

6.4: White Dwarf Stars

A main-sequence hydrogen-burning star, such as the Sun, is maintained in equilibrium via the balance of the gravitational attraction tending to make it collapse, and the thermal pressure tending to make it expand. Of course, the thermal energy of the star is generated by nuclear reactions occurring deep inside its core. Eventually, however, the star will run out of burnable fuel, and, therefore, start to collapse, as it radiates away its remaining thermal energy. What is the ultimate fate of such a star?

A burnt-out star is basically a gas of electrons and ions. As the star collapses, its density increases, and so the mean separation between its constituent particles decreases. Eventually, the mean separation becomes of order the de Broglie wavelength of the electrons, and the electron gas becomes degenerate. Note that the de Broglie wavelength of the ions is much smaller than that of the electrons, so the ion gas remains non-degenerate. Now, even at zero temperature, a degenerate electron gas exerts a substantial pressure, because the Pauli exclusion principle prevents the mean electron separation from becoming significantly smaller than the typical de Broglie wavelength. (See the previous section.) Thus, it is possible for a burnt-out star to maintain itself against complete collapse under gravity via the degeneracy pressure of its constituent electrons. Such stars are termed *white-dwarfs*. Let us investigate the physics of white-dwarfs in more detail.

The total energy of a white-dwarf star can be written

$$\mathcal{E} = K + U, \quad (6.4.1)$$

where K is the kinetic energy of the degenerate electrons (the kinetic energy of the ions is negligible), and U is the gravitational potential energy. Let us assume, for the sake of simplicity, that the density of the star is uniform. In this case, the gravitational potential energy takes the form

$$U = -\frac{3}{5} \frac{G M^2}{R}, \quad (6.4.2)$$

where G is the gravitational constant, M is the stellar mass, and R is the stellar radius.

From the previous section, the kinetic energy of a degenerate electron gas is simply

$$K = N \bar{E} = \frac{3}{5} N E_F = \frac{3}{5} N \frac{\pi^2 \hbar^2}{2 m_e} \left(\frac{3 N}{\pi V} \right)^{2/3}, \quad (6.4.3)$$

where N is the number of electrons, V the volume of the star, and m_e the electron mass.

The interior of a white-dwarf star is composed of atoms like C^{12} and O^{16} which contain equal numbers of protons, neutrons, and electrons. Thus,

$$M = 2 N m_p, \quad (6.4.4)$$

where m_p is the proton mass.

Equations (6.4.1)–(6.4.4) can be combined to give

$$\mathcal{E} = \frac{A}{R^2} - \frac{B}{R}, \quad (6.4.5)$$

where

$$A = \frac{3}{20} \left(\frac{9\pi}{8} \right)^{2/3} \frac{\hbar^2}{m_e} \left(\frac{M}{m_p} \right)^{5/3},$$

$$B = \frac{3}{5} G M^2.$$

The equilibrium radius of the star, R_* , is that which minimizes the total energy \mathcal{E} . In fact, it is easily demonstrated that

$$R_* = \frac{2 A}{B}, \quad (6.4.6)$$

which yields

$$R_* = \frac{(9\pi)^{2/3}}{8} \frac{\hbar^2}{G m_e m_p^{5/3} M^{1/3}}. \quad (6.4.7)$$

The previous formula can also be written

$$(6.4.8)$$

where $R_\odot = 7 \times 10^5$ km is the solar radius, and $M_\odot = 2 \times 10^{30}$ kg the solar mass. It follows that the radius of a typical solar mass white-dwarf is about 7000 km: that is, about the same as the radius of the Earth. The first white-dwarf to be discovered (in 1862) was the companion of Sirius. Nowadays, thousands of white-dwarfs have been observed, all with properties similar to those described previously.

Note from Equations ([e8wd4]), ([e8wd6]), and ([e8wd9]) that $\bar{E} \propto M^{4/3}$. In other words, the mean energy of the electrons inside a white dwarf increases as the stellar mass increases. Hence, for a sufficiently massive white dwarf, the electrons can become relativistic. It turns out that the degeneracy pressure for relativistic electrons only scales as R^{-1} , rather than R^{-2} , and thus is unable to balance the gravitational pressure [which also scales as R^{-1} —see Equation ([e7.52])]. It follows that electron degeneracy pressure is only able to halt the collapse of a burnt-out star provided that the stellar mass does not exceed some critical value, known as the *Chandrasekhar limit*, which turns out to be about 1.4 times the mass of the Sun. Stars whose mass exceeds the Chandrasekhar limit inevitably collapse to produce extremely compact objects, such as neutron stars (which are held up by the degeneracy pressure of their constituent neutrons), or black holes.

Contributors and Attributions

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