13.2: Black Hole Thermodynamics

Introduction:

"Time and space are modes in which we think and not conditions in which we live." -- Einstein

"Space is the order of coexistence, and time is the order of succession of phenomena." -- Leibniz

"For the sage, time is only of significance in that within it the steps of becoming can unfold in clearest sequence." -- **I Ching**

Background Information

One of the features of Hawking and Bekenstein's development of black hole thermodynamics is that it ties many many pieces of physics together. Among those pieces are:

- The realisation from Quantum Mechanics that we can think of all matter-energy as waves. A document on this appears here.
- The realisation from classical physics that in a confined region, waves exist as *standing waves*. A document on this appears here.
- The realisation from thermodynamics that the *entropy* can be viewed as a measure of the number of combinations or permutations of an ensemble that are equivalent. This is equivalent to viewing the entropy more conventionally as a measure of the heat divided by the temperature of a body. According to the Second Law of Thermodynamics, in a closed system the entropy never decreases. A document on Entropy appears here.
- The realisation from Heisenberg's Uncertainty Principle that we can violate the principle of Conservation of Energy so long as we do it for only a short period of time. This is discussed in the next section of this document.
- The realisation from classical physics that all objects with a temperature above absolute zero radiate away energy as electromagnetic radiation. This is briefly discussed below.
- Feynman's theory of antimatter as regular matter going backwards in time. A document on antimatter appears here.

Finally, background information on black holes can be found here.

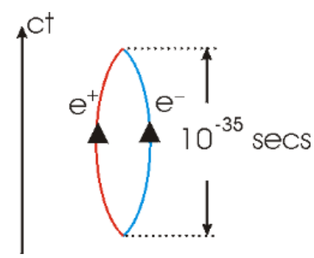
Virtual Pair Production

Recall Heisenberg's Uncertainty Principle. It basically puts a limit on how much we can reduce the disturbance we introduce in a system by doing a measurement on it. There are a number of forms of the principle, and here we shall use only one of them:

The uncertainty in any measurement of the energy of an object times the uncertainty in when the object had that energy will always be at least equal to a universal constant.

Technical note: The universal constant is *Planck's constant h* divided by 2 pi.

A moment's reflection on the implications of this form of the Uncertainty Principle may convince you that this means that the energy does not even *have* a definite value but only a lower and upper bound.
Thus the principle of conservation of energy can be violated so long as the violation occurs for only a brief period of time.
Now consider Dirac's infinite sea of negative energy electrons. One of those electrons can violate conservation of energy by spontaneously jumping into a positive energy state provided it falls back into the hole quickly enough. You will recall that we interpret the hole in the sea as a positron. Thus, we believe that this *virtual pair production* is occurring everywhere in the universe. The pair can only exist for a time of about 10^{-35} seconds, i.e. 34 zeroes followed by a 1 to the right of the decimal point; this is called the *Planck time*.



Similarly we believe virtual pairs of proton-antiprotons, neutron-antineutrons etc. are continually being formed and disappearing everywhere in the universe. Wheeler, then, characterizes the vacuum at a scale of very small distances as being *quantum foam*.

Thermal Radiation

Here is yet another "little" fact that we will need soon: Thermodynamics says that any body with a temperature above absolute zero will radiate its energy away.

Recall that heat is just the internal energy of motion and vibration of the molecules of the substance. And also recall that whenever a body with electric charge vibrates it radiates energy as electromagnetic radiation. These two facts explain the radiation process.

Your body is radiating at a rate of about 60 Watts, the same as a 60W light bulb. Most of that radiation is in the infrared region.

The higher the temperature of a body the faster it radiates energy away.

Technical note: for a perfect radiating body the rate of energy radiation is a universal constant times the fourth power of the absolute temperature.

Whether the radiation is mostly as infrared or visible light or X-rays etc. depends on the temperature of the body. For example, if we heat up a steel bar it starts to glow "red hot." Increasing the temperature shifts the radiation spectrum and it glows "white hot." Raise the temperature further and the radiation becomes blue.

Black Hole Thermodynamics

We consider here one of Stephen Hawking's contributions to Physics. A reference is his famous book **A Brief History of Time**. Sadly, a former tutor in JPU200Y was completely correct when he proposed the following multiple choice question for a test:

Stephen Hawking is:
A. a lousy writer.

Lost in the media blitz surrounding Hawking is that, working independently, Bekenstein also came to many of the realisations we describe here.

We imagine a black hole as the singularity in the center surrounded by a spherical *event horizon*.We know that when a black hole is created by a collapsing neutron star that the neutrons are crushed out of existence; by this I mean that all their *neutron*ness is wiped out. However their total mass-energy remains.

Another way of stating this is that outside of the event horizon all properties of the matter that formed it are gone except for the total mass-energy, rotation, and electric charge: this is sometimes called the *Black Hole Has No Hair* theorem.

