9.E: Waves (Exercises)

9.1 **Sound waves in a spring.** In Section 9.2, we found that the speed of a wave in a string is given by $v = T/\mu$, with T the tension in the string and μ its mass density (Equation 9.2.7).

a. A spring of mass m and spring constant k has an unstretched length L_0. Find an expression for the speed of transverse waves on this spring when it's been stretched to a length L.
b. You measure the speed of transverse waves in an ideal spring under stretch. You find that at a certain length L_1 it has a value v, and at length $2L_1$ the wavespeed has value $3v$. Find an expression for the unstretched length of the spring in terms of L_1.
c. A uniform cable hangs vertically under its own weight. Show that the speed of waves on the cable is given by $v = \sqrt{zg}$, where z is the distance from the bottom of the cable. You may assume that the stretching of the cable is small enough that its mass density can be taken to be uniform.
d. Show that the time it takes a wave to propagate up the cable in (1c) is $t = 2\sqrt{\frac{L}{g}}$, with L the cable length.

9.2 In deep water, the speed of surface waves depends on their wavelength:

$$v = \sqrt{\frac{\lambda g}{2\pi}} \tag{9.E.1}$$

a. Apart from satellite images, offshore storms can also be detected by watching the waves at the beach. Equation 9.E.1 tells us that the longest-wavelength waves will travel the fastest, so the arrival of such waves, if their amplitude is high, is a foreboding of the possible arrival of a storm (the friction between the wind and the water being the source of the waves). A typical storm may be thus detected from a distance of 500 km, and travel at 50 km/h. Suppose the detected waves have crests 200 m apart. Estimate the time interval between the detection of these waves and the arrival of the storm (in the case the storm moves straight towards the beach).
b. In shallow water, the speed of surface waves becomes (to first order) independent of the wavelength, but scales with the depth of the water instead

$$v = \sqrt{gd} \tag{9.E.2}$$

. Next to storms, a possible source of surface waves in the ocean are underwater earthquakes. While storms are typically more dangerous at sea, the waves generated by earthquakes are more dangerous on land, as they may result in tsunamis: huge wavecrests that carry a lot of energy. At open sea, the amplitude of the waves that will create the tsunami may be modest, on the order of 1 m. What will happen with this wave's speed, amplitude, and wavelength when it approaches the land?

9.3 Because the wave equation is linear, any linear combination of solutions is again a solution; this is known as the principle of superposition, see Section 9.4. We will consider several examples of superposition in this problem. First, consider the two one-dimensional sinusoidal traveling waves $u_{\pm}(x,t) = A\sin(kx \pm \omega t)$

a. Which wave is traveling in which direction?
b. Find an expression for the combined wave, $u(x,t) = u_{+}(x,t) + u_{-}(x,t)$. You may use that $\sin(\alpha) + \sin(\beta) = 2\sin((\alpha+\beta)/2)\cos((\alpha-\beta)/2)$.
c. The combined wave is a standing wave - how can you tell?
d. Find the positions at which $u(x,t) = 0$ for all t. These are known as the nodes of the standing wave.
e. Find the positions at which $u(x,t)$ reaches its maximum value. These are known as the antinodes of the standing wave.

Next, consider two sinusoidal waves which have the same angular frequency ω, wave number k, and amplitude A, but they differ in phase:

$$u_1(x,t) = A\cos(kx - \omega t) \quad \text{and} \quad u_2(x,t) = A\cos(kx - \omega t + \phi) \tag{9.E.3}$$

a. Show that the superposition of these two waves is also a simple harmonic (i.e., sinusoidal) wave, and determine its amplitude as a function of the phase difference ϕ.

Finally consider two sources of sound that have slightly different frequencies. If you listen to these, you'll notice that the sound increases and decreases in intensity periodically: it exhibits a beating pattern, due to interference of the two waves in time. In case the two sources can be described as emitting sound according to simple harmonics with identical amplitudes, their waves at your position can be described by $u_1(t) = A\cos(\omega_1 t)$ and $u_2(t) = A\cos(\omega_2 t)$.

a. Find an expression for the resulting wave you're hearing.
b. What is the frequency of the beats you're hearing? NB: because the human ear is not sensitive to the phase, only to the amplitude or intensity of the sound, you only hear the absolute value of the envelope. What effect does this have on the observed frequency?
c. You put some water in a glass soda bottle, and put it next to a 440 Hz tuning fork. When you strike both, you hear a beat frequency of 4 Hz. After adding a little water to the soda bottle, the beat frequency has increased to 5 Hz. What are the initial and final frequencies of the bottle?

9.4 One of your friends stands in the middle of a rectangular 10.0×6.0 m swimming pool, his hands 1.0 meter apart in the direction parallel to the long edge of the pool. He produces surface waves in the water of the pool by oscillating his hands. At the edge, you find that at the point closest to your friend, the water is rough, then if you move to the side, it gets quiet, rough again, and quiet again. That point, where the water gets quiet for the second time, lies 1.0 m from your starting point (facing your friend).

a. What is the wavelength of the surface waves in the pool?
b. At which distance does the water get quiet for the first time?
c. And at which distance do you find rough water for the third time (counting the initial point)?

9.5 The Doppler effect is the shift in observed frequency of a wave due to either a moving observer or moving source, as discussed in Section 9.7. We will consider a sound wave emitted by some noisy source and observed by you.

a. If you are standing still and the source is moving towards you, will the frequency you hear be higher or lower than the frequency emitted by the source?
b. If you move towards a stationary source, will the frequency you hear be higher or lower than the frequency emitted by the source?
c. The observed frequency f_{obs} depends on the actual frequency emitted by the source f_{source} (obviously), the speed of the source v_{source}, the speed of the observer v_{obs} and the speed of sound v_{sound}. Take the observer to be stationary. What happens if the source is stationary also? And what if the source moves at the speed of sound?
d. Based on your answers to the previous items, guess a functional form for fobs as a function of f_{source}, v_{source}, and v_{sound}.