4.4: Barrier Penetration

In order to understand quantum mechanical tunnelling in fission it makes sense to look at the simplest fission process: the emission of a He nucleus, so called α radiation (Figure $4.4.1$).

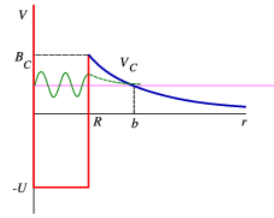

Figure $4.4.1$:The potential energy for alpha decay

Suppose there exists an α particle inside a nucleus at an (unbound) energy >0. Since it isn't bound, why doesn't it decay immediately? This must be tunnelling. In Figure $4.4.1$) we have once again shown the nuclear binding potential as a square well (red curve), but we have included the Coulomb tail (blue curve),

$$V_{\text{Coulomb}}(r)=\frac{(Z-2)2e^2}{4\pi\epsilon_0 r}.$$

The height of the barrier is exactly the coulomb potential at the boundary, which is the nuclear radius, $R_C=1.2A^{1.3}\text{ fm}$, and thus $B_C=2.4(Z-2)A^{-1/3}$. The decay probability across a barrier can be given by the simple integral expression $P=e^{-2\gamma}$, with

$$\begin{aligned}\gamma &= \frac{(2\mu_\alpha)^{1/2}}{\hbar}\int_{R_C}^{b}[V(r)-E_\alpha]^{1/2}dr \\ &= \frac{(2\mu_\alpha)^{1/2}}{\hbar}\int_{R_C}^{b}\left[\frac{2(Z-2)e^2}{4\pi\epsilon_0 r}-E_\alpha\right]^{1/2}dr \\ &= \frac{2(Z-2)e^2}{2\pi\epsilon_0\hbar v}[\arccos(E_\alpha/B_C)-(E_\alpha/B_C)(1-E_\alpha/B_C)],\end{aligned}$$

where v is the velocity associated with E_α. In the limit that $B_C\gg E_\alpha$ we find

$$P=\exp\left[-\frac{2(Z-2)e^2}{2\epsilon_0\hbar v}\right].$$

This shows how sensitive the probability is to Z and v!