# 11.1: The momentum operator as a vector

First of all we know from classical mechanics that velocity and momentum, as well as position, are represented by vectors. Thus we need to represent the momentum operator by a vector of operators as well,

$$\hat{p}=\left(\frac{\hbar}{i}\frac{\partial}{\partial x},\frac{\hbar}{i}\frac{\partial}{\partial y},\frac{\hbar}{i}\frac{\partial}{\partial z}\right). \tag{11.1.1}$$

There exists a special notation for the vector of partial derivatives, which is usually called the gradient, and one writes

$$\hat{p}=\frac{\hbar}{i}\nabla. \tag{11.1.2}$$

We now that the energy, and Hamiltonian, can be written in classical mechanics as

$$E=\frac{1}{2}mv^2+V(x)=\frac{1}{2m}p^2+V(x), \tag{11.1.3}$$

where the square of a vector is defined as the sum of the squares of the components,

$$(v_1,v_2,v_3)^2=v_1^2+v_2^2+v_3^2. \tag{11.1.4}$$

The Hamiltonian operator in quantum mechanics can now be read of from the classical one,

$$\hat{H}=\frac{1}{2m}\hat{p}^2+V(x)=-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)+V(x). \tag{11.1.5}$$

Let me introduce one more piece of notation: the square of the gradient operator is called the Laplacian, and is denoted by $\Delta$.