9.2: Invariant Mass

One of the key numbers we can extract from mass and momentum is the *invariant mass*, a number independent of the Lorentz frame we are in

$$W^2c^4 = (\sum_i E_i)^2 - (\sum_i \vec{p}_i)^2 c^2 .$$

This quantity takes it most transparent form in the center-of-mass, where $\sum_i \vec{p}_i = 0$. In that case

$$W = E_{\rm CM}/c^2 ,$$

and is thus, apart from the factor $1/c^2$, nothing but the energy in the CM frame. For a single particle $W = m_0$, the rest mass.

Most considerations about processes in high energy physics are greatly simplified by concentrating on the invariant mass. This removes the Lorentz-frame dependence of writing four momenta. I

As an example we look at the collision of a proton and an antiproton at rest, where we produce two quanta of electromagnetic radiation (γ's), see Figure $9.2.1$, where the anti proton has three-momentum $(p, 0, 0)$, and the proton is at rest.

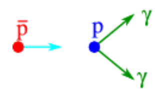

Figure $9.2.1$: A sketch of a collision between a proton with velocity v and an antiproton at rest producing two γ quanta.

The four-momenta are

$$\begin{aligned} p_{\rm p} &= (p_{\rm lab}, 0, 0, \sqrt{m_p^2c^4 + p_{\rm lab}^2c^2}) \\ p_{\bar{\rm p}} &= (0, 0, 0, m_pc^2). \end{aligned}$$

From this we find the invariant mass

$$W = \sqrt{2m_p^2 + 2m_p\sqrt{m_p^2 + p_{\rm lab}^2/c^2}}$$

If the initial momentum is much larger than m_p, more accurately

$$p_{\rm lab} \gg m_pc,$$

we find that

$$W \approx \sqrt{2m_pp_{\rm lab}/c},$$

which energy needs to be shared between the two photons, in equal parts. We could also have chosen to work in the CM frame, where the calculations get a lot easier.