1.1: Fundamentals of Sound

Why Sound?

Physics 9D is a class about modern physics, so why is the first chapter in the textbook dedicated to the phenomenon of sound? Three reasons:

1. There is typically a long period of time that elapses between taking Physics 9B (where general wave phenomena are first studied) and taking Physics 9D, where wave physics is used extensively. By returning to the specific wave phenomenon of sound, you are given an opportunity to review some of the general features of waves, while applying them to the specific physical conditions present for sound.
2. Studying sound provides a useful historical context for modern physics. Sound was a wave phenomenon that was studied extensively prior to the modern era of physics, and this knowledge was both a help and a hindrance to physicists trying to unravel the mysteries that came around in the transition period between the 19^{th} and 20^{th} centuries. We can benefit from following their journey.
3. Examining properties of sound provides a useful contrast to the unusual aspects of waves that arise in relativity and quantum physics.

As Physics 9D progresses, the reader is encouraged to sort out which characteristics of sound waves are closely paralleled in modern physics, and which are vastly different (but may nevertheless be useful as an analogous model).

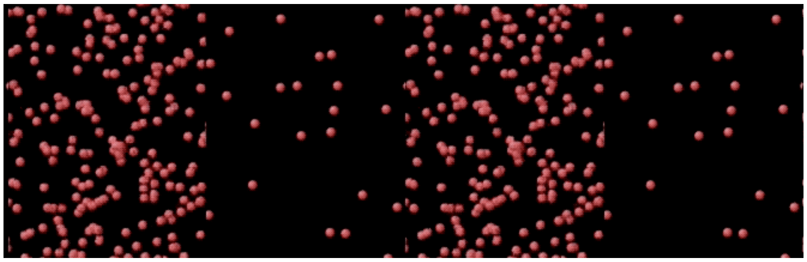
Sound Waves in Air

Sound can travel through any phase of matter – solid, liquid, or gas. Like other mechanical waves, it depends upon a restoring mechanism that returns particles in the medium to equilibrium after they are displaced. But unlike a wave in a string, for which the restoring mechanism is perpendicular to the direction of wave motion, the direction of sound's restoring mechanism is parallel to the direction of the wave motion – sound waves are longitudinal rather than transverse. As we discussed in Physics 9B, this comes about as a result of compressions and rarefactions. In one-dimension, this can be visualized as compressing and expanding coils of a slinky, but sound waves can travel in three dimensions, which makes the picture slightly more challenging to grasp.

We will primarily focus our exploration on sound waves in air, mainly because that is the way that we usually encounter them. What is not commonly noted is that sound waves in air are fundamentally different from sound waves in liquids and solids. As we noted in Physics 9B, gases are collections of particles that, to a very good approximation, don't interact with each other. If one particle in a medium doesn't interact with a neighboring particle, it seems strange that a wave can propagate through that medium at all. After all, waves require some sort of "restoring force" that returns the medium to its equilibrium state after it is displaced away from it. So how does the gas medium oscillate as a wave goes by, if the particles are not experiencing forces to make them oscillate?

The answer is "probability and statistics." Whenever there is an imbalance in the populations of particles in a gas (as there is when there is a low-density rarefaction next to a high-density compression), on average more randomly-moving particles enter the low-density region than leave it, and more particles leave the high-density region than enter it. So the low-density region naturally (through sheer probability) grows in density, while the high-density region drops in density, providing a statistical rather than mechanical "restoring force." Note that for solids (and to a lesser extent, liquids), the particles *do* interact with each other, and the restoring forces are exactly that – forces, but not so for sound through gases. As you can see below, the sound wave of compressions and rarefactions propagates along, even as the particles of the gas fly around randomly.

Figure 1.1.1 – Sound Wave in Air



Alert

Frequently textbooks and other sources, in their discussion of sound in air, refer to the oscillatory displacement of particles in the medium. It is okay to refer to the "average displacement" of a particle from the equilibrium point, but individual particles in a gas are flying all over the place, not actually bouncing back-and-forth. The distinction between these can easily be lost, leading to a lot of confusion.

For these reasons, when we express a wave function for sound in air, it will have units of either density or pressure. Note that when we detect a sound with our ears, it is the variations in pressure in the medium that causes our eardrums to vibrate.

Sound Wave Properties

The speed of sound in air at standard temperature and pressure is around $343 m/s$. For air and other fluids, the sound wave velocity dependence on the medium is very similar to that which we found for a transverse wave on a string. The density changes from a linear density to a volume density (which we denote with a ρ), and the tension is replaced by a constant known as *bulk modulus*. The velocity relation looks like:

$$v_{sound\ in\ fluid} = \sqrt{\frac{B}{\rho}} \tag{1.1.1}$$

Sound will also travel through a solid, but in that case the interactions of the particles are different than in a fluid, and the constant that takes the place of tension is a different one: *Young's modulus*. But the formula looks the same:

$$v_{sound\ in\ solid} = \sqrt{\frac{Y}{\rho}} \tag{1.1.2}$$

We will not explore the exact nature of the bulk and Young's moduli – simply knowing that they play the same role for fluids and solids respectively as the tension plays for a transverse wave on a string will suffice for our purposes.

