2.9: Particles

Classical Particles

In this book, we are going to concentrate, almost exclusively, on the behavior of non-relativistic particles of non-zero mass (e.g., electrons). In the absence of external forces, such particles, of mass m, energy E, and momentum p, move classically in a straight-line with velocity

$$v = \frac{p}{m}, \tag{2.9.1}$$

and satisfy

$$E = \frac{p^2}{2\,m}. \tag{2.9.2}$$

Quantum Particles

Just as light-waves sometimes exhibit particle-like properties, it turns out that massive particles sometimes exhibit wave-like properties. For instance, it is possible to obtain a double-slit interference pattern from a stream of mono-energetic electrons passing through two closely-spaced narrow slits . The effective wavelength of the electrons can be determined by measuring the width of the light and dark bands in the interference pattern. [See Equation (2.7.6).] It is found that

$$\lambda = \frac{h}{p}. \tag{2.9.3}$$

The same relation is found for other types of particles. The previous wavelength is called the *de Broglie wavelength*, after Louis de Broglie, who first suggested that particles should have wave-like properties in 1923 . Note that the de Broglie wavelength is generally very small. For instance, that of an electron is

$$\lambda_e = 1.2\times 10^{-9}\,[E({\rm eV})]^{-1/2}\,{\rm m}, \tag{2.9.4}$$

where the electron energy is conveniently measured in units of electron-volts (eV). (An electron accelerated from rest through a potential difference of $1000\,{\rm V}$ acquires an energy of $1000\,{\rm eV}$, and so on.) The de Broglie wavelength of a proton is

$$\lambda_p = 2.9\times 10^{-11}\,[E({\rm eV})]^{-1/2}\,{\rm m}. \tag{2.9.5}$$

Given the smallness of the de Broglie wavelengths of common particles, it is actually quite difficult to perform particle interference experiments. In general, in order to perform an effective interference experiment, the spacing of the slits must not be too much greater than the wavelength of the wave. Hence, particle interference experiments require either very low-energy particles (because $\lambda\propto E^{\,-1/2}$), or very closely-spaced slits. Usually the "slits" consist of crystals, which act a bit like diffraction gratings with a characteristic spacing of order the inter-atomic spacing (which is generally about 10^{-9} m).

Equation (2.9.3) can be rearranged to give

$$p = \hbar\,k, \tag{2.9.6}$$

which is exactly the same as the relation between momentum and wavenumber that we obtained earlier for photons. [See Equation ([e2.19b]).] For the case of a particle moving the three dimensions, the previous relation generalizes to give

$${\bf p} = \hbar\,{\bf k}, \tag{2.9.7}$$

where ${\bf p}$ is the particle's vector momentum, and ${\bf k}$ its wavevector. It follows that the momentum of a quantum particle, and, hence, its velocity, is always parallel to its wavevector.

Because the relation ([e2.19b]) between momentum and wavenumber applies to both photons and massive particles, it seems plausible that the closely-related relation (2.6.1) between energy and wave angular frequency should also apply to both photons and particles. If this is the case, and we can write

$$E = \hbar\,\omega \tag{2.9.8}$$

for particle waves, then Equations (2.9.2) and (2.9.6) yield the following dispersion relation for such waves:

$$\omega = \frac{\hbar\, k^{\,2}}{2\, m}. \tag{2.9.9}$$

We saw earlier that a plane-wave propagates at the so-called phase-velocity,

$$v_p = \frac{\omega}{k}. \tag{2.9.10}$$

However, according to the previous dispersion relation, a particle plane-wave propagates at

$$v_p = \frac{p}{2\, m}. \tag{2.9.11}$$

Note, from Equation (2.9.1), that this is only half of the classical particle velocity. Does this imply that the dispersion relation (2.9.9) is incorrect? Let us investigate further.

Contributors and Attributions

- Richard Fitzpatrick (Professor of Physics, The University of Texas at Austin)