1.1: Dimensions and Units

In physics in general, we are interested in relating different physical quantities to one another - we want to answer questions like 'how much work do I need to do to get this box up to the third floor'? In order to be able to give an answer, we need certain measurable quantities as input - in the present case, the mass of the box and the height of a floor. Then, using our laws of physics, we will be able to produce another measurable quantity as our answer - here the amount of work needed. Of course, you could check this answer, and thus validate our physical model of reality, by measuring the quantity in question.

Measurable (or 'physical', or 'observational') quantities aren't just numbers - the fact that they correspond to something physical matters, and 10 seconds is something very different from 10 meters, or 10 kilograms. The term we use to express this is, rather unfortunately, to say that physical quantities have a dimension - not to be confused with length, height and width. Anything that has a dimension can be measured, and to do so we use units - though there may be different units in which we measure the same quantity, such as centimeters and inches for length. When measuring the same quantity in different units, you can always convert between them - there are 2.54 centimeters in an inch - but it's meaningless to try to convert centimeters into seconds, because length and time are different quantities - they have different dimensions.

Table 1.1.1: Overview of the SI quantities and units, and the physical constants they are (or are proposed to be) based on.

quantity	symbol	unit	symbol	based on
length	L	meter	m	speed of light
time	T	second	s	caesium atom oscillation
mass	M	kilogram	kg	Planck's constant[1]
current	I	Ampere	A	electron charge
temperature	T	Kelvin	K	Boltzmann's constant
luminosity	J	candela	cd	monochromatic radiation
particle count	N	mole	mol	Avogadro's constant

We will encounter only three different basic quantities, which have the dimensions of length (L), time (T), and mass (M). Thanks to the Napoleonic conquest of Europe in the early 1800s, we have a basic unit for each of these: meters (m) for length, seconds (s) for time, and kilograms (kg) for mass. Although we won't encounter them here, the standard system of units (called the Système International, or SI) has four more of these basic pairs: (electric) current I, measured in Ampères (A), temperature T, measured in Kelvin (K), luminosity J, measured in candelas (cd), and 'amount of stuff', measured in moles (mol), see Table 1.1.1. Unfortunately, although this system is commonly used in (continental) Europe and in many other parts of the world, it is not everywhere, notably in the US, where people persist in using such things as inches and pounds, so you'll often have to convert between units.

From the seven basic quantities in the SI, all others can be derived. For example, speed is defined as the distance traveled (length) divided by the time it took, so speed has the dimension of L/T and is measured in units of m/s. Note that in order to be able to compare two quantities, they must have the same dimension. This simple observation has an important consequence: in any physics equation, the dimensions on both sides of the equality sign always have to be the same. There's no bargaining on this point: equating two quantities with different dimensions does not make any kind of sense, so if you find that that's what you're doing at any point, backtrack and find where things went wrong.

[1] At the time of writing, the unit of mass is still determined using a prototype in Paris, however, a redefined unit based on the value of Planck's constant is expected to be adopted on May 20, 2019.