12.5.3: The Zeeman Effect

When considering the Zeeman effect, it is easiest first to consider the hydrogen atom without hyperfine structure. Then m_j is a good quantum number, and the atom has a $2j+1$ degeneracy associated with the different possible values of m_j. In the presence of an external magnetic field, these different states will have different energies due to having different orientations of the magnetic dipoles in the external field. The splitting of these energy levels is called the Zeeman effect.

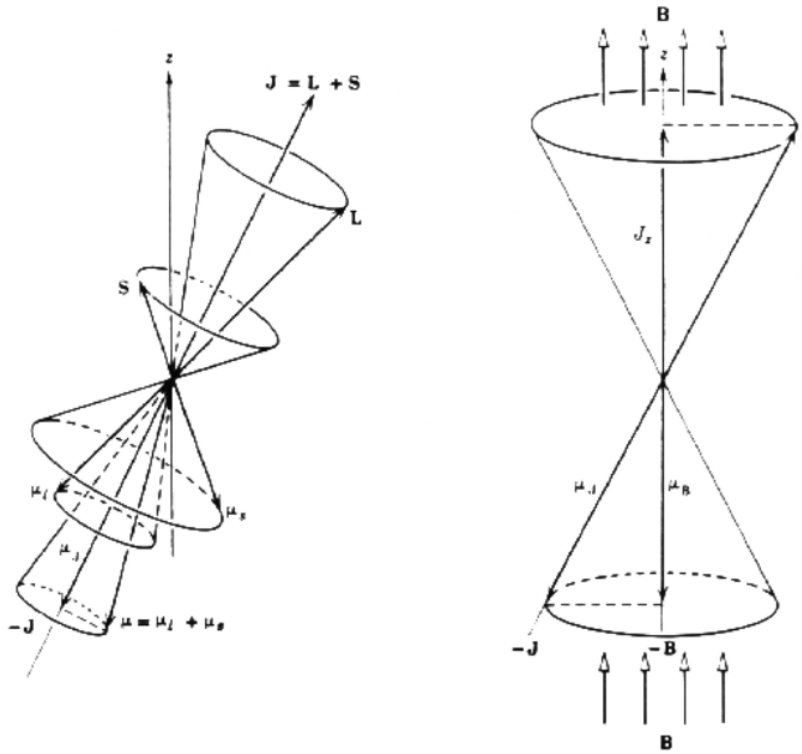

Figure 12.5.3.1:Geometry of the Zeeman effect. On the left, the total dipole moment μ precesses around the total angular momentum J. On the right, $\mathbf{J}$ precesses much more slowly about the magnetic field.

Figure 12.5.3.1illustrates the geometry of the Zeeman effect. The total magnetic dipole moment of the electron is

$$\mu=\mu_{\mathrm{l}}+\mu_{\mathrm{S}}=-\frac{\mu_b}{\hbar}(\mathbf{L}+2\mathbf{S}) \tag{12.5.3.1}$$

where $g_l=1$ and $g_s=2$ have been used. Because of the difference in the orbital and spin gyromagnetic ratios of the electron, this is not in general parallel to

$$\mathbf{J}=\mathbf{L}+\mathbf{S} \tag{12.5.3.2}$$

So, as $\mathbf{L}$ and $\mathbf{S}$ precess about $\mathbf{J}$, the total dipole moment μ also precesses about J. Assuming the external field to be in the z direction, this field causes $\mathbf{J}$ to precess about the z-axis. Typical internal magnetic fields in the hydrogen atom can be shown to be of the order 1 Tesla. If the external field is much weaker than 1 Tesla, which it is for almost all practical purposes, then the precession of $\mathbf{J}$ around the z-axis will take place much more slowly than the precession of μ around $\mathbf{J}$. The Hamiltonian of the Zeeman effect is

$$\Delta H_z=-\mu\cdot\mathbf{B}=-\mu_B B, \tag{12.5.3.3}$$

where μ_B is the projection of the dipole moment onto the direction of the field, the z-axis. Because of the difference in the precession rates, it is reasonable to evaluate μ_b by first evaluating the projection of μ onto $\mathbf{J}$, called μ_J, and then evaluating the projection of this onto $\mathbf{B}$, thus giving some average projection of μ onto $\mathbf{B}$. First, the projection of μ onto $\mathbf{J}$ is

$$\mu_J=\frac{\mu\cdot\mathbf{J}}{J}=-\frac{\mu_b}{\hbar}\frac{(\mathbf{L}+2\mathbf{S})\cdot(\mathbf{L}+\mathbf{S})}{J}. \tag{12.5.3.4}$$

Then

$$\mu_B = \mu_J \frac{\mathbf{J} \cdot \mathbf{B}}{JB} = \mu_J \frac{J_z}{J} = -\frac{\mu_b}{\hbar} \frac{(\mathbf{L} + 2\mathbf{S}) \cdot (\mathbf{L} + \mathbf{S}) J_z}{J^2}. \tag{12.5.3.5}$$

Evaluating the dot product using again that $J^2 = L^2 + S^2 + 2\mathbf{L} \cdot \mathbf{S}$, this becomes

$$\mu_B = -\frac{\mu_b}{\hbar} \frac{\left(3J^2 + S^2 - L^2\right)}{2J^2} J_z. \tag{12.5.3.6}$$

