9.1: Harmonic oscillators

One of the major playing fields for operatorial methods is the harmonic oscillator. Even though they look very artificial, harmonic potentials play an extremely important role in many areas of physics. This is due to the fact that around an equilibrium point, where the forces vanish, any potential behaves as an harmonic one (plus small corrections). This can best be seen by making a Taylor series expansion about such a point,

$$V(x)=V_0+\frac{1}{2}m\omega^2x^2+O(x^3). \tag{9.1.1}$$

Question

Why is there no linear term in Equation 9.1.1?

For small enough x the quadratic term dominates, and we can ignore other terms. Such situations occur in many physical problems, and make the harmonic oscillator such an important problem.

As explained in our first discussion of harmonic oscillators, we scale to dimensionless variables ("pure numbers")

$$y=\sqrt{\frac{m\omega}{\hbar}}x \tag{9.1.2}$$

with $\epsilon=E/\hbar\omega$.

In these new variables the Schrödinger equation becomes

$$\frac{1}{2}\left(\frac{-d^2}{dy^2}+y^2\right)u(y)=\epsilon u(y). \tag{9.1.3}$$