4.5: Alpha Decay

Many types of heavy atomic nucleus spontaneously decay to produce daughter nucleii via the emission of α-particles (i.e., helium nucleii) of some characteristic energy. This process is know as α-decay. Let us investigate the α-decay of a particular type of atomic nucleus of radius R, charge-number Z, and mass-number A. Such a nucleus thus decays to produce a daughter nucleus of charge-number $Z_1 = Z - 2$ and mass-number $A_1 = A - 4$, and an α-particle of charge-number $Z_2 = 2$ and mass-number $A_2 = 4$. Let the characteristic energy of the α-particle be E. Incidentally, nuclear radii are found to satisfy the empirical formula

$$R = 1.5\times 10^{-15}\,A^{1/3}\,\mathrm{m} = 2.0\times 10^{-15}\,Z_1^{1/3}\,\mathrm{m} \tag{4.5.1}$$

for $Z \gg 1$.

In 1928, George Gamow proposed a very successful theory of α-decay, according to which the α-particle moves freely inside the nucleus, and is emitted after tunneling through the potential barrier between itself and the daughter nucleus . In other words, the α-particle, whose energy is E, is trapped in a potential well of radius R by the potential barrier

$$V(r) = \frac{Z_1\,Z_2\,e^{2}}{4\pi\,\epsilon_0\,r} \tag{4.5.2}$$

for $r > R$.

Making use of the WKB approximation (and neglecting the fact that r is a radial, rather than a Cartesian, coordinate), the probability of the α-particle tunneling through the barrier is

$$|T|^{2} = \exp\left(-\frac{2\sqrt{2\,m}}{\hbar}\int_{r_1}^{r_2}\sqrt{V(r)-E}\,dr\right), \tag{4.5.3}$$

where $r_1 = R$ and $r_2 = Z_1\,Z_2\,e^{2}/(4\pi\,\epsilon_0\,E)$. Here, $m = 4\,m_p$ is the α-particle mass. The previous expression reduces to

$$|T|^{2} = \exp\left(-2\sqrt{2}\,\beta\int_{1}^{E_c/E}\left[\frac{1}{y}-\frac{E}{E_c}\right]^{1/2}dy\right), \tag{4.5.4}$$

where

$$\beta = \left(\frac{Z_1\,Z_2\,e^{2}\,m\,R}{4\pi\,\epsilon_0\,\hbar^{2}}\right)^{1/2} = 0.74\,Z_1^{2/3} \tag{4.5.5}$$

is a dimensionless constant, and

$$E_c = \frac{Z_1\,Z_2\,e^{2}}{4\pi\,\epsilon_0\,R} = 1.44\,Z_1^{2/3}\ \mathrm{MeV} \tag{4.5.6}$$

is the characteristic energy the α-particle would need in order to escape from the nucleus without tunneling. Of course, $E \ll E_c$. It is easily demonstrated that

$$\int_{1}^{1/\epsilon}\left(\frac{1}{y}-\epsilon\right)^{1/2}dy \simeq \frac{\pi}{2\sqrt{\epsilon}} - 2 \tag{4.5.7}$$

when $\epsilon \ll 1$. Hence.

$$|T|^{2} \simeq \exp\left(-2\sqrt{2}\,\beta\left[\frac{\pi}{2}\sqrt{\frac{E_c}{E}} - 2\right]\right). \tag{4.5.8}$$

Now, the α-particle moves inside the nucleus with the characteristic velocity $v = \sqrt{2\,E/m}$. It follows that the particle bounces backward and forward within the nucleus at the frequency $\nu \simeq v/R$, giving

$$\nu \simeq 2\times 10^{28}\ \mathrm{yr}^{-1} \tag{4.5.9}$$

for a 1 MeV α-particle trapped inside a typical heavy nucleus of radius 10^{-14} m. Thus, the α-particle effectively attempts to tunnel through the potential barrier ν times a second. If each of these attempts has a probability $|T|^{2}$ of succeeding then the probability of decay per unit time is $\nu\,|T|^{\,2}$. Hence, if there are $N(t)\gg 1$ undecayed nuclii at time t then there are only $N+dN$ at time $t+dt$, where

$$dN = -N\,\nu\,|T|^{\,2}\,dt. \tag{4.5.10}$$

This expression can be integrated to give

$$N(t) = N(0)\,\exp(-\nu\,|T|^{\,2}\,t). \tag{4.5.11}$$

Now, the *half-life*, τ, is defined as the time which must elapse in order for half of the nuclii originally present to decay. It follows from the previous formula that

$$\tau = \frac{\ln 2}{\nu\,|T|^{\,2}}. \tag{4.5.12}$$

Note that the half-life is independent of $N(0)$.

