometers as Ammeters

The same galvanometer can also function as an ammeter when it is placed in parallel with a small resistance *R*, often called the shunt resistance. Since the shunt resistance is small, most of the current passes through it, allowing an ammeter to measure currents much greater than those that would produce a full-scale deflection of the galvanometer.

Suppose, for example, we need an ammeter that gives a full-scale deflection for 1.0 A and that contains the same 25-Ω galvanometer with 50-μA sensitivity. Since *R* and *r* are in parallel, the voltage across them is the same.

These IR drops are: $IR = I_G r$

so that: $IR = \frac{I_G}{I} = \frac{R}{r}$.

Solving for R, and noting that IG is 50 μA and I is 0.999950 A, we have:

$$R = r\frac{I_G}{I} = (25\Omega)\frac{50\mu A}{0.999950 A} = 1.25 \times 10^{-3}\Omega \tag{20.4.3}$$

Null Measurements

Null measurements balance voltages so there is no current flowing through the measuring devices that would interfere with the measurement.

learning objectives

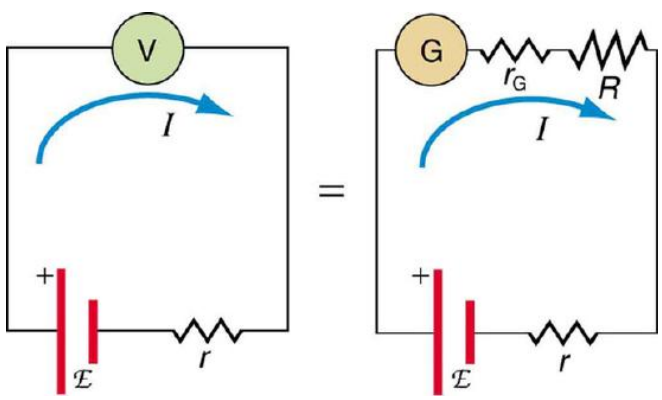
- Explain why null measurements are employed

Null Measurements

Standard measurements of voltage and current alter circuits, introducing numerical uncertainties. Voltmeters draw some extra current, whereas ammeters reduce current flow. Null measurements balance voltages, so there is no current flowing through the measuring device and the circuit is unaltered. Null measurements are generally more accurate but more complex than standard voltmeters and ammeters. Their precision is still limited.

The Potentiometer

When measuring the EMF of a battery and connecting the battery directly to a standard voltmeter, as shown in, the actual quantity measured is the terminal voltage *V*. Voltage is related to the EMF of the battery by *V=emf−Ir*, where *I* is the current that flows and *r* is the internal resistance of the battery.

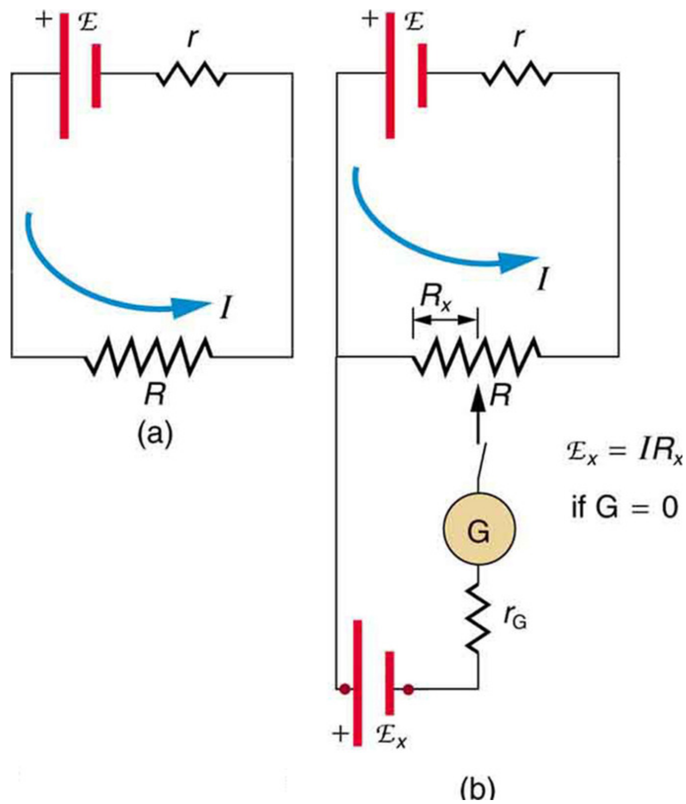


Voltmeter Connected to Battery: An analog voltmeter attached to a battery draws a small but nonzero current and measures a terminal voltage that differs from the EMF of the battery. (Note that the script capital E symbolizes electromotive force, or EMF.) Since the internal resistance of the battery is not known precisely, it is not possible to calculate the EMF precisely.

The EMF could be accurately calculated if *r* were known, which is rare. If the current *I* could be made zero, then *V=emf*, and EMF could be directly measured. However, standard voltmeters need a current to operate.

A potentiometer is a null measurement device for measuring potentials (voltages). A voltage source is connected to resistor *R*, passing a constant current through it. There is a steady drop in potential (IR drop) along the wire, so a variable potential is obtained through contact along the wire.

An unknown emf_x (represented by script E_x) connected in series with a galvanometer is shown in. Note that emf_x opposes the other voltage source. The location of the contact point is adjusted until the galvanometer reads zero. When the galvanometer reads zero, $emf_x=IR_x$, where $R_x$is the resistance of the wire section up to the contact point. Since no current flows through the galvanometer, none flows through the unknown EMF, and emf_x is sensed.



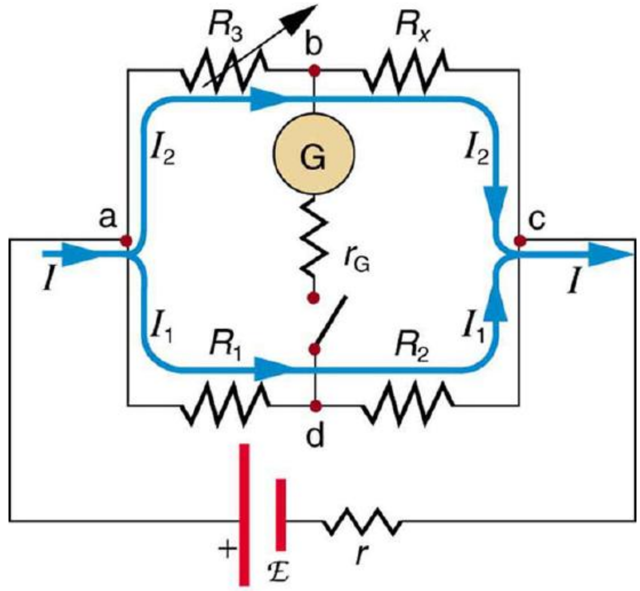
Potentiometer: The potentiometer is a null measurement device. (a.) A voltage source connected to a long wire resistor passes a constant current I through it. (b.) An unknown EMF (labeled script Ex) is connected as shown, and the point of contact along R is adjusted until the galvanometer reads zero. The segment of wire has a resistance Rx and script Ex=IRx, where I is unaffected by the connection, since no current flows through the galvanometer. The unknown EMF is thus proportional to the resistance of the wire segment.

Standard EMF is substituted for emf_x, and the contact point is adjusted until the galvanometer reads zero, so that $emf_s=IR_s$. In both cases, no current passes through the galvanometer. The current Ithrough the long wire is identical. Taking the ratio emf_x/emf_s, Icancels, and solving for emf_x gives what is seen in.

Because a long uniform wire is used for R, the ratio of resistances R_x/R_s is the same as the ratio of the lengths of wire that zero the galvanometer for each EMF. The three quantities on the right-hand side of the equation are now known or measured, and emf_x can be calculated. There is often less uncertainty in this calculation than when using a voltmeter directly, but it is not zero. There is always some uncertainty in the ratio of resistances $R_x/R_s$and in the standard EMFs. Furthermore, it is not possible to tell when the galvanometer reads exactly zero, which introduces error into both R_x and R_s, and may also affect the current I.

Resistance Measurements

Many so-called ohmmeters measure resistance. Most common ohmmeters apply a voltage to a resistance, measure the current, and calculate the resistance using Ohm ‘s law. Their readout is this calculated resistance. Simple configurations using standard voltmeters and ammeters have limited accuracy, because the meters alter both the voltage applied to the resistor and the current flowing through it. The Wheatstone bridge is a null measurement device for calculating resistance by balancing potential drops in a circuit. The device is called a bridge because the galvanometer forms a bridge between two branches. A variety of bridge devicesare used to make null measurements in circuits. Resistors R_1 and R_2 are precisely known, while the arrow through R_3indicates that it is a variable resistance. The value of R_3 can be precisely read. With the unknown resistance Rx in the circuit, R_3 is adjusted until the galvanometer reads zero.



Wheatstone Bridge: The Wheatstone bridge is used to calculate unknown resistances. The variable resistance R3 is adjusted until the galvanometer reads zero with the switch closed. This simplifies the circuit, allowing Rx to be calculated based on the IR drops.

The potential difference between points *b* and *d* is then zero, meaning that *b* and *d* are at the same potential. With no current running through the galvanometer, it has no effect on the rest of the circuit. So the branches *abc* and *adc* are in parallel, and each branch has the full voltage of the source. Since *b* and *d* are at the same potential, the IR drop along *ad* must equal the IR drop along *ab*. Again, since *b* and *d* are at the same potential, the IR drop along *dc*must equal the IR drop along *bc*. This equation is used to calculate the unknown resistance when current through the galvanometer is zero. This method can be very accurate, but it is limited by two factors. First, it is not possible for the current through the galvanometer to be exactly zero. Second, there are always uncertainties in R_1, R_2, and R_3, which contribute to the uncertainty in R_x.

Key Points

- A voltmeter is an instrument used for measuring electrical potential difference between two points in an electric circuit.
- An ammeter is a measuring device used to measure the electric current in a circuit.
- A voltmeter is connected in parallel with a device to measure its voltage, while an ammeter is connected in series with a device to measure its current.
- At the heart of most analog meters is a galvanometer, an instrument that measures current flow using the movement, or deflection, of a needle. The needle deflection is produced by a magnetic force acting on a current-carrying wire.
- Measurements of voltages and current with standard voltmeters and ammeters alter the circuit being measured, introducing uncertainties. Voltmeters draw some extra current, whereas ammeters reduce current flow.
- Null measurements are employed to reduce the uncertainty in the measured voltage and current.
- The potentiometer and the Wheatstone bridge are two methods for making null measurements.
- The potentiometer is an instrument that measures an unknown voltage by opposing with a known voltage, without drawing current from the voltage source being measured.
- A Wheatstone bridge is an electrical circuit used to measure an unknown electrical resistance by balancing two legs of a bridge circuit, one leg of which includes the unknown component.

Key Terms

- **shunt resistance**: a small resistance R placed in parallel with a galvanometer G to produce an ammeter; the larger the current to be measured, the smaller R must be; most of the current flowing through the meter is shunted through R to protect the galvanometer
- **galvanometer**: An analog measuring device, denoted by G, that measures current flow using a needle deflection caused by a magnetic field force acting upon a current-carrying wire.
- **null measurements**: methods of measuring current and voltage more accurately by balancing the circuit so that no current flows through the measurement device

- **potentiometer**: an instrument that measures a voltage by opposing it with a precise fraction of a known voltage, and without drawing current from the unknown source.
- **Wheatstone bridge**: An instrument used to measure an unknown electrical resistance by balancing two legs of a bridge circuit, one leg of which includes the unknown component.

20.5: RC Circuits

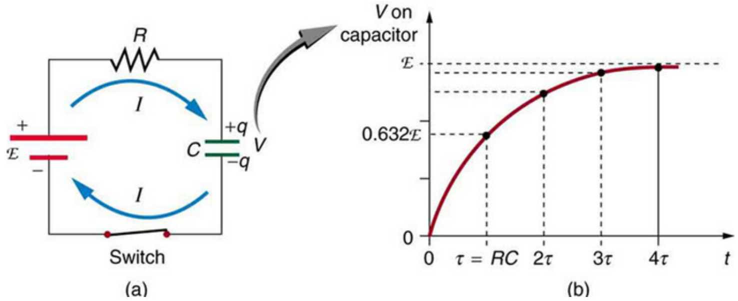
learning objectives

- Describe the components and function of an RC circut, noting especially the time-dependence of the capacitor's charge

An RC circuit is one containing a resistor R and a capacitor C. The capacitor is an electrical component that houses electric charge. In this Atom, we will study how a series RC circuit behaves when connected to a DC voltage source. (In subsequent Atoms, we will study its AC behavior.)

Charging

Fig 1 shows a simple RC circuit that employs a DC voltage source. The capacitor is initially uncharged. As soon as the switch is closed, current flows to and from the initially uncharged capacitor. As charge increases on the capacitor plates, there is increasing opposition to the flow of charge by the repulsion of like charges on each plate.



Charging an RC Circuit: (a) An RC circuit with an initially uncharged capacitor. Current flows in the direction shown as soon as the switch is closed. Mutual repulsion of like charges in the capacitor progressively slows the flow as the capacitor is charged, stopping the current when the capacitor is fully charged and Q=C⋅emf. (b) A graph of voltage across the capacitor versus time, with the switch closing at time t=0. (Note that in the two parts of the figure, the capital script E stands for emf, q stands for the charge stored on the capacitor, and τ is the RC time constant.)

In terms of voltage, across the capacitor voltage is given by V_c=Q/C, where Q is the amount of charge stored on each plate and C is the capacitance. This voltage opposes the battery, growing from zero to the maximum emf when fully charged. Thus, the current decreases from its initial value of I_0=emf/R to zero as the voltage on the capacitor reaches the same value as the emf. When there is no current, there is no IR drop, so the voltage on the capacitor must then equal the emf of the voltage source.

Initially, voltage on the capacitor is zero and rises rapidly at first since the initial current is a maximum. Fig 1 (b) shows a graph of capacitor voltage versus time (t) starting when the switch is closed at t=0. The voltage approaches emf asymptotically since the closer it gets to emf the less current flows. The equation for voltage versus time when charging a capacitor C through a resistor R, is:

$$\mathrm{V(t) = emf\left(1 - e^{t/RC}\right)} \tag{20.5.1}$$

where V(t) is the voltage across the capacitor and emf is equal to the emf of the DC voltage source. (The exact form can be derived by solving a linear differential equation describing the RC circuit, but this is slightly beyond the scope of this Atom.) Note that the unit of RC is second. We define the time constant τ for an RC circuit as $\tau = \mathrm{RC}$. τ shows how quickly the circuit charges or discharges.

Discharging

Discharging a capacitor through a resistor proceeds in a similar fashion, as illustrates. Initially, the current is $I_0=V_0/R$, driven by the initial voltage V_0 on the capacitor. As the voltage decreases, the current and hence the rate of discharge decreases, implying another exponential formula for V. Using calculus, the voltage V on a capacitor C being discharged through a resistor R is found to be

$$\mathrm{V(t) = V_0 e^{-t/RC}} \tag{20.5.2}$$

Impedance

Impedance is the measure of the opposition that a circuit presents to the passage of a current when a voltage is applied.

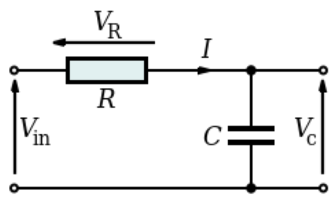
learning objectives

- Express the relationship between the impedance, the resistance, and the capacitance of a series RC circuit in a form of equation

Rather than solving the differential equation relating to circuits that contain resistors and capacitors, we can imagine all sources in the circuit are complex exponentials having the same frequency. This technique is useful in solving problems in which phase relationship is important. The phase of the complex impedance is the phase shift by which the current is ahead of the voltage.

Complex Analysis

For an RC circuit in, the AC source driving the circuit is given as:



Series RC Circuit: Series RC circuit.

$$v_{in}(t) = Ve^{j\omega t} \tag{20.5.3}$$

where V is the amplitude of the AC voltage, j is the imaginary unit (j^2=-1), and ωω is the angular frequency of the AC source. Two things to note:

1. We use lower case alphabets for voltages and sources to represent that they are alternating (i.e., we use $v_{in}(t)$ instead of $V_{in}(t)$).
2. The imaginary unit is given the symbol "j", not the usual "i". "i" is reserved for alternating currents.

Complex Impedance

The advantage of assuming sources take this form is that all voltages and currents in the circuit are also complex exponentials (having the same frequency as the source). To appreciate the reason for this, we can investigate how each circuit element behaves when either the voltage or current is a complex exponential. For the resistor, $v = Ri$. From our voltage given above, $i = \frac{V}{R}e^{j\omega t}$. Thus the resistor's voltage is a complex, as is the current with an amplitude $I = \frac{V}{R}$. For a capacitor, $i = C\frac{dv}{dt}$. Letting the voltage be a complex exponential we have $i = j\omega CVe^{j\omega t}$. The amplitude of this complex exponential is $I = j\omega CV$.

The major consequence of assuming complex exponential voltage and currents is that the ratio $Z = \frac{V}{I}$ for rather than depending on time each element depends on source frequency. This quantity is known as the element's (complex) impedance. The magnitude of the complex impedance is the ratio of the voltage amplitude to the current amplitude. Just like resistance in DC cases, impedance is the measure of the opposition that a circuit presents to the passage of a current when a voltage is applied. The impedance of a resistor is R, while that of a capacitor (C) is $\frac{1}{j\omega C}$. In the case of the circuit in, to find the complex impedance of the RC circuit, we add the impedance of the two components, just as with two resistors in series: $Z = R + \frac{1}{j\omega C}$.

Finding Real Currents and Voltages

Since $e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$, to find the real currents and voltages we simply need to take the real part of the i(t) and v(t). The (real value) impedance is the real part of the complex impedance Z. For a series RC circuit, we get $Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$. We see that the amplitude of the current will be $\frac{V}{Z} = \frac{V}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}$.

Phase Angle and Power Factor

In a series RC circuit connected to an AC voltage source, voltage and current maintain a phase difference.

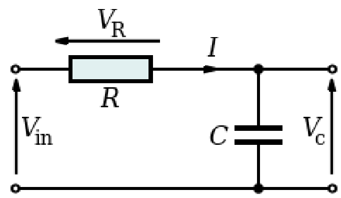
learning objectives

- Compare the currents in the resistor and capacitor in a series RC circuit connected to an AC voltage source

Phase Angle

Impedance is an AC (alternating current) analogue to resistance in a DC circuit. As we studied in a previously Atom ("Impedance"), current, voltage and impedance in an RC circuit are related by an AC version of Ohm 's law: $\mathrm{I}=\frac{\mathrm{V}}{\mathrm{Z}}$, where I and V are peak current and peak voltage respectively, and Z is the impedance of the circuit.

In a series RC circuit connected to an AC voltage source as shown in, conservation of charge requires current be the same in each part of the circuit at all times. Therefore we can say: *the currents in the resistor and capacitor are equal and in phase*. (We will represent instantaneous current as i(t).)



Series RC Circuit: Series RC circuit.

On the other hand, because the total voltage should be equal to the sum of voltages on the resistor and capacitor, so we have:

$$\begin{aligned}\mathrm{v}(\mathrm{t}) &=\mathrm{v}_{\mathrm{R}}(\mathrm{t})+\mathrm{v}_{\mathrm{C}}(\mathrm{t}) \\ &=\mathrm{i}(\mathrm{t}) \mathrm{R}+\mathrm{i}(\mathrm{t}) \frac{1}{\mathrm{j} \omega C} \\ &=\mathrm{i}(\mathrm{t})\left(\mathrm{R}+\frac{1}{\mathrm{j} \omega C}\right)\end{aligned}$$

where ωω is the angular frequency of the AC voltage source and j is the imaginary unit; j^2=-1. Since the complex number $\mathrm{Z}=\mathrm{R}+\frac{1}{\mathrm{j}\omega C}=\sqrt{\mathrm{R}^2+\left(\frac{1}{\omega \mathrm{C}}\right)^2} \mathrm{e}^{\mathrm{j}\phi}$ has a phase angle ϕ that satisfies $\cos\phi=\frac{\mathrm{R}}{\sqrt{\mathrm{R}^2+\left(\frac{1}{\omega\mathrm{C}}\right)^2}}$,

we notice that voltage $\mathrm{v}(\mathrm{t})$ and current $\mathrm{i}(\mathrm{t})$ has a phase difference of ϕ.

For $\mathrm{R}=0, \phi=90^{\circ}$. As learned from the preceding series of Atoms—the voltage across the capacitor V_C follows the current by one-fourth of a cycle (or 90º).

Power Factor

Because voltage and current are out of phase, power dissipated by the circuit is not equal to: *(peak voltage) times* (*peak current)*. The fact that source voltage and current are out of phase affects the power delivered to the circuit. It can be shown that the average power is $I_{rms}V_{rms}\cos\phi$, where I_{rms} and V_{rms} are the root mean square (rms) averages of the current and voltage, respectively. For this reason, cosϕ is called the *power factor*, which can range from 0 to 1.

Key Points

- In an RC circuit connected to a DC voltage source, the current decreases from its initial value of I_0=emf/R to zero as the voltage on the capacitor reaches the same value as the emf.
- In an RC circuit connected to a DC voltage source, voltage on the capacitor is initially zero and rises rapidly at first since the initial current is a maximum: $\mathrm{V}(\mathrm{t})=\mathrm{emf}\left(1-\mathrm{e}^{\mathrm{t}/\mathrm{RC}}\right)$.
- The time constant τ for an RC circuit is defined to be RC. It's unit is in seconds and shows how quickly the circuit charges or discharges.
- The advantage of assuming that sources have complex exponential form is that all voltages and currents in the circuit are also complex exponentials, having the same frequency as the source.
- The major consequence of assuming complex exponential voltage and currents is that the ratio (Z = V/I) for each element does not depend on time, but does depend on source frequency.

- For a series RC circuit, the impedance is given as $Z=\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}$.
- In a series RC circuit connected to an AC voltage source, the currents in the resistor and capacitor are equal and in phase.
- In a series RC circuit connected to an AC voltage source, the total voltage should be equal to the sum of voltages on the resistor and capacitor.
- In a series RC circuit connected to an AC voltage source, voltage and current have a phase difference of ϕ, where $\cos\phi=\frac{R}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}}$. cosϕ is called the power factor.

Key Terms

- **DC**: Direct current; the unidirectional flow of electric charge.
- **capacitor**: An electronic component capable of storing an electric charge, especially one consisting of two conductors separated by a dielectric.
- **differential equation**: An equation involving the derivatives of a function.
- **impedance**: A measure of the opposition to the flow of an alternating current in a circuit; the aggregation of its resistance, inductive and capacitive reactance. Represented by the symbol Z.
- **resistor**: An electric component that transmits current in direct proportion to the voltage across it.
- **alternating current**: (AC)—An electric current in which the direction of flow of the electrons reverses periodically having an average of zero, with positive and negative values (with a frequency of 50 Hz in Europe, 60 Hz in the US, 400 Hz for airport lighting, and some others); especially such a current produced by a rotating generator or alternator.
- **rms**: Root mean square: a statistical measure of the magnitude of a varying quantity.

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CHAPTER OVERVIEW

21: Magnetism

Topic hierarchy

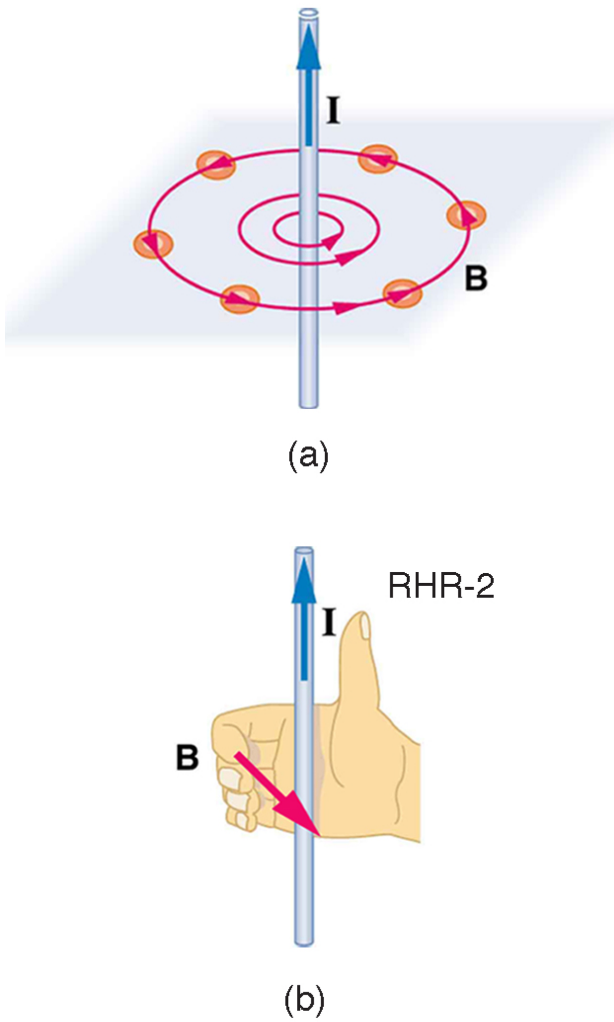
21.1: Magnetism and Magnetic Fields

learning objectives

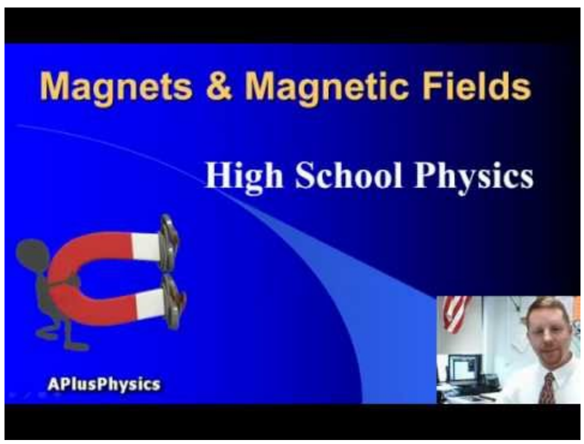
- Describe shape of a magnetic field produced by an electric current flowing through a wire

Electric Current and Magnetic Fields

Electric current produces a magnetic field. This magnetic field can be visualized as a pattern of circular field lines surrounding a wire. One way to explore the direction of a magnetic field is with a compass, as shown by a long straight current-carrying wire in. Hall probes can determine the magnitude of the field. Another version of the right hand rule emerges from this exploration and is valid for any current segment—*point the thumb in the direction of the current, and the fingers curl in the direction of the magnetic field loops* created by it.



Magnetic Field Generated by Current: (a) Compasses placed near a long straight current-carrying wire indicate that field lines form circular loops centered on the wire. (b) Right hand rule 2 states that, if the right hand thumb points in the direction of the current, the fingers curl in the direction of the field. This rule is consistent with the field mapped for the long straight wire and is valid for any current segment.



Magnets and Magnetic Fields: A brief introduction to magnetism for introductory physics students.

Magnitude of Magnetic Field from Current

The equation for the magnetic field strength (magnitude) produced by a long straight current-carrying wire is:

$$\mathrm{B} = \frac{\mu_0 \mathrm{I}}{2\pi \mathrm{r}} \tag{21.1.1}$$

For a long straight wire where *I* is the current, *r* is the shortest distance to the wire, and the constant $_0 = 4\pi 10^{-7}$ T·m/A is the permeability of free space. (μ_0 is one of the basic constants in nature, related to the speed of light.) Since the wire is very long, the magnitude of the field depends only on distance from the wire *r*, not on position along the wire. This is one of the simplest cases to calculate the magnetic field strenght from a current.

The magnetic field of a long straight wire has more implications than one might first suspect. *Each segment of current produces a magnetic field like that of a long straight wire, and the total field of any shape current is the vector sum of the fields due to each segment.* The formal statement of the direction and magnitude of the field due to each segment is called the Biot-Savart law. Integral calculus is needed to sum the field for an arbitrary shape current. The Biot-Savart law is written in its complete form as:

$$B = \frac{\mu_0 I}{4\pi} \int \frac{dl \times \hat{r}}{r^2} \tag{21.1.2}$$

where the integral sums over the wire length where vector $d\ell$ is the direction of the current; r is the distance between the location of $d\ell$, and the location at which the magnetic field is being calculated; and $\hat{r}$is a unit vector in the direction of r. The reader may apply the simplifications in calculating the magnetic field from an infinite straight wire as above and see that the Biot-Savart law reduces to the first, simpler equation.

Ampere's Law

A more fundamental law than the Biot-Savart law is Ampere's Law, which relates magnetic field and current in a general way. In SI units, the integral form of the original Ampere's circuital law is a line integral of the magnetic field around some closed curve C (arbitrary but must be closed). The curve C in turn bounds both a surface S through which the electric current passes through (again arbitrary but not closed—since no three-dimensional volume is enclosed by S), and encloses the current. You can think of the "surface" as the cross-sectional area of a wire carrying current.

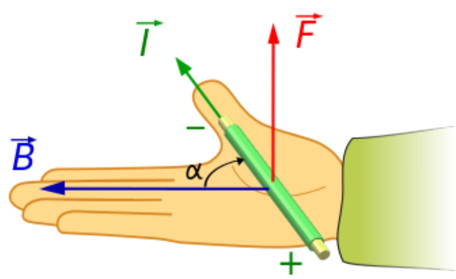
The mathematical statement of the law states that the total magnetic field around some path is directly proportional to the current which passes through that enclosed path. It can be written in a number of forms, one of which is given below.

$$\oint B \cdot dl = \mu_0 \iint_S J \cdot dS = \mu_0 I_{enc} \tag{21.1.3}$$

where the magnetic field is integrated over a curve (circumfrence of a wire), equivalent to integrating the current density (in amperes per square meter, Am^{-2}) over the cross section area of the wire (which is equal to the permeability constant times the enclosed current $I_{enc)}$. Ampere's law is always valid for steady currents and can be used to calculate the B-field for certain highly symmetric situations such as an infinite wire or an infinite solenoid. Ampere's Law is also a component of Maxwell's Equations.

Force on a Current-Carrying Wire

The force on a current carrying wire (as in) is similar to that of a moving charge as expected since a charge carrying wire is a collection of moving charges. A current-carrying wire feels a force in the presence of a magnetic field. Consider a conductor (wire) of length ℓ, cross section A, and charge q which is due to electric current i. If this conductor is placed in a magnetic field of magnitude B which makes an angle with the velocity of charges (current) in the conductor, the force exerted on a single charge q is



Force on a Current-Carrying Wire: The right hand rule can be used to determine the direction of the force on a current-carrying wire placed in an external magnetic field.

$$F = qvB\sin\theta \tag{21.1.4}$$

So, for N charges where

$$N = nlA \tag{21.1.5}$$

the force exerted on the conductor is

$$f = FN = qvBnlA\sin\theta = Bil\sin\theta \tag{21.1.6}$$

where $i = nqvA$. The right hand rule can give you the direction of the force on the wire, as seen in the above figure. Note that the B-field in this case is the *external* field.

Permanent Magnets

Permanent magnets are objects made from ferromagnetic material that produce a persistent magnetic field.

learning objectives

- Give examples and counterexamples of permanent magnets

Permanent Magnets

Overview

Recall that a magnet is a material or object that generates a magnetic field. This magnetic field is invisible but is responsible for the most notable property of a magnet: a force that pulls on other ferromagnetic materials, such as iron, and attracts or repels other magnets.

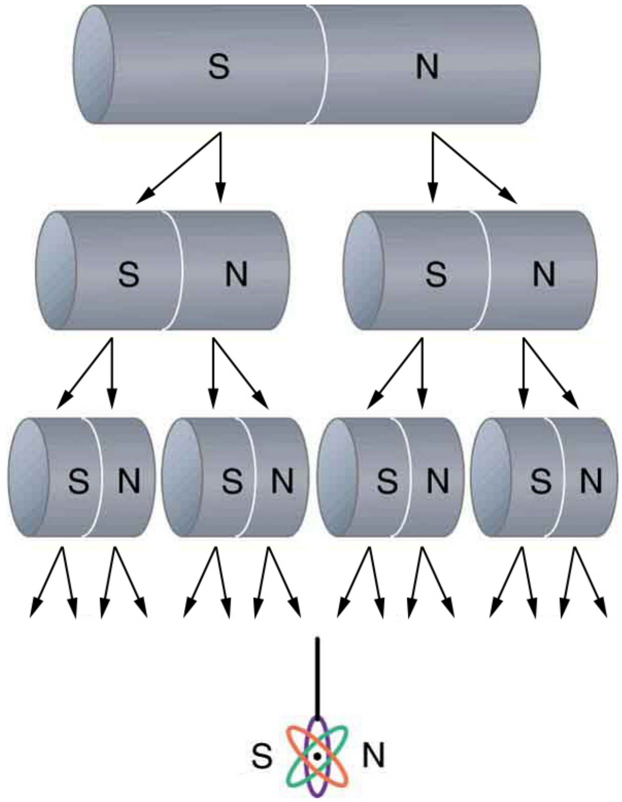
Types of Magnets

A permanent magnet is an object made from a material that is magnetized and creates its own persistent magnetic field. An everyday example is a refrigerator magnet used to hold notes on a refrigerator door. Materials that can be magnetized, which are also the ones that are strongly attracted to a magnet, are called ferromagnetic. These include iron, nickel, cobalt, some alloys of rare earth metals, and some naturally occurring minerals such as lodestone. Although ferromagnetic materials are the only ones attracted to a magnet strongly enough to be commonly considered magnetic, all other substances respond weakly to a magnetic field, by one of several other types of magnetism. The counterexample to a permanent magnet is an electromagnet, which only becomes magnetized when an electric current flows through it.

Example of a Permanent Magnet: An example of a permanent magnet: a “horseshoe magnet” made of alnico, an iron alloy. The magnet is made in the shape of a horseshoe to bring the two magnetic poles close to each other, to create a strong magnetic field there that can pick up heavy pieces of iron.

Polarity

All magnets have two poles, one called the north pole and one called the south pole. Like poles repel and unlike polls attract (in analogy to positive and negative charges in electrostatics). North and south poles always exist in pairs (there are no magnetic monopoles in nature), so if one were to split a permanent magnet in half, two smaller magnets would be created, each with a north pole and south pole.

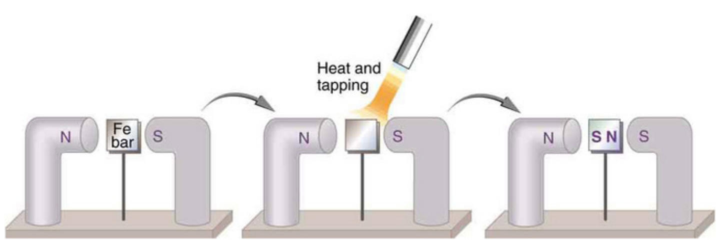


North and South Poles Always Come in Pairs: North and south poles always occur in pairs. Attempts to separate them result in more pairs of poles. If we continue to split the magnet, we will eventually get down to an iron atom with a north pole and a south pole—these, too, cannot be separated.

Manufacturing Permanent Magnets

Ferromagnetic materials can be divided into magnetically “soft” materials like annealed iron, which can be magnetized but do not tend to stay magnetized, and magnetically “hard” materials, which do. Permanent magnets are made from “hard” ferromagnetic materials such as alcino and ferrite that are subjected to special processing in a powerful magnetic field during manufacture, to align their internal microcrystalline structure, making them very hard to demagnetize.

When a magnet is brought near a previously unmagnetized ferromagnetic material, it causes local magnetization of the material with unlike poles closest. (This results in the attraction of the previously unmagnetized material to the magnet.) On the microscopic scale, there are regions in the unmagnetized ferromagnetic material that act like small bar magnets. In each region the poles of individual atoms are aligned. However, before magnetization these regions are small and randomly oriented throughout the unmagnetized ferromagnetic objects, so there is no net magnetic field. In response to an external magnetic field like the one applied in the above figure, these regions grow and become aligned. This arrangement can become permanent when the ferromagnetic material is heated and then cooled.



Making a Ferromagnet: An unmagnetized piece of iron is placed between two magnets, heated, and then cooled, or simply tapped when cold. The iron becomes a permanent magnet with the poles aligned as shown: its south pole is adjacent to the north pole of the original magnet, and its north pole is adjacent to the south pole of the original magnet. Note that there are attractive forces between the magnets.

Magnetic Field Lines

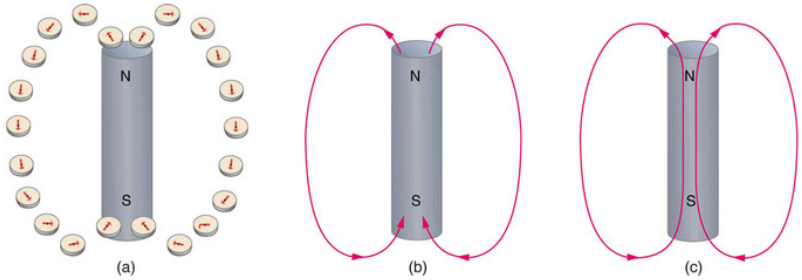
Magnetic field lines are useful for visually representing the strength and direction of the magnetic field.

learning objectives

- Relate the strength of the magnetic field with the density of the magnetic field lines

Magnetic Field Lines

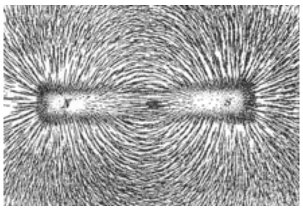
Einstein is said to have been fascinated by a compass as a child, perhaps musing on how the needle felt a force without direct physical contact. His ability to think deeply and clearly about action at a distance, particularly for gravitational, electric, and magnetic forces, later enabled him to create his revolutionary theory of relativity. Since magnetic forces act at a distance, we define a magnetic field to represent magnetic forces. A pictorial representation of magnetic field lines is very useful in visualizing the strength and direction of the magnetic field. The direction of magnetic field lines is defined to be the direction in which the north end of a compass needle points. The magnetic field is traditionally called the B-field.



Visualizing Magnetic Field Lines: Magnetic field lines are defined to have the direction that a small compass points when placed at a location. (A) If small compasses are used to map the magnetic field around a bar magnet, they will point in the directions shown: away from the north pole of the magnet, toward the south pole of the magnet (recall that Earth's north magnetic pole is really a south pole in terms of definitions of poles on a bar magnet.) (B) Connecting the arrows gives continuous magnetic field lines. The strength of the field is proportional to the closeness (or density) of the lines. (C) If the interior of the magnet could be probed, the field lines would be found to form continuous closed loops.

Mapping the magnetic field of an object is simple in principle. First, measure the strength and direction of the magnetic field at a large number of locations (or at every point in space). Then, mark each location with an arrow (called a vector) pointing in the direction of the local magnetic field with its magnitude proportional to the strength of the magnetic field (producing a vector field). You can "connect" the arrows to form magnetic field lines. The direction of the magnetic field at any point is parallel to the direction of nearby field lines, and the local density of field lines can be made proportional to its strength.

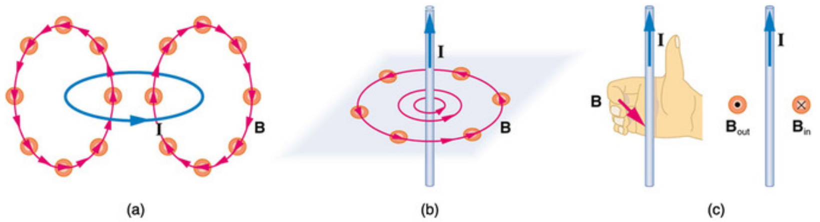
Magnetic field lines are like the contour lines (constant altitude) on a topographic map in that they represent something continuous, and a different mapping scale would show more or fewer lines. An advantage of using magnetic field lines as a representation is that many laws of magnetism (and electromagnetism) can be stated completely and concisely using simple concepts such as the "number" of field lines through a surface. These concepts can be quickly translated to their mathematical form. For example, the number of field lines through a given surface is the surface integral of the magnetic field.



Bar Magnet and Magnetic Field Lines: The direction of magnetic field lines represented by the alignment of iron filings sprinkled on paper placed above a bar magnet.

Various phenomena have the effect of "displaying" magnetic field lines as though the field lines are physical phenomena. For example, iron filings placed in a magnetic field line up to form lines that correspond to "field lines. " Magnetic fields' lines are also visually displayed in polar auroras, in which plasma particle dipole interactions create visible streaks of light that line up with the local direction of Earth's magnetic field.

Small compasses used to test a magnetic field will not disturb it. (This is analogous to the way we tested electric fields with a small test charge. In both cases, the fields represent only the object creating them and not the probe testing them.) Figure 15051 shows how the magnetic field appears for a current loop and a long straight wire, as could be explored with small compasses. A small compass placed in these fields will align itself parallel to the field line at its location, with its north pole pointing in the direction of *B*. Note the symbols used for field into and out of the paper. We'll explore the consequences of these various sources of magnetic fields in further sections.



Mapping Magnetic Field Lines: Small compasses could be used to map the fields shown here. (A) The magnetic field of a circular current loop is similar to that of a bar magnet. (B) A long and straight wire creates a field with magnetic field lines forming circular loops. (C) When the wire is in the plane of the paper, the field is perpendicular to the paper. Note that the symbols used for the field pointing inward (like the tail of an arrow) and the field pointing outward (like the tip of an arrow).

Extensive exploration of magnetic fields has revealed a number of hard-and-fast rules. We use magnetic field lines to represent the field (the lines are a pictorial tool, not a physical entity in and of themselves). The properties of magnetic field lines can be summarized by these rules:

1. The direction of the magnetic field is tangent to the field line at any point in space. A small compass will point in the direction of the field line.
2. The strength of the field is proportional to the closeness of the lines. It is exactly proportional to the number of lines per unit area perpendicular to the lines (called the areal density).
3. Magnetic field lines can never cross, meaning that the field is unique at any point in space.
4. Magnetic field lines are continuous, forming closed loops without beginning or end. They go from the north pole to the south pole.

The last property is related to the fact that the north and south poles cannot be separated. It is a distinct difference from electric field lines, which begin and end on the positive and negative charges. If magnetic monopoles existed, then magnetic field lines would begin and end on them.

Geomagnetism

Earth's magnetic field is caused by electric currents in the molten outer core and varies with time.

learning objectives

- Explain the origin of the Earth's magnetic field and its importance for the life on Earth

Geomagnetism

The Structure of Earth's Magnetic Field

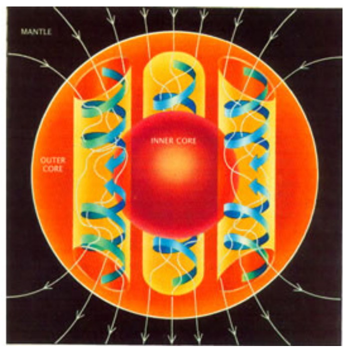
Earth is largely protected from the solar wind, a stream of energetic charged particles emanating from the sun, by its magnetic field, which deflects most of the charged particles. These particles would strip away the ozone layer, which protects Earth from harmful ultraviolet rays. The region above the ionosphere, and extending several tens of thousands of kilometers into space, is called the magnetosphere. This region protects Earth from cosmic rays that would strip away the upper atmosphere, including the ozone layer that protects our planet from harmful ultraviolet radiation. The magnetic field strength ranges from approximately 25 to 65 microteslas (0.25 to 0.65 G; by comparison, a strong refrigerator magnet has a field of about 100 G). The intensity of the field is greatest near the poles and weaker near the equator. An isodynamic chart of Earth's magnetic field, shows a minimum intensity over South America while there are maxima over northern Canada, Siberia, and the coast of Antarctica south of Australia. Near the surface of Earth, its magnetic field can be closely approximated by the field of a magnetic dipole positioned at the center of Earth and tilted at an angle of about 10° with respect to the rotational axis of Earth.

Physical Origin

Earth’s magnetic field is mostly caused by electric currents in the liquid outer core, which is composed of highly conductive molten iron. A magnetic field is generated by a feedback loop: Current loops generate magnetic fields (Ampère’s law); a changing magnetic field generates an electric field (Faraday’s law); and the electric and magnetic fields exert a force on the charges that are flowing in currents (the Lorentz force). These effects can be combined into a partial differential equation called the magnetic induction equation:

$$\frac{\partial \mathrm{B}}{\partial \mathrm{t}} = \eta \nabla^2 \mathrm{B} + \nabla \times (\mathrm{u} \times \mathrm{B}) \tag{21.1.7}$$

In this equation u is the velocity of the fluid, B is the magnetic field, and eta is the magnetic diffusivity. The first term on the right hand side of the induction equation is a diffusion term. If Earth’s dynamo shut off, the dipole part would disappear in a few tens of thousands of years. The motion of the molten outer iron core is sustained by convection, or motion driven by buoyancy. The temperature increases toward the center of Earth, and the higher temperature of the fluid lower down makes it buoyant. The Coriolis effect, caused by the overall planetary rotation, tends to organize the flow into rolls aligned along the north-south polar axis.



Origin of Earth’s Magnetic Field: A schematic illustrating the relationship between motion of conducting fluid, organized into rolls by the Coriolis force, and the magnetic field the motion generates.

Electric currents induced in the ionosphere generate magnetic fields (ionospheric dynamo region). Such a field is always generated near where the atmosphere is closest to the sun, causing daily alterations that can deflect surface magnetic fields by as much as one degree. Typical daily variations of field strength are about 25 nanoteslas (nT), with variations over a few seconds of typically around 1 nT.

Time Variations

The geomagnetic field changes on time scales from milliseconds to millions of years. Shorter time scales mostly arise from currents in the ionosphere (ionospheric dynamo region) and magnetosphere, and some changes can be traced to geomagnetic storms or daily variations in currents. Changes over time scales of a year or more mostly reflect changes in Earth’s interior, particularly the iron-rich core. Frequently, Earth’s magnetosphere is hit by solar flares causing geomagnetic storms, provoking displays of aurorae. At present, the overall geomagnetic field is becoming weaker; the present strong deterioration corresponds to a 10 to 15 percent decline over the last 150 years and has accelerated in the past several years. Geomagnetic intensity has declined almost continuously from a maximum 35 percent above the modern value achieved approximately 2,000 years ago. Earth’s magnetic North Pole is drifting from northern Canada toward Siberia with a presently accelerating rate—10 km per year at the beginning of the 20th century, up to 40 km per year in 2003, and since then has only accelerated.

Although Earth’s field is generally well approximated by a magnetic dipole with its axis near the rotational axis, there are occasional dramatic events where the North and South geomagnetic poles trade places. These events are called geomagnetic reversals. Evidence for these events can be found worldwide in basalts, sediment cores taken from the ocean floors, and seafloor magnetic anomalies. Reversals occur at apparently random intervals ranging from less than 0.1 million years to as much as 50 million years. The most recent such event, called the Brunhes-Matuyama reversal, occurred about 780,000 years ago.

Key Points

- A wire carrying electric current will produce a magnetic field with closed field lines surrounding the wire.
- Another version of the right hand rules can be used to determine the magnetic field direction from a current—point the thumb in the direction of the current, and the fingers curl in the direction of the magnetic field loops created by it. See.
- The Biot-Savart Law can be used to determine the magnetic field strength from a current segment. For the simple case of an infinite straight current-carrying wire it is reduced to the form $\mathrm{B}=\frac{\mu_0 \mathrm{I}}{2\pi \mathrm{r}}$.
- A more fundamental law than the Biot-Savart law is Ampere 's Law, which relates magnetic field and current in a general way. It is written in integral form as $\oint \mathrm{B}\cdot\mathrm{dl}=\mu_0 \mathrm{I}_{\mathrm{enc}}$, where $\mathrm{I}_{\mathrm{enc}}$ is the enclosed current and μ_0 is a constant.
- A current-carrying wire feels a force in the presence of an external magnetic field. It is found to be $\mathrm{F}=\mathrm{Bilsin}\theta$, where ℓ is the length of the wire, i is the current, and θ is the angle between the current direction and the magnetic field.
- Permanent magnets are objects made from magnetized material and produce continual magnetic fields. Everyday examples include refrigerator magnets used to hold notes on a refrigerator door.
- Materials that can be magnetized, which are also the ones that are strongly attracted to a magnet, are called ferromagnetic. Examples of these materials include iron, nickel, and cobalt.
- The counterexample to a permanent magnet is an electromagnet, which only becomes magnetized when an electric current flows through it.
- Magnets always have a north pole and a south pole, so if one were to split a permanent magnet in half, two smaller magnets would be created, each with a north pole and south pole.
- Permanent magnets are made from ferromagnetic materials that are exposed to a strong external magnetic field and heated to align their internal microcrystalline structure, making them very hard to demagnetize.
- The magnetic field direction is the same direction a compass needle points, which is tangent to the magnetic field line at any given point.
- The strength of the B-field is inversely proportional to the distance between field lines. It is exactly proportional to the number of lines per unit area perpendicular to the lines.
- A magnetic field line can never cross another field line. The magnetic field is unique at every point in space.
- Magnetic field lines are continuous and unbroken, forming closed loops. Magnetic field lines are defined to begin on the north pole of a magnet and terminate on the south pole.
- Earth is largely protected from the solar wind, a stream of energetic charged particles emanating from the sun, by its magnetic field. These particles would strip away the ozone layer, which protects Earth from harmful ultraviolet rays.
- Earth's magnetic field is generated by a feedback loop in the liquid outer core: Current loops generate magnetic fields; a changing magnetic field generates an electric field; and the electric and magnetic fields exert a force on the charges that are flowing in currents (the Lorentz force).
- The geomagnetic field varies with time. Currents in the ionosphere and magnetosphere cause changes over short time scales, while dramatic geomagnetic reversald (where the North and South poles switch locations) occur at apparently random intervals ranging from 0.1 to 50 million years.

Key Terms

- **Biot-Savart Law**: An equation that describes the magnetic field generated by an electric current. It relates the magnetic field to the magnitude, direction, length, and proximity of the electric current. The law is valid in the magnetostatic approximation, and is consistent with both Ampère's circuital law and Gauss's law for magnetism.
- **Ampere's Law**: An equation that relates magnetic fields to electric currents that produce them. Using Ampere's law, one can determine the magnetic field associated with a given current or current associated with a given magnetic field, providing there is no time changing electric field present.
- **permanent magnet**: A material, or piece of such material, which retains its magnetism even when not subjected to any external magnetic fields.
- **ferromagnetic**: Of a material, such as iron or nickel, that is easily magnetized.
- **electromagnet**: A magnet which attracts metals only when electrically activated.
- **B-field**: A synonym for the magnetic field.
- **magnetic field lines**: A graphical representation of the magnitude and the direction of a magnetic field.
- **dynamo**: A mechanism by which a celestial body such as Earth or a star generates a magnetic field over astronomical timescales via a rotating, convecting, and electrically conducting fluid.

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21.2: Magnets

learning objectives

- Identify two types of magnets

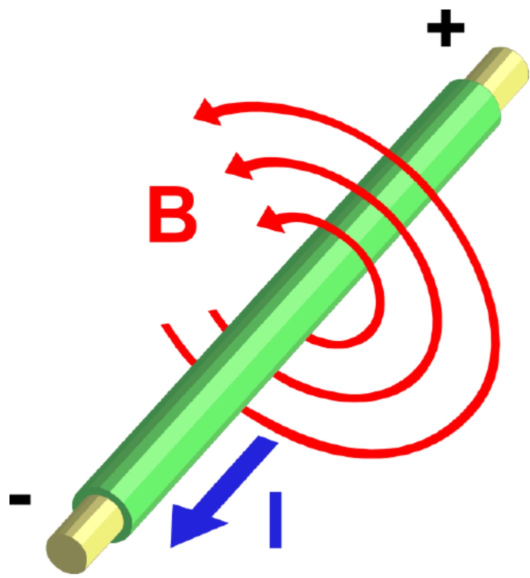
Ferromagnets and Electromagnets

In common language it is often understood that ‘magnet’ refers to a permanent magnet, like one that might adorn a family’s refrigerator, or function as the needle in a hiker’s compass. Such magnets are called ferromagnets. In the second class of magnets—known as electromagnets—the magnetic field is generated through the use of electric current. These magnets can be found in all types of electronic devices. We’ll explore these two types of magnets below.

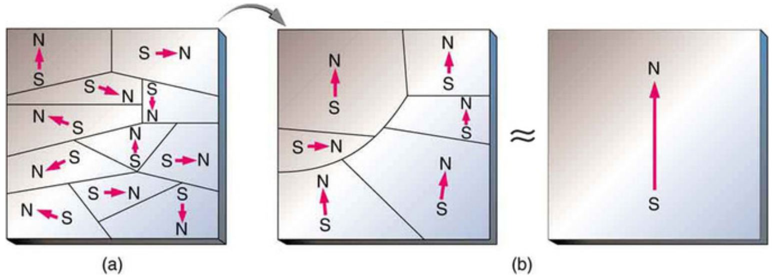
Ferromagnets

Only certain materials (e.g., iron, cobalt, nickel, and gadolinium) exhibit strong magnetic effects. These materials are called ferromagnetic, after the Latin word *ferrum* (iron). A group of materials made from the alloys of the rare earth elements are also used as strong and permanent magnets (neodymium is a common one). Other materials exhibit weak magnetic effects detectable only with sensitive instruments. Not only do ferromagnetic materials respond strongly to magnets (the way iron is attracted to magnets), they can also be magnetized themselves—that is, they can be induced to become magnetic or made into permanent magnets.

When a magnet is brought near a previously unmagnetized ferromagnetic material, it causes local magnetization of the material with unlike poles closest, as in. This results in the attraction of the previously unmagnetized material to the magnet as diagramed in. When current produces a magnetic field on a microscopic scale, as illustrated in, the regions within the material called magnetic domainsact like small bar magnets. Within domains, the poles of individual atoms are aligned, and each atom acts like a tiny bar magnet. In an unmagnetized ferromagnetic object, domains are small and randomly oriented. In response to an external magnetic field, the domains may grow to millimeter size, aligning themselves as shown in part (b) of the second figure. This induced magnetization can become permanent if the material is heated and then cooled, or simply tapped in the presence of other magnets.



Current Produces a Magnetic Field: Current (I) through a wire produces a magnetic field (B). The field is oriented according to the right-hand rule.



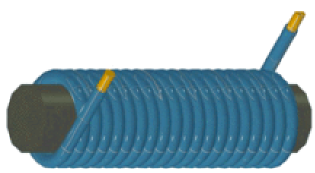
Unmagnetized to Magnetized Iron: (a) An unmagnetized piece of iron (or other ferromagnetic material) has randomly oriented domains. (b) When magnetized by an external field, the domains show greater alignment, and some grow at the expense of others. Individual atoms are aligned within domains; each atom acts like a tiny bar magnet.

Conversely, a permanent magnet can be demagnetized by hard blows or by heating it in the absence of another magnet. Increased thermal motion at higher temperature can disrupt and randomize the orientation and the size of the domains. There is a well-defined temperature for ferromagnetic materials called the Curie temperature, above which they cannot be magnetized. The Curie temperature for iron is well above room temperature at 1043 K (770ºC). Several elements and alloys have Curie temperatures much lower than room temperature, and are ferromagnetic only below those temperatures.

Electromagnets

In an electromagnet the magnetic field is produced by the flow of electric current. If the current disappears, the magnetic field is turned off. Electromagnets are widely used as components of electrical devices, such as motors, generators, relays, loudspeakers, hard disks, MRI machines, scientific instruments and magnetic separation equipment; they are also employed as industrial lifting electromagnets for picking up and moving heavy iron objects like scrap iron.

An electric current flowing in a wire creates a magnetic field around the wire. To concentrate the magnetic field, the wire is wound into a coil with many turns of wire lying side by side. The magnetic field from all the turns of wire passes through the center of the coil creating a strong magnetic field there. The coil forming the shape of a straight tube (a helix) is called a solenoid, as shown in. Much stronger magnetic fields can be produced if a “core” of ferromagnetic material (such as soft iron) is placed inside the coil. Due to the high magnetic permeability μ of the ferromagnetic material, the ferromagnetic core increases the magnetic field to thousands of times the strength of the field of the coil alone. This is called a ferromagnetic-core or iron-core electromagnet.



Electromagnet (Solenoid): A simple electromagnet consisting of a coil of insulated wire wrapped around an iron core. The strength of magnetic field generated is proportional to the amount of current.

The direction of the magnetic field through a coil of wire can be likend to a form of the right-hand rule. If the fingers of the right hand are curled around the coil in the direction of current flow (conventional current, flow of positive charge) through the windings, the thumb points in the direction of the field inside the coil. The side of the magnet from which the field lines emerge is defined as the north pole.The main advantage of an electromagnet over a permanent magnet is that the magnetic field can be rapidly manipulated over a wide range by controlling the amount of electric current; a continuous supply of electrical energy is required to maintain the field.

Key Points

- Only certain materials, such as iron, cobalt, nickel, and gadolinium, exhibit strong magnetic effects. These materials are called ferromagnetic. Ferromagnetic materials will respond strongly to magnets and can also be magnetized themselves.
- Regions of uniform called magnetic domains are randomly oriented in unmagnetized ferromagnetic material, but may become aligned under the influence of an external magnetic field. This process may become permanent if heated and cooled in the presence of a magnetic field.
- A ferromagnetic will lose its magnetism if heated about its Curie temperature.

- Electromagnets are a type of magnet in which the magnetic field is produced by the flow of current.
- A strong electromagnet called a solenoid may be produced by wrapping wires into a coil and passing a current through them. The magnetic field of all the turns of wire passes through the center of the coil, creating a strong magnetic field there.

Key Terms

- **magnetic domain**: A region within a magnetic material which has uniform magnetization. This means that the individual magnetic moments of the atoms are aligned with one another and they point in the same direction.
- **Curie temperature**: The temperature above which a material will lose its magnetism.
- **solenoid**: A coil of wire that acts as a magnet when an electric current flows through it.

21.3: Magnetic Force on a Moving Electric Charge

Magnitude of the Magnetic Force

How does one magnet attracts another? The answer relies on the fact that all magnetism relies on current, the flow of charge. *Magnetic fields exert forces on moving charges*, and so they exert forces on other magnets, all of which have moving charges.

The magnetic force on a moving charge is one of the most fundamental known. The magnetic force is as important as the electrostatic or Coulomb force. Yet the magnetic force is more complex, in both the number of factors that affects it and in its direction, than the relatively simple Coulomb force. The magnitude of the magnetic force F on a charge q moving at a speed v in a magnetic field of strength B is given by:

$$\mathrm{F} = \mathrm{qvB} \sin(\theta) \tag{21.3.1}$$

where θ is the angle between the directions of v and B. This formula is used to define the magnetic strength B in terms of the force on a charged particle moving in a magnetic field. The SI unit for magnitude of the magnetic field strength is called the tesla (T) in honor of the brilliant and eccentric inventor Nikola Tesla (1856–1943), who made great contributions to our understanding of magnetic fields and their practical applications. To determine how the tesla relates to other SI units, we solve $\mathrm{F} = \mathrm{qvB} \sin(\theta)$ for B*:*

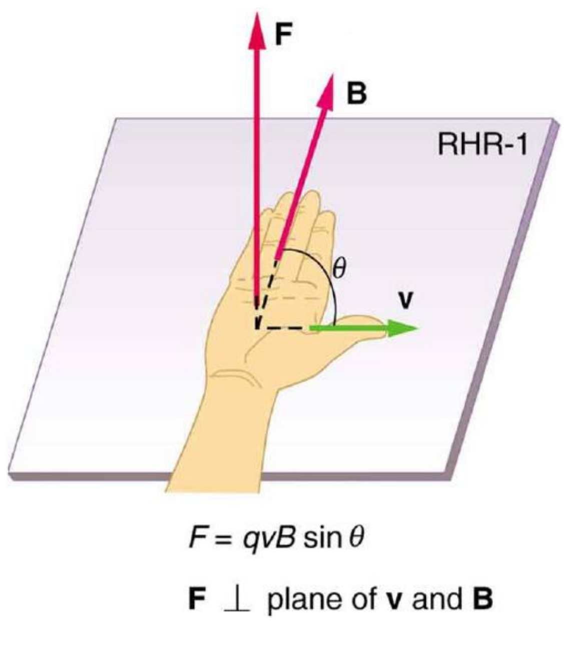
$$\mathrm{B} = \frac{\mathrm{F}}{\mathrm{qvsin}(\theta)} \tag{21.3.2}$$

Because sinθ is unitless, the tesla is

$$1\mathrm{T} = \frac{1\mathrm{N}}{\mathrm{C} \times \mathrm{m/s}} = \frac{1\mathrm{N}}{\mathrm{A} \times \mathrm{m}} \tag{21.3.3}$$

Another smaller unit, called the gauss (G), where 1 G=10^{-4} T, is sometimes used. The strongest permanent magnets have fields near 2 T; superconducting electromagnets may attain 10 T or more. The Earth's magnetic field on its surface is only about 5×10^{-5} T, or 0.5 G.

The *direction* of the magnetic force F is perpendicular to the plane formed by v and B as determined by the right hand rule, which is illustrated in Figure 1. It states that, to determine the direction of the magnetic force on a positive moving charge, you point the thumb of the right hand in the direction of v, the fingers in the direction of B, and a perpendicular to the palm points in the direction of F. One way to remember this is that there is one velocity, and so the thumb represents it. There are many field lines, and so the fingers represent them. The force is in the direction you would push with your palm. The force on a negative charge is in exactly the opposite direction to that on a positive charge.



Right Hand Rule: Magnetic fields exert forces on moving charges. This force is one of the most basic known. The direction of the magnetic force on a moving charge is perpendicular to the plane formed by v and B and follows right hand rule–1 (RHR-1) as shown. The magnitude of the force is proportional to q, v, B, and the sine of the angle between v and B.

Direction of the Magnetic Force: The Right Hand Rule

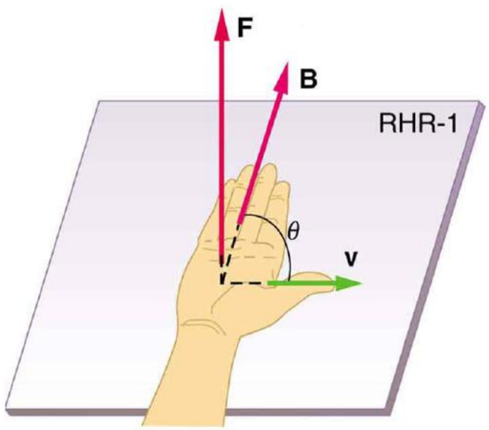
The right hand rule is used to determine the direction of the magnetic force on a positive charge.

learning objectives

- Apply the right hand rule to determine the direction of the magnetic force on a charge

Direction of the Magnetic Force: The Right Hand Rule

So far we have described the magnitude of the magnetic force on a moving electric charge, but not the direction. The magnetic field is a vector field, thus the force applied will be oriented in a particular direction. There is a clever way to determine this direction using nothing more than your right hand. The direction of the magnetic force F is perpendicular to the plane formed by v and B, as determined by the right hand rule, which is illustrated in the figure above. The right hand rule states that: to determine the direction of the magnetic force on a positive moving charge, f, point the thumb of the right hand in the direction of v, the fingers in the direction of B, and a perpendicular to the palm points in the direction of F.



$$F = qvB \sin\theta$$

$$\mathbf{F} \perp \text{ plane of } \mathbf{v} \text{ and } \mathbf{B}$$

Right Hand Rule: Magnetic fields exert forces on moving charges. This force is one of the most basic known. The direction of the magnetic force on a moving charge is perpendicular to the plane formed by v and B and follows right hand rule–1 (RHR-1) as shown. The magnitude of the force is proportional to q, v, B, and the sine of the angle between v and B.

One way to remember this is that there is one velocity, represented accordingly by the thumb. There are many field lines, represented accordingly by the fingers. The force is in the direction you would push with your palm. The force on a negative charge is in exactly the opposite direction to that on a positive charge. Because the force is always perpendicular to the velocity vector, a pure magnetic field will not accelerate a charged particle in a single direction, however will produce circular or helical motion (a concept explored in more detail in future sections). It is important to note that magnetic field will not exert a force on a static electric charge. These two observations are in keeping with the rule that *magnetic fields do no work*.

Key Points

- Magnetic fields exert forces on charged particles in motion.
- The *direction* of the magnetic force F is perpendicular to the plane formed by v and B as determined by the right hand rule.
- The SI unit for magnitude of the magnetic field strength is called the tesla (T), which is equivalent to one Newton per ampere-meter. Sometimes the smaller unit gauss (10^{-4} T) is used instead.
- When the expression for the magnetic force is combined with that for the electric force, the combined expression is known as the Lorentz force.

- When considering the motion of a charged particle in a magnetic field, the relevant vectors are the magnetic field B, the velocity of the particle v, and the magnetic force exerted on the particle F. These vectors are all perpendicular to each other.
- The right hand rule states that, to find the direction of the magnetic force on a positive moving charge, the thumb of the right hand point in the direction of v, the fingers in the direction of B, and the force (F) is directed perpendicular to the right hand palm.
- The direction of the force F on a negative charge is in the opposite sense to that above (so pointed away from the back of your hand).

Key Terms

- **Coulomb force**: the electrostatic force between two charges, as described by Coulomb's law
- **magnetic field**: A condition in the space around a magnet or electric current in which there is a detectable magnetic force, and where two magnetic poles are present.
- **tesla**: In the International System of Units, the derived unit of magnetic flux density or magnetic inductivity. Symbol: T
- **right hand rule**: Direction of angular velocity ω and angular momentum L in which the thumb of your right hand points when you curl your fingers in the direction of rotation.

21.4: Motion of a Charged Particle in a Magnetic Field

learning objectives

- Compare the effects of the electric and the magnetic fields on the charged particle

Electric vs. Magnetic Forces

Force due to both electric and magnetic forces will influence the motion of charged particles. However, the resulting change to the trajectory of the particles will differ qualitatively between the two forces. Below we will quickly review the two types of force and compare and contrast their effects on a charged particle.

Electrostatic Force and Magnetic Force on a Charged Particle

Recall that in a static, unchanging electric field *E* the force on a particle with charge q will be:

$$F = qE \tag{21.4.1}$$

Where *F* is the force vector, q is the charge, and *E* is the electric field vector. Note that the direction of *F* is identical to *E* in the case of a positivist charge q, and in the opposite direction in the case of a negatively charged particle. This electric field may be established by a larger charge, Q, acting on the smaller charge q over a distance *r* so that:

$$E = \left|\frac{F}{q}\right| = k\left|\frac{qQ}{qr^2}\right| = k\frac{|Q|}{r^2} \tag{21.4.2}$$

It should be emphasized that the electric force *F* acts parallel to the electric field *E*. The curl of the electric force is zero, i.e.:

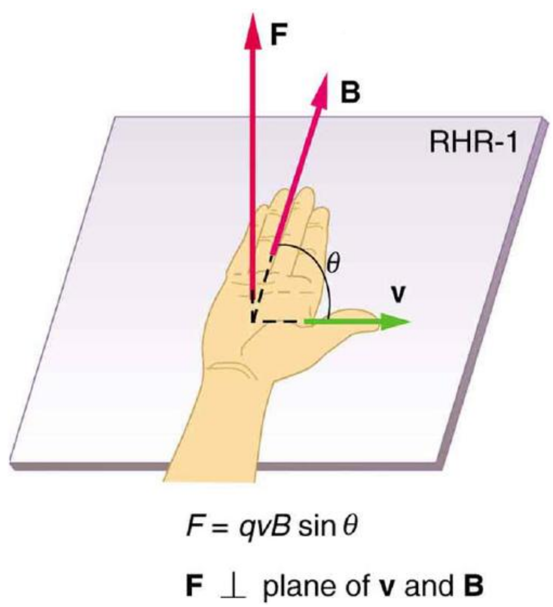
$$\nabla \times E = 0 \tag{21.4.3}$$

A consequence of this is that the electric field may do work and a charge in a pure electric field *will follow the tangent of an electric field line*.

In contrast, recall that the magnetic force on a charged particle is orthogonal to the magnetic field such that:

$$F = qv \times B = qvB\sin\theta \tag{21.4.4}$$

where *B* is the magnetic field vector, *v* is the velocity of the particle and θ is the angle between the magnetic field and the particle velocity. The direction of F can be easily determined by the use of the right hand rule.



Right Hand Rule: Magnetic fields exert forces on moving charges. This force is one of the most basic known. The direction of the magnetic force on a moving charge is perpendicular to the plane formed by v and B and follows right hand rule–1 (RHR-1) as shown. The magnitude of the force is proportional to q, v, B, and the sine of the angle between v and B.

If the particle velocity happens to be aligned parallel to the magnetic field, or is zero, the magnetic force will be zero. This differs from the case of an electric field, where the particle velocity has no bearing, on any given instant, on the magnitude or direction of the electric force.

The angle dependence of the magnetic field also causes charged particles to move perpendicular to the magnetic field lines in a circular or helical fashion, while a particle in an electric field will move in a straight line along an electric field line.

A further difference between magnetic and electric forces is that magnetic fields *do not net work*, since the particle motion is circular and therefore ends up in the same place. We express this mathematically as:

$$\mathrm{W} = \oint \mathrm{B} \cdot \mathrm{dr} = 0 \tag{21.4.5}$$

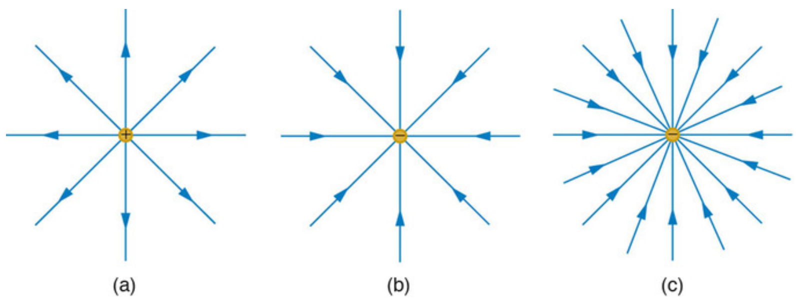
Lorentz Force

The Lorentz force is the combined force on a charged particle due both electric and magnetic fields, which are often considered together for practical applications. If a particle of charge *q* moves with velocity *v* in the presence of an electric field *E* and a magnetic field *B*, then it will experience a force:

$$\mathrm{F} = \mathrm{q}[\mathrm{E} + \mathrm{vB}\sin\theta] \tag{21.4.6}$$

Electric and Magnetic Field Lines

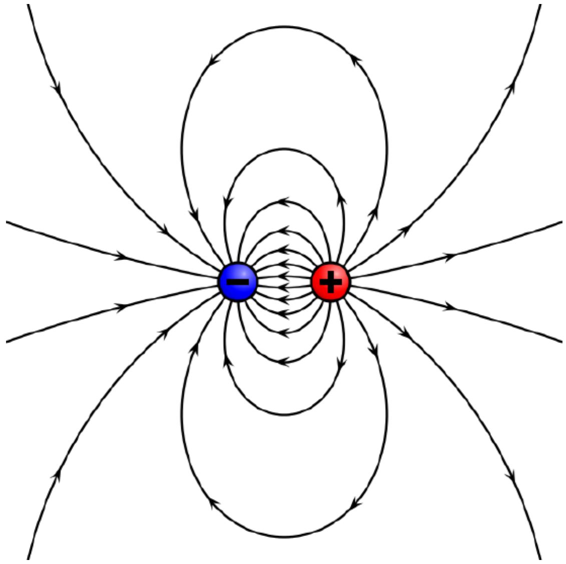
We mentioned briefly above that the motion of charged particles relative to the field lines differs depending on whether one is dealing with electric or magnetic fields. There are some notable differences between how electric and magnetic field lines are conceptualized. The electric field lines from a positive isolated charge are simply a sequence of evenly-spaced, radially directed lines pointed outwards from the charge. In the case of a negative charge, the direction of the field is reversed. The electric field is directed tangent to the field lines. Of course, we imagine the field lines are more densely packed the larger the charges are. One can see clearly that the curl of the electric force is zero.



Electric Field Generated by Point Charges: The electric field surrounding three different point charges: (a) A positive charge; (b) a negative charge of equal magnitude; (c) a larger negative charge.

If multiple charges are involved, field lines are generated on positive charges, and terminate on negative ones.

In the case of magnets, field lines are generated on the north pole (+) and terminate on the south pole (-) – see the below figure. Magnetic ‘charges’, however, always come in pairs – there are no magnetic monopoles (isolated north or south poles). The curl of a magnetic field generated by a conventional magnet is therefore always non zero. Charged particles will spiral around these field lines, as long as the particles have some non-zero component of velocity directed perpendicular to the field lines.



Magnetic Pole Model: The magnetic pole model: two opposing poles, North (+) and South (−), separated by a distance d produce an H-field (lines).

A magnetic field may also be generated by a current with the field lines envisioned as concentric circles around the current-carrying wire.The magnetic force at any point in this case can be determined with the right hand rule, and will be perpendicular to both the current and the magnetic field.

Constant Velocity Produces a Straight-Line

If a charged particle's velocity is parallel to the magnetic field, there is no net force and the particle moves in a straight line.

learning objectives

- Identify conditions required for the particle to move in a straight line in the magnetic field

Constant Velocity Produces Straight-Line Motion

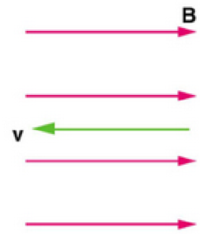
Recall Newton's first law of motion. If an object experiences no net force, then its velocity is constant: the object is either at rest (if its velocity is zero), or it moves in a straight line with constant speed (if its velocity is nonzero).

There are many cases where a particle may experience no net force. The particle could exist in a vacuum far away from any massive bodies (that exert gravitational forces) and electromagnetic fields. Or there could be two or more forces on the particle that are balanced such that the net force is zero. This is the case for, say, a particle suspended in an electric field with the electric force exactly counterbalancing gravity.

If the net force on a particle is zero, then the acceleration is necessarily zero from Newton's second law: F=ma. If the acceleration is zero, any velocity the particle has will be maintained indefinitely (or until such time as the net force is no longer zero). Because velocity is a vector, the direction remains unchanged along with the speed, so the particle continues in a single direction, such as with a straight line.

Charged Particles Moving Parallel to Magnetic Fields

The force a charged particle "feels" due to a magnetic field is dependent on the angle between the velocity vector and the magnetic field vector B . Recall that the magnetic force is:



Zero Force When Velocity is Parallel to Magnetic Field: In the case above the magnetic force is zero because the velocity is parallel to the magnetic field lines.

$$\mathrm{F=qvB\sin\theta} \tag{21.4.7}$$

If the magnetic field and the velocity are parallel (or antiparallel), then sinθ equals zero and there is no force. In this case a charged particle can continue with straight-line motion even in a strong magnetic field. If is between 0 and 90 degrees, then the component of *v* parallel to *B* remains unchanged.

Circular Motion

Since the magnetic force is always perpendicular to the velocity of a charged particle, the particle will undergo circular motion.

learning objectives

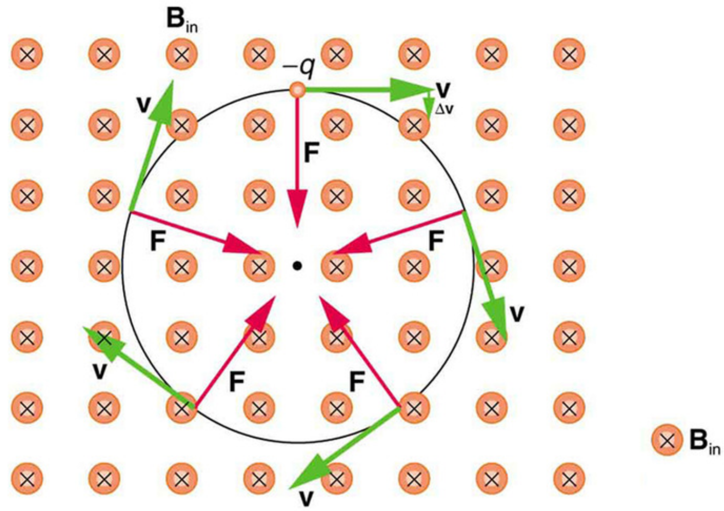
- Describe conditions that lead to the circular motion of a charged particle in the magnetic field

Circular Motion of a Charged particle in a Magnetic Field

Magnetic forces can cause charged particles to move in circular or spiral paths. Particle accelerators keep protons following circular paths with magnetic force. Cosmic rays will follow spiral paths when encountering the magnetic field of astrophysical objects or planets (one example being Earth's magnetic field). The bubble chamber photograph in the figure below shows charged particles moving in such curved paths. The curved paths of charged particles in magnetic fields are the basis of a number of phenomena and can even be used analytically, such as in a mass spectrometer. shows the path traced by particles in a bubble chamber.

Bubble Chamber: Trails of bubbles are produced by high-energy charged particles moving through the superheated liquid hydrogen in this artist's rendition of a bubble chamber. There is a strong magnetic field perpendicular to the page that causes the curved paths of the particles. The radius of the path can be used to find the mass, charge, and energy of the particle.

So, does the magnetic force cause circular motion? Magnetic force is always perpendicular to velocity, so that it does no work on the charged particle. The particle's kinetic energy and speed thus remain constant. The direction of motion is affected, but not the speed. This is typical of uniform circular motion. The simplest case occurs when a charged particle moves perpendicular to a uniform B-field, such as shown in. (If this takes place in a vacuum, the magnetic field is the dominant factor determining the motion.) Here, the magnetic force (Lorentz force) supplies the centripetal force



Circular Motion of Charged Particle in Magnetic Field: A negatively charged particle moves in the plane of the page in a region where the magnetic field is perpendicular into the page (represented by the small circles with x's—like the tails of arrows). The magnetic force is perpendicular to the velocity, and so velocity changes in direction but not magnitude. Uniform circular motion results.

$$\mathrm{F_c = \frac{mv^2}{r}} \tag{21.4.8}$$

Noting that

$$\sin \theta = 1 \tag{21.4.9}$$

we see that

$$\mathrm{F = qvB} \tag{21.4.10}$$

The Lorentz magnetic force supplies the centripetal force, so these terms are equal:

$$\mathrm{qvB = \frac{mv^2}{r}} \tag{21.4.11}$$

solving for r yields

$$\mathrm{r = \frac{mv}{qB}} \tag{21.4.12}$$

Here, *r*, called the gyroradius or cyclotron radius, is the radius of curvature of the path of a charged particle with mass *m* and charge *q*, moving at a speed *v* perpendicular to a magnetic field of strength *B*. In other words, it is the radius of the circular motion of a charged particle in the presence of a uniform magnetic field. If the velocity is not perpendicular to the magnetic field, then *v* is the component of the velocity perpendicular to the field. The component of the velocity parallel to the field is unaffected, since the magnetic force is zero for motion parallel to the field. We'll explore the consequences of this case in a later section on spiral motion.

A particle experiencing circular motion due to a uniform magnetic field is termed to be in a *cyclotron resonance*. The term comes from the name of a cyclic particle accelerator called a cyclotron, showed in. The cyclotron frequency (or, equivalently, gyrofrequency) is the number of cycles a particle completes around its circular circuit every second and can be found by solving for *v* above and substituting in the circulation frequency so that

Cyclotron: A French cyclotron, produced in Zurich, Switzerland in 1937

$$f = \frac{v}{2\pi r} \tag{21.4.13}$$

becomes

$$f = \frac{qB}{2\pi m} \tag{21.4.14}$$

The cyclotron frequency is trivially given in radians per second by

$$\omega = \frac{qB}{m} \tag{21.4.15}$$

Helical Motion

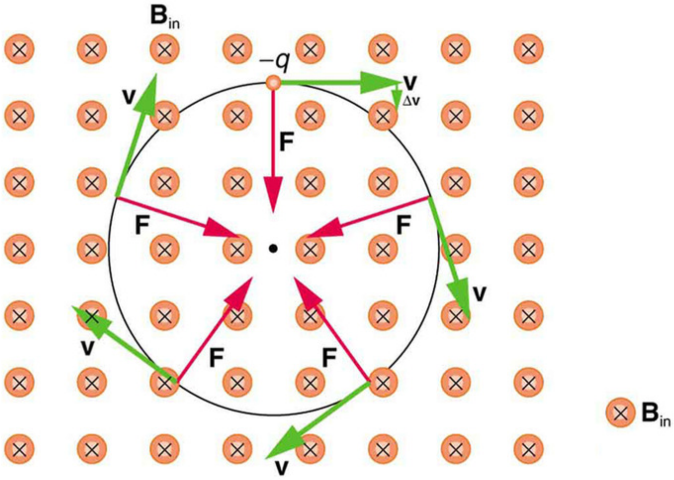
Helical motion results when the velocity vector is not perpendicular to the magnetic field vector.

learning objectives

- Describe conditions that lead to the helical motion of a charged particle in the magnetic field

Helical Motion

In the section on circular motion we described the motion of a charged particle with the magnetic field vector aligned perpendicular to the velocity of the particle. In this case, the magnetic force is also perpendicular to the velocity (and the magnetic field vector, of course) at any given moment resulting in circular motion. The speed and kinetic energy of the particle remain constant, but the direction is altered at each instant by the perpendicular magnetic force. quickly reviews this situation in the case of a negatively charged particle in a magnetic field directed into the page.



Circular Motion of Charged Particle in Magnetic Field: A negatively charged particle moves in the plane of the page in a region where the magnetic field is perpendicular into the page (represented by the small circles with x's—like the tails of arrows). The magnetic force is perpendicular to the velocity, and so velocity changes in direction but not magnitude. Uniform circular motion results.

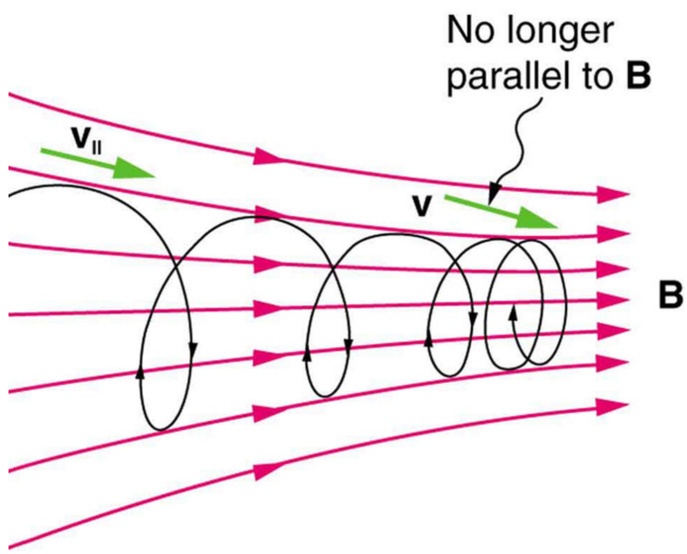
What if the velocity is not perpendicular to the magnetic field? Then we consider only the *component* of *v* that is perpendicular to the field when making our calculations, so that the equations of motion become:

$$\mathrm{F_c = \frac{mv_\perp^2}{r}} \tag{21.4.16}$$

$$\mathrm{F = qvB\sin\theta = qv_\perp B} \tag{21.4.17}$$

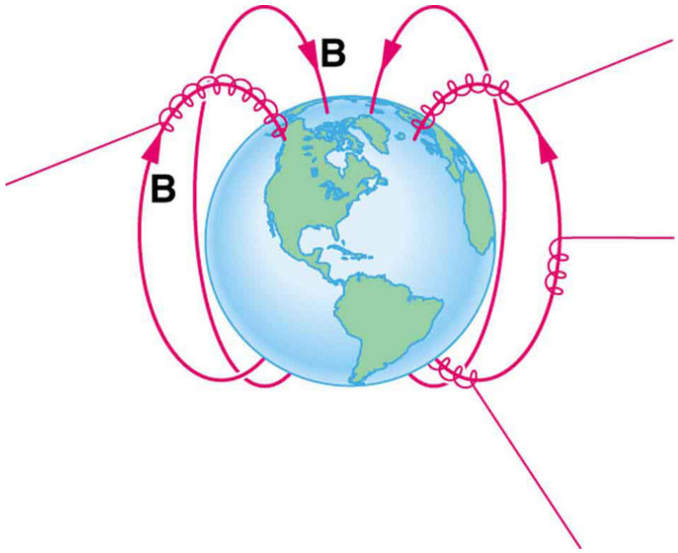
The component of the velocity parallel to the field is unaffected, since the magnetic force is zero for motion parallel to the field. This produces helical motion (i.e., spiral motion) rather than a circular motion.

shows how electrons not moving perpendicular to magnetic field lines follow the field lines. The component of velocity parallel to the lines is unaffected, and so the charges spiral along the field lines. If field strength increases in the direction of motion, the field will exert a force to slow the charges (and even reverse their direction), forming a kind of magnetic mirror.



Helical Motion and Magnetic Mirrors: When a charged particle moves along a magnetic field line into a region where the field becomes stronger, the particle experiences a force that reduces the component of velocity parallel to the field. This force slows the motion along the field line and here reverses it, forming a "magnetic mirror. "

The motion of charged particles in magnetic fields are related to such different things as the Aurora Borealis or Aurora Australis (northern and southern lights) and particle accelerators. *Charged particles approaching magnetic field lines may get trapped in spiral orbits about the lines rather than crossing them*, as seen above. Some cosmic rays, for example, follow the Earth's magnetic field lines, entering the atmosphere near the magnetic poles and causing the southern or northern lights through their ionization of molecules in the atmosphere. Those particles that approach middle latitudes must cross magnetic field lines, and many are prevented from penetrating the atmosphere. Cosmic rays are a component of background radiation; consequently, they give a higher radiation dose at the poles than at the equator.



Charged Particles Spiral Along Earth's Magnetic Field Lines: Energetic electrons and protons, components of cosmic rays, from the Sun and deep outer space often follow the Earth's magnetic field lines rather than cross them. (Recall that the Earth's north magnetic pole is really a south pole in terms of a bar magnet.)

Examples and Applications

Cyclotrons, magnetrons, and mass spectrometers represent practical technological applications of electromagnetic fields.

learning objectives

- Discuss application of mass spectrometers, movement of charged particles in a cyclotron, and how microwaves are generated in the cavity magnetron

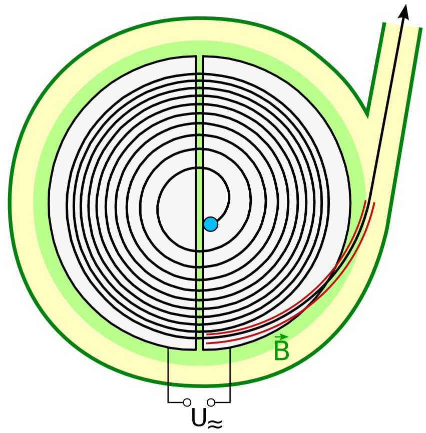
Examples and Applications – Motion of a Charged Particle in a Magnetic Field

Overview

Recall that the charged particles in a magnetic field will follow a circular or spiral path depending on the alignment of their velocity vector with the magnetic field vector. The consequences of such motion can have profoundly practical applications. Many technologies are based on the motion of charged particles in electromagnetic fields. We will explore some of these, including the cyclotron and synchrotron, cavity magnetron, and mass spectrometer.

Cyclotrons and Synchrotrons

A cyclotron is a type of particle accelerator in which charged particles accelerate outwards from the center along a spiral path. The particles are held to a spiral trajectory by a static magnetic field and accelerated by a rapidly varying (radio frequency) electric field.



Cyclotron Sketch: Sketch of a particle being accelerated in a cyclotron, and being ejected through a beamline.

Cyclotrons accelerate charged particle beams using a high frequency alternating voltage which is applied between two “D”-shaped electrodes (also called “dees”). An additional static magnetic field is applied in perpendicular direction to the electrode plane, enabling particles to re-encounter the accelerating voltage many times at the same phase. To achieve this, the voltage frequency must match the particle’s cyclotron resonance frequency,

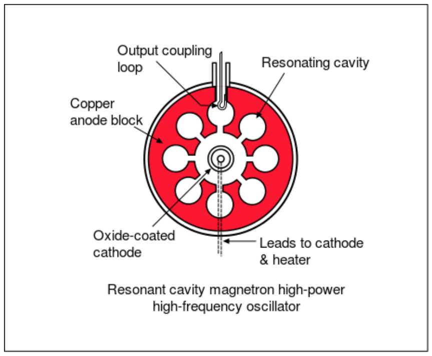
$$f = \frac{qB}{2\pi m} \tag{21.4.18}$$

with the relativistic mass *m* and its charge *q*. This frequency is given by equality of centripetal force and magnetic Lorentz force. The particles, injected near the center of the magnetic field, increase their kinetic energy only when recirculating through the gap between the electrodes; thus they travel outwards along a spiral path. Their radius will increase until the particles hit a target at the perimeter of the vacuum chamber, or leave the cyclotron using a beam tube, enabling their use. The particles accelerated by the cyclotron can be used in particle therapy to treat some types of cancer. Additionally, cyclotrons are a good source of high-energy beams for nuclear physics experiments.

A synchrotron is an improvement upon the cyclotron in which the guiding magnetic field (bending the particles into a closed path) is time-dependent, being *synchronized* to a particle beam of increasing kinetic energy. The synchrotron is one of the first accelerator concepts that enable the construction of large-scale facilities, since bending, beam focusing and acceleration can be separated into different components.

Cavity Magnetron

The cavity magnetron is a high-powered vacuum tube that generates microwaves using the interaction of a stream of electrons with a magnetic field. All cavity magnetrons consist of a hot cathode with a high (continuous or pulsed) negative potential created by a high-voltage, direct-current power supply. The cathode is built into the center of an evacuated, lobed, circular chamber. A magnetic field parallel to the filament is imposed by a permanent magnet. The magnetic field causes the electrons, attracted to the (relatively) positive outer part of the chamber, to spiral outward in a circular path, a consequence of the Lorentz force. Spaced around the rim of the chamber are cylindrical cavities. The cavities are open along their length and connect the common cavity space. As electrons sweep past these openings, they induce a resonant, high-frequency radio field in the cavity, which in turn causes the electrons to bunch into groups.



Cavity Magnetron Diagram: A cross-sectional diagram of a resonant cavity magnetron. Magnetic lines of force are parallel to the geometric axis of this structure.

The sizes of the cavities determine the resonant frequency, and thereby the frequency of emitted microwaves. The magnetron is a self-oscillating device requiring no external elements other than a power supply. The magnetron has practical applications in radar, heating (as the primary component of a microwave oven), and lighting.

Mass Spectrometry

Mass spectrometry is an analytical technique that measures the mass-to-charge ratio of charged particles. It is used for determining masses of particles and determining the elemental composition of a sample or molecule.

Mass analyzers separate the ions according to their mass-to-charge ratio. The following two laws govern the dynamics of charged particles in electric and magnetic fields in a vacuum:

$$\mathrm{F=Q(E+v\times B)\ (Lorentz\ force)} \tag{21.4.19}$$

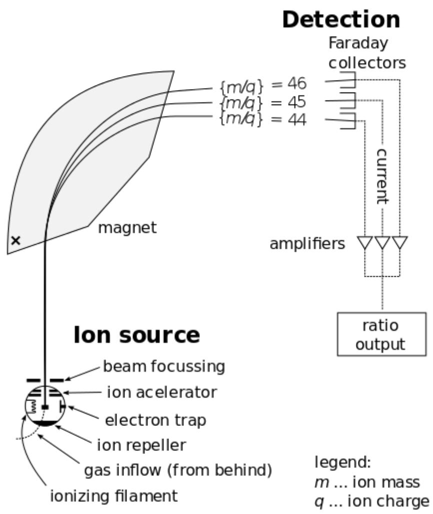
$$\mathrm{F=ma} \tag{21.4.20}$$

Equating the above expressions for the force applied to the ion yields:

$$\mathrm{(m/Q)a=E+v\times B} \tag{21.4.21}$$

This differential equation along with initial conditions completely determines the motion of a charged particle in terms of m/Q. There are many types of mass analyzers, using either static or dynamic fields, and magnetic or electric fields, but all operate according to the above differential equation.

The following figure illustrates one type of mass spectrometer. The deflections of the particles are dependent on the mass-to-charge ratio. In the case of isotopic carbon dioxide, each molecule has the same charge, but different masses. The mass spectrometer will segregate the particles spatially allowing a detector to measure the mass-to-charge ratio of each particle. Since the charge is known, the absolute mass can be determined trivially. The relative abundances can be inferred from counting the number of particles of each given mass.



Mass Spectrometry: Schematics of a simple mass spectrometer with sector type mass analyzer. This one is for the measurement of carbon dioxide isotope ratios (IRMS) as in the carbon-13 urea breath test.

Key Points

- The force on a charged particle due to an electric field is directed parallel to the electric field vector in the case of a positive charge, and anti-parallel in the case of a negative charge. It does not depend on the velocity of the particle.
- In contrast, the magnetic force on a charge particle is orthogonal to the magnetic field vector, and depends on the velocity of the particle. The right hand rule can be used to determine the direction of the force.
- An electric field may do work on a charged particle, while a magnetic field does no work.
- The Lorentz force is the combination of the electric and magnetic force, which are often considered together for practical applications.
- Electric field lines are generated on positive charges and terminate on negative ones. The field lines of an isolated charge are directly radially outward. The electric field is tangent to these lines.
- Magnetic field lines, in the case of a magnet, are generated at the north pole and terminate on a south pole. Magnetic poles do not exist in isolation. Like in the case of electric field lines, the magnetic field is tangent to the field lines. Charged particles will spiral around these field lines.
- Newton's first law of motion states that if an object experiences no net force, then its velocity is constant.
- A particle with constant velocity will move along a straight line through space.
- If a charged particle's velocity is completely parallel to the magnetic field, the magnetic field will exert no force on the particle and thus the velocity will remain constant.
- In the case that the velocity vector is neither parallel nor perpendicular to the magnetic field, the component of the velocity parallel to the field will remain constant.
- The magnetic field does no work, so the kinetic energy and speed of a charged particle in a magnetic field remain constant.
- The magnetic force, acting perpendicular to the velocity of the particle, will cause circular motion.
- The centripetal force of the particle is provided by magnetic Lorentzian force so that $\mathrm{qvB} = \frac{\mathrm{mv}^2}{\mathrm{r}}$.
- Solving for r above yields the gryoradius, or the radius of curvature of the path of a particle with charge q and mass m moving in a magnetic field of strength B. The gryoradius is then given by $\Gamma = \frac{\mathrm{mv}}{\mathrm{qB}}$.
- The cyclotron frequency (or, equivalently, gyrofrequency) is the number of cycles a particle completes around its circular circuit every second and is given by $\mathrm{f} = \frac{\mathrm{qB}}{2\pi\mathrm{m}}$.
- Previously, we have seen that circular motion results when the velocity of a charged particle is perpendicular to the magnetic field. The speed and kinetic energy of the particle remain constant, but the direction is altered at each instant by the perpendicular magnetic force.
- If the velocity is not perpendicular to the magnetic field, we consider only the component of v that is perpendicular to the field when making our calculations.
- The component of the velocity parallel to the field is unaffected, since the magnetic force is zero for motion parallel to the field. This produces helical motion.

- Charges may spiral along field lines. If the strength of the magnetic field increases in the direction of motion, the field will exert a force to slow the charges and even reverse their direction. This is known as a magnetic mirror.
- A cyclotron is a type of particle accelerator in which charged particles accelerate outwards from the center along a spiral path. The particles are held to a spiral trajectory by a static magnetic field and accelerated by a rapidly varying electric field.
- The cavity magnetron is a high-powered vacuum tube that generates microwaves using the interaction of a stream of electrons with a magnetic field. The magnetron has applications in radar, heating, and lighting.
- Mass spectrometers measure the mass-to-charge ratio of charged particles through the use of electromagnetic fields to segregate particles with different masses and/or charges. It can be used to determine the elemental composition of a molecule or sample.

Key Terms

- **orthogonal**: Of two objects, at right angles; perpendicular to each other.
- **straight-line motion**: motion that proceeds in a single direction.
- **gyroradius**: The radius of the circular motion of a charged particle in the presence of a uniform magnetic field.
- **cyclotron frequency**: The frequency of a charged particle moving perpendicular to the direction of a uniform magnetic field B (constant magnitude and direction). Given by the equality of the centripetal force and magnetic Lorentz force.
- **helical motion**: The motion that is produced when one component of the velocity is constant in magnitude and direction (i.e., straight-line motion) while the other component is constant in speed but uniformly varies in direction (i.e., circular motion). It is the superposition of straight-line and circular motion.
- **magnetic mirror**: A magnetic field configuration where the field strength changes when moving along a field line. The mirror effect results in a tendency for charged particles to bounce back from the high field region.
- **cyclotron**: An early particle accelerator in which charged particles were generated at a central source and accelerated spirally outward through a fixed magnetic and alternating electric fields.
- **mass spectrometer**: A device used in mass spectrometry to discover the mass composition of a given substance.
- **magnetron**: A device in which electrons are made to resonate in a specially shaped chamber and thus produce microwave radiation; used in radar, and in microwave ovens.

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21.5: Magnetic Fields, Magnetic Forces, and Conductors

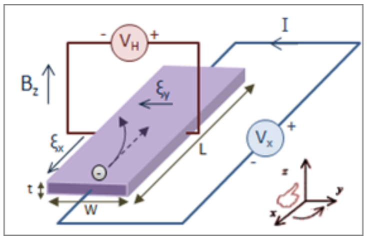
learning objectives

- Express Hall voltage for a a metal containing only one type of charge carriers

The Hall effect is the phenomenon in which a voltage difference (called the Hall voltage) is produced across an electrical conductor, transverse to the conductor's electric current when a magnetic field perpendicular to the conductor's current is applied.

When a magnetic field is present that is not parallel to the motion of moving charges within a conductor, the charges experience the Lorentz force. In the absence of such a field, the charges follow a roughly straight path, occasionally colliding with impurities.

In the presence of a magnetic field with a perpendicular component, the paths charges take becomes curved such that they accumulate on one face of the material. On the other face, there is an excess of opposite charge remaining. Thus, an electric potential is created so long as the charge flows. This opposes the magnetic force, eventually to the point of cancelation, resulting in electron flow in a straight path.



Hall Effect for Electrons: Initially, the electrons are attracted by the magnetic force and follow the curved arrow. Eventually, when electrons accumulate in excess on the left side and are in deficit on the right, an electric field ξy is created. This force becomes strong enough to cancel out the magnetic force, so future electrons follow a straight (rather than curved) path.

For a metal containing only one type of charge carrier (electrons), the Hall voltage (V_H) can be calculated as a factor of current (I), magnetic field (B), thickness of the conductor plate (t), and charge carrier density (n) of the carrier electrons:

$$V_H = -\frac{IB}{net} \tag{21.5.1}$$

In this formula, e represents the elementary charge.

The Hall coefficient (R_H) is a characteristic of a conductor's material, and is defined as the ratio of induced electric field (E_y) to the product of current density (j_x) and applied magnetic field (B):

$$R_H = \frac{E_y}{j_{xB}} = \frac{V_{Ht}}{IB} = -\frac{1}{ne} \tag{21.5.2}$$

The Hall effect is a rather ubiquitous phenomenon in physics, and appears not only in conductors, but semiconductors, ionized gases, and in quantum spin among other applications.

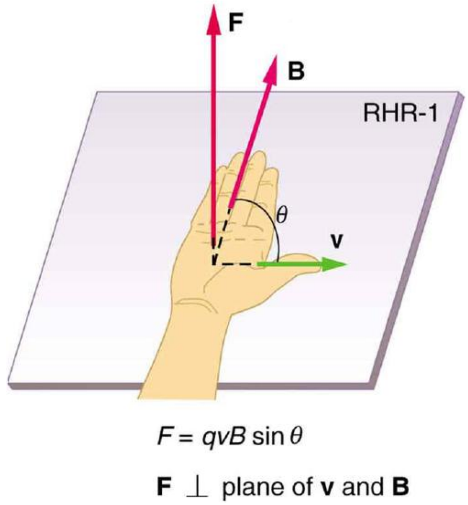
Magnetic Force on a Current-Carrying Conductor

When an electrical wire is exposed to a magnet, the current in that wire will experience a force—the result of a magnet field.

learning objectives

- Express equation used to calculate the magnetic force for an electrical wire exposed to a magnetic field

When an electrical wire is exposed to a magnet, the current in that wire will be affected by a magnetic field. The effect comes in the form of a force. The expression for magnetic force on current can be found by summing the magnetic force on each of the many individual charges that comprise the current. Since they all run in the same direction, the forces can be added.



Right Hand Rule: Used to determine direction of magnetic force.

The force (F) a magnetic field (B) exerts on an individual charge (q) traveling at drift velocity v_d is:

$$F = qv_{dB}\sin\theta \tag{21.5.3}$$

In this instance, θ represents the angle between the magnetic field and the wire (magnetic force is typically calculated as a cross product). If B is constant throughout a wire, and is 0 elsewhere, then for a wire with N charge carriers in its total length l, the total magnetic force on the wire is:

$$F = Nqv_{dB}\sin\theta \tag{21.5.4}$$

Given that N=nV, where n is the number of charge carriers per unit volume and V is volume of the wire, and that this volume is calculated as the product of the circular cross-sectional area A and length (V=Al), yields the equation:

$$F = (nqAv_d)\,lB\sin\theta \tag{21.5.5}$$

The terms in parentheses are equal to current (I), and thus the equation can be rewritten as:

$$F = IlB\sin\theta \tag{21.5.6}$$

The direction of the magnetic force can be determined using the *right hand rule*, demonstrated in. The thumb is pointing in the direction of the current, with the four other fingers parallel to the magnetic field. Curling the fingers reveals the direction of magnetic force.

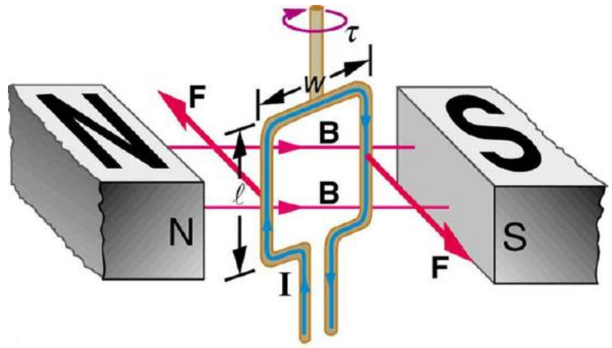
Torque on a Current Loop: Rectangular and General

A current-carrying loop exposed to a magnetic field experiences a torque, which can be used to power a motor.

learning objectives

- Identify the general quation for the torque on a loop of any shape

When a current travels in a loop that is exposed to a magnetic field, that field exerts torque on the loop. This principle is commonly used in motors, in which the loop is connected to a shaft that rotates as a result of the torque. Thus, the electrical energy from the current is converted to mechanical energy as the loop and shaft rotate, and this mechanical energy is then used to power another device.

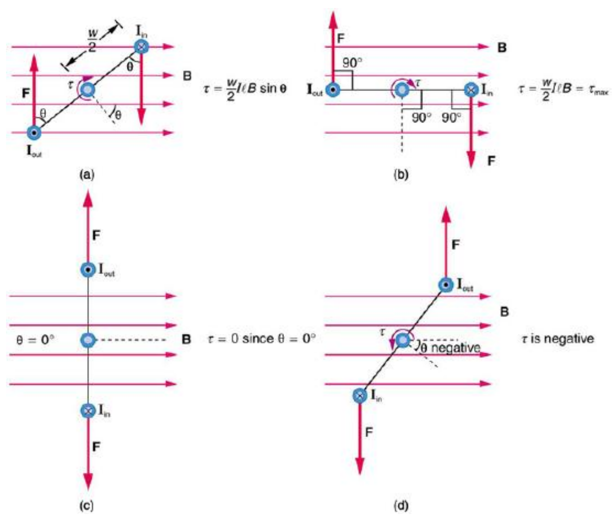


Torque on a Current Loop: Electrical energy from the current is converted to mechanical energy as the loop and shaft rotate, and this mechanical energy is then used to power another device.

In this model, the north and south poles of magnets are denoted by N and S, respectively. In the center is a rectangular wire loop of length l and width w, carrying current I. The effect of magnetic field B on the current-carrying wire exerts torque τ.

To understand the torque, we must analyze the forces acting on each segment of the loop. Assuming a constant magnetic field, we can conclude that the forces on the top and bottom parts of the loop are equal in magnitude and opposite in direction, and thus produce no net force. Incidentally, those forces are vertical and thus parallel to the shaft.

However, as illustrated by (a) in the figure below, the equal but opposite forces produce a torque that acts clockwise.



Varying torque on a charged loop in a magnetic field: Maximum torque occurs in (b), when is 90 degrees. Minimum torque is 0, and occurs in (c) when θ is 0 degrees. When loop rotates past =0, the torque reverses (d).

Given that torque is calculated from the equation:

$$\tau = \mathrm{rF}\sin\theta \tag{21.5.7}$$

where F is force on the rotating object, r is the distance from the pivot point that the force is applied, and θ is the angle between r and F, we can use the sum of two torques (the forces act on either side of the loop) to find the total torque:

$$\tau = \frac{\mathrm{w}}{2}\mathrm{F}\sin\theta + \frac{\mathrm{w}}{2}\mathrm{F}\sin\theta = \mathrm{wF}\sin\theta \tag{21.5.8}$$

Note that r is equal to w/2, as illustrated.

