3.3: Analysis of the wave equation

One of the important aspects of the Schrödinger equation(s) is its linearity. For the time independent Schrödinger equation, which is usually called an eigenvalue problem, the only consequence we shall need here, is that if $\phi_i(x)$ is an eigenfunction (a solution for E_i) of the Schrödinger equation, so is $A\phi_i(x)$. This is useful in defining a probability, since we would like

$$\int_{-\infty}^{\infty} |A|^2 |\phi_i(x)|^2 \, dx = 1 \tag{3.3.1}$$

Given $\phi_i(x)$ we can thus use this freedom to ”normalize” the wavefunction! (If the integral over $|\phi(x)|^2$ is finite, i.e., if $\phi(x)$ is “normalizable”; not all functions are).

Example 3.3.1

As an example suppose that we have a Hamiltonian that has the function $\psi_i(x) = e^{-x^2/2}$ as eigenfunction. This function is not normalized since

$$\int_{-\infty}^{\infty} |\phi_i(x)|^2 \, dx = \sqrt{\pi}. \tag{3.3.2}$$

The normalized form of this function is

$$\frac{1}{\pi^{1/4}} e^{-x^2/2}. \tag{3.3.3}$$

We need to know a bit more about the structure of the solution of the Schrödinger equation – boundary conditions and such. Here I shall postulate the boundary conditions, without any derivation.

1. $\phi(x)$ is a continuous function, and is single valued.
2. $$\int_{-\infty}^{\infty} |\phi(x)|^2 \, dx \tag{3.3.4}$$

   must be finite, so that

   $$P(x) = |\phi(x)|^2 \int_{-\infty}^{\infty} |\psi(x)|^2 \, dx \tag{3.3.5}$$

   is a probability density.
3. $\frac{\partial \phi(x)}{\partial x}$ is continuous except where $V(x)$ has an infinite discontinuity.

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