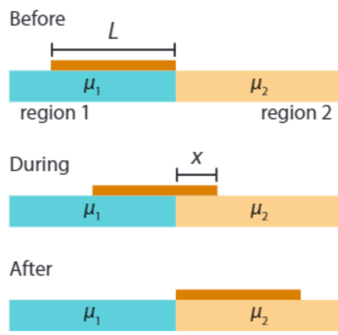
3.E: Energy (Exercises)

3.1

a. Show that, if you ignore drag, a projectile fired at an initial velocity v_0 and angle θ has a range R given by
b. A target is situated 1.5 km away from a cannon across a flat field. Will the target be hit if the firing angle is 42° and the cannonball is fired at an initial velocity of 121 m/s? (Cannonballs, as you know,do not bounce).
c. To increase the cannon's range, you put it on a tower of height h_0. Find the maximum range in this case, as a function of the firing angle and velocity, assuming the land around is still flat.

3.2 You push a box of mass m up a slope with angle θ and kinetic friction coefficient μ. Find the minimum initial speed v you must give the box so that it reaches a height h.

3.3 A uniform board of length L and mass M lies near a boundary that separates two regions. In region 1, the coefficient of kinetic friction between the board and the surface is μ_1, and in region 2, the coefficient is μ_2. Our objective is to find the net work W done by friction in pulling the board directly from region 1 to region 2, under the assumption that the board moves at constant velocity.

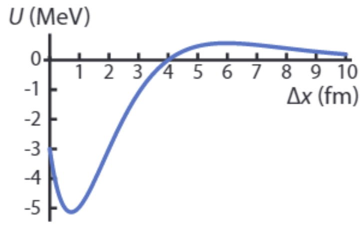


a. Suppose that at some point during the process, the right edge of the board is a distance x from the boundary, as shown. When the board is at this position, what is the magnitude of the force of friction acting on the board, assuming that it's moving to the right? Express your answer in terms of all relevant variables (L, M, g, x, μ_1, and μ_2).
b. As we've seen in Section 3.1, when the force is not constant, you can determine the work by integrating the force over the displacement, $W = \int F(x)dx$. Integrate your answer from (a) to get the net work you need to do to pull the board from region 1 to region 2.

3.4 The government wishes to secure votes from car-owners by increasing the speed limit on the highway from 120 to 140 km/h. The opposition points out that this is both more dangerous and will cause more pollution. Lobbyists from the car industry tell the government not to worry: the drag coefficients of the cars have gone down significantly and their construction is a lot more solid than in the time that the 120 km/h speed limit was set.

a. Suppose the 120 km/h limit was set with a Volkswagen Beetle ($c_d = 0.48$) in mind, and the lobbyist'scar has a drag coefficient of 0.19. Will the new car need to do more or less work to maintain a constant speed of 140 km/h than the Beetle at 120 km/h?
b. What is the ratio of the total kinetic energy released in a full head-on collision (resulting in an immediate standstill) between two cars both at 140 km/h and two cars both at 120 km/h?
c. The government dismisses the opposition's objections on safety by stating that on the highway, all cars move in the same direction (opposite direction lanes are well separated), so if they all move at 140 km/h, it would be just as safe as all at 120 km/h. The opposition then points out that running a Beetle (those are still around) at 120 km/h is already challenging, so there would be speed differences between newer and older cars. The government claims that the 20 km/h difference won't matter, as clearly even a Beetle can survive a 20 km/h collision. Explain why their argument is invalid.

3.5 Nuclear fusion, the process that powers the Sun, occurs when two low-mass atomic nuclei fuse together to make a larger nucleus, releasing substantial energy. Fusion is hard to achieve because atomic nuclei carry positive electric charge, and their electrical repulsion makes it difficult to get them close enough for the short-range nuclear force to bind them into a single nucleus. The figure below shows the potential-energy curve for fusion of two deuterons (heavy hydrogen nuclei, consisting of a proton and a neutron).The energy is measured in million electron volts ($MeV, 1eV = 1.6 \cdot 10^{-19} J$), a unit commonly used in nuclear physics, and the separation is in femtometers ($1 fm = 10^{-15} m$).



a. Find the position(s) (if any) at which the force between two deuterons is zero.
b. Find the kinetic energy two initially widely separated deuterons need to have to get close enough to fuse.
c. The energy available in fusion is the energy difference between that of widely separated deuterons and the bound deutrons after they've 'fallen' into the deep potential well shown in the figure. About how big is that energy?
d. Determine whether the force between two deuterons that are 4 fm apart is repulsive, attractive, or zero.

