13.10: Quantum Mechanics- a Poor Person's Guide

The Wave Function

The observation of the world lets us know that something very odd is going on. The Double Slit Experiment is the prototypical experiment of Quantum Physics. Guided by this experiment and others like it, a mathematical theory, called Quantum Theory has been developed to give results which are in agreement with our observations; the "weirdness" we find in Nature is reflected in the way in which the Quantum Theory is constructed. It goes a bit like this.

The state of a physical system is described by a "wave function", usually denoted by the symbolΨ.. In particular cases, we will know more or less exactly what function this is - e.g a sine or a cosine, a quadratic expression, etc. This wave function can depend on time, spatial coordinates, etc. Quantum Theory tells us that to make calculations about real measurements that could be made on the system, we must **take the square of the wave function.** The value so obtained will give us the **probability** of obtaining, through measurement on the system, a particular value of the quantity we are interested in. We would expect a theory from Classical Physics to give us an **exact** value of the quantity we were interested in; here, however, the best we can do is calculate a probability of obtaining the value. The wave function is also called a Probability Amplitude, for this reason.

For example, if, on the basis of our knowledge of conditions in which a particle might find itself (in a box, with a magnetic field, for example) we knew how to write down the particle's wave function; and let's say this wave function depended on its position (call that x) and the time measured from some starting time (call that t). In that case, we would write its wave function as $\Psi(x,t)$. x could take any values of position that the particle could reach. If then, we wanted to know the chance, or probability, of finding the particle at a particular value of x, say x = 45 cm, at a particular time of, say 7 seconds, Quantum Theory tells us that the answer is$\boldsymbol{\Psi(45,7)^2}$. Note that this is very different from Classical Physics; there, we might know that the "equation of motion" of the particle was, e.g. $x = 6t$; then the answer to our question would be that the position of the particle at time 7 seconds, would be the exactly x = 42 cm.

Now let us look at the odd way in which Quantum Theory does its calculations about the world. Suppose we have an experiment about a physical process which can happen in more than one way, and we know the Probability Amplitude (or wave function) for each way. To calculate what results we would expect in an experiment which does not distinguish which way actually happens, we have to **first** add the Probability Amplitudes; **then** we square the result of this addition to get the answer to compare to measurement. If, on the other hand, the experiment does distinguish which way actually happens, we **square** the Probability Amplitudes **before** adding them. To see how this works, let's look at the Double Slit Experiment for electrons.

The Double Slit Experiment Again

Suppose thatΨ_1 is the Probability Amplitude for the electron's going through one slit, andΨ_2 is the Probability Amplitude for its going through the other slit; then the Probability Amplitude to calculate the results of an experiment which does not determine through which slit the electron goes (call it Experiment **I**) is written as $\boldsymbol{\Psi_I = \Psi_1 + \Psi_2}$. [This is called a "(linear) superposition of probable states"]. Now, if we want to make a theoretical calculation of the results of a real experiment we might carry out (e.g. the distribution of the electrons on the detecting screen), we have to take the **square** of this total Probability Amplitude, i.e. $\boldsymbol{\Psi_I^2 = \{\Psi_1 + \Psi_2\}^2}$. Multiplying out, this result can be written as $\Psi_I^2 = \Psi_1^2 + \Psi_2^2 + 2\Psi_1\Psi_2$. (In this not-quite-correct formulation, Ψ_1 can equal $-\Psi_2$).

Suppose we have a set-up which has equal size slits, located at the same distance from the source of electrons, then the probability that the electron goes through slit number 1 is equal to the probability that it goes through slit number 2. We express this fact by writing $\boldsymbol{\Psi_1^2} = \boldsymbol{\Psi_2^2}$ = 0.5 (or 50%).

Then : EITHER$\boldsymbol{\Psi_1} = +\boldsymbol{\Psi_2}$ and the result is **1** (or 100%);
OR$\boldsymbol{\Psi_1} = -\boldsymbol{\Psi_2}$, and the result is **0** (or 0%).

For the Double Slit Experiment, this is obviously (??) a calculation of the interference pattern, with its maxima (**1**, in some arbitrary units) and minima (**0**) which we observe. However, if our experiment has some means for detecting, even in principle, which hole the electron goes through (Experiment **II**), the result of **this** experiment must be written as $\boldsymbol{\Psi_{II}^2 = \Psi_1^2 + \Psi_2^2}$. This is clearly (??) the case in which **no** interference is observed.

Thus the Quantum Theory has managed to come up with a recipe to give calculations which agree with the observations we make on this weird world in which we live.

What can we say about the wave function (Probability Amplitude) of the electron after it has gone through the slit system, but **just before** we look at it to decide which slit it went through? In this case, Nature tells us we must write its wave function as .$\Psi = \Psi_1 + \Psi_2$, as explained above. But if we make a measurement to determine which slit the electron did go through, we know we must get the result Ψ_1 (if it went through slit number 1) ORΨ_2 (if it went through slit number 2). Then we say that the wave function has **collapsed** on to its final value.

What is an Electron?

According to Schrödinger, the electron can be represented by a wave-function, which contains all the information we can know about the particle. If an electron looks like anything we are familiar with (and it doesn't!!), it comes closest to a small "packet" of waves confined to a region of space Δx. This wave function obeys a wave equation first written down by Schrödinger. The **square** of the wave function gives the probability of finding it at a given place (and time).

(MATH NOTE: To represent such a function, we need a superposition of many wave forms, with a "spread" of wavelengths. Since $p = h/\lambda$ this implies a corresponding spread in momentum; this can be calculated to be $\Delta p = h/\Delta x$ - as we might have expected from Heisenberg's Uncertainty Principle).

Schrödinger's Cat (or Is the Moon There when Nobody Looks?)

By analogy, in the case of Schrödinger's cat, the state of the cat **before we open the box** is :
$\Psi_{cat} = \Psi_{alive} + \Psi_{dead}$. If we have designed the experiment so that there is equal probability for finding the cat alive or dead, we must have $\Psi_{alive}^2 = \Psi_{dead}^2$. When we open the box, since the cat must be alive OR dead, the total wave function, Ψ_{cat} must be EITHER $= \Psi_{alive}$ OR $= \Psi_{dead}$; we don't know which before we open the box. However, it appears that **just before** we open the box, the cat is NEITHER alive OR dead, but a superposition of the two states! Just try telling that to your grand mother!

The Implications of the Quantum

Quantum Physics forces us to the conclusion that:

a. there are no certainties, only probabilities - and the future is unpredictable.
b.Physical properties have no objective reality independent of the act of observation OR the act of measurement can, in principle, act *instantaneously* over enormous distances (i.e. non- local interactions exist). (*Bell's Theorem and the experiments of Aspect et al.*)

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