

## 8.4: Rydberg Formula

An electron in a given stationary state of a hydrogen atom, characterized by the quantum numbers  $n$ ,  $l$ , and  $m$ , should, in principle, remain in that state indefinitely. In practice, if the state is slightly perturbed—for instance, via interaction with a photon—then the electron can make a transition to another stationary state with different quantum numbers. (See Chapter [\[s13\]](#).)

Suppose that an electron in a hydrogen atom makes a transition from an initial state whose radial quantum number is  $n_i$  to a final state whose radial quantum number is  $n_f$ . According to Equation ([\[e9.55\]](#)), the energy of the electron will change by

$$\Delta E = E_0 \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right). \quad (8.4.1)$$

If  $\Delta E$  is negative then we would expect the electron to emit a photon of frequency  $\nu = -\Delta E/h$ . [See Equation ([\[ee3.15\]](#)).] Likewise, if  $\Delta E$  is positive then the electron must absorb a photon of energy  $\nu = \Delta E/h$ . Given that  $\lambda^{-1} = \nu/c$ , the possible wavelengths of the photons emitted by a hydrogen atom as its electron makes transitions between different energy levels are

$$\frac{1}{\lambda} = R \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right), \quad (8.4.2)$$

where

$$R = \frac{-E_0}{h c} = \frac{m_e e^4}{(4\pi)^3 \epsilon_0^2 \hbar^3 c} = 1.097 \times 10^7 \text{ m}^{-1}. \quad (8.4.3)$$

Here, it is assumed that  $n_f < n_i$ . Note that the emission spectrum of hydrogen is quantized: that is, a hydrogen atom can only emit photons with certain fixed set of wavelengths. Likewise, a hydrogen atom can only absorb photons that have the same fixed set of wavelengths. This set of wavelengths constitutes the characteristic emission/absorption spectrum of the hydrogen atom, and can be observed as “spectral lines” using a spectroscope.

Equation ([\[e9.77\]](#)) is known as the *Rydberg formula*. Likewise,  $R$  is called the *Rydberg constant*. The Rydberg formula was actually discovered empirically in the nineteenth century by spectroscopists, and was first explained theoretically by Bohr in 1913 using a primitive version of quantum mechanics. Transitions to the ground-state ( $n_f = 1$ ) give rise to spectral lines in the ultraviolet band—this set of lines is called the *Lyman series*. Transitions to the first excited state ( $n_f = 2$ ) give rise to spectral lines in the visible band—this set of lines is called the *Balmer series*. Transitions to the second excited state ( $n_f = 3$ ) give rise to spectral lines in the infrared band—this set of lines is called the *Paschen series*, and so on.

### Contributors and Attributions

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