3.6 A pigeon in flight experiences a drag force due to air resistance given approximately by $F = bv^2$, where v is the flight speed and b is a constant.

a. What are the units of b?
b. What is the largest possible speed of the pigeon if its maximum power output is P?
c. By what factor does the largest possible speed increase if the maximum power output is doubled

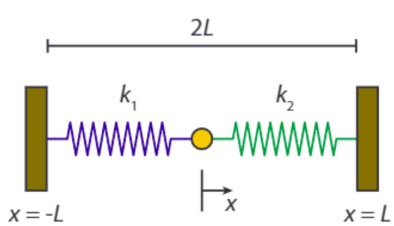
3.7

a. For which value(s) of the parameters $\alpha, \beta,$ and γ is the force given by

$$\boldsymbol{F} = \left(x^3y^3 + \alpha z^2, \beta x^4y^2, \gamma xz\right) \tag{3.E.1}$$

conservative?
b. Find the force for the potential energy given by $U(x, y, z) = \frac{xy}{z} - \frac{xz}{y}$.

3.8 A point mass is connected to two opposite walls by two springs, as shown in the figure. The distance between the walls is 2L. The left spring has rest length $l_1 = \frac{L}{2}$ and spring constant $k_1 = k$, the right spring has rest length $l_2 = \frac{3L}{4}$ and spring constant $k_2 = 3k$.



a. Determine the magnitude of the force acting on the point mass if it is at x=0.
b. Determine the equilibrium position of the point mass.
c. Find the potential energy of the point mass as a function of x. Use the equilibrium point from (b) as your point of reference.
d. If the point mass is displaced a small distance from its equilibrium position and then released, it will oscillate. By comparing the equation of the net force on the mass in this system with a simple harmonic oscillator, determine the frequency of that oscillation. (We'll return to systems oscillating about the minimum of a potential energy in Section 8.1.4, feel free to take a sneak peak ahead).

3.9 A block of mass m=3.50 kg slides from rest a distance d down a frictionless incline at angle $\theta = 30.0^{\circ}$,where it runs into a spring of spring constant 450 N/m. When the block momentarily stops, it has com-pressed the spring by 25.0 cm.

a. Find d.
b. What is the distance between the first block-spring contact and the point at which the block's speed is greatest?

3.10 Playground slides frequently have sections of varying slope: steeper ones to pick up speed, less steep ones to lose speed, so kids (and students) arrive at the bottom safely. We consider a slide with two steep sections (angle α) and two less steep ones (angle

β). Each of the sections has a width L. The slide has a coefficient of kinetic friction μ.

a. Kids start at the top of the slide with velocity zero. Calculate the velocity of a kid of mass m at the end of the first steep section.
b. Now calculate the velocity of the kid at the bottom of the entire slide.
c. If L=1.0 m, $\alpha=30^{\circ}$ and $\mu=0.5$, find the minimum value β must have so that kids up to 30 kg can enjoy the slide (Hint: what is the minimum requirement for the slide to be functional)?
d. A given slide has $\alpha=30^{\circ}$, $\beta=20^{\circ}$, and $\mu=0.5$. A young child of 10 kg slides down, while its cousin of 20 kg sits at the bottom. When the sliding kid reaches the end, the two children collide, and together slide further over the ground. The coefficient of kinetic friction with the ground is 0.70. How far do the two children slide before they come to a full stop?

3.11 In this problem, we consider the anharmonic potential given b

$$U(x)=\frac{a}{2}\left(x-x_{0}\right)^{2}+\frac{b}{3}\left(x-x_{0}\right)^{3} \tag{3.E.2}$$

where a, b, and x_0 are positive constants.

a. Find the dimensions of a, b, and x_0.
b. Determine whether the force on a particle at a position $x>>x_0$ is attractive or repulsive (taking the origin as your point of reference).
c. Find the equilibrium point(s) (if any) of this potential, and determine their stability.
d. For b=0, the potential given in Equation (3.24) becomes harmonic (i.e., the potential of a harmonic oscillator), in which case a particle that is initially located at a non-equilibrium point will oscillate. Are there initial values for x for which a particle in this anharmonic potential will oscillate? If so,find them,and find the approximate oscillation frequency; if not, explain why not. (NB: As the problem involves a third order polynomial function, you may find yourself having to solve a third order problem. When that happens, for your answer you can simply say: the solution x to the problem X).

3.12 After you have successfully finished your mechanics course, you decide to launch the book into an orbit around the Earth. However, the teacher is not convinced that you do not need it anymore and asks the following question: What is the ratio between the kinetic energy and the potential energy of the book in its orbit?

Let m be the mass of the book, $M_{\oplus}$ and $R_{\oplus}$ the mass and the radius of the Earth respectively. The gravitational pull at distance r from the center is given by Newton's law of gravitation (Equation 2.2.3):

$$F_{\mathrm{g}}(r)=-G\frac{mM_{\oplus}}{r^{2}}\hat{\boldsymbol{r}} \tag{3.E.3}$$

a. Find the orbital velocity v of an object at height h above the surface of the Earth.
b. Express the work required to get the book at height h.
c. Calculate the ratio between the kinetic and the potential energy of the book in its orbit.
d. What requires more work, getting the book to the International Space Station (orbiting at h=400 km)or giving it the same speed as the ISS?