To find torque we still must solve for F from the magnetic field B on the current I. The rectangle has length l, so F=IlB. Replacing F with IlB in the torque equation gives:

$$\tau = \mathrm{wIlB}\sin\theta \tag{21.5.9}$$

Note that the product of w and l is included in this equation; those terms can be replaced with area (A) of the rectangle. If another shape of wire is used, its area can be inserted in the equation regardless of shape (whether circular, square, or otherwise).

Also note that this equation of torque is for a single turn. Torque increases proportionally according to number of turns (N). Thus, the general equation for torque on a loop of any shape, of N turns, each of A area, carrying I current and exposed to a magnetic field B is a value that fluctuates as the loop rotates, and can be calculated by:

$$\tau = \mathrm{NIAB}\sin\theta \tag{21.5.10}$$

Ampere's Law: Magnetic Field Due to a Long Straight Wire

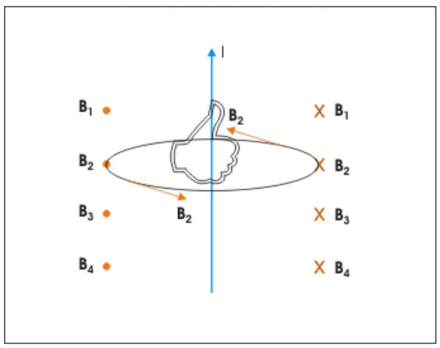
Current running through a wire will produce a magnetic field that can be calculated using the Biot-Savart Law.

learning objectives

- Express the relationship between the strength of a magnetic field and a current running through a wire in a form of equation

Current running through a wire will produce both an electric field and a magnetic field. For a closed curve of length C, magnetic field (B) is related to current (I_C) as in Ampere's Law, stated mathematically as:

$$\oint_{\mathrm{C}} \mathrm{B}d\ell = \mu_0 \mathrm{I_C} \tag{21.5.11}$$



Direction of magnetic field: The direction of the magnetic field can be determined by the right hand rule.

In this equation, dl represents the differential of length of wire in the curved wire, and μ_0 is the permeability of free space. This can be related to the Biot-Savart law. For a short, straight length of conductor (typically a wire) this law generally calculates partial magnetic field (dB) as a function of current for an infinitesimally small segment of wire (dl) at a point r distance away from the conductor:

$$\mathrm{d}\mathbf{B} = \frac{\mu_0}{4\pi}\frac{\mathrm{Idl}\times\mathbf{r}}{\mathrm{r}^3} \tag{21.5.12}$$

In this equation, the r vector can be written as $\hat{r}$ (the unit vector in direction of r), if the r^3 term in the denominator is reduced to r^2 (this is simply reducing like terms in a fraction). Integrating the previous differential equation, we find:

$$\mathbf{B} = \frac{\mu_0}{4\pi}\oint_C \frac{\mathrm{Idl}\times\hat{\mathbf{r}}}{\mathrm{r}^2} \tag{21.5.13}$$

This relationship holds for constant current in a straight wire, in which magnetic field at a point due to all current elements comprising the straight wire is the same. As illustrated in the direction of the magnetic field can be determined using the *right hand rule*—pointing one's thumb in the direction of current, the curl of one's fingers indicates the direction of the magnetic field around the straight wire.

Magnetic Force Between Two Parallel Conductors

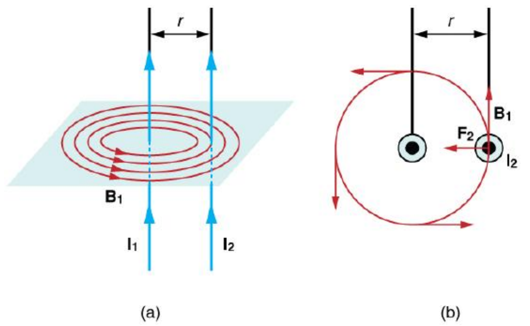
Parallel wires carrying current produce significant magnetic fields, which in turn produce significant forces on currents.

learning objectives

- Express the magnetic force felt by a pair of wires in a form of an equation

Parallel wires carrying current produce significant magnetic fields, which in turn produce significant forces on currents. The force felt between the wires is used to define the the standard unit of current, know as an amphere.

In, the field (B_1) that I_1 creates can be calculated as a function of current and wire separation (r):



Magnetic fields and force exerted by parallel current-carrying wires.: Currents I1 and I2 flow in the same direction, separated by a distance of r.

$$\mathrm{B_1} = \frac{\mu_0 \mathrm{I_1}}{2\pi \mathrm{r}} \tag{21.5.14}$$

The field B_1 exerts a force on the wire containing I_2. In the figure, this force is denoted as F_2.

The force F_2 exerts on wire 2 can be calculated as:

$$\mathrm{F_2} = \mathrm{I_2 l B_1} \sin\theta \tag{21.5.15}$$

Given that the field is uniform along and perpendicular to wire 2, sin θ = sin 90 derees = 1. Thus the force simplifies to: $F_2=I_2lB_1$

According to Newton's Third Law (F_1=-F_2), the forces on the two wires will be equal in magnitude and opposite in direction, so to simply we can use F instead of F_2. Given that wires are often very long, it's often convenient to solve for force per unit length. Rearranging the previous equation and using the definition of B_1gives:

$$\frac{\mathrm{F}}{1} = \frac{\mu_0 \mathrm{I_1 I_2}}{2\pi \mathrm{r}} \tag{21.5.16}$$

If the currents are in the same direction, the force attracts the wires. If the currents are in opposite directions, the force repels the wires.

The force between current-carrying wires is used as part of the operational definition of the ampere. For parallel wires placed one meter away from one another, each carrying one ampere, the force per meter is:

$$\frac{\mathrm{F}}{1} = \frac{\left(4\pi \cdot 10^{-7} \mathrm{T \cdot m/A}\right)(1\mathrm{A})^2}{(2\pi)(1\mathrm{m})} = 2 \cdot 10^{-7} \mathrm{N/m} \tag{21.5.17}$$

The final units come from replacing T with 1N/(A×m).

Incidentally, this value is the basis of the operational definition of the ampere. This means that one ampere of current through two infinitely long parallel conductors (separated by one meter in empty space and free of any other magnetic fields) causes a force of 2×10^{-7}N/m on each conductor.

Key Points

- The Hall effect is the phenomenon in which a voltage difference (called the Hall voltage) is produced across an electrical conductor that is transverse to the conductor's electric current when a magnetic field perpendicular to the conductor's current is applied.
- Moving charges in a wire will change trajectory in the presence of a magnetic field, "bending" toward it. Thus, those charges accumulate on one face of the material. On the other face, there is left an excess of opposite charge. Thus, an electric potential

is created.

- $V_H = -\frac{IB}{net}$ is the formula for Hall voltage (V_H). It is a factor of current (I), magnetic field (B), thickness of the conductor plate (t), and charge carrier density (n) of the carrier electrons.
- Magnetic force on current can be found by summing the magnetic force on each of the individual charges that make this current.
- For a wire exposed to a magnetic field, $\tau = NIAB\sin\theta$ describes the relationship between magnetic force (F), current (I), length of wire (l), magnetic field (B), and angle between field and wire (θ).
- The direction of the magnetic force can be determined using the *right hand rule*, as in fig .
- $\tau = NIAB\sin\theta$ can be used to calculate torque (ττ) a loop of N turns and A area, carrying I current feels in the presence of a magnetic field B.
- Although the forces acting upon the loop are equal and opposite, they both act to rotate the loop in the same direction.
- Torque experienced is independent of the loop's shape. What matters is the area of the loop.
- Ampere 's Law states that for a closed curve of length C, magnetic field (B) is related to current (I_C): $\oint_C Bd\ell = \mu_0 I_C$. In this equation, dl represents the differential of length of wire in the curved wire, and μ_0 is the permeability of free space.
- Ampere's Law can be related to the Biot-Savart law, which holds for a short, straight length of conductor: $dB = \frac{\mu_0}{4\pi}\frac{Id\times \mathbf{lr}}{r^3}$. In this equation, partial magnetic field (dB) is expressed as a function of current for an infinitesimally small segment of wire (dl) at a point r distance away from the conductor.
- After integrating, the direction of the magnetic field according to the Biot-Savart Law can be determined using the right hand rule.
- The field (B_1) that that current (I_1) from a wire creates can be calculated as a function of current and wire separation (r): $B_1 = \frac{\mu_0 I_1}{2\pi r}$ μ_0 is a constant.
- $F = IlB\sin\theta$ describes the magnetic force felt by a pair of wires. If they are parallel the equation is simplified as the sine function is 1.
- The force felt between two parallel conductive wires is used to define the ampere —the standard unit of current.

Key Terms

- **elementary charge**: The electric charge on a single proton.
- **transverse**: Not tangent, so that a nondegenerate angle is formed between the two things intersecting.
- **drift velocity**: The average velocity of the free charges in a conductor.
- **magnetic field**: A condition in the space around a magnet or electric current in which there is a detectable magnetic force, and where two magnetic poles are present.
- **torque**: A rotational or twisting effect of a force; (SI unit newton-meter or Nm; imperial unit foot-pound or ft-lb)
- **electric field**: A region of space around a charged particle, or between two voltages; it exerts a force on charged objects in its vicinity.
- **ampere**: A unit of electrical current; the standard base unit in the International System of Units. Abbreviation: amp. Symbol: A.
- **current**: The time rate of flow of electric charge.

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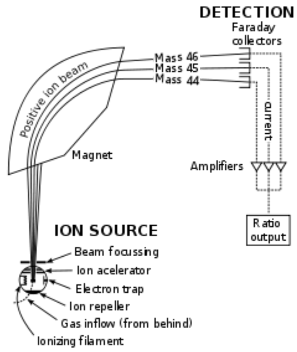
21.6: Applications of Magnetism

learning objectives

- Explain why magnetic field is utilized in mass spectrometers

Mass Spectrometry

Mass spectrometry (MS) is the art of displaying the spectra (singular spectrum) of the masses of a sample of material. It is used for determining the elemental composition of a sample, and properties of the particles and molecules (the chemical structures of molecules, such as peptides and other chemical compounds).



Schematic of Mass Spectrometer: Schematics of a simple mass spectrometer with sector type mass analyzer. This one is for the measurement of carbon dioxide isotope ratios as in the carbon-13urea breath test.

Mass spectrometers, as diagramed in, separate compounds based on a property known as the mass-to-charge ratio. The sample to be identified is first ionized, and then passed through some form of magnetic field. Based on parameters, such as how long it takes the molecule to travel a certain distance or the amount of deflection caused by the field, a mass can be calculated for the ion.

How it Works

First, the sample undergoes vaporization by intense heat. The gaseous components of the sample are each ionized (turned into ions) with the same quantity of charge. The ions are then grouped by applying a similar magnetic force to each. Since the acceleration of a charge is dependent on the mass and strength of the charge, a lighter mass-to-charge ratio will not travel as far as a high mass-to-charge ratio, allowing for comparison of the physical properties of different particles. The groupings are detected by some quantitative signal from of ongoing probing. The detector differentiates the ions based on how far they curve in the magnetic field. The signal is processed into the spectra (singular of spectrum) of the masses of the particles of that sample. The elements or molecules are uniquely identified by correlating known masses by the identified masses.

Ferromagnetism

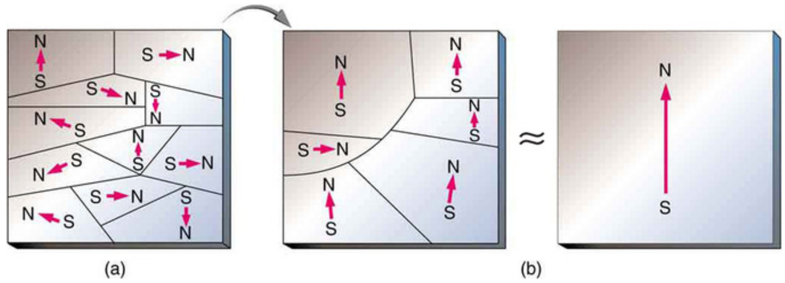
Ferromagnetism is the property of certain materials that enables them to form magnets and be attracted to magnets.

learning objectives

- Relate ferromagnetism with the electron configuration

Ferromagnetism is the basic mechanism by which certain materials (such as iron) form permanent magnets, or are attracted to magnets. In physics, several different types of magnetism are distinguished. Ferromagnetism is the strongest type—it is the only type that creates forces strong enough to be felt, and is responsible for the common phenomena of magnetism encountered in everyday life.

When a magnet comes into close proximity with a previously unmagnetized ferromagnetic material, it causes local magnetization of the material with closest unlike poles. Regions within the material (called domains) act like small bar magnets. Within domains, the poles of individual atoms are aligned. Each atom acts like a tiny bar magnet. Domains are small and randomly oriented in an unmagnetized ferromagnetic object. In response to an external magnetic field, the domains may grow to millimeter size, aligning themselves. This induced magnetization can be made permanent if the material is heated and then cooled, or simply tapped in the presence of other magnets, as shown in. Permanent magnets (materials that can be magnetized by an external magnetic field and remain magnetized after the external field is removed) are ferromagnetic, as are other materials that are noticeably attracted to them.



Unmagnetized to Magnetized Iron: (a) An unmagnetized piece of iron (or other ferromagnetic material) has randomly oriented domains. (b) When magnetized by an external field, the domains show greater alignment, and some grow at the expense of others. Individual atoms are aligned within domains; each atom acts like a tiny bar magnet.

Ferromagnetism arises from the fundamental property of an electron; it also carries charge to have a dipole moment. Thus, an electron itself behaves as a tiny magnet. This dipole moment comes from the more fundamental property of the electron—its quantum mechanical spin. The quantum mechanical nature of this spin limits the electron to only two states: with the magnetic field pointing either “up” or “down” (for any choice of up and down). When these tiny magnetic dipoles are aligned in the same direction, their individual magnetic fields combine to create a measurable macroscopic field.

However, in materials with a filled electron shell, the total dipole moment of the electrons is zero, as the spins are in up/down pairs. Only atoms with partially filled shells (i.e., unpaired spins) can have a net magnetic moment. Thus ferromagnetism only occurs in materials with partially filled shells. (According to Hund’s rules, the first few electrons in a shell tend to have the same spin, thereby increasing the total dipole moment.)

Accordingly, only certain materials (such as iron, cobalt, nickel, and gadolinium) exhibit strong magnetic effects. Such materials are called ferromagnetic, after the Latin word for iron, ferrum. A group of materials made from the alloys of the rare earth elements are also used as strong and permanent magnets (a popular one is neodymium). Other materials exhibit weak magnetic effects, detectable only with sensitive instruments. Not only do ferromagnetic materials respond strongly to magnets (the way iron is attracted to magnets), they can also be magnetized themselves—that is, they can be induced to be magnetic or made into permanent magnets.

Ferromagnetism is very important in industry and modern technology, and is the basis for many electrical and electromechanical devices such as: electromagnets, electric motors, generators, transformers, and magnetic storage (e.g., tape recorders and hard disks).

Refrigerator Magnets: Different magnets attached to the doors of a refrigerator.

Paramagnetism and Diamagnetism

Paramagnetism is the attraction of material while in a magnetic field, and diamagnetism is the repulsion of magnetic fields.

learning objectives

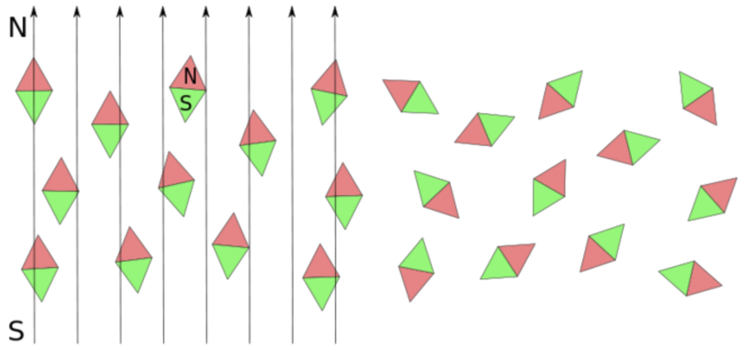
- Describe properties of diamagnetic and paramagnetic materials

Paramagnetism

Paramagnetism is a form of magnetism whereby the paramagnetic material is only attracted when in the presence of an externally applied magnetic field. Paramagnetic materials have a relative magnetic permeability greater or equal to unity (i.e., a positive magnetic susceptibility) and hence are attracted to magnetic fields. The magnetic moment induced by the applied field is linear in the field strength; it is also rather weak.

Constituent atoms or molecules of paramagnetic materials have permanent magnetic moments (dipoles), even in the absence of an applied field. Generally, the permanent moment is caused by the spin of unpaired electrons in atomic or molecular electron orbitals. In pure paramagnetism, the dipoles do not interact with each other and are randomly oriented in the absence of an external field due to thermal agitation; this results in a zero net magnetic moment. When a magnetic field is applied, the dipoles will tend to align with the applied field, resulting in a net magnetic moment in the direction of the applied field.

Paramagnetic materials have a small, positive susceptibility to magnetic fields. These materials are slightly attracted by a magnetic field and the material does not retain the magnetic properties when the external field is removed, as illustrated in. Paramagnetic properties are due to the presence of some unpaired electrons, and from the realignment of the electron paths caused by the external magnetic field.

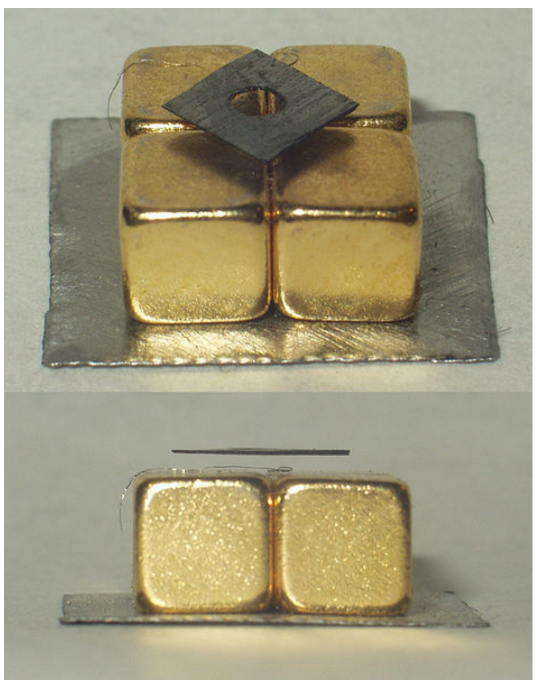


Paramagnetic Materials and Electric Fields: Orientation in paramagnetic material when electric field is applied (right image) and removed (left image).

Paramagnetic materials include magnesium, molybdenum, lithium and tantalum. Unlike ferromagnets, paramagnets do not retain any magnetization in the absence of an externally applied magnetic field, because thermal motion randomizes the spin orientations responsible for magnetism. Some paramagnetic materials retain spin disorder at absolute zero (meaning they are paramagnetic in the ground state). Thus the total magnetization drops to zero when the applied field is removed. Even in the presence of the field there is only a small induced magnetization because only a small fraction of the spins will be oriented by the field.

Diamagnetism

Diamagnetism is the property of an object or material that causes it to create a magnetic field in opposition to an externally applied magnetic field. Thus, unlike paramagnets, diamagnets are repelled by magnetic fields, which can lead to its unusual effects, such as levitation of diamagnetic material when located above powerful magnet (as shown in).



Levitating Carbon: Pyrolytic carbon levitating over permanent magnets

Diamagnetism, to a greater or lesser degree, is a property of all materials and it always makes a weak contribution to the material's response to a magnetic field. However, for materials that display some other form of magnetism (such as ferromagnetism or paramagnetism), the diamagnetic contribution becomes negligible. In addition, all conductors exhibit an effective diamagnetism when they experience a changing magnetic field. For example, the Lorentz force on electrons causes them to circulate around forming eddy currents. The eddy currents then produce an induced magnetic field opposite the applied field, resisting the conductor's motion.

Solenoids, Current Loops, and Electromagnets

Solenoids are loops of wire around a metallic core, and can be used to create controlled magnetic fields.

learning objectives

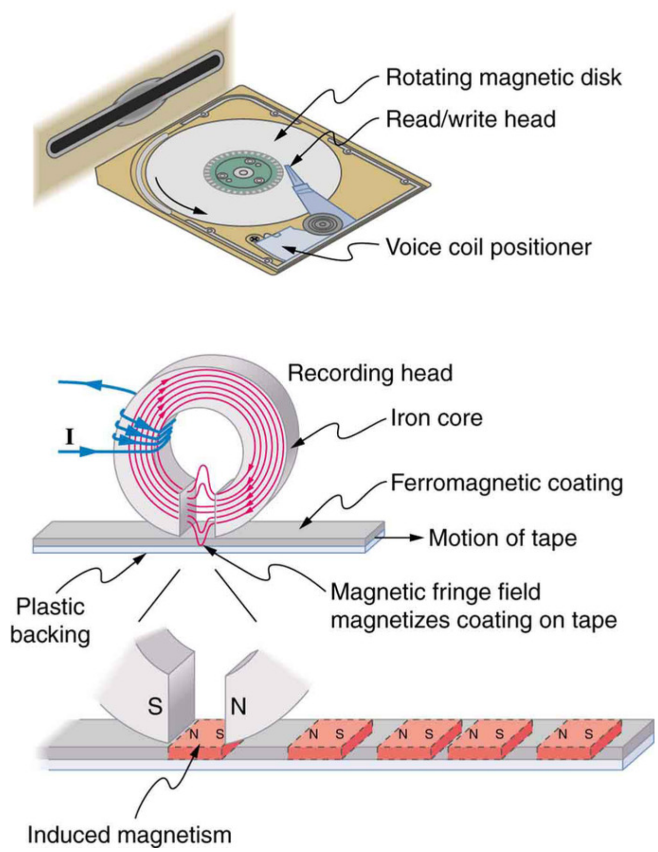
- Explain why solenoids are important and their structure

Solenoids

A solenoid is a coil wound into a tightly packed helix. In physics, the term solenoid refers to a long, thin loop of wire, often wrapped around a metallic core; it produces a magnetic field when an electric current is passed through it. Solenoids are important because they can create controlled magnetic fields and can be used as electromagnets. The term solenoid refers specifically to a coil designed to produce a uniform magnetic field in a volume of space (in which some experiment might be performed).

Electromagnets and Current Loops

Early in the 19th century, it was discovered that electrical currents cause magnetic effects. The first significant observation was made by the Danish scientist Hans Christian Oersted (1777–1851), who found that a compass needle was deflected by a current-carrying wire. This was the first significant evidence that the movement of charges had any connection with magnets. Electromagnetism is the use of electric current to make magnets. These temporarily induced magnets are called electromagnets. Electromagnets are employed for many uses: from a wrecking yard crane that lifts scrapped cars, to controlling the beam of a 90-km-circumference particle accelerator, to the magnets in medical imaging machines (for other examples see).



Uses of Electromagnets: An electromagnet induces regions of permanent magnetism on a floppy disk coated with a ferromagnetic material. The information stored here is digital (a region is either magnetic or not); in other applications, it can be analog (with a varying strength), such as on audiotapes.

Combining a ferromagnet with an electromagnet can produce particularly strong magnetic effects. Whenever strong magnetic effects are needed (such as lifting scrap metal, or in particle accelerators) electromagnets are enhanced by ferromagnetic materials.

An electromagnet creates magnetism with an electric current. In later sections we explore this more quantitatively, finding the strength and direction of magnetic fields created by various currents. Currents, including those associated with other submicroscopic particles like protons, allow us to explain ferromagnetism and all other magnetic effects. Ferromagnetism, for example, results from an internal cooperative alignment of electron spins, possible in some materials but not in others.

Key Points

- By vaporizing a material into ions, mass spectrometers can tell what elements make up that material.
- When a magnetic field is applied to ions, they deflect or accelerate various amounts depending on their electric charge to mass ratio.
- By observing how much the ions deflect, the mass spectrometer can identify the charge to mass ratio of each ion and thus which element it is.
- Ferromagnetism is the basic mechanism by which certain materials (such as iron) form permanent magnets, or are attracted to magnets.
- When a ferromagnetic material is brought close to a magnet, the poles of the individual atoms of the material align along the magnetic field lines. If made permanent, this alignment can create a permanent magnet.
- Ferromagnetism only occurs in materials with partially filled electron shells.
- Paramagnets act like magnets while in the presence of an externally applied magnetic field.
- Diamagnets create a magnetic field in opposition to an externally applied magnetic field. Thus, they repulse magnets.
- Diamagnetism is a property of all materials and always makes a weak contribution to the material’s response to a magnetic field. However, for materials that show ferromagnetism or paramagnetism, the diamagnetic contribution becomes negligible.
- Loops of wire in a magnetic field create current.
- Electromagnetism is the use of electric current to make magnets. Electromagnets are temporary magnets which keep their magnetic properties only when current is passing through them.
- Solenoids and electromagnets have many uses in physics and engineering because they allow the control of magnetic fields.

Key Terms

- **electric charge**: A quantum number that determines the electromagnetic interactions of some subatomic particles; by convention, the electron has an electric charge of -1 and the proton +1, and quarks have fractional charge.
- **mass**: The quantity of matter which a body contains, irrespective of its bulk or volume. It is one of four fundamental properties of matter. It is measured in kilograms in the SI system of measurement.
- **magnetic field**: A condition in the space around a magnet or electric current in which there is a detectable magnetic force, and where two magnetic poles are present.
- **spin**: A quantum angular momentum associated with subatomic particles; it also creates a magnetic moment.
- **dipole moment**: The vector product of the charge on either pole of a dipole and the distance separating them.
- **electron shell**: The collective states of all electrons in an atom having the same principal quantum number (visualized as an orbit in which the electrons move).
- **ferromagnetism**: The phenomenon whereby certain substances can become permanent magnets when subjected to a magnetic field.
- **paramagnetism**: The tendency of magnetic dipoles to align with an external magnetic field; materials that exhibit this tendency become temporary magnets.
- **diamagnetism**: A weak form of magnetism that is only observed in the presence of an external magnetic field; due to an induced magnetic field in an opposite direction.
- **ferromagnetic**: Of a material, such as iron or nickel, that is easily magnetized.
- **magnetic field**: A condition in the space around a magnet or electric current in which there is a detectable magnetic force, and where two magnetic poles are present.

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CHAPTER OVERVIEW

22: Induction, AC Circuits, and Electrical Technologies

Topic hierarchy

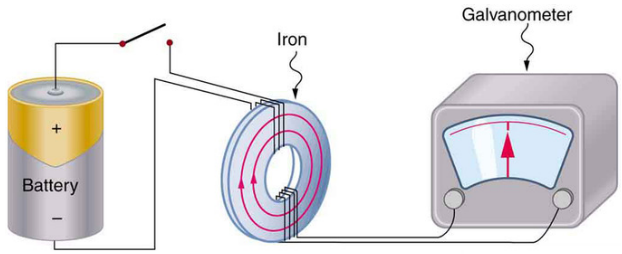
22.1: Magnetic Flux, Induction, and Faraday's Law

learning objectives

- Explain the relationship between the magnetic field and the electromotive force

Induced EMF

The apparatus used by Faraday to demonstrate that magnetic fields can create currents is illustrated in the following figure. When the switch is closed, a magnetic field is produced in the coil on the top part of the iron ring and transmitted (or guided) to the coil on the bottom part of the ring. The galvanometer is used to detect any current induced in a separate coil on the bottom.



Faraday's Apparatus: This is Faraday's apparatus for demonstrating that a magnetic field can produce a current. A change in the field produced by the top coil induces an EMF and, hence, a current in the bottom coil. When the switch is opened and closed, the galvanometer registers currents in opposite directions. No current flows through the galvanometer when the switch remains closed or open.

It was found that each time the switch is closed, the galvanometer detects a current in one direction in the coil on the bottom. Each time the switch is opened, the galvanometer detects a current in the opposite direction. Interestingly, if the switch remains closed or open for any length of time, there is no current through the galvanometer. Closing and opening the switch induces the current. It is the change in magnetic field that creates the current. More basic than the current that flows is the electromotive force (EMF) that causes it. The current is a result of an EMF induced by a changing magnetic field, whether or not there is a path for current to flow.

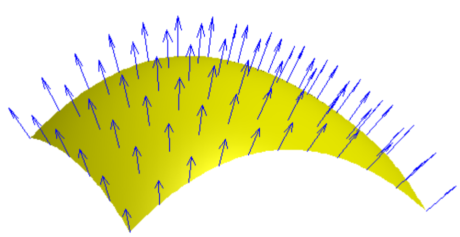
Magnetic Flux

The magnetic flux (often denoted Φ or Φ_B) through a surface is the component of the magnetic field passing through that surface. The magnetic flux through some surface is proportional to the number of field lines passing through that surface. The magnetic flux passing through a surface of vector area A is

$$\Phi_{\mathrm{B}}=\mathbf{B}\cdot\mathbf{A}=\mathrm{BA}\cos\theta \tag{22.1.1}$$

where B is the magnitude of the magnetic field (having the unit of Tesla, T), A is the area of the surface, and θ is the angle between the magnetic field lines and the normal (perpendicular) to A.

For a varying magnetic field, we first consider the magnetic flux dΦBdΦB through an infinitesimal area element dA, where we may consider the field to be constant:



Varying Magnetic Field: Each point on a surface is associated with a direction, called the surface normal; the magnetic flux through a point is then the component of the magnetic field along this normal direction.

\mathrm { d } \Phi _ { \mathrm { B } } = \mathbf { B } \cdot \mathrm { d } \mathbf { A }

A generic surface, A, can then be broken into infinitesimal elements and the total magnetic flux through the surface is then the surface integral

$$\Phi_{\mathrm{B}}=\iint_{\mathrm{A}}\mathbf{B}\cdot\mathrm{d}\mathbf{A} \tag{22.1.2}$$

Faraday's Law of Induction and Lenz' Law

Faraday's law of induction states that the EMF induced by a change in magnetic flux is $\mathrm{EMF}=-\mathrm{N}\frac{\Delta\Phi}{\Delta\mathrm{t}}$, when flux changes by Δ in a time Δt.

learning objectives

- Express the Faraday's law of induction in a form of equation

Faraday's Law of Induction

Faraday's law of induction is a basic law of electromagnetism that predicts how a magnetic field will interact with an electric circuit to produce an electromotive force (EMF). It is the fundamental operating principle of transformers, inductors, and many types of electrical motors, generators, and solenoids.

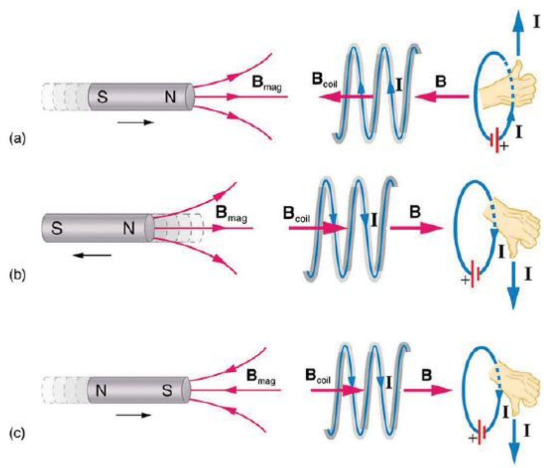
Faraday's experiments showed that the EMF induced by a change in magnetic flux depends on only a few factors. First, EMF is directly proportional to the change in flux Δ. Second, EMF is greatest when the change in time Δt is smallest—that is, EMF is inversely proportional to Δt. Finally, if a coil has N turns, an EMF will be produced that is N times greater than for a single coil, so that EMF is directly proportional to N. The equation for the EMF induced by a change in magnetic flux is

$$\mathrm{EMF} = -\mathrm{N}\frac{\Delta\Phi}{\Delta \mathrm{t}} \tag{22.1.3}$$

This relationship is known as Faraday's law of induction. The units for EMF are volts, as is usual.

Lenz' Law

The minus sign in Faraday's law of induction is very important. The minus means that the EMF creates a current I and magnetic field B that oppose the change in flux Δthis is known as Lenz' law. The direction (given by the minus sign) of the EMF is so important that it is called Lenz' law after the Russian Heinrich Lenz (1804–1865), who, like Faraday and Henry, independently investigated aspects of induction. Faraday was aware of the direction, but Lenz stated it, so he is credited for its discovery.



Lenz' Law: (a) When this bar magnet is thrust into the coil, the strength of the magnetic field increases in the coil. The current induced in the coil creates another field, in the opposite direction of the bar magnet's to oppose the increase. This is one aspect of Lenz's law—induction opposes any change in flux. (b) and (c) are two other situations. Verify for yourself that the direction of the induced Bcoil shown indeed opposes the change in flux and that the current direction shown is consistent with the right hand rule.

Energy Conservation

Lenz' law is a manifestation of the conservation of energy. The induced EMF produces a current that opposes the change in flux, because a change in flux means a change in energy. Energy can enter or leave, but not instantaneously. Lenz' law is a consequence. As the change begins, the law says induction opposes and, thus, slows the change. In fact, if the induced EMF were in the same direction as the change in flux, there would be a positive feedback that would give us free energy from no apparent source—conservation of energy would be violated.

Motional EMF

Motion in a magnetic field that is stationary relative to the Earth induces motional EMF (electromotive force).

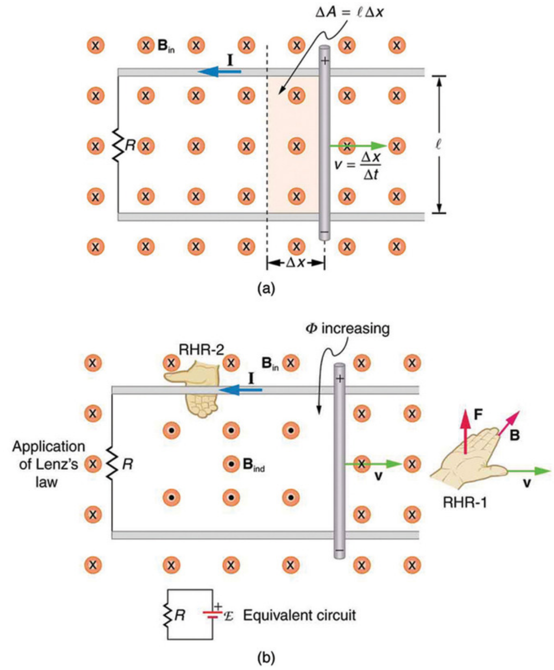
learning objectives

- Identify process that induces motional electromotive force

As seen in previous Atoms, any change in magnetic flux induces an electromotive force (EMF) opposing that change—a process known as induction. Motion is one of the major causes of induction. For example, a magnet moved toward a coil induces an EMF, and a coil moved toward a magnet produces a similar EMF. In this Atom, we concentrate on motion in a magnetic field that is stationary relative to the Earth, producing what is loosely called motional EMF.

Motional EMF

Consider the situation shown in. A rod is moved at a speed v along a pair of conducting rails separated by a distance ℓ in a uniform magnetic field B. The rails are stationary relative to B, and are connected to a stationary resistor R (the resistor could be anything from a light bulb to a voltmeter). Consider the area enclosed by the moving rod, rails and resistor. B is perpendicular to this area, and the area is increasing as the rod moves. Thus the magnetic flux enclosed by the rails, rod and resistor is increasing. When flux changes, an EMF is induced according to Faraday's law of induction.



Motional EMF: (a) A motional emf=Bℓv is induced between the rails when this rod moves to the right in the uniform magnetic field. The magnetic field B is into the page, perpendicular to the moving rod and rails and, hence, to the area enclosed by them. (b) Lenz's law gives the directions of the induced field and current, and the polarity of the induced emf. Since the flux is increasing, the induced field is in the opposite direction, or out of the page. Right hand rule gives the current direction shown, and the polarity of the rod will drive such a current.

To find the magnitude of EMF induced along the moving rod, we use Faraday's law of induction without the sign:

$$\mathrm{EMF} = \mathrm{N}\frac{\Delta\Phi}{\Delta \mathrm{t}} \tag{22.1.4}$$

In this equation, N=1 and the flux Φ=BAcosθ. We have θ=0º and cosθ=1, since B is perpendicular to A. Now $\Delta = \Delta(\mathrm{BA}) = \mathrm{B}\Delta\mathrm{A}$, since B is uniform. Note that the area swept out by the rod is $\Delta\mathrm{A} = \ell \mathrm{x}$. Entering these quantities into the expression for EMF yields:

$$\mathrm{EMF} = \frac{\mathrm{B}\Delta\mathrm{A}}{\Delta\mathrm{t}} = \mathrm{B}\frac{l\Delta\mathrm{x}}{\Delta\mathrm{t}} = \mathrm{Blv} \tag{22.1.5}$$

To find the direction of the induced field, the direction of the current, and the polarity of the induced EMF we apply Lenz' law, as explained in Faraday's Law of Induction: Lenz' Law. As seen in Fig 1 (b), F lux is increasing, since the area enclosed is increasing. Thus the induced field must oppose the existing one and be out of the page. (The right hand rule requires that I be counterclockwise, which in turn means the top of the rod is positive, as shown.)

Electric Field vs. Magnetic Field

There are many connections between the electric force and the magnetic force. That a moving magnetic field produces an electric field (and conversely that a moving electric field produces a magnetic field) is part of the reason electric and magnetic forces are now considered as *different manifestations of the same force* (first noticed by Albert Einstein). This classic unification of electric and magnetic forces into what is called the *electromagnetic force* is the inspiration for contemporary efforts to unify other basic forces.

Back EMF, Eddy Currents, and Magnetic Damping

Back EMF, eddy currents, and magnetic damping are all due to induced EMF and can be explained by Faraday's law of induction.

learning objectives

- Explain the relationship between the motional electromotive force, eddy currents, and magnetic damping

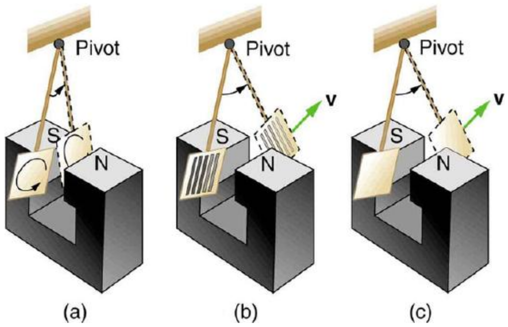
Back EMF

Motors and generators are very similar. (Read our Atoms on "Electric Generators" and "Electric Motors. ") Generators convert mechanical energy into electrical energy, whereas motors convert electrical energy into mechanical energy. Furthermore, motors and generators have the same construction. When the coil of a motor is turned, magnetic flux changes, and an electromotive force (EMF), consistent with Faraday's law of induction, is induced. The motor thus acts as a generator whenever its coil rotates. This will happen whether the shaft is turned by an external input, like a belt drive, or by the action of the motor itself. That is, when a motor is doing work and its shaft is turning, an EMF is generated. Lenz' law tells us the induced EMF opposes any change, so that the input EMF that powers the motor will be opposed by the motor's self-generated EMF, called the back EMF of the motor.

Eddy Current

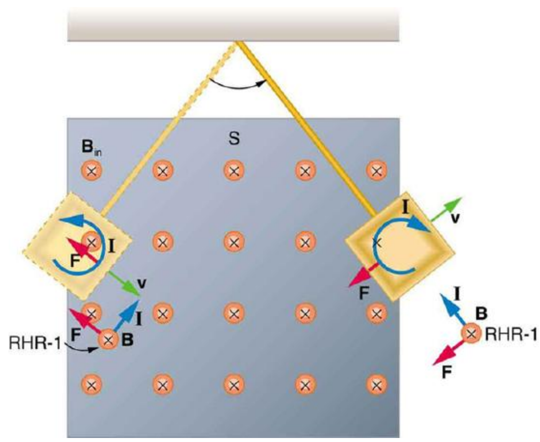
As discussed in “Motional EMF,” motional EMF is induced when a conductor moves in a magnetic field or when a magnetic field moves relative to a conductor. If motional EMF can cause a current loop in the conductor, we refer to that current as an eddy current. Eddy currents can produce significant drag, called magnetic damping, on the motion involved.

Consider the apparatus shown in, which swings a pendulum bob between the poles of a strong magnet. If the bob is metal, there is significant drag on the bob as it enters and leaves the field, quickly damping the motion. If, however, the bob is a slotted metal plate, as shown in (b), there is a much smaller effect due to the magnet. There is no discernible effect on a bob made of an insulator.



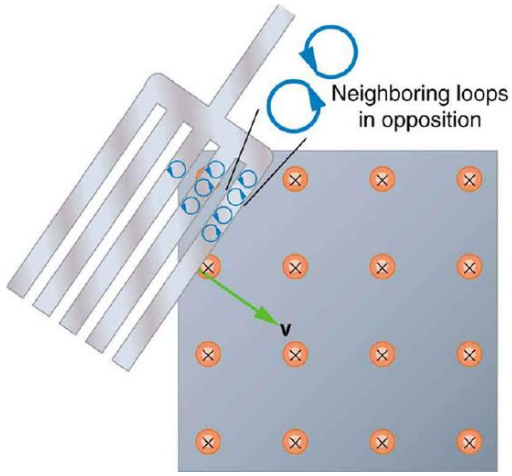
Device for Exploring Eddy Currents and Magnetic Damping: A common physics demonstration device for exploring eddy currents and magnetic damping. (a) The motion of a metal pendulum bob swinging between the poles of a magnet is quickly damped by the action of eddy currents. (b) There is little effect on the motion of a slotted metal bob, implying that eddy currents are made less effective. (c) There is also no magnetic damping on a nonconducting bob, since the eddy currents are extremely small.

shows what happens to the metal plate as it enters and leaves the magnetic field. In both cases, it experiences a force opposing its motion. As it enters from the left, flux increases, and so an eddy current is set up (Faraday’s law) in the counterclockwise direction (Lenz’ law), as shown. Only the right-hand side of the current loop is in the field, so that there is an unopposed force on it to the left (right hand rule). When the metal plate is completely inside the field, there is no eddy current if the field is uniform, since the flux remains constant in this region. But when the plate leaves the field on the right, flux decreases, causing an eddy current in the clockwise direction that, again, experiences a force to the left, further slowing the motion. A similar analysis of what happens when the plate swings from the right toward the left shows that its motion is also damped when entering and leaving the field.



Conducting Plate Passing Between the Poles of a Magnet: A more detailed look at the conducting plate passing between the poles of a magnet. As it enters and leaves the field, the change in flux produces an eddy current. Magnetic force on the current loop opposes the motion. There is no current and no magnetic drag when the plate is completely inside the uniform field.

When a slotted metal plate enters the field, as shown in, an EMF is induced by the change in flux, but it is less effective because the slots limit the size of the current loops. Moreover, adjacent loops have currents in opposite directions, and their effects cancel. When an insulating material is used, the eddy current is extremely small, and so magnetic damping on insulators is negligible. If eddy currents are to be avoided in conductors, then they can be slotted or constructed of thin layers of conducting material separated by insulating sheets.



Eddy Currents Induced in a Slotted Metal Plate: Eddy currents induced in a slotted metal plate entering a magnetic field form small loops, and the forces on them tend to cancel, thereby making magnetic drag almost zero.

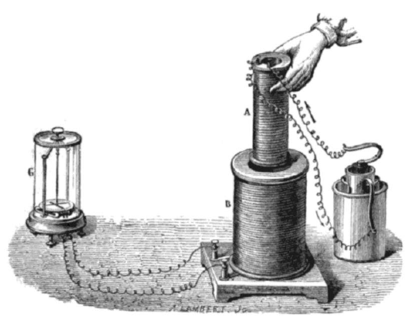
Changing Magnetic Flux Produces an Electric Field

Faraday's law of induction states that changing magnetic field produces an electric field: $\varepsilon = -\frac{\partial \Phi_B}{\partial t}$.

learning objectives

- Describe the relationship between the changing magnetic field and an electric field

We have studied Faraday's law of induction in previous atoms. We learned the relationship between induced electromotive force (EMF) and magnetic flux. In a nutshell, the law states that changing magnetic field($\frac{d\Phi_B}{dt}$) produces an electric field (ε), Faraday's law of induction is expressed as $\varepsilon = -\frac{\partial \Phi_B}{\partial t}$, where ε is induced EMF and $\frac{d\Phi_B}{dt}$ is magnetic flux. ("N" is dropped from our previous expression. The number of turns of coil is included can be incorporated in the magnetic flux, so the factor is optional.) Faraday's law of induction is a basic law of electromagnetism that predicts how a magnetic field will interact with an electric circuit to produce an electromotive force (EMF). In this Atom, we will learn about an alternative mathematical expression of the law.



Faraday's Experiment: Faraday's experiment showing induction between coils of wire: The liquid battery (right) provides a current which flows through the small coil (A), creating a magnetic field. When the coils are stationary, no current is induced. But when the small coil is moved in or out of the large coil (B), the magnetic flux through the large coil changes, inducing a current which is detected by the galvanometer (G).

Differential form of Faraday's law

The magnetic flux is $\Phi_B = \int_S \vec{B} \cdot d\vec{A}$ where $\vec{A}$ is a vector area over a closed surface S. A device that can maintain a potential difference, despite the flow of current is a source of electromotive force. (EMF) The definition is mathematically $\varepsilon = \oint_C \vec{E} \cdot d\vec{s}$, where the integral is evaluated over a closed loop C.

Faraday's law now can be rewritten $\oint_C \vec{E} \cdot d\vec{s} = -\frac{\partial}{\partial t}\left(\int \vec{B} \cdot d\vec{A}\right)$. Using the Stokes' theorem in vector calculus, the left hand side is $\oint_C \vec{E} \cdot d\vec{s} = \int_S (\nabla \times \vec{E}) \cdot d\vec{A}$. Also, note that in the right hand side $\frac{\partial}{\partial t}\left(\int \vec{B} \cdot d\vec{A}\right) = \int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}$. Therefore, we get an alternative form of the Faraday's law of induction: $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ .This is also called a differential form of the Faraday's law. It is one of the four equations in Maxwell's equations, governing all electromagnetic phenomena.

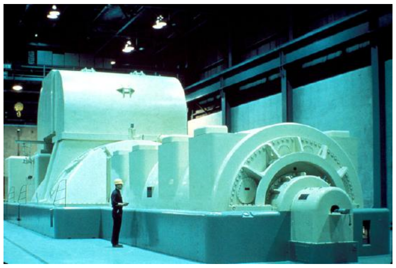
Electric Generators

Electric generators convert mechanical energy to electrical energy; they induce an EMF by rotating a coil in a magnetic field.

learning objectives

- Explain how an electromotive force is induced in electric generators

Electric generators are devices that convert mechanical energy to electrical energy. They induce an electromotive force (EMF) by rotating a coil in a magnetic field. It is a device that converts mechanical energy to electrical energy. A generator forces electric charge (usually carried by electrons) to flow through an external electrical circuit. Possible sources of mechanical energy include: a reciprocating or turbine steam engine, water falling through a turbine or waterwheel, an internal combustion engine, a wind turbine, a hand crank, compressed air, or any other source of mechanical energy. Generators supply almost all of the power for the electric power grids which provide most of the world's electric power.



Steam Turbine Generator: A modern steam turbine generator.

Basic Setup

Consider the setup shown in. Charges in the wires of the loop experience the magnetic force because they are moving in a magnetic field. Charges in the vertical wires experience forces parallel to the wire, causing currents. However, those in the top and bottom segments feel a force perpendicular to the wire; this force does not cause a current. We can thus find the induced EMF by considering only the side wires. Motional EMF is given to be EMF=Bℓv, where the velocity v is perpendicular to the magnetic field B (see our Atom on "Motional EMF"). Here, the velocity is at an angle θ with B, so that its component perpendicular to B is vsinθ.

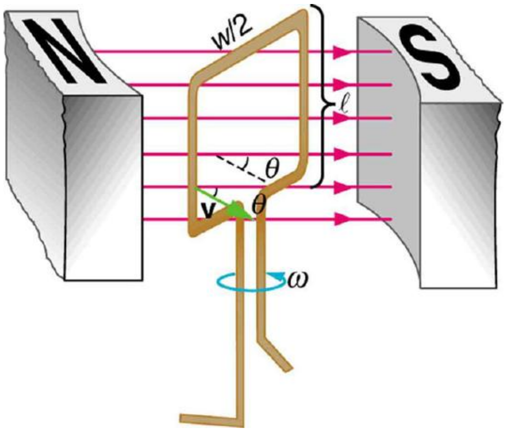


Diagram of an Electric Generator: A generator with a single rectangular coil rotated at constant angular velocity in a uniform magnetic field produces an emf that varies sinusoidally in time. Note the generator is similar to a motor, except the shaft is rotated to produce a current rather than the other way around.

Thus in this case the EMF induced on each side is EMF=Bℓvsinθ, and they are in the same direction. The total EMF εε around the loop is then:

$$\varepsilon = 2\mathrm{Blv}\sin\theta \tag{22.1.6}$$

This expression is valid, but it does not give EMF as a function of time. To find the time dependence of EMF, we assume the coil rotates at a constant angular velocity ω. The angle θ is related to angular velocity by $\theta = \omega\mathrm{t}$, so that:

$$\varepsilon = 2\mathrm{Blv}\sin\omega\mathrm{t} \tag{22.1.7}$$

Now, linear velocity v is related to angular velocity by $\mathrm{v} = \mathrm{r}\omega$. Here $\mathrm{r} = \mathrm{w}/2$, so that $\mathrm{v} = (\mathrm{w}/2)\omega$, and:

$$\varepsilon = 2\mathrm{B}l\frac{\mathrm{w}}{2}\omega\sin\omega\mathrm{t} = (\mathrm{lw})\mathrm{B}\omega\sin\omega\mathrm{t} \tag{22.1.8}$$

Noting that the area of the loop is $\mathrm{A} = \ell\mathrm{w}$, and allowing for N loops, we find that:

$\varepsilon = \mathrm{NABw}\sin\omega t$ is the EMF induced in a generator coil of N turns and area A rotating at a constant angular velocity in a uniform magnetic field B.

Generators illustrated in this Atom look very much like the motors illustrated previously. This is not coincidental. In fact, a motor becomes a generator when its shaft rotates.

Electric Motors

learning objectives

- Explain how force is generated into electric motors

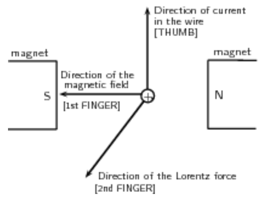
An electric motor is a device that converts electrical energy into mechanical energy.

The basic principles of operation for a motor are the same as those for a generator, except that a motor converts electrical energy into mechanical energy (motion). (Read our atom on electric generators first.) Most electric motors use the interaction of magnetic fields and current-carrying conductors to generate force. Electric motors are found in applications as diverse as industrial fans, blowers and pumps, machine tools, household appliances, power tools, and disk drives.

Lorentz Force

If you were to place a moving charged particle in a magnetic field, it would experience a force called the Lorentz force:

$$\mathrm{F} = \mathrm{q} \times \mathrm{v} \times \mathrm{B} \tag{22.1.9}$$

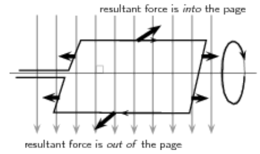


Right-Hand Rule: Right-hand rule showing the direction of the Lorentz force

where *v* is the speed of the moving charge, *q* is the charge, and *B* is the magnetic field. Current in a conductor consists of moving charges. Therefore, a current-carrying coil in a magnetic field will also feel the Lorentz force. For a straight current carrying wire that is not moving, the Lorentz force is:

$$\mathrm{F} = \mathrm{I} \times \mathrm{L} \times \mathrm{B} \tag{22.1.10}$$

where *F* is the force (in newtons, N), *I* is the current in the wire (in amperes, A), *L* is the length of the wire that is in the magnetic field (in m), and *B* is the magnetic field strength (in teslas, T). The direction of the Lorentz force is perpendicular to both the direction of the flow of current and the magnetic field and can be found using the right-hand rule, shown in. Using your right hand, point your thumb in the direction of the current, and point your first finger in the direction of the magnetic field. Your third finger will now be pointing in the direction of the force.



Torque: The force on opposite sides of the coil will be in opposite directions because the charges are moving in opposite directions. This means the coil will rotate.

Mechanics of a Motor

Both motors and generators can be explained in terms of a coil that rotates in a magnetic field. In a generator the coil is attached to an external circuit that is then turned. This results in a changing flux, which induces an electromagnetic field. In a motor, a current-carrying coil in a magnetic field experiences a force on both sides of the coil, which creates a twisting force (called a torque) that makes it turn. Any coil carrying current can feel a force in a magnetic field. This force is the Lorentz force on the moving charges in the conductor. The force on opposite sides of the coil will be in opposite directions because the charges are moving in opposite directions. This means the coil will rotate.

Inductance

Inductance is the property of a device that tells how effectively it induces an emf in another device or on itself.

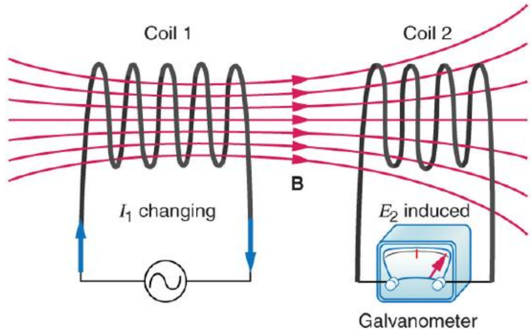
learning objectives

- Describe properties of an inductor, distinguishing mutual inductance and self-inductance

Induction is the process in which an emf is induced by changing magnetic flux. Transformers, for example, are designed to be particularly effective at inducing a desired voltage and current with very little loss of energy to other forms (see our Atom on “Transformers. “) Is there a useful physical quantity related to how “effective” a given device is? The answer is yes, and that physical quantity is called inductance.

Mutual Inductance

Mutual inductance is the effect of Faraday’s law of induction for one device upon another, such as the primary coil in transmitting energy to the secondary in a transformer. See, where simple coils induce emfs in one another.



Mutual Inductance in Coils: These coils can induce emfs in one another like an inefficient transformer. Their mutual inductance M indicates the effectiveness of the coupling between them. Here a change in current in coil 1 is seen to induce an emf in coil 2. (Note that "E2 induced" represents the induced emf in coil 2.)

In the many cases where the geometry of the devices is fixed, flux is changed by varying current. We therefore concentrate on the rate of change of current, ΔI/Δt, as the cause of induction. A change in the current I_1 in one device, coil 1, induces an EMF_2 in the other. We express this in equation form as

$$\mathrm{EMF_2 = -M\frac{\Delta I_1}{\Delta t}} \tag{22.1.11}$$

where M is defined to be the mutual inductance between the two devices. The minus sign is an expression of Lenz's law. The larger the mutual inductance M, the more effective the coupling.

Nature is symmetric here. If we change the current I2 in coil 2, we induce an emf1 in coil 1, which is given by

$$\mathrm{EMF_1 = -M\frac{\Delta I_2}{\Delta t}} \tag{22.1.12}$$

where M is the same as for the reverse process. Transformers run backward with the same effectiveness, or mutual inductance M.

Self-Inductance

Self-inductance, the effect of Faraday's law of induction of a device on itself, also exists. When, for example, current through a coil is increased, the magnetic field and flux also increase, inducing a counter emf, as required by Lenz's law. Conversely, if the current is decreased, an emf is induced that opposes the decrease. Most devices have a fixed geometry, and so the change in flux is due entirely to the change in current ΔI through the device. The induced emf is related to the physical geometry of the device and the rate of change of current. It is given by

$$\mathrm{EMF = -L\frac{\Delta I}{\Delta t}} \tag{22.1.13}$$

where L is the self-inductance of the device. A device that exhibits significant self-inductance is called an inductor. Again, the minus sign is an expression of Lenz's law, indicating that emf opposes the change in current.

A Quantitative Interpretation of Motional EMF

A a motional EMF is an electromotive force (EMF) induced by motion relative to a magnetic field B.

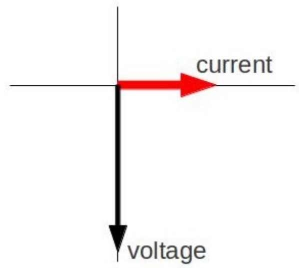
learning objectives

- Formulate two views that are applied to calculate the electromotive force

An electromotive force (EMF) induced by motion relative to a magnetic field B is called a motional EMF. You might have noticed that motional EMF is very similar to the induced EMF caused by a changing magnetic field. In this Atom we see that they are indeed the same phenomenon, shown in different frame of reference.

Motional EMF

In the case where a conductor loop is moving into magnet shown in (a), magnetic force on a moving charge in the loop is given by evBevB (Lorentz force, e: electron charge).



Conductor Loop Moving Into a Magnet: (a) Motional EMF. The current loop is moving into a stationary magnet. The direction of the magnetic field is into the screen. (b) Induced EMF. Current loop is stationary, and the magnet is moving.

Due to the force, electrons will keep building up on one side (bottom end in the figure) until enough of an electric field opposing the motion of electrons is established across the rod, which is eE. Equating the two forces, we get $\mathrm{E = vB}$.

Therefore, the motional EMF over the length L of the side of the loop is given by $\varepsilon_{\mathrm{motion}} = \mathrm{vB \times L}$ (Eq. 1), where L is the length of the object moving at speed v relative to the magnet.

Induced EMF

Since the rate of change of the magnetic flux passing through the loop is $\mathrm{B\frac{dA}{dt}}$(A: area of the loop that magnetic field pass through), the induced EMF $\varepsilon_{\mathrm{induced}} = \mathrm{BLv}$ (Eq. 2).

Equivalence of the Motional and Induced EMF

From Eq. 1 and Eq. 2 we can confirm that motional and induced EMF yield the same result. In fact, the equivalence of the two phenomena is what triggered Albert Einstein to examine special relativity. In his seminal paper on special relativity published in 1905, Einstein begins by mentioning the equivalence of the two phenomena:

"…… for example, the reciprocal electrodynamic action of a magnet and a conductor. The observable phenomenon here depends only on the relative motion of the conductor and the magnet, whereas the customary view draws a sharp distinction between the two cases in which either the one or the other of these bodies is in motion. For if the magnet is in motion and the conductor at rest, there arises in the neighbourhood of the magnet an electric field with a certain definite energy, producing a current at the places where parts of the conductor are situated. But if the magnet is stationary and the conductor in motion, no electric field arises in the neighbourhood of the magnet. In the conductor, however, we find an electromotive force, to which in itself there is no corresponding energy, but which gives rise—assuming equality of relative motion in the two cases discussed—to electric currents of the same path and intensity as those produced by the electric forces in the former case. "

Mechanical Work and Electrical Energy

Mechanical work done by an external force to produce motional EMF is converted to heat energy; energy is conserved in the process.

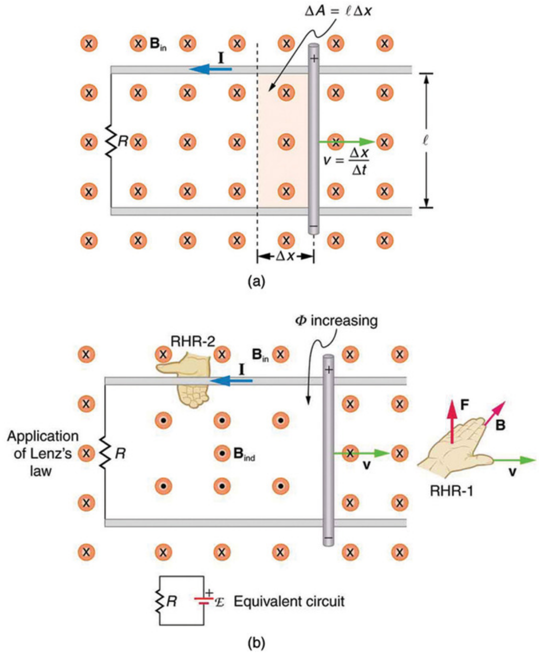
learning objectives

- Apply the law of conservation of energy to describe the production motional electromotive force with mechanical work

We learned about motional EMF previously (see our Atom on "Motional EMF"). For the simple setup shown below, motional EMF (ε)(ε) produced by a moving conductor (in a uniform field) is given as follows:

$$\varepsilon = \mathrm{Blv} \tag{22.1.14}$$

where *B* is the magnetic field, *l* is the length of the conducting rod, and *v* is the (constant) speed of its motion. (*B*, *l*, and *v* are all perpendicular to each other as shown in the image below.)



Motional EMF: (a) A motional emf=Bℓv is induced between the rails when this rod moves to the right in the uniform magnetic field. The magnetic field B is into the page, perpendicular to the moving rod and rails and, hence, to the area enclosed by them. (b) Lenz's law gives the directions of the induced field and current, and the polarity of the induced emf. Since the flux is increasing, the induced field is in the opposite direction, or out of the page. Right hand rule gives the current direction shown, and the polarity of the rod will drive such a current.

Conservation of Energy

In this atom, we will consider the system from the *energy perspective*. As the rod moves and carries current *i*, it will feel the Lorentz force

$$\mathrm{F_L = iBL} \tag{22.1.15}$$

To keep the rod moving at a constant speed *v*, we must constantly apply an external force F_{ext}(equal to magnitude of F_L and opposite in its direction) to the rod along its motion. Since the rod is moving at *v*, the power *P* delivered by the external force would be:

$$\mathrm{P = F_{ext}\,v = (iBL) \times v = i\varepsilon} \tag{22.1.16}$$

In the final step, we used the first equation we talked about. Note that this is exactly the power dissipated in the loop $(= \text{current} \times \text{voltage})$. Therefore, we conclude that the mechanical work done by an external force to keep the rod moving at a constant speed is converted to heat energy in the loop. More generally, mechanical work done by an external force to produce motional EMF is converted to heat energy. Energy is conserved in the process.

Lenz' Law

We learned in the Atom "Faraday's Law of Induction and Lenz' Law" that Lenz' law is a manifestation of the conservation of energy. As we see in the example in this Atom, Lenz' law guarantees that the motion of the rod is opposed because of nature's tendency to oppose a change in magnetic field. If the induced EMF were in the same direction as the change in flux, there would be a positive feedback causing the rod to fly away from the slightest perturbation.

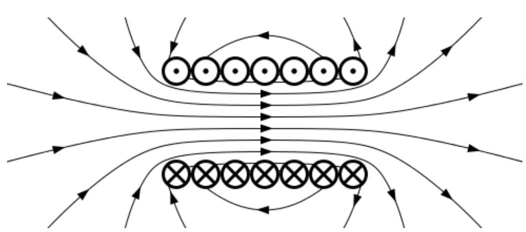
Energy in a Magnetic Field

Magnetic field stores energy. The energy density is given as $\mathbf{u} = \frac{\mathbf{B \cdot B}}{2\mu}$.

learning objectives

- Express the energy density of a magnetic field in a form of equation

Energy is needed to generate a magnetic field both to work against the electric field that a changing magnetic field creates and to change the magnetization of any material within the magnetic field. For non-dispersive materials this same energy is released when the magnetic field is destroyed. Therefore, this energy can be modeled as being "stored" in the magnetic field.



Magnetic Field Created By A Solenoid: Magnetic field created by a solenoid (cross-sectional view) described using field lines. Energy is "stored" in the magnetic field.

Energy Stored in a Magnetic Field

For linear, non-dispersive, materials (such that B = μH where μ, *called the permeability,* is frequency-independent), the energy density is:

$$u=\frac{\mathbf{B}\cdot\mathbf{B}}{2\mu}=\frac{\mu\mathbf{H}\cdot\mathbf{H}}{2} \tag{22.1.17}$$

Energy density is the amount of energy stored in a given system or region of space per unit volume. If there are no magnetic materials around, μ can be replaced by μ_0. The above equation cannot be used for nonlinear materials, though; a more general expression (given below) must be used.

In general, the incremental amount of work per unit volume δW needed to cause a small change of magnetic field δB is:

$$\delta W=\mathbf{H}\cdot\delta\mathbf{B} \tag{22.1.18}$$

Once the relationship between H and B is known this equation is used to determine the work needed to reach a given magnetic state. For hysteretic materials such as ferromagnets and superconductors, the work needed also depends on how the magnetic field is created. For linear non-dispersive materials, though, the general equation leads directly to the simpler energy density equation given above.

Energy Stored in the Field of a Solenoid

The energy stored by an inductor is equal to the amount of work required to establish the current through the inductor, and therefore the magnetic field. This is given by:

$$E_{stored}=\frac{1}{2}LI^2 \tag{22.1.19}$$

Proof: Power that should be supplied to an inductor with inductance L to run current I through it it given as

$$P=VI=L\frac{dI}{dt}\times I \tag{22.1.20}$$

Therefore

$$E_{stored}=\int_0^T P(t)dt=\int_0^I LI'dI'=\frac{1}{2}LI^2 \tag{22.1.21}$$

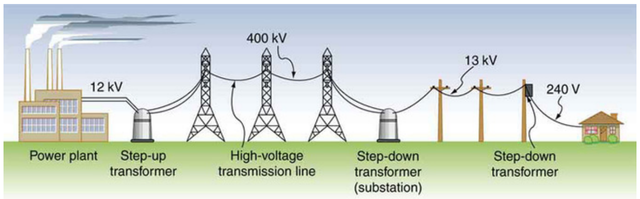
Transformers

Transformers transform voltages from one value to another; its function is governed by the transformer equation.

learning objectives

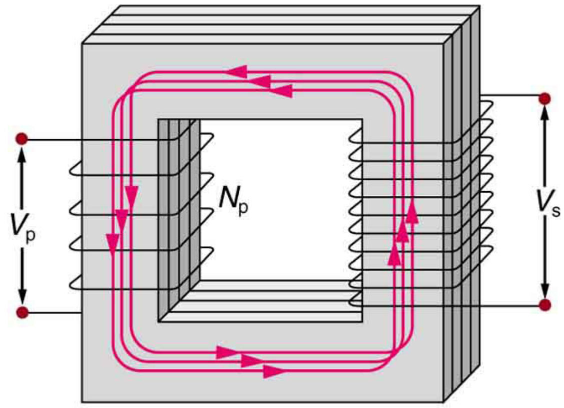
- Apply the transformer equation to compare the secondary and primary voltages

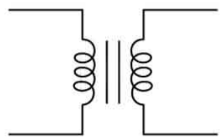
Transformers change voltages from one value to another. For example, devices such as cell phones, laptops, video games, power tools and small appliances have a transformer (built into their plug-in unit) that changes 120 V into the proper voltage for the device. Transformers are also used at several points in power distribution systems, as shown in. Power is sent long distances at high voltages, as less current is required for a given amount of power (this means less line loss). Because high voltages pose greater hazards, transformers are employed to produce lower voltage at the user's location.



Transformer Setup: Transformers change voltages at several points in a power distribution system. Electric power is usually generated at greater than 10 kV, and transmitted long distances at voltages over 200 kV—sometimes as great as 700 kV—to limit energy losses. Local power distribution to neighborhoods or industries goes through a substation and is sent short distances at voltages ranging from 5 to 13 kV. This is reduced to 120, 240, or 480 V for safety at the individual user site.

The type of transformer considered here is based on Faraday's law of induction, and is very similar in construction to the apparatus Faraday used to demonstrate that magnetic fields can create currents (illustrated in). The two coils are called the primary and secondary coils. In normal use, the input voltage is placed on the primary, and the secondary produces the transformed output voltage. Not only does the iron core trap the magnetic field created by the primary coil, its magnetization increases the field strength. Since the input voltage is AC, a time-varying magnetic flux is sent to the secondary, inducing its AC output voltage.





Simple Transformer: A typical construction of a simple transformer has two coils wound on a ferromagnetic core that is laminated to minimize eddy currents. The magnetic field created by the primary is mostly confined to and increased by the core, which transmits it to the secondary coil. Any change in current in the primary induces a current in the secondary.The figure shows a simple transformer with two coils wound on either sides of a laminated ferromagnetic core. The set of coil on left side of the core is marked as the primary and there number is given as N p. The voltage across the primary is given by V p. The set of coil on right side of the core is marked as the secondary and there number is represented as N s. The voltage across the secondary is given by V s. A symbol of the transformer is also shown below the diagram. It consists of two inductor coils separated by two equal parallel lines representing the core.

Transformer Equation

For the simple transformer shown in, the output voltage $V_s$depends almost entirely on the input voltage V_p and the ratio of the number of loops in the primary and secondary coils. Faraday's law of induction for the secondary coil gives its induced output voltage V_s as:

$$V_s = -N_s \frac{\Delta\Phi}{\Delta t} \tag{22.1.22}$$

where N_s is the number of loops in the secondary coil and Δ/Δt is the rate of change of magnetic flux. Note that the output voltage equals the induced EMF (V_s=EMF_s), provided coil resistance is small. The cross-sectional area of the coils is the same on either side, as is the magnetic field strength, so /Δt is the same on either side. The input primary voltage V_p is also related to changing flux by:

$$V_p = -N_{[} \frac{\Delta\Phi}{\Delta t} \tag{22.1.23}$$

Taking the ratio of these last two equations yields a useful relationship:

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} \tag{22.1.24}$$

This is known as the *transformer equation*, which simply states that the ratio of the secondary to primary voltages in a transformer equals the ratio of the number of loops in their coils. The output voltage of a transformer can be less than, greater than or equal to the input voltage, depending on the ratio of the number of loops in their coils. Some transformers even provide a variable output by allowing connection to be made at different points on the secondary coil. A step-up transformer is one that increases voltage, whereas a step-down transformer decreases voltage.

Assuming, as we have, that resistance is negligible, the electrical power output of a transformer equals its input. Equating the power input and output,

$$P_p = I_p V_p = I_s V_s = P_s \tag{22.1.25}$$

Combining this results with the transformer equation, we find:

$$\frac{I_s}{I_p} = \frac{N_p}{N_s} \tag{22.1.26}$$

So if voltage increases, current decreases. Conversely, if voltage decreases, current increases.

Key Points

- It is a change in the magnetic field flux that results in an electromotive force (or voltage).
- The magnetic flux (often denoted Φ or Φ_B) through a surface is the component of the magnetic field passing through that surface.

- In the most general form, magnetic flux is defined as $\Phi_B = \iint_A \mathbf{B} \cdot d\mathbf{A}$. It is the integral (sum) of all of the magnetic field passing through infinitesimal area elements dA.
- The minus in the Faraday's law means that the EMF creates a current I and magnetic field B that oppose the change in flux Δthis is known as Lenz' law.
- Faraday's law of induction is the fundamental operating principle of transformers, inductors, and many types of electrical motors, generators, and solenoids.
- Faraday's law states that the EMF induced by a change in magnetic flux depends on the change in flux Δ, time Δt, and number of turns of coils.
- Faraday's law of induction can be used to calculate the motional EMF when a change in magnetic flux is caused by a moving element in a system.
- That a moving magnetic field produces an electric field (and conversely that a moving electric field produces a magnetic field) is part of the reason electric and magnetic forces are now considered as different manifestations of the same force.
- Any change in magnetic flux induces an electromotive force (EMF) opposing that change—a process known as induction. Motion is one of the major causes of induction.
- Input EMF that powers a motor can be opposed by the motor's self-generated EMF, called the back EMF of the motor.
- If motional EMF can cause a current loop in the conductor, the current is called an eddy current.
- Eddy currents can produce significant drag, called magnetic damping, on the motion involved.
- Faraday's law of induction is a basic law of electromagnetism that predicts how a magnetic field will interact with an electric circuit to produce an electromotive force.
- An alternative, differential form of Faraday's law of induction is express in the equation $x\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$.
- Faraday's law of induction is one of the four equations in Maxwell's equations, governing all electromagnetic phenomena.
- An electric generator rotates a coil in a magnetic field, inducing an EMF given as a function of time by $\varepsilon = NABw \sin \omega t$.
- Generators supply almost all of the power for the electric power grids which provide most of the world's electric power.
- A motor becomes a generator when its shaft rotates.
- Most electric motors use the interaction of magnetic fields and current -carrying conductors to generate force.
- Current in a conductor consists of moving charges. Therefore, a current-carrying coil in a magnetic field will also feel the Lorentz force.
- In a motor, a current-carrying coil in a magnetic field experiences a force on both sides of the coil, which creates a twisting force (called a torque) that makes it turn.
- Mutual inductance is the effect of two devices in inducing emfs in each other. A change in current $\Delta I_1/\Delta t$ in one induces an emf emf2 in the seccond: $EMF_2 = -M\ \Delta I_1/\Delta t$, where M is defined to be the mutual inductance between the two devices.
- Self-inductance is the effect of the device inducing emf in itself.
- A device that exhibits significant self-inductance is called an inductor, and the EMF induced in it by a change in current through it is $EMF = -L\frac{\Delta I}{\Delta t}$.
- Motional and induced EMF are the same phenomenon, just observed in different reference frames. Equivalence of the two phenomena is what triggered Einstein to work on special relativity.
- The EMF produced due to the relative motion of the loop and magnet is given as $\varepsilon_{motion} = vB \times L$ (Eq. 1), where L is the length of the object moving at speed v relative to the magnet.
- The EMF can be calculated from two different points of view: 1) in terms of the magnetic force on moving electrons in a magnetic field, and 2) in terms of the rate of change in magnetic flux. Both yield the same result.
- Motional EMF produced by a moving conductor in a uniform field is given as follows $\varepsilon = Blv$.
- To keep the rod moving at a constant speed v, we have to apply an external force F_{ext} constantly on the rod along its motion.
- Lenz' law guarantees that the motion of the rod is opposed, and therefore the law of energy conservation is not violated.
- Energy is needed to generate a magnetic field both to work against the electric field that a changing magnetic field creates and to change the magnetization of any material within the magnetic field.
- For linear, non-dispersive, materials (such that $B = \mu H$ where μ, called the permeability, is frequency-independent), the energy density is: . $u = \frac{\mathbf{B}\cdot\mathbf{B}}{2\mu} = \frac{\mu \mathbf{H}\cdot\mathbf{H}}{2}$
- The energy stored by an inductor is $E_{stored} = \frac{1}{2}LI^2$.
- Transformers are often used at several points in the power distribution systems and also in many household power adapters.
- $Transformerequationstatesthattheratioofthesecondarytoprimaryvoltagesinatransformerequalstheratioofthenumberofloopsintheircoils$: \($\frac{V_s}{V_p} = \frac{N_s}{N_p}$
  .
- Assuming, as we have, that resistance is negligible, the electrical power output of a transformer equals its input. This leads us to another useful eqaution: $\frac{I_s}{I_p} = \frac{N_p}{N_s}$. If voltage increases, current decreases. Conversely, if voltage decreases, current increases.

Key Terms

- **vector area**: A vector whose magnitude is the area under consideration, and whose direction is perpendicular to the surface area.
- **galvanometer**: An analog measuring device, denoted by G, that measures current flow using a needle deflection caused by a magnetic field force acting upon a current-carrying wire.
- **electromotive force**: (EMF)—The voltage generated by a battery or by the magnetic force according to Faraday's Law. It is measured in units of volts, not newtons, and thus, is not actually a force.
- **solenoid**: A coil of wire that acts as a magnet when an electric current flows through it.
- **flux**: The rate of transfer of energy (or another physical quantity) through a given surface, specifically electric flux or magnetic flux.
- **magnetic flux**: A measure of the strength of a magnetic field in a given area.
- **induction**: The generation of an electric current by a varying magnetic field.
- **Faraday's law of induction**: A basic law of electromagnetism that predicts how a magnetic field will interact with an electric circuit to produce an electromotive force (EMF).

- **Maxwell’s equations**: A set of equations describing how electric and magnetic fields are generated and altered by each other and by charges and currents.
- **Stokes’ theorem**: a statement about the integration of differential forms on manifolds, which both simplifies and generalizes several theorems from vector calculus.
- **turbine**: Any of various rotary machines that use the kinetic energy of a continuous stream of fluid (a liquid or a gas) to turn a shaft.
- **Lorentz force**: The force exerted on a charged particle in an electromagnetic field.
- **torque**: A rotational or twisting effect of a force; (SI unit newton-meter or Nm; imperial unit foot-pound or ft-lb)
- **transformer**: A static device that transfers electric energy from one circuit to another by magnetic coupling. Their main use is to transfer energy between different voltage levels, which allows choosing most appropriate voltage for power generation, transmission and distribution separately.
- **special relativity**: A theory that (neglecting the effects of gravity) reconciles the principle of relativity with the observation that the speed of light is constant in all frames of reference.
- **magnetic field**: A condition in the space around a magnet or electric current in which there is a detectable magnetic force, and where two magnetic poles are present.
- **frame of reference**: A coordinate system or set of axes within which to measure the position, orientation, and other properties of objects in it.
- **motional EMF**: An EMF (electromotive force) induced by motion relative to a magnetic field.
- **permeability**: A quantitative measure of the degree of magnetization of a material in the presence of an applied magnetic field (measured in newtons per ampere squared in SI units).
- **inductor**: A passive device that introduces inductance into an electrical circuit.
- **ferromagnet**: Materials that show a permanent magnetic property.

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22.2: AC Circuits

learning objectives

- Describe properties of an inductor

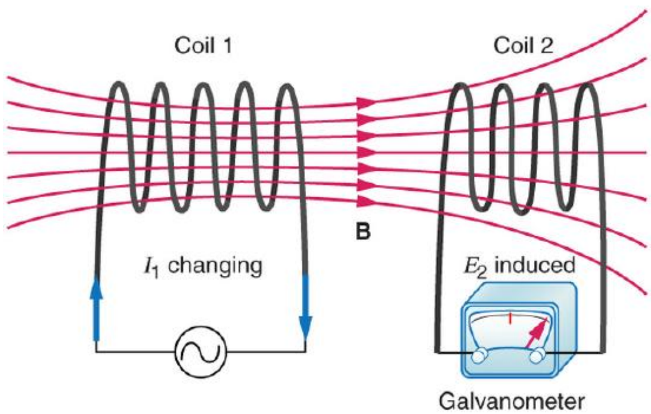
Inductance

OVERVIEW

Induction is the process in which an emf is induced by changing magnetic flux. Specifically in the case of electronics, inductance is the property of a conductor by which a change in current in the conductor creates a voltage in both the conductor itself (self-inductance) and any nearby conductors (mutual inductance). This effect derives from two fundamental observations of physics: First, that a steady current creates a steady magnetic field and second, that a time-varying magnetic field induces a voltage in a nearby conductor (Faraday's law of induction). From Lenz's law, a changing electric current through a circuit that has inductance induces a proportional voltage which opposes the change in current (if this wasn't true one can easily see how energy could not be conserved, with a changing current reinforcing the change in a positive feedback loop).

MUTUAL INDUCTANCE

Mutual inductance is the effect of Faraday's law of induction for one device upon another, such as the primary coil in transmitting energy to the secondary in a transformer. See, where simple coils induce emfs in one another.



Mutual Inductance in Coils: These coils can induce emfs in one another like an inefficient transformer. Their mutual inductance M indicates the effectiveness of the coupling between them. Here a change in current in coil 1 is seen to induce an emf in coil 2. (Note that "E2 induced" represents the induced emf in coil 2.)

In the many cases where the geometry of the devices is fixed, flux is changed by varying current. We therefore concentrate on the rate of change of current, $\Delta I/\Delta t$, as the cause of induction. A change in the current I_1 in one device, coil 1 in the figure, induces an emf_2 in the other. We express this in equation form as

[Math Processing Error]

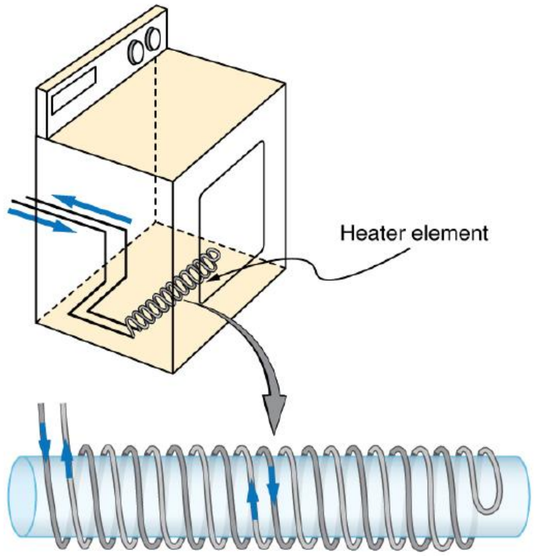
where M is defined to be the mutual inductance between the two devices. The minus sign is an expression of Lenz's law. The larger the mutual inductance M, the more effective the coupling. Units for M are (V⋅s)/A=Ωs, which is named a henry (H), after Joseph Henry (discovered of self-inductance). That is, 1 H=1Ωs.

Nature is symmetric here. If we change the current I_2 in coil 2, we induce an emf_1 in coil 1, which is given by

[Math Processing Error]

where M is the same as for the reverse process. Transformers run backward with the same effectiveness, or mutual inductance M.

A large mutual inductance M may or may not be desirable. We want a transformer to have a large mutual inductance. But an appliance, such as an electric clothes dryer, can induce a dangerous emf on its case if the mutual inductance between its coils and the case is large. One way to reduce mutual inductance M is to counterwind coils to cancel the magnetic field produced. (See).



Counterwinding: The heating coils of an electric clothes dryer can be counter-wound so that their magnetic fields cancel one another, greatly reducing the mutual inductance with the case of the dryer.

SELF-INDUCTANCE

Self-inductance, the effect of Faraday's law of induction of a device on itself, also exists. When, for example, current through a coil is increased, the magnetic field and flux also increase, inducing a counter emf, as required by Lenz's law. Conversely, if the current is decreased, an emf is induced that opposes the decrease. Most devices have a fixed geometry, and so the change in flux is due entirely to the change in current ΔI through the device. The induced emf is related to the physical geometry of the device and the rate of change of current. It is given by

[Math Processing Error]

where L is the self-inductance of the device. A device that exhibits significant self-inductance is called an inductor, and given the symbol in.

Inductor Symbol

The minus sign is an expression of Lenz's law, indicating that emf opposes the change in current. Units of self-inductance are henries (H) just as for mutual inductance. The larger the self-inductance L of a device, the greater its opposition to any change in current through it. For example, a large coil with many turns and an iron core has a large L and will not allow current to change quickly. To avoid this effect, a small L must be achieved, such as by counterwinding coils as in.

SOLENOIDS

It is possible to calculate L for an inductor given its geometry (size and shape) and knowing the magnetic field that it produces. This is difficult in most cases, because of the complexity of the field created. The inductance L is usually a given quantity. One exception is the solenoid, because it has a very uniform field inside, a nearly zero field outside, and a simple shape. The self-inductance of a solenoid of cross-sectional area A and length ℓ is

[Math Processing Error]

It is instructive to derive this equation, but this is left as an exercise to the reader. (Hint: start by noting that the induced emf is given by Faraday's law of induction as emf=−N(Δ/Δt) and, by the definition of self-inductance is given as as emf=−L(ΔI//Δt) and equate these two expressions). Note that the inductance depends only on the physical characteristics of the solenoid, consistent with its definition.

RL Circuits

An RL circuit consists of an inductor and a resistor, in series or parallel with each other, with current driven by a voltage source.

learning objectives

- Describe current-voltage relationship in the RL circuit and calculate energy that can be stored in an inductor

RL Circuits

A resistor-inductor circuit (RL circuit) consists of a resistor and an inductor (either in series or in parallel) driven by a voltage source.

Review

Recall that induction is the process in which an emf is induced by changing magnetic flux. Mutual inductance is the effect of Faraday's law of induction for one device upon another, while self-inductance is the the effect of Faraday's law of induction of a device on itself. An inductor is a device or circuit component that exhibits self-inductance.

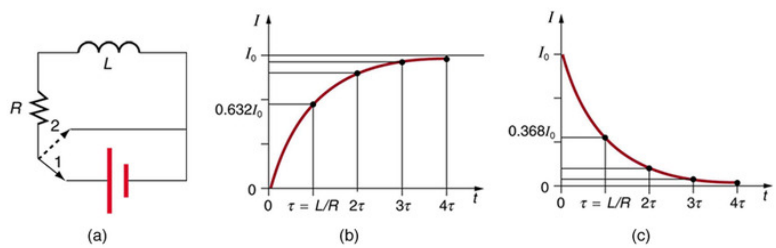
Energy of an Inductor

We know from Lenz's law that inductors oppose changes in current. We can think of this situation in terms of energy. Energy is stored in a magnetic field. It takes time to build up energy, and it also takes time to deplete energy; hence, there is an opposition to rapid change. In an inductor, the magnetic field is directly proportional to current and to the inductance of the device. It can be shown that the energy stored in an inductor E_{ind} is given by:

[Math Processing Error]

Inductors in Circuits

We know that the current through an inductor L cannot be turned on or off instantaneously. The change in current changes the magnetic flux, inducing an emf opposing the change (Lenz's law). How long does the opposition last? Current *will* flow and *can* be turned off, but how long does it take? The following figure shows a switching circuit that can be used to examine current through an inductor as a function of time.



Current in an RL Circuit: (a) An RL circuit with a switch to turn current on and off. When in position 1, the battery, resistor, and inductor are in series and a current is established. In position 2, the battery is removed and the current eventually stops because of energy loss in the resistor. (b) A graph of current growth versus time when the switch is moved to position 1. (c) A graph of current decay when the switch is moved to position 2.

When the switch is first moved to position 1 (at $t=0$), the current is zero and it eventually rises to $I_0=V/R$, where R is the total resistance of the circuitand V is the battery 's voltage. The opposition of the inductor L is greatest at the beginning, because the change in current is greatest at that time. The opposition it poses is in the form of an induced emf, which decreases to zero as the current approaches its final value. This is the hallmark of an exponential behavior, and it can be shown (with calculus) that

[Math Processing Error]

is the current in an *RL* circuit when switched on. (Note the similarity to the exponential behavior of the voltage on a charging capacitor.) The initial current is zero and approaches $I_0=V/R$ with a characteristic time constant for an *RL* circuit, given by:

[Math Processing Error]

where ττ has units of seconds, since 1H=1Ω·s1H=1Ω·s. In the first period of time ττ, the current rises from zero to $0.632I_0$, since $I=I_0(1-e^{-1})=I_0(1-0.368)=0.632I_0$. The current will be 0.632 of the remainder in the next time. A well-known property of the exponential function is that the final value is never exactly reached, but 0.632 of the remainder to that value is achieved in every characteristic time ττ. In just a few multiples of the time ττ, the final value is very nearly achieved (see part (b) of above figure).

The characteristic time ττ depends on only two factors, the inductance *L* and the resistance *R*. The greater the inductance *L*, the greater it is, which makes sense since a large inductance is very effective in opposing change. The smaller the resistance R, the greater ττ is. Again this makes sense, since a small resistance means a large final current and a greater change to get there. In both cases (large *L*and small *R)* more energy is stored in the inductor and more time is required to get it in and out.

When the switch in (a) is moved to position 2 and cuts the battery out of the circuit, the current drops because of energy dissipation by the resistor. However, this is also not instantaneous, since the inductor opposes the decrease in current by inducing an emf in the same direction as the battery that drove the current. Furthermore, there is a certain amount of energy, $(1/2)LI_0^2$, stored in the inductor, and it is dissipated at a finite rate. As the current approaches zero, the rate of decrease slows, since the energy dissipation rate is I^2R. Once again the behavior is exponential, and I is found to be

[Math Processing Error]

In (c), in the first period of time τ=L/Rτ=L/R after the switch is closed, the current falls to 0.368 of its initial value, since $I=I_0e^{-1}=0.368I_0$. In each successive time ττ, the current falls to 0.368 of the preceding value, and in a few multiples of ττ, the current becomes very close to zero.

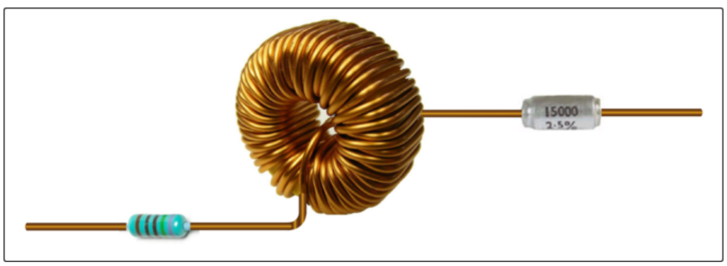
RLC Series Circuit: At Large and Small Frequencies; Phasor Diagram

Response of an RLC circuit depends on the driving frequency—at large enough frequencies, inductive (capacitive) term dominates.

learning objectives

- Distinguish behavior of RLC series circuits as large and small frequencies

In previous Atoms we learned how an RLC series circuit, as shown in, responds to an AC voltage source. By combining Ohm's law ($I_{rms}=V_{rms}/Z$; I_{rms} and V_{rms} are rms current and voltage) and the expression for impedance Z, from:



Series RLC Circuit: A series RLC circuit: a resistor, inductor and capacitor (from left).

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we arrived at: *[Math Processing Error]*.

From the equation, we studied resonance conditions for the circuit. We also learned the phase relationships among the voltages across resistor, capacitor and inductor: when a sinusoidal voltage is applied, the current lags the voltage by a 90º phase in a circuit with an inductor, while the current leads the voltage by 90∘ in a circuit with a capacitor. Now, we will examine the system's response at limits of large and small frequencies.

At Large Frequencies

At large enough frequencies (*[Math Processing Error]*), X_L is much greater than X_C. If the frequency is high enough that X_L is much larger than R as well, the impedance Z is dominated by the inductive term. When *[Math Processing Error]*, the circuit is almost equivalent to an AC circuit with just an inductor. Therefore, the rms current will be V_{rms}/X_L, and the current lags the voltage by almost 90∘. This response makes sense because, at high frequencies, Lenz's law suggests that the impedance due to the inductor will be large.

At Small Frequencies

The impedance Z at small frequencies (*[Math Processing Error]*) is dominated by the capacitive term, assuming that the frequency is high enough so that X_C is much larger than R. When *[Math Processing Error]*, the circuit is almost equivalent to an AC circuit with just a capacitor. Therefore, the rms current will be given as $V_{rms}/X_{C,}$ and the current leads the voltage by almost 90°.

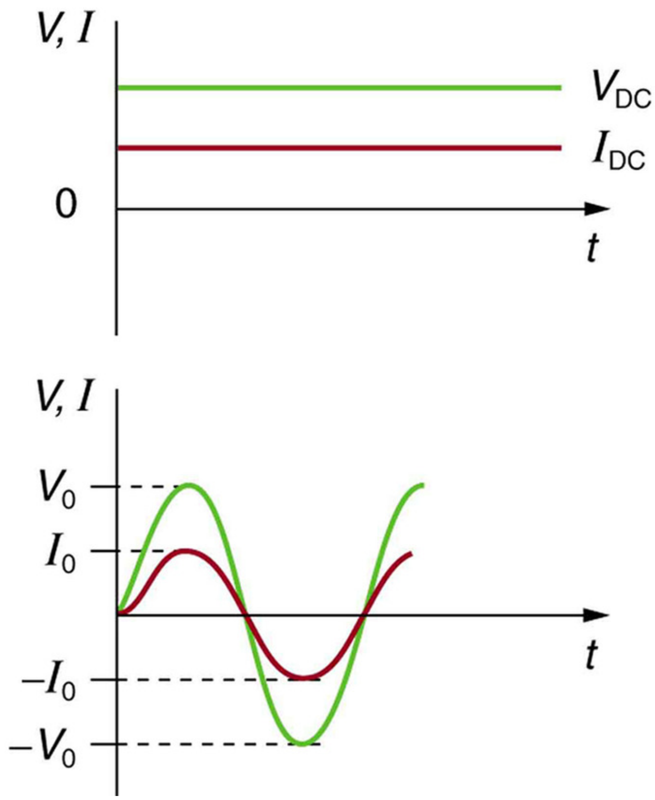
Resistors in AC Circuits

In a circuit with a resistor and an AC power source, Ohm's law still applies ($V = IR$).

learning objectives

- Apply Ohm's law to determine current and voltage in an AC circuit

Direct current (DC) is the flow of electric charge in only one direction. It is the steady state of a constant-voltage circuit. Most well known applications, however, use a time-varying voltage source. Alternating current (AC) is the flow of electric charge that periodically reverses direction. If the source varies periodically, particularly sinusoidally, the circuit is known as an alternating-current circuit. Examples include the commercial and residential power that serves so many of our needs. shows graphs of voltage and current versus time for typical DC and AC power. The AC voltages and frequencies commonly used in homes and businesses vary around the world.



Sinusoidal Voltage and Current: (a) DC voltage and current are constant in time, once the current is established. (b) A graph of voltage and current versus time for 60-Hz AC power. The voltage and current are sinusoidal and are in phase for a simple resistance circuit. The frequencies and peak voltages of AC sources differ greatly.

We have studied Ohm's law:

[Math Processing Error]

where I is the current, V is the voltage, and R is the resistance of the circuit. Ohm's law applies to AC circuits as well as to DC circuits. Therefore, with an AC voltage given by:

[Math Processing Error]

where V_0 is the peak voltage and νν is the frequency in hertz, the current in the circuit is given as:

[Math Processing Error]

In this example, in which we have a resistor and the voltage source in the circuit, the voltage and current are said to be in phase, as seen in (b). Current in the resistor alternates back and forth without any phase difference, just like the driving voltage.

Consider a perfect resistor that brightens and dims 120 times per second as the current repeatedly goes through zero. (A 120-Hz flicker is too rapid for your eyes to detect.) The fact that the light output fluctuates means that the power is fluctuating. Since the power supplied is $P = IV$, if we use the above expressions for I and V, we see that the time dependence of power is:

[Math Processing Error]

To find the average power consumed by this circuit, we need to take the time average of the function. Since:

[Math Processing Error]

we see that:

[Math Processing Error]

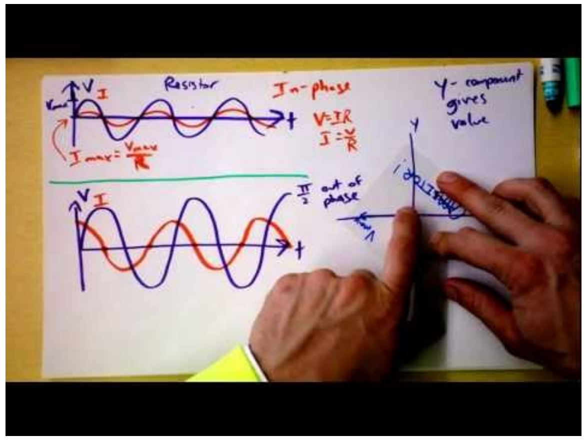
Capacitors in AC Circuits: Capacitive Reactance and Phasor Diagrams

The voltage across a capacitor lags the current. Due to the phase difference, it is useful to introduce phasors to describe these circuits.

learning objectives

- Explain the benefits of using a phasor representation

In the previous Atom on “Resistors in AC Circuits”, we introduced an AC power source and studied how resistors behave in AC circuits. There, we used the Ohm’s law (V=IR) to derive the relationship between voltage and current in AC circuits. In this and following Atoms, we will generalize the Ohm’s law so that we can use it even when we have capacitors and inductors in the circuit. To get there, we will first introduce a very general, pictorial way of representing a sinusoidal wave, using phasor.



Capacitors in AC Circuits with Phasors

Phasor

The key idea in the phasor representation is that a complex, time-varying signal may be represented as the product of a complex number (that is independent of time) and a complex signal (that is dependent on time). Phasors separate the dependencies on A (amplitude), νν (frequency), and θ (phase) into three independent factors. This can be particularly useful because the frequency factor (which includes the time-dependence of the sinusoid) is often common to all the components of a linear combination of sinusoids. In those situations, phasors allow this common feature to be factored out, leaving just the A and θ features. For example, we can represent *[Math Processing Error]* simply as a complex constant, *[Math Processing Error]*. Since phasors are represented by a magnitude (or modulus) and an angle, it is pictorially represented by a rotating arrow (or a vector) in x-y plane.

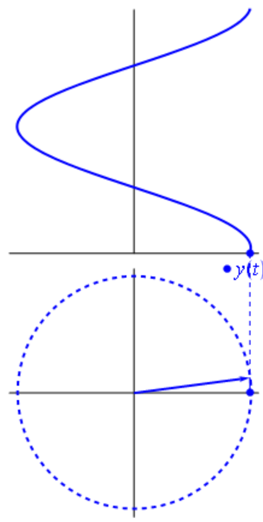


Fig 3: A phasor can be seen as a vector rotating about the origin in a complex plane. The cosine function is the projection of the vector onto the real axis. Its amplitude is the modulus of the vector, and its argument is the total phase \omega t+\theta. The phase constant \theta represents the angle that the vector forms with the real axis at t = 0.

Capacitors in AC circuits

Earlier in a previous Atom, we studied how the voltage and the current varied with time. If the AC supply is connected to a resistor, then the current and voltage will be proportional to each other. This means that the current and voltage will “peak” at the same time. We say that the current and voltage are in phase.

When a capacitor is connected to an alternating voltage, the maximum voltage is proportional to the maximum current, but the maximum voltage does not occur at the same time as the maximum current. The current has its maximum (it peaks) one quarter of a cycle before the voltage peaks. Engineers say that the “current leads the voltage by 90°“. This is shown in.

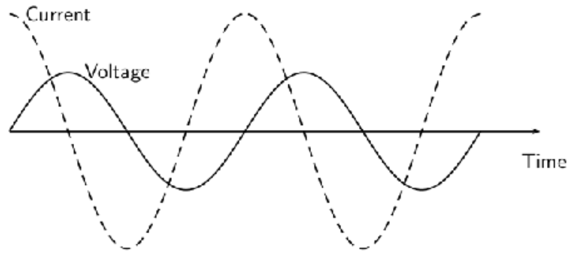


Fig 2: The current peaks (has its maximum) one quarter of a wave before the voltage when a capacitor is connected to an alternating voltage.

For a circuit with a capacitor, the instantaneous value of V/I is not constant. However, the value of V_{max}/I_{max} is useful, and is called the capacitive reactance (X_C) of the component. Because it is still a voltage divided by a current (like resistance), its unit is the ohm. The value of X_C (C standing for capacitor) depends on its capacitance (C) and the frequency (f) of the alternating current. *[Math Processing Error]*.

The capacitor is affecting the current, having the ability to stop it altogether when fully charged. Since an AC voltage is applied, there is an rms current, but it is limited by the capacitor. This is considered to be an effective resistance of the capacitor to AC, and so the rms current I_{rms} in the circuit containing only a capacitor C is given by another version of Ohm's law to be *[Math Processing Error]*, where V_{rms} is the rms voltage. Note that X_C replaces R in the DC version of the Ohm's law.

Phase representation

Since the voltage across a capacitor lags the current, the phasor representing the current and voltage would be give as in. In the diagram, the arrows rotate in counter-clockwise direction at a frequency νν. (Therefore, current leads voltage.) In the following Atoms, we will see how these phasors can be used to analyze RC, RL, LC, and RLC circuits.

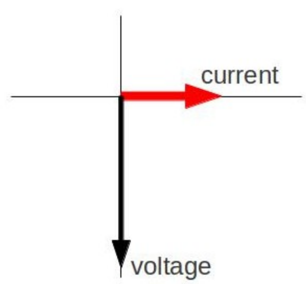


Fig 4: Phasor diagram for an AC circuit with a capacitor

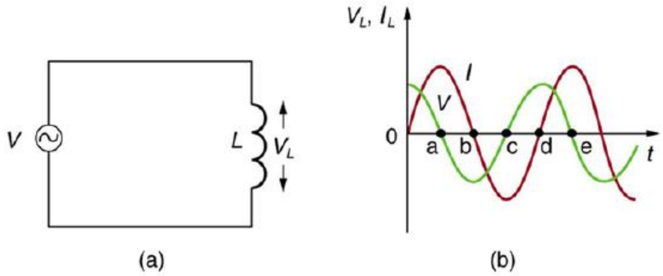
Inductors in AC Circuits: Inductive Reactive and Phasor Diagrams

In an AC circuit with an inductor, the voltage across an inductor "leads" the current because of the Lenz' law.

learning objectives

- Explain why the voltage across an inductor "leads" the current in an AC circuit with an inductor

Suppose an inductor is connected directly to an AC voltage source, as shown in. It is reasonable to assume negligible resistance because in practice we can make the resistance of an inductor so small that it has a negligible effect on the circuit. The graph shows voltage and current as functions of time. (b) starts with voltage at a maximum. Note that the current starts at zero, then rises to its peak after the voltage driving it (as seen in the preceding section when DC voltage was switched on).



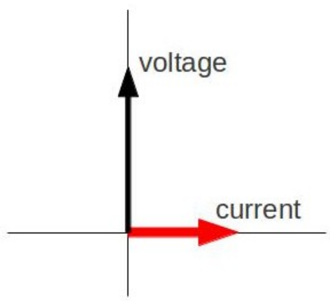
AC Voltage Source in Series with an Inductor: (a) An AC voltage source in series with an inductor having negligible resistance. (b) Graph of current and voltage across the inductor as functions of time.

When the voltage becomes negative at point a, the current begins to decrease; it becomes zero at point b, where voltage is its most negative. The current then becomes negative, again following the voltage. The voltage becomes positive at point c where it begins to make the current less negative. At point d, the current goes through zero just as the voltage reaches its positive peak to start another cycle. Hence, when a sinusoidal voltage is applied to an inductor, the voltage leads the current by one-fourth of a cycle, or by a 90^o phase angle.

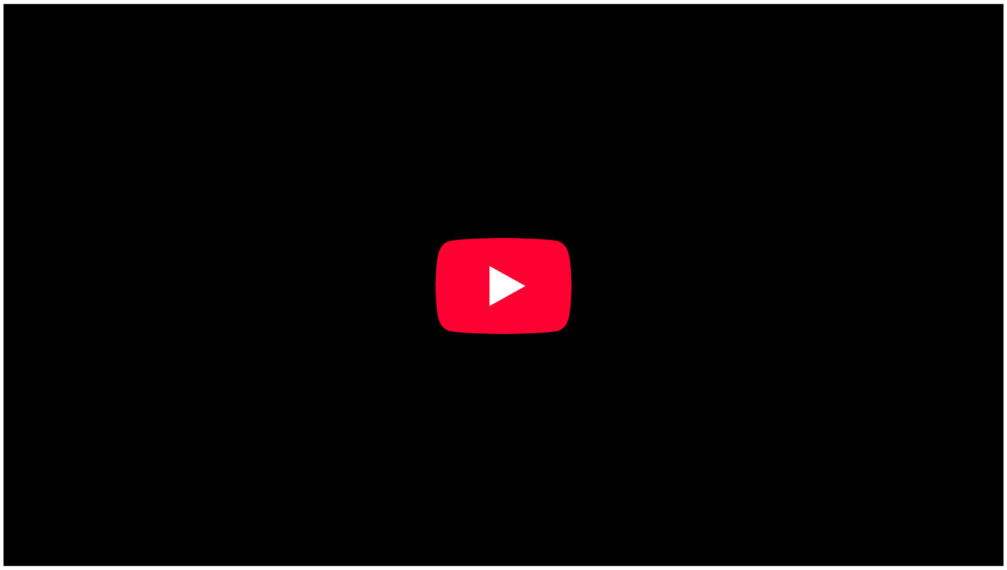
Current lags behind voltage, since inductors oppose change in current. Changing current induces an emf. This is considered an effective resistance of the inductor to AC. The rms current I_{rms} through an inductor L is given by a version of Ohm's law: *[Math Processing Error]* where V_{rms} is the rms voltage across the inductor and *[Math Processing Error]* with *[Math Processing Error]* the frequency of the AC voltage source in hertz. X_L is called the inductive reactance. Because the inductor reacts to impede the current, $X_L$has units of ohms (1 H=1 Ωs, so that frequency times inductance has units of (cycles/s)(Ωs)=Ω), consistent with its role as an effective resistance.

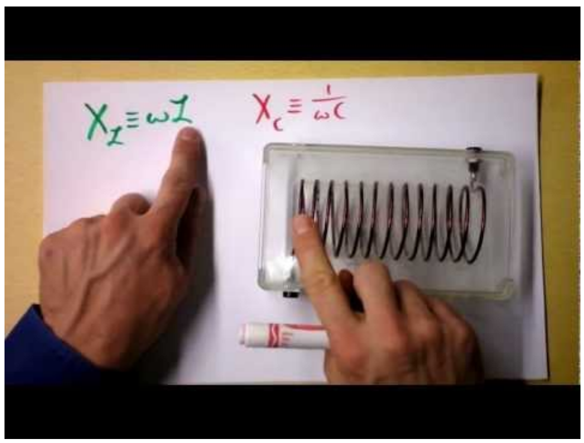
Phasor Representation

The voltage across an inductor "leads" the current because of the Lenz's law. Therefore, the phasor representing the current and voltage would be given as in. Again, the phasors are vectors rotating in counter-clockwise direction at a frequency *[Math Processing Error]* (you can see that the voltage leads the current). Subsequent Atoms will discuss how these phasors can be used to analyze RC, RL, LC, and RLC circuits.



Phasor Diagram: Phasor diagram for an AC circuit with an inductor.