Finally, making use of the previous results, we obtain

$$\log_{10}[\tau(\mathrm{yr})] = -C_1 - C_2\,Z_1^{\,2/3} + C_3\,\frac{Z_1}{\sqrt{E(\mathrm{MeV})}}, \tag{4.5.13}$$

where

$$C_1 = 28.5,$$
$$C_2 = 1.83,$$
$$C_3 = 1.73.$$

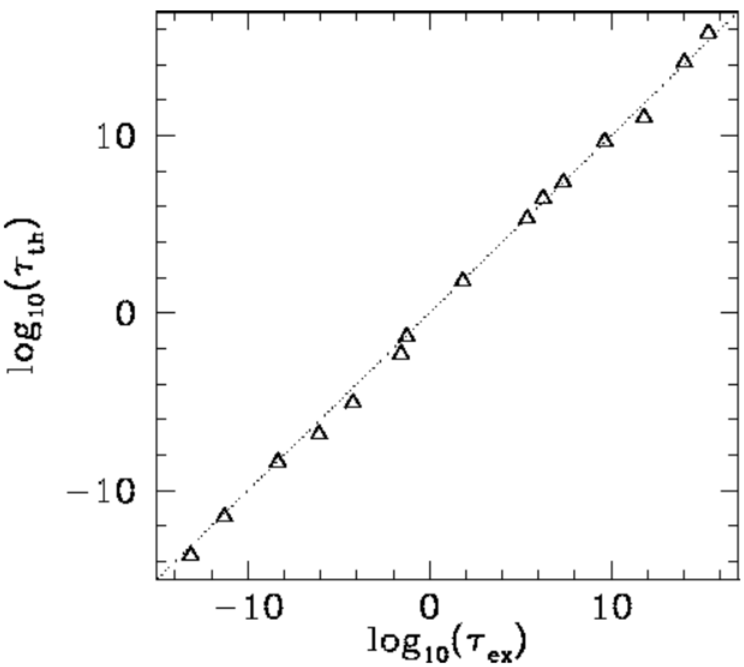

Figure 15: *The experimentally determined half-life,* $\tau_{\rm ex}$ *of various atomic nucleii which decay via* α *emission versus the best-fit theoretical half-life* $\log_{10}(\tau_{th}) = -28.9 - 1.60 Z_1^{2/3} + 1.61 Z_1/\sqrt{E}$. *Both half-lives are measured in years. Here,* $Z_1 = Z-2$. *Both half-lives are measured in years. Here,* $Z_1 = Z-2$, where Z *is the charge number of the nucleus, and* E *the characteristic energy of the emitted* α*-particle in MeV. In order of increasing half-life, the points correspond to the following nuclei: Rn 215, Po 214, Po 216, Po 197, Fm 250, Ac 225, U 230, U 232, U 234, Gd 150, U 236, U 238, Pt 190, Gd 152, Nd 144. Data obtained from IAEA Nuclear Data Centre.*

Equation ([e5.64]) is known as the *Geiger-Nuttall formula*, because it was discovered empirically by H. Geiger and J.M. Nuttall in 1911 .

The half-life, τ, the daughter charge-number, $Z_1 = Z - 2$, and the α-particle energy, E, for atomic nucleii which undergo α-decay are indeed found to satisfy a relationship of the form ([e5.64]). The best fit to the data (see Figure [fal]) is obtained using

$$\begin{aligned} C_1 &= 28.9, \\ C_2 &= 1.60, \\ C_3 &= 1.61. \end{aligned}$$

Note that these values are remarkably similar to those calculated previously.

Contributors and Attributions

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