Alert

At a very young age, children in science classes learn that sound travels faster through water than through air, and faster through solids than through water. This often leads to the erroneous conclusion that sound travels faster in media that are more dense. Indeed, the opposite is true, and in fact it is the greater bulk or Young's modulus that accounts for the faster speed of sound.

The intensity of a sound wave also obeys the rule-of-thumb for intensity – the intensity is proportional to the square of the amplitude. Specifically, it turns out that for an amplitude measured in pressure, the intensity is given by:

$$I = \frac{A^2}{2\sqrt{\rho B}} \tag{1.1.3}$$

Alert

It is important to note that the amplitude of pressure is the difference between the maximum pressure of the compression (or the minimum pressure of the rarefaction) and the ambient (equilibrium) pressure.

As with any other wave, the dimensions into which a sound wave spreads also determines how the intensity varies with distance from the source. That is, a sound that expands spherically outward has its intensity dissipate according to the inverse-square law. This explains why sound made into a closed tube (like those that can be found in playgrounds for children to play with) will remain so much louder despite the distance the sound travels – the sound is not allowed to spread out spherically, and is instead reflected back into the direction of the tube. Even shouting through a cone or cupped hands has some effect in this regard.

The Decibel Scale

The human ear is very sensitive to detecting sound. How loud a sound is depends upon the amplitude of the vibration of the eardrum, which is determined by how much energy the sound wave transfers to the eardrum per second. This of course depends upon the intensity (which is multiplied by the area of the eardrum to get the power transferred), and it turns out that the range of intensities that the ear can detect before it starts becoming painful is quite large. A healthy human ear can hear sounds with intensities as low as $10^{-12} W/m^2$ (known as the *threshold of hearing*), and starts to feel pain around $1 W/m^2$. A range of 12 orders of magnitude is quite large, making it more convenient to count the powers of ten rather than the exact values. A logarithmic scale has therefore been devised that works as follows:

We start with a benchmark value – the threshold of hearing – which will translate to a zero value in the logarithmic scale (so the power of ten will be zero). Then just convert every intensity to a ratio with this benchmark, and take the logarithm (to base 10):

$$\beta=(10\ dB)\log_{10}\left(\frac{I}{I_o}\right) \tag{1.1.4}$$

The number yielded by just the logarithm of the ratio is described as the number of "bels" of the loudness of the sound. It is traditional to multiply this by number by 10, so that the unit describing the loudness is *decibels*. Note that the threshold of hearing results in zero decibels, while the pain threshold occurs at $120\ dB$.

Alert

Sometimes the decibel level of sound is referred to as the sound's "intensity." Strictly speaking this is not accurate, but as there is a one-to-one correspondence between an intensity and a decibel level, it doesn't cause problems, especially if the context of this use of the term "intensity" includes some mention of a number of decibels.

Example 1.1.1

A medieval village has a bell located in a tower in its central square which is rung to warn the townspeople of emergencies, such as raiding parties from nearby regions. If the loudness of the bell heard by villagers in the town $500\ ft$ (about 1/10 mile) from the tower is $20\ dB$, then about how far from the tower does the sound carry (i.e. at what distance does the bell become barely audible)? Assume that there is negligible energy dissipated from the sound wave due to obstacles and the atmosphere.

Solution

The decibel scale is logarithmic, which means that every time the decibel level changes by $10\ dB$, the intensity changes by a factor of 10. The bell can barely be heard at the threshold of hearing, which is $0\ dB$, which means that the decibel level can afford to drop by $20\ dB$, and the intensity can drop by two factors of 10 (i.e. drop by a factor of 100). The sound from the bell expands outward spherically, so the intensity drops off according to the inverse-square law. Therefore to drop by a factor of 100, the distance must increase by a factor of 10. So the bell can barely be heard at a distance of one mile.

Example 1.1.2

A speaker at the north end of a round football stadium emits a sound at a single frequency. A listener in the center of the stadium hears the sound at an decibel level of $35\ dB$. Speakers in phase with the north end speaker are then turned on at the east, south, and west ends of the stadium, with all four speakers emitting sound at the same power output. Find the decibel level of the sound heard at the center of the stadium from all four speakers combined. Assume no thermal dissipation of sound wave energy into the air.

Solution

The sound waves coming from the four speakers all start in phase at the same time, and travel the same distance, so when they reach the common point at the center of the stadium, they are in phase, and interfere constructively. [Note that the direction of motion of the sound is irrelevant, as the contributions to the density of the air is what is superposing.] With four identical waves in phase, the superposed wave will have four times the amplitude of each individual wave. Multiplying the amplitude of the sound wave by 4 results in an intensity that is increased by a factor of 16. Now all we need to do is determine how much of a change this means for the decibel level (which is not a factor of 16!):

$$\begin{aligned} \beta_{four\ speakers} &= (10\ dB)\log\left(\frac{I_{four\ speakers}}{I_o}\right) \\ &= (10\ dB)\log\left(\frac{16\ I_{one\ speaker}}{I_o}\right) \\ &= (10\ dB)\log 16+(10\ dB)\log\left(\frac{I_{one\ speaker}}{I_o}\right) \\ &= 12\ dB+35\ dB=47\ dB \end{aligned}$$