3.1: Moving Charge

Electric Current

Up to now we have avoided talking about the details of moving charge, but we avoid it no longer. We begin by defining a quantity we will be using a lot – *electric current*. Simply put, this is the amount of charge that passes a fixed point in a given period of time:

$$I \equiv \frac{dq}{dt} \tag{3.1.1}$$

This has units of coulombs per second, which is given its own name: *amperes* or *amps*.

First off, we need to say that it is the *electrons* that do the moving – the protons are fixed in the nucleus of the atoms that are fixed in a lattice that constitutes the conductor. This may cause some confusion at first, since electrons are defined to have negative charge, and the current is defined to be in the direction of positive charge flow. This means that while electrons are moving in one direction, the current associated with this charge flow is in the opposite direction.

Digression: Charge Carriers

*There are other types of electrical current besides electrons moving through conductors. These currents are effectuated by other types charge carriers. One common variety of charge carrier is an ion, which is an atom that is not neutrally charged because it is either missing an electron or has an extra electron. This sort of charge carrier is most prevalent in biological systems, in fluids called electrolytes. Another charge carrier is not a true particle at all, but rather the **absence** of an electron (so it is positively-charged), called a hole. These are important in semiconductor physics, and come into play in the common electrical components of diodes and transistors.*

Second, we will discard the notion that these electrons are *completely* free to move within a conductor, as they actually will encounter something very similar to air resistance. If you recall from Physics 9A, air resistance is a dissipative force that comes about because particles that comprise the air collide with, and thereby transfer momentum to, the object experiencing moving through the air. A simple model of resistance in a conductor has the electrons colliding with the fixed atom nuclei. This is of course oversimplified, but without more advanced quantum physics, it is a model that works pretty well.

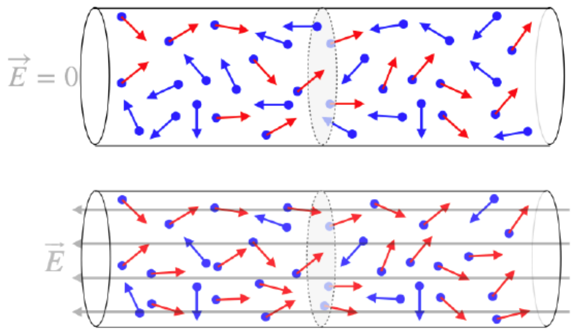
A feature of air resistance that carries over to electrical "drag" is the fact that the faster the object moves, the greater the force. This means that eventually an object moving through the air reaches a terminal velocity, and we will see the same for electrons moving through a conductor. The electric field within the conductor results in a force on the electrons, but the electrons don't keep accelerating indefinitely, just like a falling object under the influence of the gravity force doesn't keep speeding up indefinitely. If we increase the strength of the electric field, then the terminal velocity goes up, just as it would if we increased the gravitational force.

Wait, did we just say the *electric field within the conductor*?! Isn't the electric field inside a conductor always zero? In the case of electro*statics*, yes. That is, we stated previously that the electric field in the presence of a conductor causes charges to migrate, and once they have stopped moving, they produce a second field that cancels the applied field. But now we are discussing what is happening *as the charges move*, so we are now looking at the case when the electric field has not been canceled by the field of a separated charge.

Current Density

We need to take some time to determine the factors that affect the amount of current that passes through a conductor. In a conductor that has no bias placed on it by an electric field, the electrons are still able to move, but they do so randomly, consistent with thermal motion that we studied in Physics 9B. If we watch a specific place in the conductor, we will see these randomly-moving electrons passing by, but the randomness of their motion means that of all the electrons passing the observed checkpoint, half are going each way, for no net charge flow. When an electric field is applied, however, the force it exerts on electrons gives them a bias to move in a specific direction. Of course, some fraction of the electrons will have recently bounced off a nucleus and will briefly be going the opposite direction, but on average the electrons will be flowing in the direction opposite to the electric field.

