13.E: Position, Energy and Momentum in Special Relativity (Exercises)

13.1 In high-energy physics, it is customary to express the mass of elementary particles not in kilograms but in MeV/c^2, expressing the fact that (rest) mass is a form of energy. An MeV or mega-electron-Volt is one million (the 'mega') times the (kinetic) energy an electron gains when it moves through an electric field between two positions with an electric potential difference of 1 volt, or 1 joule per coulomb. One electron-volt thus corresponds to an amount of energy (in joules) equal in number to the charge of the electron in coulombs. Express the mass of both the electron and the proton in MeV/c^2; you may find the numbers in Table B.1 useful.

13.2 For an arbitrary particle of (rest) mass m, find the speed at which its kinetic energy equals its rest energy.

13.3 We constructed four vectors in such a way that their length is invariant under Lorentz transformations. The length of a four-vector is defined as the square root of its dot product with itself: $|\overline{\boldsymbol{x}}|=\sqrt{\overline{\boldsymbol{x}} \cdot \overline{\boldsymbol{x}}}=x_0^2-x_1^2-x_2^2-x_3^2$. In Equation 13.1.2 we also defined the dot product of two arbitrary four-vectors $\overline{\boldsymbol{a}}$ and $\overline{\boldsymbol{b}}$.

a. Show that the sum of two four-vectors is again a four-vector (i.e., show that the length is invariant under Lorentz transformations, and the components transform the same way that those of the position four-vector do).
b. Calculate the square of the length of the four-vectors $\overline{\boldsymbol{a}}+\overline{\boldsymbol{b}}$ and $\overline{\boldsymbol{a}}-\overline{\boldsymbol{b}}$.
c. Use your answer at (b) to write the dot product of $\overline{\boldsymbol{a}}$ and $\overline{\boldsymbol{b}}$ as a linear combination of quantities that are invariant under Lorentz transformations (thus showing that the dot product is also invariant).

13.4 A particle with mass m has three-momentum $\boldsymbol{p}$ as measured in an inertial lab frame S. Find the particle's energy as measured by an observer with three-velocity $\boldsymbol{u}$. Hint: Determine the four-vectors of the particle's momentum and the observer's motion both in the lab frame S and the observer's rest frame S', then use the fact that inner products of four-vectors are invariant under Lorentz transformations.