PGCC: PHYS 1030 - GENERAL PHYSICS

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CHAPTER OVERVIEW

01: The Basics of Physics

Topic hierarchy

1.1: The Basics of Physics

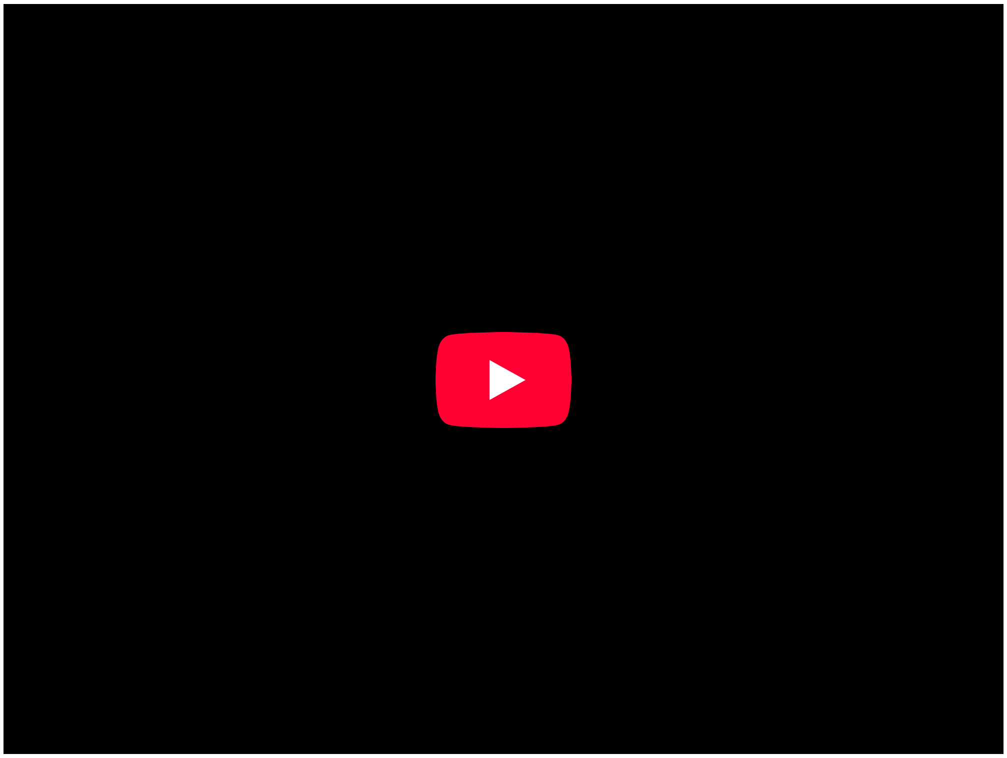
Introduction: Physics and Matter

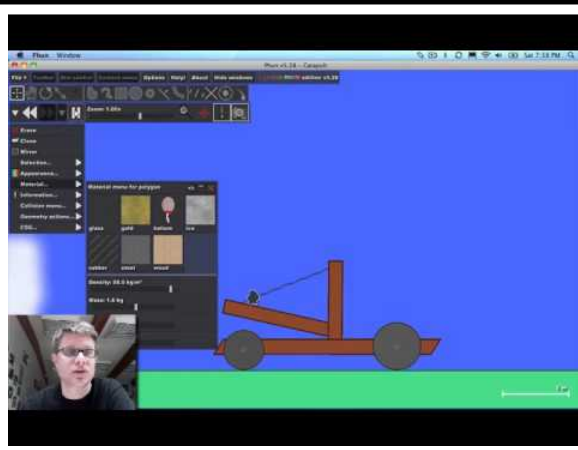
Physics is a study of how the universe behaves.

learning objectives

- Apply physics to describe the function of daily life

Physics is a natural science that involves the study of matter and its motion through space and time, along with related concepts such as energy and force. More broadly, it is the study of nature in an attempt to understand how the universe behaves.



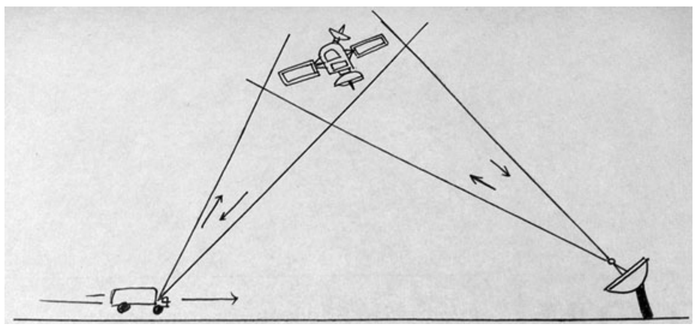


What is Physics?: Mr. Andersen explains the importance of physics as a science. History and virtual examples are used to give the discipline context.

Physics uses the scientific method to help uncover the basic principles governing light and matter, and to discover the implications of those laws. It assumes that there are rules by which the universe functions, and that those laws can be at least partially understood by humans. It is also commonly believed that those laws could be used to predict everything about the universe's future if complete information was available about the present state of all light and matter.

Matter is generally considered to be anything that has mass and volume. Many concepts integral to the study of classical physics involve theories and laws that explain matter and its motion. The law of conservation of mass, for example, states that mass cannot be created or destroyed. Further experiments and calculations in physics, therefore, take this law into account when formulating hypotheses to try to explain natural phenomena.

Physics aims to describe the function of everything around us, from the movement of tiny charged particles to the motion of people, cars, and spaceships. In fact, almost everything around you can be described quite accurately by the laws of physics. Consider a smart phone; physics describes how electricity interacts with the various circuits inside the device. This knowledge helps engineers select the appropriate materials and circuit layout when building the smart phone. Next, consider a GPS system; physics describes the relationship between the speed of an object, the distance over which it travels, and the time it takes to travel that distance. When you use a GPS device in a vehicle, it utilizes these physics equations to determine the travel time from one location to another. The study of physics is capable of making significant contributions through advances in new technologies that arise from theoretical breakthroughs.



Global Positioning System: GPS calculates the speed of an object, the distance over which it travels, and the time it takes to travel that distance using equations based on the laws of physics.

Physics and Other Fields

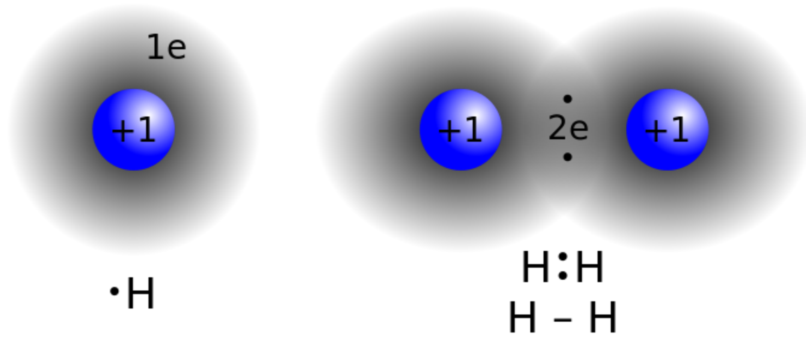
Physics is the foundation of many disciplines and contributes directly to chemistry, astronomy, engineering, and most scientific fields.

learning objectives

- Explain why the study of physics is integral to the study of other sciences

Physics and Other Disciplines

Physics is the foundation of many important disciplines and contributes directly to others. Chemistry deals with the interactions of atoms and molecules, so it is rooted in atomic and molecular physics. Most branches of engineering are applied physics. In architecture, physics is at the heart of structural stability and is involved in acoustics, heating, lighting, and the cooling of buildings. Parts of geology rely heavily on physics, such as the radioactive dating of rocks, earthquake analysis, and heat transfer in the Earth. Some disciplines, such as biophysics and geophysics, are hybrids of physics and other disciplines.



Physics in Chemistry: The study of matter and electricity in physics is fundamental towards the understanding of concepts in chemistry, such as the covalent bond.

Physics has many applications in the biological sciences. On the microscopic level, it helps describe the properties of cell walls and cell membranes. On the macroscopic level, it can explain the heat, work, and power associated with the human body. Physics is involved in medical diagnostics, such as X-rays, magnetic resonance imaging (MRI), and ultrasonic blood flow measurements. Medical therapy sometimes directly involves physics: cancer radiotherapy uses ionizing radiation, for instance. Physics can also explain sensory phenomena, such as how musical instruments make sound, how the eye detects color, and how lasers can transmit information.

The boundary between physics and the other sciences is not always clear. For instance, chemists study atoms and molecules, which are what matter is built from, and there are some scientists who would be equally willing to call themselves physical chemists or chemical physicists. It might seem that the distinction between physics and biology would be clearer, since physics seems to deal with inanimate objects. In fact, almost all physicists would agree that the basic laws of physics that apply to molecules in a test tube work equally well for the combination of molecules that constitutes a bacterium. What differentiates physics from biology is that many of the scientific theories that describe living things ultimately result from the fundamental laws of physics, but cannot be rigorously derived from physical principles.

It is not necessary to formally study all applications of physics. What is most useful is the knowledge of the basic laws of physics and skill in the analytical methods for applying them. The study of physics can also improve your problem-solving skills. Furthermore, physics has retained the most basic aspects of science, so it is used by all of the sciences. The study of physics makes other sciences easier to understand.

Models, Theories, and Laws

The terms *model*, *theory*, and *law* have exact meanings in relation to their usage in the study of physics.

learning objectives

- Define the terms model, theory, and law

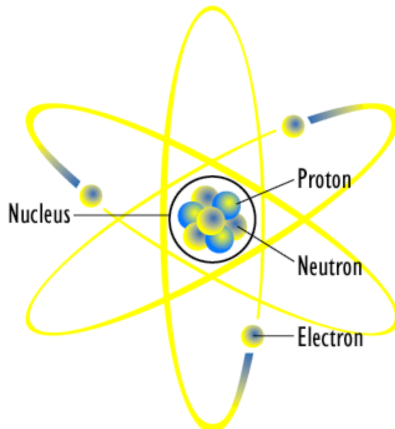
Definition of Terms: Model, Theory, Law

In colloquial usage, the terms *model*, *theory*, and *law* are often used interchangeably or have different interpretations than they do in the sciences. In relation to the study of physics, however, each term has its own specific meaning.

The *laws of nature* are concise descriptions of the universe around us. They are not explanations, but human statements of the underlying rules that all natural processes follow. They are intrinsic to the universe; humans did not create them and we cannot change them. We can only discover and understand them. The cornerstone of discovering natural laws is observation; science must describe the universe as it is, not as we may imagine it to be. Laws can never be known with absolute certainty, because it is impossible to perform experiments to establish and confirm a law in every possible scenario without exception. Physicists operate under the assumption that all scientific laws and theories are valid until a counterexample is observed. If a good-quality, verifiable experiment contradicts a well-established law, then the law must be modified or overthrown completely.

Models

A *model* is a representation of something that is often too difficult (or impossible) to display directly. While a model's design is justified using experimental information, it is only accurate under limited situations. An example is the commonly used "planetary model" of the atom, in which electrons are pictured as orbiting the nucleus, analogous to the way planets orbit the Sun. We cannot observe electron orbits directly, but the mental image helps explain the observations we can make, such as the emission of light from hot gases. Physicists use models for a variety of purposes. For example, models can help physicists analyze a scenario and perform a calculation, or they can be used to represent a situation in the form of a computer simulation.



Planetary Model of an Atom: The planetary model of the atom in which electrons are pictured as orbiting the nucleus, analogous to the way planets orbit the Sun

Theories

A *theory* is an explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers. *Some theories include models to help visualize phenomena, whereas others do not*. Newton's theory of gravity, for example, does not require a model or mental image, because we can observe the objects directly with our own senses. The kinetic theory of gases, on the other hand, makes use of a model in which a gas is viewed as being composed of atoms and molecules. Atoms and molecules are too small to be observed directly with our senses—thus, we picture them mentally to understand what our instruments tell us about the behavior of gases.

Laws

A law uses concise language to describe a generalized pattern in nature that is supported by scientific evidence and repeated experiments. Often, a law can be expressed in the form of a single mathematical equation. Laws and theories are similar in that they are both scientific statements that result from a tested hypothesis and are supported by scientific evidence. However, the designation law is reserved for a concise and very general statement that describes phenomena in nature, such as the law that energy is conserved during any process, or Newton's second law of motion, which relates force, mass, and acceleration by the simple equation $F = ma$. A theory, in contrast, is a less concise statement of observed phenomena. For example, the Theory of Evolution and the Theory of Relativity cannot be expressed concisely enough to be considered a law. The biggest difference between a law and a theory is that a law is much more complex and dynamic, and a theory is more explanatory. A law describes a single observable point of fact, whereas a theory explains an entire group of related phenomena. And, whereas a law is a postulate that forms the foundation of the scientific method, a theory is the end result of that process.

Key Points

- Physics is a natural science that involves the study of matter and its motion through space and time, along with related concepts such as energy and force.
- Matter is generally considered to be anything that has mass and volume.
- Scientific laws and theories express the general truths of nature and the body of knowledge they encompass. These laws of nature are rules that all natural processes appear to follow.
- Many scientific disciplines, such as biophysics, are hybrids of physics and other sciences.
- The study of physics encompasses all forms of matter and its motion in space and time.

- The application of physics is fundamental towards significant contributions in new technologies that arise from theoretical breakthroughs.
- Concepts in physics cannot be proven, they can only be supported or disproven through observation and experimentation.
- A model is an evidence-based representation of something that is either too difficult or impossible to display directly.
- A theory is an explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers.
- A law uses concise language, often expressed as a mathematical equation, to describe a generalized pattern in nature that is supported by scientific evidence and repeated experiments.

Key Terms

- **matter**: The basic structural component of the universe. Matter usually has mass and volume.
- **scientific method**: A method of discovering knowledge about the natural world based in making falsifiable predictions (hypotheses), testing them empirically, and developing peer-reviewed theories that best explain the known data.
- **application**: the act of putting something into operation
- **Model**: A representation of something difficult or impossible to display directly
- **Law**: A concise description, usually in the form of a mathematical equation, used to describe a pattern in nature
- **theory**: An explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers

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1.2: Units

Length

Length is a physical measurement of distance that is fundamentally measured in the SI unit of a meter.

learning objectives

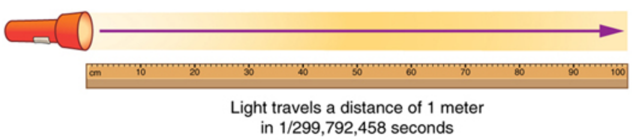
- Distinguish SI and customary units of length

Length can be defined as a measurement of the physical quantity of distance. Many qualitative observations fundamental to physics are commonly described using the measurement of length. The distance between objects, the rate at which objects are traveling, and how much force an object exerts are all dependent on length as a variable. In order to describe length in a standardized and quantitative manner, an accepted unit of measurement must be utilized.

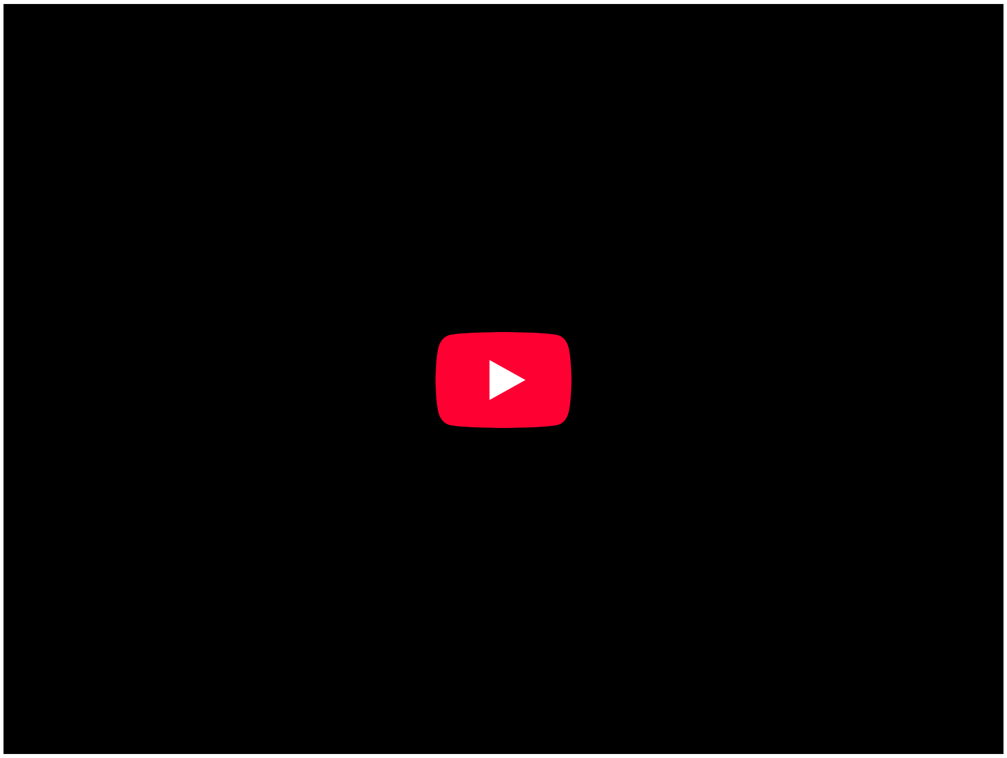
Many different units of length are used around the world. In the United States, the U.S. customary units operationally describe length in terms of the basic unit of an inch. Varying lengths are thus described in relation to the inch, such as a foot equaling 12 inches, a yard equaling three feet, and a mile equaling 1,760 yards.

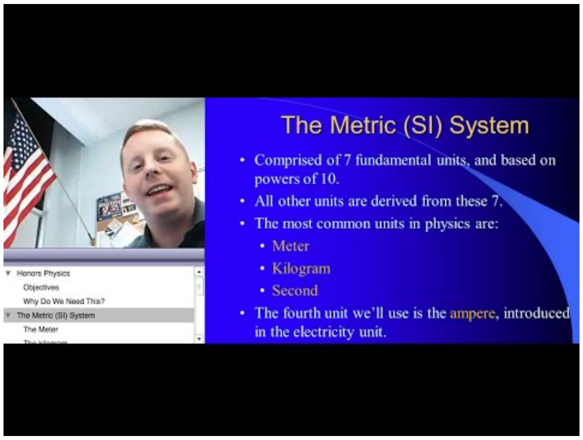
Though regional use of different measurement units is not generally problematic, it can raise issues of compatibility and understanding when working abroad or collaboratively with international partners. As such, a standard unit of measurement that is internationally accepted is needed. The basic unit of length as identified by the International System of Units (SI) is the meter. The meter is expressed more specifically in terms of speed of light.

One meter is defined as the distance that light travels in a vacuum in $\frac{1}{299,792,458}$ of a second. All lengths are measured in terms related to the meter, where its multiples are devised around the convenience of the number 10. For example, a centimeter is equal to $\frac{1}{100}$ of a meter (or 10^{-2} meters), and a kilometer is equal to 1,000 meters (or 10^{3} meters).



Meter Defined by Speed of Light: The meter is defined to be the distance that light travels in $\frac{1}{299,792,458}$ of a second in a vacuum. Distance traveled is speed multiplied by time.





Metric System – Length: A brief introduction to the metric system and unit conversions.

Mass

Mass is the quantity of matter that an object contains, as measured by its resistance to acceleration.

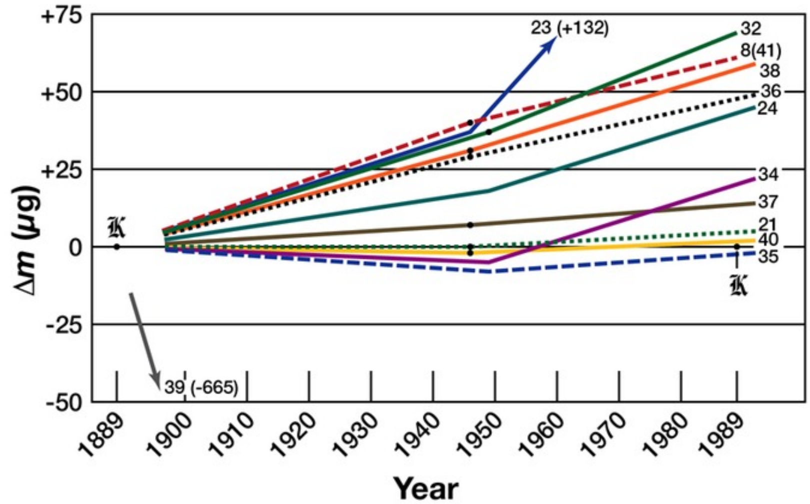
> **learning objectives**
>
> - Explain the difference between mass and weight

Mass

Mass, specifically inertial mass, is a quantitative measure of an object's resistance to acceleration. It is an intrinsic property of an object and does not change because of the environment. The SI unit of mass is the kilogram (kg).

The kilogram is defined as being equal to the mass of the International Prototype Kilogram (IPK), which is almost exactly equal to the mass of one liter of water. It is also the only SI unit that is directly defined by an artifact, rather than a fundamental physical property that can be reproduced in different laboratories. Four of the seven base units in the SI system are defined relative to the kilogram, so the stability of this measurement is crucial for accurate and consistent measurements.

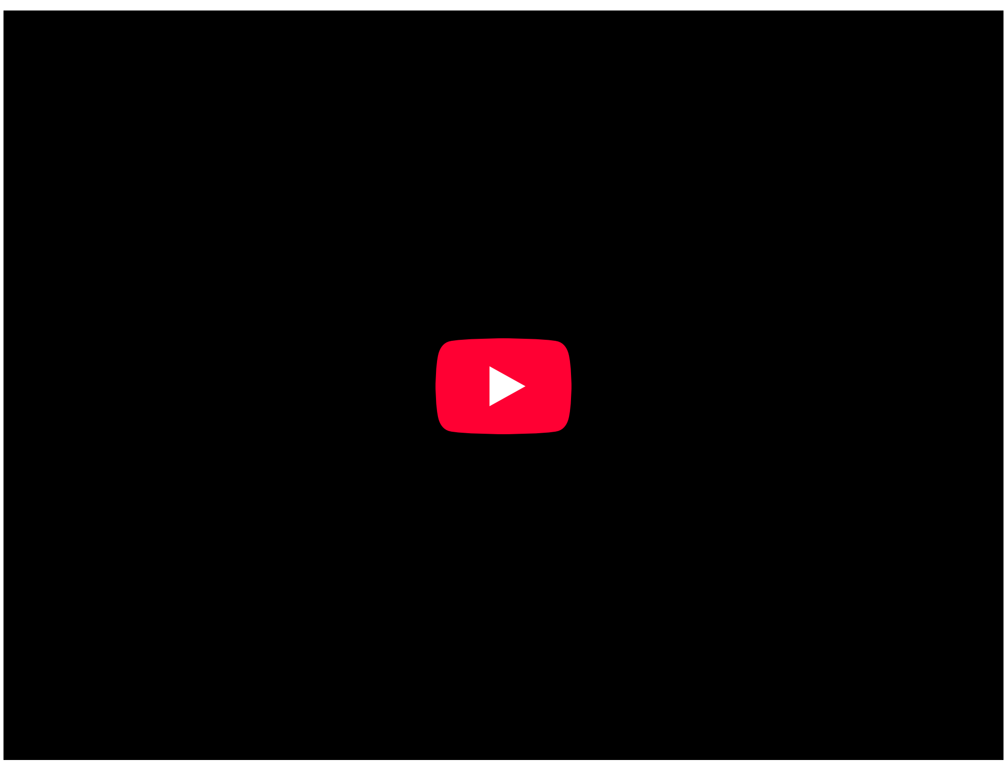
In 2005, the International Committee for Weights and Measures (CIPM) recommended that the kilogram be redefined in terms of a fundamental constant of nature, due to evidence that the International Prototype Kilogram will vary in mass over time. At its 2011 meeting, the General Conference on Weights and Measures (CGPM) agreed that the kilogram should be redefined in terms of the Planck constant. The conference deferred a final decision until its next meeting in 2014.

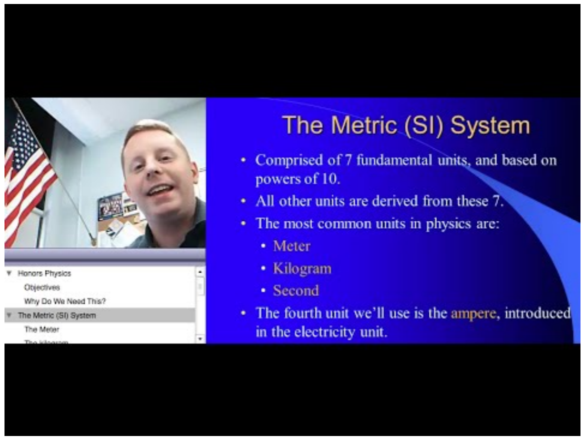


Prototype Mass Drifts: A graph of the relative change in mass of selected kilogram prototypes.

Mass and Weight

In everyday usage, the mass of an object in kilograms is often referred to as its weight. This value, though given in kilograms, is actually the non-SI unit of measure known as the kilogram- force. In scientific terms, ‘weight’ refers to the gravitational force acting on a given body. This measurement changes depending on the gravitational pull of the opposing body. For example, a person’s weight on the Earth is different than a person’s weight on the moon because of the differences in the gravitational pull of each body. In contrast, the mass of an object is an intrinsic property and remains the same regardless of gravitational fields. Accordingly, astronauts in microgravity must exert 10 times more force to accelerate a 10-kg object at the same rate as a 1-kg object, even though the differences in weight are imperceptible.





Metric System – Mass: A brief introduction to the metric system and unit conversions.

Time

Time is the fundamental physical quantity of duration and is measured by the SI Unit known as the second.

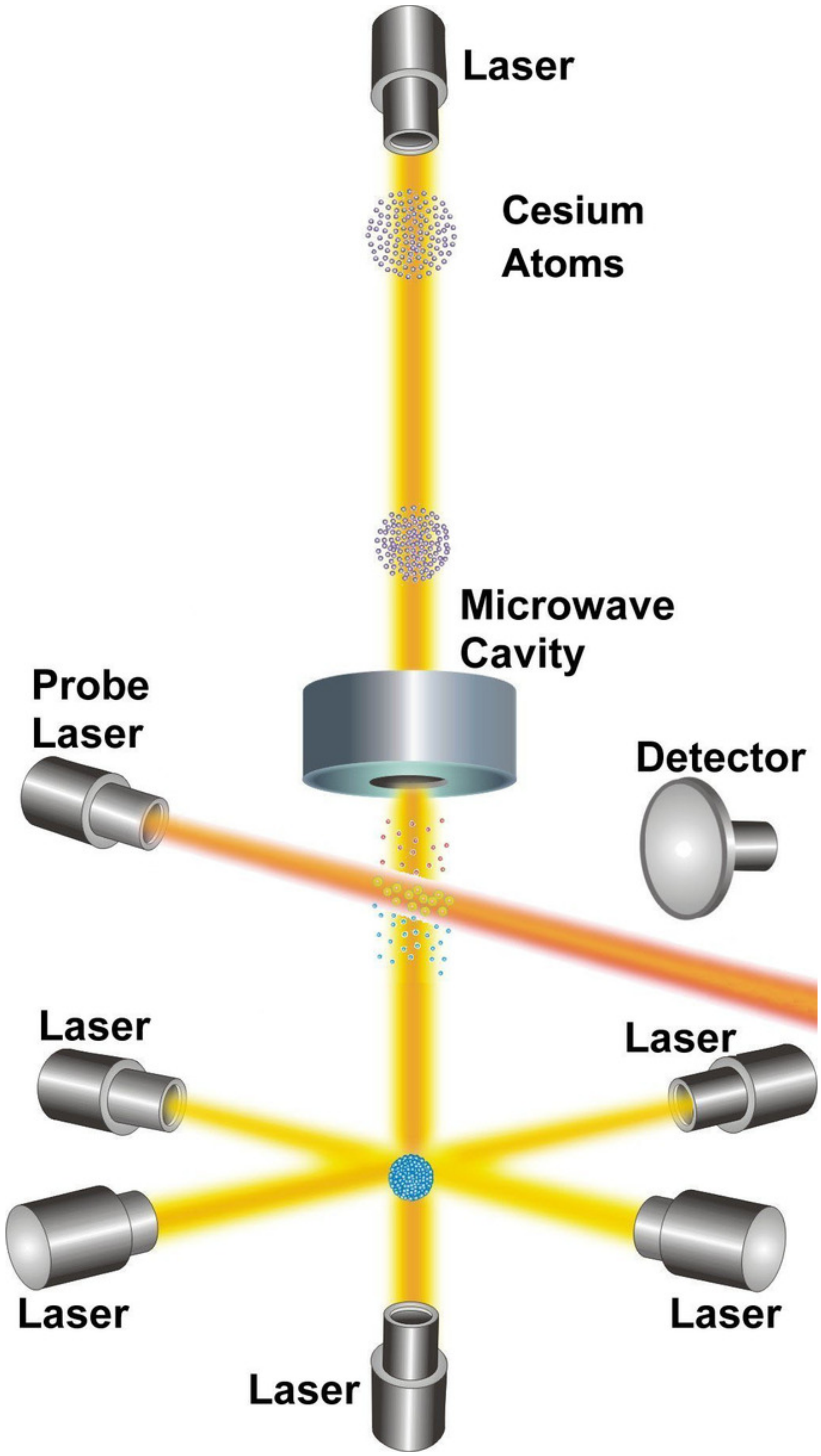
learning objectives

- Relate time with other physical quantities

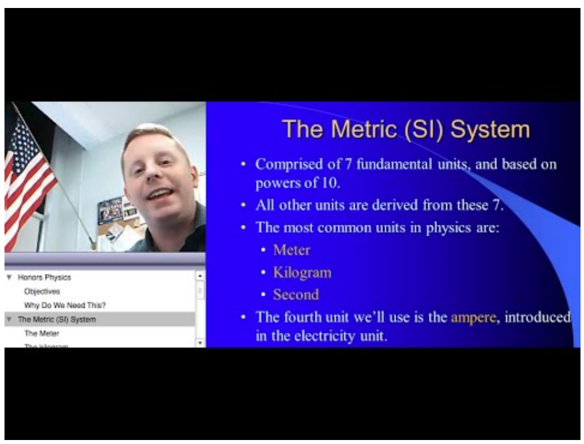
Time is one of the seven fundamental physical quantities in the International System (SI) of Units. Time is used to define other quantities, such as velocity or acceleration, and as such, it is important that it be standardized and quantified precisely. An operational definition of time is highly useful in the conduct of both advanced experiments and everyday affairs of life.

Historically, temporal measurement was a prime motivation in navigation and astronomy. Periodic events and motion have long served as standards for units of time. For example, the movement of the sun across the sky, the phases of the moon, the swing of a pendulum, and the beat of a heart have all been used as a standard for time keeping. These events and standards, however, are highly dynamic in nature and cannot reliably be utilized for accurate quantitative measures. Between 1000 and 1960 the second was defined as $\frac{1}{86,400}$ of a mean solar day. This definition changed between 1960 and 1967 and was defined in terms of the period of the Earth's orbit around the Sun in 1900. Today, the SI Unit of the second is defined in terms of radiation emitted by cesium atoms.

The second is now operationally defined as "the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom." It follows that the hyperfine splitting in the ground state of the cesium 133 atom is exactly 9,192,631,770 hertz. In other words, cesium atoms can be made to vibrate in a very steady way, and these vibrations can be readily observed and counted. The second is the time required for 9,192,631,770 of these vibrations to occur.



NIST-F1 Cesium Clock: NIST-F1 is referred to as a fountain clock because it uses a fountain-like movement of atoms to obtain its improved reckoning of time.



Metric System – Time: A brief introduction to the metric system and unit conversions.

Prefixes and Other Systems of Units

SI prefixes precede a basic unit of measure to indicate a multiple or fraction of the unit.

learning objectives

- Apply prefixes to units and distinguish between SI and customary units

Prefixes

A metric prefix, or SI prefix, is a unit prefix that precedes a basic unit of measure to indicate a multiple or fraction of the unit. Each prefix has a unique symbol that is prepended to the unit symbol. The prefix kilo-, for example, may be added to gram to indicate multiplication by one thousand; one kilogram is equal to one thousand grams (1 kg = 1000 g). The prefix centi-, likewise, may be added to meter to indicate division by one hundred; one centimeter is equal to one hundredth of a meter (1 cm = 0.01 m). Prefixes in varying multiples of 10 are a feature of all forms of the metric system, with many dating back to the system's introduction in the 1790s. Today, the prefixes are standardized for use in the International System of Units (SI) by the International Bureau of Weights and Measures.There are twenty prefixes officially specified by SI.

Metric prefixes

Prefix	Symbol	1000^m	10^n	Decimal	Short scale	Long scale
yotta	Y	1000^8	10^{24}	1000000000000000000000000	septillion	quadrillion
zetta	Z	1000^7	10^{21}	1000000000000000000000	sextillion	trilliard
exa	E	1000^6	10^{18}	1000000000000000000	quintillion	trillion
peta	P	1000^5	10^{15}	1000000000000000	quadrillion	billiard
tera	T	1000^4	10^{12}	1000000000000	trillion	billion
giga	G	1000^3	10^9	1000000000	billion	milliard
mega	M	1000^2	10^6	1000000	million	
kilo	k	1000^1	10^3	1000	thousand	
hecto	h	$1000^{2/3}$	10^2	100	hundred	
deca	da	$1000^{1/3}$	10^1	10	ten	
		1000^0	10^0	1	one	
deci	d	$1000^{-1/3}$	10^{-1}	0.1	tenth	
centi	c	$1000^{-2/3}$	10^{-2}	0.01	hundredth	
milli	m	1000^{-1}	10^{-3}	0.001	thousandth	
micro	μ	1000^{-2}	10^{-6}	0.000001	millionth	
nano	n	1000^{-3}	10^{-9}	0.000000001	billionth	milliardth
pico	p	1000^{-4}	10^{-12}	0.000000000001	trillionth	billionth
femto	f	1000^{-5}	10^{-15}	0.000000000000001	quadrillionth	billiardth
atto	a	1000^{-6}	10^{-18}	0.000000000000000001	quintillionth	trillionth
zepto	z	1000^{-7}	10^{-21}	0.000000000000000000001	sextillionth	trilliardth
yocto	y	1000^{-8}	10^{-24}	0.000000000000000000000001	septillionth	quadrillionth

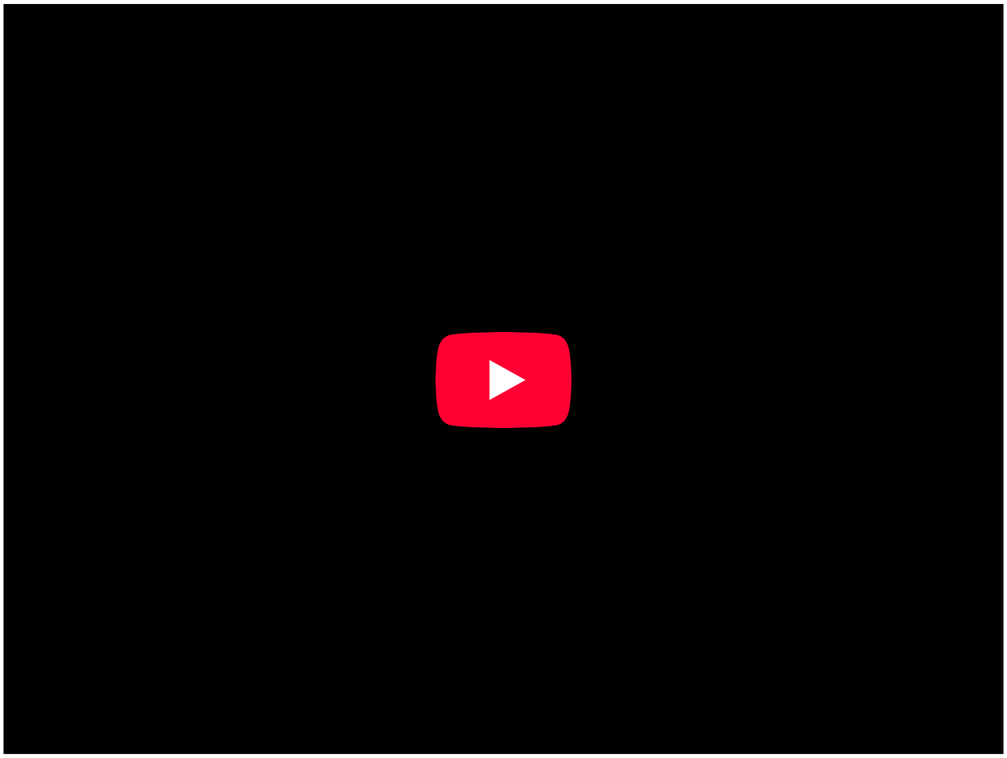
SI Unit Prefixes: The twenty prefixes officially specified by the International System of Units

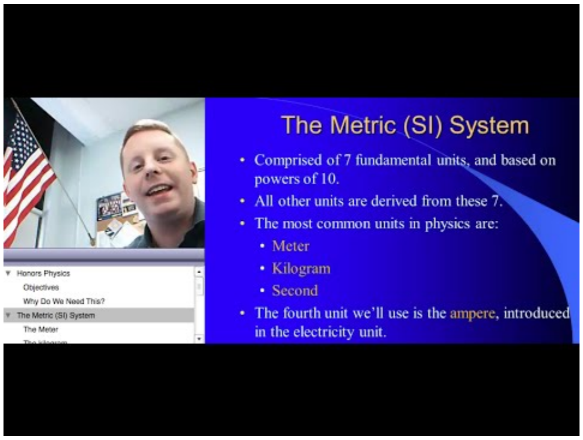
It is important to note that the kilogram is the only SI unit with a prefix as part of its name and symbol. Because multiple prefixes may not be used, in the case of the kilogram the prefix names are used with the unit name "gram" and the prefix symbols are used with the unit symbol "g. " With this exception, any SI prefix may be used with any SI unit, including the degree Celsius and its symbol °C.

Other Systems of Units

The SI Unit system, or the metric system, is used by the majority of countries in the world, and is the standard system agreed upon by scientists and mathematicians. Colloquially, however, other systems of units are used in many countries. The United States, for example, teaches and uses the *United States customary units*. This system of units was developed from the English, or Imperial, unit standards of the United Kingdom.The United States customary units define measurements using different standards than those used in SI Units. The system for measuring length using the United States customary system is based on the inch, foot, yard, and mile. Likewise, units of area are measured in terms of square feet, and units of capacity and volume are measured in terms of cubic inches, cubic feet, or cubic yards. Units of mass are commonly defined in terms of ounces and pounds, rather than the SI unit of kilograms.Other commonly used units from the United States customary system include the fluid volume units of the teaspoon, tablespoon, fluid ounce, US cup, pint, quart, and gallon, as well as the degrees Fahrenheit used to measure temperature.

Some units that are widely used are not a part of the International System of Units and are considered Non-SI Units. These units, though not officially part of SI Units, are generally accepted for use in conjunction with SI units. These can include the minute, hour, and day used in temporal measurements, the liter for volumetric measurements, and the degree, minute, and second used to measure angles.





Metric System – Prefixes: A brief introduction to the metric system and unit conversions.

Converting Units

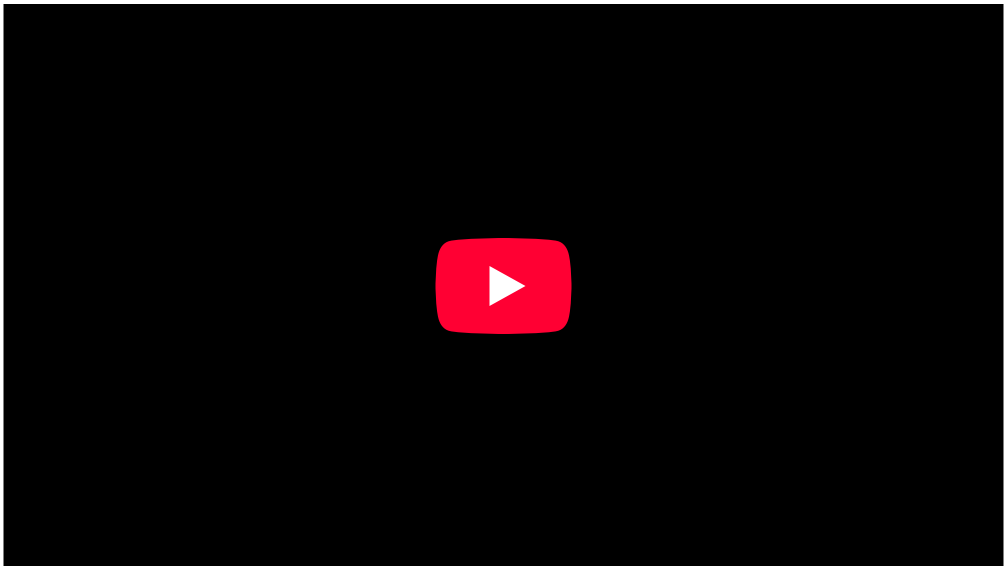
Converting between units can be done through the use of conversion factors or specific conversion formulas.

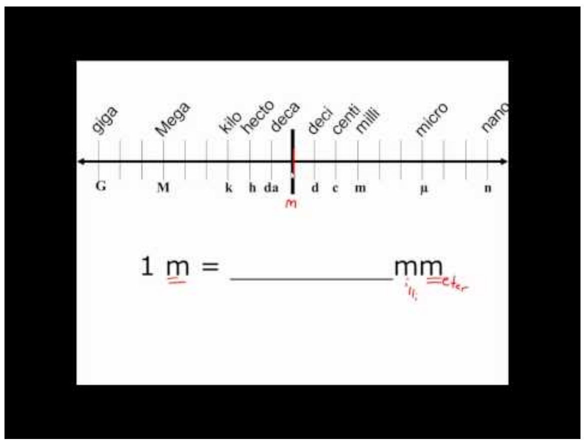
learning objectives

- Apply factor-label method for converting units

Translating Systems of Measurement

It is often necessary to convert from one type of unit to another. Conversion of units is the conversion of different units of measurement for the same quantity, typically using conversion factors. For example, if you are reading a European cookbook, some quantities may be expressed in units of liters; if you're cooking in the US in a standard kitchen with standard tools, you will need to convert those measurements to cups. Or, perhaps you are reading walking directions from one location to another and you are interested in how many miles you will be walking. In this case, you will need to convert units of feet to miles. This is a bit like translating a substitution code, using a formula that helps you understand what one measure means in terms of another system.





Unit Conversion in the Metric System: EASY Unit Conversion in the Metric System – This simple extra help video tutorial explains the metric system and how to make simple metric conversions.

Conversion Methods

There are several ways to approach doing conversions. One commonly used method is known as the Factor-label method for converting units, or the “railroad method. ”

The factor-label method is the sequential application of conversion factors expressed as fractions and arranged so that any dimensional unit appearing in both the numerator and denominator of any of the fractions can be cancelled out until only the desired set of dimensional units is obtained. For example, 10 miles per hour can be converted to meters per second by using a sequence of conversion factors.

Each conversion factor is equivalent to the value of one. For example, starting with 1 mile = 1609 meters and dividing both sides of the equation by 1 mile yields $\frac{1\text{ mile}}{1\text{ mile}} = \frac{1609\text{ meters}}{1\text{ mile}}$, which when simplified yields $1 = \frac{1609\text{ meters}}{1\text{ mile}}$. Physically crossing out the units that cancel each other out will also help visualize what’s left over.

$$1\ \cancel{\text{year}} \times \frac{365\ \cancel{\text{days}}}{1\ \cancel{\text{year}}} \times \frac{24\ \cancel{\text{hours}}}{1\ \cancel{\text{day}}} \times \frac{60\ \cancel{\text{min}}}{1\ \cancel{\text{hour}}} \times \frac{60\text{ s}}{1\ \cancel{\text{min}}} = $$
$$= 3.15 \times 10^7\text{ s}$$

Converting 1 year into seconds using the Factor-Label Method: Physically crossing out units that cancel out helps visualize the “leftover” unit(s).

So, when the units mile and hour are cancelled out and the arithmetic is done, 10 miles per hour converts to 4.47 meters per second.

A limitation of the factor-label method is that it can only convert between units that have a constant ratio that can be multiplied, or a multiplication factor. This method cannot be used between units that have a displacement, or difference factor. An example is the conversion between degrees Celsius and kelvins, or between Celsius and Fahrenheit. For these, it is best to use the specific conversion formulas.

For example, if you are planning a trip abroad in Spain and the weather forecast predicts the weather to be mostly cloudy and 16°C, you may want to convert the temperature into °F, a unit that you are more comfortable interpreting. In order to do this, you would need to know the conversion formula from Celsius to Fahrenheit. This formula is: $[^\circ \mathrm{F}] = [^\circ \mathrm{C}] \times \frac{9}{5} + 32$.

$$[^\circ \mathrm{F}] = [^\circ \mathrm{C}] \times \frac{9}{5} + 32 \tag{1.2.1}$$

$$[^\circ \mathrm{F}] = 28.8 + 32 \tag{1.2.2}$$

$$[^\circ \mathrm{F}] = 60.8 + 32 \tag{1.2.3}$$

So you would then know that 16°C is equivalent to 60.8°F and be able to pack the right type of clothing to be comfortable.

Key Points

- The SI unit for length is the meter.
- One meter is defined as the distance that light travels in a vacuum in $\frac{1}{299,792,458}$ of a second.
- Derivatives of measurement units related to the meter are devised around the convenience of the number 10.
- The kilogram is the only SI unit directly defined by the artifact itself.
- Mass is a property that does not depend on gravitational fields, unlike weight.
- One kilogram is defined as the mass of the International Prototype Kilogram (IPK), a platinum-iridium alloy cylinder.
- One kilogram is almost exactly equal to the mass of one liter of water.
- Time is a physical quantity of duration.
- The SI Unit for time is the second.
- The second is operationally defined in terms of radiation emitted by cesium atoms.
- The twenty standardized prefixes for use in the International System of Units are derived from multiples of 10.
- The kilogram is the only SI unit with a prefix as part of its name and symbol; as such, SI unit prefixes are prepended to the unit gram.
- The United States customary units define measurements based on the English, or Imperial, unit standards.
- Conversion of units is the conversion between different units of measurement for the same quantity, typically through multiplicative conversion factors.
- The factor-label method is the sequential application of conversion factors expressed as fractions in which units appearing in both the numerator and denominator can be cancelled out, leaving only the desired set of units.
- For conversions that have a difference factor, specific conversion formulas should be used.

Key Terms

- **Length**: How far apart objects are physically.
- **acceleration**: the rate at which the velocity of a body changes with time
- **inertia**: the tendency of an object to resist any change in its motion
- **Radiation**: the emission of energy as electromagnetic waves or as moving or oscillating subatomic particles.
- **prefix**: That which is prefixed; especially one or more letters or syllables added to the beginning of a word to modify its meaning; as, pre- in prefix, con- in conjure.
- **conversion**: a change between different units of measurement for the same quantity.

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1.3: Significant Figures and Order of Magnitude

Scientific Notation

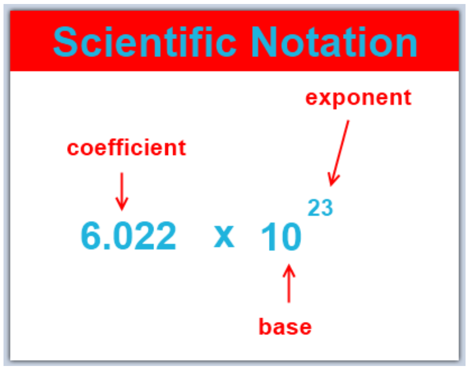
Scientific notation is a way of writing numbers that are too big or too small in a convenient and standard form.

learning objectives

- Convert properly between standard and scientific notation and identify appropriate situations to use it

Scientific Notation: A Matter of Convenience

Scientific notation is a way of writing numbers that are too big or too small in a convenient and standard form. Scientific notation has a number of useful properties and is commonly used in calculators and by scientists, mathematicians and engineers. In scientific notation all numbers are written in the form of $a \cdot 10^{b}$ (a multiplied by ten raised to the power of b), where the exponent b) is an integer, and the coefficient (a is any real number.



Scientific Notation: There are three parts to writing a number in scientific notation: the coefficient, the base, and the exponent.

Most of the interesting phenomena in our universe are not on the human scale. It would take about 1,000,000,000,000,000,000,000 bacteria to equal the mass of a human body. Thomas Young's discovery that light was a wave preceded the use of scientific notation, and he was obliged to write that the time required for one vibration of the wave was "$\frac{1}{500}$ of a millionth of a millionth of a second"; an inconvenient way of expressing the point. Scientific notation is a less awkward and wordy way to write very large and very small numbers such as these.

A Simple System

Scientific notation means writing a number in terms of a product of something from 1 to 10 and something else that is a power of ten.

For instance, $32 = 3.2 \cdot 10^{1}$

$320 = 3.2 \cdot 10^{2}$

$3200 = 3.2 \cdot 10^{3}$, and so forth…

Each number is ten times bigger than the previous one. Since 10^{1} is ten times smaller than 10^{2}, it makes sense to use the notation 10^{0} to stand for one, the number that is in turn ten times smaller than 10^{1}. Continuing on, we can write 10^{-1} to stand for 0.1, the number ten times smaller than 10^{0}. Negative exponents are used for small numbers:

$3.2 = 3.2 \cdot 10^{0}$

$0.32 = 3.2 \cdot 10^{-1}$

$0.032 = 3.2 \cdot 10^{-2}$

Scientific notation displayed calculators can take other shortened forms that mean the same thing. For example, $3.2 \cdot 10^{6}$(written notation) is the same as $3.2\mathrm{E}+6$ (notation on some calculators) and 3.2^{6} (notation on some other calculators).

Round-off Error

A round-off error is the difference between the calculated approximation of a number and its exact mathematical value.

learning objectives

- Explain the impact round-off errors may have on calculations, and how to reduce this impact

Round-off Error

A round-off error, also called a rounding error, is the difference between the calculated approximation of a number and its exact mathematical value. Numerical analysis specifically tries to estimate this error when using approximation equations, algorithms, or both, especially when using finitely many digits to represent real numbers. When a sequence of calculations subject to rounding errors is made, errors may accumulate, sometimes dominating the calculation.

Calculations rarely lead to whole numbers. As such, values are expressed in the form of a decimal with infinite digits. The more digits that are used, the more accurate the calculations will be upon completion. Using a slew of digits in multiple calculations, however, is often unfeasible if calculating by hand and can lead to much more human error when keeping track of so many digits. To make calculations much easier, the results are often 'rounded off' to the nearest few decimal places.

For example, the equation for finding the area of a circle is $\mathrm{A}=\pi \mathrm{r}^{2}$. The number π(pi) has infinitely many digits, but can be truncated to a rounded representation of as 3.14159265359. However, for the convenience of performing calculations by hand, this number is typically rounded even further, to the nearest two decimal places, giving just 3.14. Though this technically decreases the accuracy of the calculations, the value derived is typically 'close enough' for most estimation purposes.

However, when doing a series of calculations, numbers are rounded off at each subsequent step. This leads to an accumulation of errors, and if profound enough, can misrepresent calculated values and lead to miscalculations and mistakes.

The following is an example of round-off error:

$$\sqrt{4.58^{2}+3.28^{2}}=\sqrt{21.0+10.8}=5.64$$

Rounding these numbers off to one decimal place or to the nearest whole number would change the answer to 5.7 and 6, respectively. The more rounding off that is done, the more errors are introduced.

Order of Magnitude Calculations

An order of magnitude is the class of scale of any amount in which each class contains values of a fixed ratio to the class preceding it.

learning objectives

- Choose when it is appropriate to perform an order-of-magnitude calculation

Orders of Magnitude

An order of magnitude is the class of scale of any amount in which each class contains values of a fixed ratio to the class preceding it. In its most common usage, the amount scaled is 10, and the scale is the exponent applied to this amount (therefore, to be an order of magnitude greater is to be 10 times, or 10 to the power of 1, greater). Such differences in order of magnitude can be measured on the logarithmic scale in "decades," or factors of ten. It is common among scientists and technologists to say that a parameter whose value is not accurately known or is known only within a range is "on the order of" some value. The order of magnitude of a physical quantity is its magnitude in powers of ten when the physical quantity is expressed in powers of ten with one digit to the left of the decimal.

Orders of magnitude are generally used to make very approximate comparisons and reflect very large differences. If two numbers differ by one order of magnitude, one is about ten times larger than the other. If they differ by two orders of magnitude, they differ by a factor of about 100. Two numbers of the same order of magnitude have roughly the same scale — the larger value is less than ten times the smaller value.

It is important in the field of science that estimates be at least in the right ballpark. In many situations, it is often sufficient for an estimate to be within an order of magnitude of the value in question. Although making order-of-magnitude estimates seems simple and natural to experienced scientists, it may be completely unfamiliar to the less experienced.

Example 1.3.1:

Some of the mental steps of estimating in orders of magnitude are illustrated in answering the following example question: Roughly what percentage of the price of a tomato comes from the cost of transporting it in a truck?

Guessing the Number of Jelly Beans: Can you guess how many jelly beans are in the jar? If you try to guess directly, you will almost certainly underestimate. The right way to do it is to estimate the linear dimensions and then estimate the volume indirectly.

Incorrect solution: Let's say the trucker needs to make a profit on the trip. Taking into account her benifits, the cost of gas, and maintenance and payments on the truck, let's say the total cost is more like 2000. You might guess about 5000 tomatoes would fit in the back of the truck, so the extra cost per tomato is 40 cents. That means the cost of transporting one tomato is comparable to the cost of the tomato itself.

The problem here is that the human brain is not very good at estimating area or volume — it turns out the estimate of 5000 tomatoes fitting in the truck is way off. (This is why people have a hard time in volume-estimation contests, such as the one shown below.) When estimating area or volume, you are much better off estimating linear dimensions and computing the volume from there.

So, here's a better solution: As before, let's say the cost of the trip is $2000. The dimensions of the bin are probably 4m by 2m by 1m, for a volume of $8 \mathrm{~m}^3$. Since our goal is just an order-of-magnitude estimate, let's round that volume off to the nearest power of ten: $10 \mathrm{~m}^3$. The shape of a tomato doesn't follow linear dimensions, but since this is just an estimate, let's pretend that a tomato is an 0.1m by 0.1m by 0.1m cube, with a volume of $1 \cdot 10^{-3} \mathrm{~m}^3$. We can find the total number of tomatoes by dividing the volume of the bin by the volume of one tomato: $\frac{10^3 \mathrm{~m}^3}{10^{-3} \mathrm{~m}^3} = 10^6$ tomatoes. The transportation cost per tomato is $\frac{\$2000}{10^6 \text{ tomatoes}} = \$0.002$ per tomato. That means that transportation really doesn't contribute very much to the cost of a tomato. Approximating the shape of a tomato as a cube is an example of another general strategy for making order-of-magnitude estimates.

Key Points

- Scientific notation means writing a number in terms of a product of something from 1 to 10 and something else that is a power of 10.
- In scientific notation all numbers are written in the form of $a \cdot 10^b$ (a times ten raised to the power of b).
- Each consecutive exponent number is ten times bigger than the previous one; negative exponents are used for small numbers.
- When a sequence of calculations subject to rounding error is made, these errors can accumulate and lead to the misrepresentation of calculated values.
- Increasing the number of digits allowed in a representation reduces the magnitude of possible round-off errors, but may not always be feasible, especially when doing manual calculations.
- The degree to which numbers are rounded off is relative to the purpose of calculations and the actual value.
- Orders of magnitude are generally used to make very approximate comparisons and reflect very large differences.
- In the field of science, it is often sufficient for an estimate to be within an order of magnitude of the value in question.

- When estimating area or volume, you are much better off estimating linear dimensions and computing volume from those linear dimensions.

Key Terms

- **exponent**: The power to which a number, symbol or expression is to be raised. For example, the 3 in x^3.
- **Scientific notation**: A method of writing, or of displaying real numbers as a decimal number between 1 and 10 followed by an integer power of 10
- **approximation**: An imprecise solution or result that is adequate for a defined purpose.
- **Order of Magnitude**: The class of scale or magnitude of any amount, where each class contains values of a fixed ratio (most often 10) to the class preceding it. For example, something that is 2 orders of magnitude larger is 100 times larger; something that is 3 orders of magnitude larger is 1000 times larger; and something that is 6 orders of magnitude larger is one million times larger, because $10^2 = 100$, $10^3 = 1000$, and $10^6 =$ one million

1.4: Solving Physics Problems

Dimensional Analysis

Any physical quantity can be expressed as a product of a combination of the basic physical dimensions.

learning objectives

- Calculate the conversion from one kind of dimension to another

Dimensions

The dimension of a physical quantity indicates how it relates to one of the seven basic quantities. These fundamental quantities are:

- [M] Mass
- [L] Length
- [T] Time
- [A] Current
- [K] Temperature
- [mol] Amount of a Substance
- [cd] Luminous Intensity

As you can see, the symbol is enclosed in a pair of square brackets. This is often used to represent the dimension of individual basic quantity. An example of the use of basic dimensions is speed, which has a dimension of 1 in length and -1 in time; $\frac{[\mathrm{L}]}{[\mathrm{T}]}=[\mathrm{LT}^{-1}]$. Any physical quantity can be expressed as a product of a combination of the basic physical dimensions.

Dimensional Analysis

Dimensional analysis is the practice of checking relations between physical quantities by identifying their dimensions. The dimension of any physical quantity is the combination of the basic physical dimensions that compose it. Dimensional analysis is based on the fact that physical law must be independent of the units used to measure the physical variables. It can be used to check the plausibility of derived equations, computations and hypotheses.

Derived Dimensions

The dimensions of derived quantities may include few or all dimensions in individual basic quantities. In order to understand the technique to write dimensions of a derived quantity, we consider the case of force. Force is defined as:

$$\mathrm{F}=\mathrm{m}\cdot\mathrm{a} \tag{1.4.1}$$

$$\mathrm{F}=[\mathrm{M}][\mathrm{a}] \tag{1.4.2}$$

The dimension of acceleration, represented as [a], is itself a derived quantity being the ratio of velocity and time. In turn, velocity is also a derived quantity, being ratio of length and time.

$$\mathrm{F}=[\mathrm{M}][\mathrm{a}]=[\mathrm{M}][\mathrm{vT}^{-1}] \tag{1.4.3}$$

$$\mathrm{F}=[\mathrm{M}][\mathrm{LT}^{-1}\mathrm{T}^{-1}]=[\mathrm{MLT}^{-2}] \tag{1.4.4}$$

Dimensional Conversion

In practice, one might need to convert from one kind of dimension to another. For common conversions, you might already know how to convert off the top of your head. But for less common ones, it is helpful to know how to find the conversion factor:

$$\mathrm{Q}=\mathrm{n}_1\mathrm{u}_1=\mathrm{n}_2\mathrm{u}_2 \tag{1.4.5}$$

where n represents the amount per u dimensions. You can then use ratios to figure out the conversion:

$$\mathrm{n}_2=\frac{\mathrm{u}_2}{\mathrm{u}_1}\cdot\mathrm{n}_1 \tag{1.4.6}$$

Trigonometry

Trigonometry is central to the use of free body diagrams, which help visually represent difficult physics problems.

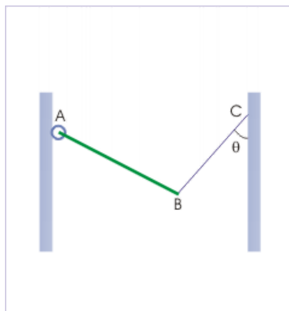
learning objectives

- Explain why trigonometry is useful in determining horizontal and vertical components of forces

Trigonometry and Solving Physics Problems

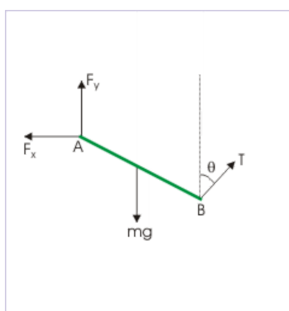
In physics, most problems are solved much more easily when a free body diagram is used. Free body diagrams use geometry and vectors to visually represent the problem. Trigonometry is also used in determining the horizontal and vertical components of forces and objects. Free body diagrams are very helpful in visually identifying which components are unknown and where the moments are applied. They can help analyze a problem, whether it is static or dynamic.

When people draw free body diagrams, often not everything is perfectly parallel and perpendicular. Sometimes people need to analyze the horizontal and vertical components of forces and object orientation. When the force or object is not acting parallel to the x or y axis, people can employ basic trigonometry to use the simplest components of the action to analyze it. Basically, everything should be considered in terms of x and y, which sometimes takes some manipulation.



Free Body Diagram: The rod is hinged from a wall and is held with the help of a string.

A rod ‘AB’ is hinged at ‘A’ from a wall and is held still with the help of a string, as shown in. This exercise involves drawing the free body diagram. To make the problem easier, the force F will be expressed in terms of its horizontal and vertical components. Removing all other elements from the image helps produce the finished free body diagram.



Free Body Diagram: The free body diagram as a finished product

Given the finished free body diagram, people can use their knowledge of trigonometry and the laws of sine and cosine to mathematically and numerical represent the horizontal and vertical components:

General Problem-Solving Tricks

Free body diagrams use geometry and vectors to visually represent the problem.

learning objectives

- Construct a free-body diagram for a physical scenario

In physics, most problems are solved much more easily when a free body diagram is used. This uses geometry and vectors to visually represent to problem, and trigonometry is also used in determining horizontal and vertical components of forces and objects.

Purpose: Free body diagrams are very helpful in visually identifying which components are unknown, where the moments are applied, and help analyze a problem, whether static or dynamic.

How to Make A Free Body Diagram

To draw a free body diagram, do not worry about drawing it to scale, this will just be what you use to help yourself identify the problems. First you want to model the body, in one of three ways:

- As a particle. This model may be used when any turning effects are zero or have zero interest even though the body itself may be extended. The body may be represented by a small symbolic blob and the diagram reduces to a set of concurrent arrows. A force on a particle is a *bound* vector.
- *rigid extended*. Stresses and strains are of no interest but turning effects are. A force arrow should lie along the line of force, but where along the line is irrelevant. A force on an extended rigid body is a*sliding* vector.
- *non-rigid extended*. The *point of application* of a force becomes crucial and has to be indicated on the diagram. A force on a non-rigid body is a *bound* vector. Some engineers use the tail of the arrow to indicate the point of application. Others use the tip.

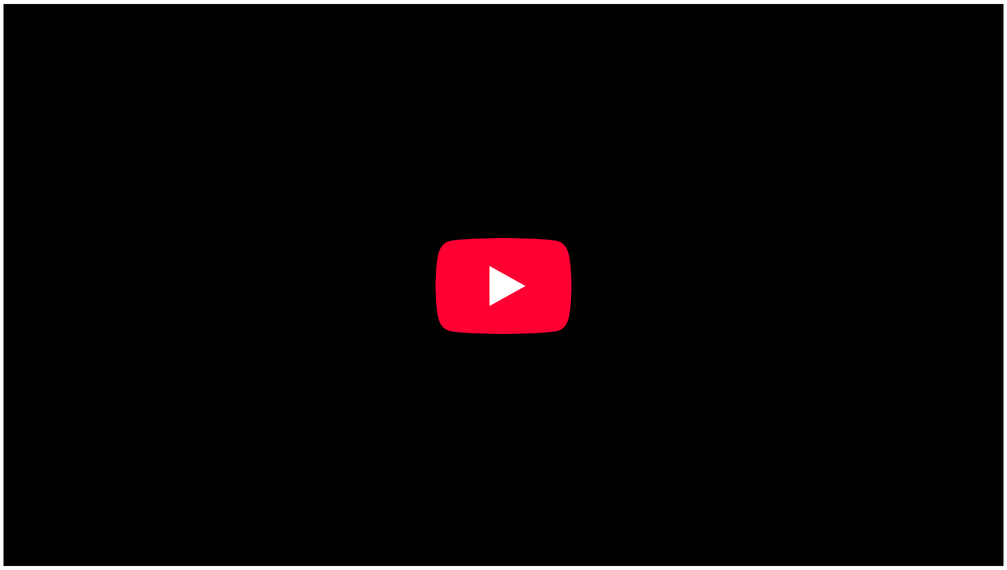
Do’s and Don’ts

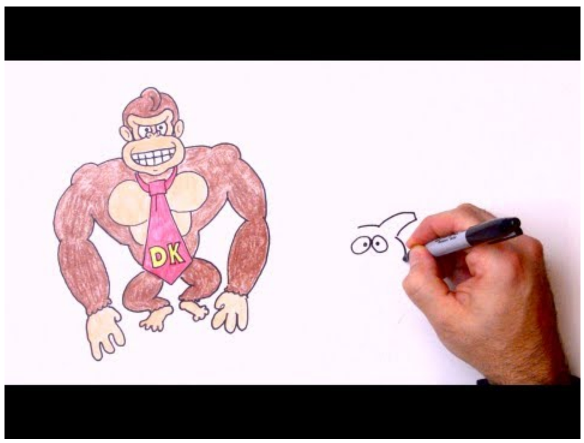
What to include: Since a free body diagram represents the body itself and the external forces on it. So you will want to include the following things in the diagram:

- The body: This is usually sketched in a schematic way depending on the body – particle/extended, rigid/non-rigid – and on what questions are to be answered. Thus if rotation of the body and torque is in consideration, an indication of size and shape of the body is needed.
- The external forces: These are indicated by labelled arrows. In a fully solved problem, a force arrow is capable of indicating the direction, the magnitude the point of application. These forces can be friction, gravity, normal force, drag, tension, etc…

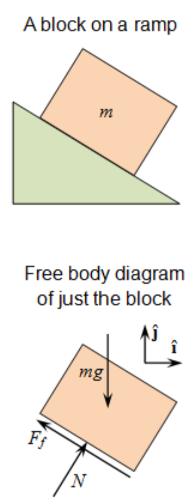
Do not include:

- Do not show bodies other than the body of interest.
- Do not show forces exerted by the body.
- Internal forces acting on various parts of the body by other parts of the body.
- Any velocity or acceleration is left out.





How To Solve Any Physics Problem: Learn five simple steps in five minutes! In this episode we cover the most effective problem-solving method I've encountered and call upon some fuzzy friends to help us remember the steps.



Free Body Diagram: Use this figure to work through the example problem.

Key Points

- Dimensional analysis is the practice of checking relations amount physical quantities by identifying their dimensions.
- It is common to be faced with a problem that uses different dimensions to express the same basic quantity. The following equation can be used to find the conversion factor between the two derived dimensions: $n_2 = \frac{u_2}{u_1} \times n_1$.
- Dimensional analysis can also be used as a simple check to computations, theories and hypotheses.
- It is important to identify the problem and the unknowns and draw them in a free body diagram.
- The laws of cosine and sine can be used to determine the vertical and horizontal components of the different elements of the diagram.
- Free body diagrams use geometry and vectors to visually represent physics problems.
- A free body diagram lets you visually isolate the problem you are trying to solve, and simplify it into simple geometry and trigonometry.
- When drawing these diagrams, it is helpful to only draw the body it self, and the forces acting on it.
- Drawing other objects and internal forces can condense the diagram and cause it to be less helpful.

Key Terms

- **dimension**: A measure of spatial extent in a particular direction, such as height, width or breadth, or depth.
- **trigonometry**: The branch of mathematics that deals with the relationships between the sides and the angles of triangles and the calculations based on them, particularly the trigonometric functions.
- **static**: Fixed in place; having no motion.
- **dynamic**: Changing; active; in motion.

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CHAPTER OVERVIEW

02: Kinematics

Topic hierarchy

2.1: Basics of Kinematics

Defining Kinematics

Kinematics is the study of the motion of points, objects, and groups of objects without considering the causes of its motion.

learning objectives

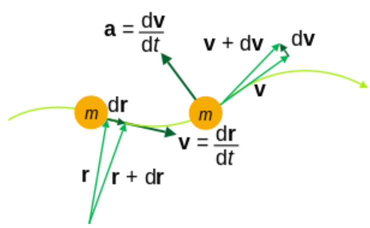
- Define kinematics

Kinematics is the branch of classical mechanics that describes the motion of points, objects and systems of groups of objects, without reference to the causes of motion (i.e., forces). The study of kinematics is often referred to as the "geometry of motion."

Objects are in motion all around us. Everything from a tennis match to a space-probe flyby of the planet Neptune involves motion. When you are resting, your heart moves blood through your veins. Even in inanimate objects there is continuous motion in the vibrations of atoms and molecules. Interesting questions about motion can arise: how long will it take for a space probe to travel to Mars? Where will a football land if thrown at a certain angle? An understanding of motion, however, is also key to understanding other concepts in physics. An understanding of acceleration, for example, is crucial to the study of force.

To describe motion, kinematics studies the trajectories of points, lines and other geometric objects, as well as their differential properties (such as velocity and acceleration). Kinematics is used in astrophysics to describe the motion of celestial bodies and systems; and in mechanical engineering, robotics and biomechanics to describe the motion of systems composed of joined parts (such as an engine, a robotic arm, or the skeleton of the human body).

A formal study of physics begins with kinematics. The word "kinematics" comes from a Greek word "kinesis" meaning motion, and is related to other English words such as "cinema" (movies) and "kinesiology" (the study of human motion). Kinematic analysis is the process of measuring the kinematic quantities used to describe motion. The study of kinematics can be abstracted into purely mathematical expressions, which can be used to calculate various aspects of motion such as velocity, acceleration, displacement, time, and trajectory.



Kinematics of a particle trajectory: Kinematic equations can be used to calculate the trajectory of particles or objects. The physical quantities relevant to the motion of a particle include: mass m, position r, velocity v, acceleration a.

Reference Frames and Displacement

In order to describe an object's motion, you need to specify its position relative to a convenient reference frame.

learning objectives

- Evaluate displacement within a frame of reference.

In order to describe the motion of an object, you must first describe its position — where it is at any particular time. More precisely, you need to specify its position relative to a convenient reference frame. Earth is often used as a reference frame, and we often describe the position of objects related to its position to or from Earth. Mathematically, the position of an object is generally represented by the variable x.

Frames of Reference

There are two choices you have to make in order to define a position variable x. You have to decide where to put $x = 0$ and which direction will be positive. This is referred to as choosing a coordinate system, or choosing a frame of reference. As long as you are consistent, any frame is equally valid. But you don't want to change coordinate systems in the middle of a calculation. Imagine sitting in a train in a station when suddenly you notice that the station is moving backward. Most people would say that they just failed to notice that the train was moving — it only *seemed* like the station was moving. But this shows that there is a *third* arbitrary choice that goes into choosing a coordinate system: valid frames of reference can differ from each other by moving relative to one another. It might seem strange to use a coordinate system moving relative to the earth — but, for instance, the frame of reference moving along with a train might be far more convenient for describing things happening inside the train. Frames of reference are particularly important when describing an object's displacement.

FRAMES OF REFERENCE by Professor Hume and Professor Donald Ivey of the University of Toronto

In this classic film, Professors Hume and Ivey cleverly illustrate reference frames and distinguish between fixed and moving frames of reference.

Frames of Reference (1960) Educational Film: Frames of Reference is a 1960 educational film by Physical Sciences Study Committee. The film was made to be shown in high school physics courses. In the film University of Toronto physics professors Patterson Hume and Donald Ivey explain the distinction between inertial and nonintertial frames of reference, while demonstrating these concepts through humorous camera tricks. For example, the film opens with Dr. Hume, who appears to be upside down, accusing Dr. Ivey of being upside down. Only when the pair flip a coin does it become obvious that Dr. Ivey — and the camera — are indeed inverted. The film's humor serves both to hold students' interest and to demonstrate the concepts being discussed. This PSSC film utilizes a fascinating set consisting of a rotating table and furniture occupying surprisingly unpredictable spots within the viewing area. The fine cinematography by Abraham Morochnik, and funny narration by University of Toronto professors Donald Ivey and Patterson Hume is a wonderful example of the fun a creative team of filmmakers can have with a subject that other, less imaginative types might find pedestrian. Producer: Richard Leacock Production Company: Educational Development Corp. Sponsor: Eric Prestamon

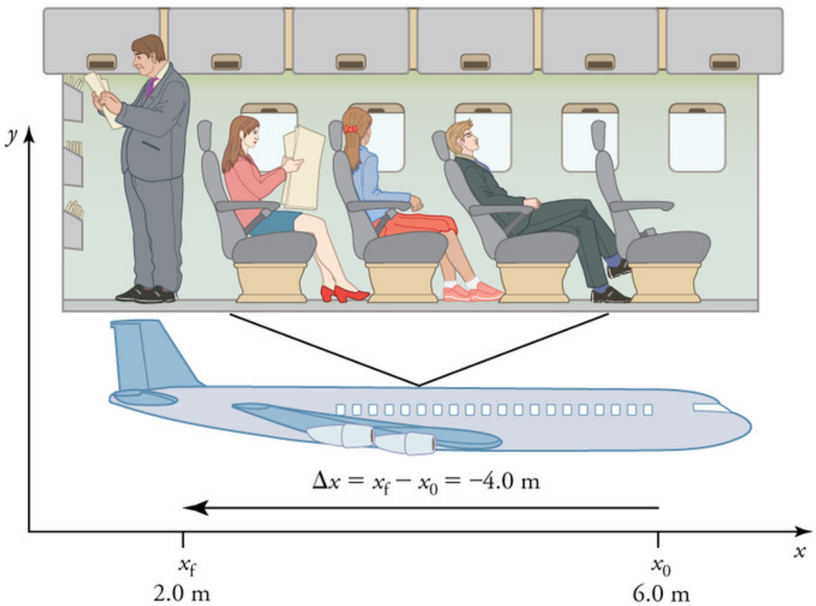
Displacement

Displacement is the change in position of an object relative to its reference frame. For example, if a car moves from a house to a grocery store, its displacement is the relative distance of the grocery store to the reference frame, or the house. The word "displacement" implies that an object has moved or has been displaced. Displacement is the change in position of an object and can be represented mathematically as follows:

$$\Delta \mathrm{x} = \mathrm{x_f} - \mathrm{x_0} \tag{2.1.1}$$

where Δx is displacement, x_f is the final position, and x_0 is the initial position.

shows the importance of using a frame of reference when describing the displacement of a passenger on an airplane.



Displacement in Terms of Frame of Reference: A passenger moves from his seat to the back of the plane. His location relative to the airplane is given by x. The -4.0m displacement of the passenger relative to the plane is represented by an arrow toward the rear of the plane. Notice that the arrow representing his displacement is twice as long as the arrow representing the displacement of the professor (he moves twice as far).

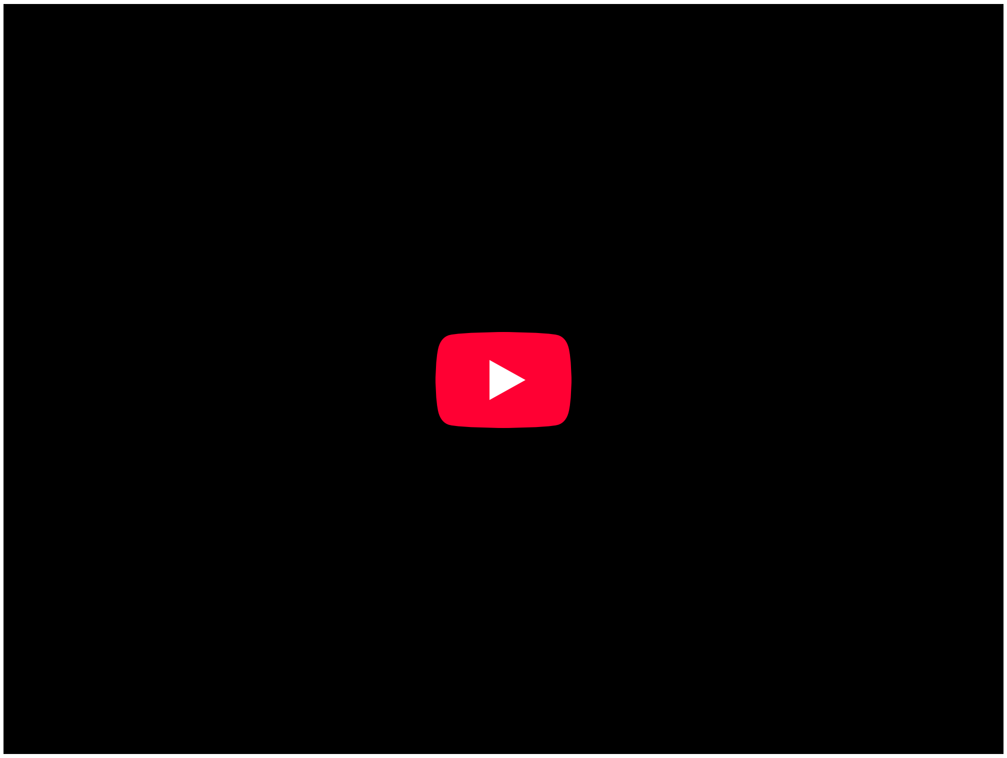
Introduction to Scalars and Vectors

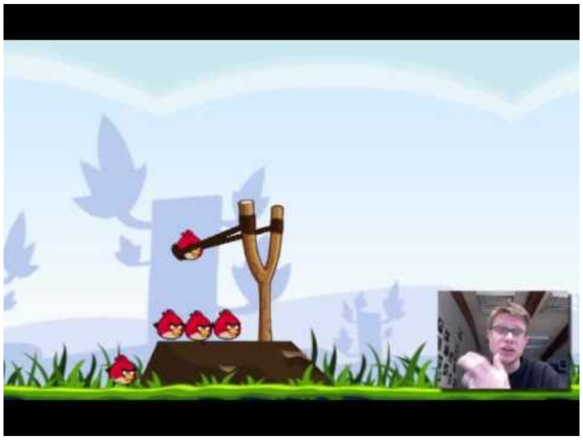
A vector is any quantity that has both magnitude and direction, whereas a scalar has only magnitude.

learning objectives

- Distinguish the difference between scalars and vectors

What is the difference between distance and displacement? Whereas displacement is defined by both direction and magnitude, distance is defined by magnitude alone. Displacement is an example of a vector quantity. Distance is an example of a scalar quantity. A vector is any quantity with both magnitude and direction. Other examples of vectors include a velocity of 90 km/h east and a force of 500 newtons straight down.





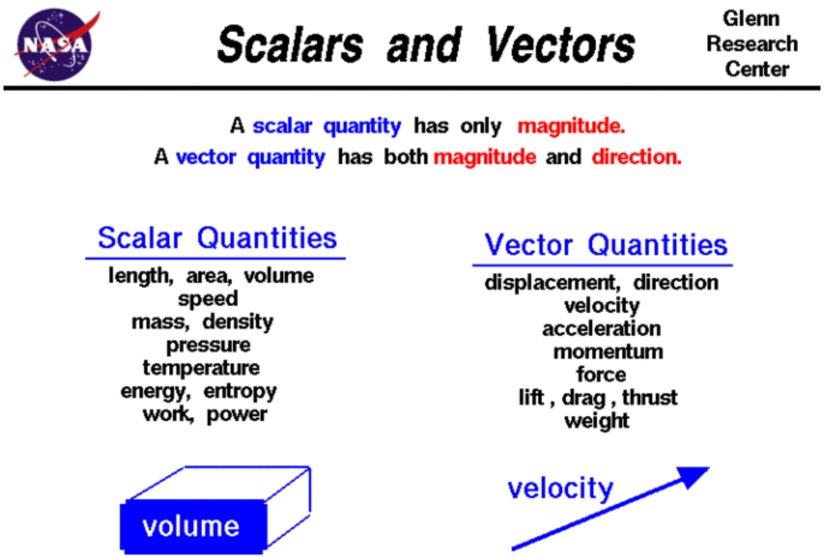
Scalars and Vectors: Mr. Andersen explains the differences between scalar and vectors quantities. He also uses a demonstration to show the importance of vectors and vector addition.

In mathematics, physics, and engineering, a vector is a geometric object that has a magnitude (or length) and direction and can be added to other vectors according to vector algebra. The direction of a vector in one-dimensional motion is given simply by a plus (+) or minus (−) sign. A vector is frequently represented by a line segment with a definite direction, or graphically as an arrow, connecting an initial point A with a terminal point B, as shown in.

$$\overrightarrow{AB}.$$

Vector representation: A vector is frequently represented by a line segment with a definite direction, or graphically as an arrow, connecting an initial point A with a terminal point B.

Some physical quantities, like distance, either have no direction or no specified direction. In physics, a scalar is a simple physical quantity that is not changed by coordinate system rotations or translations. It is any quantity that can be expressed by a single number and has a magnitude, but no direction. For example, a 20ºC temperature, the 250 kilocalories (250 Calories) of energy in a candy bar, a 90 km/h speed limit, a person’s 1.8 m height, and a distance of 2.0 m are all scalars, or quantities with no specified direction. Note, however, that a scalar can be negative, such as a −20ºC temperature. In this case, the minus sign indicates a point on a scale rather than a direction. Scalars are never represented by arrows. (A comparison of scalars vs. vectors is shown in.)



Scalars vs. Vectors: A brief list of quantities that are either scalars or vectors.

Key Points

- To describe motion, kinematics studies the trajectories of points, lines and other geometric objects.
- The study of kinematics can be abstracted into purely mathematical expressions.
- Kinematic equations can be used to calculate various aspects of motion such as velocity, acceleration, displacement, and time.
- Choosing a frame of reference requires deciding where the object's initial position is and which direction will be considered positive.
- Valid frames of reference can differ from each other by moving relative to one another.
- Frames of reference are particularly important when describing an object's displacement.
- Displacement is the change in position of an object relative to its reference frame.
- A vector is any quantity that has magnitude and direction.
- A scalar is any quantity that has magnitude but no direction.
- Displacement and velocity are vectors, whereas distance and speed are scalars.

Key Terms

- **kinematics**: The branch of mechanics concerned with objects in motion, but not with the forces involved.
- **displacement**: A vector quantity that denotes distance with a directional component.
- **frame of reference**: A coordinate system or set of axes within which to measure the position, orientation, and other properties of objects in it.
- **scalar**: A quantity that has magnitude but not direction; compare vector.
- **vector**: A directed quantity, one with both magnitude and direction; the between two points.

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2.2: Speed and Velocity

Average Velocity: A Graphical Interpretation

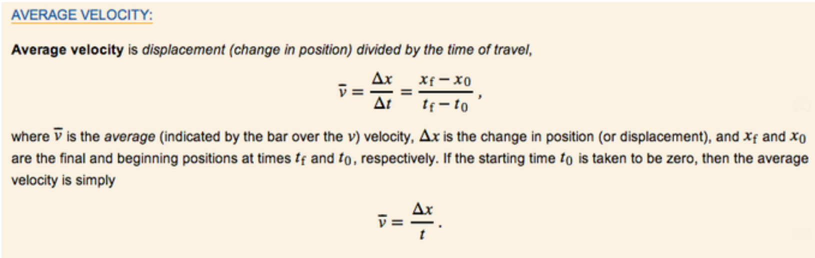
Average velocity is defined as the change in position (or displacement) over the time of travel.

learning objectives

- Contrast speed and velocity in physics

In everyday usage, the terms “speed” and “velocity” are used interchangeably. In physics, however, they are distinct quantities. Speed is a scalar quantity and has only magnitude. Velocity, on the other hand, is a vector quantity and so has both magnitude and direction. This distinction becomes more apparent when we calculate average speed and velocity.

Average speed is calculated as the distance traveled over the total time of travel. In contrast, average velocity is defined as the change in *position* (or displacement) over the total time of travel.



AVERAGE VELOCITY:

Average velocity is *displacement (change in position) divided by the time of travel*,

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_0}{t_f - t_0},$$

where $\bar{v}$ is the *average* (indicated by the bar over the v) velocity, Δx is the change in position (or displacement), and x_f and x_0 are the final and beginning positions at times t_f and t_0, respectively. If the starting time t_0 is taken to be zero, then the average velocity is simply

$$\bar{v} = \frac{\Delta x}{t}.$$

Average Velocity: The kinematic formula for calculating average velocity is the change in position over the time of travel.

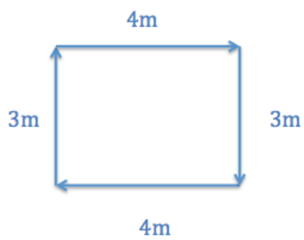
The SI unit for velocity is meters per second, or m/s, but many other units (such as km/h, mph, and cm/s) are commonly used. Suppose, for example, an airplane passenger took five seconds to move -4 m (the negative sign indicates that displacement is toward the back of the plane). His average velocity would be:

$$\mathrm{v} = \frac{\Delta \mathrm{x}}{\mathrm{t}} = \frac{-4\ \mathrm{m}}{5\ \mathrm{s}} = -0.8\frac{\mathrm{m}}{\mathrm{s}} \tag{2.2.1}$$

The minus sign indicates that the average velocity is also toward the rear of the plane.

The average velocity of an object does not tell us anything about what happens to it between the starting point and ending point, however. For example, we cannot tell from average velocity whether the airplane passenger stops momentarily or backs up before he gets to the back of the plane. To get more details, we must consider smaller segments of the trip over smaller time intervals.

To illustrate the difference between average speed and average velocity, consider the following additional example. Imagine you are walking in a small rectangle. You walk three meters north, four meters east, three meters south, and another four meters west. The entire walk takes you 30 seconds. If you are calculating average speed, you would calculate the entire distance (3 + 4 + 3 + 4 = 14 meters) over the total time, 30 seconds. From this, you would get an average speed of 14/30 = 0.47 m/s. When calculating average velocity, however, you are looking at the displacement over time. Because you walked in a full rectangle and ended up exactly where you started, your displacement is 0 meters. Therefore, your average velocity, or displacement over time, would be 0 m/s.



Average Speed vs. Average Velocity: If you started walking from one corner and went all the way around the rectangle in 30 seconds, your average speed would be 0.47 m/s, but your average velocity would be 0 m/s.

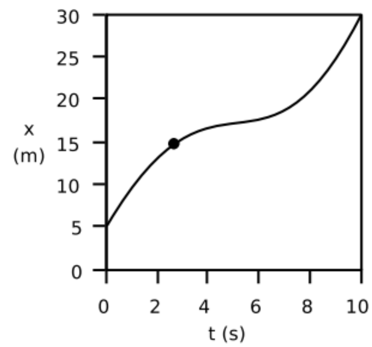
Instananeous Velocity: A Graphical Interpretation

Instantaneous velocity is the velocity of an object at a single point in time and space as calculated by the slope of the tangent line.

learning objectives

- Differentiate instantaneous velocity from other ways of determining velocity

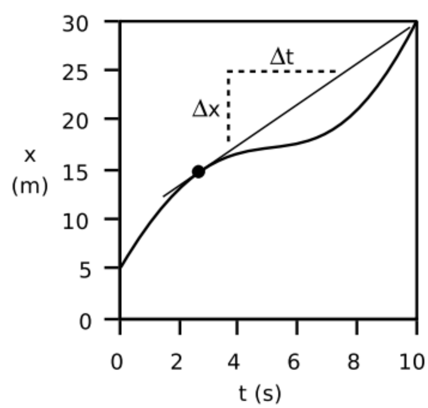
Typically, motion is not with constant velocity nor speed. While driving in a car, for example, we continuously speed up and slow down. A graphical representation of our motion in terms of distance vs. time, therefore, would be more variable or “curvy” rather than a straight line, indicating motion with a constant velocity as shown below. (We limit our discussion to one dimensional motion. It should be straightforward to generalize to three dimensional cases.)



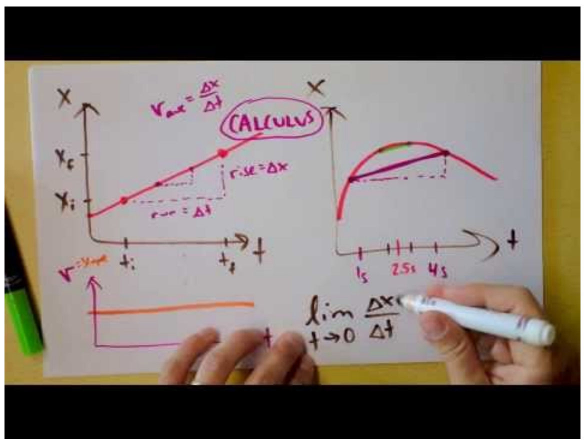
Motion with Changing Velocity: Motion is often observed with changing velocity. This would result in a curvy line when graphed with distance over time.

To calculate the speed of an object from a graph representing constant velocity, all that is needed is to find the slope of the line; this would indicate the change in distance over the change in time. However, changing velocity it is not as straightforward.

Since our velocity is constantly changing, we can estimate velocity in different ways. One way is to look at our instantaneous velocity, represented by one point on our curvy line of motion graphed with distance vs. time. In order to determine our velocity at any given moment, we must determine the slope at that point. To do this, we find a line that represents our velocity in that moment, shown graphically in. That line would be the line tangent to the curve at that point. If we extend this line, we can easily calculate the displacement of distance over time and determine our velocity at that given point. The velocity of an object at any given moment is the slope of the tangent line through the relevant point on its x vs. t graph.



Determining instantaneous velocity: The velocity at any given moment is defined as the slope of the tangent line through the relevant point on the graph



Instantaneous Velocity, Acceleration, Jerk, Slopes, Graphs vs. Time: This is how kinematics begins.

In calculus, finding the slope of curve $f(x)$ at $x=x_0$ x=x0 is equivalent to finding the first derivative:

$$\frac{df(x)}{dx}|_{x=x_0}. \tag{2.2.2}$$

One interpretation of this definition is that the velocity shows how many meters the object would travel in one second if it continues moving at the same speed for at least one second.

Key Points

- Average velocity can be calculated by determining the total displacement divided by the total time of travel.
- The average velocity of an object does not tell us anything about what happens to it between the starting point and ending point.
- Average velocity is different from average speed in that it considers the direction of travel and the overall change in position.
- When velocity is constantly changing, we can estimate our velocity by looking at instantaneous velocity.
- Instantaneous velocity is calculated by determining the slope of the line tangent to the curve at the point of interest.
- Instantaneous velocity is similar to determining how many meters the object would travel in one second at a specific moment.

Key Terms

- **velocity**: A vector quantity that denotes the rate of change of position with respect to time, or a speed with a directional component.
- **instantaneous**: (As in velocity)—occurring, arising, or functioning without any delay; happening within an imperceptibly brief period of time.

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2.3: Acceleration

Graphical Interpretation

The graphical representation of acceleration over time can be derived through the graph of an object's position over time.

learning objectives

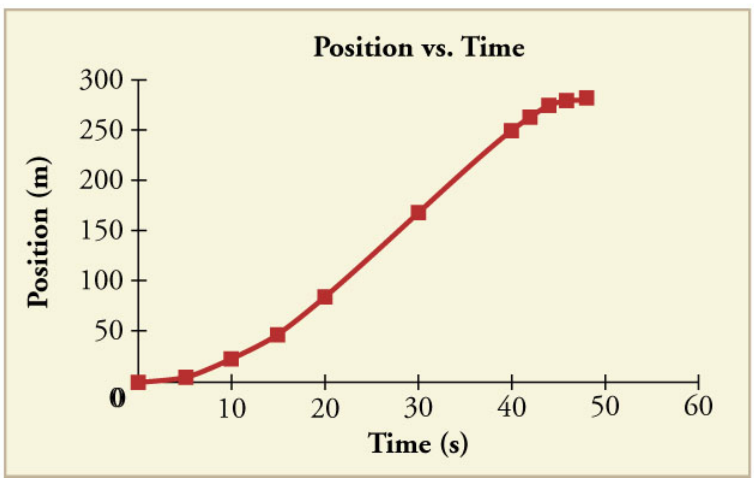
- Distinguish the difference between how to plot a velocity graph and how to plot an acceleration graph

In physics, acceleration is the rate at which the velocity of a body changes with time. It is a vector quantity with both magnitude and direction. Acceleration is accompanied by a force, as described by Newton's Second Law; the force, as a vector, is the product of the mass of the object being accelerated and the acceleration (vector), or $\mathrm{F} = \mathrm{ma}$. The SI unit of acceleration is the meter per second squared: $\frac{\mathrm{m}}{\mathrm{s}^2}$

Acceleration is a vector that points in the same direction as the change in velocity, though it may not always be in the direction of motion. For example, when an object slows down, or decelerating, its acceleration is in the opposite direction of its motion.

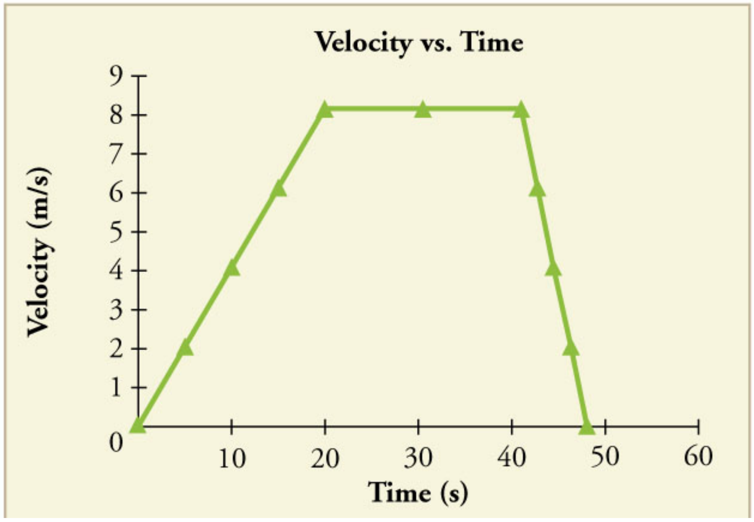
The motion of an object can be depicted graphically by plotting the position of an object over time. This distance-time graph can be used to create another graph that shows changes in velocity over time. Because acceleration is velocity in $\frac{\mathrm{m}}{\mathrm{s}}$ divided by time in s, we can further derive a graph of acceleration from a graph of an object's speed or position.

is a graph of an object's position over time. This graph is similar to the motion of a car. In the beginning, the object's position changes slowly as it gains speed. In the middle, the speed is constant and the position changes at a constant rate. As it slows down toward the end, the position changes more slowly. From this graph, we can derive a velocity vs time graph.



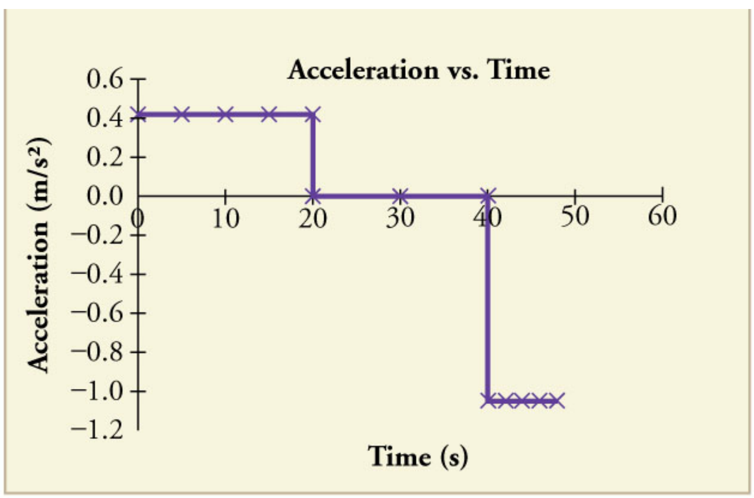
Position vs Time Graph: Notice that the object's position changes slowly at the beginning of the journey, then more and more quickly as it picks up speed. Its position then changes more slowly as it slows down at the end of the journey. In the middle of the journey, while the velocity remains constant, the position changes at a constant rate.

This shows the velocity of the object over time. The object's velocity increases in the beginning as it accelerates at the beginning, then remains constant in the middle before it slows down toward the end. Notice that this graph is a representation of the slope of the previous position vs time graph. From this graph, we can further derive an acceleration vs time graph.