Phasors for Inductors in AC Circuits

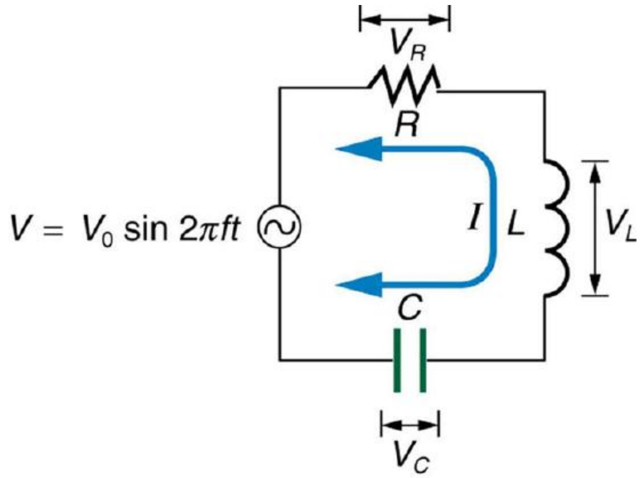
Resonance in RLC Circuits

Resonance is the tendency of a system to oscillate with greater amplitude at some frequencies—in an RLC series circuit, it occurs at $\nu_{0} = \frac{1}{2\pi\sqrt{\mathrm{LC}}}$.

learning objectives

- Compare resonance characteristics of higher- and lower-resistance circuits

Resonance is the tendency of a system to oscillate with greater amplitude at some frequencies than at others. Frequencies at which the response amplitude is a relative maximum are known as the system's resonance frequencies. To study the resonance in an RLC circuit, as illustrated below, we can see how the circuit behaves as a function of the frequency of the driving voltage source.



RLC Series Circuit: An RLC series circuit with an AC voltage source. f is the frequency of the source.

Combining Ohm ‘s law, $I_{rms}=V_{rms}/Z$, and the expression for impedance Z from

[Math Processing Error] gives

[Math Processing Error]

where I_{rms} and V_{rms} are rms current and voltage, respectively. The reactances vary with frequency νν, with X_L large at high frequencies and X_C large at low frequencies given as:

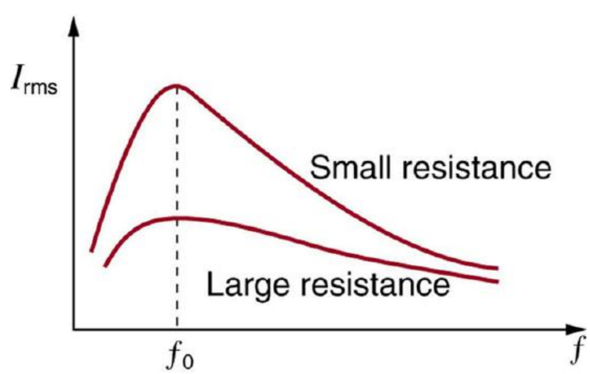
[Math Processing Error]

At some intermediate frequency *[Math Processing Error]* , the reactances will be equal and cancel, giving Z=R —this is a minimum value for impedance, and a maximum value for I_{rms} results. We can get an expression for ν0ν0 by taking $X_L=X_C$. Substituting the definitions of X_L and X_C yields:

[Math Processing Error]

[Math Processing Error] is the resonant frequency of an RLC series circuit. This is also the natural frequency at which the circuit would oscillate if not driven by the voltage source. At *[Math Processing Error]*, the effects of the inductor and capacitor cancel, so that Z=R, and I_{rms} is a maximum. Resonance in AC circuits is analogous to mechanical resonance, where resonance is defined as a forced oscillation (in this case, forced by the voltage source) at the natural frequency of the system.

The receiver in a radio is an RLC circuit that oscillates best at its *[Math Processing Error]*. A variable capacitor is often used to adjust the resonance frequency to receive a desired frequency and to reject others. is a graph of current as a function of frequency, illustrating a resonant peak in I_{rms}at *[Math Processing Error]*. The two curves are for two different circuits, which differ only in the amount of resistance in them. The peak is lower and broader for the higher-resistance circuit. Thus higher-resistance circuits do not resonate as strongly, nor would they be as selective in, for example, a radio receiver.



Current vs. Frequency: A graph of current versus frequency for two RLC series circuits differing only in the amount of resistance. Both have a resonance at f0, but that for the higher resistance is lower and broader. The driving AC voltage source has a fixed amplitude V0.

Power

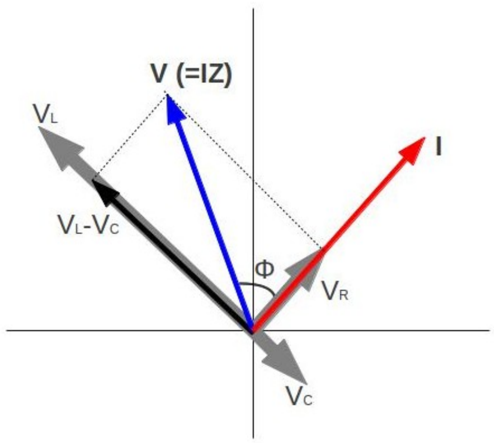
Power delivered to an RLC series AC circuit is dissipated by the resistance in the circuit, and is given as *[Math Processing Error]*. Here, *[Math Processing Error]* is called the phase angle.

learning objectives

- Calculate the power delivered to an RLC-series AC circuit given the current and the voltage

If current varies with frequency in an RLC circuit, then the power delivered to it also varies with frequency. However, the average power is not simply current times voltage, as is the case in purely resistive circuits. As seen in previous Atoms, voltage and current are out of phase in an RLC circuit. There is a phase angle ϕ between the source voltage V and the current I, given as

[Math Processing Error], as diagramed in.



Phasor Diagram for an RLC Series Circuit: Phasor diagram for an RLC series circuit. \phi is the phase angle, equal to the phase difference between the voltage and current.

For example, at the resonant frequency (*[Math Processing Error]*) or in a purely resistive circuit, Z=R, so that cosϕ=1. This implies that ϕ=0º and that voltage and current are in phase, as expected for resistors. At other frequencies, average power is less than at resonance, because voltage and current are out of phase and I_{rms} is lower.

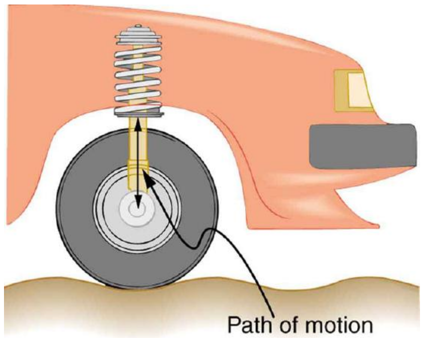
The fact that source voltage and current are out of phase affects the power delivered to the circuit. It can be shown that the average power is

[Math Processing Error]

(an equation derived by taking a time average of power, P(t) = I(t)V(t), over a period. I(t) and V(t) are current and voltage at time t). Thus cosϕ is called the power factor, which can range from 0 to 1. Power factors near 1 are desirable when designing an efficient motor, for example. At the resonant frequency, cosϕ=1.

Power delivered to an RLC series AC circuit is dissipated by the resistance alone. The inductor and capacitor have energy input and output, but do not dissipate energy out of the circuit. Rather, they transfer energy back and forth to one another, with the resistor dissipating the exact amount that the voltage source gives the circuit. This assumes no significant electromagnetic radiation from the inductor and capacitor (such as radio waves).

The circuit is analogous to the wheel of a car driven over a corrugated road, as seen in. The regularly spaced bumps in the road are analogous to the voltage source, driving the wheel up and down. The shock absorber is analogous to the resistance damping and limiting the amplitude of the oscillation. Energy within the system goes back and forth between kinetic (analogous to maximum current, and energy stored in an inductor) and potential energy stored in the car spring (analogous to no current, and energy stored in the electric field of a capacitor). The amplitude of the wheels’ motion is a maximum if the bumps in the road are hit at the resonant frequency.



Forced Damped Motion of a Wheel on a Car Spring: The forced but damped motion of the wheel on the car spring is analogous to an RLC series AC circuit. The shock absorber damps the motion and dissipates energy, analogous to the resistance in an RLC circuit. The mass and spring determine the resonant frequency.

Key Points

- In the case of electronics, inductance is the property of a conductor by which a change in current in the conductor creates a voltage in both the conductor itself, called self-inductance, and any nearby conductors, called mutual inductance.
- From Lenz's law, a changing electric current through a circuit that has inductance induces a proportional voltage which opposes the change in current.
- Mutual inductance is illustrated by. A change in the current I_1 in one device, coil 1 in the figure, induces an emf_2 in the other. We express this in equation form as *[Math Processing Error]*. M is the same for the reverse process.
- Self-inductance is the effect of Faraday's law of induction of a device on itself. The induced emf is related to the physical geometry of the device and the rate of change of current given by *[Math Processing Error]* .
- A device that exhibits significant self-inductance is called an inductor, and given the symbol in.
- The energy stored in an inductor is *[Math Processing Error]*. It takes time to build up stored energy in a conductor and time to deplete it.
- When a resistor and an inductor in series are connected to a voltage source, the time-dependent current is given by *[Math Processing Error]*. The final current after a long time is *[Math Processing Error]*.
- The characteristic time constant is given by *[Math Processing Error]*, where R is resistance and L is inductance. This represents the time necessary for the current in a circuit just closed to go from zero to *[Math Processing Error]*.
- When the voltage source is disconnected from the inductor, the current will decay according to *[Math Processing Error]*. In the first time interval τ the current falls by a factor of *[Math Processing Error]* to *[Math Processing Error]*.
- RLC circuits can be described by the (generalized) Ohm 's law. As for the phase, when a sinusoidal voltage is applied, the current lags the voltage by a 90º phase in a circuit with an inductor, while the current leads the voltage by 90∘ in a circuit with a capacitor.
- At large enough frequencies (*[Math Processing Error]*), the circuit is almost equivalent to an AC circuit with just an inductor. Therefore, the rms current will be Vrms/XL, and the current lags the voltage by almost 90∘.
- At small enough frequencies (*[Math Processing Error]*), the circuit is almost equivalent to an AC circuit with just a capacitor. Therefore, the rms current will be given as $V_{rms}/X_{C,}$ and the current leads the voltage by almost 90˚.
- With an AC voltage given by: *[Math Processing Error]* the current in the circuit is given as: *[Math Processing Error]* This expression comes from Ohm 's law: *[Math Processing Error]*.
- Most common applications use a time-varying voltage source instead of a DC source. Examples include the commercial and residential power that serves so many of our needs.
- Power dissipated by the AC circuit with a resistor in the example is: *[Math Processing Error]* Therefore, average AC power is: *[Math Processing Error]*.
- When a capacitor is connected to an alternating voltage, the maximum voltage is proportional to the maximum current, but the maximum voltage does not occur at the same time as the maximum current.
- If the AC supply is connected to a resistor, then the current and voltage will be proportional to each other. This means that the current and voltage will "peak" at the same time.
- The rms current in the circuit containing only a capacitor C is given by another version of Ohm 's law to be *[Math Processing Error]*, where *[Math Processing Error]* is the capacitive reactance.

- With an inductor in an AC circuit, the voltage leads the current by one-fourth of a cycle, or by a 90º phase angle.
- The rms current I_{rms} through an inductor L is given by a version of Ohm 's law: *[Math Processing Error]*. X_L is called the inductive reactance, given as *[Math Processing Error]*.
- Phasors are vectors rotating in counter-clockwise direction. A phasor for an inductor shows that the voltage lead the current by a 90º phase.
- Resonance condition of an RLC series circuit can be obtained by equating X_L and X_C, so that the two opposing phasors cancel each other.
- At resonance, the effects of the inductor and capacitor cancel, so that Z=R, and I_{rms} is a maximum.
- Higher- resistance circuits do not resonate as strongly compared to lower-resistance circuits, nor would they be as selective in, for example, a radio receiver.
- Phase angle ϕ is the phase difference between the source voltage V and the current I. See the phasor diagram in.
- At the resonant frequency or in a purely resistive circuit Z=R, so that cosϕ=1. This implies that ϕ=0º and that voltage and current are in phase.
- Average power dissipated in an RLC circuit can be calculated by taking a time average of power, P(t) = I(t)V(t), over a period.

Key Terms

- **mutual inductance**: The ratio of the voltage in a circuit to the change in current in a neighboring circuit.
- **self-inductance**: The ratio of the voltage to the change in current in the same circuit.
- **inductor**: A passive device that introduces inductance into an electrical circuit.
- **characteristic time constant**: Denoted by *[Math Processing Error]*, in RL circuits it is given by *[Math Processing Error]* where R is resistance and L is inductance. When a switch is closed, it is the time it takes for the current to decay by a factor of 1/e.
- **inductor**: A device or circuit component that exhibits significant self-inductance; a device which stores energy in a magnetic field.
- **Lenz's law**: A law of electromagnetic induction that states that an electromotive force, induced in a conductor, is always in such a direction that the current produced would oppose the change that caused it; this law is a form of the law of conservation of energy.
- **resonance**: The increase in the amplitude of an oscillation of a system under the influence of a periodic force whose frequency is close to that of the system's natural frequency.
- **rms**: Root mean square: a statistical measure of the magnitude of a varying quantity.
- **phasor**: A representation of a complex number in terms of a complex exponential.
- **reactance**: The opposition to the change in flow of current in an alternating current circuit, due to inductance and capacitance; the imaginary part of the impedance.
- **impedance**: A measure of the opposition to the flow of an alternating current in a circuit; the aggregation of its resistance, inductive and capacitive reactance. Represented by the symbol Z.
- **Ohm's law**: Ohm's observation is that the direct current flowing in an electrical circuit consisting only of resistances is directly proportional to the voltage applied.

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22.3: Applications of Induction and EM Waves

learning objectives

- Discuss the use of inductance in modern devices

Inductors have many uses in modern electronics. In a sound system, sound can be transmitted from a microphone to a speaker (shown in). The microphone works by induction, as the vibrating membrane induces an emf in a coil. That "signal" is then transmitted to an amplifier and then to a speaker. The speaker is then driven by modulated electrical currents (produced by an amplifier) that pass through and magnetize (by inductance) a speaker coil of copper wire, creating a magnetic field. Thus, the electrical current variations that pass through the speaker are converted to varying magnetic forces, which move the speaker diaphragm, forcing the driver to produce air motion that is similar to the original signal from the amplifier.

image

A simple, modern speaker.: A speaker with magnet and coils that is used to produce sound.

Inductance in modern electronics is also used in computer memory. Magnetic storage uses different patterns of magnetization on a magnetically coated surface to store information. Differently magnetized areas on tape (or disk) induce signals on "read-write" heads, from which the information is later accessed.

Another application is the seismograph—an instrument for detecting and recording the intensity, direction and duration of ground movement. It contains a fixed coil and a magnet hung on a spring (or vice versa). These "record" the current induced when the Earth shakes.

A ground fault circuit interrupter (GFCI) provides additional safety (that circuit breakers cannot) by stopping the current in a shorted-out circuit. This is done by inductance. If a GFCI detects that there is a leakage of current, it produces an EMF and a current in the opposite direction of the original current.

Antennae

An antenna is a device that converts electric power into radio waves, and vice versa.

learning objectives

- Describe functions and uses of antennae

Maxwell's equations predicted that all light waves have the same structure, regardless of wavelength and frequency. As a consequence, visible light and radio waves should share common characteristics. Maxwell's 1865 prediction passed an important test in 1888, when Heinrich Hertz published the results of experiments in which he showed that radio waves could be manipulated in the same ways as visible light waves. To aid in his experiment, Hertz built the first antenna.

Car Antenna: A common car antenna that converts electric power in the air into electromagnetic waves.

An antenna (or aerial) is an electrical device that converts electric power into radio waves, and vice versa. Usually, it is used with a radio transmitter or radio receiver. In transmission, a radio transmitter supplies an oscillating radio frequency electric current to the antenna's terminals, and the antenna radiates the energy from the current as electromagnetic waves (radio waves). In reception, an antenna intercepts some of the power of an electromagnetic wave in order to produce a tiny voltage at its terminals. This voltage is applied to a receiver to be amplified.

Antennas are essential components of all types of equipment that utilize radio. These include: radio broadcasting, broadcast television, two-way radio, communications receivers, radar, cell phones, and satellite communications; as well as other devices such as garage door openers, wireless microphones, bluetooth enabled devices, wireless computer networks, baby monitors, and RFID tags on merchandise.

Antennas may also include reflective or directive elements or surfaces not connected to the transmitter or receiver, such as parasitic elements, parabolic reflectors, or horns. These serve to direct the radio waves into a beam or other desired radiation pattern. Antennas can be designed to transmit or receive radio waves in all directions equally (omnidirectional antennas), or transmit them in a beam in a particular direction and receive from that one direction only (directional or high gain antennas).

Key Points

- The microphone works by induction, as the vibrating membrane induces an emf in a coil.
- A speaker produces sound by induction as varying magnetic forces move a speaker diaphragm, producing air motion that produces sound.
- Computer memory is stored by inducing signals on read/write heads.
- GFCIs induce a current to oppose sudden increases in current in a circuit, thus preventing possible electrocution.
- The first antennas were built in 1888 by German physicist Heinrich Hertz.
- Antennas are essential components of all equipment that uses radio.
- Antennas transmit or receive radio waves in all directions equally, or transmit them in a beam in a particular direction.

Key Terms

- **amplifier**: An appliance or circuit that increases the strength of a weak electrical signal without changing the other characteristics of the signal.
- **wavelength**: The length of a single cycle of a wave, as measured by the distance between one peak or trough of a wave and the next; it is often designated in physics as λ, and corresponds to the velocity of the wave divided by its frequency.
- **frequency**: The quotient of the number of times n a periodic phenomenon occurs over the time t in which it occurs: $f = n / t$.
- **transmitter**: An electronic device that generates and amplifies a carrier wave, modulates it with a meaningful signal derived from speech, music, TV or other sources, and broadcasts the resulting signal from an antenna.

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22.4: Magnetic Fields and Maxwell Revisited

learning objectives

- Describe behavior of an inductor when the current is changed, and express energy stored in a magnetic field in a form of equation

When a conductor carries a current, a magnetic field surrounding the conductor is produced. The resulting magnetic flux is proportional to the current. If the current changes, the change in magnetic flux is proportional to the time-rate of change in current by a factor called inductance (L). Since nature abhors rapid change, a voltage (*electromotive force* , *EMF*) produced in the conductor opposes the change in current, which is also proportional to the change in magnetic flux. Thus, inductors oppose change in current by producing a voltage that,in turn, creates a current to oppose the change in magnetic flux; the voltage is proportional to the change in current.

Energy Stored in Inductor

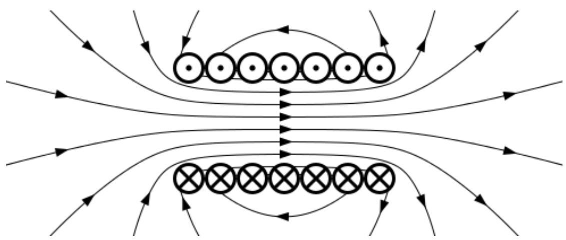
Due to energy conservation, the energy needed to drive the original current must have an outlet. For an inductor, that outlet is the magnetic field—the energy stored by an inductor is equal to the work needed to produce a current through the inductor. The formula for this energy is given as:

[Math Processing Error]

(Eq. 1), where L is the inductance in units of Henry and I is the current in units of Ampere.

Energy Stored in Magnetic Field

Let's consider Fig 1, an example of a solenoid (ℓ: length, N: number of turns, I: current, A: cross-section area) that works as an inductor. From Eq. 1, the energy stored in the magnetic field created by the solenoid is:



Magnetic Field Created By A Solenoid: Magnetic field created by a solenoid (cross-sectional view) described using field lines. Energy is "stored" in the magnetic field.

[Math Processing Error]

(We used the relation *[Math Processing Error]* and B=μNI/LB=μNI/L.)

Therefore, the energy density *[Math Processing Error]* of a magnetic field B is written as *[Math Processing Error]*

Maxwell's Predictions and Hertz' Confirmation

Maxwell's prediction of the electromagnetic force was confirmed by Hertz who generated and detected electromagnetic waves.

learning objectives

- Explain how Maxwell's prediction of the electromagnetic force was confirmed by Hertz

Maxwell's Predictions and Hertz' Confirmation

Combining the work of physicists including Oersted, Coulomb, Gauss, and Faraday, and adding his own insights, James Clerk Maxwell developed a complete and overarching theory showing electric and magnetic forces are not separate, but different forms of the same thing: the *electromagnetic* force. In 1865, he did this in the form of four equations that state the following:

1. Electric field lines originate on positive charges and terminate on negative charges, and the electric field is defined as the force per unit charge on a test charge. The strength of the force is related to the electric constant ε_0, also known as the permitivity of

free space.
2. Magnetic field lines are continuous, having no beginning or end. No magnetic monopoles are known to exist.
3. A changing magnetic field induces an electromotive force (emf) and, hence, an electric field. The direction of the emf opposes the change.
4. Magnetic fields are generated by moving charges or by changing electric fields.

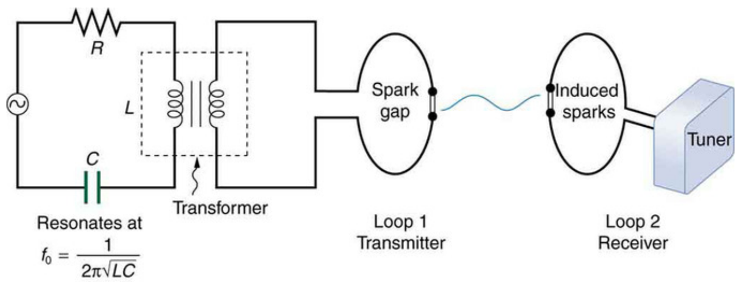
Maxwell's equations predict that regardless of wavelength and frequency, every light wave has the same structure. This means Maxwell's equations predicted that radio and x-ray waves existed, even though they hadn't actually been discovered yet.

Proving Maxwell's Equations

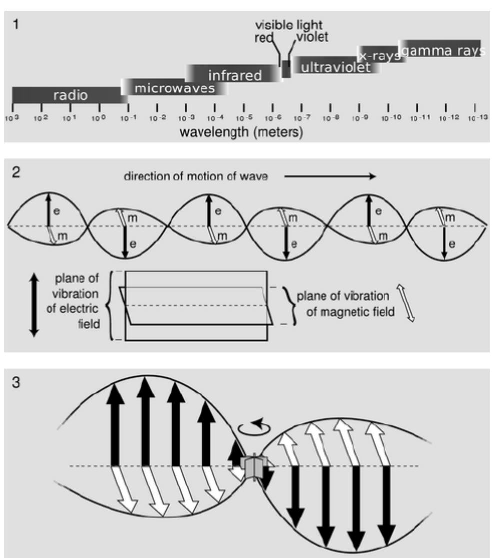
Simple and brilliant in their insight, Maxwell's famous equations would still be hard to prove. Since changing electric fields create relatively weak magnetic fields, they could not be easily detected at the time of Maxwell's hypothesis.

It was not until 1888 that Maxwell's prediction passed an important test when Heinrich Hertz generated and detected certain types of electromagnetic waves in the laboratory. He performed a series of experiments that not only confirmed the existence of electromagnetic waves, but also verified that they travel at the speed of light.

Hertz used an AC RLC (resistor-inductor-capacitor) circuit that resonates at a known frequency and connected it to a loop of wire as shown in. High voltages induced across the gap in the loop produced sparks that were visible evidence of the current in the circuit and that helped generate electromagnetic waves. Across the laboratory, Hertz had another loop attached to another RLC circuit, which could be tuned (as the dial on a radio) to the same resonant frequency as the first and could, thus, be made to receive electromagnetic waves. This loop also had a gap across which sparks were generated, giving solid evidence that electromagnetic waves had been received.



Apparatus Used by Hertz: The apparatus used by Hertz in 1887 to generate and detect electromagnetic waves. An RLC circuit connected to the first loop caused sparks across a gap in the wire loop and generated electromagnetic waves. Sparks across a gap in the second loop located across the laboratory gave evidence that the waves had been received.



EM Wave: The propogation of an electromagnetic wave as predicted by Maxwell and confirmed by Hertz.

Key Points

- The formula for the energy stored in a magnetic field is E = 1/2 LI^2.
- The energy stored in a magnetic field is equal to the work needed to produce a current through the inductor.
- Energy is stored in a magnetic field. Energy density can be written as *[Math Processing Error]*.
- Maxwell predicted that electric and magnetic forces are linked.
- Maxwell's equations predict that regardless of wavelength and frequency, every light wave has the same structure.
- Hertz was able to confirm Maxwell's equation experimentally by generating and detecting certain types of electromagnetic waves in the laboratory.

Key Terms

- **inductor**: A device or circuit component that exhibits significant self-inductance; a device which stores energy in a magnetic field.
- **electric field**: A region of space around a charged particle, or between two voltages; it exerts a force on charged objects in its vicinity.
- **magnetic field**: A condition in the space around a magnet or electric current in which there is a detectable magnetic force, and where two magnetic poles are present.
- **electromotive force**: (EMF)—The voltage generated by a battery or by the magnetic force according to Faraday's Law. It is measured in units of volts, not newtons, and thus, is not actually a force.

CHAPTER OVERVIEW

23: Electromagnetic Waves

Topic hierarchy

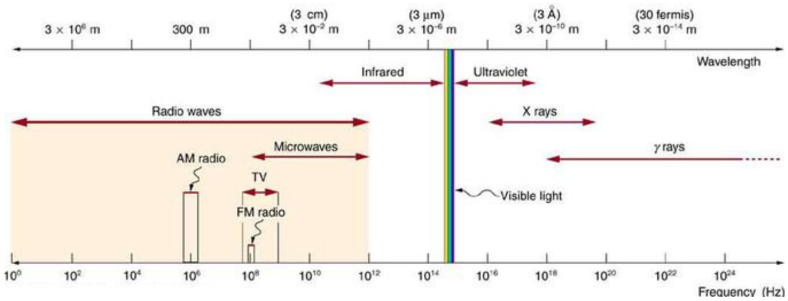
23.1: The Electromagnetic Spectrum

learning objectives

- Compare properties of AM and FM radio waves

Radio Waves

Radio waves are a type of electromagnetic (EM) radiation with wavelengths in the electromagnetic spectrum longer than infrared light. They have have frequencies from 300 GHz to as low as 3 kHz, and corresponding wavelengths from 1 millimeter to 100 kilometers. Like all other electromagnetic waves, radio waves travel at the speed of light. Naturally occurring radio waves are made by lightning or by astronomical objects. Artificially generated radio waves are used for fixed and mobile radio communication, broadcasting, radar and other navigation systems, communications satellites, computer networks and innumerable other applications. Different frequencies of radio waves have different propagation characteristics in the Earth's atmosphere—long waves may cover a part of the Earth very consistently, shorter waves can reflect off the ionosphere and travel around the world, and much shorter wavelengths bend or reflect very little and travel on a line of sight.



Electromagnetic Spectrum: The electromagnetic spectrum, showing the major categories of electromagnetic waves. The range of frequencies and wavelengths is remarkable. The dividing line between some categories is distinct, whereas other categories overlap. Microwaves encompass the high frequency portion of the radio section of the EM spectrum.

Types of Radio Waves and Applications

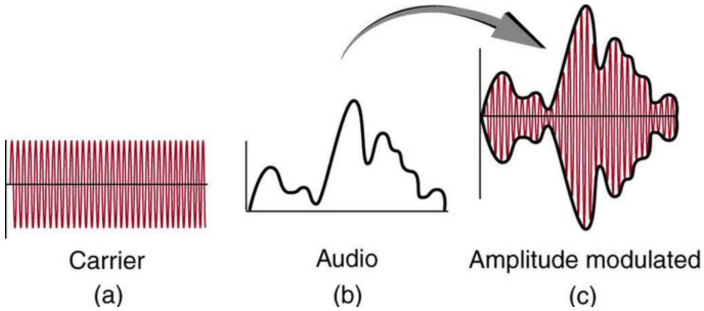
Radio waves have many uses—the category is divided into many subcategories, including microwaves and electromagnetic waves used for AM and FM radio, cellular telephones and TV.

The lowest commonly encountered radio frequencies are produced by high-voltage AC power transmission lines at frequencies of 50 or 60 Hz. These extremely long wavelength electromagnetic waves (about 6000 km) are one means of energy loss in long-distance power transmission.

Extremely low frequency (ELF) radio waves of about 1 kHz are used to communicate with submerged submarines. The ability of radio waves to penetrate salt water is related to their wavelength (much like ultrasound penetrating tissue)—the longer the wavelength, the farther they penetrate. Since salt water is a good conductor, radio waves are strongly absorbed by it; very long wavelengths are needed to reach a submarine under the surface.

AM Radio Waves

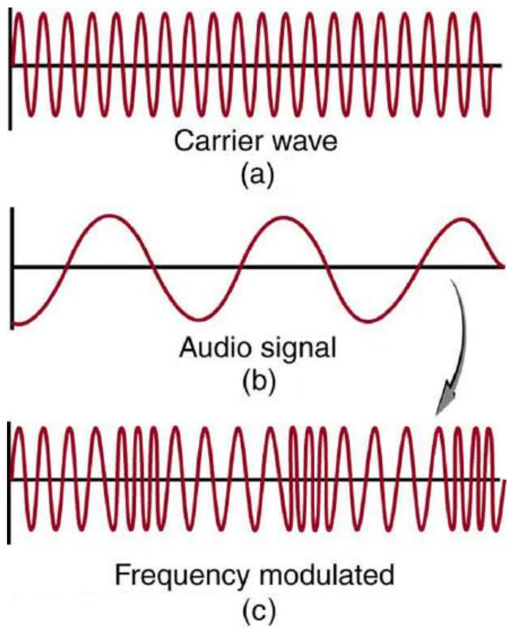
AM radio waves are used to carry commercial radio signals in the frequency range from 540 to 1600 kHz. The abbreviation AM stands for amplitude modulation—the method for placing information on these waves. A carrier wave having the basic frequency of the radio station (for instance, 1530 kHz) is varied or modulated in amplitude by an audio signal. The resulting wave has a constant frequency, but a varying amplitude.



AM Radio: Amplitude modulation for AM radio. (a) A carrier wave at the station's basic frequency. (b) An audio signal at much lower audible frequencies. (c) The amplitude of the carrier is modulated by the audio signal without changing its basic frequency.

FM Radio Waves

FM radio waves are also used for commercial radio transmission, but in the frequency range of 88 to 108 MHz. FM stands for frequency modulation, another method of carrying information. In this case, a carrier wave having the basic frequency of the radio station (perhaps 105.1 MHz) is modulated in frequency by the audio signal, producing a wave of constant amplitude but varying frequency.



FM Radio: Frequency modulation for FM radio. (a) A carrier wave at the station's basic frequency. (b) An audio signal at much lower audible frequencies. (c) The frequency of the carrier is modulated by the audio signal without changing its amplitude.

Since audible frequencies range up to 20 kHz (or 0.020 MHz) at most, the frequency of the FM radio wave can vary from the carrier by as much as 0.020 MHz. For this reason, the carrier frequencies of two different radio stations cannot be closer than 0.020 MHz. An FM receiver is tuned to resonate at the carrier frequency and has circuitry that responds to variations in frequency, reproducing the audio information.

FM radio is inherently less subject to noise from stray radio sources than AM radio because amplitudes of waves add noise. Thus, an AM receiver would interpret noise added onto the amplitude of its carrier wave as part of the information. An FM receiver can be fashioned to reject amplitudes other than that of the basic carrier wave and only look for variations in frequency. Thus, since noise produces a variation in amplitude, it is easier to reject noise from FM.

TV

Electromagnetic waves also broadcast television transmission. However, as the waves must carry a great deal of visual as well as audio information, each channel requires a larger range of frequencies than simple radio transmission. TV channels utilize frequencies in the range of 54 to 88 MHz and 174 to 222 MHz (the entire FM radio band lies between channels 88 MHz and 174

MHz). These TV channels are called VHF (very high frequency). Other channels called UHF (ultra high frequency) utilize an even higher frequency range of 470 to 1000 MHz.

The TV video signal is AM, while the TV audio is FM. Note that these frequencies are those of free transmission with the user utilizing an old-fashioned roof antenna. Satellite dishes and cable transmission of TV occurs at significantly higher frequencies, and is rapidly evolving with the use of the high-definition or HD format.

Microwaves

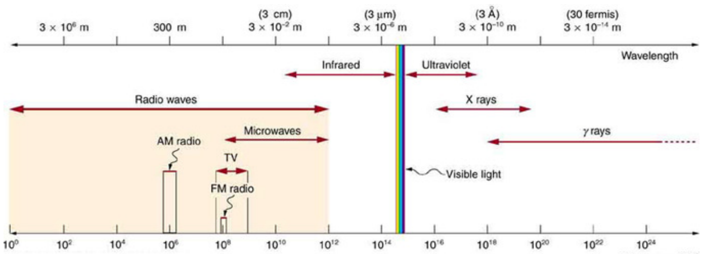
Microwaves are electromagnetic waves with wavelengths ranging from one meter to one millimeter (frequencies between 300 MHz and 300 GHz).

Learning objectives

- Distinguish three ranges of the microwave portion of the electromagnetic spectrum

Microwaves

Microwaves are electromagnetic waves with wavelengths ranging from as long as one meter to as short as one millimeter, or equivalently with frequencies between 300 MHz (0.3 GHz) and 300 GHz. The microwave region of the electromagnetic (EM) spectrum is generally considered to overlap with the highest frequency (shortest wavelength) radio waves. As is the case for all EM waves, microwaves travel in a vacuum at the speed of light. The prefix “micro-” in “microwave” is not meant to suggest a wavelength in the micrometer range. It indicates that microwaves are “small” because have shorter wavelengths as compared to waves used in typical radio broadcasting. The boundaries between far infrared light, terahertz radiation, microwaves, and ultra-high-frequency radio waves are fairly arbitrary. They are used variously between different fields of study (see figure).



Electromagnetic Spectrum: The electromagnetic spectrum, showing the major categories of electromagnetic waves. The range of frequencies and wavelengths is remarkable. The dividing line between some categories is distinct, whereas other categories overlap. Microwaves overlap with the high frequency portion of the radio section of the EM spectrum.

Subcategories of Microwaves

The microwave portion of the radio spectrum can be subdivided into three ranges, listed below from high to low frequencies.

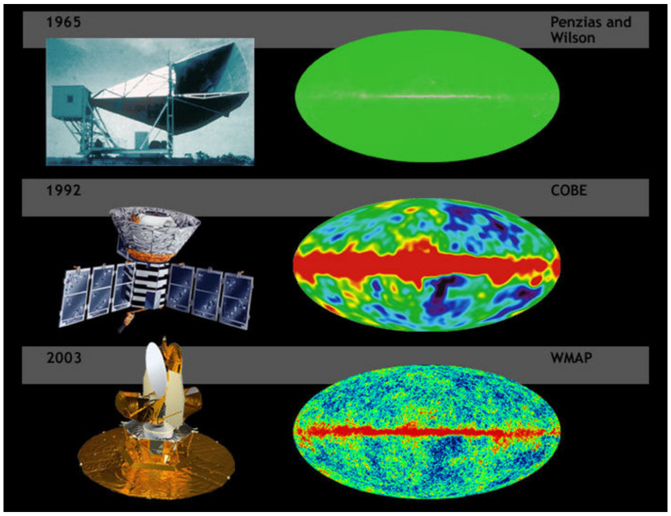
- Extremely high frequency (EHF) is the highest microwave frequency band. EHF runs the range of frequencies from 30 to 300 gigahertz, above which electromagnetic radiation is considered as far infrared light, also referred to as terahertz radiation. This frequency range corresponds to a wavelength range of 10 to 1 millimeter, so it is sometimes called the millimeter band. This band is commonly used in radio astronomy and remote sensing.
- Super high frequency (SHF) is the designation for electromagnetic wave frequencies in the range of 3 GHz to 30 GHz. This band of frequencies is known also as the centimeter band because the wavelengths range from ten to one centimeters. This frequency range is used for most radar transmitters, microwave ovens, wireless LANs, cell phones, satellite communication, microwave radio relay links, and numerous short range terrestrial data links.
- Ultra-high frequency (UHF) designates the microwave frequency range of electromagnetic waves between 300 MHz and 3 GHz, also known as the decimeter band because the wavelengths range from one to ten decimeters, or 10 centimeters to 1 meter. They are used for television broadcasting, cordless phones, walkie-talkies, satellite communication, and numerous other applications.

Sources of Microwaves

Microwaves are the highest-frequency electromagnetic waves that can be produced by currents in macroscopic circuits and devices. Microwaves can also be produced by atoms and molecules—e.g., they are a component of electromagnetic radiation generated by *thermal agitation*. The thermal motion of atoms and molecules in any object at a temperature above absolute zero causes them to emit and absorb radiation.

Since it is possible to carry more information per unit time on high frequencies, microwaves are quite suitable for communications devices. Most satellite-transmitted information is carried on microwaves, as are land-based long-distance transmissions. A clear line of sight between transmitter and receiver is needed because of the short wavelengths involved.

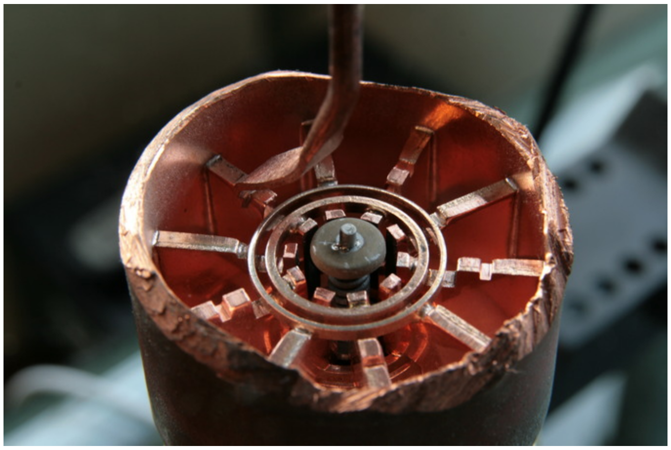
The sun also emits microwave radiation, although most of it is blocked by Earth's atmosphere. The Cosmic Microwave Background Radiation (CMBR) is microwave radiation that permeates all of space, and its discovery supports the Big Bang theory of the origin of the universe.



Cosmic Microwave Background: Cosmic background radiation of the Big Bang mapped with increasing resolution.

Devices Employing Microwaves

High-power microwave sources use specialized vacuum tubes to generate microwaves. These devices operate on different principles from low-frequency vacuum tubes, using the ballistic motion of electrons in a vacuum under the influence of controlling electric or magnetic fields, and include the magnetron (used in microwave ovens), klystron, traveling-wave tube (TWT), and gyrotron.



Cavity Magnetron: Cutaway view inside a cavity magnetron as used in a microwave oven.

Microwaves are used by microwave ovens to heat food. Microwaves at a frequency of 2.45 GHz are produced by accelerating electrons. The microwaves then induce an alternating electric field in the oven. Water and some other constituents of food have a slightly negative charge at one end and a slightly positive charge at one end (called polar molecules). The range of microwave frequencies is specially selected so that the polar molecules, in trying to maintain their orientation with the electric field, absorb these energies and increase their temperatures—a process called *dielectric heating*.

Radar, first developed in World War II, is a common application of microwaves. By detecting and timing microwave echoes, radar systems can determine the distance to objects as diverse as clouds and aircraft. A Doppler shift in the radar echo can determine the speed of a car or the intensity of a rainstorm. Sophisticated radar systems can map the Earth and other planets, with a resolution limited by wavelength. The shorter the wavelength of any probe, the smaller the detail it is possible to observe.

A *maser* is a device similar to a laser, which amplifies light energy by stimulating photons. The maser, rather than amplifying visible light energy, amplifies the lower-frequency, longer-wavelength microwaves and radio frequency emissions.

Infrared Waves

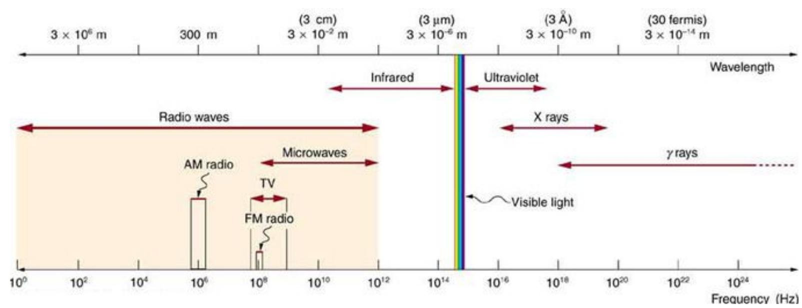
Infrared (IR) light is EM radiation with wavelengths longer than those of visible light from 0.74 µm to 1 mm (300 GHz to 1 THz).

Skills to Develop

- Distinguish three ranges of the infrared portion of the spectrum, and describe processes of absorption and emission of infrared light by molecules

Infrared Waves

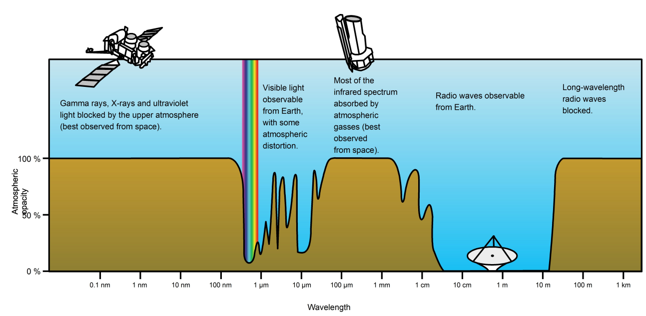
Infrared (IR) light is electromagnetic radiation with longer wavelengths than those of visible light, extending from the nominal red edge of the visible spectrum at 0.74 micrometers (µm) to 1 mm. This range of wavelengths corresponds to a frequency range of approximately 300 GHz to 400 THz, and includes most of the thermal radiation emitted by objects near room temperature. Infrared light is emitted or absorbed by molecules when they change their rotational-vibrational movements.



Electromagnetic Spectrum: The electromagnetic spectrum, showing the major categories of electromagnetic waves. The range of frequencies and wavelengths is remarkable. The dividing line between some categories is distinct, whereas other categories overlap. Microwaves encompass the high frequency portion of the radio section of the EM spectrum.

Subcategories of IR Waves

The infrared part of the electromagnetic spectrum covers the range from roughly 300 GHz (1 mm) to 400 THz (750 nm). It can be divided into three parts: It can be divided into three parts:



Atmospheric Transmittance: This is a plot of Earth's atmospheric transmittance (or opacity) to various wavelengths of electromagnetic radiation. Most UV wavelengths are absorbed by oxygen and ozone in Earth's atmosphere. Observations of astronomical UV sources must be done from space.

- Far-infrared, from 300 GHz (1 mm) to 30 THz (10 μm) – The lower part of this range may also be called microwaves. This radiation is typically absorbed by so-called rotational modes in gas-phase molecules, by molecular motions in liquids, and by phonons in solids. The water in Earth's atmosphere absorbs so strongly in this range that it renders the atmosphere in effect opaque. However, there are certain wavelength ranges ("windows") within the opaque range that allow partial transmission, and can be used for astronomy. The wavelength range from approximately 200 μm up to a few mm is often referred to as "sub-millimeter" in astronomy, reserving far infrared for wavelengths below 200 μm.
- Mid-infrared, from 30 to 120 THz (10 to 2.5 μm) – Hot objects (black-body radiators) can radiate strongly in this range, and human skin at normal body temperature radiates strongly at the lower end of this region. This radiation is absorbed by molecular vibrations, where the different atoms in a molecule vibrate around their equilibrium positions. This range is sometimes called the fingerprint region, since the mid-infrared absorption spectrum of a compound is very specific for that compound.
- Near-infrared, from 120 to 400 THz (2,500 to 750 nm) – Physical processes that are relevant for this range are similar to those for visible light. The highest frequences in this region can be detected directly by some types of photographic film, and by many types of solid state image sensors for infrared photography and videography.

Note that in some fields the boundaries of these categories differ slightly; for example, in astronomy "near-infrared" is considered to extend to 5 μm rather than 2.5 μm.

Heat and Thermal Radiation

Infrared radiation is popularly known as "heat radiation," but light and electromagnetic waves of any frequency will heat surfaces that absorb them. Infrared light from the Sun only accounts for 49% of the heating of the Earth, with the rest being caused by visible light that is absorbed then re-radiated at longer wavelengths. Visible light or ultraviolet-emitting lasers can char paper and incandescently hot objects emit visible radiation. Objects at room temperature will emit radiation mostly concentrated in the 8 to 25 μm band, but this is not distinct from the emission of visible light by incandescent objects and ultraviolet by even hotter objects (see sections on black body radiation and Wien's displacement law).

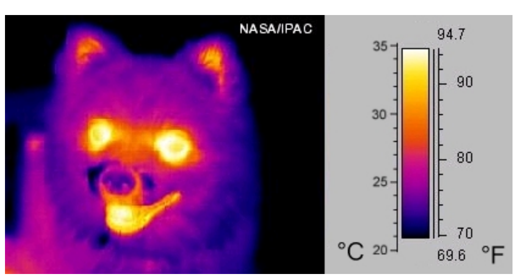
Heat is energy in transient form that flows due to temperature difference. Unlike heat transmitted by thermal conduction or thermal convection, radiation can propagate through a vacuum.

The concept of emissivity is important in understanding the infrared emissions of objects. This is a property of a surface which describes how its thermal emissions deviate from the ideal of a black body. To further explain, two objects at the same physical temperature will not "appear" the same temperature in an infrared image if they have differing emissivities.

Sources of IR Waves

As stated above, while infrared radiation is commonly referred to as heat radiation, only objects emitting with a certain range of temperatures and emissivities will produce most of their electromagnetic emission in the infrared part of the spectrum. However, this is the case for most objects and environments humans encounter in our daily lives. Humans, their surroundings, and the Earth itself emit most of their thermal radiation at wavelengths near 10 microns, the boundary between mid and far infrared according to the delineation above. The range of wavelengths most relevant to thermally emitting objects on earth is often called the thermal infrared. Many astronomical objects emit detectable amounts of IR radiation at non-thermal wavelengths.

Infrared radiation can be used to remotely determine the temperature of objects (if the emissivity is known). This is termed thermography, mainly used in military and industrial applications but the technology is reaching the public market in the form of infrared cameras on cars due to the massively reduced production costs.



Thermography: A thermographic image of a dog

Applications of IR waves extend to heating, communication, meteorology, spectroscopy, astronomy, biological and medical science, and even the analysis of works of art.

Visible Light

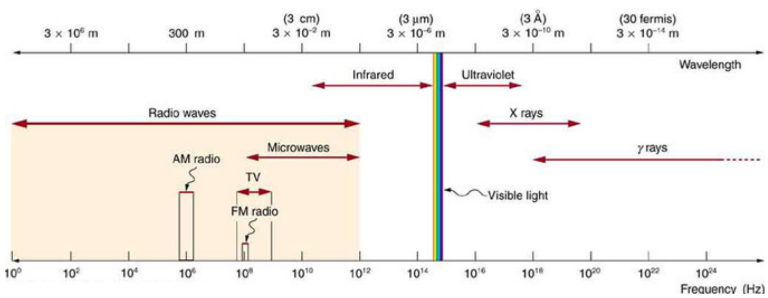
Visible light is the portion of the electromagnetic spectrum that is visible to the human eye, ranging from roughly 390 to 750 nm.

Skills to Develop

- Distinguish six ranges of the visible spectrum

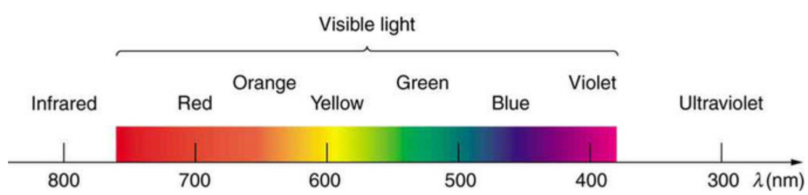
Visible Light

Visible light, as called the visible spectrum, is the portion of the electromagnetic spectrum that is visible to (can be detected by) the human eye. Electromagnetic radiation in this range of wavelengths is often simply referred to as “light”. A typical human eye will respond to wavelengths from about 390 to 750 nm (0.39 to 0.75 µm). In terms of frequency, this corresponds to a band in the vicinity of 400–790 THz. A light-adapted eye generally has its maximum sensitivity at around 555 nm (540 THz), in the green region of the optical spectrum. The spectrum does not, however, contain all the colors that the human eyes and brain can distinguish. Unsaturated colors such as pink, or purple variations such as magenta, are absent, for example, because they can be made only by a mix of multiple wavelengths.



Electromagnetic Spectrum: The electromagnetic spectrum, showing the major categories of electromagnetic waves. The range of frequencies and wavelengths is remarkable. The dividing line between some categories is distinct, whereas other categories overlap. Microwaves encompass the high frequency portion of the radio section of the EM spectrum.

Visible light is produced by vibrations and rotations of atoms and molecules, as well as by electronic transitions within atoms and molecules. The receivers or detectors of light largely utilize electronic transitions. We say the atoms and molecules are excited when they absorb and relax when they emit through electronic transitions.



Visible Spectrum: A small part of the electromagnetic spectrum that includes its visible components. The divisions between infrared, visible, and ultraviolet are not perfectly distinct, nor are those between the seven rainbow colors.

The figure above shows this part of the spectrum, together with the colors associated with particular pure wavelengths. Red light has the lowest frequencies and longest wavelengths, while violet has the highest frequencies and shortest wavelengths. Blackbody radiation from the Sun peaks in the visible part of the spectrum but is more intense in the red than in the violet, making the Sun yellowish in appearance.

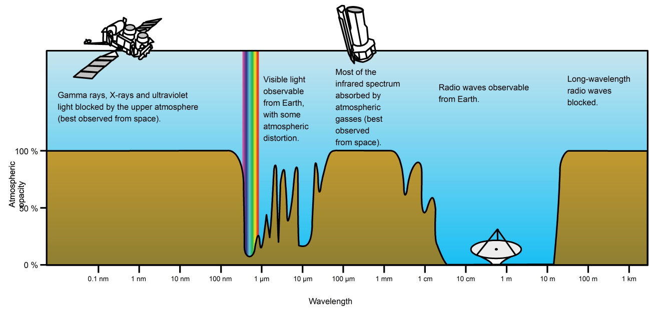
Colors that can be produced by visible light of a narrow band of wavelengths (monochromaticlight) are called pure spectral colors. Quantitatively, the regions of the visible spectrum encompassing each spectral color can be delineated roughly as:

- red – 620 to 750 nm (400-484 THz)

Note that each color can come in many shades, since the spectrum is continuous. The human eye is insensitive to electromagnetic radiation outside this range. By definition any images presented with data recorded from wavelengths other than those in the visible part of the spectrum (such as IR images of humans or animals or astronomical X-ray images) are necessarily in false color.

Visible Light and Earth's Atmosphere

Visible wavelengths pass through the "optical window", the region of the electromagnetic spectrum which allows wavelengths to pass largely unattenuated through the Earth's atmosphere (see opacity plot in. An example of this phenomenon is that clean air scatters blue light more than red wavelengths, and so the midday sky appears blue.



Atmospheric Transmittance: This is a plot of Earth's atmospheric transmittance (or opacity) to various wavelengths of electromagnetic radiation. Most UV wavelengths are absorbed by oxygen and ozone in Earth's atmosphere. Observations of astronomical UV sources must be done from space.

The optical window is also called the visible window because it overlaps the human visible response spectrum. This is not coincidental as humanity's ancestors evolved vision that could make use of the most plentiful wavelengths of light.The near infrared (NIR) window lies just out of the human vision, as well as the Medium Wavelength IR (MWIR) window and the Long Wavelength or Far Infrared (LWIR or FIR) window though other animals may experience them.

A consequence of the existence of the optical window in Earth's atmosphere is the relatively balmy temperature conditions on Earth's surface. The Sun's luminosity function peaks in the visible range and light in that range is able to travel to the surface of the planet unattenuated due to the optical window. This allows visible light to heat the surface. The surface of the planet then emits energy primarily in infrared wavelengths, which has much greater difficulty escaping (and thus causing the planet to cool) due to the opacity of the atmosphere in the infrared. Earth's surface would be much cooler without this effect.

Photosynthesis

Plants, like animals, have evolved to utilize and respond to parts of the electromagnetic spectrum they are embedded in. Plants (and many bacteria) convert the light energy captured from the Sun into chemical energy that can be used to fuel the organism's activities. In plants, algae, and cyanobacteria, photosynthesis uses carbon dioxide and water, releasing oxygen as a waste product. Photosynthesis is vital for all aerobic life on Earth (such as humans and animals). The portion of the EM spectrum used by photosynthetic organisms is called the photosynthetically active region (PAR) and corresponds to solar radiation between 400 and 700 nm, substantially overlapping with the range of human vision. This is again not coincidental; the light in this range is the most plentiful to organisms on the surface of Earth because the Sun emits about half of its luminosity in this wavelength range and it is allowed to pass freely through the optical windows in Earth's atmosphere.

Ultraviolet Light

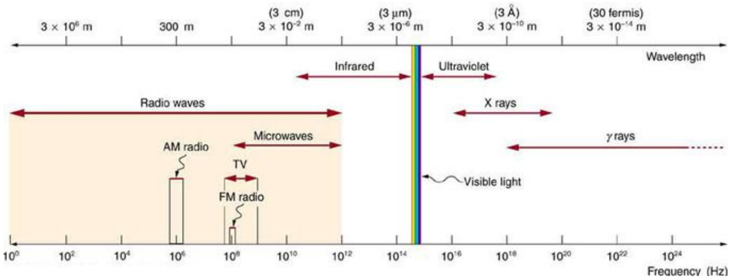
Ultraviolet (UV) light is electromagnetic radiation with a wavelength shorter than that of visible light in the range 10 nm to 400 nm.

Skills to Develop

- Identify wavelength range characteristic for ultraviolet light and its biological effects

Ultraviolet Light

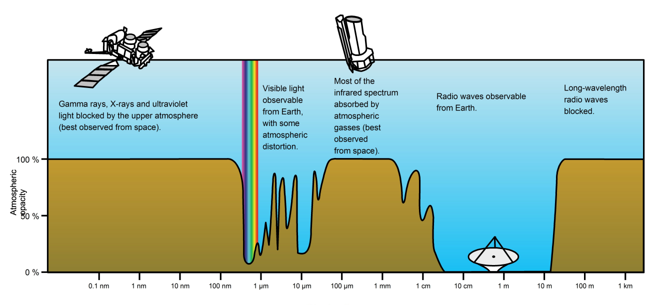
Ultraviolet (UV) light is electromagnetic radiation with a wavelength shorter than that of visible light, but longer than X-rays, that is, in the range 10 nm to 400 nm, corresponding to photon energies from 3 eV to 124 eV (1 eV = $1.6e^{-19}$ J; EM radiation with frequencies higher than those of visible light are often expressed in terms of energy rather than frequency). It is so-named because the spectrum consists of electromagnetic waves with frequencies higher than those that humans identify as the color violet. These frequencies are invisible to humans, but visible to a number of insects and birds.



Electromagnetic Spectrum: The electromagnetic spectrum, showing the major categories of electromagnetic waves. The range of frequencies and wavelengths is remarkable. The dividing line between some categories is distinct, whereas other categories overlap. Microwaves encompass the high frequency portion of the radio section of the EM spectrum.

UV light is found in sunlight (where it constitutes about 10% of the energy in vacuum) and is emitted by electric arcs and specialized lights such as black lights. It can cause chemical reactions, and causes many substances to glow or fluoresce. Most ultraviolet is classified as non-ionizing radiation. The higher energies of the ultraviolet spectrum from wavelengths about 10 nm to 120 nm ('extreme' ultraviolet) are ionizing, but this type of ultraviolet in sunlight is blocked by normal molecular oxygen (O_2) in air, and does not reach the ground. However, the entire spectrum of ultraviolet radiation has some of the biological features of ionizing radiation, in doing far more damage to many molecules in biological systems than is accounted for by simple heating effects (an example is sunburn). These properties derive from the ultraviolet photon's power to alter chemical bonds in molecules, even without having enough energy to ionize atoms.

Although ultraviolet radiation is invisible to the human eye, most people are aware of the effects of UV on the skin, called suntan and sunburn. In addition to short wave UV blocked by oxygen, a great deal (>97%) of mid-range ultraviolet (almost all UV above 280 nm and most up to 315 nm) is blocked by the ozone layer, and like ionizing short wave UV, would cause much damage to living organisms if it penetrated the atmosphere. After atmospheric filtering, only about 3% of the total energy of sunlight at the zenith is ultraviolet, and this fraction decreases at other sun angles. Much of it is near-ultraviolet that does not cause sunburn, but is still capable of causing long term skin damage and cancer. An even smaller fraction of ultraviolet that reaches the ground is responsible for sunburn and also the formation of vitamin D (peak production occurring between 295 and 297 nm) in all organisms that make this vitamin (including humans). The UV spectrum thus has many effects, both beneficial and damaging, to human health.



Atmospheric Transmittance: This is a plot of Earth's atmospheric opacity (opposite of transmittance) to various wavelengths of electromagnetic radiation, including visible light. Visible light passes relatively unimpeded through the atmosphere in the "optical window." Most UV wavelengths are absorbed by oxygen and ozone in Earth's atmosphere. Observations of astronomical UV sources must be done from space.

Subcategories of UV Light

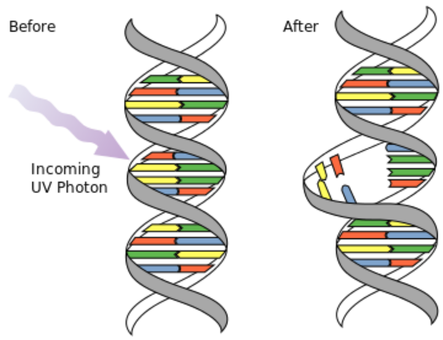
Solar UV radiation is commonly subdivided into three regions: UV-A (320–400 nm), UV-B (290–320 nm), and UV-C (220–290 nm), ranked from long to shorter wavelengths (from smaller to larger energies). Most UV-B and all UV-C is absorbed by ozone

(O_3) molecules in the upper atmosphere. Consequently, 99% of the solar UV radiation reaching the Earth's surface is UV-A.

There are other schemes for dividing UV into different categories, another common one is: near-ultraviolet (NUV – 300-400 nm), middle ultraviolet (MUV – 200-300 nm), far ultraviolet (FUV – 200-122 nm), and extreme ultraviolet (EUV- 121-10 nm).

Harmful Effects

An overexposure to UVB radiation can cause sunburn and some forms of skin cancer. In humans, prolonged exposure to solar UV radiation may result in acute and chronic health effects on the skin, eye, and immune system. Moreover, UVC can cause adverse effects that can variously be mutagenic or carcinogenic.



DNA UV Mutation: Ultraviolet photons harm the DNA molecules of living organisms in different ways. In one common damage event, adjacent thymine bases bond with each other, instead of across the "ladder. " This "thymine dimer" makes a bulge, and the distorted DNA molecule does not function properly.

The International Agency for Research on Cancer of the World Health Organization has classified all categories and wavelengths of ultraviolet radiation as a Group 1 carcinogen. This is the highest level designation for carcinogens and means that "there is enough evidence to conclude that it can cause cancer in humans. "

Beneficial Effects

UVB exposure induces the production of vitamin D in the skin. The majority of positive health effects are related to this vitamin. It has regulatory roles in calcium metabolism (which is vital for normal functioning of the nervous system, as well as for bone growth and maintenance of bone density), immunity, cell proliferation, insulin secretion, and blood pressure.

X-Rays

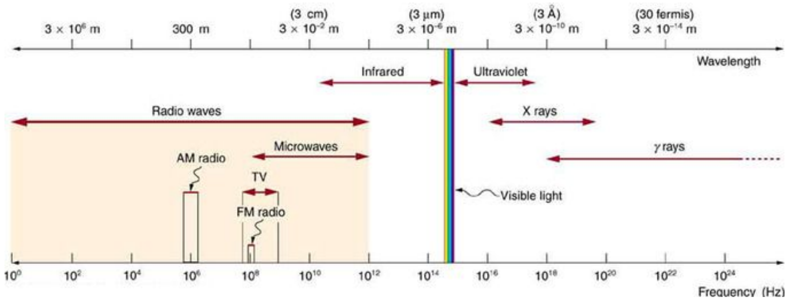
X-rays are electromagnetic waves with wavelengths in the range of 0.01 to 10 nanometers and energies in the range of 100 eV to 100 keV.

Skills to Develop

- Distinguish two categories of X-rays and their biological effects

X-Rays

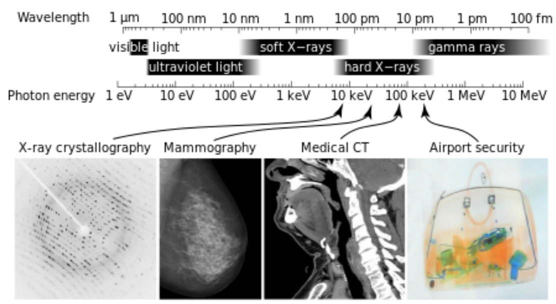
X-rays are electromagnetic waves with wavelengths in the range of 0.01 to 10 nanometers, corresponding to frequencies in the range 30 petahertz to 30 exahertz (3×10^{16} Hz to 3×10^{19} Hz) and energies in the range 100 eV to 100 keV. They are shorter in wavelength than UV rays and longer than gamma rays. In many languages, X-radiation is called Röntgen radiation, after Wilhelm Röntgen, who is usually credited as its discoverer, and who had named it X-radiation to signify an unknown type of radiation.



Electromagnetic Spectrum: The electromagnetic spectrum, showing the major categories of electromagnetic waves. The range of frequencies and wavelengths is remarkable. The dividing line between some categories is distinct, whereas other categories overlap. Microwaves encompass the high frequency portion of the radio section of the EM spectrum.

Properties and Applications

X-ray photons carry enough energy to ionize atoms and disrupt molecular bonds. This makes it a type of ionizing radiation and thereby harmful to living tissue. A very high radiation dose over a short amount of time causes radiation sickness, while lower doses can give an increased risk of radiation-induced cancer. In medical imaging this increased cancer risk is generally greatly outweighed by the benefits of the examination. The ionizing capability of X-rays can be utilized in cancer treatment to kill malignant cells using radiation therapy. It is also used for material characterization using X-ray spectroscopy.



X-Ray Spectrum and Applications: X-rays are part of the electromagnetic spectrum, with wavelengths shorter than those of visible light. Different applications use different parts of the X-ray spectrum.

X-rays with photon energies above 5 to 10 keV (below 0.2-0.1 nm wavelength), are called hard X-rays, while those with lower energy are called soft X-rays. Due to their penetrating ability, hard X-rays are widely used to image the inside of objects (e.g., in medical radiography and airport security). As a result, the term X-ray is metonymically used to refer to a radiographic image produced using this method, in addition to the method itself. Since the wavelength of hard X-rays are similar to the size of atoms, they are also useful for determining crystal structures by X-ray crystallography. By contrast, soft X-rays are easily absorbed in air and the attenuation length of 600 eV (~2 nm) X-rays in water is less than 1 micrometer.

In medical diagnostic applications, the low energy (soft) X-rays are unwanted, since they are totally absorbed by the body, increasing the radiation dose without contributing to the image. Hence, a thin metal sheet, often of aluminum, called an X-ray filter, is usually placed over the window of the X-ray tube, absorbing the low energy part in the spectrum. This is called hardening the beam since it shifts the center of the spectrum towards higher energy (or harder) X-rays.

Distinction Between X-Rays and Gamma Rays

The distinction between X-rays and gamma rays is somewhat arbitrary. The most frequent method of distinguishing between X- and gamma radiation is the basis of wavelength, with radiation shorter than some arbitrary wavelength, such as 10^{-11} m, defined as gamma rays. The electromagnetic radiation emitted by X-ray tubes generally has a longer wavelength than the radiation emitted by radioactive nuclei. Historically, therefore, an alternative means of distinguishing between the two types of radiation has been by their origin: X-rays are emitted by electrons outside the nucleus, while gamma rays are emitted by the nucleus. There is overlap between the wavelength bands of photons emitted by electrons outside the nucleus, and photons emitted by the nucleus. Like all electromagnetic radiation, the properties of X-rays (or gamma rays) depend only on their wavelength and polarization.

Gamma Rays

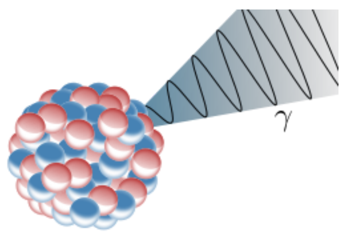
Gamma rays are very high frequency electromagnetic waves usually emitted from radioactive decay with frequencies greater than 10^{19}Hz.

Skills to Develop

- Identify wavelength range characteristic for gamma rays, noting their biological effects and distinguishing them from gamma rays

Gamma Rays

Gamma radiation, also known as gamma rays or hyphenated as gamma-rays and denoted as γ, is electromagnetic radiation of high frequency and therefore high energy. Gamma rays typically have frequencies above 10 exahertz (or $>10^{19}$ Hz), and therefore have energies above 100 keV and wavelengths less than 10 picometers (less than the diameter of an atom). However, this is not a hard and fast definition, but rather only a rule-of-thumb description for natural processes. Gamma rays from radioactive decay are defined as gamma rays no matter what their energy, so that there is no lower limit to gamma energy derived from radioactive decay. Gamma decay commonly produces energies of a few hundred keV, and almost always less than 10 MeV.



Gamma Decay: Illustration of an emission of a gamma ray (γ) from an atomic nucleus

Gamma rays are ionizing radiation and are thus biologically hazardous. They are classically produced by the decay from high energy states of atomic nuclei, a process called gamma decay, but are also created by other processes. Paul Villard, a French chemist and physicist, discovered gamma radiation in 1900, while studying radiation emitted from radium during its gamma decay. Villard's radiation was named "gamma rays" by Ernest Rutherford in 1903.

Gamma Ray Sources

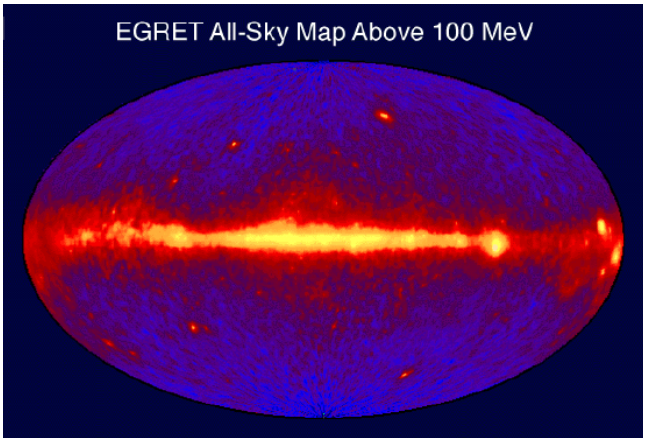
Natural sources of gamma rays on Earth include gamma decay from naturally occurring radioisotopes such as potassium-40, and also as a secondary radiation from various atmospheric interactions with cosmic ray particles. Some rare terrestrial natural sources that produce gamma rays that are not of a nuclear origin, are lightning strikes and terrestrial gamma-ray flashes, which produce high energy emissions from natural high-energy voltages. Gamma rays are produced by a number of astronomical processes in which very high-energy electrons are produced. Such electrons produce secondary gamma rays by the mechanisms of bremsstrahlung, inverse Compton scattering and synchrotron radiation. A large fraction of such astronomical gamma rays are screened by Earth's atmosphere and must be detected by spacecraft. Notable artificial sources of gamma rays include fission such as occurs in nuclear reactors, and high energy physics experiments, such as neutral pion decay and nuclear fusion.

Gamma Rays vs. X-Rays

Gamma rays have characteristics identical to X-rays of the same frequency—they differ only in source. At higher frequencies, γ rays are more penetrating and more damaging to living tissue. They have many of the same uses as X-rays, including cancer therapy. Gamma radiation from radioactive materials is used in nuclear medicine.

The distinction between X-rays and gamma rays has changed in recent decades. Originally, the electromagnetic radiation emitted by X-ray tubes almost invariably had a longer wavelength than the radiation (gamma rays) emitted by radioactive nuclei. Older literature distinguished between X- and gamma radiation on the basis of wavelength, with radiation shorter than some arbitrary wavelength, such as 10^{-11} m, defined as gamma rays. However, with artificial sources now able to duplicate any electromagnetic radiation that originates in the nucleus, as well as far higher energies, the wavelengths characteristic of radioactive gamma ray sources vs. other types, now completely overlap. Thus, gamma rays are now usually distinguished by their origin: X-rays are emitted by definition by electrons outside the nucleus, while gamma rays are emitted by the nucleus.

Exceptions to this convention occur in astronomy, where gamma decay is seen in the afterglow of certain supernovas, but other high energy processes known to involve other than radioactive decay are still classed as sources of gamma radiation. A notable example is extremely powerful bursts of high-energy radiation normally referred to as long duration gamma-ray bursts, which produce gamma rays by a mechanism not compatible with radioactive decay. These bursts of gamma rays, thought to be due to the collapse of stars called hypernovas, are the most powerful events so far discovered in the cosmos. Astrophysical processes are the only sources for very high energy gamma rays (~100 MeV).



Gamma Ray Sky Map: This is an image of the entire sky in 100 MeV or greater gamma rays as seen by the EGRET instrument aboard the CGRO spacecraft. Bright spots within the galactic plane are pulsars (spinning neutron stars with strong magnetic fields), while those above and below the plane are thought to be quasars (galaxies with supermassive black holes actively accreting matter).

Health Effects

All ionizing radiation causes similar damage at a cellular level, but because rays of alpha particles and beta particles are relatively non-penetrating, external exposure to them causes only localized damage (e.g., radiation burns to the skin). Gamma rays and neutrons are more penetrating, causing diffuse damage throughout the body (e.g., radiation sickness, cell's DNA damage, cell death due to damaged DNA, increasing incidence of cancer) rather than burns. External radiation exposure should also be distinguished from internal exposure, due to ingested or inhaled radioactive substances, which, depending on the substance's chemical nature, can produce both diffuse and localized internal damage. The most biological damaging forms of gamma radiation occur at energies between 3 and 10 MeV.

Key Points

- The lowest frequency portion of the electromagnetic spectrum is designated as "radio," generally considered to have wavelengths within 1 millimeter to 100 kilometers or frequencies within 300 GHz to 3 kHz.
- There is a wide range of subcategories contained within radio including AM and FM radio. Radio waves can be generated by natural sources such as lightning or astronomical phenomena; or by artificial sources such as broadcast radio towers, cell phones, satellites and radar.
- AM radio waves are used to carry commercial radio signals in the frequency range from 540 to 1600 kHz. The abbreviation AM stands for amplitude modulation—the method for placing information on these waves. AM waves have constant frequency, but a varying amplitude.
- FM radio waves are also used for commercial radio transmission in the frequency range of 88 to 108 MHz. FM stands for frequency modulation, which produces a wave of constant amplitude but varying frequency.
- The microwave region of the electromagnetic (EM) spectrum is generally considered to overlap with the highest frequency (shortest wavelength) radio waves.
- The prefix "micro-" in "microwave" is not meant to suggest a wavelength in the micrometer range. It indicates that microwaves are "small" compared to waves used in typical radio broadcasting in that they have shorter wavelengths.
- The microwave portion of the electromagnetic spectrum can be subdivided into three ranges listed below from high to low frequencies: extremely high frequency (30 to 300 GHz), super high frequency (3 to 30 GHz), and ultra-high frequency (300 MHz to 3 GHz).
- Microwave sources include artificial devices such as circuits, transmission towers, radar, masers, and microwave ovens, as well as natural sources such as the Sun and the Cosmic Microwave Background.

- Microwaves can also be produced by atoms and molecules. They are, for example, a component of electromagnetic radiation generated by thermal agitation. The thermal motion of atoms and molecules in any object at a temperature above absolute zero causes them to emit and absorb radiation.
- Infrared light includes most of the thermal radiation emitted by objects near room temperature. Infrared light is emitted or absorbed by molecules when they change their rotational-vibrational movements.
- The infrared portion of the spectrum can be divided into three regions in wavelength: far-infrared, from 300 GHz (1 mm) to 30 THz (10 μm); mid-infrared, from 30 to 120 THz (10 to 2.5 μm); and near-infrared, from 120 to 400 THz (2,500 to 750 nm).
- Infrared radiation is popularly known as " heat radiation," but light and electromagnetic waves of any frequency will heat surfaces that absorb them.
- The concept of emissivity is important in understanding the infrared emissions of objects. This is a property of a surface which describes how its thermal emissions deviate from the ideal of a black body.
- Infrared radiation can be used to remotely determine the temperature of objects (if the emissivity is known). This is termed thermography, mainly used in military and industrial applications.
- Visible light is produced by vibrations and rotations of atoms and molecules, as well as by electronic transitions within atoms and molecules. We say the atoms and molecules are excited when they absorb and relax when they emit through electronic transitions.
- This figure shows the visible part of the spectrum, together with the colors associated with particular pure wavelengths. Red light has the lowest frequencies and longest wavelengths, while violet has the highest frequencies and shortest wavelengths.
- Colors that can be produced by visible light of a narrow band of wavelengths are called pure spectral colors. They can be delineated roughly in wavelength as: violet (380-450 nm), blue (450-495 nm), green (495-570 nm), yellow (570-590 nm), orange (590-620 nm), and red (620 to 750 nm).
- Visible wavelengths pass through the optical window, the Earth's atmosphere allows this region of the electromagnetic spectrum to pass through largely unattenuated (see opacity plot in.
- The portion of the EM spectrum used by photosynthesic organisms is called the photosynthetically active region (PAR) and corresponds to solar radiation between 400 and 700 nm, substantially overlapping with the range of human vision.
- Ultraviolet light gets its name because the spectrum consists of electromagnetic waves with frequencies higher than those that humans identify as the color violet.
- Most UV is non- ionizing radiation, though UV with higher energies (10-120 nm) is ionizing. All UV can have harmful effects on biological matter (such as causing cancers) with the highest energies causing the most damage.
- The danger posed by lower energy UV radiation is derived from the ultraviolet photon 's power to alter chemical bonds in molecules, even without having enough energy to ionize atoms.
- Solar UV radiation is commonly subdivided into three regions: UV-A (320–400 nm), UV-B (290–320 nm), and UV-C (220–290 nm), ranked from long to shorter wavelengths (from smaller to larger energies).
- Most UV-B and all UV-C is absorbed by ozone (O_3) molecules in the upper atmosphere. Consequently, 99% of the solar UV radiation reaching the Earth's surface is UV-A.
- X-rays have shorter wavelengths (higher energy) than UV waves and, generally, longer wavelengths (lower energy) than gamma rays. Sometimes X-rays are called Röntgen radiation, after Wilhelm Röntgen, who is usually credited as their discoverer.
- Because X-rays have very high energy they are known as ionizing radiation and can harm living tissue. A very high radiation dose over a short amount of time causes radiation sickness, while lower doses can give an increased risk of radiation-induced cancer.
- Lower doses of X-ray radiation can be very effectively used in medical radiography and X-ray spectroscopy. In the case of medical radiography, the benefits of using X-rays for examination far outweighs the risk.
- X-rays are broken up into broad two categories: hard X-rays with energies above 5-10 keV (below 0.2-0.1 nm wavelength) and soft X-rays with energies 100 eV – 5 keV (10 – 0.1 nm wavelength). Hard X-rays are more useful for radiography because they pass through tissue.
- The distinction between X-rays and gamma rays is somewhat arbitrary and there is substantial overlap at the high energy boundary. However, in general they are distinguished by their source, with gamma rays originating from the nucleus and X-rays from the electrons in the atom.
- Gamma rays are the highest energy EM radiation and typically have energies greater than 100 keV, frequencies greater than 10^{19} Hz, and wavelengths less than 10 picometers.

- Gamma rays from radioactive decay are defined as gamma rays no matter what their energy, so that there is no lower limit to gamma energy derived from radioactive decay. Gamma decay commonly produces energies of a few hundred keV, and almost always less than 10 MeV.
- Gamma rays have characteristics identical to X-rays of the same frequency—they differ only in source. Gamma rays are usually distinguished by their origin: X-rays are emitted by definition by electrons outside the nucleus, while gamma rays are emitted by the nucleus.
- Natural sources of gamma rays include gamma decay from naturally occurring radioisotopes such as potassium-40, and also as a secondary radiation from atmospheric interactions with cosmic ray particles. Exotic astrophysical processes will also produce gamma rays.
- Gamma rays are ionizing radiation and are thus biologically hazardous. The most biological damaging forms of gamma radiation occur at energies between 3 and 10 MeV.

Key Terms

- **AM radio waves**: Waves used to carry commercial radio signals between 540 and 1600 kHz. Information is carried by amplitude variation, while the frequency remains constant.
- **FM radio waves**: Waves used to carry commercial radio signals between 88 and 108 MHz. Information is carried by frequency modulation, while the signal amplitude remains constant.
- **radio waves**: Designates a portion of the electromagnetic spectrum having frequencies ranging from 300 GHz to 3 kHz, or equivalently, wavelengths from 1 millimeter to 100 kilometers.
- **terahertz radiation**: Electromagnetic waves with frequencies around one terahertz.
- **thermal agitation**: The thermal motion of atoms and molecules in any object at a temperature above absolute zero, causing them to emit and absorb radiation.
- **radar**: A method of detecting distant objects and determining their position, velocity, or other characteristics by analysis of sent radio waves (usually microwaves) reflected from their surfaces.
- **emissivity**: The energy-emitting propensity of a surface, usually measured at a specific wavelength.
- **thermography**: Any of several techniques for the remote measurement of the temperature variations of a body, especially by creating images produced by infrared radiation.
- **thermal radiation**: The electromagnetic radiation emitted from a body as a consequence of its temperature; increasing the temperature of the body increases the amount of radiation produced, and shifts it to shorter wavelengths (higher frequencies) in a manner explained only by quantum mechanics.
- **spectral color**: a color that is evoked by a single wavelength of light in the visible spectrum, or by a relatively narrow band of wavelengths. Every wavelength of light is perceived as a spectral color, in a continuous spectrum; the colors of sufficiently close wavelengths are indistinguishable.
- **optical window**: the optical portion of the electromagnetic spectrum that passes through the atmosphere all the way to the ground. The window runs from around 300 nanometers (ultraviolet-C) at the short end up into the range the eye can use, roughly 400-700 nm and continues up through the visual infrared to around 1100 nm, which is thermal infrared.
- **visible light**: the part of the electromagnetic spectrum, between infrared and ultraviolet, that is visible to the human eye
- **ozone layer**: A region of the stratosphere, between 15 and 30 kilometres in altitude, containing a relatively high concentration of ozone; it absorbs most solar ultraviolet radiation.
- **ionizing radiation**: high-energy radiation that is capable of causing ionization in substances through which it passes; also includes high-energy particles
- **non-ionizing radiation**: Radiation that does not cause atmospheric ionization; electrically neutral radiation.
- **X-ray spectroscopy**: The use of an X-ray spectrometer for chemical analysis.
- **x-ray crystallography**: A technique in which the patterns formed by the diffraction of X-rays on passing through a crystalline substance yield information on the lattice structure of the crystal, and the molecular structure of the substance.
- **radiograph**: An image, often a photographic negative, produced by radiation other than normal light; especially an X-ray photograph.
- **gamma ray**: A very high frequency (and therefore very high energy) electromagnetic radiation emitted as a consequence of radioactivity.
- **gamma decay**: A nuclear reaction with the emission of a gamma ray.

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23.2: Electromagnetic Waves and their Properties

learning objectives

- Explain the meaning and importance of Maxwell’s equations

Maxwell’s Equations

Maxwell’s equations are a set of four partial differential equations that, along with the Lorentz force law, form the foundation of classical electrodynamics, classical optics, and electric circuits.

Named after esteemed physicist James Clerk Maxwell, the equations describe the creation and propagation of electric and magnetic fields. Fundamentally, they describe how electric charges and currents create electric and magnetic fields, and how they affect each other.

Maxwell’s equations can be divided into two major subsets. The first two, Gauss’s law and Gauss’s law for magnetism, describe how fields emanate from charges and magnets respectively. The other two, Faraday’s law and Ampere’s law with Maxwell’s correction, describe how induced electric and magnetic fields circulate around their respective sources.

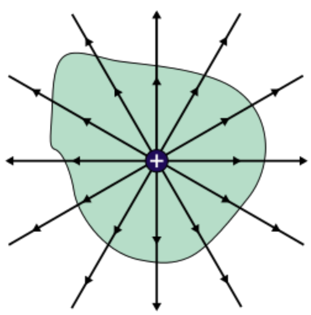
Each of Maxwell’s equations can be looked at from the “microscopic” perspective, which deals with total charge and total current, and the “macroscopic” set, which defines two new auxiliary fields that allow one to perform calculations without knowing microscopic data like atomic-level charges.

Gauss’s Law

Gauss’s law relates an electric field to the charge(s) that create(s) it. The field (E) points towards negative charges and away from positive charges, and from the microscopic perspective, is related to charge density (ρ) and vaccuum permittivity (ε_0, or permittivity of free space) as:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \tag{23.2.1}$$

Gauss’s Law basically says that a net amount of charge contained within a region of space will generate an electric field that emanates through the surface that surrounds that region.

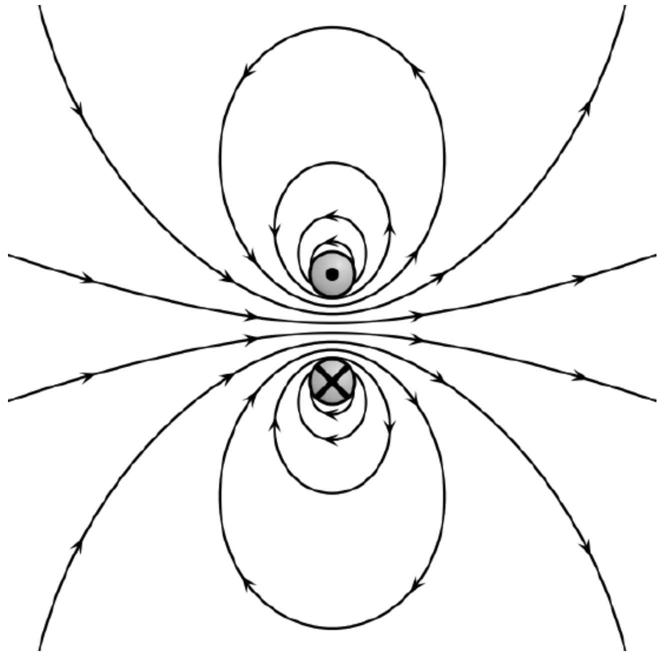


Example of Gauss’s Law: A positive charge contained within a region of space creates an electric field that emanates from the surface of that region.

Gauss’s Law for Magnetism

Gauss’s law for magnetism states that there are *no* “magnetic charges (or monopoles)” analogous to electric charges, and that magnetic fields are instead generated by magnetic *dipoles*. Such dipoles can be represented as loops of current, but in many ways are similar in appearance to positive and negative “magnetic charges” that are inseparable and thus have no formal net “magnetic charge.”

Magnetic field lines form loops such that all field lines that go into an object leave it at some point. Thus, the total magnetic flux through a surface surrounding a magnetic dipole is always zero.



Field lines caused by a magnetic dipole: The field lines created by this magnetic dipole either form loops or extend infinitely.

The differential form of Gauss's law for magnetic for magnetism is

$$\nabla \cdot \mathbf{B} = \mathbf{0} \tag{23.2.2}$$

Faraday's Law

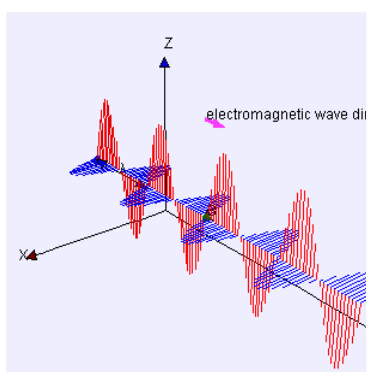
Faraday's law describes how a time-varying magnetic field (or flux) induces an electric field. The principle behind this phenomenon is used in many electric generators. Both macroscopic and microscopic differential equations are the same, relating electric field (E) to the time-partial derivative of magnetic field (B):

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial \mathbf{t}} \tag{23.2.3}$$

Ampere's Circuital Law (with Maxwell's correction)

Ampere's law originally stated that magnetic field could be created by electrical current. Maxwell added a second source of magnetic fields in his correction: a changing electric field (or flux), which would induce a magnetic field even in the absence of an electrical current. He named the changing electric field "displacement current."

Maxwell's correction shows that self-sustaining electromagnetic waves (light) can travel through empty space even in the absence of moving charges or currents, with the electric field component and magnetic field component each continually changing and each perpetuating the other.



Electromagnetic Waves: Electric (red) and magnetic (blue) waves propagate in phase sinusoidally, and perpendicularly to one another.

The microscopic approach to the Maxwell-corrected Ampere's law relates magnetic field (B) to current density (J, or current per unit cross sectional area) and the time-partial derivative of electric field (E):

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \tag{23.2.4}$$

The Production of Electromagnetic Waves

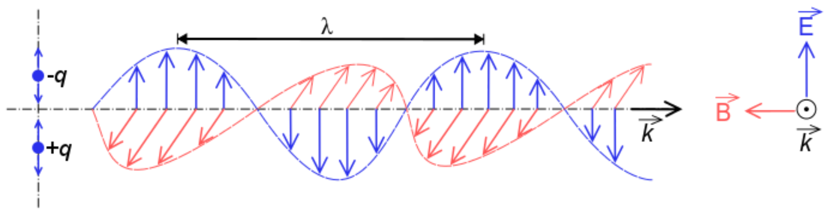
Electromagnetic waves are the combination of electric and magnetic field waves produced by moving charges.

learning objectives

- Explain the self-perpetuating behavior of an electromagnetic wave

Electromagnetic waves

Electromagnetic radiation, is a form of energy emitted by moving charged particles. As it travels through space it behaves like a wave, and has an oscillating electric field component and an oscillating magnetic field. These waves oscillate perpendicularly to and in phase with one another.



Electromagnetic Wave: Electromagnetic waves are a self-propagating transverse wave of oscillating electric and magnetic fields. The direction of the electric field is indicated in blue, the magnetic field in red, and the wave propagates in the positive x-direction. Notice that the electric and magnetic field waves are in phase.

The creation of all electromagnetic waves begins with a charged particle. This charged particle creates an electric field (which can exert a force on other nearby charged particles). When it accelerates as part of an oscillatory motion, the charged particle creates ripples, or oscillations, in its electric field, and also produces a magnetic field (as predicted by Maxwell's equations).

Once in motion, the electric and magnetic fields created by a charged particle are self-perpetuating—time-dependent changes in one field (electric or magnetic) produce the other. This means that an electric field that oscillates as a function of time will produce a magnetic field, and a magnetic field that changes as a function of time will produce an electric field. Both electric and magnetic fields in an electromagnetic wave will fluctuate in time, one causing the other to change.

Electromagnetic waves are ubiquitous in nature (i.e., light) and used in modern technology—AM and FM radio, cordless and cellular phones, garage door openers, wireless networks, radar, microwave ovens, etc. These and many more such devices use electromagnetic waves to transmit data and signals.

All the above sources of electromagnetic waves use the simple principle of moving charge, which can be easily modeled. Placing a coin in contact with both terminals of a 9-volt battery produces electromagnetic waves that can be detected by bringing the antenna of a radio (tuned to a static-producing station) within a few inches of the point of contact.

Energy and Momentum

Electromagnetic waves have energy and momentum that are both associated with their wavelength and frequency.

learning objectives

- Relate energy of an electromagnetic wave with the frequency and wavelength

Electromagnetic radiation can essentially be described as photon streams. These photons are strictly defined as massless, but have both energy and surprisingly, given their lack of mass, momentum, which can be calculated from their wave properties.

Waves were poorly understood until the 1900s, when Max Planck and Albert Einstein developed modern corrections to classical theory.

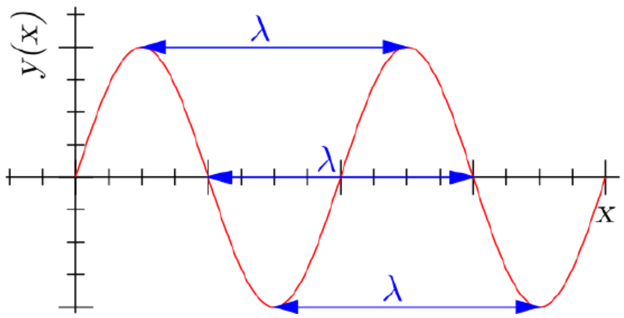
Planck theorized that “black bodies” (thermal radiators) and other forms of electromagnetic radiation existed not as spectra, but in discrete, “quantized” form. In other words, there were only certain energies an electromagnetic wave could have. In his work he developed what is now known as “Planck’s constant,” which is approximately equal to 6.626×10^{-34} J·s.

Energy

The energy (E) of a photon can be related to its frequency (f) by Planck’s constant (h):

$$\mathrm{E} = \mathrm{hf} = \frac{\mathrm{hc}}{\lambda} \tag{23.2.5}$$

The ratio of speed of light (c) to wavelength (λ) can be substituted in place of f to give the same equation to energy in different terms. Note that energy cannot take any value: it can only exist in increments of frequency times Planck’s constant (or Planck’s constant times c divided by wavelength). Energy of a wave is therefore “quantized. ”



Wavelength: Wavelength of the sinusoidal function is represented by λ.

Momentum

Momentum is classically defined as the product of mass and velocity and thus would intuitively seem irrelevant to a discussion of electromagnetic radiation, which is both massless and composed of waves.

However, Einstein proved that light can act as particles in some circumstances, and that a wave-particle duality exists. And, given that he related energy and mass ($E=mc^2$), it becomes more conceivable that a wave (which has an energy value) not only has an equation to mass but a momentum as well.

And indeed, Einstein proved that the momentum (p) of a photon is the ratio of its energy to the speed of light.

$$\mathrm{p} = \frac{\mathrm{E}}{\mathrm{c}} = \frac{\mathrm{hf}}{\mathrm{c}} = \frac{\mathrm{h}}{\lambda} \tag{23.2.6}$$

Substituting E with hc/λ cancels the c terms, making momentum also equal to the simple ratio of Planck’s constant to wavelength.

The Speed of Light

The speed of light in a vacuum is one of the most fundamental constant in physics, playing a pivotal role in modern physics.

learning objectives

- Relate speed of light with the index of refraction of the medium

The Speed of Light

The speed of light is generally a point of comparison to express that something is fast. shows a scale representation of the time it takes a beam of light to reach the moon from Earth. But what exactly is the speed of light?

Light Going from Earth to the Moon: A beam of light is depicted travelling between the Earth and the Moon in the time it takes a light pulse to move between them: 1.255 seconds at their mean orbital (surface-to-surface) distance. The relative sizes and separation of the Earth–Moon system are shown to scale.

It is just that: the speed of a photon or light particle. The speed of light in a vacuum (commonly written as c) is 299,792,458 meters per second. This is a universal physical constant used in many areas of physics. For example, you might be familiar with the equation:

$$\mathrm{E}=\mathrm{mc}^{2} \tag{23.2.7}$$

where E = Energy and m = mass. This is known as the mass-energy equivalence, and it uses the speed of light to interrelate space and time. This not only explains the energy a body of mass contains, but also explains the hindrance mass has on speed.

There are many uses for the speed of light in a vacuum, such as in special relativity, which says that c is the natural speed limit and nothing can move faster than it. However, we know from our understanding of physics (and previous atoms) that the speed at which something travels also depends on the medium through which it is traveling. The speed at which light propagates through transparent materials (air, glass, etc.,) is dependent on the refractive index of that material, n:

$$\mathrm{v}=\frac{\mathrm{c}}{\mathrm{n}} \tag{23.2.8}$$

where v = actual velocity of light moving through the medium, c = speed of light in a vacuum, and n = refractive index of medium. The refractive index of air is about 1.0003, and from this equation we can find that the speed of visible light in air is about 90 km/s slower than c.

As mentioned earlier, the speed of light (usually of light in a vacuum) is used in many areas of physics. Below is an example of an application of the constant c.

The Lorentz Factor

Fast-moving objects exhibit some properties that are counterintuitive from the perspective of classical mechanics. For example, length contracts and time dilates (runs slower) for objects in motion. The effects are typically minute, but are noticeable at sufficiently high speeds. The Lorentz factor (γ) is the factor by which length shortens and time dilates as a function of velocity (v):

$$\gamma=\left(1-\mathrm{v}^{2} / \mathrm{c}^{2}\right)^{-1 / 2} \gamma=\left(1-\mathrm{v}^{2} / \mathrm{c}^{2}\right)^{-1 / 2} \gamma=\left(1-\mathrm{v}^{2} / \mathrm{c}^{2}\right)^{-1 / 2} \tag{23.2.9}$$

At low velocities, the quotient of v^2/c^2 is sufficiently close to 0 such that γ is approximately 1. However, as velocity approaches c, γ increases rapidly towards infinity.

The Doppler Effect

The Doppler Effect is the change in a wave's perceived frequency that results from the source's motion, the observer, and the medium.

learning objectives

- Give examples of daily observations of the Doppler effect

The Doppler Effect

The Doppler effect is a periodic event's change in frequency for an observer in motion relative to the event's source. Typically, this periodic event is a wave.

Most people have experienced the Doppler effect in action. Consider an emergency vehicle in motion, sounding its siren. As it approaches an observer, the pitch of the sound (its frequency) sounds higher than it actually is. When the vehicle reaches the observer, the pitch is perceived as it actually is. When the vehicle continues away from the observer, the pitch is perceived as lower than it actually is. From the perspective of an observer inside the vehicle, the pitch of the siren is constant.

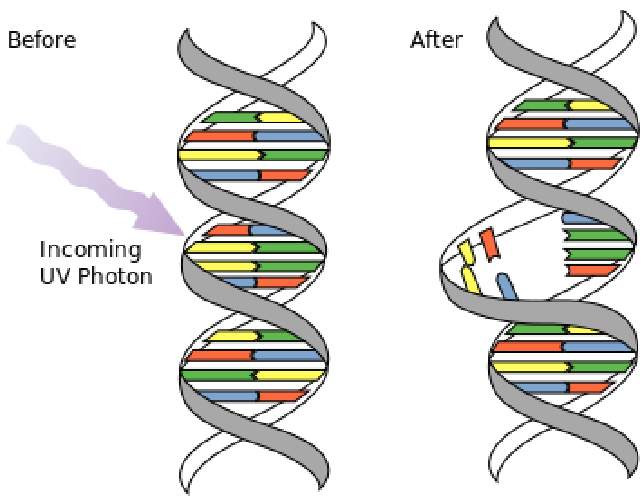
The Doppler Effect and Sirens: Waves emitted by a siren in a moving vehicle

The difference in the perceived pitch depending on observer location can be explained by the fact that the siren's position changes as it emits waves. A wave of sound is emitted by a moving vehicle every millisecond. The vehicle 'chases' each wave in one direction. By the time the next wave is emitted, it is closer (relative to an onlooker ahead of the vehicle) to the previous wave than the wave's frequency would suggest. Relative to an onlooker behind the vehicle, the second wave is further from the first wave than one would expect, which suggests a lower frequency.

The Doppler effect can be caused by any kind of motion. In the example above, the siren moved relative to a stationary observer. If the observer moves relative to the stationary siren, the observer will notice the Doppler effect on the pitch of the siren. Finally, if the medium through which the waves propagate moves, the Doppler effect will be noticed even for a stationary observer. An example of this phenomenon is wind.

Quantitatively, the Doppler effect can be characterized by relating the frequency perceived (f) to the velocity of waves in the medium (c), the velocity of the receiver relative to the medium (v_r), the velocity of the source relative to the medium (v_s), and the actual emitted frequency (f_0):

$$f = \left(\frac{c + v_r}{c + v_s}\right) f_0 \tag{23.2.10}$$



The Doppler Effect: Wavelength change due to the motion of source

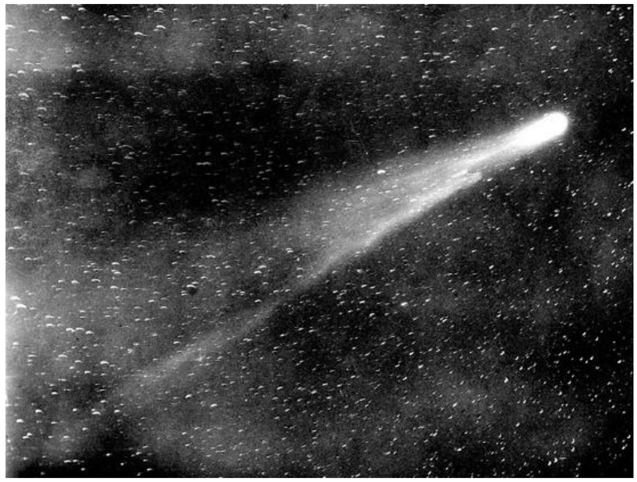
Momentum Transfer and Radiation Pressure Atom

Radiation pressure is the pressure exerted upon any surface exposed to electromagnetic (EM) radiation.

learning objectives

- Explain formation of radiation pressure

Radiation pressure is the pressure exerted upon any surface exposed to electromagnetic (EM) radiation. EM radiation (or photon, which is a quantum of light) carries momentum; this momentum is transferred to an object when the radiation is absorbed or reflected. Perhaps one of the most well know examples of the radiation pressure would be comet tails. Haley's comet is shown in.



Halley's Comet: As a comet approaches the inner Solar System, solar radiation causes the volatile materials within the comet to vaporize and stream out of the nucleus. The streams of dust and gas thus released form an atmosphere around the comet (called the coma), and the force exerted on the coma by the Sun's radiation pressure and solar wind cause the formation of an enormous tail that points away from the Sun.

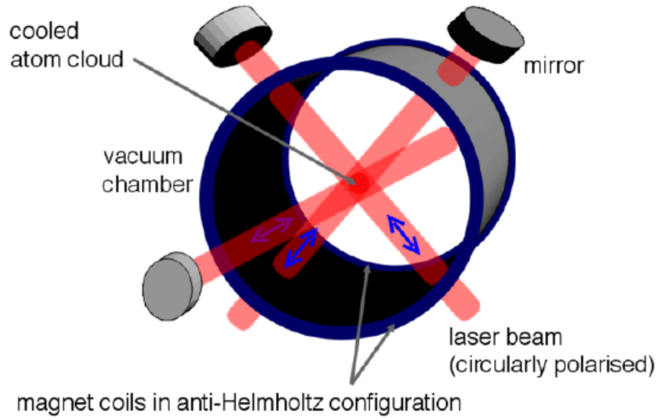
Although radiation pressure can be understood using classical electrodynamics, here we will examine the quantum mechanical argument. From the perspective of quantum theory, light is made of photons: particles with zero mass but which carry energy and – importantly in this argument – momentum. According to special relativity, because photons are devoid of mass, their energy (E) and momentum (p) are related by E=pc.

Now consider a beam of light perpendicularly incident on a surface, and let us assume the beam of light is totally absorbed. The momentum the photons carry is a conserved quantity (i.e., it cannot be destroyed) so it must be transferred to the surface; thus the absorption of the light beam causes the surface to gain momentum. Newton's Second Law tells us that force equals rate of change of momentum; thus during each second, the surface experiences a force (or pressure, as pressure is force per unit area) due to the momentum the photons transfer to it.

This gives us: pressure = momentum transferred per second per unit area = energy deposited per second per unit area / c = I/c, (where I is the intensity of the beam of light).

Laser Cooling

There are many variations of laser cooling, but they all use radiation pressure to remove energy from atomic gases (and therefore cool the sample). In laser cooling (sometimes called Doppler cooling), the frequency of light is tuned slightly below an electronic transition in the atom. Because light is detuned to the "red" (i.e., at lower frequency) of the transition, the atoms will absorb more photons if they move towards the light source, due to the Doppler effect. Thus if one applies light from two opposite directions, the atoms will always scatter more photons from the laser beam pointing opposite to their direction of motion (typical setups applies three opposing pairs of laser beams as in).



The Magneto Optical Trap: Experimental setup of Magneto Optical Trap (MOT), which uses radiation pressure to cool atomic species. Atoms are slowed down by absorbing (and emitting) photons.

In each scattering event, the atom loses a momentum equal to the momentum of the photon. If the atom (which is now in the excited state) then emits a photon spontaneously, it will be kicked by the same amount of momentum, only in a random direction. Since the initial momentum loss was opposite to the direction of motion (while the subsequent momentum gain was in a random direction), the overall result of the absorption and emission process is to reduce the speed of the atom. If the absorption and emission are repeated many times, the average speed (and therefore the kinetic energy) of the atom will be reduced. Since the temperature of a group of atoms is a measure of the average random internal kinetic energy, this is equivalent to cooling the atoms. Simple laser cooling setups can produce a cold sample of atomic gases at around 1mK (=10^{-3} K) starting from a room temperature gas.

Key Points

- Maxwell's four equations describe how electric charges and currents create electric and magnetic fields, and how they affect each other.
- Gauss's law relates an electric field to the charge(s) that create(s) it.
- Gauss's law for magnetism states that there are no "magnetic charges" analogous to electric charges, and that magnetic fields are instead generated by magnetic dipoles.
- Faraday's law describes how a time-varying magnetic field (or flux) induces an electric field. The principle behind this phenomenon is used in many electric generators.
- Ampere 's law originally stated that a magnetic field is created by an electrical current. Maxwell added that a changing electric flux can also generate a magnetic field.
- Electromagnetic waves consist of both electric and magnetic field waves. These waves oscillate in perpendicular planes with respect to each other, and are in phase.
- The creation of all electromagnetic waves begins with an oscillating charged particle, which creates oscillating electric and magnetic fields.
- Once in motion, the electric and magnetic fields that a charged particle creates are self-perpetuating: time-dependent changes in one field (electric or magnetic) produce the other.
- Max Planck proved that energy of a photon (a stream of which is an electromagnetic wave) is quantized and can exist in multiples of "Planck's constant" (denoted as h, approximately equal to 6.626×10^{-34} J·s).
- $\mathrm{E=hf=\frac{hc}{\lambda}}$ describes the energy (E) of a photon as a function of frequency (f), or wavelength (λ).
- $\mathrm{p=\frac{E}{c}=\frac{hf}{c}=\frac{h}{\lambda}}$ describes the momentum (p) of a photon as a function of its energy, frequency, or wavelength.
- The maximum possible value for the speed of light is that of light in a vacuum, and this speed is used for a constant in many area of physics.
- c is the symbol used to represent the speed of light in a vacuum, and its value is 299,792,458 meters per second.
- When light travels through medium, its speed is hindered by the index of refraction of that medium. Its actual speed can be found with: $v=\frac{c}{n}$.
- The Doppler effect is very commonly observed in action.
- The Doppler effect can be observed in the apparent change in pitch of a siren on an emergency vehicle, according to a stationary observer.
- The observer will notice the Doppler effect on the pitch of the stationary siren when moving relative to its pitch, or if the medium moves when the observer is stationary.
- Photons carry momentum (p = E/c). When photons are absorbed or reflected on a surface, the surface receives momentum kicks. This momentum transfer leads to radiation pressure.
- Electromagnetic radiation applies radiation pressure equal to the Intensity (of light beam) divided by c (speed of light).
- Laser cooling uses radiation pressure to remove energy from atomic gases. The technique can produce cold samples of gases at 1mK or so.

Key Terms

- **differential equation**: An equation involving the derivatives of a function.
- **flux**: A quantitative description of the transfer of a given vector quantity through a surface. In this context, we refer to the electric flux and magnetic flux.
- **electromagnetic wave**: A wave of oscillating electric and magnetic fields.

- **phase**: Waves are said to be "in phase" when they begin at the same part (e.g., crest) of their respective cycles.
- **photon**: The quantum of light and other electromagnetic energy, regarded as a discrete particle having zero rest mass, no electric charge, and an indefinitely long lifetime.
- **wavelength**: The length of a single cycle of a wave, as measured by the distance between one peak or trough of a wave and the next; it is often designated in physics as λ, and corresponds to the velocity of the wave divided by its frequency.
- **frequency**: The quotient of the number of times n a periodic phenomenon occurs over the time t in which it occurs: f = n / t.
- **special relativity**: A theory that (neglecting the effects of gravity) reconciles the principle of relativity with the observation that the speed of light is constant in all frames of reference.
- **refractive index**: The ratio of the speed of light in air or vacuum to that in another medium.
- **doppler effect**: Apparent change in frequency of a wave when the observer and the source of the wave move relative to each other.
- **classical electrodynamics**: A branch of theoretical physics that studies consequences of the electromagnetic forces between electric charges and currents.

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23.3: Applications of EM Waves

learning objectives

- Discuss wireless technologies and their applications

Wireless communication is the transfer of information between two or more points that are not connected by an electrical conductor. The term is commonly used in the telecommunications industry to refer to telecommunications systems (e.g., radio transmitters and receivers, remote controls, etc.) that use some form of energy (e.g., radio waves, acoustic energy, etc.) to transfer information without the use of wires. Information is transferred in this manner over both short and long distances. Wireless operations permit services, such as long-range communications, that are otherwise impossible (or impractical) to implement with the use of wires.

The most common wireless technologies use electromagnetic wireless telecommunications, such as radio or infra-red signals. With infra-red waves, distances are short (such as a few meters for television remote control) while radio waves can reach as far as thousands or even millions of kilometers for deep-space radio communications. It encompasses various types of fixed, mobile, and portable applications, including two-way radios, cellular telephones, personal digital assistants (PDAs), and wireless networking. Other examples of applications of radio wireless technology include GPS units, garage door openers, wireless computer mice, keyboards and headsets, headphones, radio receivers, satellite television, broadcast television, and cordless telephones. Less common methods of achieving wireless communications include the use of light, sound, magnetic, or electric fields.