3.13 Using dimensional arguments, in Problem 1.4 we found the scaling relation of the escape velocity (the minimal initial velocity an object must have to escape the gravitational pull of the planet/moon/other object it's on completely) with the mass of the radius of the planet. Here, we'll re-derive the result, including the numerical factor that dimensional arguments cannot give us.

a. Derive the expression of the gravitational potential energy,Ug, of an object of mass m due to a gravitational force F_g given by Newton's law of gravitation (Equation 2.2.3)

$$F_{\mathrm{g}}=-\frac{GmM}{r^{2}}\hat{r} \tag{3.E.4}$$

Set the value of the integration constant by $U_g\to 0$ as $r\to\infty$
b. Find the escape velocity on the surface of a planet of mass M and radius R by equating the initial kinetic energy of your object (when launched from the surface of the planet) to the total gravitational potential energy it has there.

3.14 A cannonball is fired upwards from the surface of the Earth with just enough speed such that it reaches the Moon. Find the speed of the cannonball as it crashes on the Moon's surface, taking the gravity of both the Earth and the Moon into account. Table B.3 contains the necessary astronomical data.

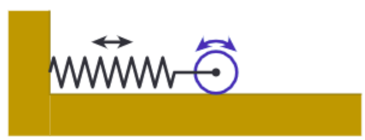
3.15 The draw force F(x) of a Turkish bow as a function of the bowstring displacement x (for x \gt 0) is approximately given by a quadrant of the ellipse

$$\left(\frac{F(x)}{F_{\max}}\right)^2+\left(\frac{x+d}{x}\right)^2=1 \tag{3.E.5}$$

In rest, the bowstring is at x=0; when pulled all the way back, it's at x=-d.

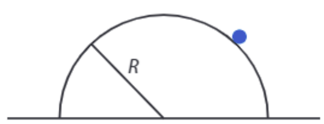
a. Calculate the work done by the bow in accelerating an arrow of mass m=37 g, for d=0.85 m, and F_{max}=360 N.
b. Assuming that all of the work is converted to kinetic energy of the arrow, find the maximum distance the arrow can fly.Hint: which variable can you control when shooting? Maximize the distance with respect to that variable.
c. Compare the result of (b) with the range of a bow that acts like a simple (Hookean) spring with the same values of F_{max} and d. How much further does the arrow shot from the Turkish bow fly than that of the simple spring bow?

3.16 A massive cylinder with mass M and radius R is connected to a wall by a spring at its center (see figure).The cylinder can roll back-and-forth without slipping.



a. Determine the total energy of the system consisting of the cylinder and the spring.
b. Differentiate the energy of problem (16a) to obtain the equation of motion of the cylinder and spring system.
c. Find the oscillation frequency of the cylinder by comparing the equation of motion at (16b) with that of a simple harmonic oscillator (a mass-spring system).

3.17 A small particle (blue dot) is placed atop the center of a hemispherical mount of ice of radius R (see figure). It slides down the side of the mount with negligible initial speed. Assuming no friction between the ice and the particle, find the height at which the particle loses contact with the ice.



Hint: To solve this problem, first draw a free body diagram, and combine what you know of energy and forces.

3.18 **Pulling membrane tubes**

The (potential) energy of a cylindrical membrane tube of length L and radius R is given by

$$\mathscr{E}_{\text{tube}}(R, L)=2\pi RL\left(\frac{\kappa}{2}\frac{1}{R^2}+\sigma\right) \tag{3.E.6}$$

Here κ is the membrane's bending modulus and σ its surface tension.

a. Find the dimensions of the bending modulus and the surface tension.
b. Find the forces acting on the tube along its radial and axial direction.
c. Membrane tubes are often pulled by membrane motors pulling along the axial direction, as sketched in Figure 3.5. For that case, we add the work done by the motors to the total energy of the tube, so we get:

$$\mathscr{E}_{\text{tube}}(R, L)=2\pi RL\left(\frac{\kappa}{2}\frac{1}{R^2}+\sigma\right)-FL \tag{3.E.7}$$

Show that for a stable tube, the motors need to exert a force of magnitude $F=2\pi\sqrt{2\kappa\sigma}$

d. Can the force of (c) be considered to be an effective spring force? If so, find its associated spring constant. If not, explain why not.

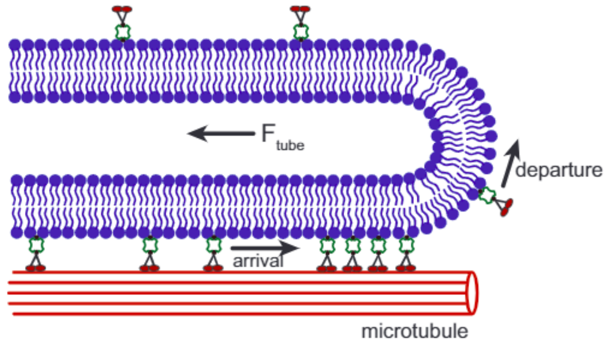


Figure $3.E.1$: Cartoon of molecular motors together pulling a membrane tube.