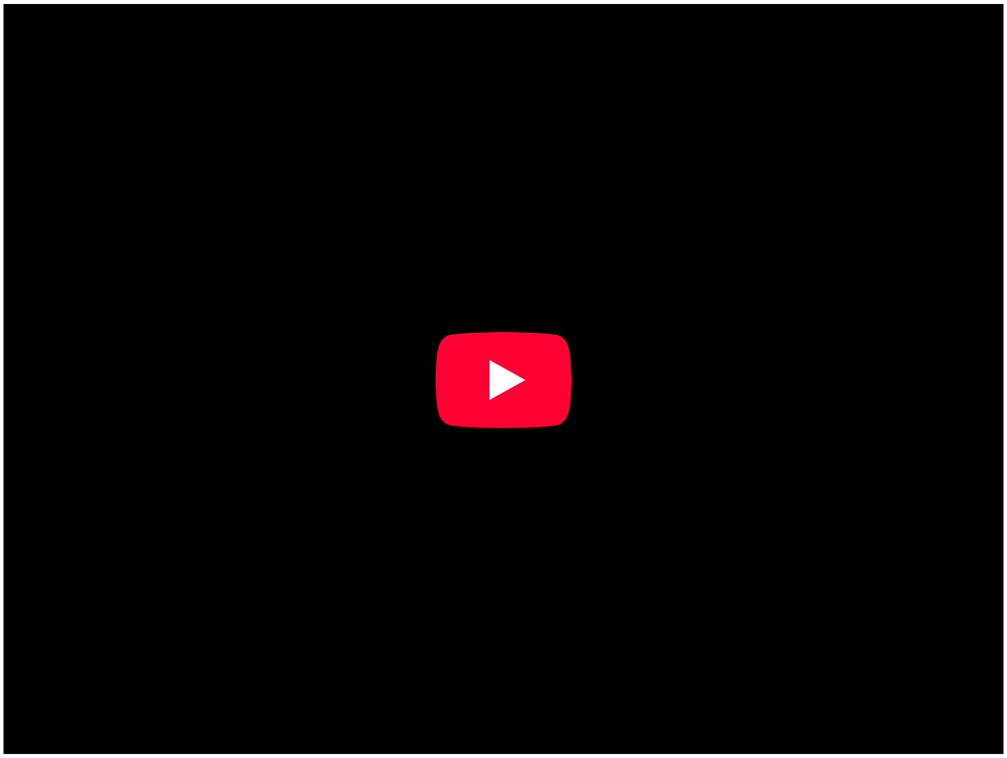


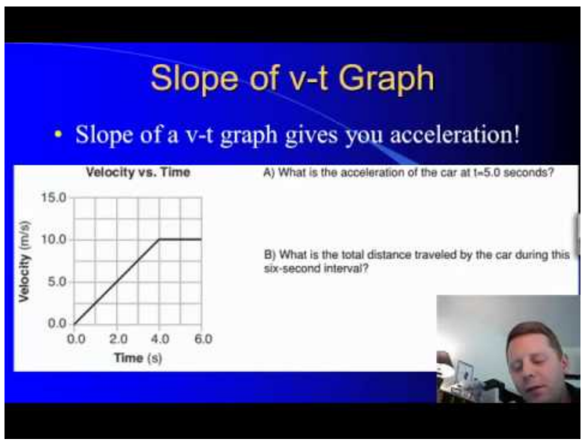
Velocity vs Time: The object's velocity increases as it accelerates at the beginning of the journey. It remains the same in the middle of the journey (where there is no acceleration). It decreases as the object decelerates at the end of the journey.

To do this, we would also plot the slope of the velocity vs time graph. In this graph, the acceleration is constant in the three different stages of motion. As we noted earlier, the object is increasing speed and changing positions slowly in the beginning. The acceleration graph shows that the object was increasing at a positive constant acceleration during this time. In the middle, when the object was changing position at a constant velocity, the acceleration was 0. This is because the object is no longer changing its velocity and is moving at a constant rate. Towards the end of the motion, the object slows down. This is depicted as a negative value on the acceleration graph. Note that in this example, the motion of the object is still forward (positive), but since it is decelerating, the acceleration is negative.



Acceleration vs Time Graph: The object has positive acceleration as it speeds up at the beginning of the journey. It has no acceleration as it travels at constant velocity in the middle of the journey. Its acceleration is negative as it slows down at the end of the journey.





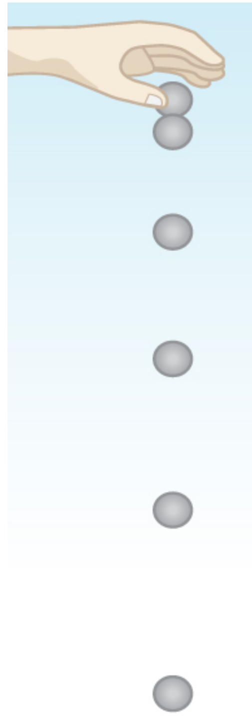
Graphing Motion: A brief introduction to particle diagrams and motion graphs.

Motion with Constant Acceleration

Constant acceleration occurs when an object’s velocity changes by an equal amount in every equal time period.

learning objectives

- Describe how constant acceleration affects the motion of an object



One-Dimensional Motion: When you drop an object, it falls vertically toward the center of the earth due to the constant acceleration of gravity.

An object experiencing constant acceleration has a velocity that increases or decreases by an equal amount for any constant period of time. Acceleration can be derived easily from basic kinematic principles. It is defined as the first time derivative of velocity (so the second derivative of position with respect to time):

$$a=\frac{\partial v}{\partial t}=\frac{\partial^2 x}{\partial t^2} \tag{2.3.1}$$

Assuming acceleration to be constant does not seriously limit the situations we can study and does not degrade the accuracy of our treatment, because in a great number of situations, acceleration *is* constant. When it is not, we can either consider it in separate parts of constant acceleration or use an average acceleration over a period of time.

The motion of falling objects is a simple, one-dimensional type of projectile motion in which there is no horizontal movement. For example, if you held a rock out and dropped it, the rock would fall only vertically downward toward the earth. If you were to throw the rock instead of just dropping it, it would follow a more projectile-like pattern, similar to the one a kicked ball follows.

Projectile motion is the motion of an object thrown or projected into the air and is subject only to the acceleration of gravity. The object thrown is called a projectile, and the object's path is called its trajectory. In two-dimensional projectile motion, there is both a vertical and a horizontal component.

Due to the algebraic properties of constant acceleration, there are kinematic equations that relate displacement, initial velocity, final velocity, acceleration, and time. A summary of these equations is given below.

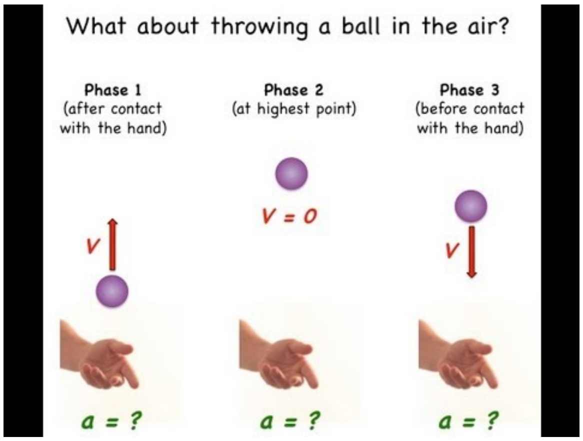
$$x=x_0+\bar{v}t \tag{2.3.2}$$

$$\bar{v}=\frac{v_0+v}{2} \tag{2.3.3}$$

$$v=v_0+at \tag{2.3.4}$$

$$x=x_0+v_0t+\frac{1}{2}at^2 \tag{2.3.5}$$

$$v^2=v_0^2+2a(x-x_0) \tag{2.3.6}$$



Constant Acceleration Explained with Vectors and Algebra: This video answers the question “what is acceleration? ".

Key Points

- Acceleration is the rate at which the velocity of a body changes with time.
- Acceleration is a vector that points in the same direction as the change in velocity, though it may not always be in the direction of motion.
- Because acceleration is velocity in m/s divided by time in s, we can derive a graph of acceleration from a graph of an object’s speed or position.
- Assuming acceleration to be constant does not seriously limit the situations we can study and does not degrade the accuracy of our treatment.
- Due to the algebraic properties of constant acceleration, there are kinematic equations that can be used to calculate displacement, velocity, acceleration, and time.
- Calculations with constant acceleration can be done in relation to one-dimensional motion as well as two-dimensional motion.

Key Terms

- **acceleration**: The amount by which a speed or velocity increases (and so a scalar quantity or a vector quantity).
- **velocity**: A vector quantity that denotes the rate of change of position with respect to time, or a speed with a directional component.
- **position**: A place or location.
- **kinematic**: of or relating to motion or kinematics

2.4: Problem-Solving for Basic Kinematics

Applications

There are four kinematic equations that describe the motion of objects without consideration of its causes.

learning objectives

- Choose which kinematics equation to use in problems in which the initial starting position is equal to zero

Kinematics is the branch of classical mechanics that describes the motion of points, bodies (objects), and systems of bodies (groups of objects) without consideration of the causes of motion. There are four kinematic equations when the initial starting position is the origin, and the acceleration is constant:

1. $v = v_0 + at$
2. $d = \frac{1}{2}(v_0 + v)t$ or alternatively $v_{average} = \frac{d}{t}$
3. $d = v_0 t + (\frac{at^2}{2})$
4. $v^2 = v_0^2 + 2ad$

Notice that the four kinematic equations involve five kinematic variables: d, v, v_0, a and t. Each of these equations contains only four of the five variables and has a different one missing. This tells us that we need the values of three variables to obtain the value of the fourth and we need to choose the equation that contains the three known variables and one unknown variable for each specific situation.

Here the basic problem solving steps to use these equations:

Step one – Identify exactly what needs to be determined in the problem (identify the unknowns).

Step two – Find an equation or set of equations that can help you solve the problem.

Step three – Substitute the knowns along with their units into the appropriate equation, and obtain numerical solutions complete with units.

Step four – Check the answer to see if it is reasonable: Does it make sense?

Problem-solving skills are obviously essential to success in a quantitative course in physics. More importantly, the ability to apply broad physical principles, usually represented by equations, to specific situations is a very powerful form of knowledge. It is much more powerful than memorizing a list of facts. Analytical skills and problem-solving abilities can be applied to new situations, whereas a list of facts cannot be made long enough to contain every possible circumstance. Such analytical skills are useful both for solving problems in a physics class and for applying physics in everyday and professional life.

Motion Diagrams

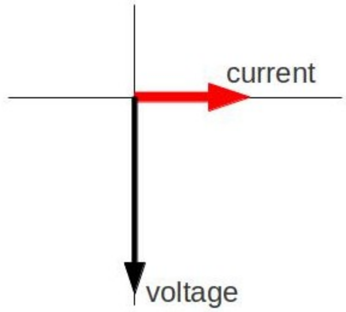
A motion diagram is a pictorial description of an object's motion and represents the position of an object at equally spaced time intervals.

learning objectives

- Construct a motion diagram

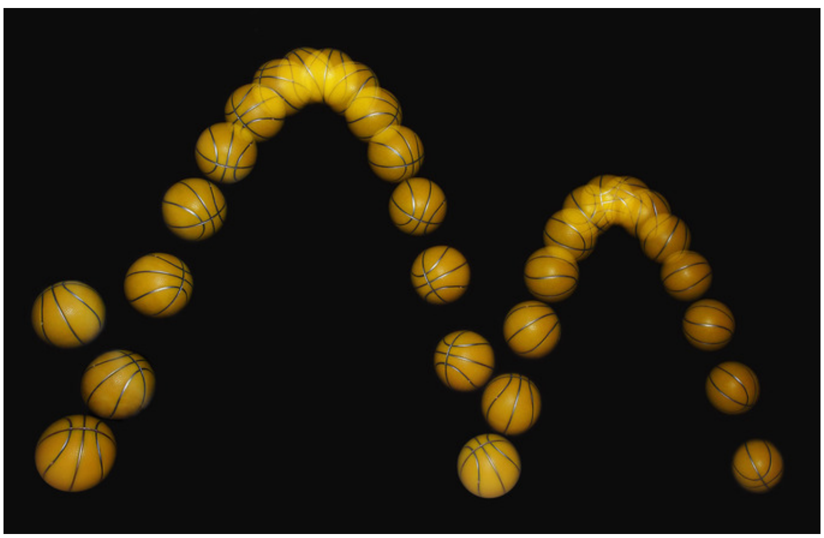
A motion diagram is a pictorial description of the motion of an object. It displays the object's location at various equally spaced times on the same diagram; shows an object's initial position and velocity; and presents several spots in the center of the diagram. These spots reveal whether or not the object has accelerated or decelerated. For simplicity, the object is represented by a simple shape, such as a filled circle, which contains information about an object's position at particular time instances. For this reason, a motion diagram is more information than a path diagram. It may also display the forces acting on the object at each time instance.

is a motion diagram of a simple trajectory. Imagine the object as a hockey puck sliding on ice. Notice that the puck covers the same distance per unit interval along the trajectory. We can conclude that the puck is moving at a constant velocity and, therefore, there is no acceleration or deceleration during the motion.



Puck Sliding on Ice: Motion diagram of a puck sliding on ice. The puck is moving at a constant velocity.

One major use of motion diagrams is the presentation of film through a series of frames taken by a camera; this is sometimes called stroboscopic technique (as seen in). Viewing an object on a motion diagram allows one to determine whether an object is speeding up or slowing down, or if it is at constant rest. As the frames are taken, we can assume that an object is at a constant rest if it occupies the same position over time. We can assume that an object is speeding up if there is a visible increase in the space between objects as time passes, and that it is slowing down if there is a visible decrease in the space between objects as time passes. The objects on the frame come very close together.



Bouncing Ball: A bouncing ball captured with a stroboscopic flash at 25 images per second.

Key Points

- The four kinematic equations involve five kinematic variables: d, v, v_0, a and t.
- Each equation contains only four of the five variables and has a different one missing.
- It is important to choose the equation that contains the three known variables and one unknown variable for each specific situation.
- Motion diagrams represent the motion of an object by displaying its location at various equally spaced times on the same diagram.
- Motion diagrams show an object's initial position and velocity and presents several spots in the center of the diagram. These spots reveal the object's state of motion.
- Motion diagrams contain information about an object's position at particular time instances and is therefore more informative than a path diagram.

Key Terms

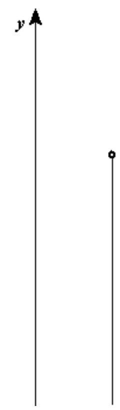
- **kinematics**: The branch of physics concerned with objects in motion.
- **stroboscopic**: Relating to an instrument used to make a cyclically moving object appear to be slow-moving, or stationary.
- **diagram**: A graph or chart.
- **motion**: A change of position with respect to time.

2.5: Free-Falling Objects

learning objectives

- Solve basic problems concerning free fall and distinguish it from other kinds of motion

The motion of falling objects is the simplest and most common example of motion with changing velocity. If a coin and a piece of paper are simultaneously dropped side by side, the paper takes much longer to hit the ground. However, if you crumple the paper into a compact ball and drop the items again, it will look like both the coin and the paper hit the floor simultaneously. This is because the amount of force acting on an object is a function of not only its mass, but also area. Free fall is the motion of a body where its weight is the only force acting on an object.



Free Fall: This clip shows an object in free fall.

Galileo also observed this phenomena and realized that it disagreed with the Aristotle principle that heavier items fall more quickly. Galileo then hypothesized that there is an upward force exerted by air in addition to the downward force of gravity. If air resistance and friction are negligible, then in a given location (because gravity changes with location), all objects fall toward the center of Earth with the *same constant acceleration, independent of their mass*, that constant acceleration is gravity. Air resistance opposes the motion of an object through the air, while friction opposes motion between objects and the medium through which they are traveling. The acceleration of free-falling objects is referred to as the acceleration due to gravity gg. As we said earlier, gravity varies depending on location and altitude on Earth (or any other planet), but the average acceleration due to gravity on Earth is 9.8 $\frac{m}{s^2}$. This value is also often expressed as a negative acceleration in mathematical calculations due to the downward direction of gravity.

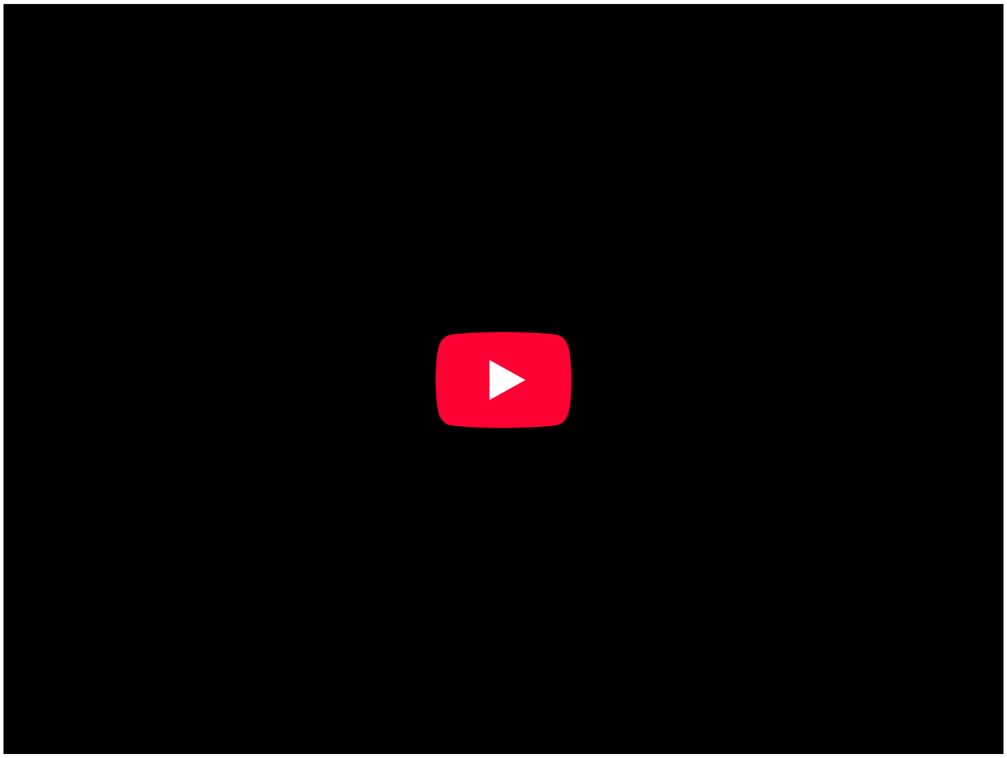
The best way to see the basic features of motion involving gravity is to start by considering straight up and down motion with no air resistance or friction. This means that if the object is dropped, we know the initial velocity is zero. Once the object is in motion, the object is in free-fall. Under these circumstances, the motion is one-dimensional and has constant acceleration, gg. The kinematic equations for objects experiencing free fall are:

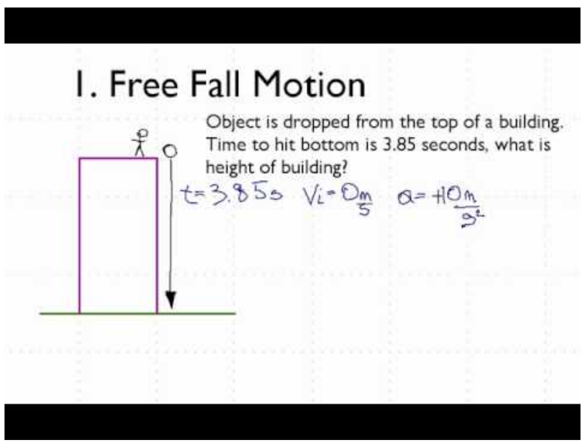
$$v = v_0 - gt \tag{2.5.1}$$

$$y = y_0 + v_0 t - \frac{1}{2} g t^2 \tag{2.5.2}$$

$$v^2 = v_0^2 - 2g(y - y_0), \tag{2.5.3}$$

where $v = \text{velocity}, g = \text{gravity}, t = \text{time}, \text{ and } y = \text{vertical displacement}.$





Video 2.5.1*: Free Fall Motion - Describes how to calculate the time for an object to fall if given the height and the height that an object fell if given the time to fall.*

Example 2.5.1:

Some examples of objects that are in free fall include:

- A spacecraft in continuous orbit. The free fall would end once the propulsion devices turned on.
- An stone dropped down an empty well.
- An object, in projectile motion, on its descent.

Key Points

- The acceleration of free-falling objects is called the acceleration due to gravity, since objects are pulled towards the center of the earth.
- The acceleration due to gravity is constant on the surface of the Earth and has the value of 9.80 $\frac{\text{m}}{\text{s}^2}$.

Glossary

Acceleration

The amount by which a speed or velocity changes within a certain period of time (and so a scalar quantity or a vector quantity).

CHAPTER OVERVIEW

03: Two-Dimensional Kinematics

3.1: Motion in Two Dimensions

Constant Velocity

An object moving with constant velocity must have a constant speed in a constant direction.

learning objectives

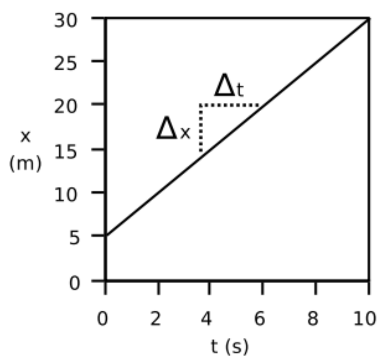
- Examine the terms for constant velocity and how they apply to acceleration

Motion with constant velocity is one of the simplest forms of motion. This type of motion occurs when an an object is moving (or sliding) in the presence of little or negligible friction, similar to that of a hockey puck sliding across the ice. To have a constant velocity, an object must have a constant speed in a constant direction. Constant direction constrains the object to motion to a straight path.

Newton's second law ($F = ma$) suggests that when a force is applied to an object, the object would experience acceleration. If the acceleration is 0, the object shouldn't have any external forces applied on it. Mathematically, this can be shown as the following:

$$a = \frac{dv}{dt} = 0 \Rightarrow v = \text{const.} \tag{3.1.1}$$

If an object is moving at constant velocity, the graph of distance vs. time (x vs. t) shows the same change in position over each interval of time. Therefore the motion of an object at constant velocity is represented by a straight line: $x = x_0 + vt$, where x_0 is the displacement when $t = 0$ (or at the y-axis intercept).



Motion with Constant Velocity: When an object is moving with constant velocity, it does not change direction nor speed and therefore is represented as a straight line when graphed as distance over time.

You can also obtain an object's velocity if you know its trace over time. Given a graph as in, we can calculate the velocity from the change in distance over the change in time. In graphical terms, the velocity can be interpreted as the slope of the line. The velocity can be positive or negative, and is indicated by the sign of our slope. This tells us in which direction the object moves.

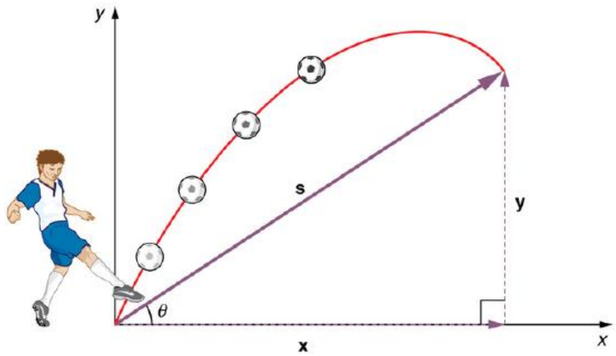
Constant Acceleration

Analyzing two-dimensional projectile motion is done by breaking it into two motions: along the horizontal and vertical axes.

learning objectives

- Analyze a two-dimensional projectile motion along horizontal and vertical axes

Projectile motion is the motion of an object thrown, or projected, into the air, subject only to the force of gravity. The object is called a projectile, and its path is called its trajectory. The motion of falling objects is a simple one-dimensional type of projectile motion in which there is no horizontal movement. In two-dimensional projectile motion, such as that of a football or other thrown object, there is both a vertical and a horizontal component to the motion.



Projectile Motion: Throwing a rock or kicking a ball generally produces a projectile pattern of motion that has both a vertical and a horizontal component.

The most important fact to remember is that motion along perpendicular axes are independent and thus can be analyzed separately. The key to analyzing two-dimensional projectile motion is to break it into two motions, one along the horizontal axis and the other along the vertical. To describe motion we must deal with velocity and acceleration, as well as with displacement.

We will assume all forces except for gravity (such as air resistance and friction, for example) are negligible. The components of acceleration are then very simple: $\mathrm{a_y = -g = -9.81 \frac{m}{s^2}}$ (we assume that the motion occurs at small enough heights near the surface of the earth so that the acceleration due to gravity is constant). Because the acceleration due to gravity is along the vertical direction *only*, $\mathrm{a_x = 0}$. Thus, the kinematic equations describing the motion along the x and y directions respectively, can be used:

$$\mathrm{x = x_0 + v_x t} \tag{3.1.2}$$

$$\mathrm{y = v_{0y} + a_y t} \tag{3.1.3}$$

$$\mathrm{y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2} \tag{3.1.4}$$

$$\mathrm{v_y^2 = v_{0y}^2 + 2a_y (y - y_0)} \tag{3.1.5}$$

We analyze two-dimensional projectile motion by breaking it into two independent one-dimensional motions along the vertical and horizontal axes. The horizontal motion is simple, because $\mathrm{a_x = 0}$ and $\mathrm{v_x}$ is thus constant. The velocity in the vertical direction begins to decrease as an object rises; at its highest point, the vertical velocity is zero. As an object falls towards the Earth again, the vertical velocity increases again in magnitude but points in the opposite direction to the initial vertical velocity. The xx and yy motions can be recombined to give the total velocity at any given point on the trajectory.

Key Points

- Constant velocity means that the object in motion is moving in a straight line at a constant speed.
- This line can be represented algebraically as: $\mathrm{x = x_0 + vt}$, where $\mathrm{x_0}$ represents the position of the object at $\mathrm{t = 0}$, and the slope of the line indicates the object's speed.
- The velocity can be positive or negative, and is indicated by the sign of our slope. This tells us in which direction the object moves.
- Constant acceleration in motion in two dimensions generally follows a projectile pattern.
- Projectile motion is the motion of an object thrown or projected into the air, subject to only the (vertical) acceleration due to gravity.
- We analyze two-dimensional projectile motion by breaking it into two independent one-dimensional motions along the vertical and horizontal axes.

Key Terms

- **constant velocity**: Motion that does not change in speed nor direction.
- **kinematic**: of or relating to motion or kinematics

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3.2: Vectors

Components of a Vector

Vectors are geometric representations of magnitude and direction and can be expressed as arrows in two or three dimensions.

learning objectives

- Contrast two-dimensional and three-dimensional vectors

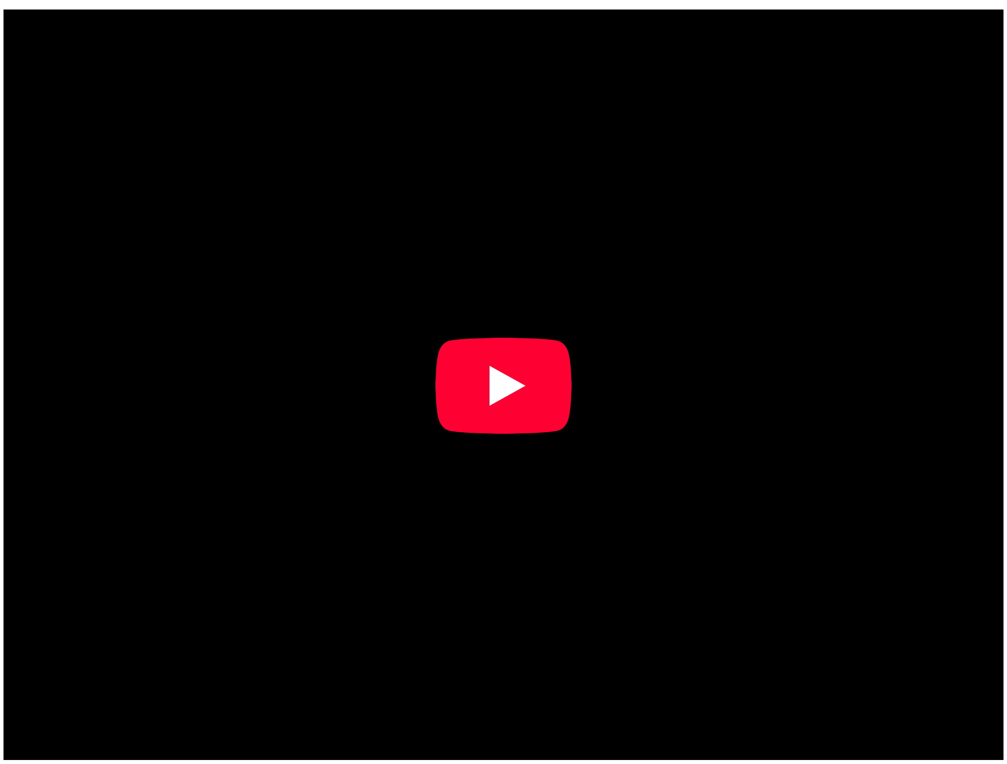
Vectors are geometric representations of magnitude and direction which are often represented by straight arrows, starting at one point on a coordinate axis and ending at a different point. All vectors have a length, called the magnitude, which represents some quality of interest so that the vector may be compared to another vector. Vectors, being arrows, also have a direction. This differentiates them from scalars, which are mere numbers without a direction.

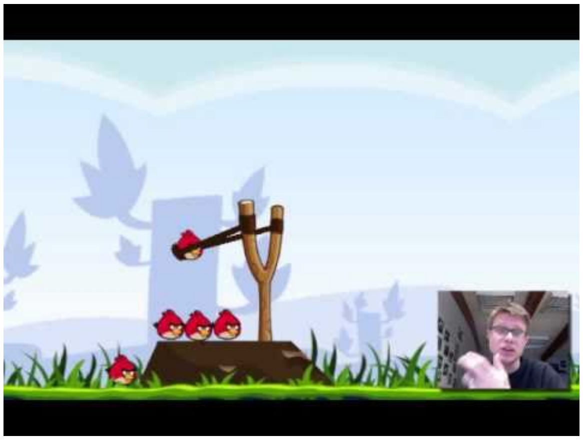
A vector is defined by its magnitude and its orientation with respect to a set of coordinates. It is often useful in analyzing vectors to break them into their component parts. For two-dimensional vectors, these components are horizontal and vertical. For three dimensional vectors, the magnitude component is the same, but the direction component is expressed in terms of xx, yy and zz.

Decomposing a Vector

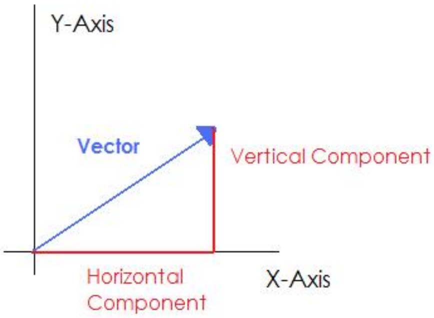
To visualize the process of decomposing a vector into its components, begin by drawing the vector from the origin of a set of coordinates. Next, draw a straight line from the origin along the x-axis until the line is even with the tip of the original vector. This is the horizontal component of the vector. To find the vertical component, draw a line straight up from the end of the horizontal vector until you reach the tip of the original vector. You should find you have a right triangle such that the original vector is the hypotenuse.

Decomposing a vector into horizontal and vertical components is a very useful technique in understanding physics problems. Whenever you see motion at an angle, you should think of it as moving horizontally and vertically at the same time. Simplifying vectors in this way can speed calculations and help to keep track of the motion of objects.





Scalars and Vectors: Mr. Andersen explains the differences between scalar and vectors quantities. He also uses a demonstration to show the importance of vectors and vector addition.



Components of a Vector: The original vector, defined relative to a set of axes. The horizontal component stretches from the start of the vector to its furthest x-coordinate. The vertical component stretches from the x-axis to the most vertical point on the vector. Together, the two components and the vector form a right triangle.

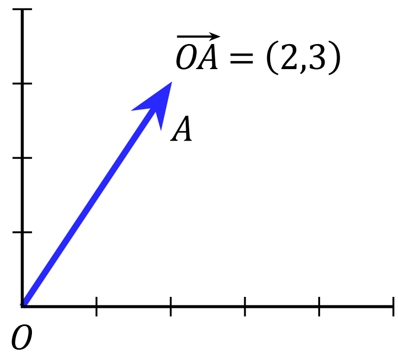
Scalars vs. Vectors

Scalars are physical quantities represented by a single number, and vectors are represented by both a number and a direction.

learning objectives

- Distinguish the difference between the quantities scalars and vectors represent

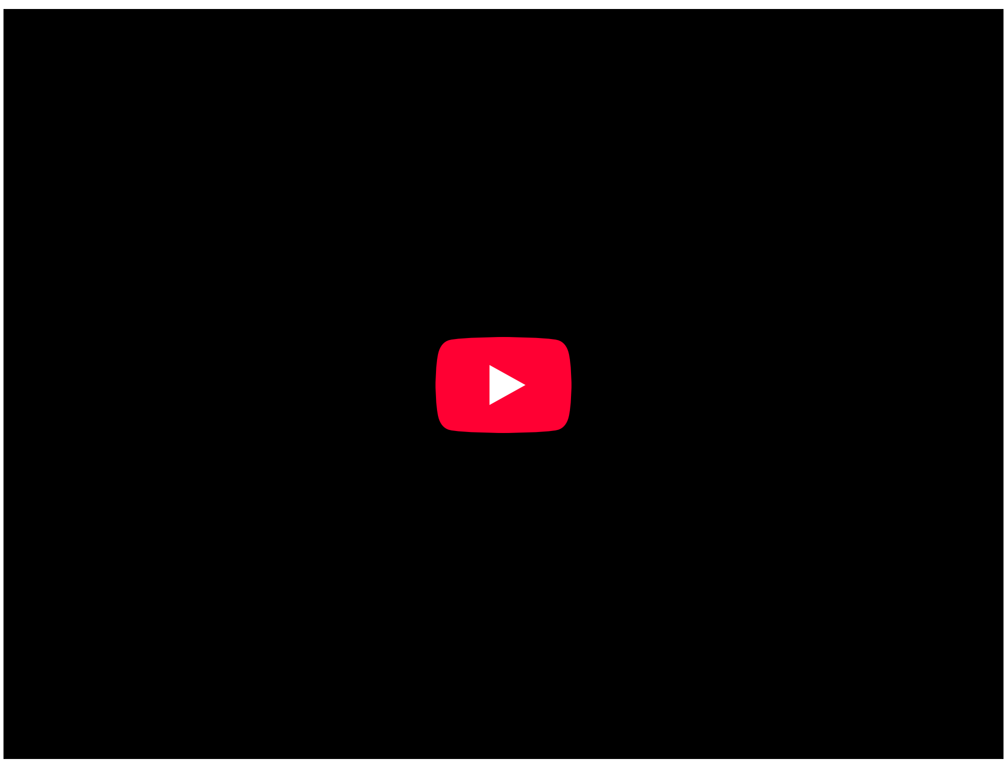
Physical quantities can usually be placed into two categories, vectors and scalars. These two categories are typified by what information they require. Vectors require two pieces of information: the magnitude and direction. In contrast, scalars require only the magnitude. Scalars can be thought of as numbers, whereas vectors must be thought of more like arrows pointing in a specific direction.

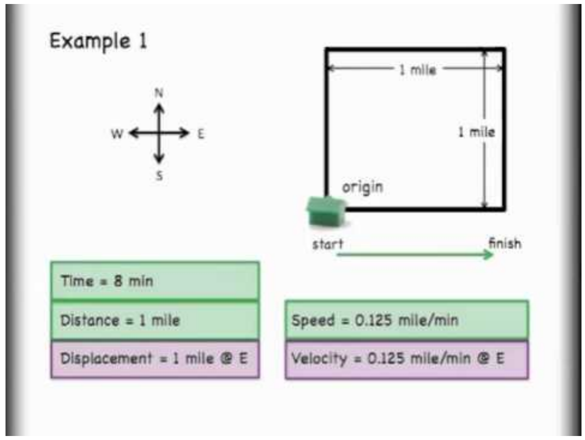


A Vector: An example of a vector. Vectors are usually represented by arrows with their length representing the magnitude and their direction represented by the direction the arrow points.

Vectors require both a magnitude and a direction. The magnitude of a vector is a number for comparing one vector to another. In the geometric interpretation of a vector the vector is represented by an arrow. The arrow has two parts that define it. The two parts are its length which represents the magnitude and its direction with respect to some set of coordinate axes. The greater the magnitude, the longer the arrow. Physical concepts such as displacement, velocity, and acceleration are all examples of quantities that can be represented by vectors. Each of these quantities has both a magnitude (how far or how fast) and a direction. In order to specify a direction, there must be something to which the direction is relative. Typically this reference point is a set of coordinate axes like the x-y plane.

Scalars differ from vectors in that they do not have a direction. Scalars are used primarily to represent physical quantities for which a direction does not make sense. Some examples of these are: mass, height, length, volume, and area. Talking about the direction of these quantities has no meaning and so they cannot be expressed as vectors.





The difference between Vectors and Scalars, Introduction and Basics: This video introduces the difference between scalars and vectors. Ideas about magnitude and direction are introduced and examples of both vectors and scalars are given.

Adding and Subtracting Vectors Graphically

Vectors may be added or subtracted graphically by laying them end to end on a set of axes.

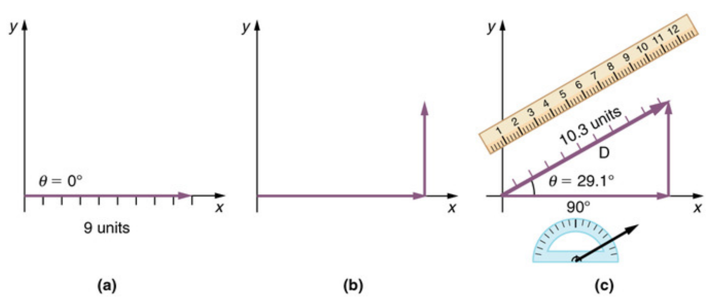
learning objectives

- Distinguish the difference between the quantities scalars and vectors represent

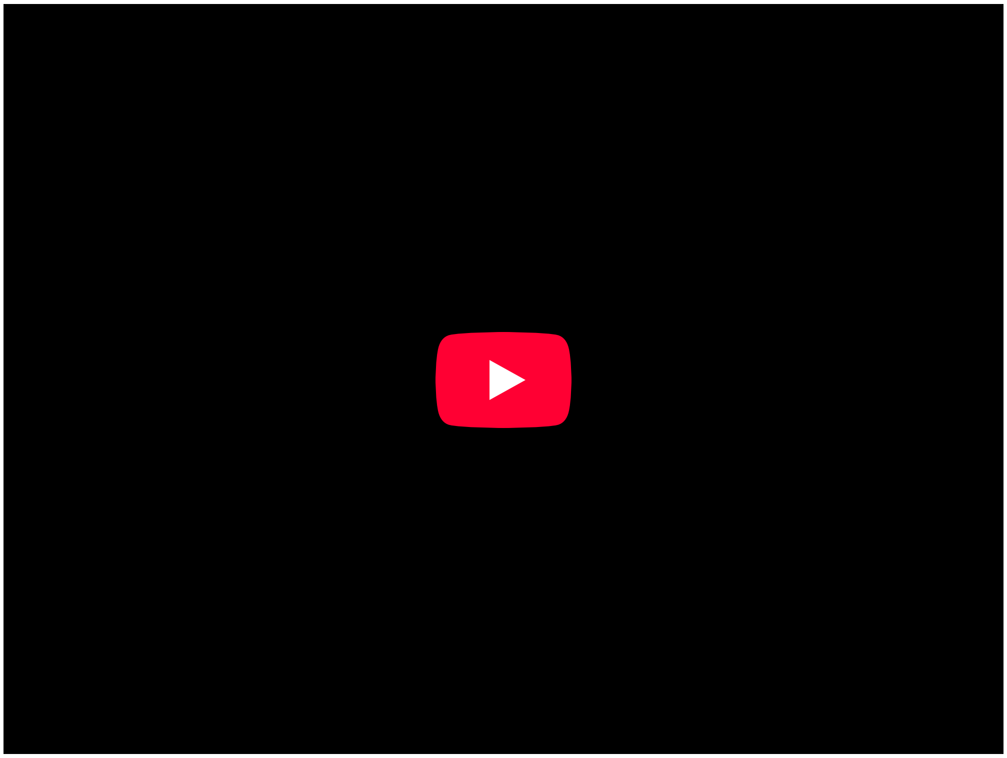
Adding and Subtracting Vectors

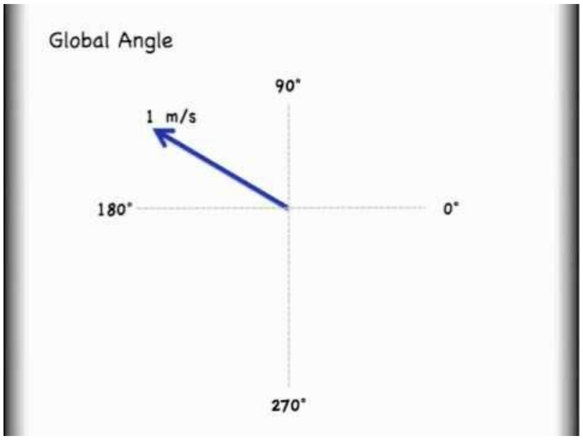
One of the ways in which representing physical quantities as vectors makes analysis easier is the ease with which vectors may be added to one another. Since vectors are graphical visualizations, addition and subtraction of vectors can be done graphically.

The graphical method of vector addition is also known as the head-to-tail method. To start, draw a set of coordinate axes. Next, draw out the first vector with its tail (base) at the origin of the coordinate axes. For vector addition it does not matter which vector you draw first since addition is commutative, but for subtraction ensure that the vector you draw first is the one you are subtracting *from*. The next step is to take the next vector and draw it such that its tail starts at the previous vector's head (the arrow side). Continue to place each vector at the head of the preceding one until all the vectors you wish to add are joined together. Finally, draw a straight line from the origin to the head of the final vector in the chain. This new line is the vector result of adding those vectors together.



Graphical Addition of Vectors: The head-to-tail method of vector addition requires that you lay out the first vector along a set of coordinate axes. Next, place the tail of the next vector on the head of the first one. Draw a new vector from the origin to the head of the last vector. This new vector is the sum of the original two.





Vector Addition Lesson 1 of 2: Head to Tail Addition Method: This video gets viewers started with vector addition and subtraction. The first lesson shows graphical addition while the second video takes a more mathematical approach and shows vector addition by components.

To subtract vectors the method is similar. Make sure that the first vector you draw is the one to be subtracted from. Then, to subtract a vector, proceed as if adding the *opposite* of that vector. In other words, flip the vector to be subtracted across the axes and then join it tail to head as if adding. To flip the vector, simply put its head where its tail was and its tail where its head was.

Adding and Subtracting Vectors Using Components

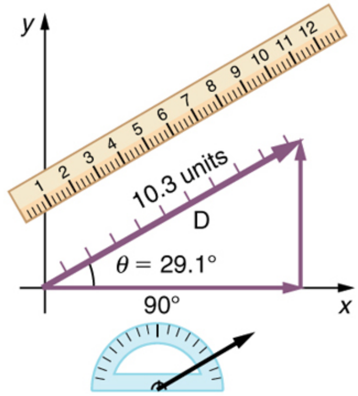
It is often simpler to add or subtract vectors by using their components.

learning objectives

- Demonstrate how to add and subtract vectors by components

Using Components to Add and Subtract Vectors

Another way of adding vectors is to add the components. Previously, we saw that vectors can be expressed in terms of their horizontal and vertical components. To add vectors, merely express both of them in terms of their horizontal and vertical components and then add the components together.

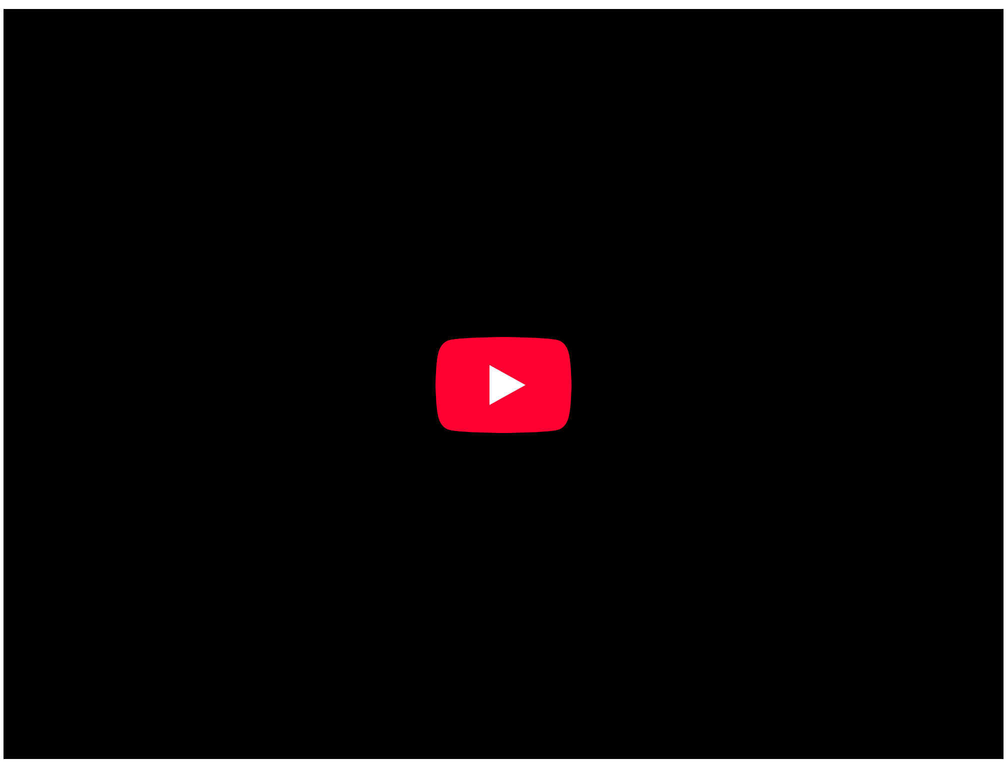


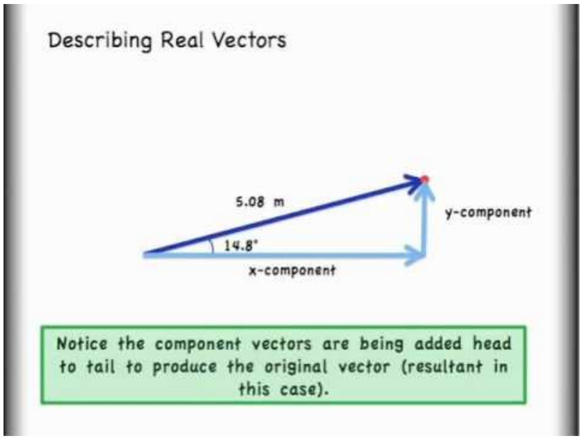
(c)

Vector with Horizontal and Vertical Components: The vector in this image has a magnitude of 10.3 units and a direction of 29.1 degrees above the x-axis. It can be decomposed into a horizontal part and a vertical part as shown.

For example, a vector with a length of 5 at a 36.9 degree angle to the horizontal axis will have a horizontal component of 4 units and a vertical component of 3 units. If we were to add this to another vector of the same magnitude and direction, we would get a vector twice as long at the same angle. This can be seen by adding the horizontal components of the two vectors $(4+4)$ and the two vertical components $(3+3)$. These additions give a new vector with a horizontal component of $8(4+4)$ and a vertical component of $6(3+3)$. To find the resultant vector, simply place the tail of the vertical component at the head (arrow side) of the horizontal component and then draw a line from the origin to the head of the vertical component. This new line is the resultant vector. It should be twice as long as the original, since both of its components are twice as large as they were previously.

To subtract vectors by components, simply subtract the two horizontal components from each other and do the same for the vertical components. Then draw the resultant vector as you did in the previous part.





Vector Addition Lesson 2 of 2: How to Add Vectors by Components: This video gets viewers started with vector addition using a mathematical approach and shows vector addition by components.

Multiplying Vectors by a Scalar

Multiplying a vector by a scalar changes the magnitude of the vector but not the direction.

learning objectives

- Summarize the interaction between vectors and scalars

Overview

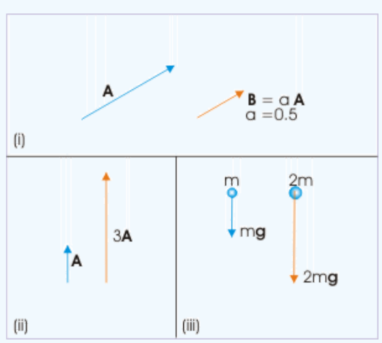
Although vectors and scalars represent different types of physical quantities, it is sometimes necessary for them to interact. While adding a scalar to a vector is impossible because of their different dimensions in space, it is possible to multiply a vector by a scalar. A scalar, however, cannot be multiplied by a vector.

To multiply a vector by a scalar, simply multiply the similar components, that is, the vector's magnitude by the scalar's magnitude. This will result in a new vector with the same direction but the product of the two magnitudes.

Example 3.2.1:

For example, if you have a vector A with a certain magnitude and direction, multiplying it by a scalar a with magnitude 0.5 will give a new vector with a magnitude of half the original. Similarly if you take the number 3 which is a pure and unit-less scalar and multiply it to a vector, you get a version of the original vector which is 3 times as long. As a more physical example take the gravitational force on an object. The force is a vector with its magnitude depending on the scalar known as mass and its direction being down. If the mass of the object is doubled, the force of gravity is doubled as well.

Multiplying vectors by scalars is very useful in physics. Most of the units used in vector quantities are intrinsically scalars multiplied by the vector. For example, the unit of meters per second used in velocity, which is a vector, is made up of two scalars, which are magnitudes: the scalar of length in meters and the scalar of time in seconds. In order to make this conversion from magnitudes to velocity, one must multiply the unit vector in a particular direction by these scalars.



Scalar Multiplication: (i) Multiplying the vector A by the scalar $\mathrm{a}=0.5$ yields the vector B which is half as long. (ii) Multiplying the vector A by 3 triples its length. (iii) Doubling the mass (scalar) doubles the force (vector) of gravity.

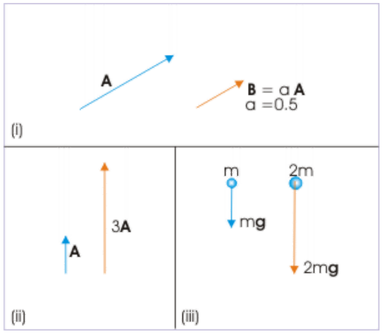
Unit Vectors and Multiplication by a Scalar

Multiplying a vector by a scalar is the same as multiplying its magnitude by a number.

learning objectives

- Predict the influence of multiplying a vector by a scalar

In addition to adding vectors, vectors can also be multiplied by constants known as scalars. Scalars are distinct from vectors in that they are represented by a magnitude but no direction. Examples of scalars include an object's mass, height, or volume.



Scalar Multiplication: (i) Multiplying the vector A by the scalar $\mathrm{a}=0.5$ yields the vector B which is half as long. (ii) Multiplying the vector A by 3 triples its length. (iii) Doubling the mass (scalar) doubles the force (vector) of gravity.

When multiplying a vector by a scalar, the direction of the vector is unchanged and the magnitude is multiplied by the magnitude of the scalar. This results in a new vector arrow pointing in the same direction as the old one but with a longer or shorter length. You can also accomplish scalar multiplication through the use of a vector's components. Once you have the vector's components, multiply each of the components by the scalar to get the new components and thus the new vector.

A useful concept in the study of vectors and geometry is the concept of a unit vector. A unit vector is a vector with a length or magnitude of one. The unit vectors are different for different coordinates. In Cartesian coordinates the directions are x and y usually denoted $\hat{\mathrm{x}}$ and $\hat{\mathrm{y}}$. With the triangle above the letters referred to as a "hat". The unit vectors in Cartesian coordinates describe a circle known as the "unit circle" which has radius one. This can be seen by taking all the possible vectors of length one at all the possible angles in this coordinate system and placing them on the coordinates. If you were to draw a line around connecting all the heads of all the vectors together, you would get a circle of radius one.

Position, Displacement, Velocity, and Acceleration as Vectors

Position, displacement, velocity, and acceleration can all be shown vectors since they are defined in terms of a magnitude and a direction.

learning objectives

- Examine the applications of vectors in analyzing physical quantities

Use of Vectors

Vectors can be used to represent physical quantities. Most commonly in physics, vectors are used to represent displacement, velocity, and acceleration. Vectors are a combination of magnitude and direction, and are drawn as arrows. The length represents the magnitude and the direction of that quantity is the direction in which the vector is pointing. Because vectors are constructed this way, it is helpful to analyze physical quantities (with both size and direction) as vectors.

Applications

In physics, vectors are useful because they can visually represent position, displacement, velocity and acceleration. When drawing vectors, you often do not have enough space to draw them to the scale they are representing, so it is important to denote somewhere what scale they are being drawn at. For example, when drawing a vector that represents a magnitude of 100, one may draw a line that is 5 units long at a scale of $\frac{1}{20}$. When the inverse of the scale is multiplied by the drawn magnitude, it should equal the actual magnitude.

Position and Displacement

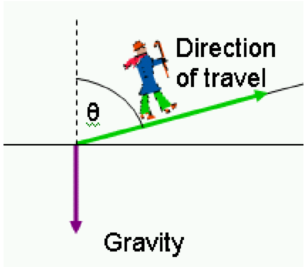
Displacement is defined as the distance, in any direction, of an object relative to the position of another object. Physicists use the concept of a position vector as a graphical tool to visualize displacements. A position vector expresses the position of an object from the origin of a coordinate system. A position vector can also be used to show the position of an object in relation to a reference point, secondary object or initial position (if analyzing how far the object has moved from its original location). The position vector is a straight line drawn from the arbitrary origin to the object. Once drawn, the vector has a length and a direction relative to the coordinate system used.

Velocity

Velocity is also defined in terms of a magnitude and direction. To say that something is gaining or losing velocity one must also say how much and in what direction. For example, an airplane flying at 200 $\frac{\mathrm{km}}{\mathrm{h}}$ to the northeast can be represented by an vector pointing in the northeast direction with a magnitude of 200 $\frac{\mathrm{km}}{\mathrm{h}}$. In drawing the vector, the magnitude is only important as a way to compare two vectors of the same units. So, if there were another airplane flying 100 $\frac{\mathrm{km}}{\mathrm{h}}$ to the southwest, the vector arrow should be half as long and pointing in the direction of southwest.

Acceleration

Acceleration, being the time rate of change of velocity, is composed of a magnitude and a direction, and is drawn with the same concept as a velocity vector. A value for acceleration would not be helpful in physics if the magnitude and direction of this acceleration was unknown, which is why these vectors are important. In a free body diagram, for example, of an object falling, it would be helpful to use an acceleration vector near the object to denote its acceleration towards the ground. If gravity is the only force acting on the object, this vector would be pointing downward with a magnitude of 9.81 $\frac{\mathrm{m}}{\mathrm{s}^2}$ of 32.2 $\frac{\mathrm{ft}}{\mathrm{s}^2}$.



Vector Diagram: Here is a man walking up a hill. His direction of travel is defined by the angle theta relative to the vertical axis and by the length of the arrow going up the hill. He is also being accelerated downward by gravity.

Key Points

- Vectors can be broken down into two components: magnitude and direction.
- By taking the vector to be analyzed as the hypotenuse, the horizontal and vertical components can be found by completing a right triangle. The bottom edge of the triangle is the horizontal component and the side opposite the angle is the vertical component.
- The angle that the vector makes with the horizontal can be used to calculate the length of the two components.
- Scalars are physical quantities represented by a single number and no direction.
- Vectors are physical quantities that require both magnitude and direction.
- Examples of scalars include height, mass, area, and volume. Examples of vectors include displacement, velocity, and acceleration.
- To add vectors, lay the first one on a set of axes with its tail at the origin. Place the next vector with its tail at the previous vector's head. When there are no more vectors, draw a straight line from the origin to the head of the last vector. This line is the sum of the vectors.
- To subtract vectors, proceed as if adding the two vectors, but flip the vector to be subtracted across the axes and then join it tail to head as if adding.

- Adding or subtracting any number of vectors yields a resultant vector.
- Vectors can be decomposed into horizontal and vertical components.
- Once the vectors are decomposed into components, the components can be added.
- Adding the respective components of two vectors yields a vector which is the sum of the two vectors.
- A vector is a quantity with both magnitude and direction.
- A scalar is a quantity with only magnitude.
- Multiplying a vector by a scalar is equivalent to multiplying the vector's magnitude by the scalar. The vector lengthens or shrinks but does not change direction.
- A unit vector is a vector of magnitude (length) 1.
- A scalar is a physical quantity that can be represented by a single number. Unlike vectors, scalars do not have direction.
- Multiplying a vector by a scalar is the same as multiplying the vector's magnitude by the number represented by the scalar.
- Vectors are arrows consisting of a magnitude and a direction. They are used in physics to represent physical quantities that also have both magnitude and direction.
- Displacement is a physics term meaning the distance of an object from a reference point. Since the displacement contains two pieces of information: the distance from the reference point and the direction away from the point, it is well represented by a vector.
- Velocity is defined as the rate of change in time of the displacement. To know the velocity of an object one must know both how fast the displacement is changing and in what direction. Therefore it is also well represented by a vector.
- Acceleration, being the rate of change of velocity also requires both a magnitude and a direction relative to some coordinates.
- When drawing vectors, you often do not have enough space to draw them to the scale they are representing, so it is important to denote somewhere what scale they are being drawn at.

Key Terms

- **coordinates**: Numbers indicating a position with respect to some axis. Ex: x and y coordinates indicate position relative to xx and yy axes.
- **axis**: An imaginary line around which an object spins or is symmetrically arranged.
- **magnitude**: A number assigned to a vector indicating its length.
- **Coordinate axes**: A set of perpendicular lines which define coordinates relative to an origin. Example: x and y coordinate axes define horizontal and vertical position.
- **origin**: The center of a coordinate axis, defined as being the coordinate 0 in all axes.
- **Component**: A part of a vector. For example, horizontal and vertical components.
- **vector**: A directed quantity, one with both magnitude and direction; the between two points.
- **magnitude**: A number assigned to a vector indicating its length.
- **scalar**: A quantity that has magnitude but not direction; compare vector.
- **unit vector**: A vector of magnitude 1.
- **velocity**: The rate of change of displacement with respect to change in time.
- **displacement**: The length and direction of a straight line between two objects.
- **acceleration**: the rate at which the velocity of a body changes with time

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3.3: Projectile Motion

Basic Equations and Parabolic Path

Projectile motion is a form of motion where an object moves in parabolic path; the path that the object follows is called its trajectory.

learning objectives

- Assess the effect of angle and velocity on the trajectory of the projectile; derive maximum height using displacement

Projectile Motion

Projectile motion is a form of motion where an object moves in a bilaterally symmetrical, parabolic path. The path that the object follows is called its trajectory. Projectile motion only occurs when there is one force applied at the beginning on the trajectory, after which the only interference is from gravity. In a previous atom we discussed what the various components of an object in projectile motion are. In this atom we will discuss the basic equations that go along with them in the special case in which the projectile initial positions are null (i.e. $x_0 = 0$ and $y_0 = 0$).

Initial Velocity

The initial velocity can be expressed as x components and y components:

$$u_x = u \cdot \cos\theta \tag{3.3.1}$$

$$u_y = u \cdot \sin\theta \tag{3.3.2}$$

In this equation, u stands for initial velocity magnitude and θ refers to projectile angle.

Time of Flight

The time of flight of a projectile motion is the time from when the object is projected to the time it reaches the surface. As we discussed previously, T depends on the initial velocity magnitude and the angle of the projectile:

$$T = \frac{2 \cdot u_y}{g} \tag{3.3.3}$$

$$T = \frac{2 \cdot u \cdot \sin\theta}{g} \tag{3.3.4}$$

Acceleration

In projectile motion, there is no acceleration in the horizontal direction. The acceleration, a, in the vertical direction is just due to gravity, also known as free fall:

$$a_x = 0 \tag{3.3.5}$$

$$a_y = -g \tag{3.3.6}$$

Velocity

The horizontal velocity remains constant, but the vertical velocity varies linearly, because the acceleration is constant. At any time, t, the velocity is:

$$u_x = u \cdot \cos\theta \tag{3.3.7}$$

$$u_y = u \cdot \sin\theta - g \cdot t \tag{3.3.8}$$

You can also use the Pythagorean Theorem to find velocity:

$$u = \sqrt{u_x^2 + u_y^2} \tag{3.3.9}$$

Displacement

At time, t, the displacement components are:

$$x = u \cdot t \cdot \cos\theta \tag{3.3.10}$$

$$y = u \cdot t \cdot \sin\theta - \frac{1}{2} g t^2 \tag{3.3.11}$$

The equation for the magnitude of the displacement is $\Delta r = \sqrt{x^2 + y^2}$.

Parabolic Trajectory

We can use the displacement equations in the x and y direction to obtain an equation for the parabolic form of a projectile motion:

$$y = \tan\theta \cdot x - \frac{g}{2 \cdot u^2 \cdot \cos^2\theta} \cdot x^2 \tag{3.3.12}$$

Maximum Height

The maximum height is reached when $v_y = 0$. Using this we can rearrange the velocity equation to find the time it will take for the object to reach maximum height

$$t_h = \frac{u \cdot \sin\theta}{g} \tag{3.3.13}$$

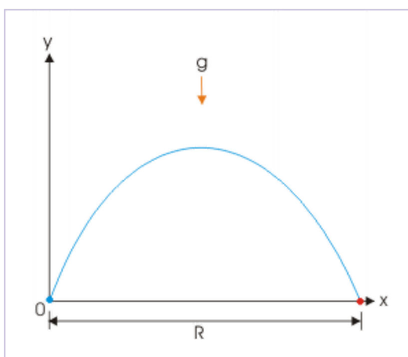
where t_h stands for the time it takes to reach maximum height. From the displacement equation we can find the maximum height

$$h = \frac{u^2 \cdot \sin^2\theta}{2 \cdot g} \tag{3.3.14}$$

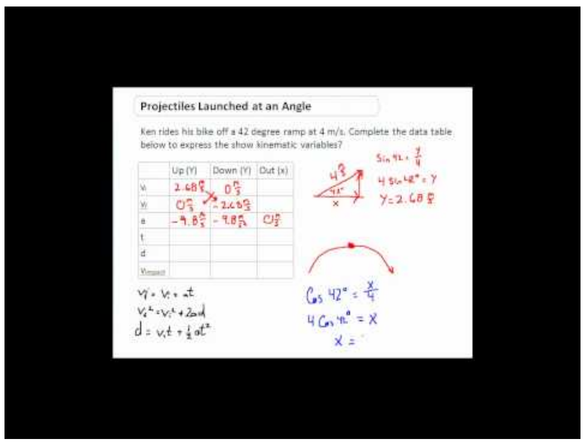
Range

The range of the motion is fixed by the condition $y = 0$. Using this we can rearrange the parabolic motion equation to find the range of the motion:

$$R = \frac{u^2 \cdot \sin 2\theta}{g}. \tag{3.3.15}$$



Range of Trajectory: The range of a trajectory is shown in this figure.



Projectiles at an Angle: This video gives a clear and simple explanation of how to solve a problem on Projectiles Launched at an Angle. I try to go step by step through this difficult problem to layout how to solve it in a super clear way. 2D kinematic problems take time to solve, take notes on the order of how I solved it. Best wishes. Tune into my other videos for more help. Peace.

Solving Problems

In projectile motion, an object moves in parabolic path; the path the object follows is called its trajectory.

learning objectives

- Identify which components are essential in determining projectile motion of an object

We have previously discussed projectile motion and its key components and basic equations. Using that information, we can solve many problems involving projectile motion. Before we do this, let's review some of the key factors that will go into this problem-solving.

What is Projectile Motion?

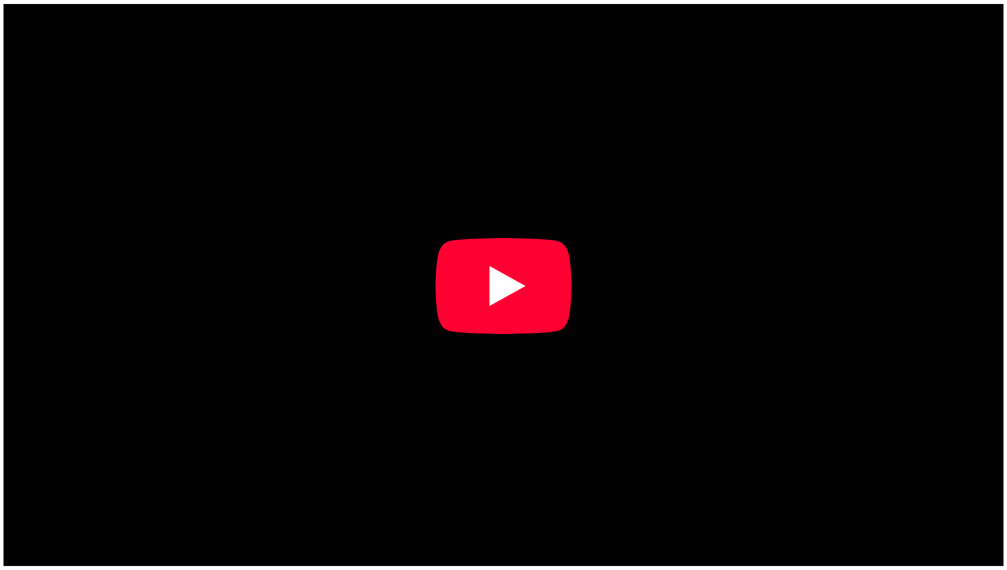
Projectile motion is when an object moves in a bilaterally symmetrical, parabolic path. The path that the object follows is called its trajectory. Projectile motion only occurs when there is one force applied at the beginning, after which the only influence on the trajectory is that of gravity.

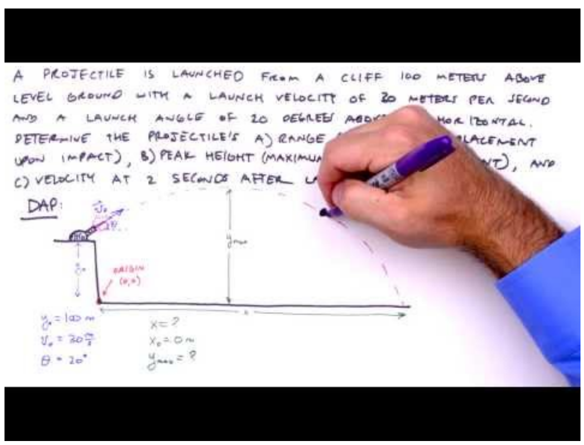
What are the Key Components of Projectile Motion?

The key components that we need to remember in order to solve projectile motion problems are:

- Initial launch angle, θ

- Initial velocity, u
- Time of flight, T
- Acceleration, a
- Horizontal velocity, v_x
- Vertical velocity, v_y
- Displacement, d
- Maximum height, H
- Range, R





How To Solve Any Projectile Motion Problem (The Toolbox Method): Introducing the “Toolbox” method of solving projectile motion problems! Here we use kinematic equations and modify with initial conditions to generate a “toolbox” of equations with which to solve a classic three-part projectile motion problem.

Now, let’s look at two examples of problems involving projectile motion.

Example 3.3.1:

Example 1

Let’s say you are given an object that needs to clear two posts of equal height separated by a specific distance. Refer to for this example. The projectile is thrown at $25\sqrt{2}$ m/s at an angle of 45°. If the object is to clear both posts, each with a height of 30m, find the minimum: (a) position of the launch on the ground in relation to the posts and (b) the separation between the posts. For simplicity’s sake, use a gravity constant of 10. Problems of any type in physics are much easier to solve if you list the things that you know (the “givens”).

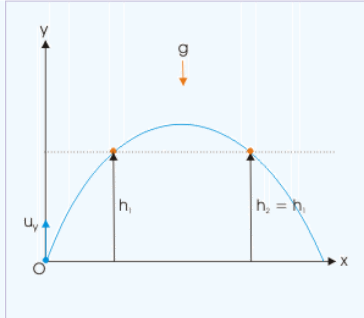


Diagram for Example 1: Use this figure as a reference to solve example 1. The problem is to make sure the object is able to clear both posts.

Solution: The first thing we need to do is figure out at what time tt the object reaches the specified height. Since the motion is in a parabolic shape, this will occur twice: once when traveling upward, and again when the object is traveling downward. For this we can use the equation of displacement in the vertical direction, $y - y_0$:

$$y - y_0 = (v_y \cdot t) - (\frac{1}{2} \cdot g \cdot t^2) \tag{3.3.16}$$

We substitute in the appropriate variables:

$$v_y = u \cdot \sin\theta = 25\sqrt{2}\frac{m}{s} \cdot \sin 45^\circ = 25\frac{m}{s} \tag{3.3.17}$$

Therefore:

$$30m = 25 \cdot t - \frac{1}{2} \cdot 10 \cdot t^2 \tag{3.3.18}$$

We can use the quadratic equation to find that the roots of this equation are 2s and 3s. This means that the projectile will reach 30m after 2s, on its way up, *and* after 3s, on its way down.

Example 2

An object is launched from the base of an incline, which is at an angle of 30°. If the launch angle is 60° *from the horizontal* and the launch speed is 10 m/s, what is the total flight time? The following information is given: $u = 10\frac{m}{s}; \theta = 60^\circ; g = 10\frac{m}{s^2}$.

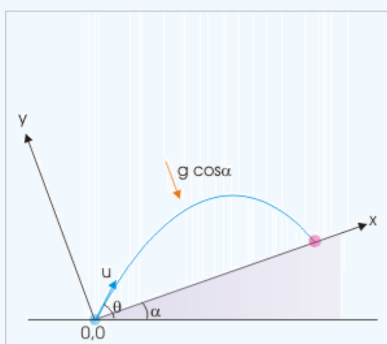


Diagram for Example 2: When dealing with an object in projectile motion on an incline, we first need to use the given information to reorient the coordinate system in order to have the object launch and fall on the same surface.

Solution: In order to account for the incline angle, we have to reorient the coordinate system so that the points of projection and return are on the same level. The angle of projection with respect to the x direction is $\theta - \alpha$, and the acceleration in the y direction is $g \cdot \cos\alpha$. We replace θ with $\theta - \alpha$ and g with $g \cdot \cos\alpha$:

$$T = \frac{2 \cdot u \cdot \sin(\theta)}{g} = \frac{2 \cdot u \cdot \sin(\theta - \alpha)}{g \cdot \cos(\alpha)} = \frac{2 \cdot 10 \cdot \sin(60 - 30)}{10 \cdot \cos(30)} = \frac{20 \cdot \sin(30)}{10 \cdot \cos(30)} \tag{3.3.19}$$

$$T = \frac{2}{\sqrt{3}}s \tag{3.3.20}$$

Zero Launch Angle

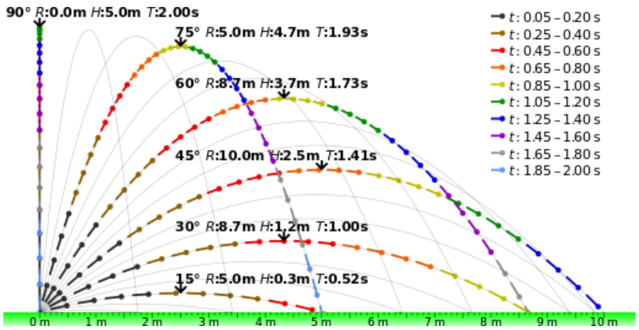
An object launched horizontally at a height H travels a range $\mathrm{v_0}\sqrt{\frac{2\mathrm{H}}{\mathrm{g}}}$ during a time of flight $\mathrm{T}=\sqrt{\frac{2\mathrm{H}}{\mathrm{g}}}$.

learning objectives

- Explain the relationship between the range and the time of flight

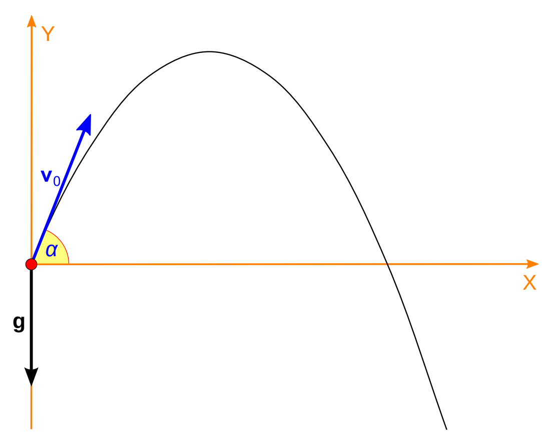
Projectile motion is a form of motion where an object moves in a parabolic path. The path followed by the object is called its trajectory. Projectile motion occurs when a force is applied at the beginning of the trajectory for the launch (after this the projectile is subject only to the gravity).

One of the key components of the projectile motion, and the trajectory it follows, is the initial launch angle. The angle at which the object is launched dictates the range, height, and time of flight the object will experience while in projectile motion. shows different paths for the same object being launched at the same initial velocity and different launch angles. As illustrated by the figure, the larger the initial launch angle and maximum height, the longer the flight time of the object.



Projectile Trajectories: The launch angle determines the range and maximum height that an object will experience after being launched.This image shows that path of the same object being launched at the same speed but different angles.

We have previously discussed the effects of different launch angles on range, height, and time of flight. However, what happens if there is no angle, and the object is just launched horizontally? It makes sense that the object should be launched at a certain height (H), otherwise it wouldn't travel very far before hitting the ground. Let's examine how an object launched horizontally at a height H travels. In our case is when α is 0.



Projectile motion: Projectile moving following a parabola.Initial launch angle is αα, and the velocity is $\mathrm{v_0}$.

Duration of Flight

There is no vertical component in the initial velocity ($\mathrm{v_0}$) because the object is launched horizontally. Since the object travels distance H in the vertical direction before it hits the ground, we can use the kinematic equation for the vertical motion:

$$(y - y_0) = -H = 0 \cdot T - \frac{1}{2} g T^2 \tag{3.3.21}$$

Here, T is the duration of the flight before the object its the ground. Therefore:

$$T = \sqrt{\frac{2H}{g}} \tag{3.3.22}$$

Range

In the horizontal direction, the object travels at a constant speed v_0 during the flight. Therefore, the range R (in the horizontal direction) is given as:

$$R = v_0 \cdot T = v_0 \sqrt{\frac{2H}{g}} \tag{3.3.23}$$

General Launch Angle

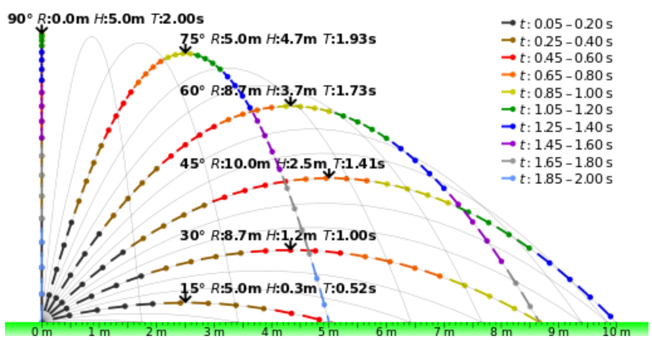
The initial launch angle (0-90 degrees) of an object in projectile motion dictates the range, height, and time of flight of that object.

learning objectives

- Choose the appropriate equation to find range, maximum height, and time of flight

Projectile motion is a form of motion where an object moves in a bilaterally symmetrical, parabolic path. The path that the object follows is called its trajectory. Projectile motion only occurs when there is one force applied at the beginning of the trajectory, after which the only interference is from gravity.

One of the key components of projectile motion and the trajectory that it follows is the initial launch angle. This angle can be anywhere from 0 to 90 degrees. The angle at which the object is launched dictates the range, height, and time of flight it will experience while in projectile motion. shows different paths for the same object launched at the same initial velocity at different launch angles. As you can see from the figure, the larger the initial launch angle, the closer the object comes to maximum height and the longer the flight time. The largest range will be experienced at a launch angle up to 45 degrees.



Launch Angle: The launch angle determines the range and maximum height that an object will experience after being launched. This image shows that path of the same object being launched at the same velocity but different angles.

The range, maximum height, and time of flight can be found if you know the initial launch angle and velocity, using the following equations:

$$R = \frac{v_i^2 \sin 2\theta_i}{g} \tag{3.3.24}$$

$$h = \frac{v_i^2 \sin^2 \theta_i}{2g} \tag{3.3.25}$$

$$T = \frac{2 v_i \sin \theta}{g} \tag{3.3.26}$$

Where R – Range, h – maximum height, T – time of flight, v_i – initial velocity, θ_i – initial launch angle, g – gravity.

Now that we understand how the launch angle plays a major role in many other components of the trajectory of an object in projectile motion, we can apply that knowledge to making an object land where we want it. If there is a certain distance, d, that you want your object to go and you know the initial velocity at which it will be launched, the initial launch angle required to get it that distance is called the angle of reach. It can be found using the following equation:

$$\theta = \frac{1}{2}\sin^{-1}\left(\frac{\mathrm{gd}}{\mathrm{v}^2}\right) \tag{3.3.27}$$

Key Points: Range, Symmetry, Maximum Height

Projectile motion is a form of motion where an object moves in parabolic path. The path that the object follows is called its trajectory.

learning objectives

- Construct a model of projectile motion by including time of flight, maximum height, and range

What is Projectile Motion ?

Projectile motion is a form of motion where an object moves in a bilaterally symmetrical, parabolic path. The path that the object follows is called its trajectory. Projectile motion only occurs when there is one force applied at the beginning on the trajectory, after which the only interference is from gravity. In this atom we are going to discuss what the various components of an object in projectile motion are, we will discuss the basic equations that go along with them in another atom, “Basic Equations and Parabolic Path”

Key Components of Projectile Motion:

Time of Flight, T:

The time of flight of a projectile motion is exactly what it sounds like. It is the time from when the object is projected to the time it reaches the surface. The time of flight depends on the initial velocity of the object and the angle of the projection, θθ. When the point of projection and point of return are on the same horizontal plane, the net vertical displacement of the object is zero.

Symmetry:

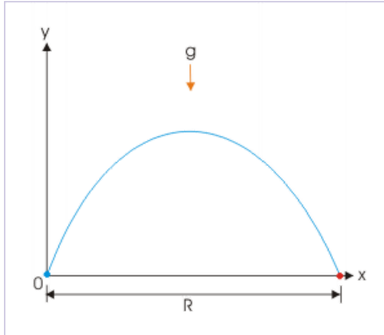
All projectile motion happens in a bilaterally symmetrical path, as long as the point of projection and return occur along the same horizontal surface. Bilateral symmetry means that the motion is symmetrical in the vertical plane. If you were to draw a straight vertical line from the maximum height of the trajectory, it would mirror itself along this line.

Maximum Height, H:

The maximum height of a object in a projectile trajectory occurs when the vertical component of velocity, vyvy, equals zero. As the projectile moves upwards it goes against gravity, and therefore the velocity begins to decelerate. Eventually the vertical velocity will reach zero, and the projectile is accelerated downward under gravity immediately. Once the projectile reaches its maximum height, it begins to accelerate downward. This is also the point where you would draw a vertical line of symmetry.

Range of the Projectile, R:

The range of the projectile is the displacement in the horizontal direction. There is no acceleration in this direction since gravity only acts vertically. shows the line of range. Like time of flight and maximum height, the range of the projectile is a function of initial speed.



Range: The range of a projectile motion, as seen in this image, is independent of the forces of gravity.

Key Points

- Objects that are projected from, and land on the same horizontal surface will have a vertically symmetrical path.
- The time it takes from an object to be projected and land is called the time of flight. This depends on the initial velocity of the projectile and the angle of projection.
- When the projectile reaches a vertical velocity of zero, this is the maximum height of the projectile and then gravity will take over and accelerate the object downward.
- The horizontal displacement of the projectile is called the range of the projectile, and depends on the initial velocity of the object.
- When solving problems involving projectile motion, we must remember all the key components of the motion and the basic equations that go along with them.
- Using that information, we can solve many different types of problems as long as we can analyze the information we are given and use the basic equations to figure it out.
- To clear two posts of equal height, and to figure out what the distance between these posts is, we need to remember that the trajectory is a parabolic shape and that there are two different times at which the object will reach the height of the posts.
- When dealing with an object in projectile motion on an incline, we first need to use the given information to reorientate the coordinate system in order to have the object launch and fall on the same surface.
- For the zero launch angle, there is no vertical component in the initial velocity.
- The duration of the flight before the object hits the ground is given as $\mathrm{T}=\sqrt{\frac{2\mathrm{H}}{\mathrm{g}}}$.
- In the horizontal direction, the object travels at a constant speed $\mathrm{v_0}$ during the flight. The range R (in the horizontal direction) is given as: $\mathrm{R}=\mathrm{v_0}\cdot\mathrm{T}=\mathrm{v_0}\sqrt{\frac{2\mathrm{H}}{\mathrm{g}}}$.
- If the same object is launched at the same initial velocity, the height and time of flight will increase proportionally to the initial launch angle.
- An object launched into projectile motion will have an initial launch angle anywhere from 0 to 90 degrees.
- The range of an object, given the initial launch angle and initial velocity is found with: $\mathrm{R}=\frac{\mathrm{v_i^2}\sin 2\theta_\mathrm{i}}{\mathrm{g}}$.
- The maximum height of an object, given the initial launch angle and initial velocity is found with: $\mathrm{h}=\frac{\mathrm{v_i^2}\sin^2\theta_\mathrm{i}}{2\mathrm{g}}$.
- The time of flight of an object, given the initial launch angle and initial velocity is found with: $\mathrm{T}=\frac{2\mathrm{v_i}\sin\theta}{\mathrm{g}}$.
- The angle of reach is the angle the object must be launched at in order to achieve a specific distance: $\theta=\frac{1}{2}\sin^{-1}(\frac{\mathrm{gd}}{\mathrm{v^2}})$.
- Objects that are projected from and land on the same horizontal surface will have a path symmetric about a vertical line through a point at the maximum height of the projectile.
- The time it takes from an object to be projected and land is called the time of flight. It depends on the initial velocity of the projectile and the angle of projection.
- The maximum height of the projectile is when the projectile reaches zero vertical velocity. From this point the vertical component of the velocity vector will point downwards.

- The horizontal displacement of the projectile is called the range of the projectile and depends on the initial velocity of the object.
- If an object is projected at the same initial speed, but two complementary angles of projection, the range of the projectile will be the same.

Key Terms

- **trajectory**: The path of a body as it travels through space.
- **symmetrical**: Exhibiting symmetry; having harmonious or proportionate arrangement of parts; having corresponding parts or relations.
- **reorientate**: to orientate anew; to cause to face a different direction
- **gravity**: Resultant force on Earth's surface, of the attraction by the Earth's masses, and the centrifugal pseudo-force caused by the Earth's rotation.
- **bilateral symmetry**: the property of being symmetrical about a vertical plane

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3.4: Multiple Velocities

Addition of Velocities

Relative velocities can be found by adding the velocity of the observed object to the velocity of the frame of reference it was measured in.

As learned in a previous atom, relative velocity is the velocity of an object as observed from a certain frame of reference.

demonstrates the concept of relative velocity. The girl is riding in a sled at 1.0 m/s, relative to an observer. When she throws the snowball forward at a speed of 1.5 m/s, relative to the sled, the velocity of the snowball to the observer is the sum of the velocity of the sled and the velocity of the snowball relative to the sled:

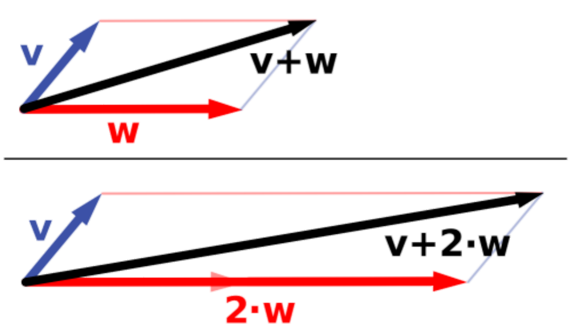
$$1.0\mathrm{m/s} + 1.5\mathrm{m/s} = 2.5\mathrm{m/s} \tag{3.4.1}$$

If the girl were to throw the snowball behind her at the same speed, the velocity of the ball relative to the observer would be:

$$1.0\mathrm{m/s} - 1.5\mathrm{m/s} = -0.5\mathrm{m/s} \tag{3.4.2}$$

The concept of relative velocity can also be demonstrated using the example of a boat in a river with a current. The boat is only trying to move forward, but since the river is in motion, it carries the boat sideways while it moves forward. The person on the boat is only observing the forward motion, while an observer on the shore will notice that the boat is moving sideways. In order to calculate the velocity that the object is moving relative to earth, it is helpful to remember that velocity is a vector. In order to analytically add these vectors, you need to remember the relationship between the magnitude and direction of the vector and its components on the x and y axis of the coordinate system:

- Magnitude: $\mathrm{v} = \sqrt{\mathrm{v_x^2} + \mathrm{v_y^2}}$
- Direction: $\theta = \tan^{-1}(\frac{\mathrm{v_y}}{\mathrm{v_x}})$
- x-component: $\mathrm{v_x} = \mathrm{v}\cos\theta$
- y-component: $\mathrm{v_y} = \mathrm{v}\sin\theta$



Vector Addition: Addition of velocities is simply the addition of vectors.

These components are shown above. The first two equations are for when the magnitude and direction are known, but you are looking for the components. The last two equations are for when the components are known, and you are looking for the magnitude and direction. The magnitude of the observed velocity from the shore is the square root sum of the squared velocity of the boat and the squared velocity of the river.

Relative Velocity

Relative velocity is the velocity of an object B measured with respect to the velocity of another object A, denoted as $\mathrm{v_{BA}}$.

Relative velocity is the velocity of an object B, in the rest frame of another object A. This is denoted as $\mathrm{v_{AB}}$, where A is the velocity in the rest frame of B.

Galileo observed the concept of relative velocity by using an example of a fly and a boat. He observed that while you are aboard the boat, if you see a fly, you can measure its velocity, u. You can then go back on land and measure the velocity of the boat, v. Is the velocity of the fly, u, the actual velocity of the fly? No, because what you measured was the velocity of the fly relative to the velocity of the boat. To obtain the velocity of the fly relative to the shore, s, you can use the vector sum as shown: $\mathrm{s} = \mathrm{u} + \mathrm{v}$

Examples of Relative Velocity

This concept is best explained using examples. Pretend you are sitting in a passenger train that is moving east. If you were to look out the window and see a man walking in the same direction, it would appear the the man is moving much slower than he actually is. Now imagine you are standing outside and observe the same man walking next to the train. It will appear the the man is walking much faster than it seemed when you were inside the train.

Now, imagine you are on a boat, and you see a man walking from one end of the deck to the other. The velocity that you observe the man walking in will be the same velocity that he would be walking in if you both were on land. Now imagine that you are on land and see the man on the moving boat, walking from one end of the deck to another. It will now appear that the man is walking much faster than it appeared when you were on the boat with him.

Why is this? The concept of relative velocity has to do with your frame of reference. When you were on the train, your frame of reference was moving in the same direction that the man was walking, so it appeared that he was walking slower. But once you were off the train, you were in a stationary frame of reference, so you were able to observe him moving at his actual speed. When you were on the boat, you were in a moving frame of reference, but so was the object you were observing, so you were able to observe the man walking at his actual velocity. Once you were back on land, you were in a stationary frame of reference, but the man was not, so the velocity you saw was his relative velocity.

Key Points

- In order to find the velocity of an object B that is moving on object A that is observed by an observer that is not moving, add the velocity of B and A together.
- Velocity is a vector quantity, so the relationships between the magnitude, direction, x- axis component and y-axis component are important.
- These vector components can be added analytically or graphically.
- In order to calculate the magnitude and direction, you must know the values of the axis components (either x and y, or x, y, and z) and to calculate the component values, you must know the magnitude and direction.
- Relative velocity is the velocity of an object in motion being observed from a frame of reference that is either also in motion or stationary.
- If the frame of reference is moving in the same direction as the object being observed, it will appear as though the object is moving slower than it actually is.
- If the object being observed is on a moving surface, the velocity observed from that surface will be less than the velocity observed from a stationary surface looking onto the moving surface.
- In Galileo's example, the observed velocity of the fly, uu, measured in reference to the velocity of the boat, v. In order to find the velocity of the fly with respect to the shore, S, he had to add the velocity of the boat to the observed velocity of the fly: $s=u+v$.

Key Terms

- **relative**: Expressed in relation to another item, rather than in complete form.

CHAPTER OVERVIEW

04: The Laws of Motion

Topic hierarchy

4.1: Introduction

Newton and His Laws

There are three laws of motion that describe the relationship between forces, mass, and acceleration.

learning objectives

- Apply three Newton's laws of motion to relate forces, mass, and acceleration

Newton's laws of motion describe the relationship between the forces acting on a body and its motion due to those forces. For example, if your car breaks down and you need to push it, you must exert a force with your hands on the car in order for it to move. The laws of motion will tell you how quickly the car will move from your pushing. There are three laws of motion:

First law: If an object experiences no net force, then its velocity is constant: the object is either at rest (if its velocity is zero), or it moves in a straight line with constant speed (if its velocity is nonzero). For example, if you don't push the car (no force), then it doesn't move.

Second law: The acceleration aa of a body is parallel and directly proportional to the net force F acting on the body, is in the direction of the net force, and is inversely proportional to the mass mm of the body:

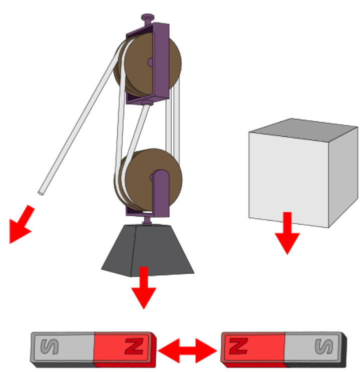
$$\mathrm{F}=\mathrm{m}\cdot\mathrm{a}\ \mathrm{or}\ \mathrm{a}=\frac{\mathrm{F}}{\mathrm{m}} \tag{4.1.1}$$

For example, if you push the car with a greater force it will accelerate more. But, if the car is more massive (mm is larger) then it won't accelerate as much from the same size force as a lighter car.

Third law: When a first body exerts a force F_1 on a second body, the second body simultaneously exerts a force $\mathrm{F}_2=-\mathrm{F}_1$ on the first body. This means that F_1 and F_2 are equal in magnitude and opposite in direction. For example, when you push a car, if it is exerting the same force on you that you are exerting on it, you might wonder why you don't move backwards? The answer is there are also forces from the ground on your feet pushing you forward. So, in fact, the car is pushing a force back on you that is of the same magnitude that you are using to push it forward.

In the figure below there are some practical examples illustrating the concept of force:

- Strain: by using a machine known as pulley you can easily raise or lower a massive body
- Gravitational Force: a massive body is attracted downward by the gravitational force practiced by the Earth
- Magnetic Force: two magnets repel each other when the same poles get closer



Examples of Force: Some situations in which forces are at play.

Key Points

- Acceleration of an object is proportional to the force on it.
- Force causes an object to move.
- Objects with more mass require more force to move.

Key Terms

- **force**: Any influence that causes an object to undergo a certain change, either concerning its movement, direction or geometrical construction.

4.2: Force and Mass

Force

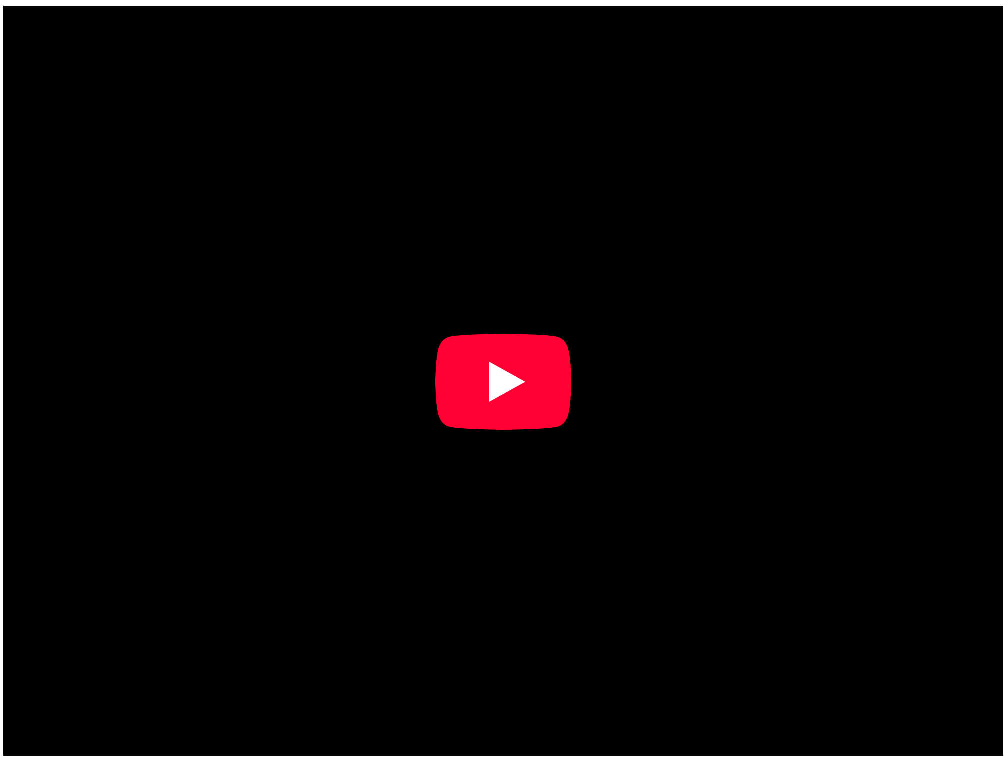
Force is any influence that causes an object to change, either concerning its movement, direction, or geometrical construction.

learning objectives

- Develop the relationship between mass and acceleration in determining force

Overview of Forces

In physics, a force is any influence that causes an object to undergo a certain change, either concerning its movement, direction, or geometrical construction. It is measured with the SI unit of Newtons. A force is that which can cause an object with mass to change its velocity, i.e., to accelerate, or which can cause a flexible object to deform. Force can also be described by intuitive concepts such as a push or pull. A force has both magnitude and direction, making it a vector quantity.



What is a force?: Describes what forces are and what they do.

Qualities of Force

The original form of Newton's second law states that the net force acting upon an object is equal to the rate at which its momentum changes. This law is further given to mean that the acceleration of an object is directly proportional to the net force acting on the object, is in the direction of the net force, and is inversely proportional to the mass of the object.

As we mentioned, force is a vector quantity. A vector is a one dimensional array with elements of both magnitude and direction. In a force vector, the mass, m, is the magnitude component and the acceleration, a, is the directional component. The equation for force is written:

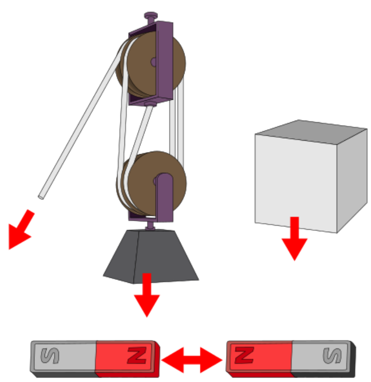
$$F = m \cdot a \tag{4.2.1}$$

Related concepts to force include thrust, which increases the velocity of an object; drag, which decreases the velocity of an object; and torque which produces changes in rotational speed of an object. Forces which do not act uniformly on all parts of a body will also cause mechanical stresses, a technical term for influences which cause deformation of matter. While mechanical stress can remain embedded in a solid object, gradually deforming it, mechanical stress in a fluid determines changes in its pressure and volume.

Dynamics

Dynamics is the study of the forces that cause objects and systems to move. To understand this, we need a working definition of force. Our intuitive definition of force — that is, a push or a pull —is a good place to start. We know that a push or pull has both magnitude and direction (therefore, it is a vector quantity) and can vary considerably in each regard.

shows a few examples of the "push-pull" nature of force. The top left example is that of a pulley system. The force that someone would have to pull down on the cable would have to equal and exceed the force made by the mass the object and the effects of gravity on those object in order for the system to move up. The top right example shows that any object resting on a surface will still exert force on that surface. The bottom example is that of two magnets being attracted to each other due to magnetic force.



Examples of Force: Some situations in which forces are at play.

Mass

Mass is a physical property of matter that depends on size and shape of matter, and is expressed as kilograms by the SI system.

learning objectives

- Justify the significance of understanding mass in physics

What is Mass?

All elements have physical properties whose values can help describe an elements physical state. Changes to these properties can describe elemental transformations. Physical properties do not change the chemical nature of matter. The physical property we are covering in this atom is called mass.

Mass is defined as a quantitative measure of an object's resistance to acceleration. The terms mass and weight are often interchanged, however it is incorrect to do so. Weight is a different property of matter that, while related to mass, is not mass, but rather the amount of gravitational force acting on a given body of matter. Mass is an intrinsic property that never changes.

Units of Mass

In order to measure something, a standard value must be established to use in relation to the object of measurement. This relation is called a unit. The International System of Units (SI) measures mass in kilograms, or kg. There are other units of mass, including the following (only the first two are accepted by the SI system):

- t – Tonne; 1t = 1000kg
- u – atomic mass unit; 1u ~= 1.66×10^{-27}kg
- sl – slug
- lb – pound

Concepts Using Mass

- Weight – see
- Newtons Second Law – mass has a central role in determining the behavior of bodies. Newtons Second Law relates force f, exerted in a body of mass m, to the body's acceleration a: $\mathrm{F} = \mathrm{ma}$
- Momentum – mass relates a body's momentum, p, to its linear velocity, v: $\mathrm{p} = \mathrm{mv}$
- Kinetic Energy – mass relates kinetic energy, K to velocity, v: $\mathrm{K} = \frac{1}{2}\mathrm{m}|\mathrm{v}^2|$

Key Points

- Force is stated as a vector quantity, meaning it has elements of both magnitude and direction. Mass and acceleration respectively.
- In layman's terms, force is a push or pull that can be defined in terms of various standards.
- Dynamics is the study of the force that causes objects and systems to move or deform.
- External forces are any outside forces that act on a body, and internal forces are any force acting within a body.
- Mass is defined as a quantitative measure of an object's resistance to acceleration.
- According to Newton's second law of motion, if a body of fixed mass m is subjected to a single force F, its acceleration a is given by F/m.
- Mass is central in many concepts of physics, including:weight, momentum, acceleration, and kinetic energy.
- According to Newton's second law of motion, if a body of fixed mass m is subjected to a single force F, its acceleration a is given by F/m.

