

8.E: Central Potentials (Exercises)

1. A particle of mass m is placed in a finite spherical well:

$$V(r) = \begin{cases} -V_0 & \text{for } r \leq a \\ 0 & \text{for } r > a \end{cases} \quad (8.E.1)$$

with $V_0 > 0$ and $a > 0$. Find the ground-state by solving the radial equation with $l = 0$. Show that there is no ground-state if $V_0 a^2 < \pi^2 \hbar^2 / (8m)$.

2. Consider a particle of mass m in the three-dimensional harmonic oscillator potential $V(r) = (1/2) m \omega^2 r^2$. Solve the problem by separation of variables in spherical coordinates, and, hence, determine the energy eigenvalues of the system.
3. The normalized wavefunction for the ground-state of a hydrogen-like atom (neutral hydrogen, He^+ , Li^{++} , et cetera.) with nuclear charge Ze has the form

$$\psi = A \exp(-\beta r), \quad (8.E.2)$$

where A and β are constants, and r is the distance between the nucleus and the electron. Show the following:

1. $A^2 = \beta^3 / \pi$.
 2. $\beta = Z/a_0$, where $a_0 = (\hbar^2 / m_e) (4\pi \epsilon_0 / e^2)$.
 3. The energy is $E = -Z^2 E_0$ where $E_0 = (m_e / 2 \hbar^2) (e^2 / 4\pi \epsilon_0)^2$.
 4. The expectation values of the potential and kinetic energies are $2E$ and $-E$, respectively.
 5. The expectation value of r is $(3/2) (a_0 / Z)$.
 6. The most probable value of r is a_0 / Z .
4. An atom of tritium is in its ground-state. Suddenly the nucleus decays into a helium nucleus, via the emission of a fast electron that leaves the atom without perturbing the extranuclear electron, Find the probability that the resulting He^+ ion will be left in an $n = 1, l = 0$ state. Find the probability that it will be left in a $n = 2, l = 0$ state. What is the probability that the ion will be left in an $l > 0$ state?
 5. Calculate the wavelengths of the photons emitted from the $n = 2, l = 1$ to $n = 1, l = 0$ transition in hydrogen, deuterium, and positronium.
 6. To conserve linear momentum, an atom emitting a photon must recoil, which means that not all of the energy made available in the downward jump goes to the photon. Find a hydrogen atom's recoil energy when it emits a photon in an $n = 2$ to $n = 1$ transition. What fraction of the transition energy is the recoil energy?

Contributors and Attributions

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