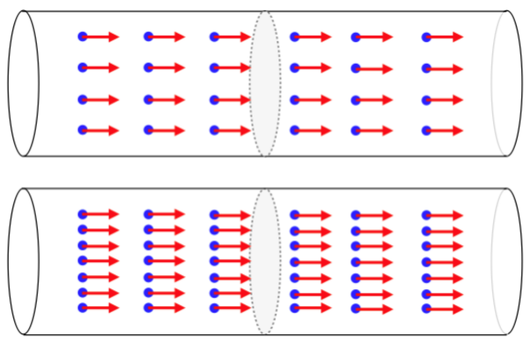
Figure 3.1.1 – Applied Electric Fields Affect Electron Motion in a Conductor



With so many particles crossing a fixed point in so many different ways (and at so many different speeds), we need a way to reconcile our microscopic picture with our simple definition of current above. As we saw in 9B, relating microscopic pictures to macroscopic ones requires speaking in terms of *averages*. In this case, the average we will introduce is called the *drift velocity*, $\vec{v}_d$. This is a *vector* average – the velocity of the average electron.

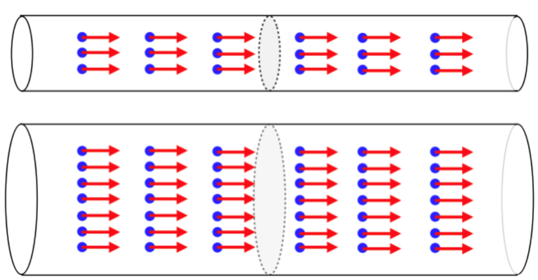
Suppose we know the drift velocity of the electrons – is this enough to give us the current? No, because knowing how fast and in which direction the electrons are moving doesn't include information about *how many* electrons are included in that average, and we need to know the amount of charge passing per second. This number goes up for a fixed drift velocity when the sheer number of electrons goes up. For a given conductor, we can get more electrons moving past a point per second when they are more densely-packed. In the figure below, the electrons in both conductors have equal drift velocities (depicted by the red arrows), but there is more charge passing the checkpoint per second in the lower conductor because the electrons are closer together.

Figure 3.1.2 – Electron Density Affects Current



There is one other consideration to take into account here: Who says that all conductors are the same thickness? The cross-sectional area of the conductor plays a role in the number of charges that can pass through the checkpoint per second.

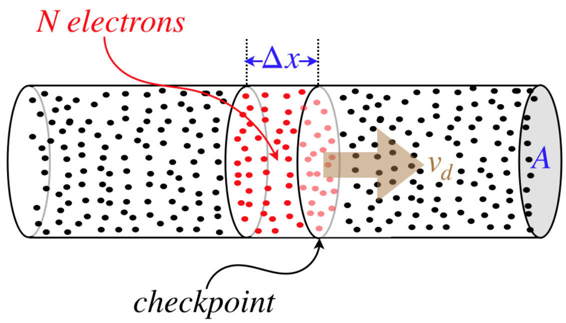
Figure 3.1.3 – Cross-Sectional Area of the Conductor Affects Current



Notice that in the figure above the electrons have the same drift velocity *and* they are equally dense in both conductors, but there is more space for the electrons to pass through the lower conductor, so more charge goes past the checkpoint per second in the lower conductor.

We can put all of these quantities together to come up with a mathematical expression for electric current. If we consider a thin segment of the conductor, as in the figure below, then we can write the rate of charge flow across the checkpoint as the amount of charge in the segment divided by the time it takes for all that charge to exit the segment.

Figure 3.1.4 – Calculating Current



The amount of charge that passes through the checkpoint in the allotted time is Nq, where N is the number of electrons in the tiny volume, and q is the charge of a single electron. So the rate of charge flow is this number divided by the time:

$$I=\frac{Nq}{\Delta t} \tag{3.1.2}$$

The number of particles is equal to the particle density (particles per unit volume, which we will call n), multiplied by the volume of the slice, which is $A\Delta x$. The length of the slice divided by the time that the last electron exits the slice is the drift velocity of the charges, so we get:

$$I=\frac{(nA\Delta x)\,q}{\Delta t}=nqv_dA \tag{3.1.3}$$

Instead of particle density, it is generally more convenient to use our old friend charge density (the three-dimensional variety, ρ), and this is simply the density of particles n multiplied by the charge per particle q, giving:

$$I=\rho v_dA \tag{3.1.4}$$

Example 3.1.1

A thin plastic circular ring is uniformly charged with a total charge of Q. The ring rotates with a rotational speed ω. Find the electric current associated with this charge motion.

