2.2: Force Laws

Newton’s second law of motion tells us what a force does: it causes a change in momentum of any particle it acts upon. It does not tell us where the force comes from, nor does it care - which is a very useful feature, as it means that the law applies to all forces. However, we do of course need to know what to put down for the force, so we need some rule to determine it independently. This is where the force laws come in.

Springs: Hooke's Law

One very familiar example of a force is the spring force: you need to exert a force on something to compress it, and (in accordance with Newton’s third law), if you press on something you’ll feel it push back on you. The simplest possible object that you can compress is an ideal spring, for which the force necessary to compress it scales linearly with the compression itself. This relation is known as **Hooke’s law:**

$$\boldsymbol{F}_{s}=-k \boldsymbol{x} \tag{2.2.1}$$

where x is now the displacement (from rest) and k is the spring constant, measured in newtons per meter.The value of k depends on the spring in question - stiffer springs having higher spring constants.

Hooke’s law gives us another way to *measure* forces. We have already defined the unit of force using Newton’s second law of motion, and we can use that to calibrate a spring, i.e., determine its spring constant, by determining the displacement due to a known force. Once we have k, we can simply measure forces by measuring displacements - this is exactly what a spring scale does.

Robert Hooke

Robert Hooke (1635-1703) was a British all-round natural scientist and architect. He discovered the force law named after him in 1660, which he published first as an anagram: ‘ceiiinosssttuv’, so he could claim the discovery without actually revealing it (a fairly common practice at the time); he only provided the solution in 1678: ‘ut tensio, sic vis’ (‘as the extension, so the force’). Hooke made many contributions to the development of microscopes, using them to reveal the structure of plants, coining the word cell for their basic units. Hooke was the curator of experiments of England’s Royal Society for over 40 years, combining this position with a professorship in geometry and the job of surveyor of the city of London after the great fire of 1666. In the latter position he got a strong reputation for a hard work and great honesty. At the same time, he was frequently at odds with his contemporaries Isaac Newton and Christiaan Huygens; it is not unlikely that they independently developed similar notions on, among others on the inverse-square law of gravity.

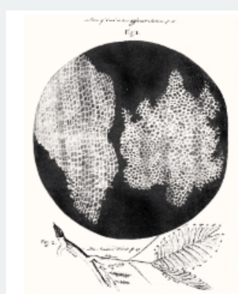

Figure 2.2.1: Drawing of the cell structure of cork by Hooke, from his 1665 book Micrographia [3]. No portraits of Hooke survive.

Gravity: Newton's Law of Gravity

A second and probably even more familiar example is force due to gravity, at the local scale, i.e., around you, in the approximation that the Earth is flat. Anything that has mass attracts everything else that has mass, and since the Earth is very massive, it attracts all objects in the space around you, including yourself. Since the force of gravity is weak, you won’t feel the pull of your book, but since the Earth is so massive, you do feel its pull. Therefore if you let go of something, it will be accelerated towards the Earth due to its attracting gravitational force. As demonstrated by Galilei (and some guys in spacesuits on a rock we call the moon[2]), the acceleration of any object due to the force of gravity is the same, and thus the force exerted by the Earth on any object equals the mass of that object times this acceleration, which we call g:

$$\boldsymbol{F}_{g}=m \boldsymbol{g} \tag{2.2.2}$$

Because the Earth's mass is not exactly uniformly distributed, the magnitude of g varies slightly from place to place, but to good approximation equals $9.81\,\frac{m}{s^2}$. It always points down.

Although Equation 2.2.2 for local gravity is handy, its range of application is limited to everyday objects at everyday altitudes - say up to a couple thousand kilograms and a couple kilometers above the surface of the Earth, which is tiny compared to Earth's mass and radius. For larger distances and bodies with larger mass- say the Earth-Moon, or Earth-Sun systems - we need something else, namelyNewton's law of gravitation between two bodies with masses m_1 and m_2 and a distance r apart:

$$\boldsymbol{F}_{\mathrm{G}}=-G\frac{m_1 m_2}{r^2}\hat{\boldsymbol{r}} \tag{2.2.3}$$

where $\hat{r}$ is the unit vector pointing along the line connecting the two masses, and the proportionality constant $G=6.67\cdot 10^{-11}\,\frac{N\cdot m^2}{kg^2}$ is known as the gravitational constant (or Newton's constant). The minus sign indicates that the force is attractive. Equation 2.2.3 allows you to actually calculate the gravitational pull that your book exerts on you, and understand why you don't feel it. It also lets you calculate the value of g - simply fill in the mass and radius of the Earth. If you wish to know the value of g on any other celestial body, you can put in its particulars, and compare with Earth. You'll find you'd 'weigh' 3 times less on Mars and 6 times less on the Moon. Most of the time we can safely assume the Earth is flat and use Equation 2.2.2, but in particular for celestial mechanics and when considering satellites we'll need to use Equation 2.2.3.

Galileo galilei

Galileo Galilei (1564-1642) was an Italian physicist and astronomer, who is widely regarded as one of the founding figures of modern science. Unlike classical philosophers, Galilei championed the use of experiments and observations to validate (or disprove) scientific theories, a practice that is the cornerstone of the scientific method. He pioneered the use of the telescope (newly invented at the time) for astronomical observations, leading to his discovery of mountains on the moon an the four largest moons of Jupiter (now known as the Galilean moons in his honor). On the theoretical side, Galilei argued that Aristotle's argument that heavy objects fall faster than light ones is incorrect, and that the acceleration due to gravity is equal for all objects (Equation 2.2.2). Galilei also strongly advocated the heliocentric worldview introduced by Copernicus in 1543, as opposed to the widely-held geocentric view. Unfortunately, the Inquisition thought otherwise, leading to his conviction for heresy with a sentence of life-long house arrest in 1633, a position that was only recanted by the church in 1995.

