9.E: Spin Angular Momentum (Exercises)

1. Find the Pauli representations of S_x, S_y, and S_z for a spin-1 particle.
2. Find the Pauli representations of the normalized eigenstates of S_x and S_y for a spin-$1/2$ particle.
3. Suppose that a spin-$1/2$ particle has a spin vector that lies in the x-z plane, making an angle θ with the z-axis. Demonstrate that a measurement of S_z yields $\hbar/2$ with probability $\cos^2(\theta/2)$, and $-\hbar/2$ with probability $\sin^2(\theta/2)$.
4. An electron is in the spin-state

$$\chi = A \begin{pmatrix} 1-2\,\mathrm{i} \\ 2 \end{pmatrix} \tag{9.E.1}$$

   in the Pauli representation. Determine the constant A by normalizing χ. If a measurement of S_z is made, what values will be obtained, and with what probabilities? What is the expectation value of S_z? Repeat the previous calculations for S_x and S_y.
5. Consider a spin-$1/2$ system represented by the normalized spinor

$$\chi = \begin{pmatrix} \cos\alpha \\ \sin\alpha\, \exp(\mathrm{i}\,\beta) \end{pmatrix} \tag{9.E.2}$$

   in the Pauli representation, where α and β are real. What is the probability that a measurement of S_y yields $-\hbar/2$?
6. An electron is at rest in an oscillating magnetic field

$$\mathbf{B} = B_0\, \cos(\omega\, t)\, \mathbf{e}_z, \tag{9.E.3}$$

   where B_0 and ω are real positive constants.
   1. Find the Hamiltonian of the system.
   2. If the electron starts in the spin-up state with respect to the x-axis, determine the spinor $\chi(t)$ which represents the state of the system in the Pauli representation at all subsequent times.
   3. Find the probability that a measurement of S_x yields the result $-\hbar/2$ as a function of time.
   4. What is the minimum value of B_0 required to force a complete flip in S_x?

Contributors and Attributions

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