02. Analysis Tools

Resistors in Circuits

The circuit at right represents a 12 V car battery and two mismatched headlights, R_1 = 1.9 W and R_2 = 2.1 W.

a. Determine the magnitude of the potential difference across and the current through each circuit component.

b. If the battery has a total stored energy of 800 W hr, and produce a constant potential difference until discharged, how long will the bulbs stay lit?

pic 1

The potential difference across the car battery is given as 12 V. This means that the electric potential in the wire coming out of the "top" of the battery is 12 V larger than the potential in the wire coming from the "bottom". Since each of the resistors are attached to these same two wires, the top of each resistor is 12 V higher in potential than the bottom. Therefore the potential difference across each resistor is 12 V. When circuit elements are connected such that the elements all have the same potential difference, the elements are said to be in *parallel*.

Since the potential difference across each resistor is known, we can use the definition of resistance to calculate the current through each branch of the circuit. Analyzing branch #1 yields

pic 2

and branch #2

pic 3

The current that flows through R_1 and the current that flows through R_2 must also flow through both the top and bottom wires connected to the battery. To complete the mental image of a closed circuit of current, we will say the current flows "through" the battery as well, although this is not technically true. Therefore, the current that flows through the battery (the total current flowing in the circuit) is:

pic 4

We can summarize this information in a simple table:

	V_{across} (V)	$i_{through}$ (A)
battery	12	12.0
R_1	12	6.32
R_2	12	5.71

To determine how long the headlights will stay lit, we must calculate the total power of the circuit (the total amount of electrical energy converted per second). We can do this separately for each headlight and then add the results:

pic 5

and

pic 6

so the total power of the circuit is:

pic 7

Therefore, the battery will last for

pic 8

Capacitor Properties

Imagine a pair of long, hollow nested cylinders of inner radius a and outer radius b. Calculate the capacitance, per meter, for these nested cylinders.

pic 9

Since capacitance is defined by the relation

pic 10

we need to determine the potential difference that would develop between these cylinders if charges Q (and -Q) where placed on the two surfaces.'To do this, imagine that a charge +Q (per meter) was placed on the inner cylinder. Using Gauss' Law, this leads to an electric field between the cylinders of:

pic 11

This field is directed radially away from the central axis of the cylinders.

Once the electric field between the cylinders is known, the magnitude of the potential difference between the cylinders can be calculated by:

pic 12

Substituting this result into definition of capacitance yields:

pic 13

Thus, the capacitance permeter of a set of nested cylinders depends on the natural logarithm of the ratio of the cylinder radii. Notice that is the cylinders are very close together (b is not much larger than a), the capacitance is very large. The capacitance of a capacitor is always enhanced by having the two charged surfaces very close together. However, as the surfaces get closer together, the possibility of electrical breakdown (charges "jumping" across the gap) becomes larger. For this reason, and several others, the space between the surfaces in a capacitor is typically filled with a type of material, called a *dielectric*, which both enhances the capacitance of the capacitor and inhibits electrical breakdown.

Capacitors in Circuits

The device at right represents a simplified camera flash circuit. With V = 3 V and R = 100 , find C such that the flash reaches 80% of its final voltage in 1.0 s.

pic 14

The circuit above, termed an *RC circuit*, can best be analyzed by considering the changes in electric potential experienced by a hypothetical charge "journeying" around the circuit:

- as it "passes through" the battery the potential increases by V,
- as it passes through the resistor the potential decreases by

pic 15

- and as it "passes through" the capacitor the potential decreases by

pic 16

Putting these changes in potential together results in:

pic 17

Note that the total change in potential (and potential energy) must be zero since the energy given to the charge by the battery is partially converted by the resistor and partially stored by the capacitor.

If we take a time derivative of the above equation (noting that V, R, and C are constants, but that Q, the charge on the capacitor, is changing) we are left with a differential equation for the current in the circuit:

pic 18

This equation says that the time derivative of the cureent is equal to the product of the current and the numerical factor pic 19. The only mathematical function that has the property that its derivative is proportional to itself is the exponential function. Therefore, the current must be given by the function:

pic 20

where i_0 is the current at t = 0 s.

If we assume that the capacitor is uncharged when the switch is first closed, then

pic 21

so the final expression for the current in the circuit as a function of time is:

pic 22

Using this expression we can determine the time-dependence of any other circuit parameter.

For example, the question asks about the voltage across the capacitor. Since the voltage across the resistor can be expressed as:

pic 23

the voltage across the capacitor is the amount of the source voltage that "remains":

pic 24

This function shows that after a long time ($t \rightarrow \infty$), the voltage across the capacitor will equal the voltage of the source.

Therefore,

pic 25

Thus, a 6.21 mF capacitor will reach 80% of its final voltage in 1.0 s.
