# 13.5: Conservation of Energy and Momentum

In classical mechanics, energy and momentum were separate entities, each obeying its own conservation law. In special relativity, they are two parts of the same quantity (the energy-momentum four-vector), just like time and space are two parts of the same position four-vector. Consequently, energy and momentum have to obey the same rules in special relativity. Fortunately, a conservation law on a vector quantity applies to each of its components, and so conservation of energy and momentum translates to conservation of the energy-momentum four-vector $\bar{\boldsymbol{p}}$. However, unlike in classical mechanics, mass is no longer conserved: since it is now interpreted as a part of the total energy of a system (Equation 13.4.2), it can be converted into or created from kinetic energy. The equivalence of mass and energy has important consequences for collision experiments, including a whole new type of 'collisions': radioactive decay of matter.

You might complain that we haven't actually proved that the energy-momentum four-vector is conserved in special relativity (and you would be right). What we have done is define the relativistic energy $E=\gamma(v) m c^2$ and three-momentum $\boldsymbol{p}=\gamma(v) m \boldsymbol{v}$, as well as the energy-momentum four-vector $\bar{\boldsymbol{p}}$. We have also shown that with these definitions, $\bar{\boldsymbol{p}}$ is a proper four-vector, in the sense that it is invariant under Lorentz transformations. Therefore, we know that if it is conserved in one inertial frame, it must also be conserved in all others. We also know that our relativistic energy and momentum revert to the classical kinetic energy (plus a constant, $mc^2$) and the classical momentum $mv$ at low velocities. The conservation laws for these classical quantities follow from Newton's second and third laws of motion, respectively. In special relativity, we no longer take these laws as our axioms, only retaining Newton's first law of motion in inertial reference frame. We therefore cannot prove conservation of the energy-momentum four-vector mathematically, and must take it as an axiom. As I've just argued, this axiom is consistent with the laws of classical mechanics in the low-velocity limit. It is also consistent with experimental data - which, like Einstein's postulates and Newton's laws in classical mechanics, is the ultimate test of our physical model.