2.1: Newton's Laws of Motion

As described in Chapter 1, classical mechanics is based on a set of axioms, which in turn are based on (repeated) physical observations. In order to formulate the first three axioms, we will need to first define three quantities: the (instantaneous) velocity, acceleration and momentum of a particle. If we denote the position of a particle as x(t) - indicating a vector[1] quantity with the dimension of length that depends on time, we define its velocity as the time derivative of the position:

$$v(t)=\dot{x}(t)=\frac{dx(t)}{dt} \tag{2.1.1}$$

Note that we use an overdot to indicate a *time* derivative, we will use this convention throughout these notes. The acceleration is the time derivative of the velocity, and thus the second derivative of the position:

$$a(t)=\ddot{x}(t)=\frac{dv(t)}{dt}=\frac{d^2x(t)}{dt^2} \tag{2.1.2}$$

Finally the momentum of a particle is its mass times its velocity:

$$p(t)=mv(t)=m\dot{x}(t) \tag{2.1.3}$$

We are now ready to give our next three axioms. You may have encountered them before; they are known as Newton's three laws of motion.

Axiom 1 (Newton's first law of motion). As long as there is no external action, a particle's velocity will remain constant. Note that the first law includes particles at rest, i.e., $v=0$. We will define the general 'external action' as a force, therefore a force is now anything that can change the velocity of a particle. The second law quantifies the force.

Axiom 2 (Newton's second law of motion). If there is a net force acting on a particle, then its instantaneous change in momentum due to that force is equal to that force:

$$F(t)=\frac{dp(t)}{dt} \tag{2.1.4}$$

Now since $p=mv$ and $a=\frac{dv}{dt}$, if the mass is constant we can also write Equation 2.1.4 as $F=ma$, or

$$F(t)=m\ddot{x}(t) \tag{2.1.5}$$

which is the form in we will use most. Based on the second law, we see that a force has the physical dimension of a mass times a length divided by a time squared - since this is quite a lot to put in every time, we define the dimension of force as such: $F=MLT^{-2}$. Likewise, we define a unit, the Newton (N), as a kilogram times a meter per second squared: $N=\frac{kg\cdot m}{s^2}$. Therefore, in principle Newton's second law of motion can also be used to measure forces, though we will often use it the other way around, and calculate changes in momentum due to a known force.

Note how Newton's first law follows from the second: if the force is zero, there is no change in momentum, and thus (assuming constant mass) a constant velocity. Note also that although the second law gives us a quantification of the force, by itself it will not help us achieve much, as we at present have no idea what the force is (though you probably have some intuitive ideas from experience) - for that we will use the force laws of the next section. Before we go there, there is another important observation on the nature of forces in general.

Axiom 3 (Newton's third law of motion). If a body exerts a force F_1 on a second body, the second body exerts an equal but opposite force F_2, on the first, i.e., the forces are equal in magnitude but opposite in direction:

$$F_1=-F_2 \tag{2.1.6}$$

Isaac Newton

Isaac Newton (1642-1727) was a British physicist, astronomer and mathematician, who is widely regarded as one of the most important scientists in history. Newton was a professor at Cambridge from 1667 till 1702, where he held the famous Lucasian chair in mathematics. Newton invented infinitesimal calculus to be able to express the laws of mechanics that now bear his name in mathematical form. He also gave a mathematical description of gravity (Equation 2.2.3), from which he could derive Kepler's laws of planetary motion (Section 6.4). In addition to his work on mechanics, Newton made key contributions to optics and invented the reflection telescope, which uses a mirror rather than a lens to gather light. Having retired from his position in Cambridge, Newton spend most of the second half of his life in London, as warden and later master of the Royal mint, and president of the Royal society.

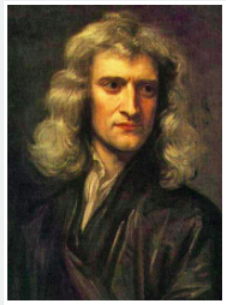

Figure 2.1.1: Portrait of Isaac Newton by Godfrey Kneller (1689) [2].

[1] Appendix A.1 lists some basic properties of vectors that you may find useful.