Two cellular phones: The Qualcomm QCP-2700, a mid-1990s candybar style phone, and an iPhone 4S, a current production smartphone.

One of the best-known examples of wireless technology is the mobile (or cellular) phone, with more than 4.6 billion mobile cellular subscriptions worldwide as of the end of 2010 (examples of such phones are shown in). These wireless devices use radio waves to enable their users to make phone calls from many locations worldwide. They can be used within range of the mobile telephone sites that house the necessary equipment to transmit and receive the radio signals these devices emit. Wireless data communications are also an essential component of mobile computing. The various available technologies differ in local availability, coverage range, and performance. In some circumstances, users must be able to employ multiple connection types and switch between them.

To simplify the experience for the user, connection manager software is available, or a mobile VPN can be utilized to handle the multiple connections as a secure, single virtual network. One popular supporting technology is Wi-Fi, a wireless local area network that enables portable computing devices to connect easily to the Internet. Standardized as IEEE 802.11 a,b,g,n, Wi-Fi approaches speeds of some types of wired Ethernet. Wi-Fi has become the de facto standard for access in private homes, within offices, and at public hotspots. Some businesses charge customers a monthly fee for the service, while others offer it for free in an effort to increase sales of their goods.

Cellular data service offers coverage within a range of 10-15 miles from the nearest cell site. Speeds have increased as technologies have evolved, from earlier technologies such as GSM, CDMA and GPRS, to 3G networks such as W-CDMA, EDGE or CDMA2000. Mobile Satellite Communications may be used where other wireless connections are unavailable, such as in largely rural areas or remote locations. Satellite communications are especially important for transportation, aviation, maritime, and military use.

Key Points

- Wireless operations permit services, such as long-range communications, that are impossible or impractical to implement with the use of wires.
- The most common wireless technologies use electromagnetic wireless telecommunications, such as radio.
- Less common methods of achieving wireless communications include the use of light, sound, magnetic, or electric fields.

Key Terms

- **radio wave**: Electromagnetic radiation having a wavelength between about.5 centimeters and 30,000 meters; used for the broadcasting of radio and television signals.
- **conductor**: A material which contains movable electric charges.
- **telecommunication**: The science and technology of the communication or messages over a distance, especially using electric, electronic or electromagnetic impulses.

CHAPTER OVERVIEW

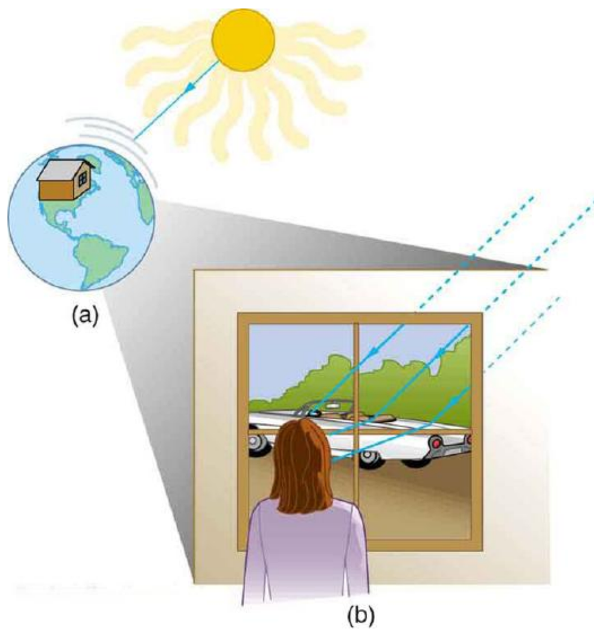
24: Geometric Optics

24.1: Overview

learning objectives

- Distinguish three ways that rays can travel

Rays, or beams of light, can travel in three ways: directly, through a material, or indirectly (reflection). These three methods of light travel are shown in this image. The word ray comes from mathematics, and refers to a straight line that originates at some point. Even when passing through a material, or bouncing off of a material in a reflection, the light continues to travel in a straight line, even if that line has changed direction. The movement of light, as a ray, can be shown with simple geometry and trigonometry. This is called geometric optics.



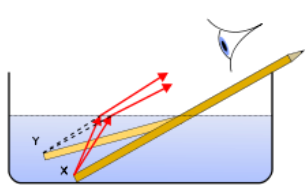
Methods of Travel by a Ray of Light: Light can travel through empty space directly from the source, through media like air and glass, reflect from an object like a mirror, or travel in a straight line.

Direct Light Travel

Direct light travel is when a ray of light starts at a source, and continues to travel from that source to its destination without encountering any interference. The light will continue in a straight line or ray until it reaches the observer. An example of this is the light that travels from the sun to the earth.

Light Travel Through a Material

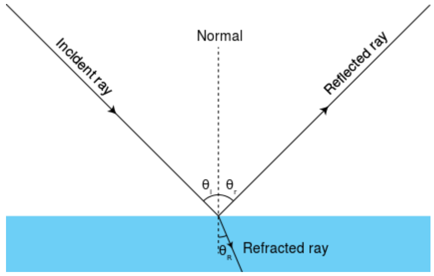
When light travels through any material, air, glass, water, etc., it encounters interference and changes direction. This is only a directional change and will continue in this new path, but still as a straight line, or ray. The law that deals with this change in direction is called the law of refraction. This change in ray direction will depend on the refractive index of the material through which the light is travelling. This concept is what led to the development of lenses and glasses. Have you ever noticed that if you put part of a pencil, spoon, or straw in a bowl of water, the object no longer appears straight, but seems to bend? This is because the index of refraction of the water is different from that of the air. This causes the light rays to change direction.



Refraction of Light Rays: The concept of refraction explains how a pencil submerged in water appears to bend.

Light Bouncing Off a Material

When light is bounced off of a material, such as a mirror, this is called a reflection. This is when a light ray, the incident ray, hits a reflective material and bounces off as the reflected ray at a specific angle. This is called the angle of reflection. Since the movement of the light rays can be shown geometrically, if a mirror is one-half your height, you could see your whole body in the reflection.



Reflected Rays: This diagram shows how light rays reflects off of a surface.

Key Points

- Direct light motion is when a ray of light travels from a source and is uninterrupted until it reaches it destination.
- Refracted light is when a light ray travels through another medium and, due to the difference in refractive indexes of the materials, changes direction slightly.
- When a light ray hits a reflective material, it bounces off as a reflected ray at a specific angle.

Key Terms

- **refraction**: Changing of a light ray's direction when it passes through variations in matter.
- **geometric optics**: Optics that describes light propagation in terms of "rays".
- **reflection**: the property of a propagated wave being thrown back from a surface (such as a mirror)

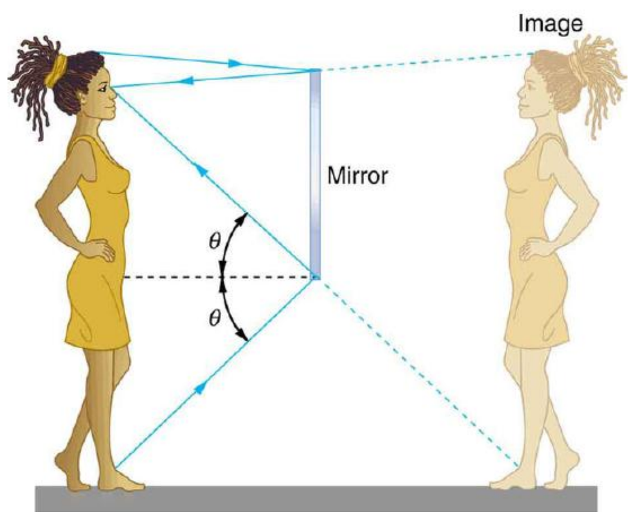
24.2: Reflection, Refraction, and Dispersion

learning objectives

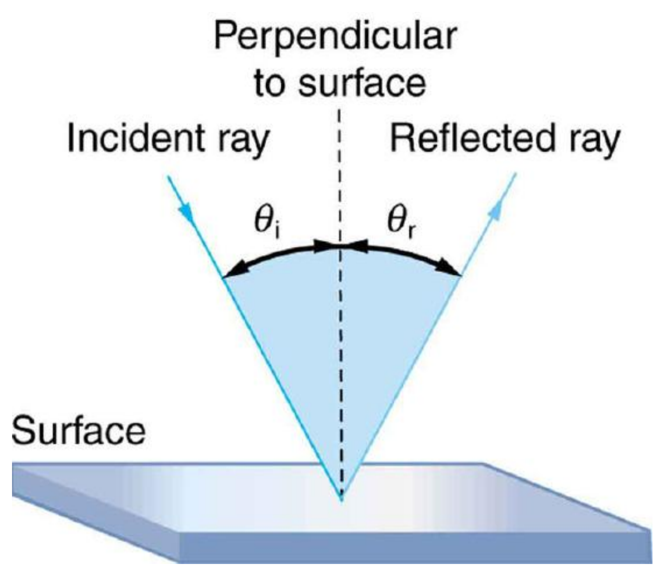
- Formulate the relationship between the angle of reflection and the angle of incidence

Whenever you look into a mirror or squint at sunlight glinting off a lake, you are seeing a reflection. When you look at the text in a book, you are actually seeing the light that is reflected from it. Large telescopes use reflections to form images of stars and other astronomical objects. In fact, the only way we can see an object that does not itself emit light is if that object reflects light.

The law of reflection is illustrated in, which also shows how the angles are measured relative to the perpendicular to the surface at the point where the light ray strikes. The law of reflection is very simple: The angle of reflection equals the angle of incidence. When we see our reflection in a mirror, it appears that our image is actually behind the mirror — we see the light coming from a direction determined by the law of reflection. The angles are such that our image appears exactly the same distance behind the mirror as we stand away from the mirror.

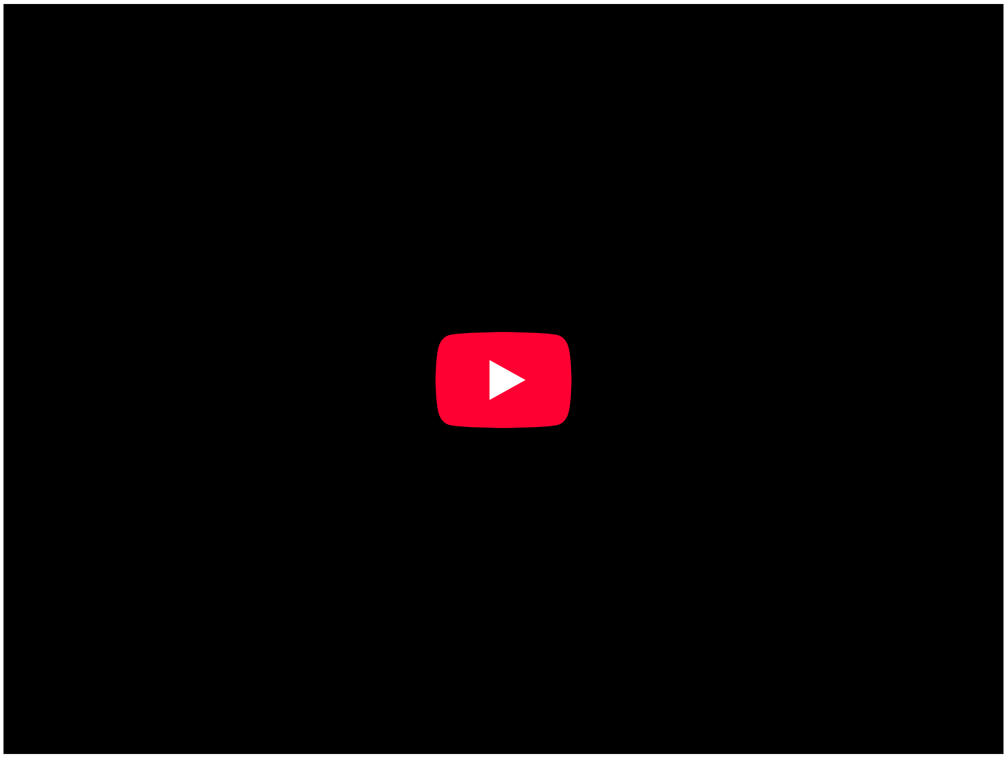


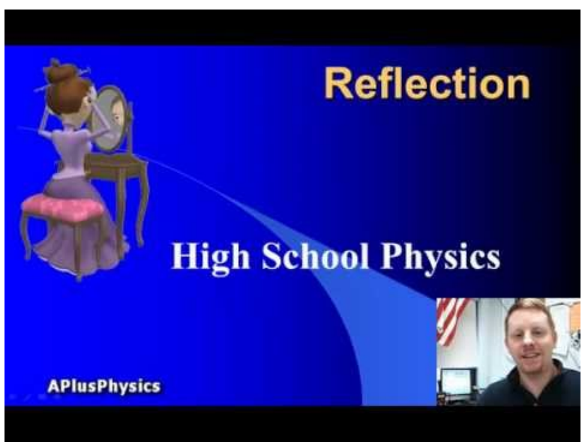
Mirror Reflection: An image in a mirror appears as though it is behind the mirror. The two rays shown are those that strike the mirror at just the correct angles to be reflected into the eyes of the viewer. The image appears to come from the direction the rays are coming from when they enter the viewer's eyes.



Law of Reflection: The law of reflection states that the angle of reflection equals the angle of incidence: θr = θi. The angles are measured relative to the perpendicular to the surface at the point where the ray strikes the surface.

We expect to see reflections off a smooth surface. However, light strikes different parts of a rough surface at different angles, and it is reflected in many different directions ("diffused"). Diffused light is what allows us to see a sheet of paper from any angle. Many objects, such as people, clothing, leaves, and walls, have rough surfaces and can be seen from all sides. A mirror, on the other hand, has a smooth surface (compared with the wavelength of light) and reflects light at specific angles. When the moon reflects off the surface of a lake, a combination of these effects takes place.





Reflection: A brief overview of reflection and the law of reflection.

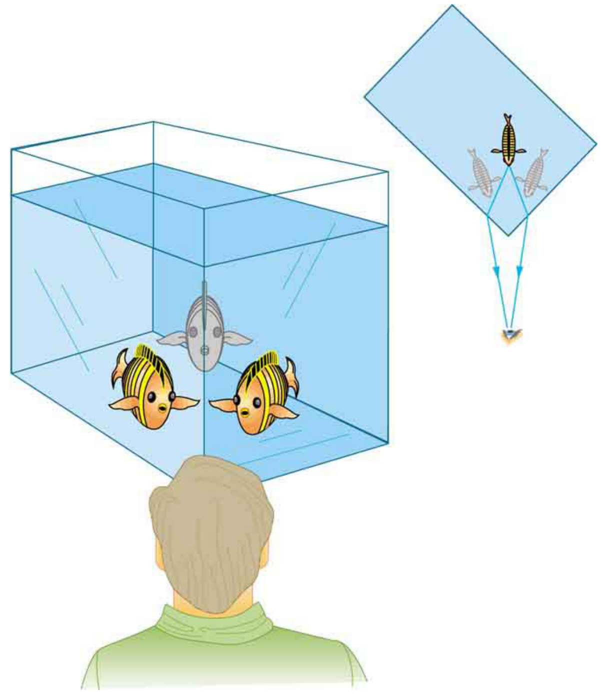
The Law of Refraction: Snell's Law and the Index of Refraction

The amount that a light ray changes its direction depends both on the incident angle and the amount that the speed changes.

learning objectives

- Formulate the relationship between the index of refraction and the speed of light

It is easy to notice some odd things when looking into a fish tank. For example, you may see the same fish appearing to be in two different places. This is because light coming from the fish to us changes direction when it leaves the tank, and in this case, it can travel two different paths to get to our eyes. The changing of a light ray's direction (loosely called bending) when it passes through variations in matter is called refraction. Refraction is responsible for a tremendous range of optical phenomena, from the action of lenses to voice transmission through optical fibers.



Law of Refraction: Looking at the fish tank as shown, we can see the same fish in two different locations, because light changes directions when it passes from water to air. In this case, the light can reach the observer by two different paths, and so the fish seems to be in two different places. This bending of light is called refraction and is responsible for many optical phenomena.

Refraction: The changing of a light ray's direction (loosely called bending) when it passes through variations in matter is called refraction.

Speed of Light

The speed of light *c* not only affects refraction, it is one of the central concepts of Einstein's theory of relativity. The speed of light varies in a precise manner with the material it traverses. It makes connections between space and time and alters our expectations that all observers measure the same time for the same event, for example. The speed of light is so important that its value in a vacuum is one of the most fundamental constants in nature as well as being one of the four fundamental SI units.

Why does light change direction when passing from one material (medium) to another? It is because light changes speed when going from one material to another.

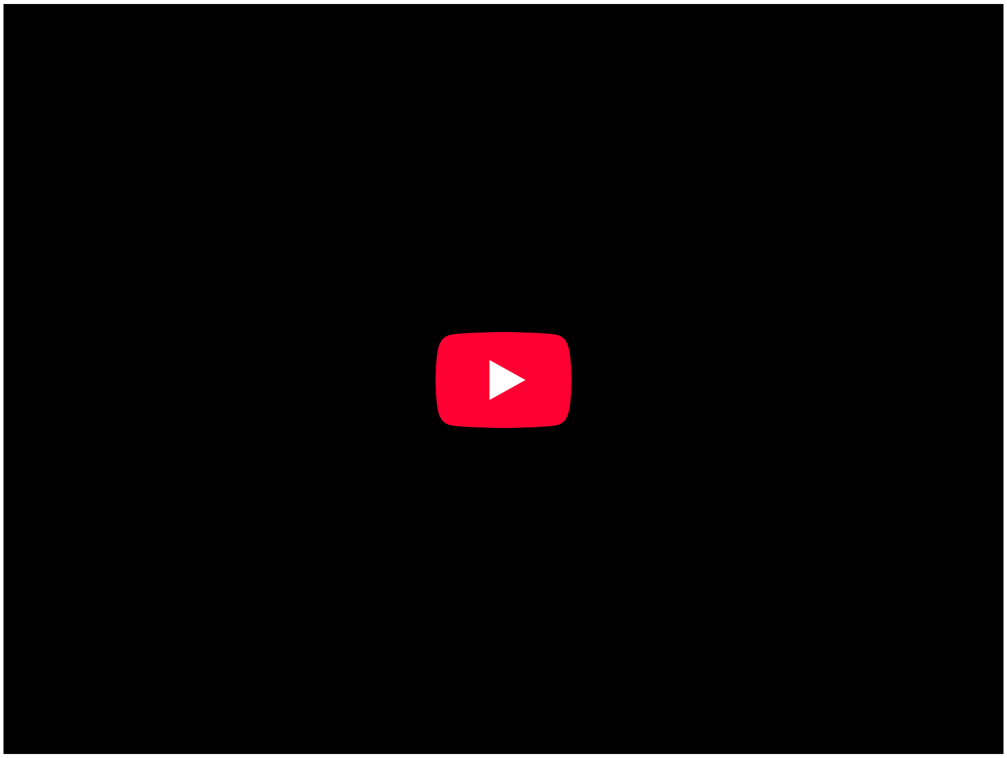
Law of Refraction

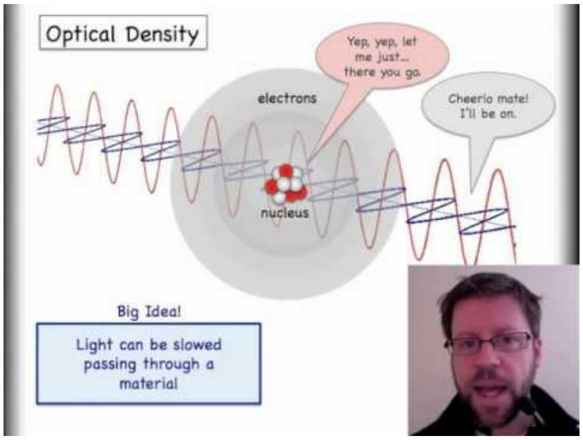
A ray of light changes direction when it passes from one medium to another. As before, the angles are measured relative to a perpendicular to the surface at the point where the light ray crosses it. The change in direction of the light ray depends on how the speed of light changes. The change in the speed of light is related to the indices of refraction of the media involved. In mediums that have a greater index of refraction the speed of light is less. Imagine moving your hand through the air and then moving it through a body of water. It is more difficult to move your hand through the water, and thus your hand slows down if you are applying the same amount of force. Similarly, light travels slower when moving through mediums that have higher indices of refraction.

The amount that a light ray changes its direction depends both on the incident angle and the amount that the speed changes. For a ray at a given incident angle, a large change in speed causes a large change in direction, and thus a large change in angle. The exact mathematical relationship is the law of refraction, or "Snell's Law," which is stated in equation form as:

[Math Processing Error]

Here n_1 and n_2 are the indices of refraction for medium 1 and 2, and θ_1 and $\theta_2$are the angles between the rays and the perpendicular in medium 1 and 2. The incoming ray is called the incident ray and the outgoing ray the refracted ray, and the associated angles the incident angle and the refracted angle. The law of refraction is also called Snell's law after the Dutch mathematician Willebrord Snell, who discovered it in 1621. Snell's experiments showed that the law of refraction was obeyed and that a characteristic index of refraction n could be assigned to a given medium.





Understanding Snell's Law with the Index of Refraction: This video introduces refraction with Snell's Law and the index of refraction.The second video discusses total internal reflection (TIR) in detail. http://www.youtube.com/watch?v=fvrvqm3Erzk

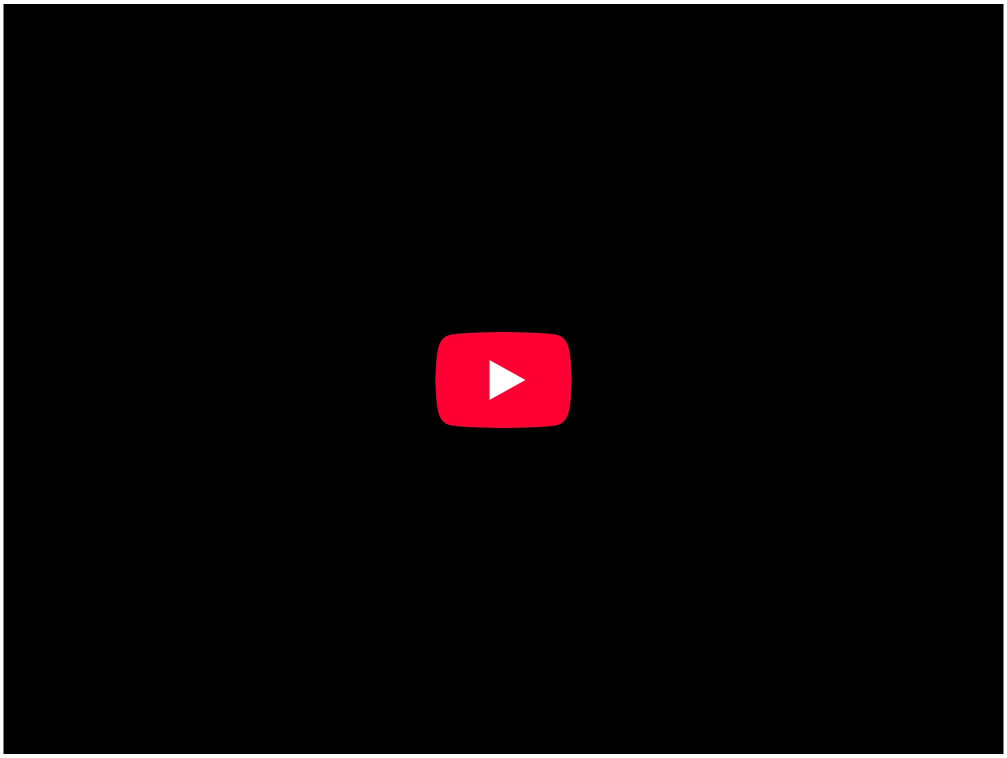
Total Internal Reflection and Fiber Optics

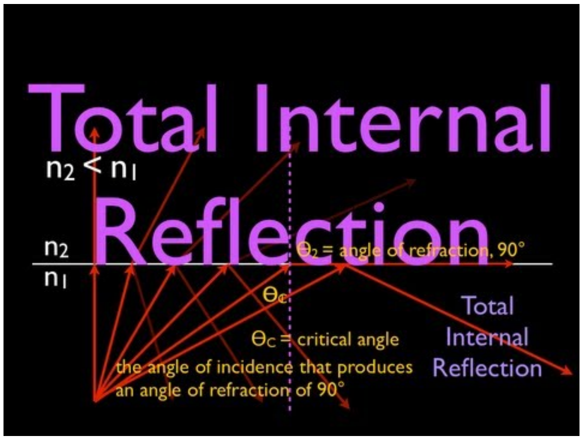
Total internal reflection happens when a propagating wave strikes a medium boundary at an angle larger than a particular critical angle.

learning objectives

- Formulate conditions required for the total internal reflection

Total internal reflection is a phenomenon that happens when a propagating wave strikes a medium boundary at an angle larger than a particular critical angle with respect to the normal to the surface. If the refractive index is lower on the other side of the boundary and the incident angle is greater than the critical angle, the wave cannot pass through and is entirely reflected. The critical angle is the angle of incidence above which the total internal reflectance occurs.





What is Total Internal Reflection?: Describes the concept of total internal reflection, derives the equation for the critical angle and shows one example.

Critical angle

The critical angle is the angle of incidence above which total internal reflection occurs. The angle of incidence is measured with respect to the normal at the refractive boundary (see diagram illustrating Snell's law). Consider a light ray passing from glass into air. The light emanating from the interface is bent towards the glass. When the incident angle is increased sufficiently, the transmitted angle (in air) reaches 90 degrees. It is at this point no light is transmitted into air. The critical angle $\theta c\theta c$ is given by Snell's law, $n1\sin\theta1=n2\sin\theta2n1\sin\theta1=n2\sin\theta2$. Here, n_1 and n_2 are refractive indices of the media, and $\theta1\theta1$ and $\theta2\theta2$are angles of incidence and refraction, respectively. To find the critical angle, we find the value for $\theta1\theta1$ when $\theta2\theta2= 90°$ and thus $\sin\theta2=1\sin\theta2=1$. The resulting value of $\theta1\theta1$ is equal to the critical angle $\theta c=\theta1=\arcsin(n2n1)\theta c=\theta1=\arcsin(n2n1)$. So the critical angle is only defined when n_2/n_1 is less than 1.

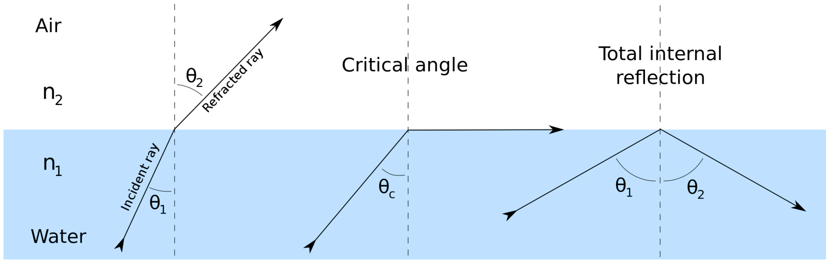


Fig 1: Refraction of light at the interface between two media, including total internal reflection.

Optical Fiber

Total internal reflection is a powerful tool since it can be used to confine light. One of the most common applications of total internal reflection is in fibre optics. An optical fibre is a thin, transparent fibre, usually made of glass or plastic, for transmitting light. The construction of a single optical fibre is shown in.

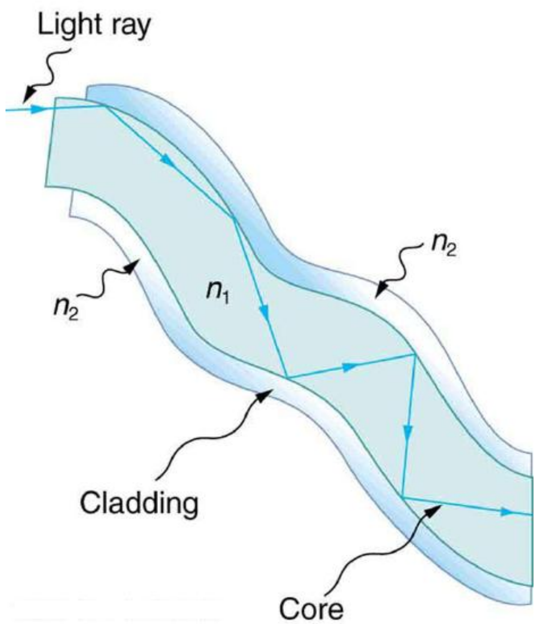


Fig 2: Fibers in bundles are clad by a material that has a lower index of refraction than the core to ensure total internal reflection, even when fibers are in contact with one another. This shows a single fiber with its cladding.

The basic functional structure of an optical fiber consists of an outer protective cladding and an inner core through which light pulses travel. The overall diameter of the fiber is about 125 μm and that of the core is just about 50 μm. The difference in refractive index of the cladding and the core allows total internal reflection in the same way as happens at an air-water surface show in. If light is incident on a cable end with an angle of incidence greater than the critical angle then the light will remain trapped inside the glass strand. In this way, light travels very quickly down the length of the cable over a very long distance (tens of kilometers). Optical fibers are commonly used in telecommunications, because information can be transported over long distances, with minimal loss of data. Another common use can be found in medicine in endoscopes. The field of applied science and engineering concerned with the design and application of optical fibers are called fiber optics.

Total Polarization

Brewster's angle is an angle of incidence at which light with a particular polarization is perfectly transmitted through a surface.

learning objectives

- Calculate the Brewster's angle from the indices of refraction and discuss its physical mechanism

Brewster's angle (also known as the polarization angle) is an angle of incidence at which light with a particular polarization is perfectly transmitted through a transparent dielectric surface, with no reflection. When unpolarized light is incident at this angle, the light that is reflected from the surface is therefore perfectly polarized. This special angle of incidence is named after the Scottish physicist Sir David Brewster (1781–1868).

The physical mechanism for this can be qualitatively understood from the manner in which electric dipoles in the media respond to p-polarized light (whose electric field is polarized in the same plane as the incident ray and the surface normal). One can imagine that light incident on the surface is absorbed, and then re-radiated by oscillating electric dipoles at the interface between the two media. The refracted light is emitted perpendicular to the direction of the dipole moment; no energy can be radiated in the direction of the dipole moment. Thus, if the angle of reflection θ_1 (angle of reflection) is equal to the alignment of the dipoles $(90 - \theta_2)$, where θ_2 is angle of refraction, no light is reflected.

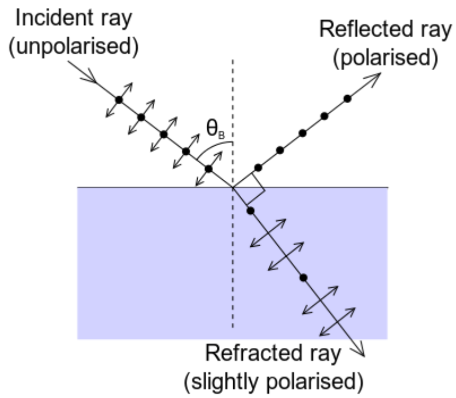
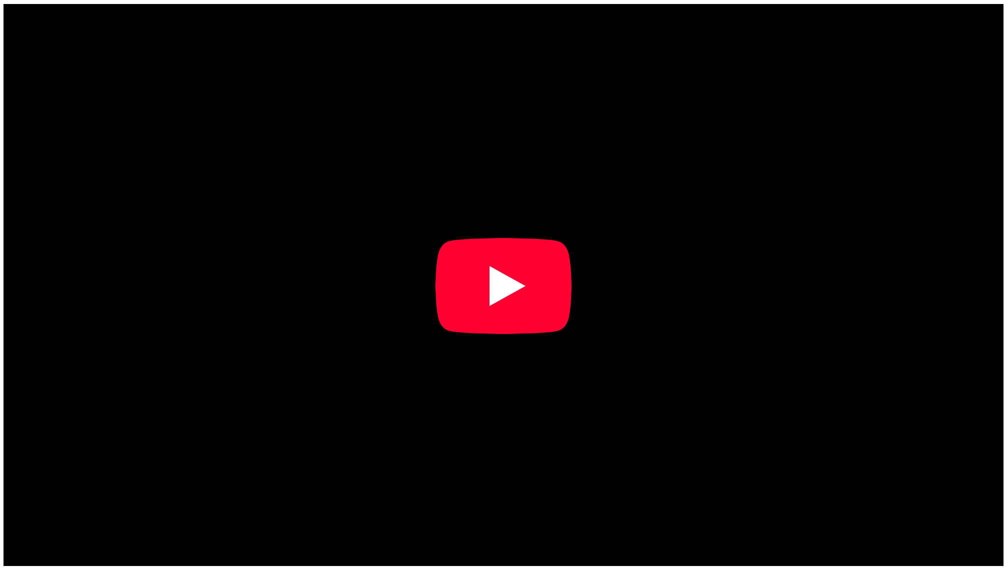


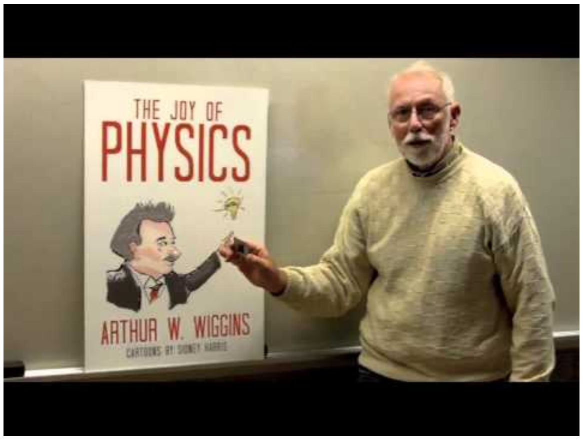
Fig 1: An illustration of the polarization of light that is incident on an interface at Brewster's angle.

This geometric condition can be expressed as *[Math Processing Error]*, where θ_1 is the angle of incidence and θ_2 is the angle of refraction. Using Snell's law ($n_1 \sin\theta 1 = n_2 \sin\theta_2$), one can calculate the incident angle $\theta_1 = {}_B$ at which no light is reflected: *[Math Processing Error]* Solving for θ_B gives *[Math Processing Error]*.

When light hits a surface at a Brewster angle, reflected beam is linearly polarized. shows an example, where the reflected beam was nearly perfectly polarized and hence, blocked by a polarizer on the right picture. Polarized sunglasses use the same principle to reduce glare from the sun reflecting off horizontal surfaces such as water or road.

Fig 2: Photograph taken of a window with a camera polarizer filter rotated to two different angles. In the picture at left, the polarizer is aligned with the polarization angle of the window reflection. In the picture at right, the polarizer has been rotated 90° eliminating the heavily polarized reflected sunlight.





Polarization Experience: A polarizing filter allows light of a particular plane of polarization to pass, but scatters the rest of the light. When two polarizing filters are crossed, almost no light gets through. Some materials have molecules that rotate the plane of polarization of light. When one of these materials is placed between crossed polarizing filters, more light is allowed to pass through.

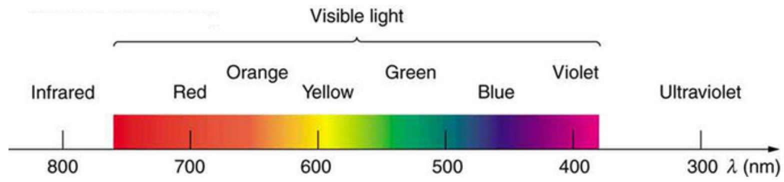
Dispersion: Rainbows and Prisims

Dispersion is defined as the spreading of white light into its full spectrum of wavelengths.

learning objectives

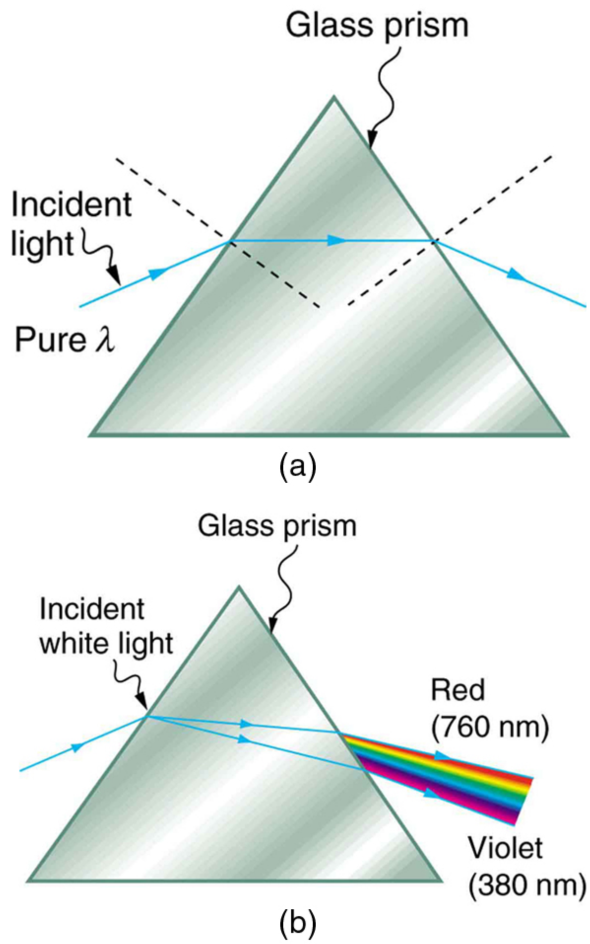
- Describe production of rainbows by a combination of refraction and reflection processes

We see about six colors in a rainbow—red, orange, yellow, green, blue, and violet; sometimes indigo is listed, too. These colors are associated with different wavelengths of light. White light, in particular, is a fairly uniform mixture of all visible wavelengths. Sunlight, considered to be white, actually appears to be a bit yellow because of its mixture of wavelengths, but it does contain all visible wavelengths. The sequence of colors in rainbows is the same sequence as the colors plotted versus wavelength. What this implies is that white light is spread out according to wavelength in a rainbow. Dispersion is defined as the spreading of white light into its full spectrum of wavelengths. More technically, dispersion occurs whenever there is a process that changes the direction of light in a manner that depends on wavelength. Dispersion, as a general phenomenon, can occur for any type of wave and always involves wavelength-dependent processes.



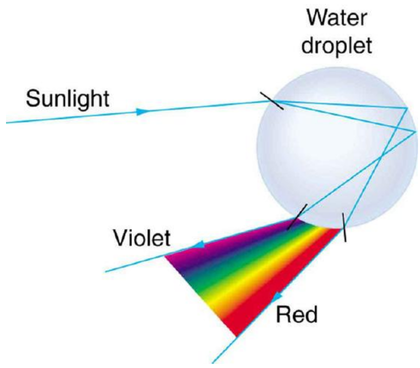
Colors of a Rainbow: Even though rainbows are associated with seven colors, the rainbow is a continuous distribution of colors according to wavelengths.

Refraction is responsible for dispersion in rainbows and many other situations. The angle of refraction depends on the index of refraction, as we saw in the Law of Refraction. We know that the index of refraction *n* depends on the medium. But for a given medium, *n* also depends on wavelength. Note that, for a given medium, *n* increases as wavelength decreases and is greatest for violet light. Thus violet light is bent more than red light and the light is dispersed into the same sequence of wavelengths.



Pure Light and Light Dispersion: (a) A pure wavelength of light falls onto a prism and is refracted at both surfaces. (b) White light is dispersed by the prism (shown exaggerated). Since the index of refraction varies with wavelength, the angles of refraction vary with wavelength. A sequence of red to violet is produced, because the index of refraction increases steadily with decreasing wavelength.

Rainbows are produced by a combination of refraction and reflection. You may have noticed that you see a rainbow only when you look away from the sun. Light enters a drop of water and is reflected from the back of the drop. The light is refracted both as it enters and as it leaves the drop. Since the index of refraction of water varies with wavelength, the light is dispersed, and a rainbow is observed. (There is no dispersion caused by reflection at the back surface, since the law of reflection does not depend on wavelength.) The actual rainbow of colors seen by an observer depends on the myriad of rays being refracted and reflected toward the observer's eyes from numerous drops of water. The arc of a rainbow comes from the need to be looking at a specific angle relative to the direction of the sun.



Light Reflecting on Water Droplet: Part of the light falling on this water drop enters and is reflected from the back of the drop. This light is refracted and dispersed both as it enters and as it leaves the drop.

Key Points

- Light strikes different parts of a rough surface at different angles and is reflected, or diffused, in many different directions.
- A mirror has a smooth surface (compared with the wavelength of light) and so reflects light at specific angles.
- We see the light reflected off a mirror coming from a direction determined by the law of reflection.
- The changing of a light ray's direction (loosely called bending) when it passes through variations in matter is called refraction.
- The index of refraction is n=c/v, where v is the speed of light in the material, c is the speed of light in vacuum, and n is the index of refraction.
- Snell's law, the law of refraction, is stated in equation form as n1sinθ1=n2sinθ2n1sinθ1=n2sinθ2.
- The critical angle is the angle of incidence above which total internal reflection occurs and given as θc=arcsin(n2n1)θc=arcsin(n2n1).
- The critical angle is only defined when n2/n1 is less than 1.
- If light is incident on an optical fiber with an angle of incidence greater than the critical angle then the light will remain trapped inside the glass strand. Light can travel over a very long distance without a significant loss.
- When light hits a surface at a Brewster angle, reflected beam is linearly polarized.
- The physical mechanism for the Brewster's angle can be qualitatively understood from the manner in which electric dipoles in the media respond to p-polarized light.
- Brewsters' angle is given as θB=arctan(n2n1)θB=arctan(n2n1).
- Dispersion occurs whenever there is a process that changes the direction of light in a manner that depends on wavelength. Dispersion can occur for any type of wave and always involves wavelength-dependent processes.
- For a given medium, n increases as wavelength decreases and is greatest for violet light. Thus violet light is bent more than red light, as can be seen with a prism.
- In a rainbow, light enters a drop of water and is reflected from the back of the drop. The light is refracted both as it enters and as it leaves the drop.

Key Terms

- **reflection**: the property of a propagated wave being thrown back from a surface (such as a mirror)
- **refraction**: Changing of a light ray's direction when it passes through variations in matter.
- **index of refraction**: For a material, the ratio of the speed of light in vacuum to that in the material.
- **Snell's law**: A formula used to describe the relationship between the angles of incidence and refraction.
- **cladding**: One or more layers of materials of lower refractive index, in intimate contact with a core material of higher refractive index.
- **dipole**: A separation of positive and negative charges.
- **dielectric**: An electrically insulating or nonconducting material considered for its electric susceptibility (i.e., its property of polarization when exposed to an external electric field).
- **polarizer**: An optical filter that passes light of a specific polarization and blocks waves of other polarizations.

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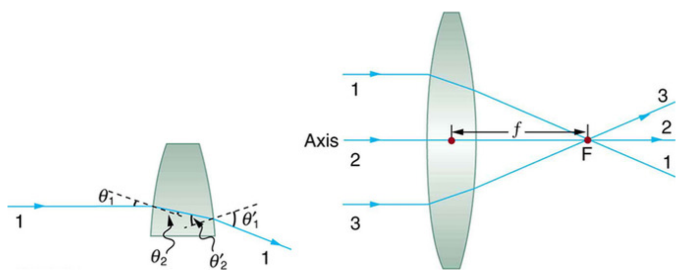
24.3: Lenses

> **learning objectives**
>
> - Describe properties of a thin lens and the purpose of ray tracing

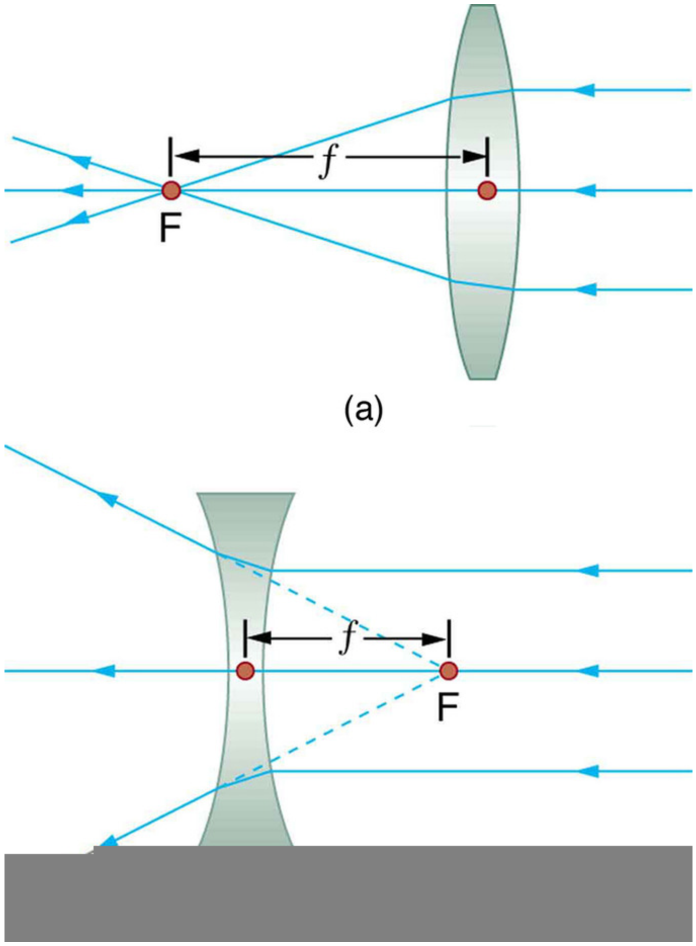
Thin Lenses and Ray Tracing

Ray tracing is the technique of determining or following (tracing) the paths that light rays take. Experiments, as well as our own experiences, show that when light interacts with objects several times as large as its wavelength, it travels in straight lines and acts like a ray. (A ray is simply a straight line that originates at a point.) Its wave characteristics are not pronounced in such situations. Since the wavelength of light is less than a micron (a thousandth of a millimeter), it acts like a ray in the many common situations in which it encounters objects larger than a micron, such as lenses.

For rays passing through matter, the law of refraction is used to trace the paths. Here we use ray tracing to help us understand the action of lenses in situations ranging from forming images on film to magnifying small print to correcting nearsightedness. While ray tracing for complicated lenses, such as those found in sophisticated cameras, may require computer techniques, there is a set of simple rules for tracing rays through thin lenses. A thin lens is defined to be one whose thickness allows rays to refract, as illustrated in, but does not allow properties such as dispersion and aberrations. An ideal thin lens has two refracting surfaces but the lens is thin enough toassume that light rays bend only once. Another way of saying this is that the lens thickness is much much smaller than the focal length of the lens. A thin symmetrical lens has two focal points, one on either side and both at the same distance from the lens. (See.) Another important characteristic of a thin lens is that light rays through its center are deflected by a negligible amount, as seen in the center rays in the first two figures. The treatment of a lens as a thin lens is known as the "thin lens approximation. "



Convex Lens: Rays of light entering a converging lens parallel to its axis converge at its focal point F. (Ray 2 lies on the axis of the lens.) The distance from the center of the lens to the focal point is the lens's focal length f. An expanded view of the path taken by ray 1 shows the perpendiculars and the angles of incidence and refraction at both surfaces.

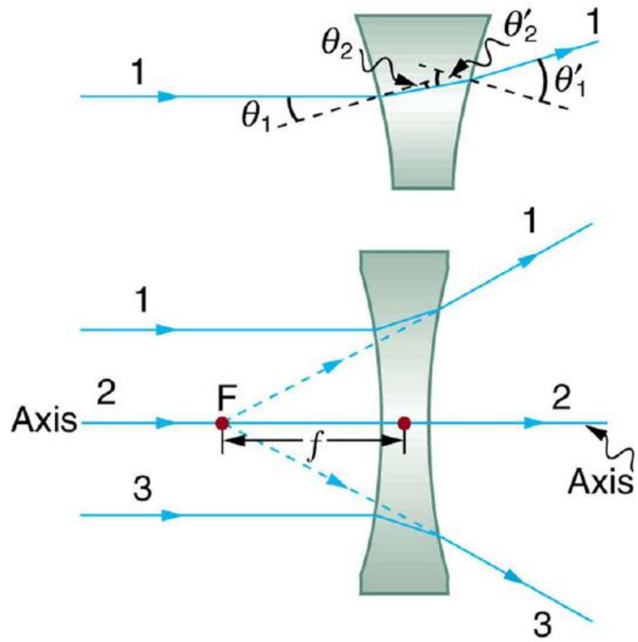


Thin Lens: Thin lenses have the same focal length on either side. (a) Parallel light rays entering a converging lens from the right cross at its focal point on the left. (b) Parallel light rays entering a diverging lens from the right seem to come from the focal point on the right.

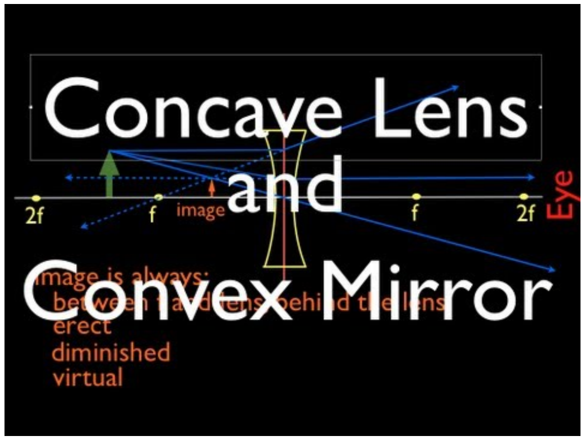
Rules for Ray Tracing

Using paper, pencil, and a straight edge, ray tracing can accurately describe the operation of a lens. The rules for ray tracing for thin lenses are based on the illustrations included in this section:

1. A ray entering a converging lens parallel to its axis passes through the focal point F of the lens on the other side. (See rays 1 and 3 in.)
2. A ray entering a diverging lens parallel to its axis seems to come from the focal point F. (See rays 1 and 3 in.)
3. A ray passing through the center of either a converging or a diverging lens does not change direction. (See ray 2 in and.)
4. A ray entering a converging lens through its focal point exits parallel to its axis. (The reverse of rays 1 and 3 in.)
5. A ray that enters a diverging lens by heading toward the focal point on the opposite side exits parallel to the axis. (The reverse of rays 1 and 3 in).



Diverging Lens: Rays of light entering a diverging lens parallel to its axis are diverged, and all appear to originate at its focal point F. The dashed lines are not rays—they indicate the directions from which the rays appear to come. The focal length f of a diverging lens is negative. An expanded view of the path taken by ray 1 shows the perpendiculars and the angles of incidence and refraction at both surfaces.



Ray Diagrams, Concave Lens and Convex Mirror: Shows how to draw the ray diagrams for locating the image produced by a concave lens and a convex mirror.

The Thin Lens Equation and Magnification

The thin lens equation relates the object distance d_o, image distance d_i, and focal length f.

learning objectives

- Formulate five basic rules of ray tracing

The Thin Lens Equation and Magnification

Image Formation by Thin Lenses

How does a lens form an image of an object? We can use the technique of ray tracing to illustrate how lenses form images. We can also develop equations to describe the images quantitatively. Recall the five basic rules of ray tracing:

1. A ray entering a converging lens parallel to its axis passes through the focal point F of the lens on the other side.
2. A ray entering a diverging lens parallel to its axis seems to come from the focal point F.
3. A ray passing through the center of either a converging or a diverging lens does not change direction.
4. A ray entering a converging lens through its focal point exits parallel to its axis.
5. A ray that enters a diverging lens by heading toward the focal point on the opposite side exits parallel to the axis.

Consider an object some distance away from a converging lens, as shown in. To find the location and size of the image formed, we trace the paths of selected light rays originating from one point on the object (in this case the top of the person's head). The figure shows three rays from the top of the object that can be traced using the five ray tracing rules. Rays leave this point going in many directions, but we concentrate on only a few with paths that are easy to trace. The first ray is one that enters the lens parallel to its axis and passes through the focal point on the other side (rule 1). The second ray passes through the center of the lens without changing direction (rule 3). The third ray passes through the nearer focal point on its way into the lens and leaves the lens parallel to its axis (rule 4). The three rays cross at the same point on the other side of the lens. The image of the top of the person's head is located at this point. All rays that come from the same point on the top of the person's head are refracted in such a way as to cross at the point shown. Rays from another point on the object, such as her belt buckle, will also cross at another common point, forming a complete image, as shown. Although three rays are traced in, only two are necessary to locate the image. It is best to trace rays for which there are simple ray tracing rules. Before applying ray tracing to other situations, let us consider the example shown in in more detail.

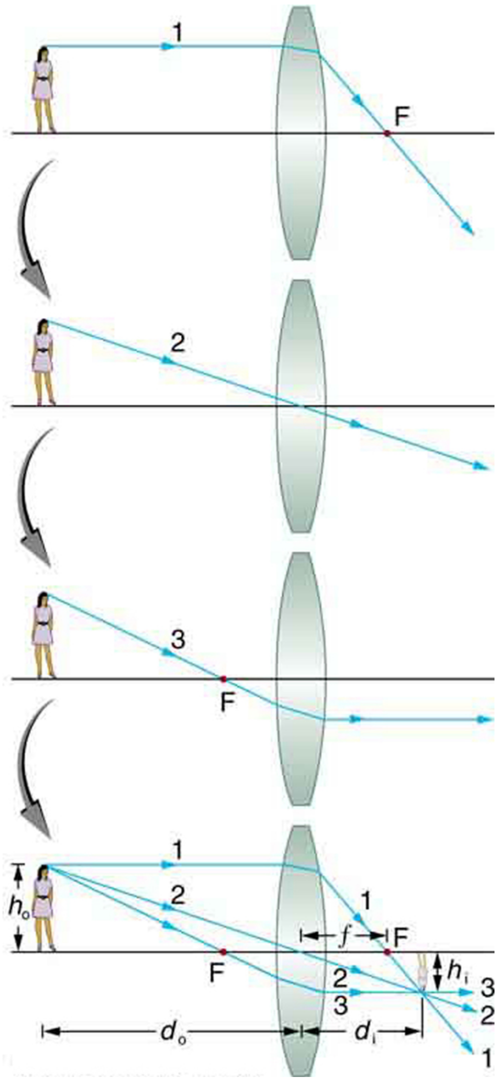


Image Formation with a Thin Lens: Ray tracing is used to locate the image formed by a lens. Rays originating from the same point on the object are traced—the three chosen rays each follow one of the rules for ray tracing, so that their paths are easy to determine. The image is located at the point where the rays cross. In this case, a real image—one that can be projected on a screen —is formed.

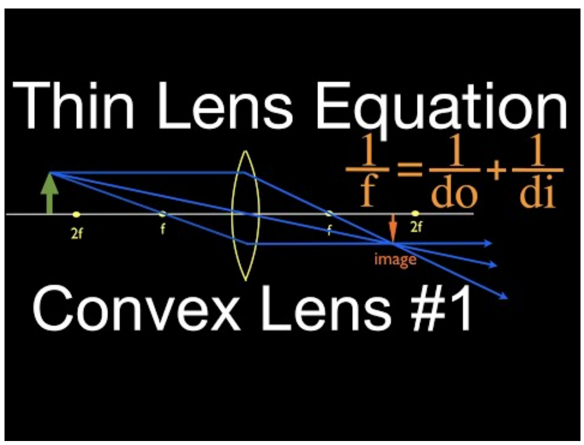
Several important distances appear in. We define d_o as the object distance—the distance of an object from the center of a lens. Image distance $d_i$is defined as the distance of the image from the center of a lens. The height of the object and height of the image are given the symbols h_o and h_i, respectively. Images that appear upright relative to the object have heights that are positive and those that are inverted have negative heights. Using the rules of ray tracing and making a scale drawing with paper and pencil, like that in, we can accurately describe the location and size of an image. But the real benefit of ray tracing is in visualizing how images are formed in a variety of situations. To obtain numerical information, we use a pair of equations that can be derived from a geometric analysis of ray tracing for thin lenses. The thin lens equation is:

$$\frac{1}{\mathrm{d}_o}+\frac{1}{\mathrm{d_i}}=\frac{1}{\mathrm{f}} \tag{24.3.1}$$

We define the ratio of image height to object height (h_i/h_o) as the magnification m. The magnification is related to d_o, d_i, h_o, and h_i by the following relation:

$$\frac{\mathrm{h_i}}{\mathrm{h_o}}=-\frac{\mathrm{d_i}}{\mathrm{d_o}}=\mathrm{m} \tag{24.3.2}$$

In many cases both of these equations are referred to together as the thin lens equations. The thin lens equations are broadly applicable to all situations involving thin lenses (and “thin” mirrors).



Thin Lens Equations for a Convex Lens: Shows how to use the thin lens equation to calculate the image distance, image height and image orientation for convex lenses when the object distance is greater the the focal length (f).

Combinations of Lenses

A compound lens is an array of simple lenses with a common axis.

learning objectives

- Calculate focal length for a compound lens from the focal lengths of simple lenses

COMPOUND LENSES

In contrast to a simple lens, which consists of only one optical element, a compound lens is an array of simple lenses (elements) with a common axis. The use of multiple elements allows for the correction of more optical aberrations, such as the chromatic aberration caused by the wavelength-dependent index of refraction in glass, than is possible using a single lens. In many cases these aberrations can be compensated for to a great extent by using a combination of simple lenses with complementary aberrations.

The simplest case is where lenses are placed in contact: if the lenses of focal lengths f_1 and f_2 are “thin”, the combined focal length f of the lenses is given by

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2} \tag{24.3.3}$$

Since $1/f$ is the power of a lens, it can be seen that the powers of thin lenses in contact are additive.

If two thin lenses are separated in air by some distance d (where d is smaller than the focal length of the first lens), the focal length for the combined system is given by

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2} \tag{24.3.4}$$

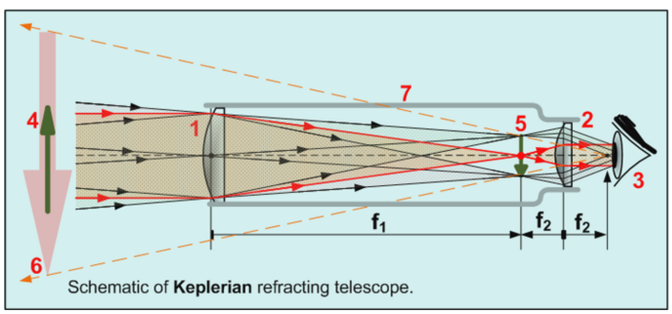
BACK FOCAL LENGTH

The distance from the second lens to the focal point of the combined lenses is called the *back focal length* (BFL).

$$\mathrm{BFL}=\frac{\mathrm{f}_2\left(\mathrm{d}-\mathrm{f}_1\right)}{\mathrm{d}-\left(\mathrm{f}_1+\mathrm{f}_2\right)} \tag{24.3.5}$$

As d tends to zero, the value of the BFL tends to the value of f given for thin lenses in contact.

If the separation distance is equal to the sum of the focal lengths ($d = f_1+f_2$), the combined focal length and BFL are infinite. This corresponds to a pair of lenses that transform a parallel (collimated) beam into another collimated beam (see). This type of system is called an afocal system, since it produces no net convergence or divergence of the beam. Two lenses at this separation form the simplest type of optical telescope. Although the system does not alter the divergence of a collimated beam, it does alter the width of the beam. The magnification of such a telescope is given by



Keplerian Telescope: All refracting telescopes use the same principles. The combination of an objective lens 1 and some type of eyepiece 2 is used to gather more light than the human eye could collect on its own, focus it 5, and present the viewer with a brighter, clearer, and magnified virtual image 6. The magnification can be found by dividing the focal length of the objective lens by the focal length of the eyepiece.

$$\mathrm{M}=-\frac{\mathrm{f}_2}{\mathrm{f}_1} \tag{24.3.6}$$

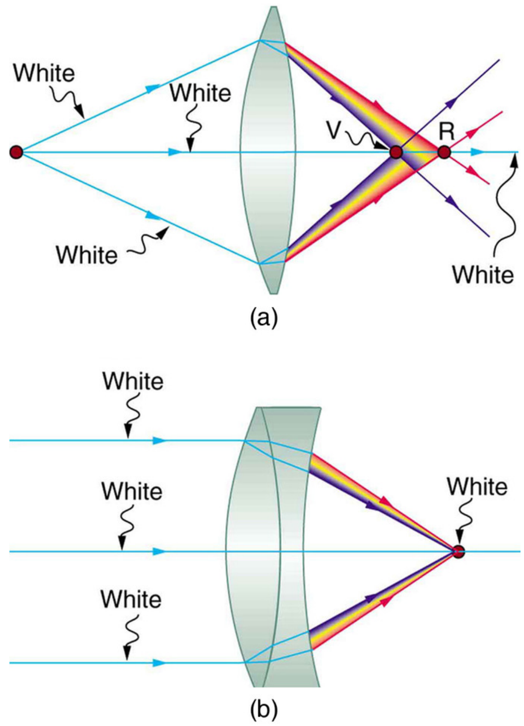
which is the ratio of the input beam width to the output beam width. Note the sign convention: a telescope with two convex lenses ($f_1 > 0$, $f_2 > 0$) produces a negative magnification, indicating an inverted image. A convex plus a concave lens ($f_1 > 0 > f_2$) produces a positive magnification and the image is upright.

ACHROMATS

An achromatic lens or achromat is a lens that is designed to limit the effects of chromatic and spherical aberration. Achromatic lenses are corrected to bring two wavelengths (typically red and blue/violet) into focus in the same plane.

The most common type of achromat is the achromatic doublet, which is composed of two individual lenses made from glasses with different amounts of dispersion Typically, one element is a negative (concave) element made out of flint, which has relatively high dispersion, and the other is a positive (convex) element made of crown glass, which has lower dispersion. The lens elements are mounted next to each other, often cemented together, and shaped so that the chromatic aberration of one is counterbalanced by that of the other.

In the most common type (shown in), the positive power of the crown lens element is not quite equaled by the negative power of the flint lens element. Together they form a weak positive lens that will bring two different wavelengths of light to a common focus. Negative doublets, in which the negative-power element predominates, are also made.



Achromatic Doublet: (a) Chromatic aberration is caused by the dependence of a lens's index of refraction on color (wavelength). The lens is more powerful for violet (V) than for red (R), producing images with different locations and magnifications. (b) Multiple-lens systems, such as this achromatic doublet, can partially correct chromatic aberrations, but they may require lenses of different materials and add to the expense of optical systems such as cameras.

The Lensmaker's Equation

The lensmaker's formula is used to relate the radii of curvature, the thickness, the refractive index, and the focal length of a thick lens.

learning objectives

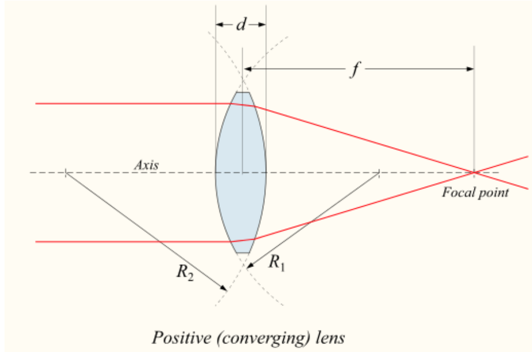
- Compare idealized thin lenses with real lenses

The Lensmaker's Equation

Thick Lenses

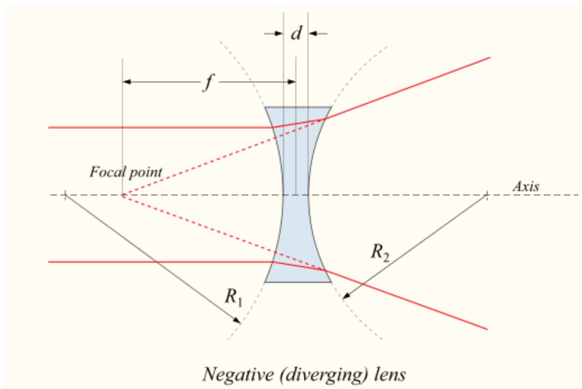
Unlike idealized thin lenses, real lenses have a finite thickness between their two surfaces of curvature. An ideal thin lens with two surfaces of equal curvature would have zero optical power, meaning that it would neither converge nor diverge light. A lens whose thickness is not negligible is called a thick lens. In this case, we can not simply assume that a light ray is only refracted once while traveling through the lens. Instead the extent of the refraction must be dependent on the thickness of the lens.

Lenses are classified by the curvature of the two optical surfaces. A lens is *biconvex* (or *double convex*, or just *convex*) if both surfaces are convex. If the lens is biconvex, a beam of light travelling parallel to the lens axis and passing through the lens will be converged (or *focused*) to a spot on the axis, at a certain distance behind the lens (i.e. the *focal length*). In this case, the lens is called a *positive* or *converging* lens. See for a diagram of a positive (converging) lens.



Thick Converging Lens: Diagram of a positive (converging) lens. The lensmaker's formula relates the radii of curvature, the index of refraction of the lens, the thickness of the lens, and the focal length.

If the lens is biconcave, a beam of light passing through the lens is diverged (spread); the lens is thus called a *negative* or *diverging* lens. The beam after passing through the lens appears to be emanating from a particular point on the axis in front of the lens; the distance from this point to the lens is also known as the focal length, although it is negative with respect to the focal length of a converging lens. See for a diagram of a negative (diverging) lens.



Negative Diverging Lens: Diagram of a negative (diverging) lens. The lensmaker's formula relates the radii of curvature, the index of refraction of the lens, the thickness of the lens, and the focal length.

The focal length of a thick lens *in air* can be calculated from the lensmaker's equation:

$$\mathrm{P}=\frac{1}{\mathrm{f}}=(\mathrm{n}-1)\left[\frac{1}{\mathrm{R}_1}-\frac{1}{\mathrm{R}_2}+\frac{(\mathrm{n}-1)\mathrm{d}}{\mathrm{nR}_1\mathrm{R}_2}\right] \tag{24.3.7}$$

where

- P is the power of the lens,
- f is the focal length of the lens,
- n is the refractive index of the lens material,
- $R_1$is the radius of curvature of the lens surface closest to the light source,
- R_2 is the radius of curvature of the lens surface farthest from the light source, d and is the thickness of the lens (the distance along the lens axis between the two surface vertices).

Sign convention of Radii R_1 and R_2

The signs of the lens' radii of curvature indicate whether the corresponding surfaces are convex or concave. The sign convention used to represent this varies, but for our treatment if R_1 is positive the first surface is convex, and if R_1 is negative the surface is concave. The signs are reversed for the back surface of the lens: if R_2 is positive the surface is concave, and if R_2 is negative the surface is convex. If either radius is infinite, the corresponding surface is flat. With this convention the signs are determined by the shapes of the lens surfaces, and are independent of the direction in which light travels through the lens.

Thin Lens Approximation

The above equation can be greatly simplified if the lens thickness *d* is very small compared to R_1 and R_2. In this case, the thin lens approximation can then be made and the lensmaker's equation can be approximated as

$$P = \frac{1}{f} \approx (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \tag{24.3.8}$$

The focal length *f* is positive for converging lenses, and negative for diverging lenses. The reciprocal of the focal length, 1/*f*, is the optical of the lens. If the focal length is in meters, this gives the optical power in diopters (inverse meters).

Lenses have the same focal length when light travels from the back to the front as when light goes from the front to the back, although other properties of the lens, such as the aberrations are not necessarily the same in both directions.

Refraction Through Lenses

Because the index of refraction of a lens is greater than air, a ray moves towards the perpendicular as it enters and away as it leaves.

learning objectives

- Compare the effect of a convex lens and a concave lens on the light rays

Refraction Through Lenses

Lenses are found in a huge array of optical instruments, ranging from the simple magnifying glass to a camera lens to the lens of the human eye. The word *lens* derives from the Latin word for lentil bean—the shape of which is similar to that of the convex lens (as shown in). The convex lens is shaped so that all light rays that enter it parallel to its axis cross one another at a single point on the opposite side of the lens. The axis is defined as a line normal to the lens at its center (as shown in). Such a lens is called a converging (or convex) lens for the corresponding effect it has on light rays. The expanded view of the path of one ray through the lens illustrates how the ray changes direction both as it enters and as it leaves the lens.

Since the index of refraction of the lens is greater than that of air, the ray moves towards the perpendicular as it enters, and away from the perpendicular as it leaves (this is in accordance with the law of refraction). Due to the lens's shape, light is thus bent toward the axis at both surfaces. The point at which the rays cross is defined as the focal point F of the lens. The distance from the center of the lens to its focal point is defined as the focal length f of the lens. shows how a converging lens, such as that in a magnifying glass, can concentrate (converge) the nearly parallel light rays from the sun towards a small spot.

Magnifying Glass: Sunlight focused by a converging magnifying glass can burn paper. Light rays from the sun are nearly parallel and cross at the focal point of the lens. The more powerful the lens, the closer to the lens the rays will cross.

The greater effect a lens has on light rays, the more powerful it is said to be. For example, a powerful converging lens will focus parallel light rays closer to itself and will have a smaller focal length than a weak lens. The light will also focus into a smaller, more intense spot for a more powerful lens. The power P of a lens is defined as the inverse of its focal length. In equation form:

$$\mathrm{P}=\frac{1}{\mathrm{f}} \tag{24.3.9}$$

shows the effect of a concave lens on rays of light entering it parallel to its axis (the path taken by ray 2 in the figure is the axis of the lens). The concave lens is a *diverging lens,* because it causes the light rays to bend away (diverge) from its axis. In this case, the lens is shaped so that all light rays entering it parallel to its axis appear to originate from the same point F, defined as the focal point of a diverging lens. The distance from the center of the lens to the focal point is again called the focal length f of the lens. Note that the focal length and power of a diverging lens are defined as negative. For example, if the distance to F in is 5.00 cm, then the focal length is f=–5.00 cm and the power of the lens is P=–20 D. The expanded view of the path of one ray through the lens illustrates how the shape of the lens (given the law of refraction) causes the ray to follow its particular path and be diverged.

In subsequent sections we will examine the technique of ray tracing to describe the formation of images by lenses. Additionally, we will explore how image locations and characteristics can be quantified with the help of a set of geometric optics equations.

Key Points

- When light interacts with objects several times as large as its wavelength, it travels in straight lines and acts like a ray. A ray is simply a straight line that originates at a point.
- Ray tracing is the method for determining the paths light takes through matter, such as optical systems that include lenses.
- A thin lens is defined as one with a thickness that allows rays to refract, as illustrated in, but that does not allow properties such as dispersion and aberrations. An ideal thin lens has two refracting surfaces but the lens is thin enough to assume that light rays bend only once.
- There are five basic rules for tracing rays through a lens.
- Ray tracing can be used to construct an image from the light rays originating from an object that pass through a lens. The image is located at the point where the rays cross. By choosing several points from an object the entire image can be constructed.
- We define do to be the object distance, the distance of an object from the center of a lens. Image distance di is defined to be the distance of the image from the center of a lens. The height of the object and height of the image are given the symbols ho and hi, respectively.
- The thin lens equation quickly provides the relation between d_i, d_o, and the focal length f. It can be derived from a geometric analysis of ray tracing for thin lenses and is given by $\frac{1}{\mathrm{d_o}}+\frac{1}{\mathrm{d_i}}=\frac{1}{\mathrm{f}}$.
- The magnification m of an image is the ratio between the image and object height (h_i/h_o). The magnification is related to d_o, d_i, h_o, and h_i by the following relation: $\frac{\mathrm{h_i}}{\mathrm{h_o}}=-\frac{\mathrm{d_i}}{\mathrm{d_o}}=\mathrm{m}$.
- The use of multiple elements allows for the correction of more optical aberrations, such as the chromatic abberation caused by the wavelength -dependent index of refraction in glass, than is possible using a single lens.
- If the lenses of focal lengths f_1 and $f_2$are “thin”, the combined focal length f of the lenses is given by $\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$ while if the lenses are separated by some distance d then the combined focal length is given by $\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2}$.
- If the separation distance is equal to the sum of the focal lengths ($d = f_1+f_2$), the combined focal length is infinite. This corresponds to a pair of lenses that transform a collimated beam into another collimated beam. This type of system is called an afocal system (a simple optical telescope).
- An achromatic doublet is a kind of compound lens designed to bring two wavelengths (typically red and blue/violet) into focus in the same plane. This (partially) corrects for the chromatic aberration found in a single simple lens. See.
- If a lens is biconvex, a beam of light travelling parallel to the lens axis and passing through the lens will be focused to a spot on the axis, at a certain distance behind the lens (i.e. the focal length). In this case, the lens is called a positive or converging lens. See.
- If a lens is biconcave, a beam of light passing through the lens is diverged (spread); the lens is thus called a negative or diverging lens. See.
- The focal length of a thick lens in air can be calculated from the lensmaker’s equation: $\mathrm{P}=\frac{1}{\mathrm{f}}=(\mathrm{n}-1)\left[\frac{1}{\mathrm{R_1}}-\frac{1}{\mathrm{R_2}}+\frac{(\mathrm{n}-1)\mathrm{d}}{\mathrm{nR_1R_2}}\right]$.
- The signs of the lens’ radii of curvature indicate whether the corresponding surfaces are convex or concave. The signs are reversed for the back surface of the lens: if R_2 is positive the surface is concave, and if R_2 is negative the surface is convex.
- The lensmaker’s equation can be greatly simplified if the lens thickness d is very small compared to R_1 and R_2. In this case, the thin lens approximation can then be made and the lensmaker’s equation can be approximated as $P=\frac{1}{f}\approx(n-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$.

- Recall that the a ray will bend as it enters a medium with a different refractive index. Since the refractive index of a lens is greater than air, a light ray will move towards the perpendicular as it enters and away as it leaves.
- A convex lens has been shaped so that all light rays that enter it parallel to its axis cross one another at a single point on the opposite side of the lens (the focal point). Such a lens is called a converging (or convex) lens for the converging effect it has on light rays. See.
- A concave lens is a diverging lens, because it causes the light rays to bend away (diverge) from its axis. shows the effect it has on rays of light that enter it parallel to its axis (the path taken by ray 2 in the figure is the axis of the lens).
- The greater effect a lens has on light rays, the more powerful it is said to be. A powerful converging lens will focus parallel light rays closer to itself and will have a smaller focal length than a weak lens. The power of a lens is given by the equation $\mathrm{P}=\frac{1}{\mathrm{f}}$.

Key Terms

- **focal point**: A focus—a point at which rays of light or other radiation converge.
- **ray tracing**: A technique used in optics for analysis of optical systems.
- **thin lens**: A thin lens is defined to be one whose thickness allows rays to refract but does not allow properties such as dispersion and aberrations.
- **thin lens equation**: Relates object distance do, image distance di, and focal length f: $\frac{1}{\mathrm{d_o}}+\frac{1}{\mathrm{d_i}}=\frac{1}{\mathrm{f}}$
- **image distance**: The distance of the image from the center of the lens.
- **magnification**: The apparent enlargement of an object in an image.
- **aberration**: The convergence to different foci, by a lens or mirror, of rays of light emanating from one and the same point, or the deviation of such rays from a single focus; a defect in a focusing mechanism that prevents the intended focal point.
- **afocal system**: An optical system that produces no net convergence or divergence of the beam, i.e. has an infinite effective focal length. This type of system can be created with a pair of optical elements where the distance between the elements is equal to the sum of each element's focal length (d = f1+f2).
- **achromatic doublet**: A type of lens made up of two simple lenses paired together designed so that the chromatic aberration of each lens partially offsets the other; in this way light in a range of wavelengths may be brought to the same focus.
- **thick lens**: Lenses whose thicknesses are not negligible (i.e., one cannot make the simple assumption that a light ray is refracted only once in the lens).
- **surface vertices**: The points where each surface crosses the optical axis. They are important primarily because they are the physically measurable parameters for the position of the optical elements, and so the positions of the other cardinal points must be known with respect to the vertices to describe the physical system.
- **convex lens**: A lens having at least one convex surface, such that light passing through it, may be brought to a focus.
- **concave lens**: A lens having at least one concave surface, such that light rays passing through it bend away from its optical axis.

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24.4: Mirrors

learning objectives

- Describe interaction of the light with a mirror surface

Plane Mirrors and Reflection

A mirror is a reflective surface that does not allow the passage of light and instead bounces it off, thus producing an image. The most common mirrors are flat and called plane mirrors. These mirrors are made by putting a thin layer of silver nitrate or aluminium behind a flat piece of glass.

When you place an object in front of a mirror, you see an image of the same object in the mirror. The object is the source of the incident rays, and the image is formed by the reflected rays. An image formed by reflection may be real or virtual. A “real” image occurs when light rays actually intersect at the image, and become inverted, or turned upside down. A “virtual” image occurs when light rays do not actually meet at the image. Instead, you “see” the image because your eye projects light rays backward. You are fooled into seeing an image! A virtual image is right side up (upright).

In flat, or plane mirrors, the image is a virtual image, and is the same distance behind the mirror as the object is in front of the mirror. The image is also the same size as the object. These images are also parity inverted, which means they have a left-right inversion.

Ray Diagrams

The way that we can predict how a reflection will look is by drawing a ray diagram. These diagrams can be used to find the position and size of the image and whether that image is real or virtual. These are the steps you follow to draw a ray diagram:

1. Draw the plane mirror as a straight line on a principal axis. The principal axis is an imaginary line that is drawn perpendicular to the mirror.
2. Draw the object as an arrow in front of the mirror.
3. Draw the image of the object, by using the principle that the image is placed at the same distance behind the mirror that the object is in front of the mirror. The image size is also the same as the object size. shows these first three steps.
4. Place a dot at the point the eye is located.
5. Pick one point on the image and draw the reflected ray that travels to the eye as it sees this point. Remember to add an arrowhead.
6. Draw the incident ray for light traveling from the corresponding point on the object to the mirror, such that the law of reflection is obeyed.
7. Continue for other extreme points on the object (i.e. the tip and base of the arrow). A completed ray diagram is shown in

The angle in which a light ray hits the mirror is the same angle in which it will be reflected back. If, for example, a light ray leaves the top of an object travelling parallel to the principal axis, it will hit the mirror at a 0 degree angle, and be reflected back at 0 degrees. When this happens, we say the ray hit the mirror normally. If the light ray hit the object at a 30 degree angle, it will be reflected back at a 30 degree angle.

Image Formation by Spherical Mirrors: Reflection and Sign Conventions

A mirror is a reflective surface that light does not pass through, made by a layer of silver nitrate or aluminium behind piece of glass.

learning objectives

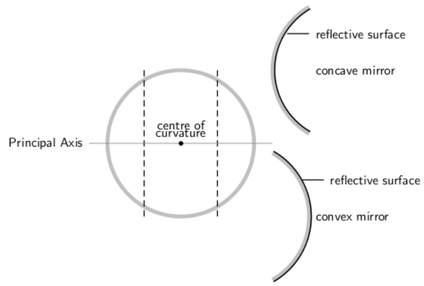
- Distinguish properties of the concave and the convex mirrors

Overview

A mirror is a reflective surface that light does not pass through, but bounces off of and this produces an image. Mirrors are made by putting a thin layer of silver nitrate or aluminium behind a flat piece of glass.

When you place an object in front of a mirror, you see the same object in the mirror. This image that appears to be behind the mirror is called the image. The object is the source of the incident rays, and the image is formed by the reflected rays. An image formed by reflection may be real or virtual. A real image occurs when light rays actually intersect at the image, and is inverted, or upside down. A virtual image occurs when light rays do not actually meet at the image. Instead, you “see” the image because your eye projects light rays backward. A virtual image is right side up (upright).

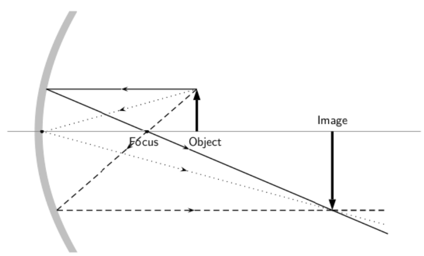
This section will cover spherical mirrors. Spherical mirrors can be either concave or convex. The center of curvature is the point at the center of the sphere and describes how big the sphere is. These concepts are shown in.



Spherical Mirrors: This figure shows the difference between a concave and convex mirror.

Concave Mirrors

In a concave mirror, the principal axis is a line that is perpendicular to the center of the mirror. The easiest way to visualize what a image will look like in this type of mirror is a ray diagram. Before that can be done, the focal point must first be defined. This point is half way between the mirror and the center of curvature on the principal axis. The distance to the focal point from the mirror is called the focal length. We can see from the figure that this focal length is also equal to half of the radius of the curvature. shows the ray diagram of a concave mirror.



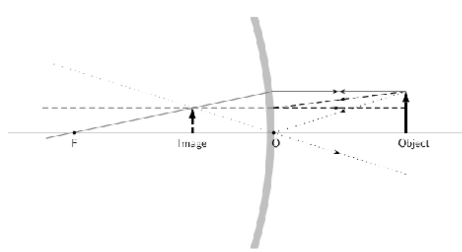
Concave Ray Diagram: This is a ray diagram of a concave mirror. The steps taken to draw are the same as those in a plane mirror.

A summary of the properties of concave mirrors is shown below:

- converging
- real image
- inverted
- image in front of mirror

Convex Mirrors

In convex mirrors, the principal axis is the same as in a plane or concave mirror, perpendicular to the center of the mirror. In this case, the focal point is behind the mirror. A convex mirror has a negative focal length because of this. The focal point is the same distance from the mirror as in a concave mirror. This is shown in.



Convex Mirror Ray Diagram: A convex mirror with three rays drawn to locate the image. Each incident ray is reflected according to the Law of Reflection. The reflected rays diverge. If the reflected rays are extended behind the mirror, then their intersection gives the location of the image behind the mirror. For a convex mirror, the image is virtual and upright.

A summary of the properties of convex mirrors is shown below:

- diverging
- virtual image
- upright
- image behind mirror

Key Points

- Reflected images can be either real or virtual. In a plane mirror, the images are virtual.
- The virtual images in a plane mirror have a left-right inversion.
- Drawing a ray diagram is a way to predict what a reflected image will look like.
- Images in mirrors can be either real or virtual.
- A summary of the properties of the concave mirrors are shown below: converging real image inverted image in front of mirror.
- A summary of the properties of the convex mirrors are shown below: diverging virtual image upright image behind mirror.

Key Terms

- **virtual image**: A virtual image occurs when light rays do not actually meet at the image
- **concave**: curved like the inner surface of a sphere or bowl
- **convex**: curved or bowed outward like the outside of a bowl or sphere or circle

CHAPTER OVERVIEW

25: Vision and Optical Instruments

Topic hierarchy

25.1: The Human Eye

learning objectives

- Identify parts of human eye and their functions

The human eye is the gateway to one of our five senses. The human eye is an organ that reacts with light. It allows light perception, color vision and depth perception. A normal human eye can see about 10 million different colors! There are many parts of a human eye, and that is what we are going to cover in this atom.

Properties

Contrary to what you might think, the human eye is not a perfect sphere, but is made up of two differently shaped pieces, the cornea and the sclera. These two parts are connected by a ring called the limbus. The part of the eye that is seen is the iris, which is the colorful part of the eye. In the middle of the iris is the pupil, the black dot that changes size. The cornea covers these elements, but is transparent. The fundus is on the opposite of the pupil, but inside the eye and can not be seen without special instruments. The optic nerve is what conveys the signals of the eye to the brain. is a diagram of the eye. The human eye is made up of three coats:

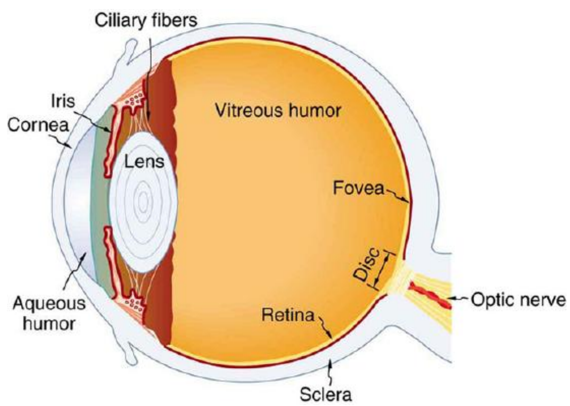


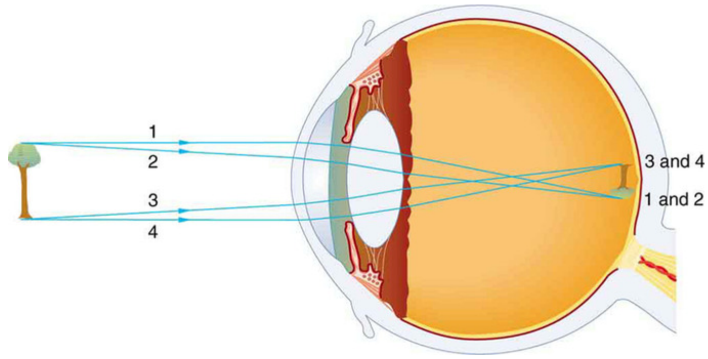
Diagram of the Human Eye: The cornea and lens of an eye act together to form a real image on the light-sensing retina, which has its densest concentration of receptors in the fovea and a blind spot over the optic nerve. The power of the lens of an eye is adjustable to provide an image on the retina for varying object distances. Layers of tissues with varying indices of refraction in the lens are shown here. However, they have been omitted from other pictures for clarity.

1. Outermost Layer – composed of the cornea and the sclera.
2. Middle Layer – composed of the choroid, ciliary body and iris.
3. Innermost Layer – the retina, which can be seen with an instrument called the ophthalmoscope.

Once you are inside these three layers, there is the aqueous humor (clear fluid that is contained in the anterior chamber and posterior chamber), vitreous body (clear jelly that is much bigger than the aqueous humor), and the flexible lens. All of these are connected by the pupil.

Dynamics

Whenever the eye moves, even just a little, it automatically readjusts the exposure by adjusting the iris, which regulates the size of the pupil. This is what helps the eye adjust to dark places or really bright lights. The lens of the eye is similar to one in glasses or cameras. The human eye is had an aperture, just like a camera. The pupil serves this function, and the iris is the aperture stop. The different parts of the eye has different refractive indexes, and this is what bends the rays to form an image. The cornea provides two-thirds of the power to the eye. The lens provides the remaining power. The image passes through several layers of the eye, but happens in a way very similar to that of a convex lens. When the image finally reaches the retena, it is inverted, but the brain will correct this. shows what happens.



Vision Diagram: An image is formed on the retina with light rays converging most at the cornea and upon entering and exiting the lens. Rays from the top and bottom of the object are traced and produce an inverted real image on the retina. The distance to the object is drawn smaller than scale.

Eye Movement

Each eye has six muscles; lateral rectus, medial rectus, inferior rectus, superior rectus, inferior oblique, and superior oblique. All of these muscles provide differnt tensions and torques to control the movement of the eye. These are a few examples of types of eye movement:

- Rapid Eye Movement – Often referred to as REM, this happens in the sleep stage when most vivid dreams occur.
- Saccade – These are quick, simultaneous movements of both eyes, and is controlled by the frontal lobe of the brain.
- Vestibulo-ocular Reflex – This is the eye movement which is opposite to the movement of the head and keeps the object you are looking at in the center of vision.
- Pursuit Movement – This is the tracking movement when you are following a moving object. It is less accurate than the vestibulo-ocular reflex.

Color Vision

Using the cone cells in the retina, we perceive images in color; each type of cone specifically sees in regions of red, green, or blue.

learning objectives

- Explain how the human eye perceives colors

With human eyesight, cone cells are responsible for color vision. From there, it is important to understand how color is perceived. Using the cone cells in the retina, we perceive images in color. Each type of cone specifically sees in regions of red, green, or blue, (RGB), in the color spectrum of red, orange, yellow, green, blue, indigo, violet.

The colors in between these absolutes are seen as different linear combinations of RGB. This is why TVs and computer screens are made up of thousands of little red, green, or blue lights, and why colors in electronic form are represented by different values of RGB. These values are usually given in the value of their frequency in log form.

YUV Color Space

The human eye is more sensitive to intensity changes than color changes, which is why it is acceptable to use black and white photography in place of color and why people can still distinguish everything in the photo without colors. The intensity, or luminance Y, can be found from the following equation:

$$\mathrm{Y = 0.3R + 0.6G + 0.1B}$$
$$\mathrm{Y = 0.3R + 0.6G + 0.1B} \tag{25.1.1}$$

The prior equation deals with the luminance, but the chrominance (dealing with colors) can be found from the following equations:

$$\mathrm{U = 0.5(BY)}$$
$$\mathrm{U = 0.5(B - Y)} \tag{25.1.2}$$
$$\mathrm{V = 0.625(R - Y)}$$
$$\mathrm{V = 0.625(RY)} \tag{25.1.3}$$

You can go from RGB to YUV color spaces with the following matrix operation:

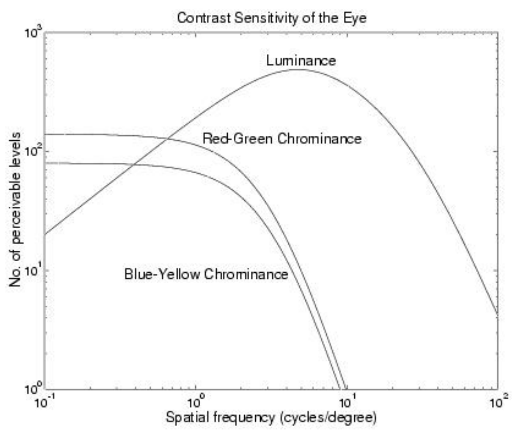
$$\begin{pmatrix} \mathrm{Y} \\ \mathrm{U} \\ \mathrm{V} \end{pmatrix} = \mathrm{C} * \begin{pmatrix} \mathrm{R} \\ \mathrm{G} \\ \mathrm{B} \end{pmatrix} \tag{25.1.4}$$

Where C is equal to:

$$\begin{pmatrix} 0.3 & 0.6 & 0.1 \\ -0.15 & -0.3 & 0.45 \\ 0.4375 & -0.3750 & -0.0625 \end{pmatrix} \tag{25.1.5}$$

Visual Sensitivity

In, we can see that



Visual Sensitivity: This graph shows the sensitivity of the eye to luminance (Y) and chrominance (U, V) components of images. The horizontal scale is spatial frequency, and represents the frequency of an alternating pattern of parallel stripes with sinusoidally varying intensity. The vertical scale is the contrast sensitivity of human vision, which is the ratio of the maximum visible range of intensities to the minimum discernible peak-to-peak intensity variation at the specified frequency.

- the maximum sensitivity to Y occurs for spatial frequencies around 5 cycles / degree, which corresponds to striped patterns with a half-period (stripe width) of 1.8 mm at a distance of 1 m (~arm's length).
- The eye has very little response above 100 cycles / degree, which corresponds to a stripe width of 0.1 mm at 1 m. On a standard PC display of width 250 mm, this would require 2500 pels per line! Hence the current SVGA standard of 1024×768 pels still falls somewhat short of the ideal and is limited by CRT spot size. Modern laptop displays have a pel size of about 0.3 mm, but are pleasing to view because the pel edges are so sharp (and there is no flicker).
- The sensitivity to luminance drops off at low spatial frequencies, showing that we are not very good at estimating absolute luminance levels as long as they do not change with time – the luminance sensitivity to temporal fluctuations (flicker) does not fall off at low spatial frequencies.
- The maximum chrominance sensitivity is much lower than the maximum luminance sensitivity with blue-yellow (U) sensitivity being about half of red-green (V) sensitivity and about 16 of the maximum luminance sensitivity.
- The chrominance sensitivities fall off above 1 cycle / degree, requiring a much lower spatial bandwidth than luminance.

We can now see why it is better to convert to the YUV domain before attempting image compression. The U and V components may be sampled at a lower rate than Y (due to narrower bandwidth) and may be quantified more coarsely (due to lower contrast sensitivity).

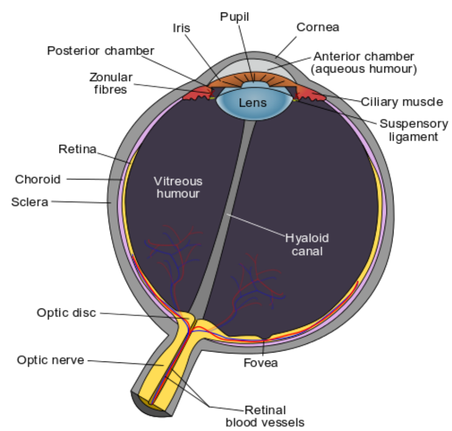
Resolution of the Human Eye

The human eye is a sense organ that allows vision and is capable to distinguish about 10 million colors.

learning objectives

- Describe field of view and color sensitivity of the human eye

The human eye is an organ that reacts to light in many circumstances. As a conscious sense organ the human eye allows vision; rod and cone cells in the retina allow conscious light perception and vision, including color differentiation and the perception of depth. The human eye can distinguish about 10 million colors. A model of the human eye can be seen in.



Schematic Diagram of the Human Eye: Structure of the eye and closeup of the retina.

The retina of human eye has a static contrast ratio of around 100:1 (about 6.5 f-stops). As soon as the eye moves, it re-adjusts its exposure, both chemically and geometrically, by adjusting the iris (which regulates the size of the pupil). Initial dark adaptation takes place in approximately four seconds of profound, uninterrupted darkness; full adaptation, through adjustments in retinal chemistry, is mostly complete in thirty minutes. Hence, a dynamic contrast ratio of about 1,000,000:1 (about 20 f-stops) is possible. The process is nonlinear and multifaceted, so an interruption by light starts the adaptation process over again. Full adaptation is dependent on good blood flow (thus dark adaptation may be hampered by poor circulation, and vasoconstrictors like tobacco).

The eye includes a lens not dissimilar to lenses found in optical instruments (such as cameras). The same principles can be applied. The pupil of the human eye is its aperture. The iris is the diaphragm that serves as the aperture stop. Refraction in the cornea causes the effective aperture (the entrance pupil) to differ slightly from the physical pupil diameter. The entrance pupil is typically about 4 mm in diameter, although it can range from 2 mm (f/8.3) in a brightly lit place to 8 mm (f/2.1) in the dark. The latter value decreases slowly with age; older people's eyes sometimes dilate to not more than 5-6mm.

The approximate field of view of an individual human eye is 95° away from the nose, 75° downward, 60° toward the nose, and 60° upward, allowing humans to have an almost 180-degree forward-facing horizontal field of view. With eyeball rotation of about 90° (head rotation excluded, peripheral vision included), horizontal field of view is as high as 170°. About 12–15° temporal and 1.5° below the horizontal is the optic nerve or blind spot which is roughly 7.5° high and 5.5° wide.

Nearsightedness, Farsidedness, and Vision Correction

In order for the human eye to see clearly, the image needs to be formed directly on the retina; if it is not, the image is blurry.

learning objectives

- Identify factors responsible for nearsightedness and farsightedness vision defects

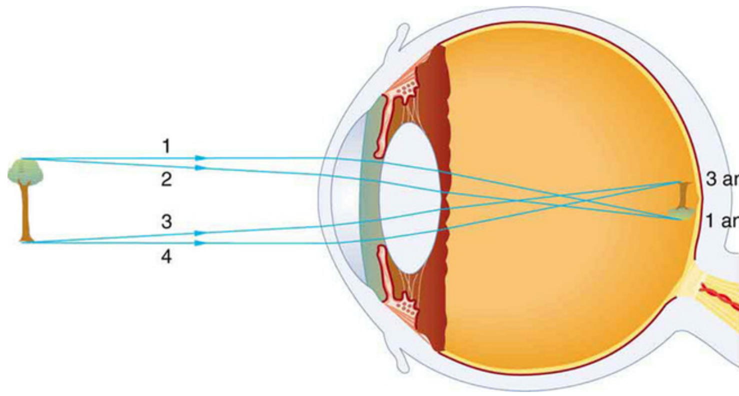
The human eye is the gateway to one of our five senses. The human eye is an organ that reacts with light. It allows light perception, color vision, and depth perception, but not all eyes are perfect. A normal human eye can see about 10 million different colors!

Properties

Contrary to what you might think, the human eye is not a perfect sphere, but is made up of two differently shaped pieces, the cornea and the sclera. These two parts are connected by a ring called the limbus. The part of the eye that is seen is the iris, which is the colorful part of the eye. In the middle of the iris is the pupil, which is the black dot that changes size. The cornea covers these elements, but it is transparent. The fundus is on the opposite of the pupil, but inside the eye and cannot be seen without special instruments. The optic nerve is what conveys the signals of the eye to the brain. shows a diagram of the eye.

Vision

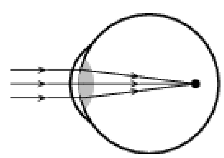
The different parts of the eye have different refractive indexes, and this is what bends the rays to form an image. The cornea provides two-thirds of the power to the eye. The lens provides the remaining power. The image passes through several layers of the eye, but this happens in a way very similar to that of a convex lens. When the image finally reaches the retena, it is inverted, but the brain will correct this. For the vision to be clear, the image has to be formed directly on the retina. The focus needs to be changed, much like a camera, depending on the distance and size of the object. The eye's lens is flexible, and changes shape. This changes the focal length. The eye's ciliary muscles control the shape of the lens. When you focus on something, you squeeze or relax these muscles.



Vision Diagram: An image is formed on the retina with light rays converging most at the cornea and upon entering and exiting the lens. Rays from the top and bottom of the object are traced and produce an inverted real image on the retina. The distance to the object is drawn smaller than scale.

Near Sighted Vision

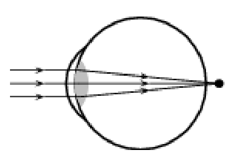
Nearsightedness, or myopia is a vision defect that occurs when the focus of the image is in front of the retina. This is shown in. Close objects are seen fine, but distant objects are blurry. This can be corrected by placing diverging lenses in front of the eye. This will cause the light rays to spread out before they enter the eye.



Near Sighted Vision: This occurs when the image is formed before the retina

Far Sighted Vision

Farsightedness, or hyperopia, is a vision defect that occurs when the focus of the image is behind the retina. This is shown in. Distant objects are seen fine, but closer objects are blurry. This can be corrected by placing converging lenses in front of the eye. This will cause the light rays to slightly converge together before they enter the eye.



Far Sighted Vision: This occurs when the image is formed behind the retina

Key Points

- The eye is made up of a number of parts, including the iris, pupil, cornea, and retina.
- The eye has six muscles which control the eye movement, all providing different tension and torque.
- The eye works a lot like a camera, the pupil provides the f-stop, the iris the aperture stop, the cornea resembles a lens. The way that the image is formed is much like the way a convex lens forms an image.
- The cones in the retina are responsible for seeing colors. There are three types of cones, each type can only pick up one color: red, green, or blue. This is why TVs and computer screens are made up of thousands of little red, green, or blue lights.
- The human eye is more sensitive to intensity changes than color changes, which is why it is acceptable to use black and white photography in place of color and why people can still distinguish everything in the photo without colors.
- Colors are usually written in different values of red, green, or blue. Each value is the log form of that frequency.
- The retina of human eye has a static contrast ratio of around 100:1 and a dynamic contrast ratio of about 1,000,000:1.
- The eye includes a lens not dissimilar to lenses found in optical instruments, such as cameras.
- The approximate field of view of an individual human eye is 95° away from the nose, 75° downward, 60° toward the nose, and 60° upward, allowing humans to have an almost 180-degree forward-facing horizontal field of view.
- The focal point of the image will change depending on how the lens is shaped. Your lens changes depending on the distance of the object, by the relaxation or contraction of the muscles, and this controls the focal length.
- Near sightedness occurs when the image is formed before the retina.
- Farsightedness occurs when the image is formed behind the retina.

Key Terms

- **pupil**: The hole in the middle of the iris of the eye, through which light passes to be focused on the retina.
- **aperture**: The diameter of the aperture that restricts the width of the light path through the whole system. For a telescope, this is the diameter of the objective lens (e.g., a telescope may have a 100 cm aperture).
- **luminance**: The intensity of an object, independent from its color.
- **static contrast ratio**: Luminosity ratio of the brightest and darkest color the system is capable of processing simultaneously at any instant of time.
- **dynamic contrast ratio**: Luminosity ratio of the brightest and darkest color the system is capable of processing over time (while the picture is moving).
- **field of view**: The angular extent of what can be seen, either with the eye or with an optical instrument or camera.
- **myopia**: A disorder of the vision where distant objects appear blurred because the eye focuses their images in front of the retina instead of on it.
- **hyperopia**: A disorder of the vision where the eye focusses images behind the retina instead of on it, so that distant objects can be seen better than near objects.

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25.2: Other Optical Instruments

A magnifying glass is a convex lens that lets the observer see a larger image of the object under observation. The lens is usually mounted in a frame with a handle, as shown below.

Magnifying Glass: A magnifying glass is a convex lens that lets the observer see a larger image of the object under observation.

The magnification of a magnifying glass depends upon where the instrument is placed between the user's eye and the object being viewed and upon the total distance between eye and object. The magnifying power is the ratio of the sizes of the images formed on the user's retina with and without the magnifying glass. When not using the lens, the user would typically bring the object as close to the eye as possible without it becoming blurry. (This point, known as the near point, varies with age. In a young child its distance can be as short as five centimeters, while in an elderly person its distance may be as long as one or two meters.) Magnifiers are typically characterized using a "standard" value of 0.25m.

The highest magnifying power is obtained by putting the lens very close to the eye and moving both the eye and the lens together to obtain the best focus. When the lens is used this way, the magnifying power can be found with the following equation:

$$\mathrm{MP}_0 = \frac{1}{4} \cdot \Phi + 1 \tag{25.2.1}$$

where Φ= optical power. When the magnifying glass is held close to the object and the eye is moved away, the magnifying power is approximated by:

$$\mathrm{MP}_0 = \frac{1}{4} \cdot \Phi \tag{25.2.2}$$

Typical magnifying glasses have a focal length of 25cm and an optical power of four diopters. This type of glass would be sold as a 2x magnifier, but a typical observer would see about one to two times magnification depending on the lens position.

The earliest evidence of a magnifying device was Aristophanes's "lens" from 424 BC, a glass globe filled with water. (Seneca wrote that it could be used to read letters "no matter how small or dim.") Roger Bacon described the properties of magnifying glasses in the 13th century, and eyeglasses were also developed in 13th-century Italy.

The Camera

Cameras are optical devices that allow a user to record an image of an object, either on photo paper or digitally.

What is a Camera?

A camera is a device that allows you to record images, either on film or digitally. Cameras can record images as well as movies; movies themselves got their name from moving pictures. The word camera comes from the Latin phrase *camera obscura*, which means "dark chamber. " The camera obscura was an early instrument for projecting images from slides. The camera that you use today is an evolution of the camera obscura.

A camera is usually comprised of an opening, or aperture, that allows light to enter into a hollow area and a surface that records the light at the other end. In the 20th century, these images would be stored on photographic paper that then had to be developed, but now most cameras store images digitally.

How does a Camera Work?

Cameras have a lot of components that allow them to work. Let's look at them one at a time.

The Lens

The camera lens allows the light to enter into the camera and is typically convex. There are many types of lenses that can be used, each for a different type of photography. There are lenses for close-ups, for sports, for architecture, and for portraits.

The two major features of a lens are focal length and aperture. The focal length determines the magnification of the image, and the aperture controls the light intensity. The f-number on a camera controls the shutter speed. This is the speed at which the shutter, which acts as its "eyelid," opens and closes. The larger the aperture, the smaller the f-number must be in order to get the shutter opened and closed fully. The time it takes to open and close the shutter is called the exposure. shows an example of two lenses of the same size but with different apertures.

Focus

Some cameras have a fixed focus, and only objects of a certain size at a certain distance from the camera will be in focus. Other cameras allow you to manually or automatically adjust the focus. shows a picture taken with a camera with manual focus; this allows the user to determine which objects will be in focus and which will not. The range of distance within which objects appear sharp and clear is called the depth of field.

Exposure

The aperture controls the intensity of the light entering the camera, and the shutter controls the exposure — the amount of time that the light is allowed into the camera.

Shutter

The shutter is what opens and closes to allow light through the aperture. The speed at which it opens and closes is called the f-number. For a larger aperture, the f-number is generally small for a quick shutter speed. For a smaller aperture, the f-number is larger, allowing for a slower shutter speed.

The Compound Microscope

A compound microscope is made of two convex lenses; the first, the ocular lens, is close to the eye, and the second is the objective lens.

A compound microscope uses multiple lenses to magnify an image for an observer. It is made of two convex lenses: the first, the ocular lens, is close to the eye; the second is the objective lens.

Compound microscopes are much larger, heavier and more expensive than simple microscopes because of the multiple lenses. The advantages of these microscopes, due to the multiple lenses, are the reduced chromatic aberrations and exchangeable objective lenses to adjust magnification.

shows a diagram of a compound microscope made from two convex lenses. The first lens is called the objective lens and is closest to the object being observed. The distance between the object and the objective lens is slightly longer than the focal length, f_0. The objective lens creates an enlarged image of the object, which then acts as the object for the second lens. The second or ocular lens is the eyepiece. The distance between the objective lens and the ocular lens is slightly shorter than the focal length of the ocular lens, f_e. This causes the ocular lens to act as a magnifying glass to the first image and makes it even larger. Because the final image is inverted, it is farther away from the observer's eye and thus much easier to view.

