5.3: Torque

Anyone who has ever used a lever - that is everyone, presumably - knows how useful they are at augmenting force: you push with a small force at the long end, to produce a large force at the short end, and make the crank turn, stone lift or bottle cap pop off. If the force is at straight angles with the lever arm (the line connecting the point at which you exert the force to the pivot around which your lever rotates), the effect is largest. In that case we define the torque (or moment of force) τ as the product of the force and the lever arm length. As only the perpendicular component of the force counts (you'll simply push or pull on your lever with a parallel component, not make it turn), in a vector setting you need to project on that perpendicular component, so if r (from the pivot to the point at which the force acts) makes an angle θ with the force vector F, the magnitude of the torque becomes $\tau = rF\sin\theta$. That's exactly the magnitude of the cross product of r and F, which also has directional information - useful as a torque can be clockwise or counterclockwise. In general we'll therefore define the torque by:

$$\boldsymbol{\tau} = \boldsymbol{r} \times \boldsymbol{F} \tag{5.3.1}$$

which makes a counterclockwise torque positive, in correspondence with the definition of the rotation vector $\boldsymbol{\omega}$.