Key Terms

- **force**: A force is any influence that causes an object to undergo a certain change, either concerning its movement, direction or geometrical construction.
- **velocity**: A vector quantity that denotes the rate of change of position with respect to time, or a speed with a directional component.
- **vector**: A directed quantity, one with both magnitude and direction; the between two points.
- **mass**: The quantity of matter which a body contains, irrespective of its bulk or volume. It is one of four fundamental properties of matter. It is measured in kilograms in the SI system of measurement.

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4.3: Newton's Laws

The First Law: Inertia

Newton's first law of motion describes inertia. According to this law, a body at rest tends to stay at rest, and a body in motion tends to stay in motion, unless acted on by a net external force.

learning objectives

- Define the First Law of Motion

History

Sir Isaac Newton was an English scientist who was interested in the motion of objects under various conditions. In 1687, he published a work called *Philosophiae Naturalis Principla Mathematica*, which described his three laws of motion. Newton used these laws to explain and explore the motion of physical objects and systems. These laws form the basis for mechanics. The laws describe the relationship between forces acting on a body and the motions experienced due to these forces. The three laws are as follows:

1. If an object experiences no net force, its velocity will remain constant. The object is either at rest and the velocity is zero or it moves in a straight line with a constant speed.
2. The acceleration of an object is parallel and directly proportional to the net force acting on the object, is in the direction of the net force, and is inversely proportional to the mass of the object.
3. When a first object exerts a force on a second object, the second object simultaneously exerts a force on the first object, meaning that the force of the first object and the force of the second object are equal in magnitude and opposite in direction.

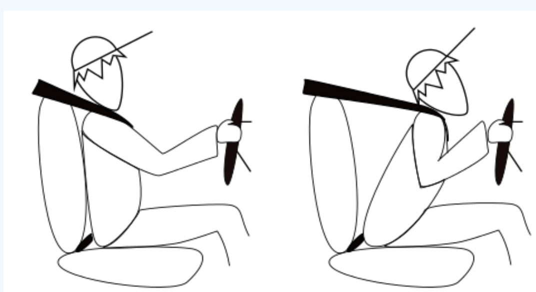
The First Law of Motion

You have most likely heard Newton's first law of motion before. If you haven't heard it in the form written above, you have probably heard that "a body in motion stays in motion, and a body at rest stays at rest." This means that an object that is in motion will not change its velocity unless an unbalanced force acts upon it. This is called uniform motion. It is easier to explain this concept through examples.

Example $4.3.1$:

If you are ice skating, and you push yourself away from the side of the rink, according to Newton's first law you will continue all the way to the other side of the rink. But, this won't actually happen. Newton says that a body in motion will stay in motion until an outside force acts upon it. In this and most other real world cases, this outside force is friction. The friction between your ice skates and the ice is what causes you to slow down and eventually stop.

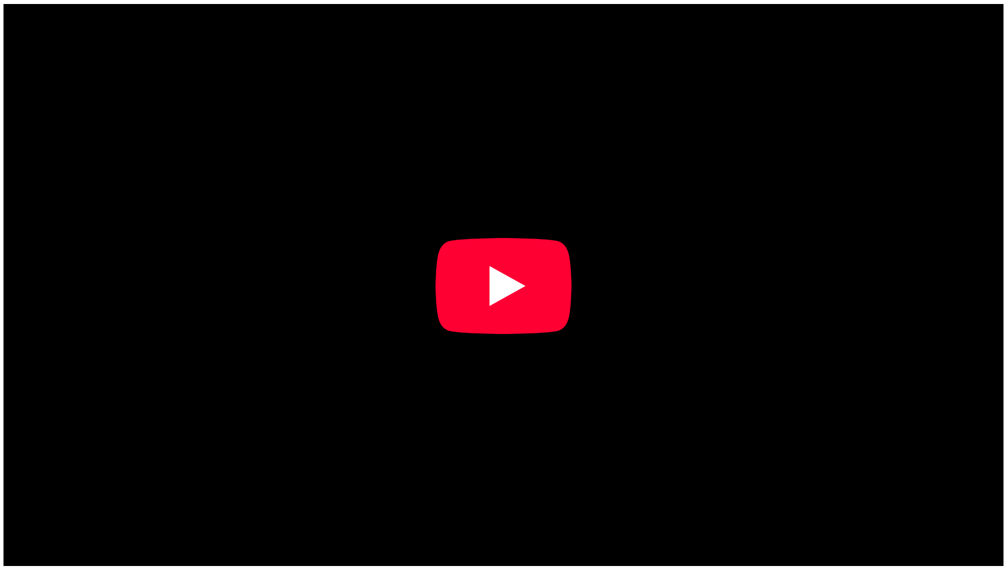
Let's look at another situation. Refer to for this example. Why do we wear seat belts? Obviously, they're there to protect us from injury in case of a car accident. If a car is traveling at 60 mph, the driver is also traveling at 60 mph. When the car suddenly stops, an external force is applied to the car that causes it to slow down. But there is no force acting on the driver, so the driver continues to travel at 60 mph. The seat belt is there to counteract this and act as that external force to slow the driver down along with the car, preventing them from being harmed.

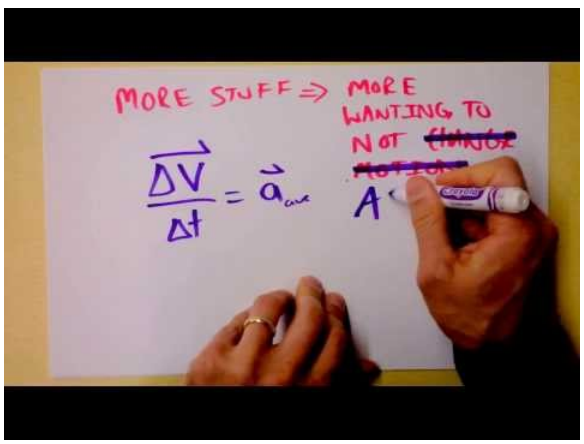


Newton's First Law: Newton's first law in effect on the driver of a car

Inertia

Sometimes this first law of motion is referred to as the law of inertia. Inertia is the property of a body to remain at rest or to remain in motion with constant velocity. Some objects have more inertia than others because the inertia of an object is equivalent to its mass. This is why it is more difficult to change the direction of a boulder than a baseball.





Doc Physics – Newton: Newton's first law is hugely counterintuitive. You may have learned it in gradeschool, though. Let's see it for the mind-blowing conclusion it really is.

The Second Law: Force and Acceleration

The second law states that the net force on an object is equal to the rate of change, or derivative, of its linear momentum.

learning objectives

- Define the Second Law of Motion

English scientist Sir Isaac Newton examined the motion of physical objects and systems under various conditions. In 1687, he published his three laws of motion in *Philosophiae Naturalis Principla Mathematica*. The laws form the basis for mechanics—they describe the relationship between forces acting on a body, and the motion experienced due to these forces. These three laws state:

1. If an object experiences no net force, its velocity will remain constant. The object is either at rest and the velocity is zero, or it moves in a straight line with a constant speed.
2. The acceleration of an object is parallel and directly proportional to the net force acting on the object, is in the direction of the net force and is inversely proportional to the mass of the object.

3. When a first object exerts a force on a second object, the second object simultaneously exerts a force on the first object, meaning that the force of the first object and the force of the second object are equal in magnitude and opposite in direction.

The first law of motion defines only the natural state of the motion of the body (i.e., when the net force is zero). It does not allow us to quantify the force and acceleration of a body. The acceleration is the rate of change in velocity; it is caused only by an external force acting on it. The second law of motion states that the net force on an object is equal to the rate of change of its linear momentum.

Linear Momentum

Linear momentum of an object is a vector quantity that has both magnitude and direction. It is the product of mass and velocity of a particle at a given time:

$$\mathrm{p}=\mathrm{mv} \tag{4.3.1}$$

where, $\mathrm{p}=\text{momentum}, \mathrm{m}=\text{mass}, \text{ and } \mathrm{v}=\text{velocity}$. From this equation, we see that objects with more mass will have more momentum.

The Second Law of Motion

Picture two balls of different mass, traveling in the same direction at the same velocity. If they both collide with a wall at the same time, the heavier ball will exert a larger force on the wall. This concept, illustrated below, explains Newton’s second law, which emphasizes the importance of force and motion, over velocity alone. It states: the net force on an object is equal to the rate of change of its linear momentum. From calculus we know that the rate of change is the same as a derivative. When we the linear momentum of an object we get:

Force and Mass: This animation demonstrates the connection between force and mass.

$$\mathrm{F}=\frac{\mathrm{dp}}{\mathrm{dt}} \tag{4.3.2}$$

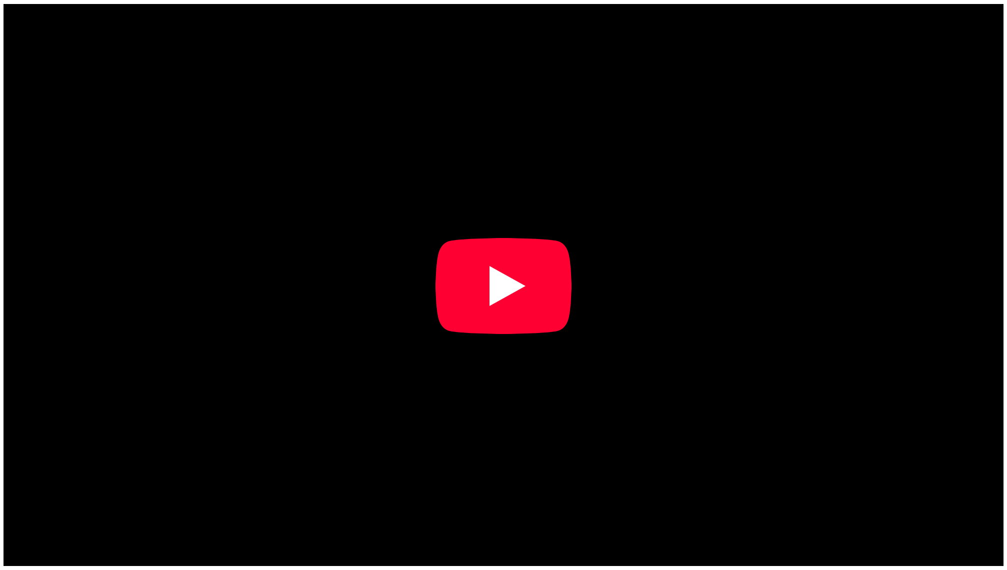
$$\mathrm{F}=\frac{\mathrm{d}(\mathrm{m}\cdot\mathrm{v})}{\mathrm{dt}} \tag{4.3.3}$$

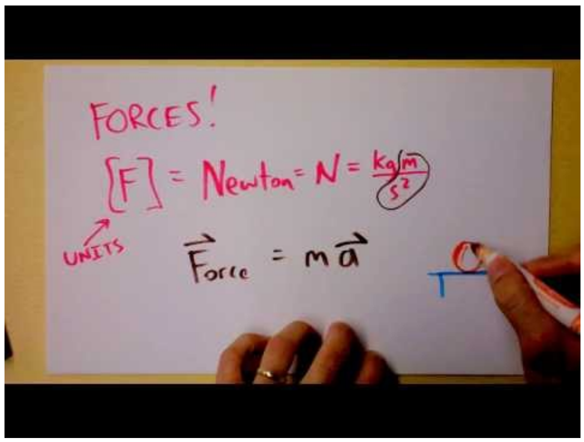
where, $\mathrm{F}=\text{Force}$ and $\mathrm{t}=\text{time}$. From this we can further simplify the equation:

$$\mathrm{F}=\mathrm{m}\frac{\mathrm{d}(\mathrm{v})}{\mathrm{dt}} \tag{4.3.4}$$

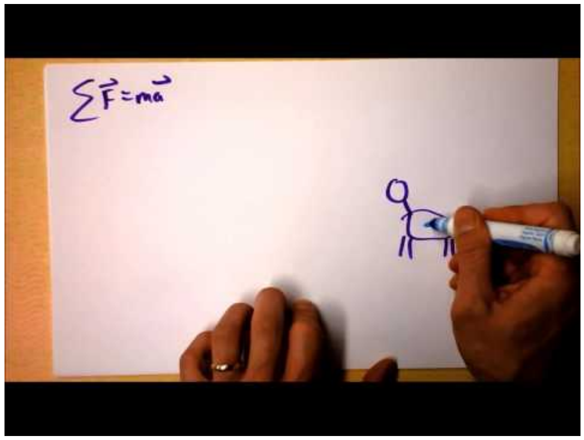
$$\mathrm{F}=\mathrm{m}\cdot\mathrm{a} \tag{4.3.5}$$

where, a=accelerationa=acceleration. As we stated earlier, acceleration is the rate of change of velocity, or velocity divided by time.





Newton’s Three Laws of Mechanics – Second Law – Part 1: Here we’ll see how many people can confuse your understanding of Newton’s 2nd law of motion through oversight, sloppy language, or cruel intentions.



Newton's Three Laws of Mechanics – Second Law – Part Two: Equilibrium is investigated and Newton's 1st law is seen as a special case of Newton's 2nd law!

The Third Law: Symmetry in Forces

The third law of motion states that for every action, there is an equal and opposite reaction.

learning objectives

- Define the Third Law of Motion

Sir Isaac Newton was a scientist from England who was interested in the motion of objects under various conditions. In 1687, he published a work called *Philosophiae Naturalis Principla Mathematica*, which contained his three laws of motion. Newton used these laws to explain and explore the motion of physical objects and systems. These laws form the bases for mechanics. The laws describe the relationship between forces acting on a body, and the motion is an experience due to these forces. Newton's three laws are:

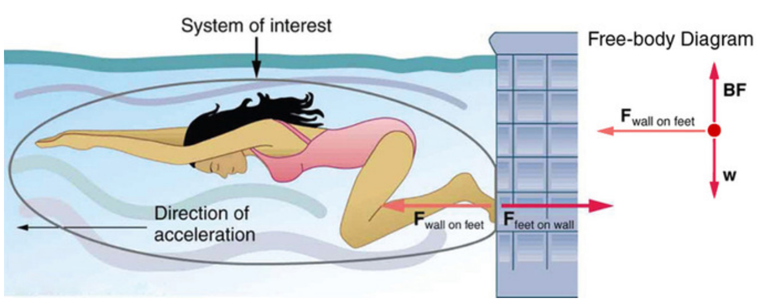
1. If an object experiences no net force, its velocity will remain constant. The object is either at rest and the velocity is zero or it moves in a straight line with a constant speed.
2. The acceleration of an object is parallel and directly proportional to the net force acting on the object, is in the direction of the net force and is inversely proportional to the mass of the object.
3. When a first object exerts a force on a second object, the second object simultaneously exerts a force on the first object, meaning that the force of the first object and the force of the second object are equal in magnitude and opposite in direction.

Newton's Third Law of Motion

Newton's third law basically states that for every action, there is an equal and opposite reaction. If object A exerts a force on object B, because of the law of symmetry, object B will exert a force on object A that is equal to the force acted on it:

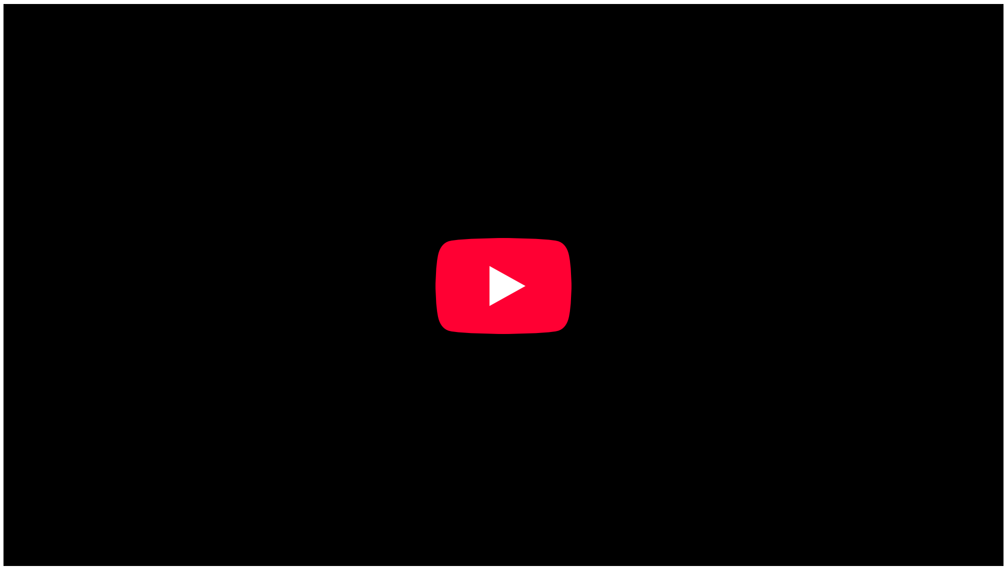
$$\mathrm{FA} = -\mathrm{FB} \tag{4.3.6}$$

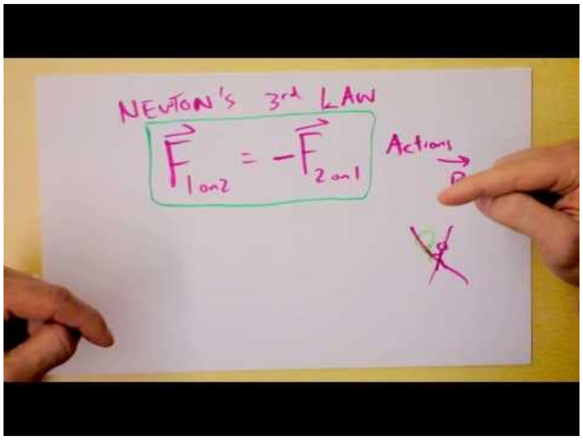
In this example, F_A is the action and F_B is the reaction. You have undoubtedly witnessed this law of motion. For example, take a swimmer who uses her feet to push off the wall in order to gain speed. The more force she exerts on the wall, the harder she pushes off. This is because the wall exerts the same force on her that she forces on it. She pushes the wall in the direction behind her, therefore the wall will exert a force on her that is in the direction in front of her and propel her forward.



Newton’s Third Law of Motion: When a swimmer pushes off the wall, the swimmer is using the third law of motion.

Take as another example, the concept of thrust. When a rocket launches into outer space, it expels gas backward at a high velocity. The rocket exerts a large backward force on the gas, and the gas exerts and equal and opposite reaction force forward on the rocket, causing it to launch. This force is called thrust. Thrust is used in cars and planes as well.





Newton’s Third Law: The most fundamental statement of basic physical reality is also the most often misunderstood. As your mom if she’s clear on Newton’s Third. Then ask her why things can move if every force has a paired opposite force all the time, forever.

Key Points

- Newton’s three laws of physics are the basis for mechanics.
- The first law states that a body at rest will stay at rest until a net external force acts upon it and that a body in motion will remain in motion at a constant velocity until acted on by a net external force.
- Net external force is the sum of all of the forcing acting on an object.
- Just because there are forces acting on an object doesn’t necessarily mean that there is a net external force; forces that are equal in magnitude but acting in opposite directions can cancel one another out.
- Friction is the force between an object in motion and the surface on which it moves. Friction is the external force that acts on objects and causes them to slow down when no other external force acts upon them.
- Inertia is the tendency of a body in motion to remain in motion. Inertia is dependent on mass, which is why it is harder to change the direction of a heavy body in motion than it is to change the direction of a lighter object in motion.
- Newton’s three laws of motion explain the relationship between forces acting on an object and the motion they experience due to these forces. These laws act as the basis for mechanics.

- The second law explains the relationship between force and motion, as opposed to velocity and motion. It uses the concept of linear momentum to do this.
- Linear momentum p, is the product of mass m, and velocity v : $p = mv$.
- The second law states that the net force is equal to the derivative, or rate of change of its linear momentum.
- By simplifying this relationship and remembering that acceleration is the rate of change of velocity, we can see that the second law of motion is where the relationship between force and acceleration comes from.
- If an object A exerts a force on object B, object B exerts an equal and opposite force on object A.
- Newton's third law can be seen in many everyday circumstances. When you walk, the force you use to push off the ground backwards makes you move forward.
- Thrust is an application of the third law of motion. A helicopter uses thrust to push the air under the propeller down, and therefore lift off the ground.

Key Terms

- **inertia**: The property of a body that resists any change to its uniform motion; equivalent to its mass.
- **friction**: A force that resists the relative motion or tendency to such motion of two bodies in contact.
- **uniform motion**: Motion at a constant velocity (with zero acceleration). Note that an object in motion will not change its velocity unless an unbalanced force acts upon it.
- **net force**: The combination of all the forces that act on an object.
- **momentum**: (of a body in motion) the product of its mass and velocity.
- **acceleration**: The amount by which a speed or velocity increases (and so a scalar quantity or a vector quantity).
- **symmetry**: Exact correspondence on either side of a dividing line, plane, center or axis.
- **thrust**: The force generated by propulsion, as in a jet engine.

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4.4: Other Examples of Forces

Weight

Weight is taken as the force on an object due to gravity, and is different than the mass of an object.

learning objectives

- Infer what factors other than gravity will contribute to the apparent weight of an object

In physics, it is important to differentiate the weight of an object from its mass. The mass of an object is an intrinsic quantity, independent of the location of the object. On the other hand, the weight of an object is an extrinsic quantity. It is considered as the force on an object due to gravity. Since gravitational acceleration changes depending on the location in the universe, weight does as well.

Mathematically, the weight of an object (*W*) can be found by multiplying its mass (*m*) by the acceleration due to gravity (*g*): $\mathrm{W = M \cdot g}$. The strength of gravity varies very little over the surface of the Earth. In fact, the greatest percent difference in the value of the acceleration due to gravity on Earth is 0.5%.

For most calculations involving the weight of an object on Earth, it is sufficient to assume that $\mathrm{g = 9.8 \frac{m}{s^2}}$.

The weight of an object has the same SI unit as force—the Newton ($\mathrm{1N = 1kg \cdot \frac{m}{s^2}}$).

In US customary units, the weight of an object can be expressed in pounds. Keep in mind that in US units the pound is either a unit of force or of mass. If one must find the weight (as opposed to the mass) of an object in US units, it can be calculated in terms of pounds of force.

It is important to note that the apparent weight of an object (i.e., the weight of an object determined by a scale) will vary if forces other than gravity are acting upon the object. For example, if you weigh a given mass underwater you will find a different result than if you weigh that mass in air. In this case, the weight of the object varies due to the force of buoyancy. While the mass is in the water it displaces fluid, resulting in an upward force upon it. This upward force affects the net force that the mass exerts on the scale, and thus alters its “apparent” weight.

Spring Scale: A spring scale measures weight by finding the extent to which a spring is compressed.This is proportional to the force that a mass exerts on the scale due to its weight.

Normal Forces

The normal force comes about when an object contacts a surface; the resulting force is always perpendicular to the surface of contact.

learning objectives

- Evaluate Newton's Second and Third Laws in determining the normal force on an object

Overview

The normal force, F_N, comes about when an object contacts a surface. According to Newton's third law, when one object exerts a force on a second object, the second object always exerts a force that is equal in magnitude and opposite in direction on the first object. This is the reason that the normal force exists.

A common situation in which a normal force exists is when a person stands on the ground. Because of Newton's third law, the ground exerts a force on the person that is equal in magnitude to the person's weight. In this simple case, the weight of the person and the opposing normal force are the only two forces considered on the person. The person remains still because the forces due to weight and the normal force create a net force of zero on the person.

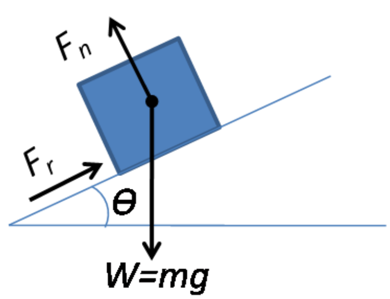
Forces on Inclined Planes

A more complex example of a situation in which a normal force exists is when a mass rests on an inclined plane. In this case, the normal force is not in the exact opposite direction as the force due to the weight of the mass. This is because the mass contacts the surface at an angle. By taking this angle into account, the magnitude of the normal force (F_N) can be found from:

$$F_N = mg\cos(\theta), \tag{4.4.1}$$

where:

- m is the mass under consideration,
- g is the acceleration due to gravity,
- and θ is the angle between the inclined surface and the horizontal.



Inclined Plane: A mass rests on an inclined plane that is at an angle θ to the horizontal. The following forces act on the mass: the weight of the mass ($m \cdot g$),the force due to friction (Fr),and the normal force (Fn).

Another interesting example involving normal forces is when a person stands in an elevator. When the elevator goes up, the normal force is actually greater than the force due to gravity. In this situation there are only two forces acting on the person. The first is the force of gravity on the person, which does not change. The second is the normal force. By summing the forces and setting them equal to m·am·a (utilizing Newton's second law), we find:

$$F_N - m \cdot g = m \cdot a \tag{4.4.2}$$

where:

- F_N is the normal force,
- $m \cdot g$ is the force due to gravity,
- m is the mass of the person,
- and a is the acceleration.

Since acceleration is positive, the normal force must actually be greater than the force due to gravity (the weight of the person).

Key Points

- Weight is taken to be the force on an object due to gravity.
- Weight and mass are not the same thing!
- The weight of a given mass will be different when the acceleration due to gravity is different.
- Apparent weight can change because of the effect of buoyancy.
- The strength of gravity is almost the same everywhere on the surface of the Earth.
- The normal force, F_N, comes about when an object contacts a surface.
- The normal force exists because for every force, there is always an equal and opposite force.
- The normal force is always perpendicular to the plane that the object contacts or rests on.

Key Terms

- **Gravitational acceleration**: Gravitational acceleration is the acceleration that an object undergoes due solely to gravity
- **perpendicular**: at or forming a right angle (to).
- **normal**: A line or vector that is perpendicular to another line, surface, or plane.

4.5: Problem-Solving

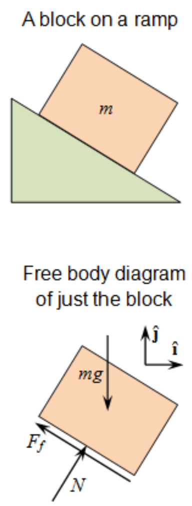
A General Approach

Basic problem-solving techniques can aid in the solution of problems involving motion (i.e., the laws of motion).

learning objectives

- Assess the laws of motion through practiced problem solving techniques

When dealing with the laws of motion, although knowledge of concepts and equations is important, understanding basic problem solving techniques can simplify the process of solving problems that may appear difficult. Your approach to problem solving can involve several key steps.



Free body diagram: An example of a drawing to help identify forces and directions.

First, gather all relevant information from the problem. Identify all quantities that are given (the *knowns*), then do the same for all quantities needed (the *unknowns*). Also, identify the physical principles involved (e.g., force, gravity, friction, etc.).

Next, a drawing may be helpful. Sometimes a drawing can even help determine the known and unknown quantities. It need not be a work of art, but it should be clear enough to illustrate proper dimension, (meaning one, two, or three dimensions). You can then use this drawing to determine which direction is positive and which is negative (making note of this on the drawing).

A next step is to use what is known to find the appropriate equation to find what is unknown. While it is easiest to find an equation that leaves only one unknown, sometimes this is not possible. In these situations, you can solve multiple equations to find the right answer. Remember that equations represent physical principles and relationships, so use the equations and drawings in tandem.

You may then substitute the knowns into the appropriate equations and find a numerical solution.

Check the answer to see if it is reasonable and makes sense. Your judgment will improve and fine tune as you solve more problems of this nature. This “judgement” step helps intuit the problem in terms of its conceptual meaning. If you can judge whether the answer is reasonable, you have a deeper understanding of physics than simply the mechanics of problem solving.

When solving problems, we tend to perform these steps in different order, as well as do several steps simultaneously. There is no rigid procedure that will work every time. Creativity and insight grow with experience. In time, the basics of problem solving can become relatively automatic.

Key Points

- Gathering all relevant information and identifying knowns and unknowns is an important first step.
- Always make a drawing to help identify directions of forces and to establish x, y, and z axes.
- Choose the correct equations, solve the problem, and check that the answer fits expectations numerically.

Key Terms

- **equation**: An assertion that two expressions are equal, expressed by writing the two expressions separated by an equal sign; from which one is to determine a particular quantity.

4.6: Vector Nature of Forces

Forces in Two Dimensions

Forces act in a particular direction and have sizes dependent upon how strong the push or pull is.

learning objectives

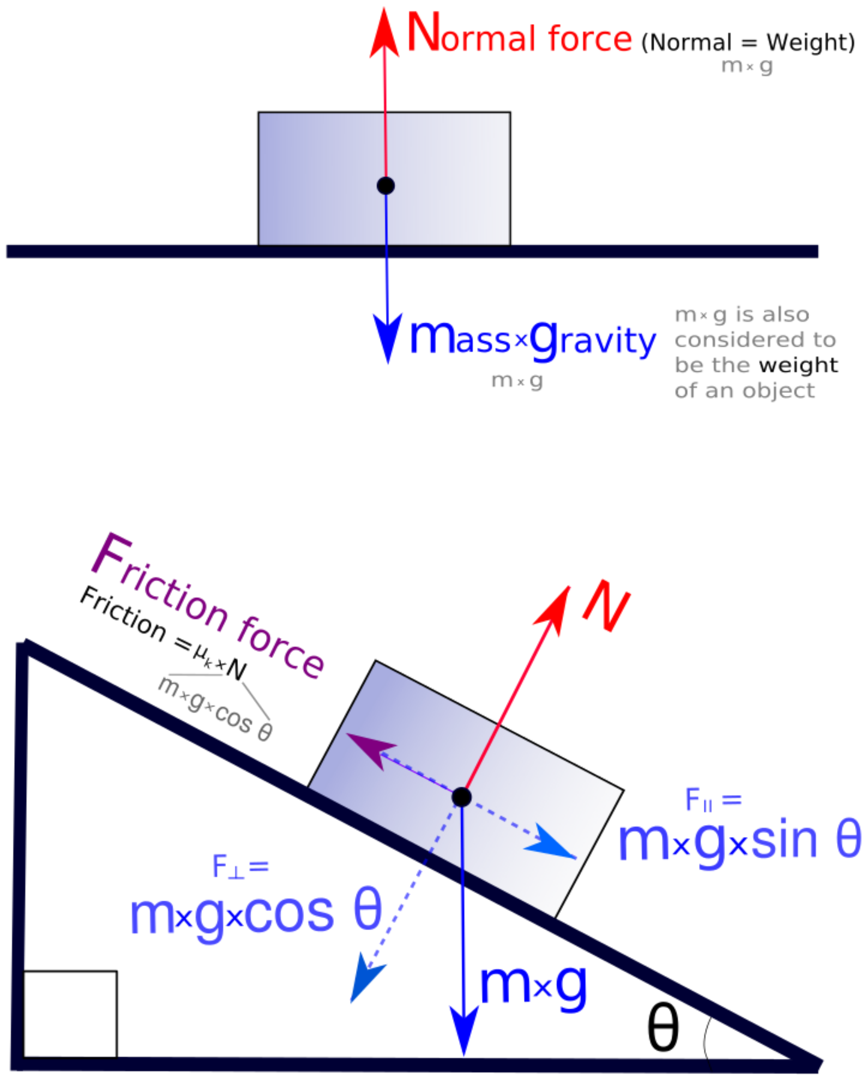
- Explain why forces are classified as “vector quantities”

Forces act in a particular direction and have sizes dependent upon how strong the push or pull is. Because of these characteristics, forces are classified as “vector quantities. ” This means that forces follow a different set of mathematical rules than physical quantities that do not have direction (denoted scalar quantities).

For example, when determining what happens when two forces act on the same object, it is necessary to know both the magnitude and the direction of both forces to calculate the result. If both of these pieces of information are not known for each force, the situation is ambiguous. For example, if you know that two people are pulling on the same rope with known magnitudes of force but you do not know which direction either person is pulling, it is impossible to determine what the acceleration of the rope will be. The two people could be pulling against each other as in tug of war or the two people could be pulling in the same direction. In this simple one-dimensional example, without knowing the direction of the forces it is impossible to decide whether the net force is the result of adding the two force magnitudes or subtracting one from the other. Associating forces with vectors avoids such problems.

When two forces act on a point particle, the resulting force or the resultant (also called the net force) can be determined by following the parallelogram rule of vector addition: the addition of two vectors represented by sides of a parallelogram gives an equivalent resultant vector which is equal in magnitude and direction to the transversal of the parallelogram. The magnitude of the resultant varies from the difference of the magnitudes of the two forces to their sum, depending on the angle between their lines of action.

Free-body diagrams can be used as a convenient way to keep track of forces acting on a system. Ideally, these diagrams are drawn with the angles and relative magnitudes of the force vectors preserved so that graphical vector addition can be done to determine the net force.



Forces as Vectors: Free-body diagrams of an object on a flat surface and an inclined plane. Forces are resolved and added together to determine their magnitudes and the net force.

Key Points

- When determining what happens when two forces act on the same object, it is necessary to know both the magnitude and the direction of both forces to calculate the result.
- When two forces act on a point particle, the resulting force or the resultant (also called the net force), can be determined by following the parallelogram rule of vector addition.
- Free-body diagrams can be used as a convenient way to keep track of forces acting on an object.

Key Terms

- **vector**: A directed quantity, one with both magnitude and direction; the between two points.
- **free-body diagram**: A free body diagram, also called a force diagram, is a pictorial representation often used by physicists and engineers to analyze the forces acting on a body of interest.
- **resultant**: A vector that is the vector sum of multiple vectors

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4.7: Further Applications of Newton's Laws

Applications of Newton's Laws

Net force affects the motion, postion and/or shape of objects (some important and commonly used forces are friction, drag and deformation).

learning objectives

- Explain the effect of forces on an object's motion and shape

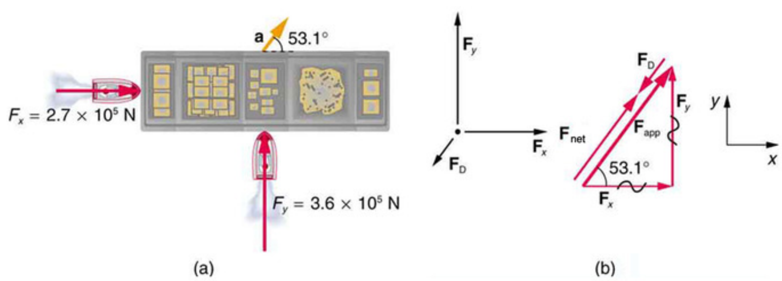
We know that a net force affects the motion, position and shape of an object. It is useful at this point to look at some particularly interesting and common forces that will provide further applications of Newton's laws of motion. Specifically, we will discuss the forces of friction, air or liquid drag, and deformation.

Friction

Friction is a force that resists movement between two surfaces sliding against each other. When surfaces in contact move relative to each other, the friction between the two surfaces converts kinetic energy into heat. This property can have a dramatic effect, as seen in the use of friction created by rubbing pieces of wood together to start a fire. Friction is not itself a fundamental force, but arises from fundamental electromagnetic forces between the charged particles constituting the two contacting surfaces.

Drag

Another interesting force in everyday life is the force of drag on an object when it is moving in a fluid (either gas or liquid). You feel this drag force when you move your hand through water, or through the wind. Like friction, the force of drag is a force that resists motion. As we will discuss in later units, the drag force is proportional to the velocity of the object moving through it. We see an illustrated example of drag force in.



Drag Force on a Barge: (a) A view from above of two tugboats pushing on a barge. (b) The free-body diagram for the ship contains only forces acting in the plane of the water. It omits the two vertical forces—the weight of the barge and the buoyant force of the water supporting it cancel and are not shown. Since the applied forces are perpendicular, the x– and y-axes are in the same direction as $\mathrm{F_x}$ and $\mathrm{F_y}$. The problem quickly becomes a one-dimensional problem along the direction of $\mathrm{F_{app}}$, since friction is in the direction opposite to $\mathrm{F_{app}}$.

Deformation

We now move from consideration of forces that affect the motion of an object (such as friction and drag) to those that affect an object's shape. If a bulldozer pushes a car into a wall, the car will not move but it will noticeably change shape. The change in shape of an object due to the application of a force is a deformation. Even very small forces are known to cause some deformation. For small deformations, two important characteristics are observed. First, the object returns to its original shape when the force is removed (that is, the deformation is elastic for small deformations). Second, the size of the deformation is proportional to the force.

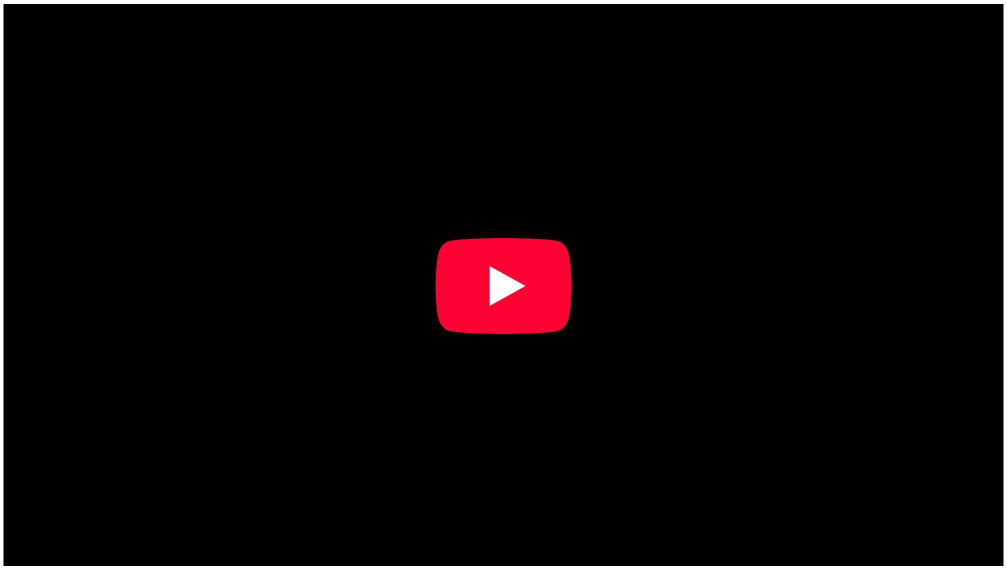
Friction: Kinetic

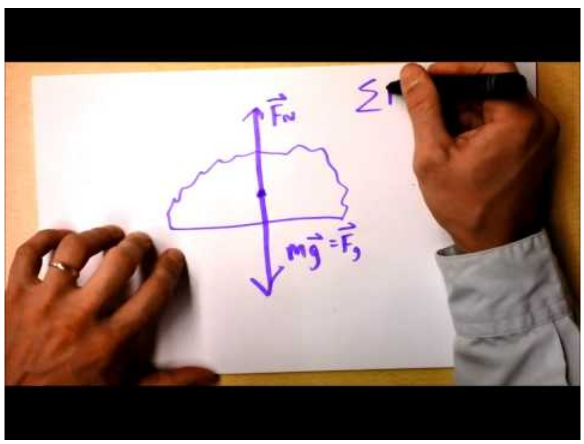
If two systems are in contact and moving relative to one another, then the friction between them is called kinetic friction.

learning objectives

- Explain the dynamics of energy for friction between two surfaces

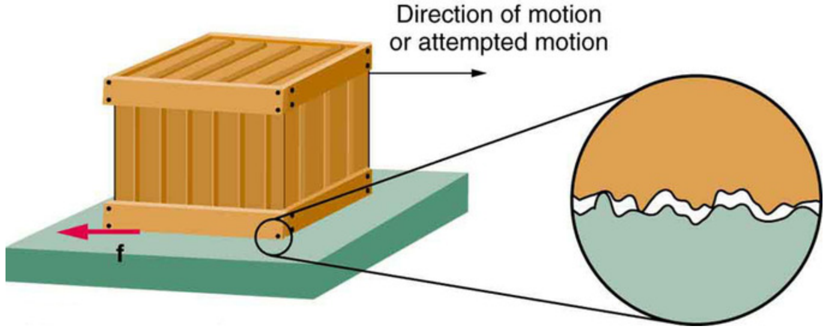
When surfaces in contact move relative to each other, the friction between the two surfaces converts kinetic energy into heat. This property can have dramatic consequences, as illustrated by the use of friction created by rubbing pieces of wood together to start a fire. Kinetic energy is converted to heat whenever motion with friction occurs, for example when a viscous fluid is stirred.





Kinetic Friction Introduction: Here, I’ll explain the microscopic justification of friction and what we can know about it. The coefficient of friction, too!

Kinetic (or dynamic) friction occurs when two objects are moving relative to each other and rub together; a sled on the ground would be a good example of kinetic friction.



Friction: Frictional forces always oppose motion or attempted motion between objects in contact. Friction arises in part because of the roughness of the surfaces in contact, as seen in the expanded view. In order for the object to move, it must rise to where the peaks can skip along the bottom surface. Thus, a force is required just to set the object in motion. Some of the peaks will be broken off, also requiring a force to maintain motion. Much of the friction is actually due to attractive forces between molecules making up the two objects, so that even perfectly smooth surfaces are not friction-free. Such adhesive forces also depend on the substances the surfaces are made of, explaining, for example, why rubber-soled shoes slip less than those with leather soles.

The force of friction is what slows an object sliding over a surface. This force is what makes the brakes on cars work or causes resistance when you slide your hand across a surface. The force of friction can be represented by an equation: $\mathrm{F_{friction}} = \mu \mathrm{F_n}$. In this equation μ is something called the coefficient of friction. This is a unitless number that represents the strength of the friction of the object. A very "grippy" surface like rubber might have a high coefficient of friction, whereas a slippery surface like ice has a much lower coefficient. $\mathrm{F_n}$ is called the normal force and is the force of the surface pushing up on the object. In most cases on level ground, the normal force will be the equal and opposite of the object's weight. In other words, it is the force that the surface must exert to keep the object from falling through.

The coefficient of kinetic friction is typically represented as μ_k and is usually less than the coefficient of static friction for the same materials.

Friction: Static

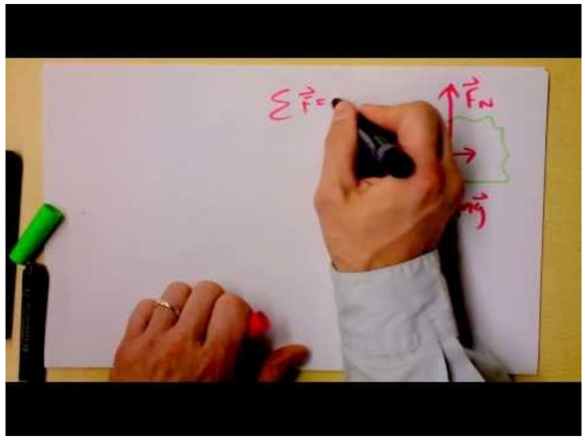
Static friction is a type of friction that occurs to resist motion when two objects are at rest against each other.

learning objectives

- Demonstrate the relationship of maximum force of static friction

Static Friction

Another type of frictional force is static friction, otherwise known as stiction. Like all friction, it acts to resist the motion of an object moving over a surface. Unlike kinetic friction, however, static friction acts to resist the start of motion.



Static Friction and some friction challenges: Here, I talk about sneaky ol' static friction.

Static friction is friction between two objects that are not moving relative to each other. This frictional force is what prevents a parked car from sliding down a hill, for example. Before an object at rest on a surface can move, it must overcome the force of static friction.

Static friction originates from multiple sources. For any given material on another material of the same composition, friction will be greater as the material surfaces become rougher (consider sandpaper) on the macroscopic level. Additionally, intermolecular forces can greatly influence friction when two materials are put into contact. When surface area is below the micrometer range, Van der Waals' forces, electrostatic interactions and hydrogen bonding can cause two materials to adhere to one another. A force is required to overcome these interactions and cause the surfaces to move across one another.

Like kinetic friction, the force of static friction is given by a coefficient multiplied by the normal force. The normal force is the force of the surface pushing up on the object, which is usually equal to the object's weight. The coefficient of static friction is usually greater than the coefficient of kinetic friction and is usually represented by μ_s.

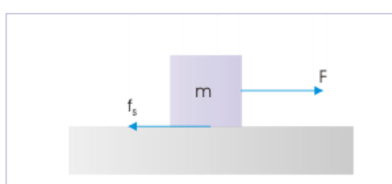
Putting these elements together gives the maximum force of static friction as:

$$\mathrm{F_s} = \mu_s \mathrm{F_n} \tag{4.7.1}$$

In general, the force of static friction can be represented as:

$$\mathrm{F_s} \leq \mu_s \mathrm{F_n} \tag{4.7.2}$$

As with all frictional forces, the force of friction can never exceed the force applied. Thus the force of static friction will vary between 0 and ${}_s\mathrm{F_n}$ depending on the strength of the applied force. Any force smaller than ${}_s\mathrm{F_n}$ attempting to slide one surface over the other is opposed by a frictional force of equal magnitude and opposite direction. Any force larger than that overcomes the force of static friction and causes sliding to occur. The instant sliding occurs, static friction is no longer applicable—the friction between the two surfaces is then called kinetic friction.



Static Friction: To move a block at rest on a surface, a force must be applied which is great enough to overcome the force of static friction.

Problem-Solving With Friction and Inclines

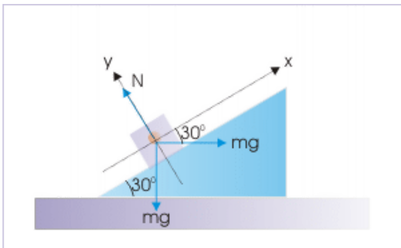
Combining motion on inclines with friction uses such concepts as equilibrium and contact force on an incline.

learning objectives

- Calculate the force of friction on an incline

Contact Force on an Incline

The incline plane has two contact or interface surfaces. One is the incline surface, where the block is placed and the other is the base of the incline, which is in contact with the surface underneath. The motion of the block, therefore, may depend on the motion of the incline itself.



Block and incline system: Forces on the block

When on an incline, calculating the force of friction is different than when the object is on a level surface. Recall that the force of friction depends on both the coefficient of friction and the normal force. $\mathrm{F_f} = \mu \mathrm{F_n}$ When on an incline with an angle θ, the normal force becomes $\mathrm{F_n} = \mathrm{mg}\cos(\theta)$

As always, the frictional force resists motion. If the block is being pushed up the incline the friction force points down the incline. If the block is being pulled down the incline, the friction force will hold the block up.

Equilibrium of Forces on an Incline

When not acted on by any other forces, only by gravity and friction, the frictional force will resist the tendency of the block to slide down the incline. If the frictional force is equal to the gravitational force the block will not slide down the incline. The block is said to be in equilibrium since the sum of the forces on it is 0.

Gravitational force down an incline is given by $\mathrm{mg}\sin(\theta)$.

Where θ is the angle the incline makes with the horizontal. For the block to be in equilibrium, the maximum force of friction $\mathrm{F_f} = \mu \mathrm{mg}\cos(\theta)$ must be greater than or equal to $\mathrm{F_G} = \mathrm{mg}\sin(\theta)$ FG=mgsin(θ). If the maximum frictional force is greater than the force of gravity, the sum of the forces is still 0. The force of friction can never exceed the other forces acting on it. The frictional forces only act to counter motion.

Drag

The drag force is the resistive force felt by objects moving through fluids and is proportional to the square of the object’s speed.

learning objectives

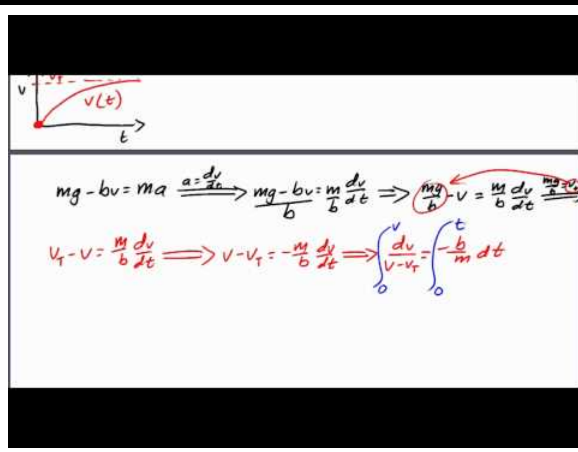
- Relate the magnitude of drag force to the speed of an object

Another interesting force in everyday life is the force of drag on an object when it is moving in a fluid (either a gas or a liquid). You feel the drag force when you move your hand through water. You might also feel it if you move your hand during a strong wind. The faster you move your hand, the harder it is to move. You feel a smaller drag force when you tilt your hand so only the side goes through the air—you have decreased the area of your hand that faces the direction of motion.

Like friction, the drag force always opposes the motion of an object. Unlike simple friction, the drag force is proportional to some function of the velocity of the object in that fluid. This functionality is complicated and depends upon the shape of the object, its size, its velocity, and the fluid it is in. Aerodynamic objects tend to have small surface areas and be designed to have low drag coefficients.

For most large objects such as bicyclists, cars, and baseballs not moving too slowly, the magnitude of the drag force $\mathrm{F_D}$ is found to be proportional to the square of the speed of the object. We can write this relationship mathematically as $\mathrm{FD} \propto \mathrm{v}^2$. When taking into account other factors, this relationship becomes $F_D = \frac{\frac{1}{2C}}{\text{rhoAv}^2}$, where C is known as the drag coefficient, a unit-less number that represents the aerodynamic properties of the object, A is the cross-sectional area of the object which is facing the direction of motion, and ρ is the density of the fluid the object is moving through.

Aerodynamic Shape: From racing cars to bobsled racers, aerodynamic shaping is crucial to achieving top speeds. Bobsleds are designed for speed. They are shaped like a bullet with tapered fins. (credit: U.S. Army, via Wikimedia Commons)



Retarding and Drag Forces: A brief look at retarding (drag) forces in physics, for students in introductory physics classes that use calculus. This video walks through a single scenario of an object experiencing a drag force where the drag force is proportional to the object’s velocity.

Stress and Strain

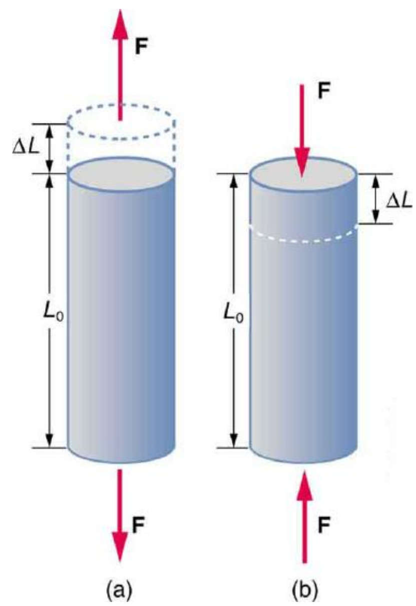
The ratio of force to area $\frac{F}{A}$ is called stress and the ratio of change in length to length $\frac{\Delta L}{L}$ is called the strain.

learning objectives

- Explain how forces affects the shape of an object

We now move from consideration of forces that affect the motion of an object (such as friction and drag) to those that affect an object's shape. If a bulldozer pushes a car into a wall, the car will not move past the wall but it will noticeably change shape. A change in shape due to the application of a force is a deformation. Even very small forces are known to cause some deformation. For small deformations, two important characteristics are observed. First, the object returns to its original shape when the force is removed—that is, the deformation is elastic for small deformations. Second, the size of the deformation is proportional to the force —that is, for small deformations, Hooke's law is obeyed. In equation form, Hooke's law is given by $F = k \cdot \Delta L$ where ΔL is the change in length and kk is a constant which depends on the material properties of the object.

Deformations come in several types: changes in length (tension and compression), sideways shear (stress), and changes in volume.



Tension/Compression: Tension: The rod is stretched a length ΔL when a force is applied parallel to its length. (b) Compression: The same rod is compressed by forces with the same magnitude in the opposite direction. For very small deformations and uniform materials, ΔL is approximately the same for the same magnitude of tension or compression. For larger deformations, the cross-sectional area changes as the rod is compressed or stretched.

The ratio of force to area $\frac{F}{A}$ is called stress and the ratio of change in length to length $\frac{\Delta L}{L}$ is called the strain.

Stress and strain are related to each other by a constant called Young's Modulus or the elastic modulus which varies depending on the material. Using Young's Modulus the relation between stress and strain is given by: $\text{stress} = Y \cdot \text{strain}$.

A material with a high elastic modulus is said to have high tensile strength. Such materials are very resistant to being stretched and require a large amount of force to deform a small amount.

Translational Equilibrium

An object is said to be in equilibrium when there is no external net force acting on it.

learning objectives

- Assess the role each type of equilibrium plays in mechanical devices

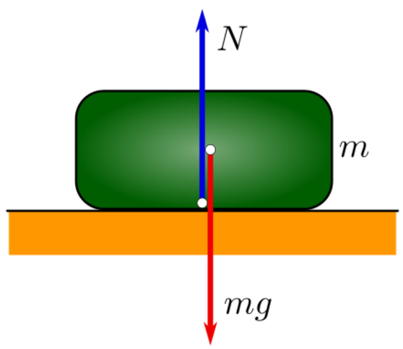
We are surrounded by great engineering architectures and mechanical devices, which are at rest in the frame of reference of Earth. A large part of engineering creations are static objects. Yet we also seek equilibrium of moving objects like that of floating ship, airplane cruising at high speed, and such other moving mechanical devices. In both cases – static or dynamic – net external forces and torques are zero.

A body is said to be in mechanical equilibrium when net external force is equal to zero and net external torque is also zero. Mathematically,

$$\Sigma \overrightarrow{\mathrm{F}} \text{ext} = 0 \text{ and } \Sigma \overrightarrow{\tau} \text{ext} = 0 \tag{4.7.3}$$

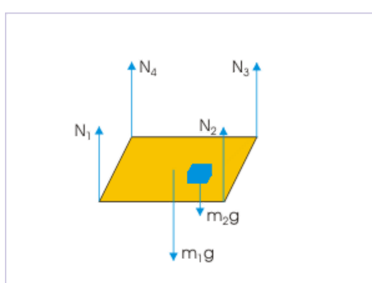
Since there is no net force on the object, the object does not accelerate. This implies two types of possible equilibrium. The first type, where all particles in the system are at rest and do have not velocity, is known as static equilibrium. In the second type, the object has a velocity, but since there are no net forces acting on it, the velocity remains constant. In the second case, the particle is said to be in dynamic equilibrium. Static or dynamic, these kinds of equilibrium can be categorized as translational equilibrium.

Examples of translational equilibrium are all around us. A book resting on a table is pushing down on the table with the force of its weight. The table, in turn, is pushing back on the book, keeping the book from falling through the table. Since neither the table nor the book are moving, this is an example of static equilibrium. The force of gravity on the book is perfectly counteracted by the force of the table pushing on it.



Forces Acting on an Object at Rest: A force diagram showing the forces acting on an object at rest on a surface. Notice that the amount of force that the table is pushing upward on the object (the N vector) is equal to the downward force of the object's weight (shown here as mg, as weight is equal to the object's mass multiplied by the acceleration due to gravity): because these forces are equal, the object is in a state of equilibrium (all the forces acting on it balance to zero).

An example of dynamic (or mechanical) equilibrium is an object sliding down a wedge. The force of gravity pulls the object down the wedge, but it is counteracted by the force of friction between the wedge and the object. If the force of friction is equal to the force of gravity, the object will proceed at a constant velocity.



Forces on a Table: These six forces are in equilibrium. The four forces of the table leg counteract the force of the table and the object pushing on them.

Connected Objects

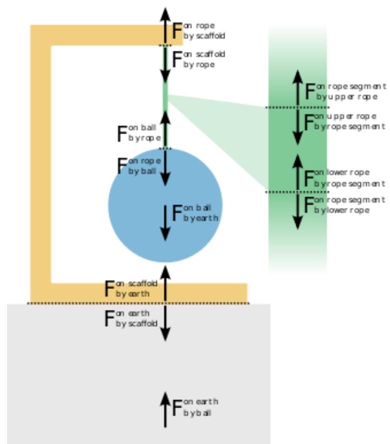
Forces can be transferred from one object to another through connections.

> **learning objectives**
>
> - Analyze the affect a rigid connection has on the movement of objects

The physics of connected objects is very similar to physics of simple objects. There are a variety of ways objects can be connected to each other, and a corresponding variety of mathematical ways to model such connections.

The simplest form of connection is a perfectly rigid connection. If two objects are connected by a perfectly rigid connector then they may be thought of as the same object. Perfectly rigid connectors cannot stretch nor deform, and transfer forces instantaneously from one side of the connection to the other. For example, given two blocks (both of mass 1 kg) connected by a perfectly rigid bar, if the first block is pulled with a force of 1 Newton, then both blocks will accelerate at the same time and the same acceleration. In this case the acceleration is $\frac{1}{2}\mathrm{m/s^2}$ —the same as if a force of mass 2 kg is exerted on one object. Thus it can be said that a perfectly rigid connection makes two objects into one large object. Of course, perfectly rigid connections do not exist in nature. Some deformation will always exist in any object as force travels along it. However, many materials are sufficiently rigid, so that using the perfectly rigid approximation is useful for simplicity's sake.

One can think of the force transferring through the connection by means of the "tension" force. Tension is the pulling force exerted by a string, chain, or similar connector on another object. If two objects are connected by a string, a force exerted on one is balanced by a tension force in the string which pulls on the other. Of course, if the tension force is greater than the rope can withstand, the rope will break.



Tension Forces: The forces involved in supporting a ball by a rope. Tension is the force of the rope on the scaffold, the force of the rope on the ball, and the balanced forces acting on and produced by segments of the rope.

Circular Motion

An object in circular motion undergoes acceleration due to centripetal force in the direction of the center of rotation.

> **learning objectives**
>
> - Develop an understanding of uniform circular motion as an indicator for net external force

Uniform circular motion describes the motion of an object along a circle or a circular arc at constant speed. It is the basic form of rotational motion in the same way that uniform linear motion is the basic form of translational motion. However, the two types of motion are different with respect to the force required to maintain the motion.

Let us consider Newton's first law of motion. It states that an object will maintain a constant velocity unless a net external force is applied. Therefore, uniform linear motion indicates the absence of a net external force. On the other hand, uniform circular motion requires that the velocity vector of an object constantly change direction. Since the velocity vector of the object is changing, an acceleration is occurring. Therefore, uniform *circular* motion indicates the *presence* of a net external force.

In uniform circular motion, the force is always perpendicular to the direction of the velocity. Since the direction of the velocity is continuously changing, the direction of the force must be as well.

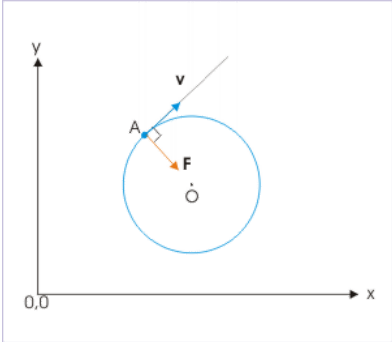
The direction of the velocity along the circular trajectory is tangential. The perpendicular direction to the circular trajectory is, therefore, the radial direction. Therefore, the force (and therefore the acceleration) in uniform direction motion is in the radial direction. For this reason, acceleration in uniform circular motion is recognized to “seek the center” — i.e., centripetal force.

The equation for the acceleration aa required to sustain uniform circular motion is:

$$a = \frac{v^2}{r} \tag{4.7.4}$$

where mm is the mass of the object, v is the velocity of the object, and r is the radius of the circle. Consequently, the net external force F_{net} required to sustain circular motion is:

$$F_{net} = \frac{m \cdot v^2}{r} \tag{4.7.5}$$



Uniform Circular Motion: In uniform circular motion, the centripetal force is perpendicular to the velocity. The centripetal force points toward the center of the circle, keeping the object on the circular track.

Key Points

- Friction is the force that resists relative motion between two surfaces sliding across each other. Friction converts kinetic energy into heat.
- Drag force is the force that resists motion of an object traveling through a fluid such as air or water. Drag force is proportional to the velocity of the object traveling.
- Deformation forces are forces caused by stretching or compressing a material. Some examples would be springs or elastics.
- Kinetic (or dynamic) friction occurs when two objects are moving relative to each other and rub together (like a sled on the ground).
- The force of friction can be represented by an equation $F_{friction} = \mu F_n$ where μ is the coefficient of friction and is a unitless number that represents the strength of the friction of the surface.
- Kinetic friction and static (stationary) friction use two different coefficients for the same material.
- Static friction is a force that acts to resist the start of motion. It is borne of macroscopic inconsistencies in the surfaces of materials in contact as well as intermolecular interactions between the materials, such as hydrogen bonding, Van der Waal’s interactions and electrostatic interactions.
- Static friction uses a different, usually higher, coefficient than kinetic friction does.
- The force of static friction is $F_{fs} = \mu_s F_n$. Where μ_s is the coefficient of static friction which varies by material and FnFn is the normal force.
- Motion on an incline is resisted by friction.
- The frictional force on an incline is dependent on the angle of the incline. $F_f = \mu mg \cos(\theta)$ is the maximum friction force on an incline.
- If the friction force is greater than or equal to the forces in the direction of motion, then the net force is 0 and the object is in equilibrium.
- Objects moving through a fluid feel a force which resists their motion. This force is known as the drag force.
- The drag force is proportional to the square of the velocity of the object relative to the fluid.
- The equation for drag is $F_D = \frac{1}{2} C_\rho A v^2$. C is a constant called the drag coefficient. ρρ is the density of the fluid. A is the surface area in the direction of motion.

- The ratio of force to area $\frac{F}{A}$ is called stress and the ratio of change in length to length $\frac{\Delta L}{L}$ is called the strain.
- Stress and strain are related to each other by a constant called Young's Modulus or the elastic modulus which varies depending on the material. Using Young's Modulus the relation between stress and strain is given by: $\text{stress} = Y \cdot \text{strain}$.
- A material with a high elastic modulus is said to have high tensile strength. Such materials are very resistant to being stretched and require a large amount of force to deform a small amount.
- When there is no external net force on an object, the object is said to be in equilibrium.
- When an object is in equilibrium, it does not accelerate. If it had a velocity, the velocity remains constant; if it was at rest, it remains at rest.
- An equilibrium in motion is known as dynamic equilibrium; an equilibrium at rest is a static equilibrium.
- If two objects are connected, a force on one has an effect on the other.
- Connections can often be approximated as completely rigid. In completely rigid cases, the connection does not deform and the force is transferred instantaneously.
- Tension is the force of a rope or cable or other connector on the object it is connected to. It is one way force is transferred between objects.
- An object that is undergoing circular motion has a velocity vector that is constantly changing direction.
- The force that is needed to maintain circular motion points toward the center of the circular path. It is therefore known as the centripetal force.
- The velocity of an object in circular motion is always tangent to the circle, and the centripetal force is always perpendicular to the velocity.

Key Terms

- **kinetic energy**: The energy possessed by an object because of its motion, equal to one half the mass of the body times the square of its velocity.
- **static**: Fixed in place; having no motion.
- **kinetic**: Of or relating to motion
- **friction**: A force that resists the relative motion or tendency to such motion of two bodies in contact.
- **incline**: A slope.
- **equilibrium**: The state of a body at rest or in uniform motion, the resultant of all forces on which is zero.
- **fluid**: Any substance which can flow with relative ease, tends to assume the shape of its container, and obeys Bernoulli's principle; a liquid, gas or plasma.
- **strain**: The amount by which a material deforms under stress or force, given as a ratio of the deformation to the initial dimension of the material and typically symbolized by ϵϵ is termed the engineering strain. The true strain is defined as the natural logarithm of the ratio of the final dimension to the initial dimension.
- **stress**: The internal distribution of force per unit area (pressure) within a body reacting to applied forces which causes strain or deformation and is typically symbolized by σ.
- **dynamic**: Changing; active; in motion.
- **torque**: A rotational or twisting effect of a force; (SI unit newton-meter or Nm; imperial unit foot-pound or ft-lb)
- **rigid**: Stiff, rather than flexible.
- **tangent**: a straight line touching a curve at a single point without crossing it at that point
- **perpendicular**: at or forming a right angle (to).

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CHAPTER OVERVIEW

05: Uniform Circular Motion and Gravitation

5.1: Introduction to UCM and Gravitation

Kinematics of UCM

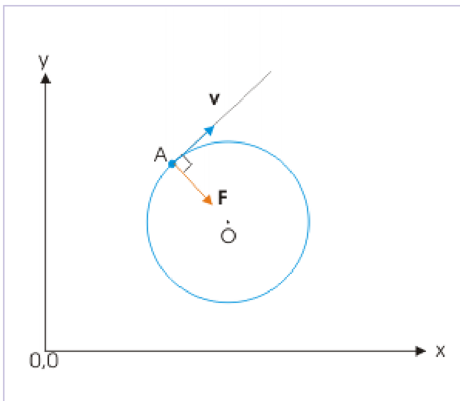
Uniform circular motion is a motion in a circular path at constant speed.

> **learning objectives**
>
> - Relate centripetal force and centripetal acceleration to uniform circular motion

Angular Quantities

Under uniform circular motion, angular and linear quantities have simple relations. When objects rotate about some axis, each point in the object follows a circular arc. The rotation angle is the amount of rotation and is analogous to linear distance. We define the rotation angle $\Delta\theta$ to be the ratio of the arc length to the radius of curvature:

$$\Delta\theta = \frac{\Delta \mathrm{s}}{\mathrm{r}} \tag{5.1.1}$$



Angle θθ and Arc Length ss: The radius of a circle is rotated through an angle $\Delta\theta$. The arc length $\Delta \mathrm{s}$ is described on the circumference.

We define angular velocity ωω as the rate of change of an angle. In symbols, this is $\omega = \frac{\Delta\theta}{\Delta \mathrm{t}}$, where an angular rotation $\Delta\theta$ takes place in a time $\Delta \mathrm{t}$. From the relation of s and $(\Delta \mathrm{s} = \mathrm{r}\Delta\theta)$, we see:

$$\mathrm{v} = \frac{\Delta \mathrm{s}}{\Delta \mathrm{t}} = \mathrm{r}\frac{\Delta\theta}{\Delta \mathrm{t}} = \mathrm{r}\omega \tag{5.1.2}$$

Under uniform circular motion, the angular velocity is constant. The acceleration can be written as:

$$\mathrm{ac} = \frac{\mathrm{dv}}{\mathrm{dt}} = \omega\frac{\mathrm{dr}}{\mathrm{dt}} = \omega \mathrm{v} = \mathrm{r}\omega^2 = \frac{\mathrm{v}^2}{\mathrm{r}} \tag{5.1.3}$$

This acceleration, responsible for the uniform circular motion, is called centripetal acceleration.

Centripetal Force

Any force or combination of forces can cause a centripetal or radial acceleration. Just a few examples are the tension in the rope on a tether ball, the force of Earth's gravity on the Moon, friction between roller skates and a rink floor, a banked roadway's force on a car, and forces on the tube of a spinning centrifuge.

Any net force causing uniform circular motion is called a centripetal force. The direction of a centripetal force is toward the center of curvature, the same as the direction of centripetal acceleration. According to Newton's second law of motion, net force is mass times acceleration. For uniform circular motion, the acceleration is the centripetal acceleration: $\mathrm{a} = \mathrm{a_c}$. Thus, the magnitude of centripetal force $\mathrm{F_c}$ is:

$$\mathrm{F_c} = \mathrm{ma_c} = \mathrm{m}\frac{\mathrm{v}^2}{\mathrm{r}} = \mathrm{mr}\omega^2 \tag{5.1.4}$$

Dynamics of UCM

Newton's universal law of gravitation states that every particle attracts every other particle with a force along a line joining them.

learning objectives

- Relate Kepler's laws to Newton's universal law of gravitation

Newton's Universal Law of Gravitation

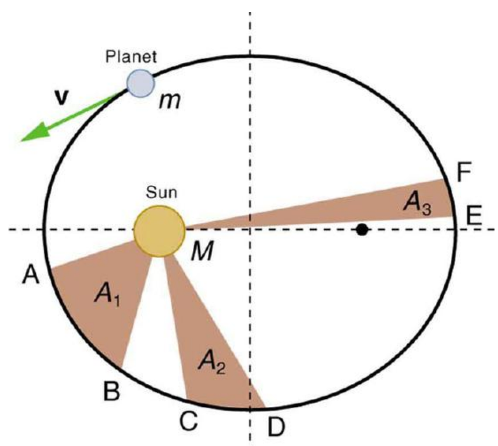
Newton's universal law of gravitation states that every particle in the universe attracts every other particle with a force along a line joining them. The force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. For two bodies having masses mm and MM with a distance rr between their centers of mass, the equation for Newton's universal law of gravitation is:

$$F = G\frac{mM}{r^2} \tag{5.1.5}$$

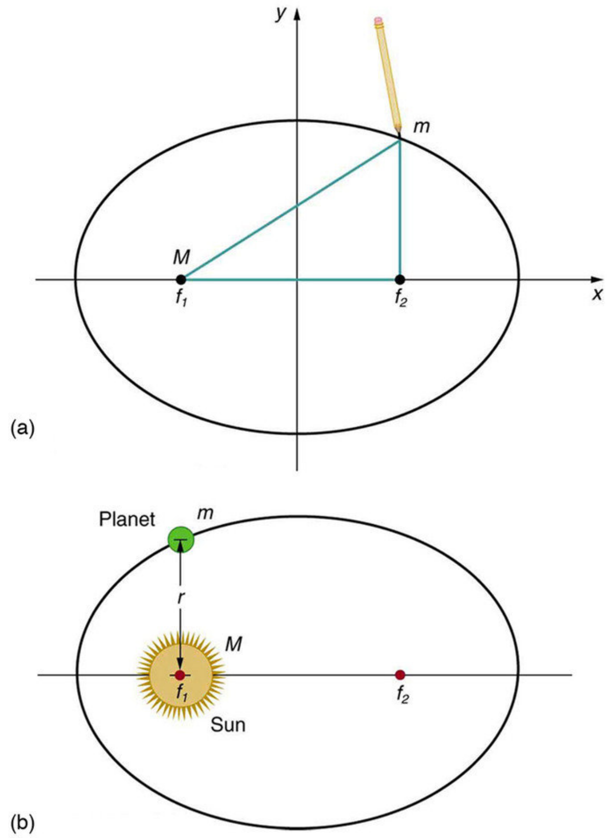
The gravitational force is responsible for artificial satellites orbiting the Earth. The Moon's orbit about Earth, the orbits of planets, asteroids, meteors, and comets about the Sun are other examples of gravitational orbits. Historically, Kepler discovered his 3 laws (called Kepler's law of planetary motion) long before the days of Newton. Kepler devised his laws after careful study (over some 20 years) of a large amount of meticulously recorded observations of planetary motion done by Tycho Brahe (1546–1601).

Kepler's Laws

- The orbit of each planet about the Sun is an ellipse with the Sun at one focus.
- Each planet moves so that an imaginary line drawn from the Sun to the planet sweeps out equal areas in equal times.
- The ratio of the squares of the periods of any two planets about the Sun is equal to the ratio of the cubes of their average distances from the Sun.



Kepler's Second Law: The shaded regions have equal areas. It takes equal times for mm to go from A to B, from C to D, and from E to F. The mass mm moves fastest when it is closest to M. Kepler's second law was originally devised for planets orbiting the Sun, but it has broader validity.



Ellipses and Kepler's First Law: (a) An ellipse is a closed curve such that the sum of the distances from a point on the curve to the two foci (f_1 and f_2) is a constant. You can draw an ellipse as shown by putting a pin at each focus, and then placing a string around a pencil and the pins and tracing a line on paper. A circle is a special case of an ellipse in which the two foci coincide (thus any point on the circle is the same distance from the center). (b) For any closed gravitational orbit, m follows an elliptical path with M at one focus. Kepler's first law states this fact for planets orbiting the Sun.

Derivation of Kepler's Third Law For Circular Orbits

Kepler's 3rd law is equivalent to:

$$\frac{T_1^2}{T_2^2} = \frac{r_3^1}{r_3^2} \tag{5.1.6}$$

T is the period (time for one orbit) and r is the average radius. We shall derive Kepler's third law, starting with Newton's laws of motion and his universal law of gravitation. We will assume a circular path (not an elliptical one) for simplicity.

Let us consider a circular orbit of a small mass mm around a large mass M, satisfying the two conditions stated at the beginning of this section. Gravity supplies the centripetal force to mass m. Therefore, for a uniform circular motion:

$$G\frac{mM}{r^2} = ma_c = m\frac{v^2}{r} \tag{5.1.7}$$

The mass m cancels, yielding:

$$G\frac{mM}{r^2} = v^2 \tag{5.1.8}$$

Now, to get at Kepler's third law, we must get the period T into the equation. By definition, period T is the time for one complete orbit. Now the average speed v is the circumference divided by the period:

$$v = \frac{2\pi r}{T} \tag{5.1.9}$$

Substituting this into the previous equation gives:

$$G\frac{M}{r} = \frac{4\pi^2 r^2}{T^2} \tag{5.1.10}$$

Solving for T^2 yields:

$$T^2 = \frac{4\pi^2}{GM} r^3 \tag{5.1.11}$$

Since T^2 is proportional to r^3, their ratio is constant. This is Kepler's 3rd law.

Banked and Unbacked Highway Curves

In an "ideally banked curve," the angle θ is chosen such that one can negotiate the curve at a certain speed without the aid of friction.

learning objectives

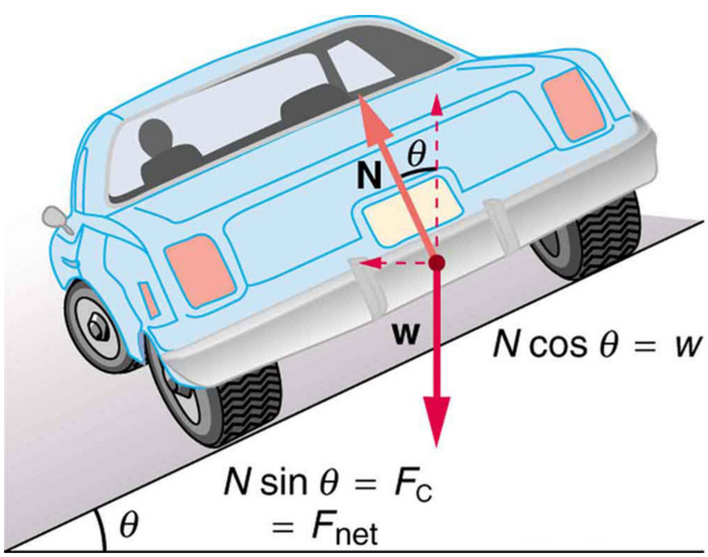
- Derive θ for an ideally banked curve for speed

Overview

As an example of a uniform circular motion and its application, let us now consider banked curves, where the slope of the road helps you negotiate the curve. The greater the angle θ, the faster you can take the curve. Race tracks for bikes as well as cars, for example, often have steeply banked curves. In an "ideally banked curve," the angle θ is such that you can negotiate the curve at a certain speed without the aid of friction between the tires and the road. We will derive an expression for θ for an ideally banked curve for speed $v$and consider an example related to it.

Uniform Circular Motion and Determining Ideal Banking Conditions

For ideal banking, the net external force equals the horizontal centripetal force in the absence of friction. The components of the normal force N in the horizontal and vertical directions must equal the centripetal force and the weight of the car, respectively. In cases in which forces are not parallel, it is most convenient to consider components along perpendicular axes—in this case, the vertical and horizontal directions.



Car on a Banked Curve: The car on this banked curve is moving away and turning to the left.

Above is a free body diagram for a car on a frictionless banked curve. The only two external forces acting on the car are its weight ww and the normal force of the road N. (A frictionless surface can only exert a force perpendicular to the surface—that is, a normal force.) These two forces must add to give a net external force that is horizontal toward the center of curvature and has magnitude $\frac{mv^2}{r}$. Only the normal force has a horizontal component, and so this must equal the centripetal force—that is:

$$N\sin\theta = \frac{mv^2}{r} \tag{5.1.12}$$

Because the car does not leave the surface of the road, the net vertical force must be zero, meaning that the vertical components of the two external forces must be equal in magnitude and opposite in direction. From the figure, we see that the vertical component of the normal force is $N\cos\theta$, and the only other vertical force is the car's weight. These must be equal in magnitude, thus:

$$N\cos\theta = mg \tag{5.1.13}$$

Dividing the above equations yields:

$$\tan\theta = \frac{v^2}{rg} \tag{5.1.14}$$

Taking the inverse tangent gives:

$$\theta = \tan^{-1}(\frac{v^2}{rg}) \tag{5.1.15}$$

for an ideally banked curve with no friction.

This expression can be understood by considering how θ depends on v and r. A large θ will be obtained for a large $v$and a small r. That is, roads must be steeply banked for high speeds and sharp curves. Friction helps, because it allows you to take the curve at greater or lower speed than if the curve is frictionless. Note that θ does not depend on the mass of the vehicle.

Key Points

- Under uniform circular motion, angular and linear quantities have simple relations. The length of an arc is proportional to the rotation angle and the radius. Also, $v = r\omega$.
- The acceleration responsible for the uniform circular motion is called centripetal acceleration. It is given as $a_c = r\omega^2 = \frac{v^2}{r}$.
- Any net force causing uniform circular motion is called a centripetal force. The direction of a centripetal force is toward the center of curvature and its magnitude is $m\frac{v^2}{r} = mr\omega^2$.
- The gravitational force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.
- Kepler discovered laws describing planetary motion long before the days of Newton, purely based on the observations of Tycho Brahe.
- Kepler's laws can be derived from the Newton's universal law of gravitation and his equation of motion.
- For ideal banking, the net external force equals the horizontal centripetal force in the absence of friction.
- For ideal banking, the components of the normal force NN in the horizontal and vertical directions must equal the centripetal force and the weight of the car, respectively.
- The ideal banking condition is given as $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$.

Key Terms

- **centripetal**: Directed or moving towards a center.
- **asteroid**: A naturally occurring solid object, which is smaller than a planet and is not a comet, that orbits a star.
- **planet**: A large body which directly orbits any star (or star cluster) but which has not attained nuclear fusion.
- **normal force**: Any force acting normal, to a surface, or perpendicular to the tangent plane.

5.2: Non-Uniform Circular Motion

Overview of Non-Uniform Circular Motion

Non-uniform circular motion denotes a change in the speed of a particle moving along a circular path.

learning objectives

- Explain when a particle undergoes non-uniform circular motion

What do we mean by non-uniform circular motion? The answer lies in the definition of uniform circular motion, which is a circular motion with constant speed. It follows then that non-uniform circular motion denotes a change in the speed of the particle moving along the circular path. Note especially the change in the velocity vector sizes, denoting change in the magnitude of velocity.

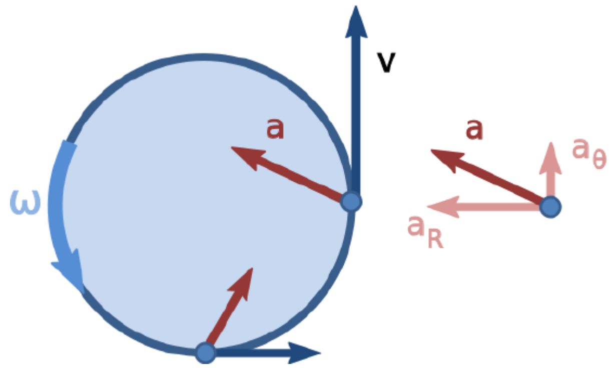


Diagram of non-uniform circular motion: In non-uniform circular motion, the magnitude of the angular velocity changes over time.

The change in direction is accounted by radial acceleration (centripetal acceleration), which is given by following relation: $a_r = \frac{v^2}{r}$. The change in speed has implications for radial (centripetal) acceleration. There are two possibilities:

1: The radius of circle is constant (like in the motion along a circular rail or motor track). A change in v will change the magnitude of radial acceleration. This means that the centripetal acceleration is not constant, as is the case with uniform circular motion. The greater the speed, the greater the radial acceleration. A particle moving at higher speed will need a greater radial force to change direction and vice-versa when the radius of the circular path is constant.

2: The radial (centripetal) force is constant (like a satellite rotating about the earth under the influence of a constant force of gravity). The circular motion adjusts its radius in response to changes in speed. This means that the radius of the circular path is variable, unlike the case of uniform circular motion. In any eventuality, the equation of centripetal acceleration in terms of “speed” and “radius” must be satisfied. The important thing to note here is that, although change in speed of the particle affects radial acceleration, the change in speed is not affected by radial or centripetal force. We need a tangential force to affect the change in the magnitude of a tangential velocity. The corresponding acceleration is called tangential acceleration.

In either case, the angular velocity in non-uniform circular motion is not constant as $\omega = \frac{v}{r}$ and v is varying.

Key Points

- In non- uniform circular motion, the size of the velocity vector (speed) changes, denoting change in the magnitude of velocity.
- The change in speed has implications for radial (centripetal) acceleration. There are two possibilities: 1) the radius of the circle is constant; or 2) the radial (centripetal) force is constant.
- In either case, the angular velocity in non-uniform circular motion is not constant, as $\omega = \frac{v}{r}$, and v varies.

Key Terms

- **radial**: Moving along a radius.
- **centripetal**: Directed or moving towards a center.

LICENSES AND ATTRIBUTIONS

5.3: Velocity, Acceleration, and Force

Rotational Angle and Angular Velocity

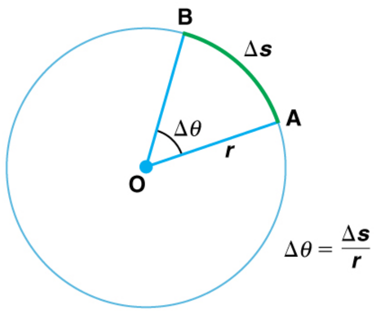
The rotational angle is a measure of how far an object rotates, and angular velocity measures how fast it rotates.

learning objectives

- Express the relationship between the rotational angle and the distance

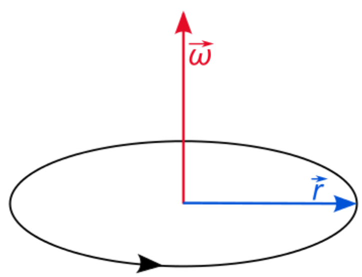
Rotational Angle and Angular Velocity

When an object rotates about an axis, as with a tire on a car or a record on a turntable, the motion can be described in two ways. A point on the edge of the rotating object will have some velocity and will be carried through an arc by riding the spinning object. The point will travel through a distance of $\Delta \mathrm{S}$, but it is often more convenient to talk about the extent the object has rotated. The amount the object rotates is called the rotational angle and may be measured in either degrees or radians. Since the rotational angle is related to the distance $\Delta \mathrm{S}$ and to the radius r by the equation $\Delta\theta = \frac{\Delta \mathrm{S}}{\mathrm{R}}$, it is usually more convenient to use radians.



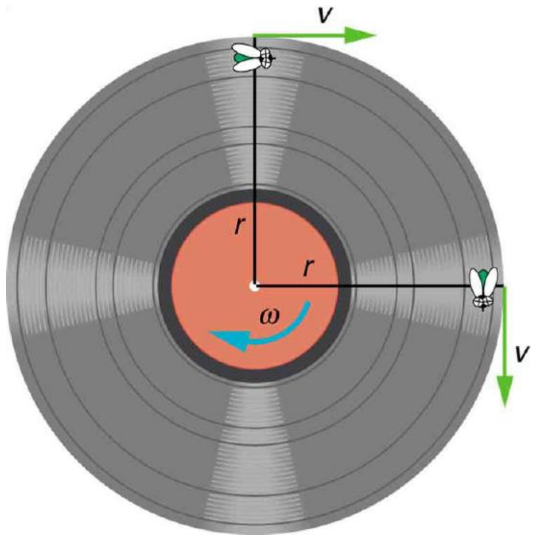
Angle θ and Arc Length s: The radius of a circle is rotated through an angle $\Delta\theta$. The arc length Δs is described on the circumference.

The speed at which the object rotates is given by the angular velocity, which is the rate of change of the rotational angle with respect to time. Although the angle itself is not a vector quantity, the angular velocity is a vector. The direction of the angular velocity vector is perpendicular to the plane of rotation, in a direction which is usually specified by the right-hand rule. Angular acceleration gives the rate of change of angular velocity. The angle, angular velocity, and angular acceleration are very useful in describing the rotational motion of an object.



The Direction of Angular Velocity: The angular velocity describes the speed of rotation and the orientation of the instantaneous axis about which the rotation occurs. The direction of the angular velocity will be along the axis of rotation. In this case (counter-clockwise rotation), the vector points upwards.

When the axis of rotation is perpendicular to the position vector, the angular velocity may be calculated by taking the linear velocity v of a point on the edge of the rotating object and dividing by the radius. This will give the angular velocity, usually denoted by ω, in terms of radians per second.



Angular Velocity: A fly on the edge of a rotating object records a constant velocity v. The object is rotating with an angular velocity equal to $\frac{v}{r}$.

Centripetial Acceleration

Centripetal acceleration is the constant change in velocity necessary for an object to maintain a circular path.

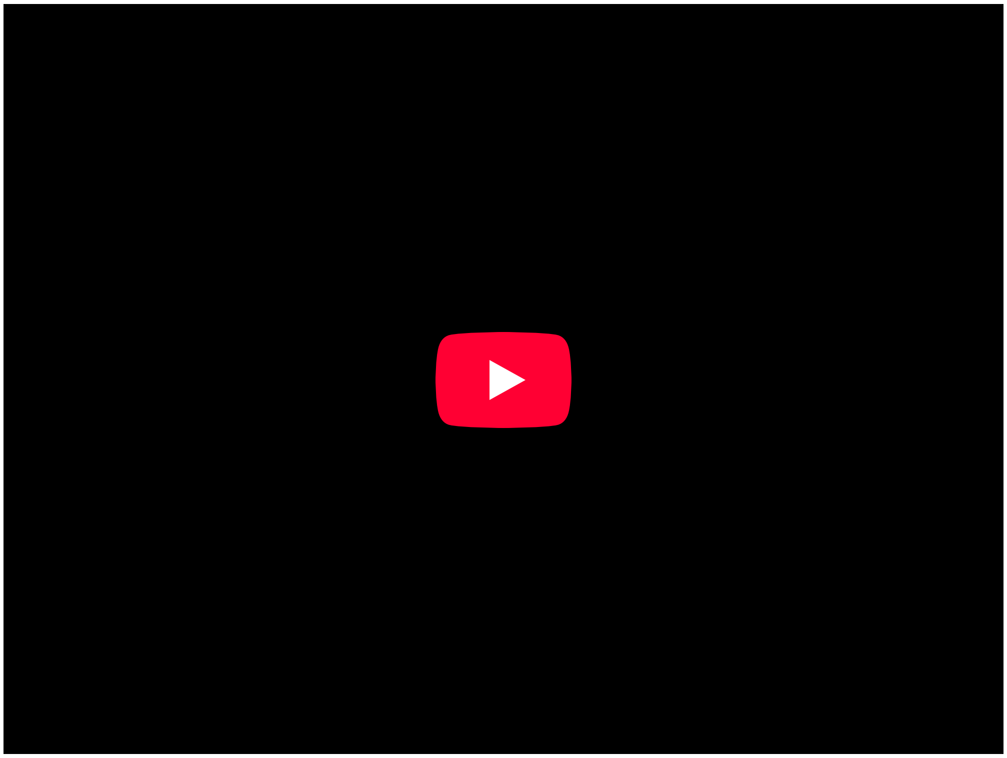
learning objectives

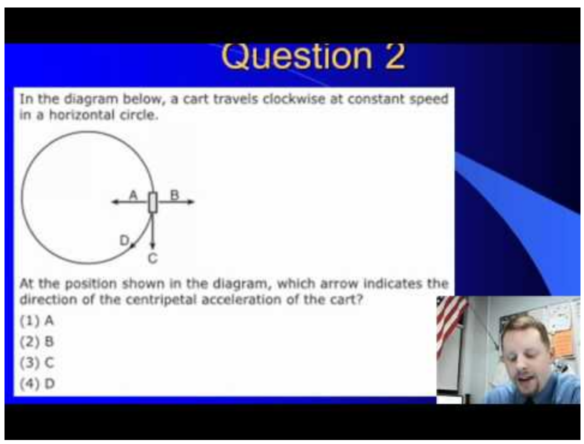
- Express the centripetal acceleration in terms of rotational velocity

Overview

As mentioned in previous sections on kinematics, any change in velocity is given by an acceleration. Often the changes in velocity are changes in magnitude. When an object speeds up or slows down this is a change in the objects velocity. Changes in the magnitude of the velocity match our intuitive and every day usage of the term accelerate. However, because velocity is a vector, it also has a direction. Therefore, any change in the direction of travel of an object must also be met with an acceleration.

Uniform circular motion involves an object traveling a circular path at constant speed. Since the speed is constant, one would not usually think that the object is accelerating. However, the direction is constantly changing as the object traverses the circle. Thus, it is said to be accelerating. One can feel this acceleration when one is on a roller coaster. Even if the speed is constant, a quick turn will provoke a feeling of force on the rider. This feeling is an acceleration.





Centripetal Acceleration: A brief overview of centripetal acceleration for high school physics students.

Calculating Centripetal Acceleration

To calculate the centripetal acceleration of an object undergoing uniform circular motion, it is necessary to have the speed at which the object is traveling and the radius of the circle about which the motion is taking place. The simple equation is:

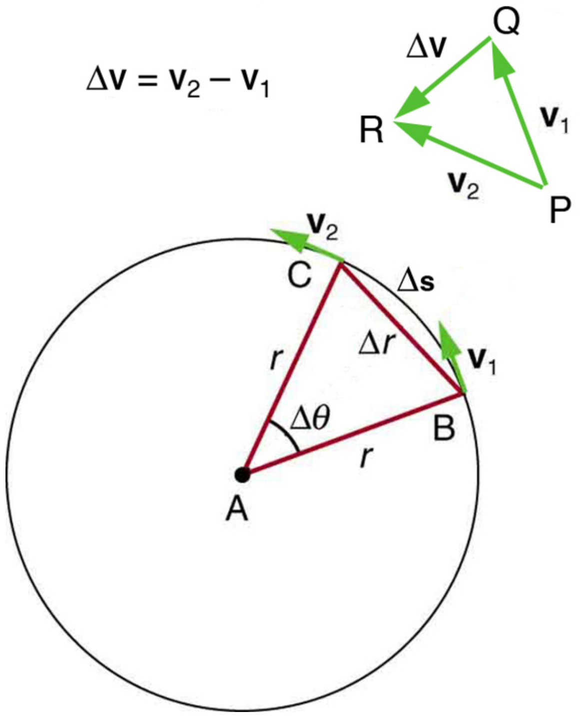
$$a_c = \frac{v^2}{r} \tag{5.3.1}$$

where v is the linear velocity of the object and r is the radius of the circle.

The centripetal acceleration may also be expressed in terms of rotational velocity as follows:

$$a_c = \omega^2 r \tag{5.3.2}$$

with omega being the rotational velocity given by $\frac{v}{r}$.



Centripetal Acceleration: As an object moves around a circle, the direction of the velocity vector constantly changes.

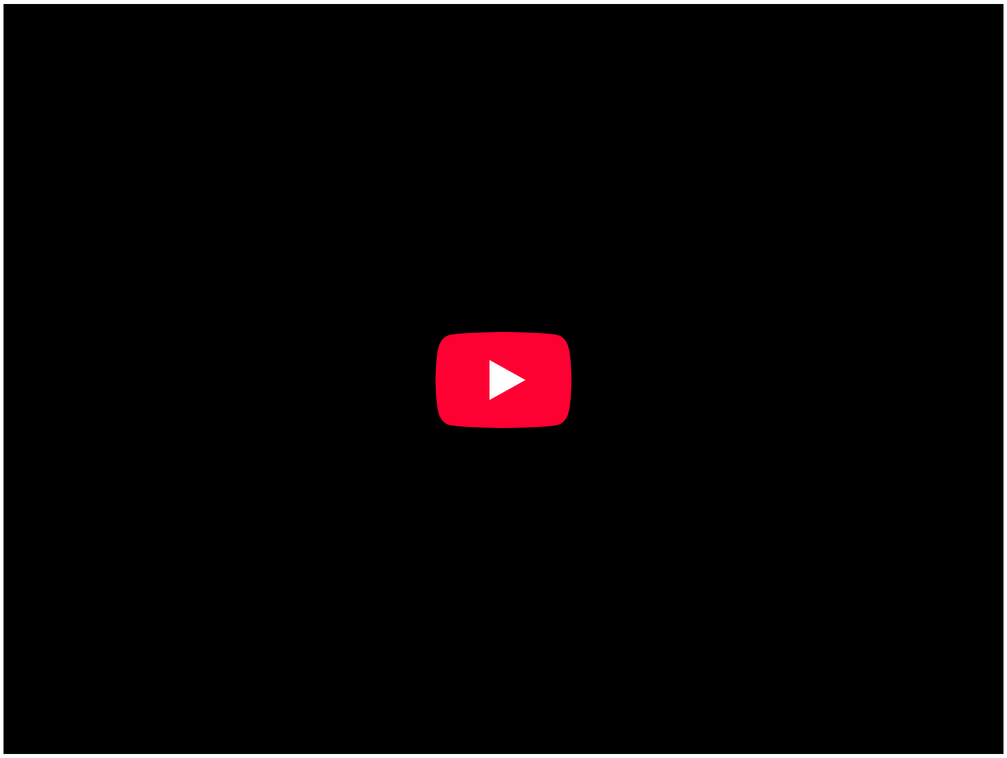
Centripetal Force

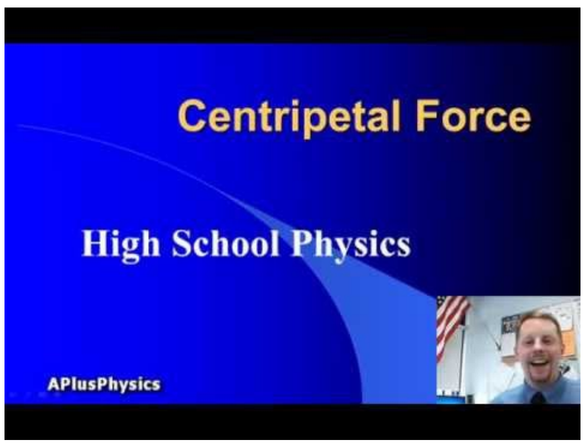
A force which causes motion in a curved path is called a centripetal force (uniform circular motion is an example of centripetal force).

learning objectives

- Express the equations for the centripetal force and acceleration

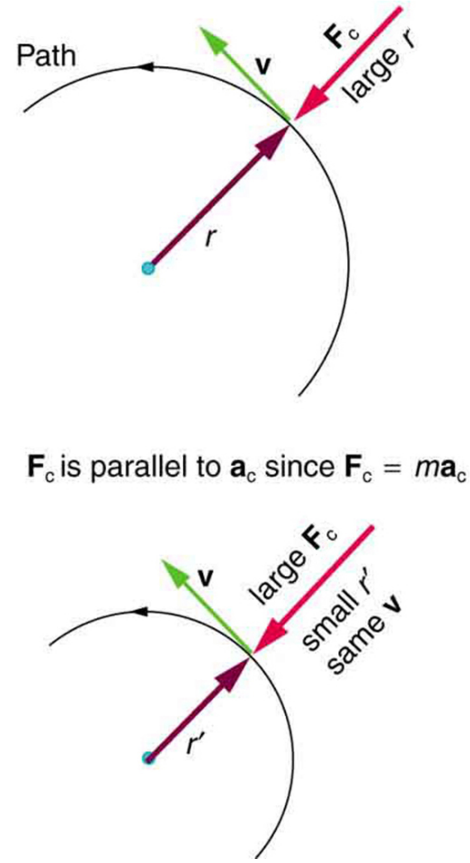
A force that causes motion in a curved path is called a centripetal force. Uniform circular motion is an example of centripetal force in action. It can be seen in the orbit of satellites around the earth, the tension in a rope in a game of tether ball, a roller coaster loop de loop, or in a bucket swung around the body.





Overview of centripetal force: A brief overview of centripetal force.

Previously, we learned that any change in a velocity is an acceleration. As the object moves through the circular path it is constantly changing direction, and therefore accelerating—causing constant force to be acting on the object. This centripetal force acts toward the center of curvature, toward the axis of rotation. Because the object is moving perpendicular to the force, the path followed by the object is a circular one. It is this force that keeps a ball from falling out of a bucket if you swing it in circular continuously.



Centripetal force: As an object travels around a circular path at a constant speed, it experiences a centripetal force accelerating it toward the center.

The equation for centripetal force is as follows:

$$\mathrm{Fc} = \frac{\mathrm{mv}^2}{\mathrm{r}} \tag{5.3.3}$$

where: $\mathrm{F_c}$ is centripetal force, m is mass, v is velocity, and r is the radius of the path of motion.

From Newton's second law $\mathrm{F = m \cdot a}$, we can see that centripetal acceleration is:

$$\mathrm{a_c} = \frac{\mathrm{v}^2}{\mathrm{r}} \tag{5.3.4}$$

Centripetal force can also be expressed in terms of angular velocity. Angular velocity is the measure of how fast an object is traversing the circular path. As the object travels its path, it sweeps out an arc that can be measured in degrees or radians. The equation for centripetal force using angular velocity is:

$$\mathrm{F_c} = \mathrm{mr}\omega^2 \tag{5.3.5}$$

Key Points

- When an object rotates about an axis, the points on the edge of the object travel in arcs.
- The angle these arcs sweep out is called the rotational angle, and it is usually represented by the symbol *theta*.
- A measure of how quickly the object is rotating, with respect to time, is called the angular velocity. It is usually represented by a Greek *omega* symbol. Like its counterpart linear velocity, it is a vector.
- For an object to maintain circular motion it must constantly change direction.
- Since velocity is a vector, changes in direction constitute changes in velocity.
- A change in velocity is known as an acceleration. The change in velocity due to circular motion is known as centripetal acceleration.