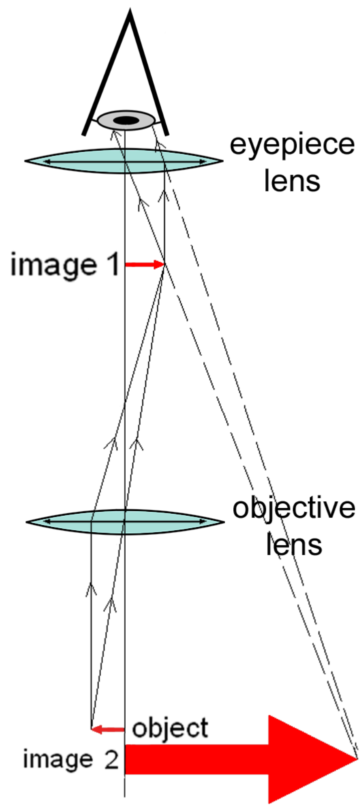


Diagram of a Compound Microscope: This diagram shows the setup of mirrors that allow for the magnification of images.

Since each lens produces a magnification that multiplies the height of the image, the total magnification is a product of the individual magnifications. The equation for calculating this is as follows:

$$\mathrm{m} = \mathrm{m_o m_e} \tag{25.2.3}$$

where m is total magnification, $\mathrm{m_o}$ is objective lens magnification, $\mathrm{m_e}$ is ocular lens magnification.

The Telescope

The telescope aids in observation of remote objects by collecting electromagnetic radiation, such as visible light.

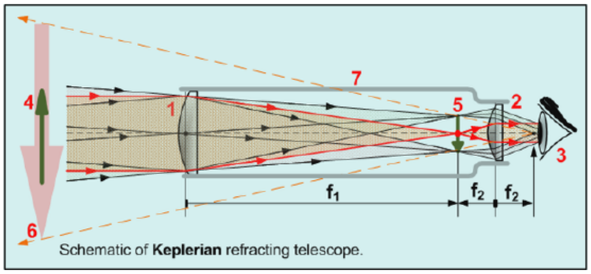
The telescope aids in observation of remote objects by collecting electromagnetic radiation, such as x-rays, visible light, infrared, and submillimeter rays. The first telescopes were invented in the Netherlands in the 1600s and used glass lenses. Shortly after, people began to build them using mirrors and called them reflecting telescopes.

History

The first telescope was a refracting telescope made by spectacle makers in the Netherlands in 1608. In 1610, Galileo made his own improved design. After the refracting telescope was invented, people began to explore the idea of a telescope that used mirrors. The potential advantages of using mirrors instead of lenses were a reduction in spherical aberrations and the elimination of chromatic aberrations. In 1668, Newton built the first practical reflecting telescope. With the invention of achromatic lenses in 1733, color aberrations were partially corrected, and shorter, more functional refracting telescopes could be constructed. Reflecting telescopes were not practical because of the highly corrosive metals used to make mirrors until the introduction of silver-coated glass mirrors in 1857.

Types of Telescopes

Refracting Telescopes



Schematic of **Keplerian** refracting telescope.

Schematic of Keplerian Refracting Telescope: All refracting telescopes use the same principles. The combination of an objective lens 1 and some type of eyepiece 2 is used to gather more light than the human eye is able to collect on its own, focus it 5, and present the viewer with a brighter, clearer, and magnified virtual image 6.

The figure above is a diagram of a refracting telescope. The objective lens (at point 1) and the eyepiece (point 2) gather more light than a human eye can collect by itself. The image is focused at point 5, and the observer is shown a brighter, magnified virtual image at point 6. The objective lens refracts, or bends, light. This causes the parallel rays to converge at a focal point, and those that are not parallel converge on a focal plane.

Reflecting Telescopes

Reflecting telescopes, such as the one shown in, use either one or a combination of curved mirrors that reflect light to form an image. They allow an observer to view objects that have very large diameters and are the primary type of telescope used in astronomy. The object being observed is reflected by a curved primary mirror onto the focal plane. (The distance from the mirror to the focal plane is called the focal length.) A sensor could be located here to record the image, or a secondary mirror could be added to redirect the light to an eyepiece.

Catadioptric Telescopes

Catadioptric telescopes, such as the one shown in, combine mirrors and lenses to form an image. This system has a greater degree of error correction than other types of telescopes. The combination of reflective and refractive elements allows for each element to correct the errors made by the other.

X-Ray Diffraction

The principle of diffraction is applied to record interference on a subatomic level in the study of x-ray crystallography.

X-ray diffraction was discovered by Max von Laue, who won the Nobel Prize in physics in 1914 for his mathematical evaluation of observed x-ray diffraction patterns.

Diffraction is the irregularities caused when waves encounter an object. You have most likely observed the effects of diffraction when looking at the bottom of a CD or DVD. The rainbow pattern that appears is a result of the light being interfered by the pits and lands on the disc that hold the data. shows this effect. Diffraction can happen to any type of wave, not just visible light waves.

Bragg Diffraction

In x-ray crystallography, the term for diffraction is Bragg diffraction, which is the scattering of waves from a crystalline structure. William Lawrence Bragg formulated the equation for Bragg's law, which relates wavelength to the angle of incidence and lattice spacing. Refer to for a diagram of the following equation: $n\lambda=2d\sin(\theta)n\lambda=2d\sin(\theta)$

- n – numeric constant known as the order of the diffracted beam
- λ – wavelength
- d – distance between lattice planes
- θ – angle of diffracted wave

The waves will experience either constructive interference or destructive interference. Similarly, the x-ray beam that is diffracted off a crystal will have some parts that have stronger energy, and others that lose energy. This depends on the wavelength and the lattice spacing.

The X-ray Diffractometer

The XRD machine uses copper metal as the element for the x-ray source. Diffraction patterns are recorded over an extended period of time, so it is very important that the beam intensity remains constant. Film used to be used to record the data, but that was inconvenient because it had to be replaced often. Now the XRD machines are equipped with semiconductor detectors. These XRD machines record images in two ways, either continuous scans or step scanning. In continuous scans, the detector moves in circular motions around the object, while a beam of x-ray is constantly shot at the detector. Pulses of energy are plotted with respect to diffraction angle. The step scan method is the more popular method. It is much more efficient than continuous scans. In this method, the detector collects data at a single fixed angle at a time. To ensure that the incident beam is continuous, XRD machines are equipped with a Soller slit. This acts like polarized sunglasses by organizing random x-ray beams into a stack of neatly arranged waves parallel to the plane of the detector.

X-Ray Imaging and CT Scans

Radiography uses x-rays to view material that cannot be seen by the human eye by identifying areas of different density and composition.

Overview

X-ray imaging, or radiography, used x-rays to view material within the body that cannot be seen by the human eye by identifying areas of different density and composition. CT Scans use the assistance of a computer to take this information, and generate 3 dimensional images.

X-ray Imaging

X-ray radiographs are produced by projecting a beam of X-rays toward an object, in medical cases, a part of the human body. Depending on the physical properties of the object (density and composition), some of the X-rays can be partially absorbed. The portion of the rays that are not absorbed then pass through the object and are recorded by either film or a detector, like in a camera. This provides the observer with a 2 dimensional representation of all the components of that object superimposed on each other. shows an image of a human elbow.

X-Ray Radiography: Radiography of the knee in a modern X-ray machine.

Tomography

Tomography refers to imaging by sections, or sectioning. demonstrates this concept. The three-dimensional image is broken down into sections. (S_1) shows a section from the left and (S_2) shows a section from the right.

CT Scans

CT scans, or computed tomography scans use a combination of X-ray radiography and tomography to produce slices of areas of the human body. Doctors can analyze the area, and based on the ability of the material to block the X-ray beam, understand more about the material. shows a CT Scan of a human brain. Doctors can cross reference the images with known properties of the same material and determine if there are any inconsistencies or problems. Although generally these scans are shown as in, the information recorded can be used to create a 3 dimensional image of the area. shows a three dimensional image of a brain that was made by compiling CT Scans.

Specialty Microscopes and Contrast

Microscopes are instruments that let the human eye see objects that would otherwise be too small.

Microscopes are instruments that let the human eye see objects that would otherwise be too small. There are many types of microscopes: optical microscopes, transmission electron microscopes, scanning electron microscopes and scanning probe microscopes.

Microscope Classes

One way to group microscopes is based on how the image is generated through the microscope. Here are three ways we can classify microscopes:

1.) Light or Photon – optical microscopes

2.) Electrons – electron microscopes

3.) Probe – scanning probe microscopes.

Microscopes can also be classified based on whether they analyze the sample by scanning a point at a time (scanning electron microscopes), or by analyzing the entire sample at once (transmission electron microscopes).

Types of Microscopes

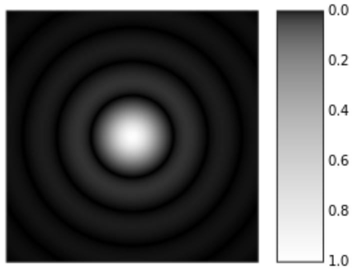
- In optical microscopes, the better the contrast between the image and the surface it is being viewed on, the better the resolution will be to the viewer. There are many illumination techniques to generate improved contrast. These techniques include “dark field” and “bright field.” With the dark field technique the light is scattered by the object and the image appears to the observer on a dark background. With the bright field technique the object is illuminated from below to increase the contrast in the image seen by viewers.
- Transmission Electron Microscope: The TEM passes electrons through the sample, and allows people to see objects that are normally not seen by the naked eye. A beam of electrons is transmitted through an ultra thin specimen, interacting with the specimen as it passes through. This interaction forms an image that is magnified and focused onto an imaging device.
- Scanning Electron Microscopes: Referred to as SEM, these microscopes look at the surface of objects by scanning them with a fine electron beam. The electron beam of the microscope interacts with the electrons in the sample and produces signals that can be detected and have information about the topography and composition.
- Atomic Force Microscopy: The AFM is a scanning probe type of microscopy with very high resolution and is one of the foremost tools for imaging at the nanoscale. The mechanical probe feels the surface with a cantilever with a sharp tip. The deflection of the tip is then measured using a laser spot that is reflected from the surface of the cantilever.

Limits of Resolution and Circular Aperatures

In optical imaging, there is a fundamental limit to the resolution of any optical system that is due to diffraction.

The resolution of an optical imaging system (e.g., a microscope, telescope, or camera) can be limited by factors such as imperfections in the lenses or misalignment. However, there is a fundamental maximum to the resolution of any optical system that is due to diffraction (a wave nature of light). An optical system with the ability to produce images with angular resolution as good as the instrument’s theoretical limit is said to be diffraction limited.

For telescopes with circular apertures, the size of the smallest feature in an image that is diffraction limited is the size of the Airy disc, as shown in. As one decreases the size of the aperture in a lens, diffraction increases and the ring features from diffraction become more prominent. Similarly, when imaged objects get smaller, features from diffraction begin to blur the boundary of the object. Since effects of diffraction become most prominent for waves whose wavelength is roughly similar to the dimensions of the diffracting objects, the wavelength of the imaging beam sets a fundamental limit on the resolution of any optical system.



Airy Disk: Computer-generated image of an Airy disk. The gray scale intensities have been adjusted to enhance the brightness of the outer rings of the Airy pattern.

The Abbe Diffraction Limit for a Microscope

The observation of sub-wavelength structures with microscopes is difficult because of the Abbe diffraction limit. In 1873, Ernst Abbe found that light, with wavelength λ, traveling in a medium with refractive index n, cannot be converged to a spot with a radius less than:

$$\mathrm{d}=\frac{\lambda}{2(\mathrm{n}\sin\theta)} \tag{25.2.4}$$

The denominator nsinθnsinθ is called the numerical aperture and can reach about 1.4 in modern optics, hence the Abbe limit is roughly d=λ/2. With green light around 500 nm, the Abbe limit is 250 nm which is large compared to most nanostructures, or biological cells with sizes on the order of 1μm and internal organelles which are much smaller. Using a 500 nm beam, you cannot (in principle) resolve any features with size less than around 250 nm.

Improving Resolution

To increase the resolution, shorter wavelengths can be used such as UV and X-ray microscopes. These techniques offer better resolution but are expensive, suffer from lack of contrast in biological samples and may damage the sample. There are techniques for producing images that appear to have higher resolution than allowed by simple use of diffraction-limited optics. Although these techniques improve some aspect of resolution, they generally involve an enormous increase in cost and complexity. Usually the technique is only appropriate for a small subset of imaging problems.

Aberrations

An aberration, or distortion, is a failure of rays to converge at one focus because of limitations or defects in a lens or mirror.

The Basics of Aberrations

An aberration is the failure of rays to converge at one focus because of limitations or defects in a lens or mirror. Basically, an aberration is a distortion of an image due to the fact that lenses will never behave exactly according to the way they were modeled. Types of aberrations vary due to the size, material composition, or thickness of a lens, or the position of an object.

Chromatic Aberration

A chromatic aberration, also called achromatism or chromatic distortion, is a distortion of colors. This aberration happens when the lens fails to focus all the colors on the same convergence point. This happens because lenses have a different index of refraction for different wavelengths of light. The refractive index decreases with increasing wavelength. These aberrations or distortions occur on the edges of color boundaries between bright and dark areas of an image. Since the index of refraction of lenses depends on color or wavelength, images are produced at different places and with different magnifications for different colors. shows chromatic aberration for a single convex lens. Since violet rays have a higher refractive index than red, they are bent more and focused closed to the lens. shows a two-lens system using a diverging lens to partially correct for this, but it is nearly impossible to do so completely.

The law of reflection is independent of wavelength, and therefore mirrors do not have this problem. This is why it is advantageous to use mirrors in telescopes and other optical systems.

Comatic Aberration

A comatic aberration, or coma, occurs when the object is off-center. Different parts of a lens of a mirror do not refract or reflect the image to the same point, as shown in. They can also be result of an imperfection in the lens or other component and result in off-axis point sources. These aberrations can cause objects to appear pear-shaped. They can also cause stars to appear distorted or appear to have tails, as with comets.

Other Aberrations

Spherical aberrations are a form of aberration where rays converging from the outer edges of a lens converge to a focus closer to the lens, and rays closer to the axis focus further. Astigmatisms are also a form of aberration in the lenses of the eyes where rays that propagate in two perpendicular planes have different foci. This can eventually cause a monochromatic image to distort vertically or horizontally. Another aberration or distortion is a barrel distortion where image magnification decreases with the distance from the optical axis. The apparent effect is that of an image which has been mapped around a sphere, like in a fisheye lens.

Key Points

- The magnification of a magnifying glass depends upon where it is placed between the user's eye and the object being viewed and upon the total distance between eye and object.
- The magnifying power is the ratio of the sizes of the images formed on the user's retina with and without the lens.
- The highest magnifying power is obtained by putting the lens very close to the eye and moving both the eye and the lens together to obtain the best focus.
- Cameras work very similarly to how the human eye works. The iris is similar to the lens; the pupil is similar to the aperture; and the eyelid is similar to the shutter.
- Cameras are a modern evolution of the camera obscura. The camera obscura was a device used to project images.
- The most important part of a camera is the lens, which allows the image to be magnified and focused. This can be done manually on some cameras and automatically on newer cameras.
- Movie cameras work by taking many pictures each second and then showing each image in order very quickly to give the effect that the pictures are moving. This is where the name "movie" comes from.
- A compound microscope uses multiple lenses to create an enlarged image that is easier for a human eye to see; this is due to the fact that the final image is farther away from the observer, and therefore the eye is more relaxed when viewing the image.
- The object is placed just beyond to focal length of the objective lens. An enlarged image is then captured by the objective lens, which acts as the object for the ocular lens. The ocular lens is closer to the new image than its focal length, causing it to act as a magnifying glass.
- Since the final image is just a multiple of the size of the first image, the final magnification is a product of both magnifications from each lens.
- Until the invention of silver-backed mirrors, refractive mirrors were the standard for use in telescopes. This was because of the highly corrosive nature of the metals used in older mirrors. Since then, reflective mirrors have replaced refractive mirrors in astronomy.
- There are three main types of optical telescopes: refractive, reflective, and catadioptric.
- Refractive telescopes, such as the one invented by Galileo, use an objective lens and an eyepiece. The image is focused at the focal point and allows the observer to see a brighter, larger image than they would with their own eye.
- Reflective telescopes use curved mirrors that reflect light to form an image. Sometimes a secondary mirror redirects the image to an eyepiece. Other times the image is recorded by a sensor and observed on a computer screen.
- Catadioptric telescopes combine mirrors and lenses to form an image. This system has a greater degree of error correction than other types of telescopes. The combination of reflective and refractive elements allows for each element to correct the errors made by the other.
- Diffraction is what happens when waves encounter irregularities on a surface or object and are caused to interfere with each other, either constructively or destructively.
- The Bragg law pertains to applying the laws of diffraction to crystallography in order to obtain precise images of the lattice structures in atoms.
- The x-ray diffractometer is the machine used to scan the object by shooting a wave at it and recording the interference it encounters.

- Most XRDs are equipped with a Soller slit, which acts like a polarizer for the incident beam. It makes sure that the incident beam being recorded is perfectly parallel to the object being analyzed.
- Radiography uses x-rays to take pictures of materials with in an object that can not be seen. They shoot x-ray beams through the object, and collect the rays on film or a detector on the other side. Some of the rays are absorbed into the denser materials, and this is how the image is produced.
- X-ray radiographs take images of all the materials within an object superimposed on each other.
- The traditional, superimposed images can be helpful for a number of applications, but CT scans enable the observer to see just the desired sections of a material.
- Modern CT scans can even take all the slices, or layers, and arrange them into a three-dimensional representation of the object.
- For better resolution, it is important that there is a lot of contrast between the image and the background.
- Microscopes are classified by what interacts with the object, such as light or electrons. They are also classified by whether they take images by scanning a little at a time or by taking images of the entire object at once.
- Some common types of specialty microscopes are scanning electron micrscopes (SEM), transmission electron microscopes (TEM), both of which are electron microscopes, and atomic force microscopes (ATM) which is a scanning probe microscope.
- Since effects of diffraction become most prominent for waves whose wavelength is roughly similar to the dimensions of the diffracting objects, the wavelength of the imaging beam sets a fundamental limit on the resolution of any optical system.
- The Abbe diffraction limit for a microscope is given as $\mathrm{d}=\frac{\lambda}{2(\mathrm{n}\sin\theta)}$.
- Since the diffraction limit is proportional to wavelength, to increase the resolution, shorter wavelengths can be used such as UV and X-ray microscopes.
- There are many types of aberrations, including chromatic, spherical, comatic, astigmatism, and barrel distortion.
- Chromatic aberrations occur due to the fact that lenses have different refractive indexes for different wavelengths, and therefore colors. These aberrations occur right on the edges of images between light and dark areas of the picture.
- Mirrors do not have chromatic aberrations because they do not rely on the index of refraction, but rather the index of reflection, which is independent of wavelength.
- Comatic aberrations are due to imperfections in lenses and cause the point source to be off-center. This can cause images to appear pear-shaped, or cause images to have tails, as with comets.

Key Terms

- **lens**: an object, usually made of glass, that focuses or defocuses the light that passes through it
- **diopter**: a unit of measure of the power of a lens or mirror, equal to the reciprocal of its focal length in meters. Myopia is diagnosed and measured in diopters
- **convex**: curved or bowed outward like the outside of a bowl or sphere or circle
- **shutter speed**: The duration of time for which the shutter of a camera remains open when exposing photographic film or other photosensitive material to light for the purpose of recording an image
- **chromatic aberration**: an optical aberration, in which an image has colored fringes, caused by differential refraction of light of different wavelengths
- **spherical aberration**: a type of lens aberration that causes blurriness, particularly away from the center of the lens
- **achromatic**: free from color; transmitting light without color-related distortion
- **constructive interference**: Occurs when waves interfere with each other crest to crest and the waves are exactly in phase with each other.
- **crystallography**: The experimental science of determining the arrangement of atoms in solids.
- **destructive interference**: Occurs when waves interfere with each other crest to trough (peak to valley) and are exactly out of phase with each other.
- **radiography**: The use of X-rays to view a non-uniformly composed material such as the human body.
- **tomography**: Imaging by sections or sectioning.
- **superimposed**: Positioned on or above something else, especially in layers
- **contrast**: A difference in lightness, brightness and/or hue between two colors that makes them more or less distinguishable
- **diffraction**: The bending of a wave around the edges of an opening or an obstacle.
- **nanostructure**: Any manufactured structure having a scale between molecular and microscopic.
- **aperture**: The diameter of the aperture that restricts the width of the light path through the whole system. For a telescope, this is the diameter of the objective lens (e.g., a telescope may have a 100 cm aperture).
- **distortion**: (optics) an aberration that causes magnification to change over the field of view.

- **refraction**: Changing of a light ray's direction when it passes through variations in matter.
- **aberration**: The convergence to different foci, by a lens or mirror, of rays of light emanating from one and the same point, or the deviation of such rays from a single focus; a defect in a focusing mechanism that prevents the intended focal point.

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CHAPTER OVERVIEW

26: Wave Optics

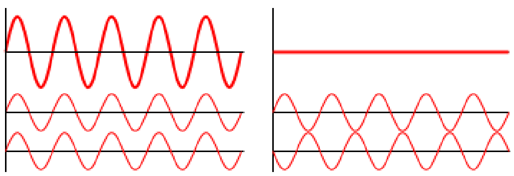
26.1: Superposition and Interference

learning objectives

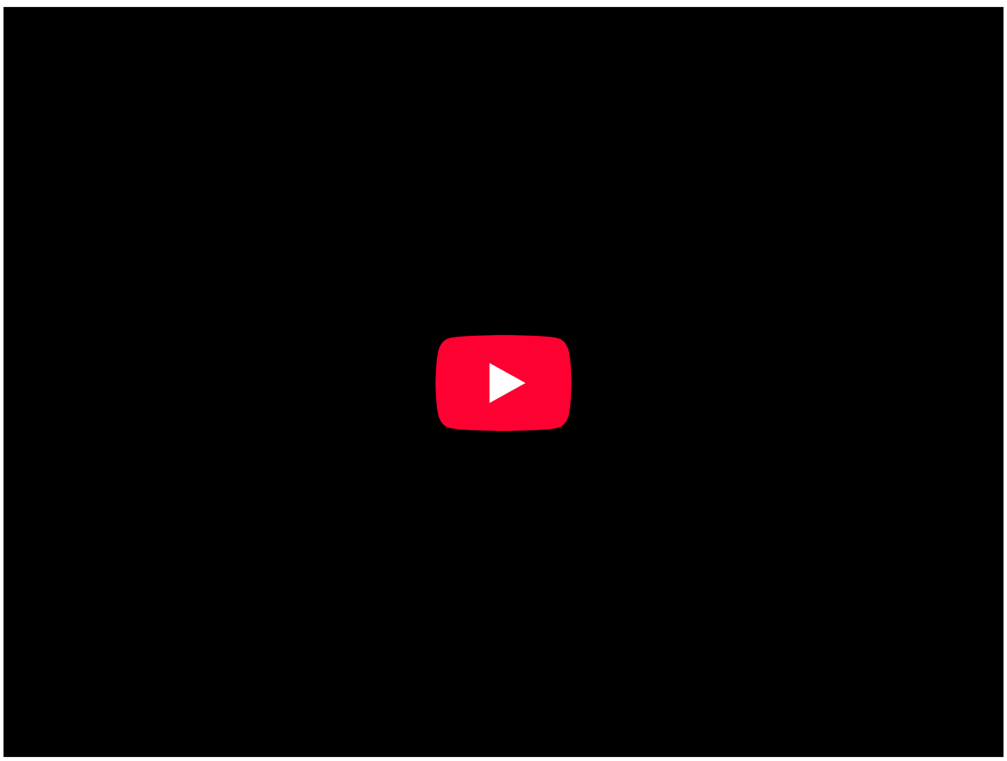
- Contrast the effects of constructive and destructive interference

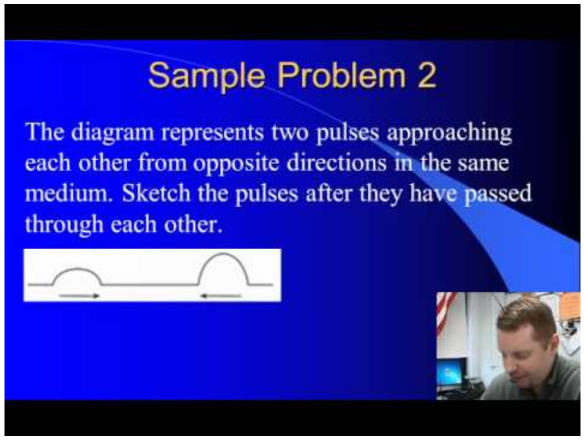
Conditions for Wave Interference

Interference is a phenomenon in which two waves superimpose to form a resultant wave of greater or lesser amplitude. Its effects can be observed in all types of waves (for example, light, acoustic waves and water waves). Interference usually refers to the interaction of waves that are correlated (coherent) with each other because they originate from the same source, or they have the same or nearly the same frequency. When two or more waves are incident on the same point, the total displacement at that point is equal to the vector sum of the displacements of the individual waves. If a crest of one wave meets a crest of another wave of the same frequency at the same point, then the magnitude of the displacement is the sum of the individual magnitudes. This is *constructive interference* and occurs when the phase difference between the waves is a multiple of 2π. *Destructive interference* occurs when the crest of one wave meets a trough of another wave. In this case, the magnitude of the displacements is equal to the difference in the individual magnitudes, and occurs when this difference is an odd multiple of π. Examples of constructive and destructive interference are shown in. If the difference between the phases is intermediate between these two extremes, then the magnitude of the displacement of the summed waves lies between the minimum and maximum values.



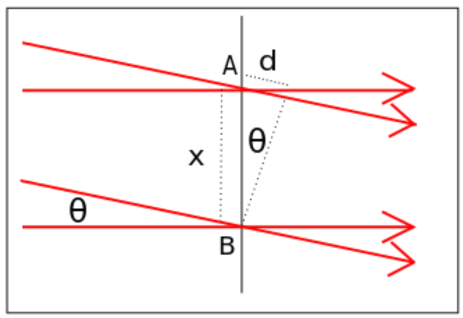
Wave Interference: Examples of constructive (left) and destructive (right) wave interference.





Wave Interference: A brief introduction to constructive and destructive wave interference and the principle of superposition.

A simple form of wave interference is observed when two waves of the same frequency (also called a plane wave) intersect at an angle, as shown in. Assuming the two waves are in phase at point B, then the relative phase changes along the *x*-axis. The phase difference at point A is given by:



Interference of Plane Waves: Geometrical arrangement for two plane wave interference.

$$\Delta\varphi = \frac{2\pi \mathrm{d}}{\lambda} = \frac{2\pi \mathrm{x} \sin\theta}{\lambda} \tag{26.1.1}$$

Constructive interference occurs when the waves are in phase, or

$$\frac{x \sin\theta}{\lambda} = 0, \pm 1, \pm 2, \ldots \tag{26.1.2}$$

Destructive interference occurs when the waves are half a cycle out of phase, or

$$\frac{x \sin\theta}{\lambda} = \pm\frac{1}{2}, \pm\frac{3}{2}, \ldots \tag{26.1.3}$$

Reflection Due to Phase Change

Light exhibits wave characteristics in various media as well as in a vacuum. When light goes from a vacuum to some medium (like water) its speed and wavelength change, but its frequency *f* remains the same. The speed of light in a medium is $v = c/n$, where *n* is the index of refraction. For example, water has an index of refraction of $n = 1.333$. When light is reflected off a medium with a higher index of refraction, crests get reflected as troughs and troughs get reflected as crests. In other words, the wave undergoes a 180 degree change of phase upon reflection, and the reflected ray “jumps” ahead by half a wavelength.

Air Wedge

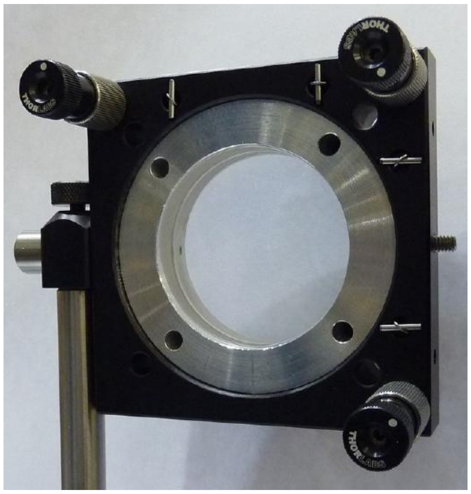
An air wedge is a simple interferometer used to visualize the disturbance of the wave front after propagation through a test object.

learning objectives

- Describe how an air wedge is used to visualize the disturbance of a wave front after proagation

Air Wedge

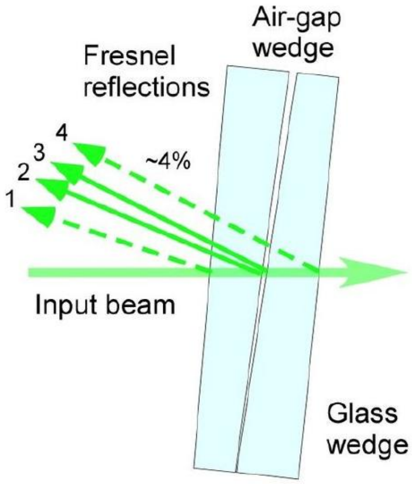
An air wedge is one of the simplest designs of shearing interferometers used to visualize the disturbance of the wave front after propagation through a test object. An air wedge can be used with nearly any light source, including non-coherent white light. The interferometer consists of two optical glass wedges (~2-5 degrees), pushed together and then slightly separated from one side to create a thin air-gap wedge. An example of an air wedge interferometer is shown in.



Air Wedge: Example of air wedge interferometer

The air gap between the two glass plates has two unique properties: it is very thin (micrometer scale) and has perfect flatness. Because of this extremely thin air-gap, the air wedge interferometer has been successfully used in experiments with femto-second high-power lasers.

An incident beam of light encounters four boundaries at which the index of refraction of the media changes, causing four reflected beams (or Fresnel reflections) as shown in. The first reflection occurs when the beam enters the first glass plate. The second reflection occurs when the beam exits the first plate and enters the air wedge, and the third reflection occurs when the beam exits the air wedge and enters the second glass plate. The fourth beam is reflected when it encounters the boundary of the second glass plate. The air wedge angle, between the second and third Fresnel reflections, can be adjusted, causing the reflected light beams to constructively and destructively interfere and create a fringe pattern. To minimize image aberrations of the resulting fringes, the angle plane of the glass wedges has to be placed orthogonal to the angle plane of the air-wedge.



Light Reflections Inside an Air Wedge Interferometer: Beam path inside of air wedge interferometer

Newton's Rings

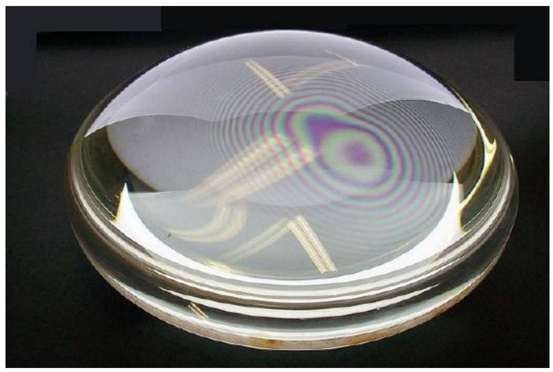
Newton's rings are a series of concentric circles centered at the point of contact between a spherical and a flat surface.

learning objectives

- Apply Newton's rings to determine light characteristics of a lens

Newton's Rings

In 1717, Isaac Newton first analyzed an interference pattern caused by the reflection of light between a spherical surface and an adjacent flat surface. Although first observed by Robert Hooke in 1664, this pattern is called Newton's rings, as Newton was the first to analyze and explain the phenomena. Newton's rings appear as a series of concentric circles centered at the point of contact between the spherical and flat surfaces. When viewed with monochromatic light, Newton's rings appear as alternating bright and dark rings; when viewed with white light, a concentric ring pattern of rainbow colors is observed. An example of Newton's rings when viewed with white light is shown in the figure below.



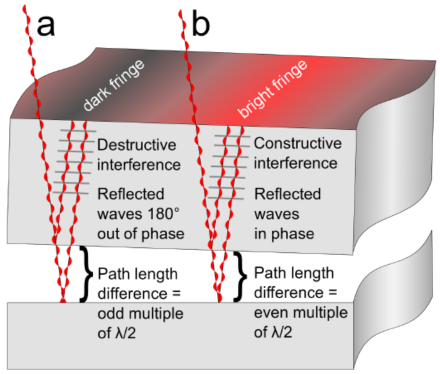
Newton's Rings in a drop of water: Newton's rings seen in two plano-convex lenses with their flat surfaces in contact. One surface is slightly convex, creating the rings. In white light, the rings are rainbow-colored, because the different wavelengths of each color interfere at different locations.

The light rings are caused by constructive interference between the light rays reflected from both surfaces, while the dark rings are caused by destructive interference. The outer rings are spaced more closely than the inner ones because the slope of the curved lens surface increases outwards. The radius of the Nth bright ring is given by:

$$\mathrm{r_N} = \left[\left(\mathrm{N} - \frac{1}{2}\lambda\mathrm{R}\right)\right]^{1/2} \tag{26.1.4}$$

where N is the bright-ring number, R is the radius of curvature of the lens the light is passing through, and λ is the wavelength of the light passing through the glass.

A spherical lens is placed on top of a flat glass surface. An incident ray of light passes through the curved lens until it comes to the glass-air boundary, at which point it passes from a region of higher refractive index *n* (the glass) to a region of lower *n* (air). At this boundary, some light is transmitted into the air, while some light is reflected. The light that is transmitted into the air does not experience a change in phase and travels a a distance, *d*, before it is reflected at the flat glass surface below. This second air-glass boundary imparts a half-cycle phase shift to the reflected light ray because air has a lower *n* than the glass. The two reflected light rays now travel in the same direction to be detected. As one gets farther from the point at which the two surfaces touch, the distance *d*increases because the lens is curving away from the flat surface.



Formation of Interference Fringes: This figure shows how interference fringes form.

If the path length difference between the two reflected light beams is an odd multiple of the wavelength divided by two, $\lambda/2$, the reflected waves will be 180 degrees out of phase and destructively interfere, causing a dark fringe. If the path-length difference is an even multiple of $\lambda/2$, the reflected waves will be in phase with one another. The constructive interference of the two reflected waves creates a bright fringe.

Key Points

- When two or more waves are incident on the same point, the total displacement at that point is equal to the vector sum of the displacements of the individual waves.
- Light exhibits wave characteristics in various media as well as in a vacuum. When light goes from a vacuum to some medium, like water, its speed and wavelength change, but its frequency f remains the same.
- When light is reflected off a medium with a higher index of refraction, crests get reflected as troughs and troughs get reflected as crests. In other words, the wave undergoes a 180 degree change of phase upon reflection, and the reflected ray "jumps" ahead by half a wavelength.
- An air wedge interferometer consists of two optical glass wedges (~2-5 degrees), pushed together and then slightly separated from one side to create a thin air-gap wedge.
- The air gap between the two glass plates has two unique properties: it is very thin (micrometer scale) and has perfect flatness.
- To minimize image aberrations of the resulting fringes, the angle plane of the glass wedges has to be placed orthogonal to the angle plane of the air wedge.
- When viewed with monochromatic light, Newton's rings appear as alternating bright and dark rings; when viewed with white light, a concentric ring pattern of rainbow colors is observed.
- If the path length difference between the two reflected light beams is an odd multiple of the wavelength divided by two, $\lambda/2$, the reflected waves will be 180 degrees out of phase and destructively interfere, causing a dark fringe.
- If the path length difference is an even multiple of $\lambda/2$, the reflected waves will be in-phase with one another. The constructive interference of the two reflected waves creates a bright fringe.

Key Terms

- **coherent**: Of waves having the same direction, wavelength and phase, as light in a laser.
- **plane wave**: A constant-frequency wave whose wavefronts (surfaces of constant phase) are infinite parallel planes of constant peak-to-peak amplitude normal to the phase velocity vector.
- **orthogonal**: Of two objects, at right angles; perpendicular to each other.
- **interferometer**: Any of several instruments that use the interference of waves to determine wavelengths and wave velocities, determine refractive indices, and measure small distances, temperature changes, stresses, and many other useful measurements.
- **wavelength**: The length of a single cycle of a wave, as measured by the distance between one peak or trough of a wave and the next; it is often designated in physics as λ, and corresponds to the velocity of the wave divided by its frequency.
- **lens**: an object, usually made of glass, that focuses or defocuses the light that passes through it
- **monochromatic**: Describes a beam of light with a single wavelength (i.e., of one specific color or frequency).

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26.2: Diffraction

learning objectives

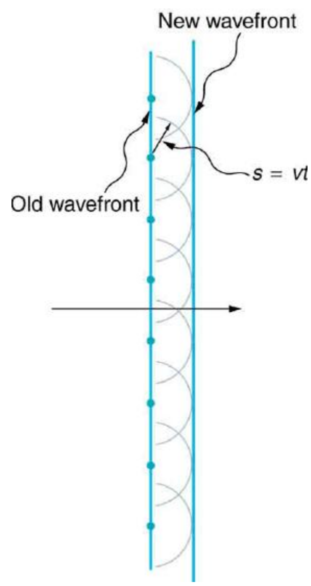
- Formulate Huygens's Principle

Overview

The Huygens-Fresnel principle states that every point on a wavefront is a source of wavelets. These wavelets spread out in the forward direction, at the same speed as the source wave. The new wavefront is a line tangent to all of the wavelets.

Background

Christiaan Huygens was a Dutch scientist who developed a useful technique for determining how and where waves propagate. In 1678, he proposed that every point that a luminous disturbance touches becomes itself a source of a spherical wave. The sum of the secondary waves (waves that are a result of the disturbance) determines the form of the new wave. shows secondary waves traveling forward from their point of origin. He was able to come up with an explanation of the linear and spherical wave propagation, and derive the laws of reflection and refraction (covered in previous atoms) using this principle. He could not, however, explain what is commonly known as diffraction effects. Diffraction effects are the deviations from rectilinear propagation that occurs when light encounters edges, screens and apertures. These effects were explained in 1816 by French physicist Augustin-Jean Fresnel.



Straight Wavefront: Huygens's principle applied to a straight wavefront. Each point on the wavefront emits a semicircular wavelet that moves a distance s=vt. The new wavefront is a line tangent to the wavelets.

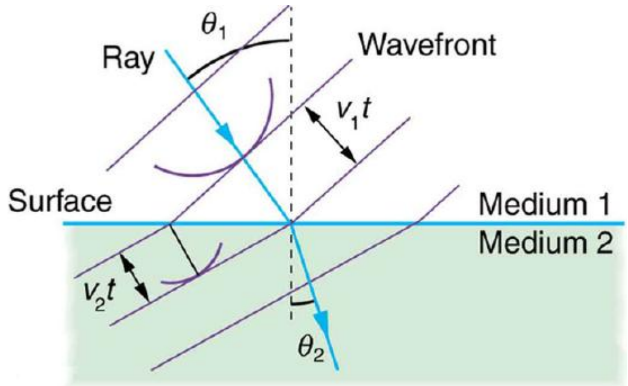
Huygens's Principle

Figure 1 shows a simple example of the Huygens's Principle of diffraction. The principle can be shown with the equation below:

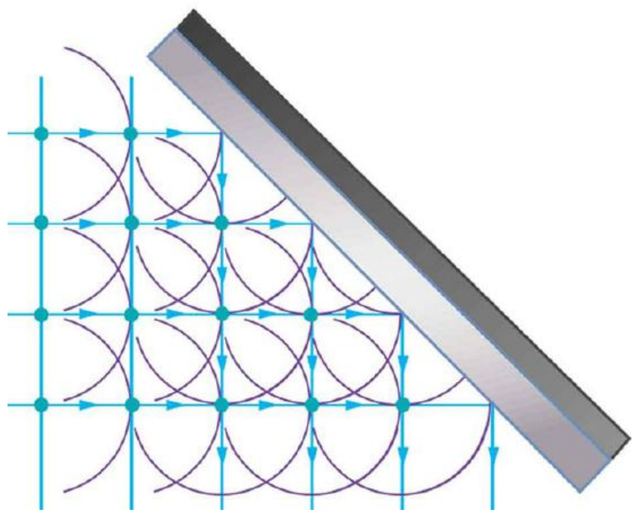
$$\mathrm{s} = \mathrm{vt} \tag{26.2.1}$$

where s is the distance, v is the propagation speed, and t is time.

Each point on the wavefront emits a wave at speed, v. The emitted waves are semicircular, and occur at t, time later. The new wavefront is tangent to the wavelets. This principle works for all wave types, not just light waves. The principle is helpful in describing reflection, refraction and interference. shows visually how Huygens's Principle can be used to explain reflection, and shows how it can be applied to refraction.



Huygens's Refraction: Huygens's principle applied to a straight wavefront traveling from one medium to another where its speed is less. The ray bends toward the perpendicular, since the wavelets have a lower speed in the second medium.



Reflection: Huygens's principle applied to a straight wavefront striking a mirror. The wavelets shown were emitted as each point on the wavefront struck the mirror. The tangent to these wavelets shows that the new wavefront has been reflected at an angle equal to the incident angle. The direction of propagation is perpendicular to the wavefront, as shown by the downward-pointing arrows.

Example $26.2.1$:

This principle is actually something you have seen or experienced often, but just don't realize. Although this principle applies to all types of waves, it is easier to explain using sound waves, since sound waves have longer wavelengths. If someone is playing music in their room, with the door closed, you might not be able to hear it while walking past the room. However, if that person where to open their door while playing music, you could hear it not only when directly in front of the door opening, but also on a considerable distance down the hall to either side. is a direct effect of diffraction. When light passes through much smaller openings, called slits, Huygens's principle shows that light bends similar to the way sound does, just on a much smaller scale. We will examine in later atoms single slit diffraction and double slit diffraction, but for now it is just important that we understand the basic concept of diffraction.

Diffraction

As we explained in the previous paragraph, diffraction is defined as the bending of a wave around the edges of an opening or an obstacle.

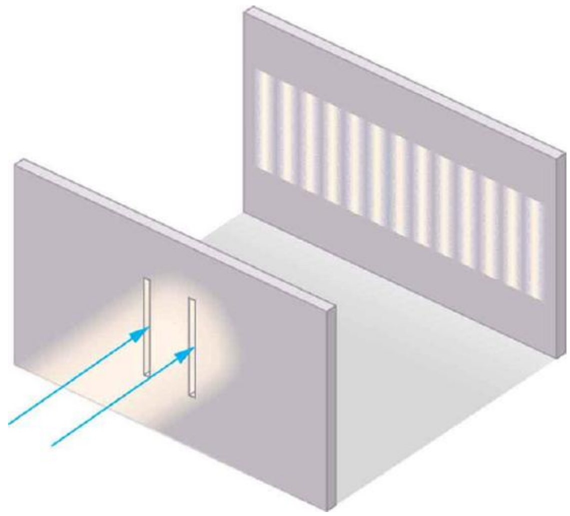
Young's Double Slit Experiment

The double-slit experiment, also called Young's experiment, shows that matter and energy can display both wave and particle characteristics.

learning objectives

- Explain why Young's experiment more credible than Huygens'

The double-slit experiment, also called Young's experiment, shows that matter and energy can display both wave and particle characteristics. As we discussed in the atom about the Huygens principle, Christiaan Huygens proved in 1628 that light was a wave. But some people disagreed with him, most notably Isaac Newton. Newton felt that color, interference, and diffraction effects needed a better explanation. People did not accept the theory that light was a wave until 1801, when English physicist Thomas Young performed his double-slit experiment. In his experiment, he sent light through two closely spaced vertical slits and observed the resulting pattern on the wall behind them. The pattern that resulted can be seen in.



Young's Double Slit Experiment: Light is sent through two vertical slits and is diffracted into a pattern of vertical lines spread out horizontally. Without diffraction and interference, the light would simply make two lines on the screen.

Wave-Particle Duality

The wave characteristics of light cause the light to pass through the slits and interfere with itself, producing the light and dark areas on the wall behind the slits. The light that appears on the wall behind the slits is scattered and absorbed by the wall, which is a characteristic of a particle.

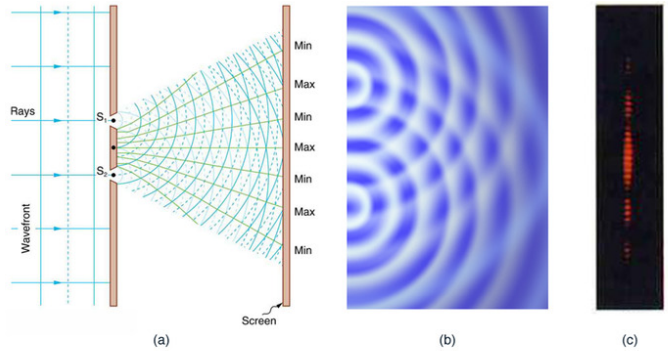
Young's Experiment

Why was Young's experiment so much more credible than Huygens'? Because, while Huygens' was correct, he could not demonstrate that light acted as a wave, while the double-slit experiment shows this very clearly. Since light has relatively short wavelengths, to show wave effects it must interact with something small — Young's small, closely spaced slits worked.

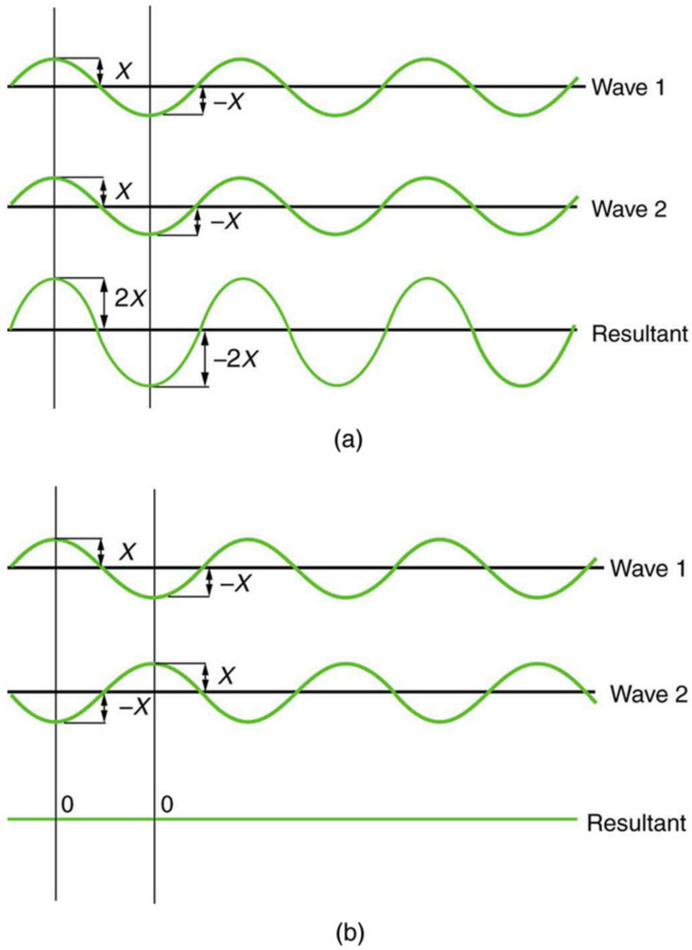
The example in uses two coherent light sources of a single monochromatic wavelength for simplicity. (This means that the light sources were in the same phase.) The two slits cause the two coherent light sources to interfere with each other either constructively or destructively.

Constructive and Destructive Wave Interference

Constructive wave interference occurs when waves interfere with each other crest-to-crest (peak-to-peak) or trough-to-trough (valley-to-valley) and the waves are exactly in phase with each other. This *amplifies* the resultant wave. Destructive wave interference occurs when waves interfere with each other crest-to-trough (peak-to-valley) and are exactly out of phase with each other. This cancels out any wave and results in no light. These concepts are shown in. It should be noted that this example uses a single, monochromatic wavelength, which is not common in real life; a more practical example is shown in.



Practical Constructive and Destructive Wave Interference: Double slits produce two coherent sources of waves that interfere. (a) Light spreads out (diffracts) from each slit because the slits are narrow. These waves overlap and interfere constructively (bright lines) and destructively (dark regions). We can only see this if the light falls onto a screen and is scattered into our eyes. (b) Double-slit interference pattern for water waves are nearly identical to that for light. Wave action is greatest in regions of constructive interference and least in regions of destructive interference. (c) When light that has passed through double slits falls on a screen, we see a pattern such as this.



Theoretical Constructive and Destructive Wave Interference: The amplitudes of waves add together. (a) Pure constructive interference is obtained when identical waves are in phase. (b) Pure destructive interference occurs when identical waves are exactly out of phase (shifted by half a wavelength).

The pattern that results from double-slit diffraction is not random, although it may seem that way. Each slit is a different distance from a given point on the wall behind it. For each different distance, a different number of wavelengths fit into that path. The waves all start out in phase (matching crest-to-crest), but depending on the distance of the point on the wall from the slit, they could be in phase at that point and interfere constructively, or they could end up out of phase and interfere with each other destructively.

Diffraction Gratings: X-Ray, Grating, Reflection

Diffraction grating has periodic structure that splits and diffracts light into several beams travelling in different directions.

learning objectives

- Describe function of the diffraction grating

Diffraction Grating

A diffraction grating is an optical component with a periodic structure that splits and diffracts light into several beams travelling in different directions. The directions of these beams depend on the spacing of the grating and the wavelength of the light so that the grating acts as the dispersive element. Because of this, gratings are often used in monochromators, spectrometers, wavelength division multiplexing devices, optical pulse compressing devices, and many other optical instruments.

A photographic slide with a fine pattern of purple lines forms a complex grating. For practical applications, gratings generally have ridges or rulings on their surface rather than dark lines. Such gratings can be either transmissive or reflective. Gratings which modulate the phase rather than the amplitude of the incident light are also produced, frequently using holography.

Ordinary pressed CD and DVD media are every-day examples of diffraction gratings and can be used to demonstrate the effect by reflecting sunlight off them onto a white wall. (see). This is a side effect of their manufacture, as one surface of a CD has many small pits in the plastic, arranged in a spiral; that surface has a thin layer of metal applied to make the pits more visible. The structure of a DVD is optically similar, although it may have more than one pitted surface, and all pitted surfaces are inside the disc. In a standard pressed vinyl record when viewed from a low angle perpendicular to the grooves, one can see a similar, but less defined effect to that in a CD/DVD. This is due to viewing angle (less than the critical angle of reflection of the black vinyl) and the path of the light being reflected due to being changed by the grooves, leaving a rainbow relief pattern behind.

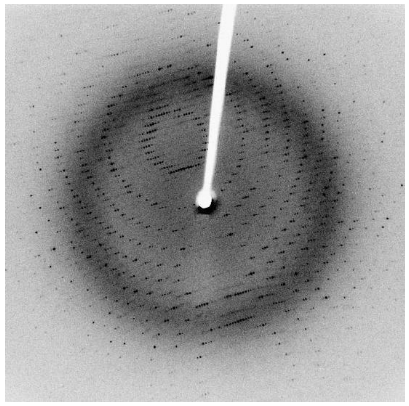
Readable Surface of a CD: The readable surface of a Compact Disc includes a spiral track wound tightly enough to cause light to diffract into a full visible spectrum.

Some bird feathers use natural diffraction grating which produce constructive interference, giving the feathers an iridescent effect. Iridescence is the effect where surfaces seem to change color when the angle of illumination is changed. An opal is another example of diffraction grating that reflects the light into different colors.

X-Ray Diffraction

X-ray crystallography is a method of determining the atomic and molecular structure of a crystal, in which the crystalline atoms cause a beam of X-rays to diffract into many specific directions. By measuring the angles and intensities of these diffracted beams, a crystallographer can produce a three-dimensional picture of the density of electrons within the crystal. From this electron density, the mean positions of the atoms in the crystal can be determined, as well as their chemical bonds, their disorder and various other information.

In an X-ray diffraction measurement, a crystal is mounted on a goniometer and gradually rotated while being bombarded with X-rays, producing a diffraction pattern of regularly spaced spots known as reflections (see). The two-dimensional images taken at different rotations are converted into a three-dimensional model of the density of electrons within the crystal using the mathematical method of Fourier transforms, combined with chemical data known for the sample.



Reflections in Diffraction Patterns: Each dot, called a reflection, in this diffraction pattern forms from the constructive interference of scattered X-rays passing through a crystal. The data can be used to determine the crystalline structure.

Single Slit Diffraction

Single slit diffraction is the phenomenon that occurs when waves pass through a narrow gap and bend, forming an interference pattern.

learning objectives

- Formulate the Huygens's Principle

Diffraction

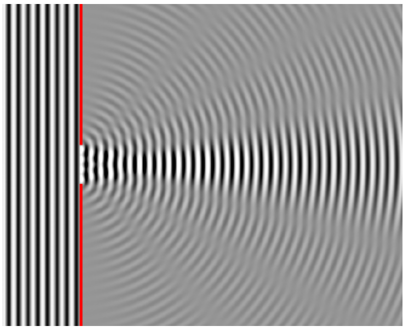
As we explained in a previous atom, diffraction is defined as the bending of a wave around the edges of an opening or obstacle. Diffraction is a phenomenon all wave types can experience. It is explained by the *Huygens-Fresnel Principle*, and the principal of superposition of waves. The former states that every point on a wavefront is a source of wavelets. These wavelets spread out in the forward direction, at the same speed as the source wave. The new wavefront is a line tangent to all of the wavelets. The superposition principle states that at any point, the net result of multiple stimuli is the sum of all stimuli.

Single Slit Diffraction

In single slit diffraction, the diffraction pattern is determined by the wavelength and by the length of the slit. Figure 1 shows a visualization of this pattern. This is the most simplistic way of using the Huygens-Fresnel Principle, which was covered in a previous atom, and applying it to slit diffraction. But what happens when the slit is NOT the exact (or close to exact) length of a single wave?

Single Slit Diffraction – One Wavelength: Visualization of single slit diffraction when the slit is equal to one wavelength.

A slit that is wider than a single wave will produce interference -like effects downstream from the slit. It is easier to understand by thinking of the slit not as a long slit, but as a number of point sources spaced evenly across the width of the slit. This can be seen in Figure 2.



Single Slit Diffraction – Four Wavelengths: This figure shows single slit diffraction, but the slit is the length of 4 wavelengths.

To examine this effect better, lets consider a single monochromatic wavelength. This will produce a wavefront that is all in the same phase. Downstream from the slit, the light at any given point is made up of contributions from each of these point sources. The resulting phase differences are caused by the different in path lengths that the contributing portions of the rays traveled from the slit.

The variation in wave intensity can be mathematically modeled. From the center of the slit, the diffracting waves propagate radially. The angle of the minimum intensity (θ_{min}) can be related to wavelength (λ) and the slit's width (d) such that:

$$\mathrm{d}\sin\theta_{\min} = \lambda \tag{26.2.2}$$

The intensity (I) of waves at any angle can also be calculated as a relation to slit width, wavelength and intensity of the original waves before passing through the slit:

$$\mathrm{I}(\theta) = \mathrm{I}_0\left(\frac{\sin(\pi \mathrm{x})}{\pi \mathrm{x}}\right)^2 \tag{26.2.3}$$

where x is equal to:

$$\frac{\mathrm{d}}{\lambda}\sin\theta \tag{26.2.4}$$

The Rayleigh Criterion

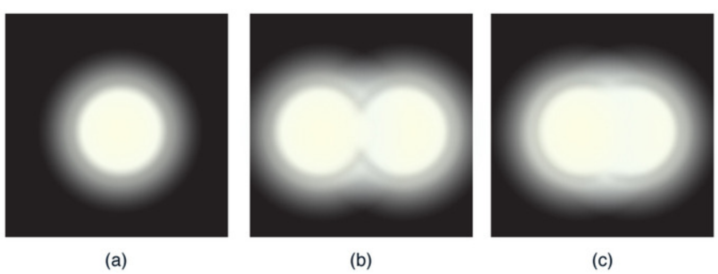
The Rayleigh criterion determines the separation angle between two light sources which are distinguishable from each other.

learning objectives

- Explain meaning of the Rayleigh criterion

Resolution Limits

Along with the diffraction effects that we have discussed in previous atoms, diffraction also limits the detail that we can obtain in images. shows three different circumstances of resolution limits due to diffraction:



Resolution Limits: (a) Monochromatic light passed through a small circular aperture produces this diffraction pattern. (b) Two point light sources that are close to one another produce overlapping images because of diffraction. (c) If they are closer together, they cannot be resolved or distinguished.

- (a) shows a light passing through a small circular aperture. You do not see a sharp circular outline, but a spot with fuzzy edges. This is due to diffraction similar to that through a single slit.
- (b) shows two point sources close together, producing overlapping images. Due to the diffraction, you can just barely distinguish between the two point sources.
- (c) shows two point sources which are so close together that you can no longer distinguish between them.

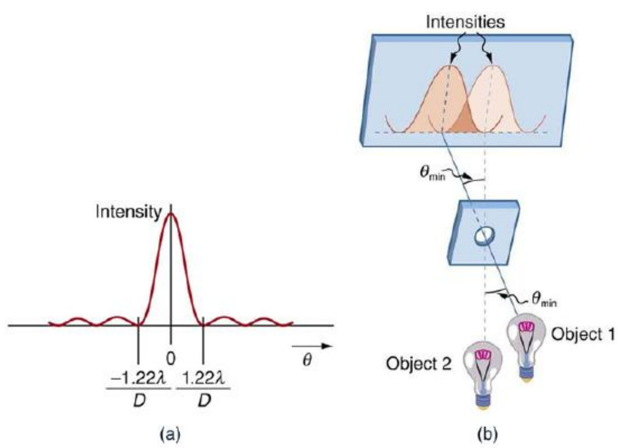
This effect can be seen with light passing through small apertures or larger apertures. This same effect happens when light passes through our pupils, and this is why the human eye has limited acuity.

Rayleigh Criterion

In the 19th century, Lord Rayleigh invented a criteria for determining when two light sources were distinguishable from each other, or resolved. According to the criteria, two point sources are considered just resolved (just distinguishable enough from each other to recognize two sources) if the center of the diffraction pattern of one is directly overlapped by the first minimum of the diffraction pattern of the other. If the distance is greater between these points, the sources are well resolved (i.e., they are easy to distingiush from each other). If the distance is smaller, they are not resolved (i.e., they cannot be distinguished from each other). The equation to determine this is:

$$\theta = 1.22 \frac{\lambda}{\mathrm{D}} \tag{26.2.5}$$

θ – angle the objects are separated by, in radian λ – wavelength of light D – aperture diameter. shows this concept visually. This equation also gives the angular spreading of a source of light having a diameter D.



Rayleigh Criterion: (a) This is a graph of intensity of the diffraction pattern for a circular aperture. Note that, similar to a single slit, the central maximum is wider and brighter than those to the sides. (b) Two point objects produce overlapping diffraction patterns. Shown here is the Rayleigh criterion for being just resolvable. The central maximum of one pattern lies on the first minimum of the other.

Key Points

- Diffraction is the concept that is explained using Huygens's Principle, and is defined as the bending of a wave around the edges of an opening or an obstacle.
- This principle can be used to define reflection, as shown in the figure. It can also be used to explain refraction and interference. Anything that experiences this phenomenon is a wave. By applying this theory to light passing through a slit, we can prove it is a wave.
- The principle can be shown with the equation below: s=vt s – distance v – propagation speed t – time Each point on the wavefront emits a wave at speed, v. The emitted waves are semicircular, and occur at t, time later. The new wavefront is tangent to the wavelets.
- The wave characteristics of light cause the light to pass through the slits and interfere with each other, producing the light and dark areas on the wall behind the slits. The light that appears on the wall behind the slits is partially absorbed by the wall, a characteristic of a particle.

- Constructive interference occurs when waves interfere with each other crest-to-crest and the waves are exactly in phase with each other. Destructive interference occurs when waves interfere with each other crest-to-trough (peak-to-valley) and are exactly out of phase with each other.
- Each point on the wall has a different distance to each slit; a different number of wavelengths fit in those two paths. If the two path lengths differ by a half a wavelength, the waves will interfere destructively. If the path length differs by a whole wavelength the waves interfere constructively.
- The directions of the diffracted beams depend on the spacing of the grating and the wavelength of the light so that the grating acts as the dispersive element.
- Gratings are commonly used in monochromators, spectrometers, wavelength division multiplexing devices, optical pulse compressing devices, and other optical instruments.
- Diffraction of X-ray is used in crystallography to produce the three-dimensional picture of the density of electrons within the crystal.
- The Huygens's Principle states that every point on a wavefront is a source of wavelets. These wavelets spread out in the forward direction, at the same speed as the source wave. The new wavefront is a line tangent to all of the wavelets.
- If a slit is longer than a single wavelength, think of it instead as a number of point sources spaced evenly across the width of the slit.
- Downstream from a slit that is longer than a single wavelength, the light at any given point is made up of contributions from each of these point sources. The resulting phase differences are caused by the different in path lengths that the contributing portions of the rays traveled from the slit.
- Diffraction plays a large part in the resolution at which we are able to see things. There is a point where two light sources can be so close to each other that we cannot distinguish them apart.
- When two light sources are close to each other, they can be: unresolved (i.e., not able to distinguish one from the other), just resolved (i.e., only able to distinguish them apart from each other), and a little well resolved (i.e., easy to tell apart from one another).
- In order for two light sources to be just resolved, the center of one diffraction pattern must directly overlap with the first minimum of the other diffraction pattern.

Key Terms

- **diffraction**: The bending of a wave around the edges of an opening or an obstacle.
- **constructive interference**: Occurs when waves interfere with each other crest to crest and the waves are exactly in phase with each other.
- **destructive interference**: Occurs when waves interfere with each other crest to trough (peak to valley) and are exactly out of phase with each other.
- **interference**: An effect caused by the superposition of two systems of waves, such as a distortion on a broadcast signal due to atmospheric or other effects.
- **iridescence**: The condition or state of being iridescent; exhibition of colors like those of the rainbow; a prismatic play of color.
- **diffraction**: The bending of a wave around the edges of an opening or an obstacle.
- **monochromatic**: Describes a beam of light with a single wavelength (i.e., of one specific color or frequency).
- **resolution**: The degree of fineness with which an image can be recorded or produced, often expressed as the number of pixels per unit of length (typically an inch).

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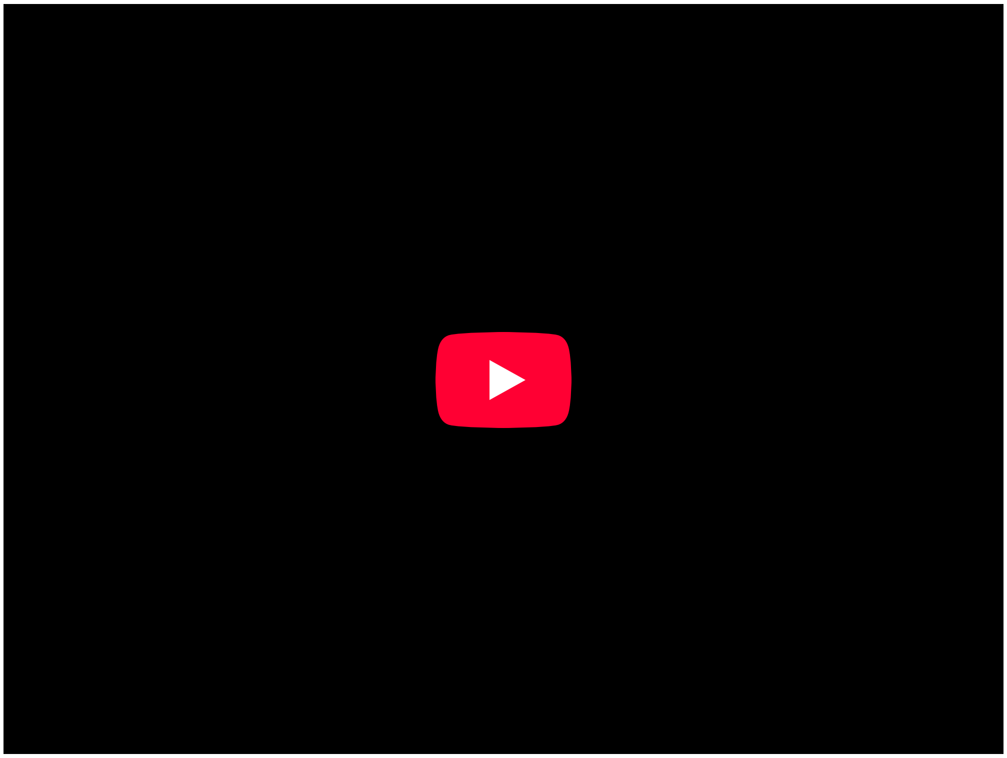
26.3: Further Topics

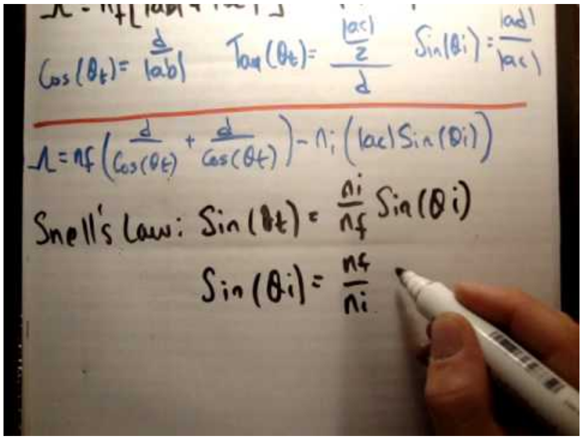
learning objectives

- Describe process that leads to the thin film interference

Thin Film Interference

This is a phenomenon that occurs when incident rays reflected by the upper and lower boundaries of a thin film interfere with one another and form a new wave. A material is considered to be a thin film if its thickness is in the sub-nanometer to micron range, *e.g.* a soap bubble. Studying the new wave can shed light on properties of the film, such as thickness or refractive index. Interference effects are most prominent when the light interacts with something that has a size similar to its own wavelength. The thickness of a thin film is a few times smaller than the wavelength of the light, λ. Color is indirectly associated with wavelength. The interference ratio of wavelength to size of the object causes the appearance of colors.





Thin Film Interference: In this video I continue with my tutorials on Electromagnetism to Optics which is pitched at university undergraduate level. I have intended for a long time to record videos which describe the transition made from classical electromagnetism to optics. In many respects these videos will cover 'wave' optics. I devote much time to discussing the complex exponential representation of waves, Maxwell's Equations, the wave equation etc.Specifically here, I derive the formula for the optical path difference and the phase difference for a 'wave' of light propagating through a thin film. This expression can be used for anti reflective coatings. The phase difference is the product of the optical path differene and the wave vector k.I hope it's of use!! Thank you for watching and I hope that this matches your requirements.

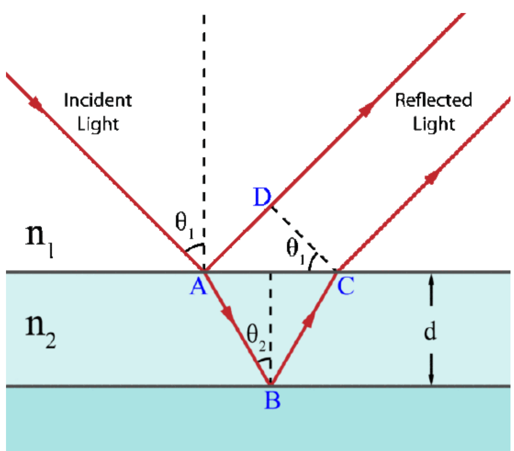
Thin Film Interference in Oil: Thin film interference can be seen in this oil slick.

Examples of Thin Film Interference

You have probably witnessed thin film interference in your every day life and just not realized it. Whenever you see the bright, rainbow like colors in oil floating in water, shown in, this is thin film interference. The colors that appear in bubbles that kids play with are also a result of thin film interference. Thin film interference can have commercial applications, such as anti- reflection coatings and optical filters.

How it Works

shows a diagram of how thin film interference works. As light strikes the surface of a film it is either transmitted or reflected at the upper surface. Light that is transmitted reaches the bottom surface and may once again be transmitted or reflected. The light reflected from the upper and lower surfaces will interfere. The degree of constructive or destructive interference between the two light waves is dependent upon the difference in their phase. This difference is dependent upon the thickness of the film layer, the refractive index of the film, and the angle of incidence of the original wave on the film. Additionally, a phase shift of 180° or ππ radians may be introduced upon reflection at a boundary depending on the refractive indices of the materials on either side said boundary. This phase shift occurs if the refractive index of the medium the light is travelling through is less than the refractive index of the material it is striking. In other words, if n1<n2n1<n2and the light is travelling from material 1 to material 2, then a phase shift will occur upon reflection. The pattern of light that results from this interference can appear either as light and dark bands or as colorful bands depending upon the source of the incident light.



Light on a Thin Film: Light incident on a thin film. Demonstration of the optical path length difference for light reflected from the upper and lower boundaries.

Interference will be constructive if the optical path difference is equal to an integer multiple of the wavelength of light:

[Math Processing Error]

where mm is the integer, dd is the thickness of the film, and λλ is the wavelength of light. However, this condition may change if phase shifts occur upon reflection.

Polarization By Passing Light Through Polarizers

Polarization is the attribute that wave oscillations have a definite direction relative to the direction of propagation of the wave.

learning objectives

- Discuss polarization of electromagnetic waves

Definition of Polarization

As we discusses in previous atoms, light waves are a type of electromagnetic waves, in the visible spectrum. These electromagnetic (EM) waves are transverse waves. Figure 1 demonstrates that a transverse wave is one oscillates perpendicular to the direction of the energy transfer. If the wave is traveling from left to right, it is oscillating up and down. Polarization is the property of waves that allow them to oscillate in more than one direction, but that direction is relative to that of the direction the wave is traveling in. For an EM wave, the direction of polarization is the direction parallel to the electric field. In Figure 2 you can see that the EM and magnetic fields are perpendicular to the path of travel. Since the direction of polarization is parallel to the electric field, you can consider the blue arrows to be the direction of polarization.

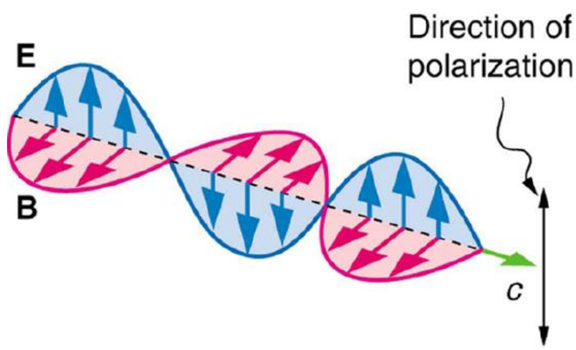


Figure 2: An EM wave, such as light, is a transverse wave. The electric and magnetic fields are perpendicular to the direction of propagation.

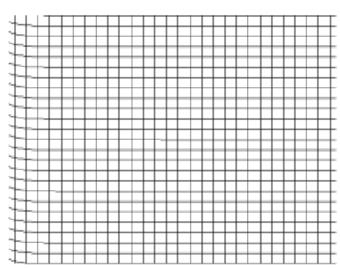


Figure 1: Transverse Waves

How it Works

Now, to examine the effects of passing light through a polarizer, lets look at Figure 3. shows a stationary vertical slot, which will act as a polarizer, and two waves traveling in the same direction, but one is oscillating vertically,a, (and therefore vertically polarized), and the other, b, horizontally. What happens to these waves as they pass through the polarizer?When wave a, the vertically oscillating wave is passed through the vertically polarized slot, nothing happens. The wave passes through untouched or manipulated. When wave b, the horizontally oscillating wave is passed through, the vertically polarized slot blocks the wave, and it is not passed through at all. Now that we understand the concept of polarization, and how it works, how can we apply this to make it useful? Look at Figure 4. The image on the left is full of glare, which makes it hard to see what we are looking at. The image on the right was taken with a polarized lens, so you only see the image, and none of the annoying glare. How this works is diagramed in Figure 5. Many light sources are unpolarized, and are comprised of many waves in all possible directions. Polarized lenses only allow one direction of light to pass through, minimizing the unwanted aspects of the light rays, such as glare. Simply, passing light through a polarized material changes the intensity of the light.

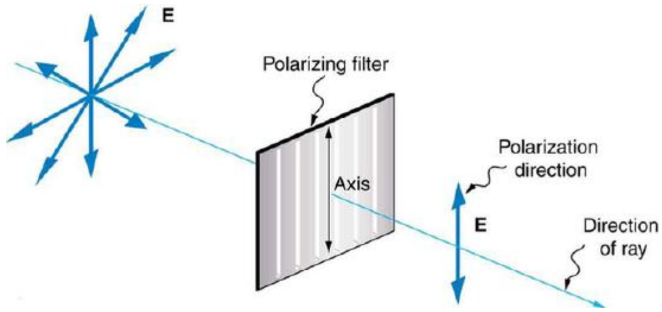


Figure 5: A polarizing filter has a polarization axis that acts as a slit passing through electric fields parallel to its direction. The direction of polarization of an EM wave is defined to be the direction of its electric field.

Figure 4: These two photographs of a river show the effect of a polarizing filter in reducing glare in light reflected from the surface of water. Part (b) of this figure was taken with a polarizing filter and part (a) was not. As a result, the reflection of clouds and sky observed in part (a) is not observed in part (b). Polarizing sunglasses are particularly useful on snow and water. (credit: Amithshs, Wikimedia Commons)

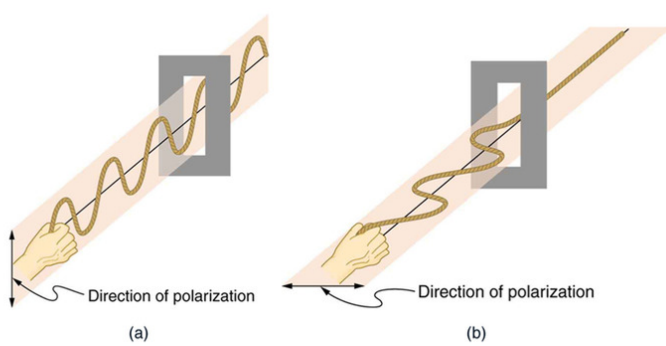


Figure 3: Example of passing light through a polarizer

Intensity

Lets call the angle between the direction of polarization and the axis of the polarization filter θ. The original intensity of the light before it is passed through a filter is denoted by I_0. In order to find the new intensity of light after traveling through the material is shown by the following equation:

[Math Processing Error]

If you pass light through two polarizing filters, you will get varied effects of polarization. If the two filters are oriented exactly perpendicular to each other, no light will pass through at all. If they are exactly parallel to each other, there will be no additional affect from the additional filter.

Polarization By Scattering and Reflecting

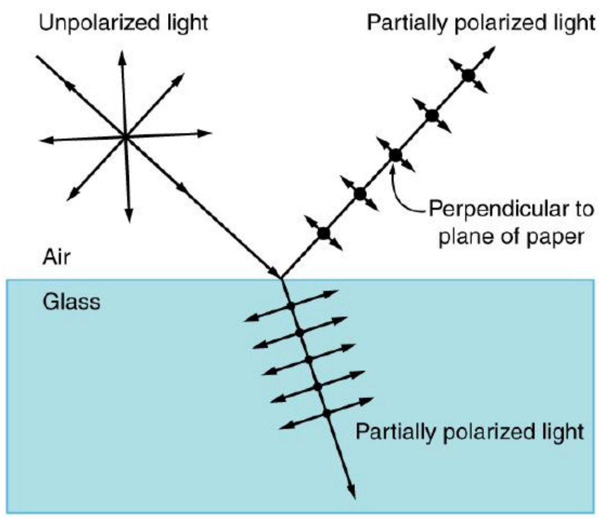
Unpolarized light can be polarized artificially, as well as by natural phenomenon like reflection and scattering.

learning objectives

- Calculate angle of reflection of complete polarization from indices of refraction

Polarization by Reflection

In the previous atom we discussed how polarized lenses work. In the case of polarized sunglasses, for example, when you look through them, reflected light is not entirely filtered out; reflected light can be slightly polarized by the reflection process (as shown in). Most light sources produce unpolarized light. When light hits a reflective surface, the vertically polarized aspects of that light are refracted at that surface. The reflected light is more horizontally polarized. To better remember this, we can think of light as an arrow and the reflective surface as a target. If the arrow hits the target perpendicularly (vertically polarized), it is going to stick in the target (be refracted into the surface). If the arrow hits the target on its side (horizontally polarized) then it will bounce right off (be reflected).



Polarization by Reflection: Unpolarized light has equal amounts of vertical and horizontal polarization. After interaction with a surface, the vertical components are preferentially absorbed or refracted, leaving the reflected light more horizontally polarized. This is akin to arrows striking on their sides bouncing off, whereas arrows striking on their tips go into the surface.

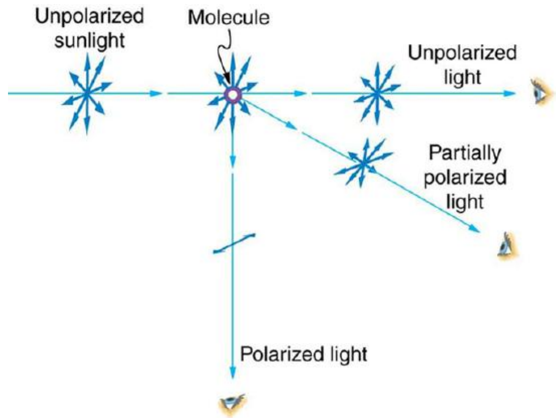
Since the light is split into two, and part of it is refracted, the amount of polarization to the reflected light depends on the index of refraction of the reflective surface. We can use the following equation to determine the angle of reflection at which light will be completely polarized:

[Math Processing Error]

where: θ_b = angle of reflection of complete polarization (also known as Brewster's angle); n_1 = index of refraction of medium in which reflected light will travel; and n_2 = index of refraction of medium by which light is reflected.

Polarization by Scattering

Just as unpolarized light can be partially polarized by reflecting, it can also be polarized by scattering (also known as Rayleigh scattering; illustrated in). Since light waves are electromagnetic (EM) waves (and EM waves are transverse waves) they will vibrate the electrons of air molecules perpendicular to the direction in which they are traveling. The electrons then produce radiation (acting like small antennae) that is polarized perpendicular to the direction of the ray. The light parallel to the original ray has no polarization. The light perpendicular to the original ray is completely polarized. In all other directions, the light scattered by air will be partially polarized.



Polarization by Scattering: Also known as Rayleigh scattering. Unpolarized light scattering from air molecules shakes their electrons perpendicular to the direction of the original ray. The scattered light therefore has a polarization perpendicular to the original direction and none parallel to the original direction.

Scattering of Light by the Atmosphere

Rayleigh scattering describes the air's gas molecules scattering light as it enters the atmosphere; it also describes why the sky is blue.

learning objectives

- Describe wave-particle relationship that leads to Rayleigh scattering and apply it to explain common phenomena

Rayleigh Scattering

Rayleigh scattering is the elastic scattering of waves by particles that are much smaller than the wavelengths of those waves. The particles that scatter the light also need to have a refractive index close to 1. This law applies to all electromagnetic radiation, but in this atom we are going to focus specifically on why the atmosphere scatters the visible spectrum of electromagnetic waves, also known as visible light. In this case, the light is scattered by the gas molecules of the atmosphere, and the refractive index of air is 1.

Rayleigh scattering is due to the polarizability of an individual molecule. This polarity describes how much the electric charges in the molecule will vibrate in an electric field. The formula to calculate the intensity of the scattering for a single particle is as follows:

[Math Processing Error]

where I is the resulting intensity, I_0 is the original intensity, α is the polarizability, λ is the wavelength, R is the distance to the particle, and θ is the scattering angle.

While you will probably not need to use this formula, it is important to understand that scattering has a strong dependence on wavelength. From the formula, we can see that a shorter wavelength will be scattered more strongly than a longer one.(The longer the wavelength, the larger the denominator, and from algebra we know that a larger denominator in a fraction means a smaller number.)

Why is the Sky Blue?

As we just learned, light scattering is inversely proportional to the fourth power of the light wavelength. So, the shorter the wavelength, the more it will get scattered. Since green and blue have relatively short wavelengths, you see a mixture of these colors in the sky, and the sky appears to be blue. When you look closer and closer to the sun, the light is not being scattered because it is approaching a 90-degree angle with the scattering particles. Since the light is being scattered less and less, you see the longer wavelengths, like red and yellow. This is why the sun appears to be a light yellow color.

Why are Sunsets Colorful?

shows a sunset. We know why the sky is blue, but why are there all those colors in a sunset? The reddening that occurs near the horizon is because the light has to pass through a significantly higher volume of air than when the sun is high in the sky. This increases the Rayleigh scattering effect and removes all blue light from the direct path of the observer. The remaining unscattered light is of longer wavelengths and so appears orange.

Sunset: A gradient of colors in the sky during sunset

Dispersion of the Visible Spectrum

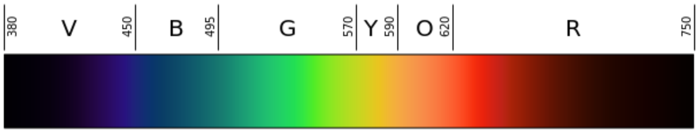
Dispersion is the spreading of white light into its full spectrum of wavelengths; this phenomenon can be observed in prisms and rainbows.

learning objectives

- Describe process of dispersion

The Visible Spectrum

Within the electromagnetic spectrum, there is only a portion that is visible to the human eye. Visible light is the range of wavelengths of electromagnetic radiation that humans can see. For a typical human eye, this ranges from 390 nm to 750 nm. shows this range and the colors associated with it:



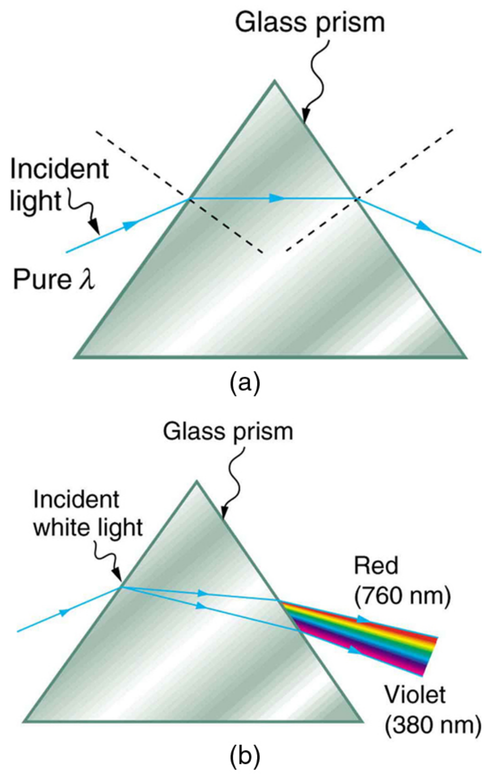
The Visible Spectrum: Visible Spectrum, represented linearly

- Violet: 380-450 nm
- Blue: 450-495 nm
- Green: 495-570 nm
- Yellow: 570-590 nm
- Orange: 590-620 nm
- Red: 620-750 nm

As you can see from, these are the colors of a rainbow and it is no coincidence.

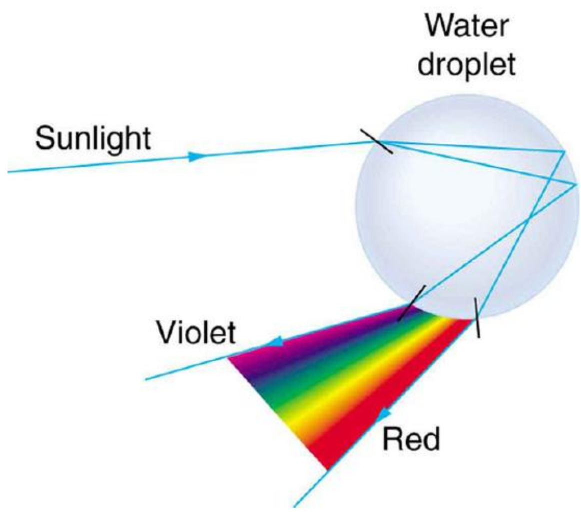
Dispersion

Dispersion is the spreading of white light into its full spectrum of wavelengths. How does this happen? The index of refraction is different for every medium that light travels through, as we learned in previous atoms. When a light ray enters a medium with a different index of refraction, the light is dispersed, as shown in with a prism. When white light enters the prism, it spreads. Since the index of refraction varies with wavelength, the light refracts at different angles as it exits, causing the exiting light rays to appear as a rainbow, or as a sequence of decreasing wavelengths, from red to violet.



Light and a Glass Prism: (a) A pure wavelength of light falls onto a prism and is refracted at both surfaces. (b) White light is dispersed by the prism (shown exaggerated). Since the index of refraction varies with wavelength, the angles of refraction vary with wavelength. A sequence of red to violet is produced, because the index of refraction increases steadily with decreasing wavelength.

This same principle can be applied to rainbows. Refer to. Rainbows are not only caused by refraction, like prisms, but also reflection. Light enters a drop of water and is reflected from the back of the droplet. The light is refracted once as it enters the drop, and again as is exits the drop. In water, the refractive index varies with wavelength, so the light is dispersed.



Light and a Water Droplet: Part of the light falling on this water drop enters and is reflected from the back of the drop. This light is refracted and dispersed both as it enters and as it leaves the drop.

Key Points

- When the incident light reaches the thin film, it is partially reflected and passed through to the bottom layer of the film. The difference in refractive indices of the air and film causes the light to change direction and interfere with the reflected portion of the ray when it emerges.
- This interference can be constructive, producing bright colors, or destructive, producing darker colors.
- Interference will be constructive if the optical path difference is equal to an integer multiple of the wavelength of light.
- The direction of polarization is parallel to the direction of the electric field associated with an electromagnetic wave, which includes light waves.
- By employing the polarization properties of waves, companies like Polariod how been able to produce materials that filter out unwanted waves of light, and minimize light intensity.
- Polarization materials can be oriented at different angles to produce different effects. The new intensity of light after passing through these materials can be found using the following formula: $I=I_0\cos 2\theta$.
- When unpolarized light hits a reflective surface, the vertically polarized aspects are refracted into the surface. The horizontally polarized aspects are reflected off the surface and the light is now perceived as partially polarized.
- When light is reflected, there is an angle at which this light is completely polarized. This is called Brewster's angle, after the Scottish physicst who discovered the law.
- Unpolarized light can also become polarized when it is scattered in air (also known as Rayleigh scattering). This occurs due to the fact that the EM waves cause the electron in air to vibrate, producing radiation and causing polarization of the light.
- The phenomenon of light scattering is called Rayleigh scattering; it can happen to any electromagnetic waves. It only occurs when waves encounter particles that are much smaller than the wavelengths of the waves.
- The amount that light is scattered is inversely proportional to the fourth power of the light wavelength. For this reason, shorter-wavelength light like greens and blues scatter more easily than longer wavelengths like yellows and reds.
- As you look closer to the sky's light source, the sun, the light is scattered less and less because the angle between the sun and the scattering particles approaches 90 degrees. This is why the sun has a yellowish color when we look at it from Earth while the rest of the sky appears blue.
- In outer space, where there is no atmosphere and therefore no particles to scatter the light, the sky appears black and the sun appears white.
- During a sunset, the light must pass through an increased volume of air. This increases the scattering effect, causing the light in the direct path of the observer to appear orange rather than blue.
- Dispersion is a side effect of the law of refraction. As refraction angles depend on wavelength, when light enters a medium with a different index of refraction, it can be dispersed, like a prism.
- The dispersion of white light can often cause the refracted light to be observed is in order of either increasing or decreasing wavelength, causing a rainbow effect.
- As you can see from the visible spectrum, there are some colors that the brain perceives that are not included. This is because some colors are a mixture of different wavelengths, like pink and magenta.

Key Terms

- **incident ray**: The ray of light that strikes the surface.
- **interference**: An effect caused by the superposition of two systems of waves, such as a distortion on a broadcast signal due to atmospheric or other effects.
- **wavelength**: The length of a single cycle of a wave, as measured by the distance between one peak or trough of a wave and the next; it is often designated in physics as λ, and corresponds to the velocity of the wave divided by its frequency.
- **oscillate**: To swing back and forth, especially if with a regular rhythm.
- **index of refraction**: For a material, the ratio of the speed of light in vacuum to that in the material.
- **polarization**: The production of polarized light; the direction in which the electric field of an electromagnetic wave points.
- **electromagnetic radiation**: radiation (quantized as photons) consisting of oscillating electric and magnetic fields oriented perpendicularly to each other, moving through space
- **polarizability**: The relative tendency of a system of electric charges to become polarized in the presence of an external electric field
- **refraction**: Changing of a light ray's direction when it passes through variations in matter.
- **reflection**: the property of a propagated wave being thrown back from a surface (such as a mirror)
- **dispersion**: The separation of visible light by refraction or diffraction.

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26.4: Applications of Wave Optics

learning objectives

- Compare optical and electron microscopy

Microscopes are used to view objects that cannot be seen with the naked eye. In this section we will discuss both optical and electron microscopy.

Optical Microscopy

You have probably used an optical microscope in a high school science class. In optical microscopy, light reflected from an object passes through the microscope's lenses; this magnifies the light. The resultant, magnified image is then seen by the eye. Although this type of microscopy has many limitations, there are several techniques that use properties of light and optics to enhance the magnified image:

- Bright field: This technique increases the contrast by illuminating the surface on which the objects sit from below.
- Oblique illumination: This technique illuminates the object from the side, giving it a three-dimensional appearance and highlighting features that would otherwise not be visible.
- Dark field: This technique is good for improving the contrast of transparent objects. A carefully aligned light source minimizes the un-scattered light entering the object plane and so only collects the light that is scattered by the object itself.
- Dispersion staining: This results in a colored image of a colorless object; it does not actually require that the object be stained.
- Phase contrast: This uses the refractive index of an object to show differences in optical density as a difference in contrast. provides a demonstration of this technique.

Electron Microscopy

Electron microscopes use electron beams to achieve higher resolutions than are possible in optical microscopy. Two kinds of electron microscopes are:

- Transmission electron microscope (TEM): The TEM sends an electron beam through a thin slice of a specimen. The electron interacts with the specimen and is then transmitted onto photographic paper or a screen. Since electron beams have a much smaller wavelength than traditional light, the resolution of the resulting image is much higher.
- Scanning electron microscope (SEM): The SEM shows details on the surface of a specimen and produces a three-dimensional view by scanning the specimen. shows an SEM image of pollen.

The Spectrometer

A spectrometer uses properties of light to identify atoms by measuring wavelength and frequency, which are functions of radiated energy.

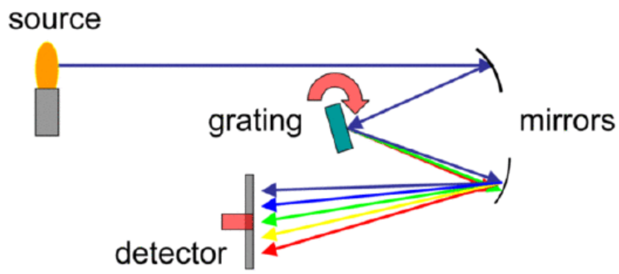
learning objectives

- Compare design and function of early and modern spectrometers

The Spectrometer

A spectrometer is an instrument used to intensely measure light over a specific portion of the electromagnetic spectrum, to identify materials. The instrument produces lines, much like those produced from diffraction grating as covered in a previous atom, and then measures the wavelengths and intensities of those lines.

shows a diagram of how a spectrometer works. The source is placed in front of a mirror, which reflects the light emitted from that object onto a diffraction grating. This grating then disperses the emitted light to anther mirror which spreads the different resultant wavelengths and reflects them onto a detector which records the findings. This type of instrument is used in spectroscopy.



Spectrometer Diagram: This diagram shows the light pathways in a spectrometer.

Spectroscopy

Spectroscopy studies the interaction between matter and radiated energy. This radiated energy is a function of wavelength and frequency. Every type of atom has its own frequency. When the spectrometer produces a reading, the observer can then use spectroscopy to identify the atoms and therefore molecules that make up that object.

Spectroscopes

Spectroscopes are used in a variety of fields, such as astronomy and chemistry. They use a diffraction grating, movable slit, and a photodetector. All of these elements are controlled by a computer, which records the findings. A material is heated to incandescence and it emits a light that is characteristic of its atomic makeup. Each atom has its own spectroscopic 'fingerprint'. In you can see a very simple spectroscope based on a prism. As another example, Sodium produces a double yellow band.

A simple spectroscope: A very simple spectroscope based on a prism

The Michelson Interferometer

The Michelson interferometer is the most common configuration for optical interferometry.

learning objectives

- Explain how the Michelson interferometer works

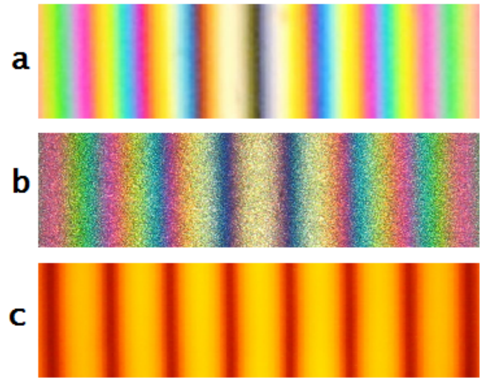
Interferometry

Before we can discuss the Michelson Interferometer, it is important we first understand interferometry—which refers to techniques that use superimposed waves to extract information about the waves. More simply, it uses the interference these waves experience to make accurate measurements of the waves. It is used in many areas of science, such as astronomy, engineering, oceanography, physics, and fiber optics.

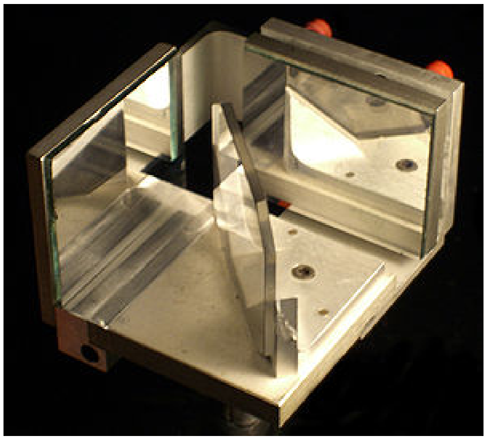
Popular applications of interferometry in industry include the measurement of small displacements, refractive index changes, and surface irregularities. As shown in previous atoms, when two waves with the same frequency combine, the resulting pattern is determined by the phase difference between the two. Constructive interference occurs when the waves are in phase, and destructive interference occurs when they are out of phase. Interferometry uses this principle to combine waves and study the resulting wave in order to obtain information about the original state of the waves.

The Michelson Interferometer

The most common tool in interferometry, the Michelson Interferometer, shown in Figure 1, was invented by Albert Abraham Michelson, the first American to win a Nobel Prize for science. The interferometer works by splitting a beam of light into two paths, bouncing them back and then recombining them to create an interference pattern. To create interference fringes on a detector (see Figure 2), the paths may be different lengths or composed of different materials.



Fringes in a Michelson Interferometer: Colored and monochromatic fringes in a Michelson interferometer: (a) White light fringes where the two beams differ in the number of phase inversions; (b) White light fringes where the two beams have experienced the same number of phase inversions; and (c) Fringe pattern using monochromatic light (sodium D lines).



Michelson Interferometer: A Michelson Interferometer.

Figure 3 shows a diagram of how a Michelson Interferometer works. M_1 and M_2 are two highly polished mirrors, S is the light source, M is a half silvered mirror that acts as a beam splitter when the light hits the surface, and C is a point on M that is partially reflective. When the beam S hits this point on M it is split into two beams. One beam is reflected in the direction of A and the other is transmitted through the surface of M to the point B. A and B are both points on the highly polished (and therefore reflective) mirrors M_1 and M_2. When the beams hit these points, they are then reflected back to point C', where they recombine to produce an interference pattern. At point E, the interference pattern produced at point C' is visible to an observer.

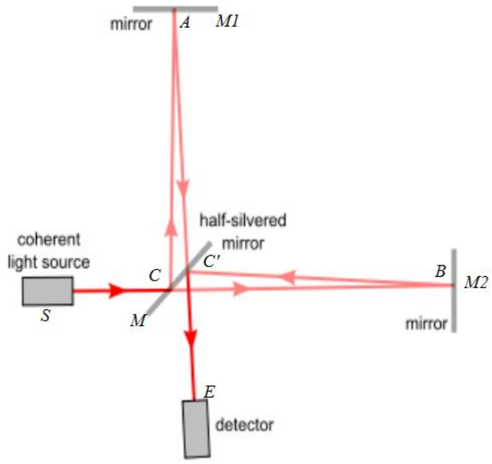


Figure 3: This diagram of a Michelson Interferometer shows the path that the light waves travels in the instrument.

Applications

The Michelson Interferometer has been used for the detection of gravitational waves, as a tunable narrow band filter, and as the core of Fourier transform spectroscopy. It has played an important role in studies of the upper atmosphere, revealing temperatures and winds (employing both space-borne and ground-based instruments) by measuring the Doppler widths and shifts in the spectra of airglow and aurora. The best known application of the Michelson Interferometer is the Michelson-Morley experiment—a failed attempt to demonstrate the effect of the hypothetical " aether wind" on the speed of light. Their experiment left theories of light based on the existence of a luminiferous aether without experimental support, and served ultimately as an inspiration for special relativity.

LCDs

Liquid crystal displays use liquid crystals which do not emit light, but use the light modulating properties of the crystals.

learning objectives

- Explain how liquid crystal displays produce images and discuss their benefits and deficiencies

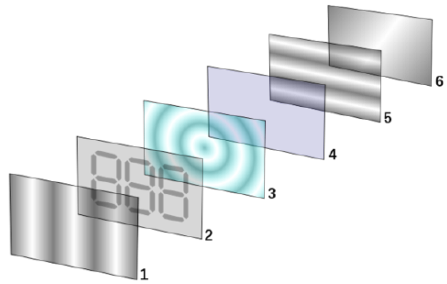
LCDs

LCD stands for a liquid crystal display. The liquid crystals themselves do not emit light, but the display uses the light modulating properties of the crystals. LCDs can be used to display arbitrary images, such as in a computer monitor or television, by using a large number of very small pixels, or they can be used to display fixed images, like a digital clock, such as in.

Digital Clock: A digital clock which uses LCD to either hide or display fixed images.

Unlike the newer cathode ray tube (CRT) and plasma displays, LCDS do not use phosphors. For this reason they do not suffer image burn-in. They do however suffer image persistence. Image burn-in occurs when an image is displayed so many times, or for so long, that an outline of image can be seen even when the display is turned off. Image persistence is similar, but the outline fades away shortly after the display is turned off and is not permanent.

LCD displays are made up of numerous layers. A typical layer is diagrammed in. Each pixel of an LCD consists of a layer of molecules aligned between two transparent electrodes and two polarizing films, and the actual liquid crystals are between these polarizing filters. The light passes through the first filter, and is blocked by the second. The electrodes are used to align the crystals in a particular direction, which produces the image seen on the screen. The crystals do not emit any light, but rather give the light a specific shape to be emitted in.



Layers of LCD Displays: Polarizing filter film with a vertical axis to polarize light as it enters.Glass substrate with ITO electrodes. The shapes of these electrodes will determine the shapes that will appear when the LCD is turned on. Vertical ridges etched on the surface are smooth.Twisted nematic liquid crystal.Glass substrate with common electrode film (ITO) with horizontal ridges to line up with the horizontal filter.Polarizing filter film with a horizontal axis to block/pass light.Reflective surface to send light back to viewer. (In a backlit LCD, this layer is replaced with a light source.)

Twisted Nematic Devices

The twisted nematic device is the most common LCD application. When no electric field is being applied, the surface alignment directions at the electrodes are perpendicular to each other. The molecules arrange themselves in a helical structure (twisted structure). Some light is able to pass through and some is not, so the result is that the screen appears gray. When the electric field is applied, the crystals in the center layer untwist, and the light is completely blocked from passing through and those pixels will appear black.

Using Interference to Read CDs and DVDs

Optical discs are digital storing media read in an optical disc drive using laser beam.

learning objectives

- Explain how information is stored on the optical disks

Overview

Compact disks (CDs) and digital video disks (DVDs) are examples of optical discs. They are read in an optical disc drive which directs a laser beam at the disc. The reader then detects whether the beam has been reflected or scattered.

Function of Digital Discs

Optical discs are digital storing media. They can store music, files, movies, pictures etc.. These discs are flat, usually made of aluminum, and have microscopic pits and lands on one of the flat surfaces (as shown in). The information on these discs are read by a computer in the form of binary data. First, a laser beam is shot at the disc. If the beam hits a land, it gets reflected back and is recorded as a value of 1. If the beam hits a pit, it gets scattered and is recorded as a value of zero.

Early Version of an Optical Disc: In this early version of an optical disc, you can see the pits and lands which either reflect back light or scatter it.

These microscopic pits and lands cover the entire surface of the disc in a spiral path, starting in the center and working its way outward. The data is stored either by a stamping machine or laser and is read when the data is illuminated by a laser diode in the disc drive. The disc spins at a faster speed when it is being read in the center track, and slower for an outer track. This is because the center tracks are smaller in circumference and therefore can be read quicker.

These pits also act as slits and cause the light to be diffracted as it is reflected back, which causes an iridescent effect. This explains the rainbow pattern that you see on the back of a CD, as shown in.

Compact Disc: The bottom surface of a compact disc showing characteristic iridescence.

Key Points

- In optical microscopy, light reflected from an object passes through the microscope's lenses; this magnifies the light. The resultant, magnified image is then seen by the eye. This technique has many limitations but can be enhanced in various ways to create more contrast.
- The transmission electron microscope (TEM) sends an electron beam through a thin slice of a specimen. The electron is then transmitted onto photographic paper or a screen. Since electron beams have a much smaller wavelength than traditional light, the resolution of this image is much higher.
- The scanning electron microscope (SEM) shows details on the surface of a specimen and produces a three-dimensional view by scanning the specimen.
- The source is placed in front of a mirror, which reflects the light emitted from that object onto a diffraction grating. This grating then disperses the emitted light to anther mirror which spreads the different resultant wavelengths and reflects them onto a detector which records the findings.
- Early forms of spectrometers were simple prisms, but modern spectrometers are automated by a computer and can record a much broader range of frequencies.
- Spectrometers are used in spectroscopy. Spectroscopy studies the interaction between matter and radiated energy. This radiated energy is a function of wavelength and frequency. Every type of atom has its own frequency.
- Interferometry refers to techniques that use superimposed waves to extract information about the waves.
- Michelson interferometer works by splitting a beam of light into two paths, bouncing them back and recombining them to create an interference pattern. To create interference fringes on a detector, the paths may be different lengths or composed of different materials.
- The best known application of the Michelson interferometer is the Michelson-Morley experiment, the unexpected null result of which was an inspiration for special relativity.
- LCDs use an electric field to arrange the liquid crystals into the desired pattern, and then pass light through these layers to produce an image on the screen.
- LCDs can be used to display an arbitrary image made up of tiny fixed pixels, or can be used to display a fixed image, as in on a digital clock.
- A twisted nematic display is the most common LCD in use. This type of display is on calculators, digital watches, and clocks. When no electric field is applied, the molecules are twisted, and let some light through. When the field is applied, they untwist, blocking the light and are seen as black.

- The iridescent layer of the disc is imprinted with tiny pits and lands. Pits scatter light when illuminated, and produce a reading of 0; lands reflect light back and produce a reading of 1.
- The optical disc drive records the 0 and 1 readings and translates them into binary data which is used to relay whatever information is recorded on the disc.
- Rainbow pattern on the back of a CD is due diffraction of the reflected light by pits.

Key Terms

- **microscopy**: using microscopes to view objects that cannot be seen with the naked eye
- **contrast**: A difference in lightness, brightness and/or hue between two colors that makes them more or less distinguishable
- **incandescence**: Incandescence is the emission of light (visible electromagnetic radiation) from a hot body as a result of its temperature.
- **special relativity**: A theory that (neglecting the effects of gravity) reconciles the principle of relativity with the observation that the speed of light is constant in all frames of reference.
- **superimposed**: Positioned on or above something else, especially in layers
- **interference**: An effect caused by the superposition of two systems of waves, such as a distortion on a broadcast signal due to atmospheric or other effects.
- **LCD**: a liquid crystal display.
- **helical**: In the shape of a helix, twist.
- **nematic**: Describing the structure of some liquid crystals whose molecules align in loose parallel lines.
- **binary data**: Data which can take on only two possible values, traditionally termed 0 and 1.
- **pit**: An imprint on an optical disc that scatters light when illuminated.
- **land**: A flat area on an optical disc that reflects light when illuminated.

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CHAPTER OVERVIEW

27: Special Relativity

27.1: Introduction

learning objectives

- Explain why the Galilean invariance didn't work in Maxwell's equations

Galilean invariance or Galilean relativity states that the laws of motion are the same in all inertial (or non-accelerating) frames. Galileo Galilei first described this principle in 1632 using the example of a ship travelling at constant velocity, without rocking, on a smooth sea; any observer doing experiments below the deck would not be able to tell whether the ship was moving or stationary. The fact that the Earth orbits around the sun at approximately 30 km/s offers a somewhat more dramatic example, though it is technically not an inertial reference frame.

