

11.3: Solutions independent of angular variables

Initially we shall just restrict ourselves to those cases where the wave function is independent of θ and φ , i.e.,

$$\phi(r, \theta, \varphi) = R(r). \quad (11.3.1)$$

In that case the Schrödinger equation becomes (why?)

$$-\frac{\hbar^2}{2m} \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} R(r) \right) + V(r)R(r) = ER(r) \quad (11.3.2)$$

One often simplifies life even further by substituting $u(r)/r = R(r)$, and multiplying the equation by r at the same time,

$$-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial r^2} u(r) + V(r)u(r) = Eu(r) \quad (11.3.3)$$

Of course we shall need to normalise solutions of this type. Even though the solution are independent of θ and φ , we shall have to integrate over these variables. Here a geometric picture comes in handy. For each value of r , the allowed values of x range over the surface of a sphere of radius r . The area of such a sphere is $4\pi r^2$. Thus the integration over r, θ, φ can be reduced to

$$\int_{\text{all space}} f(r) dx dy dz = \int_0^\infty f(r) 4\pi r^2 dr. \quad (11.3.4)$$

Especially, the normalisation condition translates to

$$\int_0^\infty |R(r)|^2 4\pi r^2 dr = \int_0^\infty |u(r)|^2 4\pi dr = 1 \quad (11.3.5)$$

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