4.2: Conservation of Momentum

In Equation (4.1.1), what is the total force acting on all the particles? Well, that's the sum of all the forces the particles exert on each other, plus all external forces: $\boldsymbol{F}_{\text{total}}=\sum_i \boldsymbol{F}_{\text{int},i}+\sum_i \boldsymbol{F}_{\text{ext},i}$. Now Newton's third law of motion tells us that the internal forces come in opposite pairs, so when we sum them, they all cancel, and the total net force acting on the particles is equal to the sum of the external forces acting on the particles.Therefore, by Equation (4.1.1), *the center of mass of a system of particles obeys Newton's second law of motion.* What about the momentum of the center of mass? Like the force, the total momentum of the system of the system is given by the vector sum of the individual particle momenta:

$$\boldsymbol{P}_{\text{total}}=\sum_\alpha \boldsymbol{p}_\alpha=\sum_\alpha m_\alpha \dot{\boldsymbol{r}}_\alpha=\frac{\mathrm{d}}{\mathrm{d}t}\sum_\alpha m_\alpha \boldsymbol{r}_\alpha=\frac{\mathrm{d}}{\mathrm{d}t} M \boldsymbol{r}_{\text{cm}} \tag{4.2.1}$$

so the total momentum of the system equals that of the center of mass. Moreover, as long as the mass of the system is conserved, we can rewrite Equation (4.1.1) as

$$\boldsymbol{F}_{\text{total}}=\frac{\mathrm{d}\boldsymbol{P}_{\text{total}}}{\mathrm{d}t} \tag{4.2.2}$$

Not only does the center of mass of a system of particles obey Newton's second law of motion, its total momentum does too. Moreover, unlike in the single-particle case, Equation (4.2.2) has an important consequence for the case that there is no external force acting on the system. For one particle, that would simply mean that the momentum does not change - Newton's first law of motion. But for multiple particles, Equation (4.2.2) tells us that no external forces means that the total momentum does not change. We have therefore arrived at our second conservation law:

Theorem 4.1 (Law of conservation of momentum). *When no external forces act on a system of particles, the total momentum of the system is conserved.*

We derived the law of conservation of momentum by applying both Newton's second and third laws of motion, so like conservation of energy, it is not an independent result, but follows from our axioms. Note that the law allows for the momenta of the individual particles in the system to change, as long as their total stays the same - this is what happens when you play billiards, and why the number of balls bouncing in a Newton's cradle is fixed.
