

11.1: The momentum operator as a vector

First of all we know from classical mechanics that velocity and momentum, as well as position, are represented by vectors. Thus we need to represent the momentum operator by a vector of operators as well,

$$\hat{p} = \left(\frac{\hbar}{i} \frac{\partial}{\partial x}, \frac{\hbar}{i} \frac{\partial}{\partial y}, \frac{\hbar}{i} \frac{\partial}{\partial z} \right). \quad (11.1.1)$$

There exists a special notation for the vector of partial derivatives, which is usually called the gradient, and one writes

$$\hat{p} = \frac{\hbar}{i} \nabla. \quad (11.1.2)$$

We now that the energy, and Hamiltonian, can be written in classical mechanics as

$$E = \frac{1}{2} m v^2 + V(x) = \frac{1}{2m} p^2 + V(x), \quad (11.1.3)$$

where the square of a vector is defined as the sum of the squares of the components,

$$(v_1, v_2, v_3)^2 = v_1^2 + v_2^2 + v_3^2. \quad (11.1.4)$$

The Hamiltonian operator in quantum mechanics can now be read of from the classical one,

$$\hat{H} = \frac{1}{2m} \hat{p}^2 + V(x) = -\frac{\hbar^2}{2m} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) + V(x). \quad (11.1.5)$$

Let me introduce one more piece of notation: the square of the gradient operator is called the Laplacian, and is denoted by Δ .

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