5.5: Kinetic Energy of Rotation

Naturally, a rotating object has kinetic energy - its parts are moving after all (even if they're just rotating around a fixed axis). The total kinetic energy of rotation is simply the sum of the kinetic energies of all rotating parts, just like the total translational kinetic energy was the sum of the individual kinetic energies of the constituent particles in Section 4.5. Using that $v=\omega r$, we can write for a discrete collection of particles:

$$K_{\mathrm{rot}}=\sum_{\alpha} \frac{1}{2} m_{\alpha} v_{\alpha}^{2}=\sum_{\alpha} \frac{1}{2} m_{\alpha} r_{\alpha}^{2} \omega^{2}=\frac{1}{2} I \omega^{2} \tag{5.5.1}$$

by the definition 5.4.2 of the moment of inertia I. Analogously we find for a continuous object, using 5.4.3:

$$K_{\mathrm{rot}}=\int_{V} \frac{1}{2} v^{2} \rho \mathrm{d} V=\int_{V} \frac{1}{2} \omega^{2} r^{2} \rho \mathrm{d} V=\frac{1}{2} I \omega^{2} \tag{5.5.2}$$

so we arrive at the general rule:

$$K_{\mathrm{rot}}=\frac{1}{2} I \omega^{2} \tag{5.5.3}$$

Naturally, the work-energy theorem (Equation 3.2.3) still holds, so we can use it to calculate the work necessary to effect a change in rotational velocity, which by Equation 5.4.1 can also be expressed in terms of the torque (in 2D):

$$W=\Delta K_{\mathrm{rot}}=\frac{1}{2} I\left(\omega_{\text {final }}^{2}-\omega_{\text {initial }}^{2}\right)=\int_{\theta_{\text {initial }}}^{\theta_{\text {final }}} \tau \mathrm{d} \theta \tag{5.5.4}$$