2.E: Forces (Exercises)

2.1 The terminal velocity is the maximum (constant) velocity a dropping object reaches. In this problem, we use Equation (2.2.6) for the drag force.

a. Use dimensional analysis to relate the terminal velocity of a falling object to the various relevant parameters.
b. Estimate the terminal velocity of a paraglider (Figure 2.3.1c).
c. Use the concept of terminal velocity to predict whether a mouse (without a parachute) is likely to survive a fall from a high tower.

2.2 When you cook rice, some of the dry grains always stick to the measuring cup. A common way to get them out is to turn the measuring cup upside-down and hit the bottom (now on top) with your hand so that the grains come off [32].

a. Explain why static friction is irrelevant here.
b. Explain why gravity is negligible.
c. Explain why hitting the cup works, and why its success depends on hitting the cup hard enough.

2.3 A ball is thrown at speed v from zero height on level ground. We want to find the angle θ at which it should be thrown so that the area under the trajectory is maximized.

a. Sketch of the trajectory of the ball.
b. Use dimensional analysis to relate the area to the initial speed v and the gravitational acceleration g.
c. Write down the x and y coordinates of the ball as a function of time.
d. Find the total time the ball is in the air.
e. The area under the trajectory is given by $A=\int y \mathrm{d} x$. Make a variable transformation to express this integral as an integration over time.
f. Evaluate the integral. Your answer should be a function of the initial speed v and angle θ.
g. From your answer at (f), find the angle that maximizes the area, and the value of that maximum area. Check that your answer is consistent with your answer at (b).

2.4 If a mass m is attached to a given spring, its period of oscillation is T. If two such springs are connected end to end, and the same mass m is attached, find the new period T', in terms of the old period T.

2.5 Two blocks, of mass m and 2m, are connected by a massless string and slide down an inclined plane at angle θ. The coefficient of kinetic friction between the lighter block and the plane is μ, and that between the heavier block and the plane is 2μ. The lighter block leads.

a. Find the magnitude of the acceleration of the blocks.
b. Find the tension in the taut string.

2.6 A 1000 kg boat is traveling at 100 $\frac{km}{h}$ when its engine is shut off. The magnitude F_d of the drag force between the boat and the water is proportional to the speed v of the boat, with a drag coefficient $\zeta=70 \frac{N \cdot s}{m}$. Find the time it takes the boat to slow to 45 $\frac{km}{h}$.

2.7 Two particles on a line are mutually attracted by a force F=-ar, where a is a constant and r the distance of separation. At time t=0, particle A of mass m is located at the origin, and particle B of mass $\frac{m}{4}$ is located at r=5.0 cm.

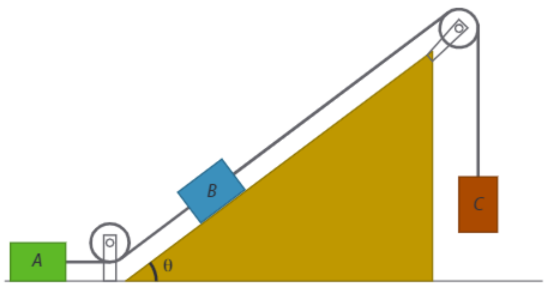
a. If the particles are at rest at t=0, at what value of r do they collide?
b. What is the relative velocity of the two particles at the moment the collision occurs?

2.8 In drag racing, specially designed cars maximize the friction with the road to achieve maximum acceleration. Consider a drag racer (or 'dragster') as shown in Figure 2.E.1, for which the center of mass is close to the rear wheels.

Figure $2.E.1$: A drag racer or dragster [8], CC BY-SA 3.0.

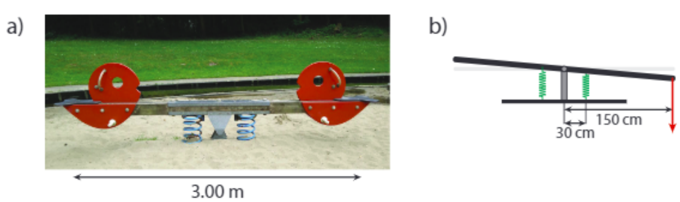
a. Draw a free-body diagram of the dragster in side view. Draw the wheels as circles, and approximate the shape of the dragster body as a triangle with a horizontal line between the wheels, a vertical line going up from the rear axis, and a diagonal line connecting the top to the front wheels. NB: consider carefully the direction of the friction force!
b. On which of the wheels is the frictional force the largest?
c. The frictional force is maximized if the wheels just don't slip (because, as usual, the coefficient of kinetic friction is smaller than that of static friction). Find the maximal possible frictional force on the rear wheels.
d. Find the maximal possible acceleration of the dragster.(e) For a coefficient of (static) friction of 1.0 (a fairly realistic value for rubber and concrete) and a track of 500 m, find the maximal velocity a drag racer can achieve at the end of the track when starting from rest.