- Centripetal acceleration can be calculated by taking the linear velocity squared divided by the radius of the circle the object is traveling along.
- When an object is in uniform circular motion, it is constantly changing direction, and therefore accelerating. This is angular acceleration.
- A force acting on the object in uniform circular motion (called centripetal force) is acting on the object from the center of the circle.

Key Terms

- **radians**: The angle subtended at the centre of a circle by an arc of the circle of the same length as the circle's radius.
- **acceleration**: The amount by which a speed or velocity increases (and so a scalar quantity or a vector quantity).
- **circular motion**: Motion in such a way that the path taken is that of a circle.
- **velocity**: A vector quantity that denotes the rate of change of position with respect to time, or a speed with a directional component.
- **centripetal**: Directed or moving towards a center.
- **angular velocity**: A vector quantity describing an object in circular motion; its magnitude is equal to the speed of the particle and the direction is perpendicular to the plane of its circular motion.

5.4: Types of Forces in Nature

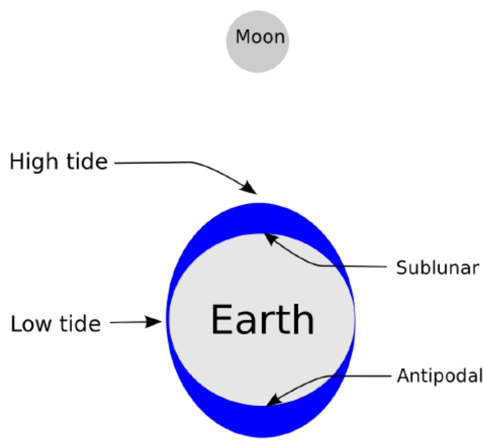
Tides

Tides are the rise and fall of sea levels due to the effects of the gravity exerted by the moon and the sun, and the rotation of the Earth.

learning objectives

- Explain factors that influence the times and amplitude of the tides at a locale

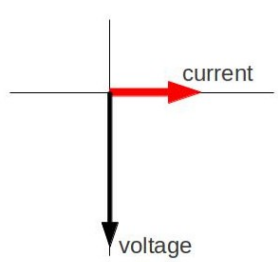
Tides are the rise and fall of sea levels due to the effects of gravitational forces exerted by the moon and the sun when combined with the rotation of the Earth. Tides occur to varying degrees and frequency, depending on location. Shorelines where two almost equally high tides and two low tides occur each day experience a semi- diurnal tide. The occurrence of only one high and one low tide each day is known as diurnal tide. A mixed tide referes to the daily occurrence of two uneven tides, or perhaps one high and one low tide. The times and amplitude of tides at various locales are influenced by the alignment of the sun and moon, the pattern of tides in the deep ocean, the shape of the coastline, and other forces.



Earth's tides.: Schematic of the lunar portion of earth's tides showing (exaggerated) high tides at the sublunar and antipodal points for the hypothetical case of an ocean of constant depth with no land. There would also be smaller, superimposed bulges on the sides facing toward and away from the sun.

Tidal Force

If we want to know the acceleration "felt" by an observer living on Earth due to the moon, a tricky part is that the Earth is not an inertial frame of reference because it is in "free fall" with respect to the moon. Given this, in order to figure out the force observed, we must subtract the acceleration of the (Earth) frame itself. The tidal force produced by the moon on a small particle located on Earth is the vector difference between the gravitational force exerted by the moon on the particle, and the gravitational force that would be exerted if it were located at the Earth's center of mass.



Moon's Gravity on the Earth: Top picture shows the gravity force due to the Moon at different locations F_r on Earth. Bottom picture shows the differential force $F_r - F_{center}$. This is the acceleration "felt" by an observer living on Earth.

As diagramed below, this is equivalent to subtracting the "red" vector from the "black" vectors on the surface of the Earth in the top picture, leading to the "differential" force represented by the bottom picture. Thus, the tidal force depends not on the strength of the lunar gravitational field, but on its gradient (which falls off approximately as the inverse cube of the distance to the originating gravitational body).

On average, the solar gravitational force on the Earth is 179 times stronger than the lunar, but because the sun is on average 389 times farther from the Earth its field gradient is weaker. The solar tidal force is 46% as large as the lunar. More precisely, the lunar tidal acceleration (along the moon-Earth axis, at the Earth's surface) is about $1.1 \cdot 10^{-7}$ g, while the solar tidal acceleration (along the sun-Earth axis, at the Earth's surface) is about $0.52 \cdot 10^{-7}$ g, where g is the gravitational acceleration at the Earth's surface. Venus has the largest effect of the other planets, at 0.000113 times the solar effect.

Tidal Energy

Energy of tides can be extracted by two means: inserting a water turbine into a tidal current, or building ponds that release/admit water through a turbine. In the first case, the energy amount is entirely determined by the timing and tidal current magnitude, but the best currents may be unavailable because the turbines would obstruct ships. In the second case, impoundment dams are expensive to construct, natural water cycles are completely disrupted, as is ship navigation. However, with multiple ponds, power can be generated at chosen times. Presently, there are few installed systems for tidal power generation (most famously, La Rance by Saint Malo, France), as many difficulties are involved. Aside from environmental issues, simply withstanding corrosion and biological fouling pose engineering challenges.

Tidal Energy Generator: Tidal energy generator that works like a wind turbine, but with the ocean currents providing the energy. The circle in the middle is the turbine. The contraption travels up and down the two legs just like a lift and sits on the sea floor when in use.

Unlike with wind power systems, tidal power proponents point out that generation levels can be reliably predicted (save for weather effects). While some generation is possible for most of the tidal cycle, in practice, turbines lose efficiency at lower operating rates. Since the power available from a flow is proportional to the cube of the flow speed, the times during which high power generation is possible are brief.

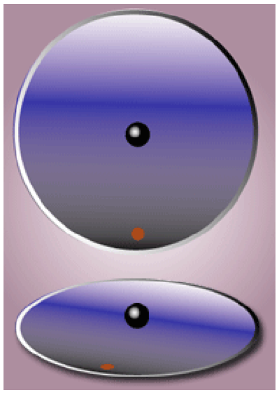
The Coriolois Force

The Coriolis effect is a deflection of moving objects when they are viewed in a rotating reference frame.

learning objectives

- Formulate relationship between the Coriolis force, mass of an object, and speed in the rotating frame

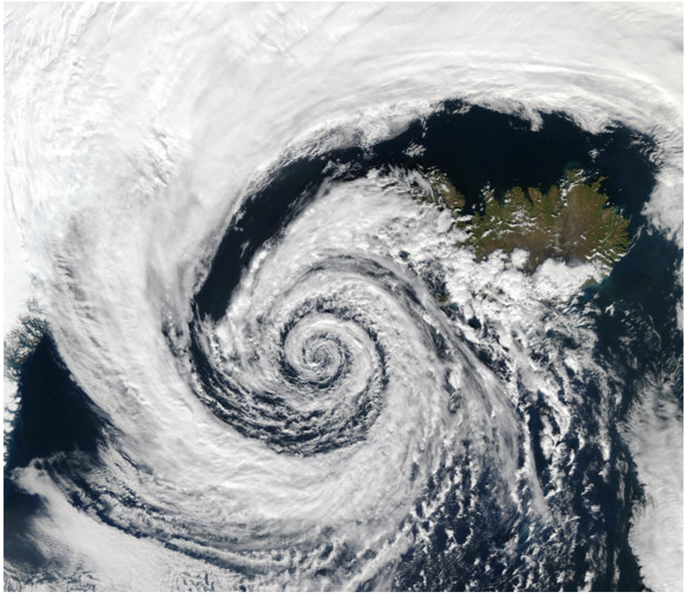
The Coriolis effect is a deflection of moving objects when they are viewed in a rotating reference frame. In a reference frame with clockwise rotation, the deflection is to the left of the motion of the object; in one with counter-clockwise rotation, the deflection is to the right. Although recognized previously by others, the mathematical expression for the Coriolis force appeared in an 1835 paper by French scientist Gaspard-Gustave Coriolis, in connection with the theory of water wheels. Early in the 20th century, the term "Coriolis force" began to be used in connection with meteorology.



Frames of Reference: In the inertial frame of reference (upper part of the picture), the black object moves in a straight line. However, the observer (red dot) who is standing in the rotating/non-inertial frame of reference (lower part of the picture) sees the object as following a curved path due to the Coriolis and centrifugal forces present in this frame.

Newton's laws of motion govern the motion of an object in a (non-accelerating) inertial frame of reference. When Newton's laws are transformed to a uniformly rotating frame of reference, the Coriolis and centrifugal forces appear. Both forces are proportional to the mass of the object. The Coriolis force is proportional to the rotation rate, and the centrifugal force is proportional to its square. The Coriolis force acts in a direction perpendicular to the rotation axis and to the velocity of the body in the rotating frame. It is proportional to the object's speed in the rotating frame. These additional forces are termed inertial forces, fictitious forces, or pseudo-forces. They allow the application of Newton's laws to a rotating system. They are correction factors that do not exist in a non-accelerating or inertial reference frame.

Perhaps the most commonly encountered rotating reference frame is the Earth. The Coriolis effect is caused by the rotation of the Earth and the inertia of the mass experiencing the effect. Because the Earth completes only one rotation per day, the Coriolis force is quite small. Its effects generally become noticeable only for motions occurring over large distances and long periods of time, such as large-scale movements of air in the atmosphere or water in the ocean. Such motions are constrained by the surface of the earth, so generally only the horizontal component of the Coriolis force is important. This force causes moving objects on the surface of the Earth to be deflected in a clockwise sense (with respect to the direction of travel) in the northern hemisphere and in a counter-clockwise sense in the southern hemisphere. Rather than flowing directly from areas of high pressure to low pressure, as they would in a non-rotating system, winds and currents tend to flow to the right of this direction north of the equator and to the left of this direction south of it. This effect is responsible for the rotation of large cyclones.



Coriolis Force: This low-pressure system over Iceland spins counter-clockwise due to balance between the Coriolis force and the pressure gradient force.

Other Geophysical Applications

Tidal and Coriolis forces may not be obvious over a small time-space scale, but they are important in meteorology, navigation, and fishing.

learning objectives

- Identify fields that have to take into account the tidal and Coriolis forces

We have studied tidal and Coriolis forces previously. To review, the tidal force is responsible for the tides — it is a “differential force,” due to a secondary effect of the force of gravity. The Coriolis force is a fictitious force, representing a deflection of moving objects when they are viewed in a rotating reference frame of the Earth. Although their effects may not be obvious over a small time-space scale, these forces are important in such contexts as meteorology, navigation, fishing, and others.

The Tides

Tidal flows are important for marine navigation, and significant errors in position occur if they are not accounted for. Tidal heights are also important; for example, many rivers and harbors have a shallow “bar” at the entrance to prevent boats with significant draft from entering at low tide. Until the advent of automated navigation, competence in calculating tidal effects was important to naval officers. The certificate of examination for lieutenants in the Royal Navy once declared that the prospective officer was able to “shift his tides. ”

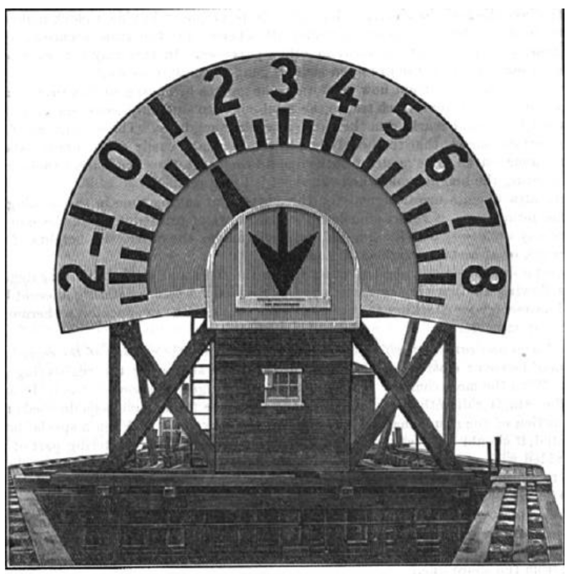
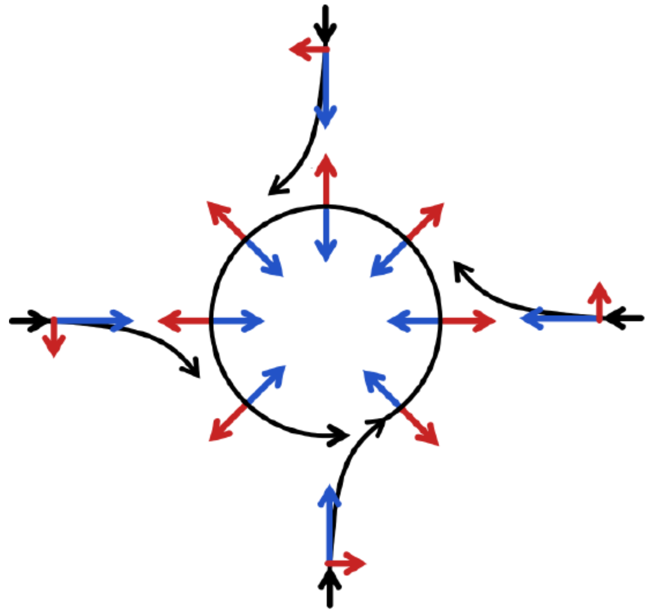


Fig. 5.—Tidal indicator, Delaware River, Delaware.

Tidal Indicator: Tidal Indicator, Delaware River, Delaware c. 1897. In the moment pictured, the tide is 1.25 feet above mean low water and is still falling, as indicated by the pointing of the arrow. The indicator is powered by a system of pulleys, cables, and a float

The Coriolis Force

The Coriolis force is quite small, and its effects generally become noticeable only when we are dealing with motions occurring over large distances and long periods of time, such as large-scale movements of air in the atmosphere or water in the ocean. The Coriolis effects also became important in ballistics calculations — for example, calculating the trajectories of very long-range artillery shells. The most famous historical example is the Paris gun, used by the Germans during World War I to bombard Paris from a range of about 120 km.



Flow Representation: A schematic representation of flow around a low-pressure area in the Northern Hemisphere. The pressure-gradient force is represented by blue arrows and the Coriolis acceleration (always perpendicular to the velocity) by red arrows

Key Points

- The tidal force depends not on the strength of the lunar gravitational field itself, but on its gradient, which falls off approximately as the inverse cube of the distance to the originating gravitational body. This is because the tidal force felt by an observer on Earth is a differential force.
- The times and amplitude of the tides at a locale are influenced by several factors, such as the alignment of the sun and moon, the pattern of tides in the deep ocean, the shape of the coastline, and others forces.
- Energy of tides can be extracted by two means: inserting a water turbine into a tidal current, or building ponds that release/admit water through a turbine.
- When Newton's laws are transformed to a uniformly rotating frame of reference, the Coriolis and centrifugal forces appear.
- The Coriolis force acts in a direction perpendicular to the rotation axis and to the velocity of the body in the rotating frame; it is proportional to the object's mass and speed in the rotating frame.
- The Coriolis effect is caused by the rotation of the Earth and the inertia of the mass experiencing the effect.
- Tidal flows are important for marine navigation, and significant errors in position occur if they are not accounted for.
- The Coriolis force is quite small, and its effects generally become noticeable only when we are dealing with motions occurring over large distances and long periods of time, such as large-scale movements of air in the atmosphere or water in the ocean.
- The tidal force is responsible for the tides. It is a "differential force," due to a secondary effect of the force of gravity. The Coriolis force is a fictitious force, representing a deflection of moving objects when they are viewed in a rotating reference frame of the Earth.

Key Terms

- **inertial frame**: A frame of reference that describes time and space homogeneously, isotropically, and in a time-independent manner.
- **diurnal**: Having a daily cycle that is completed every 24 hours, usually referring to tasks, processes, tides, or sunrise to sunset.
- **gradient**: The rate at which a physical quantity increases or decreases relative to change in a given variable, especially distance.
- **fictitious force**: an apparent force that acts on all masses in a non-inertial frame of reference, such as a rotating reference frame
- **centrifugal force**: the apparent outward force that draws a rotating body away from the center of rotation
- **ballistics**: the science of mechanics that deals with the flight, behavior, and effects of projectiles, especially bullets, gravity bombs, rockets, or the like
- **meteorology**: the interdisciplinary scientific study of the atmosphere

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5.5: Newton's Law of Universal Gravitation

The Law of Universal Gravitation

Objects with mass feel an attractive force that is proportional to their masses and inversely proportional to the square of the distance.

learning objectives

- Express the Law of Universal Gravitation in mathematical form

While an apple might not have struck Sir Isaac Newton's head as myth suggests, the falling of one did inspire Newton to one of the great discoveries in mechanics: *The Law of Universal Gravitation*. Pondering why the apple never drops sideways or upwards or any other direction except perpendicular to the ground, Newton realized that the Earth itself must be responsible for the apple's downward motion.

Theorizing that this force must be proportional to the masses of the two objects involved, and using previous intuition about the inverse-square relationship of the force between the earth and the moon, Newton was able to formulate a general physical law by induction.

The Law of Universal Gravitation states that every point mass attracts every other point mass in the universe by a force pointing in a straight line between the centers-of-mass of both points, and this force is proportional to the masses of the objects and inversely proportional to their separation This attractive force always points inward, from one point to the other. The Law applies to all objects with masses, big or small. Two big objects can be considered as point-like masses, if the distance between them is very large compared to their sizes or if they are spherically symmetric. For these cases the mass of each object can be represented as a point mass located at its center-of-mass.

While Newton was able to articulate his Law of Universal Gravitation and verify it experimentally, he could only calculate the relative gravitational force in comparison to another force. It wasn't until Henry Cavendish's verification of the gravitational constant that the Law of Universal Gravitation received its final algebraic form:

$$\mathrm{F}=\mathrm{G}\frac{\mathrm{Mm}}{\mathrm{r}^2} \tag{5.5.1}$$

where F represents the force in Newtons, M and m represent the two masses in kilograms, and r represents the separation in meters. G represents the gravitational constant, which has a value of $6.674\cdot 10^{-11}\mathrm{N}(\mathrm{m}/\mathrm{kg})^2$. Because of the magnitude of G, gravitational force is very small unless large masses are involved.

image

Forces on two masses: All masses are attracted to each other. The force is proportional to the masses and inversely proportional to the square of the distance.

Gravitational Attraction of Spherical Bodies: A Uniform Sphere

The Shell Theorem states that a spherically symmetric object affects other objects as if all of its mass were concentrated at its center.

learning objectives

- Formulate the Shell Theorem for spherically symmetric objects

Universal Gravitation for Spherically Symmetric Bodies

The Law of Universal Gravitation states that the gravitational force between two points of mass is proportional to the magnitudes of their masses and the inverse-square of their separation, d:

$$\mathrm{F}=\frac{\mathrm{GmM}}{\mathrm{d}^2} \tag{5.5.2}$$

However, most objects are not point particles. Finding the gravitational force between three-dimensional objects requires treating them as points in space. For highly symmetric shapes such as spheres or spherical shells, finding this point is simple.

The Shell Theorem

Isaac Newton proved the Shell Theorem, which states that:

1. A spherically symmetric object affects other objects gravitationally as if all of its mass were concentrated at its center,
2. If the object is a spherically symmetric shell (i.e., a hollow ball) then the net gravitational force on a body *inside* of it is zero.

Since force is a vector quantity, the vector summation of all parts of the shell/sphere contribute to the net force, and this net force is the equivalent of one force measurement taken from the sphere’s midpoint, or center of mass (COM). So when finding the force of gravity exerted on a ball of 10 kg, the distance measured from the ball is taken from the ball’s center of mass to the earth’s center of mass.

Given that a sphere can be thought of as a collection of infinitesimally thin, concentric, spherical shells (like the layers of an onion), then it can be shown that a corollary of the Shell Theorem is that the force exerted in an object inside of a solid sphere is only dependent on the mass of the sphere inside of the radius at which the object is. That is because shells at a greater radius than the one at which the object is, do *not* contribute a force to an object inside of them (Statement 2 of theorem).

When considering the gravitational force exerted on an object at a point *inside* or *outside* a uniform spherically symmetric object of radius RR, there are two simple and distinct situations that must be examined: the case of a hollow spherical shell, and that of a solid sphere with uniformly distributed mass.

Case 1: A hollow spherical shell

The gravitational force acting by a spherically symmetric shell upon a point mass *inside* it, is the vector sum of gravitational forces acted by each part of the shell, and this vector sum is equal to zero. That is, a mass mm *within* a spherically symmetric shell of mass M, will feel no net force (Statement 2 of Shell Theorem).

The net gravitational force that a spherical shell of mass M exerts on a body *outside* of it, is the vector sum of the gravitational forces acted by each part of the shell on the outside object, which add up to a net force acting as if mass M is concentrated on a point at the center of the sphere (Statement 1 of Shell Theorem).

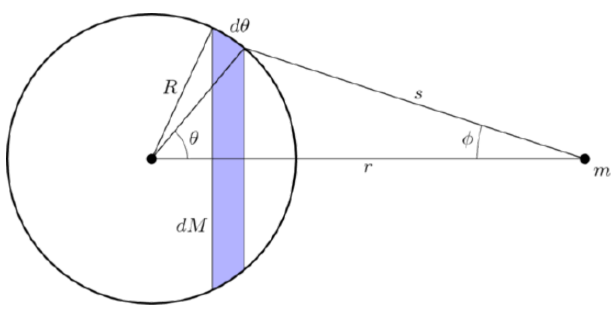


Diagram used in the proof of the Shell Theorem: This diagram outlines the geometry considered when proving The Shell Theorem. In particular, in this case a spherical shell of mass M (left side of figure) exerts a force on mass m (right side of the figure) outside of it. The surface area of a thin slice of the sphere is shown in color. (Note: The proof of the theorem is not presented here. Interested readers can explore further using the sources listed at the bottom of this article.)

Case 2: A solid, uniform sphere

The second situation we will examine is for a solid, uniform sphere of mass M and radius R, exerting a force on a body of mass m at a radius d *inside* of it (that is, $d < R$). We can use the results and corollaries of the Shell Theorem to analyze this case. The contribution of all shells of the sphere at a radius (or distance) greater than dd from the sphere’s center-of-mass can be ignored (see above corollary of the Shell Theorem). Only the mass of the sphere within the desired radius $M < d$ (that is the mass of the sphere inside dd) is relevant, and can be considered as a point mass at the center of the sphere. So, the gravitational force acting upon point mass mm is:

$$F = \frac{GmM_{<d}}{d^2} \tag{5.5.3}$$

where it can be shown that $M_{<d}=\frac{4}{3}\pi d^3\rho$

(ρ is the mass density of the sphere and we are assuming that it does not depend on the radius. That is, the sphere's mass is uniformly distributed.)

Therefore, combining the above two equations we get:

$$F=\frac{4}{3}\pi G m\rho d \tag{5.5.4}$$

which shows that mass mm feels a force that is linearly proportional to its distance, dd, from the sphere's center of mass.

As in the case of hollow spherical shells, the net gravitational force that a solid sphere of uniformly distributed mass M exerts on a body *outside* of it, is the vector sum of the gravitational forces acted by each shell of the sphere on the outside object. The resulting net gravitational force acts as if mass M is concentrated on a point at the center of the sphere, which is the center of mass, or COM (Statement 1 of Shell Theorem). More generally, this result is true even if the mass M is *not* uniformly distributed, but its density varies radially (as is the case for planets).

Weight of the Earth

When the bodies have spatial extent, gravitational force is calculated by summing the contributions of point masses which constitute them.

learning objectives

- Describe how gravitational force is calculated for the bodies with spatial extent

Newton's law of universal gravitation states that every point mass in the universe attracts every other point mass with a force that is directly proportional to the product of their masses, and inversely proportional to the square of the distance between them.

In modern language, the law states the following: *Every point mass attracts every single other point mass by a force pointing along the line intersecting both points*. The force is proportional to the product of the two masses and inversely proportional to the square of the distance between them:

$$F=G\frac{m_1 m_2}{r^2} \tag{5.5.5}$$

where F is the force between the masses, G is the gravitational constant, m_1 is the first mass, m_2 is the second mass and r is the distance between the centers of the masses.

If the bodies in question have spatial extent (rather than being theoretical point masses), then the gravitational force between them is calculated by summing the contributions of the notional point masses which constitute the bodies. In the limit, as the component point masses become "infinitely small", this entails integrating the force (in vector form, see below) over the extents of the two bodies.

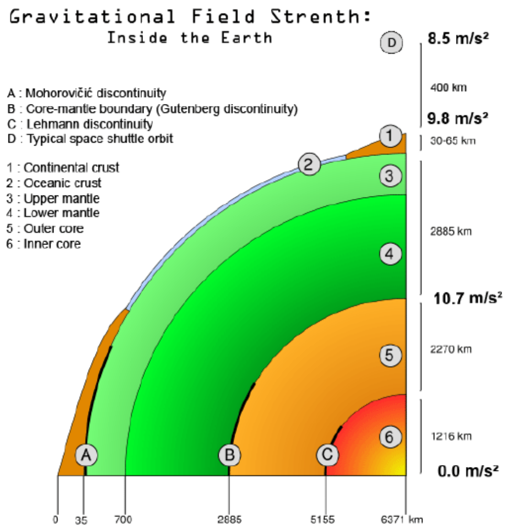
In this way it can be shown that an object with a spherically-symmetric distribution of mass exerts the same gravitational attraction on external bodies as if all the object's mass were concentrated at a point at its center.

For points inside a spherically-symmetric distribution of matter, Newton's Shell theorem can be used to find the gravitational force. The theorem tells us how different parts of the mass distribution affect the gravitational force measured at a point located a distance r_0 from the center of the mass distribution:

1. The portion of the mass that is located at radii $r<r_0$ causes the same force at r_0 as if all of the mass enclosed within a sphere of radius r_0 was concentrated at the center of the mass distribution (as noted above).
2. The portion of the mass that is located at radii $r>r_0$ exerts no net gravitational force at the distance r_0 from the center. That is, the individual gravitational forces exerted by the elements of the sphere out there, on the point at r_0 , cancel each other out.

As a consequence, for example, within a shell of uniform thickness and density there is no net gravitational acceleration anywhere within the hollow sphere. Furthermore, inside a uniform sphere the gravity increases linearly with the distance from the center; the increase due to the additional mass is 1.5 times the decrease due to the larger distance from the center. Thus, if a spherically symmetric body has a uniform core and a uniform mantle with a density that is less than $\frac{2}{3}$ of that of the core, then the gravity initially decreases outwardly beyond the boundary, and if the sphere is large enough, further outward the gravity increases again, and eventually it exceeds the gravity at the core/mantle boundary.

The gravity of the Earth may be highest at the core/mantle boundary, as shown in Figure 1:



Gravitational Field of Earth: Diagram of the gravitational field strength within the Earth.

Key Points

- Sir Isaac Newton's inspiration for the Law of Universal Gravitation was from the dropping of an apple from a tree.
- Newton's insight on the inverse-square property of gravitational force was from intuition about the motion of the earth and the moon.
- The mathematical formula for gravitational force is $\mathrm{F}=\mathrm{G}\frac{\mathrm{Mm}}{\mathrm{r}^2}$ where G is the gravitational constant.
- Since force is a vector quantity, the vector summation of all parts of the shell contribute to the net force, and this net force is the equivalent of one force measurement taken from the sphere's midpoint, or center of mass (COM).
- The gravitational force on an object within a hollow spherical shell is zero.
- The gravitational force on an object within a uniform spherical mass is linearly proportional to its distance from the sphere's center of mass (COM).
- Newton's law of universal gravitation states that every point mass in the universe attracts every other point mass with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.
- The second step in calculating earth's mass came with the development of Newton's law of universal gravitation.
- By equating Newton's second law with his law of universal gravitation, and inputting for the acceleration a the experimentally verified value of 9.8 $\frac{\mathrm{m}}{\mathrm{s}^2}$, the mass of earth is calculated to be $5.96\times 10^{24}\mathrm{kg}$, making the earth's weight calculable given any gravitational field.
- The gravity of the Earth may be highest at the core/mantle boundary

Key Terms

- **induction**: Use inductive reasoning to generalize and interpret results from applying Newton's Law of Gravitation.
- **inverse**: Opposite in effect or nature or order.
- **center of mass**: The center of mass (COM) is the unique point at the center of a distribution of mass in space that has the property that the weighted position vectors relative to this point sum to zero.
- **point mass**: A theoretical point with mass assigned to it.
- **weight**: The force on an object due to the gravitational attraction between it and the Earth (or whatever astronomical object it is primarily influenced by).
- **gravitational force**: A very long-range, but relatively weak fundamental force of attraction that acts between all particles that have mass; believed to be mediated by gravitons.

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5.6: Kepler's Laws

Kepler's First Law

Kepler's first law is: *The orbit of every planet is an ellipse with the Sun at one of the two foci*.

learning objectives

- Apply Kepler's first law to describe planetary motion

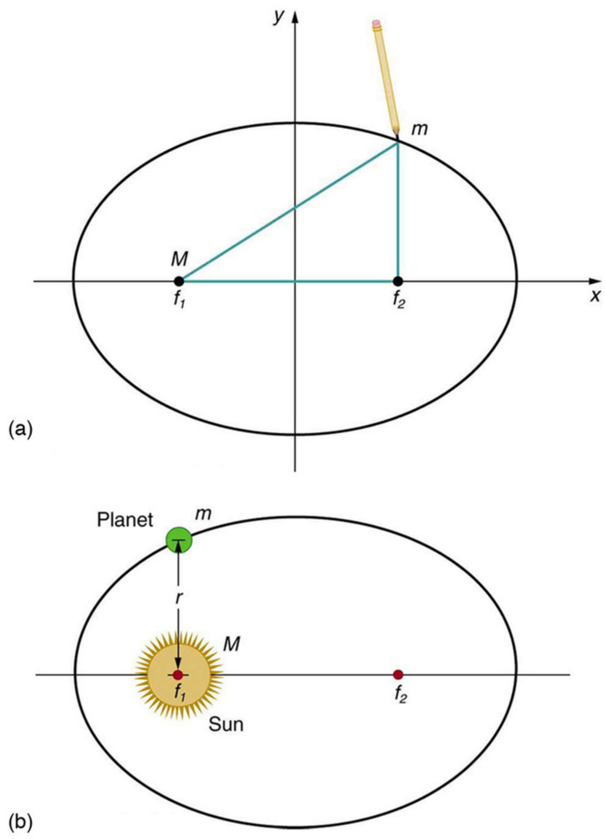
Kepler's First Law

Kepler's first law states that

Definition

The orbit of every planet is an ellipse with the Sun at one of the two foci.

An ellipse is a closed plane curve that resembles a stretched out circle. Note that the Sun is not at the center of the ellipse, but at one of its foci. The other focal point, f_2, has no physical significance for the orbit. The center of an ellipse is the midpoint of the line segment joining its focal points. A circle is a special case of an ellipse where both focal points coincide.



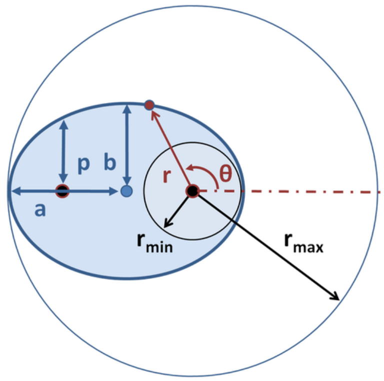
Ellipses and Kepler's First Law: (a) An ellipse is a closed curve such that the sum of the distances from a point on the curve to the two foci (f_1 and f_2) is a constant. You can draw an ellipse as shown by putting a pin at each focus, and then placing a string around a pencil and the pins and tracing a line on paper. A circle is a special case of an ellipse in which the two foci coincide (thus any point on the circle is the same distance from the center). (b) For any closed gravitational orbit, m follows an elliptical path with M at one focus. Kepler's first law states this fact for planets orbiting the Sun.

How stretched out an ellipse is from a perfect circle is known as its eccentricity: a parameter that can take any value greater than or equal to 0 (a circle) and less than 1 (as the eccentricity tends to 1, the ellipse tends to a parabola). The eccentricities of the planets known to Kepler varied from 0.007 (Venus) to 0.2 (Mercury). Minor bodies such as comets an asteroids (discovered after Kepler's time) can have very large eccentricities. The dwarf planet Pluto, discovered in 1929, has an eccentricity of 0.25.

Symbolically, an ellipse can be represented in polar coordinates as:

$$r=\frac{p}{1+\epsilon\cos\theta} \tag{5.6.1}$$

where (r,θ) are the polar coordinates (from the focus) for the ellipse, p is the semi-latus rectum, and ϵ is the eccentricity of the ellipse. For a planet orbiting the Sun, r is the distance from the Sun to the planet and θ is the angle between the planet's current position and its closest approach, with the Sun as the vertex.



Orbit As Ellipse: Heliocentric coordinate system (r,θ) for ellipse. Also shown are: semi-major axis a, semi-minor axis b and semi-latus rectum p; center of ellipse and its two foci marked by large dots. For $\theta=0^\circ$, $r=r_{\min}$ and for $\theta=180^\circ$, $r=r_{\max}$.

At $\theta=0^\circ$, perihelion, the distance is minimum

$$r_{\min}=\frac{p}{1+\epsilon}. \tag{5.6.2}$$

At $\theta=90^\circ$ and at $\theta=270^\circ$, the distance is p.

At $\theta=180^\circ$, aphelion, the distance is maximum

$$r_{\max}=\frac{p}{1-\epsilon}. \tag{5.6.3}$$

The semi-major axis a is the arithmetic mean between $r_{\min}$ and $r_{\max}$:

$$r_{\max}-a=a-r_{\min} \tag{5.6.4}$$

$$a=\frac{p}{1-\epsilon^2}. \tag{5.6.5}$$

The semi-minor axis b is the geometric mean between $r_{\min}$ and $r_{\max}$:

$$\frac{r_{\max}}{b}=\frac{b}{r_{\min}} \tag{5.6.6}$$

$$b=\frac{p}{\sqrt{1-\epsilon^2}} \tag{5.6.7}$$

The semi-latus rectum p is the harmonic mean between $r_{\min}$ and $r_{\max}$:

$$\frac{1}{r_{\min}}-\frac{1}{p}=\frac{1}{p}-\frac{1}{r_{\max}} \tag{5.6.8}$$

$$pa=r_{\max}\cdot r_{\min}=b^2. \tag{5.6.9}$$

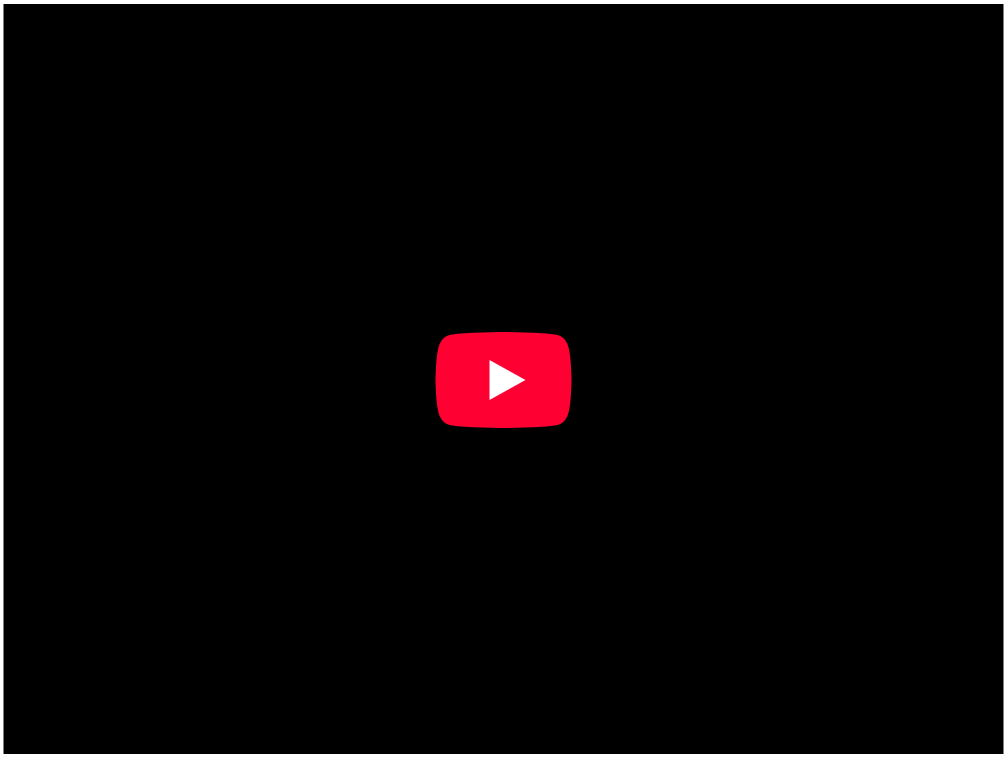
The eccentricity ϵ is the coefficient of variation between $r_{\min}$ and $r_{\max}$:

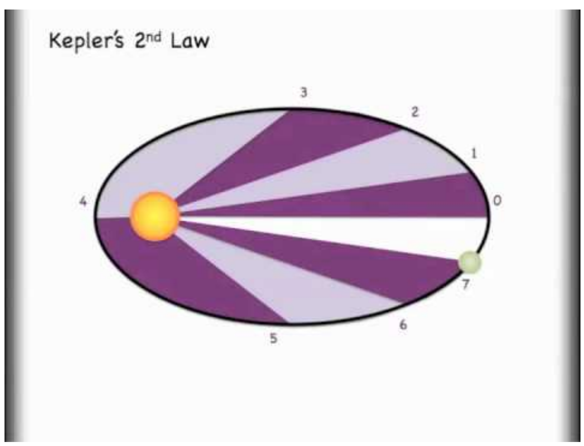
$$\epsilon=\frac{r_{\max}-r_{\min}}{r_{\max}+r_{\min}} \tag{5.6.10}$$

The area of the ellipse is

$$A = \pi ab \tag{5.6.11}$$

The special case of a circle is $\epsilon = 0$, resulting in $r = p = r_{min} = r_{max} = a = b$ and $A = \pi r^2$. The orbits of planets with very small eccentricities can be approximated as circles.





Understanding Kepler's 3 Laws and Orbits: In this video you will be introduced to Kepler's 3 laws and see how they are relevant to orbiting objects.

Kepler's Second Law

Kepler's second law states: A line joining a planet and the Sun sweeps out equal areas during equal intervals of time.

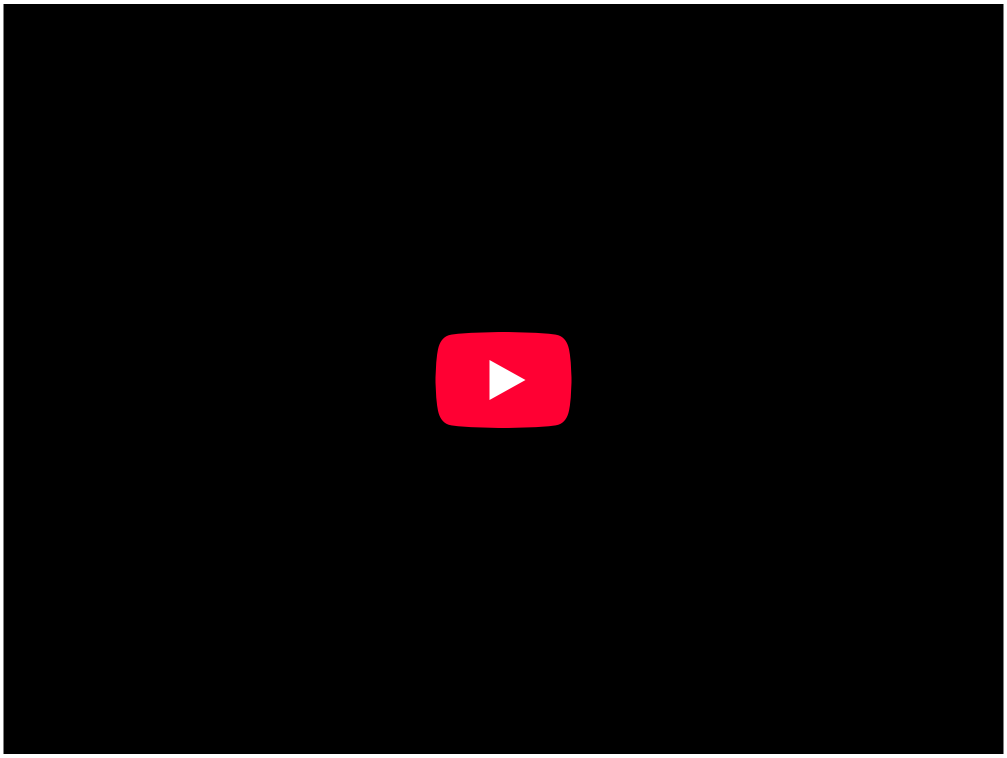
learning objectives

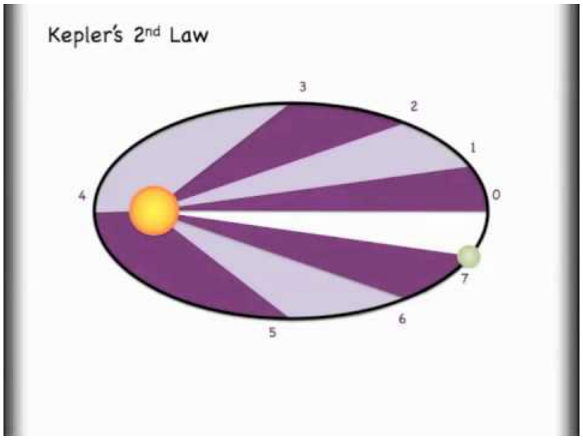
- Apply Kepler's second law to describe planetary motion

Kepler's second law states:

Definition

A line joining a planet and the Sun sweeps out equal areas during equal intervals of time.





Understanding Kepler's 3 Laws and Orbits: In this video you will be introduced to Kepler's 3 laws and see how they are relevant to orbiting objects.

In a small time the planet sweeps out a small triangle having base line and height. The area of this triangle is given by:

$$\mathrm{dA} = \frac{1}{2} \cdot \mathrm{r} \cdot \mathrm{r} d\theta \tag{5.6.12}$$

and so the constant areal velocity is:

$$\frac{\mathrm{dA}}{\mathrm{dt}} = \frac{1}{2} \mathrm{r}^2 \frac{\mathrm{d}\theta}{\mathrm{dt}} \tag{5.6.13}$$

Now as the first law states that the planet follows an ellipse, the planet is at different distances from the Sun at different parts in its orbit. So the planet has to move faster when it is closer to the Sun so that it sweeps equal areas in equal times.

The total area enclosed by the elliptical orbit is:

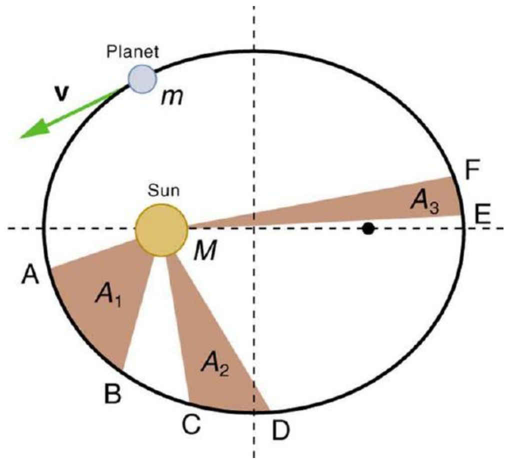
$$A = \pi ab \tag{5.6.14}$$

Therefore the period P satisfies:

$$\pi ab = P \cdot \frac{1}{2} r^2 \dot{\theta} \text{ or } r^2 \dot{\theta} = nab \tag{5.6.15}$$

Where $\dot{\theta} = \frac{d\theta}{dt}$ is the angular velocity, (using Newton notation for differentiation), and $n = \frac{2\pi}{P}$ is the mean motion of the planet around the Sun.

See below for an illustration of this effect. The planet traverses the distance between A and B, C and D, and E and F in equal times. When the planet is close to the Sun it has a larger velocity, making the base of the triangle larger, but the height of the triangle smaller, than when the planet is far from the Sun. One can see that the planet will travel fastest at perihelion and slowest at aphelion.



Kepler's Second Law: The shaded regions have equal areas. It takes equal times for m to go from A to B, from C to D, and from E to F. The mass m moves fastest when it is closest to M. Kepler's second law was originally devised for planets orbiting the Sun, but it has broader validity.

Kepler's Third Law

Kepler's third law states that *the square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit*.

learning objectives

- Apply Kepler's third law to describe planetary motion

Kepler's Third Law

Kepler's third law states:

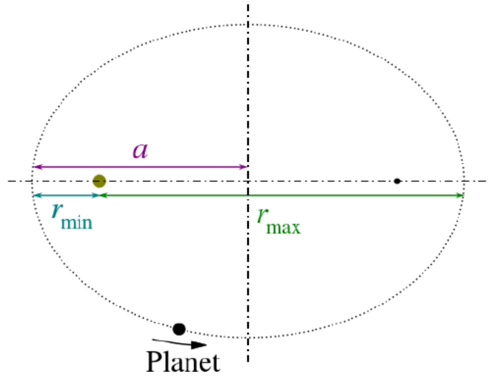
Definition

The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit.

The third law, published by Kepler in 1619, captures the relationship between the distance of planets from the Sun, and their orbital periods. Symbolically, the law can be expressed as

$$P^2 \propto a^3, \tag{5.6.16}$$

where P is the orbital period of the planet and a is the semi-major axis of the orbit (see).



Kepler's Third Law: Kepler's third law states that the square of the period of the orbit of a planet about the Sun is proportional to the cube of the semi-major axis of the orbit.

The constant of proportionality is

$$\frac{P^2_{planet}}{a^3_{planet}} = \frac{P^2_{earth}}{a^3_{earth}} = 1\frac{yr^2}{AU^3} \tag{5.6.17}$$

for a sidereal year (yr), and astronomical unit (AU).

Kepler enunciated this third law in a laborious attempt to determine what he viewed as the "music of the spheres" according to precise laws, and express it in terms of musical notation. Therefore, it used to be known as the harmonic law.

Derivation of Kepler's Third Law

We can derive Kepler's third law by starting with Newton's laws of motion and the universal law of gravitation. We can therefore demonstrate that the force of gravity is the cause of Kepler's laws.

Consider a circular orbit of a small mass m around a large mass M. Gravity supplies the centripetal force to mass m. Starting with Newton's second law applied to circular motion,

$$F_{net} = ma_c = m\frac{v^2}{r}. \tag{5.6.18}$$

The net external force on mass m is gravity, and so we substitute the force of gravity for F_{net}:

$$G\frac{mM}{r^2} = m\frac{v^2}{r} \tag{5.6.19}$$

The mass m cancels, as well as an r, yielding

$$G\frac{M}{r} = v^2 \tag{5.6.20}$$

The fact that m cancels out is another aspect of the oft-noted fact that at a given location all masses fall with the same acceleration. Here we see that at a given orbital radius r, all masses orbit at the same speed. This was implied by the result of the preceding worked example. Now, to get at Kepler's third law, we must get the period P into the equation. By definition, period P is the time for one complete orbit. Now the average speed v is the circumference divided by the period—that is,

$$v = \frac{2\pi r}{P}. \tag{5.6.21}$$

Substituting this into the previous equation gives

$$G\frac{M}{r} = \frac{4\pi^2 r^2}{P^2} \tag{5.6.22}$$

Solving for P^2 yields

$$P^2 = \frac{4\pi^2 r^3}{GM}. \tag{5.6.23}$$

Using subscripts 1 and 2 to denote two different satellites, and taking the ratio of the last equation for satellite 1 to satellite 2 yields

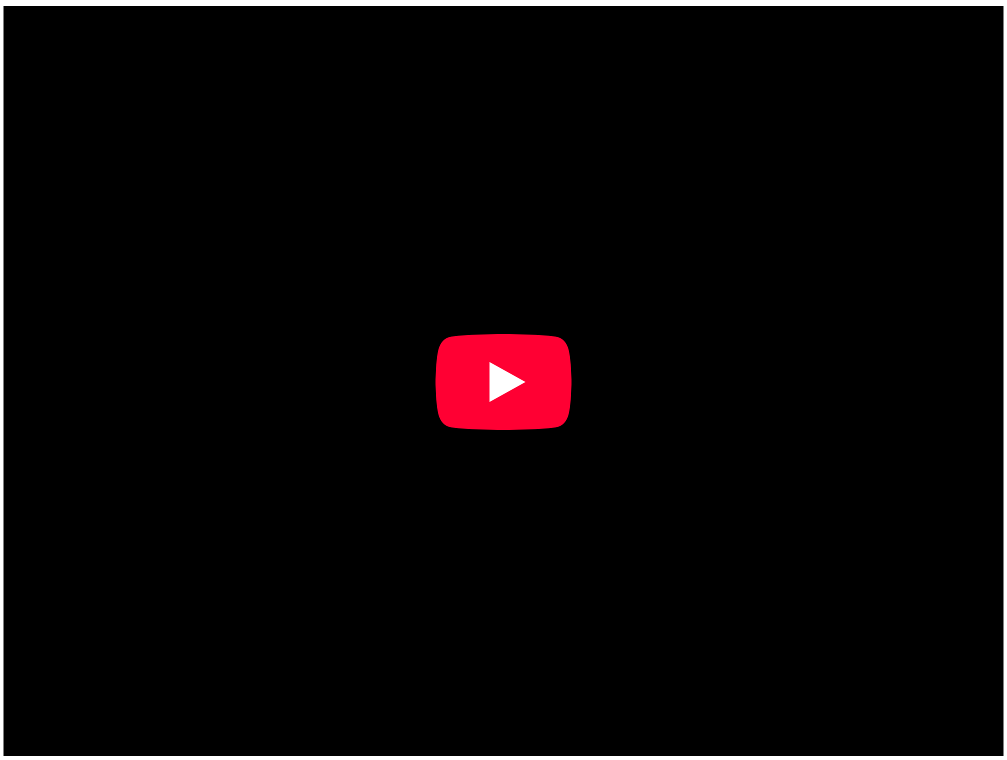
$$\frac{P_1^2}{P_2^2} = \frac{r_1^3}{r_2^3} \tag{5.6.24}$$

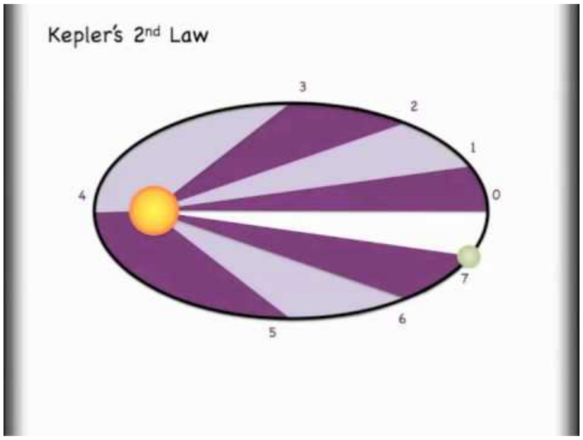
This is Kepler's third law. Note that Kepler's third law is valid only for comparing satellites of the same parent body, because only then does the mass of the parent body M cancel.

Now consider what one would get when solving $P^2 = \frac{4\pi^2 GM}{r^3}$ for the ratio $\frac{r^3}{P^2}$. We obtain a relationship that can be used to determine the mass M of a parent body from the orbits of its satellites:

$$M = \frac{4\pi^2 r^3}{GP^2} \tag{5.6.25}$$

If r and P are known for a satellite, then the mass M of the parent can be calculated. This principle has been used extensively to find the masses of heavenly bodies that have satellites. Furthermore, the ratio $\frac{r^3}{T^2}$ should be a constant for all satellites of the same parent body (because $\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$).





Understanding Kepler's 3 Laws and Orbits: In this video you will be introduced to Kepler's 3 laws and see how they are relevant to orbiting objects.

Orbital Maneuvers

An orbital maneuver is the use of propulsion systems to change the orbit of a spacecraft (the rest of the flight is called "coasting").

learning objectives

- Explain purpose of an orbital maneuver

Orbital Maneuvers

In spaceflight, an orbital maneuver is the use of propulsion systems to change the orbit of a spacecraft. The rest of the flight, especially in a transfer orbit, is called coasting.

Rocket Equation

The Tsiolkovsky rocket equation or *ideal rocket equation* is an equation useful for considering vehicles that follow the basic principle of a rocket: a device that can apply acceleration to itself (a thrust) by expelling part of its mass with high speed and moving due to the conservation of momentum. Specifically, it is a mathematical equation relating the delta-v with the effective exhaust velocity and both the initial and final mass of a rocket (or other reaction engine).

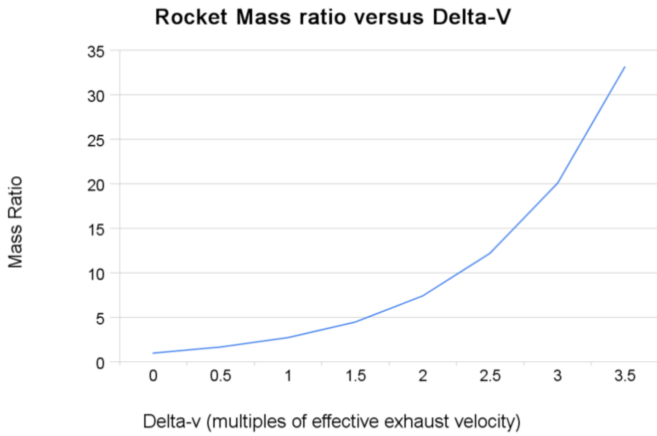
For any such maneuver (or journey involving a number of such maneuvers):

$$\Delta \mathrm{v} = \mathrm{v_e} \ln(\frac{\mathrm{m_0}}{\mathrm{m_1}}), \tag{5.6.26}$$

where:

- m_0 is the initial total mass, including propellant;
- m_1 is the final total mass;
- v_e is the effective exhaust velocity ($v_e = I_{sp} \cdot g_0$ where I_{sp} is the specific impulse expressed as a time period and g_0 is the gravitational constant); and
- Δv is delta-v the maximum change of speed of the vehicle (with no external forces acting).

See for an illustration plotting the relationship between final velocity and rocket mass ratios (according to the rocket equation).



Rocket Equation: Rocket mass ratios versus final velocity calculated from the rocket equation

Delta-v Budget:

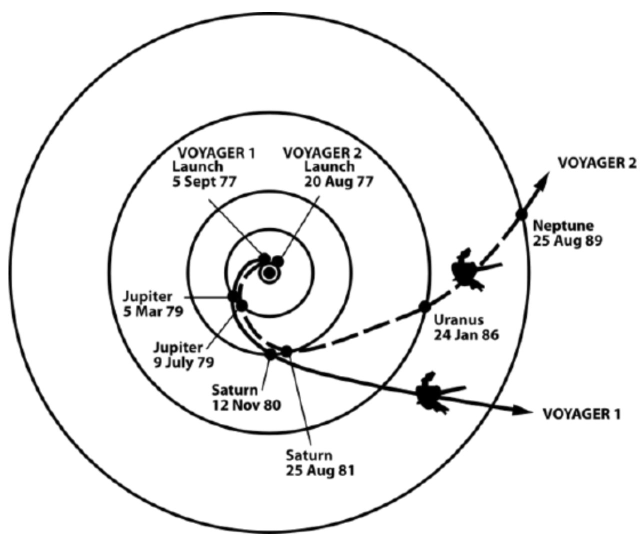
The total delta-v for each maneuver estimated for a mission is called a *delta-v budget*. With a good approximation of the delta-v budget, designers can estimate the fuel to payload requirements of the spacecraft using the rocket equation.

Oberth Effect and Gravitational Assist

In astronautics, the Oberth effect occurs when the use of a rocket engine travelling at high speed generates much more useful energy than one at low speed. This effect is the result of propellant having more usable energy (due to its kinetic energy on top of its chemical potential energy). The vehicle is able to employ this kinetic energy to generate more mechanical power.

Oberth effect is used in a powered flyby or Oberth maneuver in which the application of an impulse (typically from the use of a rocket engine) close to a gravitational body (where the gravity potential is low and the speed is high) allows for more change in kinetic energy and final speed (i.e. higher specific energy) than the same impulse applied further from the body for the same initial orbit.

In orbital mechanics, a gravitational slingshot (or gravity assist maneuver) is the use of the relative movement and gravity of a planet or other celestial body to alter the path and speed of a spacecraft, typically in an effort to save propellant, time, and expense. Gravity assistance can be used to accelerate, decelerate and/or re-direct the path of a spacecraft. This technique was used by the Voyager probes in their fly-bys of Jupiter and Saturn (see).

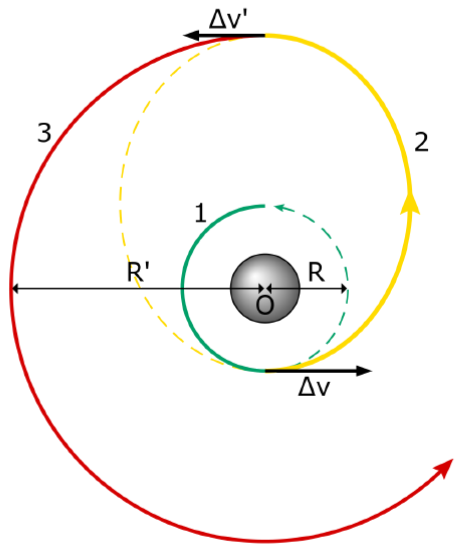


Voyager Path Using Gravity Assists: The trajectories that enabled NASA's twin Voyager spacecraft to tour the four gas giant planets and achieve velocity to escape our solar system

Transfer Orbits

Orbit insertion is a general term used for a maneuver when it is more than a small correction. It may be used in a maneuver to change a transfer orbit or an ascent orbit into a stable one, but also to change a stable orbit into a descent (i.e., descent orbit insertion). Also, the term *orbit injection* is used, especially for changing a stable orbit into a transfer orbit—e.g., trans-lunar injection (TLI), trans-Mars injection (TMI) and trans-Earth injection (TEI).

The Hohmann transfer orbit is an elliptical orbit used to transfer between two circular orbits of different altitudes in the same plane. The orbital maneuver to perform the Hohmann transfer uses two engine impulses that move aspacecraft onto and off the transfer orbit, as diagramed in. Hohmann transfer orbits are the most efficient with fuel. Other non-Hohmann types of transfer orbits that are less efficient with fuel exist, but these may be more efficient with other resources (such as time).



Hohmann Transfer Orbit: A diagram of the Hohmann Transfer Orbit.

Orbital inclination change is an orbital maneuver aimed at changing an orbiting body's orbit inclination (this maneuver is also known as an orbital plane change as the plane of the orbit is tipped). The maneuver requires a change in the orbital velocity vector (delta-v) at the orbital nodes (i.e., the point at which the initial and desired orbits intersect: the line of orbital nodes is defined by the intersection of the two orbital planes).

Satellites

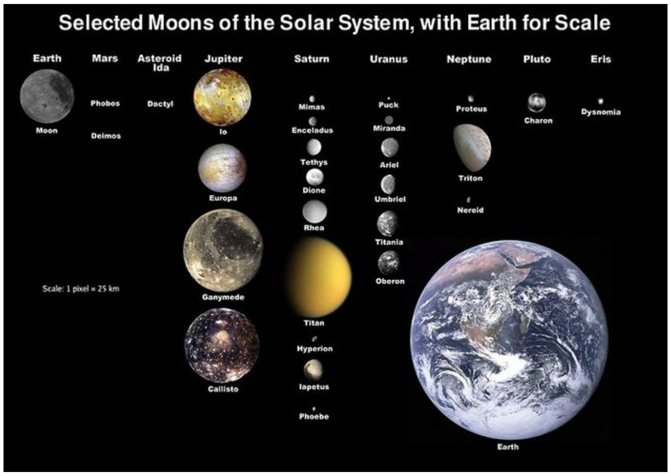
Natural satellites are celestial objects that orbit a larger body; artificial satellites are manmade objects put in the orbit of the Earth.

learning objectives

- Define the concept of a satellite, in the broadest possible terms

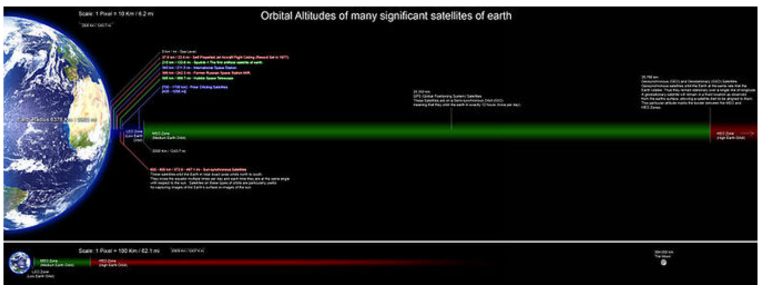
Satellites

The word "satellite" has a somewhat ambiguous definition. The broadest possible definition of a satellite is an object that orbits a larger one due to the force of gravity. Natural satellites, often called moons (see), are celestial bodies that orbit a larger body call a *primary* (often planet, though there are binary asteroids, too). It is technically correct to refer to a planet as a "satellite" of its parent star, though this is not common.



Moons of the Solar System: Nineteen natural satellites are large enough to be round, and one, Saturn's Titan, has a substantial atmosphere.

Artificial satellites (see) are man made objects put in orbit about the Earth or another planet in the Solar System. All satellites follow the laws of orbital mechanics, which can almost always be approximated with Newtonian physics.



Orbital Altitudes: Orbital Altitudes of several significant satellites of earth.

Natural satellites are often classified in terms of their size and composition, while artificial satellites are categorized in terms of their orbital parameters.

Natural Satellites

Formally classified natural satellites, or moons, include 176 planetary satellites orbiting six of the eight planets, and eight orbiting three of the five IAU-listed dwarf planets. As of January 2012, over 200 minor-planet moons have been discovered. There are 76 objects in the asteroid belt with satellites (five with two satellites each), four Jupiter trojans, 39 near-Earth objects, and 14 Mars-crossers. There are also 84 known natural satellites of trans-Neptunian objects. Planets around other stars are likely to have natural satellites as well, although none have yet been observed.

Of the inner planets, Mercury and Venus have no natural satellites; Earth has one large natural satellite, known as the Moon; and Mars has two tiny natural satellites, Phobos and Deimos. The large gas giants have extensive systems of natural satellites, including half a dozen comparable in size to Earth's Moon: the four Galilean moons, Saturn's Titan, and Neptune's Triton. Saturn has an additional six mid-sized natural satellites massive enough to have achieved hydrostatic equilibrium, and Uranus has five. It has been suggested that some satellites may potentially harbor life, though there is currently no direct evidence.

The Earth–Moon system is unique in that the ratio of the mass of the Moon to the mass of the Earth is much greater than that of any other natural satellite to planet ratio in the Solar System. Additionally the Moon's orbit with respect to the Sun is always concave.

The seven largest natural satellites in the Solar System (those bigger than 2,500 km across) are Jupiter's Galilean moons (Ganymede, Callisto, Io, and Europa), Saturn's moon Titan, Earth's moon, and Neptune's captured natural satellite Triton.

Artificial Satellites

The first satellite, Sputnik 1, was put into orbit around Earth and was therefore in geocentric orbit. By far this is the most common type of orbit with about 2,500 artificial satellites orbiting the Earth. Geocentric orbits may be further classified by their altitude, inclination and eccentricity.

The commonly used altitude classifications are Low Earth orbit (LEO), Medium Earth orbit (MEO) and High Earth orbit (HEO). Low Earth orbit is any orbit below 2000 km, and Medium Earth orbit is any orbit higher than that but still below the altitude for geosynchronous orbit at 35,786 km. High Earth orbit is any orbit higher than the altitude for geosynchronous orbit.

Altitude classifications

- Low Earth orbit (LEO): Geocentric orbits ranging in altitude from 0–2000 km (0–1240 miles)
- Medium Earth orbit (MEO): Geocentric orbits ranging in altitude from 2,000 km (1,200 mi) to just below geosynchronous orbit at 35,786 km (22,236 mi). Also known as an intermediate circular orbit.
- High Earth orbit (HEO): Geocentric orbits above the altitude of geosynchronous orbit 35,786 km (22,236 mi).

Inclination Classifications

- Inclined orbit: An orbit whose inclination in reference to the equatorial plane is not zero degrees.
- Polar orbit: An orbit that passes above or nearly above both poles of the planet on each revolution. Therefore it has an inclination of (or very close to) 90 degrees.
- Polar sun synchronous orbit: A nearly polar orbit that passes the equator at the same local time on every pass. Useful for image taking satellites because shadows will be nearly the same on every pass.

Eccentricity Classifications

- Circular orbit: An orbit that has an eccentricity of 0 and whose path traces a circle.
- Hohmann transfer orbit: An orbital maneuver that moves a spacecraft from one circular orbit to another using two engine impulses.
- Elliptic orbit: An orbit with an eccentricity greater than 0 and less than 1 whose orbit traces the path of an ellipse.
- Geosynchronous transfer orbit: An elliptic orbit where the perigee is at the altitude of a Low Earth orbit (LEO) and the apogee at the altitude of a geosynchronous orbit.
- Geostationary transfer orbit: An elliptic orbit where the perigee is at the altitude of a Low Earth orbit (LEO) and the apogee at the altitude of a geostationary orbit.

Key Points

- An ellipse is a closed plane curve that resembles a stretched out circle (The Sun is at one focus while the other focus has no physical significance. A circle is a special case of an ellipse where both focal points coincide.
- How stretched out an ellipse is from a perfect circle is known as its eccentricity: a parameter that can take any value greater than or equal to 0 (a circle) and less than 1 (as the eccentricity tends to 1, the ellipse tends to a parabola).
- Symbolically, an ellipse can be represented in polar coordinates as: $\mathrm{r}=\frac{\mathrm{p}}{1+\epsilon \cos \theta}$, where (r, θ) are the polar coordinates (from the focus) for the ellipse, p is the semi-latus rectum, and ϵ is the eccentricity of the ellipse.
- Perihelion is minimum distance from the Sun a planet achieves in its orbit and is given by$\mathrm{r}_{\max }=\frac{\mathrm{p}}{1-\epsilon}$. Aphelion is the largest distance from the Sun a planet reaches in his orbit and is given by $\mathrm{r}_{\max }=\dfrac{\mathrm{p}}{1-\epsilon}$.
- In a small time the planet sweeps out a small triangle having base line and height. The area of this triangle is given by $\mathrm{dA}=\frac{1}{2} \cdot \mathrm{r} \cdot \mathrm{r} d \theta$ and so the constant areal velocity is: $\frac{\mathrm{dA}}{\mathrm{dt}}=\dfrac{1}{2} \mathrm{r}^{2} \dfrac{\mathrm{d} \theta}{\mathrm{dt}}$
- The period P satisfies: $\pi \mathrm{ab}=\mathrm{P} \cdot \frac{1}{2} \mathrm{r}^{2} \dot{\theta}$. One can see that the product of r^{2} and must be constant, so that when the planet is further from the Sun it travels at a slower rate and vise versa.
- A planet travels fastest at perihelion and slowest at aphelion.
- Kepler's third law can be represented symbolically as $\mathrm{P}^{2} \propto \mathrm{a}^{3}$, where P is the orbital period of the planet and a is the semi-major axis of the orbit (see.
- The constant of proportionality is $\dfrac{\mathrm{P}_{\text {planet }}^{2}}{\mathrm{a}_{\text {planet }}^{3}}=\frac{\mathrm{P}_{\text {earth }}^{2}}{\mathrm{a}_{\text {earth }}^{3}}=1 \frac{\mathrm{yr}^{2}}{\mathrm{AU}^{3}}$ for a sidereal year (yr), and astronomical unit (AU).
- Kepler's third law can be derived from Newton's laws of motion and the universal law of gravitation. Set the force of gravity equal to the centripetal force. After substituting an expression for the velocity of the planet, one can obtain: $\mathrm{G} \frac{\mathrm{M}}{\mathrm{r}}=\frac{4 \pi^{2}}{\mathrm{P}^{2}}$ which can also be written $\mathrm{P}^{2}=\frac{4 \pi^{2} \mathrm{a}^{3}}{\mathrm{GM}}$.
- Using the expression above we can obtain the mass of the parent body from the orbits of its satellites: $\mathrm{M}=\frac{4 \pi^{2} \mathrm{r}^{3}}{\mathrm{GP}^{2}}$

- The ideal rocket equation related the maximum change in velocity attainable by a rocket (delta-v or Δv) as a function of the exhaust velocity (v_e) and the ratio between the mass of the rocket with and without the propellant (m_0/m_1). The equation is given by $\Delta v = v_e \ln(\frac{m_0}{m_1})$.
- The Oberth effect: where the use of a rocket engine travelling at high speed generates more useful energy than one at low speed. Thus it is more efficient to apply thrust when the spacecraft is nearest to the planet (periastron).
- A gravity assist maneuver is the use of the relative movement and gravity of a planet (or other celestial body) to alter the velocity of a spacecraft—typically in order to save propellant, time, and expense. This technique was employed by the Voyager probes (see.
- The Hohmann transfer orbit is an elliptical orbit used to transfer between two circular orbits of different altitudes, in the same plane. The orbital maneuver to perform the Hohmann transfer uses two engine impulses which move aspacecraft onto and off the transfer orbit. See.
- The broadest possible definition of a satellite is an object that orbits a larger one due to the force of gravity.
- All satellites follow the laws of orbital mechanics, which can almost always be approximated with Newtonian physics.
- Natural satellites are often classified in terms of their size and composition, while artificial satellites are categorized in terms of their orbital parameters.
- Artificial Earth-orbiting satellites have orbits categorized by their altitudes, inclinations, and eccentricities.

Key Terms

- **eccentricity**: The coefficient of variation between r_{min} and r_{max}: $\epsilon = \frac{r_{max} - r_{min}}{r_{max} + r_{min}}$ The further appart the foci are, the stronger the eccentricity.
- **perihelion**: The point in the elliptical orbit of a planet or comet etc. where it is nearest to the Sun. The point farthest from the Sun is called aphelion.
- **semi-latus rectum**: The latus rectum is a chord perpendicular to the major axis and passing through the focus. The semi-latus rectum is half the latus rectrum. See distance p in.
- **angular velocity**: A vector quantity describing an object in circular motion; its magnitude is equal to the speed of the particle and the direction is perpendicular to the plane of its circular motion.
- **mean motion**: An angle of 2π (radians) divided by the orbital period (of a celestial body in an elliptic orbit).
- **astronomical unit**: The mean distance from the Earth to the Sun (the semi-major axis of Earth’s orbit), approximately 149,600,000 kilometres (symbol AU), used to measure distances in the solar system.
- **sidereal year**: The orbital period of the Earth; a measure of the time it takes for the Sun to return to the same position with respect to the stars of the celestial sphere. A sidereal year is about 20.4 minutes longer than the tropical year due to precession of the equinoxes.
- **Hohmann transfer orbit**: The Hohmann transfer orbit is an elliptical orbit used to transfer between two circular orbits of different altitudes, in the same plane. The orbital maneuver to perform the Hohmann transfer uses two engine impulses, one to move a spacecraft onto the transfer orbit and a second to move off it.
- **delta-v**: The maximum change in the scalar speed of a rocket if the rocket were operated in a vacuum away from external forces (i.e., if no other external forces act).
- **natural satellite**: A natural satellite, moon, or secondary planet is a celestial body that orbits a planet or smaller body, which is called its primary.
- **artificial satellite**: In the context of spaceflight, a satellite is an object which has been placed into orbit by human endeavour.

5.7: Gravitational Potential Energy

Defining Graviational Potential Energy

Gravitational energy is the potential energy associated with gravitational force, such as elevating objects against the Earth's gravity.

learning objectives

- Express gravitational potential energy for two masses

Gravitational energy is the potential energy associated with gravitational force, as work is required to elevate objects against Earth's gravity. The potential energy due to elevated positions is called gravitational potential energy, and is evidenced by water in an elevated reservoir or kept behind a dam. If an object falls from one point to another point inside a gravitational field, the force of gravity will do positive work on the object, and the gravitational potential energy will decrease by the same amount.

Consider a book placed on top of a table. As the book is raised from the floor to the table, some external force works against the gravitational force. If the book falls back to the floor, the "falling" energy the book receives is provided by the gravitational force. Thus, if the book falls off the table, this potential energy goes to accelerate the mass of the book and is converted into kinetic energy. When the book hits the floor, this kinetic energy is converted into heat and sound by the impact.

The factors that affect an object's gravitational potential energy are its height relative to some reference point, its mass, and the strength of the gravitational field it is in. Thus, a book lying on a table has less gravitational potential energy than the same book on top of a taller cupboard, and less gravitational potential energy than a heavier book lying on the same table. An object at a certain height above the Moon's surface has less gravitational potential energy than at the same height above the Earth's surface because the Moon's gravity is weaker. Note that "height" in the common sense of the term cannot be used for gravitational potential energy calculations when gravity is not assumed to be a constant. The following sections provide more detail.

Local Approximation

The strength of a gravitational field varies with location. However, when the change of distance is small in relation to the distances from the center of the source of the gravitational field, this variation in field strength is negligible and we can assume that the force of gravity on a particular object is constant. Near the surface of the Earth, for example, we assume that the acceleration due to gravity is a constant $\mathrm{g = 9.8 m/s^2}$ ("standard gravity"). In this case, a simple expression for gravitational potential energy can be derived using the $\mathrm{W = Fd}$ equation for work. The upward force required while moving at a constant velocity is equal to the weight, mg, of an object, so the work done in lifting it through a height h is the product mgh. Thus, when accounting only for mass, gravity, and altitude, the equation is:

$$\mathrm{U = mgh} \tag{5.7.1}$$

where U is the potential energy of the object relative to its being on the Earth's surface, m is the mass of the object, g is the acceleration due to gravity, and h is the altitude of the object. If m is expressed in kilograms, g in $\mathrm{m/s^2}$ and h in meters then U will be calculated in joules. In most situations, the change in potential energy is the relevant quantity:

$$\mathrm{\Delta U = mg\Delta h} \tag{5.7.2}$$

General Formula

However, over large variations in distance, the approximation that gg is constant is no longer valid, and we have to use calculus and the general mathematical definition of work to determine gravitational potential energy. For the computation of the potential energy, we can integrate the gravitational force, whose magnitude is given by Newton's law of gravitation, with respect to the distance r between the two bodies. Using that definition, the gravitational potential energy of a system of masses $\mathrm{m_1}$ and $\mathrm{M_2}$ at a distance r using gravitational constant G is

$$\mathrm{U = -G\frac{m_1M_2}{r} + K} \tag{5.7.3}$$

where K is the constant of integration. Choosing the convention that $\mathrm{K = 0}$ makes calculations simpler, albeit at the cost of making U negative. Note that in this case the potential energy becomes zero when r is infinite, and approaches negative infinity as r goes to zero.

Trebuchet: A trebuchet uses the gravitational potential energy of the counterweight to throw projectiles over long distances.

Key Points

- If an object falls from one point to another point inside a gravitational field, the force of gravity will do positive work on the object, and the gravitational potential energy will decrease by the same amount.
- Near the surface of the Earth, the work done in lifting an object through a height h is the product mgh, so $U = mgh$.
- The gravitational potential energy, U, of a system of masses m_1 and M_2 at a distance r using gravitational constant G is $U = -G\frac{m_1 M_2}{r} + K$.

Key Terms

- **Newton's law of gravitation**: This law states that every point mass in the universe attracts every other point mass with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.
- **potential energy**: The energy an object has because of its position (in a gravitational or electric field) or its condition (as a stretched or compressed spring, as a chemical reactant, or by having rest mass)
- **gravity**: Resultant force on Earth's surface, of the attraction by the Earth's masses, and the centrifugal pseudo-force caused by the Earth's rotation.

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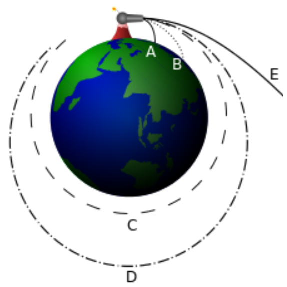
5.8: Energy Conservation

learning objectives

- Calculate the escape speed of an object given its kinetic energy and the gravitational potential energy

Escape speed is the required starting speed required by an object to go from a starting point in a gravitational potential field to an ending point that is infinitely far away. It is assumed that the velocity of the object at the ending point will be zero.



Isaac Newton's Analysis of Escape Speed: In this figure, Objects A and B don't have the required escape speed and so they fall back to Earth after launch. Objects C and D don't either, they achieve a circular and an elliptical orbit respectively. Object E is launched with sufficient escape velocity and escapes the Earth.

Imagine a situation in which a spaceship that does not have a propulsion system is launched straight away from a planet. (It is moot to discuss escape speed for objects with propulsion systems.) Let us assume that the only significant force that is acting on the spaceship is the force of gravity from the planet. The escape speed of the spaceship can calculated through a simple analysis of conservation of energy. The gravitational potential energy of the spaceship is:

$$U = -\frac{GMm}{r} \tag{5.8.1}$$

Where G is the universal gravitational constant ($G = 6.67 \cdot 10^{-11} m^3 kg^{-1} s^{-2}$), M is the mass of the planet, m is the mass of the spaceship, and r is the distance of the spaceship from the planet's center of gravity.

At the ending point of the spaceship, r goes to infinity. As r goes to infinity, the value of the gravitational potential energy expression goes to 0.

The kinetic energy of the spaceship can be found from:

$$\frac{1}{2}mv^2 \tag{5.8.2}$$

Where mm is the mass of the spaceship and v is the velocity of the spaceship.

At the starting point of the spaceship, the velocity must have a magnitude equal to the escape speed (s_e). The velocity of the spaceship is 0 at its ending point, and so consequently its kinetic energy is 0 in the end as well.

Summarizing the kinetic energy (K) and potential energy (U) of the spaceship at it's initial (i) and final (f) states:

$$(K+U)_i = \frac{1}{2}ms_e^2 + \frac{-GMm}{r} \tag{5.8.3}$$

$$(K+U)_f = 0 + 0 \tag{5.8.4}$$

Due to conservation of energy, the initial energy must equal the final energy and so we can solve for s_e:

$$s_e = \sqrt{\frac{2GM}{r}} \tag{5.8.5}$$

Interestingly, if the spaceship were to fall to the planet from a point infinitely far away it would obtain a final speed of s_e at the planet.

It should be noted that if an object is launched from a rotating body, such as the Earth, the speed at which the body rotates will affect the required velocity that an object must have relative to the surface of the body. If a rocket is launched tangentially from the Earth's equator in the same direction that the Earth is turning, it will require a lower velocity relative to the Earth than if it were launched in the opposite direction to meet escape speed requirements.

Additionally, it is a misconception that powered vehicles (such as rockets) require escape speed to leave orbit and travel through outer-space. If the vehicle has a propulsion system to provide it with energy once it has left the surface of the planet, it is not necessary to initially meet escape speed requirements.

Key Points

- It is assumed that the velocity of the object at the ending point will be zero.
- The requisite escape speed (se)(se) of an object to escape a spherically symmetric body is given by: $\mathrm{s_e} = \sqrt{\frac{2\mathrm{GM}}{\mathrm{r}}}$, where G is the universal gravitational constant, M is the mass of the body, and r is the distance of the object from the body's center of gravity.
- Escape speed is the required speed that an object has to have to go from a starting point in a gravitational potential field to an ending point that is infinitely far away.
- The speed at which a body rotates will affect the required velocity that an object must have relative to the surface of the body.
- Objects that have propulsion systems do not need to reach escape velocity.

Key Terms

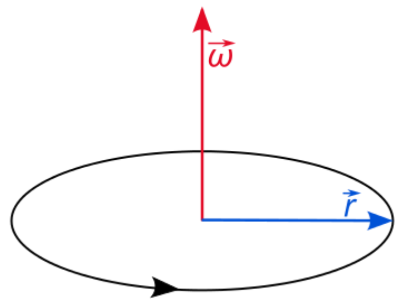
- **propulsion**: Force causing movement.
- **potential energy**: The energy an object has because of its position (in a gravitational or electric field) or its condition (as a stretched or compressed spring, as a chemical reactant, or by having rest mass)
- **kinetic energy**: The energy possessed by an object because of its motion, equal to one half the mass of the body times the square of its velocity.

5.9: Angular vs. Linear Quantities

learning objectives

- Describe properties characteristic to angular velocity and angular momentum

Linear motion is motion in a straight line. This type of motion has several familiar vector quantities associated with it, including linear velocity and momentum. These vector quantities each have a magnitude (a scalar, or number) and direction associated with them. Similarly, circular motion is motion in a circle. It has the same set of vector quantities associated with it, including angular velocity and angular momentum.



Angular velocity diagram*: A vector diagram illustrating circular motion. The blue vector connects the origin (center) of the motion to the position of the particle. The red vector is the angular velocity vector, pointing perpendicular to the plane of motion and with magnitude equal to the instantaneous velocity. File:Angular velocity.svg - Wikipedia, the free encyclopedia.* ***Provided by****: Wikipedia.* ***Located at****:* ***en.Wikipedia.org/w/index.php?title=File:Angular_velocity.svg&page=1****.* ***License****:* ***CC BY-SA: Attribution-ShareAlike***

Imagine a particle moving in a circle around a point at a constant speed. We will call that point the origin. At any instant in time, the particle is moving in a particular straight-line direction with that speed. In the next instant, the particle has the same speed, but the direction of its velocity has changed.

We recall from our study of linear velocity that a change in the direction of the velocity vector, is a change in velocity and a change in velocity is acceleration. However, we can define an angular momentum vector which is constant throughout this motion. The angular velocity has a direction perpendicular to the plane of circular motion, just like a bike axle points perpendicularly to the rotating wheel. This direction never changes as the object moves in its circle. The magnitude of the angular momentum is equal to the rate at which the angle of the particle advances:

$$\omega=\frac{\mathrm{d}\phi}{\mathrm{dt}} \tag{5.9.1}$$

Note that there are two vectors that are perpendicular to any plane. For example, imagine a vector pointing into your table and the opposite one pointing out of it. To remove this ambiguity, the convention in physics is to use the right hand rule: curl the fingers of your right hand in the direction of the circular motion, and your thumb will point toward the direction of the angular velocity and momentum vectors.

Right hand rule*: When determining the direction of an angular vector, use the right hand rule: curl the fingers of your right hand in the direction of the circular motion and your thumb points in the vector direction. File:Right-hand grip rule.svg - Wikipedia, the* *free encyclopedia.* ***Provided by****: Wikipedia.* ***Located at****:* ***en.Wikipedia.org/w/index.php?title=File:Right-hand_grip_rule.svg&page=1****.* ***License****: CC BY-SA: Attribution-ShareAlike*

The units of angular velocity are radians per second. Radian describes the plane angle subtended by a circular arc as the length of the arc divided by the radius of the arc. One radian is the angle subtended at the center of a circle by an arc that is equal in length to the radius of the circle. More generally, the magnitude in radians of such a subtended angle is equal to the ratio of the arc length to the radius of the circle; that is, $\theta = \frac{\mathrm{s}}{\mathrm{r}}$, where θ is the subtended angle in radians, s is arc length, and r is radius.

Thus, while the object moves in a circle at constant speed, it undergoes constant linear acceleration to keep it moving in a circle. However, it's angular velocity is constant since it continually sweeps out a constant arc length per unit time. Constant angular velocity in a circle is known as uniform circular motion.

Just as there is an angular version of velocity, so too is there an angular version of acceleration. When the object is going around a circle but its speed is changing, the object is undergoing angular acceleration. Just like with linear acceleration, angular acceleration is a change in the angular velocity vector. This change could be a change in the speed of the object or in the direction. Angular velocity can be clockwise or counterclockwise.

Key Points

- The direction of angular quantity vectors points perpendicular to the plane of the motion. You can determine this direction using the right hand rule.
- The direction of linear quantities such as velocity and momentum change as an object moves in a circle. We can instead define angular versions of these quantities which are constant throughout the circular motion.
- The units of angular quantities are per radian, a measurement of angle, rather than per linear distance (e.g. meter).

Key Terms

- **vector**: A directed quantity, one with both magnitude and direction; the between two points.
- **angular momentum**: A vector quantity describing an object in circular motion; its magnitude is equal to the momentum of the particle, and the direction is perpendicular to the plane of its circular motion.
- **angular velocity**: A vector quantity describing an object in circular motion; its magnitude is equal to the speed of the particle and the direction is perpendicular to the plane of its circular motion.

LICENSES AND ATTRIBUTIONS

CHAPTER OVERVIEW

06: Work and Energy

Topic hierarchy

6.1: Introduction

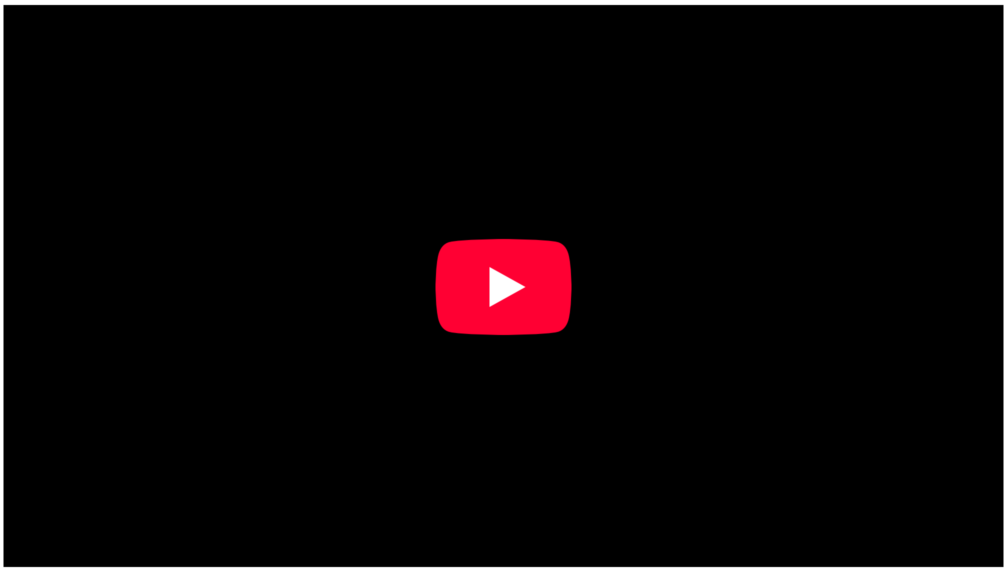
Introduction to Work and Energy

Work is the energy associated with the action of a force.

learning objectives

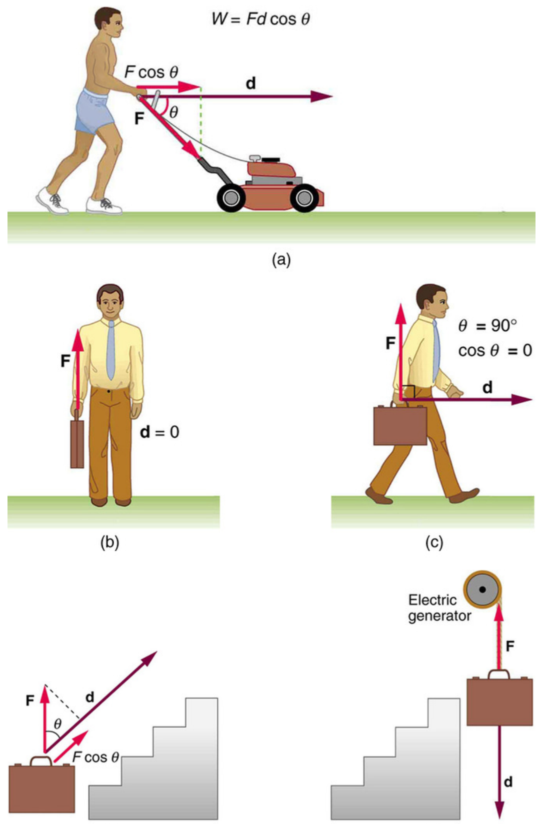
- Describe relationship between work, energy, and force

The work done on a system by a constant force is the product of the component of the force in the direction of motion times the distance through which the force acts. For one-way motion in one dimension, this is expressed in equation form as $\mathrm{W} = \mathrm{Fd}\cos\theta$, where W is work, F is the magnitude of the force on the system, d is the magnitude of the displacement of the system, and θ is the angle between the force vector F and the displacement vector d.



Work, Power, and Energy: Biology is useful.

Take this example of work in action from: (A) The work done by the force F on this lawn mower is $\mathrm{W} = \mathrm{Fd}\cos\theta$. Note that $\mathrm{W} = \mathrm{Fd}\cos\theta$ is the component of the force in the direction of motion. (B) A person holding a briefcase does no work on it, because there is no motion. No energy is transferred to or from the briefcase. (C) The person moving the briefcase horizontally at a constant speed does no work on it, and transfers no energy to it. (D) Work is done on the briefcase by carrying it up stairs at constant speed, because there is necessarily a component of force F in the direction of the motion. Energy is transferred to the briefcase and could, in turn, be used to do work. (E) When the briefcase is lowered, energy is transferred out of the briefcase and into an electric generator. Here the work done on the briefcase by the generator is negative, removing energy from the briefcase, because F and d are in opposite directions.



Examples of Work: This is how work in progress and energy co-exist and operate. Work is the energy associated with the action of a force.

In physics, a force is said to do work when it acts on a body so that there is a displacement of the point of application, however small, in the direction of the force. Thus a force does work when there is movement under the action of the force. The work done by a constant force of magnitude F on a point that moves a distance d in the direction of the force is the product:

$$\mathrm{W=Fd} \tag{6.1.1}$$

For example, if a force of 10 newton (F = 10 N) acts along point that travels two meters (d = 2 m), then it does the work W = (10 N)(2 m) = 20 N m = 20 J. This is approximately the work done lifting a 1 kg weight from the ground to over a person's head against the force of gravity. Notice that the work is doubled either by lifting twice the weight in the same distance or by lifting the same weight twice the distance.

Work is closely related to energy. The conservation of energy states that the change in total internal energy of a system equals the added heat minus the work performed by the system (see the first law of thermodynamics, and):

Baseball Pitcher: A baseball pitcher does work on a baseball by throwing the ball at some force, F, over some distance d, which for the average baseball field, is about 60 feet.

$$\delta E = \delta Q - \delta W \tag{6.1.2}$$

Also, from Newton's second law for rigid bodies, it can be shown that work on an object is equal to the change in kinetic energy of that object:

$$W = \Delta KE \tag{6.1.3}$$

The work of forces generated by a potential function is known as potential energy and the forces are said to be conservative. Therefore work on an object moving in a conservative force field is equal to minus the change of potential energy of the object:

$$W = -\Delta PE \tag{6.1.4}$$

This shows that work is the energy associated with the action of a force, and so has the physical dimensions and units of energy.

Key Points

- Work is the transfer of energy by a force acting on an object as it is displaced.
- The work done by a force is zero if the displacement is either zero or perpendicular to the force.
- The work done is positive if the force and displacement have the same direction, and the work done is negative if they have opposite direction.

Key Terms

- **energy**: A quantity that denotes the ability to do work and is measured in a unit dimensioned in mass × distance^2/time^2 (ML^2/T^2) or the equivalent.

6.2: Work Done by a Constant Force

Force in the Direction of Displacement

The work done by a constant force is proportional to the force applied times the displacement of the object.

learning objectives

- Contrast displacement and distance in constant force situations

Work Done by a Constant Force

When a force acts on an object over a distance, it is said to have done work on the object. Physically, the work done on an object is the change in kinetic energy that that object experiences. We will rigorously prove both of these claims.

The term work was introduced in 1826 by the French mathematician Gaspard-Gustave Coriolis as “weight lifted through a height,” which is based on the use of early steam engines to lift buckets of water out of flooded ore mines. The SI unit of work is the newton-meter or joule (J).

Units

One way to validate if an expression is correct is to perform dimensional analysis. We have claimed that work is the change in kinetic energy of an object and that it is also equal to the force times the distance. The units of these two should agree. Kinetic energy – and all forms of energy – have units of joules (J). Likewise, force has units of newtons (N) and distance has units of meters (m). If the two statements are equivalent they should be equivalent to one another.

$$\mathrm{N \cdot m = kg \frac{m}{s^2} \cdot m = kg \frac{m^2}{s^2} = J} \tag{6.2.1}$$

Displacement versus Distance

Often times we will be asked to calculate the work done by a force on an object. As we have shown, this is proportional to the force and the distance which the object is displaced, not moved. We will investigate two examples of a box being moved to illustrate this.

Example Problems

Here are a few example problems:

(1.a) Consider a constant force of two newtons (F = 2 N) acting on a box of mass three kilograms (M = 3 kg). Calculate the work done on the box if the box is displaced 5 meters.

(1.b) Since the box is displaced 5 meters and the force is 2 N, we multiply the two quantities together. The object’s mass will dictate how fast it is accelerating under the force, and thus the time it takes to move the object from point a to point b. Regardless of how long it takes, the object will have the same displacement and thus the same work done on it.

(2.a) Consider the same box (M = 3 kg) being pushed by a constant force of four newtons (F = 4 N). It begins at rest and is pushed for five meters (d = 5m). Assuming a frictionless surface, calculate the velocity of the box at 5 meters.

(2.b) We now understand that the work is proportional to the change in kinetic energy, from this we can calculate the final velocity. What do we know so far? We know that the block begins at rest, so the initial kinetic energy must be zero. From this we algebraically isolate and solve for the final velocity.

$$\mathrm{Fd = \Delta KE = KE_f - 0 = \frac{1}{2} m v_f^2} \tag{6.2.2}$$

$$\mathrm{v_f = \sqrt{2 \frac{Fd}{m}} = \sqrt{2 \frac{4N \cdot 5m}{2kg}} = \sqrt{10} m/s} \tag{6.2.3}$$

We see that the final velocity of the block is approximately 3.15 m/s.

Force at an Angle to Displacement

A force does not have to, and rarely does, act on an object parallel to the direction of motion.

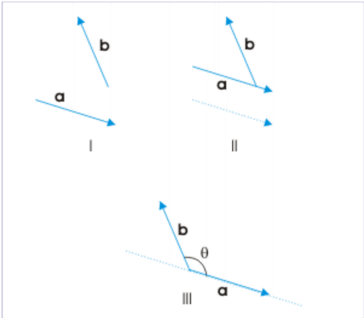
learning objectives

- Infer how to adjust one-dimensional motion for our three-dimensional world

The Fundamentals

Up until now, we have assumed that any force acting on an object has been parallel to the direction of motion. We have considered our motion to be one dimensional, only acting along the x or y axis. To best examine and understand how nature operators in our three-dimensional world, we will first discuss work in two dimensions in order to build our intuition.

A force does not have to, and rarely does, act on an object parallel to the direction of motion. In the past, we derived that $\mathrm{W}=\mathrm{Fd}$; such that the work done on an object is the force acting on the object multiplied by the displacement. But this is not the whole story. This expression contains an assumed cosine term, which we do not consider for forces parallel to the direction of motion. “Why would we do such a thing? ” you may ask. We do this because the two are equivalent. If the angle of the force along the direction of motion is zero, such that the force is parallel to the direction of motion, then the cosine term equals one and does not change the expression. As we increase the force’s angle with respect to the direction of motion, less and less work is done along the direction that we are considering; and more and more work is being done in another, perpendicular, direction of motion. This process continues until we are perpendicular to our original direction of motion, such that the angle is 90, and the cosine term would equal zero; resulting in zero work being done along our original direction. Instead, we are doing work in another direction!



Angle: Recall that both the force and direction of motion are vectors. When the angle is 90 degrees, the cosine term goes to zero. When along the same direction, they equal one.

Let’s show this explicitly and then look at this phenomena in terms of a box moving along the x and y directions.

We have discussed that work is the integral of the force and the dot product respect to x. But in fact, dot product of force and a very small distance is equal to the two terms times cosine of the angle between the two. $\mathrm{F}\times\mathrm{dx}=\mathrm{Fd}\cos(\theta)$. Explicitly,

$$\int_{\mathrm{t}_2}^{\mathrm{t}_1}\mathrm{F}\cdot\mathrm{dx}=\int_{\mathrm{t}_2}^{\mathrm{t}_1}\mathrm{Fd}\cos\theta\mathrm{dx}=\mathrm{Fd}\cos\theta \tag{6.2.4}$$

A Box Being Pushed

Consider a coordinate system such that we have x as the abscissa and y as the ordinate. More so, consider a box being pushed along the x direction. What happens in the following three scenarios?

- The box is being pushed parallel to the x direction?
- The box is being pushed at an angle of 45 degrees to the x direction?
- The box is being pushed at an angle of 60 degrees to the x direction?
- The box is being pushed at an angle of 90 degrees to the x direction?

In the first scenario, we know that all of the force is acting on the box along the x-direction, which means that work will only be done along the x-direction. More so, a vertical perspective the box is not moving – it is unchanged in the y direction. Since the force is acting parallel to the direction of motion, the angle is equal to zero and our total work is simply the force times the displacement in the x-direction.

