6.1: Projectile Motion

The simplest case of two-dimensional motion occurs when a particle experiences a force only in one direction. The prime example of this case is the motion of a projectile in Earth's (or any other planet's) gravitational field as locally described by Galilean gravity (Equation 2.2.2): $\boldsymbol{F}=m\boldsymbol{g}$. Once a projectile has been fired with a certain initial velocity $\boldsymbol{v}_0$, we can find its trajectory by solving the equation of motion that follows from Newton's second law: $m\boldsymbol{g}=m\ddot{\boldsymbol{r}}$. We can decompose $\boldsymbol{r}$ and $\boldsymbol{v}_0$ in horizontal (x) and vertical (z) components; each of them has its own one-dimensional equation of motion, which we already solved in Section 2.3. The horizontal component experiences no force and thus executes a simple linear motion with uniform velocity $v_0\cos\theta_0$, where $\theta_0=\arccos(\boldsymbol{v}_0\cdot\hat{\boldsymbol{x}})/v_0$ is the angle with the horizontal under which the projectile was fired and $v_0=|\boldsymbol{v}_0|$ the initial speed. Likewise, because the acceleration due to gravitation is constant, our projectile will execute a uniformly accelerated motion in the vertical direction with initial velocity $v_0\sin\theta_0$. If the projectile's initial position is (x_0, z_0), its motion is thus described by:

$$\boldsymbol{r}(t)=\begin{pmatrix} x(t) \\ z(t) \end{pmatrix}=\begin{pmatrix} x_0 \\ z_0 \end{pmatrix}+v_0\begin{pmatrix} \cos\theta_0 \\ \sin\theta_0 \end{pmatrix}t-\begin{pmatrix} 0 \\ g \end{pmatrix}\frac{1}{2}t^2 \tag{6.1.1}$$