10.E: Addition of Angular Momentum (Exercises)

1. An electron in a hydrogen atom occupies the combined spin and position state

$$R_{2,1}(r)\,\left[\sqrt{1/3}\,Y_{1,0}(\theta,\phi)\,\chi_+ + \sqrt{2/3}\,Y_{1,1}(\theta,\phi)\,\chi_-\right]. \tag{10.E.1}$$

   1. What values would a measurement of L^2 yield, and with what probabilities?
   2. Same for L_z.
   3. Same for S^2.
   4. Same for S_z.
   5. Same for J^2.
   6. Same for J_z.
   7. What is the probability density for finding the electron at r, θ, ϕ?
   8. What is the probability density for finding the electron in the spin up state (with respect to the z-axis) at radius r?

2. In a low energy neutron-proton system (with zero orbital angular momentum), the potential energy is given by

$$V(r) = V_1(r) + V_2(r)\left[3\,\frac{(\sigma_1\cdot\mathbf{r})\,(\sigma_2\cdot\mathbf{r})}{r^2} - \sigma_1\cdot\sigma_2\right] + V_3(r)\,\sigma_1\cdot\sigma_2, \tag{10.E.2}$$

   where σ_1 denotes the vector of the Pauli matrices of the neutron, and σ_2 denotes the vector of the Pauli matrices of the proton. Calculate the potential energy for the neutron-proton system:

   1. In the spin singlet state.
   2. In the spin triplet state.

3. Consider two electrons in a spin singlet state.

   1. If a measurement of the spin of one of the electrons shows that it is in the state with $S_z = \hbar/2$, what is the probability that a measurement of the z-component of the spin of the other electron yields $S_z = \hbar/2$?
   2. If a measurement of the spin of one of the electrons shows that it is in the state with $S_y = \hbar/2$, what is the probability that a measurement of the x-component of the spin of the other electron yields $S_x = -\hbar/2$?

   Finally, if electron 1 is in a spin state described by $\cos\alpha_1\,\chi_+ + \sin\alpha_1\,\mathrm{e}^{\,\mathrm{i}\,\beta_1}\,\chi_-$, and electron 2 is in a spin state described by $\cos\alpha_2\,\chi_+ + \sin\alpha_2\,\mathrm{e}^{\,\mathrm{i}\,\beta_2}\,\chi_-$, what is the probability that the two-electron spin state is a triplet state?

Contributors and Attributions

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