In the second scenario, the box is being pushed at an angle of 45 degrees to the x-direction; and thus also a 45 degree angle to the y-direction. When evaluated, the cosine of 45 degrees is equal to $\frac{1}{\sqrt{2}}$, or approximately 0.71. This means is that 71% of the force is contributing to the work along the x-direction. The other 29% is acting along the y-direction.

In the third scenario, we know that the force is acting at a 60 degree angle to the x-direction; and thus also a 30 degree angle to the y-direction. When evaluated, cosine of 60 degrees is equal to 1/2. This means that the force is equally acting in the x and y-direction! The work done is linear with respect to both x and y.

In the last scenario, the box is being pushed at an angle perpendicular to the x direction. In other words, we are pushing the box in the y-direction! Thus, the box's position will be unchanged and experience no displacement along the x-axis. The work done in the x direction will be zero.

Key Points

- Understanding work is quintessential to understanding systems in terms of their energy, which is necessary for higher level physics.
- Work is equivalent to the change in kinetic energy of a system.
- Distance is not the same as displacement. If a box is moved 3 meters forward and then 4 meters to the left, the total displacement is 5 meters, not 7 meters.
- Work done on an object along a given direction of motion is equal to the force times the displacement times the cosine of the angle.
- No work is done along a direction of motion if the force is perpendicular.
- When considering force parallel to the direction of motion, we omit the cosine term because it equals 1 which does not change the expression.

Key Terms

- **work**: A measure of energy expended in moving an object; most commonly, force times displacement. No work is done if the object does not move.
- **dot product**: A scalar product.

- **6.2: Work Done by a Constant Force** by Boundless has no license indicated.

6.3: Work Done by a Variable Force

Work Done by a Variable Force

Integration is used to calculate the work done by a variable force.

learning objectives

- Describe approaches used to calculate work done by a variable force

Using Integration to Calculate the Work Done by Variable Forces

A force is said to do work when it acts on a body so that there is a displacement of the point of application in the direction of the force. Thus, a force does work when it results in movement.

The work done by a constant force of magnitude F on a point that moves a displacement Δx in the direction of the force is simply the product

$$W = F \cdot \Delta x \tag{6.3.1}$$

In the case of a variable force, integration is necessary to calculate the work done. For example, let's consider work done by a spring. According to the Hooke's law the restoring force (or spring force) of a perfectly elastic spring is proportional to its extension (or compression), but opposite to the direction of extension (or compression). So the spring force acting upon an object attached to a horizontal spring is given by:

$$Fs = -kx \tag{6.3.2}$$

that is proportional to its displacement (extension or compression) in the x direction from the spring's equilibrium position, but its direction is opposite to the x direction. For a variable force, one must add all the infinitesimally small contributions to the work done during infinitesimally small time intervals dt (or equivalently, in infinitely small length intervals $dx=v_x dt$). In other words, an integral must be evaluated:

$$W_s = \int_0^t Fs \cdot vdt = \int_0^t -kxv_x dt = \int_{x_o}^{x} -kxdx = -\frac{1}{2}k\Delta x^2 \tag{6.3.3}$$

This is the work done by a spring exerting a variable force on a mass moving from position x_o to x (from time 0 to time t). The work done is positive if the applied force is in the same direction as the direction of motion; so the work done by the object on spring from time 0 to time t, is:

$$W_a = \int_0^t Fa \cdot vdt = \int_0^t -F_s \cdot vdt = \frac{1}{2}k\Delta x^2 \tag{6.3.4}$$

in this relation F_a is the force acted upon spring by the object. F_a and F_s are in fact action- reaction pairs; and W_a is equal to the elastic potential energy stored in spring.

Using Integration to Calculate the Work Done by Constant Forces

The same integration approach can be also applied to the work done by a constant force. This suggests that *integrating* the product of force and distance is the general way of determining the work done by a force on a moving body.

Consider the situation of a gas sealed in a piston, the study of which is important in Thermodynamics. In this case, the Pressure (Pressure =Force/Area) is constant and can be taken out of the integral:

$$W = \int_a^b PdV = P\int_a^b dV = P\Delta V \tag{6.3.5}$$

Another example is the work done by gravity (a constant force) on a free-falling object (we assign the y-axis to vertical motion, in this case):

$$W = \int_{t_1}^{t_2} F \cdot vdt = \int_{t_1}^{t_2} mgv_y dt = mg\int_{y_1}^{y_2} dy = mg\Delta y \tag{6.3.6}$$

Notice that the result is *the same* as we would have obtained by simply evaluating the product of force and distance.

Units Used for Work

The SI unit of work is the joule (J), which is defined as the work done by a force of one newton moving an object through a distance of one meter.

Non-SI units of work include the erg, the foot-pound, the foot-pound, the kilowatt hour, the liter-atmosphere, and the horsepower-hour.

Key Points

- The work done by a constant force of magnitude F on a point that moves a displacement d in the direction of the force is the product: $W = Fd$.
- Integration approach can be used both to calculate work done by a variable force and work done by a constant force.
- The SI unit of work is the joule; non- SI units of work include the erg, the foot-pound, the foot-poundal, the kilowatt hour, the litre-atmosphere, and the horsepower-hour.

Key Terms

- **work**: A measure of energy expended in moving an object; most commonly, force times displacement. No work is done if the object does not move.
- **force**: A physical quantity that denotes ability to push, pull, twist or accelerate a body, which is measured in a unit dimensioned in mass × distance/time² (ML/T^2): SI: newton (N); CGS: dyne (dyn)

6.4: Work-Energy Theorem

Kinetic Energy and Work-Energy Theorem

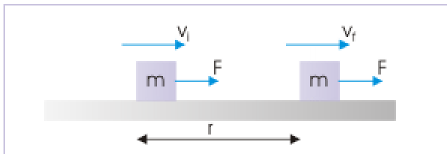
The work-energy theorem states that the work done by all forces acting on a particle equals the change in the particle's kinetic energy.

learning objectives

- Outline the derivation of the work-energy theorem

The Work-Energy Theorem

The principle of work and kinetic energy (also known as the work-energy theorem) states that the work done by the sum of all forces acting on a particle equals the change in the kinetic energy of the particle. This definition can be extended to rigid bodies by defining the work of the torque and rotational kinetic energy.



Kinetic Energy: A force does work on the block. The kinetic energy of the block increases as a result by the amount of work. This relationship is generalized in the work-energy theorem.

The work *W* done by the net force on a particle equals the change in the particle's kinetic energy K*E*:

$$\mathrm{W}=\Delta \mathrm{KE}=\frac{1}{2}\mathrm{mv}_{\mathrm{f}}^{2}-\frac{1}{2}\mathrm{mv}_{\mathrm{i}}^{2} \tag{6.4.1}$$

where v_i and v_f are the speeds of the particle before and after the application of force, and *m* is the particle's mass.

Derivation

For the sake of simplicity, we will consider the case in which the resultant force *F* is constant in both magnitude and direction and is parallel to the velocity of the particle. The particle is moving with constant acceleration *a* along a straight line. The relationship between the net force and the acceleration is given by the equation *F* = *ma* (Newton's second law), and the particle's displacement *d*, can be determined from the equation:

$$\mathrm{v}_{\mathrm{f}}^{2}=\mathrm{v}_{\mathrm{i}}^{2}+2\mathrm{ad} \tag{6.4.2}$$

obtaining,

$$\mathrm{d}=\frac{\mathrm{v}_{\mathrm{f}}^{2}-\mathrm{v}_{\mathrm{i}}^{2}}{2\mathrm{a}} \tag{6.4.3}$$

The work of the net force is calculated as the product of its magnitude (F=ma) and the particle's displacement. Substituting the above equations yields:

$$\mathrm{W}=\mathrm{Fd}=\mathrm{ma}\frac{\mathrm{v}_{\mathrm{f}}^{2}-\mathrm{v}_{\mathrm{i}}^{2}}{2\mathrm{a}}=\frac{1}{2}\mathrm{mv}_{\mathrm{f}}^{2}-\frac{1}{2}\mathrm{mv}_{\mathrm{i}}^{2}=\mathrm{KE}_{\mathrm{f}}-\mathrm{KE}_{\mathrm{i}}=\Delta \mathrm{KE} \tag{6.4.4}$$

Key Points

- The work *W* done by the net force on a particle equals the change in the particle's kinetic energy K*E*: $\mathrm{W}=\Delta\mathrm{KE}=\frac{1}{2}\mathrm{mv}_{\mathrm{f}}^{2}-\frac{1}{2}\mathrm{mv}_{\mathrm{i}}^{2}$.
- The work-energy theorem can be derived from Newton's second law.
- Work transfers energy from one place to another or one form to another. In more general systems than the particle system mentioned here, work can change the potential energy of a mechanical device, the heat energy in a thermal system, or the electrical energy in an electrical device.

Key Terms

- **torque**: A rotational or twisting effect of a force; (SI unit newton-meter or Nm; imperial unit foot-pound or ft-lb)

6.5: Potential Energy and Conservation of Energy

Conservative and Nonconservative Forces

Conservative force—a force with the property that the work done in moving a particle between two points is independent of the path it takes.

learning objectives

- Describe properties of conservative and nonconservative forces

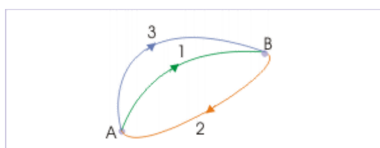
A conservative force is a force with the property that the work done in moving a particle between two points is independent of the path taken. Equivalently, if a particle travels in a closed loop, the net work done (the sum of the force acting along the path multiplied by the distance travelled) by a conservative force is zero.

A conservative force is dependent only on the position of the object. If a force is conservative, it is possible to assign a numerical value for the potential at any point. When an object moves from one location to another, the force changes the potential energy of the object by an amount that does not depend on the path taken. Gravity and spring forces are examples of conservative forces.

If a force is *not conservative*, then defining a scalar potential is not possible, because taking different paths would lead to conflicting potential differences between the start and end points. Nonconservative forces transfer energy *from* the object in motion (just like conservative force), but they do not transfer this energy back *to* the potential energy of the system to regain it during reverse motion. Instead, they transfer the energy from the system in an energy form which can not be used by the force to transfer it back to the object in motion. Friction is one such nonconservative force.

Path Independence of Conservative Force

Work done by the gravity in a closed path motion is zero. We can extend this observation to other conservative force systems as well. We imagine a closed path motion. We imagine this closed path motion be divided in two motions between points A and B as diagramed in Fig 1. Starting from point A to point B and then ending at point A via two work paths named 1 and 2 in the figure. The total work by the conservative force for the round trip is zero:



Motion Along Different Paths: Motion along different paths. For a conservative force, work done via different path is the same.

$$\mathrm{W}=\mathrm{W}_{\mathrm{AB1}}+\mathrm{W}_{\mathrm{BA2}}=0. \tag{6.5.1}$$

Let us now change the path for motion from A to B by another path, shown as path 3. Again, the total work by the conservative force for the round trip via new route is zero: $\mathrm{W}=\mathrm{W}_{\mathrm{AB3}}+\mathrm{W}_{\mathrm{BA2}}=0.$.

Comparing two equations, $\mathrm{W}_{\mathrm{AB1}}=\mathrm{W}_{\mathrm{AB3}}$. This is true for an arbitrary path. Therefore, work done for motion from A to B by conservative force along any paths are equal.

What is Potential Energy?

Potential energy is the energy difference between the energy of an object in a given position and its energy at a reference position.

learning objectives

- Relate the potential energy and the work

Potential energy is often associated with restoring forces such as a spring or the force of gravity. The action of stretching the spring or lifting the mass of an object is performed by an external force that works against the force field of the potential. This work is stored in the force field as potential energy. If the external force is removed the force field acts on the body to perform the work as it moves the body back to its initial position, reducing the stretch of the spring or causing the body to fall. The more formal definition is that potential energy is the energy difference between the energy of an object in a given position and its energy at a reference position.

Potential Energy in a Bow and Arrow: In the case of a bow and arrow, the energy is converted from the potential energy in the archer's arm to the potential energy in the bent limbs of the bow when the string is drawn back. When the string is released, the potential energy in the bow limbs is transferred back through the string to become kinetic energy in the arrow as it takes flight.

If the work for an applied force is independent of the path, then the work done by the force is evaluated at the start and end of the trajectory of the point of application. This means that there is a function *U*(*x*), called a "potential," that can be evaluated at the two points $x(t = t_1)$ and $x(t_2)$ to obtain the work over any trajectory between these two points. It is tradition to define this function with a negative sign so that positive work is represented as a reduction in the potential:

$$W = \int_C F \cdot dx = \int_{x(t_1)}^{x(t_2)} F \cdot dx \tag{6.5.2}$$

$$= U(x(t_1)) - U(x(t_2)) = -\Delta U. \tag{6.5.3}$$

Examples of Potential Energy

There are various types of potential energy, each associated with a particular type of force. More specifically, every conservative force gives rise to potential energy. For example, the work of an elastic force is called elastic potential energy; work done by the gravitational force is called gravitational potential energy; and work done by the Coulomb force is called electric potential energy.

Gravity

Gravitational energy is the potential energy associated with gravitational force, as work is required to move objects against gravity.

learning objectives

- Generate an equation that can be used to express the gravitational potential energy near the earth

Gravitational energy is the potential energy associated with gravitational force (a conservative force), as work is required to elevate objects against Earth's gravity. The potential energy due to elevated positions is called gravitational potential energy, evidenced, for example, by water held in an elevated reservoir or behind a dam (as an example, shows Hoover Dam). If an object falls from one point to another point inside a gravitational field, the force of gravity will do positive work on the object, and the gravitational potential energy will decrease by the same amount.

Hoover Dam: Hoover dam uses the stored gravitational potential energy to generate electricity.

Potential Near Earth

Gravitational potential energy near the Earth can be expressed with respect to the height from the surface of the Earth. (The surface will be the zero point of the potential energy.) We can express the potential energy (gravitational potential energy) as:

$$\mathrm{PE}=\mathrm{mgh}, \tag{6.5.4}$$

where PE = potential energy measured in joules (J), m = mass of the object (measured in kg), and h = perpendicular height from the reference point (measured in m); g = gravitational acceleration ($9.8 m/s^2$). Near the surface of the Earth, g can be considered constant.

General Formula

However, over large variations in distance, the approximation that g is constant is no longer valid. Instead, we must use calculus and the general mathematical definition of work to determine gravitational potential energy. For the computation of the potential energy we can integrate the gravitational force, whose magnitude is given by Newton's law of gravitation (with respect to the distance r between the two bodies). Using that definition, the gravitational potential energy of a system of masses m and M at a distance r using gravitational constant G is:

$$\mathrm{U(r)}=\int_{\mathrm{r}}(\mathrm{G}\frac{\mathrm{mM}}{\mathrm{r}'^2})\mathrm{dr}'=-\mathrm{G}\frac{\mathrm{mM}}{\mathrm{r}}+\mathrm{K}, \tag{6.5.5}$$

where *K* is the constant of integration. Choosing the convention that *K*=0 makes calculations simpler, albeit at the cost of making *U* negative. For this choice, the potential at infinity is defined as 0.

Springs

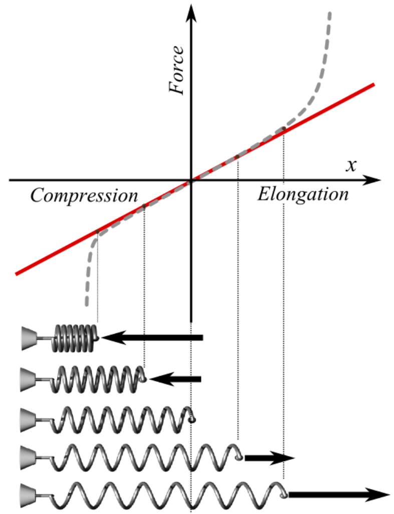
When a spring is stretched/compressed from its equilibrium position by x, its potential energy is give as $\mathrm{U}=\frac{1}{2}\mathrm{kx}^2$.

learning objectives

- Explain how potential energy is stored in springs

Spring force is conservative force, given by the Hooke's law: $\mathrm{F}=-\mathrm{kx}$, where k is spring constant, measured experimentally for a particular spring and x is the displacement. We would like to obtain an expression for the work done to the spring. From the conservation of mechanical energy (Check our Atom on "Conservation of Mechanical Energy), the work should be equal to the potential energy stored in spring. The displacement x is usually measured from the position of "neutral length" or "relaxed length"

– the length of spring corresponding to situation when spring is neither stretched nor compressed. We shall identify this position as the origin of coordinate reference (x=0).



Hooke's Law: Plot of applied force F vs. elongation X for a helical spring according to Hooke's law (solid line) and what the actual plot might look like (dashed line). Red is used extension, blue for compression. At bottom, schematic pictures of spring states corresponding to some points of the plot; the middle one is in the relaxed state (no force applied).

Let $\mathrm{x}=0$ and $\mathrm{x}=\mathrm{x_f}(>0)$ be the initial and final positions of the block attached to the string. As the block slowly moves, we do work W on the spring: $\mathrm{W}=\int_0^{\mathrm{x_f}}(\mathrm{kx})\mathrm{dx}=\frac{1}{2}\mathrm{kx_f^2}$. When we stretch the spring. We have to apply force in the same direction as the displacement. (Technically, work is given as the inner product of the two vectors: force and displacement. $\mathrm{W}=\mathbf{F}\cdot\Delta\mathrm{x}$). Therefore, the overall sign in the integral is +, not -.

If the block is gently released from the stretched position (\mathrm{x = x_f}\), the stored potential energy in the spring will start to be converted to the kinetic energy of the block, and vice versa. Neglecting frictional forces, Mechanical energy conservation demands that, at any point during its motion,

$$\text{Total Energy}=\frac{1}{2}\mathrm{mv}^2+\frac{1}{2}\mathrm{kx}^2 \tag{6.5.6}$$

$$=\frac{1}{2}\mathrm{kx_f^2}=\text{constant}. \tag{6.5.7}$$

From the energy conservation, we can estimate that, by the time the block reaches x=0 position, its speed will be $\mathrm{v}(\mathrm{x}=0)=\sqrt{\frac{\mathrm{k}}{\mathrm{m}}}\mathrm{x_f}$. The block will keep oscillating between x = -x_f and x_f.

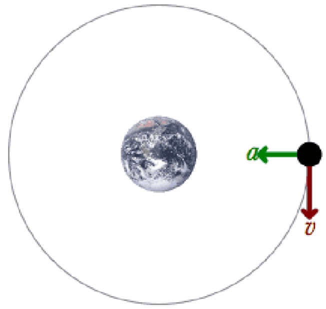
Conservation of Mechanical Energy

Conservation of mechanical energy states that the mechanical energy of an isolated system remains constant without friction.

learning objectives

- Formulate the principle of the conservation of the mechanical energy

Conservation of mechanical energy states that the mechanical energy of an isolated system remains constant in time, as long as the system is free of all frictional forces. In any real situation, frictional forces and other non-conservative forces are always present, but in many cases their effects on the system are so small that the principle of conservation of mechanical energy can be used as a fair approximation. An example of a such a system is shown in. Though energy cannot be created nor destroyed in an isolated system, it can be internally converted to any other form of energy.



A Mechanical System: An example of a mechanical system: A satellite is orbiting the Earth only influenced by the conservative gravitational force and the mechanical energy is therefore conserved. This acceleration is represented by a green acceleration vector and the velocity is represented by a red velocity vector.

Derivation

Let us consider what form the work -energy theorem takes when only conservative forces are involved (leading us to the conservation of energy principle). The work-energy theorem states that the net work done by all forces acting on a system equals its change in kinetic energy (KE). In equation form, this is:

$$\mathrm{W_{net}} = \frac{1}{2}\mathrm{mv}^2 - \frac{1}{2}\mathrm{mv}_0^2 = \Delta\mathrm{KE}. \tag{6.5.8}$$

If only conservative forces act, then $\mathrm{W_{net}} = \mathrm{W_c}$, where $\mathrm{W_c}$ is the total work done by all conservative forces. Thus, $\mathrm{W_c} = \Delta\mathrm{KE}$.

Now, if the conservative force, such as the gravitational force or a spring force, does work, the system loses potential energy (PE). That is, $\mathrm{W_c} = -\mathrm{PE}$. Therefore,

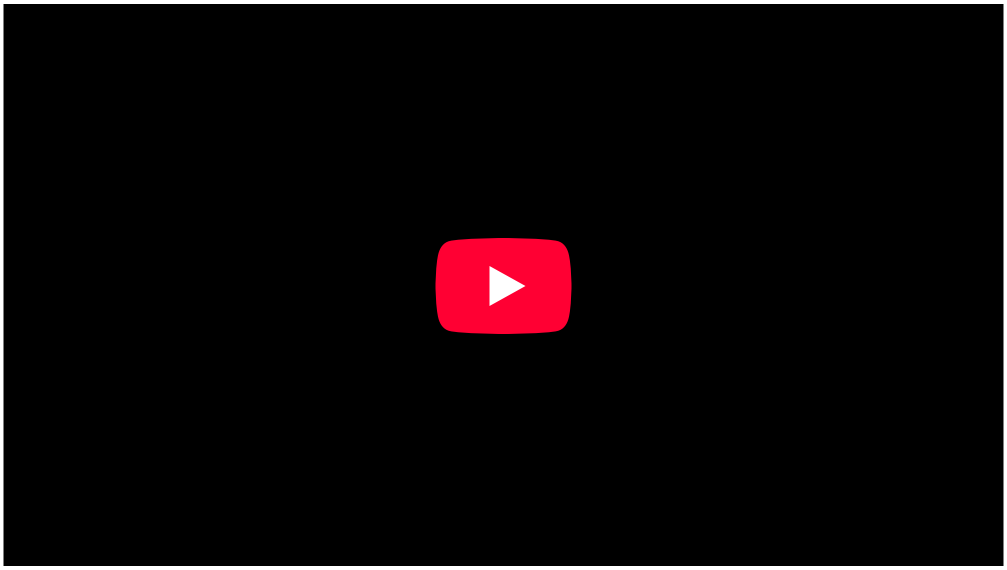
$$-\Delta\mathrm{PE} = \Delta\mathrm{KE} \tag{6.5.9}$$

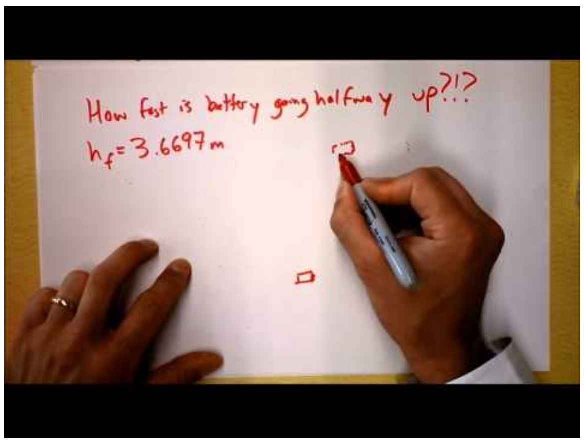
This equation means that the total kinetic and potential energy is constant for any process involving only conservative forces. That is,

$$\mathrm{KE} + \mathrm{PE} = \mathrm{const\ or\ KE_i} + \mathrm{PE_i} = \mathrm{KE_f} + \mathrm{PE_f}, \tag{6.5.10}$$

where i and f denote initial and final values. This equation is a form of the work-energy theorem for conservative forces; it is known as the conservation of mechanical energy principle.

Remember that the law applies to the extent that all the forces are conservative, so that friction is negligible. The total kinetic plus potential energy of a system is defined to be its mechanical energy $(\mathrm{KE} + \mathrm{PE})$. In a system that experiences only conservative forces, there is a potential energy associated with each force, and the energy only changes form between KE and various types of PE (with the total energy remaining constant).





Conservation of Mechanical Energy: Worked example.

Problem Solving With the Conservation of Energy

To solve a conservation of energy problem determine the system of interest, apply law of conservation of energy, and solve for the unknown.

learning objectives

- Identify steps necessary to solve a conservation of energy problem

Problem-solving Strategy

You should follow a series of steps whenever you are problem solving:

Step One

Determine the system of interest and identify what information is given and what quantity is to be calculated. For example, let's assume you have the problem with car on a roller coaster. You know that the cars of a roller coaster reach their maximum kinetic energy (KEKE) when at the bottom of their path. When they start rising, the kinetic energy begins to be converted to gravitational potential energy (PEgPEg). The sum of kinetic and potential energy in the system should remain constant, if losses to friction are ignored.

Determining Energy: The cars of a roller coaster reach their maximum kinetic energy when at the bottom of their path. When they start rising, the kinetic energy begins to be converted to gravitational potential energy. The sum of kinetic and potential energy in the system remains constant, ignoring losses to friction.

Step Two

Examine all the forces involved and determine whether you know or are given the potential energy from the work done by the forces. Then use step three or step four.

Step Three

If you know the potential energies (PE) for the forces that enter into the problem, then forces are all conservative, and you can apply conservation of mechanical energy simply in terms of potential and kinetic energy. The equation expressing conservation of energy is:

$$\mathrm{KE_i + PE_i = KE_f + PE_f}. \tag{6.5.11}$$

Step Four

If you know the potential energy for only some of the forces, then the conservation of energy law in its most general form must be used:

$$\mathrm{KE_i + PE_i + W_{nc} + OE_i = KE_f + PE_f + OE_f} \tag{6.5.12}$$

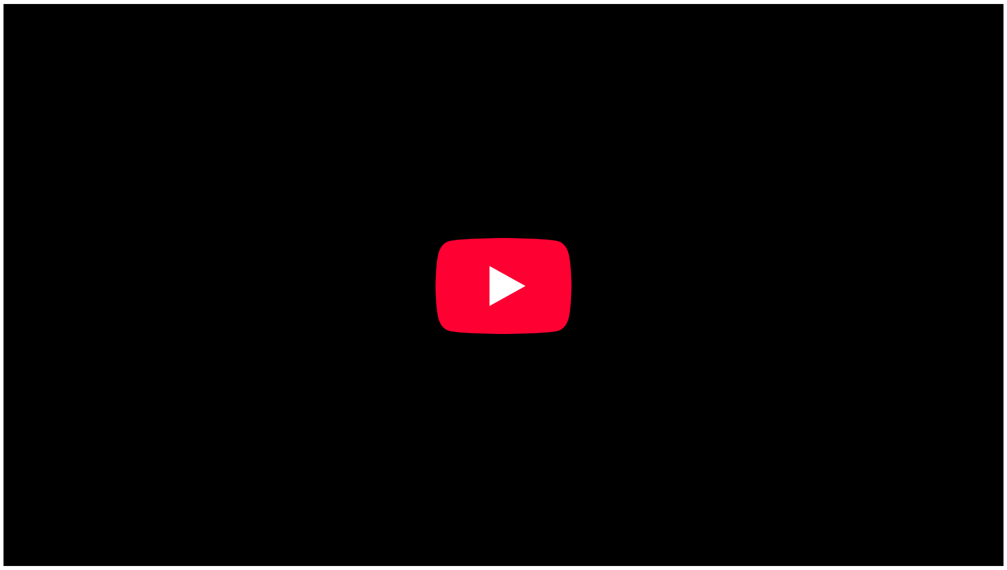
where $\mathrm{O_E}$ stand for all other energies, and $\mathrm{W_{nc}}$ stands for work done by non-conservative forces. In most problems, one or more of the terms is zero, simplifying its solution. Do *not* calculate $\mathrm{W_c}$, the work done by conservative forces; it is already incorporated in the PE terms.

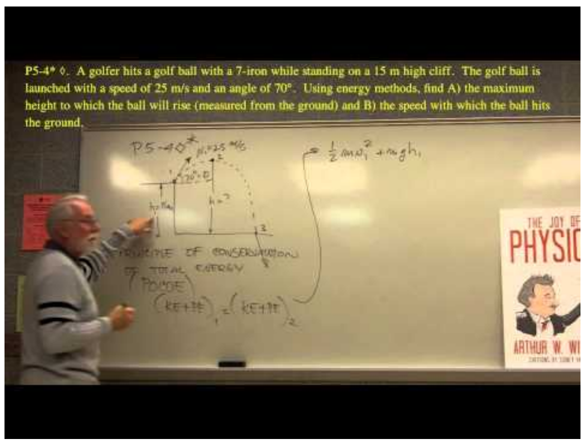
Step Five

You have already identified the types of work and energy involved (in step two). Before solving for the unknown, eliminate terms wherever possible to simplify the algebra. For example, choose height $\mathrm{h = 0}$ at either the initial or final point—this will allow to set PEg at zero. Then solve for the unknown in the customary manner.

Step Six

Check the answer to see if it is reasonable. Once you have solved a problem, reexamine the forms of work and energy to see if you have set up the conservation of energy equation correctly. For example, work done against friction should be negative, potential energy at the bottom of a hill should be less than that at the top, and so on.





Energy conservation: Part of a series of videos on physics problem-solving. The problems are taken from "The Joy of Physics." This one deals with energy conservation. The viewer is urged to pause the video at the problem statement and work the problem before watching the rest of the video.

Problem Solving with Dissipative Forces

In the presence of dissipative forces, total mechanical energy changes by exactly the amount of work done by nonconservative forces (W_c).

learning objectives

- Express the energy conservation relationship that can be applied to solve problems with dissipative forces

INTRODUCTION

We have seen a problem-solving strategy with the conservation of energy in the previous section. Here we will adopt the strategy for problems with dissipative forces. Since the work done by nonconservative (or dissipative) forces will irreversibly alter the energy of the system, the total mechanical energy (KE + PE) changes by exactly the amount of work done by nonconservative forces (W_c). Therefore, we obtain $\mathrm{KE_i + PE_i + W_{nc} = KE_f + PE_f}$, where KE and PE represent kinetic and potential energies respectively. Therefore, using the new energy conservation relationship, we can apply the same problem-solving strategy as with the case of conservative forces.

EXAMPLE

Consider the situation shown in, where a baseball player slides to a stop on level ground. Using energy considerations, calculate the distance the 65.0-kg baseball player slides, given that his initial speed is 6.00 m/s and the force of friction against him is a *constant 450 N.*

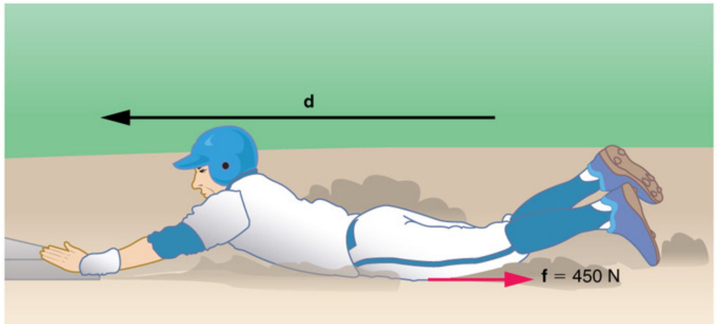


Fig 1: The baseball player slides to a stop in a distance d. In the process, friction removes the player's kinetic energy by doing an amount of work fd equal to the initial kinetic energy.

Strategy: Friction stops the player by converting his kinetic energy into other forms, including thermal energy. In terms of the work-energy theorem, the work done by friction (f), which is negative, is added to the initial kinetic energy to reduce it to zero. The work done by friction is negative, because f is in the opposite direction of the motion (that is, $\theta = 180^{\circ}$, and so $\cos\theta = -1$). Thus $W_{nc} = -fd$. The equation simplifies to $\frac{1}{2}mv_i^2 - fd = 0$.

Solution: Solving the previous equation for d and substituting known values yields, we get d = 2.60 m. The most important point of this example is that the amount of nonconservative work equals the change in mechanical energy.

Key Points

- If a particle travels in a closed loop, the net work done (the sum of the force acting along the path multiplied by the distance travelled) by a conservative force is zero.
- Conservative force is dependent only on the position of the object. If a force is conservative, it is possible to assign a numerical value for the potential at any point.
- Nonconservative force transfer the energy from the system in an energy form which can not be used by the force to transfer back to the object in motion.
- If the work for an applied force is independent of the path, then the work done by the force is evaluated at the start and end of the trajectory of the point of application. This means that there is a function U(x), called a " potential ".
- It is tradition to define the potential function with a negative sign so that positive work is represented as a reduction in the potential.
- Every conservative force gives rise to potential energy. Examples are elastic potential energy, gravitational potential energy, and electric potential energy.
- Gravitational potential energy near the earth can be expressed with respect to the height from the surface of the Earth as PE = mgh. g = gravitational acceleration ($9.8 m/s^2$). Near the surface of the Earth, g can be considered constant.
- Over large variations in distance, the approximation that g is constant is no longer valid and a general formula should be used for the potential. It is given as: $U(r) = \int_r (G\frac{mM}{r'^2})dr' = -G\frac{mM}{r} + K$.
- Choosing the convention that the constant of integration K=0 assumes that the potential at infinity is defined to be 0.
- The displacement of spring x is usually measured from the position of "neutral length " or "relaxed length". Often, it is most convenient to identify this position as the origin of coordinate reference (x=0).
- If the block is gently released from the stretched position ($x = x_f$), energy conservation tells us that $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kx_f^2 = \text{constant}$.
- If the block is released from the stretched position ($x = x_f$), by the time the block reaches x=0 position, its speed will be $v(x=0) = \sqrt{\frac{k}{m}}x_f$. The block will keep oscillating between $x = -x_f$ and x_f.
- The conservation of mechanical energy can be written as "$KE + PE = \text{const}$ ".
- Though energy cannot be created nor destroyed in an isolated system, it can be internally converted to any other form of energy.
- In a system that experiences only conservative forces, there is a potential energy associated with each force, and the energy only changes form between KE and various types of PE, with the total energy remaining constant.
- If you know the potential energies for the forces that enter into the problem, then forces are all conservative, and you can apply conservation of mechanical energy simply in terms of potential and kinetic energy. The equation expressing conservation of

energy is: $KE_i + PE_i = KE_f + PE_f$. .

- If you know the potential energy for only some of the forces, then the conservation of energy law in its most general form must be used: $KE_i + PE_i + W_{nc} + OE_i = KE_f + PE_f + OE_f$, where OE stands for all other energies.
- Once you have solved a problem, always check the answer to see if it is reasonable.
- Using the new energy conservation relationship

$$KE_i + PE_i + W_{nc} = KE_f + PE_f \tag{6.5.13}$$

, we can apply the same problem-solving strategy as with the case of conservative forces.
- The most important point is that the amount of nonconservative work equals the change in mechanical energy.
- The work done by nonconservative (or dissipative) forces will irreversibly dissipated in the system.

Key Terms

- **potential**: A curve describing the situation where the difference in the potential energies of an object in two different positions depends only on those positions.
- **Coulomb force**: the electrostatic force between two charges, as described by Coulomb's law
- **potential**: A curve describing the situation where the difference in the potential energies of an object in two different positions depends only on those positions.
- **conservative force**: A force with the property that the work done in moving a particle between two points is independent of the path taken.
- **Hooke's law**: the principle that the stress applied to a solid is directly proportional to the strain produced. This law describes the behavior of springs and solids stressed within their elastic limit.
- **conservation**: A particular measurable property of an isolated physical system does not change as the system evolves.
- **isolated system**: A system that does not interact with its surroundings, that is, its total energy and mass stay constant.
- **frictional force**: Frictional force is the force resisting the relative motion of solid surfaces, fluid layers, and material elements sliding against each other.
- **kinetic energy**: The energy possessed by an object because of its motion, equal to one half the mass of the body times the square of its velocity.
- **potential energy**: The energy an object has because of its position (in a gravitational or electric field) or its condition (as a stretched or compressed spring, as a chemical reactant, or by having rest mass)
- **conservative force**: A force with the property that the work done in moving a particle between two points is independent of the path taken.
- **dissipative force**: A force resulting in dissipation, a process in which energy (internal, bulk flow kinetic, or system potential) is transformed from some initial form to some irreversible final form.

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6.6: Power

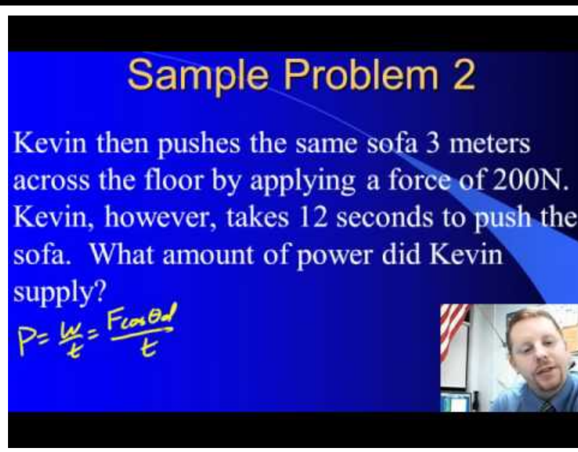
What is Power?

In physics, power is the rate of doing work—the amount of energy consumed per unit time.

learning objectives

- Relate power to the transfer, use, and transformation of different types of energy

In physics, power is the rate of doing work. It is the amount of energy consumed per unit time. The unit of power is the joule per second (J/s), known as the watt (in honor of James Watt, the eighteenth-century developer of the steam engine). For example, the rate at which a lightbulb transforms electrical energy into heat and light is measured in watts (W)—the more wattage, the more power, or equivalently the more electrical energy is used per unit time.



Power: A brief overview of power in an algebra-based physics course.

Energy transfer can be used to do work, so power is also the rate at which this work is performed. The same amount of work is done when carrying a load up a flight of stairs whether the person carrying it walks or runs, but more power is expended during the running because the work is done in a shorter amount of time. The output power of an electric motor is the product of the torque the motor generates and the angular velocity of its output shaft. The power expended to move a vehicle is the product of the traction force of the wheels and the velocity of the vehicle.

Examples of power are limited only by the imagination, because there are as many types as there are forms of work and energy. Sunlight reaching Earth's surface carries a maximum power of about 1.3 kilowatts per square meter (kW/m^2). A tiny fraction of this is retained by Earth over the long term. Our consumption rate of fossil fuels is far greater than the rate at which they are stored, so it is inevitable that they will be depleted. Power implies that energy is transferred, perhaps changing form. It is never possible to change one form completely into another without losing some of it as thermal energy. For example, a 60-W incandescent bulb converts only 5 W of electrical power to light, with 55 W dissipating into thermal energy. Furthermore, the typical electric power plant converts only 35 to 40 percent of its fuel into electricity. The remainder becomes a huge amount of thermal energy that must be dispersed as heat transfer, as rapidly as it is created. A coal-fired power plant may produce 1,000 megawatts; 1 megawatt (MW) is 106 W of electric power. But the power plant consumes chemical energy at a rate of about 2,500 MW, creating heat transfer to the surroundings at a rate of 1,500 MW.

Coal-fired Power Plant: Tremendous amounts of electric power are generated by coal-fired power plants such as this one in China, but an even larger amount of power goes into heat transfer to the surroundings. The large cooling towers here are needed to transfer heat as rapidly as it is produced. The transfer of heat is not unique to coal plants but is an unavoidable consequence of generating electric power from any fuel—nuclear, coal, oil, natural gas, or the like.

Humans: Work, Energy, and Power

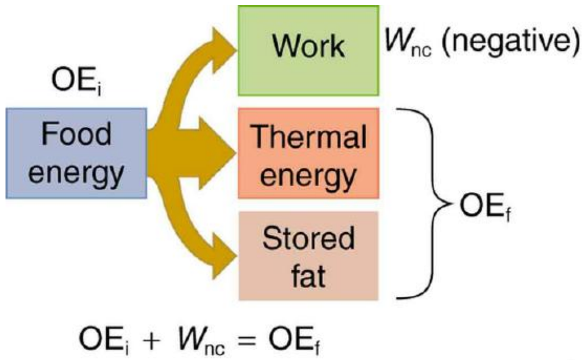
The human body converts energy stored in food into work, thermal energy, and/or chemical energy that is stored in fatty tissue.

learning objectives

- Identify what factors play a role in basal metabolic rate (BMR)

Humans: Work, Energy, and Power

Our own bodies, like all living organisms, are energy conversion machines. Conservation of energy implies that the chemical energy stored in food is converted into work, thermal energy, or stored as chemical energy in fatty tissue, as shown in. Energy consumed by humans is converted to work, thermal energy, and stored fat. By far the largest fraction goes to thermal energy, although the fraction varies depending on the type of physical activity. The fraction going into each form depends both on how much we eat and on our level of physical activity. If we eat more than is needed to do work and stay warm, the remainder goes into body fat.



Energy Conversion in Humans: Energy consumed by humans is converted to work, thermal energy, and stored fat. By far the largest fraction goes to thermal energy, although the fraction varies depending on the type of physical activity.

Functions that Require Energy

All bodily functions, from thinking to lifting weights, require energy. The many small muscle actions accompanying all quiet activity, from sleeping to head scratching, ultimately become thermal energy, as do less visible muscle actions by the heart, lungs, and digestive tract. Shivering, in fact, is an involuntary response to low body temperature that pits muscles against one another to produce thermal energy in the body (and do no work). The kidneys and liver consume a surprising amount of energy, but the biggest surprise of all it that a full 25% of all energy consumed by the body is used to maintain electrical potentials in all living cells. (Nerve cells use this electrical potential in nerve impulses.) This bioelectrical energy ultimately becomes mostly thermal energy, but some is utilized to power chemical processes such as in the kidneys and liver, and in fat production.

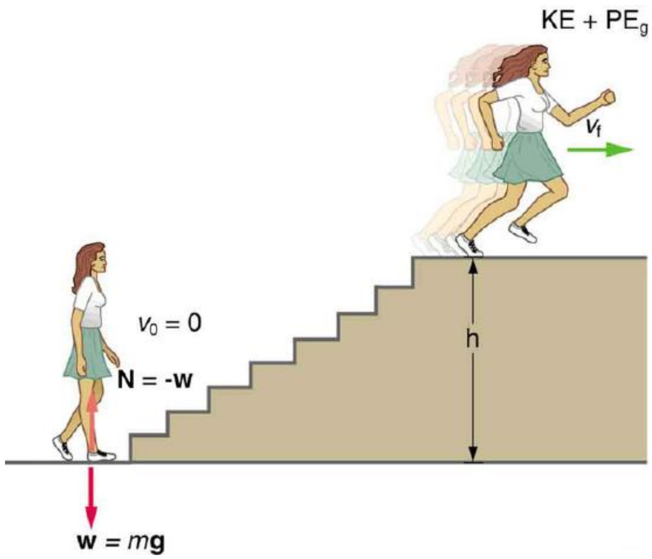
Basal Metabolic Rate

The rate at which the body uses food energy to sustain life and to do different activities is called the metabolic rate. The total energy conversion rate of a person at rest is called the basal metabolic rate (BMR) and is divided among various systems in the body. The largest fraction goes to the liver and spleen, with the brain coming next. Of course, during vigorous exercise, the energy consumption of the skeletal muscles and heart increase markedly. About 75% of the calories burned in a day go into these basic functions. The BMR is a function of age, gender, total body weight, and amount of muscle mass (which burns more calories than body fat). Athletes have a greater BMR due to this last factor.

Useful Work

Work done by a person is sometimes called useful work, which is work done on the outside world, such as lifting weights. Useful work requires a force exerted through a distance on the outside world, and so it excludes internal work, such as that done by the heart when pumping blood. Useful work does include that done in climbing stairs or accelerating to a full run, because these are accomplished by exerting forces on the outside world. Forces exerted by the body are nonconservative, so that they can change the mechanical energy (KE+PE) of the system worked upon, and this is often the goal.

For example, what is the power output for a 60.0-kg woman who runs up a 3.00 m high flight of stairs in 3.50 s, starting from rest but having a final speed of 2.00 m/s?.



Woman Running Up Stairs: When this woman runs upstairs starting from rest, she converts the chemical energy originally from food into kinetic energy and gravitational potential energy. Her power output depends on how fast she does this.

Her power output depends on how fast she does this. The work going into mechanical energy is W= KE + PE. At the bottom of the stairs, we take both KE and PE_g as initially zero; thus,

$$W = KE_f + PE_g = \frac{1}{2}mv_f^2 + mgh \tag{6.6.1}$$

where h is the vertical height of the stairs. Because all terms are given, we can calculate W and then divide it by time to get power. Substituting the expression for W into the definition of power given in the previous equation, $P = \frac{W}{t}$ yields

$$P = \frac{W}{t} = \frac{\frac{1}{2}mv_f^2 + mgh}{t} \tag{6.6.2}$$

Entering known values yields

$$P = \frac{0.5(60.0kg)(2.00m/s)^2 + (60.0kg)(9.80m/s^2)(3.00m)}{(3.50s)} = \frac{120J + 1764J}{3.50s} = 538W \tag{6.6.3}$$

The woman does 1764 J of work to move up the stairs compared with only 120 J to increase her kinetic energy; thus, most of her power output is required for climbing rather than accelerating.

Energy consumption is directly proportional to oxygen consumption because the digestive process is basically one of oxidizing food. We can measure the energy people use during various activities by measuring their oxygen use. Approximately 20 kJ of energy are produced for each liter of oxygen consumed, independent of the type of food.

Key Points

- Power implies that energy is transferred, perhaps changing form.
- Energy transfer can be used to do work, so power is also the rate at which this work is performed.
- The unit of power is the joule per second (J/s), known as the watt.
- The rate at which the body uses food energy to sustain life and to do different activities is called the metabolic rate, and the corresponding rate when at rest is called the basal metabolic rate (BMR).
- The energy included in the basal metabolic rate is divided among various systems in the body, with the largest fraction going to the liver and spleen, and the brain coming next.
- About 75% of food calories are used to sustain basic body functions included in the basal metabolic rate.
- Work done by a person is sometimes called useful work, which is work done on the outside world, such as lifting weights.
- The energy consumption of people during various activities can be determined by measuring their oxygen use, because the digestive process is basically one of oxidizing food.

Key Terms

- **power**: A measure of the rate of doing work or transferring energy.
- **watt**: In the International System of Units, the derived unit of power; the power of a system in which one joule of energy is transferred per second.
- **basal metabolic rate**: The amount of energy expended while at rest in a neutrally temperate environment, in the post-absorptive state.

6.7: CASE STUDY: World Energy Use

World Energy Use

The most prominent sources of energy used in the world are non-renewable (i.e., unsustainable).

learning objectives

- Explain why renewable energy sources must be found and utilized

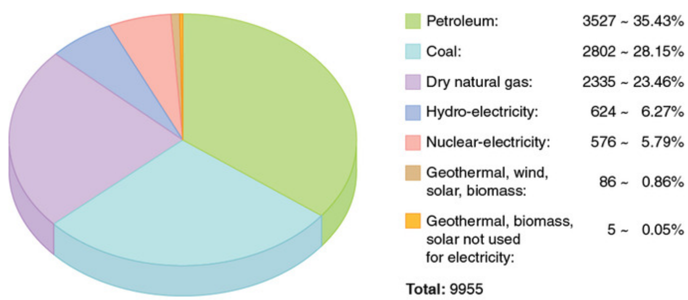
Energy Use

World energy consumption is the total amount of energy used by all humans on the planet (measured on a per-year basis). This measurement is the sum of all energy sources (and purposes) in use. Who measures this? Several organizations publish this data, including the International Energy Agency (IEA), the US Energy Information Administration (EIA), and the European Environment Agency. This data is useful because evaluating this information to discover trends might yield energy issues not currently being addressed, thereby encouraging the search for solutions. The IEA established a goal of limiting global warming to 2 degrees Celsius, but this goal is becoming more difficult to reach each year that the necessary action is not taken. In global energy use, fossil fuels make up a substantial portion. In 2011 they received over $500 billion in subsidies—six times more than that received by renewable energy sources.

Implementing new practices that will utilize different, renewable energy sources is important because having access to energy is important—it maintains our quality of life. Fossil fuels, however, are not sustainable at the rate they are currently used. About 40% of the world's energy comes from oil, but oil prices are dependent on uncertain factors (such as availability, politics, and world events). The United States alone uses 24% of the world's oil per year, yet it makes up only 4.5% of the world's population! In 2008, total worldwide energy consumption was 474 exajoules (474×10^{18} J=132,000 TWh)—equivalent to an average power usage of 15 terawatts (1.504×10^{13} W). Potential renewable energy sources include: solar energy at 1600 EJ (444,000 TWh), wind power at 600 EJ (167,000 TWh), geothermal energy at 500 EJ (139,000 TWh), biomass at 250 EJ (70,000 TWh), hydropower at 50 EJ (14,000 TWh) and ocean energy at 1 EJ (280 TWh).

Types of Energy

shows a pie chart of world energy usage by category—both renewable and nonrenewable sources. Renewable energy comes from sources with an unlimited supply. This includes energy from water, wind, the sun, and biomass. In the US, only 10% of energy comes from renewable sources (mostly hydroelectric energy). Nonrenewable sources makes up 85% of worldwide energy usage—from sources that eventually will be depleted, such as oil, natural gases and coal.



World Energy Use: This chart shows that the primary worldwide energy sources nonrenewable. If new practices are not put in place now, this model will not be sustainable.

Energy Needs

In the last 50 years, the global energy demand has tripled due to the number of developing countries and innovations in technology. It is projected to triple again over the next 30 years. In Europe, many in such developing areas recognize that the need for renewable energy sources, as the present course of energy usage cannot be sustained indefinitely. While renewable energy development makes up a only small percentage of the field, strides are being made in natural energy, particularly wind energy.

For example, by the year 2020 Germany plans to meet 10% of their total energy usage and 20% of its electricity usage with renewable resources. While some countries are making improvements in this field, coal usage is still a huge problem. In China, two thirds of the energy used each year is from commercial coal energy. India imports 50% of its oil, and 70% of its electricity is produced from coal, which is highly polluting.

Key Points

- The energy consumption increases with the increasing number of developing areas. In order for this development to continue, while maintaining quality of life, new and renewable energy sources must be found and utilized.
- Renewable energy comes from sources that will never deplete, no matter how much is used. An example of this is wind energy, which had been growing in popularity in countries like India and Germany.
- Nonrenewable energy makes up 85% the energy used on earth—the most popular form of energy being oil.

Key Terms

- **fossil fuel**: Any fuel derived from hydrocarbon deposits such as coal, petroleum, natural gas and, to some extent, peat; these fuels are irreplaceable, and their burning generates the greenhouse gas carbon dioxide.
- **renewable energy**: Energy that can be replenished at the same rate as it is used.

6.8: Further Topics

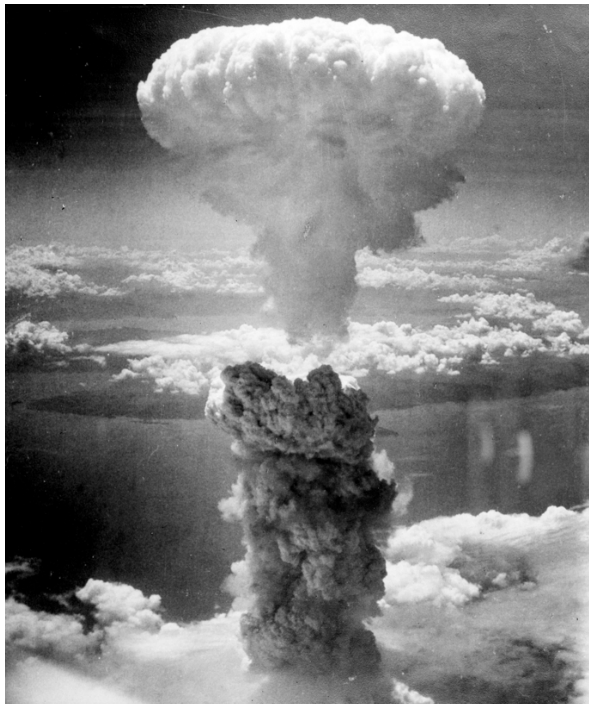
learning objectives

- Compare the different forms of energy interrelate to one another

Other Forms of Energy

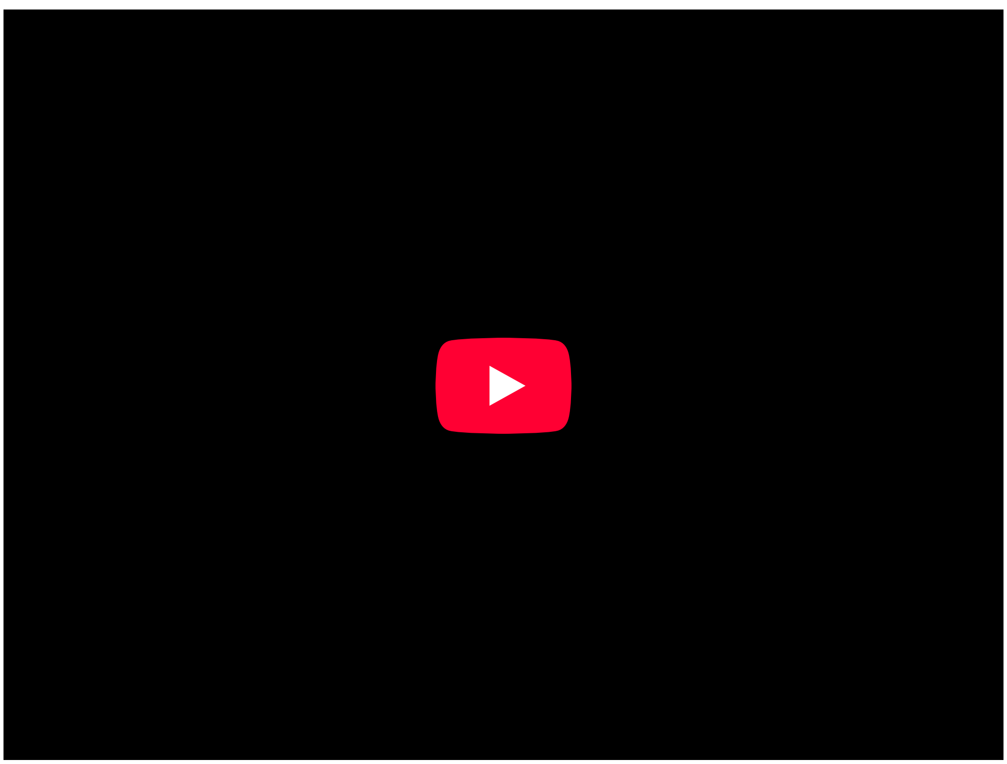
Thermal, chemical, electric, radiant, nuclear, magnetic, elastic, sound, mechanical, luminous, and mass are forms that energy can exist in. Energy can come in a variety of forms. These forms include:

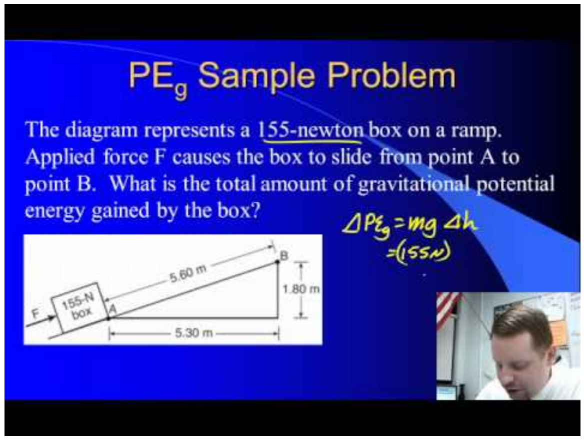
- Thermal Energy: This is energy associated with the microscopic random motion of particles in the media under consideration. An example of something that stores thermal energy is warm bath water.
- Chemical Energy: This is energy due to the way that atoms are arranged in molecules and various other collections of matter. An example of something that stores chemical energy is food. When your body digests and metabolizes food it utilizes its chemical energy.
- Electric Energy: This is energy that is from electrical potential energy, a result of Coulombic forces. Electrical potential energy is associated with the way that point charges in a system are arranged. An example of something that stores electric energy is a capacitor. A capacitor collects positive charge on one plate and negative charge on the other plate. Energy is thus stored in the resulting electrostatic field.
- Radiant Energy: This is any kind of electromagnetic radiation (see key term). An example of an electromagnetic wave is light.
- Nuclear Energy: This type of energy is liberated during the nuclear reactions of fusion and fission. Examples of things that utilize nuclear energy include nuclear power plants and nuclear weapons.
- Magnetic Energy: Technically magnetic energy is electric energy; the two are related by Maxwell's equations. An example of something that stores magnetic energy is a superconducting magnet used in an MRI.
- Elastic Energy: This is potential mechanical energy that is stored in the configuration of a material or physical system as work is performed to distort its volume or shape. An example of something that stores elastic energy is a stretched rubber band.
- Sound Energy: This is energy that is associated with the vibration or disturbance of matter. An example of something that creates sound energy is your voice box (larynx).
- Mechanical Energy: This is energy that is associated with the motion and position of an object. It is the sum of all of the kinetic and potential energy that the object has. An example of something that utilizes mechanical energy is a pendulum.
- Luminous Energy: This is energy that can be seen because it is visible light. An example of luminous energy is light from a flashlight.
- Mass: Can be converted to energy via: $\mathrm{E} = \mathrm{mc}^2$. For example, mass is converted into energy when a nuclear bomb explodes.



Atomic bomb explosion: The mushroom cloud of the atomic bombing of Nagasaki, Japan

In each of the aforementioned forms, energy exists as either kinetic energy, potential energy, or a combination of both. It is important to note that the above list is not necessarily complete as we may discover new forms of energy in the future such as “dark energy. ” Also, each of the forms that energy can take on (as listed above) are not necessarily mutually exclusive. For example, luminous energy is radiant energy.





Types of Energy: A brief overview of energy, kinetic energy, gravitational potential energy, and the work-energy theorem for algebra-based physics students.

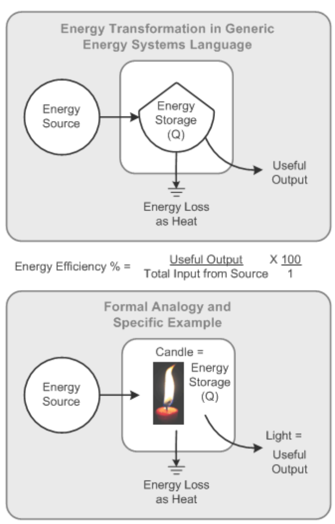
Energy Transformations

Energy transformation occurs when energy is changed from one form to another, and is a consequence of the first law of thermodynamics.

learning objectives

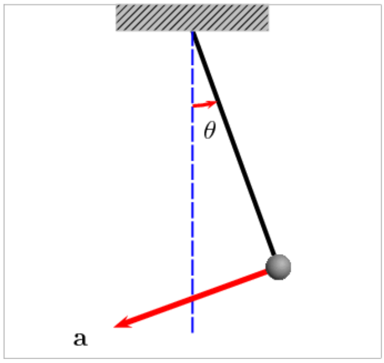
- Summarize the consequence of the first law of thermodynamics on the total energy of a system

Energy transformation occurs when energy is changed from one form to another. It is a consequence of the first law of thermodynamics that the total energy of a given system can only be changed when energy is added or subtracted from the system. Often it appears that energy has been lost from a system when it simply has been transformed. For example, an internal combustion engine converts the potential chemical energy in gasoline and oxygen into heat energy. This heat energy is then converted to kinetic energy, which is then used to propel the vehicle that is utilizing the engine. The technical term for a device that converts energy from one form to another is a *transducer*.



Energy Transformation: These figures illustrate the concepts of energy loss and useful energy output.

When analyzing energy transformations, it is important to consider the efficiency of conversion. The efficiency of conversions describes the ratio between the useful output and input of an energy conversion machine. Some energy transformations can occur with an efficiency of essentially 100%. For example, imagine a pendulum in a vacuum. As illustrated in, when the pendulum's mass reaches its maximum height, all if its energy exists in the form of potential energy. However, when the pendulum is at its lowest point, all of its energy exists in the form of kinetic energy.



Pendulum: This animation shows the velocity and acceleration vectors for a pendulum. One may note that at the maximum height of the pendulum's mass, the velocity is zero. This corresponds to zero kinetic energy and thus all of the energy of the pendulum is in the form of potential energy. When the pendulum's mass is at its lowest point, all of its energy is in the form of kinetic energy and we see its velocity vector has a maximum magnitude here.

Other energy transformations occur with a much lower efficiency of conversion. For example, the theoretical limit of the energy efficiency of a wind turbine (converting the kinetic energy of the wind to mechanical energy) is 59%. The process of photosynthesis is able to transform the light energy of the sun into chemical energy that can be used by a plant with an efficiency of conversion of a mere 6%.

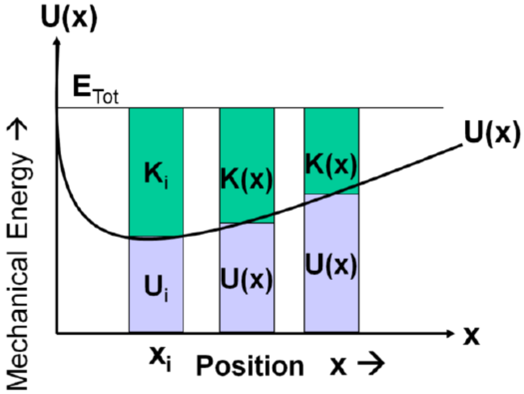
Potential Energy Curves and Equipotentials

A potential energy curve plots potential energy as a function of position; equipotential lines trace lines of equal potential energy.

learning objectives

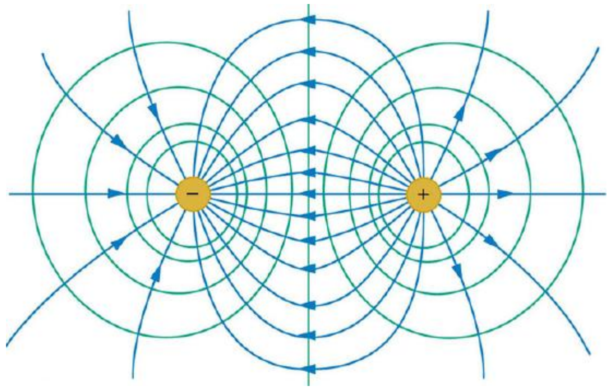
- Derive the potential of a point charge

A potential energy curve plots the potential energy of an object as a function of that object's position. For example, see. The system under consideration is a closed system, so the total energy of the system remains constant. This means that the kinetic and potential energy always have to sum to be the same value. We observe that the potential energy increases as the kinetic energy decreases and vice versa. The utility of a potential energy curve is that we can quickly determine the potential energy of the object in question at a given position.



Potential Energy Curve: This figure illustrates the potential energy of a particle as a function of position. The kinetic energy is also shown and is abbreviated K.

Equipotential lines trace lines of equal potential energy. In, if you were to draw a straight horizontal line through the center, that would be an equipotential line. In and, if you travel along an equipotential line, the electric potential will be constant.



Equipotential Lines for Two Equal and Opposite Point Charges: Electric field (blue) and equipotential lines (green) for two equal and opposite charges

Let us examine the physical explanation for the equipotentials lines in. The equation for the potential of a point charge is $\mathrm{V}=\frac{\mathrm{kQ}}{\mathrm{r}}$, where *V* is the potential, *k* is a constant with a value of $8.99 \cdot 10^9$ N m^2/C^2, *Q* is the magnitude of the point charge, and *r*is the distance from the charge. So, every point that is the same distance from the point charge will have the same electric potential energy. Therefore, if we draw a circle around the point charge, every point on the circle will have the same potential energy.

Work (*W*) is a measure of the change in potential energy $(\Delta \mathrm{PE}): \mathrm{W}=-$\DeltaPE. Since the potential energy does not change along an equipotential line, you do not need to do any work to move along one. However, you *do* need to do work to move from one equipotential line to another. Recall that work is zero if force is perpendicular to motion; in the figures shown above, the forces resulting from the electric field are in the same direction as the electric field itself. So we note that each of the equipotential lines must be perpendicular to the electric field at every point.

Key Points

- Thermal, chemical, electric, radiant, nuclear, magnetic, elastic, sound, mechanical, luminous, and mass are forms that energy can exist in.
- Energy exists as either kinetic energy, potential energy, or a combination of both.
- We may discover new forms of energy (like "dark energy") in the future.
- The total energy of a given system can only be changed when energy is added or subtracted from the system.
- Often it appears that energy has been lost from a system when it simply has been transformed.
- The efficiency of conversions describes the ratio between the useful output of an energy conversion machine and the input.
- A potential energy curve plots the potential energy of an object as a function of its position.
- Equipotential lines trace lines of equal potential energy.
- You do not need to do any work to move along an equipotential line.

Key Terms

- **fusion**: A nuclear reaction in which nuclei combine to form more massive nuclei with the concomitant release of energy.
- **electromagnetic radiation**: radiation (quantized as photons) consisting of oscillating electric and magnetic fields oriented perpendicularly to each other, moving through space
- **fission**: The process of splitting the nucleus of an atom into smaller particles; nuclear fission.
- **pendulum**: A body suspended from a fixed support so that it swings freely back and forth under the influence of gravity; it is commonly used to regulate various devices such as clocks.
- **first law of thermodynamics**: A version of the law of conservation of energy, specialized for thermodynamical systems. It is usually formulated by stating that the change in the internal energy of a closed system is equal to the amount of heat supplied to the system, minus the amount of work done by the system on its surroundings.
- **potential energy**: The energy an object has because of its position (in a gravitational or electric field) or its condition (as a stretched or compressed spring, as a chemical reactant, or by having rest mass)
- **kinetic energy**: The energy possessed by an object because of its motion, equal to one half the mass of the body times the square of its velocity.

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CHAPTER OVERVIEW

07: Linear Momentum and Collisions

Thumbnail: A pool break-off shot. Image used with permission (CC-SA-BY; No-w-ay).

7.1: Introduction

Linear Momentum

Linear momentum is the product of the mass and velocity of an object, it is conserved in elastic and inelastic collisions.

learning objectives

- Calculate the momentum of two colliding objects

In classical mechanics, linear momentum, or simply momentum (SI unit kg m/s, or equivalently N s), is the product of the mass and velocity of an object. Mathematically it is stated as:

$$\mathrm{p=mv} \tag{7.1.1}$$

(Note here that p and v are vectors.) Like velocity, linear momentum is a vector quantity, possessing a direction as well as a magnitude. Linear momentum is particularly important because it is a conserved quantity, meaning that in a closed system (without any external forces) its total linear momentum cannot change.

Because momentum has a direction, it can be used to predict the resulting direction of objects after they collide, as well as their speeds. Momentum is conserved in both inelastic and elastic collisions. (Kinetic energy is not conserved in inelastic collisions but is conserved in elastic collisions.) It important to note that if the collision takes place on a surface with friction, or if there is air resistance, we would need to account for the momentum of the bodies that would be transferred to the surface and/or air.

Let's take a look at a simple, one-dimensional example: The momentum of a system of two particles is the sum of their momenta. If two particles have masses m_1 and m_2, and velocities v_1 and v_2, the total momentum is:

$$\mathrm{p=p_1+p_2=m_1v_1+m_2v_2.} \tag{7.1.2}$$

Keep in mind that momentum and velocity are vectors. Therefore, in 1D, if two particles are moving in the same direction, v1 and v2 have the same sign. If the particles are moving in opposite directions they will have opposite signs.

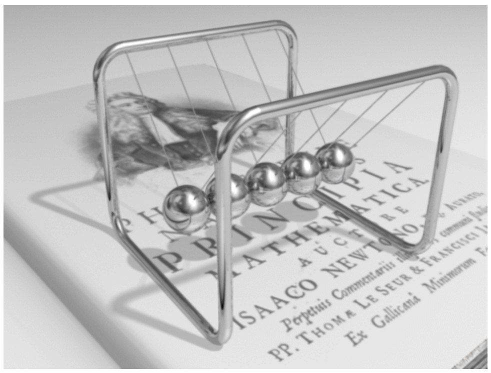
If two particles were moving on a plane we would choose our xy-plane to be on the plane of motion. We can then write the x and y component of the total momentum as:

$$\mathrm{p_x=p_{1x}+p_{2x}=m_1v_{1x}+m_2v_{2x}} \tag{7.1.3}$$

$$\mathrm{p_y=p_{1y}+p_{2y}=m_1v_{1y}+m_2v_{2y}.} \tag{7.1.4}$$

If the 2D momentum vector is decomposed into two components, the equations for each component are reduced to its 1D equivalents.

Momentum, like energy, is important because it is conserved. "Newton's cradle" shown in is an example of conservation of momentum. As we will discuss in the next concept (on Momentum, Force, and Newton's Second Law), in classical mechanics, conservation of linear momentum is implied by Newton's laws. Only a few physical quantities are conserved in nature. Studying these quantities yields fundamental insight into how nature works.



Newton's Cradle: Total momentum of the system (or Cradle) is conserved. (neglecting frictional loss in the system.)

Momentum, Force, and Newton’s Second Law

In the most general form, Newton’s 2nd law can be written as $F = \frac{dp}{dt}$.

learning objectives

- Relate Newton’s Second Law to momentum and force

In a closed system (one that does not exchange any matter with the outside and is not acted on by outside forces), the total momentum is constant. This fact, known as the law of conservation of momentum, is implied by Newton’s laws of motion. Suppose, for example, that two particles interact. Because of the third law, the forces between them are equal and opposite. If the particles are numbered 1 and 2, the second law states that

$$\frac{dp_1}{dt} = -\frac{dp_2}{dt} \tag{7.1.5}$$

or

$$\frac{d}{dt}(p_1 + p_2) = 0 \tag{7.1.6}$$

Therefore, total momentum (p_1+p_2) is constant. If the velocities of the particles are u_1 and u_2 before the interaction, and afterwards they are v_1 and v_2, then

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \tag{7.1.7}$$

This law holds regardless of the nature of the interparticle (or internal) force, no matter how complicated the force is between particles. Similarly, if there are several particles, the momentum exchanged between each pair of particles adds up to zero, so the total change in momentum is zero.

Newton’s Second Law

Newton actually stated his second law of motion in terms of momentum: The net external force equals the change in momentum of a system divided by the time over which it changes. Using symbols, this law is

$$F_{net} = \frac{\Delta p}{\Delta t}, \tag{7.1.8}$$

where F_{net} is the net external force, Δp is the change in momentum, and Δt is the change in time.

This statement of Newton’s second law of motion includes the more familiar $F_{net} = ma\backslash(asaspecialcase. Wecanderivethisformasfollows. First, notethatthechangeinmomentum\backslash(\Delta p$ is given by $\Delta p = \Delta(mv)$. If the mass of the system is constant, then $\Delta(mv) = m\Delta v$. So for constant mass, Newton’s second law of motion becomes

$$F_{net} = \frac{\Delta p}{\Delta t} = \frac{m\Delta v}{\Delta t}. \tag{7.1.9}$$

Because $\frac{\Delta v}{\Delta t} = a$, we get the familiar equation $F_{net} = ma$ when the mass of the system is constant. Newton’s second law of motion stated in terms of momentum is more generally applicable because it can be applied to systems where the mass is changing, such as rockets, as well as to systems of constant mass.

Momentum in a Closed System: In a game of pool, the system of entire balls can be considered a closed system. Therefore, the total momentum of the balls is conserved.

Impulse

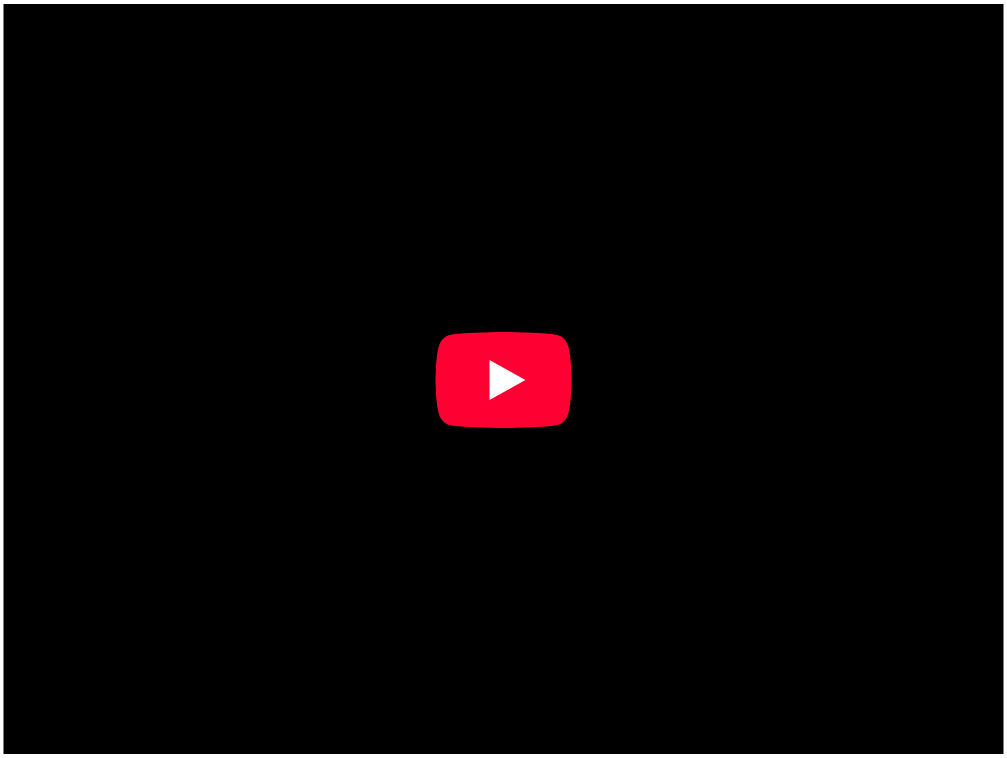
Impulse, or change in momentum, equals the average net external force multiplied by the time this force acts.

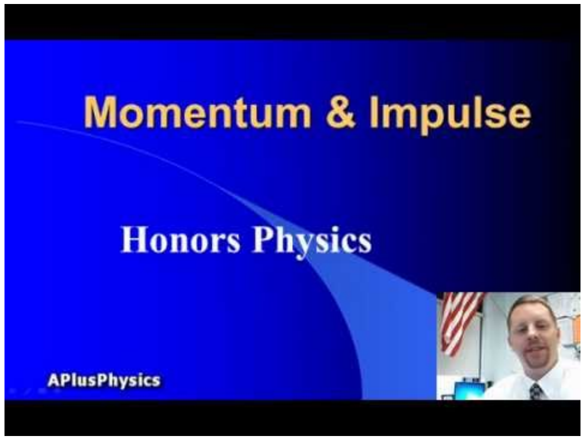
learning objectives

- Explain the relationship between change in momentum and the amount of time a force acts

Impulse

Forces produce either acceleration or deceleration on moving bodies, and the greater the force acting on an object, the greater its change in velocity and, hence, the greater its change in momentum. However, changing momentum is also related to how long a time the force acts. If a brief force is applied to a stalled automobile, a change in its momentum is produced. The same force applied over an extended period of time produces a greater change in the automobile's momentum. The quantity of impulse is *force* × *time interval*, or in shorthand notation:





Momentum & Impulse: A brief overview of momentum and impulse for high school physics students.

$$\mathrm{Impulse} = \mathrm{F}\Delta\mathrm{t}, \tag{7.1.10}$$

where F is the net force on the system, and Δt is the duration of the force.

From Newton's 2nd law:

$$\mathrm{F} = \frac{\Delta\mathrm{p}}{\Delta\mathrm{t}}\ (\Delta\mathrm{p} : \text{change in momentum}), \tag{7.1.11}$$

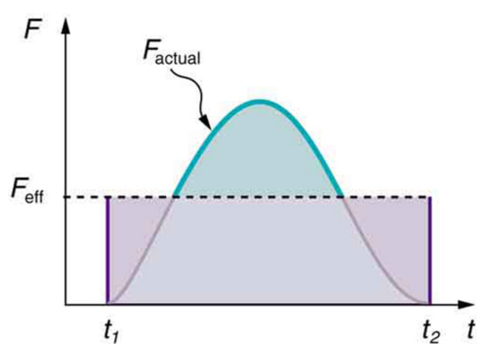
change in momentum equals the average net external force multiplied by the time this force acts.

$$\Delta\mathrm{p} = \mathrm{F}\Delta\mathrm{t}. \tag{7.1.12}$$

Therefore, impulse as defined in the previous paragraph is simply equivalent to p.

A force sustained over a long time produces more change in momentum than does the same force applied briefly. A small force applied for a long time can produce the same momentum change as a large force applied briefly because it is the product of the force and the time for which it is applied that is important. Impulse is always equal to change in momentum and is measured in Ns (Newton seconds), as both force and the time interval are important in changing momentum.

Our definition of impulse includes an assumption that the force is constant over the time interval $\Delta\mathrm{t}$. Forces are usually not constant. Forces vary considerably even during the brief time intervals considered. It is, however, possible to find an average effective force $\mathrm{F_{eff}}$ that produces the same result as the corresponding time-varying force. shows a graph of what an actual force looks like as a function of time for a ball bouncing off the floor. The area under the curve has units of momentum and is equal to the impulse or change in momentum between times $\mathrm{t_1}$ and $\mathrm{t_2}$. That area is equal to the area inside the rectangle bounded by $\mathrm{F_{eff}}, \mathrm{t_1}$, and $\mathrm{t_2}$. Thus, the impulses and their effects are the same for both the actual and effective forces. Equivalently, we can simple find the area under the curve F(t) between $\mathrm{t_1}$ and $\mathrm{t_2}$ to compute the impulse in mathematical form:



Force vs. Time: A graph of force versus time with time along the x-axis and force along the y-axis for an actual force and an equivalent effective force. The areas under the two curves are equal.

$$\mathrm{Impulse} = \int_{\mathrm{t_1}}^{\mathrm{t_2}} \mathrm{F(t)dt}. \tag{7.1.13}$$

Key Points

- Like velocity, linear momentum is a vector quantity, possessing a direction as well as a magnitude.
- Momentum, like energy, is important because it is a conserved quantity.
- The momentum of a system of particles is the sum of their momenta. If two particles have masses m_1 and m_2, and velocities v_1 and v_2, the total momentum is $p = p_1 + p_2 = m_1 v_1 + m_2 v_2.$.
- In a closed system, without any external forces, the total momentum is constant.
- The familiar equation $F = ma$ is a special case of the more general form of the 2nd law when the mass of the system is constant.
- Momentum conservation holds (in the absence of external force) regardless of the nature of the interparticle (or internal) force, no matter how complicated the force is between particles.
- A small force applied for a long time can produce the same momentum change as a large force applied briefly, because it is the product of the force and the time for which it is applied that is important.
- A force produces an acceleration, and the greater the force acting on an object, the greater its change in velocity and, hence, the greater its change in momentum. However, changing momentum is also related to how long a time the force acts.
- In case of a time-varying force, impulse can be calculated by integrating the force over the time duration. $\text{Impulse} = \int_{t_1}^{t_2} F(t)dt.$.

Key Terms

- **inelastic**: (As referring to an inelastic collision, in contrast to an elastic collision.) A collision in which kinetic energy is not conserved.
- **elastic collision**: An encounter between two bodies in which the total kinetic energy of the two bodies after the encounter is equal to their total kinetic energy before the encounter. Elastic collisions occur only if there is no net conversion of kinetic energy into other forms.
- **conservation**: A particular measurable property of an isolated physical system does not change as the system evolves.
- **closed system**: A physical system that doesn't exchange any matter with its surroundings and isn't subject to any force whose source is external to the system.
- **momentum**: (of a body in motion) the product of its mass and velocity.
- **impulse**: The integral of force over time.

7.2: Conservation of Momentum

Internal vs. External Forces

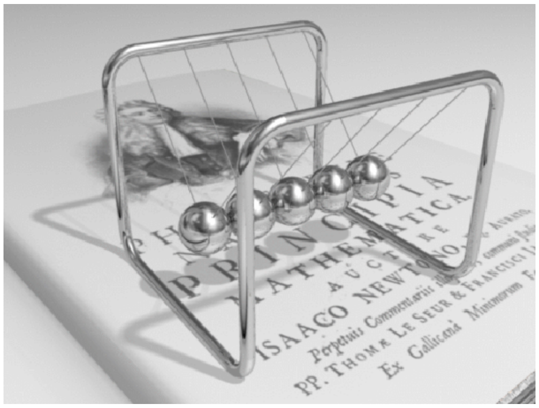
Net external forces (that are nonzero) change the total momentum of the system, while internal forces do not.

learning objectives

- Contrast the effects of external and internal forces on linear momentum and collisions

Linear Momentum and Collisions

Newton's 2nd law, applied to an isolated system composed of particles, $\mathrm{F_{tot}} = \frac{\mathrm{dp_{tot}}}{\mathrm{dt}} = 0$ indicates that the total momentum of the entire system $\mathrm{p_{tot}}$ should be constant in the absence of net external forces. Forces external to the system may change the total momentum when their sum is not 0, but internal forces, regardless of the nature of the forces, will not contribute to the change in the total momentum. To analyze a mechanical system, it is important to recognize which forces are internal and which are external. Once a mechanical system is clearly defined, it's not hard to understand what part should be considered external.



Newton's Cradle: Total momentum of the system (or Cradle) is conserved. (neglecting frictional loss in the system.)

- External forces: forces caused by external agent outside of the system.
- Internal forces: forces exchanged by the particles in the system.

To give you a better idea, let's consider a simple example. We have two hockey pucks sliding across a frictionless surface, and we neglect air resistance for simplicity. They collide with each other at t=0.

Let's first list all the forces present in the system. There are mainly three kinds of forces: Gravity, normal force (between ice & pucks), and frictional forces during the collision between the pucks

How should we define our system? In most cases, we would be interested in the motion of the pucks (and nothing else). Therefore, our system consists of two pucks (and nothing else). All the rest of the universe becomes external. With this in mind, we can see that gravity and normal forces are external, while the frictional forces between pucks are internal. Since all the external forces cancel out with each other, there are no net external forces. (Gravity and normal force on each puck have the same magnitude, but are in the opposite directions) Therefore, we conclude that the total momentum of the two pucks should be a conserved quantity.

- In the previous example, it is worthwhile to note that we didn't assume anything about the nature of the collision between the two pucks. Without knowing anything about the internal forces (frictional forces during contact), we learned that the total momentum of the system is a conserved quantity (p1 and p2 are momentum vectors of the pucks.) In fact, this relation holds true both in elastic or inelastic collisions. Whether the total kinetic energy of the pucks is conserved or not, total momentum is conserved.
- Also note that, in the previous example, if we include the rest of the Earth in our system, the gravity and normal forces themselves become internal.

Key Points

- External forces are forces caused by external agent outside of the system.
- Internal forces are forces exchanged by the objects in the system.
- To determine what part should be considered external and internal, mechanical system should be clearly defined.

Key Terms

- **inelastic**: (As referring to an inelastic collision, in contrast to an elastic collision.) A collision in which kinetic energy is not conserved.
- **elastic**: referring to elastic collision, in contrast to inelastic collision. A collision in which kinetic energy is conserved

7.3: Collisions

Conservation of Energy and Momentum

In an inelastic collision the total kinetic energy after the collision is not equal to the total kinetic energy before the collision.

learning objectives

- Assess the conservation of total momentum in an inelastic collision

At this point we will expand our discussion of inelastic collisions in one dimension to inelastic collisions in multiple dimensions. It is still true that the total kinetic energy after the collision is not equal to the total kinetic energy before the collision. While inelastic collisions may not conserve total kinetic energy, they do conserve total momentum.

We will consider an example problem in which one mass ($\mathrm{m_1}$) slides over a frictionless surface into another initially stationary mass ($\mathrm{m_2}$). Air resistance will be neglected. The following things are known:

$$\mathrm{m_1 = 0.250 kg,} \tag{7.3.1}$$

$$\mathrm{m_2 = 0.400 kg,} \tag{7.3.2}$$

$$\mathrm{v_1 = 2.00 m/s,} \tag{7.3.3}$$

$$\mathrm{v_1\,' = 1.50 m/s,} \tag{7.3.4}$$

$$\mathrm{v_2 = 0 m/s,} \tag{7.3.5}$$

$$\mathrm{\theta_1\,' = 45.0 \circ,} \tag{7.3.6}$$

where $\mathrm{v_1}$ is the initial velocity of the first mass, $\mathrm{v_1\,'}$ is the final velocity of the first mass, $\mathrm{v_2}$ is the initial velocity of the second mass, and $\mathrm{\theta_1\,'}$ is the angle between the velocity vector of the first mass and the x-axis.

The object is to calculate the magnitude and direction of the velocity of the second mass. After this, we will calculate whether this collision was inelastic or not.

Since there are no net forces at work (frictionless surface and negligible air resistance), there must be conservation of total momentum for the two masses. Momentum is equal to the product of mass and velocity. The initially stationary mass contributes no initial momentum. The components of velocities along the x-axis have the form $\mathrm{v \cdot \cos \theta}$, where θ is the angle between the velocity vector of the mass of interest and the x-axis.

Expressing these things mathematically:

$$\mathrm{m_1 v_1 = m_1 v_1\,' \cdot \cos(\theta_1) + m_2 v`_2 \cdot \cos(\theta_2). (Eq.\ 2)} \tag{7.3.7}$$

The components of velocities along the y-axis have the form $\mathrm{v \cdot \sin \theta}$, where θ is the angle between the velocity vector of the mass of interest and the x-axis. By applying conservation of momentum in the y-direction we find:

$$\mathrm{0 = m_1 v_1\,' \cdot \sin(\theta_1) + m_2 v`_2 \cdot \sin(\theta_2). (Eq.\ 3)} \tag{7.3.8}$$

If we divide Eq. 3 by Eq. 2, we will find:

$$\mathrm{\tan \theta_2 = \frac{v_1\,' \cdot \sin \theta_1}{v_1\,' \cos \theta \theta_1 - v_1} (Eq.\ 4)} \tag{7.3.9}$$

Eq. 4 can then be solved to find $\theta_2$approx. 312°.

Now let' use Eq. 3 to solve for $\mathrm{v\,'_2}$. Re-arranging Eq. 3, we find:

$$\mathrm{v_2\,' = \frac{-m_1 v_1\,' \cdot \sin \theta_1}{m_2 \cdot \sin \theta_2.}} \tag{7.3.10}$$

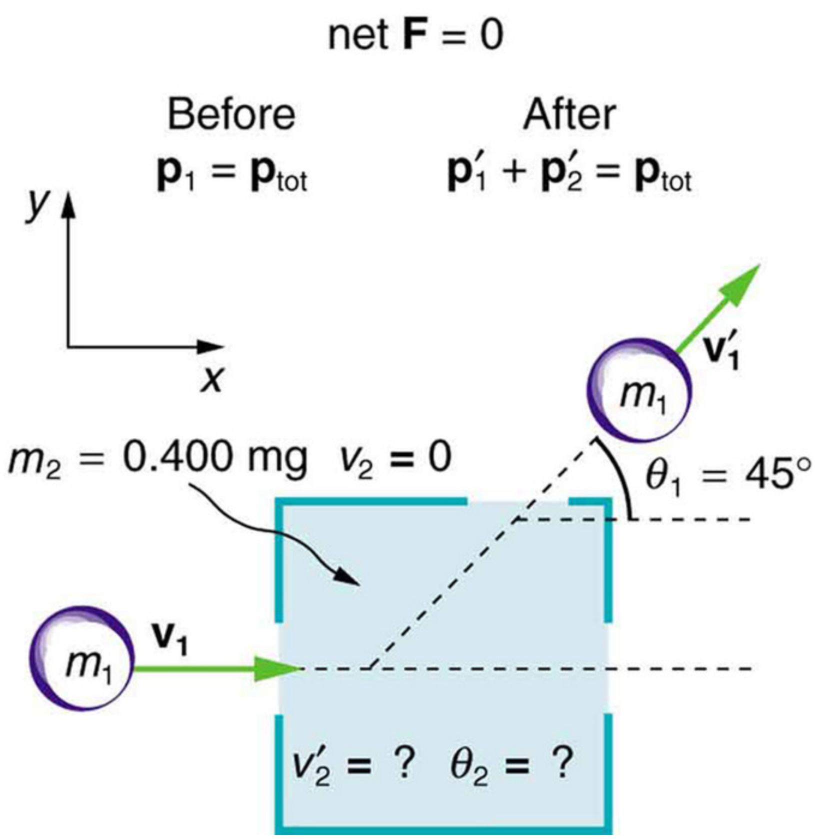
After plugging in our known values, we find that $\mathrm{v_2\,' = 0.886 m/s}$.

We can now calculate the initial and final kinetic energy of the system to see if it the same.

$$\text{Initial Kinetic Energy} = \frac{1}{2}\mathrm{m}_1 \cdot \mathrm{v}_1^2 + \frac{1}{2}\mathrm{m}_2 \cdot \mathrm{v}_2^2 = 0.5\mathrm{J}. \tag{7.3.11}$$

$$\text{Final Kinetic Energy} = \frac{1}{2}\mathrm{m}_1 \cdot \mathrm{v}_1{'}^2 + \frac{1}{2}\mathrm{m}_2 \cdot \mathrm{v}_2{'}^2 \approx 0.43\mathrm{J}. \tag{7.3.12}$$

Since these values are not the same we know that it was an inelastic collision.



Collision Example: This illustrates the example problem in which one mass collides into another mass that is initially stationary.

Glancing Collisions

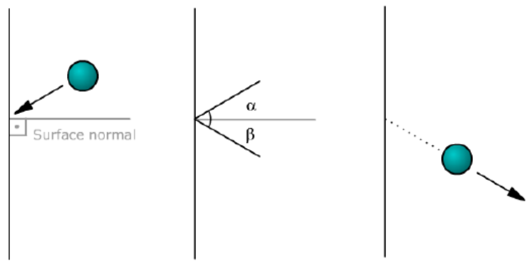
Glancing collision is a collision that takes place under a small angle, with the incident body being nearly parallel to the surface.

learning objectives

- Identify necessary conditions for a “glancing collision”

A collision is short duration interaction between two bodies or more than two bodies simultaneously causing change in motion of bodies involved due to internal forces acted between them during this. Collisions involve forces (there is a change in velocity). The magnitude of the velocity difference at impact is called the closing speed. All collisions conserve momentum. What distinguishes different types of collisions is whether they also conserve kinetic energy. Line of impact – It is the line which is common normal for surfaces are closest or in contact during impact. This is the line along which internal force of collision acts during impact and Newton’s coefficient of restitution is defined only along this line.

When dealing with an incident body that is nearly parallel to a surface, it is sometimes more useful to refer to the angle between the body and the surface, rather than that between the body and the surface normal (see), in other words 90° minus the angle of incidence. This small angle is called a glancing angle. Collision at glancing angle is called “glancing collision”.



Collision: Object is deflected after the collision withthe surface. The angles between the body and the surface normal areindicated as α and β. The angles between the body and the surface are $90-\alpha$ and $90-\beta$.

Collisions can either be elastic, meaning they conserve both momentum and kinetic energy, or inelastic, meaning they conserve momentum but not kinetic energy. An inelastic collision is sometimes also called a plastic collision.

A "perfectly-inelastic" collision (also called a "perfectly-plastic" collision) is a limiting case of inelastic collision in which the two bodies stick together after impact.

The degree to which a collision is elastic or inelastic is quantified by the coefficient of restitution, a value that generally ranges between zero and one. A perfectly elastic collision has a coefficient of restitution of one; a perfectly-inelastic collision has a coefficient of restitution of zero.

Elastic Collisions in One Dimension

An elastic collision is a collision between two or more bodies in which kinetic energy is conserved.

learning objectives

- Assess the relationship among the collision equations to derive elasticity

An elastic collision is a collision between two or more bodies in which the total kinetic energy of the bodies before the collision is equal to the total kinetic energy of the bodies after the collision. An elastic collision will not occur if kinetic energy is converted into other forms of energy. It important to understand how elastic collisions work, because atoms often undergo essentially elastic collisions when they collide. On the other hand, molecules do not undergo elastic collisions when they collide. In this atom we will review case of collision between two bodies.

The mathematics of an elastic collision is best demonstrated through an example. Consider a first particle with mass $\mathrm{m_1}$ and velocity $\mathrm{v_{1i}}$ and a second particle with mass $\mathrm{m_2}$ and velocity $\mathrm{v_{2i}}$. If these two particles collide, there must be conservation of momentum before and after the collision. If we know that this is an elastic collision, there must be conservation of kinetic energy by definition. Therefore, the velocities of particles 1 and 2 after the collision ($\mathrm{v_{1f}}$ and $\mathrm{v_{2f}}$ respectively) will be related to the initial velocities by:

$\frac{1}{2}\mathrm{m_1\cdot v_{1i}^2}+\frac{1}{2}\mathrm{m_2\cdot v_{2i}^2}=\frac{1}{2}\mathrm{m_1\cdot v_{1f}^2}+\frac{1}{2}\mathrm{m_2\cdot v_{2f}^2}$ (due to conservation of kinetic energy)

and

$\mathrm{m_1\cdot v_{1i}+m_2\cdot v_{2i}=m_1\cdot v_{1f}+m_2\cdot v_{2f}}$ (due to conservation of momentum).

Since we have two equations, we are able to solve for any two unknown variables. In our case, we will solve for the final velocities of the two particles.

By grouping like terms and canceling out the ½ terms, we can rewrite our conservation of kinetic energy equation as:

$$\mathrm{m_1\cdot(v_{1i}^2-v_{1f}^2)=m_2\cdot(v_{2f}^2-v_{2i}^2).\ (Eq.1)} \tag{7.3.13}$$

By grouping like terms from our conservation of momentum equation we can find:

$$\mathrm{m_1\cdot(v_{1i}-v_{1f})=m_2\cdot(v_{2f}-v_{2i}).\ (Eq.\ 2)} \tag{7.3.14}$$

If we then divide Eq. 1 by Eq. 2 and perform some cancelations we will find:

$$\mathrm{v_{1i}+v_{1f}=v_{2f}+v_{2i}.\ (Eq.\ 3)} \tag{7.3.15}$$

We can solve for v_{1f} as:

$$v_{1f} = v_{2f} + v_{2i} - v_{1i}. \text{ (Eq. 4)} \tag{7.3.16}$$

At this point we see that v_{2f} is still an unknown variable. So we can fix this by plugging Eq. 4 into our initial conservation of momentum equation. Our conservation of momentum equation with Eq. 4 substituted in looks like:

$$m_1 \cdot v_{1i} + m_2 \cdot v_{2i} = m_1 \cdot (v_{2f} + v_{2i} - v_{1i}) + m_2 \cdot v_{2f}. \text{ (Eq.5)} \tag{7.3.17}$$

After doing a little bit of algebra on Eq. 5 we find:

$$v_{2f} = \frac{2 \cdot m_1}{(m_2 + m_1)} v_{1i} + \frac{(m_2 - m_1)}{(m_2 + m_1)} v_{2i}. \text{ (Eq.6)} \tag{7.3.18}$$

At this point we have successfully solved for the final velocity of the second particle. We still need to solve for the velocity of the first particle, so let us do that by plugging Eq. 6 into Eq. 4.

$$v_{1f} = [\frac{2 \cdot m_1}{(m_2 + m_1)} v_{1i} + \frac{(m_2 - m_1)}{(m_2 + m_1)} v_{2i}] + v_{2i} - v_{1i}. \text{ (Eq. 7)} \tag{7.3.19}$$

After performing some algebraic manipulation of Eq. 7, we finally find:

$$v_{1f} = \frac{(m_1 - m_2)}{(m_2 + m_1)} v_{1i} + \frac{2 \cdot m_2}{(m_2 + m_1)} v_{2i}. \text{ (Eq. 8)} \tag{7.3.20}$$

Elastic Collision of Two Unequal Masses: In this animation, two unequal masses collide and recoil.

Elastic Collisions in Multiple Dimensions

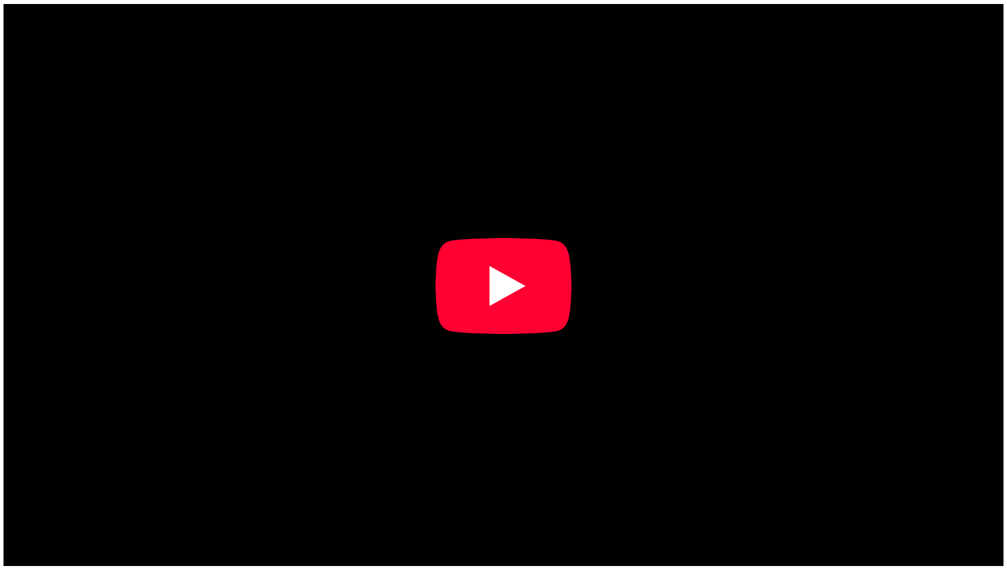
To solve a two dimensional elastic collision problem, decompose the velocity components of the masses along perpendicular axes.

