12.5.2: The Lamb Shift

According to Dirac and Schrödinger theory, states with the same n and j quantum numbers but different l quantum numbers ought to be degenerate. However, a famous experiment by Lamb and Retherford in 1947 showed that the $2s_{1/2}(n=2, l=0, j=1/2)$ and $2p_{1/2}(n=2, l=1, j=1/2)$ states of the hydrogen atom were not degenerate, but that the s state had slightly higher energy by an amount now known to be $E/h = 1057.864\text{MHz}$. The effect is explained by the theory of quantum electrodynamics, in which the electromagnetic interaction itself is quantized. Some of the effects of this theory which cause the Lamb shift are shown in the Feynman diagrams of figure 5. Table 3 shows how much each of these contribute to the splitting of $2s_{1/2}$ and $2p_{1/2}$. The most important effect is illustrated by the center diagram, which is a result of the fact that the ground state of the electromagnetic field is not zero, but rather the field undergoes "vacuum fluctuations" that interact with the electron. Any discussion of the calculation is beyond the scope of this paper, so the answers will merely be given. For $l = 0$,

$$\Delta E_{Lamb} = \alpha^5 mc^2 \frac{1}{4n^3}\{k(n,0)\} \tag{12.5.2.1}$$

where $k(n,0)$ is a numerical factor which varies slightly with n from 12.7 to 13.2. For $l \neq 0$,

$$\Delta E_{Lamb} = \alpha^5 mc^2 \frac{1}{4n^3}\left\{k(n,l) \pm \frac{1}{\pi\left(j+\frac{1}{2}\right)\left(l+\frac{1}{2}\right)}\right\} \tag{12.5.2.2}$$

for $j = l \pm 1/2$, where $k(n,l)$ is a small numerical factor < 0.05 which varies slightly with n and l. Notice that the Lamb shift is very small except for $l = 0$.

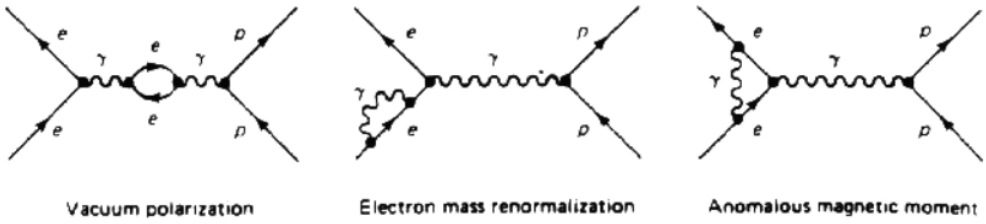

Figure 12.5.2.1:Feynman loop diagrams showing some effects that contribute to the Lamb shift

Table 12.5.2.1: Contribution of different effects to the energy splitting of $2s_{1/2}$ and $2p_{1/2}$ in hydrogen. Numbers are given in units of frequency $\nu = E/h$.

Effect	Energy contribution
Vacuum polarization	-27 MHz
Electron mass renormalization	+1017 MHz
Anomalous magnetic moment	+68 MHz
Total	+1058 MHz

Contributors and Attributions

- Randal Telfer (JWST Astronomical Optics Scientist, Space Telescope Science Institute)

This page titled 12.5.2: The Lamb Shift is shared under a CC BY-NC-SA 2.0 license and was authored, remixed, and/or curated by Niels Walet via source content that was edited to the style and standards of the LibreTexts platform.