7.2: Dimensionless Coordinates

The classical energy (Hamiltonian) is

$$E=\frac{1}{2}m\bar{x}^2+\frac{1}{2}m\omega^2x^2=\frac{1}{2m}p^2+\frac{1}{2}m\omega^2x^2 \tag{7.2.1}$$

The quantum Hamiltonian operator is thus

$$\widehat{H}=\frac{1}{2m}\frac{1}{2m}\hat{p}^2+\frac{1}{2}m\omega^2x^2=-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+\frac{1}{2}m\omega^2x^2. \tag{7.2.2}$$

And we thus have to solve Schrödinger's equation

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\phi(x)+\frac{1}{2}m\omega^2x^2\phi(x)=E\phi(x) \tag{7.2.3}$$

In order to treat this equation it is better to remove all the physical constants, and go over to dimensionless coordinates

$$y=\sqrt{\frac{m\omega}{\hbar}}x,\quad \epsilon=\frac{E}{\hbar\omega}. \tag{7.2.4}$$

When we substitute these new variables into the Schrödinger equation we get, using

$$\frac{d}{dx}f(y)=\frac{dy}{dx}\frac{d}{dy}f(y)=\sqrt{\frac{m\omega}{\hbar}}\frac{d}{dy}f(y), \tag{7.2.5}$$

that $(\phi(x)=u(y))$

$$-\frac{\hbar^2}{2m}\frac{m\omega}{\hbar}\frac{d^2}{dy^2}u(y)+\frac{1}{2}m\omega^2\frac{\hbar}{m\omega}y^2u(y)=\epsilon\hbar\omega u(y) \tag{7.2.6}$$

Cancelling the common factor

$$\hbar\omega \text{ we find}$$
$$\tfrac{d^2}{dy^2}u(y)+\left(2\epsilon-y^2\right)u(y)=0 \tag{7.2.7}$$

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