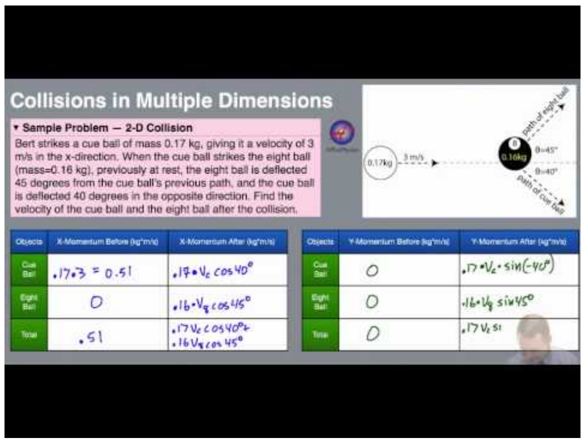
learning objectives

- Construct an equation for elastic collision

Overview

As stated previously, there is conservation of total kinetic energy before and after an elastic collision. If an elastic collision occurs in two dimensions, the colliding masses can travel side to side after the collision (not just along the same line as in a one dimensional collision). The general approach to solving a two dimensional elastic collision problem is to choose a coordinate system in which the velocity components of the masses can be decomposed along perpendicular axes.





Collisions in Multiple Dimensions: A brief introduction to problem solving of collisions in two dimensions using the law of conservation of momentum.

Example 7.3.1:

In this example, we consider only point masses. These are structure-less particles that cannot spin or rotate. We will consider a case in which no outside forces are acting on the system, meaning that momentum is conserved. We will consider a situation in which one particle is initially at rest. This situation is illustrated in.

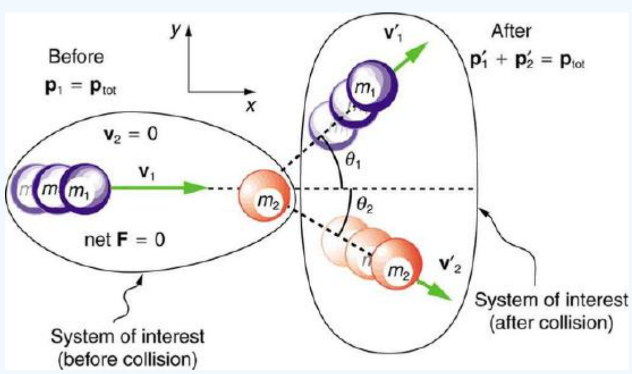


Illustration of Elastic Collision in Two Dimensions: In this illustration, we see the initial and final configurations of two masses that undergo an elastic collision in two dimensions.

By defining the x-axis to be along the direction of the incoming particle, we save ourselves time in breaking that velocity vector into its x- and y- components. Now let us consider conservation of momentum in the x direction:

$$p_{1x} + p_{2x} = p_{1x}{}' + p_{2x}{}' (\mathrm{Eq.}\ 1) \tag{7.3.21}$$

In Eq. 1, the initial momentum of the incoming particle is represented by $p_{1x,}$ the initial momentum of the stationary particle is represented by $p_{2x,}$ the final momentum of the incoming particle is represented by $p'_{1x.}$ and the final momentum of the initially stationary particle is represented by $p'_{2x.}$

We can expand Eq. 1 by taking into account that momentum is equal to the product of mass and velocity. Also, we know that $p_{2x} = 0$ because the initial velocity of the stationary particle is 0.

The components of velocities along the x-axis have the form $v \cdot \cos\theta$, where θ is the angle between the velocity vector of the particle of interest and the x-axis.

Therefore:

$$m_1 v_1 = m_1 v_1{}' \cdot \cos(\theta_1) + m_2 v'_2 \cdot \cos(\theta_2) (\mathrm{Eq.}\ 2) \tag{7.3.22}$$

The components of velocities along the y-axis have the form $v \cdot \sin\theta$, where θ is the angle between the velocity vector of the particle of interest (denoted in the following equations by subscript 1 or 2) and the x-axis. We can apply conservation of momentum in the y-direction in a similar way to yield:

$$0 = m_1 v'_1 \cdot \sin(\theta_1) + m_2 v'_2 \cdot \sin(\theta_2) (\mathrm{Eq.}\ 3) \tag{7.3.23}$$

In finding Eq. 3, it was taken into consideration that the incoming particle had no component of velocity along the y-axis.

Solving for Two Unknowns

Now we have gotten to a point where we have two equations, this means that we can solve for any two unknowns that we want. We also know that because the collision is elastic that there must be conservation of kinetic energy before and after the collision. This means that we may also write Eq. 4, which gives us three equations to solve for three unknowns:

$$\frac{1}{2} m_1 \cdot v_1^2 + \frac{1}{2} m_2 \cdot v_2^2 = \frac{1}{2} m_1 \cdot v_1{}'^2 + \frac{1}{2} m_2 \cdot v_2{}'^2 \tag{7.3.24}$$

The general approach to finding the defining equations for an n-dimensional elastic collision problem is to apply conservation of momentum in each of the n- dimensions. You can generate an additional equation by utilizing conservation of kinetic energy.

Inelastic Collisions in One Dimension

Collisions may be classified as either inelastic or elastic collisions based on how energy is conserved in the collision.

learning objectives

- Distinguish examples of inelastic collision from elastic collisions

Overview

In an inelastic collision the total kinetic energy after the collision is not equal to the total kinetic energy before the collision. This is in contrast to an elastic collision in which conservation of total kinetic energy applies. While inelastic collisions may not conserve total kinetic energy, they do conserve total momentum.

Collisions

If two objects collide, there are many ways that kinetic energy can be transformed into other forms of energy. For example, in the collision of macroscopic bodies, some kinetic energy is turned into vibrational energy of the constituent atoms. This causes a heating effect and results in deformation of the bodies. Another example in which kinetic energy is transformed into another form of energy is when the molecules of a gas or liquid collide. When this happens, kinetic energy is often exchanged between the molecules' translational motion and their internal degrees of freedom.

A perfectly inelastic collision happens when the maximum amount of kinetic energy in a system is lost. In such a collision, the colliding particles stick together. The kinetic energy is used on the bonding energy of the two bodies.

Sliding Block Example

Let us consider an example of a two-body sliding block system. The first block slides into the second (initially stationary block). In this perfectly inelastic collision, the first block bonds completely to the second block as shown. We assume that the surface over which the blocks slide has no friction. We also assume that there is no air resistance. If the surface had friction or if there was air resistance, one would have to account for the bodies' momentum that would be transferred to the surface and/or air.

Inelastic Collision: In this animation, one mass collides into another initially stationary mass in a perfectly inelastic collision.

Writing about the equation for conservation of momentum, one finds:

$$\mathrm{m_a u_a + m_b u_b = (m_a + m_b) v} \tag{7.3.25}$$

where $m_a$is the mass of the incoming block, u_a is the velocity of the incoming block, $m_b$is the mass of the initially stationary block, $u_b$is the velocity of initially stationary block (0 m/s), and v is the final velocity the two body system. Solving for the final velocity,

$$\mathrm{v = \frac{m_a u_a + m_b u_b}{m_a + m_b}}. \tag{7.3.26}$$

Taking into account that the blocks have the same mass and that the one of the blocks is initially stationary, the expression for the final velocity of the system may be defined as:

$$\mathrm{v = \frac{u_a}{2}}. \tag{7.3.27}$$

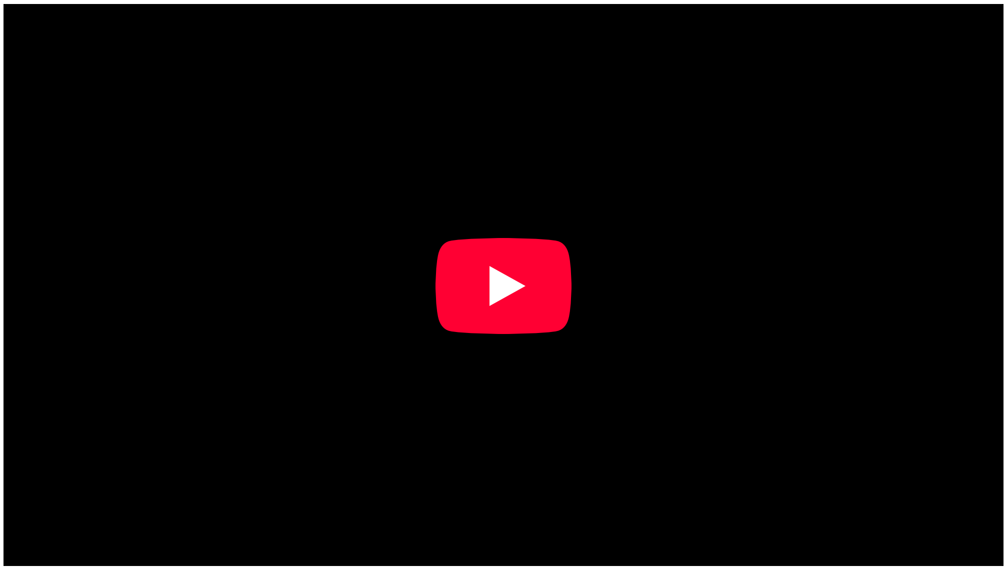
Inelastic Collisions in Multiple Dimensions

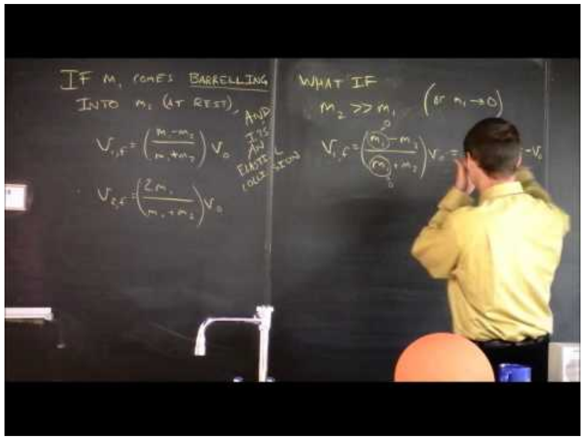
While inelastic collisions may not conserve total kinetic energy, they do conserve total momentum.

learning objectives

- Relate inelastic collision multiple dimension equations to the one dimension collisions you learned earlier

At this point we will expand our discussion of inelastic collisions in one dimension to inelastic collisions in multiple dimensions. It is still true that the total kinetic energy after the collision is not equal to the total kinetic energy before the collision. While inelastic collisions may not conserve total kinetic energy, they do conserve total momentum.

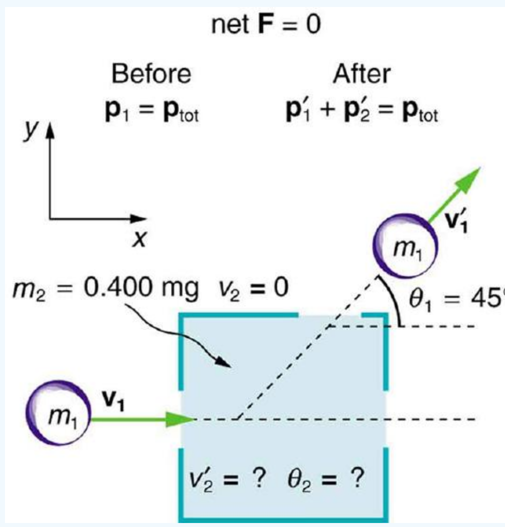




Example 7.3.2:

Examples of Collisions

We will consider an example problem, illustrated in, in which one mass (m1m1) slides over a frictionless surface into another initially stationary mass (m2m2). Air resistance will be neglected. The following quantities are known:



Collision Example: This illustrates the example problem in which one mass collides into another mass that is initially stationary.

$$m_1 = 0.250 kg, \tag{7.3.28}$$

$$m_2 = 0.400 kg, \tag{7.3.29}$$

$$v_1 = 2.00 m/s, \tag{7.3.30}$$

$$v_1{}' = 1.50 m/s, \tag{7.3.31}$$

$$v_2 = 0 m/s, \tag{7.3.32}$$

$$\theta_1{}' = 45.0^{\circ}, \tag{7.3.33}$$

where v1v1 is the initial velocity of the first mass, v′1v1′ is the final velocity of the first mass, v2v2is the initial velocity of the second mass, and θ′1θ1′ is the angle between the velocity vector of the first mass and the x-axis.

The object is to calculate the magnitude and direction of the velocity of the second mass. After this, we will calculate whether this collision was inelastic or not.

Since there are no net forces at work (frictionless surface and negligible air resistance), there must be conservation of total momentum for the two masses. Momentum is equal to the product of mass and velocity. The (initially) stationary mass contributes no initial momentum. The components of velocities along the x-axis have the form v·cosθv·cosθ, where θ is the angle between the velocity vector of the mass of interest and the x-axis.

Expressing these things mathematically:

$$\mathrm{m_1 v_1 = m_1 v_1{}' \cdot \cos(\theta_1) + m_2 v`_2 \cdot \cos(\theta_2). \,(Eq.\, 2)} \tag{7.3.34}$$

The components of velocities along the y-axis have the form $\mathrm{v} \cdot \sin\theta$, where θ is the angle between the velocity vector of the mass of interest and the x-axis. By applying conservation of momentum in the y-direction we find:

$$\mathrm{0 = m_1 v_1{}' \cdot \sin(\theta_1) + m_2 v`_2 \cdot \sin(\theta_2). \,(Eq.\, 3)} \tag{7.3.35}$$

If we divide Eq. 3 by Eq. 2, we will find:

$$\tan\theta_2 = \frac{\mathrm{v_1}{}' \cdot \sin\theta_1}{\mathrm{v_1}{}' \cos\theta\theta_1 - \mathrm{v_1}} \,(\mathrm{Eq.\, 4}) \tag{7.3.36}$$

Eq. 4 can then be solved to find $\theta_2$approx. 312º.

Now let’ use Eq. 3 to solve for v'_2. Re-arranging Eq. 3, we find:

$$\mathrm{v_2}{}' = \frac{-\mathrm{m_1 v_1}{}' \cdot \sin\theta_1}{\mathrm{m_2} \cdot \sin\theta_2.} \tag{7.3.37}$$

After plugging in our known values, we find that $\mathrm{v_2}{}' = 0.886 \mathrm{m/s}$.

We can now calculate the initial and final kinetic energy of the system to see if it the same.

$$\text{Initial Kinetic Energy} = \frac{1}{2}\mathrm{m_1} \cdot \mathrm{v}_1^2 + \frac{1}{2}\mathrm{m_2} \cdot \mathrm{v}_2^2 = 0.5\mathrm{J}. \tag{7.3.38}$$

$$\text{Final Kinetic Energy} = \frac{1}{2}\mathrm{m_1} \cdot \mathrm{v_1}{}'^2 + \frac{1}{2}\mathrm{m_2} \cdot \mathrm{v_2}{}'^2 \approx 0.43\mathrm{J}. \tag{7.3.39}$$

As these values are not the same, we know this was an inelastic collision.

Key Points

- In an inelastic collision the total kinetic energy after the collision is not equal to the total kinetic energy before the collision.
- If there are no net forces at work (collision takes place on a frictionless surface and there is negligible air resistance), there must be conservation of total momentum for the two masses.
- The variable θ is the angle between the velocity vector of the mass of interest and the x-axis in traditional Cartesian coordinate systems.
- Collision is short duration interaction between two bodies or more than two bodies simultaneously causing change in motion of bodies involved due to internal forces acted between them during this.
- Collisions can either be elastic, meaning they conserve both momentum and kinetic energy, or inelastic, meaning they conserve momentum but not kinetic energy.
- When dealing with an incident body that is nearly parallel to a surface, it is sometimes more useful to refer to the angle between the body and the surface, rather than that between the body and the surface normal.
- An elastic collision will not occur if kinetic energy is converted into other forms of energy.
- While molecules do not undergo elastic collisions, atoms often undergo elastic collisions when they collide.
- If two particles are involved in an elastic collision, the velocity of the first particle after collision can be expressed as: $\mathrm{v_{1f}} = \frac{(\mathrm{m_1 - m_2})}{(\mathrm{m_2 + m_1})}\mathrm{v_{1i}} + \frac{2 \cdot \mathrm{m_2}}{(\mathrm{m_2 + m_1})}\mathrm{v_{2i}}$.
- If two particles are involved in an elastic collision, the velocity of the second particle after collision can be expressed as: $\mathrm{v_{2f}} = \frac{2 \cdot \mathrm{m_1}}{(\mathrm{m_2 + m_1})}\mathrm{v_{1i}} + \frac{(\mathrm{m_2 - m_1})}{(\mathrm{m_2 + m_1})}\mathrm{v_{2i}}$.
- If an elastic collision occurs in two dimensions, the colliding masses can travel side to side after the collision.
- By defining the x- axis to be along the direction of the incoming particle, we can simplify the defining equations.
- The general approach to finding the defining equations for an n-dimensional elastic collision problem is to apply conservation of momentum in each of the n- dimensions. You can generate an additional equation by utilizing conservation of kinetic energy.
- In an inelastic collision, the total kinetic energy after the collision is not equal to the total kinetic energy before the collision.
- While inelastic collisions may not conserve total kinetic energy, they do conserve total momentum.

- A perfectly inelastic collision happens when the maximum amount of kinetic energy in a system is lost.
- In an inelastic collision the total kinetic energy after the collision is not equal to the total kinetic energy before the collision.
- If there are no net forces at work (i.e., collision takes place on a frictionless surface and there is negligible air resistance), there must be conservation of total momentum for the two masses.
- The variable θ is the angle between the velocity vector of the mass of interest and the x-axis in traditional Cartesian coordinate systems.

Key Terms

- **kinetic energy**: The energy possessed by an object because of its motion, equal to one half the mass of the body times the square of its velocity.
- **momentum**: (of a body in motion) the product of its mass and velocity.
- **force**: A physical quantity that denotes ability to push, pull, twist or accelerate a body which is measured in a unit dimensioned in mass × distance/time^2 (ML/T^2): SI: newton (N); CGS: dyne (dyn)
- **elastic collision**: An encounter between two bodies in which the total kinetic energy of the two bodies after the encounter is equal to their total kinetic energy before the encounter. Elastic collisions occur only if there is no net conversion of kinetic energy into other forms.
- **dimension**: A measure of spatial extent in a particular direction, such as height, width or breadth, or depth.
- **degrees of freedom**: A degree of freedom is an independent physical parameter, often called a dimension, in the formal description of the state of a physical system. The set of all dimensions of a system is known as a phase space.
- **friction**: A force that resists the relative motion or tendency to such motion of two bodies in contact.

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7.4: Rocket Propulsion

Rocket Propulsion, Changing Mass, and Momentum

In rocket propulsion, matter is forcefully ejected from a system, producing an equal and opposite reaction on what remains.

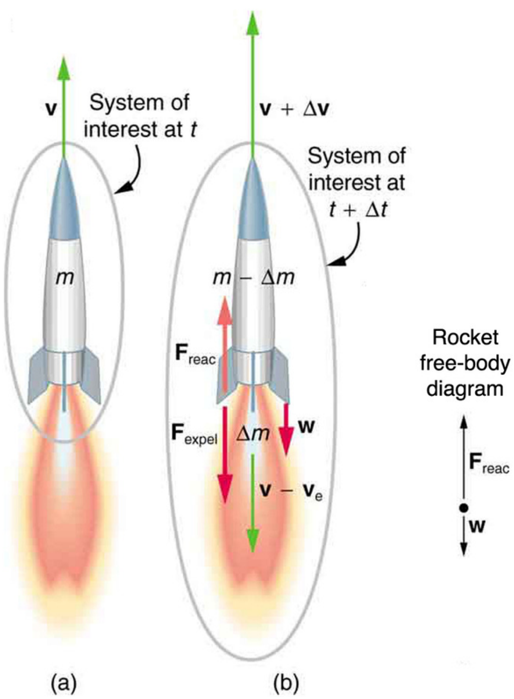
learning objectives

- Identify physical principles of rocket propulsion

Rocket Propulsion, Changing Mass, and Momentum

Rockets range in size from fireworks so small that ordinary people use them to immense Saturn Vs that once propelled massive payloads toward the Moon. The propulsion of all rockets, jet engines, deflating balloons, and even squids and octopuses is explained by the same physical principle: Newton's third law of motion. Matter is forcefully ejected from a system, producing an equal and opposite reaction on what remains. Another common example is the recoil of a gun. The gun exerts a force on a bullet to accelerate it and consequently experiences an equal and opposite force, causing the gun's recoil or kick.

shows a rocket accelerating straight up. In part (a), the rocket has a mass m and a velocity v relative to Earth, and hence a momentum mv. In part (b), a time Δt has elapsed in which the rocket has ejected a mass Δm of hot gas at a velocity v_e relative to the rocket. The remainder of the mass $(m - m)$ now has a greater velocity $(v + \Delta v)$. The momentum of the entire system (rocket plus expelled gas) has actually decreased because the force of gravity has acted for a time Δt, producing a negative impulse $\Delta p = -mg\Delta t$. (Remember that impulse is the net external force on a system multiplied by the time it acts, and it equals the change in momentum of the system.) So the center of mass of the system is in free fall but, by rapidly expelling mass, part of the system can accelerate upward. It is a commonly held misconception that the rocket exhaust pushes on the ground. If we consider thrust; that is, the force exerted on the rocket by the exhaust gases, then a rocket's thrust is greater in outer space than in the atmosphere or on the launch pad. In fact, gases are easier to expel into a vacuum.



Free-body diagram of rocket propulsion: (a) This rocket has a mass m and an upward velocity v. The net external force on the system is −mg, if air resistance is neglected. (b) A time Δt later the system has two main parts, the ejected gas and the remainder of the rocket. The reaction force on the rocket is what overcomes the gravitational force and accelerates it upward.

By calculating the change in momentum for the entire system over Δt, and equating this change to the impulse, the following expression can be shown to be a good approximation for the acceleration of the rocket.

$$a=\frac{v_e}{m}\frac{\Delta m}{\Delta t}-g \tag{7.4.1}$$

where a is the acceleration of the rocket, v_e is the escape velocity, m is the mass of the rocket, Δm is the mass of the ejected gas, and Δt is the time in which the gas is ejected.

Factors of Acceleration

A rocket's acceleration depends on three major factors, consistent with the equation for acceleration of a rocket. First, the greater the exhaust velocity of the gases relative to the rocket, v_e, the greater the acceleration is. The practical limit for v_e is about 2.5×10^3m/s for conventional (non-nuclear) hot-gas propulsion systems. The second factor is the rate at which mass is ejected from the rocket. This is the factor $\frac{\Delta m}{\Delta t}$ in the equation. The quantity $(\frac{\Delta m}{\Delta t})v_e$, with units of newtons, is called "thrust." The faster the rocket burns its fuel, the greater its thrust, and the greater its acceleration. The third factor is the mass m of the rocket. The smaller the mass is (all other factors being the same), the greater the acceleration. The rocket mass m decreases dramatically during flight because most of the rocket is fuel to begin with, so that acceleration increases continuously, reaching a maximum just before the fuel is exhausted.

To achieve the high speeds needed to hop continents, obtain orbit, or escape Earth's gravity altogether, the mass of the rocket other than fuel must be as small as possible. It can be shown that, in the absence of air resistance and neglecting gravity, the final velocity of a one-stage rocket initially at rest is

$$v=v_e\ln\frac{m_0}{m_r} \tag{7.4.2}$$

where $\ln(m_0/m_r)$ is the natural logarithm of the ratio of the initial mass of the rocket (m_0) to what is left (m_r) after all of the fuel is exhausted. (Note that v is actually the change in velocity, so the equation can be used for any segment of the flight. If we start from rest, the change in velocity equals the final velocity.)

Key Points

- The propulsion of all rockets is explained by the same physical principle: Newton's third law of motion.
- A rocket's acceleration depends on three major factors: the exhaust velocity, the rate the exhaust is ejected, and the mass of the rocket.
- To achieve the high speeds needed to hop continents, obtain orbit, or escape Earth's gravity altogether, the mass of the rocket other than fuel must be as small as possible.

Key Terms

- **Newton's third law of motion**: states that all forces exist in pairs: if one object A exerts a force FA on a second object B, then B simultaneously exerts a force FB on A, and the two forces are equal and opposite: $FA=-FB$.

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7.5: Center of Mass

learning objectives

- Identify the center of mass for an object with continuous mass distribution

In the previous modules on "Center of Mass and Translational Motion," we learned why the concept of center of mass (COM) helps solving mechanics problems involving a rigid body. Here, we will study the rigorous definition of COM and how to determine the location of it. The position of COM is mass weighted average of the positions of particles.

Definition: center of mass

The *center of mass* is a statement of spatial arrangement of mass (i.e. distribution of mass within the system). The position of COM is given a mathematical formulation which involves distribution of mass in space:

$$\mathrm{r_{COM}} = \frac{\sum_i \mathrm{m_i r_i}}{\mathrm{M}}, \tag{7.5.1}$$

where $\mathrm{r_{COM}}$ and $\mathrm{r_i}$ are vectors representing the position of COM and i-th particle respectively, and M and mi are the total mass and mass of the i-th particle, respectively. This mean means that position of COM is mass weighted average of the positions of particles.

Object with Continuous Mass Distribution

If the mass distribution is continuous with the density ρ(r) within a volume V, the position of COM is given as

$$\mathrm{r_{COM}} = \frac{1}{\mathrm{M}} \int_{\mathrm{V}} \rho(\mathrm{r})\mathrm{r}\mathrm{dV}, \tag{7.5.2}$$

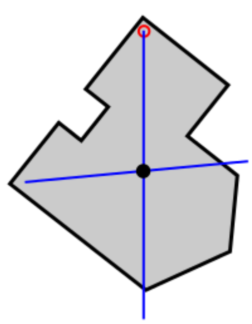
where M is the total mass in the volume. If a continuous mass distribution has uniform density, which means ρ is constant, then the center of mass is the same as the center of the volume.

Locating the Center of Mass

The experimental determination of the center of mass of a body uses gravity forces on the body and relies on the fact that in the parallel gravity field near the surface of Earth the center of mass is the same as the center of gravity.

The center of mass of a body with an axis of symmetry and constant density must lie on this axis. Thus, the center of mass of a circular cylinder of constant density has its center of mass on the axis of the cylinder. In the same way, the center of mass of a spherically symmetric body of constant density is at the center of the sphere. In general, for any symmetry of a body, its center of mass will be a fixed point of that symmetry.

In two dimensions: An experimental method for locating the center of mass is to suspend the object from two locations and to drop plumb lines from the suspension points. The intersection of the two lines is the center of mass.



Plumb Line Method for Center of Mass: Suspend the object from two locations and to drop plumb lines from the suspension points. The intersection of the two lines is the center of mass.

In three dimensions: By supporting an object at three points and measuring the forces that resist the weight of the object, COM of the three-dimensional coordinates of the center of mass can be determined.

Motion of the Center of Mass

We can describe the translational motion of a rigid body as if it is a point particle with the total mass located at the COM—center of mass.

learning objectives

- Derive the center of mass for the translational motion of a rigid body

We can describe the translational motion of a rigid body as if it is a point particle with the total mass located at the center of mass (COM). In this Atom. we will prove that the total mass (M) times the acceleration of the COM (a_{COM}), indeed, equals the sum of external forces. That is,

$$\mathrm{M} \cdot \mathrm{a_{COM}} = \sum \mathrm{F_{ext}}. \tag{7.5.3}$$

You can see that the Newton's 2nd law applies as if we are describing the motion of a point particle (with mass M) under the influence of the external force.

Derivation

From the definition of the center of mass,

$$\mathrm{r_{COM}} = \frac{\sum_{\mathrm{i}} \mathrm{m_i r_i}}{\mathrm{M}}, \tag{7.5.4}$$

we get $\mathrm{M} \cdot \mathrm{a_{COM}} = \sum \mathrm{m_i a_i}$ by taking time derivative twice on each side.

Note that $\sum \mathrm{m_i a_i} = \sum \mathrm{F_i}$.

In a system of particles, each particle may feel both external and internal forces. Here, external forces are forces from external sources, while internal forces are forces between particles in the system. Since the sum of all internal forces will be 0 due to the Newton's 3rd law,

$\sum \mathrm{F_i} = \sum \mathrm{F_{i,ext}}$. Therefore, we get $\mathrm{M} \cdot \mathrm{a_{COM}} = \sum \mathrm{F_{ext}}$.

For example, when we confine our system to the Earth and the Moon, the gravitational force due to the Sun would be external, while the gravitational force on the Earth due to the Moon (and vice versa) would be internal. Since the gravitational forces between the Earth and the Moon are equal in magnitude and opposite in direction, they will cancel out each other in the sum (see).

COM of the Earth and Moon: Earth and Moon orbiting a COM inside the Earth. The red cross represents the COM of the two-body system. The COM will orbit around the Sun as if it is a point particle.

Corollary

When there is no external force, the COM momentum is conserved.

Proof: Since there is no external force,$\mathrm{M} \cdot \mathrm{a_{COM}} = 0$. Therefore,

$\mathrm{M} \cdot \mathrm{v_{COM}} = \mathrm{constant}.$

Proof

Since there is no external force,

$$M \cdot a_{COM} = 0. \tag{7.5.5}$$

Therefore,

$M \cdot v_{COM} = constant$.

□

Center of Mass of the Human Body

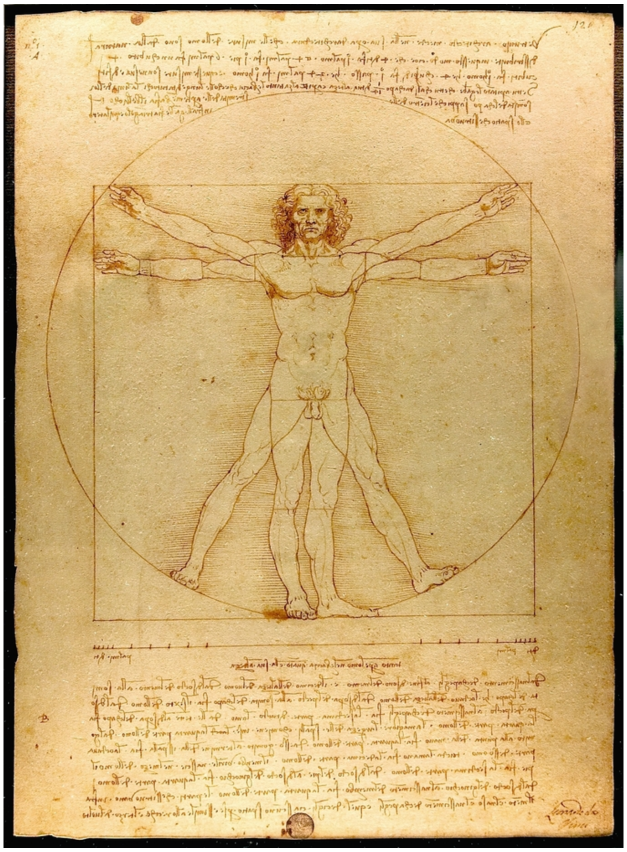
The center of mass (COM) is an important physical concept—it is the point about which objects rotate.

learning objectives

- Estimate the COM of a given object

The center of mass (COM) is an important physical concept. It is the point on an object at which the weighted relative position of the distributed mass sums to zero—the point about which objects rotate.

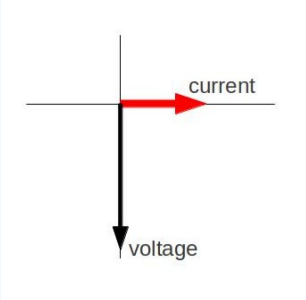
Human proportions have been important in art, measurement, and medicine (a well known drawing of the human body is seen in). Although the human body has complicated features, the location of the center of mass (COM) could be a good indicator of body proportions. The center of mass of the human body depends on the gender and the position of the limbs. In a standing posture, it is typically about 10 cm lower than the navel, near the top of the hip bones. In this Atom, we will learn how to measure the COM of a human body.



Leonardo da Vinci's "The Vitruvian Man": Vitruvian Man: A drawing created by Leonardo da Vinci. The drawing is based on the correlations of ideal human proportions with geometry described[4] by the ancient Roman architect Vitruvius in Book III of his treatise De Architectura.

Example 7.5.1:

First, let's take two scales and a wooden beam (H meter long), long enough to contain the entire body of the subject. Put the scales H meters apart, and place the beam across the scales, as illustrated in. Now, let the subject lie on the beam. Make sure that his/her heels are aligned with one end of the beam. Measure the readings (F_1, F_2) on the scale.



The COM of a Human Body: This figure demonstrates measuring the COM of a human body.

The system (person+beam) has three external forces: gravity on the subject (F_{CM}), and normal forces from the scales F_1 and F_2. The equation of motion for force (F=ma) will give us:

$$\mathrm{F1+F2=Mg,} \tag{7.5.6}$$

where M is mass of the subject. (We assume that the wooden beam has no mass.) This equation doesn't provide all the information to locate the COM. However, the equation of motion for torque $(\tau = \mathrm{I}\alpha)$ helps.

Since the net torque of the system is zero (hence no rotational acceleration),

$$\mathrm{hF_2-(H-h)F_1=0.(h:COM\ height)} \tag{7.5.7}$$

The COM is chosen as the origin for the torque. Therefore, gravity contributes nothing as a torque. Solving for h and using the equation of motion for force, we get

$$\mathrm{h=\frac{HF_1}{Mg}.} \tag{7.5.8}$$

Center of Mass and Translational Motion

The COM (center of mass) of a system of particles is a geometric point that assumes all the mass and external force(s) during motion.

learning objectives

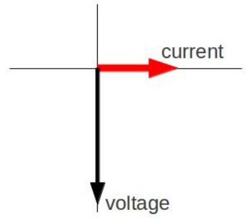
- Support the presence of COM in three dimensional bodies in motion

Introduction: COM, Linear Momentum, and Collisions

Our study of motion has been limited up to this point. We have referred to particle, object and body in the same way. We considered that actual three dimensional rigid bodies move such that all constituent particles had the same motion (i.e., same trajectory, velocity and acceleration). By doing this, we have essentially considered a rigid body as a point particle.

Center of Mass (COM)

An actual body, however, can move differently than this simplified paradigm. Consider a ball rolling down an incline plane or a stick thrown into air. Different parts of a body have different motions. While translating in the air, the stick rotates about a moving axis, as shown in. This means that such bodies may not behave like a point particle, as earlier suggested.



Forces on the COM: Left: The force appears to operate on the COM is “mgsinθ. Right: The force appears to operate on the COM is “mg”.

Describing motions of parts or particles that have different motions would be quite complicated to do in an integrated manner. However, such three dimensional bodies in motion have one surprising, simplifying characteristic—a geometric point that behaves like a particle. This point is known as center of mass, abbreviated COM (the mathematical definition of COM will be introduced in the next Atom on “Locating the Center of Mass”). It has the following two characterizing aspects:

- The center of mass appears to carry the whole mass of the body.
- At the center of mass, all external forces appear to apply.

Significantly, the center of a ball (the COM of a rolling ball) follows a straight linear path; whereas the COM of a stick follows a parabolic path (as shown in the figure above). Secondly, the forces appear to operate on the COMs in two cases (“mgsinθ and “mg”) as if they were indeed particle-like objects. This concept of COM, therefore, eliminate the complexities otherwise present in attempting to describe motions of rigid bodies.

Describing Motion in a Rigid Body

We can describe general motion of an object (with mass m) as follows:

- We describe the *translational motion* of a rigid body as if it is a point particle with mass m located at COM.
- *Rotation* of the particle, with respect to the COM, is described independently.

We “separate” the translational part of the motion from the rotational part. By introducing the concept of COM, the translational motion becomes that of a point particle with mass m. This simplifies significantly the mathematical complexity of the problem.

Key Points

- The center of mass (COM) is a statement of spatial arrangement of mass (i.e. distribution of mass within the system).
- The experimental determination of the center of mass of a body uses gravity forces on the body and relies on the fact that in the parallel gravity field near the surface of the earth the center of mass is the same as the center of gravity.
- For a 2D object, an experimental method for locating the center of mass is to suspend the object from two locations and to drop plumb lines from the suspension points. The intersection of the two lines is the center of mass.
- The total mass times the acceleration of the center of mass equals the sum of external forces.
- For the translational motion of a rigid body with mass M, Newton’s 2nd law applies as if we are describing the motion of a point particle (with mass M) under the influence of the external force.
- When there is no external force, the center of mass momentum is conserved.
- Although a human body has complicated features, the location of the center of mass (COM) could be a good indicator of the body proportions.
- We can measure the location of COM with two scales and a wooden beam. The linear and rotational equations of motion gives us the location.
- The center of mass of the human body depends on the gender and the position of the limbs. In a standing posture, it is typically about 10 cm lower than the navel, near the top of the hip bones.
- In a motion of a rigid body, different parts of the body have different motions.This means that these bodies may not behave like a point particle.
- There is a characteristic geometric point of the three dimensional body in motion. This point behaves as a particle, and is known as center of mass, abbreviated COM. COM appears to carry the whole mass of the body. All external forces appear to apply at COM.

- To describe the motion of a rigid body (with possibly a complicated geometry), we separate the translational part of the motion from the rotational part.

Key Terms

- **plumb line**: A cord with a weight attached, used to produce a vertical line.
- **rigid body**: An idealized solid whose size and shape are fixed and remain unaltered when forces are applied; used in Newtonian mechanics to model real objects.
- **center of mass**: The center of mass (COM) is the unique point at the center of a distribution of mass in space that has the property that the weighted position vectors relative to this point sum to zero.
- **torque**: A rotational or twisting effect of a force; (SI unit newton-meter or Nm; imperial unit foot-pound or ft-lb)
- **point particle**: An idealization of particles heavily used in physics. Its defining feature is that it lacks spatial extension, meaning that geometrically the particle is equivalent to a point.

CHAPTER OVERVIEW

08: Static Equilibrium, Elasticity, and Torque

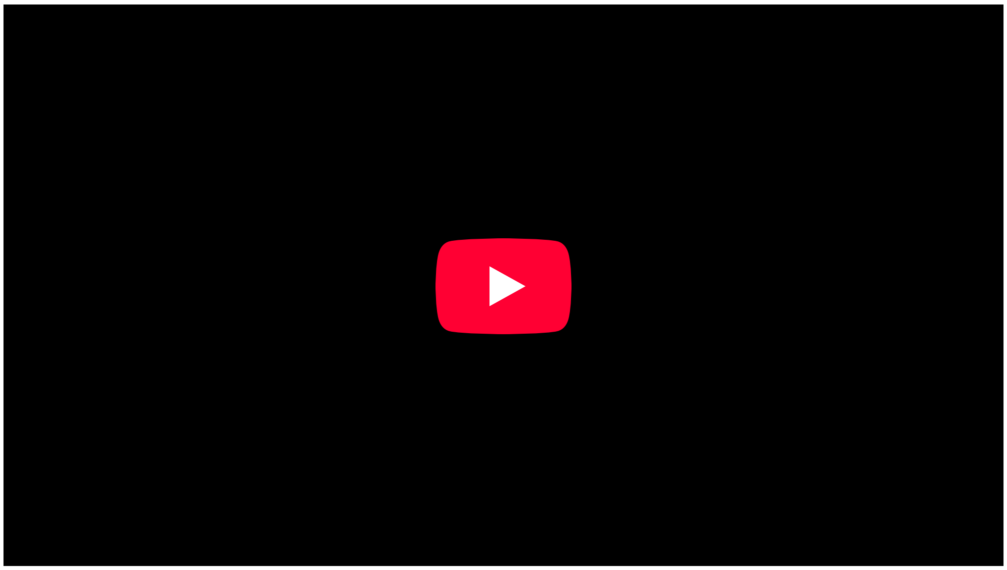
Topic hierarchy

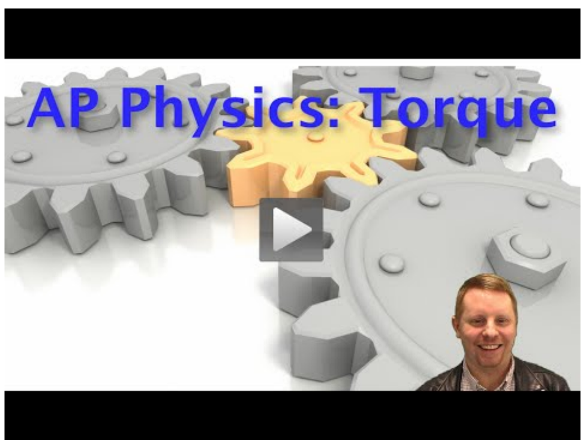
8.1: Introduction

learning objectives

- Describe the effect of the torque on an object

Torque about a point is a concept that denotes the tendency of force to turn or rotate an object in motion. This tendency is measured in general about a point, and is termed as *moment of force*. The torque in angular motion corresponds to force in translation. It is the "cause" whose effect is either angular acceleration or angular deceleration of a particle in general motion. Quantitatively, it is defined as a vector given by:





Torque: A brief introduction to torque for students studying rotational motion in algebra-based physics courses such as AP Physics 1 and Honors Physics.

$$mathrmT = r \times F \tag{8.1.1}$$

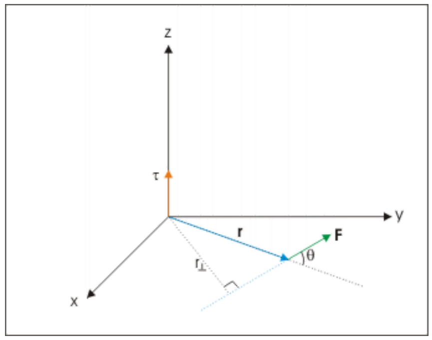
Rotation is a special case of angular motion. In the case of rotation, torque is defined with respect to an axis such that vector "r" is constrained as perpendicular to the axis of rotation. In other words, the plane of motion is perpendicular to the axis of rotation. Clearly, the torque in rotation corresponds to force in translation.

Torque is the cross product of force cross length of the moment arm; it is involved whenever there is a rotating object. Torque can also be expressed in terms of the angular acceleration of the object.

The determination of torque's direction is relatively easier than that of angular velocity. The reason for this is simple: the torque itself is equal to vector product of two vectors, unlike angular velocity which is one of the two operands of the vector product.

Clearly, if we know the directions of two operands here, the direction of torque can easily be interpreted.

Since torque depends on both the force and the distance from the axis of rotation, the SI units of torque are newton-meters.



Torque: Torque in terms of moment arm.

Key Points

- Torque is found by multiplying the applied force by the distance to the axis of rotation, called the moment arm.
- Torque is to rotation as force is to motion.
- The unit of torque is the newton-meter.

Key Terms

- **vector**: A directed quantity, one with both magnitude and direction; the between two points.
- **angular velocity**: A vector quantity describing an object in circular motion; its magnitude is equal to the speed of the particle and the direction is perpendicular to the plane of its circular motion.
- **angular motion**: The motion of a body about a fixed point or fixed axis (as of a planet or pendulum). It is equal to the angle passed over at the point or axis by a line drawn to the body.

8.2: Conditions for Equilibrium

learning objectives

- Identify the first condition of equilibrium

First Condition of Equilibrium

For an object to be in equilibrium, it must be experiencing no acceleration. This means that both the net force and the net torque on the object must be zero. Here we will discuss the first condition, that of zero net force.

In the form of an equation, this first condition is:

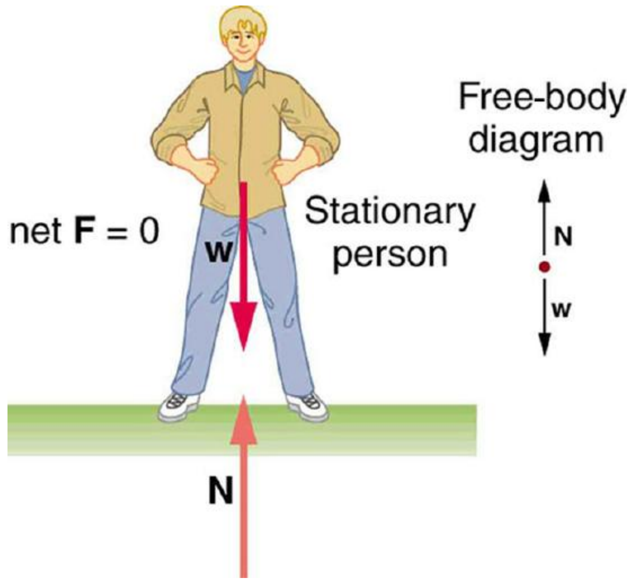
$$\mathrm{F_{net}} = 0.$$

In order to achieve this conditon, the forces acting along *each* axis of motion must sum to zero. For example, the net external forces along the typical *x*– and *y*-axes are zero. This is written as

$$\text{net } \mathrm{F_x} = 0 \text{ and net } \mathrm{F_y} = 0.$$

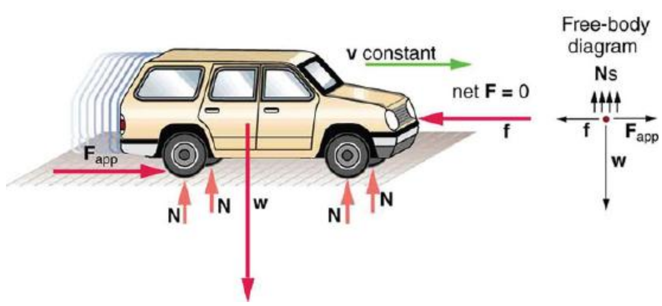
The condition $\mathrm{F_{net}} = 0$ must be true for both static equilibrium, where the object's velocity is zero, and dynamic equilibrium, where the object is moving at a constant velocity.

Below, the motionless person is in static equilibrium. The forces acting on him add up to zero. Both forces are vertical in this case.



Person in Static Equilibrium: This motionless person is in static equilibrium.

Below, the car is in dynamic equilibrium because it is moving at constant velocity. There are horizontal and vertical forces, but the net external force in any direction is zero. The applied force between the tires and the road is balanced by air friction, and the weight of the car is supported by the normal forces, here shown to be equal for all four tires.



A Car in Dynamic Equilibrium: This car is in dynamic equilibrium because it is moving at constant velocity. The forces in all directions are balanced.

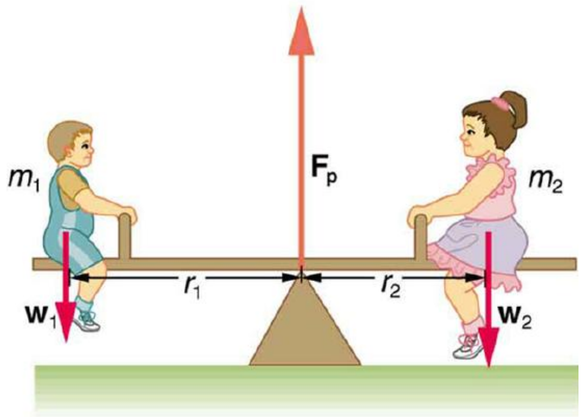
Second Condition

The second condition of static equilibrium says that the net torque acting on the object must be zero.

learning objectives

- Identify the second condition of static equilibrium

A child's seesaw, shown in, is an example of static equilibrium. An object in static equilibrium is one that has no acceleration in any direction. While there might be motion, such motion is constant.



Two children on a seesaw: The system is in static equilibrium, showing no acceleration in any direction.

If a given object is in static equilibrium, both the net force and the net torque on the object must be zero. Let's break this down:

Net Force Must Be Zero

The net force acting on the object must be zero. Therefore all forces balance in each direction. For example, a car moving along a highway at a constant speed is in equilibrium, as it is not accelerating in any forward or vertical direction. Mathematically, this is stated as $\mathrm{F_{net}} = \mathrm{ma} = 0$.

Net Torque Must Be Zero

The second condition necessary to achieve equilibrium involves avoiding accelerated rotation (maintaining a constant angular velocity). A rotating body or system can be in equilibrium if its rate of rotation is constant and remains unchanged by the forces acting on it.

To understand what factors affect rotation, let us think about what happens when you open an ordinary door by rotating it on its hinges.The magnitude, direction, and point of application of the force are incorporated into the definition of the physical quantity called torque—the rotational equivalent of a force. It is a measure of the effectiveness of a force in changing or accelerating a rotation (changing the angular velocity over a period of time).

In equation form, the magnitude of torque is defined to be $\tau = \mathrm{rF}\sin\theta$ where τ (the Greek letter tau) is the symbol for torque, r is the distance from the pivot point to the point where the force is applied, F is the magnitude of the force, and θ is the angle between the force and the vector directed from the point of application to the pivot point.

Two-Component Forces

In equilibrium, the net force and torque in any particular direction equal zero.

learning objectives

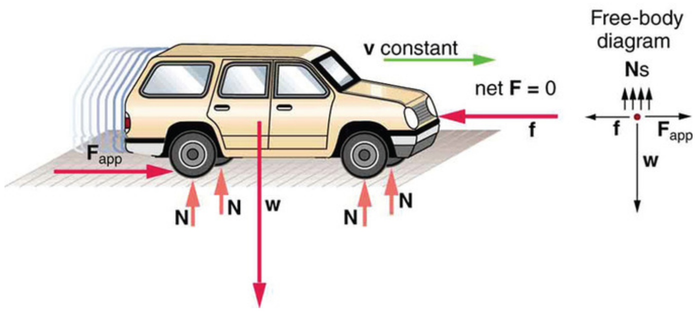
- Calculate the net force and the net torque for an object in equilibrium

An object with constant velocity has zero acceleration. A motionless object still has constant (zero) velocity, so motionless objects also have zero acceleration. Newton's second law states that:

$$\sum \mathrm{F}=\mathrm{ma} \tag{8.2.1}$$

so objects with constant velocity also have zero net external force. This means that all the forces acting on the object are balanced — that is to say, they are in equilibrium.

This rule also applies to motion in a specific direction. Consider an object moving along the *x*-axis. If no net force is applied to the object along the *x*-axis, it will continue to move along the *x*-axis at a constant velocity, with no acceleration.



Car Moving at Constant Velocity: A moving car for which the net x and y force components are zero

We can easily extend this rule to the *y*-axis. In any system, unless the applied forces cancel each other out (i.e., the resultant force is zero), there will be acceleration in the direction of the resultant force. In static systems, in which motion does not occur, the sum of the forces in all directions always equals zero. This concept can be represented mathematically with the following equations:

$$\sum \mathrm{F_x}=\mathrm{ma_x}=0 \tag{8.2.2}$$

$$\sum \mathrm{F_y}=\mathrm{ma_y}=0 \tag{8.2.3}$$

This rule also applies to rotational motion. If the resultant moment about a particular axis is zero, the object will have no rotational acceleration about the axis. If the object is not spinning, it will not start to spin. If the object is spinning, it will continue to spin at the same constant angular velocity. Again, we can extend this to moments about the *y*-axis as well. We can represent this rule mathematically with the following equations:

$$\sum \tau_{\mathrm{x}}=\mathrm{I}\alpha_{\mathrm{x}}=0 \tag{8.2.4}$$

$$\sum \tau_{\mathrm{y}}=\mathrm{I}\alpha_{\mathrm{y}}=0 \tag{8.2.5}$$

Key Points

- There are two conditions that must be met for an object to be in equilibrium.
- The first condition is that the net force on the object must be zero for the object to be in equilibrium.
- If net force is zero, then net force along any direction is zero.
- The second condition necessary to achieve equilibrium involves avoiding accelerated rotation.
- A rotating body or system can be in equilibrium if its rate of rotation is constant and remains unchanged by the forces acting on it.
- The magnitude of torque about a axis of rotation is defined to be $\tau=\mathrm{rF}\sin\theta$.
- In equilibrium, the net force in all directions is zero.
- If the net moment of inertia about an axis is zero, the object will have no rotational acceleration about the axis.
- In each direction, the net force takes the form: $\sum \mathrm{F}=\mathrm{ma}=0$ and the net torque take the form: $\sum \tau=\mathrm{I}\alpha=0$ where the sum represents the vector sum of all forces and torques acting.

Key Terms

- **force**: A physical quantity that denotes ability to push, pull, twist or accelerate a body which is measured in a unit dimensioned in mass × distance/time² (ML/T^2): SI: newton (N); CGS: dyne (dyn)
- **torque**: A rotational or twisting effect of a force; (SI unit newton-meter or Nm; imperial unit foot-pound or ft-lb)
- **translation**: Motion of a body on a linear path, without deformation or rotation, i.e. such that every part of the body moves at the same speed and in the same direction; also (in physics), the linear motion of a body considered independently of its rotation.
- **equilibrium**: The state of a body at rest or in uniform motion, the resultant of all forces on which is zero.

8.3: Stability

learning objectives

- Explain the relationship between how center of mass is defined and static equilibrium

For an object to be in static equilibrium, we expect it to stay in the same state indefinitely. If it starts accelerating away from its current position, it would hardly be in equilibrium. To quantify equilibrium for a single object, there are two conditions:

1. The net external force on the object is zero: $\sum_i \mathrm{F_i} = \mathrm{F_{net}} = 0$
2. The net external torque, regardless of choice of origin, is also zero: $\sum_i \mathrm{r_i} \times \mathrm{F_i} = \sum_i \tau_i = \tau_{net} = 0$

Those two conditions hold regardless of whether the object we are talking about is a single point particle, a rigid body, or a collection of discrete particles. Being in equilibrium means that we expect no changes to the linear momentum or the angular momentum. Note that this does not mean that the system is not moving or rotating; instead it simply means that its movement will not change as time goes on.

In a special case when the external forces are governed by some potential (e.g. gravitational potential) we can gain insight into the nature of the equilibrium. From the definition of a potential we know that $\mathrm{F_{ext}} = -\frac{\mathrm{dU(x)}}{\mathrm{dx}}|_{\mathrm{x_0}}$. When the first derivative is zero, we can take the second derivative to find whether the equilibrium is stable or unstable. Explicitly, if the potential is concave-up at $\mathrm{x_0}$, $\frac{\mathrm{d^2U(x)}}{\mathrm{dx^2}}|_{\mathrm{x_0}} > 0$, then the system is stable; conversely, if the potential is concave-down, then the equilibrium is unstable. If the second derivative is zero or does not exist, then the equilibrium is neutral—neither stable nor unstable.

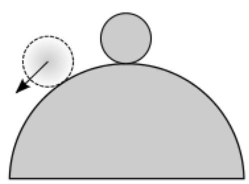
Mathematically, we can view this as a Taylor series expansion on the force slightly away from equilibrium,

$$\mathrm{F(x_0 + \delta x) = F(x_0) + \frac{dF(x)}{dx}|_{x_0}\ \delta x F(x0 + \delta x) = -\frac{dU(x)}{dx}|_{x_0} + (-\frac{d^2U(x)}{dx^2}|_{x_0})\delta x} \quad ,$$

and when it is initially at equilibrium,

$$\mathrm{F(x_0) = 0F(x_0) = 0F(x_0 + \delta x) = -\frac{dU(x)}{dx}|_{x_0} + (-\frac{d^2U(x)}{dx^2}|_{x_0})\delta x U(y) = mgy} \quad .$$

If the ball is at the top of the hill (where the potential is concave-down) it is possible for it to be perfectly balanced, and therefore at equilibrium. But if it gets pushed just slightly to the side, then it will roll down the hill with increasing speed, and the equilibrium is unstable.



Unstable Equilibrium: A ball on top of a hill can initially be balanced, but if it moves slightly left or right, it gets pushed further and further away from the initial equilibrium position. This is an example of unstable equilibrium.

Our notion of "balance" comes directly from the formulation of equilibrium. For something to be "balanced" means that the net external forces are zero. For example, a coin could balance standing up on a table. Initially the coin will feel no net external force or torque; it is in equilibrium. But if pushed slightly to the side, it will become "off-balance," experiencing both a force and a torque causing it to fall to the table. It might have been initially "balanced" and at equilibrium, but it was an unstable equilibrium, prone to being disturbed. But why all this talk of external forces, with no mention of internal forces? The reason is that all the internal forces must sum to zero. This follows directly from Newton's Third Law, $\mathrm{F_{12}} = -\mathrm{F_{21}}$. Every time we consider a force from particle 1 on particle 2 inside of a system, we know that it will later be cancelled out by the corresponding force from particle 2 on particle 1. We could include those forces in the sum, but it is unnecessary and internal forces are often more complicated than internal forces.

This differentiation between internal and external forces is a powerful one. It also implies that you can trace the motion of the system as a whole (ignoring motion inside the system) through the net external force acting on a center of mass. A center of mass acts as if it has the entire mass of the system, located at one point, and only feels external forces. Its position is defined as the weighted average of all the particles in the system: $\frac{R=\sum_i m_i r_i}{\sum_i m_i}$ or if we have a continuous density of mass, $\rho(r)$, then we can integrate: $R = \frac{\int V\rho(r) r dV}{\int V\rho(r) dV}$. The power of the center of mass is that it hides all the details of what is happening internally. We do not always want to lose the information of what is happening internally, but it is a useful tool to remember, when dealing with a number of complicated interactions.

Key Points

- Equilibrium is defined by no net forces or torques.
- Stability of an equilibrium can be determined by the second derivative of the potential.
- Defining a center of mass allows a simple way to study the behavior of a system or object as a whole.
- Stable equilibrium requires a restoring force. This restoring force can be derived by a Taylor expansion of the force, F(x).

Key Terms

- **stable equilibrium**: The response [of a system in static equilibrium] to a small perturbation is forces that tend to restore the equilibrium.
- **center of mass**: The center of mass (COM) is the unique point at the center of a distribution of mass in space that has the property that the weighted position vectors relative to this point sum to zero.
- **static equilibrium**: the physical state in which all components of a system are at rest and the net force is equal to zero throughout the system

8.4: Solving Statics Problems

learning objectives

- Formulate and apply six steps to solve static problems

Statics is the study of forces in equilibrium. Recall that Newton's second law states:

$$\sum \mathrm{F} = \mathrm{ma} \tag{8.4.1}$$

Therefore, for all objects moving at constant velocity (including a velocity of 0 — stationary objects), the net external force is zero. There are forces acting, but they are balanced — that is to say, they are "in equilibrium."

When solving equilibrium problems, it might help to use the following steps:

1. First, ensure that the problem you're solving is in fact a static problem—i.e., that no acceleration (including angular acceleration) is involved Remember: $\sum \mathrm{F} = \mathrm{ma} = 0$ for these situations. If rotational motion is involved, the condition $\sum \tau = \mathrm{I}\alpha = 0$ must also be satisfied, where is torque, is the moment of inertia, and is the angular acceleration.
2. Choose a pivot point. Often this is obvious because the problem involves a hinge or a fixed point. If the choice is not obvious, pick the pivot point as the location at which you have the most unknowns. This simplifies things because forces at the pivot point create no torque because of the cross product: $\tau = \mathrm{rF}$
3. Write an equation for the sum of torques, and then write equations for the sums of forces in the *x* and *y* directions. Set these sums equal to 0. Be careful with your signs.
4. Solve for your unknowns.
5. Insert numbers to find the final answer.
6. Check if the solution is reasonable by examining the magnitude, direction, and units of the answer. The importance of this last step cannot be overstated, although in unfamiliar applications, it can be more difficult to judge reasonableness. However, these judgments become progressively easier with experience.

Key Points

- First, ensure that the problem you're solving is in fact a static problem—i.e., that no acceleration (including angular acceleration) is involved.
- Choose a pivot point — use the location at which you have the most unknowns.
- Write equations for the sums of torques and forces in the *x* and *y* directions.
- Solve the equations for your unknowns algebraically, and insert numbers to find final answers.

Key Terms

- **torque**: A rotational or twisting effect of a force; (SI unit newton-meter or Nm; imperial unit foot-pound or ft-lb)
- **moment of inertia**: A measure of a body's resistance to a change in its angular rotation velocity

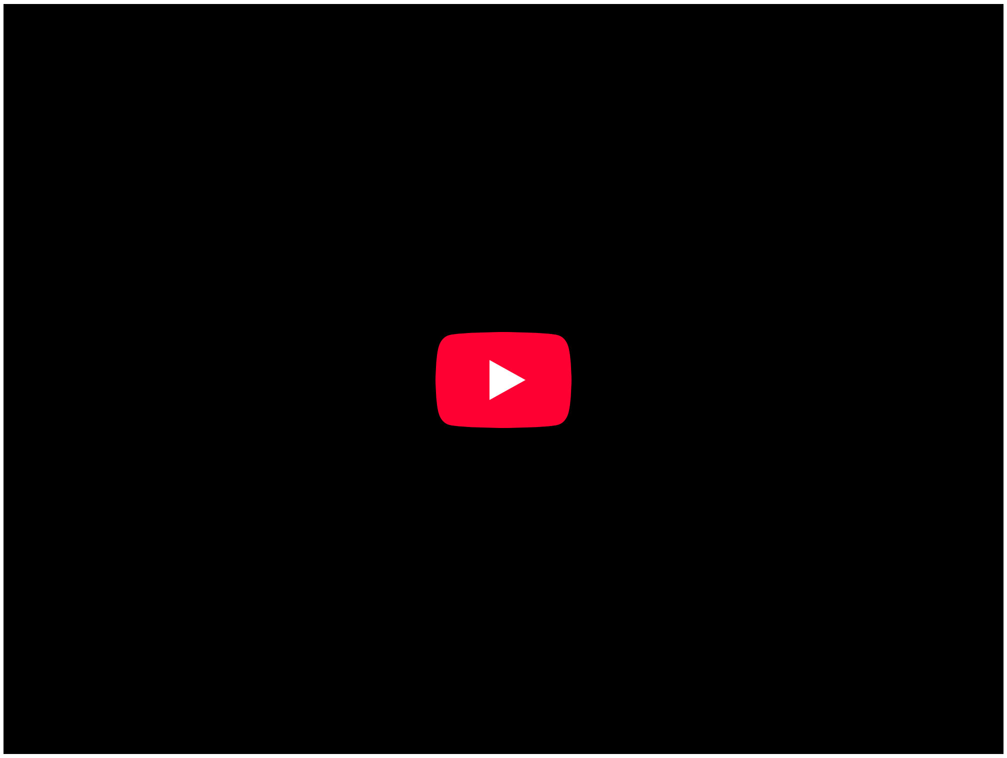
8.5: Applications of Statics

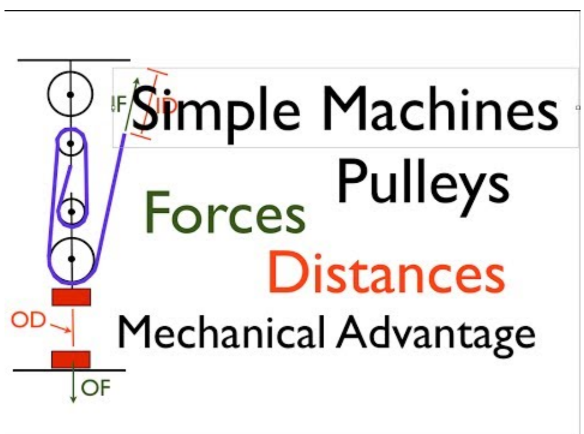
learning objectives

- Develop an understanding of how a machine applies force to work against a load force

Simple Machines

A simple machine is a device that changes the direction or magnitude of a force. They can be described as the simplest mechanisms that use mechanical advantage (or leverage) to multiply force. Usually, the term “simple machine” is referring to one of the six classical simple machines, defined by Renaissance scientists.





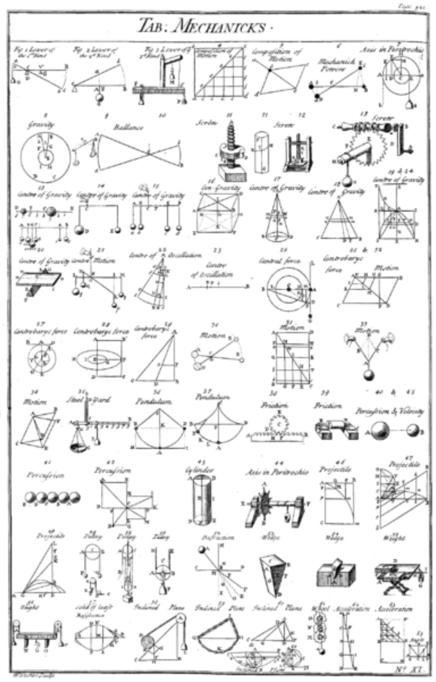
Simple Machines, Pulleys; Forces, Distances and MA: Describes the following terms as they relate to simple machine; input force, output force, input distance, output distance, mechanical advantage.

Simple machines are devices used to multiply or augment a force that we apply—often at the expense of a distance through which we apply the force. Some common examples include:

- Lever

- Wheel and Axle
- Pulley
- Inclined Plane
- Wedge
- Screw

When a device with a specific movement, called a mechanism, is joined with others to form a machine, these machines can be broken down into elementary movements. For example, a bicycle is a mechanism made up of wheels, levers, and pulleys.

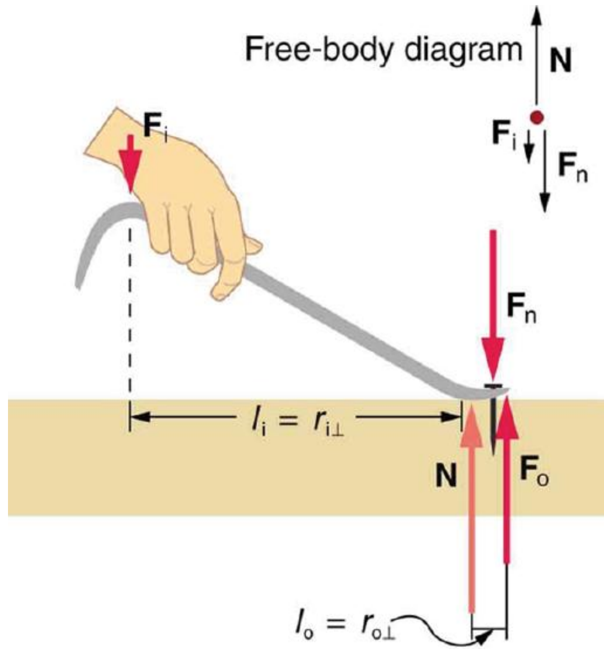


Simple Machines: Table of simple mechanisms, from Chambers' Cyclopedia, 1728. [1] Simple machines provide a "vocabulary" for understanding more complex machines.

Mechanics

A simple machine has an applied force that works against a load force. If there are no frictional losses, the work done on the load is equal to the work done by the applied force. This allows an increase in the output force at the cost of a proportional decrease in distance moved by the load. The ratio of the output force to the input force is the mechanical advantage of the machine. If the machine does not absorb energy, its mechanical advantage can be calculated from the machine's geometry. For instance, the mechanical advantage of a lever is equal to the ratio of its lever arms.

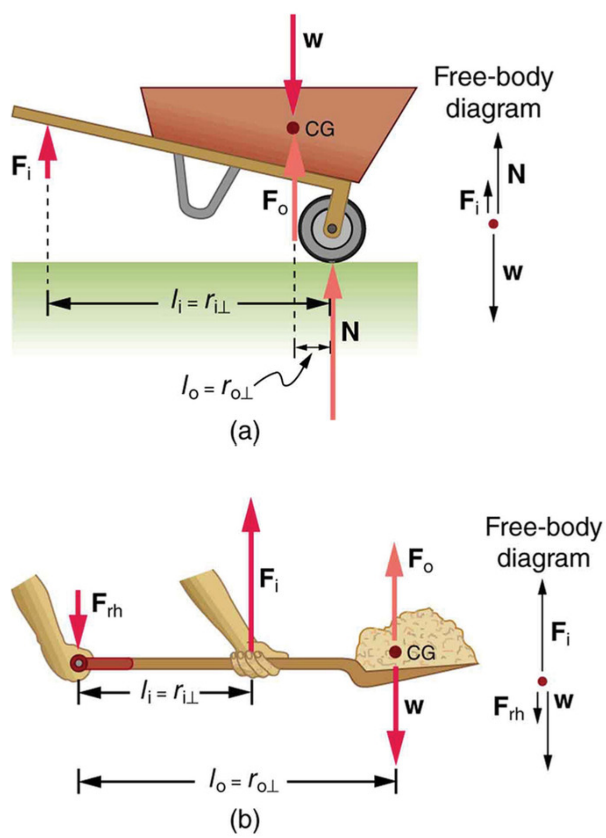
Simple machines which do not experience frictional losses are called ideal machines. For these ideal machines, the power in (rate of energy input) in equal to the power out (rate of energy output): Pin=PoutPin=Pout.



Lever: The amount of force produced by a machine can not be greater than the amount of force put into it.

Further Examples

Wheelbarrows and shovels are also examples of simple machines (these utilize levers). They use only three forces: the input force, output force, and force on the pivot. In the case of wheelbarrows, the output force is between the pivot (wheel's axle) and the input force. In the shovel, the input force is between the pivot and the load.



Examples of Simple Machines: Both of these machines use the concept of levers.

Arches and Domes

Arches and domes are structures that exhibit structural strength and can span large areas with no intermediate supports.

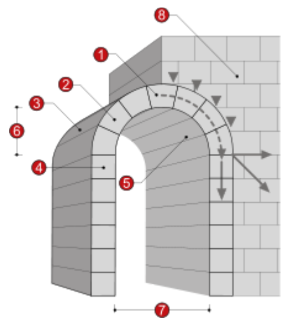
learning objectives

- Explain how an arch exhibits structural strength and how a dome can span a large area without intermediate supports

Arches and domes are structures that exhibit structural strength and can span large areas with no intermediate supports. In this atom, we will discuss the history and physics behind arches and domes.

Arches

An arch is a structure that spans a space, and supports structure and weight above it. Arches have been being built from as long ago as the second millennium, but were not used for a variety of structures until the Romans took advantage of their capabilities. Arches are a pure compression form. They span large areas by resolving forces into compressive stresses and eliminating tensile stresses (referred to as arch action). As the forces in an arch are carried toward the ground, the arch will push outward at the base (called thrust). As the height of the arch decreases, the outward thrust increases. To prevent the arch from collapsing, the thrust needs to be restrained, either with internal ties or external bracing. This external bracing is often called an abutment, as shown in.



Arches: A masonry arch1. Keystone 2. Voussoir 3. Extrados 4. Impost 5. Intrados 6. Rise 7. Clear span 8. Abutment

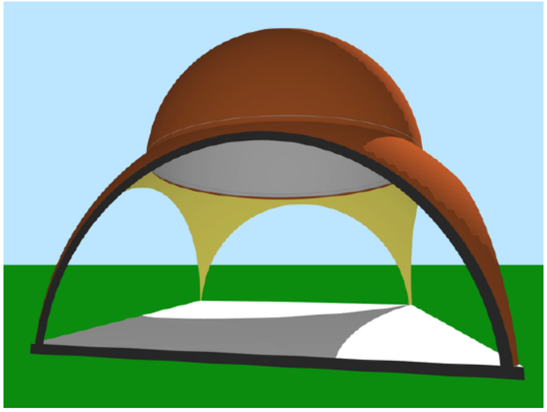
The most common true arch configurations are the fixed arch, the two-hinged arch and the three-hinged arch. The fixed arch is most often used in reinforced concrete bridge and tunnel construction, where the spans are short. Because it is subject to additional internal stress caused by thermal expansion and contraction, this type of arch is considered to be statically indeterminate. The two-hinged arch is most often used to bridge long spans. This type of arch has pinned connections at the base. Unlike the fixed arch, the pinned base is able to rotate, allowing the structure to move freely and compensate for the thermal expansion and contraction caused by changes in outdoor temperature. Because the structure is pinned between the two base connections, which can result in additional stresses, the two-hinged arch is also statically indeterminate, although not to the degree of the fixed arch.

Domes

A dome is an element of architecture that resembles the hollow upper half of a sphere. Dome structures made of various materials (from mud to stone, wood, brick, concrete, metal, glass and plastic) and have a long architectural lineage extending into prehistory.

A dome is basically an arch that has been rotated around its central vertical axis. Domes have the same properties and capabilities of arches, they can span large areas without intermediate supports and have a great deal of structural strength. When the base of a dome is not the same shape as its supporting walls, for example when a circular dome is on a square structure, techniques are employed to transition between the two. Pendentives are triangular sections of a sphere used to transition from the flat surfaces of supporting walls to the round base of a dome.

Domes can be divided into two kinds, simple and compound. Simple domes use pendentives that are part of the same sphere as the dome itself. Compound domes are part of the structure of a large sphere below that of the dome itself, forming a circular base, as shown in.



Compound Dome: A compound dome (red) with pendentives (yellow) from a sphere of greater radius than the dome.

Muscles and Joints

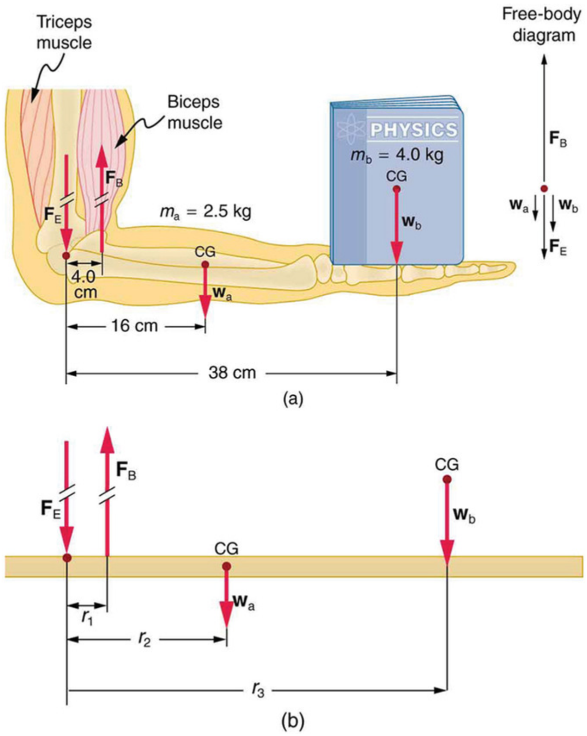
Most skeletal muscles and joints exert much larger forces within the body than the limbs will apply to the outside world.

learning objectives

- Explain the forces exerted by muscles

Muscles and Joints

Muscles and joints involve very interesting applications of statics. Muscles can only contract, so they occur in pairs. In the arm, the biceps muscle is a flexor: it closes the limb. The triceps muscle is an extensor that opens the limb. This configuration is typical of skeletal muscles, bones, and joints in humans and other vertebrates. Most skeletal muscles exert much larger forces within the body than the limbs will apply to the outside world. The reason is clear, since most muscles are attached to bones via tendons close to joints, causing these systems to have mechanical advantages much less than one. Viewing them as simple machines, the input force is much greater than the output force, as seen in.



The Forearm of a Person Holding a Book: (a.) The biceps exert a force FB to support the weight of the forearm and the book. The triceps are assumed to be relaxed. (b.) An approximately equivalent mechanical system with the pivot at the elbow joint

Very large forces are also created in the joints. Because muscles can contract but not expand beyond their resting length, joints and muscles often exert forces that act in opposite directions, and thus subtract. Forces in muscles and joints are largest when their load is far from the joint. For example, in racquet sports like tennis, the constant extension of the arm during game play creates large forces. The mass times the lever arm of a tennis racquet is an important factor, and many players use the heaviest racquet they can handle. It is no wonder that joint deterioration and damage to the tendons in the elbow, such as 'tennis elbow,' can result from repetitive motion, undue torques, and possible poor racquet selection in such sports.

Various tried techniques for holding and using a racquet, bat, or stick can not only increase sporting prowess but can minimize fatigue and long-term damage to the body. Training coaches and physical therapists use the knowledge of the relationships between forces and torques in the treatment of muscles and joints. In physical therapy, an exercise routine can apply a particular force and torque, which can revive muscles and joints in time. Some exercises should be performed under water, thus requiring the exertion of more force and further strengthening muscles.

Key Points

- The six classifications of simple machines were established by renaissance scientists; they are as follows: lever, wheel and axle, pulley, inclined plane, wedge and screw.
- Simple machines can be joined with other devices to create a more complicated machine. These building blocks are used to explain how machines work.
- The force output by a simple machine can exceed the force that was put into the machine.
- Arches span large areas by resolving forces into compressive stresses and eliminating tensile stresses.
- The most common true arch configurations are the fixed arch, the two-hinged arch, and the three-hinged arch.
- A dome is basically an arch that has been rotated around its central vertical axis.
- Domes are basically arches that have been rotated on their vertical axis, and have the same capabilities and properties of arches.
- Domes can be divided into two kinds, simple and compound.
- It is helpful to view muscles as a simple machines and draw them as free body diagrams.
- In muscles, the input force is often much greater than the output force.
- Very large forces are also created in the joints. Because muscles can contract but not expand beyond their resting length, joints and muscles often exert forces that act in opposite directions, and thus subtract.

Key Terms

- **machine**: A mechanical or electrical device that performs or assists in the performance of human tasks, whether physical or computational, laborious or for entertainment.
- **leverage**: A force amplified by means of a lever rotating around a pivot.
- **mechanical advantage**: In a simple machine, the ratio of the output force to the input force.
- **compressive stress**: Stress on materials that leads to a smaller volume.
- **tensile stress**: Stress state leading to expansion; that is, the length of a material tends to increase in the tensile direction while the volume remains constant.
- **pendentive**: The concave triangular sections of vaulting that provide the transition between a dome and the square base on which it is set and transfer the weight of the dome.
- **muscle**: A contractile form of tissue which animals use to effect movement.
- **joint**: Any part of the body where two bones join, in most cases allowing that part of the body to be bent or straightened.

8.6: Elasticity, Stress, Strain, and Fracture

learning objectives

- Identify properties of elastic objects

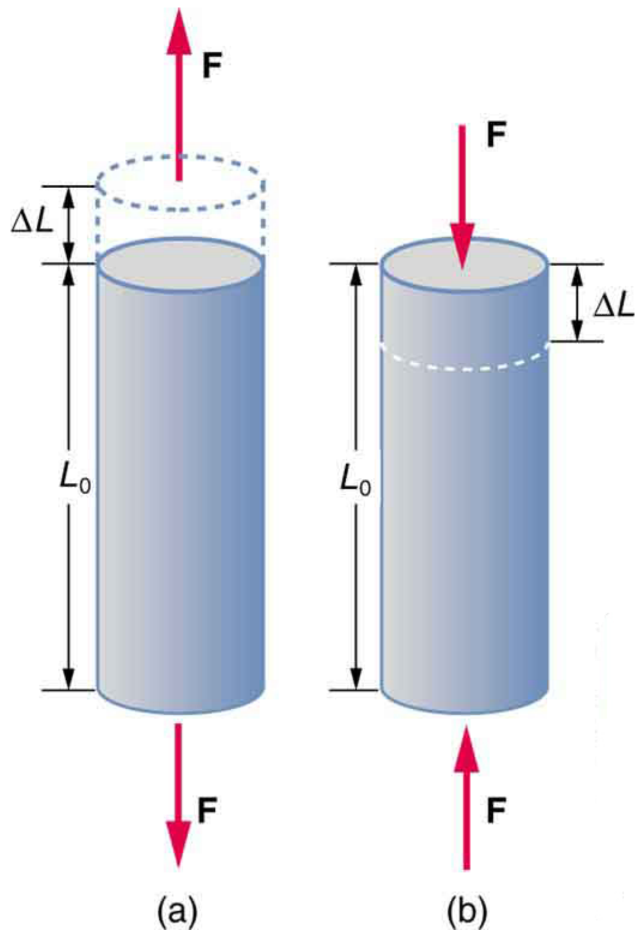
We now move from consideration of forces that affect the motion of an object (such as friction and drag) to those that affect an object's shape. If a bulldozer pushes a car into a wall, the car will not move once it hits the wall, but it will noticeably change shape. A change in shape due to the application of a force is a deformation. Even very small forces are known to cause some deformation. For small deformations, two important characteristics are observed. First, the object returns to its original shape when the force is removed—that is, the deformation is elastic for small deformations. Second, the size of the deformation is proportional to the force—that is, for small deformations, Hooke's law is obeyed. In equation form, Hooke's law is given by $\mathrm{F}=\mathrm{k}\Delta\mathrm{L}$, where $\Delta\mathrm{L}$ is the change in length.

Elasticity is a measure of how difficult it is to stretch an object. In other words it is a measure of how small kk is. Very elastic materials like rubber have small kk and thus will stretch a lot with only a small force.

Stress is a measure of the force put on the object over the area.

Strain is the change in length divided by the original length of the object.

Experiments have shown that the change in length (ΔL) depends on only a few variables. As already noted, ΔL is proportional to the force F and depends on the substance from which the object is made. Additionally, the change in length is proportional to the original length L0 and inversely proportional to the cross-sectional area of the wire or rod. For example, a long guitar string will stretch more than a short one, and a thick string will stretch less than a thin one.



Tension/Compression: Tension: The rod is stretched a length ΔL when a force is applied parallel to its length. (b) Compression: The same rod is compressed by forces with the same magnitude in the opposite direction. For very small deformations and uniform materials, ΔL is approximately the same for the same magnitude of tension or compression. For larger deformations, the cross-sectional area changes as the rod is compressed or stretched.

Fracture

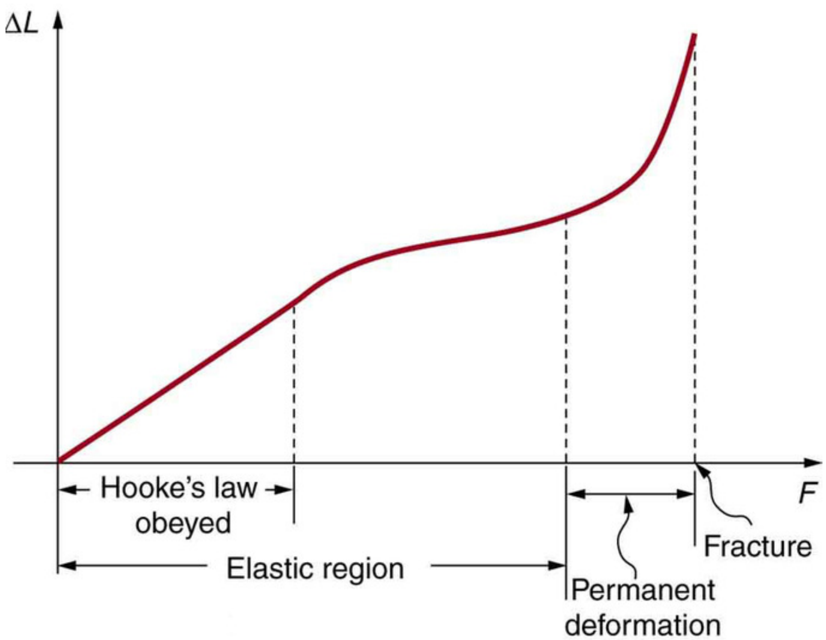
Fracture is caused by a strain placed on an object such that it deforms beyond its elastic limit and breaks.

learning objectives

- Relate fracture with the elastic limit of a material

Materials cannot stretch forever. When a strain is applied to a material it deforms elastically proportional to the force applied. However, after it has deformed a certain amount, the object can no longer take the strain and will break or fracture. The zone in which it bends under strain is called the elastic region. In that region the object will bend and then return to its original shape when the force is abated. Past that point, if more strain is added, the object may permanently deform and eventually fracture.

Fracture strength, also known as breaking strength, is the stress at which a specimen fails via fracture. This is usually determined for a given specimen by a tensile test, which charts the stress-strain curve. The final recorded point is the fracture strength.



Fracture: This is a graph of deformation ΔL versus applied force F. The straight segment is the linear region where Hooke's law is obeyed. The slope of the straight region is 1k. For larger forces, the graph is curved but the deformation is still elastic—L will return to zero if the force is removed. Still greater forces permanently deform the object until it finally fractures. The shape of the curve near fracture depends on several factors, including how the force F is applied. Note that in this graph the slope increases just before fracture, indicating that a small increase in F is producing a large increase in L near the fracture.

Bones, on the whole, do not fracture due to tension or compression. Rather they generally fracture due to sideways impact or bending, resulting in the bone shearing or snapping. The behavior of bones under tension and compression is important because it determines the load the bones can carry. Bones are classified as weight-bearing structures such as columns in buildings and trees. Weight-bearing structures have special features; columns in building have steel-reinforcing rods while trees and bones are fibrous. The bones in different parts of the body serve different structural functions and are prone to different stresses. Thus, the bone in the top of the femur is arranged in thin sheets separated by marrow while, in other places, the bones can be cylindrical and filled with marrow or just solid. Overweight people have a tendency toward bone damage due to sustained compressions in bone joints and tendons.

Key Points

- Elasticity is a measure of the deformation of an object when a force is applied. Objects that are very elastic like rubber have high elasticity and stretch easily.
- Stress is force over area.
- Strain is change in length over original length.
- Most objects behave elastically for small strains and return to their original shape after being bent.
- If the strain on an object is greater than the elastic limit of the object, it will permanently deform or eventually fracture.
- Fracture strength is a measure of the force needed to break an object.

Key Items

- **deformation**: A transformation; change of shape.
- **strain**: The amount by which a material deforms under stress or force, given as a ratio of the deformation to the initial dimension of the material and typically symbolized by ε is termed the engineering strain. The true strain is defined as the natural logarithm of the ratio of the final dimension to the initial dimension.
- **elastic**: Capable of stretching; particularly, capable of stretching so as to return to an original shape or size when force is released.

8.7: The Center of Gravity

learning objectives

- Describe how the center of mass of an oddly shaped object is found

Center of Gravity

When people think of objects, they think of them as singular particles of matter. In fact, every object is made up of millions of particles, all of which behave differently when moved. When people observe a stick being thrown in the air, it seems as though the entire object is moving at the same trajectory and velocity, but each particle is being subjected to a different motion in space and acceleration, depending on its place. The different parts of the body have different motions. shows the motion of a stick in the air: it seems to rotate around a single point. Three-dimensional bodies have a property called the center of mass, or center of gravity. This center of mass's main characteristic is that it appears to carry the whole mass of the body.

Center of Gravity: Although the center of mass is in the midpoint of the stick, all of the particles are moving as well.

The center of mass does not actually carry all the mass, despite appearances. Given a hollow sphere, the center is the center of mass, even though it does not actually have anything in it. As seen in, it looks as if the external forces of gravity appear to be working only on the center of mass, but each particle is being pushed or pulled by gravity. The center of mass is much easier to use when discussing bodies, because no one has to analyze each individual particle.

Mathematical Expression: The mathematical relation of center of gravity is read as: 'the position of the center of mass and weighted average of the position of the particles. '

Specifically: 'the total mass x the position of the center of mass= ∑ the mass of the individual particle x the position of the particle. ' The center of mass is a geometric point in three-dimensional volume. When using the definition above, it yields the following equation for center of mass:

$$\mathrm{r_{COM}} = \frac{\Sigma \mathrm{m_i r_i}}{\mathrm{M}} \tag{8.7.1}$$

where r is the reference axis x, y, or z; *m* is individual mass;$\mathrm{r_i}$ is the individual position; and M *M* is the total mass.

When taking the center of mass of an oddly shaped object, it is helpful to break it down into smaller sections whose mass and properties are easier to evaluate, and then add the products of the individual masses and positions and divide by the total mass.

Center of Mass: This child's toy uses the principles of 'center of mass' to stay balanced on a finger.

Key Points

- The center of mass 's main characteristic is that it appears to carry the whole mass of the body.
- The total mass x the position of the center of mass= ∑ mass of the individual particle x the position of the particle.
- The center of mass is a geometric point in three-dimensional volume. By using the definition above, the following equation for center of mass can be derived: $\mathrm{r_{COM}} = \Sigma \frac{\mathrm{m_i r_i}}{\mathrm{M}}$.

Key Terms

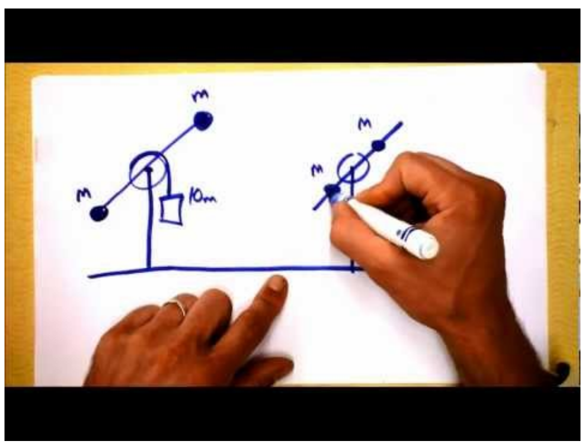
- **center of mass**: The center of mass (COM) is the unique point at the center of a distribution of mass in space that has the property that the weighted position vectors relative to this point sum to zero.

8.8: Torque and Angular Acceleration

learning objectives

- Express the relationship between the torque and the angular acceleration in a form of equation

Torque and angular acceleration are related by the following formula where is the objects moment of inertia and αα is the angular acceleration.



Torque, Angular Acceleration, and the Role of the Church in the French Revolution: Why do things change their angular velocity? Soon, you'll know.

Just like Newton's Second Law, which is force is equal to the mass times the acceleration, torque obeys a similar law. If you replace torque with force and rotational inertia with mass and angular acceleration with linear acceleration, you get Newton's Second Law back out. In fact, this equation is Newton's second law applied to a system of particles in rotation about a given axis. It makes no assumptions about constant rotational velocity.

The net torque about an axis of rotation is equal to the product of the rotational inertia about that axis and the angular acceleration, as shown in Figure 1.

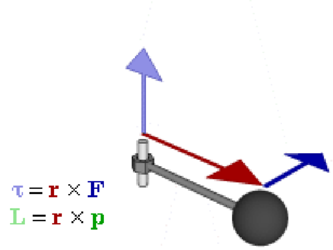


Figure 1: Relationship between force (F), torque (τ), momentum (p), and angular momentum (L) vectors in a rotating system

Similar to Newton's Second Law, angular motion also obeys Newton's First Law. If no outside forces act on an object, an object in motion remains in motion and an object at rest remains at rest. With rotating objects, we can say that unless an outside torque is applied, a rotating object will stay rotating and an object at rest will not begin rotating.

If a turntable were spinning counter clockwise (when viewed from the top), and you applied your fingers to opposite sides the turntable would begin to slow its spinning. From a translational viewpoint, at least, there would be no net force applied to the turntable. The force that points to one side would be cancelled by the force that points to the other. The forces of the two fingers would cancel. Therefore, the turntable would be in translational equilibrium. Despite that, the rotational velocity would be decreased meaning that the acceleration would no longer be zero. From this we might conclude that just because a rotating object is in translational equilibrium, it is not necessarily in rotational equilibrium.

Key Points

- When a torque is applied to an object it begins to rotate with an acceleration inversely proportional to its moment of inertia.
- This relation can be thought of as Newton's Second Law for rotation. The moment of inertia is the rotational mass and the torque is rotational force.
- Angular motion obeys Newton's First Law. If no outside forces act on an object, an object in motion remains in motion and an object at rest remains at rest.

Key Terms

- **angular acceleration**: The rate of change of angular velocity, often represented by α.
- **torque**: A rotational or twisting effect of a force; (SI unit newton-meter or Nm; imperial unit foot-pound or ft-lb)
- **rotational inertia**: The tendency of a rotating object to remain rotating unless a torque is applied to it.

CHAPTER OVERVIEW

09: Rotational Kinematics, Angular Momentum, and Energy

9.10: Conservation of Energy

> **learning objectives**
> - Conclude the interchangeability of force and radius with torque and angle of rotation in determining force

In this atom we will discuss work and energy associated with rotational motion. shows a worker using an electric grindstone propelled by a motor. Sparks are flying, and noise and vibration are created as layers of steel are pared from the pole. The stone continues to turn even after the motor is turned off, but it is eventually brought to a stop by friction. Clearly, the motor had to work to get the stone spinning. This work went into heat, light, sound, vibration, and considerable rotational kinetic energy.

Grindstone: The motor works in spinning the grindstone, giving it rotational kinetic energy. That energy is then converted to heat, light, sound, and vibration. (Credit: U.S. Navy photo by Mass Communication Specialist Seaman Zachary David Bell.)

Work must be done to rotate objects such as grindstones or merry-go-rounds. The simplest rotational situation is one in which the net force is exerted perpendicular to the radius of a disc and remains perpendicular as the disc starts to rotate. The force is parallel to the displacement, and so the net work (W) done is the product of the force (F) and the radius (r) of the disc (this is otherwise known as torque(τ)) times the angle (θ) of rotation:

$$\mathrm{W}=\mathrm{Fr}\theta=\tau\theta. \tag{9.10.1}$$

Work and energy in rotational motion are completely analogous to work and energy in translational motion and completely transferrable. Just as in translational motion (where kinetic energy equals $1/2mv^2$ where m is mass and v is velocity), energy is conserved in rotational motion. Kinetic energy (K.E.) in rotational motion is related to moment of rotational inertia (I) and angular velocity (ω):

$$\mathrm{KE}=\frac{1}{2}\mathrm{I}\omega^2. \tag{9.10.2}$$

The final rotational kinetic energy equals the work done by the torque:

$$\mathrm{W}=\tau\theta=\frac{1}{2}\mathrm{I}\omega^2=\mathrm{KE}. \tag{9.10.3}$$

This confirms that the work done went into rotational kinetic energy. To return to the grindstone example, work was done to give the grindstone rotational energy, and work is done by friction so that it loses kinetic energy. However, the energy is never destroyed; it merely changes form from rotation of the grindstone to heat when friction is applied.

Key Points

- Rotating objects have rotational kinetic energy.
- Rotational kinetic energy can change form if work is done on the object.
- Energy is never destroyed, if rotational energy is gained or lost, something must have done work on it to change the form of the energy.

Key Terms

- **work**: A measure of energy expended in moving an object; most commonly, force times displacement. No work is done if the object does not move.
- **angular velocity**: A vector quantity describing an object in circular motion; its magnitude is equal to the speed of the particle and the direction is perpendicular to the plane of its circular motion.
- **rotational inertia**: The tendency of a rotating object to remain rotating unless a torque is applied to it.

9.1: Quantities of Rotational Kinematics

learning objectives

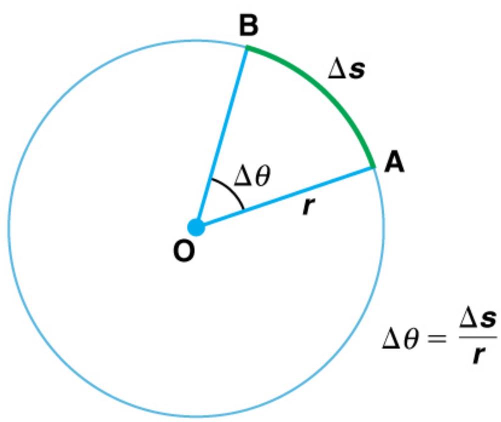
- Assess the relationship between radians the the revolution of a CD

When objects rotate about some axis—for example, when the CD (compact disc) rotates about its center—each point in the object follows a circular arc. Consider a line from the center of the CD to its edge. Each pit used to record sound along this line moves through the same angle in the same amount of time. The rotation angle is the amount of rotation, and is analogous to linear distance. We define the rotation angle $\Delta\theta$ to be the ratio of the arc length to the radius of curvature:

$\Delta\theta = \frac{\Delta \mathrm{s}}{\mathrm{r}}$ (illustrated in).

Rotation Angle: All points on a CD travel in circular arcs. The pits along a line from the center to the edge all move through the same angle Δ in a time Δt.

In mathematics, the angle of rotation (or angular position) is a measurement of the amount (i.e., the angle) that a figure is rotated about a fixed point (often the center of a circle, as shown in).



Angle θ and Arc Length s: The radius of a circle is rotated through an angle Δ. The arc length Δs is described on the circumference.

The arc length Δs is the distance traveled along a circular path. r is the radius of curvature of the circular path. We know that for one complete revolution, the arc length is the circumference of a circle of radius r. The circumference of a circle is 2πr. Thus, for one complete revolution the rotation angle is:

$$\Delta\theta = \frac{(2\pi \mathrm{r})}{\mathrm{r}} = 2\pi. \tag{9.1.1}$$

This result is the basis for defining the units used to measure rotation angles to be radians (rad), defined so that:

$$2\pi \text{ rad} = 1 \text{ revolution.} \tag{9.1.2}$$

If $\Delta\theta = 2\pi$ rad, then the CD has made one complete revolution, and every point on the CD is back at its original position. Because there are 360º in a circle or one revolution, the relationship between radians and degrees is thus $2\pi \text{ rad} = 360^{\circ}$, so that:

$$1 \text{ rad} = \frac{360^{\circ}}{2\pi} = 57.3^{\circ}. \tag{9.1.3}$$

Angular Velocity, Omega

Angular velocity ω is the rate of change of an angle, mathematically defined as $\omega = \frac{\Delta\theta}{\Delta t}$.

learning objectives

- Examine how fast an object is rotating based on angular velocity

To examine how fast an object is rotating, we define angular velocity ω as the rate of change of an angle. In symbols, this is

$$\omega = \frac{\Delta\theta}{\Delta t}, \tag{9.1.4}$$

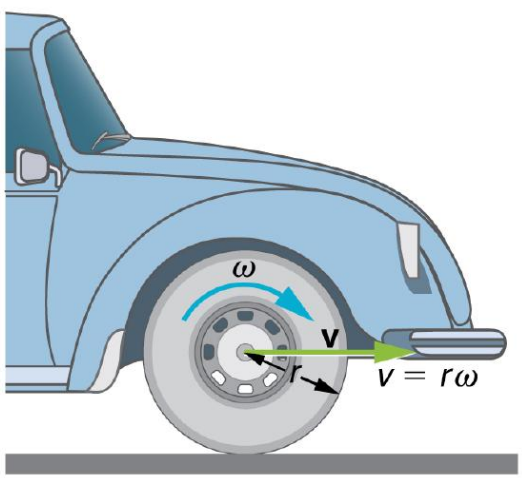
where an angular rotation Δ takes place in a time Δt. The greater the rotation angle in a given amount of time, the greater the angular velocity. The units for angular velocity are radians per second (rad/s).