Specifically, the term Galilean invariance today usually refers to this principle as applied to Newtonian mechanics—that is, Newton's laws hold in all inertial frames. In this context it is sometimes called Newtonian relativity. Among the axioms from Newton's theory are:

- There exists an absolute space in which Newton's laws are true. An inertial frame is a reference frame in relative uniform motion to absolute space.
- All inertial frames share a universal (or absolute) time.

Derivation

Galilean relativity can be shown as follows. Consider two inertial frames S and S'. A physical event in S will have position coordinates $r = (x, y, z)$ and time t; similarly for S'. By the second axiom above, one can synchronize the clock in the two frames and assume $t = t$'. Suppose S' is in relative uniform motion to S with velocity v. Consider a point object whose position is given by $r = r(t)$ in S. We see that

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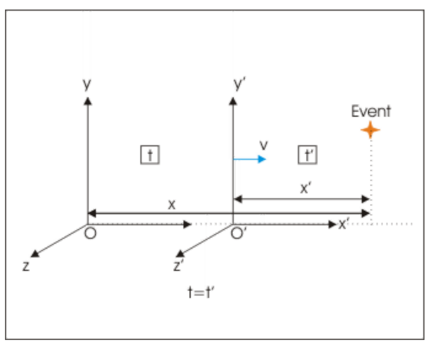
This transformation of variables between two inertial frames is called Galilean transformation. Now, the velocity of the particle is given by the time derivative of the position:



Galilean Invariance: Newtonian mechanics is invariant under a Galilean transformation between observation frames (shown). This is called Galilean invariance.

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Another differentiation gives the acceleration in the two frames:

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It is this simple but crucial result that implies Galilean relativity. Assuming that mass is invariant in all inertial frames, the above equation shows that Newton's laws of mechanics, if valid in one frame, must hold for all frames. But it is assumed to hold in absolute space, therefore Galilean relativity holds.

Issues

Both Newtonian mechanics and the Maxwell's equations were well established by the end of the 19^{th} century. The puzzle lied in the fact that the Galilean invariance didn't work in Maxwell's equations. That is, unlike Newtonian mechanics, Maxwell's equations are not invariant under a Galilean transformation. Albert Einstein's central insight in formulating special relativity was that, for full consistency with electromagnetism, mechanics must also be revised, such that Lorentz invariance (introduced later) replaces Galilean invariance. At the low relative velocities characteristic of everyday life, Lorentz invariance and Galilean invariance are nearly the same, but for relative velocities close to that of light they are very different.

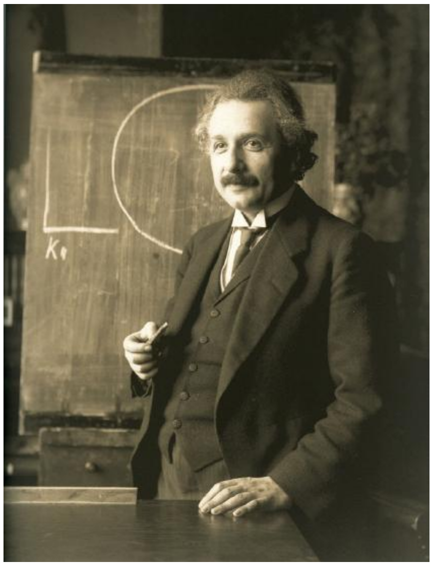
Einstein's Postulates

Special relativity is based on Einstein's two postulates: the Principle of Relativity and the Principle of Invariant Light Speed.

learning objectives

- Identify two postulates forming the foundation of special relativity

In the late 19th century, the Newtonian mechanics was considered to be valid in all inertial frames of reference, which are moving at a constant relative velocity with respect to each other. (See our previous lesson on "Galilean-Newtonian Relativity. ") One issue, however, was that another well-established theory, the laws of electricity and magnetism represented by Maxwell's equations, was not "invariant" under Galilean transformation—meaning that Maxwell's equations don't maintain the same forms for different inertial frames. In his "Special Theory of Relativity," Einstein resolved the puzzle and broadened the scope of the invariance to extend the validity of all physical laws, including electromagnetic theory, to all inertial frames of reference.



Albert Einstein: Albert Einstein, a true pioneer of modern physics. His work on relativity, gavity, quantum mechanics, and statistical physics revolutionized physics.

Einstein's Postulates

With two deceptively simple postulates and a careful consideration of how measurements are made, Einstein produced the theory of special relativity.

1. The Principle of Relativity: The laws of physics are the same and can be stated in their simplest form in all inertial frames of reference.

This postulate relates to reference frames. It says that there is no preferred frame and, therefore, no absolute motion.

2. The Principle of Invariant Light Speed: The speed of light c is a constant, independent of the relative motion of the source and observer.

The laws of electricity and magnetism predict that light travels at $c = 2.998 \times 10^8$ m/s in a vacuum, but they do not specify the frame of reference in which light has this speed. Physicists assumed that there exists a stationary medium for the propagation of light, which they called " luminiferous aether. " In 1887, Michelson and Morley attempted to detect the relative motion of the Earth through the stationary luminiferous aether, but their negative results implied the speed of light c is independent of the motion of the source relative to the observer. Einstein accepted the result of the experiment and incorporated it in his theory of relativity.

This postulate might sound easy to accept, but it is rather counterintuitive. Imagine that you can throw a baseball at a speed v (relative to you). If you are on a train moving at a speed V and throw a ball in the direction of the train's movement, the baseball will travel at a speed $v+V$ for an observer stationary on the ground.

Now, instead of a baseball, let's say you have a laser pointer. You turn on the laser pointer while you are on a moving train. What would be the speed of light from the laser pointer for a stationary observer on the ground? Our intuition says that it should be $c+V$. However, Einstein says that it should be only c!

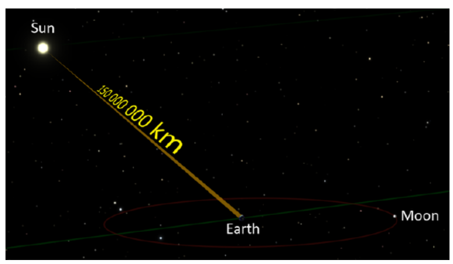
The Speed of Light

The speed of light in vacuum is a universal physical constant crucial to many areas of physics.

learning objectives

- Discuss the invariance of the speed of light and identify the value of that speed in vacuum

The speed of light in vacuum, commonly denoted c, is a universal physical constant that is crucial to many areas of physics. Its value is 299,792,458 m/s; this is a precise known value because the length of the meter is itself derived from this constant and the international standard for time. According to special relativity, c is the maximum speed at which all energy, matter, and information in the universe can travel. It is the speed at which all massless particles and associated fields (including electromagnetic radiation such as light) travel in vacuum. It is also the speed of gravity (i.e., of gravitational waves) predicted by current theories. Such particles and waves (including light) travel at c regardless of the motion of the source or the inertial frame of reference of the observer. In the theory of relativity, c interrelates space and time in the Lorentz transformation; it also appears in the famous equation of mass-energy equivalence: $E = mc^2$.



Sunlight's Flight to Earth: Sunlight takes about 8 minutes and 19 seconds to reach the earth (based on the average distance between the sun and the earth)

First Measurement

The first quantitative estimate of the speed of light was made in 1676 by Rømer. From the observation that the periods of Jupiter's innermost moon (Io) appeared to be shorter when Earth was approaching Jupiter than when it was moving away, he concluded that light travels at a finite speed. He estimated that it takes light 22 minutes to cross the diameter of Earth's orbit. Christiaan Huygens combined this estimate with an estimate for the diameter of the Earth's orbit to obtain an estimate of the speed of light of 220,000 km/s, 26 percent lower than the actual value.

Fundamental Role in Physics

The speed at which light waves propagate in vacuum is independent both of the motion of the wave source and of the inertial frame of reference of the observer. This invariance of the speed of light was postulated by Einstein in 1905 after being motivated by Maxwell's theory of electromagnetism and the lack of evidence for "luminiferous aether"; it has since been consistently confirmed by many experiments.

Key Points

- Galilean invariance states that Newton's laws hold in all inertial frames.
- Newtonian mechanics assumes that there exists an absolute space and that time is universal.
- Albert Einstein's central insight in formulating special relativity was that, for full consistency with electromagnetism, mechanics must also be revised such that Lorentz invariance (introduced later) replaces Galilean invariance.
- The special theory of relativity explores the consequences of the invariance of c with the assumption that the laws of physics are the same in all inertial frames of reference.

- Einstein's first postulate says that the laws of physics are the same and can be stated in their simplest form in all inertial frames of reference. It means that there is no preferred frame and, therefore, no absolute motion.
- The speed of light *c* is a constant, independent of the relative motion of the source and observer.
- The speed at which light waves propagate in vacuum is independent both of the motion of the wave source and of the inertial frame of reference of the observer. This invariance of the speed of light was postulated by Einstein in 1905 in his work on special relativity.
- The value of the speed of light is 299,792,458 m/s; this is a precise known value because the length of the meter is itself derived from this constant and the international standard for time.
- *c* is the maximum speed at which all energy, matter, and information in the universe can travel.

Key Terms

- **Lorentz invariance**: First introduced by Lorentz in an effort to explain how the speed of light was observed to be independent of the reference frame, and to understand the symmetries of the laws of electromagnetism.
- **absolute space**: A concept introduced by Newton that assumes space remains always similar and immovable.
- **Maxwell's equations**: A set of equations describing how electric and magnetic fields are generated and altered by each other and by charges and currents.
- **special relativity**: A theory that (neglecting the effects of gravity) reconciles the principle of relativity with the observation that the speed of light is constant in all frames of reference.
- **luminiferous aether**: Light-bearing aether; the postulated medium for the propagation of light.
- **Lorentz transformation**: a transformation relating the spacetime coordinates of one frame of reference to another in special relativity

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27.2: Consequences of Special Relativity

learning objectives

- Formulate conclusions of the theory of special relativity, noting the assumptions that were made in deriving it

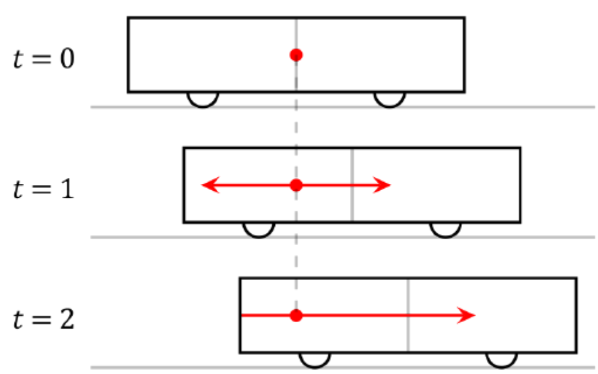
The relativity of simultaneity is the concept that simultaneity–whether two events occur at the same time–is not absolute, but depends on the observer's frame of reference.

According to the theory of special relativity, it is impossible to say in an absolute sense whether two distinct events occur at the same time if those events are separated in space, such as a car crash in London and another in New York. The question of whether the events are simultaneous is relative: in some reference frames the two accidents may happen at the same time, in other frames (in a different state of motion relative to the events) the crash in London may occur first, and still in other frames, the New York crash may occur first. If the two events are causally connected ("event A causes event B"), then the relativity of simultaneity preserves the causal order (i.e. "event A causes event B" in all frames of reference).

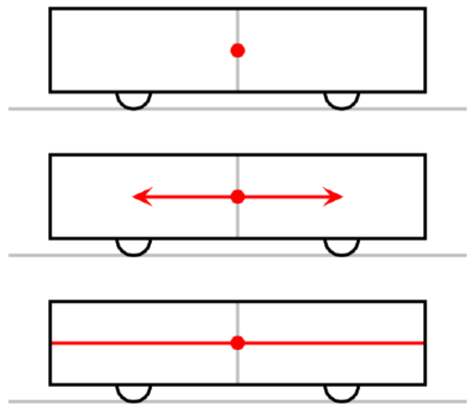
If we imagine one reference frame assigns precisely the same time to two events that are at different points in space, a reference frame that is moving relative to the first will generally assign different times to the two events. This is illustrated in the ladder paradox, a thought experiment which uses the example of a ladder moving at high speed through a garage.

A mathematical form of the relativity of simultaneity ("local time") was introduced by Hendrik Lorentz in 1892, and physically interpreted (to first order in v/c) as the result of a synchronization using light signals by Henri Poincaré in 1900. However, both Lorentz and Poincaré based their conceptions on the aether as a preferred but undetectable frame of reference, and continued to distinguish between "true time" (in the aether) and "apparent" times for moving observers.

In 1905, Albert Einstein abandoned the (classical) aether and emphasized the significance of relativity of simultaneity to our understanding of space and time. He deduced the failure of absolute simultaneity from two stated assumptions: 1) the principle of relativity–the equivalence of inertial frames, such that the laws of physics apply equally in all inertial coordinate systems; 2) the constancy of the speed of light detected in empty space, independent of the relative motion of its source.



Observer Standing on the Platform: Reference frame of an observer standing on the platform (length contraction not depicted).



Observer Onboard the Train: The train-and-platform experiment from the reference frame of an observer onboard the train.

Time Dilation

Time dilation is an actual difference of elapsed time between two events as measured by observers moving relative to each other.

learning objectives

- Explain why time dilation can be ignored in daily life

Time dilation is an actual difference of elapsed time between two events as measured by observers either moving relative to each other.

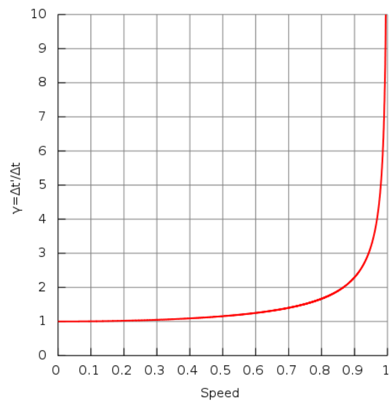
For instance, two rocket ships (A and B) speeding past one another in space would experience time dilation. If they somehow had a clear view into each other's ships, each crew would see the others' clocks and movement as going too slowly. That is, inside the frame of reference of Ship A, everything is moving normally, but everything over on Ship B appears to be moving slower (and vice versa).

From a local perspective, time registered by clocks that are at rest with respect to the local frame of reference (and far from any gravitational mass) always appears to pass at the same rate. In other words, if a new ship, Ship C, travels alongside Ship A, it is "at rest" relative to Ship A. From the point of view of Ship A, new Ship C's time would appear normal too.

The formula for determining time dilation is: *[Math Processing Error]*

where Δt is the time interval between two co-local events (i.e. happening at the same place) for an observer in some inertial frame (e.g. ticks on his clock), this is known as the proper time, Δt' is the time interval between those same events, as measured by another observer, inertially moving with velocity v with respect to the former observer, v is the relative velocity between the observer and the moving clock, c is the speed of light, and *[Math Processing Error]*

is the Lorentz factor. Thus the duration of the clock cycle of a moving clock is found to be increased: it is measured to be "running slow". Note that for speeds below 1/10 the speed of light, Lorentz factor is approximately 1. Thus, time dilation effects and extremely small and can be safely ignored in a daily life. They become important only when an object approaches speeds on the order of 30,000 km/s (1/10 the speed of light).



Lorentz Factor: Lorentz factor as a function of speed (in natural units where c = 1). Notice that for small speeds (less than 0.1), γ is approximately 1.

Effects of Time Dilation: The Twin Paradox and the Decay of the Muon

The twin paradox is a thought experiment: one twin makes a journey into space and returns home to find that twin remained aged more.

learning objectives

- Explain the twin paradox within the standard framework of special relativity

The twin paradox is a thought experiment in special relativity involving identical twins, one of whom makes a journey into space in a high-speed rocket and returns home to find that the twin who remained on Earth has aged more.

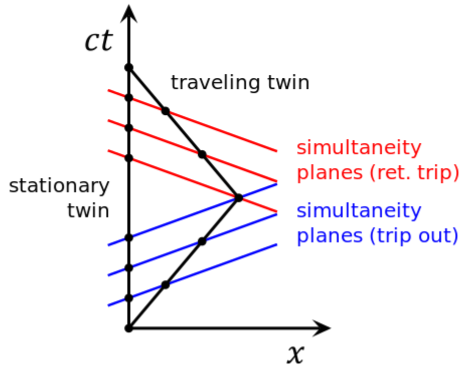
This occurs because special relativity shows that the faster one travels, the slower time moves for them.

This result appears puzzling because each twin sees the other twin as traveling, and so, according to a naive application of time dilation, each should paradoxically find the other to have aged more slowly. In other words, from the perspective of the rocketship, the earth is traveling away from the ship and from the perspective of the earth, the rocket is traveling away.

However, this scenario can be resolved within the standard framework of special relativity (because the twins are not equivalent; the space twin experienced additional, asymmetrical acceleration when switching direction to return home), and therefore is not a paradox in the sense of a logical contradiction.

The Earth and the ship are not in a symmetrical relationship: regardless of whether we view the situation from the perspective of the Earth or the ship, the ship experiences additional acceleration forces. The ship has a turnaround in which it accelerates and changes direction whereas the earth does not. Since there is no symmetry, it is not paradoxical if one twin is younger than the other. Nevertheless twin paradox is useful as a demonstration that special relativity is self-consistent.

In the spacetime diagram, drawn for the reference frame of the Earth-based twin, that twin's world line coincides with the vertical axis (his position is constant in space, moving only in time). On the first leg of the trip, the second twin moves to the right (black sloped line); and on the second leg, back to the left. Blue lines show the planes of simultaneity for the traveling twin during the first leg of the journey; red lines, during the second leg. Just before turnaround, the traveling twin calculates the age of the Earth-based twin by measuring the interval along the vertical axis from the origin to the upper blue line. Just after turnaround, if he recalculates, he'll measure the interval from the origin to the lower red line. In a sense, during the U-turn the plane of simultaneity jumps from blue to red and very quickly sweeps over a large segment of the world line of the Earth-based twin. The traveling twin reckons that there has been a jump discontinuity in the age of the Earth-based twin.



Spacetime Diagram of the Twin Paradox: Spacetime diagram of the twin paradox. Time is relative, but both twins are not equivalent (the ship experiences additional acceleration to changes the direction of travel).

Length Contraction

Objects that are moving undergo a length contraction along the dimension of motion; this effect is only significant at relativistic speeds.

learning objectives

- Explain why length contraction can be ignored in daily life

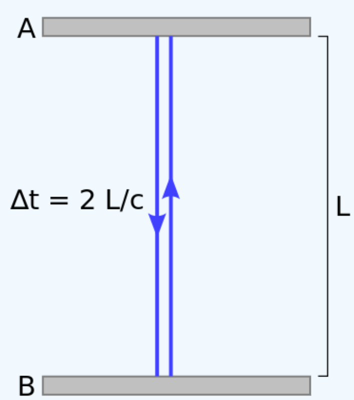
Length contraction is the physical phenomenon of a decrease in length detected by an observer of objects that travel at any non-zero velocity relative to that observer. Length contraction arises due to the fact that the speed of light in a vacuum is constant in any frame of reference. By taking this into account, as well as some geometrical considerations, we will show how perceived time and length are affected.

Example [Math Processing Error]:

Let us imagine a simple clock system that consists of two mirrors A and B in a vacuum. A light pulse bounces between the two mirrors. The separation of the mirrors is *L*, and the clock ticks once each time the light pulse hits a given mirror. Now imagine that the clock is at rest. The time that it will take for the light pulse to go from mirror A to mirror B and then back to mirror A be can be described by:

[Math Processing Error]

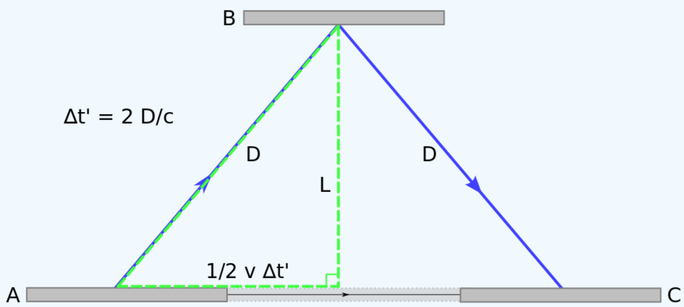
where *c* is the speed of light. Now imagine that the clock is moving in the horizontal direction relative to a stationary observer. The light pulse is emitted from mirror A. To the stationary observer, it appears that the light pulse has a longer path to travel because by the time the light reaches mirror B the clock has already moved somewhat in the horizontal direction. This is the same case for the light pulse on its way back. The stationary observer will perceive that it will take the light a total of:



Geometry for a Clock at Rest: This illustrates the path that light must traverse when the clock is at rest.

[Math Processing Error]

to traverse its path. We can see that D is longer than L, so that means that.

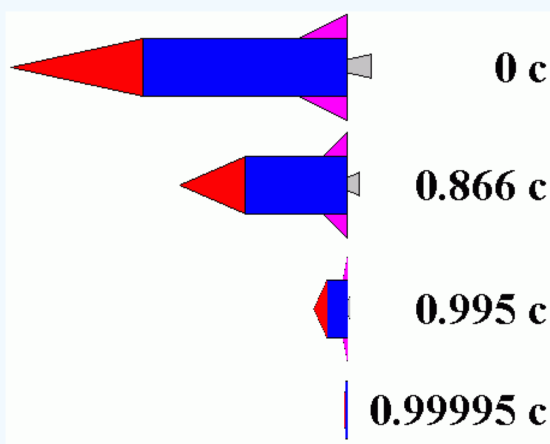


Geometry for a Moving Clock: This illustrates the path that light must traverse when the clock is moving from the perspective of a stationary observer.

Example [Math Processing Error]:

We have established that in a frame of reference that is moving relative to the clock (the stationary observer is moving in the clock's frame of reference), the clock appears to run more slowly. Now let us imagine that we want to measure the length of a ruler. This time let us imagine that you are moving with velocity v. You can mathematically determine the length of the ruler in your frame of reference (L‘) by multiplying your velocity (v) by the time that you perceive that it takes you to pass by the ruler (t‘). Expressing this in equation form, L‘ = vt‘. Now, if someone in the ruler's rest frame wanted to determine the length of the ruler, they could do the following. They could mathematically determine the length of the ruler in their frame of reference (L) by multiplying your velocity (v) by the time that they perceive that it takes you to pass by the ruler (t). This is expressed in the following equation: $L = vt$. Just as in the clock explanation, the ruler appears to be moving in your frame of reference, so t will be longer than t‘ (your time interval). Consequently, the length of the ruler will appear to be shorter in your frame of reference (the phenomenon of length contraction occurred).

The effect of length contraction is negligible at everyday speeds and can be ignored for all regular purposes. Length contraction becomes noticeable at a substantial fraction of the speed of light (as illustrated in) with the contraction only in the direction parallel to the direction in which the observed body is travelling.



Observed Length of an Object: Observed length of an object at rest and at different speeds

Example [Math Processing Error]:

For example, at a speed of 13,400,000 m/s (30 million mph,.0447c), the length is 99.9 percent of the length at rest; at a speed of 42,300,000 m/s (95 million mph, 0.141c), the length is still 99 percent. As the magnitude of the velocity approaches the speed of light, the effect becomes dominant.The mathematical formula for length contraction is:

[Math Processing Error]

where L_0 is the proper length (the length of the object in its rest frame); L is the length observed by an observer in relative motion with respect to the object; v is the relative velocity between the observer and the moving object; c is the speed of light; and the Lorentz factor is defined as:

[Math Processing Error]

In this equation it is assumed that the object is parallel with its line of movement. For the observer in relative movement, the length of the object is measured by subtracting the simultaneously measured distances of both ends of the object. An observer at rest viewing an object traveling very close to the speed of light would observe the length of the object in the direction of motion as very close to zero.

Key Points

- According to the theory of special relativity, it is impossible to say in an absolute sense whether two distinct events occur at the same time if those events are separated in space.
- A mathematical form of the relativity of simultaneity was introduced by Hendrik Lorentz and physically interpreted by Henri Poincaré. The conceptions were based on the aether as a preferred but undetectable frame of reference.
- Albert Einstein deduced the failure of absolute simultaneity from two assumptions: 1) the principle of relativity; 2) the constancy of the speed of light detected in empty space, independent of the relative motion of its source.
- Time dilation effects are extremely small for speeds below 1/10 the speed of light and can be safely ignored at daily life.
- Time dilation effects become important when an object approaches speeds on the order of 30,000 km/s (1/10 the speed of light).
- The formula for determining time dilation is: *[Math Processing Error]*.
- From a naive application of time dilation, each twin should paradoxically find the other to have aged more slowly.
- The scenario is resolved within the standard framework of special relativity: the twins are not equivalent, the space twin experiences additional, asymmetrical acceleration when switching direction to return home.
- Twin paradox is useful as a demonstration that special relativity is self-consistent.
- Length contraction is negligible at everyday speeds and can be ignored for all regular purposes.
- Length contraction becomes noticeable at a substantial fraction of the speed of light with the contraction only in the direction parallel to the direction in which the observed body is travelling.
- An observer at rest viewing an object travelling very close to the speed of light would observe the length of the object in the direction of motion as very near zero.

Key Terms

- **aether**: A space-filling substance or field, thought to be necessary as a transmission medium for the propagation of electromagnetic or gravitational forces.
- **special relativity**: A theory that (neglecting the effects of gravity) reconciles the principle of relativity with the observation that the speed of light is constant in all frames of reference.
- **speed of light**: the speed of electromagnetic radiation in a perfect vacuum: exactly 299,792,458 meters per second by definition
- **time dilation**: The slowing of the passage of time experienced by objects in motion relative to an observer; measurable only at relativistic speeds.
- **Lorentz factor**: The factor, used in special relativity, to calculate the degree of time dilation, length contraction and relativistic mass of an object moving relative to an observer.

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27.3: Relativistic Quantities

learning objectives

- Express velocity-addition formulas for objects at speeds much less and approaching the speed of light

A velocity-addition formula is an equation that relates the velocities of moving objects in different reference frames.

As Galileo Galilei observed in 17th century, if a ship is moving relative to the shore at velocity vv, and a fly is moving with velocity uu as measured on the ship, calculating the velocity of the fly as measured on the shore is what is meant by the addition of the velocities vv and uu. When both the fly and the ship are moving slowly compared to speed of light, it is accurate enough to use the vector sum s=u+vs=u+v where ss is the velocity of the fly relative to the shore.

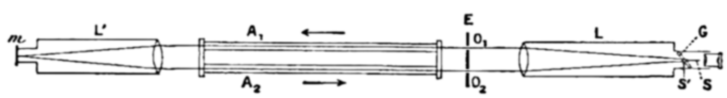
According to the theory of special relativity, the frame of the ship has a different clock rate and distance measure, and the notion of simultaneity in the direction of motion is altered, so the addition law for velocities is changed.

Since special relativity dictates that the speed of light is the same in all frames of reference, light shone from the front of a moving car can't go faster than light from a stationary lamp. Since this is counter to what Galileo used to add velocities, there needs to be a new velocity addition law.

This change isn't noticeable at low velocities but as the velocity increases towards the speed of light it becomes important. The addition law is also called a composition law for velocities. For collinear motions, the velocity of the fly relative to the shore is given by the following equation:

$$\mathrm{s}=\frac{\mathrm{v}+\mathrm{u}}{1+\mathrm{vu}/\mathrm{c}^2} \tag{27.3.1}$$

Composition law for velocities gave the first test of the kinematics of the special theory of relativity. Using a Michelson interferometer, Hyppolite Fizeau measured the speed of light in a fluid moving parallel to the light in 1851. The speed of light in the fluid is slower than the speed of light in vacuum, and it changes if the fluid is moving along with the light. The speed of light in a collinear moving fluid is predicted accurately by the collinear case of the relativistic formula.



Setup of the Fizeau Experiment: A light ray emanating from the source S' is reflected by a beam splitter G and is collimated into a parallel beam by lens L. After passing the slits O1 and O2, two rays of light travel through the tubes A1 and A2, through which water is streaming back and forth as shown by the arrows. The rays reflect off a mirror m at the focus of lens L', so that one ray always propagates in the same direction as the water stream, and the other ray opposite to the direction of the water stream. After passing back and forth through the tubes, both rays unite at S, where they produce interference fringes that can be visualized through the illustrated eyepiece. The interference pattern can be analyzed to determine the speed of light traveling along each leg of the tube.

Relativistic Momentum

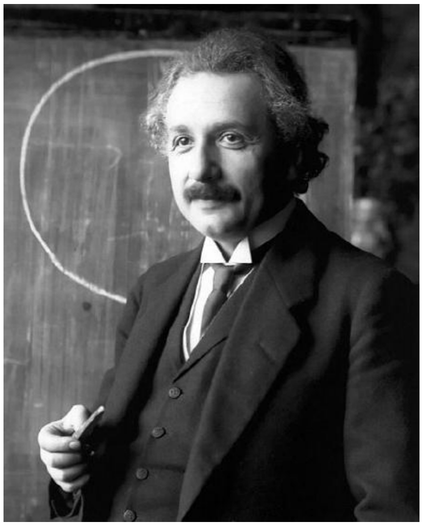
Relativistic momentum is given as $\gamma \mathrm{m}_0 \mathrm{v}$ where m_0 is the object's invariant mass and γ is Lorentz transformation.

learning objectives

- Compare Newtonian and relativistic momenta for objects at speeds much less and approaching the speed of light

Relativistic Momentum

Newtonian physics assumes that absolute time and space exist outside of any observer. This gives rise to Galilean relativity, which states that the laws of motion are the same in all inertial frames. It also results in a prediction that the speed of light can vary from one reference frame to another. However, this is contrary to observation. In the theory of special relativity, Albert Einstein keeps the postulate that the equations of motion do not depend on the reference frame, but assumes that the speed of light *c* is invariant. As a result, position and time in two reference frames are related by the Lorentz transformation instead of the Galilean transformation.



Albert Einstein: Albert Einstein in 1921

Consider, for example, a reference frame moving relative to another at velocity *v* in the *x* direction. The Galilean transformation gives the coordinates of the moving frame as

$$t' = t \tag{27.3.2}$$

$$x' = x - vt \tag{27.3.3}$$

while the Lorentz transformation gives

$$t' = \gamma\left(t - \frac{vx}{c^2}\right) \tag{27.3.4}$$

$$x' = \gamma(x - vt) \tag{27.3.5}$$

where γ is the Lorentz factor:

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} \tag{27.3.6}$$

Conservation laws in physics, such as the law of conservation of momentum, must be invariant. That is, the property that needs to be conserved should remain unchanged regardless of changes in the conditions of measurement. This means that the conservation law needs to hold in any frame of reference. Newton's second law [with mass fixed in the expression for momentum (p=m*v)], is not invariant under a Lorentz transformation. However, it can be made invariant by making the inertial mass *m* of an object a function of velocity:

$$m = \gamma m_0 \tag{27.3.7}$$

where m_0 is the object's invariant mass.

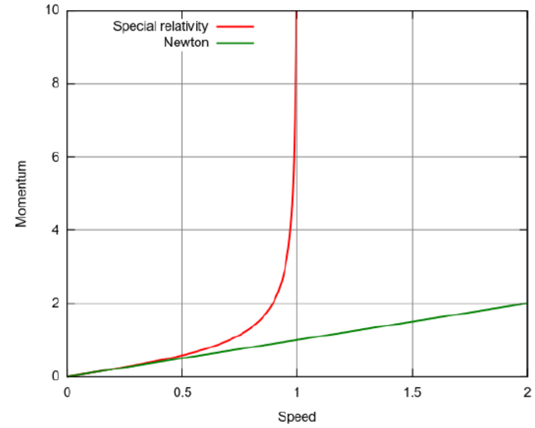
The modified momentum,

$$p = \gamma m_0 v \tag{27.3.8}$$

obeys Newton's second law:

$$F = \frac{dp}{dt} \tag{27.3.9}$$

It is important to note that for speeds much less than the speed of light, Newtonian momentum and relativistic momentum are approximately the same. As one approaches the speed of light, however, relativistic momentum becomes infinite while Newtonian momentum continues to increases linearly. Thus, it is necessary to employ the expression for relativistic momentum when one is dealing with speeds near the speed of light.



Relativistic and Newtonian Momentum: This figure illustrates that relativistic momentum approaches infinity as the speed of light is approached. Newtonian momentum increases linearly with speed.

Relativistic Energy and Mass

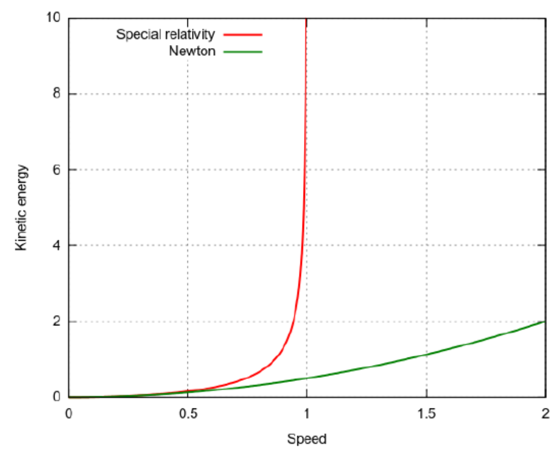
In special relativity, as the object approaches the speed of light, the object’s energy and momentum increase without bound.

learning objectives

- Evaluate possibility for an object to travel at the speed of light

Relativistic Energy and Mass

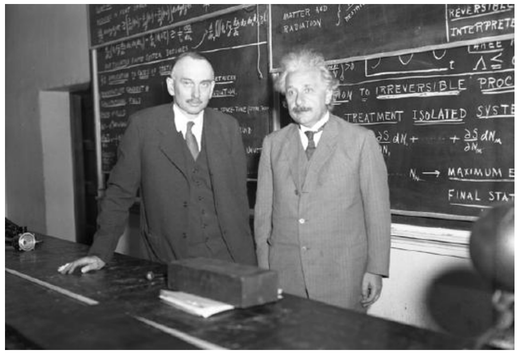
In special relativity, an object that has a mass cannot travel at the speed of light. As the object approaches the speed of light, the object’s energy and momentum increase without bound. Relativistic corrections for energy and mass need to be made because of the fact that the speed of light in a vacuum is constant in all reference frames. The conservation of mass and energy are well-accepted laws of physics. In order for these laws to hold in all reference frames, special relativity must be applied. It is important to note that for objects with speeds that are well below the speed of light that the expressions for relativistic energy and mass yield values that are approximately equal to their Newtonian counterparts.



Relativistic and Newtonian Kinetic Energy: This figure illustrates how relativistic and Newtonian Kinetic Energy are related to the speed of an object. The relativistic kinetic energy increases to infinity when an object approaches the speed of light, this indicates that no body with mass can reach the speed of light. On the other hand, Newtonian kinetic energy continues to increase without bound as the speed of an object increases.

Relativistic Mass

Relativistic mass was defined by Richard C. Tolman pictured left of Albert Einstein here in 1934 as $m = E/c^2$ which holds for all particles, including those moving at the speed of light. For a slower than light particle, a particle with a nonzero rest mass, the formula becomes $m = m_o / \gamma$ where m_o is the rest mass and γ is the Lorentz factor. The Lorentz factor is equal to: $\gamma = \frac{1}{\sqrt{1 - \mathrm{v}^2 / c^2}}$, where v is the relative velocity between inertial reference frames and c is the speed of light.



Richard C. Tolman and Albert Einstein: Richard C. Tolman (1881 – 1948) with Albert Einstein (1879 – 1955) at Caltech, 1932

When the relative velocity is zero, γ is simply equal to 1, and the relativistic mass is reduced to the rest mass. As the velocity increases toward the speed of light (c), the denominator of the right side approaches zero, and m consequently approaches infinity.

In the formula for momentum the mass that occurs is the relativistic mass. In other words, the relativistic mass is the proportionality constant between the velocity and the momentum.

While Newton's second law remains valid in the form $\mathbf{f} = \frac{d\mathbf{p}}{dt}$ the derived form is not valid because m in $\mathbf{p}$ is generally not a constant.

Relativistic Energy

Relativistic energy ($\mathrm{E_r} = \sqrt{(\mathrm{m_0 c^2})^2 + (\mathrm{pc})^2}$) is connected with rest mass via the following equation: $\mathrm{m_0} = \frac{\sqrt{(E^2 - (pc)^2}}{c^2}$. Here the term represents the square of the Euclidean norm (total vector length) of the various momentum vectors in the system, which reduces to the square of the simple momentum magnitude, if only a single particle is considered. This equation reduces to when the momentum term is zero. For photons where the equation reduces to.

Today, the predictions of relativistic energy and mass are routinely confirmed from the experimental data of particle accelerators such as the Relativistic Heavy Ion Collider. The increase of relativistic momentum and energy is not only precisely measured but also necessary to understand the behavior of cyclotrons and synchrotron, which accelerate particles to near the speed of light.

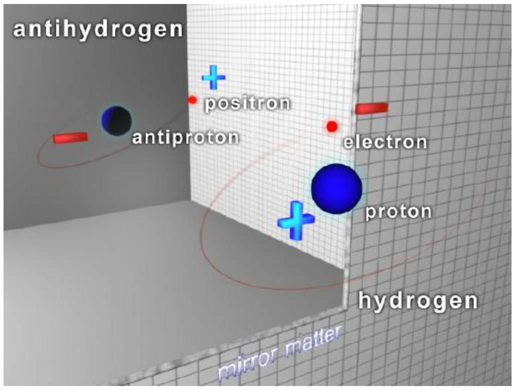
Matter and Antimatter

Antimatter is composed of antiparticles, which have the same mass as particles of ordinary matter but opposite charge and quantum spin.

learning objectives

- Describe properties of antiparticles

Antimatter is material composed of antiparticles, which have the same mass as particles of ordinary matter but have opposite charge and quantum spin. Antiparticles bind with each other to form antimatter in the same way that normal particles bind to form normal matter. For example, a positron (the antiparticle of the electron, with symbol e^+) and an antiproton (symbol p^-) can form an antihydrogen atom. Furthermore, mixing matter and antimatter can lead to the annihilation of both, in the same way that mixing antiparticles and particles does. This gives rise to high-energy photons (gamma rays) and other particle-antiparticle pairs. The end result of antimatter meeting matter is a release of energy proportional to the mass, as shown in the mass-energy equivalence equation, $E = mc^2$.



Antihydrogen and Hydrogen Atoms: Antihydrogen consists of an antiproton and a positron; hydrogen consists of a proton and an electron.

Almost all matter observable from the earth seems to be made of matter rather than antimatter. If antimatter-dominated regions of space existed, the gamma rays produced in annihilation reactions along the boundary between matter and antimatter regions would be detectable.

Antimatter may still exist in relatively large amounts in far-away galaxies due to cosmic inflation in the primordial time of the universe. Antimatter galaxies, if they exist, are expected to have the same chemistry and absorption and emission spectra as normal-matter galaxies, and their astronomical objects would be observationally identical, making them difficult to distinguish from normal-matter galaxies.

There is considerable speculation as to why the observable universe is apparently composed almost entirely of matter (as opposed to a mixture of matter and antimatter), whether there exist other places that are almost entirely composed of antimatter instead, and what sorts of technology might be possible if antimatter could be harnessed. At this time, the apparent asymmetry of matter and antimatter in the visible universe is one of the greatest unsolved problems in physics.

Relativistic Kinetic Energy

Relativistic kinetic energy can be expressed as: $\mathrm{E_k} = \frac{\mathrm{m_0c^2}}{\sqrt{1-(\mathrm{v/c})^2)}} - \mathrm{m_0c^2}$ where m_0 is rest mass, v is velocity, c is speed of light.

learning objectives

- Compare classical and relativistic kinetic energies for objects at speeds much less and approaching the speed of light

In classical mechanics, the kinetic energy of an object depends on the mass of a body as well as its speed. The kinetic energy is equal to the mass multiplied by the square of the speed, multiplied by the constant 1/2. The equation is given as:

$$\mathrm{E_k} = \frac{1}{2}\mathrm{mv^2} \tag{27.3.10}$$

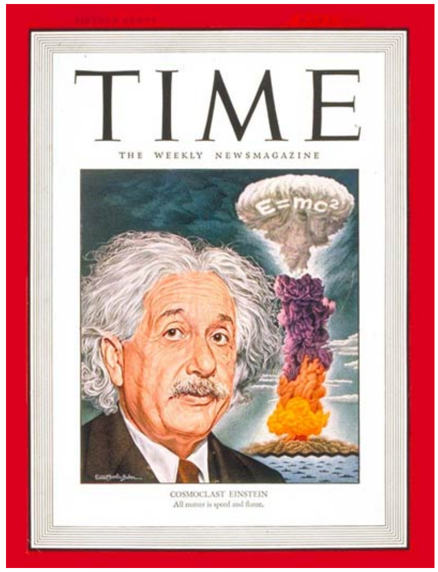
where m is the mass and v is the speed (or the velocity) of the body.

The classical kinetic energy of an object is related to its momentum by the equation:

$$\mathrm{E_k} = \frac{\mathrm{p^2}}{2\mathrm{m}} \tag{27.3.11}$$

where p is momentum.

If the speed of a body is a significant fraction of of the speed of light, it is necessary to employ special relativity to calculate its kinetic energy. It is important to know how to apply special relativity to problems with high speed particles. In special relativity, we must change the expression for linear momentum. Using m for rest mass, v and $\mathcal{V}$ for the object's velocity and speed respectively, and c for the speed of light in vacuum, the relativistic expression for linear momentum is:



Time Magazine – July 1, 1946: The popular connection between Einstein, $E = mc^2$, and the atomic bomb was prominently indicated on the cover of Time magazine (July 1946) by the writing of the equation on the mushroom cloud itself.

$p = m_0 \gamma v$, where γ is the Lorentz factor:

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$$

Since the kinetic energy of an object is related to its momentum, we intuitively know that the relativistic expression for kinetic energy will also be different from its classical counterpart. Indeed, the relativistic expression for kinetic energy is:

$$E_k = \frac{m_0 c^2}{\sqrt{1-(v/c)^2)}} - m_0 c^2 \tag{27.3.12}$$

The equation shows that the energy of an object approaches infinity as the velocity v approaches the speed of light c. Thus it is impossible to accelerate an object across this boundary.

The mathematical by-product of this calculation is the mass-energy equivalence formula (referred to in). The body at rest must have energy content equal to:

$$E_{rest} = E_0 = m_0 c^2 \tag{27.3.13}$$

The general expression for the kinetic energy of an object that is not at rest is:

$KE = mc^2 - m_0 c^2$, where m is the relativistic mass of the object and m_0 is the rest mass of the object.

At a low speed (v<<cv<<c), the relativistic kinetic energy may be approximated well by the classical kinetic energy. We can show this to be true by using a Taylor expansion for the reciprocal square root and keeping first two terms of the relativistic kinetic energy equation. When we do this we get:

$$E_k \approx m_0 c^2 \left(1 + \frac{1}{2} v^2/c^2\right) - m_0 c^2 = \frac{1}{2} mv^2 \tag{27.3.14}$$

Thus, the total energy can be partitioned into the energy of the rest mass plus the traditional classical kinetic energy at low speeds.

Key Points

- When two objects are moving slowly compared to speed of light, it is accurate enough to use the vector sum of velocities: $s = u + v$.
- As the velocity increases towards the speed of light, the vector sum of velocities is replaced with: $s = \frac{v+u}{1+vu/c^2}$.
- Composition law for velocities gave the first test of the kinematics of the special theory of relativity when, using a Michelson interferometer, Hyppolite Fizeau measured the speed of light in a fluid moving parallel to the light.

- Newtonian physics assumes that absolute time and space exist outside of any observer, resulting in a prediction that the speed of light can vary from one reference frame to another.
- In the theory of special relativity, the equations of motion do not depend on the reference frame while the speed of light (c) is invariant.
- Within the domain of classical mechanics, relativistic momentum closely approximates Newtonian momentum. At low velocity γm0vγm0v is approximately equal to m0vm0v, the Newtonian expression for momentum.
- In special relativity, an object that has a mass cannot travel at the speed of light.
- Relativistic mass is defined as $\mathrm{m_{rel}} = \frac{\mathrm{E}}{\mathrm{c^2}}$ and can be viewed as the proportionality constant between the velocity and the momentum.
- Relativistic energy is connected with rest mass via the following equation: $\mathrm{E_r} = \sqrt{(\mathrm{m_0c^2})^2 + (\mathrm{pc})^2}$.
- The end result of antimatter meeting matter is a release of energy proportional to the mass, as shown in the mass-energy equivalence equation,$\mathrm{E} = \mathrm{mc^2}$.
- Although almost all matter observable from the Earth seems to be made of matter rather than antimatter, antimatter may still exist in relatively large amounts in far-away galaxies.
- No explanation for the apparent asymmetry of matter and antimatter in the visible universe exists.
- Relativistic kinetic energy equation shows that the energy of an object approaches infinity as the velocity approaches the speed of light. Thus it is impossible to accelerate an object across this boundary.
- Kinetic energy calculations lead to the mass -energy equivalence formula: $\mathrm{E_{rest}} = \mathrm{E_0} = \mathrm{mc^2}$.
- At a low speed ($\mathrm{v} << \mathrm{C}$), the relativistic kinetic energy may be approximated by the classical kinetic energy. Thus, the total energy can be partitioned into the energy of the rest mass plus the traditional Newtonian kinetic energy at low speeds.

Key Terms

- **special relativity**: A theory that (neglecting the effects of gravity) reconciles the principle of relativity with the observation that the speed of light is constant in all frames of reference.
- **interferometer**: Any of several instruments that use the interference of waves to determine wavelengths and wave velocities, determine refractive indices, and measure small distances, temperature changes, stresses, and many other useful measurements.
- **speed of light**: the speed of electromagnetic radiation in a perfect vacuum: exactly 299,792,458 meters per second by definition
- **Galilean transformation**: a transformation used to transform between the coordinates of two reference frames which differ only by constant relative motion within the constructs of Newtonian physics.
- **Lorentz transformation**: a transformation relating the spacetime coordinates of one frame of reference to another in special relativity
- **Lorentz factor**: The factor, used in special relativity, to calculate the degree of time dilation, length contraction and relativistic mass of an object moving relative to an observer.
- **rest mass**: the mass of a body when it is not moving relative to an observer
- **annihilation**: the process of a particle and its corresponding antiparticle combining to produce energy
- **positron**: The antimatter equivalent of an electron, having the same mass but a positive charge.
- **antimatter**: matter that is composed of the antiparticles of those that constitute normal matter
- **classical mechanics**: All of the physical laws of nature that account for the behaviour of the normal world, but break down when dealing with the very small (see quantum mechanics) or the very fast or very heavy (see relativity).

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27.4: Implications of Special Relativity

learning objectives

- Formulate major changes in the understanding of time, space, mass, and energy that were introduced by the theory of Special Relativity

The theory of Special Relativity and its implications spurred a paradigm shift in our understanding of the nature of the universe, the fundamental fabric of which being space and time. Before 1905, scientists considered space and time as completely independent objects. Time could not affect space and space could not affect time. After 1905, however, the Special Theory of Relativity destroyed this old, but intuitive, view. Specifically, Special Relativity showed us that space and time are not independent of one another but can be mixed into each other and therefore must be considered as the same object, which we shall denote as space-time. The consequences of space/time mixing are:

- time dilation
- and length contraction.

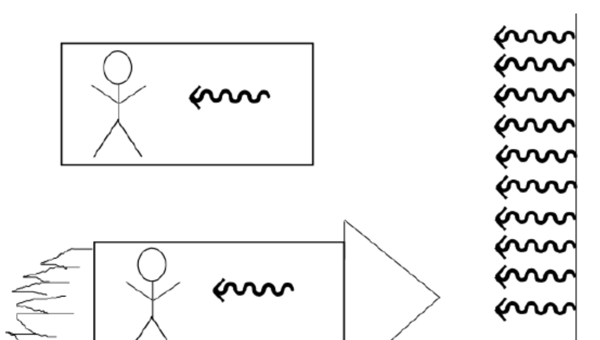
Other important consequences which will be discussed in another section are

- Relativity of Simultaneity (for certain events, the sequence in which they occur is dependent on the observer)
- Nothing can move faster than the speed of light (we shall denote the value of the speed of light as cc)

Why is there a mixing between space and time? In order to examine this we must know the founding principles of relativity. They are:

1. The Principle of Relativity: The laws of physics for *observers* which are not accelerating relative to one another should be the same.
2. The Principle of Invariant Light Speed: All observers, moving at constant speed, measure the same speed of light regardless of how fast they are moving.

If one accepts the second principle as fact then it immediately follows that space and time are not independent. Why? Let us look at the experiment in in which there is a light source, a fixed observer, and a rocket ship moving toward the light source. The two observers *are related by a coordinate , or space-time, transformation.* The second principle then tells us that no matter how fast the rocket is moving, both observers must measure the same light speed emanating from the source. The only way this can happen is if the clock of the rocket observer is ticking at a different rate than the stationary observer. How much different? This can be expressed in the time dilation equation:



Measuring Light: A stationary observer will measure the same speed of light as an observer who is moving in a rocket ship even if that rocket is moving close to light speed.

[Math Processing Error]

Where tsts is the time elapsed for the stationary observer, tmtm is the time elapsed for the moving observer, and vv is the velocity of the rocket as *measured from the stationary frame*. One can then see that as *[Math Processing Error]* then the time elapsed in the stationary frame goes to infinity. To place some numbers, let *[Math Processing Error]*, and *[Math Processing Error]* . The time elapsed in the stationary frame is then one hour. Thus for every second lived in the rocket, the stationary man lives one hour!

One of the more radical results of time dilation is the so-called “twin paradox. ” The twin paradox is a thought experiment in special relativity that involves identical twins. One of the twins goes on a journey into space in a rocket that has a velocity near the speed of light. Upon returning home the twin finds that the twin that remained on Earth has aged more. This consequence altered the perception that aging is necessarily constant.

The square root factor in the time dilation equation is very important and we denote it as:

This factor shows up frequently in special relativity. For example the length contraction formula is:

[Math Processing Error]

where *[Math Processing Error]* is the rest length, the length of an object measured in the co-moving frame of the object, and *[Math Processing Error]* is the length of the object as measured by the observer who sees the object moving at speed *[Math Processing Error]*. In our rocket example, the stationary observer measures the length of the rocket as being less than what someone who was moving with the rocket would measure. This altered the perception that the length of an object would appear the same regardless of the reference frame of the observer.

Another radical finding that was made possible by the discovery of special relativity is the equivalence of energy and mass. Combined with other laws of physics, the postulates of special relativity predict that mass and energy are related by: *[Math Processing Error]*, where c is the speed of light in vacuum. This altered the perception that mass and energy are completely different things and paved the way for nuclear power and nuclear weapons.

As a final note it is important to note how big the speed of light is compared to every day life. The speed of light is:

[Math Processing Error]

Thus in every day life *[Math Processing Error]* and we do not experience significant time dilation or length contraction. If we did, life would be very different.

Four-Dimensional Space-Time

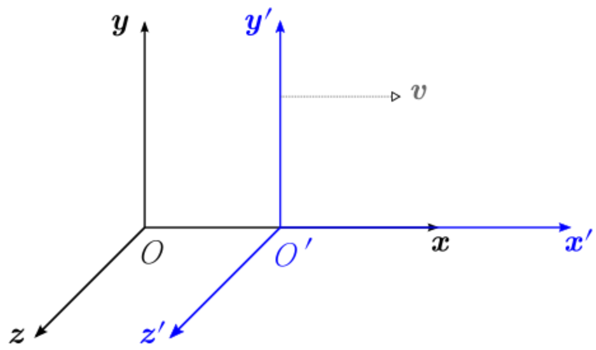
We live in four-dimensional space-time, in which the ordering of certain events can depend on the observer.

learning objectives

- Formulate major results of special relativity

Working in Four Dimensions

Let us examine two observers who are moving relative to one another at a constant velocity. We shall denote them as observer A and observer A’. Observer A sets up a space-time coordinate system (t, x, y, z); similarly, A’ sets up his own space-time coordinate system (t’, x’, y’, z’). (See for an example.) Therefore both observers live in a four-dimensional world with three space dimensions and one time dimension.



Two Coordinate Systems: Two coordinate systems in which the primed frame moves with velocity v with respect to the unprimed frame

You should not find it odd to work with four dimensions; any time you have to meet your friend somewhere you have to tell him four variables: where (three spatial coordinates) and when (one time coordinate). In other words, we have always lived in four dimensions, but so far you have probably thought of space and time as completely separate.

The Movement of Light in Four Dimensions

Back to our example: let us assume that at some point in space-time there is a beam of light that emerges. Both observers measure how far the beam has traveled at each point in time and how long it took to travel to travel that distance. That is to say, observer A measures:

[Math Processing Error]

where, for example, *[Math Processing Error]*; t is the time at which the measurement took place; and t_0 is the time at which the light was turned on.

Similarly, observer A' measures:

[Math Processing Error]

where we set up the system such that both observers agree on *[Math Processing Error]*. Due to the invariance of the speed of light both observers will agree on:

[Math Processing Error]

where *[Math Processing Error]*refers to the coordinates in either frame. Therefore there is a specific rule that all light paths must follow. For general events we can define the quantity:

[Math Processing Error]

This is known as the line element and is the same for all observers. (Using the principles of relativity, you can prove this for general separations, not just light rays). The set of all coordinate transformations that leave the above quantity invariant are known as Lorentz Transformations. It follows that the coordinate systems of all physical observers are related to each other by Lorentz Transformations. (The set of all Lorentz transformations form what mathematicians call a group, and the study of group theory has revolutionized physics). For the coordinate transformation in, the transformations are:

[Math Processing Error]

where

[Math Processing Error]

We define the separation between space-time points as follows:

[Math Processing Error]

[Math Processing Error]

[Math Processing Error]

We separate these events because they are very different. For example, for a space-like separation you can always find a coordinate transformation that reverses the time-ordering of the events (try to prove this for the example in). This phenomenon is known as the relativity of simultaneity and may be counterintuitive.

Space-Like Space-Time Separations

Let us examine two car crashes, one in New York and one in London, such that they occur in the same time in one frame. In this situation, the space-time separation between the two events is space-like. The question of whether the events are simultaneous is relative: in some reference frames the two accidents may happen at the same time; in other frames (in a different state of motion relative to the events) the crash in London may occur first; and in still other frames the New York crash may occur first. (In practice, this does not affect us because the frames that switch the order of the events must move at unreachably high speeds.)

Time-Like and Null Space-Time Separations

Events that are time-like or null do not share this property, and therefore there is a causal ordering between time-like events. In other words, if two events are time-like separated, then they can effect each other. The reason is that if two space-time points are time-like or null separated, one can always send a light signal from one point to another.

Special Relativity

Finally, let's discuss an important result of special relativity — that the energy EE of an object moving with speed vv is:

[Math Processing Error]

where m0m0 is the mass of the object at rest and m=γm0m=γm0 is the mass when the object is moving. The above formula immediately shows why it is impossible to travel faster than the speed of light. As *[Math Processing Error]* , and it takes an infinite amount of energy to accelerate the object further.

The Relativistic Universe

Gravity is a geometrical effect in which a metric matrix plays a special role, and the motion of objects are altered by curved space.

learning objectives

- Identify factors that affect motion near massive objects

Relativity

Special relativity indicates that humans live in a four-dimensional space-time where the 'distance' ssbetween points in space-time can be regarded as:

\[\mathrm{ s } ^ { 2 } = - \mathrm { c } ^ { 2 } \Delta \mathrm { t } ^ { 2 } + \Delta \mathrm { x } ^ { 2 } + \Delta \mathrm{ y } ^ { 2 } + \Delta \mathrm { z } ^ { 2 } = \mathrm{ X } ^ { \mathrm { T } } \eta \mathrm{ X }

The last equation is a matrix relation in which TT denotes transpose (for vectors *[Math Processing Error]*), *[Math Processing Error]* is the vector \)(\mathrm { c } \Delta \mathrm { t } , \Delta \mathrm { x } , \Delta \mathrm { y } , \Delta \mathrm { z })\), and *[Math Processing Error]* is the matrix:

[Math Processing Error]

A matrix that goes in between two vectors to give a length is called a metric. In mathematics, a metric or distance function is a function which defines a distance between elements of a set. In this case, the set is the space-time and the elements are points in that space-time. A space-time with the ηη metric is called Minkowski space and ηη is the Minkowski metric.

Four-dimensional Minkowski space-time is only one of many different possible space-times (geometries) which differ in their metric matrix. An arbitrary metric matrix can be denoted as gg, raising questions as to what space-times with different metrics represent. In 1916, Einstein found the importance of these space-times in his theory of general relativity.

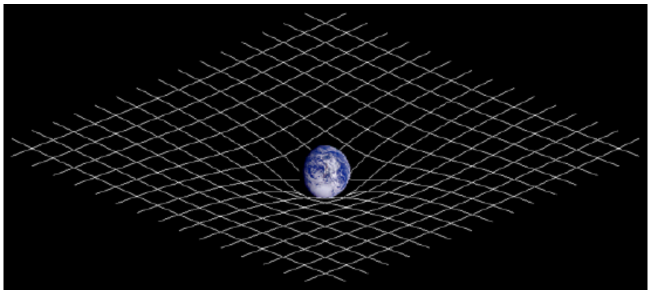
General Relativity

General relativity, or the general theory of relativity, is the geometric theory of gravitation published by Albert Einstein in 1916. It is the current description of gravitation in modern physics. General relativity generalizes special relativity and Newton's law of universal gravitation, providing a unified description of gravity as a geometric property of space and time, or space-time. In particular, the curvature of space-time is directly related to the energy and momentum of whatever matter and radiation are present. The relation is specified by the Einstein field equations, a system of partial differential equations.

[Math Processing Error]

People can use the metric to calculate curvature and then use the Einstein field equations to relate the curvature to the energy and momentum of the space-time. Going in the reverse order, energy and momentum affect the curvature and the space-time. Thus, energy and momentum curves space-time. Minkowski space is the special space devoid of matter, and as a result, it is completely flat. The precise definition of curvature requires knowledge of advanced mathematics, but an intuitive way to understand it is that the definition of a straight line changes in curved spacetime.

A pictorial example is shown in the figure below, where anything that is near the Earth has its motion altered due to the curved space-time. This altering of motion affects satellites, the moon, and even humans. If the space around the Earth were flat, humans would simply float away if they jumped upwards. Since the Earth alters the space-time, humans are pulled toward the Earth. In essence, gravity is a geometrical effect.



Curvature of space-time: The massive Earth is altering the curvature of space-time.

Key Points

- Time is relative.
- Lengths of moving objects are different than if their lengths were measured in a co-moving frame.
- Time and space are not independent.
- We live in a four-dimensional universe; the first three dimensions are spatial, and the fourth is time.
- The coordinate system of physical observers are related to one another via Lorentz transformations.
- Nothing can travel faster than the speed of light.
- The metric matrix can be used to calculate the curvature of space-time.
- Curvature can be related to energy and momentum via the Einstein equations.
- Due to the curvature of space-time, the motion of objects are altered near massive objects.

Key Terms

- **special relativity**: A theory that (neglecting the effects of gravity) reconciles the principle of relativity with the observation that the speed of light is constant in all frames of reference.
- **time dilation**: The slowing of the passage of time experienced by objects in motion relative to an observer; measurable only at relativistic speeds.
- **length contraction**: Observers measure a moving object's length as being smaller than it would be if it were stationary.
- **line element**: An invariant quantity in special relativity
- **Lorentz transformation**: a transformation relating the spacetime coordinates of one frame of reference to another in special relativity
- **relativity of simultaneity**: For space-like separated space-time points, the time-ordering between events is relative.
- **general relativity**: A theory extending special relativity and uniformly accounting for gravity and accelerated frames of reference, postulating that space-time curves in the presence of mass.
- **Minkowski space**: A four dimensional flat space-time. Because it is flat, it is devoid of matter.
- **metric**: A metric, or distance function, is a function which defines a distance between elements of a set.

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CHAPTER OVERVIEW

28: Introduction to Quantum Physics

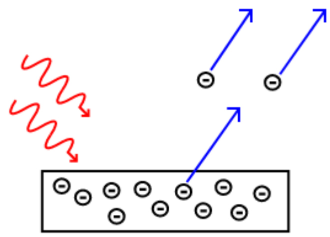
Topic hierarchy

28.1: History and Quantum Mechanical Quantities

learning objectives

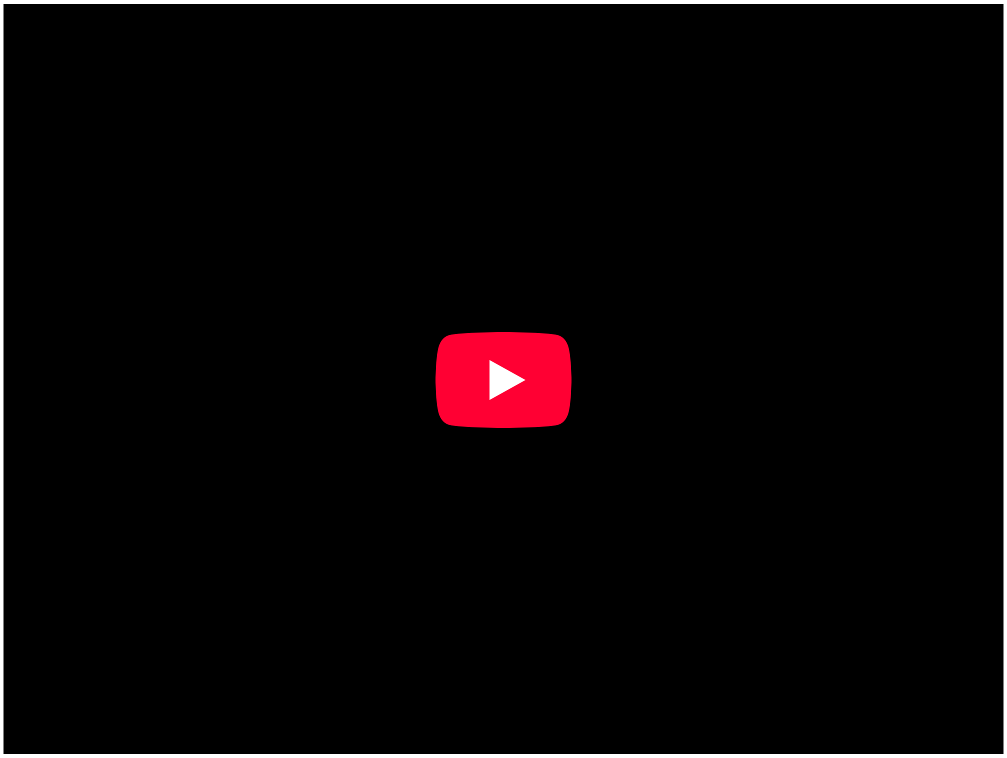
- Explain how the photoelectric effect paradox was solved by Albert Einstein.

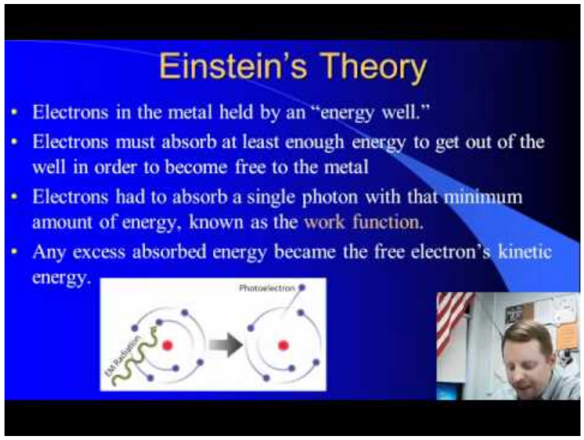
Electrons are emitted from matter when light shines on a surface . This is called the photoelectric effect, and the electrons emitted in this manner are called photoelectrons.



The Photoelectric Effect: Electrons are emitted from matter by absorbed light.

The photoelectric effect typically requires photons with energies from a few electronvolts to 1 MeV for heavier elements, roughly in the ultraviolet and X-ray range. Study of the photoelectric effect led to important steps in understanding the quantum nature of light and electrons and influenced the formation of the concept of wave-particle duality. The photoelectric effect is also widely used to investigate electron energy levels in matter.





Photoelectric Effect: A brief introduction to the Photoelectric Effect and electron photoemission.

Heinrich Hertz discovered the photoelectric effect in 1887. Although electrons had not been discovered yet, Hertz observed that electric currents were produced when ultraviolet light was shined on a metal. By the beginning of the 20th century, physicists confirmed that:

- The energy of the individual photoelectrons increased with the frequency (or color) of the light, but was independent of the intensity (or brightness) of the radiation.
- The photoelectric current was determined by the light's intensity; doubling the intensity of the light doubled the number of emitted electrons.

This observation was very puzzling to many physicists. At the time, light was accepted as a wave phenomenon. Since energy carried by a wave should only depend on its amplitude (and not on the frequency of the wave), the frequency dependence of the emitted electrons' energies didn't make sense.

In 1905, Albert Einstein solved this apparent paradox by describing light as composed of discrete quanta (now called photons), rather than continuous waves. Building on Max Planck's theory of black body radiation, Einstein theorized that the energy in each quantum of light was equal to the frequency multiplied by a constant h, later called Planck's constant. A photon above a threshold frequency has the required energy to eject a single electron, creating the observed effect. As the frequency of the incoming light increases, each photon carries more energy, hence increasing the energy of each outgoing photoelectron. By doubling the number of photons as the intensity is doubled, the number of photelectrons should double accordingly.

According to Einstein, the maximum kinetic energy of an ejected electron is given by *[Math Processing Error]*, where *[Math Processing Error]* is the Planck constant and f is the frequency of the incident photon. The term *[Math Processing Error]* is known as the work function, the minimum energy required to remove an electron from the surface of the metal. The work function satisfies *[Math Processing Error]*, where *[Math Processing Error]* is the threshold frequency for the metal for the onset of the photoelectric effect. The value of work function is an intrinsic property of matter.

Is light then composed of particles or waves? Young's experiment suggested that it was a wave, but the photoelectric effect indicated that it should be made of particles. This question would be resolved by de Broglie: light, and all matter, have both wave-like and particle-like properties.

Photon Energies of the EM Spectrum

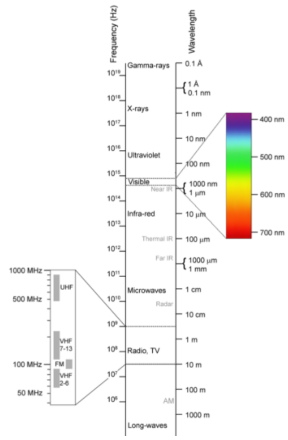
The electromagnetic (EM) spectrum is the range of all possible frequencies of electromagnetic radiation.

learning objectives

- Compare photon energy with the frequency of the radiation

The Electromagnetic Spectrum

The electromagnetic (EM) spectrum is the range of all possible frequencies of electromagnetic radiation . The electromagnetic spectrum extends from below the low frequencies used for modern radio communication to gamma radiation at the short-wavelength (high-frequency) end, thereby covering wavelengths of thousands of kiilometers down to those of a fraction of the size of an atom (approximately an angstrom). The limit for long wavelengths is the size of the universe itself.



Electromagnetic spectrum: This shows the electromagnetic spectrum, including the visible region, as a function of both frequency (left) and wavelength (right).

Maxwell’s equations predicted an infinite number of frequencies of electromagnetic waves, all traveling at the speed of light. This was the first indication of the existence of the entire electromagnetic spectrum. Maxwell’s predicted waves included waves at very low frequencies compared to infrared, which in theory might be created by oscillating charges in an ordinary electrical circuit of a certain type. In 1886, the physicist Hertz built an apparatus to generate and detect what are now called radio waves, in an attempt to prove Maxwell’s equations and detect such low-frequency electromagnetic radiation. Hertz found the waves and was able to infer (by measuring their wavelength and multiplying it by their frequency) that they traveled at the speed of light. Hertz also demonstrated that the new radiation could be both reflected and refracted by various dielectric media, in the same manner as light.

Filling in the Electromagnetic Spectrum

In 1895, Wilhelm Röntgen noticed a new type of radiation emitted during an experiment with an evacuated tube subjected to a high voltage. He called these radiations ‘X-rays’ and found that they were able to travel through parts of the human body but were reflected or stopped by denser matter such as bones. Before long, there were many new uses for them in the field of medicine.

The last portion of the electromagnetic spectrum was filled in with the discovery of gamma rays. In 1900, Paul Villard was studying the radioactive emissions of radium when he identified a new type of radiation that he first thought consisted of particles similar to known alpha and beta particles, but far more penetrating than either. However, in 1910, British physicist William Henry Bragg demonstrated that gamma rays are electromagnetic radiation, not particles. In 1914, Ernest Rutherford (who had named them gamma rays in 1903 when he realized that they were fundamentally different from charged alpha and beta rays) and Edward Andrade measured their wavelengths, and found that gamma rays were similar to X-rays, but with shorter wavelengths and higher frequencies.

The relationship between photon energy and the radiation’s frequency and wavelength is illustrated as the following equilavent equation: *[Math Processing Error]*, or *[Math Processing Error]* or *[Math Processing Error]* where νν is the frequency, *[Math Processing Error]* is the wavelength, *[Math Processing Error]* is photon energy, *[Math Processing Error]* is the speed of light, and hh is the Planck constant. Generally, electromagnetic radiation is classified by wavelength into radio waves, microwaves, terahertz (or sub-millimeter) radiation, infrared, the visible region humans perceive as light, ultraviolet, X-rays, and gamma rays. The behavior of EM radiation depends on its wavelength. When EM radiation interacts with single atoms and molecules, its behavior also depends on the amount of energy per quantum (photon) it carries.

Most parts of the electromagnetic spectrum are used in science for spectroscopic and other probing interactions as ways to study and characterize matter. Also, radiation from various parts of the spectrum has many other uses in communications and manufacturing.

Energy, Mass, and Momentum of Photon

A photon is an elementary particle, the quantum of light, which carries momentum and energy.

learning objectives

- State physical properties of a photon

A photon is an elementary particle, the quantum of light. It has no rest mass and has no electric charge. The modern photon concept was developed gradually by Albert Einstein to explain experimental observations of the photoelectric effect, which did not fit the classical wave model of light. In particular, the photon model accounted for the frequency dependence of light's energy. Max Planck explained black body radiation using semiclassical models, in which light is still described by Maxwell's equations, but the material objects that emit and absorb light, do so in amounts of energy that are quantized.

Photons are emitted in many natural processes. They are emitted from light sources such as floor lamps or lasers . For example, when a charge is accelerated it emits photons, a phenomenon known as synchrotron radiation. During a molecular, atomic or nuclear transition to a lower or higher energy level, photons of various energy will be emitted or absorbed respectively. A photon can also be emitted when a particle and its corresponding antiparticle are annihilated. During all these processes, photons will carry energy and momentum.

laser: Photons emitted in a coherent beam from a laser.

Energy of photon: From the studies of photoelectric effects, energy of a photon is directly proportional to its frequency with the Planck constant being the proportionality factor. Therefore, we already know that *[Math Processing Error]* (Eq. 1), where *[Math Processing Error]* is the energy and νν is the frequency.

Momentum of photon: According to the theory of Special Relativity, energy and momentum (p) of a particle with rest mass m has the following relationship: *[Math Processing Error]*, where *[Math Processing Error]* is the speed of light. In the case of a photon with zero rest mass, we get *[Math Processing Error]*. Combining this with Eq. 1, we get *[Math Processing Error]*. Here, *[Math Processing Error]* is the wavelength of the light. Since momentum is a vector quantity and p points in the direction of the photon's propagation, we can write *[Math Processing Error]*, where *[Math Processing Error]* and is *[Math Processing Error]* a wave vector.

You may wonder how an object with zero rest mass can have nonzero momentum. This confusion often arises because of the commonly used form of momentum (mvmv in non-relativistic mechanics and γmvγmv in relativistic mechanics, where vv is velocity and *[Math Processing Error]*.) This formula, obviously, shouldn't be used in the case *[Math Processing Error]*.

Implications of Quantum Mechanics

Quantum mechanics has had enormous success in explaining microscopic systems and has become a foundation of modern science and technology.

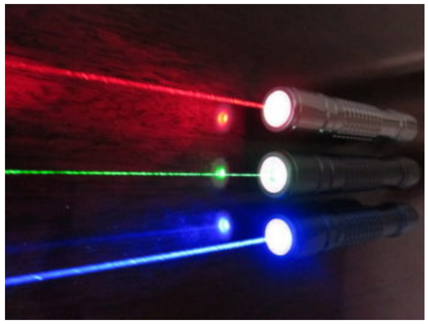
learning objectives

- Explain importance of quantum mechanics for technology and other branches of science

The field of quantum mechanics has been enormously successful in explaining many of the features of our world. The behavior of the subatomic particles (electrons, protons, neutrons, photons, and others) that make up all forms of matter can often be satisfactorily described only using quantum mechanics. Quantum mechanics has also strongly influenced string theory.

Quantum mechanics is also critically important for understanding how individual atoms combine covalently to form molecules. The application of quantum mechanics to chemistry is known as quantum chemistry. Relativistic quantum mechanics can, in principle, mathematically describe most of chemistry. Quantum mechanics can also provide quantitative insight into ionic and covalent bonding processes by explicitly showing which molecules are energetically favorable to which other molecules and the magnitudes of the energies involved. Furthermore, most of the calculations performed in modern computational chemistry rely on quantum mechanics.

A great number of modern technological inventions operate on a scale where quantum effects are significant. Examples include the laser , the transistor (and thus the microchip), the electron microscope, and magnetic resonance imaging (MRI). The study of semiconductors led to the invention of the diode and the transistor, which are indispensable parts of modern electronic systems and devices.



Laser: Red (635-nm), green (532-nm), and blue-violet (445-nm) lasers

Researchers are currently seeking robust methods of directly manipulating quantum states. Efforts are being made to more fully develop quantum cryptography, which will theoretically allow guaranteed secure transmission of information. A more distant goal is the development of quantum computers, which are expected to perform certain computational tasks exponentially faster than classical computers. Another topic of active research is quantum teleportation, which deals with techniques to transmit quantum information over arbitrary distances.

Particle-Wave Duality

Wave–particle duality postulates that all physical entities exhibit both wave and particle properties.

learning objectives

- Describe experiments that demonstrated wave-particle duality of physical entities

Wave–particle duality postulates that all physical entities exhibit both wave and particle properties. As a central concept of quantum mechanics, this duality addresses the inability of classical concepts like “particle” and “wave” to fully describe the behavior of (usually) microscopic objects.

From a classical physics point of view, particles and waves are distinct concepts. They are mutually exclusive, in the sense that a particle doesn’t exhibit wave-like properties and vice versa. Intuitively, a baseball doesn’t disappear via destructive interference, and our voice cannot be localized in space. Why then is it that physicists believe in wave-particle duality? Because that’s how mother Nature operates, as they have learned from several ground-breaking experiments. Here is a short, chronological list of those experiments:

- Young’s double-slit experiment: In the early Nineteenth century, the double-slit experiments by Young and Fresnel provided evidence that light is a wave. In 1861, James Clerk Maxwell explained light as the propagation of electromagnetic waves according to the Maxwell’s equations.
- Black body radiation: In 1901, to explain the observed spectrum of light emitted by a glowing object, Max Planck assumed that the energy of the radiation in the cavity was quantized, contradicting the established belief that electromagnetic radiation is a wave.
- Photoelectric effect: Classical wave theory of light also fails to explain photoelectric effect. In 1905, Albert Einstein explained the photoelectric effects by postulating the existence of photons, quanta of light energy with particulate qualities.
- De Broglie’s wave (matter wave): In 1924, Louis-Victor de Broglie formulated the de Broglie hypothesis, claiming that all matter, not just light, has a wave-like nature. His hypothesis was soon confirmed with the observation that electrons (matter) also displays diffraction patterns, which is intuitively a wave property.

From these historic achievements, physicists now accept that all entities in nature behave as both a particle and a wave, depending on the specifics of the phenomena under consideration. Because of its counter-intuitive aspect, the meaning of the particle-wave duality is still a point of debate in quantum physics. The standard interpretation is that the act of measurement causes the set of probabilities, governed by a probability distribution function acquired from a “wave”, to immediately and randomly assume one of the possible values, leading to a “particle”-like result.

So, why do we not notice a baseball acting like a wave? The wavelength of the matter wave associated with a baseball, say moving at 95 miles per hour, is extremely small compared to the size of the ball so that wave-like behavior is never noticeable.

Diffraction Revisited

De Broglie’s hypothesis was that particles should show wave-like properties such as diffraction or interference.

learning objectives

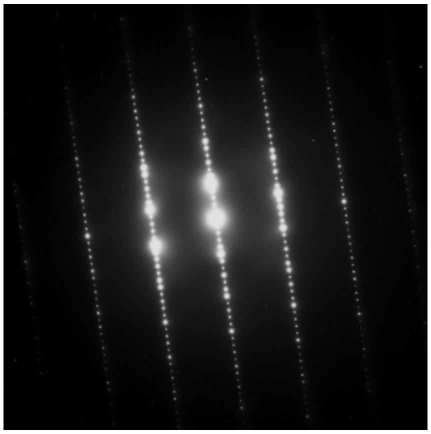
- Compare application of X-ray, electron, and neutron diffraction for materials research

The de Broglie hypothesis, formulated in 1924, predicts that particles should also behave as waves. The wavelength of an electron is given by the de Broglie equation *[Math Processing Error]*. Here *[Math Processing Error]* is Planck’s constant and pp the relativistic momentum of the electron. *[Math Processing Error]* is called the de Broglie wavelength.

From the work by Planck (black body radiation) and Einstein (photoelectric effect), physicists understood that electromagnetic waves sometimes behaved like particles. De Broglie’s hypothesis is complementary to this idea: particles should also show wave-like properties such as diffraction or interference. De Broglie’s formula was confirmed three years later for electrons (which have a rest-mass) with the observation of electron diffraction in two independent experiments. George Paget Thomson passed a beam of electrons through a thin metal film and observed the predicted interference patterns. Clinton Joseph Davisson and Lester Halbert Germerguided their beam through a crystalline grid to observe diffraction patterns.

X-ray diffraction is a commonly used tool in materials research. Thanks to the wave-particle duality, matter wave diffraction can also be used for this purpose. The electron, which is easy to produce and manipulate, is a common choice. A neutron is another particle of choice. Due to the different kinds of interactions involved in the diffraction processes, the three types of radiation (X-ray, electron, neutron) are suitable for different kinds of studies.

Electron diffraction is most frequently used in solid state physics and chemistry to study the crystalline structure of solids. Experiments are usually performed using a transmission electron microscope or a scanning electron microscope. In these instruments, electrons are accelerated by an electrostatic potential in order to gain the desired energy and, thus, wavelength before they interact with the sample to be studied. The periodic structure of a crystalline solid acts as a diffraction grating, scattering the electrons in a predictable manner. Working back from the observed diffraction pattern, it is then possible to deduce the structure of the crystal producing the diffraction pattern. Unlike other types of radiation used in diffraction studies of materials, such as X-rays and neutrons, electrons are charged particles and interact with matter through the Coulomb forces. This means that the incident electrons feel the influence of both the positively charged atomic nuclei and the surrounding electrons. In comparison, X-rays interact with the spatial distribution of the valence electrons, while neutrons are scattered by the atomic nuclei through the strong nuclear force.



Electron Diffraction Pattern: Typical electron diffraction pattern obtained in a transmission electron microscope with a parallel electron beam.

Neutrons have also been used for studying crystalline structures. They are scattered by the nuclei of the atoms, unlike X-rays, which are scattered by the electrons of the atoms. Thus, neutron diffraction has some key differences compared to more common methods using X-rays or electrons. For example, the scattering of X-rays is highly dependent on the atomic number of the atoms (i.e., the number of electrons), whereas neutron scattering depends on the properties of the nuclei. In addition, the magnetic moment of the neutron is non-zero, and can thus also be scattered by magnetic fields. This means that neutron scattering is more useful for determining the properties of atomic nuclei, despite the fact that neutrons are significantly harder to create, manipulate, and detect compared to X-rays and electrons.

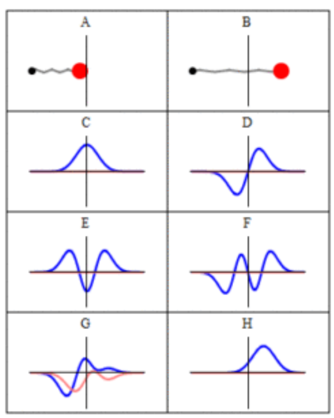
The Wave Function

A wave function is a probability amplitude in quantum mechanics that describes the quantum state of a particle and how it behaves.

learning objectives

- Relate the wave function with the probability density of finding a particle, commenting on the constraints the wave function must satisfy for this to make sense

In quantum mechanics, a wave function is a probability amplitude describing the quantum state of a particle and how it behaves. Typically, its values are complex numbers. For a single particle, it is a function of space and time. The most common symbols for a wave function are *[Math Processing Error]* or *[Math Processing Error]* (lowercase or uppercase psi, respectively), when the wave function is given as a function of position *[Math Processing Error]*. Although *[Math Processing Error]* is a complex number, *[Math Processing Error]* is a real number and corresponds to the probability density of finding a particle in a given place at a given time, if the particle's position is measured.



Trajectories of a Harmonic Oscillator: This figure shows some trajectories of a harmonic oscillator (a ball attached to a spring) in classical mechanics (A-B) and quantum mechanics (C-H). In quantum mechanics (C-H), the ball has a wave function, which is shown with its real part in blue and its imaginary part in red. The trajectories C-F are examples of standing waves, or "stationary states. " Each standing-wave frequency is proportional to a possible energy level of the oscillator. This "energy quantization" does not occur in classical physics, where the oscillator can have any energy.

The laws of quantum mechanics (the Schrödinger equation) describe how the wave function evolves over time. The wave function behaves qualitatively like other waves, such as water waves or waves on a string, because the Schrödinger equation is mathematically a type of wave equation. This explains the name "wave function" and gives rise to wave-particle duality.

The wave function must satisfy the following constraints for the calculations and physical interpretation to make sense:

- It must everywhere be finite.
- It must everywhere be a continuous function and continuously differentiable.
- It must everywhere satisfy the relevant normalization condition so that the particle (or system of particles) exists somewhere with 100-percent certainty.

If these requirements are not met, it's not possible to interpret the wave function as a probability amplitude. This is because the values of the wave function and its first order derivatives may not be finite and definite (having exactly one value), which means that the probabilities can be infinite and have multiple values at any one position and time, which is nonsense. Furthermore, when we use the wave function to calculate an observation of the quantum system without meeting these requirements, there will not be finite or definite values to use (in this case the observation can take a number of values and can be infinite). This is not a possible occurrence in a real-world experiment. Therefore, a wave function is meaningful only if these conditions are satisfied.

de Broglie and the Wave Nature of Matter

The concept of “matter waves” or “de Broglie waves” reflects the wave-particle duality of matter.

learning objectives

- Formulate the de Broglie relation as an equation

In quantum mechanics, the concept of matter waves (or de Broglie waves) reflects the wave-particle duality of matter. The theory was proposed by Louis de Broglie in 1924 in his PhD thesis. The de Broglie relations show that the wavelength is inversely proportional to the momentum of a particle, and is also called de Broglie wavelength.

Einstein derived in his theory of special relativity that the energy and momentum of a photon has the following relationship:

[Math Processing Error] (*[Math Processing Error]*: energy, *[Math Processing Error]*: momentum, *[Math Processing Error]*: speed of light).

He also demonstrated, in his study of photoelectric effects, that energy of a photon is directly proportional to its frequency, giving us this equation:

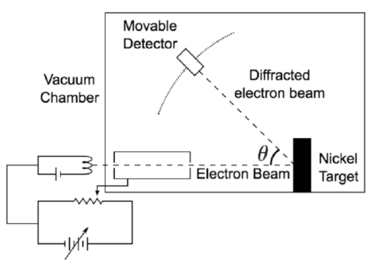
[Math Processing Error] (*[Math Processing Error]*: Planck constant, *[Math Processing Error]*: frequency).

Combining the two equations, we can derive a relationship between the momentum and wavelength of light:

[Math Processing Error]. Therefore, we arrive at *[Math Processing Error]*.

De Broglie’s hypothesis is that this relationship *[Math Processing Error]*, derived for electromagnetic waves, can be adopted to describe matter (e.g. electron, neutron, etc.) as well.

De Broglie didn’t have any experimental proof at the time of his proposal. It took three years for Clinton Davisson and Lester Germer to observe diffraction patterns from electrons passing a crystalline metallic target (see). Before the acceptance of the de Broglie hypothesis, diffraction was a property thought to be exhibited by waves only. Therefore, the presence of any diffraction effects by matter demonstrated the wave-like nature of matter. This was a pivotal result in the development of quantum mechanics. Just as the photoelectric effect demonstrated the particle nature of light, the Davisson–Germer experiment showed the wave-nature of matter, thus completing the theory of wave-particle duality.



Davisson-Germer Experimental Setup: The experiment included an electron gun consisting of a heated filament that released thermally excited electrons, which were then accelerated through a potential difference (giving them a certain amount of kinetic energy towards the nickel crystal). To avoid collisions of the electrons with other molecules on their way towards the surface, the experiment was conducted in a vacuum chamber. To measure the number of electrons that were scattered at different angles, an electron detector that could be moved on an arc path about the crystal was used. The detector was designed to accept only elastically scattered electrons.

Experiments with Fresnel diffraction and specular reflection of neutral atoms confirm the application to atoms of the de Broglie hypothesis. Further, recent experiments confirm the relations for molecules and even macromolecules, normally considered too large to undergo quantum mechanical effects. In 1999, a research team in Vienna demonstrated diffraction for molecules as large as fullerenes. The researchers calculated a De Broglie wavelength of the most probable *[Math Processing Error]* velocity as 2.5 pm.

The Heisenberg Uncertainty Principle

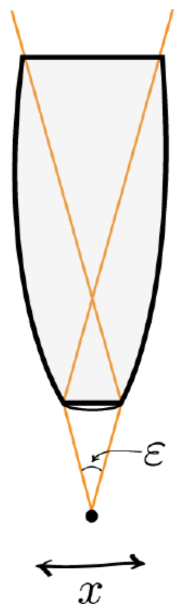
The uncertainty principle asserts a basic limit to the precision with which some physical properties of a particle can be known simultaneously.

learning objectives

- Relate the Heisenberg uncertainty principle with the matter wave nature of all quantum objects

The uncertainty principle is any of a variety of mathematical inequalities, asserting a fundamental limit to the precision with which certain pairs of physical properties of a particle, such as position *[Math Processing Error]* and momentum pp or energy *[Math Processing Error]* and time *[Math Processing Error]*, can be known simultaneously. The more precisely the position of some particle is determined, the less precisely its momentum can be known, and vice versa. This can be formulated as the following inequality: *[Math Processing Error]*, where *[Math Processing Error]* is the standard deviation of position, σp is the standard deviation of momentum, and *[Math Processing Error]*. The uncertainty principle is inherent in the properties of all wave-like systems, and it arises in quantum mechanics simply due to the matter wave nature of all quantum objects. Thus, the uncertainty principle actually states a fundamental property of quantum systems, and is not a statement about the observational success of current technology.

The principle is quite counterintuitive, so the early students of quantum theory had to be reassured that naive measurements to violate it were bound always to be unworkable. One way in which Heisenberg originally illustrated the intrinsic impossibility of violating the uncertainty principle is by using an imaginary microscope as a measuring device.



Heisenberg Microscope: Heisenberg's microscope, with cone of light rays focusing on a particle with angle *[Math Processing Error]* He imagines an experimenter trying to measure the position and momentum of an electron by shooting a photon at it.

Example *[Math Processing Error]*:

Example One

If the photon has a short wavelength and therefore a large momentum, the position can be measured accurately. But the photon scatters in a random direction, transferring a large and uncertain amount of momentum to the electron. If the photon has a long wavelength and low momentum, the collision does not disturb the electron's momentum very much, but the scattering will reveal its position only vaguely.

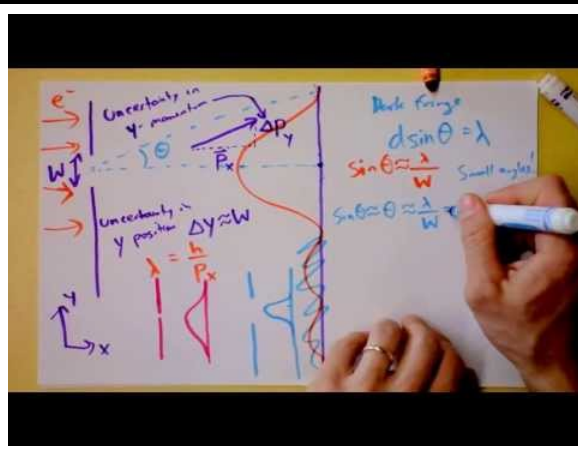
Example Two

If a large aperture is used for the microscope, the electron's location can be well resolved (see Rayleigh criterion); but by the principle of conservation of momentum, the transverse momentum of the incoming photon and hence the new momentum of the electron resolves poorly. If a small aperture is used, the accuracy of both resolutions is the other way around.

Heisenberg’s Argument

Heisenberg’s argument is summarized as follows. He begins by supposing that an electron is like a classical particle, moving in the xx direction along a line below the microscope, as in the illustration to the right. Let the cone of light rays leaving the microscope lens and focusing on the electron makes an angle εε with the electron. Let *[Math Processing Error]* be the wavelength of the light rays. Then, according to the laws of classical optics, the microscope can only resolve the position of the electron up to an accuracy of *[Math Processing Error]* When an observer perceives an image of the particle, it’s because the light rays strike the particle and bounce back through the microscope to their eye. However, we know from experimental evidence that when a photon strikes an electron, the latter has a recoil with momentum proportional to *[Math Processing Error]*, where *[Math Processing Error]* is Planck’s constant.

It is at this point that Heisenberg introduces objective indeterminacy into the thought experiment. He writes that “the recoil cannot be exactly known, since the direction of the scattered photon is undetermined within the bundle of rays entering the microscope”. In particular, the electron’s momentum in the xx direction is only determined up to δpx≈hλsin(ε/2)δpx≈hλsin(ε/2). Combining the relations for *[Math Processing Error]* and *[Math Processing Error]*, we thus have that *[Math Processing Error]*, which is an approximate expression of Heisenberg’s uncertainty principle.



Heisenberg Uncertainty Principle Derived and Explained

One of the most-oft quoted results of quantum physics, this doozie forces us to reconsider what we can know about the universe. Some things cannot be known simultaneously. In fact, if anything about a system is known perfectly, there is likely another characteristic that is completely shrouded in uncertainty. So significant figures ARE important after all!

Philosophical Implications

Since its inception, many counter-intuitive aspects of quantum mechanics have provoked strong philosophical debates.

learning objectives

- Formulate the Copenhagen interpretation of the probabilistic nature of quantum mechanics

Since its inception, many counter-intuitive aspects and results of quantum mechanics have provoked strong philosophical debates and many interpretations. Even fundamental issues, such as Max Born's basic rules interpreting *[Math Processing Error]* as a probability density function took decades to be appreciated by society and many leading scientists. Indeed, the renowned physicist Richard Feynman once said, "I think I can safely say that nobody understands quantum mechanics. "

The Copenhagen Interpretation

The Copenhagen interpretation—due largely to the Danish theoretical physicist Niels Bohr, shown in —remains a quantum mechanical formalism that is widely accepted amongst physicists, some 75 years after its enunciation. According to this interpretation, the probabilistic nature of quantum mechanics is not a temporary feature which will eventually be replaced by a deterministic theory, but instead must be considered a final renunciation of the classical idea of causality.

Niels Bohr and Albert Einstein: Niels Bohr (left) and Albert Einstein (right). Despite their pioneering contributions to the inception of the quantum mechanics, they disagreed on its interpretation.

The Copenhagen interpretation has philosophical implications to the concept of determinism. According to the theory of determinism, for everything that happens there are conditions such that, given those conditions, nothing else could happen. Determinism and free-will seem to be mutually exclusive. If the universe, and any person in it are governed by strict and universal laws , then that means that a person's behavior could be predicted based on sufficient knowledge of the circumstances obtained prior to that person's behavior. However, the Copenhagen interpretation suggests a universe in which outcomes are not fully determined by prior circumstances but also by probability. This gave thinkers alternatives to strictly bound possibilities, proposing a model for a universe that follows general rules but never had a predetermined future.

Philosophical Implications

It is also believed therein that any well-defined application of the quantum mechanical formalism must always make reference to the experimental arrangement. This is due to the quantum mechanical principle of wave function collapse. That is, a wave function which is initially in a superposition of several different possible states appears to reduce to a single one of those states after interaction with an observer. In simplified terms, it is the reduction of the physical possibilities into a single possibility as seen by an observer. This raises philosophical questions about whether something that is never observed actually exists.

Einstein-Podolsky-Rosen (EPR) Paradox

Albert Einstein (shown in , himself one of the founders of quantum theory) disliked this loss of determinism in measurement in the Copenhagen interpretation. Einstein held that there should be a local hidden variable theory underlying quantum mechanics and, consequently, that the present theory was incomplete. He produced a series of objections to the theory, the most famous of which has become known as the Einstein-Podolsky-Rosen (EPR) paradox. John Bell showed by Bell's theorem that this "EPR" paradox led to experimentally testable differences between quantum mechanics and local realistic theories. Experiments have been performed confirming the accuracy of quantum mechanics, thereby demonstrating that the physical world cannot be described by any local realistic theory. The Bohr-Einstein debates provide a vibrant critique of the Copenhagen Interpretation from an epistemological point of view.

Quantum Entanglement

One of the most bizarre aspect of the quantum mechanics is known as quantum entanglement. Quantum entanglement occurs when particles interact physically and then become separated, while isloated from the rest of the universe to prevent any deterioration of the quantum state. According to the Copenhagen interpretation of quantum mechanics, their shared state is indefinite until measured. Once a particle in the entangled state is measured and its state is determined, the Copenhagen interpretation demands that the other particles' state is also determined instantaneously. This bizarre nature of action at a distance (which seemingly violate the speed limit on the transmission of information implicit in the theory of relativity) is what bothered Einstein the most. (According to the theory of relativity, nothing can travel faster than the speed of light in a vacuum. This seemingly puts a limit on the speed at which information can be transmitted.) Quantum entanglement is the key element in proposals for quantum computers and quantum teleportation.

Key Points

- The energy of the emitted electrons depends only on the frequency of the incident light, and not on the light intensity.
- Einstein explained the photoelectric effect by describing light as composed of discrete particles.
- Study of the photoelectric effect led to important steps in understanding the quantum nature of light and electrons, which would eventually lead to the concept of wave-particle duality.
- Electromagnetic radiation is classified according to wavelength, divided into radio waves, microwaves, terahertz (or sub-millimeter) radiation, infrared, the visible region humans perceive as light, ultraviolet, X-rays, and gamma rays.
- Photon energy is proportional to the frequency of the radiation.
- Most parts of the electromagnetic spectrum are used in science for spectroscopic and other probing interactions as ways to study and characterize matter.
- *[Math Processing Error]* Energy of photon is proportional to its frequency.
- *[Math Processing Error]* Momentum of photon is proportional to the wave vector.
- Photon's rest mass is 0.
- A great number of modern technological inventions are based on quantum mechanics, including the laser, the transistor, the electron microscope, and magnetic resonance imaging.
- Quantum mechanics is also critically important for understanding how individual atoms combine covalently to form molecules. The application of quantum mechanics to chemistry is known as quantum chemistry.
- Researchers are currently seeking robust methods of directly manipulating quantum states for applications in computer and information science.
- All entities in Nature behave as both a particle and a wave, depending on the specifics of the phenomena under consideration.
- Particle-wave duality is usually hidden in macroscopic phenomena, conforming to our intuition.
- In the double-slit experiment of electrons, individual event displays a particle-like property of localization (or a "dot"). After many repetitions, however, the image shows an interference pattern, which indicates that each event is in fact governed by a probability distribution.
- The wavelength of an electron is given by the de Broglie equation *[Math Processing Error]*.
- Because of different forms of interaction involved, X-ray, electron, and neutron are suitable for different studies of material properties.
- De Broglie's idea completed the wave-particle duality.
- *[Math Processing Error]* corresponds to the probability density of finding a particle in a given location x at a given time.
- The laws of quantum mechanics (the Schrödinger equation) describe how the wave function evolves over time. The Schrödinger equation is a type of wave equation, which explains the name "wave function".

- A wave function must satisfy a set of mathematical constraints for the calculations and physical interpretation to make sense.
- de Broglie relations show that the wavelength is inversely proportional to the momentum of a particle.
- The Davisson-Germer experiment demonstrated the wave-nature of matter and completed the theory of wave-particle duality.
- Experiments demonstrated that de Broglie hypothesis is applicable to atoms and macromolecules.
- The uncertainty principle is inherent in the properties of all wave-like systems, and that it arises in quantum mechanics is simply due to the matter wave nature of all quantum objects.
- The uncertainty principle is not a statement about the observational success of current technology.
- The more precisely the position of some particle is determined, the less precisely its momentum can be known, and vice versa. This can be formulated as the following inequality: *[Math Processing Error]*.
- According to the Copenhagen interpretation, the probabilistic nature of quantum mechanics is intrinsic in our physical universe.
- When quantum wave function collapse occurs, physical possibilities are reduced into a single possibility as seen by an observer.
- Once a particle in an entangled state is measured and its state is determined, the Copenhagen interpretation demands that the state of the other entangled particle is also determined instantaneously.

Key Terms

- **black body radiation**: The type of electromagnetic radiation within or surrounding a body in thermodynamic equilibrium with its environment, or emitted by a black body (an opaque and non-reflective body) held at constant, uniform temperature.
- **photoelectron**: Electrons emitted from matter by absorbing energy from electromagnetic radiation.
- **wave-particle duality**: A postulation that all particles exhibit both wave and particle properties. It is a central concept of quantum mechanics.
- **Planck constant**: a physical constant that is the quantum of action in quantum mechanics. It has a unit of angular momentum. The Planck constant was first described as the proportionality constant between the energy of a photon (unit of electromagnetic radiation) and the frequency of its associated electromagnetic wave in his derivation of the Planck’s law
- **Maxwell’s equations**: A set of equations describing how electric and magnetic fields are generated and altered by each other and by charges and currents.
- **elementary particle**: a particle not known to have any substructure
- **photoelectric effect**: The occurrence of electrons being emitted from matter (metals and non-metallic solids, liquids, or gases) as a consequence of their absorption of energy from electromagnetic radiation.
- **cryptography**: the practice and study of techniques for secure communication in the presence of third parties
- **relativistic quantum mechanics**: a theoretical framework for constructing quantum mechanical models of fields and many-body systems
- **string theory**: an active research framework in particle physics that attempts to reconcile quantum mechanics and general relativity
- **black body**: An idealized physical body that absorbs all incident electromagnetic radiation, regardless of frequency or angle of incidence. Although black body is a theoretical concept, you can find approximate realizations of black body in nature.
- **grating**: Any regularly spaced collection of essentially identical, parallel, elongated elements.
- **Schrödinger equation**: A partial-differential that describes how the quantum state of some physical system changes with time. It was formulated in late 1925 and published in 1926 by the Austrian physicist Erwin Schrödinger
- **harmonic oscillator**: a system that, when displaced from its equilibrium position, experiences a restoring force *[Math Processing Error]* proportional to the displacement x
- **diffraction**: The bending of a wave around the edges of an opening or an obstacle.
- **special relativity**: A theory that (neglecting the effects of gravity) reconciles the principle of relativity with the observation that the speed of light is constant in all frames of reference.
- **wave-particle duality**: A postulation that all particles exhibit both wave and particle properties. It is a central concept of quantum mechanics.
- **matter wave**: A concept reflects the wave-particle duality of matter. The theory was proposed by Louis de Broglie.
- **Rayleigh criterion**: The angular resolution of an optical system can be estimated from the diameter of the aperture and the wavelength of the light, which was first proposed by Lord Rayleigh.
- **probability density function**: Any function whose integral over a set gives the probability that a random variable has a value in that set.
- **Bell’s theorem**: A no-go theorem famous for drawing an important line in the sand between quantum mechanics (QM) and the world as we know it classically. In its simplest form, Bell’s theorem states: No physical theory of local hidden variables can

ever reproduce all of the predictions of quantum mechanics.

- **epistemological**: Of or pertaining to epistemology or theory of knowledge, as a field of study.

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28.2: Applications of Quantum Mechanics

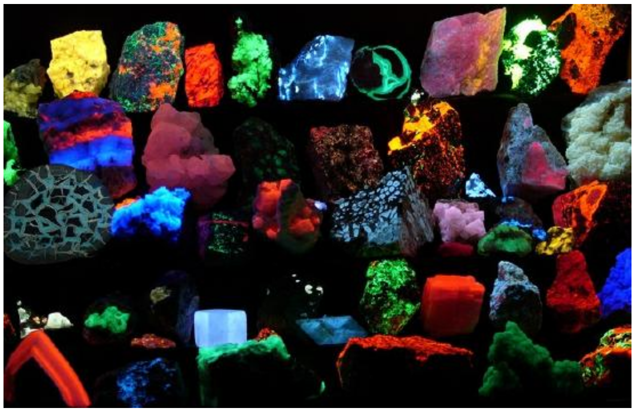
learning objectives

- Compare mechanisms of fluorescence and phosphorescence light emission

Fluorescence and Phosphorescence

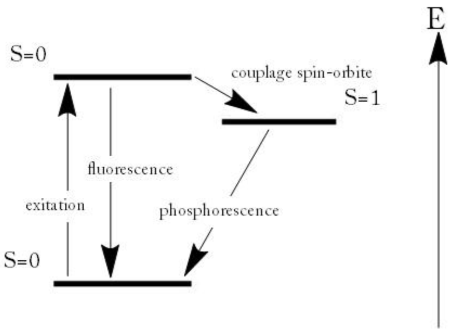
Fluorescence is the emission of light by a substance that has absorbed light or other electromagnetic radiation. It is a form of photoluminescence. In most cases, the emitted light has a longer wavelength, and therefore lower energy, than the absorbed radiation. However, when the absorbed electromagnetic radiation is intense, it is possible for one electron to absorb two photons; this two-photon absorption can lead to emission of radiation having a shorter wavelength than the absorbed radiation. The emitted radiation may also be of the same wavelength as the absorbed radiation, termed “resonance fluorescence”.

Fluorescence occurs when an orbital electron of a molecule or atom relaxes to its ground state by emitting a photon of light after being excited to a higher quantum state by some type of energy. The most striking examples of fluorescence occur when the absorbed radiation is in the ultraviolet region of the spectrum, and thus invisible to the human eye, and the emitted light is in the visible region.



Fluorescence: Fluorescent minerals emit visible light when exposed to ultraviolet light

Phosphorescence is a specific type of photoluminescence related to fluorescence. Unlike fluorescence, a phosphorescent material does not immediately re-emit the radiation it absorbs. Excitation of electrons to a higher state is accompanied with the change of a spin state. Once in a different spin state, electrons cannot relax into the ground state quickly because the re-emission involves quantum mechanically forbidden energy state transitions. As these transitions occur very slowly in certain materials, absorbed radiation may be re-emitted at a lower intensity for up to several hours after the original excitation.



Fluorescence and Phosphorescence: Energy scheme used to explain the difference between fluorescence and phosphorescence

Commonly seen examples of phosphorescent materials are the glow-in-the-dark toys, paint, and clock dials that glow for some time after being charged with a bright light such as in any normal reading or room light. Typically the glowing then slowly fades out within minutes (or up to a few hours) in a dark room.

Phosphorescence: Phosphorescent material glowing in the dark.

Lasers

A laser is a device that emits monochromatic light through a process of optical amplification based on the stimulated emission of photons.

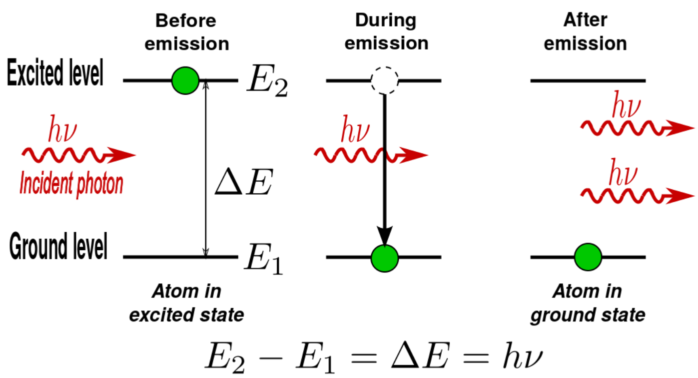
learning objectives

- Identify process that generates laser emission and the defining characteristics of laser light

A laser is a device that emits monochromatic light (electromagnetic radiation). It does so through a process of optical amplification based on the stimulated emission of photons. The term “laser” originated as an acronym for Light Amplification by Stimulated Emission of Radiation. Laser is distinct from other light sources for its high degree of spatial and temporal coherence, which means that laser outputs a narrow beam that maintains its temporal-phase relationship.

Principles of laser operation are largely based on quantum mechanics. (One exception would be free-electron lasers, whose operation can be explained solely by classical electrodynamics.) When an electron is excited from a lower-energy to a higher-energy level, it will not stay that way forever. An electron in an excited state may decay to an unoccupied lower-energy state according to a particular time constant characterizing that transition. When such an electron decays without external influence, it emits a photon; this process is called “spontaneous emission. ” The phase associated with the emitted photon is random. A material with many atoms in an excited state may thus result in radiation that is very monochromatic, but the individual photons would have no common phase relationship and would emanate in random directions. This is the mechanism of fluorescence and thermal emission.

