

11.2: Spherical Coordinates

The solution to Schrödinger's equation in three dimensions is quite complicated in general. Fortunately, nature lends us a hand, since most physical systems are "rotationally invariant", i.e., $V(x)$ depends on the size of x , but not its direction! In that case it helps to introduce spherical coordinates, as denoted in Fig. 11.2.1.

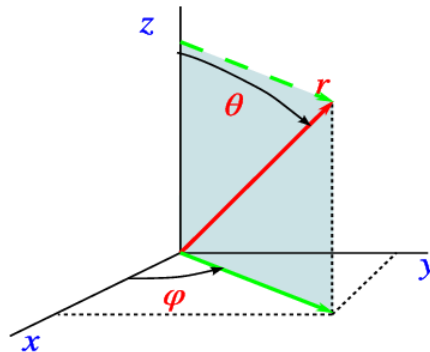


Figure 11.2.1: The spherical coordinates r, θ, ϕ .

The coordinates r, θ and ϕ are related to the standard ones by

$$\begin{aligned} x &= r \cos \phi \sin \theta \\ y &= r \sin \phi \sin \theta \\ z &= r \cos \theta \end{aligned}$$

where $0 < r < \infty$, $0 < \theta < \pi$ and $0 < \phi < 2\pi$. In these new coordinates we have

$$\Delta f(r, \theta, \phi) = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} f(r, \theta, \phi) \right) - \frac{1}{r^2} \left[\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} f(r, \theta, \phi) \right) + \frac{\partial^2}{\partial \phi^2} f(r, \theta, \phi) \right]. \quad (11.2.1)$$

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