2.9 Blocks A, B and C are placed as shown in the figure, and connected by ropes of negligible mass. Both A and B weigh 20.0 N each, and the coefficient of kinetic friction between each block and the surface is 0.3. The slope's angle θ equals 42.0°. The disks in the pulleys are of negligible mass. After the blocks are released, block C descends with constant velocity.



a. Find the tension in the rope connecting blocks A and B.
b. What is the weight of block C?
c. If the rope connecting blocks A and B were cut, what would be the acceleration of C?

2.10 The figure below shows a common present-day seesaw design. In addition to a beam with two seats, this seesaw also contains two identical springs that connect the beam to the ground. The distance between the pivot and each of the springs is 30.0 cm, the distance between the pivot and each of the seats is 1.50 m.



a. A 4-year-old weighing 20.0 kg sits on one of the seats, causing it to drop by 20.0 cm. Draw a free-body diagram of the seesaw with the child, in which you include all relevant forces (to scale).
b. Use your diagram and the provided data to calculate the spring constant of the two springs present in the seesaw.

2.11 Two marbles of identical mass m and radius r are dropped in a cylindrical container with radius 3r, as shown in the figure. Find the force exerted by the marbles on points A, B and C, and the force the marbles exert on each other.

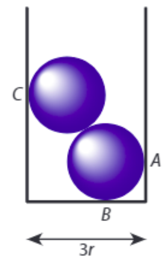


Figure 2.E.2. Suppose you have a crate with a square base that is exactly five oranges wide. You stack 25 oranges in the crate, then put another 16 on top in the holes, and then add a second layer of 25 oranges, held in place by the sides of the crate. Find the total force on the sides of the crate in this configuration. Assume all oranges are spheres with a diameter of 8.0 cm and a mass of 250 g.

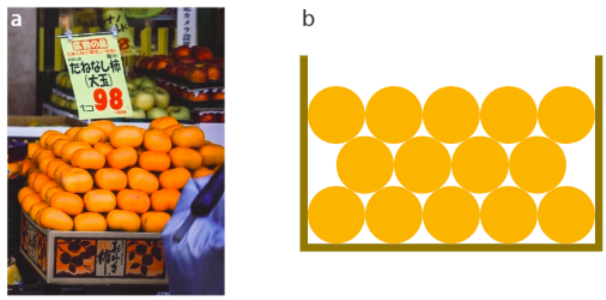


Figure $2.E.2$: Stacked fruit. (a) Stacked mandarins at a fruit stand [9]. (b) Cross-section of stacked oranges in a crate.

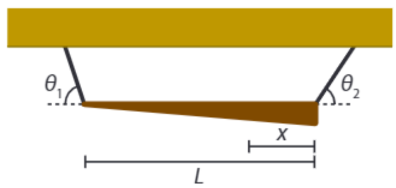
2.13 Objects with densities less than that of water float, and even objects that have higher densities are 'lighter' in the water. The force that's responsible for this is known as the buoyancy force, which is equal but opposite to the gravitational force on the displaced water: $F_{\text{buoyancy}} = \rho_w g V_w$, where ρ_w is the water's density and V_w the displaced volume. In parts (a) and (b), we consider a block of wood with density $\rho < \rho_w$ which is floating in water.

a. Which fraction of the block of wood is submerged when floating?
b. You push down the block somewhat more by hand, then let go. The block then oscillates on the surface of the water. Explain why, and calculate the frequency of the oscillation.
c. You take out the piece of wood, and now float a piece of ice in a bucket of water. On top of the ice, you place a small stone. When everything has stopped moving, you mark the water level. Then you wait till the ice has melted, and the stone has dropped to the bottom of the bucket. What has happened to the water level? Explain your answer (you can do so either qualitatively through an argument or quantitatively through a calculation).
d. Rubber ducks also float, but, despite the fact that they have a flat bottom, they usually do not stay upright in water. Explain why.
e. You drop a 5.0 kg ball with a radius of 10 cm and a drag coefficient c_d of 0.20 in water (viscosity $1.002 mPa \cdot s$). This ball has a density higher than that of water, so it sinks. After a while, it reaches a constant velocity, known as its terminal velocity. What is the value of this terminal velocity?
f. When the ball in (e) has reached terminal velocity, what is the value of its Reynolds number (see Problem 1.3)?