However, an external photon at a frequency associated with the atomic transition can affect the quantum mechanical state of the atom. As the incident photon passes by, the rate of transitions of the excited atom can be significantly enhanced beyond that due to spontaneous emission. This “induced” decay process is called stimulated emission. In stimulated emission, the decaying atom produces an identical “copy” of the incoming photon. Therefore, after the atom decays, we have two identical outgoing photons. Since there was only one *incoming* photon, we amplified the intensity of light by a factor of 2!



Stimulated Photon Emission: In stimulated emission process, a photon (with a frequency equal to the atomic transition) encounters an excited atom, and a new photon identical to the incoming photon is produced. The result is an atom in the ground state with two outgoing photons.

Holography

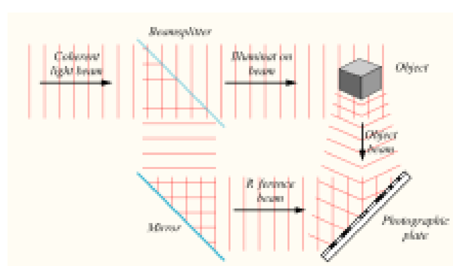
Holography is an optical technique which enables three-dimensional images to be made.

learning objectives

- Explain how holographic images are recorded and their properties

Holography is a technique which enables three-dimensional images to be made. It involves the use of a laser, interference, diffraction, light intensity recording and suitable illumination of the recording. The image changes as the position and orientation of the viewing system changes in exactly the same way as if the object were still present, thus making the image appear three-dimensional.

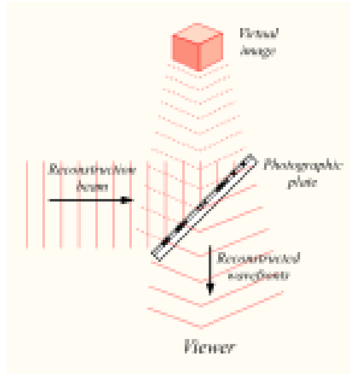
Laser: Holograms are recorded using a flash of light that illuminates a scene and then imprints on a recording medium, much in the way a photograph is recorded. In addition, however, part of the light beam must be shone directly onto the recording medium – this second light beam is known as the reference beam (]). A hologram requires a laser as the sole light source. Laser is required as a light source to produce an interference pattern on the recording plate. To prevent external light from interfering, holograms are usually taken in darkness, or in low level light of a different color from the laser light used in making the hologram. Holography requires a specific exposure time, which can be controlled using a shutter, or by electronically timing the laser



Recording a hologram: Holograms are recorded using a flash of light that illuminates a scene and then imprints on a recording medium, much in the way a photograph is recorded. In addition, however, part of the light beam must be shone directly onto the recording medium – this second light beam is known as the reference beam.

Apparatus: A hologram can be made by shining part of the light beam directly onto the recording medium, and the other part onto the object in such a way that some of the scattered light falls onto the recording medium. A more flexible arrangement for recording a hologram requires the laser beam to be aimed through a series of elements that change it in different ways. The first element is a beam splitter that divides the beam into two identical beams, each aimed in different directions:

- One beam (known as the illumination or object beam) is spread using lenses and directed onto the scene using mirrors. Some of the light scattered (reflected) from the scene then falls onto the recording medium.
- The second beam (known as the reference beam) is also spread through the use of lenses, but is directed so that it doesn't come in contact with the scene, and instead travels directly onto the recording medium.



Reconstructing a hologram: An interference pattern can be considered an encoded version of a scene, requiring a particular key – the original light source – in order to view its contents. This missing key is provided later by shining a laser, identical to the one used to record the hologram, onto the developed film. When this beam illuminates the hologram, it is diffracted by the hologram's surface pattern. This produces a light field identical to the one originally produced by the scene and scattered onto the hologram

Several different materials can be used as the recording medium. One of the most common is a film very similar to photographic film (silver halide photographic emulsion), but with a much higher concentration of light-reactive grains, making it capable of the much higher resolution that holograms require. A layer of this recording medium (e.g. silver halide) is attached to a transparent substrate, which is commonly glass, but may also be plastic.

Process: When the two laser beams reach the recording medium, their light waves intersect and interfere with each other. It is this interference pattern that is imprinted on the recording medium. The pattern itself is seemingly random, as it represents the way in which the scene's light interfered with the original light source – but not the original light source itself. The interference pattern can be considered an encoded version of the scene, requiring a particular key – the original light source – in order to view its contents.

This missing key is provided later by shining a laser, identical to the one used to record the hologram, onto the developed film. When this beam illuminates the hologram, it is diffracted by the hologram's surface pattern. This produces a light field identical to the one originally produced by the scene and scattered onto the hologram. The image this effect produces in a person's retina is known as a virtual image.

The Periodic Table of Elements

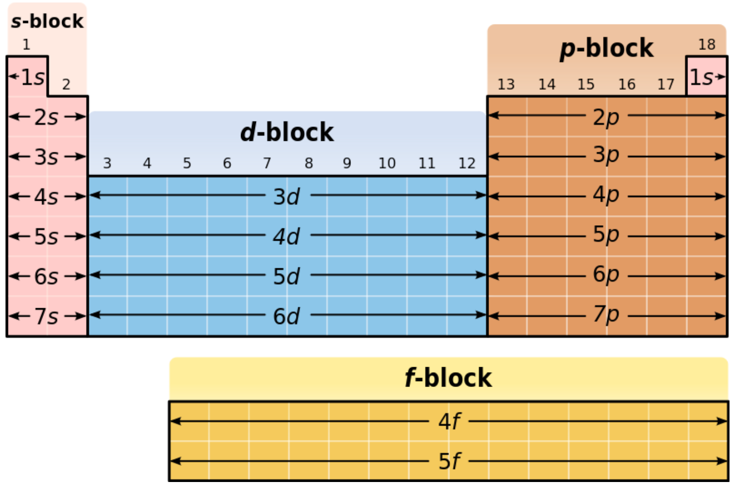
A periodic table is a tabular display of elements organized by their atomic numbers, electron configurations, and chemical properties.

learning objectives

- Explain how properties of elements vary within groups and across periods in the periodic table

The periodic table is a tabular display of the chemical elements. The elements are organized based on their atomic numbers, electron configurations, and recurring chemical properties.

In the periodic table, elements are presented in order of increasing atomic number (the number of protons). The rows of the table are called periods; the columns of the s- (columns 1-2 and He), d- (columns 3-12), and p-blocks (columns 13-18, except He) are called groups. (The terminology of s-, p-, and d- blocks originate from the valence atomic orbitals the element's electrons occupy.) Some groups have specific names, such as the halogens or the noble gases. Since, by definition, a periodic table incorporates recurring trends, any such table can be used to derive relationships between the properties of the elements and predict the properties of new, yet-to-be-discovered, or synthesized elements. As a result, the periodic table provides a useful framework for analyzing chemical behavior, and such tables are widely used in chemistry and other sciences.



Blocks in the Periodic Table: A diagram of the periodic table, highlighting the different blocks

History of the Periodic Table

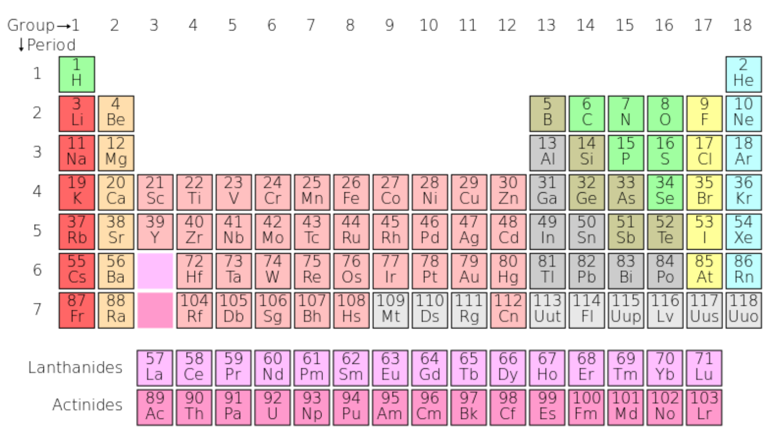
Although precursors exist, Dmitri Mendeleev is generally credited with the publication, in 1869, of the first widely recognized periodic table. Mendeleev designed the table in such a way that recurring ("periodic") trends in the properties of the elements could be shown. Using the trends he observed, he even left gaps for those elements that he thought were "missing. " He even predicted the properties that he thought the missing elements would have when they were discovered. Many of these elements were indeed later discovered, and Mendeleev's predictions were proved to be correct.

Groups

Agroup, or family, is a vertical column in the periodic table. Groups usually have more significant periodic trends than do periods and blocks, which are explained below. Modern quantum mechanical theories of atomic structure explain group trends by proposing that elements in the same group generally have the same electron configurations in their valence (or outermost, partially filled) shell. Consequently, elements in the same group tend to have shared chemistry and exhibit a clear trend in properties with increasing atomic number. However, in some parts of the periodic table, such as the d-block and the f-block, horizontal similarities can be as important as, or more pronounced than, vertical similarities.

Periods

A period is a horizontal row in the periodic table. Although groups generally have more significant periodic trends, there are regions where horizontal trends are more significant than vertical group trends, such as in the f-block, where the lanthanides and actinides form two substantial horizontal series of elements. Elements in the same period show trends in atomic radius, ionization energy, and electron affinity. Atomic radius usually decreases from left to right across a period. This occurs because each successive element has an added proton and electron, which causes the electron to be drawn closer to the nucleus, decreasing the radius.



The periodic table: Here is the complete periodic table with atomic numbers, groups, and periods. Each entry on the periodic table represents one element, and compounds are made up of several of these elements.

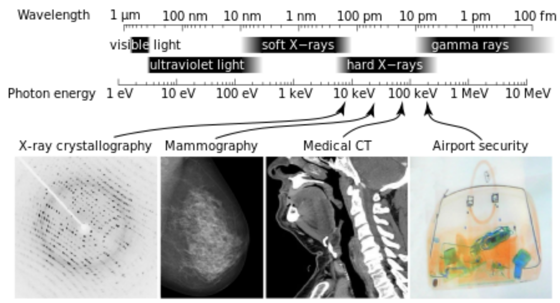
X-Rays

X-rays are a form of electromagnetic radiation and have wavelengths in the range of 0.01 to 10 nanometers.

learning objectives

- Describe the properties of X-rays and how can be generated

X-radiation (composed of x-rays) is a form of electromagnetic radiation. X-rays have wavelengths in the range of 0.01 to 10 nanometers, which corresponds to frequencies in the range of 30 petahertz to 30 exahertz ($3 \cdot 10^{16}$ Hz to $3 \cdot 10^{19}$ Hz) and energies in the of range 100 eV to 100 keV.



X-Ray Spectrum and Applications: X-rays are part of the electromagnetic spectrum, with wavelengths shorter than those of visible light. Different applications use different parts of the X-ray spectrum.

X-rays can be generated by an x-ray tube, a vacuum tube that uses high voltage to accelerate the electrons released by a hot cathode to a high velocity. The high-velocity electrons collide with a metal target, the anode, creating the x-rays. The maximum energy of the produced x-ray photon is limited by the energy of the incident electron, which is equal to the voltage on the tube times the electron charge, so an 80-kV tube cannot create x-rays with an energy greater than 80 keV. When the electrons hit the target, x-rays are created through two different atomic processes:

1. X-ray fluorescence, if the electron has enough energy that it can knock an orbital electron out of the inner electron shell of a metal atom. As a result, electrons from higher energy levels fill up the vacancy, and x-ray photons are emitted. This process produces an emission spectrum of x-rays at a few discrete frequencies, sometimes referred to as the spectral lines. The spectral lines generated depend on the target (anode) element used and therefore are called characteristic lines. Usually these are transitions from upper shells into the K shell (called K lines), or the L shell (called L lines), and so on.
2. *Bremsstrahlung*, literally meaning braking radiation. *Bremsstrahlung* is radiation given off by the electrons as they are scattered by the strong electric field near the high-Z (proton number) nuclei. These x-rays have a continuous spectrum. The intensity of the x-rays increases linearly with decreasing frequency, from zero at the energy of the incident electrons, the voltage on the x-ray tube.

Both of these x-ray production processes are inefficient, with a production efficiency of only about one percent. Therefore, to produce a usable flux of x-rays, most of the electric power consumed by the tube is released as heat waste. The x-ray tube must be designed to dissipate this excess heat.

A specialized source of x-rays that is becoming widely used in research is synchrotron radiation, which is generated by particle accelerators. Its unique features are x-ray outputs many orders of magnitude greater than those of x-ray tubes, wide x-ray spectra, excellent collimation, and linear polarization.

Quantum-Mechanical View of Atoms

Atom is a basic unit of matter that consists of a nucleus surrounded by negatively charged electron cloud, commonly called atomic orbitals.

learning objectives

- Identify major contributions to the understanding of atomic structure that were made by Niels Bohr, Erwin Schrödinger, and Werner Heisenberg

The atom is a basic unit of matter that consists of a nucleus surrounded by negatively charged electrons. The atomic nucleus contains a mix of positively charged protons and electrically neutral neutrons. The electrons of an atom are bound to the nucleus by the electromagnetic (Coulomb) force. Atoms are minuscule objects with diameters of a few tenths of a nanometer and tiny masses proportional to the volume implied by these dimensions. Atoms in solid states (or, to be precise, their electron clouds) can be observed individually using special instruments such as the scanning tunneling microscope.

Hydrogen-1 (one proton + one electron) is the simplest form of atoms, and not surprisingly, our quantum mechanical understanding of atoms evolved with the understanding of this species. In 1913, physicist Niels Bohr suggested that the electrons were confined into clearly defined, quantized orbits, and could jump between these, but could not freely spiral inward or outward in intermediate states. An electron must absorb or emit specific amounts of energy to transition between these fixed orbits. Bohr's model successfully explained spectroscopic data of hydrogen very well, but it adopted a semiclassical approach where electron was still considered a (classical) particle.

Adopting Louis de Broglie's proposal of wave-particle duality, Erwin Schrödinger, in 1926, developed a mathematical model of the atom that described the electrons as three-dimensional waveforms rather than point particles. A consequence of using waveforms to describe particles is that it is mathematically impossible to obtain precise values for both the position and momentum of a particle at the same time; this became known as the uncertainty principle, formulated by Werner Heisenberg in 1926. Thereafter, the planetary model of the atom was discarded in favor of one that described atomic orbital zones around the nucleus where a given electron is most likely to be observed.

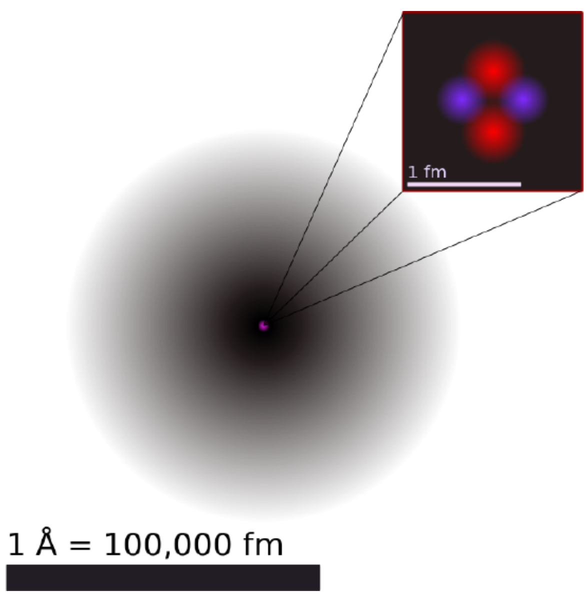


Illustration of the Helium Atom: This is an illustration of the helium atom, depicting the nucleus (pink) and the electron cloud distribution (black). The nucleus (upper right) in helium-4 is in reality spherically symmetric and closely resembles the electron cloud, although for more complicated nuclei this is not always the case. The black bar is one angstrom (10-10 m, or 100 pm).

Modern quantum mechanical view of hydrogen has evolved further after Schrödinger, by taking relativistic correction terms into account. Quantum electrodynamics (QED), a relativistic quantum field theory describing the interaction of electrically charged particles, has successfully predicted minuscule corrections in energy levels. One of the hydrogen's atomic transitions (n=2 to n=1, n: principal quantum number) has been measured to an extraordinary precision of 1 part in a hundred trillion. This kind of spectroscopic precision allows physicists to refine quantum theories of atoms, by accounting for minuscule discrepancies between experimental results and theories.

Key Points

- The emitted light usually has a longer wavelength, and therefore lower energy, than the absorbed radiation.
- Fluorescence occurs when an orbital electron of a molecule or atom relaxes to its ground state by emitting a photon of light after being excited to a higher quantum state by some type of energy.
- In a phosphorescence, excitation of electrons to a higher state is accompanied with the change of a spin state. Relaxation is a slow process since it involves energy state transitions “forbidden” in quantum mechanics.
- Principles of laser operation are largely based on quantum mechanics, most importantly on the process of the stimulated emission of photons.
- Spontaneous emission is a random decaying process. The phase associated with the emitted photon is also random.
- Atomic transition can be stimulated by the presence of an incoming photon at a frequency associated with the atomic transition. This process leads to optical amplification as an identical photon is emitted along with the incoming photon.
- When the two laser beams reach the recording medium, their light waves intersect and interfere with each other. It is this interference pattern that is imprinted on the recording medium.
- When a reconstruction beam illuminates the hologram, it is diffracted by the hologram’s surface pattern. This produces a light field identical to the one originally produced by the scene and scattered onto the hologram.
- Holographic image changes as the position and orientation of the viewing system changes in exactly the same way as if the object were still present, thus making the image appear three-dimensional.
- A periodic table is a useful framework for analyzing chemical behavior. Such tables are widely used in chemistry and other sciences.
- A group, or family, is a vertical column in the periodic table. Groups usually have more significant periodic trends than do periods and blocks.
- A period is a horizontal row in the periodic table. Elements in the same period show trends in atomic radius, ionization energy, electron affinity, and electronegativity.
- X-rays can be generated by an x-ray tube, a vacuum tube, or a particle accelerator.
- X-ray fluorescence and Bremsstrahlung are processes through which x-rays are produced.
- Synchrotron radiation is generated by particle accelerators. Its unique features are x-ray outputs many orders of magnitude greater than those of x-ray tubes, wide x-ray spectra, excellent collimation, and linear polarization.
- Niels Bohr suggested that the electrons were confined into clearly defined, quantized orbits, and could jump between these, but could not freely spiral inward or outward in intermediate states.
- Erwin Schrödinger, in 1926, developed a mathematical model of the atom that described the electrons as three-dimensional waveforms rather than point particles.
- Modern quantum mechanical view of hydrogen has evolved further after Schrödinger, by taking relativistic correction terms into account. This is referred to a quantum electrodynamics (QED).

Key Terms

- **spin**: A quantum angular momentum associated with subatomic particles; it also creates a magnetic moment.
- **photon**: The quantum of light and other electromagnetic energy, regarded as a discrete particle having zero rest mass, no electric charge, and an indefinitely long lifetime.
- **ground state**: the stationary state of lowest energy of a particle or system of particles
- **free-electron laser**: a laser that use a relativistic electron beam as the lasing medium, which moves freely through a magnetic structure
- **monochromatic**: Describes a beam of light with a single wavelength (i.e., of one specific color or frequency).
- **coherence**: an ideal property of waves that enables stationary (i.e., temporally and spatially constant) interference
- **interference**: An effect caused by the superposition of two systems of waves, such as a distortion on a broadcast signal due to atmospheric or other effects.
- **laser**: A device that produces a monochromatic, coherent beam of light.
- **silver halide**: The light-sensitive chemicals used in photographic film and pape
- **atomic orbital**: The quantum mechanical behavior of an electron in an atom describing the probability of the electron’s particular position and energy.
- **electron affinity**: the amount of energy released when an electron is added to a neutral atom or molecule to form a negative ion
- **ionization energy**: the amount of energy required to remove an electron from an atom or molecule in the gas phase

- **photon**: The quantum of light and other electromagnetic energy, regarded as a discrete particle having zero rest mass, no electric charge, and an indefinitely long lifetime.
- **particle accelerator**: A device that accelerates electrically charged particles to extremely high speeds, for the purpose of inducing high-energy reactions or producing high-energy radiation.
- **wave-particle duality**: A postulation that all particles exhibit both wave and particle properties. It is a central concept of quantum mechanics.
- **scanning tunneling microscope**: An instrument for imaging surfaces at the atomic level.
- **semiclassical approach**: A theory in which one part of a system is described quantum-mechanically whereas the other is treated classically.

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CHAPTER OVERVIEW

29: Atomic Physics

Topic hierarchy

29.1: Overview

learning objectives

- Identify assumption made by Max Planck to describe the electromagnetic radiation emitted by a black body

A black body in thermal equilibrium (i.e. at a constant temperature) emits electromagnetic radiation called black body radiation. Black body radiation has a characteristic, continuous frequency spectrum that depends only on the body's temperature. Max Planck, in 1901, accurately described the radiation by assuming that electromagnetic radiation was emitted in discrete packets (or quanta). Planck's quantum hypothesis is a pioneering work, heralding advent of a new era of modern physics and quantum theory.

Explaining the properties of black-body radiation was a major challenge in theoretical physics during the late nineteenth century. Predictions based on classical theories failed to explain black body spectra observed experimentally, especially at shorter wavelength. The puzzle was solved in 1901 by Max Planck in the formalism now known as Planck's law of black-body radiation. Contrary to the common belief that electromagnetic radiation can take continuous values of energy, Planck introduced a radical concept that electromagnetic radiation was emitted in discrete packets (or quanta) of energy. Although Planck's derivation is beyond the scope of this section (it will be covered in Quantum Mechanics), Planck's law may be written:

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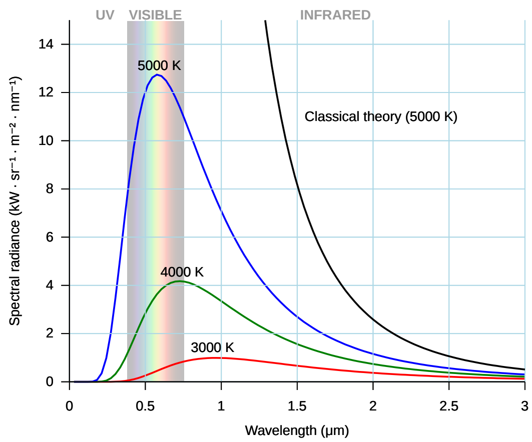
where BB is the spectral radiance of the surface of the black body, TT is its absolute temperature, λλ is wavelength of the radiation, kBkB is the Boltzmann constant, hh is the Planck constant, and cc is the speed of light. This equation explains the black body spectra shown below. Planck's quantum hypothesis is one of the breakthroughs in the modern physics. It is not a surprise that he introduced Planck constant *[Math Processing Error]* for the first time in his derivation of the Planck's law.



Black body radiation spectrum: Typical spectrum from a black body at different temperatures (shown in blue, green and red curves). As the temperature decreases, the peak of the black-body radiation curve moves to lower intensities and longer wavelengths. Black line is a prediction of a classical theory for an object at 5,000K, showing catastropic discrepancy at shorter wavelengh.

Note that the spectral radiance depends on two variables, wavelength and temperature. The radiation has a specific spectrum and intensity that depends only on the temperature of the body. Despite its simplicity, Planck's law describes radiation properties of objects (e.g. our body, planets, stars) reasonably well.

Key Points

- A black body in thermal equilibrium emits electromagnetic radiation called black body radiation.
- The radiation has a specific spectrum and intensity that depends only on the temperature of the body.
- Max Planck, in 1901, accurately described the radiation by assuming that electromagnetic radiation was emitted in discrete packets (or quanta). Planck's quantum hypothesis is a pioneering work, heralding advent of a new era of modern physics and quantum theory.

Key Terms

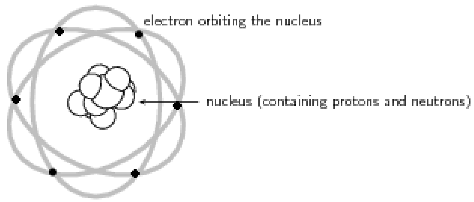
- **spectral radiance**: measures of the quantity of radiation that passes through or is emitted from a surface and falls within a given solid angle in a specified direction.
- **black body**: An idealized physical body that absorbs all incident electromagnetic radiation, regardless of frequency or angle of incidence. Although black body is a theoretical concept, you can find approximate realizations of black body in nature.
- **Planck constant**: a physical constant that is the quantum of action in quantum mechanics. It has a unit of angular momentum. The Planck constant was first described as the proportionality constant between the energy of a photon (unit of electromagnetic radiation) and the frequency of its associated electromagnetic wave in his derivation of the Planck's law

29.2: The Early Atom

learning objectives

- Discuss experiments that led to discovery of the electron and the nucleus

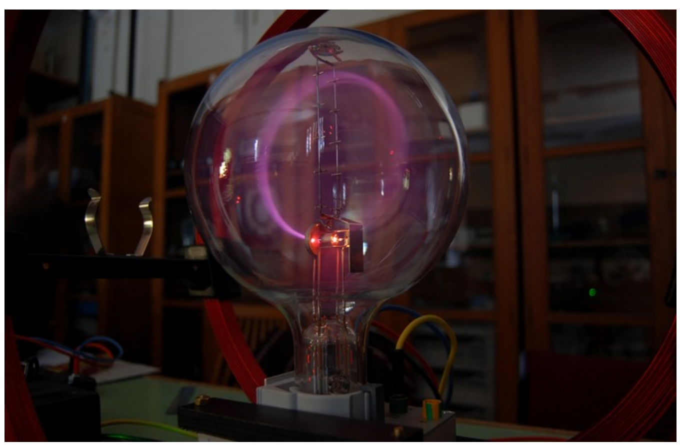
Though originally viewed as a particle that cannot be cut into smaller particles, modern scientific usage denotes the atom as composed of various subatomic particles. The constituent particles of an atom (each discovered independently) are: the electron, the proton and the neutron. (The hydrogen-1 atom, however, has no neutrons, and a positive hydrogen ion has no electrons.)



Classical Atomic Model: Atomic model before the advent of Quantum Mechanics.

Electron

The German physicist Johann Wilhelm Hittorf undertook the study of electrical conductivity in rarefied gases. In 1869, he discovered a glow emitted from the cathode that increased in size with decrease in gas pressure. In 1896, the British physicist J. J. Thomson performed experiments demonstrating that cathode rays were unique particles, rather than waves, atoms or molecules, as was believed earlier. Thomson made good estimates of both the charge ee and the mass mm, finding that cathode ray particles (which he called "corpuscles") had perhaps one thousandth the mass of hydrogen, the least massive ion known. He showed that their charge to mass ratio (e/m) was independent of cathode material. (Fig 1 shows a beam of deflected electrons.)



Electron Beam: A beam of electrons deflected in a circle by a magnetic field.

Proton

In 1917 (in experiments reported in 1919), Rutherford proved that the hydrogen nucleus is present in other nuclei, a result usually described as the discovery of the proton. Earlier, Rutherford learned to create hydrogen nuclei as a type of radiation produced as a yield of the impact of alpha particles on hydrogen gas; these nuclei were recognized by their unique penetration signature in air and their appearance in scintillation detectors. These experiments began when Rutherford noticed that when alpha particles were shot into air (mostly nitrogen), his scintillation detectors displayed the signatures of typical hydrogen nuclei as a product. After experimentation Rutherford traced the reaction to the nitrogen in air, and found that the effect was larger when alphas were produced into pure nitrogen gas. Rutherford determined that the only possible source of this hydrogen was the nitrogen, and therefore nitrogen must contain hydrogen nuclei. One hydrogen nucleus was knocked off by the impact of the alpha particle, producing oxygen-17 in the process. This was the first reported nuclear reaction, $14\mathrm{N}+\alpha \rightarrow 17\mathrm{O}+\mathrm{p}$.

Neutron

In 1920, Ernest Rutherford conceived the possible existence of the neutron. In particular, Rutherford examined the disparity found between the atomic number of an atom and its atomic mass. His explanation for this was the existence of a neutrally charged particle within the atomic nucleus. He considered the neutron to be a neutral double consisting of an electron orbiting a proton. In 1932, James Chadwick showed uncharged particles in the radiation he used. These particles had a similar mass as protons, but did not have the same characteristics as protons. Chadwick followed some of the predictions of Rutherford, the first to work in this then unknown field.

Early Models of the Atom

Dalton believed that that matter is composed of discrete units called atoms — indivisible, ultimate particles of matter.

learning objectives

- Describe postulates of Dalton's atomic theory and the atomic theories of ancient Greek philosophers

The atom is a basic unit of matter that consists of a dense central nucleus surrounded by a cloud of negatively charged electrons. The atomic nucleus contains a mix of positively charged protons and electrically neutral neutrons (except in the case of hydrogen-1, which is the only stable nuclide with no neutrons). The electrons of an atom are bound to the nucleus by the electromagnetic force. We have a detailed (and accurate) model of the atom now, but it took a long time to come up with the correct answer.

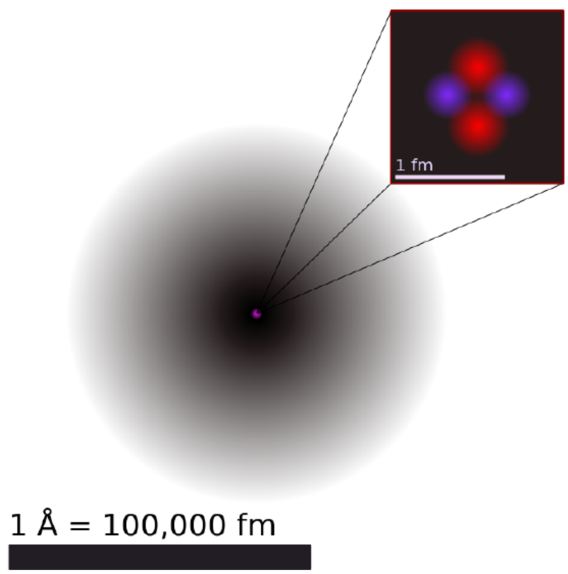


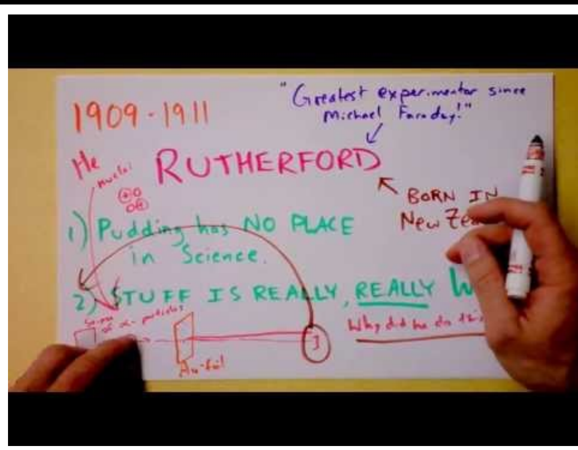
Illustration of the Helium Atom: This is an illustration of the helium atom, depicting the nucleus (pink) and the electron cloud distribution (black). The nucleus (upper right) in helium-4 is in reality spherically symmetric and closely resembles the electron cloud, although for more complicated nuclei this is not always the case. The black bar is one angstrom (10−1010−10 m, or 100 pm).

People have long speculated about the structure of matter and the existence of atoms. The earliest significant ideas to survive are from the ancient Greeks in the fifth century BC, especially from the philosophers Leucippus and Democritus. (There is some evidence that philosophers in both India and China made similar speculations at about the same time.) They considered the question of whether a substance can be divided without limit into ever smaller pieces. There are only a few possible answers to this question. One is that infinitesimally small subdivision is possible. Another is what Democritus in particular believed — that there is a smallest unit that cannot be further subdivided. Democritus called this the atom. We now know that atoms themselves can be subdivided, but their identity is destroyed in the process, so the Greeks were correct in a respect. The Greeks also felt that atoms were in constant motion, another correct notion.

The Greeks and others speculated about the properties of atoms, proposing that only a few types existed and that all matter was formed as various combinations of these types. The famous proposal that the basic elements were earth, air, fire, and water was brilliant but incorrect. The Greeks had identified the most common examples of the four states of matter (solid, gas, plasma, and liquid) rather than the basic chemical elements. More than 2000 years passed before observations could be made with equipment capable of revealing the true nature of atoms.

Over the centuries, discoveries were made regarding the properties of substances and their chemical reactions. Certain systematic features were recognized, but similarities between common and rare elements resulted in efforts to transmute them (lead into gold, in particular) for financial gain. Secrecy was commonplace. Alchemists discovered and rediscovered many facts but did not make them broadly available. As the Middle Ages ended, the practice of alchemy gradually faded, and the science of chemistry arose. It was no longer possible, nor considered desirable, to keep discoveries secret. Collective knowledge grew, and by the beginning of the 19th century, an important fact was well established: the masses of reactants in specific chemical reactions always have a particular mass ratio. This is very strong indirect evidence that there are basic units (atoms and molecules) that have these same mass ratios. English chemist John Dalton (1766-1844) did much of this work, with significant contributions by the Italian physicist Amedeo Avogadro (1776-1856). It was Avogadro who developed the idea of a fixed number of atoms and molecules in a mole. This special number is called Avogadro's number in his honor ($6.022 \cdot 1023$).

Dalton believed that matter is composed of discrete units called atoms, as opposed to the obsolete notion that matter could be divided into any arbitrarily small quantity. He also believed that atoms are the indivisible, ultimate particles of matter. However, this belief was overturned near the end of the 19th century by Thomson, with his discovery of electrons.



Intro to the History of Atomic Theory – Intro: Rutherford, Thomson, electrons, nuclei, and plums. I don't mean to be a bohr, but do you think pudding should have a role in serious scientific inquiry?

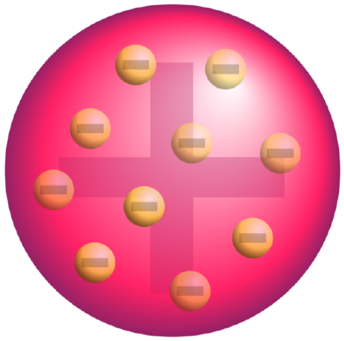
The Thomson Model

Thomson proposed that the atom is composed of electrons surrounded by a soup of positive charge to balance the electrons' negative charges.

learning objectives

- Describe model of an atom proposed by J. J. Thomson.

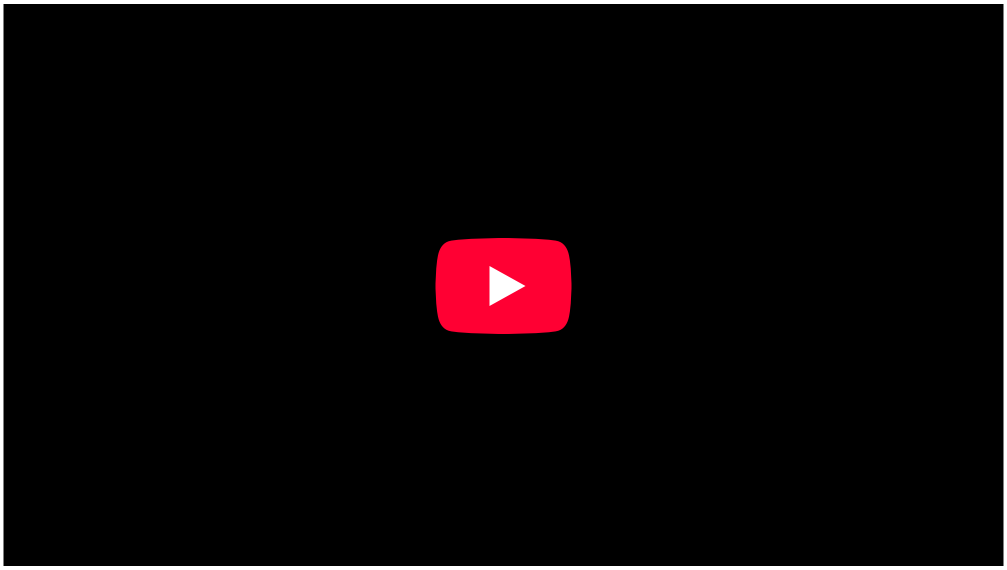
J. J. Thomson, who discovered the electron in 1897, proposed the plum pudding model of the atom in 1904 before the discovery of the atomic nucleus in order to include the electron in the atomic model. In Thomson's model, the atom is composed of electrons (which Thomson still called "corpuscles," though G. J. Stoney had proposed that atoms of electricity be called electrons in 1894) surrounded by a soup of positive charge to balance the electrons' negative charges, like negatively charged "plums" surrounded by positively charged "pudding". The electrons (as we know them today) were thought to be positioned throughout the atom in rotating rings. In this model the atom was also sometimes described to have a "cloud" of positive charge.

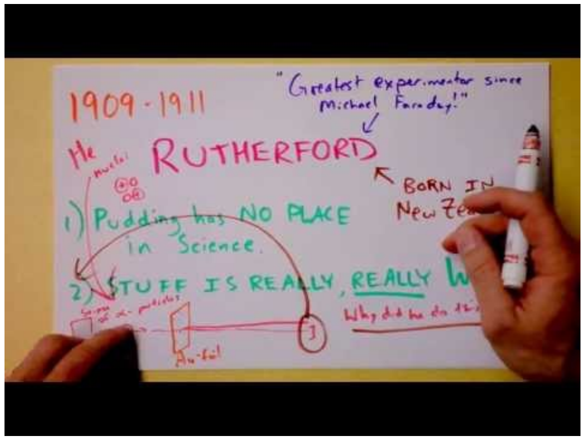


Plum pudding model of the atom: A schematic presentation of the plum pudding model of the atom; in Thomson's mathematical model the "corpuscles" (in modern language, electrons) were arranged non-randomly, in rotating rings.

With this model, Thomson abandoned his earlier "nebular atom" hypothesis, in which the atom was composed of immaterial vortices. Now, at least part of the atom was to be composed of Thomson's particulate negative corpuscles, although the rest of the positively charged part of the atom remained somewhat nebulous and ill-defined.

The 1904 Thomson model was disproved by the 1909 gold foil experiment performed by Hans Geiger and Ernest Marsden. This gold foil experiment was interpreted by Ernest Rutherford in 1911 to suggest that there is a very small nucleus of the atom that contains a very high positive charge (in the case of gold, enough to balance the collective negative charge of about 100 electrons). His conclusions led him to propose the Rutherford model of the atom.





Intro to the History of Atomic Theory – The Thomson Model: Rutherford, Thomson, electrons, nuclei, and plums. I don't mean to be a bohr, but do you think pudding should have a role in serious scientific inquiry?

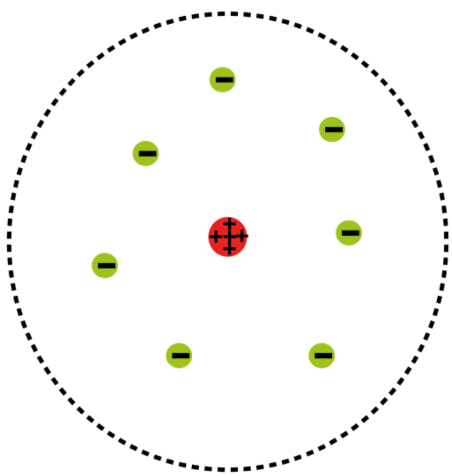
The Rutherford Model

Rutherford confirmed that the atom had a concentrated center of positive charge and relatively large mass.

learning objectives

- Describe gold foil experiment performed by Geiger and Marsden under directions of Rutherford and its implications for the model of the atom

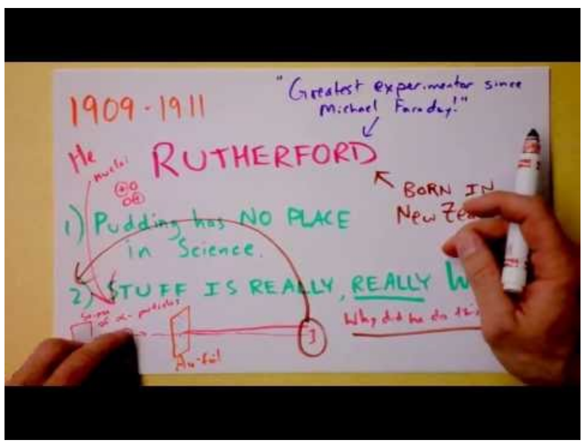
The Rutherford model is a model of the atom named after Ernest Rutherford. Rutherford directed the famous Geiger-Marsden experiment in 1909, which suggested, according to Rutherford's 1911 analysis, that J. J. Thomson's so-called "plum pudding model" of the atom was incorrect. Rutherford's new model for the atom, based on the experimental results, contained the new features of a relatively high central charge concentrated into a very small volume in comparison to the rest of the atom. This central volume also contained the bulk of the atom's mass. This region would later be named the "nucleus. "



Atomic Planetary Model: Basic diagram of the atomic planetary model; electrons are in green, and the nucleus is in red

In 1911, Rutherford designed an experiment to further explore atomic structure using the alpha particles emitted by a radioactive element. Following his direction, Geiger and Marsden shot alpha particles with large kinetic energies toward a thin foil of gold. Measuring the pattern of scattered particles was expected to provide information about the distribution of charge within the atom. Under the prevailing plum pudding model, the alpha particles should all have been deflected by, at most, a few degrees. However, the actual results surprised Rutherford. Although many of the alpha particles did pass through as expected, many others were deflected at small angles while others were reflected back to the alpha source.

From purely energetic considerations of how far particles of known speed would be able to penetrate toward a central charge of 100 e, Rutherford was able to calculate that the radius of his gold central charge would need to be less than 3.4⋅10−143.4⋅10−14 meters. This was in a gold atom known to be about 10−1010−10 meters in radius; a very surprising finding, as it implied a strong central charge less than 1300013000$^{\text{th}}$ of the diameter of the atom.



Intro to the History of Atomic Theory – The Rutherford Model: Rutherford, Thomson, electrons, nuclei, and plums. I don't mean to be a bohr, but do you think pudding should have a role in serious scientific inquiry?

The Bohr Model of the Atom

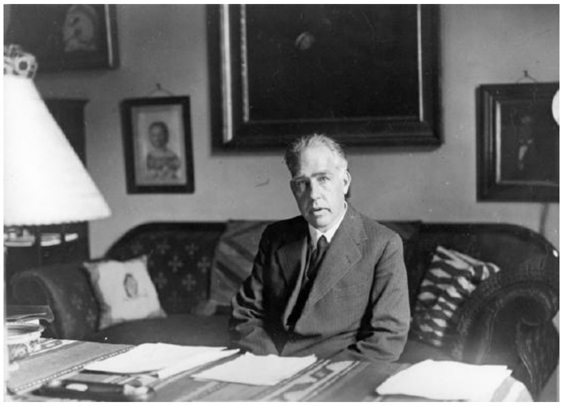
Bohr suggested that electrons in hydrogen could have certain classical motions *only when restricted by a quantum rule*.

learning objectives

- Describe model of atom proposed by Niels Bohr.

The Bohr Model of the Atom

The great Danish physicist Niels Bohr (1885–1962,) made immediate use of Rutherford's planetary model of the atom. Bohr became convinced of its validity and spent part of 1912 at Rutherford's laboratory. In 1913, after returning to Copenhagen, he began publishing his theory of the simplest atom, hydrogen, based on the planetary model of the atom.



Niels Bohr: Niels Bohr, Danish physicist, used the planetary model of the atom to explain the atomic spectrum and size of the hydrogen atom. His many contributions to the development of atomic physics and quantum mechanics; his personal influence on many students and colleagues; and his personal integrity, especially in the face of Nazi oppression, earned him a prominent place in history. (credit: Unknown Author, via Wikimedia Commons)

For decades, many questions had been asked about atomic characteristics. From their sizes to their spectra, much was known about atoms, but little had been explained in terms of the laws of physics. Bohr's theory explained the atomic spectrum of hydrogen, made him instantly famous, and established new and broadly applicable principles in quantum mechanics.

One big puzzle that the planetary-model of atom had was the following. The laws of classical mechanics predict that the electron should release electromagnetic radiation while orbiting a nucleus (according to Maxwell's equations, accelerating charge should emit electromagnetic radiation). Because the electron would lose energy, it would gradually spiral inwards, collapsing into the nucleus. This atom model is disastrous, because it predicts that all atoms are unstable. Also, as the electron spirals inward, the emission would gradually increase in frequency as the orbit got smaller and faster. This would produce a continuous smear, in frequency, of electromagnetic radiation. However, late 19th century experiments with electric discharges have shown that atoms will only emit light (that is, electromagnetic radiation) at certain discrete frequencies.

To overcome this difficulty, Niels Bohr proposed, in 1913, what is now called the Bohr model of the atom. He suggested that electrons could only have certain classical motions:

1. Electrons in atoms orbit the nucleus.
2. The electrons can only orbit stably, without radiating, in certain orbits (called by Bohr the "stationary orbits"): at a certain discrete set of distances from the nucleus. These orbits are associated with definite energies and are also called energy shells or energy levels. In these orbits, the electron's acceleration does not result in radiation and energy loss as required by classical electrodynamics.
3. Electrons can only gain and lose energy by jumping from one allowed orbit to another, absorbing or emitting electromagnetic radiation with a frequency νν determined by the energy difference of the levels according to the Planck relation:

$$\Delta \mathrm{E} = \mathrm{E}_2 - \mathrm{E}_1 = \mathrm{h}\nu \tag{29.2.1}$$

where h is Planck's constant and ν is the frequency of the radiation.

Semiclassical Model

The significance of the Bohr model is that the laws of classical mechanics apply to the motion of the electron about the nucleus *only when restricted by a quantum rule*. Therefore, his atomic model is called a semiclassical model.

Basic Assumptions of the Bohr Model

Bohr explained hydrogen's spectrum successfully by adopting a quantization condition and by introducing the Planck constant in his model.

learning objectives

- Describe basic assumptions that were applied by Niels Bohr to the planetary model of an atom

In previous modules, we have seen puzzles from classical atomic theories (e.g., the Rutherford model). Most importantly, classical electrodynamics predicts that an atom described by a (classical) planetary model would be unstable. To explain the puzzle, Bohr proposed what is now called the Bohr model of the atom in 1913. He suggested that electrons could only have certain classical motions:

1. Electrons in atoms orbit the nucleus.
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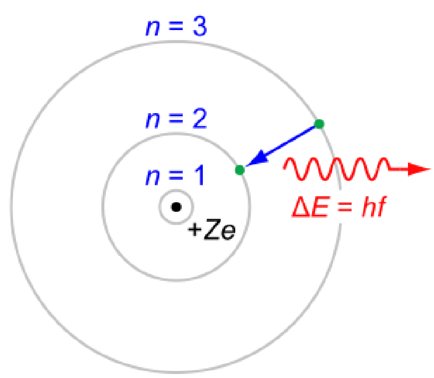
3. Electrons can only gain and lose energy by jumping from one allowed orbit to another, absorbing or emitting electromagnetic radiation with a frequency νν determined by the energy difference of the levels according to the Planck relation: $\Delta \mathrm{E}=\mathrm{E}_2-\mathrm{E}_1=\mathrm{h}\nu$, where h is the Planck constant. In addition, Bohr also assumed that the angular momentum L is restricted to be an integer multiple of a fixed unit: $\mathrm{L}=\mathrm{n}\frac{\mathrm{h}}{2\pi}=\mathrm{n}\hbar$, where $\mathrm{n}=1,2,3,\ldots$ is called the principal quantum number, and $\hbar=\frac{h}{2\pi}$.

We have seen that Planck adopted a new condition of energy quantization to explain the black body radiation, where he introduced the Planck constant hh for the first time. Soon after, Einstein resorted to this new concept of energy quantization and used the Planck constant again to explain the photoelectric effects, in which he assumed that electromagnetic radiation interact with matter as particles (later named "photons"). Here, Bohr explained the atomic hydrogen spectrum successfully for the first time by adopting a quantization condition and by introducing the Planck constant in his atomic model. Over the period of radical development in the early 20$^{\text{th}}$ century, physicists began to realize that it was essential to introduce the notion of "quantization" to explain microscopic worlds.



Rutherford-Bohr model: The Rutherford–Bohr model of the hydrogen atom ($\mathrm{Z}=1$) or a hydrogen-like ion ($\mathrm{Z}>1$), where the negatively charged electron confined to an atomic shell encircles a small, positively charged atomic nucleus, and where an electron jump between orbits is accompanied by an emitted or absorbed amount of electromagnetic energy (hνhν). The orbits in which the electron may travel are shown as gray circles; their radius increases as n^2, where n is the principal quantum number. The $3\rightarrow 2$ transition depicted here produces the first line of the Balmer series, and for hydrogen ($\mathrm{Z}=1$) it results in a photon of wavelength 656 nm (red light).

Bohr Orbits

According to Bohr, electrons can only orbit stably, in certain orbits, at a certain discrete set of distances from the nucleus.

learning objectives

- Explain relationship between the "Bohr orbits" and the quantization effect

Danish Physicist Neils Bohr was clever enough to discover a method of calculating the electron orbital energies in hydrogen. As we've seen in the previous module "The Bohr Model of Atom," Bohr assumed that the electrons can only orbit stably, without radiating, in certain orbits (named by Bohr as "stationary orbits"), at a certain discrete set of distances from the nucleus. These "Bohr orbits" have a very important feature of quantization as shown in the following. This was an important first step that has been improved upon, but it is well worth repeating here, as it correctly describes many characteristics of hydrogen. Assuming circular orbits, Bohr proposed that the angular momentum LL of an electron in its orbit is quantized, that is, has only specific, discrete values. The value for LL is given by the formula:

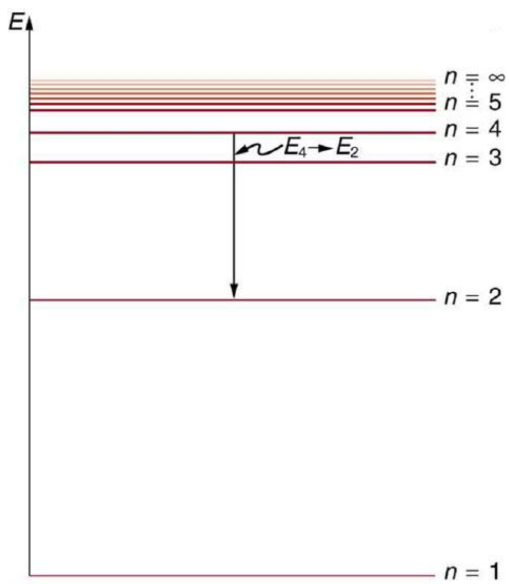
$$\mathrm{L=m_e v r_n = n\frac{h}{2\pi} = n\hbar} \tag{29.2.2}$$

where LL is the angular momentum, meme is the electron's mass, rnrn is the radius of the nn-th orbit, and hh is Planck's constant. Note that angular momentum is $\mathrm{L = I}\omega$. For a small object at a radius $\mathrm{r, I = mr^2}$ and $\omega = \frac{\mathrm{v}}{\mathrm{r}}$, so that:

$$\mathrm{L} = \left(\mathrm{mr^2}\right)\left(\frac{\mathrm{v}}{\mathrm{r}}\right) = \mathrm{mvr} \tag{29.2.3}$$

Quantization says that this value of mvrmvr can only have discrete values. At the time, Bohr himself did not know why angular momentum should be quantized, but using this assumption he was able to calculate the energies in the hydrogen spectrum, something no one else had done at the time.

Below is an energy-level diagram, which is a convenient way to display energy states—the allowed energy levels of the electron (as relative to our discussion). Energy is plotted vertically with the lowest or ground state at the bottom and with excited states above. Given the energies of the lines in an atomic spectrum, it is possible (although sometimes very difficult) to determine the energy levels of an atom. Energy-level diagrams are used for many systems, including molecules and nuclei. A theory of the atom or any other system must predict its energies based on the physics of the system.



Energy-Level Diagram Plot: An energy-level diagram plots energy vertically and is useful in visualizing the energy states of a system and the transitions between them. This diagram is for the hydrogen-atom electrons, showing a transition between two orbits having energies $\mathrm{E_4}$and $\mathrm{E_2}$.

Energy of a Bohr Orbit

Based on his assumptions, Bohr derived several important properties of the hydrogen atom from the classical physics.

learning objectives

- Apply proper equation to calculate energy levels and the energy of an emitted photon for a hydrogen-like atom

From Bohr’s assumptions, we will now derive a number of important properties of the hydrogen atom from the classical physics. We start by noting the centripetal force causing the electron to follow a circular path is supplied by the Coulomb force. To be more general, we note that this analysis is valid for any single-electron atom. So, if a nucleus has ZZ protons (Z=1Z=1 for hydrogen, Z=2Z=2 for helium, etc.) and only one electron, that atom is called a hydrogen-like atom.

The spectra of hydrogen-like ions are similar to hydrogen, but shifted to higher energy by the greater attractive force between the electron and nucleus. The magnitude of the centripetal force is m2evrnmev2rn, while the Coulomb force is Zkee2r2Zkee2r2. The tacit assumption here is that the nucleus is more massive than the stationary electron, and the electron orbits about it. This is consistent with the planetary model of the atom. Equating these:

$$\frac{m_e v^2}{r} = \frac{Zk_e e^2}{r^2} \tag{29.2.4}$$

This equation determines the electron’s speed at any radius:

$$v = \frac{\sqrt{Zk_e e^2}}{m_e r} \tag{29.2.5}$$

It also determines the electron’s total energy at any radius:

$$E = \frac{1}{2} m_e v^2 - \frac{Zk_e e^2}{r} = -\frac{Zk_e e^2}{2r} \tag{29.2.6}$$

The total energy is negative and inversely proportional to rr. This means that it takes energy to pull the orbiting electron away from the proton. For infinite values of rr, the energy is zero, corresponding to a motionless electron infinitely far from the proton.

Now, here comes the Quantum rule: As we saw in the previous module, the angular momentum L=mervL=merv is an integer multiple of ℏℏ:

$$m_e vr = n\hbar \tag{29.2.7}$$

Substituting the expression in the equation for speed above gives an equation for rr in terms of nn:

$$\sqrt{Zk_e e^2 m_e r} = n\hbar \tag{29.2.8}$$

The allowed orbit radius at any n is then:

$$r_n = \frac{n^2\hbar^2}{Zk_e e^2 m_e} \tag{29.2.9}$$

The smallest possible value of rr in the hydrogen atom is called the Bohr radius and is equal to 0.053 nm. The energy of the nn-th level for any atom is determined by the radius and quantum number:

$$E = -\frac{Zk_e e^2}{2r_n} = -\frac{Z^2\left(k_e e^2\right)^2 m_e}{2\hbar^2 n^2} \approx \frac{-13.6Z^2}{n^2} eV \tag{29.2.10}$$

Using this equation, the energy of a photon emitted by a hydrogen atom is given by the difference of two hydrogen energy levels:

$$E = E_i - E_f = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \tag{29.2.11}$$

Which is the Rydberg formula describing all the hydrogen spectrum and RR is the Rydberg constant. Bohr’s model predicted experimental hydrogen spectrum extremely well.

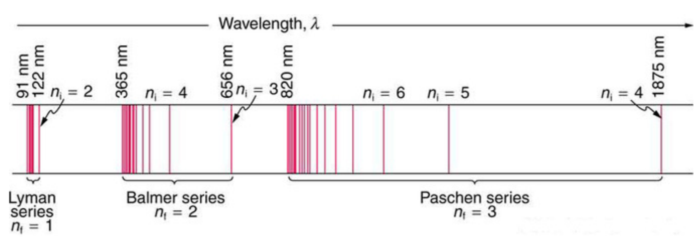


Fig 1: A schematic of the hydrogen spectrum shows several series named for those who contributed most to their determination. Part of the Balmer series is in the visible spectrum, while the Lyman series is entirely in the UV, and the Paschen series and others are in the IR. Values of nf and ni are shown for some of the lines.

Hydrogen Spectra

The observed hydrogen-spectrum wavelengths can be calculated using the following formula: $\frac{1}{\lambda} = \mathrm{R}\left(\frac{1}{\mathrm{n_f^2}} - \frac{1}{\mathrm{n_i^2}}\right)$.

learning objectives

- Explain difference between Lyman, Balmer, and Paschen series

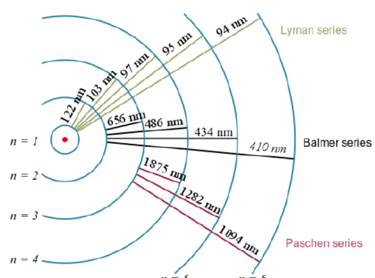
For decades, many questions had been asked about atomic characteristics. From their sizes to their spectra, much was known about atoms, but little had been explained in terms of the laws of physics. Atomic and molecular emission and absorption spectra have been known for over a century to be discrete (or quantized). Maxwell and others had realized that there must be a connection between the spectrum of an atom and its structure, something like the resonant frequencies of musical instruments. But, despite years of efforts by many great minds, no one had a workable theory. (It was a running joke that any theory of atomic and molecular spectra could be destroyed by throwing a book of data at it, so complex were the spectra.) Following Einstein's proposal of photons with quantized energies directly proportional to their wavelengths, it became even more evident that electrons in atoms can exist only in discrete orbits.

In some cases, it had been possible to devise formulas that described the emission spectra. As you might expect, the simplest atom —hydrogen, with its single electron—has a relatively simple spectrum. The hydrogen spectrum had been observed in the infrared (IR), visible, and ultraviolet (UV), and several series of spectral lines had been observed. The observed hydrogen-spectrum wavelengths can be calculated using the following formula:

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \tag{29.2.12}$$

where λ is the wavelength of the emitted EM radiation and R is the Rydberg constant, determined by the experiment to be $\mathrm{R} = 1.097 \cdot 107\ \mathrm{m}^{-1}$, and $\mathrm{n_f}, \mathrm{n_i}$ are positive integers associated with a specific series.

These series are named after early researchers who studied them in particular depth. For the Lyman series, $\mathrm{n_f} = 1$ for the Balmer series, $\mathrm{n_f} = 2$; for the Paschen series, $\mathrm{n_f} = 3$; and so on. The Lyman series is entirely in the UV, while part of the Balmer series is visible with the remainder UV. The Paschen series and all the rest are entirely IR. There are apparently an unlimited number of series, although they lie progressively farther into the infrared and become difficult to observe as nfnfincreases. The constant nini is a positive integer, but it must be greater than $\mathrm{n_f}$. Thus, for the Balmer series, $\mathrm{n_f} = 2$ and$\mathrm{n_i} = 3, 4, 5, 6 \ldots$. Note that nini can approach infinity.



Electron transitions and their resulting wavelengths for hydrogen.: Energy levels are not to scale.

While the formula in the wavelengths equation was just a recipe designed to fit data and was not based on physical principles, it did imply a deeper meaning. Balmer first devised the formula for his series alone, and it was later found to describe all the other series by using different values of nfnf. Bohr was the first to comprehend the deeper meaning. Again, we see the interplay between experiment and theory in physics. Experimentally, the spectra were well established, an equation was found to fit the experimental data, but the theoretical foundation was missing.

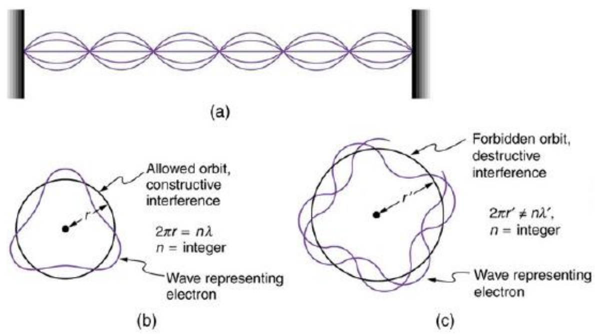
de Broglie and the Bohr Model

By assuming that the electron is described by a wave and a whole number of wavelengths must fit, we derive Bohr's quantization assumption.

learning objectives

- Describe reinterpretation of Bohr's condition by de Broglie

Bohr's condition, that the angular momentum is an integer multiple of ℏℏ, was later reinterpreted in 1924 by de Broglie as a standing wave condition. The wave-like properties of matter were subsequently confirmed by observations of electron interference when scattered from crystals. Electrons can exist only in locations where they interfere constructively. How does this affect electrons in atomic orbits? When an electron is bound to an atom, its wavelength must fit into a small space, something like a standing wave on a string.



Waves on a String: (a) Waves on a string have a wavelength related to the length of the string, allowing them to interfere constructively. (b) If we imagine the string bent into a closed circle, we get a rough idea of how electrons in circular orbits can interfere constructively. (c) If the wavelength does not fit into the circumference, the electron interferes destructively; it cannot exist in such an orbit.

Allowed orbits are those in which an electron constructively interferes with itself. Not all orbits produce constructive interference and thus only certain orbits are allowed (i.e., the orbits are quantized). By assuming that the electron is described by a wave and a whole number of wavelengths must fit along the circumference of the electron's orbit, we have the equation:

$$\mathrm{n}\lambda = 2\pi \mathrm{r} \tag{29.2.13}$$

Substituting de Broglie's wavelength of hphp reproduces Bohr's rule. Since λ=h/mevλ=h/mev, we now have:

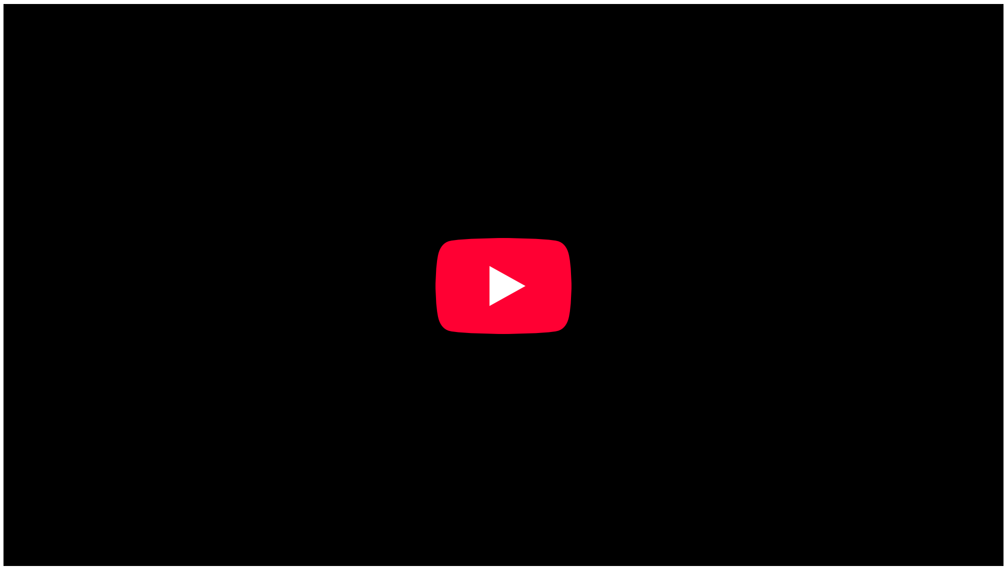
$$\frac{\mathrm{nh}}{\mathrm{m_{ev}}} = 2\pi \mathrm{r_n} \tag{29.2.14}$$

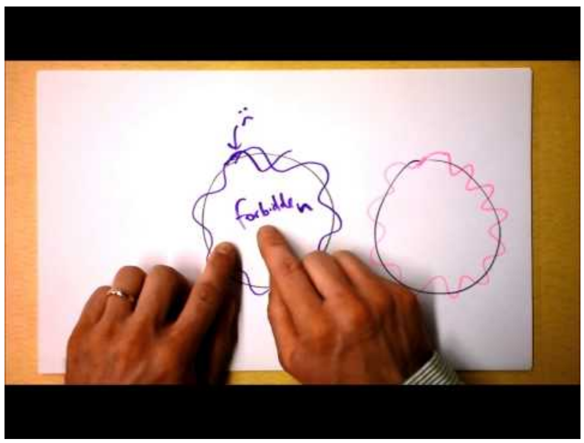
Rearranging terms, and noting that L=mvrL=mvr for a circular orbit, we obtain the quantization of angular momentum as the condition for allowed orbits:

$$\mathrm{L} = \mathrm{m_e v r_n} = \mathrm{n}\frac{\mathrm{h}}{2\pi}, (\mathrm{n} = 1, 2, 3\ldots) \tag{29.2.15}$$

As previously stated, Bohr was forced to hypothesize this rule for allowed orbits. We now realize this as the condition for constructive interference of an electron in a circular orbit.

Accordingly, a new kind of mechanics, quantum mechanics, was proposed in 1925. Bohr's model of electrons traveling in quantized orbits was extended into a more accurate model of electron motion. The new theory was proposed by Werner Heisenberg. By different reasoning, another form of the same theory, wave mechanics, was discovered independently by Austrian physicist Erwin Schrödinger. Schrödinger employed de Broglie's matter waves, but instead sought wave solutions of a three-dimensional wave equation. This described electrons that were constrained to move about the nucleus of a hydrogen-like atom by being trapped by the potential of the positive nuclear charge.





de Broglie’s Matter Waves Justify Bohr’s Magic Electron Orbital Radii: I include a summary of the hydrogen atom’s electronic structure and explain how an electron can interfere with itself in an orbit just like it can in a double-slit experiment.

X-Rays and the Compton Effect

Compton explained the X-ray frequency shift during the X-ray/electron scattering by attributing particle-like momentum to “photons”.

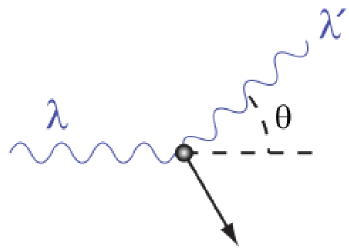
> **learning objectives**
>
> - Describe Compton effects between electrons and x-ray photons

By the early 20th century, research into the interaction of X-rays with matter was well underway. It was observed that when X-rays of a known wavelength interact with atoms, the X-rays are scattered through an angle θθ and emerge at a different wavelength related to θθ. Although classical electromagnetism predicted that the wavelength of scattered rays should be equal to the initial wavelength, multiple experiments had found that the wavelength of the scattered rays was longer (corresponding to lower energy) than the initial wavelength.

In 1923, Compton published a paper in the Physical Review which explained the X-ray shift by attributing particle-like momentum to “photons,” which Einstein had invoked in his Nobel prize winning explanation of the photoelectric effect. First postulated by Planck, these “particles” conceptualized “quantized” elements of light as containing a specific amount of energy depending only on the frequency of the light. In his paper, Compton derived the mathematical relationship between the shift in wavelength and the scattering angle of the X-rays by assuming that each scattered X-ray photon interacted with only one electron. His paper concludes by reporting on experiments which verified his derived relation:

$$\lambda' - \lambda = \frac{\mathrm{h}}{\mathrm{m_e c}}(1 - \cos\theta) \tag{29.2.16}$$

where λ\lambda is the initial wavelength, λ′\lambda’ is the wavelength after scattering, h is the Planck constant, $\mathrm{m_e}$ is the Electron rest mass, c is the speed of light, and θ\theta is the scattering angle. The quantity $\frac{\mathrm{h}}{\mathrm{m_e c}}$ is known as the Compton wavelength of the electron; it is equal to $2.43 \cdot 10^{-12}$ m. The wavelength shift $\lambda‘ - \lambda$ is at least zero (for $\theta = 0^\circ$) and at most twice the Compton wavelength of the electron (for $\theta = 0^\circ$). (The derivation of Compton’s formula is a bit lengthy and will not be covered here.)



A Photon Colliding with a Target at Rest: A photon of wavelength λλ comes in from the left, collides with a target at rest, and a new photon of wavelength λ‘λ‘ emerges at an angle θθ.

Because the mass-energy and momentum of a system must both be conserved, it is not generally possible for the electron simply to move in the direction of the incident photon. The interaction between electrons and high energy photons (comparable to the rest energy of the electron, 511 keV) results in the electron being given part of the energy (making it recoil), and a photon containing the remaining energy being emitted in a different direction from the original, so that the overall momentum of the system is conserved. If the scattered photon still has enough energy left, the Compton scattering process may be repeated. In this scenario, the electron is treated as free or loosely bound. Photons with an energy of this order of magnitude are in the x-ray range of the electromagnetic radiation spectrum. Therefore, you can say that Compton effects (with electrons) occur with x-ray photons.

If the photon is of lower energy, but still has sufficient energy (in general a few eV to a few keV, corresponding to visible light through soft X-rays), it can eject an electron from its host atom entirely (a process known as the photoelectric effect), instead of undergoing Compton scattering. Higher energy photons (1.022 MeV and above, in the gamma ray range) may be able to bombard the nucleus and cause an electron and a positron to be formed, a process called pair production.

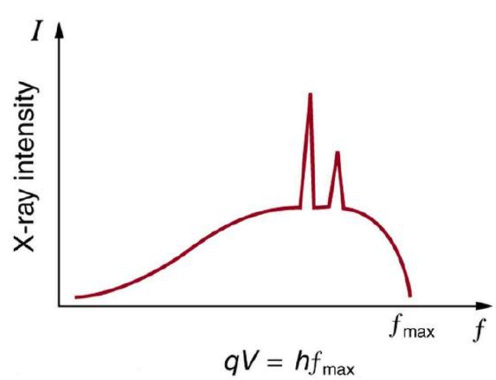
X-Ray Spectra: Origins, Diffraction by Crystals, and Importance

X-ray shows its wave nature when radiated upon atomic/molecular structures and can be used to study them.

learning objectives

- Describe interactions between X-rays and atoms

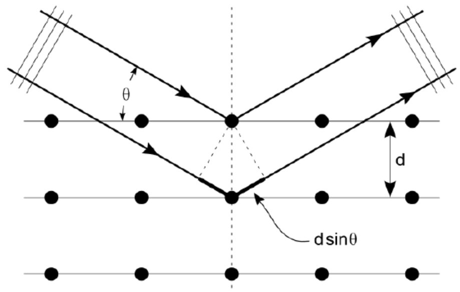
In a previous Atom on X-rays, we have seen that there are two processes by which x-rays are produced in the anode of an x-ray tube. In one process, the deceleration of electrons produces x-rays, and these x-rays are called *Bremsstrahlung*, or braking radiation. The second process is atomic in nature and produces characteristic x-rays, so called because they are characteristic of the anode material. The x-ray spectrum in is typical of what is produced by an x-ray tube, showing a broad curve of Bremsstrahlung radiation with characteristic x-ray peaks on it.



X-Ray Spectrum: X-ray spectrum obtained when energetic electrons strike a material, such as in the anode of a CRT. The smooth part of the spectrum is bremsstrahlung radiation, while the peaks are characteristic of the anode material. A different anode material would have characteristic x-ray peaks at different frequencies.

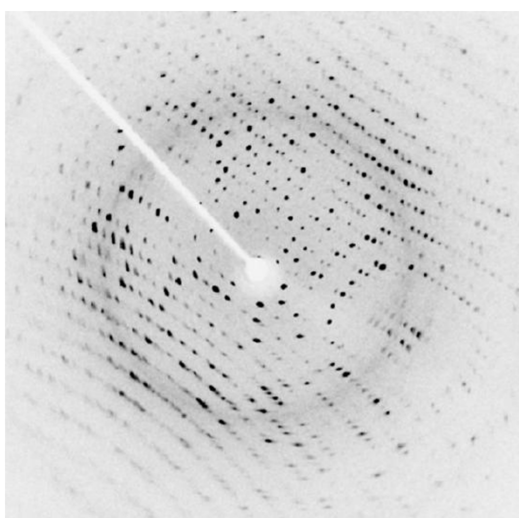
Since x-ray photons are very energetic, they have relatively short wavelengths. For example, the 54.4-keV Kα x-ray, for example, has a wavelength $\lambda = \frac{\mathrm{hc}}{\mathrm{E}} = 0.0228$ nm. Thus, typical x-ray photons act like rays when they encounter macroscopic objects, like teeth, and produce sharp shadows. However, since atoms and atomic structures have a typical size on the order of 0.1 nm, x-ray shows its wave nature with them. The process is called x-ray diffraction because it involves the diffraction and interference of x-rays to produce patterns that can be analyzed for information about the structures that scattered the x-rays.

Shown below, Bragg's Law gives the angles for coherent and incoherent scattering of light from a crystal lattice, which happens during x-ray diffraction. When x-ray are incident on an atom, they make the electronic cloud move as an electromagnetic wave. The movement of these charges re-radiate waves with the same frequency. This is called Rayleigh Scattering, which you should remember from a previous atom. A similar thing happens when neutron waves from the nuclei scatter from interaction with an unpaired electron. These re-emitted wave fields interfere with each other either constructively or destructively, and produce a diffraction pattern that is captured by a sensor or film. This is called the Braggs diffraction, and is the basis for x-ray diffraction.



X-Ray Diffraction: Bragg's Law of diffraction: illustration of how x-rays interact with crystal lattice.

Perhaps the most famous example of x-ray diffraction is the discovery of the double-helix structure of DNA in 1953. Using x-ray diffraction data, researchers were able to discern the structure of DNA shows a diffraction pattern produced by the scattering of x-rays from a crystal of protein. This process is known as x-ray crystallography because of the information it can yield about crystal structure. Not only do x-rays confirm the size and shape of atoms, they also give information on the atomic arrangements in materials. For example, current research in high-temperature superconductors involves complex materials whose lattice arrangements are crucial to obtaining a superconducting material. These can be studied using x-ray crystallography.



X-Ray Diffraction: X-ray diffraction from the crystal of a protein, hen egg lysozyme, produced this interference pattern. Analysis of the pattern yields information about the structure of the protein.

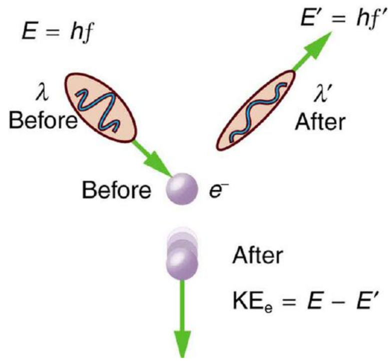
The Compton Effect

The Compton Effect is the phenomenon of the decrease in energy of photon when scattered by a free charged particle.

learning objectives

- Explain why Compton scattering is an inelastic scattering.

Compton scattering is an inelastic scattering of a photon by a free charged particle (usually an electron). It results in a decrease in energy (increase in wavelength) of the photon (which may be an X-ray or gamma ray photon), called the Compton Effect. Part of the energy of the photon is transferred to the scattering electron. Inverse Compton scattering also exists, and happens when a charged particle transfers part of its energy to a photon.



Scattering in the Compton Effect: The Compton Effect is the name given to the scattering of a photon by an electron. Energy and momentum are conserved, resulting in a reduction of both for the scattered photon. Studying this effect, Compton verified that photons have momentum.

Compton scattering is an example of inelastic scattering because the wavelength of the scattered light is different from the incident radiation. Still, the origin of the effect can be considered as an elastic collision between a photon and an electron. The amount of change in the wavelength is called the Compton shift. Although nuclear Compton scattering exists, Compton scattering usually refers to the interaction involving only the electrons of an atom.

The Compton effect is important because it demonstrates that light cannot be explained purely as a wave phenomenon. Thomson scattering, the classical theory of an electromagnetic wave scattered by charged particles, cannot explain low intensity shifts in wavelength: classically, light of sufficient intensity for the electric field to accelerate a charged particle to a relativistic speed will cause radiation-pressure recoil and an associated Doppler shift of the scattered light. However, the effect will become arbitrarily small at sufficiently low light intensities regardless of wavelength. Light must behave as if it consists of particles to explain the low-intensity Compton scattering. Compton's experiment convinced physicists that light can behave as a stream of particle-like objects (quanta) whose energy is proportional to the frequency.

Key Points

- The British physicist J. J. Thomson performed experiments studying cathode rays and discovered that they were unique particles, later named electrons.
- Rutherford proved that the hydrogen nucleus is present in other nuclei.
- In 1932, James Chadwick showed that there were uncharged particles in the radiation he was using. These particles, later called neutrons, had a similar mass of the protons but did not have the same characteristics as protons.
- The atom is a basic unit of matter that consists of a dense central nucleus surrounded by a cloud of negatively charged electrons.
- Scattered knowledge discovered by alchemists over the Middle Ages contributed to the discovery of atoms.
- Dalton established his atomic theory based on the fact that the masses of reactants in specific chemical reactions always have a particular mass ratio.
- J. J. Thomson, who discovered the electron in 1897, proposed the plum pudding model of the atom in 1904 before the discovery of the atomic nucleus in order to include the electron in the atomic model.
- In Thomson's model, the atom is composed of electrons surrounded by a soup of positive charge to balance the electrons' negative charges, like negatively charged "plums" surrounded by positively charged "pudding".
- The 1904 Thomson model was disproved by Hans Geiger's and Ernest Marsden's 1909 gold foil experiment.

- Rutherford overturned Thomson’s model in 1911 with his well-known gold foil experiment, in which he demonstrated that the atom has a tiny, high- mass nucleus.
- In his experiment, Rutherford observed that many alpha particles were deflected at small angles while others were reflected back to the alpha source.
- This highly concentrated, positively charged region is named the “nucleus” of the atom.
- According to Bohr: 1) Electrons in atoms orbit the nucleus, 2) The electrons can only orbit stably, without radiating, in certain orbits, and 3) Electrons can only gain and lose energy by jumping from one allowed orbit to another.
- The significance of the Bohr model is that the laws of classical mechanics apply to the motion of the electron about the nucleus only when restricted by a quantum rule. Therefore, his atomic model is called a semiclassical model.
- The laws of classical mechanics predict that the electron should release electromagnetic radiation while orbiting a nucleus, suggesting that all atoms should be unstable!
- Classical electrodynamics predicts that an atom described by a (classical) planetary model would be unstable.
- To explain the hydrogen spectrum, Bohr had to make a few assumptions that electrons could only have certain classical motions.
- After the seminal work by Planck, Einstein, and Bohr, physicists began to realize that it was essential to introduce the notion of ” quantization ” to explain microscopic worlds.
- The “Bohr orbits” have a very important feature of quantization: that the angular momentum L of an electron in its orbit is quantized, that is, it has only specific, discrete values. This leads to the equation $\mathrm{L}=\mathrm{m_e v r_n}=\mathrm{n}\frac{\mathrm{h}}{2\pi}=\mathrm{n}\hbar$.
- At the time of proposal, Bohr himself did not know why angular momentum should be quantized, but using this assumption he was able to calculate the energies in the hydrogen spectrum.
- A theory of the atom or any other system must predict its energies based on the physics of the system, which the Bohr model was able to do.
- According to Bohr, allowed orbit radius at any n is $\mathrm{r_n}=\frac{\mathrm{n}^2\hbar^2}{Z\mathrm{k}_e\mathrm{e}^2\mathrm{m}_e}$. The smallest possible value of rr in the hydrogen atom is called the Bohr radius and is equal to 0.053 nm.
- The energy of the nn-th level for any atom is $\mathrm{E}=\approx\frac{-13.6\mathrm{Z}^2}{\mathrm{n}^2}\mathrm{eV}$.
- The energy of a photon emitted by a hydrogen atom is given by the difference of two hydrogen energy levels: $\mathrm{E}=\mathrm{E_i}-\mathrm{E_f}=\mathrm{R}\left(\frac{1}{\mathrm{n_f^2}}-\frac{1}{\mathrm{n_i^2}}\right)$, which is known as Rydberg formula.
- Atomic and molecular emission and absorption spectra have been known for over a century to be discrete (or quantized).
- Lyman, Balmer, and Paschen series are named after early researchers who studied them in particular depth.
- Bohr was the first one to provide a theoretical explanation of the hydrogen spectra.
- Bohr’s condition, that the angular momentum is an integer multiple of $\hbar\hbar$, was later reinterpreted in 1924 by de Broglie as a standing wave condition.
- For what Bohr was forced to hypothesize as the rule for allowed orbits, de Broglie’s matter wave concept explains it as the condition for constructive interference of an electron in a circular orbit.
- Bohr’s model was only applicable to hydrogen-like atoms. In 1925, more general forms of description (now called quantum mechanics) emerged, thanks to Heisenberg and Schrodinger.
- Compton derived the mathematical relationship between the shift in wavelength and the scattering angle of the X-rays.
- Compton effects (with electrons) usually occur with x-ray photons.
- If the photon is of lower energy, in the visible light through soft X-rays range, photoelectric effects are observed. Higher energy photons, in the gamma ray range, may lead to pair production.
- X rays are relatively high- frequency EM radiation. They are produced by transitions between inner-shell electron levels, which produce x rays characteristic of the atomic element, or by accelerating electrons.
- x-ray diffraction is a technique that provides the detailed information about crystallographic structure of natural and manufactured materials.
- Current research in material science and physics involves complex materials whose lattice arrangements are crucial to obtaining a superconducting material, which can be studied using x-ray crystallography.

Key Terms

- **scintillation**: A flash of light produced in a transparent material by the passage of a particle.
- **alpha particle**: A positively charged nucleus of a helium-4 atom (consisting of two protons and two neutrons), emitted as a consequence of radioactivity.

- **cathode**: An electrode through which electric current flows out of a polarized electrical device.
- **electromagnetic force**: a long-range fundamental force that acts between charged bodies, mediated by the exchange of photons
- **Avogadro's number**: the number of constituent particles (usually atoms or molecules) in one mole of a given substance. It has dimensions of reciprocal mol and its value is equal to $6.02214129 \cdot 10^{23}\ \mathrm{mol}^{-1}$
- **nucleus**: the massive, positively charged central part of an atom, made up of protons and neutrons
- **Maxwell's equations**: A set of equations describing how electric and magnetic fields are generated and altered by each other and by charges and currents.
- **semiclassical**: a theory in which one part of a system is described quantum-mechanically whereas the other is treated classically.
- **black body**: An idealized physical body that absorbs all incident electromagnetic radiation, regardless of frequency or angle of incidence. Although black body is a theoretical concept, you can find approximate realizations of black body in nature.
- **photoelectric effect**: The occurrence of electrons being emitted from matter (metals and non-metallic solids, liquids, or gases) as a consequence of their absorption of energy from electromagnetic radiation.
- **quantization**: The process of explaining a classical understanding of physical phenomena in terms of a newer understanding known as quantum mechanics.
- **centripetal**: Directed or moving towards a center.
- **photon**: The quantum of light and other electromagnetic energy, regarded as a discrete particle having zero rest mass, no electric charge, and an indefinitely long lifetime.
- **spectrum**: A condition that is not limited to a specific set of values but can vary infinitely within a continuum. The word saw its first scientific use within the field of optics to describe the rainbow of colors in visible light when separated using a prism.
- **standing wave**: A wave form which occurs in a limited, fixed medium in such a way that the reflected wave coincides with the produced wave. A common example is the vibration of the strings on a musical stringed instrument.
- **matter wave**: A concept reflects the wave-particle duality of matter. The theory was proposed by Louis de Broglie.
- **gamma ray**: A very high frequency (and therefore very high energy) electromagnetic radiation emitted as a consequence of radioactivity.
- **photoelectric effects**: In photoelectric effects, electrons are emitted from matter (metals and non-metallic solids, liquids or gases) as a consequence of their absorption of energy from electromagnetic radiation.

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29.3: Atomic Physics and Quantum Mechanics

learning objectives

- Explain relationship between the wave nature of matter and the quantization of energy levels in bound systems

To consider why wave nature of matter in bound systems leads to quantization, let's consider an example in classical mechanics. We will look at a basic string "instrument" (a string pulled tight and fixed at both ends). If a string was free and not attached to anything, we know that it could oscillate at any driven frequency. However, the string in this example (with fixed ends and specific length) can only produce a very specific set of pitches because only waves of a certain wavelength can "fit" on the string of a given length with fixed ends. Once the string becomes a "bound system" with specific boundary restrictions, it allows waves with only a discrete set of frequencies.

This is the exact mechanism that causes quantization in atoms. The wave nature of matter is responsible for the quantization of energy levels in bound systems. Just like a free string, the matter wave of a free electron can have any wavelength, determined by its momentum. However, once an electron is "bound" by a Coulomb potential of a nucleus, it can no longer have an arbitrary wavelength as the wave needs to satisfy a certain boundary condition. Only those states where matter interferes constructively (leading to standing waves) exist, or are "allowed" (see illustration in.

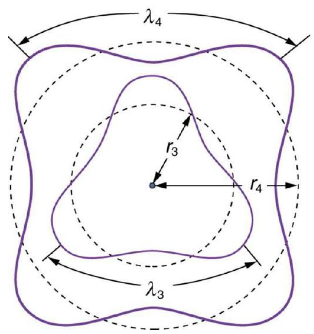


Fig 2: The third and fourth allowed circular orbits have three and four wavelengths, respectively, in their circumferences.

Assuming that an integral multiple of the electron's wavelength equals the circumference of the orbit, we have:

$$\mathrm{n}\lambda_{\mathrm{n}} = 2\pi \mathrm{r_n} (\mathrm{n} = 1, 2, 3, \ldots) \tag{29.3.1}$$

Substituting $\lambda = \frac{\mathrm{h}}{\mathrm{m_e v}}$, this becomes:

$$\frac{\mathrm{nh}}{\mathrm{m_{ev}}} = 2\pi \mathrm{r_n} \tag{29.3.2}$$

The angular momentum is $\mathrm{L} = \mathrm{m_e vr}$, therefore we obtain the quantization of angular momentum:

$$\mathrm{L} = \mathrm{m_e v r_n} = \mathrm{n}\frac{\mathrm{h}}{2\pi} (\mathrm{n} = 1, 2, 3, \ldots) \tag{29.3.3}$$

As previously discussed, Bohr was forced to hypothesize this as the rule for allowed orbits. We now realize this as a condition for constructive interference of an electron in a (bound) circular orbit.

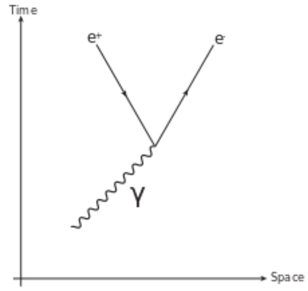
Photon Interactions and Pair Production

Pair production refers to the creation of an elementary particle and its antiparticle, usually when a photon interacts with a nucleus.

learning objectives

- Describe process of pair production as the result of photon interaction with nucleus

Below is an illustration of pair production, which refers to the creation of an elementary particle and its antiparticle, usually when a photon interacts with a nucleus. For example, an electron and its antiparticle, the positron, may be created. This is allowed, provided there is enough energy available to create the pair (i.e., the total rest mass energy of the two particles) and that the situation allows both energy and momentum to be conserved. Some other conserved quantum numbers such as angular momentum, electric charge, etc., must sum to zero as well. The probability of pair production in photon-matter interactions increases with increasing photon energy, and also increases with atomic number (Z) of the nucleus approximately as Z^2.



Pair Production: Feynman diagram for pair production. A photon decays into an electron-positron pair.

$\gamma + \gamma \rightarrow \mathbf{e}^- + \mathbf{e}^+$

In nuclear physics, this reaction occurs when a high-energy photon (gamma rays) interacts with a nucleus. The energy of this photon can be converted into mass through Einstein's equation $E = mc^2$ where E is energy, mm is mass and c is the speed of light. The photon must have enough energy to create the mass of an electron plus a positron. The mass of an electron is $9.11 \cdot 10^{-31}$ kg (equivalent to 0.511 MeV in energy), the same as a positron.

Without a nucleus to absorb momentum, a photon decaying into electron-positron pair (or other pairs for that matter) can never conserve energy and momentum simultaneously. The nucleus in the process carries away (or provides) access momentum.

The reverse process is also possible. The electron and positron can annihilate and produce two 0.511 MeV gamma photons. If all three gamma rays, the original with its energy reduced by 1.022 MeV and the two annihilation gamma rays, are detected simultaneously, then a full energy peak is observed.

These interactions were first observed in Patrick Blackett's counter-controlled cloud chamber, leading him to receive the 1948 Nobel Prize in Physics.

Key Points

- Strings in musical instruments (guitar, for example) can only produce a very specific set of pitches because only waves of a certain wavelength can "fit" on the string of a given length with fixed ends.
- Similarly, once an electron is bound by a Coulomb potential of a nucleus, it no longer can have any arbitrary wavelength because the wave should satisfy a certain boundary condition.
- Bohr's quantization assumption can be derived from the condition for constructive intereference of an electron matter wave in a circular orbit.
- The probability of pair production in photon -matter interactions increases with increasing photon energy, and also increases with atomic number of the nucleus approximately as Z.
- Energy and momentum should be conserved through the pair production process. Some other conserved quantum numbers such as angular momentum, electric charge, etc., must sum to zero as well.
- Nucleus is needed in the pair production of electron and positron to satisfy the energy and momentum conservation laws.

Key Terms

- **quantization**: The process of explaining a classical understanding of physical phenomena in terms of a newer understanding known as quantum mechanics.
- **angular momentum**: A vector quantity describing an object in circular motion; its magnitude is equal to the momentum of the particle, and the direction is perpendicular to the plane of its circular motion.
- **matter wave**: A concept reflects the wave-particle duality of matter. The theory was proposed by Louis de Broglie.
- **gamma ray**: A very high frequency (and therefore very high energy) electromagnetic radiation emitted as a consequence of radioactivity.
- **positron**: The antimatter equivalent of an electron, having the same mass but a positive charge.

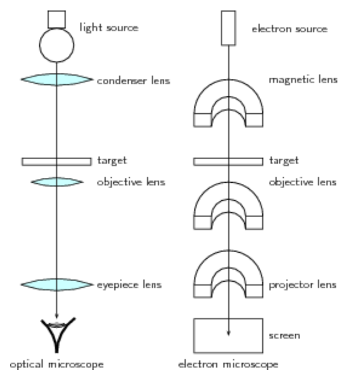
LICENSES AND ATTRIBUTIONS

29.4: Applications of Atomic Physics

We have seen that under certain circumstances particles behave like waves. This idea is used in the electron microscope which is a type that uses electrons to create an image of the target. It has much higher magnification or resolving power than a normal light microscope. It can achieve better than 50 pm resolution and magnifications of up to about 10,000,000 times, whereas ordinary, nonconfocal light microscopes are limited by diffraction to about 200 nm resolution and useful magnifications below 2000 times.

Electron Microscope Image: An image of an ant in a scanning electron microscope.

Let's first review how a regular optical microscope works. A beam of light is shone through a thin target and the image is then magnified and focused using objective and ocular lenses. The amount of light which passes through the target depends on its densities, since the less dense regions allow more light to pass through than the denser regions. This means that the beam of light which is partially transmitted through the target carries information about the inner structure of the target.



Optical and Electron Microscopes: Diagram of the basic components of an optical microscope and an electron microscope.

The original form of electron microscopy, transmission electron microscopy, works in a similar manner using electrons. In the electron microscope, electrons which are emitted by a cathode are formed into a beam using magnetic lenses (usually electromagnets). This electron beam is then passed through a very thin target. Again, the regions in the target with higher densities stop the electrons more easily. So, the number of electrons which pass through the different regions of the target depends on their densities. This means that the partially transmitted beam of electrons carries information about the densities of the inner structure of the target.

The spatial variation in this information (the "image") is then magnified by a series of magnetic lenses and it is recorded by hitting a fluorescent screen, photographic plate, or light-sensitive sensor such as a CCD (charge-coupled device) camera. The image detected by the CCD may be displayed in real time on a monitor or computer.

Electron microscopes are very useful as they are able to magnify objects to a much higher resolution. This is because their de Broglie wavelengths are so much smaller than that of visible light. You hopefully remember that light is diffracted by objects which are separated by a distance of about the same size as the wavelength of the light. This diffraction then prevents you from being able to focus the transmitted light into an image.

Therefore, the sizes at which diffraction occurs for a beam of electrons is much smaller than those for visible light. This is why you can magnify targets to a much higher order of magnification using electrons rather than visible light.

Lasers

A laser consists of a gain medium, a mechanism to supply energy to it, and something to provide optical feedback.

learning objectives

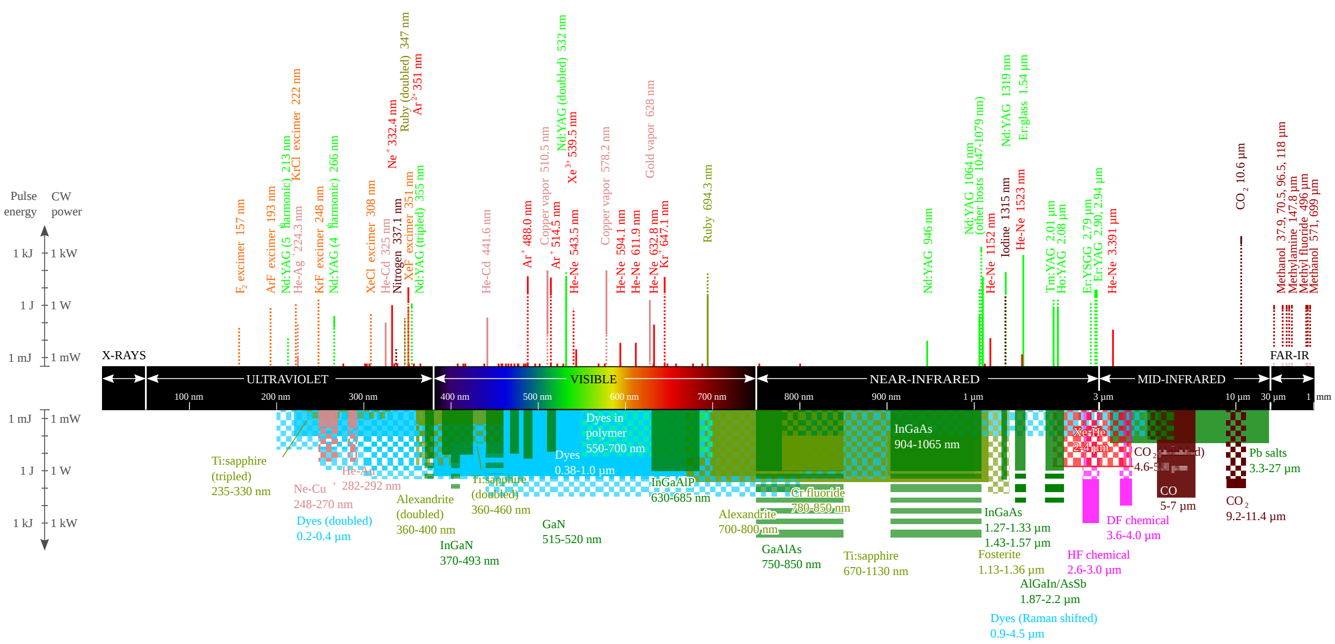
- Describe basic parts of laser

When lasers were invented in 1960, they were called “a solution looking for a problem. ” Nowadays, lasers are ubiquitous, finding utility in thousands of highly varied applications in every section of modern society, including consumer electronics, information technology, science, medicine, industry, law enforcement, entertainment, and the military.

Having examined stimulated emission and optical amplification process in the “Lasers, Applications of Quantum Mechanics” section, this atom looks at how lasers are built.

A laser consists of a gain medium, a mechanism to supply energy to it, and something to provide optical feedback (usually an optical cavity). When a gain medium is placed in an optical cavity, a laser can then produce a coherent beam of photons.

The gain medium is where the optical amplification process occurs. It is excited by an external source of energy into an excited state (called “population inversion”), ready to be fired when a photon with the right frequency enters the medium. In most lasers, this medium consists of a population of atoms which have been excited by an outside light source or an electrical field which supplies energy for atoms to absorb in order to be transformed into excited states. There are many types of lasers depending on the gain media and mode of operation. Gas and semiconductors are commonly used gain media.



Wavelengths of Commercially Available Lasers: Laser types with distinct laser lines are shown above the wavelength bar, while below are shown lasers that can emit in a wavelength range. The height of the lines and bars gives an indication of the maximal power/pulse energy commercially available, while the color codifies the type of laser material.

The most common type of laser uses feedback from an optical cavity–a pair of highly reflective mirrors on either end of the gain medium. A single photon can bounce back and forth between the mirrors many times, passing through the gain medium and being amplified each time. Typically one of the two mirrors, the output coupler, is partially transparent. Some of the light escapes through this mirror, producing a laser beam that is visible to the naked eye.

Key Points

- Electron microscopes are very useful as they are able to magnify objects to a much higher resolution than optical ones.
- Higher resolution can be achieved with electron microscopes because the de Broglie wavelengths for electrons are so much smaller than that of visible light.

- In electron microscopes, electromagnets can be used as magnetic lenses to manipulate electron beams.
- The gain medium is where the optical amplification process occurs. Gas and semiconductors are commonly used gain media.
- The most common type of laser uses feedback from an optical cavity–a pair of highly reflective mirrors on either end of the gain medium. A single photon can bounce back and forth between the mirrors many times, passing through the gain medium and being amplified each time.
- Lasers are ubiquitous, finding utility in thousands of highly varied applications in every section of modern society.

Key Terms

- **CCD**: A charge-coupled device (CCD) is a device for the movement of electrical charge, usually from within the device to an area where the charge can be manipulated, for example conversion into a digital value. The CCD is a major technology required for digital imaging.
- **de Broglie wavelength**: The wavelength of a matter wave is inversely proportional to the momentum of a particle and is called a de Broglie wavelength.
- **stimulated emission**: The process by which an atomic electron (or an excited molecular state) interacting with an electromagnetic wave of a certain frequency may drop to a lower energy level, transferring its energy to that field.

29.5: Multielectron Atoms

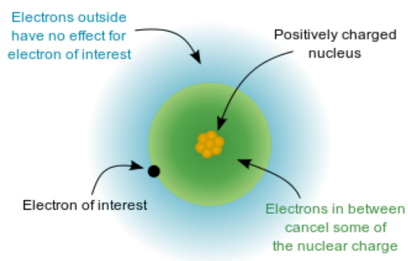
learning objectives

- Describe atomic structure and shielding in multielectron atoms

Multielectron Atoms

Atoms with more than one electron, such as Helium (He) and Nitrogen (N), are referred to as multielectron atoms. Hydrogen is the only atom in the periodic table that has one electron in the orbitals under ground state.

In hydrogen-like atoms (those with only one electron), the net force on the electron is just as large as the electric attraction from the nucleus. However, when more electrons are involved, each electron (in the nn-shell) feels not only the electromagnetic attraction from the positive nucleus, but also repulsion forces from other electrons in shells from '1' to 'nn'. This causes the net force on electrons in the outer electron shells to be significantly smaller in magnitude. Therefore, these electrons are not as strongly bonded to the nucleus as electrons closer to the nucleus. This phenomenon is often referred to as the Orbital Penetration Effect. The shielding theory also explains why valence shell electrons are more easily removed from the atom.



Electron Shielding Effect: A multielectron atom with inner electrons shielding outside electrons from the positively charged nucleus

The size of the shielding effect is difficult to calculate precisely due to effects from quantum mechanics. As an approximation, the effective nuclear charge on each electron can be estimated by: $\mathrm{Z_{eff}} = \mathrm{Z} - \sigma \mathrm{Z}\ _{\text{eff}} = \mathrm{Z} – \sigma$, where Z is the number of protons in the nucleus and σ\sigma is the average number of electrons between the nucleus and the electron in question. σ\sigma can be found by using quantum chemistry and the Schrodinger equation or by using Slater's empirical formula.

For example, consider a sodium cation, a fluorine anion, and a neutral neon atom. Each has 10 electrons, and the number of nonvalence electrons is two (10 total electrons minus eight valence electrons), but the effective nuclear charge varies because each has a different number of protons:

$$\mathrm{Z_{eff}F^{-}} = 9 - 2 = 7+ \tag{29.5.1}$$

$$\mathrm{Z_{eff}Ne} = 10 - 2 = 8+ \tag{29.5.2}$$

$$\mathrm{Z_{eff}Na^{+}} = 11 - 2 = 9+ \tag{29.5.3}$$

As a consequence, the sodium cation has the largest effective nuclear charge and, therefore, the smallest atomic radius.

The Periodic Table

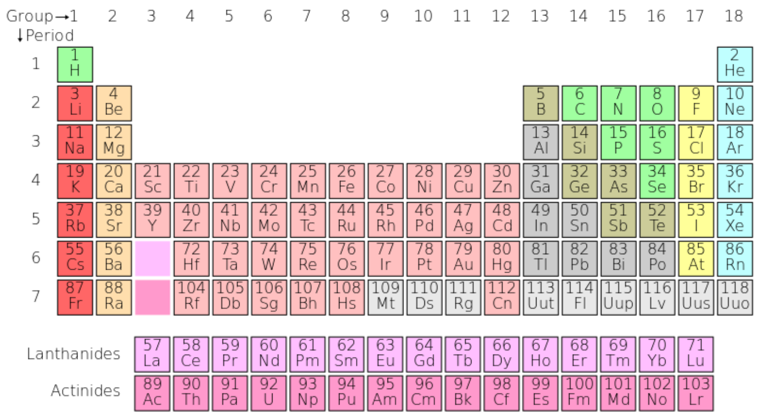
A periodic table is the arrangement of chemical elements according to their electron configurations and recurring chemical properties.

learning objectives

- Explain how elements are arranged in the Periodic Table.

A periodic table is a tabular display of the chemical elements, organized on the basis of their atomic numbers, electron configurations, and recurring chemical properties. Elements are presented according to their atomic numbers (number of protons) in increasing order. The standard form of the table comprises an eighteen by seven grid or main body of elements, positioned above a smaller double row of elements. The table can also be deconstructed into four rectangular blocks: the s-block to the left, the p-block to the right, the d-block in the middle, and the f-block below that. The rows of the table are called periods. The columns of the s-, d-, and p-blocks are called groups, some of which have names such as the halogens or the noble gases.

Since, by definition, a periodic table incorporates recurring trends, any such table can be used to derive relationships between the properties of the elements and predict the properties of new elements that are yet to be discovered or synthesized. As a result, a periodic table, in the standard form or some other variant, provides a useful framework for analyzing chemical behavior. Such tables are widely used in chemistry and other sciences.



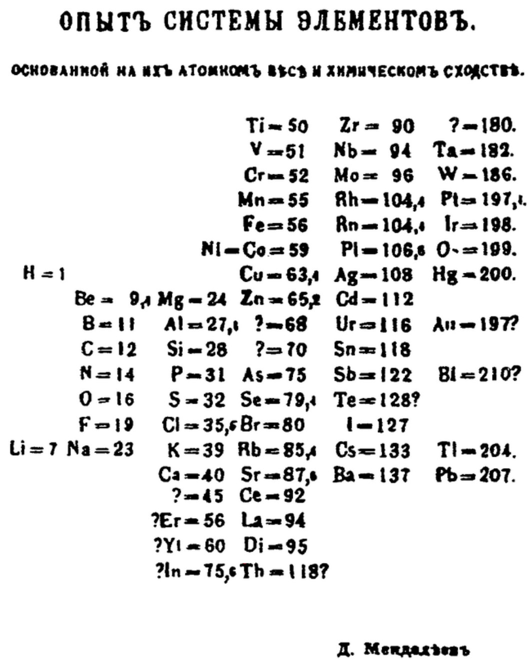
Periodic Table of Elements: The standard form of the periodic table, where the colors represent different categories of elements

The Specifics of the Periodic Table

All versions of the periodic table include only chemical elements, rather than mixtures, compounds, or subatomic particles. Each chemical element has a unique atomic number representing the number of protons in its nucleus. Most elements have differing numbers of neutrons among different atoms: these variants are referred to as isotopes. For example, carbon has three naturally occurring isotopes. All of its atoms have six protons and most have six neutrons as well, but about one percent have seven neutrons, and a very small fraction have eight neutrons. Isotopes are never separated in the periodic table. They are always grouped together under a single element. Elements with no stable isotopes have the atomic masses of their most stable isotopes listed in parentheses.

All elements from atomic numbers ‘1’ (hydrogen) to ‘118’ (ununoctium) have been discovered or synthesized. Of these, elements up through californium exist naturally; the rest have only been synthesized in laboratories. The production of elements beyond ununoctium is being pursued. The question of how the periodic table may need to be modified to accommodate any such additions is a matter of ongoing debate. Numerous synthetic radionuclides of naturally occurring elements have also been produced in laboratories.

Although precursors exist, Dmitri Mendeleev is generally credited with the publication of the first widely recognized periodic table in 1869. He developed his table to illustrate periodic trends in the properties of the elements known at the time. Mendeleev also predicted some properties of then-unknown elements that were expected to fill gaps in the table. Most of his predictions were proved correct when the elements in question were subsequently discovered. Mendeleev’s periodic table has since been expanded and refined with the discovery or synthesis of more new elements and the development of new theoretical models to explain chemical behavior.



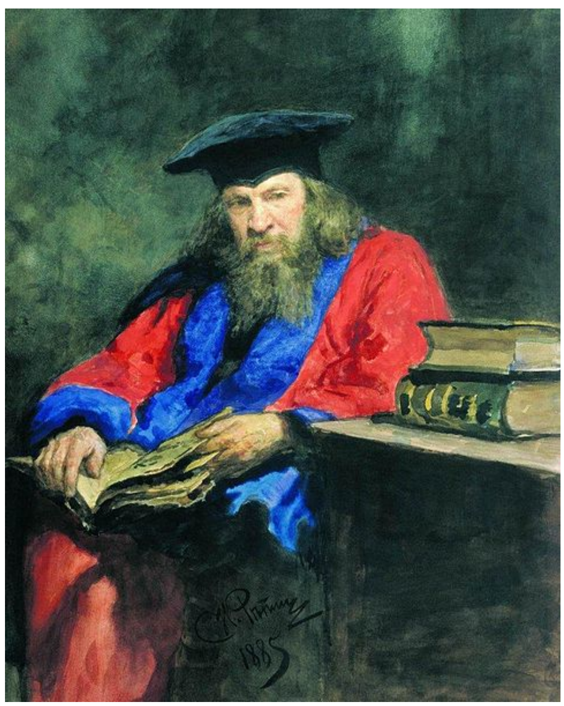
ОПЫТЪ СИСТЕМЫ ЭЛЕМЕНТОВЪ.

