3.5: Stability of Nuclei

In Figure $3.5.1$ we have color coded the nuclei of a given mass $A = N + Z$ by their mass, red for those of lowest mass through to magenta for those of highest mass. We can see that typically the nuclei that are most stable for fixed A have more neutrons than protons, more so for large A increases than for low A. This is the "neutron excess".

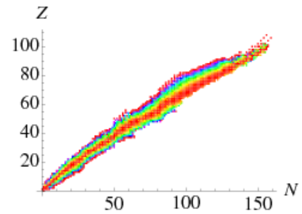

Figure $3.5.1$: The valley of stability

β decay

If we look at the mass of nuclides with fixed nucleon number A (i.e., roughly perpendicular cuts through the valley of stability in Figure $3.5.1$), we can see that the masses vary strongly,

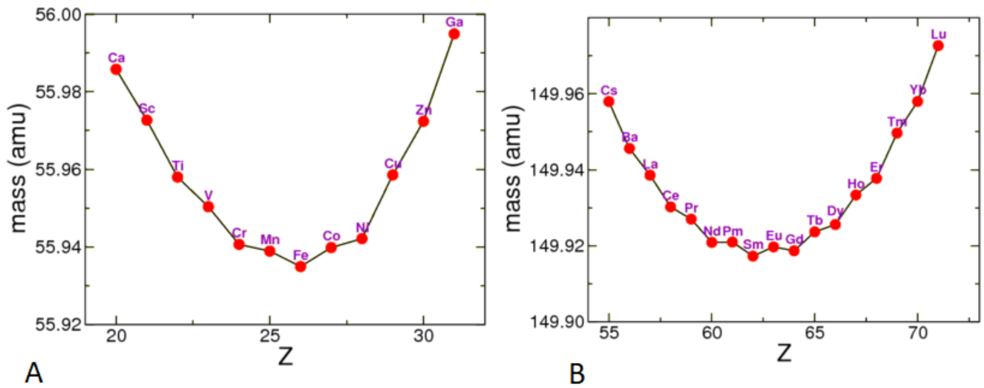

Figure $3.5.2$: The negative of binding energy per nucleon for nuclides with fixed A: (left) $A = 56$ and (right) $A = 150$. The profile of binding energy across the valley of stability is roughly a parabola (e.g., Iron-56 is stable, while Vandium-56 is unstable to β^- decay.

It is known that a *free* neutron is not a stable particle, it actually decays by emission of an electron and an antineutrino,

$$n \rightarrow p + e^- + \bar{\nu}_e.$$

The reason that this reaction can take place is that it is endothermic,

$$m_n c^2 > m_p c^2 + m_e c^2.$$

Here we assume that the neutrino has no mass.

The degree of allowance of such a reaction is usually expressed in a Q value, the amount of energy released in such a reaction,

$$Q = m_n c^2 - m_p c^2 - m_e c^2 \tag{3.5.1}$$

$$= 939.6 - 938.3 - 0.5 = 0.8 \text{ MeV}. \tag{3.5.2}$$

Generically it is found that two reaction may take place, depending on the balance of masses. Either a neutron "β decays" as sketched above, or we have the inverse reaction

$$p \rightarrow n + e^+ + \nu_e.$$

For historical reason the electron or positron emitted in such a process is called a β particle. Thus in β^- decay of a nucleus, a nucleus of Z protons and N neutrons turns into one of $Z+1$ protons and $N-1$ neutrons (moving towards the right in Figure $3.5.2A$. In β^+ decay the nucleus moves to the left. Since in that figure I am using atomic masses, the Q factor is

$$
\begin{aligned}
Q_{\beta^-} &= M(A,Z)c^2 - M(A,Z+1)c^2, \\
Q_{\beta^-} &= M(A,Z)c^2 - M(A,Z-1)c^2 - 2m_e c^2.
\end{aligned}
$$

The double electron mass contribution in this last equation because the atom looses one electron, as well as emits a positron with has the same mass as the electron.

In similar ways we can study the fact whether reactions where a single nucleon (neutron or proton) is emitted, as well as those where more complicated objects, such as Helium nuclei (α particles) are emitted. I shall return to such processed later, but let us note the Q values,

$$
\begin{aligned}
\text{neutron emission} \qquad Q &= [M(A,Z) - M(A-1,Z) - m_n]c^2, \\
\text{proton emission} \qquad Q &= [M(A,Z) - M(A-1,Z-1) - M(1,1)]c^2, \\
\alpha\text{ emission} \qquad Q &= [M(A,Z) - M(A-4,Z-2) - M(4,2)]c^2, \\
\text{break up} \qquad Q &= [M(A,Z) - M(A-A_1,Z-Z_1) - M(A_1,Z_1)]c^2.
\end{aligned}
$$