

4.7: Totally Elastic Collisions

For a totally elastic collision, we can invoke both conservation of momentum and (by definition of a totally elastic collision) of kinetic energy. We also have an additional variable, as compared to the totally inelastic case, because in this case the objects do not stick together and thus get different end speeds. The two equations governing a totally elastic collision are:

$$m_1 v_{1,i} + m_2 v_{2,i} = m_1 v_{1,f} + m_2 v_{2,f} \quad (4.7.1)$$

for momentum conservation, and

$$\frac{1}{2} m_1 v_{1,i}^2 + \frac{1}{2} m_2 v_{2,i}^2 = \frac{1}{2} m_1 v_{1,f}^2 + \frac{1}{2} m_2 v_{2,f}^2 \quad (4.7.2)$$

for kinetic energy conservation.

When the collision occurs in one dimension, we can combine equations (4.7.1) and (4.7.2) to calculate the final velocities as functions of the initial ones. We first rewrite the two equations so that everything associated with particle 1 is on the left, and the terms for particle 2 are on the right:

$$m_1 (v_{1,i} - v_{1,f}) = m_2 (v_{2,f} - v_{2,i}) \quad (4.7.3)$$

and

$$m_1 (v_{1,i}^2 - v_{1,f}^2) = m_2 (v_{2,f}^2 - v_{2,i}^2) \quad (4.7.4)$$

We can expand the terms in parentheses in Equation (4.7.4), which gives:

$$m_1 (v_{1,i} - v_{1,f}) (v_{1,i} + v_{1,f}) = m_2 (v_{2,f} - v_{2,i}) (v_{2,f} + v_{2,i}) \quad (4.7.5)$$

Dividing Equation (4.7.5) by Equation (4.7.3), we get a relation between the velocities alone:

$$v_{1,i} + v_{1,f} = v_{2,i} + v_{2,f} \quad (4.7.6)$$

From Equation (4.7.6) we can isolate $v_{2,f}$ and substitute back in (4.7.3) to find $v_{1,f}$ in terms of the initial velocities:

$$v_{1,f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1,i} + 2 \frac{m_2}{m_1 + m_2} v_{2,i} \quad (4.7.7)$$

Naturally, we could just as well have calculated $v_{2,f}$, the equation for which is just (4.7.7) with the 1's and 2's swapped:

$$v_{2,f} = 2 \frac{m_1}{m_1 + m_2} v_{1,i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2,i} \quad (4.7.8)$$

We note that in the limit case that $m_2 \gg m_1$, v_2 is hardly affected, and $v_{1,f} \simeq -v_{1,i} + 2v_{2,f}$.

This page titled 4.7: Totally Elastic Collisions is shared under a CC BY-NC-SA 4.0 license and was authored, remixed, and/or curated by Timon Idema (TU Delft Open) via source content that was edited to the style and standards of the LibreTexts platform.