5.2: Antiparticles

Both the Klein-Gordon and the Dirac equation have a really nasty property. Since the relativistic energy relation is quadratic, both equations have, for every positive energy solution, a negative energy solution. We don't really wish to see such things, do we? Energies are always positive and this is a real problem. The resolution is surprisingly simple, but also very profound – It requires us to look at the problem in a very different light.

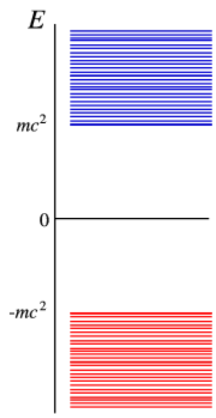

Figure $5.2.1$: A schematic picture of the levels in the Dirac equation

In Figure $5.2.1$ we have sketched the solutions for the Dirac equation for a free particle. It has a positive energy spectrum starting at mc^2 (you cannot have a particle at lower energy), but also a negative energy spectrum below $-mc^2$. The interpretation of the positive energy states is natural – each state describes a particle moving at an energy above mc^2. Since we cannot have negative energy states, their interpretation must be very different. The solution is simple: We assume that in an *empty* vacuum all negative energy states are filled (the "Dirac sea"). Excitations relative to the vacuum can now be obtained by adding particles at positive energies, or creating *holes* at negative energies. Creating a hole takes energy, so the hole states appear at positive energies. They do have opposite charge to the particle states, and thus would correspond to positrons! This shows a great similarity to the behaviour of semiconductors, as you may well know. The situation is explained in Figure $5.2.2$.

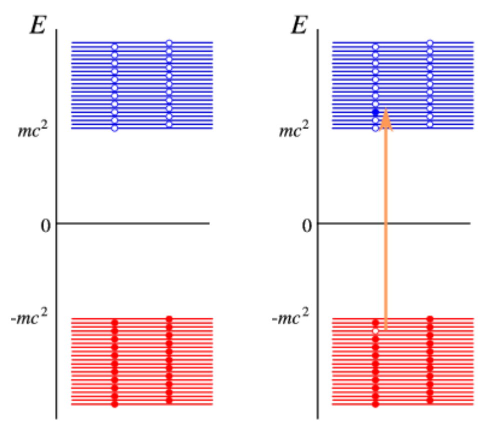

Figure $5.2.2$: A schematic picture of the occupied and empty levels in the Dirac equation. The promotion of a particle to an empty level corresponds to the creation of a positron-electron pair, and takes an energy larger than $2mc^2$.

Note that we have ignored the infinite charge of the vacuum (actually, we subtract it away assuming a constant positive background charge.) Removing infinities from calculations is a frequent occurrence in relativistic quantum theory (RQT). Many *unmeasurable* quantities become infinite, and we are only interested in the finite part remaining after removing the infinities. This process is part of what is called *renormalization*, which is a systematic procedure to extract finite information from infinite answers!