10.3: Completeness and time-dependence

In the discussion on formal aspects of quantum mechanics I have shown that the eigenfunctions to the Hamiltonian are complete, i.e., for any $\psi(x,t)$

$$\psi(x,t)=\sum_{n=1}^{\infty}c_n(t)\phi_n(x) \tag{10.3.1}$$

where

$$\hat{H}\phi_n(x)=E_n\phi_n(x). \tag{10.3.2}$$

We know, from the superposition principle, that

$$\psi(x,t)=\sum_{n=1}^{\infty}c_n(0)e^{-iEt/\hbar}\phi_n(x), \tag{10.3.3}$$

so that the time dependence is completely fixed by knowing $c(0)$ at time $t=0$ only! In other words if we know how the wave function at time $t=0$ can be written as a sum over eigenfunctions of the Hamiltonian, we can then determibe the wave function for all times.