7.9: Representations of SU(3) and Multiplication Rules

A very important group is SU(3), since it is related to the color carried by the quarks, the basic building blocks of QCD. The transformations within SU(3) are all those amongst a vector consisting of three complex objects that conserve the length of the vector. These are all three-by-three unitary matrices, which act on the complex vector ψ by

$$\begin{aligned} \psi \rightarrow & U\psi \\ = & \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \end{aligned}$$

The complex conjugate vector can be shown to transform as

$$\psi^* \rightarrow \psi^* U^\dagger,$$

with the inverse of the matrix. Clearly the fundamental representation of the group, where the matrices representing the transformation are just the matrix transformations, the vectors have length 3. The representation is usually labelled by its number of basis elements as $\mathbf{3}$. The one the transforms under the inverse matrices is usually denoted by $\bar{\mathbf{3}}$.

What happens if we combine two of these objects, ψ and χ^*? It is easy to see that the inner product of ψ and χ^* is scalar,

$$\chi^* \cdot \psi \rightarrow \chi^* U^\dagger U \psi = \chi^* \cdot \psi,$$

where we have used the unitary properties of the matrices the remaining 8 components can all be shown to transform amongst themselves, and we write

$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}.$$

Of further interest is the product of three of these vectors,

$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10}.$$