3.1: Schrodinger's Equation

Consider a dynamical system consisting of a single non-relativistic particle of mass m moving along the x-axis in some real potential $V(x)$. In quantum mechanics, the instantaneous state of the system is represented by a complex wavefunction $\psi(x,t)$. This wavefunction evolves in time according to Schrödinger's equation:

$$\mathrm{i}\,\hbar\,\frac{\partial\psi}{\partial t} = -\frac{\hbar^{\,2}}{2\,m}\,\frac{\partial^{\,2}\psi}{\partial x^{\,2}} + V(x)\,\psi. \tag{3.1.1}$$

The wavefunction is interpreted as follows: $|\psi(x,t)|^{\,2}$ is the probability density of a measurement of the particle's displacement yielding the value x. Thus, the probability of a measurement of the displacement giving a result between a and b (where $a<b$) is

$$P_{x\,\in\,a:b}(t) = \int_a^b |\psi(x,t)|^{\,2}\,dx. \tag{3.1.2}$$

Note that this quantity is real and positive definite.