Angular velocity ω is analogous to linear velocity v. To find the precise relationship between angular and linear velocity, we again consider a pit on the rotating CD. This pit moves an arc length Δs in a time Δt, and so it has a linear velocity $v = \frac{\Delta s}{\Delta t}$.

From $\Delta\theta = \frac{(\Delta s)}{r}$ we see that $\Delta s = r \cdot \Delta\theta$. Substituting this into the expression for v gives $v = \frac{(r \cdot \Delta\theta)}{(\Delta t)} = r(\frac{\Delta\theta}{\Delta t}) = r\omega$.

We can write this relationship in two different ways: $v = r\omega$ or $\omega = \frac{v}{r}$.

The first relationship states that the linear velocity v is proportional to the distance from the center of rotation, thus it is largest for a point on the rim (largest r), as you might expect. We can also call this linear speed v of a point on the rim the tangential speed. The second relationship can be illustrated by considering the tire of a moving car, as shown in the picture below. Note that the speed of the point at the center of the tire is the same as the speed v of the car. The faster the car moves, the faster the tire spins—large v means a large ω, because v=rω. Similarly, a larger-radius tire rotating at the same angular velocity (ω) will produce a greater linear speed (v) for the car.



Angular Velocity: A car moving at a velocity v to the right has a tire rotating with an angular velocity ω. The speed of the tread of the tire relative to the axle is v, the same as if the car were jacked up. Thus the car moves forward at linear velocity v=rω, where r is the tire radius. A larger angular velocity for the tire means a greater velocity for the car.

Angular Acceleration, Alpha

Angular acceleration is the rate of change of angular velocity, expressed mathematically as $\alpha=\frac{\Delta\omega}{\Delta t}$.

learning objectives

- Explain the relationship between angular acceleration and angular velocity

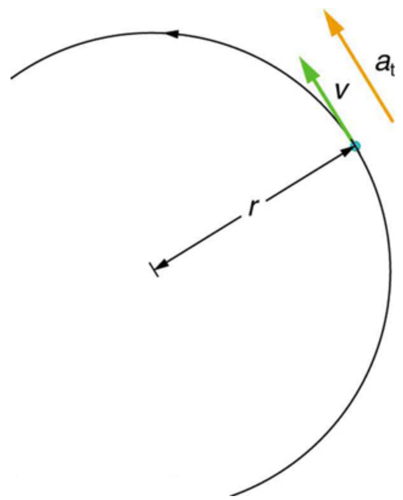
Angular acceleration is the rate of change of angular velocity. In SI units, it is measured in radians per second squared (rad/s^2), and is usually denoted by the Greek letter alpha (α).

Consider the following situations in which angular velocity is not constant: when a skater pulls in her arms, when a child starts up a merry-go-round from rest, or when a computer's hard disk slows to a halt when switched off. In all these cases, there is an angular acceleration in which ωω changes. The faster the change occurs, the greater the angular acceleration. Angular acceleration is defined as the rate of change of angular velocity. In equation form, angular acceleration is expressed as follows:

$$\alpha=\frac{\Delta\omega}{\Delta t} \tag{9.1.5}$$

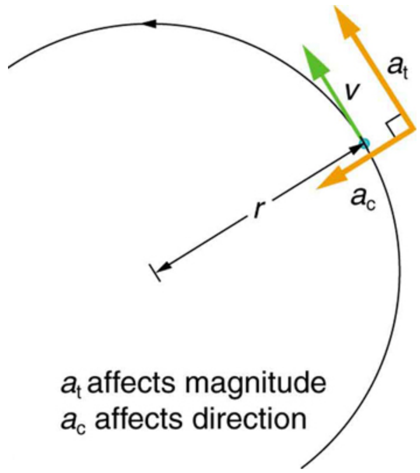
where $\Delta\omega$ is the change in angular velocity and Δt is the change in time. The units of angular acceleration are (rad/s)/s, or rad/s^2. If ωω increases, then αα is positive. If ωω decreases, then αα is negative.

It is useful to know how linear and angular acceleration are related. In circular motion, there is acceleration that is *tangent* to the circle at the point of interest (as seen in the diagram below). This acceleration is called *tangential acceleration*, a_t.



Tangential acceleration: In circular motion, acceleration can occur as the magnitude of the velocity changes: a is tangent to the motion. This acceleration is called tangential acceleration.

Tangential acceleration refers to changes in the magnitude of velocity but not its direction. In circular motion, centripetal acceleration, a_c, refers to changes in the direction of the velocity but not its magnitude. An object undergoing circular motion experiences centripetal acceleration (as seen in the diagram below.) Thus, a_t and a_c are perpendicular and independent of one another. Tangential acceleration a_t is directly related to the angular acceleration and is linked to an increase or decrease in the velocity (but not its direction).



Centripetal Acceleration: Centripetal acceleration occurs as the direction of velocity changes; it is perpendicular to the circular motion. Centripetal and tangential acceleration are thus perpendicular to each other.

Key Points

- The arc length Δs is the distance traveled along a circular path. r is the radius of curvature of the circular path.
- The rotation angle is the amount of rotation and is analogous to linear distance. We define the rotation angle $\Delta\theta$ to be the ratio of the arc length to the radius of curvature: $\Delta\theta = \frac{\Delta \mathrm{s}}{\mathrm{r}}$.
- For one complete revolution the rotation angle is 2π.
- The greater the rotation angle in a given amount of time, the greater the angular velocity.
- Angular velocity ω is analogous to linear velocity v.
- We can write the relationship between linear velocity and angular velocity in two different ways: $\mathrm{v} = \mathrm{r}\omega$ or $\omega = \frac{\mathrm{v}}{\mathrm{r}}$.
- The faster the change in angular velocity occurs, the greater the angular acceleration.
- In circular motion, linear acceleration is tangent to the circle at the point of interest, and is called tangential acceleration.
- In circular motion, centripetal acceleration refers to changes in the direction of the velocity but not its magnitude. An object undergoing circular motion experiences centripetal acceleration.

Key Terms

- **Angular position**: The angle in radians (degrees, revolutions) through which a point or line has been rotated in a specified sense about a specified axis.
- **angular velocity**: A vector quantity describing an object in circular motion; its magnitude is equal to the speed of the particle and the direction is perpendicular to the plane of its circular motion.
- **angular acceleration**: The rate of change of angular velocity, often represented by α.
- **tangential acceleration**: The acceleration in a direction tangent to the circle at the point of interest in circular motion.

9.2: Angular Acceleration

learning objectives

- Relate angle of rotation, angular velocity, and angular acceleration to their equivalents in linear kinematics

Simply by using our intuition, we can begin to see the interrelatedness of rotational quantities like θ (angle of rotation), ω(angular velocity) and α (angular acceleration). For example, if a motorcycle wheel has a large angular acceleration for a fairly long time, it ends up spinning rapidly and rotating through many revolutions. The wheel's rotational motion is analogous to the fact that the motorcycle's large translational acceleration produces a large final velocity, and the distance traveled will also be large.

Kinematic Equations

Kinematics is the description of motion. We have already studied kinematic equations governing linear motion under constant acceleration:

$$\mathrm{v}=\mathrm{v}_0+\mathrm{at} \tag{9.2.1}$$

$$\mathrm{x}=\mathrm{v}_0\mathrm{t}+\frac{1}{2}\mathrm{at}^2 \tag{9.2.2}$$

$$\mathrm{v}^2=\mathrm{v}_0^2+2\mathrm{ax} \tag{9.2.3}$$

Similarly, the kinematics of rotational motion describes the relationships among rotation angle, angular velocity, angular acceleration, and time. Let us start by finding an equation relating ω, α, and t. To determine this equation, we use the corresponding equation for linear motion:

$$\mathrm{v}=\mathrm{v}_0+\mathrm{at}. \tag{9.2.4}$$

As in linear kinematics where we assumed a is constant, here we assume that angular acceleration α is a constant, and can use the relation: $\mathrm{a}=\mathrm{r}\alpha$ Where r – radius of curve.Similarly, we have the following relationships between linear and angular values:

$$\mathrm{v}=\mathrm{r}\omega \tag{9.2.5}$$

$$\mathrm{x}=\mathrm{r}\theta \tag{9.2.6}$$

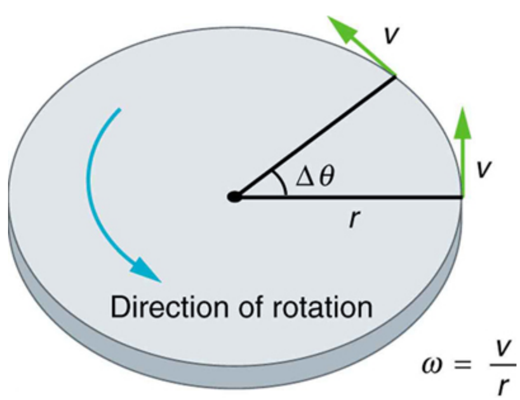
By using the relationships $\mathrm{a}=\mathrm{r}\alpha, \mathrm{v}=\mathrm{r}\omega,$ and $\mathrm{x}=\mathrm{r}\theta$, we derive all the other kinematic equations for rotational motion under constant acceleration:

$$\omega=\omega_0+\alpha\mathrm{t} \tag{9.2.7}$$

$$\theta=\omega_0\mathrm{t}+\frac{1}{2}\alpha\mathrm{t}^2 \tag{9.2.8}$$

$$\omega^2=\omega_0^2+2\alpha\theta \tag{9.2.9}$$

The equations given above can be used to solve any rotational or translational kinematics problem in which a and α are constant. shows the relationship between some of the quantities discussed in this atom.



Linear and Angular: This figure shows uniform circular motion and some of its defined quantities.

Key Points

- The kinematic equations for rotational and/or linear motion given here can be used to solve any rotational or translational kinematics problem in which a and α are constant.
- By using the relationships between velocity and angular velocity, distance and angle of rotation, and acceleration and angular acceleration, rotational kinematic equations can be derived from their linear motion counterparts.
- To derive rotational equations from the linear counterparts, we used the relationships $a = r\alpha, v = r\omega,$ and $x = r\theta$.

Key Terms

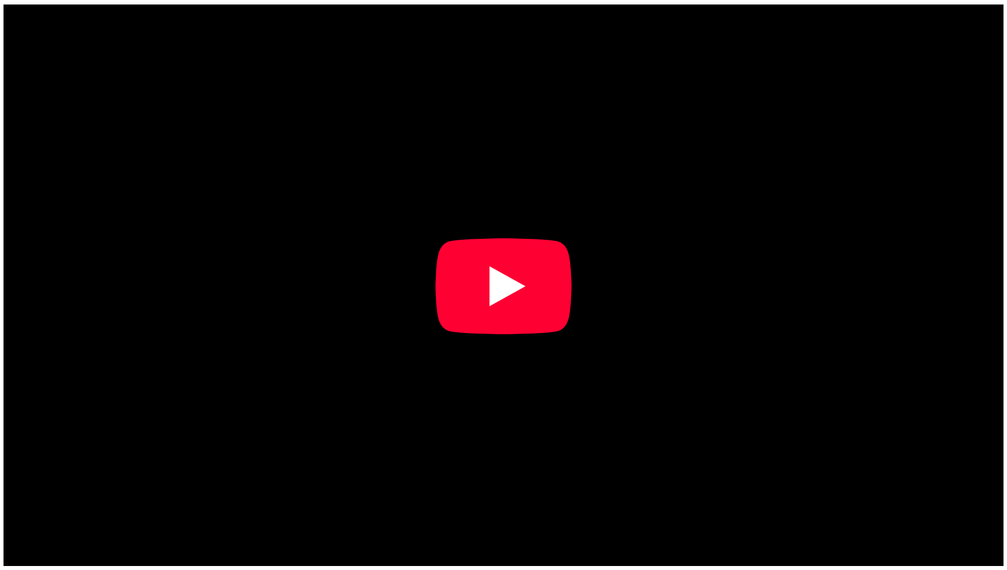
- **kinematics**: The branch of mechanics concerned with objects in motion, but not with the forces involved.
- **angular**: Relating to an angle or angles; having an angle or angles; forming an angle or corner; sharp-cornered; pointed; as in, an angular figure.

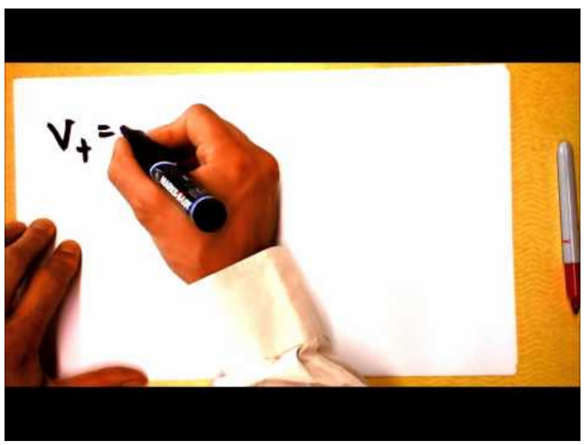
9.3: Rotational Kinematics

learning objectives

- Distinguish the two different motions in which rolling without slipping is broken down

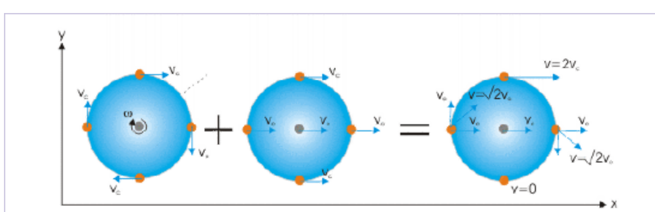
Rolling without slipping generally occurs when an object rolls without skidding. To relate this to a real world phenomenon, we can consider the example of a wheel rolling on a flat, horizontal surface.





Connecting Linear and Rotational Motion! Rolling without Slipping!: How fast does the axle of a bike wheel move? How fast does the BOTTOM of a wheel move?

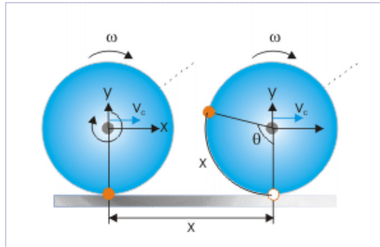
Rolling without slipping can be better understood by breaking it down into two different motions: 1) Motion of the center of mass, with linear velocity *v* (translational motion); and 2) rotational motion around its center, with angular velocity *w*.



Rolling Motion: Rolling motion is a combination of rotational and translational motion.

When an object is rolling on a plane without slipping, the point of contact of the object with the plane does not move. If we imagine a wheel moving forward by rolling on a plane at speed v, it must also be rotating about its axis at an angular speed ωω since it is rolling.

The object's angular velocity ωω is directly proportional to its velocity, because as we know, the faster we are driving in a car, the faster the wheels have to turn. So, to determine the exact relationship between linear velocity and angular velocity, we can imagine the scenario in which the wheel travels a distance of x while rotating through an angle θ.



Rolling Without Slipping: A body rolling a distance of x on a plane without slipping.

In mathematical terms, the length of the arc is equal to the angle of the segment multiplied by the object's radius, R. Therefore, we can say that the length of the arc of the wheel that has rotated an angle θ, is equal to Rθ. Furthermore, since the wheel is in constant contact with the ground, the length of the arc correlating to the angle is also equal to x. Therefore,

$$\mathrm{x}=\mathrm{R}\theta \tag{9.3.1}$$

Since xx and θθ depend on time, we can take the derivative of both sides to obtain:

$$\frac{\mathrm{dx}}{\mathrm{dt}}=\mathrm{R}\frac{d\theta}{\mathrm{dt}} \tag{9.3.2}$$

where $\frac{\mathrm{dx}}{\mathrm{dt}}$ is equal to the linear velocity v, and $\frac{d\theta}{\mathrm{dt}}$ is equal to the angular velocity ωω. We can then simplify this equation to:

$$\mathrm{v}=\omega\mathrm{R} \tag{9.3.3}$$

Key Points

- Rolling without slipping can be better understood by breaking it down into translational motion and rotational motion.
- When an object is rolling on a plane without slipping, the point of contact of the object with the plane does not move.
- A rolling object's velocity v is directly related to its angular velocity ω, and is mathematically expressed as $\mathrm{v}=\omega\mathrm{R}$, where R is the object's radius and v is its linear velocity.

Key Terms

- **angular velocity**: A vector quantity describing the motion of an object in circular motion; its magnitude is equal to the angular speed (ωω) of the particle, and the direction is perpendicular to the plane of its circular motion.
- **linear velocity**: A vector quantity that denotes the rate of change of position with respect to time of the object's center of mass.

- Sunil Kumar Singh, Rolling Motion. November 8, 2012. **Provided by**: OpenStax CNX. **Located at**: http://cnx.org/content/m14311/latest/. **License**: *CC BY: Attribution*
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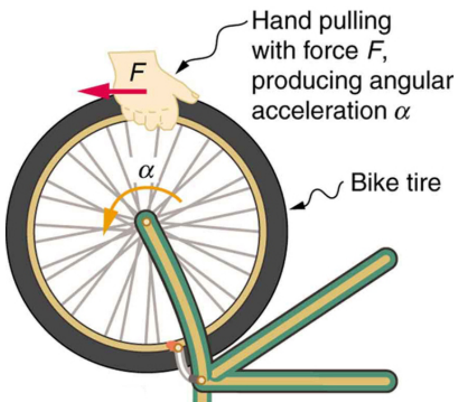
9.4: Dynamics

learning objectives

- Explain the relationship between the force, mass, radius, and angular acceleration

If you have ever spun a bike wheel or pushed a merry-go-round, you have experienced the force needed to change angular velocity. Our intuition is reliable in predicting many of the factors that are involved. For example, we know that a door opens slowly if we push too close to its hinges. Furthermore, we know that the more massive the door, the more slowly it opens. The first example implies that the farther the force is applied from the pivot, the greater the angular acceleration; another implication is that angular acceleration is inversely proportional to mass. These relationships should seem very similar to the familiar relationships among force, mass, and acceleration embodied in Newton's second law of motion. There are, in fact, precise rotational analogs to both force and mass.

Rotational inertia, as illustrated in, is the resistance of objects to changes in their rotation. In other words, a rotating object will stay rotating and a non-rotating object will stay non-rotating unless acted on by a torque. This should remind you of Newton's First Law.



Rotational Inertia: Force is required to spin the bike wheel. The greater the force, the greater the angular acceleration produced. The more massive the wheel, the smaller the angular acceleration. If you push on a spoke closer to the axle, the angular acceleration will be smaller.

To develop the precise relationship among force, mass, radius, and angular acceleration, consider what happens if we exert a force F on a point mass m that is at a distance r from a pivot point. Because the force is perpendicular to r, an acceleration $a = \frac{F}{m}$ is obtained in the direction of F. We can rearrange this equation such that F=ma and then look for ways to relate this expression to expressions for rotational quantities. We note that a=rα, and we substitute this expression into F=ma, yielding:

$$F = mr\alpha. \tag{9.4.1}$$

Recall that torque is the turning effectiveness of a force. In this case, because F is perpendicular to r, torque is simply $\tau = Fr$. So, if we multiply both sides of the equation above by r, we get torque on the left-hand side. That is,

$$rF = mr^2\alpha \tag{9.4.2}$$

, or

$$\tau = mr^2\alpha. \tag{9.4.3}$$

This equation is the rotational analog of Newton's second law ($F = ma$), where torque is analogous to force, angular acceleration is analogous to translational acceleration, and mr^2 is analogous to mass (or inertia). The quantity mr^2 is called the rotational inertia or moment of inertia of a point mass m a distance r from the center of rotation.

Different shapes of objects have different rotational inertia which depend on the distribution of their mass.

Key Points

- The farther the force is applied from the pivot, the greater the angular acceleration.
- Angular acceleration is inversely proportional to mass.
- The equation $\tau = \mathrm{mr}^2 \alpha.$ is the rotational analog of Newton's second law ($\mathrm{F} = \mathrm{ma}$), where torque is analogous to force, angular acceleration is analogous to translational acceleration, and mr^2 is analogous to mass (or inertia).

Key Terms

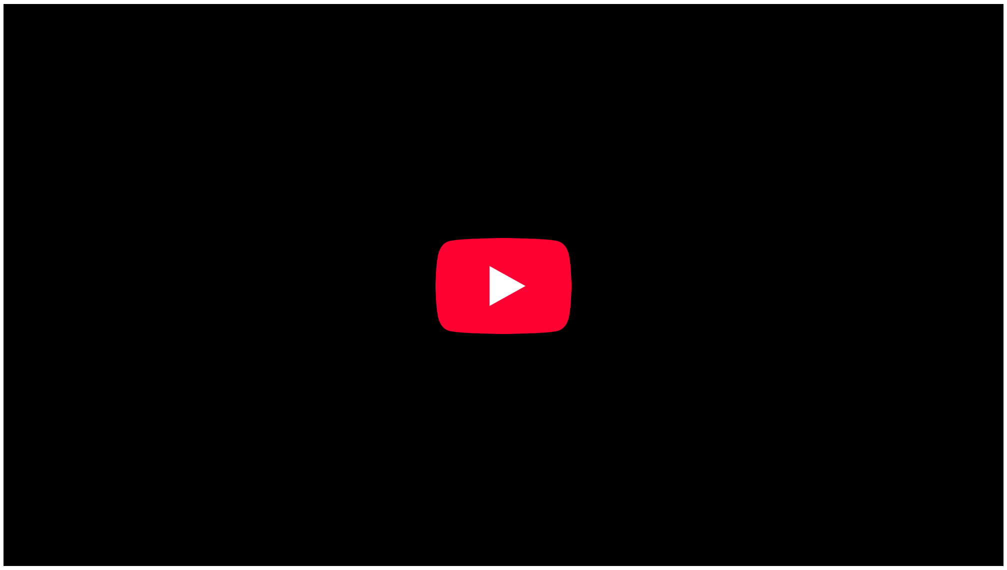
- **rotational inertia**: The tendency of a rotating object to remain rotating unless a torque is applied to it.
- **torque**: A rotational or twisting effect of a force; (SI unit newton-meter or Nm; imperial unit foot-pound or ft-lb)

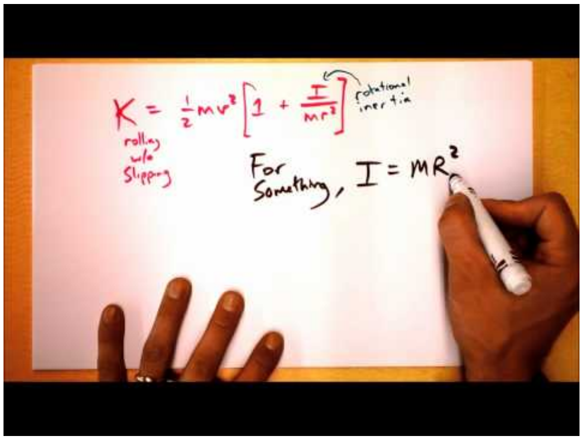
9.5: Rotational Kinetic Energy

learning objectives

- Express the rotational kinetic energy as a function of the angular velocity and the moment of inertia, and relate it to the total kinetic energy

Rotational kinetic energy is the kinetic energy due to the rotation of an object and is part of its total kinetic energy. Looking at rotational energy separately around an object's axis of rotation yields the following dependence on the object's moment of inertia:





Kinetic Energy of Rotation: Things that roll without slipping have some fraction of their energy as translational kinetic and the remainder as rotational kinetic. The ratio depends on the moment of inertia of the object that's rolling.

$$\mathrm{E_{rotational}} = \frac{1}{2}\mathrm{I}\omega^2, \tag{9.5.1}$$

where ω is the angular velocity and I is the moment of inertia around the axis of rotation.

The mechanical work applied during rotation is the torque (τ) times the rotation angle (θ) : $\mathrm{W} = \tau\theta$.

The instantaneous power of an angularly accelerating body is the torque times the angular velocity: $\mathrm{P} = \tau\omega$.

Note the close relationship between the result for rotational energy and the energy held by linear (or translational) motion:

$$\mathrm{E_{translational}} = \frac{1}{2}\mathrm{mv}^2. \tag{9.5.2}$$

In the rotating system, the moment of inertia takes the role of the mass and the angular velocity takes the role of the linear velocity.

As an example, let us calculate the rotational kinetic energy of the Earth (animated in Figure 1). As the Earth has a period of about 23.93 hours, it has an angular velocity of 7.29×10^{-5} rad/s. The Earth has a moment of inertia, $I = 8.04 \times 10^{37}$ kg·m2. Therefore, it has a rotational kinetic energy of 2.138×10^{29} J.

The Rotating Earth: The earth's rotation is a prominent example of rotational kinetic energy.

This can be partially tapped using tidal power. Additional friction of the two global tidal waves creates energy in a physical manner, infinitesimally slowing down Earth's angular velocity. Due to conservation of angular momentum this process transfers angular momentum to the Moon's orbital motion, increasing its distance from Earth and its orbital period.

Moment of Inertia

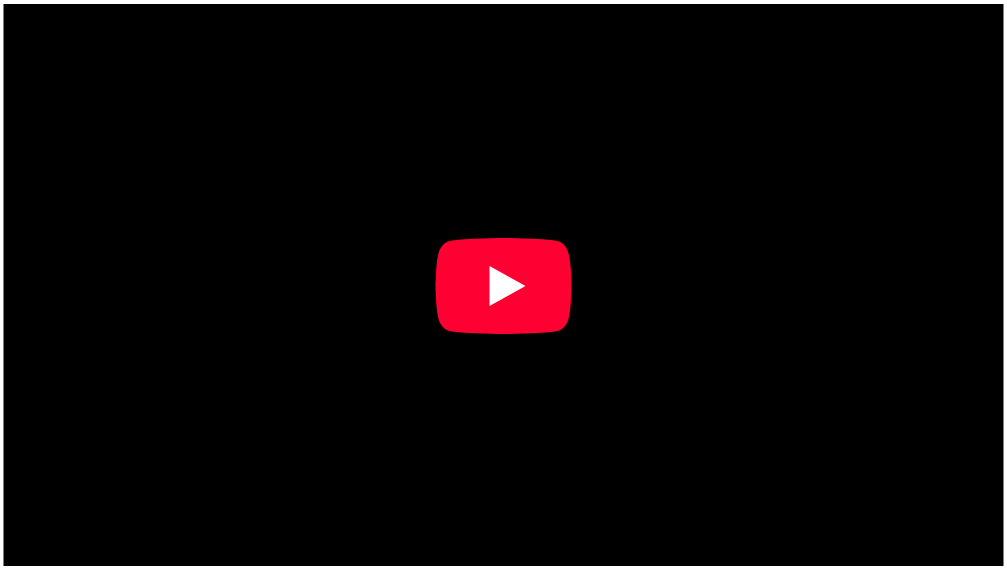
The moment of inertia is a property of a mass that measures its resistance to rotational acceleration about one or more axes.

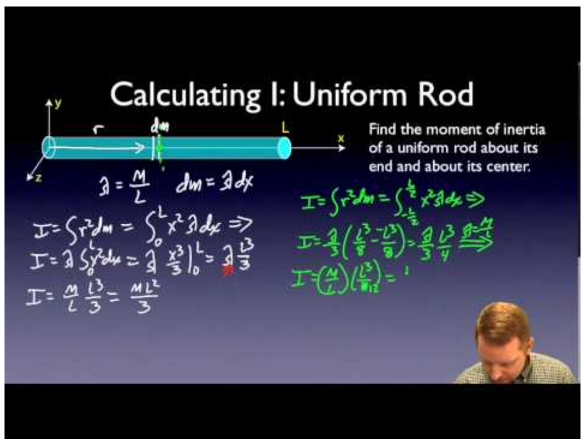
learning objectives

- Identify a property of a mass described by the moment of inertia

The Moment of Inertia

The moment of inertia is a property of the distribution of mass in space that measures mass's resistance to rotational acceleration about one or more axes. Newton's first law, which describes the inertia of a body in linear motion, can be extended to the inertia of a body rotating about an axis using the moment of inertia. That is, an object that is rotating at constant angular velocity will remain rotating unless it is acted upon by an external torque. In this way, the moment of inertia plays the same role in rotational dynamics as mass does in linear dynamics: it describes the relationship between angular momentum and angular velocity as well as torque and angular acceleration.





Moment of Inertia: A brief introduction to moment of inertia (rotational inertia) for calculus-based physics students.

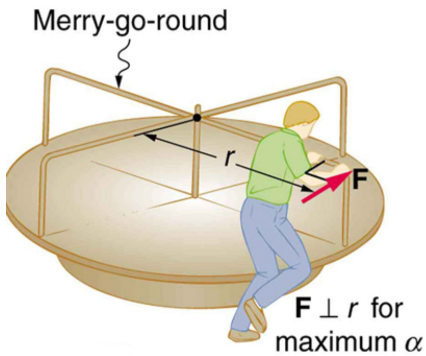
The moment of inertia *I* of an object can be defined as the sum of mr^2 for all the point masses of which it is composed, where m is the mass and r is the distance of the mass from the center of mass. It can be expressed mathematically as: $\mathrm{I} = \sum \mathrm{mr}^2$. Here, I is analogous to m in translational motion.

As an example, consider a hoop of radius r. Assuming that the hoop material is uniform, the hoop's moment of inertia can be found by summing up all the mass of the hoop and multiplying by the distance of that mass from the center of mass. Since the hoop is a circle and the mass is uniform around the circle, the moment of inertia is mr^2. All of the mass m is at a distance r from the center.

Moment of inertia also depends on the axis about which you rotate an object. Objects will usually rotate about their center of mass, but can be made to rotate about any axis. The moment of inertia in the case of rotation about a different axis other than the center of mass is given by the parallel axis theorem. The theorem states that the moment of inertia for an object rotated about a different axis parallel to the axis passing through the center of mass is $\mathrm{I_{cm}} + \mathrm{mr}^2$ where r is now the distance between the two axes and IcmIcmis the moment of inertia when rotated about the center of mass which you learned how to calculate in the previous paragraph.

A general relationship among the torque, moment of inertia, and angular acceleration is: $\mathrm{net}\ \tau = \mathrm{I}\alpha$, or $\alpha = \frac{(\mathrm{net}\ \tau)}{\mathrm{I}}$. Net τ is the total torque from all forces relative to a chosen axis. Such torques are either positive or negative and add like ordinary numbers. The relationship in $\tau = \mathrm{I}\alpha$ is the rotational analog to Newton's second law and is very applicable. This equation is actually valid for any torque, applied to any object, and relative to any axis.

As can be expected, the larger the torque, the larger the angular acceleration. For example, the harder a child pushes on a merry-go-round, the slower it accelerates for the same torque. The basic relationship between the moment of inertia and the angular acceleration is that the larger the moment of inertia, the smaller the angular acceleration. The moment of inertia depends not only on the mass of an object, but also on its distribution of mass relative to the axis around which it rotates. For example, it wil be much easier to accelerate a merry-go-round full of children if they stand close to its axis than if they all stand at the outer edge.



Moment of Inertia on a Merry-Go-Round: A father pushes a playground merry-go-round at its edge and perpendicular to its radius to achieve maximum torque.

Key Points

- Rotational kinetic energy can be expressed as: $\mathrm{E}_{\text{rotational}} = \frac{1}{2}\mathrm{I}\omega^2$ where ω is the angular velocity and I is the moment of inertia around the axis of rotation.
- The mechanical work applied during rotation is the torque times the rotation angle: (θ) : $\mathrm{W} = \tau\theta$.
- The instantaneous power of an angularly accelerating body is the torque times the angular velocity: $\mathrm{P} = \tau\omega$.
- There is a close relationship between the result for rotational energy and the energy held by linear (or translational) motion.
- Newton’s first law, which describes the inertia of a body in linear motion, can be extended to the inertia of a body rotating about an axis using the moment of inertia.
- An object that is rotating at constant angular velocity will remain rotating unless it is acted upon by an external torque.
- The larger the torque, the larger the angular acceleration.

Key Items

- **torque**: A rotational or twisting effect of a force; (SI unit newton-meter or Nm; imperial unit foot-pound or ft-lb)
- **inertia**: The property of a body that resists any change to its uniform motion; equivalent to its mass.
- **angular velocity**: A vector quantity describing an object in circular motion; its magnitude is equal to the speed of the particle and the direction is perpendicular to the plane of its circular motion.

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9.6: Conservation of Angular Momentum

learning objectives

- Evaluate the implications of net torque on conservation of energy

Let us consider some examples of momentum: the Earth continues to spin at the same rate it has for billions of years; a high-diver who is "rotating" when jumping off the board does not need to make any physical effort to continue rotating, and indeed would be unable to stop rotating before hitting the water. These examples have the hallmarks of a *conservation law*. Following are further observations to consider:

1. *A closed system is involved*. Nothing is making an effort to twist the Earth or the high-diver. They are isolated from rotation changing influences (hence the term "closed system").
2. *Something remains unchanged*. There appears to be a numerical quantity for measuring rotational motion such that the total amount of that quantity remains constant in a closed system.
3. *Something can be transferred back and forth without changing the total amount*. A diver rotates faster with arms and legs pulled toward the chest from a fully stretched posture.

Angular Momentum

The conserved quantity we are investigating is called angular momentum. The symbol for angular momentum is the letter L. Just as linear momentum is conserved when there is no net external forces, angular momentum is constant or conserved when the net torque is zero. We can see this by considering Newton's 2nd law for rotational motion:

$\vec{\tau} = \frac{d\vec{L}}{dt}$, where τ is the torque. For the situation in which the net torque is zero, $\frac{d\vec{L}}{dt} = 0$.

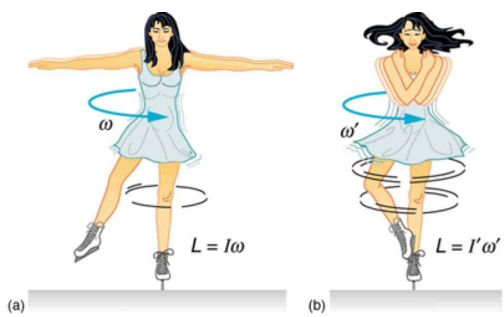
If the change in angular momentum ΔL is zero, then the angular momentum is constant; therefore,

$\vec{L} = \text{constant}$ (when net τ=0).

This is an expression for the law of conservation of angular momentum.

Example and Implications

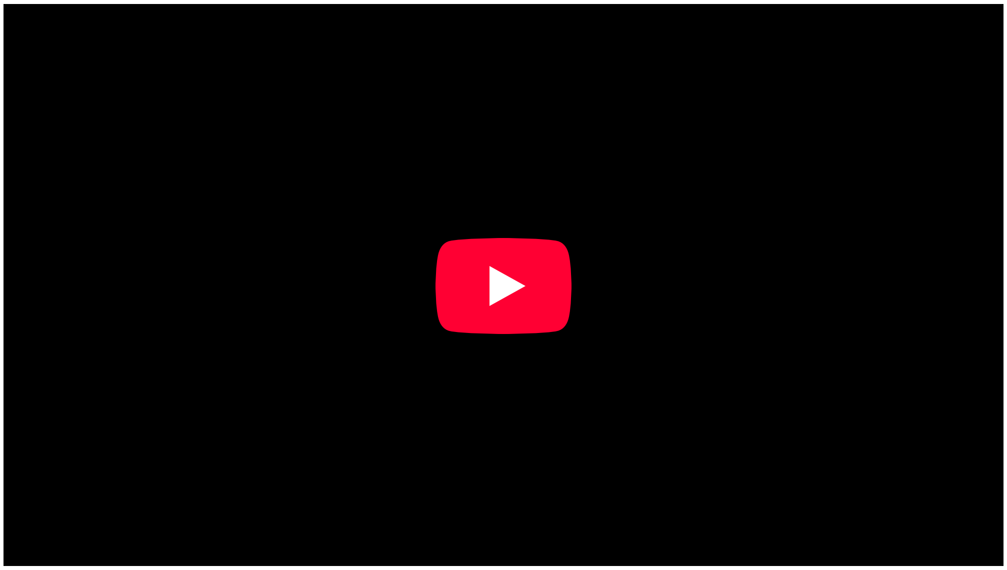
An example of conservation of angular momentum is seen in an ice skater executing a spin, as shown in. The net torque on her is very close to zero, because 1) there is relatively little friction between her skates and the ice, and 2) the friction is exerted very close to the pivot point.

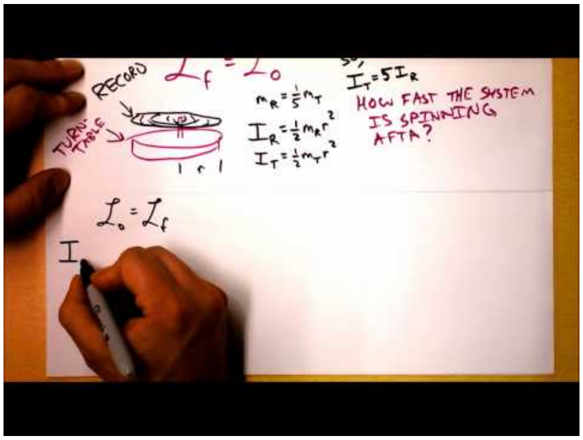


Conservation of Angular Momentum: An ice skater is spinning on the tip of her skate with her arms extended. Her angular momentum is conserved because the net torque on her is negligibly small. In the next image, her rate of spin increases greatly when she pulls in her arms, decreasing her moment of inertia. The work she does to pull in her arms results in an increase in rotational kinetic energy.

(Both F and r are small, and so $\vec{\tau} = \vec{r} \times \vec{F}$ is negligibly small.) Consequently, she can spin for quite some time. She can also increase her rate of spin by pulling in her arms and legs. When she does this, the rotational inertia decreases and the rotation rate increases in order to keep the angular momentum $L = I\omega$ constant. (I: rotational inertia, ω: angular velocity)

Conservation of angular momentum is one of the key conservation laws in physics, along with the conservation laws for energy and (linear) momentum. These laws are applicable even in microscopic domains where quantum mechanics governs; they exist due to inherent symmetries present in nature.





Conservation of Angular Momentum Theory: What it do?

Rotational Collisions

In a closed system, angular momentum is conserved in a similar fashion as linear momentum.

> **learning objectives**
>
> - Evaluate the difference in equation variables in rotational versus angular momentum

During a collision of objects in a closed system, momentum is always conserved. This fact is readily seen in linear motion. When an object of mass m and velocity v collides with another object of mass m_2 and velocity v_2, the net momentum after the collision, $mv_{1f} + mv_{2f}$, is the same as the momentum before the collision, $mv_{1i} + mv_{2i}$.

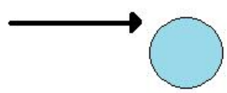
What if an rotational component of motion is introduced? Is momentum still conserved ?

Bowling ball and pi: When a bowling ball collides with a pin, linear and angular momentum is conserved

Yes. For objects with a rotational component, there exists angular momentum. Angular momentum is defined, mathematically, as $L = I\omega$, or $L = r \times p$. This equation is an analog to the definition of linear momentum as $p = mv$. Units for linear momentum are kg⋅m/s while units for angular momentum are kg⋅m^2/s. As we would expect, an object that has a large moment of inertia *I*, such as Earth, has a very large angular momentum. An object that has a large angular velocity ω, such as a centrifuge, also has a rather large angular momentum.

So rotating objects that collide in a closed system conserve not only linear momentum p in all directions, but also angular momentum L in all directions.

For example, take the case of an archer who decides to shoot an arrow of mass m_1 at a stationary cylinder of mass m_2 and radius r, lying on its side. If the archer releases the arrow with a velocity v_{1i} and the arrow hits the cylinder at its radial edge, what's the final momentum ?



Arrow hitting cyclinde: The arrow hits the edge of the cylinder causing it to roll.

Initially, the cylinder is stationary, so it has no momentum linearly or radially. Once the arrow is released, it has a linear momentum $p = m_{1i}$ and an angular component relative to the cylinders rotating axis, $L = rp = rm_1v_{1i}$. After the collision, the arrow sticks to the rolling cylinder and the system has a net angular momentum equal to the original angular momentum of the arrow before the collision.

Key Points

- When an object is spinning in a closed system and no external torques are applied to it, it will have no change in angular momentum.
- The conservation of angular momentum explains the angular acceleration of an ice skater as she brings her arms and legs close to the vertical axis of rotation.
- If the net torque is zero, then angular momentum is constant or conserved.
- Angular momentum is defined, mathematically, as $L = I\omega$, or $L = r \times p$. Which is the moment of inertia times the angular velocity, or the radius of the object crossed with the linear momentum.
- In a closed system, angular momentum is conserved in all directions after a collision.
- Since momentum is conserved, part of the momentum in a collision may become angular momentum as an object starts to spin after a collision.

Key Items

- **quantum mechanics**: The branch of physics that studies matter and energy at the level of atoms and other elementary particles; it substitutes probabilistic mechanisms for classical Newtonian ones.

- **torque**: A rotational or twisting effect of a force; (SI unit newton-meter or Nm; imperial unit foot-pound or ft-lb)
- **angular momentum**: A vector quantity describing an object in circular motion; its magnitude is equal to the momentum of the particle, and the direction is perpendicular to the plane of its circular motion.
- **momentum**: (of a body in motion) the product of its mass and velocity.
- **rotation**: The act of turning around a centre or an axis.

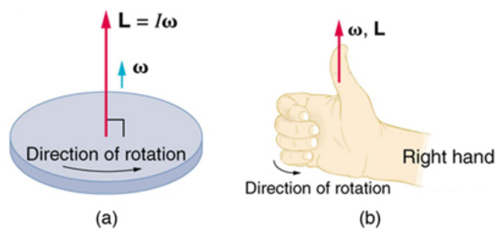
9.7: Vector Nature of Rotational Kinematics

learning objectives

- Identify the direction of a vector using the Right Hand Rule

Angular momentum and angular velocity have both magnitude and direction and, therefore, are vector quantities. The direction of these quantities is inherently difficult to track—a point on a rotating wheel is constantly rotating and changing direction. The axis of rotation of a rotating wheel is the only place that has a fixed direction. The direction of angular momentum and velocity can be determined along this axis.

Imagine the axis of rotation as a pole through the center of a wheel. The pole protrudes on both sides of the wheel and, depending on which side you're looking at, the wheel is turning either clockwise or counterclockwise. This dependency on perspective makes determining the angle of rotation slightly more difficult. As with all physical quantities, there is a standard for measurement that makes these types of quantities consistent. For angular quantities, the direction of the vector is determined using the Right Hand Rule, illustrated in.



The Right Hand Rule: Figure (a) shows a disk is rotating counterclockwise when viewed from above. Figure (b) shows the right-hand rule. The direction of angular velocity ω size and angular momentum L are defined to be the direction in which the thumb of your right hand points when you curl your fingers in the direction of the disk's rotation as shown.

The right hand rule can be used to find the direction of both the angular momentum and the angular velocity. From a spinning disc, for example, let's again imagine a pole through the center of the disc, at the axis of rotation. Using the right hand rule, your right hand would be grasping the pole so that your four fingers (index, middle, ring, and pinky) are following the direction of rotation. That is, an imaginary arrow from your wrist to your fingertips points in the same direction as the disc is rotating. In addition, your thumb is pointing straight out in the axis, perpendicular to your other fingers (or parallel to the 'pole' at the axis of rotation). Using this right hand rule, the direction of angular velocity ω and angular momentum L are defined as the direction in which the thumb of your right hand points when you curl your fingers in the direction of the disc's rotation.

Gyroscopes

A gyroscope is a spinning wheel or disk in which the axle is free to assume any orientation.

learning objectives

- Compare the concept of a rotating wheel with a gyroscope

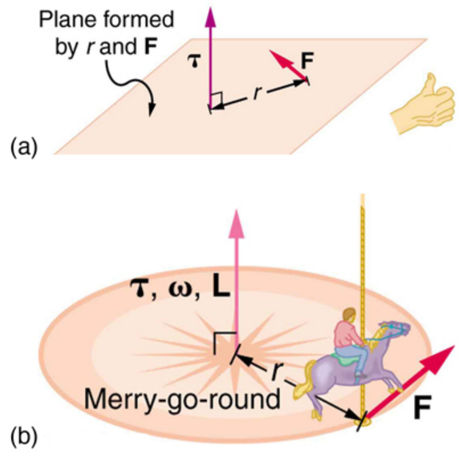
A gyroscope is a device for measuring or maintaining orientation based on the principles of angular momentum. Mechanically, a gyroscope is a spinning wheel or disk in which the axle is free to assume any orientation. Although this orientation does not remain fixed, it changes in response to an external torque much less and in a different direction than it would without the large angular momentum associated with the disk's high rate of spin and moment of inertia. The device's orientation remains nearly fixed, regardless of the mounting platform's motion, because mounting the device in a gimbal minimizes external torque.

How It Works: Examples

Torque: Torque changes angular momentum as expressed by the equation,

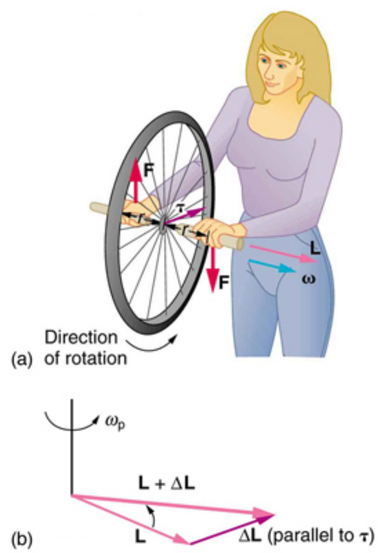
$$\tau = \frac{\Delta \mathrm{L}}{\Delta \mathrm{t}}. \tag{9.7.1}$$

This equation means that the direction of ΔL is the same as the direction of the torque that creates it, as illustrated in. This direction can be determined using the right hand rule, which says that the fingers on your hand curl towards the direction of rotation or force exerted, and your thumb points towards the direction of angular momentum, torque, and angular velocity.



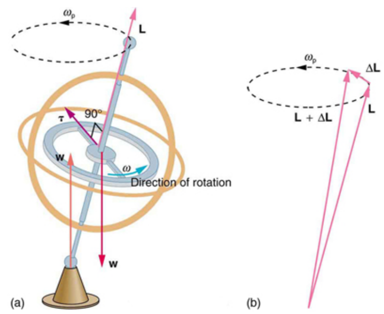
Direction of Torque and Angular Momentum: In figure (a), the torque is perpendicular to the plane formed by r and F and is the direction your right thumb would point to if you curled your fingers in the direction of F. Figure (b) shows that the direction of the torque is the same as that of the angular momentum it produces.

Rotating wheel: Consider a bicycle wheel with handles attached to it, as in. With the wheel rotating as shown, its angular momentum is to the woman's left. Suppose the person holding the wheel tries to rotate it as in the figure. Her natural expectation is that the wheel will rotate in the direction she pushes it, however, what happens is quite different. The forces exerted create a torque that is horizontal toward the person, and this torque creates a change in angular momentum L in the same direction, perpendicular to the original angular momentum L, thus changing the direction of L but not the magnitude of L. ΔL and L add, giving a new angular momentum with direction that is inclined more toward the person than before. The axis of the wheel has thus moved perpendicular to the forces exerted on it, instead of in the expected direction.



Gyroscopic Effect: In figure (a), a person holding the spinning bike wheel lifts it with her right hand and pushes down with her left hand in an attempt to rotate the wheel. This action creates a torque directly toward her. This torque causes a change in angular momentum ΔL in exactly the same direction. Figure (b) shows a vector diagram depicting how ΔL and L add, producing a new angular momentum pointing more toward the person. The wheel moves toward the person, perpendicular to the forces she exerts on it.

Gyroscope: This same logic explains the behavior of gyroscopes (see). There are two forces acting on a spinning gyroscope. The torque produced is perpendicular to the angular momentum, thus the direction of the angular momentum is changed, but not its magnitude. The gyroscope precesses around a vertical axis, since the torque is always horizontal and perpendicular to L. If the gyroscope is not spinning, it acquires angular momentum in the direction of the torque (L=ΔL), and it rotates around a horizontal axis, falling over just as we would expect.



Gyroscopes: As seen in figure (a), the forces on a spinning gyroscope are its weight and the supporting force from the stand. These forces create a horizontal torque on the gyroscope, which create a change in angular momentum ΔL that is also horizontal. In figure (b), ΔL and L add to produce a new angular momentum with the same magnitude, but different direction, so that the gyroscope precesses in the direction shown instead of falling over.

Applications

Gyroscopes serve as rotational sensors. For this reason, applications of gyroscopes include inertial navigation systems where magnetic compasses would not work (as in the Hubble telescope) or would not be precise enough (as in ICBMs). Another application is the stabilization of flying vehicles, such as radio-controlled helicopters or unmanned aerial vehicles.

Key Points

- Angular velocity and angular momentum are vector quantities and have both magnitude and direction.
- The direction of angular velocity and angular momentum are perpendicular to the plane of rotation.
- Using the right hand rule, the direction of both angular velocity and angular momentum is defined as the direction in which the thumb of your right hand points when you curl your fingers in the direction of rotation.
- Torque is perpendicular to the plane formed by r and F and is the direction your right thumb would point if you curled the fingers of your right hand in the direction of F.
- The direction of the torque is thus the same as that of the angular momentum it produces.
- The gyroscope precesses around a vertical axis, since the torque is always horizontal and perpendicular to L. If the gyroscope is not spinning, it acquires angular momentum in the direction of the torque, and it rotates about a horizontal axis, falling over just as we would expect.

Key Items

- **angular momentum**: A vector quantity describing an object in circular motion; its magnitude is equal to the momentum of the particle, and the direction is perpendicular to the plane of its circular motion.
- **right hand rule**: Direction of angular velocity ω and angular momentum L in which the thumb of your right hand points when you curl your fingers in the direction of rotation.
- **angular velocity**: A vector quantity describing an object in circular motion; its magnitude is equal to the speed of the particle and the direction is perpendicular to the plane of its circular motion.
- **gimbal**: A device for suspending something, such as a ship's compass, so that it will remain level when its support is tipped.
- **torque**: A rotational or twisting effect of a force; (SI unit newton-meter or Nm; imperial unit foot-pound or ft-lb)

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9.8: Problem Solving

learning objectives

- Develop and apply a strong problem-solving strategy for rotational kinematics

Problem-Solving Strategy For Rotational Kinematics

When solving problems on rotational kinematics:

- Examine the situation to determine that rotational kinematics (rotational motion) is involved. Rotation must be involved, but without the need to consider forces or masses that affect the motion.
- Identify exactly what needs to be determined in the problem (identify the unknowns). A sketch of the situation is useful.
- Make a list of what is given or can be inferred from the problem as stated (identify the knowns).
- Solve the appropriate equation or equations for the quantity to be determined (the unknown). It can be useful to think in terms of a translational analog because by now you are familiar with such motion.
- Substitute the known values along with their units into the appropriate equation, and obtain numerical solutions complete with units. Be sure to use units of radians for angles.
- Check your answer to see if it is reasonable: Does your answer make sense?

Example 9.8.1:

Suppose a large freight train accelerates from rest, giving its 0.350 m radius wheels an angular acceleration of 0.250 rad/s^2. After the wheels have made 200 revolutions (assume no slippage): (a) How far has the train moved down the track? (b) What are the final angular velocity of the wheels and the linear velocity of the train?

In part (a), we are asked to find x, and in (b) we are asked to find ω and v. We are given the number of revolutions θ, the radius of the wheels r, and the angular acceleration α.

The distance x is very easily found from the relationship between distance and rotation angle: $\theta = \frac{x}{r}$.

Solving this equation for x yields $x = r\theta$.

Before using this equation, we must convert the number of revolutions into radians, because we are dealing with a relationship between linear and rotational quantities:

$$\theta = (200\text{rev})\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 1257 \text{ rad}. \tag{9.8.1}$$

Substitute the known values into $x = r\theta$ to find the distance the train moved down the track:

$$x = r\theta = (0.350 \text{ m})(1257 \text{ rad}) = 440 \text{ m}. \tag{9.8.2}$$

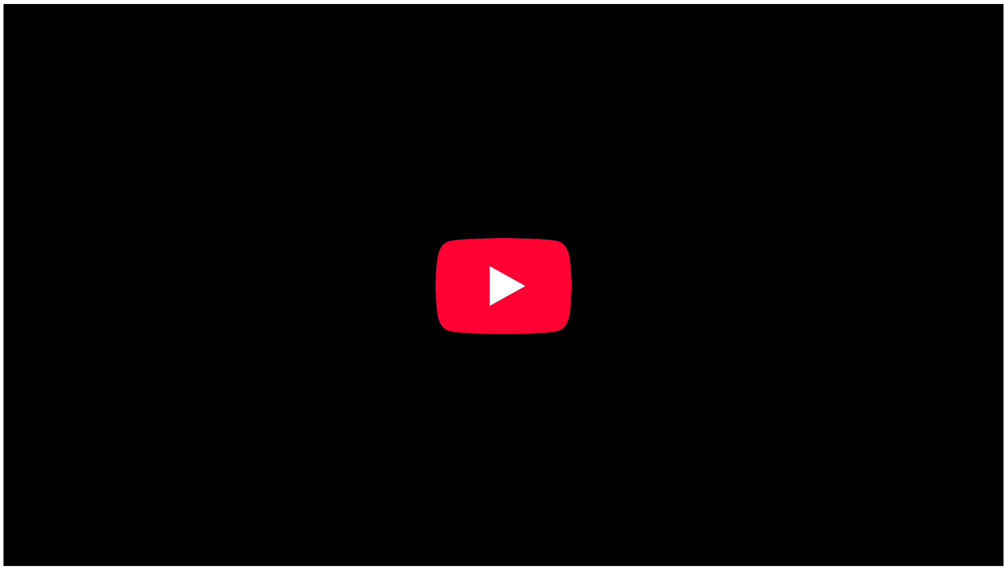
We cannot use any equation that incorporates t to find ω, because the equation would have at least two unknown values. The equation $\omega^2 = \omega_0^2 + 2\alpha\theta$ will work, because we know the values for all variables except ω. Taking the square root of this equation and entering the known values gives

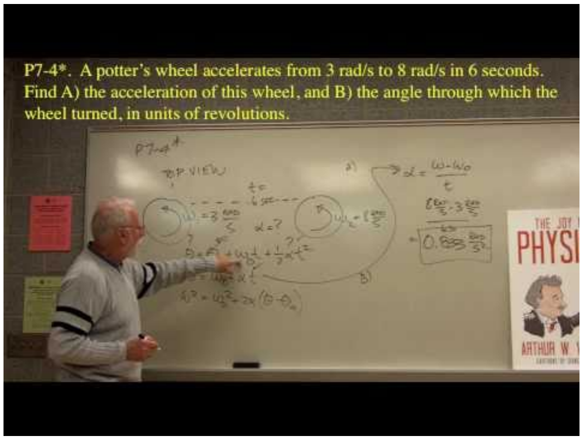
$$\omega = \sqrt{0 + 2(0.250 \text{ rad/s}^2)(1257 \text{ rad})} \tag{9.8.3}$$

$$= 25.1 \text{ rad/s} \tag{9.8.4}$$

One may find the linear velocity of the train, v, through its relationship to ω:

$$v = r\omega = (0.350 \text{ m})(25.1 \text{ rad/s}) = 8.77 \text{ m/s} \tag{9.8.5}$$





Rotational motion: Part of a series of videos on physics problem-solving. The problems are taken from "The Joy of Physics. " This one deals with angular motion. The viewer is urged to pause the video at the problem statement and work the problem before watching the rest of the video.

Rotational	Translational	
$\theta=\overline{\omega}t$	$x=\overline{v}t$	
$\omega=\omega_0+\alpha t$	$v=v_0+at$	(constant α, a)
$\theta=\omega_0 t+\frac{1}{2}\alpha t^2$	$x=v_0 t+\frac{1}{2}at^2$	(constant α, a)
$\omega^2={\omega_0}^2+2\alpha\theta$	$v^2={v_0}^2+2ax$	(constant α, a)

Table 1: **Rotational Kinematic Equations**

Equation list: Rotational and translational kinematic equations.

Key Points

- Examine the situation to determine that rotational kinematics (rotational motion) is involved, and identify exactly what needs to be determined.
- Make a list of what is given or can be inferred from the problem as stated and solve the appropriate equations.

- Substitute the known values along with their units into the appropriate equation, and obtain numerical solutions complete with units.

Key Terms

- **kinematics**: The branch of mechanics concerned with objects in motion, but not with the forces involved.

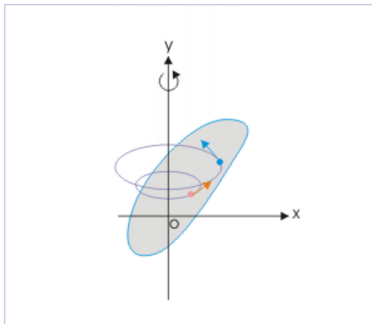
9.9: Linear and Rotational Quantities

learning objectives

- Derive uniform circular motion from linear equations

Defining Circular Motion

The description of circular motion is described better in terms of angular quantity than its linear counter part. The reasons are easy to understand. For example, consider the case of uniform circular motion. Here, the velocity of particle is changing – though the motion is "uniform". The two concepts do not go together. The general connotation of the term "uniform" indicates "constant", but the velocity is actually changing all the time.



A Rotating Body: Each particle constituting the body executes a uniform circular motion about the fixed axis. For the description of the motion, angular quantities are the better choice.

When we describe the uniform circular motion in terms of angular velocity, there is no contradiction. The velocity (i.e. angular velocity) is indeed constant. This is the first advantage of describing uniform circular motion in terms of angular velocity.

Second advantage is that angular velocity conveys the physical sense of the rotation of the particle as against linear velocity, which indicates translational motion. Alternatively, angular description emphasizes the distinction between two types of motion (translational and rotational).

Relationship Between Linear and Angular Speed

For simplicity, let's consider a uniform circular motion. For the length of the arc subtending angle " at the origin and "r" is the radius of the circle containing the position of the particle, we have $\mathrm{s=r\theta}$.

Differentiating with respect to time, we have

$$\mathrm{\frac{ds}{dt}=\frac{dr}{dt}\theta+r\frac{d\theta}{dt}}. \tag{9.9.1}$$

Because $\mathrm{\frac{dr}{dt}}=0$ for a uniform circular motion, we get $\mathrm{v=\omega r}$. Similarly, we also get $\mathrm{a=\alpha r}$ where a stands for linear acceleration, while α refers to angular acceleration (In a more general case, the relationship between angular and linear quantities are given as $\mathrm{v=\omega\times r, a=\alpha\times r+\omega\times v}$

Rotational Kinematic Equations

With the relationship of the linear and angular speed/acceleration, we can derive the following four rotational kinematic equations for constant aa and αα:

$$\mathrm{\omega=\omega_0+\alpha t:v=v_0+at} \tag{9.9.2}$$

$$\mathrm{\theta=\omega_0 t+(\frac{1}{2})\alpha t^2:x=v_0 t+(\frac{1}{2})at^2} \tag{9.9.3}$$

$$\mathrm{\omega^2=\omega_0^2+2\alpha\theta:v^2=v_0^2+2ax} \tag{9.9.4}$$

Mass, Momentum, Energy, and Newton's Second Law

As we use mass, linear momentum, translational kinetic energy, and Newton's 2nd law to describe linear motion, we can describe a general rotational motion using corresponding scalar/vector/tensor quantities:

- Mass/ Rotational inertia:
- Linenar/angular momentum:
- Force/ Torque:
- Kinetic energy:

For example, just as we use the equation of motion F=maF=ma to describe a linear motion, we can use its counterpart $\tau = \frac{dL}{dt} = r \times F$ to describe an angular motion. The descriptions are equivalent, and the choice can be made purely for the convenience of use.

Key Points

- As we use mass, linear momentum, translational kinetic energy, and Newton's 2nd law to describe linear motion, we can describe a general rotational motion using corresponding scalar/vector/tensor quantities.
- Angular and linear velocity have the following relationship: $v = \omega \times r$.
- As we use the equation of motion $F = ma$ to describe a linear motion, we can use its counterpart $\tau = \frac{dL}{dt} = r \times F$, to describe angular motion. The descriptions are equivalent, and the choice can be made purely for the convenience of use.

Key Items

- **uniform circular motion**: Movement around a circular path with constant speed.
- **torque**: A rotational or twisting effect of a force; (SI unit newton-meter or Nm; imperial unit foot-pound or ft-lb)
- **rotational inertia**: The tendency of a rotating object to remain rotating unless a torque is applied to it.

CHAPTER OVERVIEW

10: Fluids

Topic hierarchy

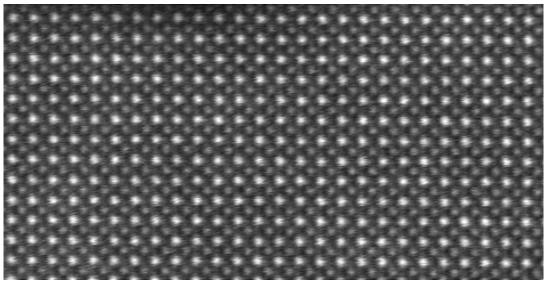
10.1: Introduction

learning objectives

- Assess the distinguishing characteristics of the four states of matter

There are a number of properties that can be observed in a material that identify what state the matter is in — solid, liquid, gas, or plasma.

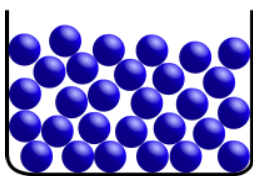
Solids



Solid: Solids are in a state of matter that maintains a fixed volume and shape.

A solid is in a state of matter that maintains a fixed volume and shape. A solid's particles fit closely together. The forces between the particles are so strong that the particles can not move freely; they can only vibrate. This causes a solid to be a stable, non-compressible shape with definite volume.

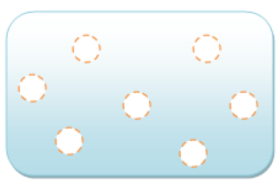
Liquids



Liquid: Liquids maintain a fixed volume, but their shape will mold to the shape of the container they are being held in.

A liquid maintains a fixed volume, but its shape will mold to the shape of the container it is being held in. In, you can see that even though the liquid's shape is determined by the container, it has a free surface that is not controlled by the container. The particles are close together but not as close as in solids; they are still able to move around, which causes the liquid to flow. Liquids usually have a higher volume than their solid counterparts, except for certain molecules, such as H_2O (water).

Gases

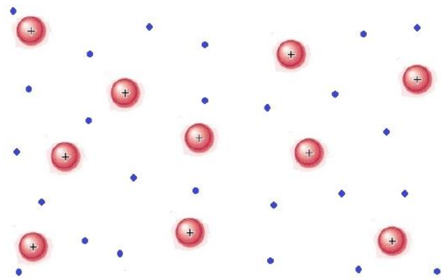


Gas: The particles are much farther from each other, usually a farther length than the size of the particles, and move a lot

shows a gas, whose particles move around a lot and are much farther apart from each other, usually farther apart than the diameter of the particles themselves. The gas behaves like a liquid; the particles are moving but are still attracted to each other, so they still flow. Unlike a solid or a liquid, the gas will try to fill whatever container it is in, adapting its volume accordingly.

Plasma

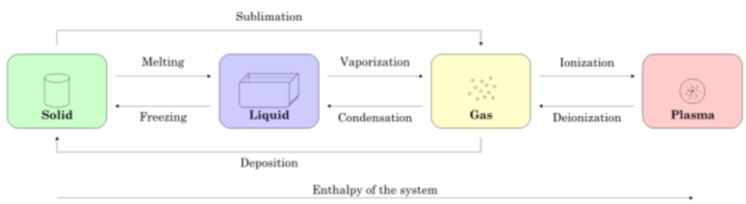
Plasma is a gas that has been ionized. That is to say, sufficient energy has been supplied to the gas such that the electrons have enough energy to escape their atoms or molecules. Plasma contains ions and electrons that can move around freely. Matter in the plasma state has variable volume and shape. Plasma is the most common form of visible matter in the universe. Lightning, sparks, neon lights, and the sun are all examples of matter in the plasma state.



Plasma: Matter in the plasma state has variable volume and shape, but as well as neutral atoms, it contains a significant number of ions and electrons, both of which can move around freely.

Phase Transitions

shows the different states of matter and how they can change from one to another as a function of enthalpy and pressure and temperature changes. A solid can change to a liquid with an enthalpy increase. The process of a liquid going to a solid is known as melting. A liquid can change into a gas when it hits its boiling point or can even enter a plasma state if the enthalpy is increased enough. When the enthalpy is lowered, a liquid can transform into a solid through freezing. Sometimes, a solid can skip freezing and go directly to a gaseous state; this process is called sublimation.



States of Matter: This figure illustrates the relationship between the enthalpy of a system and the state of matter that the system is in.

What is a Fluid?

A fluid is a substance that continually deforms (flows) under an applied shear stress.

learning objectives

- Explain the properties of substances under an applied shear stress

A fluid is a substance that continually deforms (flows) under an applied shear stress. Fluids are a subset of the states of matter and include three of the four states—liquids, gases, and plasma (shown in).

Four Fundamental States of Matter: Four fundamental states of matter: 1) top left corner corresponds to solid; 2) top right corner corresponds to liquid; 3) bottom left corner corresponds to gas; 4) bottom right corner corresponds to plasma.

Liquids form a free surface (i.e., a surface not created by the container) while gases do not. The distinction between solids and fluid is not entirely obvious. The distinction is made by evaluating the viscosity of the substance. Silly Putty can be considered to behave like a solid or a fluid, depending on the time period over which it is observed. It is best described as a viscoelastic fluid.

Fluids display properties such as:

- a) not resisting deformation or resisting it only lightly (viscosity), and
- b) the ability to flow (also described as the ability to take on the shape of the container).

This also means that all fluids have the property of fluidity. These properties are typically a function of their inability to support a shear stress in static equilibrium.

Solids can be subjected to shear stresses, and normal stresses—both compressive and tensile. In contrast, ideal fluids can only be subjected to normal, compressive stress (called pressure). Real fluids display viscosity and so are capable of being subjected to low levels of shear stress.

Although the term *fluid* includes both the liquid and gas phases, it is also commonly used as a synonym for *liquid*, with no implication that gas could also be present. For example, “brake fluid” is hydraulic oil and will not perform its required function if there is gas in it. This colloquial usage of the term is also common in the fields of medicine and nutrition (e.g., “take plenty of fluids”).

Key Points

- Solids are non-compressible and have constant volume and constant shape.
- Liquids are non-compressible and have constant volume but can change shape. A liquid’s shape is dictated by the shape of the container it is in.
- Gases do not have a constant volume or shape; they not only take the shape of the container they are in, they try to fill the entire container.
- Matter in the plasma state has variable volume and shape. Plasma contains ions and electrons, both of which can move around freely.
- Fluids are a subset of the states of matter and include liquids, gases, and plasma.
- Fluids display properties such as: not resisting deformation or resisting it only lightly (viscosity), and the ability to flow (also described as the ability to take on the shape of the container).
- Ideal fluids can only be subjected to normal, compressive stress which is called pressure. Real fluids display viscosity and thus are capable of being subjected to low levels of shear stress.

Key Items

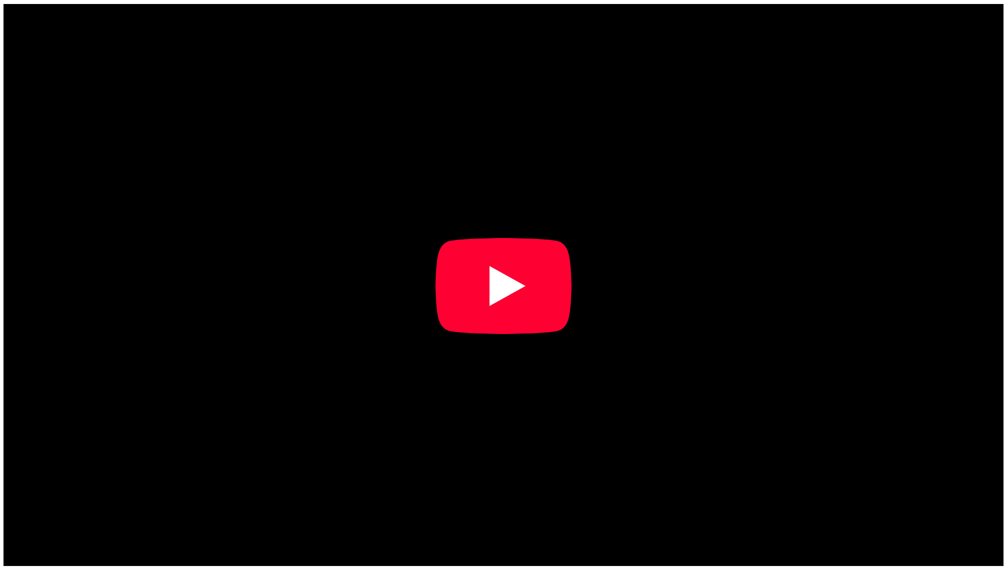
- **plasma**: a state of matter consisting of partially ionized gas
- **enthalpy**: the total amount of energy in a system, including both the internal energy and the energy needed to displace its environment
- **sublimation**: the transition of a substance from the solid phase directly to the vapor state such that it does not pass through the intermediate, liquid phase
- **fluidity**: A measure of the extent to which something is fluid. The reciprocal of its viscosity.
- **viscosity**: A quantity expressing the magnitude of internal friction in a fluid, as measured by the force per unit area resisting uniform flow.
- **shear stress**: The component of stress that causes parallel layers of a material to move relative to each other in their own planes.

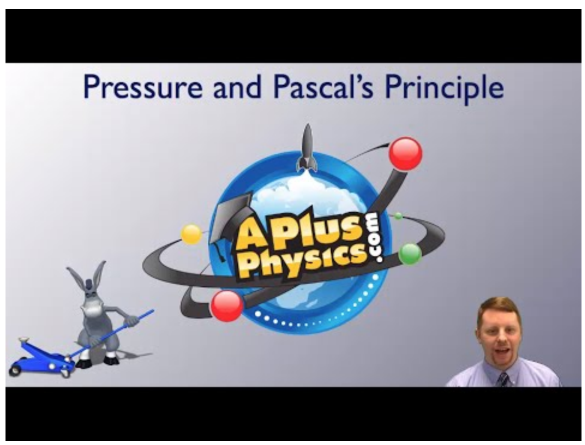
10.2: Density and Pressure

learning objectives

- Identify factors that determine the pressure exerted by the gas

Pressure is an important physical quantity—it plays an essential role in topics ranging from thermodynamics to solid and fluid mechanics. As a scalar physical quantity (having magnitude but no direction), pressure is defined as the force per unit area applied perpendicular to the surface to which it is applied. Pressure can be expressed in a number of units depending on the context of use.





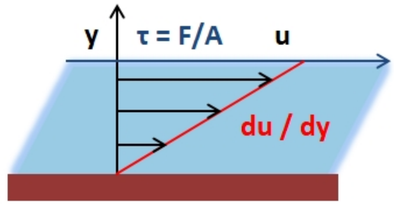
Pressure and Pascal's Principle: A brief introduction to pressure and Pascal's Principle, including hydraulics.

Units, Equations and Representations

In SI units, the unit of pressure is the Pascal (Pa), which is equal to a Newton / meter^2 (N/m^2). Other important units of pressure include the pound per square inch (psi) and the standard atmosphere (atm). The elementary mathematical expression for pressure is given by:

$$\text{pressure} = \frac{\text{Force}}{\text{Area}} = \frac{\mathrm{F}}{\mathrm{A}} \tag{10.2.1}$$

where p is pressure, F is the force acting perpendicular to the surface to which this force is applied, and A is the area of the surface. Any object that possesses weight, whether at rest or not, exerts a pressure upon the surface with which it is in contact. The magnitude of the pressure exerted by an object on a given surface is equal to its weight acting in the direction perpendicular to that surface, divided by the total surface area of contact between the object and the surface. shows the graphical representations and corresponding mathematical expressions for the case in which a force acts perpendicular to the surface of contact, as well as the case in which a force acts at angle θ relative to the surface.

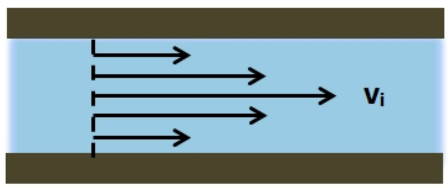


$$\frac{F}{A} = \tau = \eta \frac{du}{dy}$$

Representation of Pressure: This image shows the graphical representations and corresponding mathematical expressions for the case in which a force acts perpendicular to the surface of contact, as well as the case in which a force acts at angle θ relative to the surface.

Pressure as a Function of Surface Area

Since pressure depends only on the force acting perpendicular to the surface upon which it is applied, only the force component perpendicular to the surface contributes to the pressure exerted by that force on that surface. Pressure can be increased by either increasing the force or by decreasing the area or can oppositely be decreased by either decreasing the force or increasing the area. illustrates this concept. A rectangular block weighing 1000 N is first placed horizontally. It has an area of contact (with the surface upon which it is resting) of 0.1 m^2, thus exerting a pressure of 1,000 Pa on that surface. That same block in a different configuration (also in Figure 2), in which the block is placed vertically, has an area of contact with the surface upon which it is resting of 0.01 m^2, thus exerting a pressure of 10,000 Pa—10 times larger than the first configuration due to a decrease in the surface area by a factor of 10.



Pressure as a Function of Surface Area: Pressure can be increased by either increasing the force or by decreasing the area or can oppositely be decreased by either decreasing the force or increasing the area.

A good illustration of this is the reason a sharp knife is far more effective for cutting than a blunt knife. The same force applied by a sharp knife with a smaller area of contact will exert a much greater pressure than a blunt knife having a considerably larger area of contact. Similarly, a person standing on one leg on a trampoline causes a greater displacement of the trampoline than that same person standing on the same trampoline using two legs—not because the individual exerts a larger force when standing on one leg, but because the area upon which this force is exerted is decreased, thus increasing the pressure on the trampoline. Alternatively, an object having a weight larger than another object of the same dimensionality and area of contact with a given surface will exert a greater pressure on that surface due to an increase in force. Finally, when considering a given force of constant magnitude acting on a constant area of a given surface, the pressure exerted by that force on that surface will be greater the larger the angle of that force as it acts upon the surface, reaching a maximum when that force acts perpendicular to the surface.

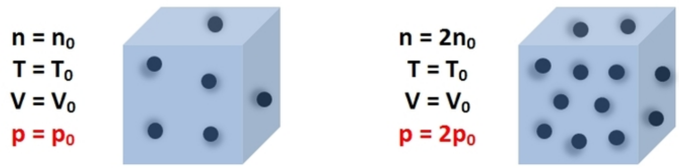
Liquids and Gases: Fluids

Just as a solid exerts a pressure on a surface upon which it is in contact, liquids and gases likewise exert pressures on surfaces and objects upon which they are in contact with. The pressure exerted by an ideal gas on a closed container in which it is confined is best analyzed on a molecular level. Gas molecules in a gas container move in a random manner throughout the volume of the container, exerting a force on the container walls upon collision. Taking the overall average force of all the collisions of the gas molecules confined within the container over a unit time allows for a proper measurement of the effective force of the gas molecules on the container walls. Given that the container acts as a confining surface for this net force, the gas molecules exert a pressure on the container. For such an ideal gas confined within a rigid container, the pressure exerted by the gas molecules can be calculated using the ideal gas law:

$$\mathrm{p} = \frac{\mathrm{nRT}}{\mathrm{V}} \tag{10.2.2}$$

where n is the number of gas molecules, R is the ideal gas constant (R = 8.314 J mol^{-1} K^{-1}), T is the temperature of the gas, and V is the volume of the container.

The pressure exerted by the gas can be increased by: increasing the number of collisions of gas molecules per unit time by increasing the number of gas molecules; increasing the kinetic energy of the gas by increasing the temperature; or decreasing the volume of the container. offers a representation of the ideal gas law, as well as the effect of varying the equation parameters on the gas pressure. Another common type of pressure is that exerted by a static liquid or hydrostatic pressure. Hydrostatic pressure is most easily addressed by treating the liquid as a continuous distribution of matter, and may be considered a measure of energy per unit volume or energy density. We will further discuss hydrostatic pressure in other sections.



$$p = \frac{nRT}{V}$$

n	n_0	$2n_0$	n_0	n_0
T	T_0	T_0	T_0	T_0
V	V_0	V_0	V_0	$2V_0$
p	p_0	$2p_0$	$2p_0$	$½p_0$

Pressure of an Ideal Gas: This image is a representation of the ideal gas law, as well as the effect of varying the equation parameters on the gas pressure.

Variation of Pressure With Depth

Pressure within static fluids depends on the properties of the fluid, the acceleration due to gravity, and the depth within the fluid.

learning obectives

- Identify factors that determine the pressure exerted by static liquids and gases

Pressure is defined in simplest terms as force per unit area. However, when dealing with pressures exerted by gases and liquids, it is most convenient to approach pressure as a measure of energy per unit volume by means of the definition of work (W = F·d). The derivation of pressure as a measure of energy per unit volume from its definition as force per unit area is given in. Since, for gases and liquids, the force acting on a system contributing to pressure does not act on a specific point or particular surface, but rather as a distribution of force, analyzing pressure as a measure of energy per unit volume is more appropriate. For liquids and gases at rest, the pressure of the liquid or gas at any point within the medium is called the hydrostatic pressure. At any such point within a medium, the pressure is the same in all directions, as if the pressure was not the same in all directions, the fluid, whether it is a gas or liquid, would not be static. Note that the following discussion and expressions pertain only to incompressible fluids at static equilibrium.

$$\Delta p = \frac{8\eta Q \Delta x}{\pi r^4}$$

Energy per Unit Volume: This equation is the derivation of pressure as a measure of energy per unit volume from its definition as force per unit area.

The pressure exerted by a static liquid depends only on the depth, density of the liquid, and the acceleration due to gravity. gives the expression for pressure as a function of depth within an incompressible, static liquid as well as the derivation of this equation from the definition of pressure as a measure of energy per unit volume (ρ is the density of the gas, g is the acceleration due to gravity, and h is the depth within the liquid). For any given liquid with constant density throughout, pressure increases with increasing depth. For example, a person under water at a depth of h_1 will experience half the pressure as a person under water at a depth of $h_2 = 2h_1$. For many liquids, the density can be assumed to be nearly constant throughout the volume of the liquid and, for virtually all practical applications, so can the acceleration due to gravity (g = 9.81 m/s^2). As a result, pressure within a liquid is therefore a function of depth only, with the pressure increasing at a linear rate with respect to increasing depth. In practical applications involving calculation of pressure as a function of depth, an important distinction must be made as to whether the absolute or relative pressure within a liquid is desired. Equation 2 by itself gives the pressure exerted by a liquid relative to atmospheric pressure, yet if the absolute pressure is desired, the atmospheric pressure must then be added to the pressure exerted by the liquid alone.

$$R_e = \frac{V}{I} \quad \rightarrow \quad \begin{cases} R_h \rightarrow R_e \\ \Delta p \rightarrow V \\ I \rightarrow Q \end{cases} \quad \rightarrow \quad \Delta p = \frac{8\eta Q \Delta x}{\pi r^4} \quad \rightarrow \quad R_h = \frac{\Delta p}{Q} = \frac{8\eta \Delta x}{\pi r^4}$$

Pressure as Energy per Unit Volume: This equation gives the expression for pressure as a function of depth within an incompressible, static liquid as well as the derivation of this equation from the definition of pressure as a measure of energy per unit volume (ρ is the density of the gas, g is the acceleration due to gravity, and h is the depth within the liquid).

When analyzing pressure within gases, a slightly different approach must be taken as, by the nature of gases, the force contributing to pressure arises from the average number of gas molecules occupying a certain point within the gas per unit time. Thus the force contributing to the pressure of a gas within the medium is not a continuous distribution as for liquids and the barometric equation given in must be utilized to determine the pressure exerted by the gas at a certain depth (or height) within the gas (p_0 is the pressure at h = 0, M is the mass of a single molecule of gas, g is the acceleration due to gravity, k is the Boltzmann constant, T is the temperature of the gas, and h is the height or depth within the gas). Equation 3 assumes that the gas is incompressible and that the pressure is hydrostatic.

$$p = p_0 e^{-\frac{Mgh}{kT}}$$

Pressure within a gas: The force contributing to the pressure of a gas within the medium is not a continuous distribution as for liquids and the barometric equation given in this figure must be utilized to determine the pressure exerted by the gas at a certain depth (or height) within the gas (p0 is the pressure at h = 0, M is the mass of a single molecule of gas, g is the acceleration due to gravity, k is the Boltzmann constant, T is the temperature of the gas, and h is the height or depth within the gas)

Static Equilibrium

Any region or point, or any static object within a static fluid is in static equilibrium where all forces and torques are equal to zero.

learning obectives

- Identify required conditions for a fluid to be in rest

Static equilibrium is a particular state of a physical system. It is qualitatively described by an object at rest and by the sum of all forces, with the sum of all torques acting on that object being equal to zero. Static objects are in static equilibrium, with the net force and net torque acting on that object being equal to zero; otherwise there would be a driving mechanism for that object to undergo movement in space. The analysis and study of objects in static equilibrium and the forces and torques acting on them is called statics—a subtopic of mechanics. Statics is particularly important in the design of static and load bearing structures. As it pertains to fluidics, static equilibrium concerns the forces acting on a static object within a fluid medium.

Fluids

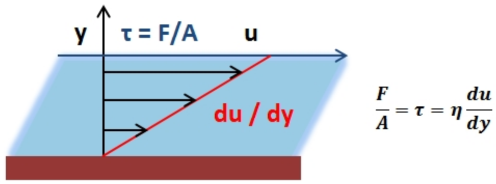
For a fluid at rest, the conditions for static equilibrium must be met at any point within the fluid medium. Therefore, the sum of the forces and torques at any point within the static liquid or gas must be zero. Similarly, the sum of the forces and torques of an object at rest within a static fluid medium must also be zero. In considering a stationary object within a liquid medium at rest, the forces acting at any point in time and at any point in space within the medium must be analyzed. For a stationary object within a static liquid, there are no torques acting on the object so the sum of the torques for such a system is immediately zero; it need not concern analysis since the torque condition for equilibrium is fulfilled.

Density

At any point in space within a static fluid, the sum of the acting forces must be zero; otherwise the condition for static equilibrium would not be met. In analyzing such a simple system, consider a rectangular region within the fluid medium with density ρ_L (same density as the fluid medium), width w, length l, and height h, as shown in. Next, the forces acting on this region within the medium are taken into account. First, the region has a force of gravity acting downwards (its weight) equal to its density object, times its volume of the object, times the acceleration due to gravity. The downward force acting on this region due to the fluid above the region is equal to the pressure times the area of contact. Similarly, there is an upward force acting on this region due to the fluid below the region equal to the pressure times the area of contact. For static equilibrium to be achieved, the sum of these forces must be zero, as shown in. Thus for any region within a fluid, in order to achieve static equilibrium, the pressure from the fluid below the region must be greater than the pressure from the fluid above by the weight of the region. This force which counteracts the weight of a region or object within a static fluid is called the buoyant force (or buoyancy).

$$\Delta p = \frac{8\eta Q \Delta x}{\pi r^4}$$

Static Equilibrium of a Region Within a Fluid: This figure shows the equations for static equilibrium of a region within a fluid.



Region Within a Static Fluid: This figure is a free body diagram of a region within a static fluid.

In the case on an object at stationary equilibrium within a static fluid, the sum of the forces acting on that object must be zero. As previously discussed, there are two downward acting forces, one being the weight of the object and the other being the force exerted by the pressure from the fluid above the object. At the same time, there is an upwards force exerted by the pressure from the fluid below the object, which includes the buoyant force. shows how the calculation of the forces acting on a stationary object within a static fluid would change from those presented in if an object having a density ρ_S different from that of the fluid medium is surrounded by the fluid. The appearance of a buoyant force in static fluids is due to the fact that pressure within the fluid changes as depth changes. The analysis presented above can furthermore be extended to much more complicated systems involving complex objects and diverse materials.

Pascal’s Principle

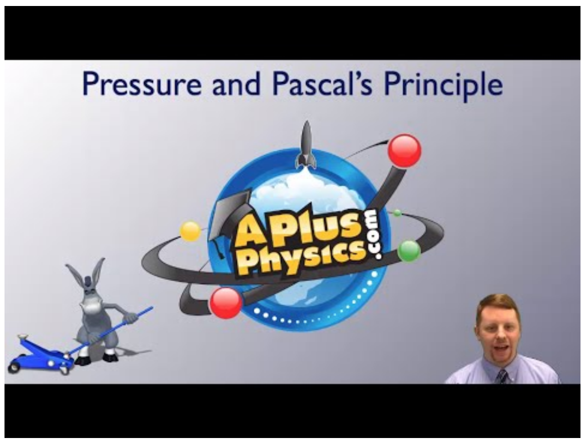
Pascal’s Principle states that pressure is transmitted and undiminished in a closed static fluid.

learning obectives

- Apply Pascal’s Principle to describe pressure behavior in static fluids

Pascal’s Principle

Pascal’s Principle (or Pascal’s Law) applies to static fluids and takes advantage of the height dependency of pressure in static fluids. Named after French mathematician Blaise Pascal, who established this important relationship, Pascal’s Principle can be used to exploit pressure of a static liquid as a measure of energy per unit volume to perform work in applications such as hydraulic presses. Qualitatively, Pascal’s Principle states that pressure is transmitted undiminished in an enclosed static liquid. Quantitatively, Pascal’s Law is derived from the expression for determining the pressure at a given height (or depth) within a fluid and is defined by Pascal’s Principle:



Pressure and Pascal’s Principle: A brief introduction to pressure and Pascal’s Principle, including hydraulics.

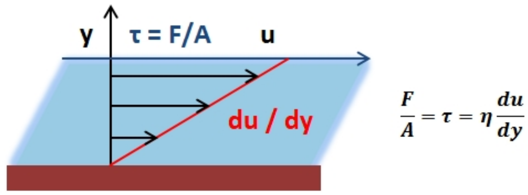
$$p_2 = p_1 + \Delta p, \Delta p = \rho g \Delta h \tag{10.2.3}$$

where p_1 is the external applied pressure, ρ is the density of the fluid, Δh is the difference in height of the static liquid, and g is the acceleration due to gravity. Pascal’s Law explicitly determines the pressure difference between two different heights (or depths) within a static liquid. As, by Pascal’s Law, a change in pressure is linearly proportional to a change in height within an incompressible, static liquid of constant density, doubling the height between the two points of reference will double the change of pressure, while halving the height between the two points will half the change in pressure.

Enclosed Static Liquids

While Pascal’s Principle applies to any static fluid, it is most useful in terms of applications when considering systems involving rigid wall closed column configurations containing homogeneous fluids of constant density. By exploiting the fact that pressure is transmitted undiminished in an enclosed static liquid, such as in this type of system, static liquids can be used to transform small amounts of force into large amounts of force for many applications such as hydraulic presses.

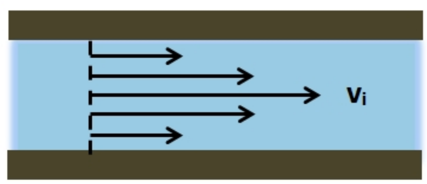
As an example, referring to, a downwards force of 10 N is applied to a bottle filled with a static liquid of constant density ρ at the spout of cross-sectional area of 5 cm^2, yielding an applied pressure of 2 N/cm^2. The cross-sectional area of the bottle changes with height so that at the bottom of the bottle the cross-sectional area is 500 cm^2. As a result of Pascal’s Law, the pressure change (pressure applied to the static liquid) is transmitted undiminished in the static liquid so that the applied pressure is 2 N/m^2 at the bottom of the bottle as well. Furthermore, the hydrostatic pressure due to the difference in height of the liquid is given by Equation 1 and yields the total pressure at the bottom surface of the bottle. Since the cross-sectional area at the bottom of the bottle is 100 times larger than at the top, the force contributing to the pressure at the bottom of the bottle is 1000 N plus the force from the weight of the static fluid in the bottle. This example shows how, through Pascal's Principle, the force exerted by a static fluid in a closed system can be multiplied by changing the height and the surface area of contact.



Pressure Applied to a Hydrostatic Fluid: A downwards force of 10 N is applied to a bottle filled with a static liquid of constant density ρ at the spout of cross-sectional area of 5 cm2, yielding an applied pressure of 2 N/cm2.

Pressure Transmitted Throughout an Entire Fluid

As stated by Pascal's Principle, the pressure applied to a static fluid in a closed container is transmitted throughout the entire fluid. Taking advantage of this phenomenon, hydraulic presses are able to exert a large amount of force requiring a much smaller amount of input force. This gives two different types of hydraulic press configurations, the first in which there is no difference in height of the static liquid and the second in which there is a difference in height Δh of the static liquid. In the first configuration, a force F_1 is applied to a static liquid of density ρ across a surface area of contact A_1, yielding an input pressure of P_2. On the other side of the press configuration, the fluid exerts an output pressure P_1 across a surface area of contact A_2, where $A_2 > A_1$. By Pascal's Principle, $P_1 = P_2$, yielding a force exerted by the static fluid of F_2, where $F_2 > F_1$. Depending on the applied pressure and geometry of the hydraulic press, the magnitude of F_2 can be changed. In the second configuration, the geometry of the system is the same, except that the height of the fluid on the output end is a height Δh less than the height of the fluid at the input end. The difference in height of the fluid between the input and the output ends contributes to the total force exerted by the fluid. For a hydraulic press, the force multiplication factor is the ratio of the output to the input contact areas.



Hydraulic Press Diagrams: Two different types of hydraulic press configurations, the first in which there is no difference in height of the static liquid and the second in which there is a difference in height Δh of the static liquid.

Gauge Pressure and Atmospheric Pressure

Pressure is often measured as gauge pressure, which is defined as the absolute pressure minus the atmospheric pressure.

learning obectives

- Explain the relationship among absolute pressure, gauge pressure, and atmospheric pressure

Atmospheric Pressure

An important distinction must be made as to the type of pressure quantity being used when dealing with pressure measurements and calculations. Atmospheric pressure is the magnitude of pressure in a system due to the atmosphere, such as the pressure exerted by air molecules (a static fluid) on the surface of the earth at a given elevation. In most measurements and calculations, the atmospheric pressure is considered to be constant at 1 atm or 101,325 Pa, which is the atmospheric pressure under standard conditions at sea level.

Atmospheric pressure is due to the force of the molecules in the atmosphere and is a case of hydrostatic pressure. Depending on the altitude relative to sea level, the actual atmospheric pressure will be less at higher altitudes and more at lower altitudes as the weight of air molecules in the immediate atmosphere changes, thus changing the effective atmospheric pressure. Atmospheric pressure is a measure of absolute pressure and can be affected by the temperature and air composition of the atmosphere but can generally be accurately approximated to be around standard atmospheric pressure of 101,325 Pa. Within the majority of earth's atmosphere, pressure varies with height according to. In this equation p_0 is the pressure at sea level (101,325 Pa), g is the acceleration due to gravity, M is the mass of a single molecule of air, R is the universal gas constant, T_0 is the standard temperature at sea level, and h is the height relative to sea level.

$$\Delta p = \frac{8\eta Q \Delta x}{\pi r^4}$$

Pressure and Height: Atmospheric pressure depends on altitude or height.

Gauge Pressure

For most applications, particularly those involving pressure measurements, it is more practical to use gauge pressure than absolute pressure as a unit of measurement. Gauge pressure is a relative pressure measurement which measures pressure relative to atmospheric pressure and is defined as the absolute pressure minus the atmospheric pressure. Most pressure measuring equipment give the pressure of a system in terms of gauge pressure as opposed to absolute pressure. For example, tire pressure and blood pressure are gauge pressures by convention, while atmospheric pressures, deep vacuum pressures, and altimeter pressures must be absolute.

For most working fluids where a fluid exists in a closed system, gauge pressure measurement prevails. Pressure instruments connected to the system will indicate pressures relative to the current atmospheric pressure. The situation changes when extreme vacuum pressures are measured; absolute pressures are typically used instead.

To find the absolute pressure of a system, the atmospheric pressure must then be added to the gauge pressure. While gauge pressure is very useful in practical pressure measurements, most calculations involving pressure, such as the ideal gas law, require pressure values in terms of absolute pressures and thus require gauge pressures to be converted to absolute pressures.

Measurements: Gauge Pressure and the Barometer

Barometers are devices used for measuring atmospheric and gauge pressure indirectly through the use of hydrostatic fluids.

learning obectives

- Compare design and operation of aneroid and hydrostatic based barometers

Gauge Pressure

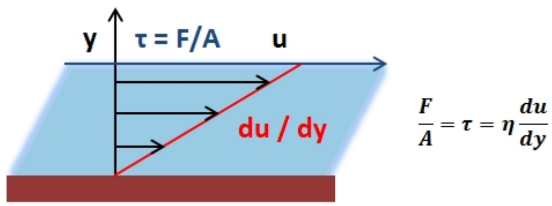
In practice, pressure is most often measured in terms of gauge pressure. Gauge pressure is the pressure of a system above atmospheric pressure. Since atmospheric pressure is mostly constant with little variation near sea level, where most practical pressure measurements are taken, it is assumed to be approximately 101,325 Pa. Modern pressure measuring devices sometimes have incorporated mechanisms to account for changes in atmospheric pressure due to elevation changes. Gauge pressure is much more convenient than absolute pressure for practical measurements and is widely used as an established measure of pressure. However, it is important to determine whether it is necessary to use absolute (gauge plus atmospheric) pressure for calculations, as is often the case for most calculations, such as those involving the ideal gas law. Pressure measurements have been accurately taken since the mid-1600s with the invention of the traditional barometer. Barometers are devices used to measure pressure and were initially used to measure atmospheric pressure.

Hydrostatic Based Barometers

Early barometers were used to measure atmospheric pressure through the use of hydrostatic fluids. Hydrostatic based barometers consist of columnar devices usually made from glass and filled with a static liquid of consistent density. The columnar section is sealed, holds a vacuum, and is partially filled with the liquid while the base section is open to the atmosphere and makes an interface with the surrounding environment. As the atmospheric pressure changes, the pressure exerted by the atmosphere on the fluid reservoir exposed to the atmosphere at the base changes, increasing as the atmospheric pressure increases and decreasing as the atmospheric pressure decreases. This change in pressure causes the height of the fluid in the columnar structure to change, increasing in height as the atmosphere exerts greater pressure on the liquid in the reservoir base and decreasing as the atmosphere exerts lower pressure on the liquid in the reservoir base. The height of the liquid within the glass column then gives a measure of the atmospheric pressure. Pressure, as determined by hydrostatic barometers, is often measured by determining the height of the liquid in the barometer column, thus the torr as a unit of pressure, but can be used to determine pressure in SI units. Hydrostatic based barometers most commonly use water or mercury as the static liquid. While the use of water is much less hazardous than mercury, mercury is often a better choice for fabricating accurate hydrostatic barometers. The density of mercury is much higher than that of water, thus allowing for higher accuracy of measurements and the ability to fabricate more compact hydrostatic barometers. In theory, a hydrostatic barometer can be placed in a closed system to measure the absolute pressure and the gauge pressure of the system by subtracting the atmospheric pressure.

Aneroid Barometer

Another type of barometer is the aneroid barometer, which consists of a small, flexible sealed metal box called an aneroid cell. The aneroid cell is made from beryllium-copper alloy and is partially evacuated. A stiff spring prevents the aneroid cell from collapsing. Small changes in external air pressure cause the cell to expand or contract. This expansion and contraction is amplified by mechanical mechanisms to give a pressure reading. Such pressure measuring devices are more practical than hydrostatic barometers for measuring system pressures. Many modern pressure measuring devices are pre-engineered to output gauge pressure measurements. While the aneroid barometer is the underlying mechanism behind many modern pressure measuring devices, pressure can also be measured using more advanced measuring mechanisms.



Hydrostatic Column Barometer: The concept of determining pressure using the fluid height in a hydrostatic column barometer

$$\Delta p = \frac{8\eta Q \Delta x}{\pi r^4}$$

Variation of Pressure with Height: The density of the liquid is p, g is the acceleration due to gravity, and h is the height of the fluid in the barometer column.

Pressure in the Body

Pressure plays an essential role in a number of critical bodily functions including respiration and blood circulation.

learning obectives

- Explain role played by pressure in the circulatory and respiratory systems

The Role of Pressure in the Circulatory System

Pressure plays an essential role in various critical bodily systems that are necessary for survival. One such critical bodily system which relies on pressure for functionality is the circulatory system, which is an example of a closed fluid system under pressure. The circulatory system is responsible for transporting oxygen and essential nutrients to all organs within the body as well as removing waste materials from these organs. Blood can be regarded as a viscous liquid contained within the circulatory system that travels throughout this closed system as a result of pressure and pressure differences within the circulatory system.

As the volume of blood within the circulatory system is confined to the veins, arteries, and capillaries there is a pressure within this closed system. Furthermore, through a complicated system of veins, arteries, and capillaries of varying diameter as well as valves and the heart acting as a continuous pump, pressure differences arise within the circulatory system that result in the potential for blood to circulate throughout the circulatory system, thus carrying out essential bodily functions for survival.

