11.5: Now where does the probability peak?

Clearly the probability density to find an electron at point x is

$$P(x) = R(r)^* R(r), \tag{11.5.1}$$

but what is the probability to find the electron at a distance r from the proton? The key point to realise is that for each value of r the electron can be anywhere on the surface of a sphere of radius r, so that for larger r more points contribute than for smaller r. This is exactly the source of the factor $4\pi r^2$ in the normalisation integral. The probability to find a certain value of r is thus

$$P(r) = 4\pi r^2 R(r)^* R(r) \tag{11.5.2}$$

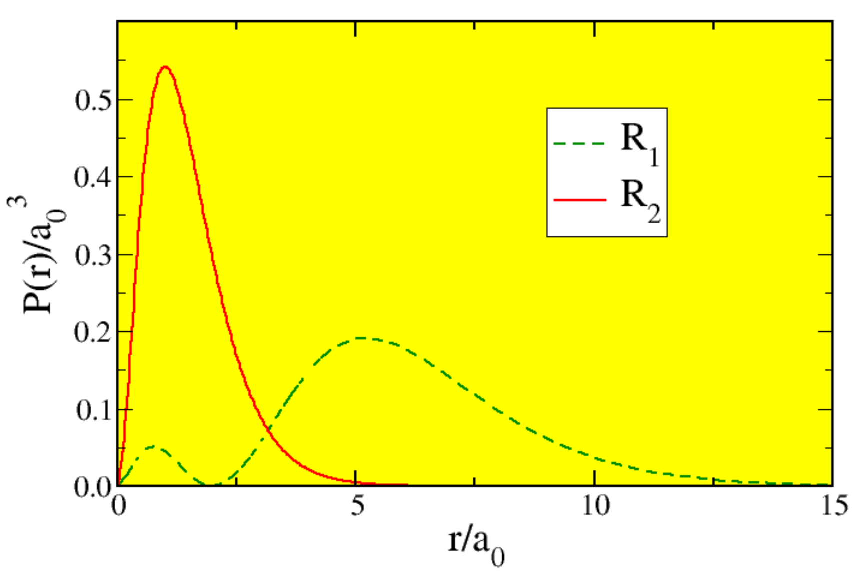

Figure 11.5.1: The probability to find a certain value of r for the first two Harmonic oscillator wave functions.

These probabilities are sketched in Fig. 11.5.1. The peaks are of some interest, since they show where the electrons are most likely to be found. Let's investigate this mathematically:

$$P_1 = 4r^2/a_0^3 e^{-2r/a_0}. \tag{11.5.3}$$

if we differentiate with respect to r, we get

$$\frac{d}{dr} P_1 = \frac{4}{a_0^3}\left(2re^{-2r/a_0} - 2r^2/a_0 e^{-2r/a_0}\right) \tag{11.5.4}$$

This is zero at $r = a_0$. For the first excited state this gets a little more complicated, and we will have to work harder to find the answer.