1.5: Exercises

1. Show that the probability of throwing 6 points or less with three (six-sided) dice is $5/54$.
2. A battery of total emf V is connected to a resistor R. As a result, an amount of power $P = V^2/R$ is dissipated in the resistor. The battery itself consists of N individual cells connected in series, so that V is equal to the sum of the emf's of all these cells. The battery is old, however, so that not all cells are in perfect condition. Thus, there is a probability p that the emf of any individual cell has its normal value v; and a probability $1-p$ that the emf of any individual cell is zero because the cell has become internally shorted. The individual cells are statistically independent of each other. Under these conditions, show that the mean power, $\langle P\rangle$, dissipated in the resistor, is

$$\langle P\rangle = \frac{p^2\,V^2}{R}\left[1+\frac{(1-p)}{N\,p}\right]. \tag{1.5.1}$$

3. In the "game" of Russian roulette, the player inserts a single bullet into the drum of a revolver, leaving the other five chambers of the drum empty. The player then spins the drum, aims at his/her head, and pulls the trigger.
   1. What is the probability of the player still being alive after playing the game N times?
   2. What is the probability of the player surviving $N-1$ turns in this game, and then being shot the Nth time he/she pulls the trigger?
   3. What is the mean number of times the player gets to pull the trigger?
4. Suppose that the probability density for the speed, s, of a car on a road is given by

$$P(s) = A\,s\,\exp\left(-\frac{s}{s_0}\right), \tag{1.5.2}$$

   where $0 \leq s \leq \infty$. Here, A and s_0 are positive constants. More explicitly, $P(s)\,ds$ gives the probability that a car has a speed between s and $s+ds$.
   1. Determine A in terms of s_0.
   2. What is the mean value of the speed?
   3. What is the "most probable" speed: that is, the speed for which the probability density has a maximum?
   4. What is the probability that a car has a speed more than three times as large as the mean value?
5. An radioactive atom has a uniform decay probability per unit time w. In other words, the probability of decay in a time interval dt is $w\,dt$. Let $P(t)$ be the probability of the atom not having decayed at time t, given that it was created at time $t=0$. Demonstrate that

$$P(t) = \mathrm{e}^{-w\,t}. \tag{1.5.3}$$

   What is the mean lifetime of the atom?

Contributors and Attributions

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