8.1: Background Material

Text References

- moment of inertia of common geometries
- unwinding spools

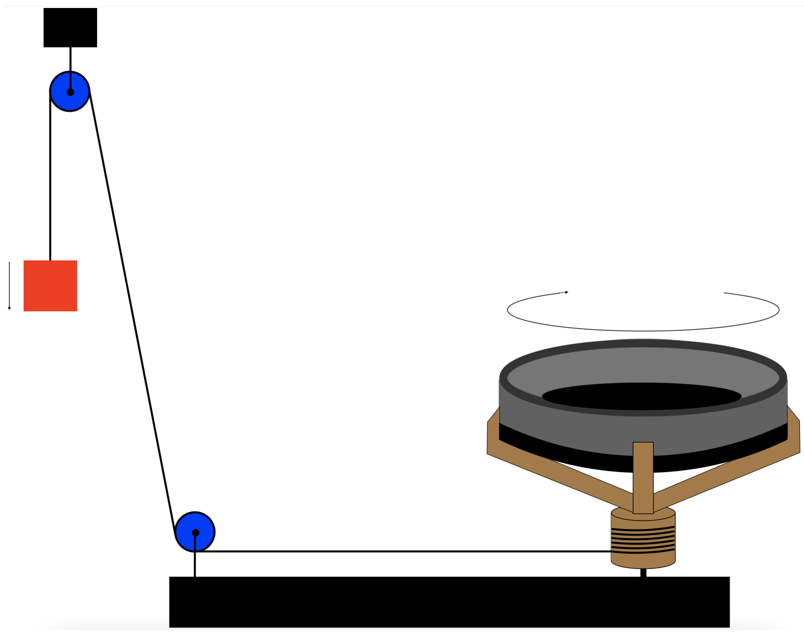
Measuring Moment of Inertia Dynamically

Half of this lab consists of using dynamics to determine the moment of inertia of a thick circular ring. The first thing that should come to mind when thinking of "dynamics" and "inertia" is Newton's second law. In this case, it is the version of the second law that applies to rotations:

$$\overrightarrow{\alpha}=\frac{\tau_{net}}{I} \tag{8.1.1}$$

Clearly if we can measure the torque and angular acceleration, we immediately have the value of the moment of inertia. But these are not the easiest values to measure directly, so we have an experimental setup that simplifies the task somewhat:

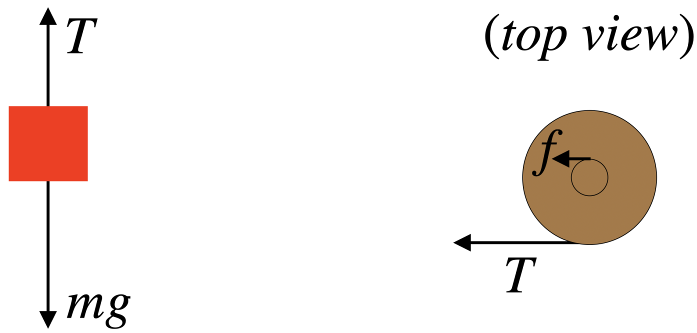
Figure 8.1.1 – Experimental Apparatus



As the mass accelerates downward, it accelerates the ring in the cradle in a rotational fashion. Their motions are linked by the fact that the string does not slip as it unwinds from the hub, and we can pretty easily measure the acceleration of the descending block. As for the torque on the rotating system, we can get this from the tension that pulls on the hub (minus a small contribution by friction on the hub by the axle), and the radius of the hub. With the tension also affecting the motion of the mass, we end up with a convenient result.

Let's solve the physics problem. Start with free-body diagrams. For the hub, we will take a top view, and since we are only interested in the torques, we will leave off the balancing force on the hub by the axle that acts through its center.

Figure 8.1.2 – Free-Body Diagrams



As the mass descends, the hub rotates clockwise according to the diagram. Summing the forces on the block and the torques (about the axle) on the hub, we get two second-law equations (f is the frictional force on the hub by the axle, N is the normal force on the hub by the axle, R is the radius of the hub, r the radius of the axle, and I is the moment of inertia of the rotating system):

$$F_{net}=mg-T=ma \qquad \tau_{net}=TR-fr=I\alpha \tag{8.1.2}$$

The "string unwinds without slipping" constraint gives a relation between the linear acceleration of the mass and the rotational acceleration of the ring & cradle:

$$a=R\alpha \tag{8.1.3}$$

Putting these three equations together such that we eliminate α and T (neither of which is easy to directly measure in an experiment), we get the following equation:

$$a=\frac{mR^2}{I}(g-a)-\frac{fRr}{I} \tag{8.1.4}$$

It might seem odd that we left the acceleration a on both sides of this equation, but there is a reason for this. When you actually perform this experiment, you will find that the acceleration a is much, *much* less than g (on the order of 0.1% of g for the weights we will hang on the string), so we should feel fairly comfortable ignoring entirely the second term on the right side of that equation. This gives us an equation for a line with two measurable quantities, a and m, as variables:

$$a=\left(\frac{gR^2}{I}\right)m-\frac{fRr}{I} \tag{8.1.5}$$

Conveniently, the moment of inertia of the rotating platform appears as a factor in the slope of that line. We don't know the friction force or the radius of the axle, so we can't really deal with the second term (nor do we need to), but it is a negative constant, which means that if we plot a vs. m, we would expect the y-intercept to be below the horizontal axis (i.e. this is not one of those cases where we can use the origin as an extra data point). [*Note: It is common to express the slope of a line as "m", as in "$y=mx+b$". In the linear equation above, the variable m represents the mass hung from the string, and it is actually the "x" in the linear equation. The slope of this line is the constant in parentheses.*]