Solution

This is not a three-dimensional distribution of charge, so determining the current requires more though than just plugging into what we have found above. The current is the rate at which charge is passing a specific position of the loop (say 12 o'clock). A small slice of the loop has a charge dq on it, and has an arclength we will call ds. These are related to each other through the charge density, in the usual way:

$$dq=\lambda\;ds$$

The charge density is uniform, which means that it equals the total charge divided by the full length over which it is distributed (the circumference of the loop). Calling the radius of the loop R gives us:

$$dq=\frac{Q}{2\pi R}ds$$

Dividing this by the small time it takes the charge to clear that tiny segment gives us the current:

$$I=\frac{dq}{dt}=\frac{Q}{2\pi R}\frac{ds}{dt}$$

The quantity $\frac{ds}{dt}$ is the linear speed of the charge, and we can relate this to the rotational speed, to give our final answer:

$$I=\frac{Q}{2\pi R}v=\frac{Q}{2\pi R}(R\omega)=\frac{Q\omega}{2\pi}$$

While it seems reasonable that the drift velocity of the electrons would be parallel to the axis of a straight conductor like the one in the diagram, in a more general case (such as when the conductor gets wider or thinner), at some positions in the stream the drift velocities could vary from one position to the next. In this case, the definition of "current" will depend upon the area we use as a checkpoint. If the drift velocity at the checkpoint cross section is not perpendicular to the area, then only the component of the drift velocity that is perpendicular will contribute to the current.

With the possibility of different drift velocities at different positions in the stream, we clearly need to add up (i.e. integrate) all of the contributions through a given area. This sounds exactly like the concept of flux we discussed in Section 1.6, except this time the vectors are not electric field vectors. If we pull the area out of Equation 3.1.3, and allow for different drift velocities at different positions (so that we have to integrate just the parts perpendicular to the surface), we get:

$$I=\int \overrightarrow{J}\cdot d\overrightarrow{A}\;,\quad where:\;\; \overrightarrow{J}(\overrightarrow{r})\equiv\rho\, v_d(\overrightarrow{r}) \tag{3.1.5}$$

$\overrightarrow{J}(\overrightarrow{r})$ is called the *current density* (at position $\overrightarrow{r}$).

Alert

Current density is a vector, but the current is not. That is, we define current simply as the rate that charge passes a certain point, and if the flow changes direction (such as in a bend of a wire), the current doesn't change, since it does not have a direction.

Charge Conservation

Consider the flow of charge out of a closed volume. The rate of this flow is related to the total flux of current density out of that volume:

$$-\frac{dQ}{dt}=\oint \overrightarrow{J}\cdot d\overrightarrow{A} \tag{3.1.6}$$

The minus sign appears because the charge within the volume goes down when the current density points out of the volume (in the same direction of the differential area vector). The charge within the volume is the integral of the charge density over the volume, as usual, so:

$$Q=\int\rho dV \quad\Rightarrow\quad -\frac{dQ}{dt}=-\frac{d}{dt}\int\rho dV=-\int\frac{d\rho}{dt}dV \tag{3.1.7}$$

Setting these last two equations equal and using the divergence theorem gives:

$$-\int\frac{d\rho}{dt}dV=\oint\overrightarrow{J}\cdot d\overrightarrow{A} \quad\Rightarrow\quad -\int\frac{d\rho}{dt}dV=\int\left(\overrightarrow{\nabla}\cdot\overrightarrow{J}\right)dV \quad\Rightarrow\quad \overrightarrow{\nabla}\cdot\overrightarrow{J}+\frac{d\rho}{dt}=0 \tag{3.1.8}$$

This is known as the *continuity equation*. It is the differential statement of what we assumed at the outset – that the rate of charge flow into a closed volume, minus the rate of charge flow out (net flux of current density out), equals the rate at which charge accumulates inside (rate of change of enclosed charge density). Put more simply, it is a differential declaration that charge is neither created nor destroyed – it is conserved. This relation is actually used in many other fields of study (such as fluid mechanics, which we briefly encountered in Physics 9B), where the current is a different kind of flow than that of electric charge. Conservation principles are ubiquitous in physics, and wherever a conservation principle applies, this equation makes an appearance.