14.2: Photons

Before we dive into the examples, there's one particle that requires special attention: the photon, or quantum of light - from that other early-20th-century theory known as quantum mechanics. Photons travel (by definition) at the speed of light, and need therefore be massless. They do carry energy though, which is related to their frequency f (or wavelength λ, or color) through

$$E_{\text{photon}} = hf = \frac{hc}{\lambda} \tag{14.2.1}$$

where h is Planck's constant. Since photons have nonzero energy, they also have nonzero momentum through Einstein's equation (13.16), despite the fact that they have no mass[1]

$$p_{\text{photon}} = \frac{E_{\text{photon}}}{c} = \frac{hf}{c} = \frac{h}{\lambda} \tag{14.2.2}$$

A photon with frequency f (and thus energy $E = hf$) traveling in the positive x direction has a very simple energy-momentum four-vector:

$$\bar{\boldsymbol{p}}_{\text{photon}} = (E/c, E/c, 0, 0) \tag{14.2.3}$$

The length of this four-vector, unsurprisingly, is zero.

[1]Note that relativistic momentum is given by $p = \gamma(v)mv$; substituting $v = c$ gives $\gamma(c) = \infty$, and so this expression gives us the product of infinity with zero for the momentum of the photon - that could be anything and thus is meaningless. The way to calculate the momentum of the photon is through Equation 14.2.2. Although the photon momentum is small, it is large enough to be measured and even useful in devices known as optical tweezers, where focused laser beams are used to move micron-sized objects around.