5.7: Conservation of Angular Momentum

Given that the torque is the rotational analog of the force, and the angular momentum is that of the linear momentum, it will not come as a surprise that Newton's second law of motion has a rotational counterpart that relates the net torque to the time derivative of the angular momentum. To see that this is true, we simply calculate that time derivative:

$$\frac{\mathrm{d}\boldsymbol{L}}{\mathrm{d}t}=\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t}\times\boldsymbol{p}+\boldsymbol{r}\times\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t}=\boldsymbol{\tau}\tag{5.7.1}$$

because $\dot{\boldsymbol{r}}\times\boldsymbol{p}=\boldsymbol{v}\times m\boldsymbol{v}=0$. Some texts even use Equation 5.7.1 as the definition of torque and work from there. Note that in the case that there is no external torque, we arrive at another conservation law:

Theorem 5.3: Law of conservation of angular momentum

When no external torques act on a rotating object, its angular momentum is conserved.

Conservation of angular momentum is why a rolling hoop keeps rolling, and why a balancing a bicycle is relatively easy once you go fast enough.

What about collections of particles? Here things are a little more subtle. Writing $\boldsymbol{L}=\Sigma_i\boldsymbol{L}_i$ and again taking the derivative, we arrive at

$$\frac{\mathrm{d}\boldsymbol{L}}{\mathrm{d}t}=\sum_i\boldsymbol{r}_i\times\boldsymbol{F}_i=\sum_i\tau_i\tag{5.7.2}$$

Now the sum on the right hand side of 5.7.2 includes both external torques exerted on the system, and internal torques exerted by the particles on each other. When we discussed conservation of linear momentum, the internal momenta all canceled pairwise because of Newton's third law of motion. For torques this is not necessarily true, and we need the additional condition that the internal forces between two particles act along the line connecting those particles - then the internal torques are zero, and Equation 5.7.1 holds for the collection as well. Consequently, if the net external torque is zero, angular momentum is again conserved.