Figure 2.2.2: Portrait of Galileo Galilei by Justus Sustermans (1636) [4].

Electrostatics: Coulomb's Law

Like two masses interact due to the gravitational force, two charged objects interact via Coulomb's force. Because charge has two possible signs, Coulomb's force can both be attractive (between opposite charges) and repulsive (between identical charges). Its mathematical form strongly resembles that of Newton's law of gravity:

$$\boldsymbol{F}_{\mathrm{C}}=k_e\frac{q_1 q_2}{r^2}\hat{\boldsymbol{r}} \tag{2.2.4}$$

where q_1 and q_2 are the signed magnitudes of the charges, r is again the distance between them, and $k_e=8.99\cdot 10^{9}\,\frac{N\cdot m^2}{C^2}$ is Coulomb's constant. For everyday length and force scales, Coulomb's force is much larger than the force of gravity.

Charles-augustin de Coulomb

Charles-Augustin de Coulomb (1736-1806) was a French physicist and military engineer. For most of his working life, Coulomb served in the French army, for which he supervised many construction projects. As part of this job, Coulomb did research, first in mechanics (leading to his law of kinetic friction, Equation 2.2.7), and later in electricity and magnetism, for which he discovered that the force between charges (and those between magnetic poles) drops off quadratically with their distance (Equation 2.2.4). Near the end of his life, Coulomb participated in setting up the SI system of units.

Figure 2.2.3: Portrait of Charles de Coulomb [5].

Friction and Drag

Why did it take the genius of Galilei and Newton to uncover Newton's first law of motion? Because everyday experience seems to contradict it: if you don't exert a force, you won't keep moving, but gradually slow down. You know of course why this is: there's drag and friction acting on a moving body, which is why it's much easier (though not necessarily handier) for a car to keep moving on ice than on a regular tarmac road (less friction on ice), and why walking through water is so much harder than walking through air (more drag in water). The medium in which you move can exert a drag force on you, and the surface over which you move exerts friction forces. These of course are the forces responsible for slowing you down when you stop exerting force yourself, so the first law doesn't apply, as there are forces acting.

For low speeds, the drag force typically scales linearly with the velocity of the moving object. Drag forces for objects moving through a (fluid) medium moreover depend on the properties of the medium (its viscosity η) and the cross-sectional area of the moving object. For a sphere of radius R moving at velocity v, the drag force is given by **Stokes' law**:

$$\boldsymbol{F}_{\mathrm{d}}=-6 \pi \eta R \boldsymbol{v} \tag{2.2.5}$$

The more general version for an object of arbitrary shape is $F_d=\zeta v$, where ζ is a proportionality constant. Stokes' law breaks down at high velocities, for which the drag force scales quadratically with the speed:

$$F_{\mathrm{d}}=\frac{1}{2} \rho c_{\mathrm{d}} A v^{2} \tag{2.2.6}$$

where ρ is the density of the fluid, A the cross-sectional area of the object, v its speed, and c_{d} its dimensionless drag coefficient, which depends on the object's shape and surface properties. Typical values for the drag coefficient are 1.0 for a cyclist, 1.2 for a running person, 0.48 for a Volkswagen Beetle, and 0.19 for a modern aerodynamic car. The direction of the drag force is still opposite that of the motion.

Frictional forces are due to two surfaces sliding past each other. It should come as no surprise that the direction of the frictional force is opposite that of the motion, and its magnitude depends on the properties of the surfaces. Moreover, the magnitude of the frictional force also depends on how strongly the two surfaces are pushed against each other - i.e., on the forces they exert on each other, perpendicular to the surface. These forces are of course equal (by Newton's third law) and are called **normal** forces, because they are normal (that is, perpendicular) to the surface. If you stand on a box, gravity exerts a force on you pulling you down, which you 'transfer' to a force you exert on the top of the box, and causes an equal but opposite normal force exerted by the top of the box on your feet. If the box is tilted, the normal force is still perpendicular to the surface (it remains normal), but is no longer equal in magnitude to the force exerted on you by gravity. Instead, it will be equal to the component of the gravitational force along the direction perpendicular to the surface (see figure 2.6). We denote normal forces as F_n. Now according to the **Coulomb friction law** (not to be confused with the Coulomb force between two charged particles), the magnitude of the frictional force between two surfaces satisfies

$$F_f \leq \mu F_n \tag{2.2.7}$$

Here μ is the coefficient of friction, which of course depends on the two surfaces, but also on the question whether the two surfaces are moving with respect to each other or not. If they are not moving, i.e., the con-figuration is static, the appropriate coefficient is called the *coefficient of static friction* and denoted by μ_s.The actual magnitude of the friction force will be such that it balances the other forces (more on that in section 2.4). Equation 2.2.7 tells us that this is only possible if the required magnitude of the friction force is less than $\mu_s F_n$. When things start moving, the static friction coefficient is replaced by the *coefficient of kinetic friction* μ_k, which is usually smaller than μ_s; also in that case the inequality in Equation 2.2.7 gets replaced by an equals sign, and we have

$$F_f = \mu_k F_n. \tag{2.2.8}$$

[2] To be precise, astronaut David Scott of the Apollo 15 mission in 1971, who dropped both a hammer and a feather and saw them fall at exactly the same rate, as shown in this NASA movie.