2.1: Wavefunctions

A *wave* is defined as a disturbance in some physical system that is periodic in both space and time. In one dimension, a wave is generally represented in terms of a *wavefunction*: for instance,

$$\psi(x,t) = A\,\cos(k\,x-\omega\,t+\varphi), \tag{2.1.1}$$

where x is a position coordinate, t represents time, and A, k, $\omega>0$. For example, if we are considering a sound wave then $\psi(x,t)$ might correspond to the pressure perturbation associated with the wave at position x and time t. On the other hand, if we are considering a light-wave then $\psi(x,t)$ might represent the wave's transverse electric field. As is well known, the cosine function, $\cos\theta$, is periodic in its argument, θ, with period 2π: in other words, $\cos(\theta+2\pi)=\cos\theta$ for all θ. The function also oscillates between the minimum and maximum values -1 and $+1$, respectively, as θ varies. It follows that the wavefunction (2.1.1) is periodic in x with period $\lambda=2\pi/k$. In other words, $\psi(x+\lambda,t)=\psi(x,t)$ for all x and t. Moreover, the wavefunction is periodic in t with period $T=2\pi/\omega$. In other words, $\psi(x,t+T)=\psi(x,t)$ for all x and t. Finally, the wavefunction oscillates between the minimum and maximum values $-A$ and $+A$, respectively, as x and t vary. The spatial period of the wave, λ, is known as its *wavelength*, and the temporal period, T, is called its *period*. Furthermore, the quantity A is termed the wave *amplitude*, the quantity k the *wavenumber*, and the quantity ω the wave *angular frequency*. Note that the units of ω are radians per second. The conventional wave *frequency*, in cycles per second (otherwise known as hertz), is $\nu=1/T=\omega/2\pi$. Finally, the quantity φ, appearing in expression (2.1.1), is termed the *phase angle*, and determines the exact positions of the wave maxima and minima at a given time. In fact, the maxima are located at $k\,x-\omega\,t+\varphi=j\,2\pi$, where j is an integer. This follows because the maxima of $\cos\theta$ occur at $\theta=j\,2\pi$. Note that a given maximum satisfies $x=(j-\varphi/2\pi)\,\lambda+v\,t$, where $v=\omega/k$. It follows that the maximum, and, by implication, the whole wave, propagates in the positive x-direction at the velocity ω/k. Analogous reasoning reveals that

$$\psi(x,t) = A\,\cos(-k\,x-\omega\,t+\varphi) = A\,\cos(k\,x+\omega\,t-\varphi), \tag{2.1.2}$$

is the wavefunction of a wave of amplitude A, wavenumber k, angular frequency ω, and phase angle φ, that propagates in the negative x-direction at the velocity ω/k.