6.2: Potential step

Consider a potential step

$$V(x)=\begin{cases}V_0 & x<a\\ V_1 & x>a\end{cases}$$

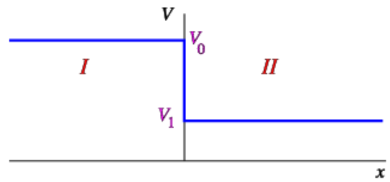

***Figure 6.2.1**: The step potential discussed in the text*

Let me define

$$k_0=\sqrt{\frac{2m}{\hbar^2}(E-V_0)}, \tag{6.2.1}$$

$$k_1=\sqrt{\frac{2m}{\hbar^2}(E-V_1)}. \tag{6.2.2}$$

I assume a beam of particles comes in from the left,

$$\phi(x)=A_0e^{ik_0x}\text{ for }x<0. \tag{6.2.3}$$

At the potential step the particles either get reflected back to region I, or are transmitted to region II. There can thus only be a wave moving to the right in region II, but in region I we have both the *incoming* and a *reflected* wave,

$$\phi_I(x)=A0e^{ik_0x}+B_0e^{-ik_0x}, \tag{6.2.4}$$

$$\phi_{II}(x)=A_1e^{ik_1x}. \tag{6.2.5}$$

We define a *transmission (T)* and *reflection (R) coefficient* as the ratio of currents between reflected or transmitted wave and the incoming wave, where we have canceled a common factor

$$R=\frac{|B_0|^2}{|A_0|^2} \tag{6.2.6}$$

$$T=\frac{k_1|A_1|^2}{k_0|A_0|^2}. \tag{6.2.7}$$

Even though we have given up normalisability, we still have the two continuity conditions. At $x=0$ these imply, using continuity of ϕ and $\frac{d}{dx}\phi$,

$$A_0+B_0=A_1, \tag{6.2.8}$$

$$ik_0(A_0-B_0)=ik_1A_1. \tag{6.2.9}$$

We thus find

$$A_1=\frac{2k_0}{k_0+k_1A_0} \tag{6.2.10}$$

$$B_0=\frac{k_0-k_1}{k_0+k_1A_0}, \tag{6.2.11}$$

and the reflection and transmission coefficients can thus be expressed as

$$R = \left(\frac{k_0 - k_1}{k_0 + k_1}\right)^2, \tag{6.2.12}$$

$$T = \frac{4k_1 k_0}{(k_0 + k_1)^2}. \tag{6.2.13}$$

Notice that $R + T = 1$!

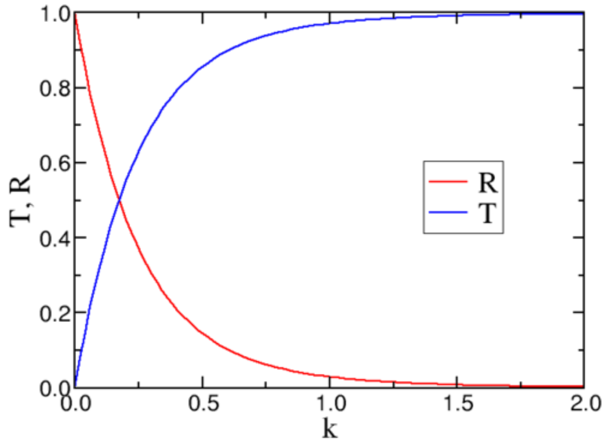

Figure $\mathbf{6.2.2}$***:*** *The transmission and reflection coefficients for a square barrier.*

In Figure $6.2.2$ we have plotted the behaviour of the transmission and reflection of a beam of Hydrogen atoms impinging on a barrier of height 2 meV.