11.3: Solutions independent of angular variables

Initially we shall just restrict ourselves to those cases where the wave function is independent of θ and φ, i.e.,

$$\phi(r, \theta, \varphi) = R(r). \tag{11.3.1}$$

In that case the Schrödinger equation becomes (why?)

$$-\frac{\hbar^2}{2m}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}R(r)\right) + V(r)R(r) = ER(r) \tag{11.3.2}$$

One often simplifies life even further by substituting $u(r)/r = R(r)$, and multiplying the equation by r at the same time,

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial r^2}u(r) + V(r)u(r) = Eu(r) \tag{11.3.3}$$

Of course we shall need to normalise solutions of this type. Even though the solution are independent of θ and φ, we shall have to integrate over these variables. Here a geometric picture comes in handy. For each value of r, the allowed values of x range over the surface of a sphere of radius r. The area of such a sphere is $4\pi r^2$. Thus the integration over r, θ, φ can be reduced to

$$\int_{\text{all space}} f(r)dxdydz = \int_0^\infty f(r)4\pi r^2 dr. \tag{11.3.4}$$

Especially, the normalisation condition translates to

$$\int_0^\infty |R(r)|^2 4\pi r^2 dr = \int_0^\infty |u(r)|^2 4\pi dr = 1 \tag{11.3.5}$$