# 3.4: Conservation of Energy

Work, kinetic energy and potential energy are all quantities with the same dimension - so we can do arithmetic with them. One particularly useful quantity is the total energy E of a system, which is simply the sum of the kinetic and potential energy:

$$E=K+U \tag{3.4.1}$$

**Theorem 3.4.1: Law of Conservation of Energy**

If all forces in a system are conservative, the total energy in that system is conserved.

*Proof*. For simplicity, we'll look at the 1D case (3D goes analogously). Conserved means not changing in time, so in order to prove the statement, we only need to calculate the time derivative of E and check that it is always zero.

$$\begin{aligned}
\frac{\mathrm{d}E}{\mathrm{d}t} &= \frac{\mathrm{d}K}{\mathrm{d}t}+\frac{\mathrm{d}U}{\mathrm{d}t} \\
&= \frac{\mathrm{d}\left(\frac{1}{2}mv^2\right)}{\mathrm{d}t}+\frac{\mathrm{d}U}{\mathrm{d}x}\frac{\mathrm{d}x}{\mathrm{d}t} \\
&= mv\frac{\mathrm{d}v}{\mathrm{d}t}-Fv \\
&= -\left(F-m\frac{\mathrm{d}v}{\mathrm{d}t}\right)v \\
&= 0 \text{ by Newton's second law.}
\end{aligned}$$

Conservation of energy means that the total energy of a system cannot change, but of course the potential and kinetic energy can - and by conservation of total energy we know that they get converted directly into one another. Exploiting this fact will allow us to analyze and easily solve many problems in classical mechanics - this conservation law is an immensely useful tool.

Note that conservation of energy is not the same as the work-energy theorem of Section 3.2. For the total energy to be conserved, all forces need to be conservative. In the work-energy theorem, this is not the case.You can therefore calculate changes in kinetic energy due to the work done by non-conservative forces using the latter.