9.1: Sinusoidal Waves

Probably the simplest kind of wave is a transverse sinusoidal wave in a one-dimensional string. In such a wave each point of the string undergoes a harmonic oscillation. We will call the displacement from equilibrium u, then we can plot u as a function of position on the string at a given point in time, Figure 9.2.1a, which is a snapshot of the wave. Alternatively, we can plot u as a function of time for a given point (with given position)on the string, Figure 9.2.1b. Because the oscillation is harmonic, the displacement as a function of time is a sine function, with an amplitude (maximum displacement)A and a period (time between maxima) T.

By definition, each point of the string undergoing a sinusoidal wave undergoes a harmonic oscillation,so for each point we can write $u(t)=A\cos(\omega t+\phi)$ (Equation 8.1.4) where as before $\omega=2\pi/T$ is the (angular) frequency and ϕ the phase. Two neighboring points on the string are slightly out of phase - if the wave is traveling to the right, then your right-hand neighbor will reach maximum slightly later than you, and thus has a slightly larger phase. The difference in phase is directly proportional to the distance between two points, coming to a full 2π (which is of course equivalent to zero) after a distance λ, the *wavelength*. The wavelength this thus the distance between two successive points with the same phase, in particular between two maxima. In between these maxima, the phase runs over the full 2π, so the wave is also a sinusoid in space, with a 'spatial frequency' or *wavenumber* $k=2\pi/\lambda$. Combining the dependencies on space and time in a single expression, we can write for the sinusoidal wave:

$$u(x,t)=A\cos(kx-\omega t) \tag{9.1.1}$$

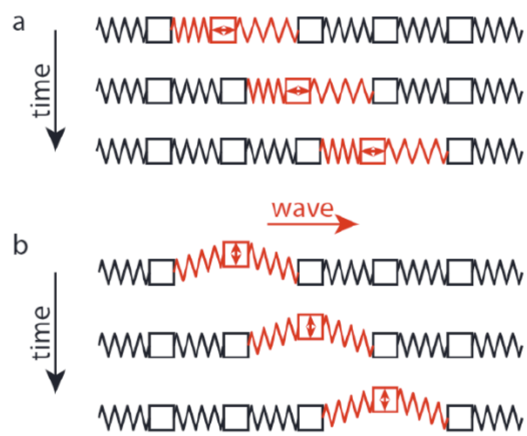

Figure 9.1.1: Two basic types of waves. (a) Longitudinal wave, where the oscillatory motion of the particles is in the same direction as that of the wave. (b) Transverse wave, where the oscillatory motion of the particles is perpendicular to that of the wave.

The speed of the wave is the distance the wave travels per unit time. A unit time for a wave is one period, as that is the time it takes the oscillation to return to its original point. The distance traveled in one period is one wavelength, as that is the distance between two maxima. The speed is therefore simply their ratio, which can also be expressed in terms of the wave number and frequency:

$$v_{\mathrm{w}}=\frac{\lambda}{T}=\frac{\omega}{k}. \tag{9.1.2}$$