03. Analysis Tools 2

Inductor Properties

Imagine a pair of long, hollow nested wires of inner radius a and outer radius b, designed to carry current into and out of the page. Calculate the inductance, per meter, for these nested wires.

pic 1

Since inductance is defined by the relation

pic 2

we need to determine the flux that would develop between these wires if current i (and -i) flowed along the two wires:

To do this, imagine that current i flowed out of the page along the inner wire. Using Ampere's Law, this leads to a magnetic field between the cylinders of:

pic 3

To help calculated the flux between the wires, the diagram at right is a top view of the nested wires. The dashed area is the area over which we will calculate the flux. (The current along the inner wire flows toward the top of the page, resulting in magnetic field pointing directly out of the page in the area of interest.)

The shaded sliver is the differential element, located a distance r from the center of the wires, with thickness dr and length l. The magnetic flux is then:

pic 4

Substituting this result into the definition of inductance yields:

pic 5

The inductance per meter is then:

pic 6

Thus, the inductance per meter of a set of nested wires depends on the natural lograithm of the ratio of the wire radii. Notice that if the wires are very far apart, the inductance is larger. However, as the wires get farther apart, the size of the device gets larger and may become impractical. For this reason, and several others, the space containing the magnetic flux in an inductor is typically filled with a material with a high magnetic permeability, like iron, in order to "concentrate" the magnetic flux into a smaller region of space.

Inductors in Circuits

The device below represents a simplified electromagnet. With V = 100 V and R = 15 W, find L such that the current reaches 5.0 A in 0.5 s.

pic 7

The circuit above, termed an *RL circuit*, can best be analyzed by considering the changes in electric potential experienced by a hypothetical charge "journeying" around the circuit:

- as it "passes through" the battery the potential increases by V,
- as it passes through the resistor the potential decreases by

pic 8

- and as it "passes through" the inductor's potential changes by

pic 9

Since by Faraday's Law of Induction,

pic 10

The emf induced by the inductor is the potential drop across it, so

pic 11

Putting these changes in potential together results in:

pic 12

Again, note that the total change in potential (and potential energy) must be zero since the energy given to the charge by the battery is partially converted by the resistor and partially stored by the inductor.

If we take a time derivative of the above equation (noting that V, R, and L are constant) we are left with a differential equation for the current in the circuit:

pic 13

This equation says that the time derivative of the *derivative* of the current is equal to the product of the *derivative* of the current and a numerical factor. This means that the derivative of the current must be exponential function. Therefore, the derivative of the current must be given by the function:

pic 14

where A is an arbitrary constant. Integrating this result leads to a current of the form:

pic 15

where B and D are arbitrary constants.

To determine these constants, consider the current in the circuit after a very long time ($t \to \infty$). After this amount of time the circuit will have reached an equilibrium value, so the change in the current will be zero. Thus,

pic 16

Therefore,

pic 17

Now consider the current in the circuit the instant you first close the switch ($t \to 0$). At this instant, no current can be flowing in the circuit. This is because if there was current flowing instantaneously after the switch was closed, this would be discontinuous change in current and the inductor would create an infinite emf to oppose this "infinite" increase in current. Therefore,

pic 18

Now that we know the values of the two constants, the final expression for the current in the circuit as a function of time is:

pic 19

Using this expression we can determine the time-dependence of any other circuit parameter.

Since the question asks about the current directly,

pic 20

Therefore, if the electromagnet has an inductance of 5.4 H, it will take 0.5 s for the current to rise to 5.0 A.
