2.1: Conservative Fields

In all our discussions I will only consider forces which are conservative, i.e., where the total energy is a constant. This excludes problems with friction. For such systems we can split the total energy in a part related to the movement of the system, called the kinetic energy (Greek κivelv=to move), and a second part called the potential energy, since it describes the potential of a system to produce kinetic energy.

An extremely important property of the potential energy is that we can derive the forces as a derivavtive of the potential energy, typically denoted by $V(r)$, as

$$F=-\left(\frac{\partial}{\partial x} V(r), \frac{\partial}{\partial y} V(r), \frac{\partial}{\partial z} V(r)\right). \tag{2.1.1}$$

A typical example of a potential energy function is the one for a particle of mass m in the earth's gravitationol field, which in the flat-earth limit is written as $V(r)=mgz$. This leads to a gravitational force

$$F=(0,0,-mg) \tag{2.1.2}$$

Of course when total energy is conserved, that doesn't define the zero of energy. The kinetic energy is easily defined to be zero when the particle is not moving, but we can add any constant to the potential energy, and the forces will not change. One typically takes $V(r)=0$ when the length of r goes to ∞.