ОСНОВАННОЙ НА ИХЪ АТОМНОМЪ ВѢСѢ И ХИМИЧЕСКОМЪ СХОДСТВѢ.

			Ti=50	Zr=90	?=180.
			V=51	Nb=94	Ta=182.
			Cr=52	Mo=96	W=186.
			Mn=55	Rh=104,4	Pt=197,4.
			Fe=56	Rn=104,4	Ir=198.
			Ni=Co=59	Pl=106,6	Os=199.
H=1			Cu=63,4	Ag=108	Hg=200.
	Be=9,4	Mg=24	Zn=65,2	Cd=112	
	B=11	Al=27,4	?=68	Ur=116	Au=197?
	C=12	Si=28	?=70	Sn=118	
	N=14	P=31	As=75	Sb=122	Bi=210?
	O=16	S=32	Se=79,4	Te=128?	
	F=19	Cl=35,5	Br=80	I=127	
Li=7	Na=23	K=39	Rb=85,4	Cs=133	Tl=204.
		Ca=40	Sr=87,6	Ba=137	Pb=207.
		?=45	Ce=92		
		?Er=56	La=94		
		?Yt=60	Di=95		
		?In=75,6	Th=118?		

Д. Менделѣевъ

Mendeleev's 1869 Periodic Table: Mendeleev's 1869 periodic table presents the periods vertically and the groups horizontally.



Dmitri Mendeleev: Dmitri Mendeleev is known for publishing a widely recognized periodic table.

Electron Configurations

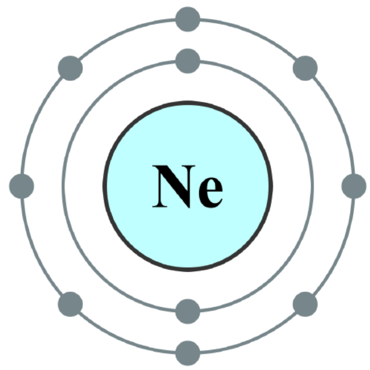
The electron configuration is the distribution of electrons of an atom or molecule in atomic or molecular orbitals.

learning objectives

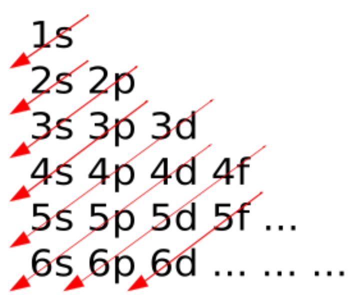
- Explain the meaning of electron configurations

The electron configuration is the distribution of electrons of an atom or molecule in atomic or molecular orbitals. Electron configurations describe electrons as each moving independently in an orbital, in an average field created by all other orbitals.

In atoms, electrons fill atomic orbitals according to the Aufbau principle (shown in), stated as: a maximum of two electrons are put into orbitals in the order of increasing orbital energy—the lowest-energy orbitals are filled before electrons are placed in higher-energy orbitals. As an example, the electron configuration of the neon atom is $1s^2$ $2s^2$ $2p^6$ or [He]$2s^2$ $2p^6$, as diagramed in. In molecules, the situation becomes more complex, as each molecule has a different orbital structure. The molecular orbitals are labelled according to their symmetry, rather than the atomic orbital labels used for atoms and monoatomic ions: hence, the electron configuration of the diatomic oxygen molecule, O_2, is $1\sigma_g^2$ $1\sigma_u^2$ $2\sigma_g^2$ $2\sigma_u^2$ $1\pi_u^4$ $3\sigma_g^2$ $1\pi_g2$.



Electron Configuration of Neon Atom: Electron configuration of neon atom showing only outer electron shell.



Aufbau Principle: In the Aufbau Principle, as electrons are added to atoms, they are added to the lowest orbitals first.

According to the laws of quantum mechanics, for systems with only one electron, an energy is associated with each electron configuration and, upon certain conditions, electrons are able to move from one configuration to another by emission or absorption of a quantum of energy, in the form of a photon.

For atoms or molecules with more than one electron, the motion of electrons are correlated and such picture is no longer exact. An infinite number of electronic configurations are needed to exactly describe any multi-electron system, and no energy can be associated with one single configuration. However, the electronic wave function is usually dominated by a very small number of configurations and therefore the notion of electronic configuration remains essential for multi-electron systems.

Electronic configuration of polyatomic molecules can change without absorption or emission of photon through vibronic couplings.

Knowledge of the electron configuration of different atoms is useful in understanding the structure of the periodic table of elements. The outermost electron shell is often referred to as the valence shell and (to a first approximation) determines the chemical properties. It should be remembered that the similarities in the chemical properties were remarked more than a century before the idea of electron configuration. The concept of electron configuration is also useful for describing the chemical bonds that hold atoms together. In bulk materials this same idea helps explain the peculiar properties of lasers and semiconductors.

Key Points

- Hydrogen is the only atom in the periodic table that has one electron in the orbitals under ground state.
- In multielectron atoms, the net force on electrons in the outer shells is reduced due to shielding.

- The effective nuclear charge on each electron can be approximated as: Zeff=Z−σZeff=Z−σ, where ZZ is the number of protons in the nucleus and σσ is the average number of electrons between the nucleus and the electron in question.
- A periodic table provides a useful framework for analyzing the chemical behavior of elements.
- A periodic table includes only chemical elements with each chemical element assigned a unique atomic number representing the number of protons in its nucleus.
- Dmitri Mendeleev is credited with the publication of the first widely recognized periodic table in 1869.
- Electrons fill atomic orbitals according to the Aufbau principle in atoms.
- For systems with only one electron, an energy is associated with each electron configuration and electrons are able to move from one configuration to another by emission or absorption of a quantum of energy, in the form of a photon.
- For atoms or molecules with more than one electron, an infinite number of electronic configurations are needed to exactly describe any multi-electron system, and no energy can be associated with one single configuration.

Key Terms

- **hydrogen-like**: having a single electron
- **electron shell**: The collective states of all electrons in an atom having the same principal quantum number (visualized as an orbit in which the electrons move).
- **valence shell**: the outermost shell of electrons in an atom; these electrons take part in bonding with other atoms
- **periodic table**: A tabular chart of the chemical elements according to their atomic numbers so that elements with similar properties are in the same column.
- **element**: Any one of the simplest chemical substances that cannot be decomposed in a chemical reaction or by any chemical means and made up of atoms all having the same number of protons.
- **atomic number**: The number, equal to the number of protons in an atom that determines its chemical properties. Symbol: Z
- **atomic orbital**: The quantum mechanical behavior of an electron in an atom describing the probability of the electron’s particular position and energy.

Effective_Nuclear_Charge.svg.png. **License**: *CC BY-SA: Attribution-ShareAlike*
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CHAPTER OVERVIEW

30: Nuclear Physics and Radioactivity

Topic hierarchy

30.1: The Nucleus

learning objectives

- Explain relationship between nuclear radius, nuclear density, and nuclear size.

Nuclear size is defined by nuclear radius, also called rms charge radius. It can be measured by the scattering of electrons by the nucleus and also inferred from the effects of finite nuclear size on electron energy levels as measured in atomic spectra.

The problem of defining a radius for the atomic nucleus is similar to the problem of atomic radius, in that neither atoms nor their nuclei have definite boundaries. However, the nucleus can be modelled as a sphere of positive charge for the interpretation of electron scattering experiments: because there is no definite boundary to the nucleus, the electrons "see" a range of cross-sections, for which a mean can be taken. The qualification of "rms" (for "root mean square") arises because it is the nuclear cross-section, proportional to the square of the radius, which is determining for electron scattering.

The first estimate of a nuclear charge radius was made by Hans Geiger and Ernest Marsden in 1909, under the direction of Ernest Rutherford at the Physical Laboratories of the University of Manchester, UK. The famous Rutherford gold foil experiment involved the scattering of α-particles by gold foil, with some of the particles being scattered through angles of more than 90°, that is coming back to the same side of the foil as the α-source, as shown in Figure 1. Rutherford was able to put an upper limit on the radius of the gold nucleus of 34 femtometers (fm).

Later studies found an empirical relation between the charge radius and the mass number, A, for heavier nuclei $(\mathrm{A}>20): \mathrm{R} \approx \mathrm{r} \cdot \mathrm{A}^{1/3}$ where r is an empirical constant of 1.2–1.5 fm. This gives a charge radius for the gold nucleus (A=197A=197) of about 7.5 fm.

Nuclear density is the density of the nucleus of an atom, averaging about $4 \cdot 10^{17} \mathrm{kg}/\mathrm{m}^3$. The nuclear density for a typical nucleus can be approximately calculated from the size of the nucleus:

$$\mathrm{n}=\frac{\mathrm{A}}{\frac{4}{3}\pi \mathrm{R}^3} \tag{30.1.1}$$

Nuclear Stability

The stability of an atom depends on the ratio and number of protons and neutrons, which may represent closed and filled quantum shells.

learning objectives

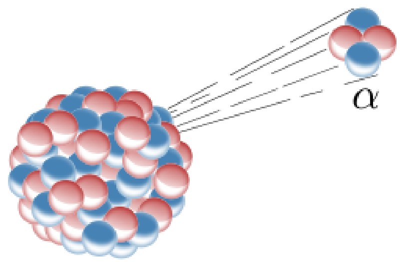
- Explain the relationship between the stability of an atom and its atomic structure.

The stability of an atom depends on the ratio of its protons to its neutrons, as well as on whether it contains a "magic number" of neutrons or protons that would represent closed and filled quantum shells. These quantum shells correspond to energy levels within the shell model of the nucleus. Filled shells, such as the filled shell of 50 protons in the element tin, confers unusual stability on the nuclide. Of the 254 known stable nuclides, only four have both an odd number of protons and an odd number of neutrons:

- hydrogen-2 (deuterium)
- lithium-6
- boron-10
- nitrogen-14

Also, only four naturally occurring, radioactive odd-odd nuclides have a half-life greater than a billion years:

- potassium-40
- vanadium-50
- lanthanum-138
- tantalum-180m



Alpha Decay: Alpha decay is one type of radioactive decay. An atomic nucleus emits an alpha particle and thereby transforms (“decays”) into an atom with a mass number smaller by four and an atomic number smaller by two. Many other types of decay are possible.

Most odd-odd nuclei are highly unstable with respect to beta decay because the decay products are even-even and therefore more strongly bound, due to nuclear pairing effects.

An atom with an unstable nucleus, called a radionuclide, is characterized by excess energy available either for a newly created radiation particle within the nucleus or via internal conversion. During this process, the radionuclide is said to undergo radioactive decay. Radioactive decay results in the emission of gamma rays and/or subatomic particles such as alpha or beta particles, as shown in. These emissions constitute ionizing radiation. Radionuclides occur naturally but can also be produced artificially.

All elements form a number of radionuclides, although the half-lives of many are so short that they are not observed in nature. Even the lightest element, hydrogen, has a well-known radioisotope: tritium. The heaviest elements (heavier than bismuth) exist only as radionuclides. For every chemical element, many radioisotopes that do not occur in nature (due to short half-lives or the lack of a natural production source) have been produced artificially.

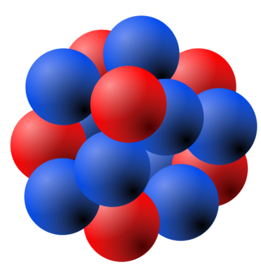
Binding Energy and Nuclear Forces

Nuclear force is the force that is responsible for binding of protons and neutrons into atomic nuclei.

learning objectives

- Explain how nuclear force varies with distance.

The nuclear force is the force between two or more component parts of an atomic nuclei. The component parts are neutrons and protons, which collectively are called nucleons. Nuclear force is responsible for the binding of protons and neutrons into atomic nuclei.



Drawing of Atomic Nucleus: A model of the atomic nucleus showing it as a compact bundle of the two types of nucleons: protons (red) and neutrons (blue).

To disassemble a nucleus into unbound protons and neutrons would require working against the nuclear force. Conversely, energy is released when a nucleus is created from free nucleons or other nuclei—known as the nuclear binding energy. The binding energy of nuclei is always a positive number, since all nuclei require net energy to separate into individual protons and neutrons. Because of mass-energy equivalence (i.e., Einstein’s famous formula $E = mc^2$), releasing this energy causes the mass of the nucleus to be lower than the total mass of the individual nucleons (leading to “mass deficit”). Binding energy is the energy used in nuclear power plants and nuclear weapons.

The nuclear force is powerfully attractive between nucleons at distances of about 1 femtometer (fm) between their centers, but rapidly decreases to relative insignificance at distances beyond about 2.5 fm. At very short distances (less than 0.7 fm) it becomes repulsive; it is responsible for the physical size of nuclei since the nucleons can come no closer than the force allows.

The nuclear force is now understood as a residual effect of an even more powerful "strong force" or strong interaction. It is the attractive force that binds together particles known as quarks (to form the nucleons themselves). This more powerful force is mediated by particles called gluons. Gluons hold quarks together with a force like that of an electric charge (but of far greater power).

The nuclear forces arising between nucleons are now seen as analogous to the forces in chemistry between neutral atoms or molecules (called London forces). Such forces between atoms are much weaker than the attractive electrical forces that hold together the atoms themselves (i.e., that bind electrons to the nucleus), and their range between atoms is shorter because they arise from a small separation of charges inside the neutral atom.

Similarly, even though nucleons are made of quarks in combinations which cancel most gluon forces (they are "color neutral"), some combinations of quarks and gluons leak away from nucleons in the form of short-range nuclear force fields that extend from one nucleon to another nucleon in close proximity. These nuclear forces are very weak compared to direct gluon forces ("color forces" or "strong forces") inside nucleons, and the nuclear forces extend over only a few nuclear diameters, falling exponentially with distance. Nevertheless, they are strong enough to bind neutrons and protons over short distances, as well as overcome the electrical repulsion between protons in the nucleus. Like London forces, nuclear forces also stop being attractive, and become repulsive when nucleons are brought too close together.

Key Points

- The first estimate of a nuclear charge radius was made by Hans Geiger and Ernest Marsden in 1909, under the direction of Ernest Rutherford, in the gold foil experiment that involved the scattering of α-particles by gold foil, as shown in Figure 1.
- An empirical relation exists between the charge radius and the mass number, A, for heavier nuclei ($\mathrm{A}>20$): where r is an empirical constant of 1.2–1.5 fm.
- The nuclear density for a typical nucleus can be approximately calculated from the size of the nucleus: $\mathrm{n}=\frac{\mathrm{A}}{\frac{4}{3}\pi\mathrm{R}^3}$
- Most odd-odd nuclei are highly unstable with respect to beta decay because the decay products are even-even and therefore more strongly bound, due to nuclear pairing effects.
- An atom with an unstable nucleus is characterized by excess energy available either for a newly created radiation particle within the nucleus or via internal conversion.
- All elements form a number of radionuclides, although the half-lives of many are so short that they are not observed in nature.
- The nuclear force is powerfully attractive at distances of about 1 femtometer (fm), rapidly decreases to insignificance at distances beyond about 2.5 fm, and becomes repulsive at very short distances less than 0.7 fm.
- The nuclear force is a residual effect of the a strong interaction that binds together particles called quarks into nucleons.
- The binding energy of nuclei is always a positive number while the mass of an atom 's nucleus is always less than the sum of the individual masses of the constituent protons and neutrons when separated.

Key Terms

- **α-particle**: two protons and two neutrons bound together into a particle identical to a helium nucleus
- **atomic spectra**: emission or absorption lines formed when an electron makes a transition from one energy level of an atom to another
- **nucleus**: the massive, positively charged central part of an atom, made up of protons and neutrons
- **nuclide**: A nuclide (from "nucleus") is an atomic species characterized by the specific constitution of its nucleus — i.e., by its number of protons (Z), its number of neutrons (N), and its nuclear energy state.
- **radionuclide**: A radionuclide is an atom with an unstable nucleus, characterized by excess energy available to be imparted either to a newly created radiation particle within the nucleus or via internal conversion.
- **radioactive decay**: any of several processes by which unstable nuclei emit subatomic particles and/or ionizing radiation and disintegrate into one or more smaller nuclei
- **nucleus**: the massive, positively charged central part of an atom, made up of protons and neutrons
- **quark**: In the Standard Model, an elementary subatomic particle that forms matter. Quarks are never found alone in nature, but combine to form hadrons, such as protons and neutrons.
- **gluon**: A massless gauge boson that binds quarks together to form baryons, mesons and other hadrons; it is associated with the strong nuclear force.

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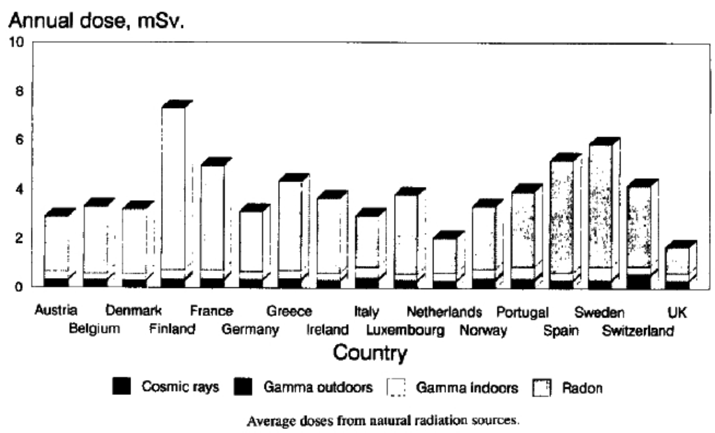
30.2: Radioactivity

Learning objectives

- Name major sources of terrestrial radiation.

Radioactive material is found throughout nature. Detectable amounts occur naturally in soil, rocks, water, air, and vegetation. From these sources it can be inhaled and ingested into the body. In addition to this internal exposure, humans also receive external exposure from radioactive materials that remain outside the body and from cosmic radiation from space. The worldwide average natural dose to humans is about 2.4 millisieverts (mSv) per year. This is four times more than the worldwide average artificial radiation exposure, which in the year 2008 amounted to about 0.6 mSv per year. In some wealthier countries, such as the US and Japan, artificial exposure is, on average, greater than the natural exposure, due to greater access to medical imaging. In Europe, the average natural background exposure by country ranges from under 2 mSv annually in the United Kingdom to more than 7 mSv annually in Finland, as shown in.



Natural Radiation Atlas of Europe: Bar chart of average annual dosages from natural radiation sources for major European countries

Natural Background Radiation

The biggest source of natural background radiation is airborne radon, a radioactive gas that emanates from the ground. Radon and its isotopes, parent radionuclides, and decay products all contribute to an average inhaled dose of 1.26 mSv/a. Radon is unevenly distributed and variable with weather, such that much higher doses occur in certain areas of the world. In these areas it can represent a significant health hazard. Concentrations over 500 times higher than the world average have been found inside buildings in Scandinavia, the United States, Iran, and the Czech Republic. Radon is a decay product of uranium, which is relatively common in the Earth's crust but more concentrated in ore-bearing rocks scattered around the world. Radon seeps out of these ores into the atmosphere or into ground water; it can also infiltrate into buildings. It can be inhaled into the lungs, along with its decay products, where it will reside for a period of time after exposure.

Radiation from Outer Space

In addition, the earth, and all living things on it, are constantly bombarded by radiation from outer space. This radiation primarily consists of positively charged ions ranging from protons to iron and larger nuclei derived from sources outside of our solar system. This radiation interacts with atoms in the atmosphere to create an air shower of secondary radiation, including x-rays, muons, protons, alpha particles, pions, electrons, and neutrons. The immediate dose from cosmic radiation is largely from muons, neutrons, and electrons, and this dose varies in different parts of the world based on the geomagnetic field and altitude. This radiation is much more intense in the upper troposphere (around 10 km in altitude) and is therefore of particular concern for airline crews and frequent passengers, who spend many hours per year in this environment. An airline crew typically gets an extra dose on the order of 2.2 mSv (220 mrem) per year.

Terrestrial Radiation

Terrestrial radiation only includes sources that remain external to the body. The major radionuclides of concern are potassium, uranium, and thorium and their decay products. Some of these decay products, like radium and radon, are intensely radioactive but occur in low concentrations. Most of these sources have been decreasing, due to radioactive decay since the formation of the earth, because there is no significant source of replacement. Because of this, the present activity on Earth from uranium-238 is only half as much as it originally was because of its 4.5-billion-year half-life. Potassium-40 (with a half-life of 1.25 billion years) is at about eight percent of its original activity. However, the effects on humans of the actual diminishment (due to decay) of these isotopes is minimal. This is because humans evolved too recently for the difference in activity over a fraction of a half-life to be significant. Put another way, human history is so short in comparison to a half-life of a billion years that the activity of these long-lived isotopes has been effectively constant throughout our time on this planet.

Many shorter-half-life and therefore more intensely radioactive isotopes have not decayed out of the terrestrial environment because they are still being produced. Examples of these are radium-226 (a decay product of uranium-238) and radon-222 (a decay product of radium-226).

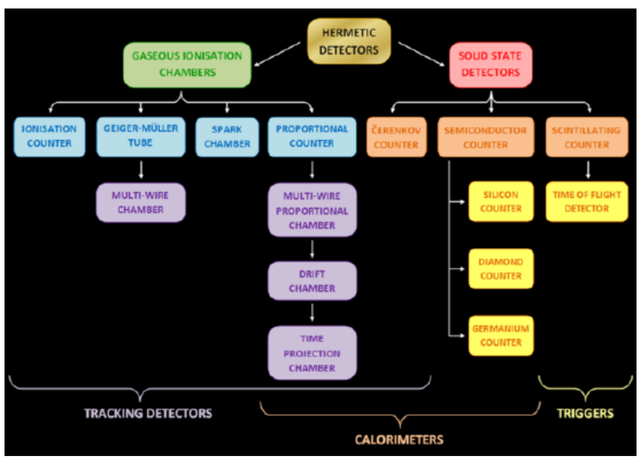
Radiation Detection

A radiation detector is a device used to detect, track, or identify high-energy particles.

Learning objectives

- Explain difference between major types of radiation detectors.

A radiation detector is a device used to detect, track, or identify high- energy particles, such as those produced by nuclear decay, cosmic radiation, and reactions in a particle accelerator. Modern detectors are also used as calorimeters to measure the energy of detected radiation. They may be also used to measure other attributes, such as momentum, spin, and charge of the particles. Different types of radiation detectors exist; gaseous ionization detectors, semiconductor detectors, and scintillation detectors are the most common.



Different Types of Radiation Detectors: different types of radiation detectors (counters)

Gaseous Ionization Detectors

Gaseous ionization detectors use the ionizing effect of radiation upon gas-filled sensors. If a particle has enough energy to ionize a gas atom or molecule, the resulting electrons and ions cause a current flow, which can be measured.

Semiconductor Detectors

A semiconductor detector uses a semiconductor (usually silicon or germanium) to detect traversing charged particles or the absorption of photons. When these detectors' sensitive structures are based on single diodes, they are called semiconductor diode detectors. When they contain many diodes with different functions, the more general term "semiconductor detector" is used. Semiconductor detectors have had various applications in recent decades, in particular in gamma and x-ray spectrometry and as particle detectors.

Scintillation Detectors

A scintillation detector is created by coupling a scintillator — a material that exhibits luminescence when excited by ionizing radiation — to an electronic light sensor, such as a photomultiplier tube (PMT) or a photodiode. PMTs absorb the light emitted by the scintillator and re-emit it in the form of electrons via the photoelectric effect. The subsequent multiplication of those electrons (sometimes called photo-electrons) results in an electrical pulse, which can then be analyzed. The pulse yields meaningful information about the particle that originally struck the scintillator.

Scintillators are used by the American government, particularly Homeland Security, as radiation detectors. Scintillators can also be used in neutron and high-energy particle physics experiments, new energy resource exploration, x-ray security, nuclear cameras, computed tomography, and gas exploration. Other applications of scintillators include CT scanners and gamma cameras in medical diagnostics, screens in computer monitors, and television sets.

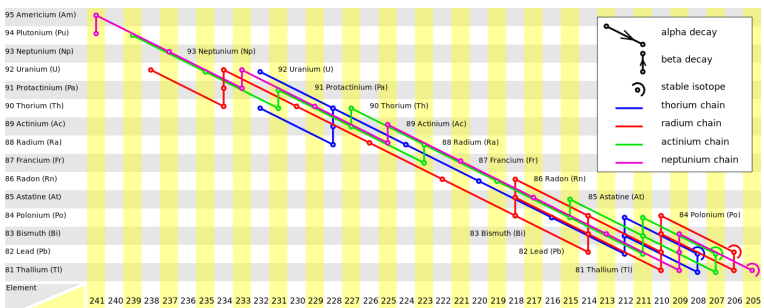
Radioactive Decay Series: Introduction

Radioactive decay series describe the decay of different discrete radioactive decay products as a chained series of transformations.

Learning objectives

- Describe importance of radioactive decay series for decay process.

Radioactive decay series, or decay chains, describe the radioactive decay of different discrete radioactive decay products as a chained series of transformations. Most radioactive elements do not decay directly to a stable state; rather, they undergo a series of decays until eventually a stable isotope is reached.



Radioactive Decay Series Diagram: This diagram provides examples of four decay series: thorium (in blue), radium (in red), actinium (in green), and neptunium (in purple).

Decay stages are referred to by their relationship to previous or subsequent stages. A parent isotope is one that undergoes decay to form a daughter isotope. The daughter isotope may be stable, or it may itself decay to form a daughter isotope of its own. The daughter of a daughter isotope is sometimes called a granddaughter isotope.

The time it takes for a single parent atom to decay to an atom of its daughter isotope can vary widely, not only for different parent-daughter chains, but also for identical pairings of parent and daughter isotopes. While the decay of a single atom occurs spontaneously, the decay of an initial population of identical atoms over time, tt, follows a decaying exponential distribution, e−te−t, where λλis called the decay constant. Because of this exponential nature, one of the properties of an isotope is its half-life, the time by which half of an initial number of identical parent radioisotopes have decayed to their daughters. Half-lives have been determined in laboratories for thousands of radioisotopes (radionuclides). These half-lives can range from nearly nonexistent spans of time to as much as 10^{19} years or more.

The intermediate stages often emit more radioactivity than the original radioisotope. When equilibrium is achieved, a granddaughter isotope is present in proportion to its half-life. But, since its activity is inversely proportional to its half-life, any nuclide in the decay chain finally contributes as much as the head of the chain. For example, natural uranium is not significantly radioactive, but pitchblende, a uranium ore, is 13 times more radioactive because of the radium and other daughter isotopes it contains. Not only are unstable radium isotopes significant radioactivity emitters, but as the next stage in the decay chain they also generate radon, a heavy, inert, naturally occurring radioactive gas. Rock containing thorium and/or uranium (such as some granites) emits radon gas, which can accumulate in enclosed places such as basements or underground mines. Radon exposure is considered the leading cause of lung cancer in non-smokers.

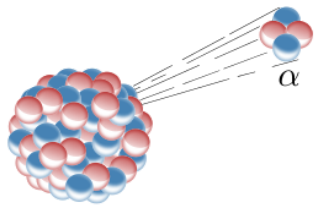
Alpha Decay

In alpha decay an atomic nucleus emits an alpha particle and transforms into an atom with smaller mass (by four) and atomic number (by two).

Learning objectives

- Describe the process, penetration power, and effects of alpha radiation

Alpha decay is a type of radioactive decay in which an atomic nucleus emits an alpha particle that consists of two protons and two neutrons, as shown in. As the result of this process, the parent atom transforms ("decays") into a new atom with a mass number smaller by four and an atomic number smaller by two.



Alpha Decay: Alpha decay is one type of radioactive decay. An atomic nucleus emits an alpha particle and thereby transforms ("decays") into an atom with a mass number smaller by four and an atomic number smaller by two. Many other types of decay are possible.

For example: $238\mathrm{U} \rightarrow 234\mathrm{Th} + \alpha$

Because an alpha particle is the same as a helium-4 nucleus, which has mass number 4 and atomic number 2, this can also be written as:

238

$92\mathrm{U} \rightarrow 234$

$90\mathrm{Th} + 4$

$2\mathrm{He}$

The alpha particle also has charge +2, but the charge is usually not written in nuclear equations, which describe nuclear reactions without considering the electrons. This convention is not meant to imply that the nuclei necessarily occur in neutral atoms.

Alpha decay is by far the most common form of cluster decay, in which the parent atom ejects a defined daughter collection of nucleons, leaving another defined product behind (in nuclear fission, a number of different pairs of daughters of approximately equal size are formed). Alpha decay is the most common cluster decay because of the combined extremely high binding energy and relatively small mass of the helium-4 product nucleus (the alpha particle).

Alpha decay typically occurs in the heaviest nuclides. In theory it can occur only in nuclei somewhat heavier than nickel (element 28), in which overall binding energy per nucleon is no longer a minimum and the nuclides are therefore unstable toward spontaneous fission-type processes. The lightest known alpha emitters are the lightest isotopes (mass numbers 106-110) of tellurium (element 52).

Alpha particles have a typical kinetic energy of 5 MeV (approximately 0.13 percent of their total energy, i.e., 110 TJ/kg) and a speed of 15,000 km/s. This corresponds to a speed of around 0.05 c. There is surprisingly small variation in this energy, due to the heavy dependence of the half-life of this process on the energy produced.

Because of their relatively large mass, +2 electric charge, and relatively low velocity, alpha particles are very likely to interact with other atoms and lose their energy, so their forward motion is effectively stopped within a few centimeters of air.

Most of the helium produced on Earth (approximately 99 percent of it) is the result of the alpha decay of underground deposits of minerals containing uranium or thorium. The helium is brought to the surface as a byproduct of natural gas production.

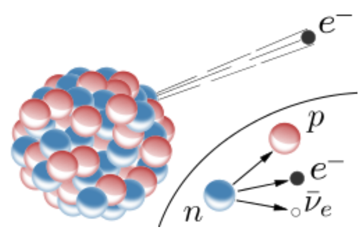
Beta Decay

Beta decay is a type of radioactive decay in which a beta particle (an electron or a positron) is emitted from an atomic nucleus.

Learning objectives

- Explain difference between beta minus and beta plus decays.

Beta decay is a type of radioactive decay in which a beta particle (an electron or a positron) is emitted from an atomic nucleus, as shown in. Beta decay is a process that allows the atom to obtain the optimal ratio of protons and neutrons.



Beta Decay: β decay in an atomic nucleus (the accompanying antineutrino is omitted). The inset shows beta decay of a free neutron

There are two types of beta decay. Beta minus (β) leads to an electron emission (e^-); beta plus (β+) leads to a positron emission (e^+). In electron emission an electron antineutrino is also emitted, while positron emission is accompanied by an electron neutrino. Beta decay is mediated by the weak force.

Emitted beta particles have a continuous kinetic energy spectrum, ranging from 0 to the maximal available energy (Q), that depends on the parent and daughter nuclear states that participate in the decay. The continuous energy spectra of beta particles occur because Q is shared between a beta particle and a neutrino. A typical Q is around 1 MeV, but it can range from a few keV to a several tens of MeV. Since the rest mass energy of the electron is 511 keV, the most energetic beta particles are ultrarelativistic, with speeds very close to the speed of light.

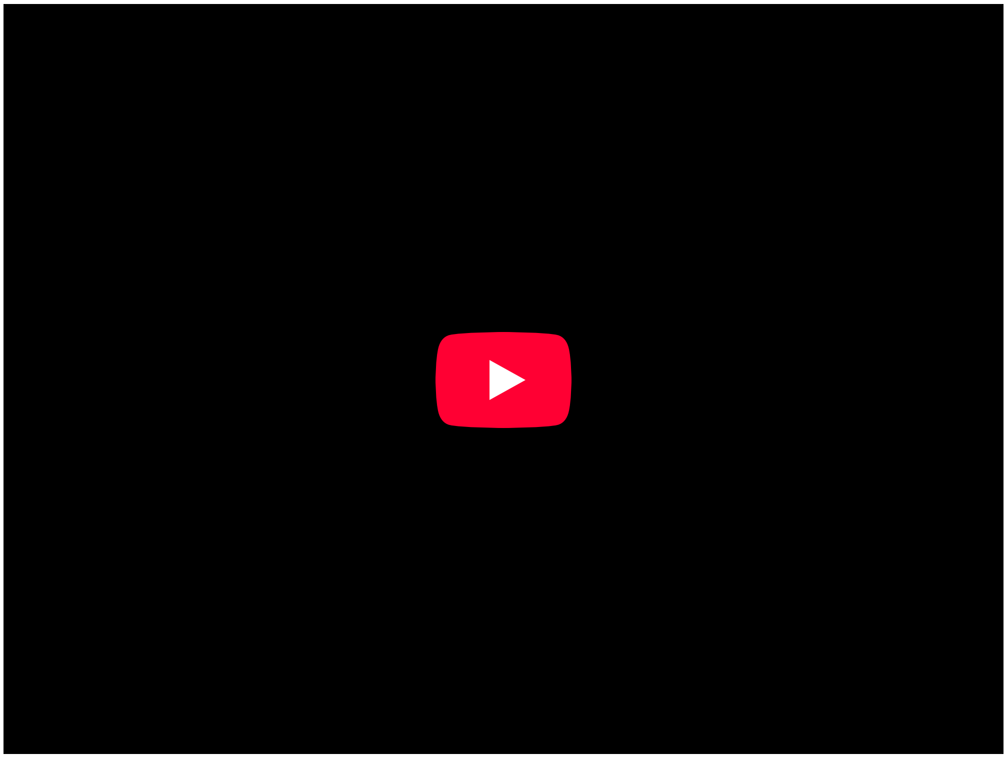
Since the proton and neutron are part of an atomic nucleus, beta decay processes result in transmutation of one chemical element into another. For example:

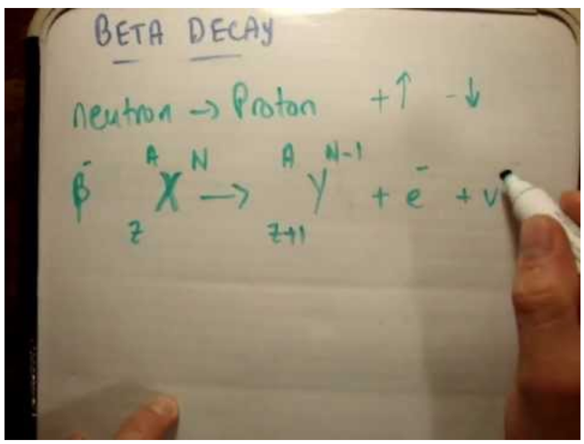
$$137\mathrm{Cs} \rightarrow 137\mathrm{Ba} + \mathrm{e}^- \tag{30.2.1}$$

$$11\mathrm{Na} \rightarrow 10\mathrm{Ne} + \mathrm{e}^+ \tag{30.2.2}$$

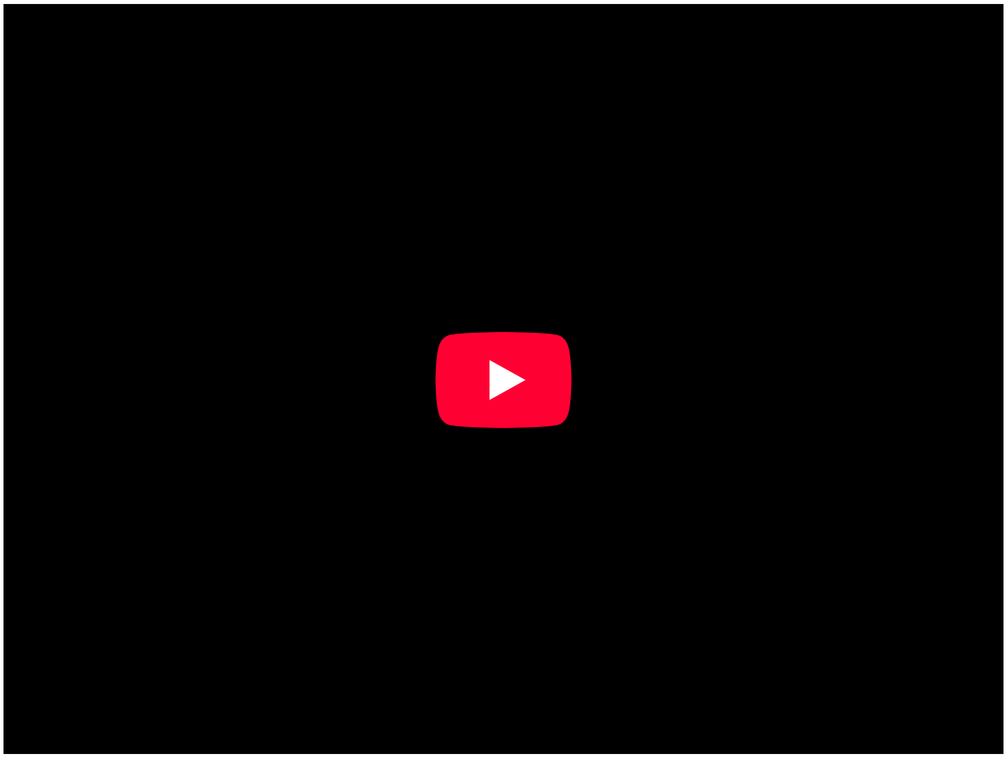
Beta decay does not change the number of nucleons, *A*, in the nucleus; it changes only its charge, *Z*. Therefore the set of all nuclides with the same *A* can be introduced; these isobaric nuclides may turn into each other via beta decay.

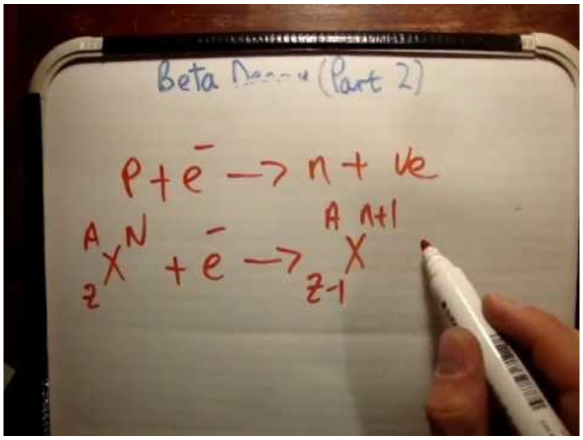
A beta-stable nucleus may undergo other kinds of radioactive decay (for example, alpha decay). In nature, most isotopes are beta-stable, but there exist a few exceptions with half-lives so long that they have not had enough time to decay since the moment of their nucleosynthesis. One example is the odd-proton odd-neutron nuclide 40 K, which undergoes both types of beta decay with a half-life of $1.277 \cdot 10^9$ years.





Beta Decay 1/2: In this video I introduce Beta decay and discuss it from an basic level to a perhaps second or third year University level.





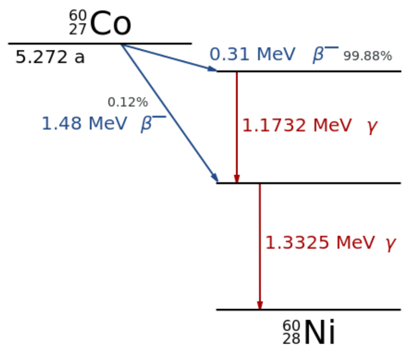
Beta Decay 2/2: In this video I introduce Beta decay and discuss it from an basic level to a perhaps second or third year University level.

Gamma Decay

Gamma decay is a process of emission of gamma rays that accompanies other forms of radioactive decay, such as alpha and beta decay.

> **Learning objectives**
>
> - Explain relationship between gamma decay and other forms of nuclear decay.

Gamma radiation, also known as gamma rays and denoted as γγ, is electromagnetic radiation of high frequency and therefore high energy. Gamma rays typically have frequencies above 10 exahertz (>1019>1019 Hz) and therefore have energies above 100 keV and wavelengths less than 10 picometers (less than the diameter of an atom). However, this is not a strict definition; rather, it is a rule-of-thumb description for natural processes. Gamma rays from radioactive decay are defined as gamma rays no matter what their energy, so there is no lower limit to gamma energy derived from radioactive decay. Gamma decay commonly produces energies of a few hundred keV and usually less than 10 MeV.



Cobalt-60 Decay Scheme: Path of decay of Co-60 to Ni-60. Excited levels for Ni-60 that drop to ground state via emission of gamma rays are indicated

Gamma decay accompanies other forms of decay, such as alpha and beta decay; gamma rays are produced after the other types of decay occur. When a nucleus emits an α or β particle, the daughter nucleus is usually left in an excited state. It can then move to a lower energy state by emitting a gamma ray, in much the same way that an atomic electron can jump to a lower energy state by emitting a photon. For example, cobalt-60 decays to excited nickel-60 by beta decay through emission of an electron of 0.31 MeV. Next, the excited nickel-60 drops down to the ground state by emitting two gamma rays in succession (1.17 MeV, then 1.33 MeV), as shown in. Emission of a gamma ray from an excited nuclear state typically requires only 10−1210−12 seconds: it is nearly instantaneous. Gamma decay from excited states may also follow nuclear reactions such as neutron capture, nuclear fission, or nuclear fusion.

In certain cases, the excited nuclear state following the emission of a beta particle may be more stable than average; in these cases it is termed a metastable excited state if its decay is 100 to 1000 times longer than the average 10−1210−12 seconds. Such nuclei have half-lives that are easily measurable; these are termed nuclear isomers. Some nuclear isomers are able to stay in their excited state for minutes, hours, or days, or occasionally far longer, before emitting a gamma ray. This phenomenon is called isomeric transition. The process of isomeric transition is therefore similar to any gamma emission; it differs only in that it involves the intermediate metastable excited states of the nuclei.

Half-Life and Rate of Decay; Carbon-14 Dating

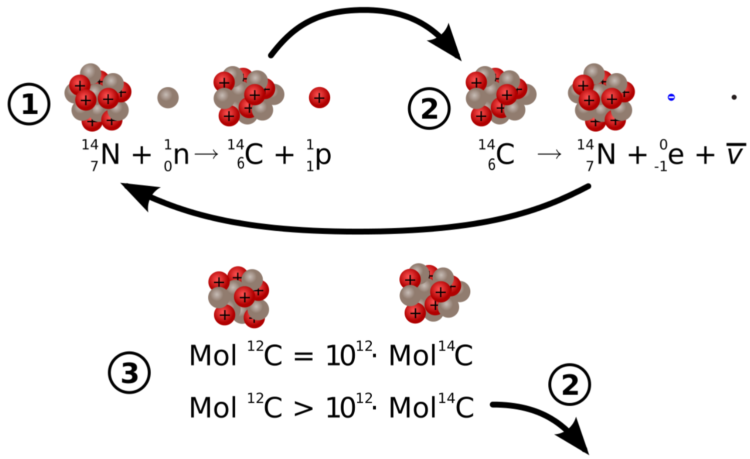
Carbon-14 dating is a radiometric dating method that uses the radioisotope carbon-14 (14C) to estimate the age of object.

Learning objectives

- Identify the age of materials that can be approximately determined using radiocarbon dating

Radiocarbon dating (usually referred to simply as carbon-14 dating) is a radiometric dating method. It uses the naturally occurring radioisotope carbon-14 (14C) to estimate the age of carbon-bearing materials up to about 58,000 to 62,000 years old.

Carbon has two stable, nonradioactive isotopes: carbon-12 (12C) and carbon-13 (13C). There are also trace amounts of the unstable radioisotope carbon-14 (14C) on Earth. Carbon-14 has a relatively short half-life of 5,730 years, meaning that the fraction of carbon-14 in a sample is halved over the course of 5,730 years due to radioactive decay to nitrogen-14. The carbon-14 isotope would vanish from Earth's atmosphere in less than a million years were it not for the constant influx of cosmic rays interacting with molecules of nitrogen (N_2) and single nitrogen atoms (N) in the stratosphere. Both processes of formation and decay of carbon-14 are shown in.

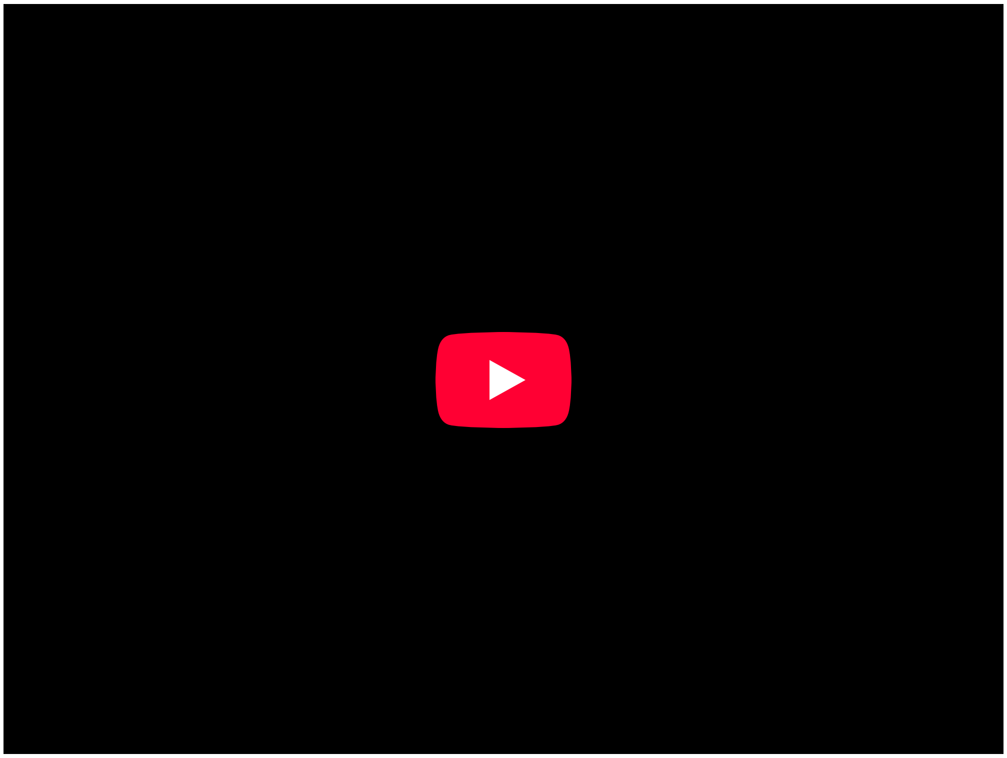


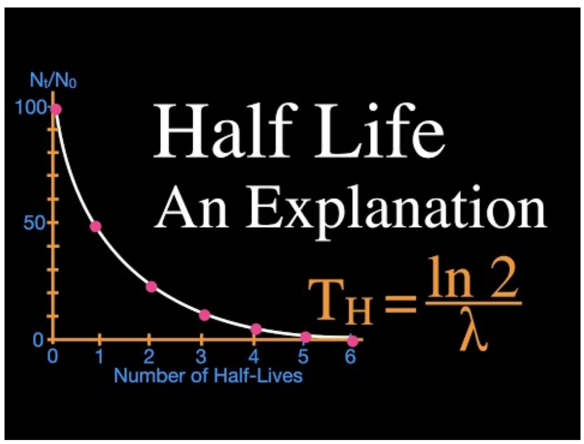
Formation and Decay of Carbon-14: Diagram of the formation of carbon-14 (1), the decay of carbon-14 (2), and equations describing the carbon-12:carbon-14 ratio in living and dead organisms

When plants fix atmospheric carbon dioxide (CO_2) into organic compounds during photosynthesis, the resulting fraction of the isotope 14C in the plant tissue will match the fraction of the isotope in the atmosphere. After plants die or are consumed by other organisms, the incorporation of all carbon isotopes, including 14C, stops. Thereafter, the concentration (fraction) of 14C declines at a fixed exponential rate due to the radioactive decay of 14C. (An equation describing this process is shown in.) Comparing the remaining 14C fraction of a sample to that expected from atmospheric 14C allows us to estimate the age of the sample.

Raw (i.e., uncalibrated) radiocarbon ages are usually reported in radiocarbon years “Before Present” (BP), with “present” defined as CE 1950. Such raw ages can be calibrated to give calendar dates. One of the most frequent uses of radiocarbon dating is to estimate the age of organic remains from archaeological sites.

The technique of radiocarbon dating was developed by Willard Libby and his colleagues at the University of Chicago in 1949. Emilio Segrè asserted in his autobiography that Enrico Fermi suggested the concept to Libby at a seminar in Chicago that year. Libby estimated that the steady-state radioactivity concentration of exchangeable carbon-14 would be about 14 disintegrations per minute (dpm) per gram. In 1960, Libby was awarded the Nobel Prize in chemistry for this work. He demonstrated the accuracy of radiocarbon dating by accurately estimating the age of wood from a series of samples for which the age was known, including an ancient Egyptian royal barge dating from 1850 BCE.





Half-life: Describes radioactive half life and how to do some simple calculations using half life.

Calculations Involving Half-Life and Decay-Rates

The half-life of a radionuclide is the time taken for half the radionuclide's atoms to decay.

Learning objectives

- Explain what is a half-life of a radionuclide.

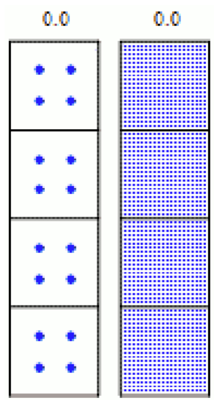
The half-life of a radionuclide is the time taken for half of the radionuclide's atoms to decay. Taking λλ to be the decay rate (number of disintegrations per unit time), and ττ the average lifetime of an atom before it decays, we have:

$$N(t) = N_0 e^{-\lambda t} = N_0 e^{-t/\tau} \tag{30.2.3}$$

The half-life is related to the decay constant by substituting the condition $N = N_o/2$ and solving for $t = t_{1/2}$:

$$t_{1/2} = \ln 2/\lambda = \tau \ln 2 \tag{30.2.4}$$

A half-life must not be thought of as the time required for exactly half of the atoms to decay.



Radioactive decay simulation: A simulation of many identical atoms undergoing radioactive decay, starting with four atoms (left) and 400 atoms (right). The number at the top indicates how many half-lives have elapsed

The following figure shows a simulation of many identical atoms undergoing radioactive decay. Note that after one half-life there are not exactly one-half of the atoms remaining; there are only *approximately*one-half left because of the random variation in the process. However, with more atoms (the boxes on the right), the overall decay is smoother and less random-looking than with fewer atoms (the boxes on the left), in accordance with the law of large numbers.

The relationship between the half-life and the decay constant shows that highly radioactive substances are quickly spent while those that radiate weakly endure longer. Half-lives of known radionuclides vary widely, from more than 10^{19} years, such as for the very nearly stable nuclide 209 Bi, to 10^{-23} seconds for highly unstable ones.

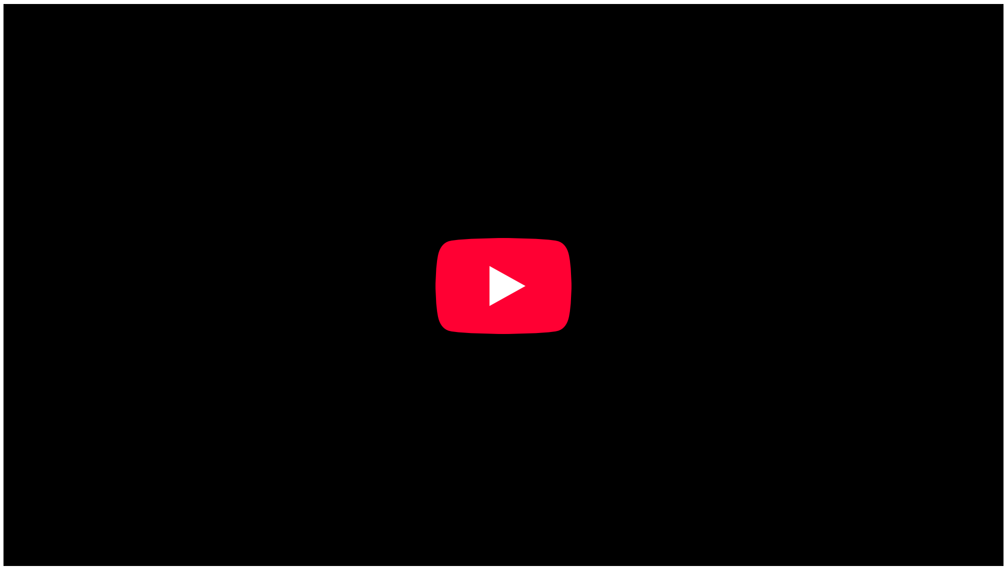
The factor of ln(2) in the above equations results from the fact that the concept of "half-life" is merely a way of selecting a different base other than the natural base *e* for the lifetime expression. The time constant $\tau$is the e^{-1}-life, the time until only 1/*e* remains — about 36.8 percent, rather than the 50 percent in the half-life of a radionuclide. Therefore, $\tau$is longer than $t_{1/2}$. The following equation can be shown to be valid:

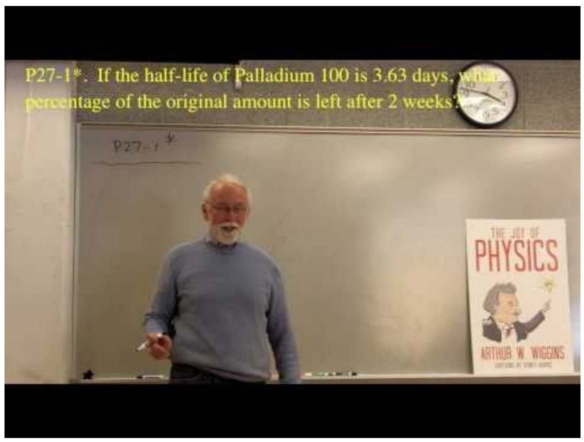
$$\mathrm{N(t) = N_0 e^{-t/\tau} = N_0 2^{-t/t_{1/2}}} \tag{30.2.5}$$

Since radioactive decay is exponential with a constant probability, each process could just as easily be described with a different constant time period that (for example) gave its 1/3-life (how long until only 1/3 is left), or its 1/10-life (how long until only 1/10 is left), and so on. Therefore, the choice of $\tau$and $t_{1/2}$ for marker-times is only for convenience and for the sake of uploading convention. These marker-times reflect a fundamental principle only in that they show that the same proportion of a given radioactive substance will decay over any time period you choose.

Mathematically, the *n*th life for the above situation would be found by the same process shown above — by setting $\mathrm{N = N_0/n}$ and substituting into the decay solution, to obtain:

$$\mathrm{t_{1/n} = \frac{\ln n}{\lambda} = \tau \ln n} \tag{30.2.6}$$





Half-life: Part of a series of videos on physics problem-solving. The problems are taken from “The Joy of Physics. ” This one deals with radioactive half-life. The viewer is urged to pause the video at the problem statement and work the problem before watching the rest of the video.

Key Points

- The biggest source of natural background radiation is airborne radon, a radioactive gas that emanates from the ground.
- The earth is constantly bombarded by radiation from outer space that consists of positively charged ions ranging from protons to iron and larger nuclei from sources outside of our solar system.
- Terrestrial radiation includes sources that remain external to the body. The major radionuclides of concern are potassium, uranium, and thorium and their decay products.
- Gaseous ionization detectors use the ionizing effect of radiation upon gas-filled sensors.
- A semiconductor detector uses a semiconductor (usually silicon or germanium) to detect traversing charged particles or the absorption of photons.
- A scintillation detector is created by coupling a scintillator to an electronic light sensor.
- Most radioactive elements do not decay directly to a stable state; rather, they undergo a series of decays until eventually a stable isotope is reached.
- Half-lives of radioisotopes range from nearly nonexistent spans of time to as much as 1019 years or more.
- The intermediate stages of radioactive decay series often emit more radioactivity than the original radioisotope.
- An alpha particle is the same as a helium-4 nucleus, which has mass number 4 and atomic number 2.
- Because of their relatively large mass, +2 electric charge, and relatively low velocity, alpha particles are very likely to interact with other atoms and lose their energy, so their forward motion is effectively stopped within a few centimeters of air.

- Most of the helium produced on Earth (approximately 99 percent of it) is the result of the alpha decay of underground deposits of minerals containing uranium or thorium.
- There are two types of beta decay: beta minus, which leads to an electron emission, and beta plus, which leads to a positron emission.
- Beta decay allows the atom to obtain the optimal ratio of protons and neutrons.
- Beta decay processes transmute one chemical element into another.
- Gamma decay accompanies other forms of decay, such as alpha and beta decay; gamma rays are produced after the other types of decay occur.
- Although emission of gamma ray is a nearly instantaneous process, it can involve intermediate metastable excited states of the nuclei.
- Gamma rays are generally the most energetic form of electromagnetic radiation.
- Carbon-14 dating can be used to estimate the age of carbon-bearing materials up to about 58,000 to 62,000 years old.
- The carbon-14 isotope would vanish from Earth's atmosphere in less than a million years were it not for the constant influx of cosmic rays interacting with atmospheric nitrogen.
- One of the most frequent uses of radiocarbon dating is to estimate the age of organic remains from archaeological sites.
- The half-life is related to the decay constant as follows: $t_{1/2} = \ln 2/\lambda$.
- The relationship between the half-life and the decay constant shows that highly radioactive substances are quickly spent while those that radiate weakly endure longer.
- Half-lives of known radionuclides vary widely, from more than 10^{19} years, such as for the very nearly stable nuclide 209 Bi, to 10^{-23} seconds for highly unstable ones.

Key Terms

- **radionuclide**: A radionuclide is an atom with an unstable nucleus, characterized by excess energy available to be imparted either to a newly created radiation particle within the nucleus or via internal conversion.
- **radon**: a radioactive chemical element (symbol Rn, formerly Ro) with atomic number 86; one of the noble gases
- **sievert**: in the International System of Units, the derived unit of radiation dose; the dose received in one hour at a distance of 1 cm from a point source of 1 mg of radium in a 0.5 mm thick platinum enclosure; symbol: Sv
- **scintillator**: any substance that glows under the action of photons or other high-energy particles
- **diode**: an electronic device that allows current to flow in one direction only; a valve
- **semiconductor**: A substance with electrical properties intermediate between a good conductor and a good insulator.
- **half-life**: the time required for half of the nuclei in a sample of a specific isotope to undergo radioactive decay
- **radioisotope**: a radioactive isotope of an element
- **decay**: to change by undergoing fission, by emitting radiation, or by capturing or losing one or more electrons
- **alpha particle**: A positively charged nucleus of a helium-4 atom (consisting of two protons and two neutrons), emitted as a consequence of radioactivity; α-particle.
- **radioactive decay**: any of several processes by which unstable nuclei emit subatomic particles and/or ionizing radiation and disintegrate into one or more smaller nuclei
- **beta decay**: a nuclear reaction in which a beta particle (electron or positron) is emitted
- **positron**: The antimatter equivalent of an electron, having the same mass but a positive charge.
- **transmutation**: the transformation of one element into another by a nuclear reaction
- **electromagnetic radiation**: radiation (quantized as photons) consisting of oscillating electric and magnetic fields oriented perpendicularly to each other, moving through space
- **gamma ray**: A very high frequency (and therefore very high energy) electromagnetic radiation emitted as a consequence of radioactivity.
- **radiometric dating**: Radiometric dating is a technique used to date objects based on a comparison between the observed abundance of a naturally occurring radioactive isotope and its decay products using known decay rates.
- **carbon-14**: carbon-14 is a radioactive isotope of carbon with a nucleus containing 6 protons and 8 neutrons.
- **radionuclide**: A radionuclide is an atom with an unstable nucleus, characterized by excess energy available to be imparted either to a newly created radiation particle within the nucleus or via internal conversion.

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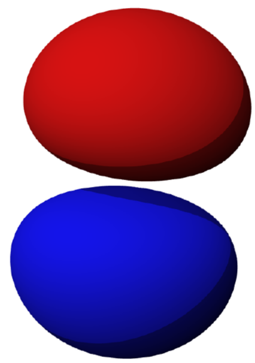
30.3: Quantum Tunneling and Conservation Laws

learning objectives

- Identify factors that affect the tunneling probability

Imagine throwing a ball at a wall and having it disappear the instant before it makes contact and appear on the other side. The wall remains intact; the ball did not break through it. Believe it or not, there is a finite (if extremely small) probability that this event would occur. This phenomenon is called quantum tunneling.

While the possibility of tunneling is essentially ignorable at macroscopic levels, it occurs regularly on the nanoscale level. Consider, for example, a p-orbital in an atom. Between the two lobes there is a nodal plane. By definition there is precisely 0 probability of finding an electron anywhere along that plane, and because the plane extends infinitely it is impossible for an electron to go around it. Yet, electrons commonly cross from one lobe to the other via quantum tunneling. They never exist in the nodal area (this is forbidden); instead they travel through imaginary space.



P-Orbital: The red and blue lobes represent the volume in which there is a 90 percent probability of finding an electron at any given time if the orbital is occupied.

Imaginary space is not real, but it is explicitly referenced in the time-dependent Schrödinger equation, which has a component of ii (the square root of −1−1, an imaginary number):

$$i\hbar\frac{\partial}{\partial t}\Psi = \hat{H}\Psi \tag{30.3.1}$$

And because all matter has a wave component (see the topic of wave-particle duality), all matter can in theory exist in imaginary space. But what accounts for the difference in probability of an electron tunneling over a nodal plane and a ball tunneling through a brick wall? The answer is a combination of the tunneling object's mass (mm) and energy (EE) and the energy height (U0U0) of the barrier through which it must travel to get to the other side.

When it reaches a barrier it cannot overcome, a particle's wave function changes from sinusoidal to exponentially diminishing in form. The solution for the Schrödinger equation in such a medium is:

$$\Psi = Ae^{-\alpha x} \tag{30.3.2}$$

where:

$$\alpha = \sqrt{\frac{2m(U_0 - E)}{h^2}} \tag{30.3.3}$$

Therefore, the probability of an object tunneling through a barrier decreases with the object's increasing mass and with the increasing gap between the energy of the object and the energy of the barrier. And although the wave function never quite reaches 0 (as can be determined from the e−xe−x functionality), this explains how tunneling is frequent on nanoscale but negligible at the macroscopic level.

Conservation of Nucleon Number and Other Laws

Through radioactive decay, nuclear fusion and nuclear fission, the number of nucleons (sum of protons and neutrons) is always held constant.

learning objectives

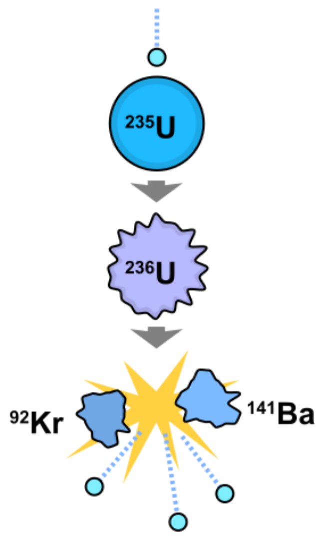
- Define the Law of Conservation of Nuclear Number

In physics and chemistry there are many conservation laws—among them, *the Law of Conservation of Nucleon Number*, which states that the total number of nucleons (nuclear particles, specifically protons and neutrons) cannot change by any nuclear reaction.

Radioactive Decay

Consider the three modes of decay. In gamma decay, an excited nucleus releases gamma rays, but its proton (Z) and neutron (A-Z) count remain the same:

$$ {}^{A}_{Z}N \rightarrow {}^{A}_{Z}N + \gamma \tag{30.3.4}$$



Nuclear fission of U-235: If U-235 is bombarded with a neutron (light blue small circe), the resulting U-236 produced is unstable and undergoes fission. The resulting elements (shown here as Kr-92 and Ba-141) do not contain as many nucleons as U-236, with the remaining three neutrons being released as high-energy particles, able to bombard another U-235 atom and maintain a chain reaction.

In beta decay, a nucleus releases energy and either an electron or a positron. In the case of an electron being released, atomic mass (A) remains the same as a neutron is converted into a proton, raising atomic number by 1:

$$ {}^{A}_{Z}N \rightarrow {}^{A}_{Z+1}N + e^{-} + \nu \tag{30.3.5}$$

In the case of a positron being released, atomic mass remains constant as a proton is converted to a neutron, lowering atomic number by 1:

$$ {}^{A}_{Z}N \rightarrow {}^{A}_{Z-1}N + e^{+} + \nu \tag{30.3.6}$$

Electron capture has the same effect on the number of protons and neutrons in a nucleus as positron emission.

Alpha decay is the only type of radioactive decay that results in an appreciable change in an atom ‘s atomic mass. However, rather than being destroyed, the two protons and two neutrons an atom loses in alpha decay are released as a helium nucleus.

Nuclear Fission

Chain reactions of nuclear fission release a tremendous amount of energy, but follow the Law of Conservation of Nucleon Number. Consider, for example, the multistep reaction that occurs when a U-235 nucleus accepts a neutron, as in:

$$ {}_{0}^{1}\mathrm{n} + {}_{92}^{235}\mathrm{U} \rightarrow {}_{92}^{236}\mathrm{U} \rightarrow {}_{56}^{144}\mathrm{Ba} + {}_{36}^{89}\mathrm{Kr} + 3{}_{0}^{1}\mathrm{n} + \gamma \tag{30.3.7} $$

In each step, the total atomic mass of all species is a constant value of 236. This is the same with all fission reactions.

Nuclear Fusion

Finally, nuclear fusion follows the Law of Conservation of Nucleon Number. Consider the fusion of deuterium and tritium (both hydrogen isotopes):

$$ {}_{1}^{2}\mathrm{H} + {}_{1}^{3}\mathrm{H} \rightarrow {}_{2}^{4}\mathrm{He} + {}_{0}^{1}\mathrm{n} \tag{30.3.8} $$

It is well understood that the tremendous amounts of energy released by nuclear fission and fusion can be attributed to the conversion of mass to energy. However, the mass that is converted to energy is rather small compared to any sample, and never includes the conversion of a proton or neutron to energy. Thus, the number of nucleons before and after fission and fusion is always constant.

Key Points

- Quantum tunneling applies to all objects facing any barrier. However, the probability of its occurrence is essentially negligible for macroscopic purposes; it is only ever observed to any appreciable degree on the nanoscale level.
- Quantum tunneling is explained by the imaginary component of the Schrödinger equation. Because the wave function of any object contains an imaginary component, it can exist in imaginary space.
- Tunneling decreases with the increasing mass of the object that must tunnel and with the increasing gap between the object's energy and the energy of the barrier it must overcome.
- The law of Conservation of Nuclear Number states that the sum of protons and neutrons among species before and after a nuclear reaction will be the same.
- In radioactive decay, a proton can be converted to a neutron and a neutron can be converted to a proton (beta-decay).
- Nuclear fusion and fission involve the conversion of matter to energy, but the matter that is converted is never a full nucleon.

Key Terms

- **tunneling**: the quantum-mechanical passing of a particle through an energy barrier
- **fusion**: A nuclear reaction in which nuclei combine to form more massive nuclei with the concomitant release of energy.
- **fission**: The process of splitting the nucleus of an atom into smaller particles; nuclear fission.
- **nucleon**: One of the subatomic particles of the atomic nucleus (i.e., a proton or a neutron).

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30.4: Applications of Nuclear Physics

Example 30.4.1:

The nuclear medicine whole body bone scan is generally used in evaluations of various bone related pathology, such as for bone pain, stress fracture, nonmalignant bone lesions, bone infections, or the spread of cancer to the bone.

Radiation therapy involves the application of ionizing radiation to treat conditions such as hyperthyroidism, thyroid cancer, and blood disorders. Radiation therapy is particularly effective as a treatment of a number of types of cancer if they are localized to one area of the body. It may also be used as part of curative therapy, to prevent tumor recurrence after surgery, or to remove a primary malignant tumor. Radiation therapy is synergistic with chemotherapy and has been used before, during, and after chemotherapy in susceptible cancers.

Ionizing radiation works by damaging the DNA of exposed tissue, leading to cellular death. When external beam therapy is used, shaped radiation beams are aimed from several angles of exposure to intersect at the tumor, providing a much larger absorbed dose there than in the surrounding, healthy tissue.

External Beam Therapy: Radiation therapy of the pelvis. Lasers and a mold under the legs are used for precise positioning

Brachytherapy is another form of radiation therapy, in which a therapeutic radioisotope is injected into the body to chemically localize to the tissue that requires destruction . A key feature of brachytherapy is that the irradiation affects only a very localized area around the radiation sources. Exposure to radiation of healthy tissues further away from the sources is therefore reduced in this technique.

Radiation therapy is in itself painless. Many low-dose palliative treatments (for example, radiation therapy targeting bony metastases) cause minimal or no side effects, although short-term pain flare-ups can be experienced in the days following treatment due to edemas compressing nerves in the treated area. Higher doses can cause varying side effects during treatment (acute), in the months or years following treatment (long-term), or after re-treatment (cumulative). The nature, severity, and longevity of side effects depend on the organs that receive the radiation, the treatment itself (type of radiation, dose, fractionation, concurrent chemotherapy), and the individual patient.

Dosimetry

Radiation dosimetry is the measurement and calculation of the absorbed dose resulting from the exposure to ionizing radiation.

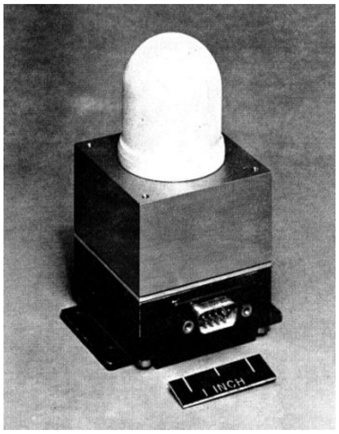
learning objectives

- Explain difference between absorbed dose and dose equivalent.

Radiation dosimetry is the measurement and calculation of the absorbed dose in matter and tissue resulting from the exposure to indirect and direct ionizing radiation.

Measuring Radiation

There are several ways of measuring the dose of ionizing radiation. Workers who come in contact with radioactive substances or who may be exposed to radiation routinely carry personal dosimeters . In the United States, these dosimeters usually contain materials that can be used in thermoluminescent dosimetry or optically stimulated luminescence. Outside the United States, the most widely used type of personal dosimeter is the film badge dosimeter, which uses photographic emulsions that are sensitive to ionizing radiation. The equipment used in radiotherapy (a linear particle accelerator in external beam therapy) is routinely calibrated using ionization chambers or the new and more accurate diode technology. Internal dosimetry is used to evaluate the intake of particles inside a human being.



Ionization Chamber: This ionization chamber was used in the South Atlantic Anomaly Probe project.

Dose is reported in grays (Gy) for absorbed doses or sieverts (Sv) for dose equivalents, where 1 Gy or 1 Sv is equal to 1 joule per kilogram. Non-SI units are still prevalent as well: absorbed dose is often reported in rads and dose equivalent in rems. By definition, 1 Gy = 100 rad, and 1 Sv = 100 rem.

Biological Effects

The distinction between absorbed dose (Gy/rad) and dose equivalent (Sv/rem) is based upon the biological effects. The weighting factor (wr) and tissue/organ weighting factor (WT) have been established. They compare the relative biological effects of various types of radiation and the susceptibility of different organs.

The weighting factor for the whole body is 1, such that 1 Gy of radiation delivered to the whole body is equal to one sievert. Therefore, the WT for all organs in the whole body must sum to 1.

By definition, X-rays and gamma rays have a wr of unity, such that 1 Gy = 1 Sv (for whole-body irradiation). Values of wr are as high as 20 for alpha particles and neutrons. That is to say, for the same absorbed dose in Gy, alpha particles are 20 times as biologically potent as x-rays or gamma rays.

Dose is a measure of deposited dose and therefore can never decrease: removal of a radioactive source can reduce only the rate of increase of absorbed dose — never the total absorbed dose.

Biological Effects of Radiation

Ionizing radiation is generally harmful, even potentially lethal, to living organisms.

learning objectives

- Describe effects of ionizing radiation on living organisms.

Example 30.4.2:

The Radium Girls were female factory workers who contracted radiation poisoning from painting watch dials with glow-in-the-dark paint at the United States Radium factory in Orange, New Jersey around 1917. The women, who had been told the paint was harmless, ingested deadly amounts of radium by licking their paintbrushes to give them a fine point; some also painted their fingernails and teeth with the glowing substance.

Ionizing radiation is generally harmful, even potentially lethal, to living organisms. Although radiation was discovered in the late 19th century, the dangers of radioactivity and of radiation were not immediately recognized. The acute effects of radiation were first observed in the use of x-rays when Wilhelm Röntgen intentionally subjected his fingers to x-rays in 1895. The genetic effects of radiation, including the effects on cancer risk, were recognized much later. In 1927, Hermann Joseph Muller published research showing genetic effects.

Some effects of ionizing radiation on human health are stochastic, meaning that their probability of occurrence increases with dose, while the severity is independent of dose. Radiation-induced cancer, teratogenesis, cognitive decline, and heart disease are all examples of stochastic effects. Other conditions, such as radiation burns, acute radiation syndrome, chronic radiation syndrome, and radiation-induced thyroiditis are deterministic, meaning they reliably occur above a threshold dose and their severity increases with dose. Deterministic effects are not necessarily more or less serious than stochastic effects; either can ultimately lead to damage ranging from a temporary nuisance to death.

Radium Girls: Radium dial painters working in a factory

Quantitative data on the effects of ionizing radiation on human health are relatively limited compared to other medical conditions because of the low number of cases to date and because of the stochastic nature of some of the effects. Stochastic effects can only be measured through large epidemiological studies in which enough data have been collected to remove confounding factors such as smoking habits and other lifestyle factors. The richest source of high-quality data is the study of Japanese atomic bomb survivors.

Two pathways of exposure to ionizing radiation exist. In the case of external exposure, the radioactive source is outside (and remains outside) the exposed organism. Examples of external exposure include a nuclear worker whose hands have been dirtied with radioactive dust or a person who places a sealed radioactive source in his pocket. External exposure is relatively easy to estimate, and the irradiated organism does not become radioactive, except if the radiation is an intense neutron beam that causes activation. In the case of internal exposure, the radioactive material enters the organism, and the radioactive atoms become incorporated into the organism. This can occur through inhalation, ingestion, or injection. Examples of internal exposure include potassium-40 present within a normal person or the ingestion of a soluble radioactive substance, such as strontium-89 in cows' milk. When radioactive compounds enter the human body, the effects are different from those resulting from exposure to an external radiation source. Especially in the case of alpha radiation, which normally does not penetrate the skin, the exposure can be much more damaging after ingestion or inhalation.

Therapeutic Uses of Radiation

Radiation therapy uses ionizing radiation to treat conditions such as hyperthyroidism, cancer, and blood disorders.

learning objectives

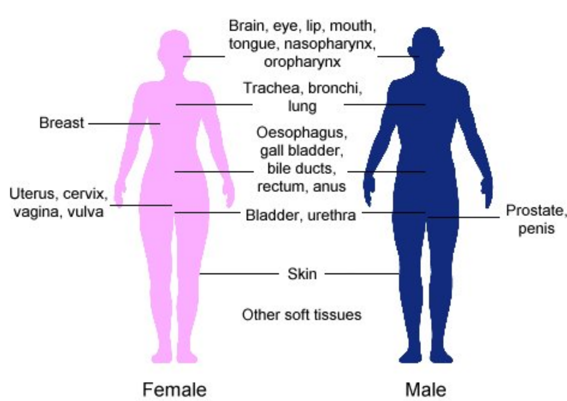
- Explain difference between external beam radiotherapy and brachytherapy.

Radiation therapy involves the application of ionizing radiation to treat conditions such as hyperthyroidism, thyroid cancer, and blood disorders. Radiation therapy is particularly effective as a treatment of a number of types of cancer if they are localized to one area of the body. It may also be used as part of curative therapy, to prevent tumor recurrence after surgery, or to remove a primary malignant tumor. Radiation therapy is synergistic with chemotherapy and has been used before, during, and after chemotherapy in susceptible cancers.

Ionizing radiation works by damaging the DNA of exposed tissue, leading to cellular death. When external beam therapy is used, shaped radiation beams are aimed from several angles of exposure to intersect at the tumor, providing a much larger absorbed dose there than in the surrounding, healthy tissue .

External Beam Therapy: Radiation therapy of the pelvis. Lasers and a mold under the legs are used for precise positioning

Brachytherapy is another form of radiation therapy, in which a therapeutic radioisotope is injected into the body to chemically localize to the tissue that requires destruction . A key feature of brachytherapy is that the irradiation affects only a very localized area around the radiation sources. Exposure to radiation of healthy tissues further away from the sources is therefore reduced in this technique.



Clinical Applications of Brachytherapy: Body sites in which brachytherapy can be used to treat cancer

Radiation therapy is in itself painless. Many low-dose palliative treatments (for example, radiation therapy targeting bony metastases) cause minimal or no side effects, although short-term pain flare-ups can be experienced in the days following treatment due to edemas compressing nerves in the treated area. Higher doses can cause varying side effects during treatment (acute), in the months or years following treatment (long-term), or after re-treatment (cumulative). The nature, severity, and longevity of side effects depend on the organs that receive the radiation, the treatment itself (type of radiation, dose, fractionation, concurrent chemotherapy), and the individual patient.

Radiation from Food

Food irradiation is a process of treating a food to a specific dosage of ionizing radiation for a predefined length of time.

learning objectives

- Explain how food irradiation is performed, commenting on its purpose and safety

Food irradiation is a process of treating a food to a specific dosage of ionizing radiation for a predefined length of time. This process slows or halts spoilage that is due to the growth of pathogens. Food irradiation is currently permitted by over 50 countries, and the volume of food treated is estimated to exceed 500,000 metric tons annually worldwide. Irradiated food is sold in regular stores, often in specially marked packages .

Radura Logo: The Radura logo, required by U.S. Food and Drug Administration regulations to show a food has been treated with ionizing radiation

By irradiating food, depending on the dose, some or all of the microorganisms, bacteria, viruses, and insects present are killed. This prolongs the shelf-life of the food in cases where pathogenic spoilage is the limiting factor. Some foods, e.g., herbs and spices, are irradiated at sufficient doses (five kilograys or more) to reduce the microbial counts by several orders of magnitude. Such ingredients do not carry spoilage or pathogen microorganisms into the final product. It has also been shown that irradiation can delay the ripening of fruits and the sprouting of vegetables.

Food irradiation using cobalt-60 is the preferred method by most processors. This is because the deep penetration of gamma rays allows for the treatment of entire industrial pallets or totes at once, which reduces the need for material handling. A pallet or tote is typically exposed for several minutes to several hours, depending on the dose. Radioactive material must be monitored and carefully stored to shield workers and the environment from its gamma rays. During operation this is achieved using concrete shields. With most designs the radioisotope can be lowered into a water-filled source storage pool to allow maintenance personnel to enter the radiation shield. In this mode the water in the pool absorbs the radiation.

X-ray irradiators are considered an alternative to isotope-based irradiation systems. X-rays are generated by colliding accelerated electrons with a dense material (the target), such as tantalum or tungsten, in a process known as bremsstrahlung-conversion. X-ray irradiators are scalable and have deep penetration comparable to Co-60, with the added benefit that the electronic source stops radiating when switched off. They also permit dose uniformity, but these systems generally have low energetic efficiency during the conversion of electron energy to photon radiation, so they require much more electrical energy than other systems. X-ray systems also rely on concrete shields to protect the environment and workers from radiation.

Irradiated food does not become radioactive, since the particles that transmit radiation are not themselves radioactive. Still, there is some controversy in the application of irradiation due to its novelty, the association with the nuclear industry, and the potential for the chemical changes to be different than the chemical changes due to heating food (since ionizing radiation produces a higher energy transfer per collision than conventional radiant heat).

Tracers

A radioactive tracer is a chemical compound in which one or more atoms have been replaced by a radioisotope.

learning objectives

- Explain structure and use of radioactive tracers

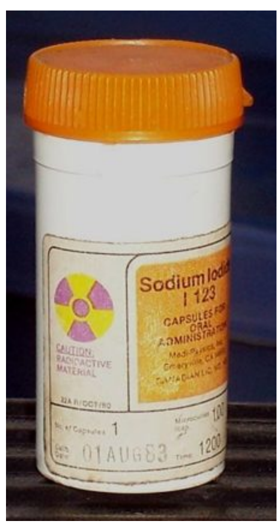
A radioactive tracer is a chemical compound in which one or more atoms have been replaced by a radioisotope. By virtue of its consequent radioactive decay, this compound can be used to explore the mechanism of chemical reactions by tracing the path that the radioisotope follows from reactants to products.

The underlying principle in the creation of a radioactive tracer is that an atom in a chemical compound is replaced by another atom of the same chemical element. In a tracer, this substituting atom is a radioactive isotope. This process is often called radioactive labeling. Radioactive decay is much more energetic than chemical reactions. Therefore, the radioactive isotope can be present in low concentration and its presence still detected by sensitive radiation detectors such as Geiger counters and scintillation counters.

Geiger Counter: Image of a Geiger counter with pancake-type probe

There are two main ways in which radioactive tracers are used:

When a labeled chemical compound undergoes chemical reactions, one or more of the products will contain the radioactive label. Analysis of what happens to the radioactive isotope provides detailed information about the mechanism of the chemical reaction.
A radioactive compound can be introduced into a living organism. The radio-isotope provides a way to build an image showing how that compound and its reaction products are distributed around the organism.
All the commonly used radioisotopes (Tritium $(^{3}H),^{11}C,^{13}N,^{15}O,^{18}F,^{32}P,^{35}S,^{99m}Tc$, and ^{123}I) have short half-lives. They do not occur in nature and are produced through nuclear reactions.



Iodine; 123 Radioisotope: Lead container containing iodine-123 radioisotope

Nuclear Fusion

In nuclear fusion two or more atomic nuclei collide at very high speed and join, forming a new nucleus.

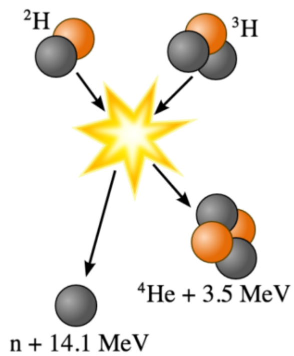
learning objectives

- Analyze possibility of the use of nuclear fusion for the production of electricity.

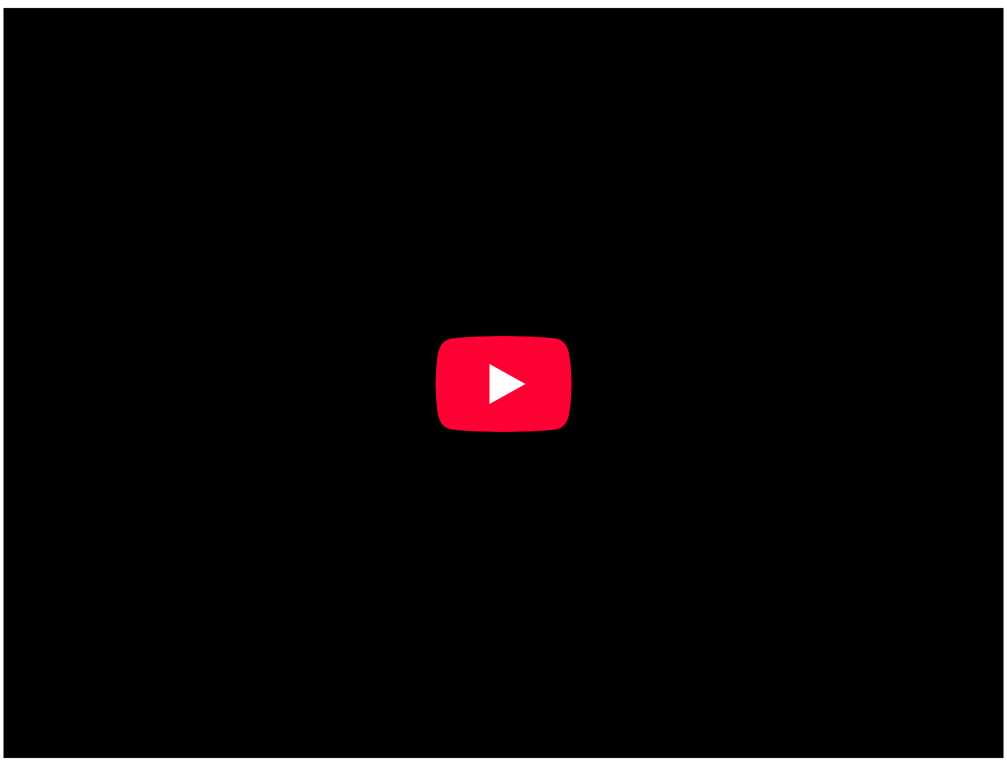
Example 30.4.3:

The sun is a main-sequence star and therefore generates its energy through nuclear fusion of hydrogen nuclei into helium. In its core, the sun fuses 620 million metric tons of hydrogen each second.

Nuclear fusion is a nuclear reaction in which two or more atomic nuclei collide at very high speed and join to form a new type of atomic nucleus. During this process, matter is not conserved because some of the mass of the fusing nuclei is converted into energy.



Fusion of Deuterium with Tritium: Fusion of deuterium with tritium creating helium-4, freeing a neutron, and releasing 17.59 MeV of energy; some mass changes form to appear as the kinetic energy of the products



Fission and Fusion

Describes the difference between fission and fusion

Fusion reactions of light elements power the stars and produce virtually all elements in a process called nucleosynthesis. The fusion of lighter elements in stars releases energy and mass. For example, in the fusion of two hydrogen nuclei to form helium, 0.7 percent of the mass is carried away from the system in the form of kinetic energy or other forms of energy (such as electromagnetic radiation).

It takes considerable energy to force nuclei to fuse, even nuclei of the lightest element, hydrogen. This is because all nuclei have a positive charge due to their protons, and since like charges repel, nuclei strongly resist being put close together. Accelerated to high speeds, they can overcome this electrostatic repulsion and be forced close enough for the attractive nuclear force to be sufficiently strong to achieve fusion. The fusion of lighter nuclei, which creates a heavier nucleus and often a free neutron or proton, generally releases more energy than it takes to force the nuclei together. This is an exothermic process that can produce self-sustaining reactions.

Research into controlled fusion, with the aim of producing fusion power for the production of electricity, has been conducted for over 60 years. It has been accompanied by extreme scientific and technological difficulties, but it has resulted in progress. At present, controlled fusion reactions have been unable to produce self-sustaining controlled fusion reactions. Researchers are working on a reactor that theoretically will deliver 10 times more fusion energy than the amount needed to heat up plasma to required temperatures. Workable designs of this reactor were originally scheduled to be operational in 2018; however, this has been delayed, and a new date has not been released.

Nuclear Fission in Reactors

Nuclear reactors convert the thermal energy released from nuclear fission into electricity.

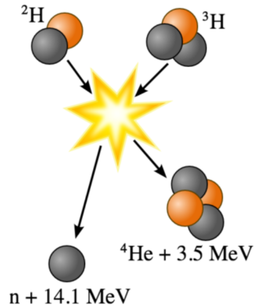
learning objectives

- Explain how nuclear chain reactions can be controlled.

Example 30.4.4:

Some serious nuclear and radiation accidents have occurred. In 2011, three of the reactors at Fukushima I overheated, causing meltdowns that eventually led to explosions, which released large amounts of radioactive material into the air.

Nuclear fission is a nuclear reaction in which the nucleus of an atom splits into smaller (lighter) nuclei. This reaction often produces free neutrons and photons (in the form of gamma rays) and releases a very large amount of energy, even by the standards of radioactive decay . The two nuclei produced are most often of comparable but slightly different sizes, typically with a mass ratio of products of about 3 to 2, for common fissile nuclides.



Nuclear Fission Reaction: An induced nuclear fission event. A neutron is absorbed by the nucleus of a uranium-235 atom, which in turn splits into fast-moving lighter elements (fission products) and free neutrons

For example, when a large fissile atomic nucleus such as uranium-235 or plutonium-239 absorbs a neutron, it may undergo nuclear fission. The heavy nucleus splits into two or more lighter nuclei (the fission products), releasing kinetic energy, gamma radiation, and free neutrons. A portion of these neutrons may later be absorbed by other fissile atoms and trigger further fission events, which release more neutrons, and so on. This is known as a nuclear chain reaction.

Just as conventional power stations generate electricity by harnessing the thermal energy released from burning fossil fuels, the thermal energy released from nuclear fission can be converted in electricity by nuclear reactors. A nuclear chain reaction can be controlled by using neutron poisons and neutron moderators to change the percentage of neutrons that will go on to cause more fissions. Nuclear reactors generally have automatic and manual systems to shut the fission reaction down if unsafe conditions are detected.

The reactor core generates heat in a number of ways. The kinetic energy of fission products is converted to thermal energy when these nuclei collide with nearby atoms. Some of the gamma rays produced during fission are absorbed by the reactor, and their energy is converted to heat. Heat is produced by the radioactive decay of fission products and materials that have been activated by neutron absorption. This decay heat source will remain for some time even after the reactor is shut down.

A nuclear reactor coolant — usually water, but sometimes a gas, liquid metal, or molten salt — is circulated past the reactor core to absorb the heat that it generates. The heat is carried away from the reactor and is then used to generate steam.

The power output of the reactor is adjusted by controlling how many neutrons are able to create more fissions. Control rods that are made of a neutron poison are used to absorb neutrons. Absorbing more neutrons in a control rod means that there are fewer neutrons available to cause fission, so pushing the control rod deeper into the reactor will reduce the reactor's power output, and extracting the control rod will increase it.

Control Rod Assembly: Control rod assembly, above fuel element

Some serious nuclear and radiation accidents have occurred. Nuclear power plant accidents include the Chernobyl disaster (1986), the Fukushima Daiichi nuclear disaster (2011), the Three Mile Island accident (1979), and the SL-1 accident (1961).

Nuclear safety involves the actions taken to prevent nuclear and radiation accidents or to limit their consequences. The nuclear power industry has improved the safety and performance of reactors and has proposed new safer (but generally untested) reactor designs. However, there is no guarantee that these reactors will be designed, built, and operated correctly.

Fukushima Daiichi Nuclear Disaster: Satellite image taken March 16, 2011 of the four damaged reactor buildings

Emission Topography

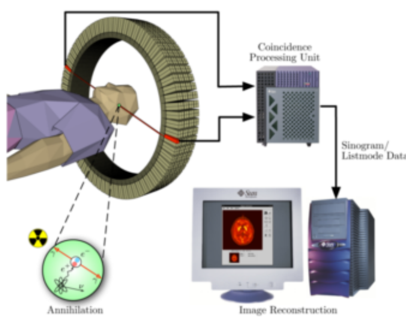
Positron emission tomography is a nuclear medical imaging technique that produces a three-dimensional image of processes in the body.

learning objectives

- Discuss possibility of uses of positron emission tomography with other diagnostic techniques.

Positron emission tomography (PET) is a nuclear medical imaging technique that produces a three-dimensional image or picture of functional processes in the body. The system detects pairs of gamma rays emitted indirectly by a positron-emitting radionuclide (tracer), which is introduced into the body on a biologically active molecule. Three-dimensional images of tracer concentration within the body are then constructed by computer analysis.

PET acquisition process occurs as the radioisotope undergoes positron emission decay (also known as positive beta decay), it emits a positron, an antiparticle of the electron with opposite charge. The emitted positron travels in tissue for a short distance (typically less than 1 mm, but dependent on the isotope), during which time it loses kinetic energy, until it decelerates to a point where it can interact with an electron. The encounter annihilates both electron and positron, producing a pair of annihilation (gamma) photons moving in approximately opposite directions. These are detected when they reach a scintillator in the scanning device, creating a burst of light which is detected by photomultiplier tubes or silicon avalanche photodiodes . The technique depends on simultaneous or coincident detection of the pair of photons moving in approximately opposite directions (it would be exactly opposite in their center of mass frame, but the scanner has no way to know this, and so has a built-in slight direction-error tolerance). Photons that do not arrive in temporal “pairs” (i.e. within a timing-window of a few nanoseconds) are ignored.



Positron Emission Tomography Acquisition Process: Schema of a PET acquisition process.

A technique much like the reconstruction of computed tomography (CT) and single-photon emission computed tomography (SPECT) data is more commonly used, although the data set collected in PET is much poorer than CT, so reconstruction techniques are more difficult.

PET scans are increasingly read alongside CT or magnetic resonance imaging (MRI) scans, with the combination giving both anatomic and metabolic information. Because PET imaging is most useful in combination with anatomical imaging, such as CT, modern PET scanners are now available with integrated high-end multi-detector-row CT scanners . Because the two scans can be performed in immediate sequence during the same session, with the patient not changing position between the two types of scans, the two sets of images are more-precisely registered, so that areas of abnormality on the PET imaging can be more perfectly correlated with anatomy on the CT images. This is very useful in showing detailed views of moving organs or structures with higher anatomical variation, which is more common outside the brain.

PET/CT-System: PET/CT-System with 16-slice CT; the ceiling mounted device is an injection pump for CT contrast agent.

PET scanning is non-invasive, but it does involve exposure to ionizing radiation. The total dose of radiation is significant, usually around 5–7 mSv. However, in modern practice, a combined PET/CT scan is almost always performed, and for PET/CT scanning, the radiation exposure may be substantial—around 23–26 mSv (for a 70 kg person—dose is likely to be higher for higher body weights). When compared to the classification level for radiation workers in the UK of 6 mSv, it can be seen that use of a PET scan needs proper justification.

Nuclear Weapons

A nuclear weapon is an explosive device that derives its destructive force from nuclear reactions—either fission, fusion, or a combination.

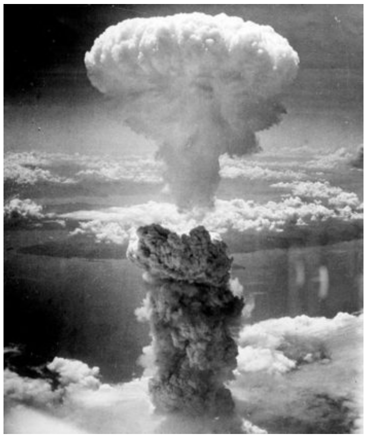
learning objectives

- Explain the difference between an "atomic" bomb and a "hydrogen" bomb, discussing their history

A nuclear weapon is an explosive device that derives its destructive force from nuclear reactions, either fission or a combination of fission and fusion. Both reactions release vast quantities of energy from relatively small amounts of matter. The first fission (i.e., "atomic") bomb test released the same amount of energy as approximately 20,000 tons of trinitrotoluene (TNT). The first fusion (i.e., thermonuclear "hydrogen") bomb test released the same amount of energy as approximately 10,000,000 tons of TNT.

A modern thermonuclear weapon weighing little more than 2,400 pounds (1,100 kg) can produce an explosive force comparable to the detonation of more than 1.2 million tons (1.1 million tonnes) of TNT. Thus, even a small nuclear device no larger than traditional bombs can devastate an entire city by blast, fire and radiation. Nuclear weapons are considered weapons of mass destruction, and their use and control have been a major focus of international relations policy since their inception.

Only two nuclear weapons have been used in the course of warfare, both by the United States near the end of World War II. On August 6, 1945, a uranium gun-type fission bomb code-named "Little Boy" was detonated over the Japanese city of Hiroshima. Only three days later a plutonium implosion-type fission bomb code-named "Fat Man" (as illustrated in) was exploded over Nagasaki, Japan. The resulting mushroom cloud is shown in . The death toll from the two bombings was estimated at approximately 200,000 people—mostly civilians, and mainly from acute injuries sustained from the explosions. The role of the bombings in Japan's surrender, and their ethical implications, remain the subject of scholarly and popular debate.



Nagasaki Atomic Bombing: The mushroom cloud of the atomic bombing of Nagasaki, Japan (August 9,1945) rose some 18 kilometers (11 mi) above the bomb's hypocenter.

Fat Man Atomic Bomb: The first nuclear weapons were gravity bombs, such as this "Fat Man" weapon dropped on Nagasaki, Japan. They were very large and could only be delivered by heavy bomber aircraft.

Since the bombings of Hiroshima and Nagasaki, nuclear weapons have been detonated on over two thousand occasions for testing purposes and demonstrations. Only a small number of nations either possess such weapons, or are suspected of trying to acquire and/or develop them. The only countries known to have detonated nuclear weapons (and that acknowledge possessing such weapons) are, as listed chronologically by date of first test: the United States, the Soviet Union (succeeded as a nuclear power by Russia), the United Kingdom, France, the People's Republic of China, India, Pakistan, and North Korea. In addition, it is also widely believed that Israel possesses nuclear weapons (though they have not admitted to it).

The Federation of American Scientists estimates that as of 2012, there are more than 17,000 nuclear warheads in the world, with around 4,300 considered "operational"—as in ready for use.

NMR and MRIs

Magnetic resonance imaging is a medical imaging technique used in radiology to visualize internal structures of the body in detail.

learning objectives

- Explain the difference between magnetic resonance imaging and computed tomography.

Magnetic resonance imaging (MRI), also called nuclear magnetic resonance imaging (NMRI) or magnetic resonance tomography (MRT), is a medical imaging technique used in radiology to visualize internal structures of the body in detail. MRI utilized the property of nuclear magnetic resonance (NMR) to image the nuclei of atoms inside the body.

MRI machines (as pictured in) make use of the fact that body tissue contains a large amount of water and therefore protons (1H nuclei), which get aligned in a large magnetic field. Each water molecule has two hydrogen nuclei or protons. When a person is inside the scanner's powerful magnetic field, the hydrogen protons in their body align with the direction of the field. A radio frequency current is briefly activated, producing a varying electromagnetic field. This electromagnetic field has just the right frequency (known as the resonance frequency) to become absorbed and then reverse the rotation of the hydrogen protons in the magnetic field.

MRI Scanner: Phillips MRI scanner in Gothenburg, Sweden.

After the electromagnetic field is turned off, the rotations of the hydrogen protons return to thermodynamic equilibrium, and then realign with the static magnetic field. During this relaxation, a radio frequency signal (electromagnetic radiation in the RF range) is generated; this signal can be measured with receiver coils. Hydrogen protons in different tissues return to their equilibrium state at different relaxation rates. Images are then constructed by performing a complex mathematical analysis of the signals emitted by the hydrogen protons.

MRI shows a marked contrast between the different soft tissues of the body, making it especially useful in imaging the brain, the muscles, the heart, and cancerous tissue—as compared with other medical imaging techniques such as computed tomography (CT) or X-rays. MRI contrast agents may be injected intravenously to enhance the appearance of blood vessels, tumors or inflammation.

Unlike CT, MRI does not use ionizing radiation and is generally a very safe procedure. The strong magnetic fields and radio pulses can, however, affect metal implants (including cochlear implants and cardiac pacemakers).

Key Points

- Ionizing radiation works by damaging the DNA of exposed tissue, leading to cellular death.
- In external beam radiotherapy, shaped radiation beams are aimed from several angles of exposure to intersect at the tumor, providing a much larger absorbed dose there than in the surrounding, healthy tissue.
- In brachytherapy, a therapeutic radioisotope is injected into the body to chemically localize to the tissue that requires destruction.
- There are several ways of measuring doses of ionizing radiation: personal dosimeters, ionization chambers, and internal dosimetry.
- The distinction between absorbed dose (Gy/rad) and dose equivalent (Sv/rem) is based upon the biological effects.
- Dose is a measure of deposited dose and therefore can never decrease: removal of a radioactive source can reduce only the rate of increase of absorbed dose — never the total absorbed dose.
- The effects of ionizing radiation on human health are separated into stochastic effects (the probability of occurrence increases with dose) and deterministic effects (they reliably occur above a threshold dose, and their severity increases with dose).
- Quantitative data on the effects of ionizing radiation on human health are relatively limited compared to other medical conditions because of the low number of cases to date and because of the stochastic nature of some of the effects.
- Two pathways (external and internal) of exposure to ionizing radiation exist.
- Ionizing radiation works by damaging the DNA of exposed tissue, leading to cellular death.
- In external beam radiotherapy, shaped radiation beams are aimed from several angles of exposure to intersect at the tumor, providing a much larger absorbed dose there than in the surrounding, healthy tissue.
- In brachytherapy, a therapeutic radioisotope is injected into the body to chemically localize to the tissue that requires destruction.
- Food irradiation kills some of the microorganisms, bacteria, viruses, and insects found in food. It prolongs shelf-life in cases where pathogenic spoilage is the limiting factor.
- Food irradiation using cobalt-60 is the preferred method by most processors.
- Irradiated food does not become radioactive, since the particles that transmit radiation are not themselves radioactive.
- Radioactive tracers are used to explore the mechanism of chemical reactions by tracing the path that the radioisotope follows from reactants to products.
- The radioactive isotope can be present in low concentration and its presence still detected by sensitive radiation detectors.
- All the commonly used radioisotopes have short half-lives, do not occur in nature, and are produced through nuclear reactions.
- The fusion of lighter elements releases energy.
- Matter is not conserved during fusion reactions.
- Fusion reactions power the stars and produce virtually all elements in a process called nucleosynthesis.
- Nuclear fission is a nuclear reaction in which the nucleus of an atom splits into smaller parts, releasing a very large amount of energy.
- Nuclear chain reactions can be controlled using neutron poisons and neutron moderators.
- Although the nuclear power industry has improved the safety and performance of reactors and has proposed new, safer reactor designs, there is no guarantee that serious nuclear accidents will not occur.
- PET scanning utilizes detection of gamma rays emitted indirectly by a positron-emitting radionuclide (tracer), which is introduced into the body on a biologically active molecule.
- PET scans are increasingly read alongside CT or magnetic resonance imaging (MRI) scans, with the combination giving both anatomic and metabolic information.
- PET scanning is non-invasive, but it does involve exposure to ionizing radiation.
- Nuclear weapons utilize either fission (“atomic” bomb) or combination of fission and fusion (“hydrogen” bomb).

- Nuclear weapons are considered weapons of mass destruction.
- The use and control of nuclear weapons is a major focus of international relations policy since their first use.
- MRI makes use of the property of nuclear magnetic resonance to image nuclei of atoms inside the body.
- MRI provides good contrast between the different soft tissues of the body (making it especially useful in imaging the brain, the muscles, the heart, and cancerous tissue).
- Although MRI uses non-ionizing radiation, the strong magnetic fields and radio pulses can affect metal implants, including cochlear implants and cardiac pacemakers.

Key Terms

- **external beam therapy**: Radiotherapy that directs the radiation at the tumour from outside the body.
- **ionizing radiation**: high-energy radiation that is capable of causing ionization in substances through which it passes; also includes high-energy particles
- **brachytherapy**: Radiotherapy using radioactive sources positioned within (or close to) the treatment volume.
- **diode**: an electronic device that allows current to flow in one direction only; a valve
- **dosimeter**: A dosimeter is a device used to measure a dose of ionizing radiation. These normally take the form of either optically stimulated luminescence (OSL), photographic-film, thermoluminescent (TLD), or electronic personal dosimeters (PDM).
- **gamma ray**: A very high frequency (and therefore very high energy) electromagnetic radiation emitted as a consequence of radioactivity.
- **x-ray**: Short-wavelength electromagnetic radiation usually produced by bombarding a metal target in a vacuum. Used to create images of the internal structure of objects; this is possible because x-rays pass through most objects and can expose photographic film
- **radioactive tracer**: a radioactive isotope that, when injected into a chemically similar substance, or artificially attached to a biological or physical system, can be traced by radiation detection devices
- **isotope**: any of two or more forms of an element where the atoms have the same number of protons but a different number of neutrons within their nuclei. As a consequence, atoms for the same isotope will have the same atomic number but different mass numbers (atomic weights)
- **radioactive decay**: any of several processes by which unstable nuclei emit subatomic particles and/or ionizing radiation and disintegrate into one or more smaller nuclei
- **nucleosynthesis**: any of several processes that lead to the synthesis of heavier atomic nuclei
- **fusion**: A nuclear reaction in which nuclei combine to form more massive nuclei with the concomitant release of energy.
- **control rod**: any of a number of steel tubes, containing boron or another neutron absorber, that is inserted into the core of a nuclear reactor in order to control its rate of reaction
- **nuclear reactor**: any device in which a controlled chain reaction is maintained for the purpose of creating heat (for power generation) or for creating neutrons and other fission products for experimental, medical, or other purposes
- **fission**: The process of splitting the nucleus of an atom into smaller particles; nuclear fission.
- **tracer**: A chemical used to track the progress or history of a natural process.
- **positron**: The antimatter equivalent of an electron, having the same mass but a positive charge.
- **tomography**: Imaging by sections or sectioning.
- **warfare**: The waging of war or armed conflict against an enemy.
- **computed tomography**: (CT) – A form of radiography which uses computer software to create images, or slices, at various planes of depth from images taken around a body or volume of interest.
- **nuclear magnetic resonance**: (NMRI) – The absorption of electromagnetic radiation (radio waves), at a specific frequency, by an atomic nucleus placed in a strong magnetic field; used in spectroscopy and in magnetic resonance imaging.
- **magnetic resonance imaging**: Commonly referred to as MRI; a technique that uses nuclear magnetic resonance to form cross sectional images of the human body for diagnostic purposes.

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