14.E: Relativistic Collisions (Exercises)

14.1 A photon with frequency f collides with a stationary atom with rest mass m. In the collision, the photon is absorbed by the atom. Determine the mass and speed of the atom after the collision.

14.2 A particle with mass m and kinetic energy $2mc^2$ collides with a stationary particle with mass $2m$. After the collision, the two particles are fused into a single particle. Find both the mass and the speed of this new particle.

14.3 A stationary atomic nucleus undergoes a radioactive process known as β-decay, in which one of its neutrons (with rest mass $m_{\mathrm{n}} = 939.6\mathrm{MeV}$) falls apart into a proton (which remains in the nucleus, rest mass $m_{\mathrm{p}} = 938.3\mathrm{MeV}$), an electron (rest mass $m_{\mathrm{e}} = 0.5\mathrm{MeV}$), and an anti-neutrino. Neutrinos are very light particles; we'll take the emitted neutrino to be effectively massless and thus travel at the speed of light with momentum p_ν. The nucleus remains stationary. Find the momenta p_ν and p_e of the emitted neutrino and electron, as well as the speed of the emitted electron.

14.4 A proton with rest mass mp and momentum p_p is moving in the positive x-direction. A photon with frequency f is traveling in the negative x-direction and collides head-on with the proton. After the collision, both proton and photon are traveling in the positive x-direction. Show that the frequency f' of the photon after the collision is given by

$$f' = \frac{E_{\mathrm{p}} + cp_{\mathrm{p}}}{E_{\mathrm{p}} - cp_{\mathrm{p}} + 2hf} f \tag{14.E.1}$$

where E_p is the energy of the proton before the collision.

14.5 Particles like the electrons in atomic orbitals can be in a low-energy ground state (with energy E_0), or, by absorbing a photon, be put in a higher-energy excited state (with energy E_1). The particle can return to the ground state by emitting another photon. Quantum mechanics tells us that only very specific states with very specific, discrete (or 'quantized') energies, are allowed.

a. If the particle is initially at rest, the energy of an incoming photon with frequency ν (and energy $E = h\nu$) has to be slightly larger than the energy difference $\Delta E = E_1 - E_0$ between the particle's ground and excited states if the particle is to absorb the photon. Explain why.
b. Show that for an incoming photon that is absorbed by an initially stationary particle, we have

$$hv_{\mathrm{a}} = \Delta E\left(1 + \frac{\Delta E}{2E_0}\right) \tag{14.E.2}$$

c. Likewise, show that for an initially stationary particle in the excited state with energy E1, the energy of the emitted photon is given by

$$hv_{\mathrm{e}} = \Delta E\left(1 - \frac{\Delta E}{2E_0}\right) \tag{14.E.3}$$

d. Suppose we have two identical atoms, one of which contains an electron in the excited state, and the other only electrons in the ground state. The atom with the electron in the excited state emits a photon. Is there a possible scenario in which the other atom absorbs the photon (resonant absorption)?

14.6 **Matter-antimatter annihilation and creation.** As you may have heard, for every elementary particle of 'ordinary' matter, there exists an antiparticle of 'antimatter', which shares many characteristics with its ordinary counterpart (such as the mass), whereas others are opposite (such as the charge). When a particle and its antiparticle meet, they completely annihilate, converting all of their combined mass into pure energy, in the form of radiation (i.e. photons). The most common antiparticle is that of the electron, which is known as the positron. First, we consider an experiment in which an ordinary electron of mass m_e with momentum p_e hits a positron (mass m_e) at rest, at which point the two annihilate, producing two photons.

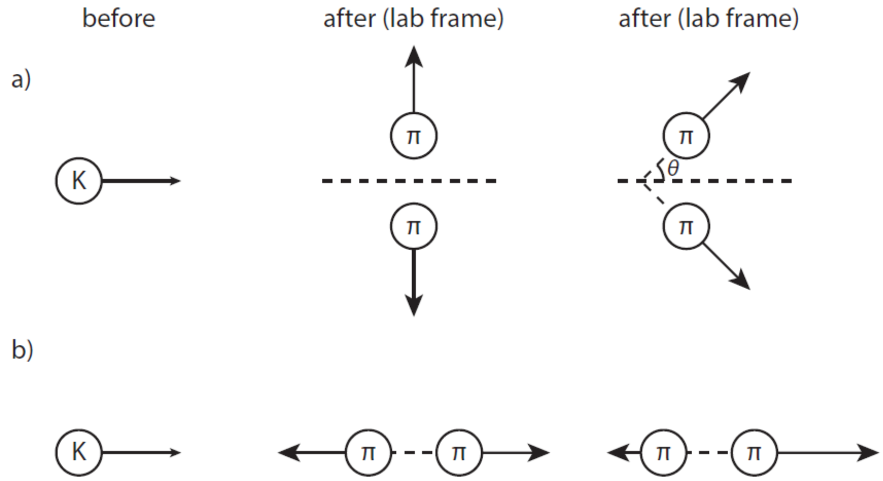
a. Argue why such an annihilation must produce at least two photons.
b. One of the two produced photons emerges at an angle of 60^o to the direction of the incident electron. What is its energy?
c. Find the angle (with the direction of the incident electron) at which the other photon emerges.

The opposite of annihilation, spontaneous creation of matter, can also happen: then a photon spontaneously converts to a particle-antiparticle pair.

d. Why must the photon convert to a particle-antiparticle pair, rather than simply convert to a single particle?

e. Find the minimum wavelength a photon must have to create an electron-positron pair. Where is this photon in the electromagnetic spectrum?

14.7 A neutral kaon (or K meson) with a mass of $498 \mathrm{MeV}/c^2$ and an initial velocity of $c/2$ decays into two pions (one with a positive and one with a negative charge), each of which has a mass of $135 \mathrm{MeV}/c^2$.



a. Find the speeds and the angles of the pions in the lab frame if, in the rest frame of the kaon, they are emitted in opposite directions, whose line makes an angle of 90^o with the propagation direction of the kaon?
b. Answer the same question as in (a), for the case that the pions are emitted one in the same and one in the opposite direction as the kaon.
c. Sometimes a kaon decays into more than two pions (there are also neutral pions; the charges, of course, need to add up to the kaon charge). Determine the maximum number of pions that our kaon can decay into.
d. Prove that in any situation, the trajectories of the created pions are in one plane. *Hint:* do this in the kaon's rest frame first.