So when first order perturbation theory is applied, the energy shift is

$$\Delta E_z = \mu_b B g m_j, \tag{12.5.3.7}$$

where

$$g = 1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)} \tag{12.5.3.8}$$

is called the Landé g factor for the particular state being considered. Note that if $s = 0$, then $j = l$ so $g = 1$, and if $l = 0, j = s$ so $g = 2$. The Landé g factor thus gives some effective gyromagnetic ratio for the electron when the total dipole moment is partially from orbital angular momentum and partially from spin. From equation 97 , it can be seen that the energy shift caused by the Zeeman effect is linear in B and m_j, so for a set of states with particular values of n, l, and j, the individual states with different m_j will be equally spaced in energy, separated by $\mu_b B g$. However, the spacing will in general be different for a set of states with different n, l, and j due to the difference in the Landé g factor.

Including hyperfine structure with the Zeeman effect is more difficult, since the field associated with the proton magnetic dipole moment is weak, and hence it does not take a particularly strong external field to make the Zeeman effect comparable in magnitude to the strength of the hyperfine interactions. The approximation of small external field is thus not practical when discussing the Zeeman splitting of hyperfine structure. However, it can be treated, and the result for the most important case of the Zeeman splitting of the hyperfine levels in the ground state of hydrogen [1] is shown in Figure 12.5.3.2 The degeneracy of the triplet state is lifted, the three states of $m_f = -1, 0, +1$ having different energies in the external field. Notice how the splitting is linear for small external field, but then deviates as the field gets larger. The " 21 cm " transitions shown on the right will have slightly different energies, and measuring the amount of this splitting is a good tool for radio astronomers to measure magnetic fields in the interstellar medium.

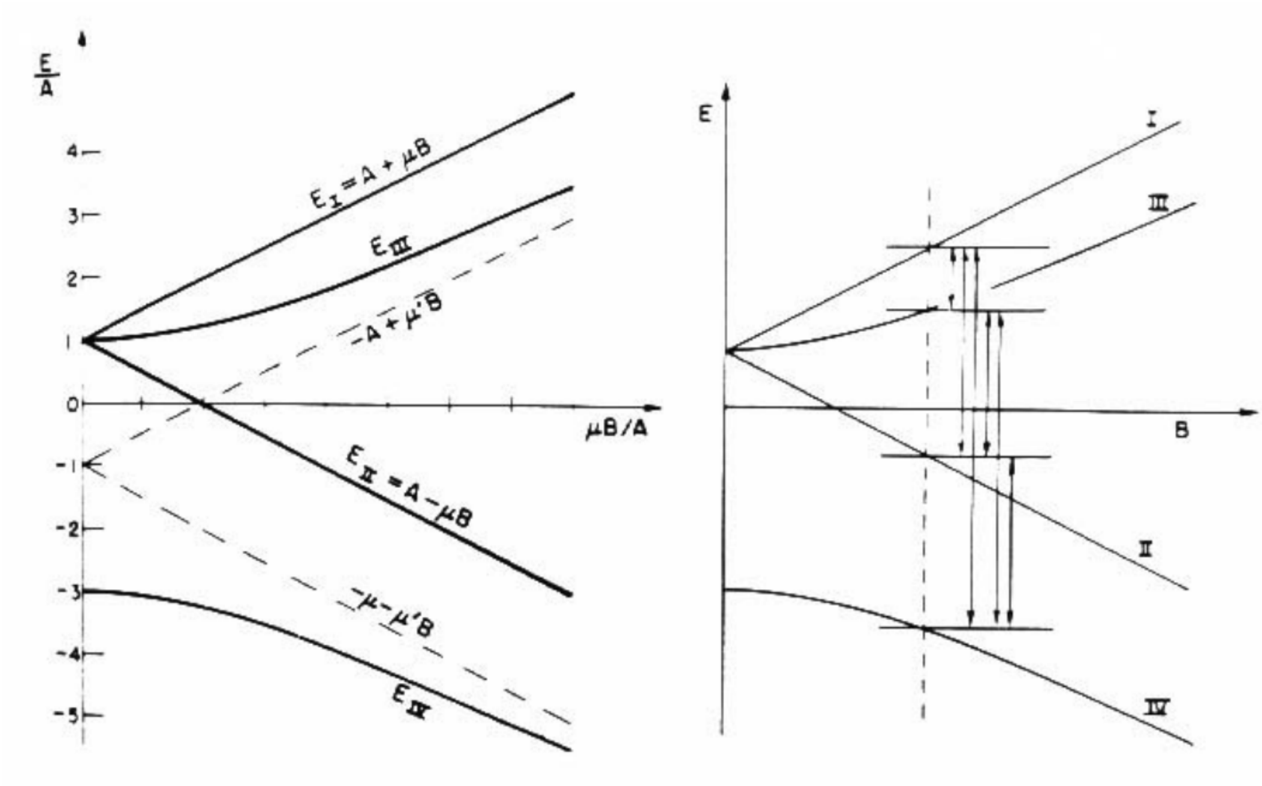

Figure 12.5.3.2:On left, Zeeman splitting of the hyperfine levels in the ground state $(1s_{1/2})$ of hydrogen. On right, some possible transitions between these states.

Footnote

1. See Feynman volume III chapter for a discussion of the calculation of

Contributors and Attributions

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