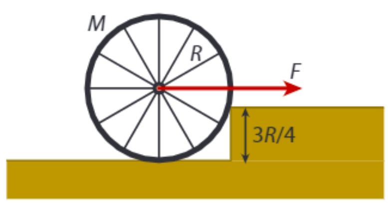
2.14 A uniform stick of mass M and length L=1.00 m has a weight of mass m hanging from one end. The stick and the weight hang in balance on a force scale at a point x=20.0 cm from the end of the stick. The measured force equals 3.00 N. Find both the mass M of the stick and m of the weight.

2.15 A uniform rod with a length of 4.25 m and a mass of 47.0 kg is attached to a wall with a hinge at one end. The rod is held in a horizontal position by a wire attached to its other end. The wire makes an angle of 30.0° with the horizontal and is bolted to the wall directly above the hinge. If the wire can support a maximum tension of 1250 N before breaking, how far from the wall can a 75.0 kg person sit without breaking the wire?

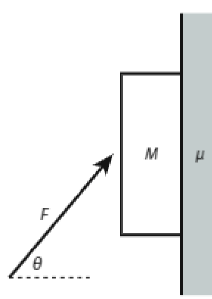
2.16 A wooden bar of uniform density but varying thickness hangs suspended on two strings of negligible mass. The strings make angles θ_1 and θ_2 with the horizontal, as shown. The bar has total mass m and length L. Find the distance x between the center of mass of the bar and its (thickest) end.



2.17 A bicycle wheel of radius R and mass M is at rest against a step of height $\frac{3R}{4}$, as shown in the figure. Find the minimum horizontal force F that must be applied to the axle to make the wheel start to rise over the step.



2.18 A block of mass M is pressed against a vertical wall, with a force F applied at an angle θ with respect to the horizontal ($-\frac{\pi}{2} < \theta < \frac{\pi}{2}$), as shown in the figure. The friction coefficient of the block and the wall is μ. We start with the case $\theta = 0$, i.e., the force is perpendicular to the wall.



a. Draw a free-body diagram showing all forces.
b. If the block is to remain stationary, the net force on it should be zero. Write down the equations for force balance (i.e., the sum of all forces is zero, or forces in one direction equal the forces in the opposite direction) for the x and y directions.
c. From the two equations you found in (b), solve for the force F needed to keep the book in place.
d. Now repeat the steps you took in (a)-(c) for a force under a given angle θ, and find the required force F.
e. For what angle θ is this minimum force F the smallest? What is the corresponding minimum value of F?

f. What is the limiting value of θ, below which it is not possible to keep the block up (independent of the magnitude of the force)?

2.19 A spherical stone of mass m=0.250 kg and radius R=5.0 cm is launched vertically from ground level with an initial speed of $v_0 = 15.0\frac{m}{s}$. As it moves upwards, it experiences drag from the air as approximated by Stokes drag, $F = 6\pi\eta Rv$, where the viscosity η of air is $1.002 mPa \cdot s$.

a. Which forces are acting on the stone while it moves upward?
b. Using Newton's second law of motion, write down an equation of motion for the stone (this is a differential equation). Be careful with the signs. Hint: Newton's second law of motion relates force and acceleration, and the drag force is in terms of the velocity. What is the relation between the two? Simplify the equation by introducing the characteristic time $\tau = \frac{m}{6\pi\eta R}$.
c. Find a particular solution $v_p(t)$ of your inhomogeneous differential equation from (19b).
d. Find the solution $v_h(t)$ of the homogeneous version of your differential equation.
e. Use the results from (19c) and (19d) and the initial condition to find the general solution v(t) of your differential equation.
f. From (19e), find the time at which the stone reaches its maximum height.
g. From v(t), find h(t) for the stone (height as a function of time).(h) Using your answers to (19f) and (19g), find the maximum height the stone reaches.