The total mass-energy is manifested as the curvature of spacetime around the singularity.

We have seen that all matter has a wave aspect, and Quantum Mechanics describes the behavior of these waves. So, we shall think about representing the mass-energy inside the event horizon as waves.

Now, what kind of waves are possible inside the black hole? The answer is *standing waves*, waves that "fit" inside the black hole with a *node* at the event horizon. The possible wave states are very similar to the standing waves on a circular drum head that we saw earlier; they aren't exactly the same because the waves exist in three dimensions instead of just the two of the drumhead, but they are very close to the same.

Note that I just said "three dimensions." This is correct; we are using non-relativistic quantum mechanics.

The energy represented by a particular wave state is related to the frequency and amplitude of its oscillation. As we saw for the standing waves on a drumhead, the higher "overtones" have a higher frequency and thus these Quantum Mechanical waves contain more energy.

Assume that the total mass-energy inside the event horizon is fixed. So, we have various standing waves, each with a certain amount of energy, and the sum of the energy of all these waves equals the total mass-energy of the black hole. There are a large number of ways that the total mass-energy can distribute itself among the standing waves. We could have it in only a few high energy waves or a larger number of low energy waves.

It turns out that all the possible standing wave states are equally probable. Thus, we can calculate the *probability* of a particular combination of waves containing the total mass-energy of the black hole the same way we calculated the probability of getting various combinations for dice. Just as for the dice, the state with the most total combinations will be the most probable state.

But we have seen that the *entropy* is just a measure of the probability. Thus we can calculate the entropy of a black hole.

We have also seen that the entropy measures the heat divided by the absolute temperature. The "heat" here is just the total mass-energy of the black hole, and if we know that and we know the entropy, we can calculate a *temperature* for the black hole.

So, as Hawking realised, we can apply all of *Thermodynamics* to a black hole.

In a previous section we saw that any body with a temperature above absolute zero will radiate energy. And we have just seen that a black hole has a non-zero temperature. Thus thermodynamics says it will radiate energy and evaporate. We can calculate the rate of radiation for a given temperature from classical thermodynamics.

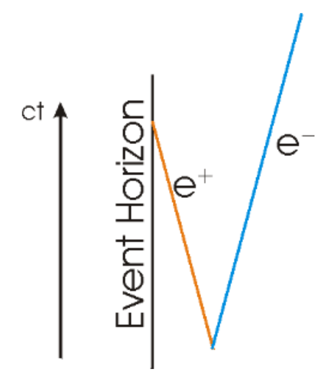
How is this possible? Nothing can get across the event horizon, so how can the black hole radiate? The answer is via virtual pair production.

Consider a virtual electron-positron pair produced just outside the event horizon. Once the pair is created, the intense curvature of spacetime of the black hole can put energy into the pair. Thus the pair can become non-virtual; the electron does not fall back into the hole.

There are many possible fates for the pair. Consider one of them: the positron falls into the black hole and the electron escapes. According to Feynman's view we can describe this as follows:

The electron crosses the event horizon travelling backwards in time, scatters, and then radiates away from the black hole travelling forwards in time.

Using the field of physics that calculates virtual pair production etc., called *Quantum Electrodynamics*, we can calculate the rate at which these electrons etc. will be radiating away from the black hole. The result is the same as the rate of radiation that we calculate using classical thermodynamics.



The fact that we can get the radiation rate in two independent ways, from classical Thermodynamics or from Quantum Electrodynamics, strengthens our belief that black holes radiate their energy away and evaporate.

Technical note: if we measure the mass-energy **M** of a black hole in units where the mass of our Sun is one, then the absolute temperature of the black hole is $\mathbf{6 \times 10^{-8} / M}$ Kelvin and its lifetime, in seconds, is: $\mathbf{10^{71}\ M^3}$.

Black Holes and Information

Above we mentioned the *Black Hole Has No Hair* theorem, which states that no matter what falls into a black hole, the only properties that remain are the total mass, charge, and angular momentum of the object. Thus if, say, an encyclopedia falls into a black hole all the *information*in the encyclopedia is lost.

We can state this circumstance in another way using Quantum Mechanical terminology. Before it falls into a black hole the encyclopedia has in principle a single well defined *wave function*. This is called a *pure state*. After it falls into the hole, however, we have seen that the description of its mass-energy becomes a combination of the possible standing wave states that can exist with nodes on the event horizon. This is called a*mixed* state.

However, Quantum Mechanics provides no mechanism by which a pure state can become a mixed one. This is usually called the "Information Problem" with black holes.

Hawking, Kip Thorne and others believe that when this problem is resolved, it will turn out that the information really has been irretrievably lost. However, John Preskill and others firmly believe that a mechanism for the information to be released by the evaporating black hole must and will be found in a correct theory of quantum gravity.

