

8.2: Expectation Values of x^2 and p^2 for the Harmonic Oscillator

As an example of all we have discussed let us look at the harmonic oscillator. Suppose we measure the average deviation from equilibrium for a harmonic oscillator in its ground state. This corresponds to measuring \bar{x} . Using

$$1. \quad \phi_0(x) = \left(\frac{m\omega}{\hbar\pi}\right)^{1/4} \exp\left(-\frac{m\omega}{2\hbar}x^2\right). \quad (8.2.1)$$

we find that

$$\langle x \rangle = \int_{-\infty}^{\infty} \left(\frac{m\omega}{\hbar\pi}\right)^{1/2} x \exp\left(-\frac{m\omega}{\hbar}x^2\right) dx = 0. \quad (8.2.2)$$

Qn Why is it 0? Similarly, using $\hat{p} = -i\hbar \frac{d}{dx}$ and

$$\hat{p} \left(\frac{m\omega}{\hbar\pi}\right)^{1/4} \exp\left(-\frac{m\omega}{2\hbar}x^2\right) = im\omega x \left(\frac{m\omega}{\hbar\pi}\right)^{1/4} \exp\left(-\frac{m\omega}{2\hbar}x^2\right) \quad (8.2.3)$$

we find

$$\langle p \rangle = 0 \quad (8.2.4)$$

More challenging are the expectation values of x^2 and p^2 . Let me look at the first one first:

$$\begin{aligned} \langle x^2 \rangle &= \int_{-\infty}^{\infty} \left(\frac{m\omega}{\hbar\pi}\right)^{1/2} x^2 \exp\left(-\frac{m\omega}{\hbar}x^2\right) dx \\ &= \left(\frac{\hbar}{m\omega}\right) \pi^{-1/2} \int_{-\infty}^{\infty} \exp(-y^2) dy \\ &= \left(\frac{\hbar}{m\omega}\right) \frac{1}{2} \end{aligned} \quad (8.2.5)$$

Now for \hat{p}^2 ,

$$\hat{p}^2 \exp\left(-\frac{m\omega}{2\hbar}x^2\right) = (-(m\omega x)^2 + \hbar m\omega) \exp\left(-\frac{m\omega}{2\hbar}x^2\right). \quad (8.2.6)$$

Thus,

$$\begin{aligned} \langle \hat{p}^2 \rangle &= \int_{-\infty}^{\infty} \left(\frac{m\omega}{\hbar\pi}\right)^{1/2} (-(m\omega x)^2 + \hbar m\omega) \exp\left(-\frac{m\omega}{\hbar}x^2\right) dx \\ &= \hbar m\omega \pi^{-1/2} \int_{-\infty}^{\infty} (1 - y^2) \exp(-y^2) dy \\ &= \hbar m\omega \frac{1}{2}. \end{aligned} \quad (8.2.7)$$

This is actually a form of the uncertainty relation, and shows that

$$\Delta x \Delta p \geq \frac{1}{2} \hbar \quad (8.2.8)$$

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