Pressure within the circulatory system is referred to as blood pressure, and is a primary and crucial vital sign which can be used to diagnose or indicate a number of medical conditions. Blood pressure varies throughout the body as well as from one individual to another and depends on a number of factors such as heart rate, blood volume, resistance of the circulatory system (veins, arteries, and capillaries), and the viscosity of blood. Any medical conditions affecting any of these factors will have an effect on blood pressure and the overall health of the circulatory system.

$$R_e = \frac{V}{I} \quad \rightarrow \quad \begin{cases} R_h \rightarrow R_e \\ \Delta p \rightarrow V \\ I \rightarrow Q \end{cases} \quad \rightarrow \quad \Delta p = \frac{8\eta Q \Delta x}{\pi r^4} \quad \rightarrow \quad R_h = \frac{\Delta p}{Q} = \frac{8\eta \Delta x}{\pi r^4}$$

Approximation for Mean Arterial Pressure: In practice, the mean arterial pressure (MAP) can be approximated from easily obtainable blood pressure measurements.

The mean arterial pressure (MAP) is the average pressure over a cardiac cycle and is determined by, where CO is the cardiac outputs, SVR is the systemic vascular resistance, and CVP is the central venous pressure (CVP). In practice, the mean arterial pressure (MAP) can be approximated from easily obtainable blood pressure measurements in, where P_{sys} is the measured systolic pressure and P_{dias} is the measured diastolic pressure. One particularly common and dangerous circulatory system condition is partial blockage of blood vessels due to a number of factors, such as plaque build-up from high cholesterol, which results in a reduction of the effective blood vessel cross-sectional diameter and a corresponding reduction in blood flow rate and thus an increase in blood pressure to restore normal blood flow according to Poiseuille's Law.

$$\Delta p = \frac{8\eta Q \Delta x}{\pi r^4}$$

Equation for Mean Arterial Pressure: The mean arterial pressure (MAP) is the average pressure over a cardiac cycle and is determined this equation, where CO is the cardiac outputs, SVR is the systemic vascular resistance, and CVP is the central venous pressure (CVP).

The Role of Pressure in the Respiratory System

Pressure also plays an essential role in the respiratory system, as it is responsible for the breathing mechanism. Pressure differences between the lungs and the atmosphere create a potential for air to enter the lungs, resulting in inhalation. The mechanism resulting in inhalation is due to lowering of the diaphragm, which increases the volume of the thoracic cavity surrounding the lungs, thus lowering its pressure as determined by the ideal gas law. The reduction in pressure of the thoracic cavity, which normally has a negative gauge pressure, thus keeping the lungs inflated, pulls air into the lungs, inflating the alveoli and resulting in oxygen transport needed for respiration. As the diaphragm restores and moves upwards, pressure within the thoracic cavity increases, resulting in exhalation. The cycle repeats itself, resulting in the respiration which as discussed is mechanically due to pressure changes. Without pressure in the body, and the corresponding potential that it has for dynamic bodily processes, essential functions such as blood circulation and respiration would not be possible.

Key Points

- Pressure is a scalar quantity defined as force per unit area. Pressure only concerns the force component perpendicular to the surface upon which it acts, thus if the force acts at an angle, the force component along the direction perpendicular to the surface must be used to calculate pressure.
- The pressure exerted on a surface by an object increases as the weight of the object increases or the surface area of contact decreases. Alternatively the pressure exerted decreases as the weight of the object decreases or the surface area of contact increases.
- Pressure exerted by ideal gases in confined containers is due to the average number of collisions of gas molecules with the container walls per unit time. As such, pressure depends on the amount of gas (in number of molecules), its temperature, and the volume of the container.
- Hydrostatic pressure refers to the pressure exerted by a fluid (gas or liquid) at any point in space within that fluid, assuming that the fluid is incompressible and at rest.
- Pressure within a liquid depends only on the density of the liquid, the acceleration due to gravity, and the depth within the liquid. The pressure exerted by such a static liquid increases linearly with increasing depth.
- Pressure within a gas depends on the temperature of the gas, the mass of a single molecule of the gas, the acceleration due to gravity, and the height (or depth) within the gas.
- Hydrostatic balance is the term used for a region or stationary object within a static fluid which is at static equilibrium, and for which the sum of all forces and sum of all torques is equal to zero.
- A region or static object within a stationary fluid experiences downward forces due to the weight of the region or object, and the pressure exerted from the fluid above the region or object, as well as an upward force due to the pressure exerted from the fluid

below the region or object.
- For a region or static object within a static fluid, the downward force due to the weight of the region or object is counteracted by the upward buoyant force, which is equal to the weight of the fluid displaced by the region or object.
- Pascal's Principle is used to quantitatively relate the pressure at two points in an incompressible, static fluid. It states that pressure is transmitted, undiminished, in a closed static fluid.
- The total pressure at any point within an incompressible, static fluid is equal to the sum of the applied pressure at any point in that fluid and the hydrostatic pressure change due to a difference in height within that fluid.
- Through the application of Pascal's Principle, a static liquid can be utilized to generate a large output force using a much smaller input force, yielding important devices such as hydraulic presses.
- Atmospheric pressure is a measure of absolute pressure and is due to the weight of the air molecules above a certain height relative to sea level, increasing with decreasing altitude and decreasing with increasing altitude.
- Gauge pressure is the additional pressure in a system relative to atmospheric pressure. It is a convenient pressure measurement for most practical applications.
- While gauge pressure is more convenient for practical measurements, absolute pressure is necessary for most pressure calculations, thus the atmospheric pressure must be added to the gauge pressure for calculations.
- Gauge pressure is the pressure of a system above atmospheric pressure, which must be converted to absolute pressure for most calculations.
- The barometer is a device which uses hydrostatic fluids to directly determine atmospheric pressure and may be used to indirectly measure the gauge pressure of systems.
- The hydrostatic column barometer uses a liquid like water or mercury for functionality, while the aneroid barometer uses an evacuated flexible metal cell.
- Pressure, along with the potential for work arising from differences in pressure, plays an essential role in the functionality of several critical bodily functions and systems necessary for survival.
- The circulatory system relies on pressure differences for circulating blood, along with oxygen, necessary nutrients, and waste products throughout the body.
- Respiration is made possible as a result of pressure differences between the thoracic cavity, the lungs, and the environment and is largely regulated by movement of the diaphragm.

Key Terms

- **ideal gas**: Theoretical gas characterized by random motion whose individual molecules do not interact with one another and are chemically inert.
- **kinetic energy**: The energy associated with a moving particle or object having a certain mass.
- **incompressible**: Unable to be compressed or condensed.
- **static equilibrium**: the physical state in which all components of a system are at rest and the net force is equal to zero throughout the system
- **Buoyancy**: The power of supporting a body so that it floats; upward pressure exerted by the fluid in which a body is immersed.
- **torque**: Something that produces or tends to produce torsion or rotation; the moment of a force or system of forces tending to cause rotation.
- **equilibrium**: A state of rest or balance due to the equal action of opposing forces.
- **hydraulic press**: Device that uses a hydraulic cylinder (closed static fluid) to generate a compressive force.
- **Gauge Pressure**: The pressure of a system above atmospheric pressure.
- **Torr**: A unit of pressure equal to one millimeter of mercury (760 torr = 101,325 Pa).
- **Aneroid Barometer**: A device for measuring pressure, often specially calibrated for use as an altimeter, consisting of a box or chamber partially exhausted of air, having an elastic top and a pointer to indicate the degree of compression of the top caused by the external air.
- **Thoracic Cavity**: A hollow place or space, or a potential space, within the body or one of its organs.
- **Poiseuille's Law**: The law that the velocity of a liquid flowing through a capillary is directly proportional to the pressure of the liquid and the fourth power of the radius of the capillary and is inversely proportional to the viscosity of the liquid and the length of the capillary.
- **Alveoli**: Small air sacs or cavities in the lung that give the tissue a honeycomb appearance and expand its surface area for the exchange of oxygen and carbon dioxide.

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10.3: Archimedes' Principle

learning objectives

- Calculate the direction of the buoyancy force

When you rise from soaking in a warm bath, your arms may feel strangely heavy. This effect is due to the loss of the buoyant support of the water. What creates this buoyant force ? Why is it that some things float and others do not? Do objects that sink get any support at all from the fluid? Is your body buoyed by the atmosphere, or are only helium balloons affected?

Buoyant Force: Cause and Calculation

We find the answers to the above questions in the fact that in any given fluid, pressure increases with depth. When an object is immersed in a fluid, the upward force on the bottom of an object is greater than the downward force on the top of the object. The result is a net upward force (a buoyant force) on any object in any fluid. If the buoyant force is greater than the object's weight, the object will rise to the surface and float. If the buoyant force is less than the object's weight, the object will sink. If the buoyant force equals the object's weight, the object will remain suspended at that depth. The buoyant force is always present in a fluid, whether an object floats, sinks or remains suspended.

The buoyant force is a result of pressure exerted by the fluid. The fluid pushes on all sides of an immersed object, but as pressure increases with depth, the push is stronger on the bottom surface of the object than in the top (as seen in).

You can calculate the buoyant force on an object by adding up the forces exerted on all of an object's sides. For example, consider the object shown in.

The top surface has area A and is at depth $\mathrm{h_1}$; the pressure at that depth is:

$$\mathrm{P_1 = h_1 \rho g}, \tag{10.3.1}$$

where ρρ is the density of the fluid and $\mathrm{g} \approx 9.81 \frac{\mathrm{m}}{\mathrm{s}^2}$ is the gravitational acceleration. The magnitude of the force on the top surface is:

$$\mathrm{F_1 = P_1 A = h_1 \rho g A}. \tag{10.3.2}$$

This force points downwards. Similarly, the force on the bottom surface is:

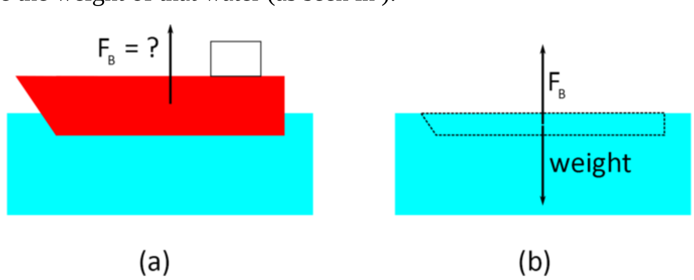
$$\mathrm{F_2 = P_2 A = h_2 \rho g A} \tag{10.3.3}$$

and points upwards. Because it is cylindrical, the net force on the object's sides is zero—the forces on different parts of the surface oppose each other and cancel exactly. Thus, the net upward force on the cylinder due to the fluid is:

$$\mathrm{F_B = F_2 - F_1 = \rho g A (h_2 - h_1)} \tag{10.3.4}$$

The Archimedes Principle

Although calculating the buoyant force in this way is always possible it is often very difficult. A simpler method follows from the Archimedes principle, which states that the buoyant force exerted on a body immersed in a fluid is equal to the weight of the fluid the body displaces. In other words, to calculate the buoyant force on an object we assume that the submersed part of the object is made of water and then calculate the weight of that water (as seen in).



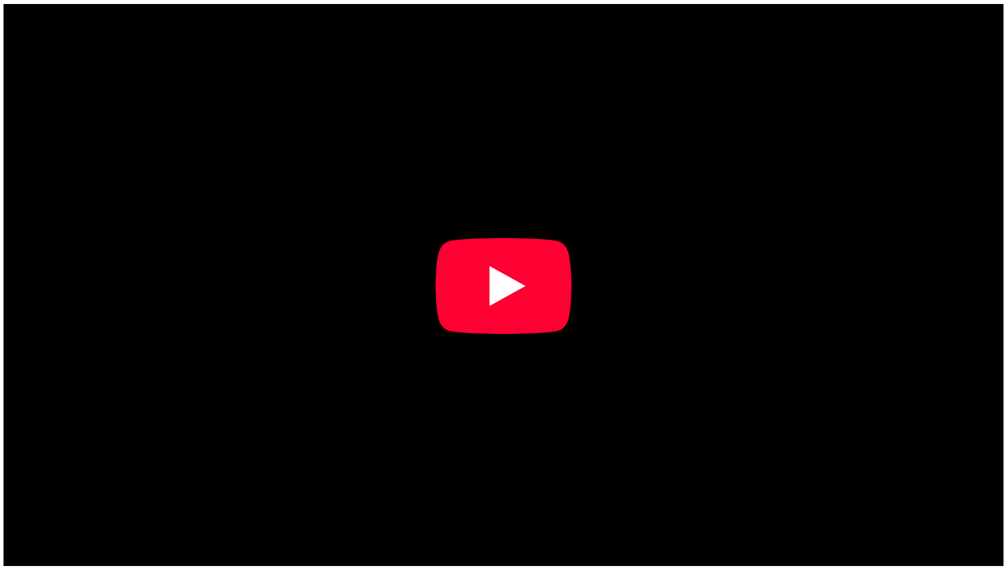
Archimedes principle: The buoyant force on the ship (a) is equal to the weight of the water displaced by the ship—shown as the dashed region in (b).

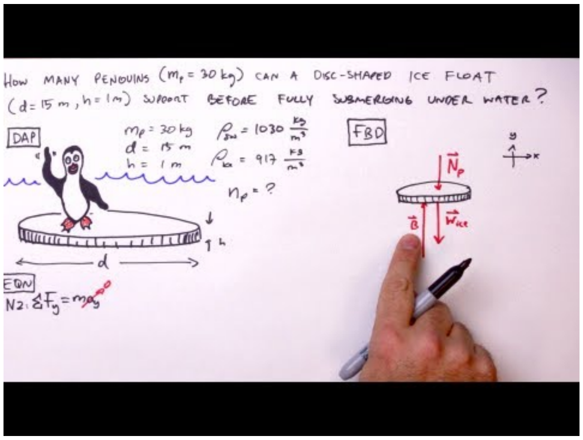
The principle can be stated as a formula:

$$F_B = w_{fl} \tag{10.3.5}$$

The reasoning behind the Archimedes principle is that the buoyancy force on an object depends on the pressure exerted by the fluid on its submerged surface. Imagine that we replace the submerged part of the object with the fluid in which it is contained, as in (b). The buoyancy force on this amount of fluid must be the same as on the original object (the ship). However, we also know that the buoyancy force on the fluid must be equal to its weight, as the fluid does not sink in itself. Therefore, the buoyancy force on the original object is equal to the weight of the “displaced fluid” (in this case, the water inside the dashed region (b)).

The Archimedes principle is valid for any fluid—not only liquids (such as water) but also gases (such as air). We will explore this further as we discuss applications of the principle in subsequent sections.





Archimedes’ Principle – Simple Example: We use Archimedes’ Principle to determine the number of penguins an ice float can dryly support.

Complete Submersion

The buoyancy force on a completely submerged object of volume is $F_B = V\rho g$.

learning objectivies

- Identify factors determining the buoyancy force on a completely submerged object

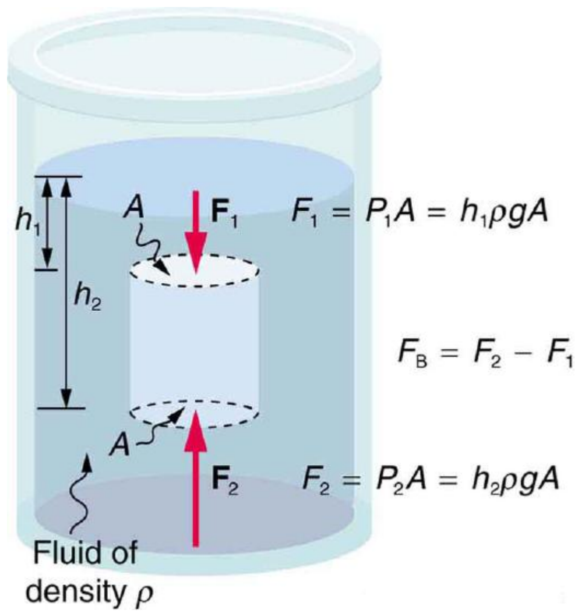
The Archimedes principle is easiest to understand and apply in the case of entirely submersed objects. In this section we discuss a few relevant examples. In general, the buoyancy force on a completely submerged object is given by the formula:

$$F_B = V\rho g, \tag{10.3.6}$$

where V is the volume of the object, ρ is the density of the fluid, and g is gravitational acceleration. This follows immediately from the Archimedes' principle, and the fact that the object is completely submerged (and so the volume of the fluid displaced is just the volume of the object).

Cylinder

In the previous section, we calculated the buoyancy force on a cylinder (shown in) by considering the force exerted on each of the cylinder's sides. Now, we'll calculate this force using Archimedes' principle. The buoyancy force on the cylinder is equal to the weight of the displaced fluid. This weight is equal to the mass of the displaced fluid multiplied by the gravitational acceleration:



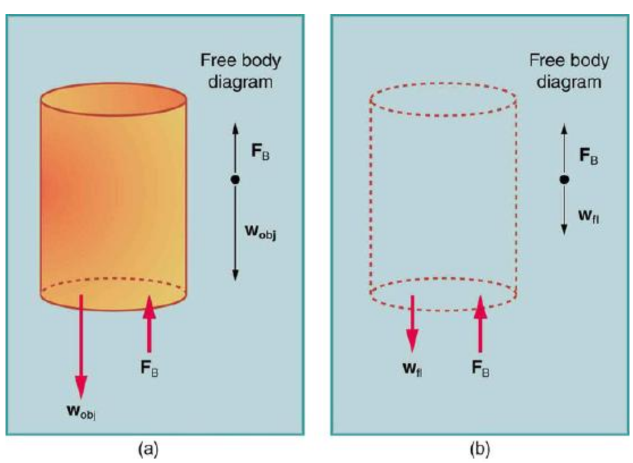
Buoyant force: The fluid pushes on all sides of a submerged object. However, because pressure increases with depth, the upward push on the bottom surface (F2) is greater than the downward push on the top surface (F1). Therefore, the net buoyant force is always upwards.

$$F_B = w_{fl} = m_{fl}g \tag{10.3.7}$$

The mass of the displaced fluid is equal to its volume multiplied by its density:

$$m_{fl} = V_{fl}\rho \tag{10.3.8}$$

However (*and this is the crucial point*), the cylinder is entirely submerged, so the volume of the displaced fluid is just the volume of the cylinder (see), and:



Archimedes principle: The volume of the fluid displaced (b) is the same as the volume of the original cylinder (a).

$$m_{fl} = V_{fl}\rho = V_{cylinder}\rho. \tag{10.3.9}$$

The volume of a cylinder is the area of its base multiplied by its height, or in our case:

$$V_{cylinder} = A(h_2 - h_1). \tag{10.3.10}$$

Therefore, the buoyancy force on the cylinder is:

$$F_B = m_{fl}g = V_{cylinder}\rho g = (h_1 - h_2)\rho g A. \tag{10.3.11}$$

This is the same result obtained in the previous section by considering the force due to the pressure exerted by the fluid.

Helium Airship

Consider the USS Macon, a helium-filled airship (shown in). Its envelope (the "balloon") contained 184,059.5 cubic meters of helium. Ignoring the small volume of the gondola, what was the buoyancy force on this airship? If the airship weighed 108,000 kg, how much cargo could it carry? Assume the density of air is 1.225 kg per meter cubed. The buoyancy force on an airship is due to the air in which it is immersed. Although we don't know the exact shape of the airship, we know its volume and the density of the air, and thus we can calculate the buoyancy force:

Helium airship: The USS Macon, a 1930s helium-filled airship.

$$F_B = V_{\rho g} = 184,059.5\ \text{kg} \times 1.225\frac{\text{kg}}{\text{m}^3} \times 9.81\frac{\text{m}}{\text{s}^2} \approx 2.212 \times 10^6\ \text{N} \tag{10.3.12}$$

To find the cargo capacity of the airship, we subtract the weight of the airship from the buoyancy force:

$$F_{cargo} = F_B - mg = 2.21 \times 10^6\ \text{N} - 1.08 \times 10^5\ \text{kg} \times 9.81\frac{\text{m}}{\text{s}^2} = 1.15 \times 10^6\ \text{N} \tag{10.3.13}$$

The mass the airship can carry is:

$$m_{cargo} = \frac{F_{cargo}}{g} = 1.2 \times 10^5\ \text{kg} = 120\ \text{tons}. \tag{10.3.14}$$

Flotation

An object floats if the buoyancy force exerted on it by the fluid balances its weight.

learning objectives

- Express the relationship between the buoyancy force and the weight for a floating object

Why do some objects float, but others don't? If you put a metal coin into a glass of water it will sink. But most ships are built of metal, and they float. So how is this possible?

Condition for Flotation

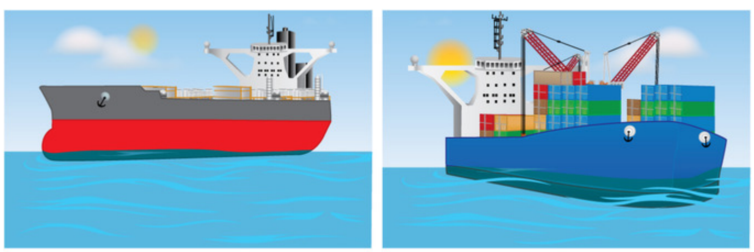
An object will float if the buoyancy force exerted on it by the fluid balances its weight, i.e. if FB=mgFB=mg.

But the Archimedes principle states that the buoyant force is the weight of the fluid displaced. So, for a floating object on a liquid, the weight of the displaced liquid is the weight of the object. Thus, only in the special case of floating does the buoyant force acting on an object equal the object's weight. Consider a one-ton block of solid iron. As iron is nearly eight times denser than water, it displaces only 1/8 ton of water when submerged, which is not enough to keep it afloat. Suppose the same iron block is reshaped into a bowl. It still weighs one ton, but when it is put in water, it displaces a greater volume of water than when it was a block. The deeper the iron bowl is immersed, the more water it displaces, and the greater the buoyant force acting on it. When the buoyant force equals one ton, it will sink no further.

When any boat displaces a weight of water equal to its own weight, it floats. This is often called the "principle of flotation" where a floating object displaces a weight of fluid equal to its own weight. Every ship, submarine, and dirigible must be designed to displace a weight of fluid equal to its own weight. A 10,000-ton ship must be built wide enough to displace 10,000 tons of water before it sinks too deep in the water. The same is true for vessels in air (as air is a fluid): A dirigible that weighs 100 tons displaces at least 100 tons of air; if it displaces more, it rises; if it displaces less, it falls. If the dirigible displaces exactly its weight, it hovers at a constant altitude.

Flotation and Density

Density plays a crucial role in Archimedes' principle. The average density of an object is what ultimately determines whether it floats. If its average density is less than that of the surrounding fluid, it will float. This is because the fluid, having a higher density, contains more mass and thus more weight in the same volume. The buoyant force, which equals the weight of the fluid displaced, is thus greater than the weight of the object. Likewise, an object denser than the fluid will sink. The extent to which a floating object is submerged depends on how the object's density is related to that of the fluid. For example, an unloaded ship has a lower density, and less of it is submerged compared with the same ship loaded with cargo. We can derive a quantitative expression for the fraction submerged by considering density. The fraction submerged is the ratio of the volume submerged to the volume of the object, or



Density and Submersion: An unloaded ship (a) floats higher in the water than a loaded ship (b).

$$\text{fraction submerged} = \frac{V_{sub}}{V_{obj}} = \frac{V_{fl}}{V_{obj}} \tag{10.3.15}$$

The volume submerged equals the volume of fluid displaced, which we call V_{fl}. Now we can obtain the relationship between the densities by substituting $\rho = mV$ into the expression. This gives

$$\text{fraction submerged} = \frac{m_{fl}/\rho_{fl}}{m_{obj}/\bar{\rho}_{obj}} \tag{10.3.16}$$

where $\bar{\rho}_{obj}$ is the average density of the object and ρflρflis the density of the fluid. Since the object floats, its mass and that of the displaced fluid are equal, and so they cancel from the equation, leaving

$$\text{fraction submerged} = \frac{\bar{\rho}_{obj}}{\rho_{fl}.} \tag{10.3.17}$$

There are a couple things to note about this expression:

1. Note that it mentions the average density of the object. This can be much less than the density of the material the object is made of. For instance, a steel ship is actually mostly filled with air (think of the corridors, cargo holds, etc.), so its average density is

between that of air and steel. To be more precise, the average density is defined as the total mass of an object divided by its total volume: $\bar{\rho} = \frac{m}{V}$.
2. This formula makes sense only if the density of the object is smaller than the density of the fluid. Otherwise, the fraction submerged becomes greater than one—a sign that the object does not float at all, but it sinks!

Key Points

- The buoyancy force is caused by the pressure exerted by the fluid in which an object is immersed.
- The buoyancy force always points upwards because the pressure of a fluid increases with depth.
- You can calculate the buoyancy force either directly by computing the force exerted on each of the object's surfaces, or indirectly by finding the weight of the displaced fluid.
- If an object is completely submerged, the volume of the fluid displaced is equal to the volume of the object.
- The buoyancy force on hot-air balloons, dirigibles and other objects can be calculated by assuming that they are entirely submerged in air.
- The buoyancy force does not depend on the shape of the object, only on its volume.
- The buoyancy force experienced by an object depends on its shape.
- The fraction of an object's volume that's submerged is given by the ratio of its average density to that of the fluid: $\frac{\bar{\rho}_{obj}}{\rho_{fl}}$.
- An object floats if the buoyancy force exerted on it by the fluid balances its weight.

Key Terms

- **buoyant force**: An upward force exerted by a fluid that opposes the weight of an immersed object.
- **Archimedes principle**: The buoyant force exerted on a body immersed in a fluid is equal to the weight of the fluid the body displaces.

10.4: Cohesion and Adhesion

learning objectives

- Explain the phenomena of surface tension and capillary action

Attractive forces between molecules of the same type are called cohesive forces. Liquids can, for example, be kept in open containers because cohesive forces hold the molecules together. Attractive forces between molecules of different types are called adhesive forces. Such forces cause liquid drops to cling to window panes, for example. In this section we examine effects of cohesive and adhesive forces in liquids.

Surface Tension

Surface tension is a contractive tendency of the surface of a liquid that allows it to resist an external force. It is shown, for example, in the floating of some objects on the surface of water, even though they are denser than water, and in the ability of some insects (e.g., water striders) to run on water's surface. This property is caused by cohesion of similar molecules and is responsible for many of the behaviors of liquids.

The cohesive forces among liquid molecules are responsible for the phenomenon of surface tension, as shown in. In the bulk of the liquid, each molecule is pulled equally in every direction by neighboring liquid molecules, resulting in a net force of zero. The molecules at the surface do not have other molecules on all sides of them and therefore are pulled inwards. This creates some internal pressure and forces liquid surfaces to contract to the minimal area.

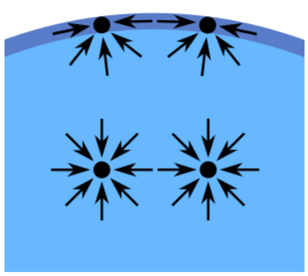


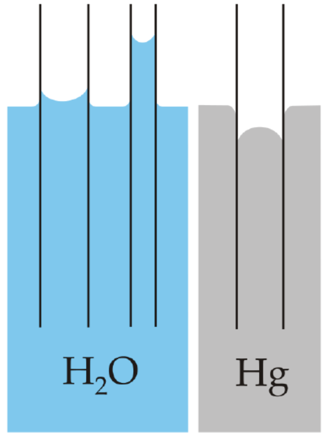
Diagram of Surface-Tension Forces: Diagram of the forces on molecules of a liquid

Surface tension has the unit of force per unit length, or of energy per unit area. The two units are equivalent. However, when we refer to energy per unit of area, we use the term surface energy, which is more general in that it applies to solids as well as liquids.

Capillary Action

Capillary action, or capillarity, is the ability of a liquid to flow in narrow spaces without the assistance of, and in opposition to, external forces like gravity. The effect can be seen in the drawing-up of liquids between the hairs of a paint-brush, in a thin tube, in porous materials such as paper, in some non-porous materials such as liquified carbon fiber, and in a cell. It occurs because of intermolecular attractive forces between the liquid and solid surrounding surfaces. If the diameter of the tube is sufficiently small, then the combination of surface tension (which is caused by cohesion within the liquid) and adhesive forces between the liquid and the container act to lift the liquid.

With some pairs of materials, such as mercury and glass (see), the intermolecular forces within the liquid exceed those between the solid and the liquid, so a convex meniscus forms, and capillary action works in reverse.



Capillarity: Capillary action of water compared to mercury, in each case with respect to glass

Key Points

- Attractive forces between molecules of the same type are called cohesive forces.
- Attractive forces between molecules of different types are called adhesive forces.
- Surface tension is a contractive tendency of the surface of a liquid that allows it to resist an external force.
- Capillary action, or capillarity, is the ability of a liquid to flow in narrow spaces without the assistance of, and in opposition to, external forces such as gravity.

Key Term

- **Pressure**: the amount of force that is applied over a given area divided by the size of that area
- **intermolecular**: from one molecule to another; between molecules

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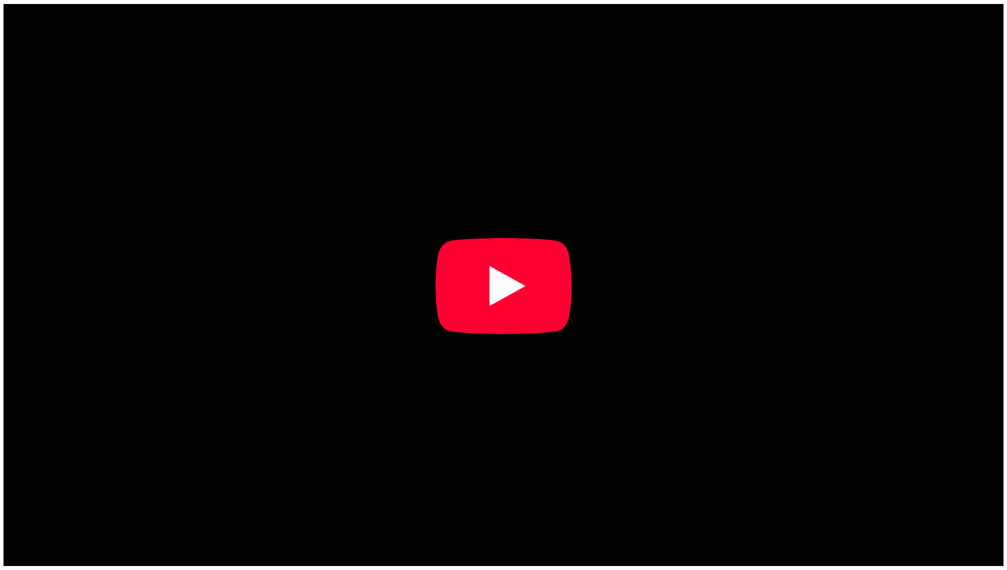
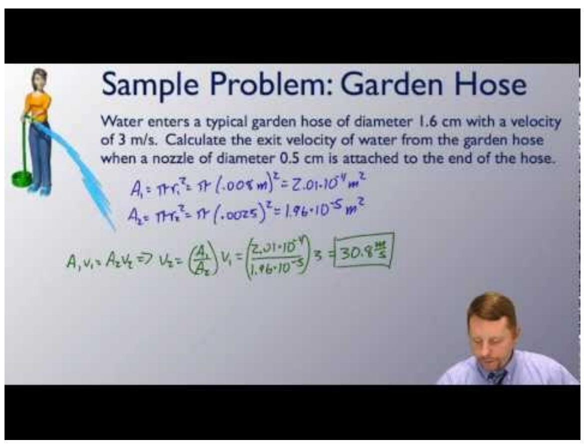
10.5: Fluids in Motion

learning objectives

- Determine the flow rate based on velocity and area or elapsed time and justify the use of continuity in expressing properties of a fluid and its motion

The flow rate of a fluid is the volume of fluid which passes through a surface in a given unit of time. It is usually represented by the symbol Q.





Continuity Equation for Fluids: A brief introduction to the Continuity Equation for Fluids.

Flow Rate

Volumetric flow rate is defined as

$$Q = v \times a, \tag{10.5.1}$$

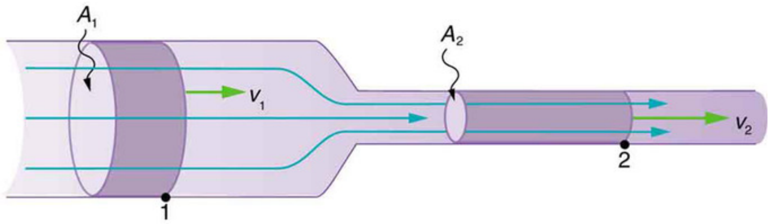
where Q is the flow rate, v is the velocity of the fluid, and a is the area of the cross section of the space the fluid is moving through. Volumetric flow rate can also be found with

$$Q = \frac{V}{t} \tag{10.5.2}$$

where Q is the flow rate, V is the Volume of fluid, and t is elapsed time.

Continuity

The equation of continuity works under the assumption that the flow in will equal the flow out. This can be useful to solve for many properties of the fluid and its motion:



Flow in = Flow out: Using the known properties of a fluid in one condition, we can use the continuity equation to solve for the properties of the same fluid under other conditions.

$$Q_1 = Q_2 \tag{10.5.3}$$

This can be expressed in many ways, for example: $A_1 v_1 = A_2 v_2$. The equation of continuity applies to any incompressible fluid. Since the fluid cannot be compressed, the amount of fluid which flows into a surface must equal the amount flowing out of the surface.

Applying the Continuity Equation

You can observe the continuity equation's effect in a garden hose. The water flows through the hose and when it reaches the narrower nozzle, the velocity of the water increases. Speed increases when cross-sectional area decreases, and speed decreases when cross-sectional area increases. This is a consequence of the continuity equation. If the flow Q is held constant, when the area A decreases, the velocity v must increase proportionally. For example, if the nozzle of the hose is half the area of the hose, the velocity must double to maintain the continuous flow.

Key Points

- Flow rate can be expressed in either terms of cross sectional area and velocity, or volume and time.
- Because liquids are incompressible, the rate of flow into an area must equal the rate of flow out of an area. This is known as the equation of continuity.
- The equation of continuity can show how much the speed of a liquid increases if it is forced to flow through a smaller area. For example, if the area of a pipe is halved, the velocity of the fluid will double.
- Although gases often behave as fluids, they are not incompressible the way liquids are and so the continuity equation does not apply.

Key Terms

- **incompressible**: Unable to be compressed or condensed.
- **continuity**: Lack of interruption or disconnection; the quality of being continuous in space or time.

10.6: Deformation of Solids

Learning objectives

- Explain how length of an object is determined

Length

In geometric measurements, length is the longest dimension of an object. In other contexts “length” is the measured dimension of an object. For example: it is possible to cut a length of a wire which is shorter than wire thickness. Length may be distinguished from height, which is vertical extent, and width or breadth, which are the distance from side to side, measuring across the object at right angles to the length.

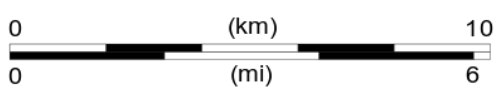
Length is a measure of one dimension, whereas area is a measure of two dimensions (length squared) and volume is a measure of three dimensions (length cubed). In most systems of measurement, the unit of length is a fundamental unit, from which other units are defined.

After Albert Einstein’s Special Relativity Theory, length can no longer be thought of being constant in all reference frames. Thus, a ruler that is one meter long in one frame of reference will not be one meter long in a reference frame that is travelling at a velocity relative to the first frame. This means that the length of an object is variable depending on the observer.

Units

One of the oldest units of length measurement used in the ancient world was the ‘cubit,’ which was the length of the arm from the tip of the finger to the elbow. This could then be subdivided into shorter units like the foot, hand (which at 4 inches is still used today for expressing the height of horses) or finger, or added together to make longer units like the stride. The cubit could vary considerably due to the different sizes of people.

In the physical sciences and engineering, when one speaks of “units of length”, the word “length” is synonymous with “distance”. There are several units that are used to measure length. Units of length may be based on lengths of human body parts, the distance traveled in a number of paces, the distance between landmarks or places on the Earth, or arbitrarily on the length of some fixed object.In the International System of Units (SI), the basic unit of length is the meter and is now defined in terms of the speed of light. The centimeter and the kilometer, derived from the meter, are also commonly used units. In U.S. customary units, English or Imperial system of units, commonly used units of length are the inch, the foot, the yard, and the mile. Units used to denote distances in the vastness of space, as in astronomy, are much longer than those typically used on Earth and include the astronomical unit, the light-year, and the parsec.



Length: The metric length of one kilometre is equivalent to the imperial measurement of 0.62137 miles.

Shape

The shape of an object is a description of space that the object takes up; the shape can change if the object is deformed.

learning objectives

- Describe effects of deformations, rotations, and magnifications

Shape

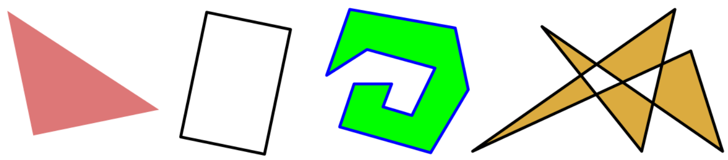
The shape of an object located in some space is a geometrical description of the part of that space occupied by the object, as determined by its external boundary – abstracting from location and orientation in space, size, and other properties such as color, content, and material composition

Simple and Complex Shapes

Simple shapes can be described by basic geometry objects such as a set of two or more points, a line, a curve, a plane, a plane figure (e.g. square or circle), or a solid figure (e.g. cube or sphere). Most shapes occurring in the physical world are complex. Some, such as plant structures and coastlines, may be so arbitrary as to defy traditional mathematical description – in which case they may be analyzed by differential geometry, or as fractals.

In geometry, two subsets of a Euclidean space have the same shape if one can be transformed to the other by a combination of translations, rotations (together also called rigid transformations), and uniform scalings. In other words, the shape of a set of points is all the geometrical information that is invariant to translations, rotations, and size changes. Having the same shape is an equivalence relation, and accordingly a precise mathematical definition of the notion of shape can be given as being an equivalence class of subsets of a Euclidean space having the same shape.

Shapes of physical objects are equal if the subsets of space these objects occupy satisfy the definition above. In particular, the shape does not depend on the size and placement in space of the object.



Shapes: Examples of shapes.

Volume

Volume is a measure of the three-dimensional space an object occupies, *usually* taken in terms of length, width and height.

learning objectives

- Explain how is volume measured geometrically

Volume is the quantity of three-dimensional space contained by a closed boundary; it is the space that a substance (solid, liquid, gas or plasma) or shape occupies or contains. Volume is often quantified numerically using an SI derived unit, the cubic meter. However, for liquids the unit of volume used is known as the liter (equivalent to 0.001 cubic meters).

Measuring Volume: A measuring cup can be used to measure volumes of liquids. This cup measures volume in units of cups, fluid ounces and millilitres.

Volume is measured geometrically by multiplying an object's three dimensions—usually taken as length, width and height. Some common volumes are taken as follows:

- The volume of a cube: length times width times height.
- The volume of a cylinder: the cross-sectional area times the height of the cylinder.
- The volume of a sphere: 4/3 times the radius cubed times pi.

The volume of a solid can be determined by the volume of liquid it displaces when submerged.

The volume of a container is generally understood as the capacity of the container, meaning the amount of fluid (gas or liquid) the container can hold, rather than the amount of space the container itself displaces. Measuring cups, as seen in, work by taking a known cross sectional area of a cup and multiplying that by a variable height. Since liquid will always cover the cross section (if there is enough liquid), adding more liquid will increase the height inside the container.

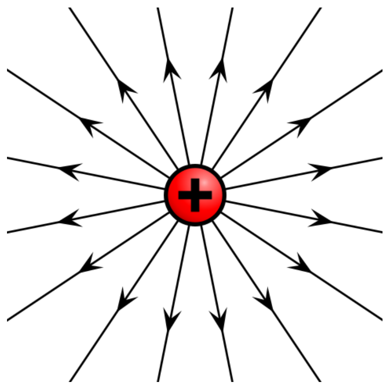
Liquids take the shape of their container, filling up the minimum height needed. Gases, on the other hand, take up the maximum amount of volume possible. Thus a measuring cup can accurately measure the volume of a liquid, whereas a gas will always fill the entire container, more or less uniformly, no matter how little gas there is.

Stress and Strain

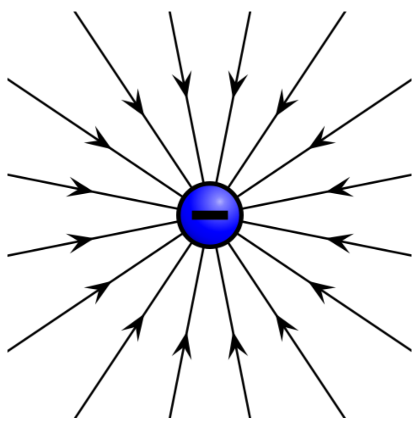
A point charge creates an electric field that can be calculated using Coulomb's Law.

The electric field of a point charge is, like any electric field, a vector field that represents the effect that the point charge has on other charges around it. The effect is felt as a force and when charged particles are not in motion this force is known as the electrostatic force. The electrostatic force is, much like gravity, a force that acts at a distance. Therefore, we rationalize this action at a distance by saying that charges create fields around them that have effects on other charges.

Given a point charge, or a particle of infinitesimal size that contains a certain charge, electric field lines emanate radially in all directions. If the charge is positive, field lines point radially away from it; if the charge is negative, field lines point radially towards it.



Electric field of positive point charge: The electric field of a positively charged particle points radially away from the charge.

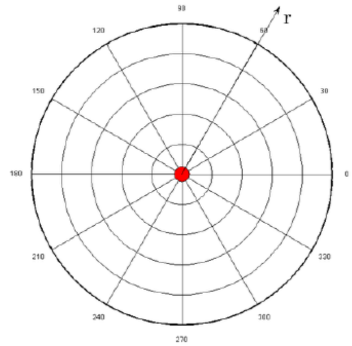


Electric field of negative point charge: The electric field of a negatively charged particle points radially toward the particle.

The reason for these directions can be seen in the derivation of the electric field of a point charge. Let's first take a look at the definition of electric field of a point particle:

$$\vec{E} = \frac{1}{4\pi\epsilon_o}\frac{q}{r^2}\hat{r} = k\frac{q}{r^2}\hat{r}. \tag{10.6.1}$$

In the above equation, q represents the charge of the particle creating the electric field and the constant k is a result of simply lumping the constants together. This charge is either positive or negative. If the charge is positive, as shown above, the electric field will be pointing in a positive radial direction from the charge q (away from the charge) and the following text explains why. The above equation is defined in radial coordinates which can be seen in.



Radial Coordinate System: The electric field of a point charge is defined in radial coordinates. The positive r direction points away from the origin, and the negative r direction points toward the origin. The electric field of a point charge is symmetric with respect to the θ direction.

If we now place another positive charge, Q (called the test charge), at some radial distance, R, away from the original particle, the test charge will feel a force given by

$$\vec{F} = Q\vec{E} = Q\frac{1}{4\pi\epsilon_o}\frac{q}{R^2}\hat{r} \tag{10.6.2}$$

The thing to keep in mind is that the force above is acting on the test charge Q, in the positive radial direction as defined by the original charge q. This means that because the charges are both positive and will repel one another, the force on the test charge points away from the original charge.

If the test charge were negative, the force felt on that charge would be

$$\vec{F} = Q\vec{E} = -Q\frac{1}{4\pi\epsilon_o}\frac{q}{R^2}\hat{r} \tag{10.6.3}$$

Notice that this points in the negative $\hat{r}$ direction, which is toward the original charge. This makes sense because opposite charges attract and the force on the test charge will tend to push it toward the original positive charge creating the field. The above mathematical description of the electric field of a point charge is known as Coulomb ‘s Law.

Key Points

- Length is typically a measure of the longest dimension of an object.
- The deformation of an object is typically a change in length.
- The SI unit of length is the meter.
- The shape of an object is a representation of the space taken up by the object.
- Deformations can change the shape of an object.
- Objects that have the same shape can be transformed into each other by rotation or magnification.
- Volume is often quantified numerically using an SI derived unit, the cubic meter. However, for liquids the unit of volume used is known as the liter (equivalent to 0.001 cubic meters).
- This can also be understood as the amount of fluid a submerged object displaces.
- Volume can be measured for geometrically regular objects by simple formulas. However, more complicated objects are easier to measure with fluid displacement.
- The electric field is a vector field around a charged particle that represents the force that other charged particles would feel when placed near the particle creating the electric field.
- Given a point charge, or a particle of infinitesimal size, that contains a certain charge, electric field lines emanate from equally in all radial directions.
- If the point charge is positive, field lines point away from it; if the charge is negative, field lines point towards it.

Key Terms

- **dimension**: A measure of spatial extent in a particular direction, such as height, width or breadth, or depth.
- **special relativity**: A theory that (neglecting the effects of gravity) reconciles the principle of relativity with the observation that the speed of light is constant in all frames of reference.
- **plane**: A level or flat surface.
- **Euclidean**: Adhering to the principles of traditional geometry, in which parallel lines are equidistant.
- **cross section**: A section formed by a plane cutting through an object, usually at right angles to an axis.
- **dimension**: A measure of spatial extent in a particular direction, such as height, width or breadth, or depth.
- **coulomb's law**: the mathematical equation calculating the electrostatic force vector between two charged particles
- **vector field**: a construction in which each point in a Euclidean space is associated with a vector; a function whose range is a vector space

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CHAPTER OVERVIEW

11: Fluid Dynamics and Its Applications

11.1: Overview

learning objectives

- Interpret the circulatory system in terms of your knowledge of fluid dynamics

We have discussed many situations in which fluids are static, though there are many situations where fluids flow. For example, a column of smoke rises from a camp fire, water streams from a fire hose, blood courses through your veins. Why does rising smoke curl and twist? How does a nozzle increase the speed of water emerging from a hose? How does the body regulate blood flow? Fluid dynamics, the physics of fluids in motion, allows us to answer these and many other questions.

Application in the Circulatory System

For example, consider the circulatory system—a connected series of tubes with fluid flowing through them. The heart is the driver of the circulatory system, generating cardiac output (CO) by rhythmically contracting and relaxing. This creates changes in regional pressures and (combined with a complex valvular system in the heart and the veins) ensures that the blood moves around the circulatory system in one direction. The “beating” of the heart generates pulsatile blood flow, conducted into the arteries across the micro-circulation and then back via the venous system to the heart.

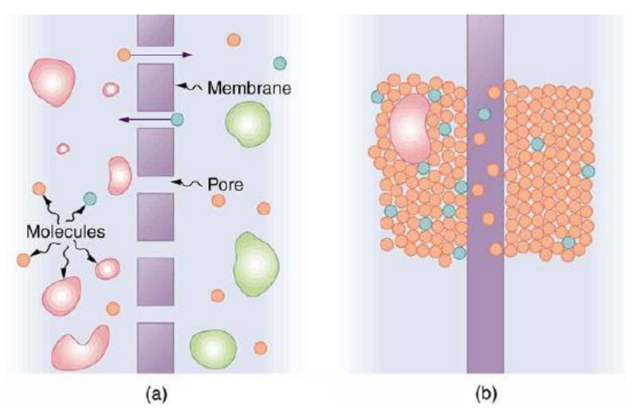
The aorta, the main artery, leaves the left side of the heart and proceeds to divide into smaller and smaller arteries that first become arterioles and eventually become capillaries, through which oxygen transfer occurs. The capillaries connect to venules, into which the deoxygenated blood passes from the cells back into the blood. The blood then travels back through the network of veins to the right heart. The micro-circulation (arterioles, capillaries and venules) constitutes most of the area of the vascular system and is the site of the transfer of O_2 into the cells.

The venous system returns the de-oxygenated blood to the right heart where it is pumped into the lungs to become oxygenated. This is also where CO_2 and other gaseous wastes are exchanged and expelled during breathing. Blood then returns to the left side of the heart where it begins the process again. The heart, vessels and lungs are all actively involved in maintaining healthy cells and organs, and all influence the fluid dynamics of the blood.

Fluids and Diffusion

Now consider how nutrients are transported through a human body. Diffusion is the movement of substances due to random thermal molecular motion. Fluids can even diffuse through solids (such as fumes or odors entering ice cubes). Diffusion is the dominant mechanism by which the exchange of nutrients and waste products occurs between the blood and tissue, and between air and blood in the lungs. In the evolutionary process, as organisms became larger they needed quicker methods of transportation than net diffusion, due to the larger distances involved in the transport. This factor lead to the development of circulatory systems. Less sophisticated, single-celled organisms still rely totally on diffusion for the removal of waste products and the uptake of nutrients.

Another important form of fluid movement is osmosis—the transport of water through a semipermeable membrane (shown in) from a region of high concentration to a region of low concentration. It is driven by the imbalance in water concentration. Similarly, dialysis is the transport of any other molecule through a semipermeable membrane due to its concentration difference. Both osmosis and dialysis are used by the kidneys to cleanse the blood, and the medical application of dialysis through machinery is important in the treatment of individuals with failing kidney function.



A Semipermeable Membrane: A semipermeable membrane with small pores that allow only small molecules to pass through.

Flow Rate and Velocity

Flow velocity and volumetric flow rates are important quantities in fluid dynamics used to quantify motion of a fluid and are interrelated.

learning objectives

- Assess the significance of studying volumetric flow in addition to flow velocity

Fluid dynamics is the study of fluids in motion and corresponding phenomena. A fluid in motion has a velocity, just as a solid object in motion has a velocity. Like the velocity of a solid, the velocity of a fluid is the rate of change of position per unit of time. In mathematical terms, the velocity of a fluid is the derivative of the position vector of the fluid with respect to time, and is therefore itself a vector quantity. The flow velocity vector is a function of position, and if the velocity of the fluid is not constant then it is also a function of time. Equation 1 shows the mathematical expression for the velocity of a fluid in motion. As a vector quantity, fluid velocity must have at least one non-zero directional component and may have up to three non-zero directional components. The velocity vector has non-zero components in any orthogonal direction along which motion of the fluid occurs.

$$\Delta p = \frac{8\eta Q \Delta x}{\pi r^4}$$

Flow Velocity: Mathematical Expression for Flow Velocity

Turbulent Flow vs. Laminar Flow

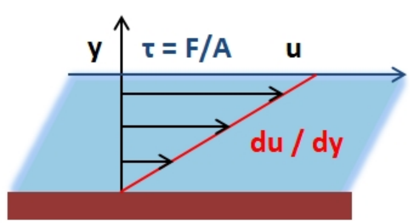
Fluid velocity can be affected by the pressure of the fluid, the viscosity of the fluid, and the cross-sectional area of the container in which the fluid is travelling. These factors affect fluid velocity depending on the nature of the fluid flow—particularly whether the flow is turbulent or laminar in nature. In the case of turbulent flow, the flow velocity is complex in nature and thus hard to predict; it must be analyzed on a system per system basis. In the case of Laminar flow, however, fluid flow is much simpler and flow velocity can be accurately calculated using Poiseuille's Law. In SI units, fluid flow velocity is expressed in terms of meters per seconds. The magnitude of the fluid flow velocity is the fluid flow speed. Fluid flow velocity effectively describes everything about the motion of a fluid.

Volumetric Flow

In addition to flow velocity, volumetric flow rate is an important quantity in fluid dynamics analysis. Volumetric flow is defined as the volume of fluid that passes through a given surface per unit time. Qualitatively, Figure 1 shows the notion of volumetric flow rate regarding a cross-sectional surface of area A. Mathematically, volumetric flow rate is the derivative of the volume of fluid that passes through a given surface with respect to time; in SI units this is expressed as meterscubed per second. Volumetric flow rate is related to the flow velocity vector as the surface integral with respect to the surface in question. If the surface area in question is a flat, plane cross-section, the surface integral reduces as shown in Equation 2, where A is the surface area of the surface in question and v is the flow velocity of the fluid.

$$R_e = \frac{V}{I} \quad \rightarrow \quad \begin{cases} R_h \rightarrow R_e \\ \Delta p \rightarrow V \\ I \rightarrow Q \end{cases} \quad \rightarrow \quad \Delta p = \frac{8\eta Q \Delta x}{\pi r^4} \quad \rightarrow \quad R_h = \frac{\Delta p}{Q} = \frac{8\eta \Delta x}{\pi r^4}$$

Volumetric Flow Rate: Volumetric Flow Rate Surface Integral and its simplification



$$\frac{F}{A} = \tau = \eta \frac{du}{dy}$$

Flow Velocity – Volumetric Flow Rate Relation: This figure shows the relation between flow velocity and volumetric flow rate.

Moreover, only the flow velocity component parallel to the surface normal of the surface in question, or alternatively the flow velocity component perpendicular to the surface in question contributes to the volumetric flow rate. Figure 1 and Equation 2 illustrate decomposition of the flow velocity vector, making an angle θ with respect to the normal of the surface plane in order to calculate volumetric flow rate through that surface. Thus, volumetric flow rate for a given fluid velocity and cross-sectional surface area increases as θ decreases, and is maximized when $\theta = 0$. Volumetric flow rate is an important scalar quantity in fluid dynamics and is used widely in fluid flow measurements. Volumetric flow rate can be converted to mass flow rate if the density of the fluid is known. Flow of fluids through a closed system is often analyzed as a hydraulic circuit analogous to electron flow in an electronic circuit where: 1) the volumetric fluid flow is analogous to the electric current, 2) pressure is analogous to the voltage, and 3) fluid velocity is analogous to current density.

Key Points

- There are many fluids in biology and understanding their behavior in motion is crucial to effective medicine.
- The heart pumps a fluid, blood, throughout a series of tubes in the body.
- Circulation may be understood through a study of fluid dynamics.
- Diffusion is the dominant mechanism by which the exchange of nutrients and waste products occur between the blood and tissue, and between air and blood in the lungs.
- Both osmosis and dialysis are used by the kidneys to cleanse the blood, and the medical application of dialysis through machinery is important in the treatment of individuals with failing kidney function.
- Flow velocity is a vector quantity used to describe the motion of a fluid. It can be easily determined for laminar flow but complex to determine for turbulent flow.
- Volumetric flow rate is the volume of a liquid that passes through a given surface per unit time. It is found from the flow velocity and the surface area of the surface through which the fluid passes.
- Fluid flow through a closed hydraulic system is analyzed much like electron flow through an electronic circuit —where volumetric flow rate is analogous to current, flow velocity is analogous to current density, and pressure is analogous to voltage (electrical potential).

Key Terms

- **vascular**: Of, pertaining to, or containing vessels that conduct or circulate fluids (such as blood, lymph, or sap) through the body of an animal or plant.
- **osmosis**: The net movement of solvent molecules from a region of high solvent potential to a region of lower solvent potential through a partially permeable membrane.
- **dialysis**: A method of separating molecules or particles of different sizes by differential diffusion through a semipermeable membrane.
- **Laminar Flow**: Non-turbulent motion of a fluid in which parallel layers have different velocities relative to each other.
- **Turbulent Flow**: The motion of a fluid having local velocities and pressures that fluctuate randomly.

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11.2: Flow in Tubes

learning objectives

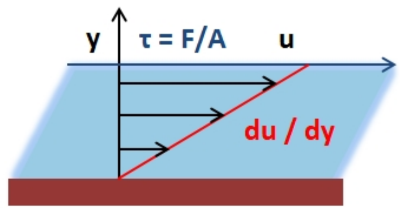
- Contrast turbulent and laminar flow in constant velocity

Virtually all moving fluids exhibit viscosity, which is a measure of the resistance of a fluid to flow. Viscosity is a basic property necessary for the analysis of fluid flow.

Measure of Fluid Friction

It describes a fluid's internal resistance to movement and can be thought of as a measure of fluid friction. The greater the viscosity, the 'thicker' the fluid and the more the fluid will resist movement.

Mathematically, viscosity is a proportionality constant relating an applied shear stress to the resulting shear velocity and is given, along with a representative diagram, (see). As shown, when a force is applied to a fluid, creating a shear stress, the fluid will undergo a certain displacement. The viscosity of the fluid is then its inherent resistance to undergo this displacement.



$$\frac{F}{A} = \tau = \eta \frac{du}{dy}$$

Representation of Viscosity: A proportionality constant relating an applied shear stress to the resulting shear velocity.

Different fluids exhibit different viscous behavior yet, in this analysis, only Newtonian fluids (fluids with constant velocity independent of applied shear stress) will be considered. Viscosity in fluids generally decreases with increasing temperature. The study of the viscous nature of fluids is called *rheology*.

In analyzing the properties of moving fluids, it is necessary to determine the nature of flow of the fluid. This is generally split into two categories, laminar and turbulent flow.

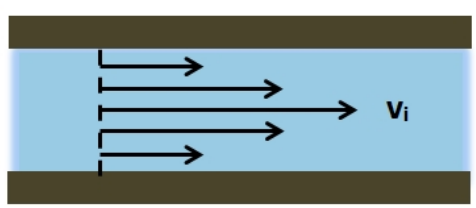
Turbulent Flow

Turbulent flow is characterized by irregular flow of a fluid in which there are both inconsistent flow patterns and velocity variations throughout the volume of the fluid in motion. Analysis of turbulent flow can be very complex and often requires advanced mathematical analysis to simulate flow in systems on a near case-by-case basis.

It occurs when the Reynolds number is above a certain critical threshold while mixed turbulent–laminar flow occurs within a range of Reynolds number below this threshold value. At the lower limit of this mixed turbulent–laminar flow Reynolds number region there is another critical threshold value, below which only laminar flow is possible.

Laminar Flow

Laminar flow consists of a regular-flow pattern with constant-flow velocity throughout the fluid volume and is much easier to analyze than turbulent flow.



Relative Magnitudes of Velocity Vectors: Laminar fluid flow in a circular pipe at the same direction.

Laminar flow is often encountered in common hydraulic systems, such as where fluid flow is through an enclosed, rigid pipe; the fluid is incompressible, has constant viscosity, and the Reynolds number is below this lower critical threshold value. It is characterized by the flow of a fluid in parallel layers, in which there is no disruption or interaction between the different layers, and in which each layer flows at a different velocity along the same direction. The variation in velocity between adjacent parallel layers is due to the viscosity of the fluid and resulting shear forces.

This figure (see) gives a representation of the relative magnitudes of the velocity vectors of each of these layers for laminar fluid flow through a circular pipe, in a direction parallel to the pipe axis.

$$\Delta p = \frac{8\eta Q \Delta x}{\pi r^4}$$

Poiseuille's Equation: Can be used to determine the pressure drop of a constant viscosity fluid exhibiting laminar flow through a rigid pipe.

Considering laminar flow of a constant density, incompressible fluid such as for a Newtonian fluid traveling in a pipe, with a Reynolds number below the upper limit level for fully laminar flow, the pressure difference between two points along the pipe can be found from the volumetric flow rate, or vice versa. For such a system with a pipe radius of r, fluid viscosity η, distance between the two points along the pipe $\Delta x = x_2 - x_1$, and the volumetric flow rate Q, of the fluid, the pressure difference between the two points along the pipe Δp is given by Poiseuille's equation (see).

This equation is valid for laminar flow of incompressible fluids only, and may be used to determine a number of properties in the hydraulic system, if the others are known or can be measured. In practice, Poiseuille's equation holds for most systems involving laminar flow of a fluid, except at regions where features disrupting laminar flow, such as at the ends of a pipe, are present.

Poiseuille's equation as given in this example (see) is analogous to Ohm 's equation for determining the resistance in an electronic circuit and is of great practical use in hydraulic-circuit analysis.

$$R_e = \frac{V}{I} \quad \rightarrow \quad \begin{cases} R_h \rightarrow R_e \\ \Delta p \rightarrow V \\ I \rightarrow Q \end{cases} \quad \rightarrow \quad \Delta p = \frac{8\eta Q \Delta x}{\pi r^4} \quad \rightarrow \quad R_h = \frac{\Delta p}{Q} = \frac{8\eta \Delta x}{\pi r^4}$$

Poiseuille's Equation: Analogous to Ohm's Law Analogy

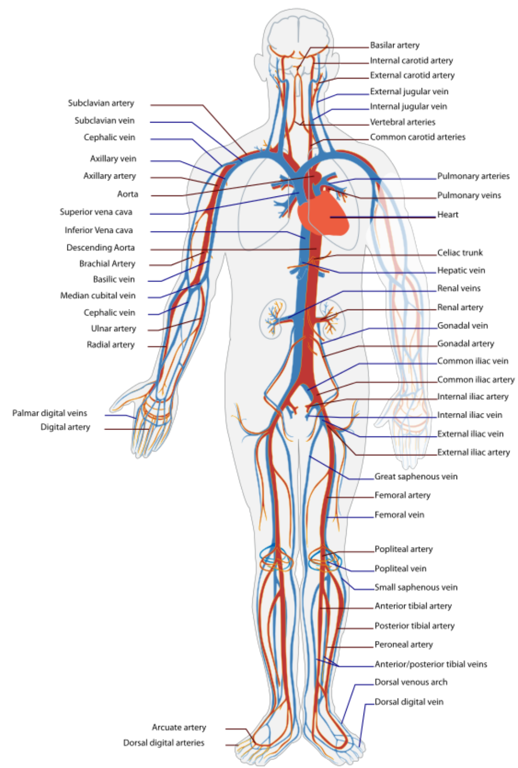
Blood Flow

Blood flow is the continuous running of blood through the cardiovascular system, which consists of the vessels and the heart.

learning objectives

- Outline how normal plasma behaves in a mammalian cardiovascular system

Blood flow is the continuous running of blood through vessels in the cardiovascular system (the mammalian cardiovascular system is shown in). Blood is the viscous fluid composed of plasma and cells. The composition of the blood includes plasma, red blood cells, white blood cells and platelets. In microcirculation, the properties of the blood cells have an important influence on flow.



An illustrative overview of the mammalian cardiovascular system: Keep in mind that both circular paths are working simultaneously and not in a sequential manner as the numbering in the illustration might suggest. Both the ventricles are working together in harmony; as tiny amounts of blood are moving in the pulmonary circuit, the remainder of the blood moves through the systemic circuit.

The cardiovascular system, which consists of blood vessels and the heart, helps to distribute nutrients, O_2, and other products of metabolism. The blood moves in the blood vessels, while the heart serves as the pump for the blood. The vessel walls of the heart are elastic and movable, therefore causing the blood and the wall to exert forces on each other and in turn influencing their respective motion.

The major quantity of interest in describing the motion of blood particles is velocity—the rate of change of the position of an object with time:

$$v=\frac{\Delta x}{\Delta t} \tag{11.2.1}$$

Blood velocities in arteries are higher during systole than during diastole. One parameter to quantify this difference is pulsatility index (PI), which is equal to the difference between the peak systolic velocity and the minimum diastolic velocity divided by the mean velocity during the cardiac cycle.

Another important parameter is the acceleration—the rate of change of velocity: $a=\frac{\Delta v}{\Delta t}$

Normal plasma behaves like a Newtonian fluid at rates of shear. Typical values for the viscosity of normal human plasma at 37°C is $1.2 Nsm^{-2}$. The viscosity of normal plasma varies with temperature in the same way as does that of its solvent, water. (a 5°C increase of temperature in the physiological range reduces plasma viscosity by about 10%).

The osmotic pressure of the plasma affects the mechanics of the circulation in several ways. An alteration of the osmotic pressure difference across the membrane of a blood cell causes a shift of water and a change in cell volume. The change, both in shape and flexibility, affects the mechanical properties of whole blood. Therefore, a change in plasma osmotic pressure alters the hematocrit (the volume concentration of red cells in the whole blood) by redistributing water between the intravascular and extravascular spaces. This in turn affects the mechanics of the whole blood.

Key Points

- Viscosity is the resistance of a fluid to flow. Virtually all fluids have viscosity which generally changes as a function of temperature; although different types of fluids exhibit different types of fluid–shear velocity dependencies.
- Laminar flow of a fluid is characterized by its flow in parallel layers in which there is no disruption or interaction between the different layers, and in which each layer flows at a different velocity along the same direction.
- Poiseuille's equation pertains to moving incompressible fluids exhibiting laminar flow. It relates the difference in pressure at different spatial points to volumetric flow rate for fluids in motion in certain cases, such as in the flow of fluid through a rigid pipe.
- The major quantity of interest in describing the motion of blood particles is the velocity – the rate of change of the position of an object with time: $\mathrm{v}=\frac{\Delta \mathrm{x}}{\Delta \mathrm{t}}$.
- Blood velocities in arteries are higher during systole than during diastole.
- The mechanics of the circulation depends on osmotic pressure of plasma.

Key Terms

- **viscosity**: The property of a fluid that resists the force which tends to cause it to flow.
- **shear stress**: The external force acting on an object or surface parallel to the slope or plane in which it lies; the stress tending to produce shear.
- **Reynolds Number**: A dimensionless number, $\frac{\mathrm{v}\rho \mathrm{l}}{\eta}$, where v is the fluid velocity, ρ the density, η the viscosity and l a dimension of the system. The value of the number indicates the type of fluid flow.
- **systole**: The rhythmic contraction of the heart, by which blood is driven through the arteries.
- **vessel**: A tube or canal that carries fluid in an animal or plant.
- **diastole**: The phase or process of relaxation and dilation of the heart chambers, between contractions, during which they fill with blood; an instance of the process.

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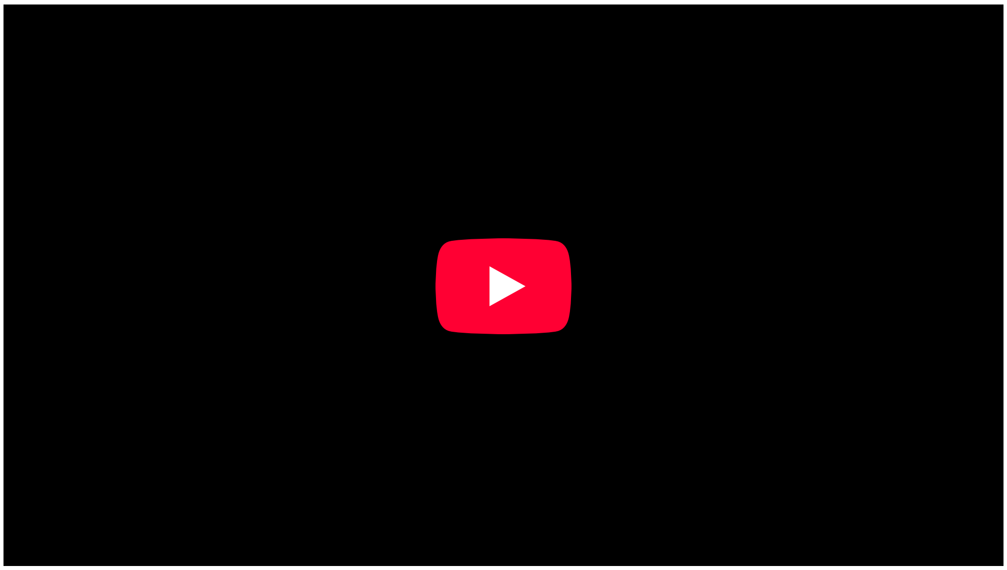
11.3: Bernoulli’s Equation

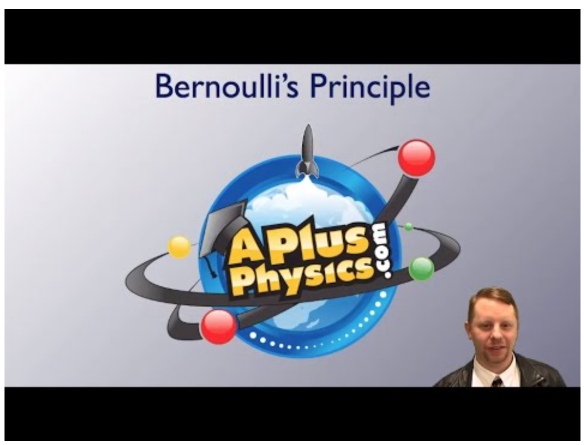
learning objectives

- Adapt Bernoulli’s equation for flows that are either unsteady or compressible

Application of Bernoulli’s Equation

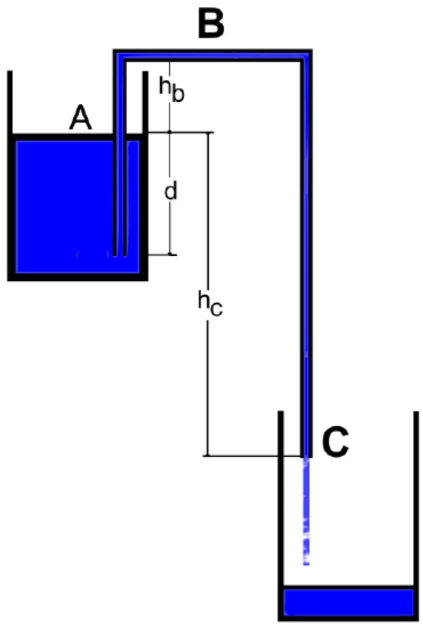
The relationship between pressure and velocity in ideal fluids is described quantitatively by Bernoulli’s equation, named after its discoverer, the Swiss scientist Daniel Bernoulli (1700–1782). Bernoulli’s equation states that for an incompressible and inviscid fluid, the total mechanical energy of the fluid is constant. (An inviscid fluid is assumed to be an ideal fluid with no viscosity.)





***Bernoulli’s Principle**: A brief introduction to Bernoulli’s Principle for students studying fluids.*

The total mechanical energy of a fluid exists in two forms: potential and kinetic. The kinetic energy of the fluid is stored in static pressure, psps, and dynamic pressure, 12ρV212ρV2, where \rho is the fluid density in (SI unit: kg/m^3) and V is the fluid velocity (SI unit: m/s). The SI unit of static pressure and dynamic pressure is the pascal.



Syphoning: *Syphoning fluid between two reservoirs. The flow rate out can be determined by drawing a streamline from point (A) to point (C).*

Static pressure is simply the pressure at a given point in the fluid, dynamic pressure is the kinetic energy per unit volume of a fluid particle. Thus, a fluid will not have dynamic pressure unless it is moving. Therefore, if there is no change in potential energy along a streamline, Bernoulli's equation implies that the total energy along that streamline is constant and is a balance between static and dynamic pressure. Mathematically, the previous statement implies:

$$\mathrm{p_s}+\frac{1}{2}\rho \mathrm{V}^2=\text{constant} \tag{11.3.1}$$

along a streamline. If changes there are significant changes in height or if the fluid density is high, the change in potential energy should not be ignored and can be accounted for with,

$$\Delta \mathrm{PE}=\rho \mathrm{g} \Delta \mathrm{h}. \tag{11.3.2}$$

This simply adds another term to the above version of the Bernoulli equation and results in

$$\mathrm{p_s}+\frac{1}{2}\rho \mathrm{V}^2+\rho \mathrm{g} \Delta \mathrm{h}=\text{constant}. \tag{11.3.3}$$

Deriving Bernoulli's Equation

The Bernoulli equation can be derived by integrating Newton's 2nd law along a streamline with gravitational and pressure forces as the only forces acting on a fluid element. Given that any energy exchanges result from conservative forces, the total energy along a streamline is constant and is simply swapped between potential and kinetic.

Applying Bernoulli's Equation

Bernoulli's equation can be applied when syphoning fluid between two reservoirs. Another useful application of the Bernoulli equation is in the derivation of Torricelli's law for flow out of a sharp edged hole in a reservoir. A streamline can be drawn from the top of the reservoir, where the total energy is known, to the exit point where the static pressure and potential energy are known but the dynamic pressure (flow velocity out) is not.

Adapting Bernoulli's Equation

The Bernoulli equation can be adapted to flows that are both unsteady and compressible. However, the assumption of inviscid flow remains in both the unsteady and compressible versions of the equation. Compressibility effects depend on the speed of the flow relative to the speed of sound in the fluid. This is determined by the dimensionless quantity known as the Mach number. The Mach number represents the ratio of the speed of an object moving through a medium to the speed of sound in the medium.

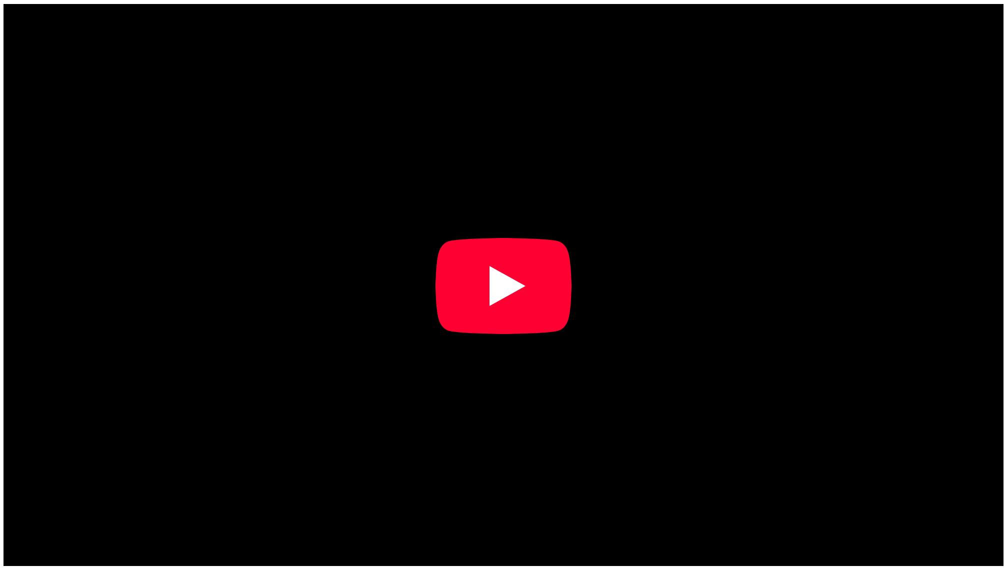
Torricelli’s Law

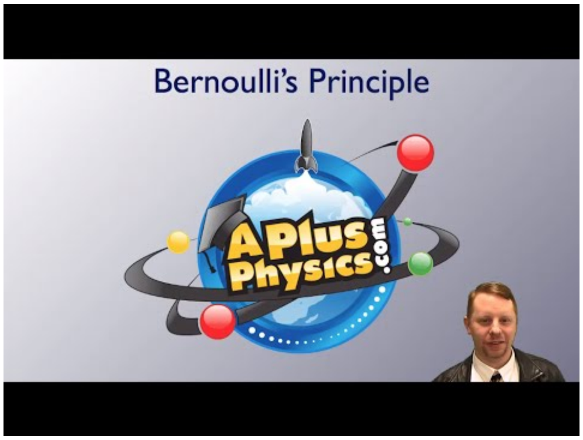
Torricelli’s law is theorem about the relation between the exit velocity of a fluid from a hole in a reservoir to the height of fluid above the hole.

learning objectives

- Infer the exit velocity through examining the Bernoulli equation

Torricelli’s law is theorem in fluid dynamics about the relation between the exit velocity of a fluid from a sharp-edged hole in a reservoir to the height of the fluid above that exit hole. This relationship applies for an “ideal” fluid (inviscid and incompressible) and results from an exchange of potential energy,





Torricelli’s Principle: A brief introduction to Torricelli’s Principle for students studying fluids.

mghmgh, for kinetic energy,

$\frac{1}{2}\rho v^2$, at the exit.

This relationship can be derived by applying the Bernoulli equation between the top of the reservoir and the exit hole. Applying Bernoulli between the top of a reservoir and an exit hole at a height h below the top of the reservoir results in,

$$PE \leftrightarrow KE$$

Exchange of Energy*: Potential energy at the top of the reservoir becomes kinetic energy at the exit.*

$$p_t + \frac{1}{2}\rho v_t^2 + \rho g h_t = p_e + \frac{1}{2}\rho v_e^2 + \rho g h_e \tag{11.3.4}$$

where subscript t implies evaluation at the top of the reservoir and subscript e implies evaluation at the exit. If we assume both the top of reservoir and the exit are open to the atmosphere, the zero for potential energy is at the exit hole, and the fluid velocity at the top of the reservoir is essentially zero (large reservoir, small hole), we arrive at

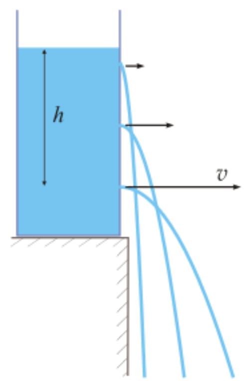
$$\rho g h_t = \frac{1}{2}\rho v_e^2 \tag{11.3.5}$$

This can be solved for the exit velocity, resulting in,

$$v_e = \sqrt{2 g h_t} \tag{11.3.6}$$

where again h_t is the height difference between the top of the reservoir and the exit hole. Due to the assumption of an ideal fluid, all forces acting on the fluid are conservative and thus there is an exchange between potential and kinetic energy. The result is that the velocity acquired by the fluid is the same that a body would acquire when simply dropped from the height h_t.

A simple experiment to test Torricelli's law involves filling a soda bottle with water and puncturing the bottom with a small hole (about 1 cm in diameter). As the height in the reservoir decreases, the exit velocity will decrease as well. The exit velocity can be increased by capping the top of the reservoir and pressurizing it.



Toricelli's Law: The exit velocity depends on the height of the fluid above the exit hole.

$$\mu = 0$$

$$\nabla \cdot \vec{u} = 0$$

Ideal Fluid: Applies to an ideal fluid (inviscid, incompressible)

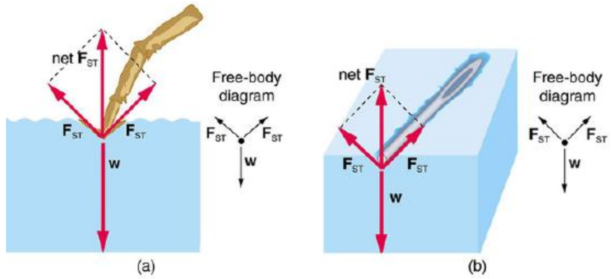
Surface Tension

The tendency of the surface of a liquid to resist a force and behave like a membrane and is a result of cohesion between liquid molecules.

learning objectives

- Summarize the cause for different surface tensions at a liquid's surface

Surface tension is the tendency of a liquid surface to resist forces applied to it. This effect is a result of cohesion of the molecules of the liquid causing the surface of the liquid to contract to the smallest area possible. This effect is visible in nature with water strider insects that are able to walk on water. Also, a paper clip or pin can be supported by the surface tension at a water air interface.

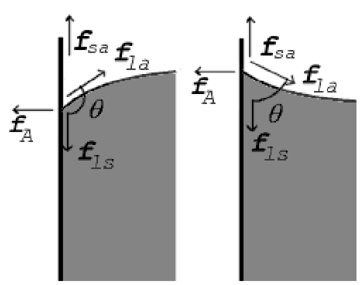


***Surface Tension FBD**: Force diagrams showing the direction of forces for water supporting a water strider (insect) foot and a pin. In both cases, the vertical component of the surface tension is enough to support the weight of the object.*

In the bulk of the liquid, the molecules are pulled equally in all directions. The molecules at the surface feel a greater attractive force toward the bulk material than the interface material.

The surface of a liquid is an interface between another fluid, a solid body, or both. Therefore, the surface tension will be a property of the interface rather than simply the liquid. Adhesion describes the attractive force between molecules of different types. The surface of a liquid in a container is an interface between the liquid, the air, and the container. Where the surfaces meet, forces must be in equilibrium. This results in a contact angle at the interface. The contact angle is measured in the liquid and depends on the relative strength of cohesive forces in the liquid and adhesive forces between the liquid and interface materials. If liquid molecules are strongly attracted to the molecules of the solid surface (adhesive forces > cohesive forces), the drop will tend to spread out and the contact angle will be close to zero degrees. If the cohesive forces are greater than the adhesive forces, the resulting contact angles will be large and will form a more circular drop.

***Water Droplet on Leaf**: When a water droplet forms on a leaf, the cohesive forces between the water molecules are greater than the adhesive forces between the water and leaf surface. The leaf is a hydrophobic surface.*



***Contact Angle**: The contact angle is the angle, measured in the fluid, that results when a liquid-gas interface, meets a solid surface.*

When the liquid is water, a surface where the contact angle is small is said to be hydrophilic. Large contact angles are present on hydrophobic surfaces. The contact angle determines the wettability of the surface.

Key Points

- The simplest form of Bernoulli’s equation (steady and incompressible flow) states that the sum of mechanical energy, potential energy and kinetic energy, along a streamline is constant. Therefore, any increase in one form results in a decrease in the other.
- Bernoulli’s equation considers only pressure and gravitational forces acting on the fluid particles. Therefore, if there is no change in height along a streamline, Bernoulli’s equation becomes a balance between static pressure and velocity.
- The steady-state, incompressible Bernoulli equation, can be derived by integrating Newton’s 2nd law along a streamline.
- Torricelli’s law applies to an inviscid, incompressible fluid (“ideal” fluid).
- You can ascertain results from applying the Bernoulli equation between the top of the reservoir and the exit hole.
- The relationship arises from an exchange of potential energy at the top of the reservoir to kinetic energy at the exit.
- The final kinetic energy is equivalent to what a solid body would acquire when falling from height h.
- Surface tension is a result of cohesion between the molecules of the liquid. The molecules at the surface of the liquid feel an attractive force pulling them toward the bulk of the liquid more than the solid or fluid at the interface.
- When a liquid-solid-gas interface is encountered, the contact angle represents a measure of the relative strength of adhesive and cohesive forces.
- The contact angle determines the wettability of a surface.

Key Terms

- **viscosity**: A quantity expressing the magnitude of internal friction in a fluid, as measured by the force per unit area resisting uniform flow.
- **Ideal Fluid**: An inviscid and incompressible fluid
- **incompressible**: Unable to be compressed or condensed.
- **inviscid**: A fluid with zero viscosity (internal friction). In reality viscosity is always present. However, it is often very small compared with other forces (e.g. gravity, pressure) and for common fluids (water and air) the fluid can be approximated as having zero viscosity.
- **cohesion**: Various intermolecular forces that hold solids and liquids together.
- **wettability**: The ability of a solid surface to reduce the surface tension of a liquid in contact with it such that it spreads over the surface and wets it.
- **adhesion**: The ability of a substance to stick to an unlike substance.

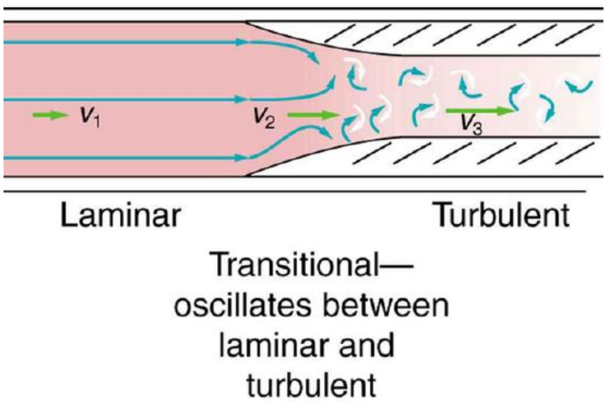
11.4: Other Applications

learning objectives

- Predict if flow will be laminar or turbulent

It is possible to predict if flow will be laminar or turbulent. At low velocity, flow in a very smooth tube or around a smooth, streamlined object will be laminar. At high velocity, even the flow in a smooth tube or around a smooth object will experience turbulence. However, between low and high velocity, flow is more difficult to predict. In fact, at intermediate velocities, flow may oscillate back and forth indefinitely between laminar and turbulent.

An occlusion (narrowing) of an artery, such as shown in, is likely to cause turbulence because of the irregularity of the blockage, as well as the complexity of blood as a fluid. Turbulence in the circulatory system (such as aneurysms, or ballooning of arteries) is noisy and can sometimes be detected with a stethoscope (such as when measuring diastolic pressure in the upper arm's partially collapsed brachial artery). These turbulent sounds, at the onset of blood flow when the cuff pressure becomes sufficiently small, are called *Korotkoff* sounds. Heart murmurs, consistent with their name, are sounds produced by turbulent flow around damaged and insufficiently closed heart valves. Another method of detecting this type of turbulence is ultrasound, used as a medical indicator in a process analogous to Doppler-shift radar (used to detect storms).



Turbulent Flow in an Artery: Flow is laminar in the large part of this blood vessel and turbulent in the part narrowed by plaque, where velocity is high. In the transition region, the flow can oscillate chaotically between laminar and turbulent flow.

Turbulence manifests in other areas, with varying causes. During an airplane flight, for example, the turbulence experienced is due to the mixing of warm and cold air in the atmosphere, causing the airplane to shake. The mixing currents in oceans creates a similar effect.

The phenomenon of turbulent air flow must be accounted for in many applications. For example, race cars are unable to follow each other around fast corners because the leading car creates turbulent air flow in its wake (this can lead to under-steering).

Industrial equipment, such as pipes, ducts, and heat exchangers are often designed to induce the flow regime of interest (laminar or turbulent). When flow is turbulent, particles exhibit additional transverse motion. This enhances the rate of energy and momentum exchange between them, increasing the heat transfer. Turbulent flow is thus desirable in applications where a relatively cool fluid is mixed with a warmer fluid to reduce the temperature of the warmer fluid.

It is imperative to take into account turbulent flow when designing certain structures, such as a bridge support, as shown in. In the late summer and fall, when river flow is slow, water flows smoothly around the support legs. In the spring, when the flow is faster, the flow may start off laminar but it is quickly separated from the leg and becomes turbulent. The bridge supports must be designed so that they can withstand the turbulent flow of the water in the spring.

Longtown Bridge: Turbulent flow is visible around the bridge supports of the Longtown bridge.

Motionof an Object in a Viscous Field

Objects moving in a viscous fluid feel a resistive force proportional to the viscosity of the fluid.

learning objectives

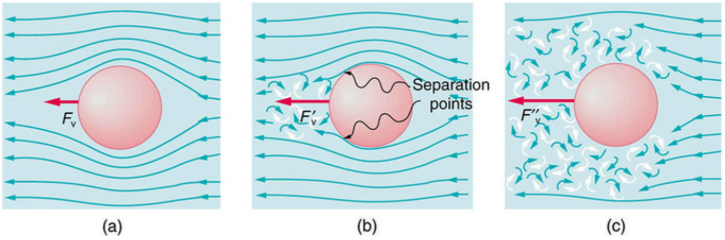
- Assess the relationship of the parameters to one another in determining the inertia of an object moving in fluid

Overview

A moving object in a viscous fluid is equivalent to a stationary object in a flowing fluid stream. (For example, when you ride a bicycle at 10 m/s in still air, you feel the air in your face exactly as if you were stationary in a 10-m/s wind.) Flow of the stationary fluid around a moving object may be laminar, turbulent, or a combination of the two. Just as with flow in tubes, it is possible to predict when a moving object creates turbulence. We use another form of the Reynolds number N′R, defined for an object moving in a fluid to be

$$\mathrm{N`R} = \frac{\rho \mathrm{vL}}{\eta} \tag{11.4.1}$$

where L is a characteristic length of the object (a sphere's diameter, for example), the fluid density, its viscosity, and v the object's speed in the fluid. If N′R is less than about 1, flow around the object can be laminar, particularly if the object has a smooth shape. The transition to turbulent flow occurs for N′R between 1 and about 10, depending on surface roughness and so on. Depending on the surface, there can be a *turbulent wake* behind the object with some laminar flow over its surface. For an N′R between 10 and 10^6, the flow may be either laminar or turbulent and may oscillate between the two. For N′R greater than about 10^6, the flow is entirely turbulent, even at the surface of the object. (See.) Laminar flow occurs mostly when the objects in the fluid are small, such as raindrops, pollen, and blood cells in plasma.



Motion of an object in a viscous fluid.: (a) Motion of this sphere to the right is equivalent to fluid flow to the left. Here the flow is laminar with N′R less than 1. There is a force, called viscous drag FV, to the left on the ball due to the fluid's viscosity. (b) At a higher speed, the flow becomes partially turbulent, creating a wake starting where the flow lines separate from the surface. Pressure in the wake is less than in front of the sphere, because fluid speed is less, creating a net force to the left F′V that is significantly greater than for laminar flow. Here N′R is greater than 10. (c) At much higher speeds, where N′R is greater than 10^6, flow becomes turbulent everywhere on the surface and behind the sphere. Drag increases dramatically.

Viscous Drag

One of the consequences of viscosity is a resistance force called viscous drag FVFV that is exerted on a moving object. This force typically depends on the object's speed (in contrast with simple friction). Experiments have shown that for laminar flow (N′R less than about one) viscous drag is proportional to speed, whereas for N′R between about 10 and 106, viscous drag is proportional to speed squared. (This relationship is a strong dependence and is pertinent to bicycle racing, where even a small headwind causes significantly increased drag on the racer. Cyclists take turns being the leader in the pack for this reason.) For N′R greater than 106, drag increases dramatically and behaves with greater complexity. For laminar flow around a sphere, FVFV is proportional to fluid viscosity, the object's characteristic size L, and its speed v. All of which makes sense—the more viscous the fluid and the larger the object, the more drag we expect. Recall Stoke's law FS=6πrηvFS=6πrηv. For the special case of a small sphere of radius R, moving slowly in a fluid of viscosity, the drag force FSFS is given by

FS=6πRηvFS=6πRηv.

Molecular Transport Phenomena

Molecular transport phenomena are ways in which molecules are transported from one region to another. These include diffusion and osmosis.

learning objectives

- Predict the role diffusion plays in blood transport throughout the body

Diffusion

Atoms and molecules are in constant motion at any temperature. In fluids they move about randomly even in the absence of macroscopic flow.

Diffusion is the movement of substances due to random thermal molecular motion. Fluids, like fish fumes or odors entering ice cubes, can even diffuse through solids. Diffusion is a slow process over macroscopic distances. The densities of common materials are great enough that molecules cannot travel very far before having a collision that can scatter them in any direction, including straight backward. More massive molecules diffuse more slowly.

Another interesting point is that the diffusion rate for oxygen in air is much greater than for oxygen in water. In water, an oxygen molecule makes many more collisions in its random walk and is slowed considerably. In water, an oxygen molecule moves only about 40μm in 1 s. (Each molecule actually collides about 1010 times per second!). Finally, note that diffusion constants increase with temperature, because average molecular speed increases with temperature. This is because the average kinetic energy of molecules, $1/2mv^2$, is proportional to absolute temperature. Because diffusion is typically very slow, its most important effects occur over small distances. For example, the cornea of the eye gets most of its oxygen by diffusion through the thin tear layer covering it.

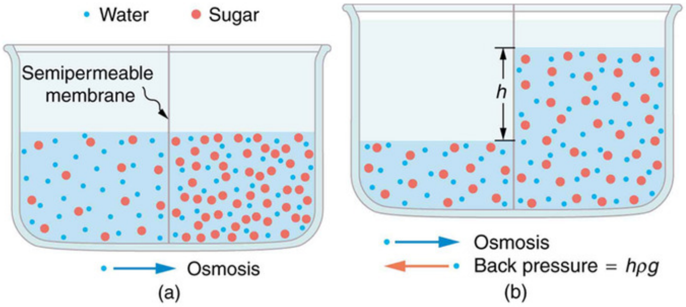
If you very carefully place a drop of food coloring in a still glass of water, it will slowly diffuse into the colorless surroundings until its concentration is the same everywhere. This type of diffusion is called free diffusion, because there are no barriers inhibiting it. Let us examine its direction and rate. Molecular motion is random in direction, and so simple chance dictates that more molecules will move out of a region of high concentration than into it. The net rate of diffusion is higher initially than after the process is partially completed. The rate of diffusion is proportional to the concentration difference. Many more molecules will leave a region of high concentration than will enter it from a region of low concentration. In fact, if the concentrations were the same, there would be no net movement. The rate of diffusion is also proportional to the diffusion constant D, which is determined experimentally. Many of the factors that affect the rate are hidden in the diffusion constant D. For example, temperature and cohesive and adhesive forces all affect values of D. Diffusion is the dominant mechanism by which the exchange of nutrients and waste products occur between the blood and tissue, and between air and blood in the lungs. In the evolutionary process, as organisms became larger, they needed quicker methods of transportation than net diffusion, because of the larger distances involved in the transport, leading to the development of circulatory systems. Less sophisticated, single-celled organisms still rely totally on diffusion for the removal of waste products and the uptake of nutrients.

Food Coloring: Food coloring spreading on a thin water film.

Osmosis and Dialysis – Diffusion Across Different Membranes

Some of the most interesting examples of diffusion occur through barriers that affect the rates of diffusion. For example, when you soak a swollen ankle in Epsom salt, water diffuses through your skin. Many substances regularly move through cell membranes; oxygen moves in, carbon dioxide moves out, nutrients go in, and wastes go out, for example. Because membranes are thin structures (typically 6.5×10^{-9} to 10×10^{-9} m across) diffusion rates through them can be high.

Diffusion through membranes is an important method of transport. Membranes are generally selectively permeable, or semipermeable. In other types of membranes, the molecules may actually dissolve in the membrane or react with molecules in the membrane while moving across. Membrane function, in fact, is the subject of much current research, involving not only physiology but also chemistry and physics. Osmosis is driven by the imbalance in water concentration. For example, water is more concentrated in your body than in Epsom salt. When you soak a swollen ankle in Epsom salt, the water moves out of your body into the lower-concentration region in the salt. Similarly, dialysis is the transport of any other molecule through a semipermeable membrane due to its concentration difference. Both osmosis and dialysis are used by the kidneys to cleanse the blood.



Diffusion: (a) Two sugar-water solutions of different concentrations, separated by a semipermeable membrane that passes water but not sugar. Osmosis will be to the right, since water is less concentrated there. (b) The fluid level rises until the back pressure ρgh equals the relative osmotic pressure; then, the net transfer of water is zero.

Pumps and the Heart

The heart pumps blood through the body by contracting and relaxing, increasing and decreasing the pressure.

learning objectives

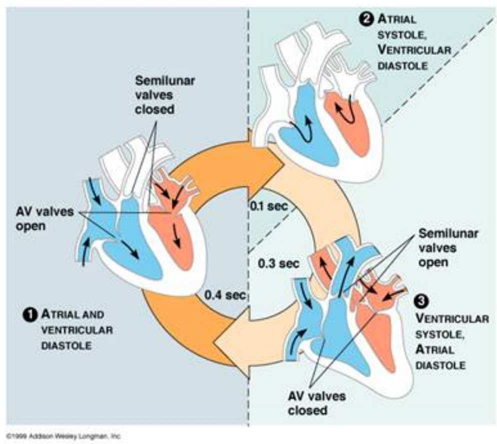
- Contrast systole and diastole in cardiovascular circulation

The Heart and its Parts

The heart is made up of four chambers. Two atria at the top of the heart receive blood and two ventricles at the bottom of the heart pump blood out of the heart. The septum divides the left and right side of the heart, while the valves of the heart ensure that blood only flows in one direction.They include the tricuspid valve-found between the right atrium and the right ventricle-and the mitral valve-found between the left atrium and the left ventricle. The list of heart valves also includes the semi-lunar valves, which are located at the bottom of the aorta and pulmonary artery. Strong tendinous chords attached to valves prevent them from turning inside out when they close.

The human heart will undergo over 3 billion contraction cycles during a normal lifetime. A complete cardiac cycle is one round of the heart pumping blood and consists of two parts: systole (contraction of the heart muscle) and diastole (relaxation of the heart muscle). During the cycle, the top half of the heart works as one unit, while the bottom half of the heart works as one unit.

The heart beat can be heard as a sound that the valves make when they close. The ‘lub’ sound is made when the atrio ventricular valves close and the ‘dub’ sound is made when the semi lunar valves close. Blood pressure is produced by the left ventricle contractions.The rhythm of ventricle diastole, often just referred to as diastole, causes the pulse, which can be felt by holding two fingers to the side of the throat.



Cardiac Cycle: The heart pumps blood through the body.

Key Points

- For low velocity, flow in a smooth tube will be laminar.
- At higher velocities or if there are obstructions, the flow turns turbulent.
- Turbulent flow is very chaotic, with rapid variations in velocity and pressure.
- Viscous fluids exert a resitive force on objects attempting to move through them.
- This resistive force is called viscous drag and is proportional to the viscosity of the fluid and the motion of the object.
- An object moving in a fluid can be thought of as a stationary object in a moving fluid.
- Diffusion is the movement of molecules due to random thermal motion.
- Osmosis is the movement of molecules due to different concentrations. Molecules will move from regions of high concentrations to lower concentrations.
- These transport phenomena can take place through membranes if the pressure is great enough.

Key Terms

- **turbulent**: Being in, or causing, disturbance or unrest.
- **streamlined**: Designed to offer little resistance to the flow of fluid, especially by having sleek, graceful lines.
- **laminar**: Of fluid motion, smooth and regular, flowing as though in different layers.
- **viscosity**: A quantity expressing the magnitude of internal friction in a fluid, as measured by the force per unit area resisting uniform flow.
- **turbulence**: Disturbance in a gas or fluid, characterized by evidence of internal motion or unrest.
- **diffusion**: the intermingling of the molecules of a fluid due to random thermal agitation
- **ventricle**: One of two lower chambers of the heart.
- **contraction**: A reversible reduction in size.
- **atrium**: An upper chamber of the heart that receives blood from the veins and forces it into a ventricle. In higher vertebrates, the right atrium receives blood from the superior vena cava and inferior vena cava, and the left atrium receives blood from the left and right pulmonary veins.

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