Thus, in February 1997 Preskill offered a bet to Hawking and Thorne that:

"When an initial pure quantum state undergoes gravitational collapse to form a black hole, the final state at the end of black hole evaporation will always be a pure quantum state."

Hawking and Thorne accepted the bet. The wager is:

"The loser(s) will reward the winner(s) with an encyclopedia of the winner's choice, from which information can be recovered at will."

Thermodynamics of the Universe

Consider the universe. It has a size of about 15 billion light years or so. It also has a total amount of mass-energy. If we represent this mass-energy as quantum mechanical standing waves, just as we did for black holes, we can calculate the total entropy of the universe.

It turns out that the entropy of either a black hole or the universe is proportional to its size squared.

Thus for a given amount of total mass-energy, the larger the object the higher the entropy.

But the universe is expanding, so its size is increasing. Thus the total entropy of the universe is also increasing.

This leads us to the idea that the Second Law of Thermodynamics may be a consequence of the expanding universe. Thus cosmology explains this nineteenth century principle.

Put another way, recall that we have realised that the direction of time, "time's arrow," can come either from the fact that the universe is expanding or from the Second Law of Thermodynamics. We have now found a relationship between these two indicators of the direction of time.

It is amusing to speculate about what will happen to the Second Law of Thermodynamics if the universe is closed, so that at some point the expansion stops and reverses.

Even more wild is the idea that if the expansion of the universe determines the direction of time's arrow, then if the universe starts to contract the direction of time will also reverse.

Sentient Beings

Hawking and Bekenstein did much of the above work on the thermodynamics of black holes and the universe. In this section we consider a speculation for which I must take the blame.

Reference: D. Harrison, "Entropy and the Number of Sentient Beings in the Universe," Speculations in Science and Technology **5** (1982) 43.

First, we must think a bit about *information*.

The search for extra-terrestrial life has concentrated on scanning the universe for radio waves and trying to see if the patterns of the radiation could contain evidence of intelligence. When the *pulsars*, the radio sources that send blips at highly regular intervals, were first discovered some people got very excited and thought perhaps the search had yielded a positive result; now we believe that the pulsars are the radiation from rapidly rotating neutron stars.

If we receive a radio transmission that is just static, there is very little information in it. The information content of a signal depends on that signal being ordered, not random. Thus if the extra-terrestrial beings are sending information to us in radio waves the signal will be ordered in some way.

Thus, information must have low *entropy*. You may recall that earlier we mentioned the *negentropy*, which is the negative of the entropy: it measures the amount of order in a system. People who work in information theory customarily think about the negentropy.

We are, hopefully, acquiring information about the world, ourselves, our friends all the time. Thus we are creating negentropy in our mental system.

Now, the Second Law of Thermodynamics says that the entropy is increasing. This is a sort of strange law for a physicist: it says that the entropy is never conserved. This is as opposed to the types of laws that we are used to, which talk about conditions under which things are conserved.

You will also recall that Quantum Mechanics seems to say that there are no *observers* in the universe, only *participators*. Thus the universe is in some sense brought into being by communicating participators acquiring information about it (and vice versa).

What if the Second Law of Thermodynamics is not quite complete as stated? My speculation is that it could be extended to read:

The rate of production of physical entropy by the universe equals the rate of production of negentropy by sentient beings in the universe.

Now we have a conservation law.

However, the total physical entropy of the universe is increasing, and we can calculate the rate of that increase. If we could calculate the rate at which a typical human-like creature acquires information throughout its lifetime, then a simple division will allow us to calculate the number of such sentient beings there are in the universe.

To guess at the rate at which we produce negentropy we take our memory system to be essentially digital, with each of the 10^{14} synapses in our brain in either an *on* or *off* state. The combinations of synapse states are just the same as the combinations of black marble-white marble states that I insisted we think about when we discussed entropy in an earlier class. So, just as always, we count the number of combinations to calculate the entropy, whose negative is the negentropy.

We assume evolution is efficient, so the memory store is full after 100 years.

The result of dividing this rate of negentropy production into the rate at which physical entropy is being produced by our expanding universe is a number on the order of 10^{102} sentient human-like beings in the universe. To put this number into context, there are on the order of 10^{80}protons plus neutrons in the universe.

So, perhaps this is a failed speculation. Other alternatives include:

- Neutrons, protons, etc. are sentient.
- The human memory system contains a great deal more potential that we have allowed for.
- We have not included the negentropy production due to the communication that occurs between sentient beings.
- We, along with Hawking and Bekenstein, have calculated the rate of physical entropy production in the universe using equilibrium thermodynamics. A self-organising universe with negentropy production through dissipative structures makes our calculation of the rate of physical entropy production incorrect.

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