3.4: Nuclear mass formula

There is more structure in Figure 4.3.1 than just a simple linear dependence on A. A naive analysis suggests that the following terms should play a role:

1. *Bulk energy*: This is the term studied above, and saturation implies that the energy is proportional to $B_{\text{bulk}} = \alpha A$.
2. *Surface energy*: Nucleons at the surface of the nuclear sphere have less neighbors, and should feel less attraction. Since the surface area goes with R^2, we find $B_{\text{surface}} = -\beta A$.
3. *Pauli or symmetry energy*: nucleons are fermions (will be discussed later). That means that they cannot occupy the same states, thus reducing the binding. This is found to be proportional to $B_{\text{symm}} = -\gamma(N/2 - Z/2)^2/A^2$.
4. *Coulomb energy*: protons are charges and they repel. The average distance between is related to the radius of the nucleus, the number of interaction is roughly Z^2 (or $Z(Z-1)$). We have to include the term $B_{\text{Coul}} = -\epsilon Z^2/A$.

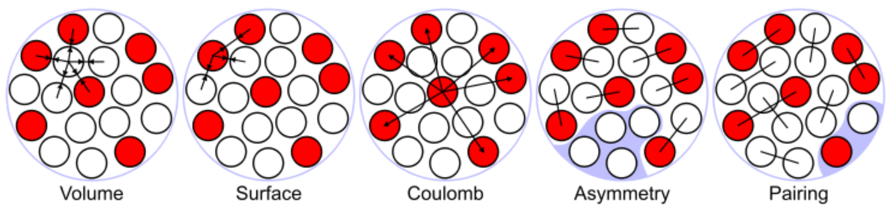

Figure 3.4.1: Illustration of the terms of the semi-empirical mass formula in the liquid drop model of the atomic nucleus. (CC BY-SA; Daniel FR).

Taking all this together we fit the formula

$$B(A, Z) = \alpha A - \beta A^{2/3} - \gamma(A/2 - Z)^2 A^{-1} - \epsilon Z^2 A^{-1/3} \tag{3.4.1}$$

to all know nuclear binding energies with $A \geq 16$ (the formula is not so good for light nuclei). The fit results are given in Table 3.4.1.

Table 3.4.1: Fit of masses to Equation 3.4.1.

parameter	value
α	15.36 MeV
β	16.32 MeV
γ	90.45 MeV
ϵ	0.6928 MeV

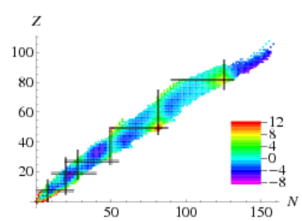

Figure 3.4.2: Difference between fitted binding energies and experimental values (color), as a function of N and Z.

In Table 3.4.1 we show how well this fit works. There remains a certain amount of structure, see below, as well as a strong difference between neighbouring nuclei. This is due to the superfluid nature of nuclear material: nucleons of opposite momenta tend to anti-align their spins, thus gaining energy. The solution is to add a pairing term to the binding energy,

$$B_{\text{pair}} = \begin{cases} A^{-1/2} & \text{for } N \text{ odd, } Z \text{ odd} \\ -A^{-1/2} & \text{for } N \text{ even, } Z \text{ even} \end{cases}$$

The results including this term are significantly better, even though all other parameters remain at the same position (Table 3.4.2). Taking all this together we fit the formula

$$B(A,Z) = \alpha A - \beta A^{2/3} - \gamma (A/2-Z)^2 A^{-1} - \delta B_{\text{pair}}(A,Z) - \epsilon Z^2 A^{-1/3} \tag{3.4.2}$$

Table 3.4.2: Fit of masses to Equation 3.4.2.

parameter	value
α	15.36 MeV
β	16.32 MeV
γ	90.46 MeV
δ	11.32 MeV
ϵ	0.6929 MeV

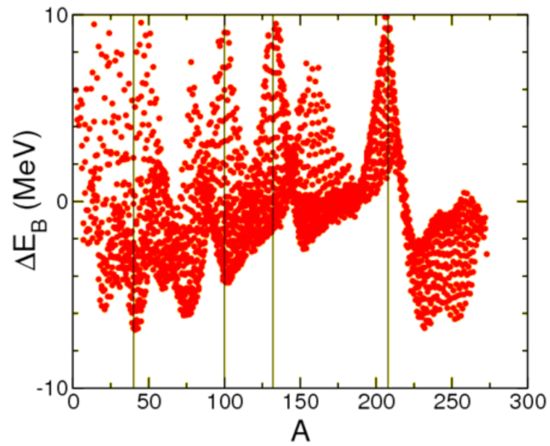

Figure 3.4.3: B/A versus A, mass formula subtracted.