3.2: Kinetic Energy

Newton's first law told us that a moving object will stay moving unless a force is acting on it - which holds for moving with any speed, including zero. Now if you want to start moving something that is initially at rest, you'll need to accelerate it, and Newton's second law tells you that this requires a force - and moving something means that you're displacing it. Therefore, there is work involved in getting something moving. We define the kinetic energy (K) of a moving object to be equal to the work required to bring the object from rest to that speed, or equivalently, from that speed to rest:

$$K=\frac{1}{2}mv^2 \tag{3.2.1}$$

Because the kinetic energy is equal to an amount of work, it is also a scalar quantity, has the same dimension, and is measured in the same unit. The factor v^2 is the square of the magnitude of the velocity of the moving object, which you can calculate with the dot product: $v^2=v\cdot v$. You may wonder where Equation (3.2.1) comes from. Newton's second law tells us that $F=m\frac{dv}{dt}$, relating the force to an infinitesimal change in the velocity. In the definition for work, Equation (3.1.3), we multiply the force with an infinitesimal change in the position dr. That infinitesimal displacement takes an infinitesimal amount of time dt, which is related to the displacement by the instantaneous velocity v: $dr=vdt$. We can now calculate the work necessary to accelerate from zero to a finite speed:

$$K=\int F\cdot \mathrm{d}r=\int m\frac{\mathrm{d}v}{\mathrm{d}t}\cdot v\mathrm{d}t=\int mv\cdot\frac{\mathrm{d}v}{\mathrm{d}t}\mathrm{d}t=\int mv\cdot\mathrm{d}v=\frac{m}{2}\int \mathrm{d}(v\cdot v)=\frac{1}{2}mv^2 \tag{3.2.2}$$

where we used that the dot product is commutative and the fact that the integral over the derivative of a function is the function itself.

Of course, now that we know that the kinetic energy is given by Equation (3.2.1), we no longer need to use a complicated integral to calculate it. However, because the kinetic energy is ultimately given by this integral,which is equal to a net amount of work, we arrive at the following statement, sometimes referred to as the **Work-energy theorem**: the change in kinetic energy of a system equals the net amount of work done on or by it (in case of increase/decrease of K):

$$\Delta K=W_{\mathrm{net}} \tag{3.2.3}$$

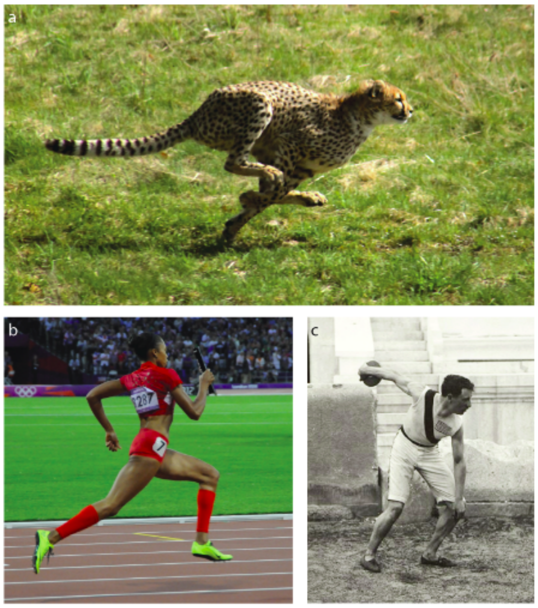

Figure 3.2.1: *Examples of high power resulting in high kinetic energy. (a) Running cheetah, the fastest land animal, which can reach speeds over 100 km/h in 2-3 seconds, corresponding to an enormous increase in its kinetic energy [10], CC BY-SA 3.0. (b) Allyson Felix running second in the women's 4×400 relay of the 2012 London Summer Olympics [11], CC BY-SA 3.0. (c) Robert Garrett preparing to throw the discus at the 1896 Athens Summer Olympics [12]. Unlike the runners, the goal of discus throwing is to maximize the distance, not the speed, but to get the largest possible distance, the discus must still get the maximal possible kinetic energy.*