

9.3: Eigenfunctions of H through ladder operations

If we start with the ground state we would expect that we can't go any lower,

$$\hat{a}u_0(y) = 0. \quad (9.3.1)$$

This can of course be checked explicitly,

$$\begin{aligned} \hat{a}e^{-y^2/2} &= \frac{1}{\sqrt{2}} \left(\hat{y} + \frac{d}{dy} \right) e^{-y^2/2} \\ &= \frac{1}{\sqrt{2}} \left(ye^{-y^2/2} - ye^{-y^2/2} \right) \\ &= 0 \end{aligned}$$

Quiz Can you show that $\epsilon_0 = 1/2$ using the operators \hat{a} ?

Once we know that $\epsilon_0 = 1/2$, repeated application of

$$(a^\dagger a + 1/2)a^\dagger u(y) = (\epsilon + 1/2)a^\dagger u(y) \quad (9.3.2)$$

from the previous page shows that $\epsilon_n = n + 1/2$, which we know to be correct from our previous treatment.

Actually, once we know the ground state, we can now easily determine all the Hermite polynomials up to a normalisation constant:

$$\begin{aligned} u_1(y) &\propto a^\dagger e^{-y^2/2} \\ &= \frac{1}{\sqrt{2}} \left(\hat{y} - \frac{d}{dy} \right) e^{-y^2/2} \\ &= \frac{1}{\sqrt{2}} \left(ye^{-y^2/2} + ye^{-y^2/2} \right) \\ &= \sqrt{2}ye^{-y^2/2} \end{aligned}$$

Indeed $H_1(y) \propto y$.

From math books we can learn that the standard definition of the Hermite polynomials corresponds to

$$H_n(y)e^{-y^2/2} = (\sqrt{2})^n (\hat{a}^\dagger)^n e^{-y^2/2} \quad (9.3.3)$$

We thus learn $H_1(y) = 2y$ and $H_2(y) = (4y^2 - 2)$.

Question: Prove this last relation.

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