11.2: Spherical Coordinates

The solution to Schrödinger's equation in three dimensions is quite complicated in general. Fortunately, nature lends us a hand, since most physical systems are "rotationally invariant", i.e., $V(x)$ depends on the size of x, but not its direction! In that case it helps to introduce spherical coordinates, as denoted in Fig. $11.2.1$.

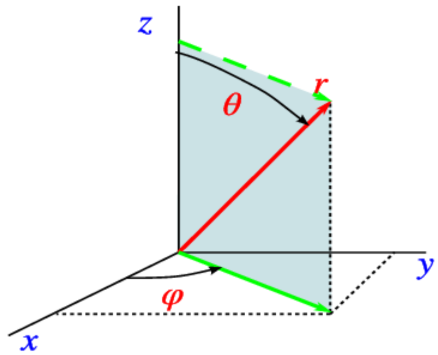

Figure $11.2.1$: The spherical coordinates r, θ, φ .

The coordinates r, θ and ϕ are related to the standard ones by

$$\begin{aligned} x &= r\cos\varphi\sin\theta \\ y &= r\sin\varphi\sin\theta \\ &= r\cos\theta \end{aligned}$$

where $0 < r < \infty, 0 < \theta < \pi$ and $0 < \phi < 2\pi$. In these new coordinates we have

$$\Delta f(r,\theta,\varphi) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}f(r,\theta,\varphi)\right) - \frac{1}{r^2}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}f(r,\theta,\varphi)\right) + \frac{\partial^2}{\partial\varphi^2}f(r,\theta,\varphi)\right]. \tag{11.2.1}$$