11.1: Classical Case- Galilean Transformations

To figure out how velocities add in our new reality set by the light postulate, we need to reconsider the world-view of a stationary and moving observer, each in their own inertial reference frame. In classical mechanics, for an observer moving at speed u in the x-direction, we can find the coordinates of this observer's reference frame with respect to those of a stationary observer using a simple set of transformation rules:

$$\begin{aligned} x' &= x - ut, \\ y' &= y, \\ z' &= z, \\ t' &= t. \end{aligned} \tag{11.1.1}$$

Here the primed variables denote the coordinates of the moving observer, and the unprimed variables the stationary ones. We'll call the stationary frame S, and the moving frame S'. Of course we could also express the coordinates of S in those of S' - that is just equation (11.1.1) with the sign of u flipped. Note that we included the observation that time, as measured by both observers, is the same, as well as the y and z coordinates (since the train moves in the x direction - and we can just pick the x direction to be the one the train moves in). Equation (11.1.1) is known as the *Galilean coordinate transformation*. Note that it fits with the classical statement that accelerations are the same as measured in any reference frame:

$$a = \frac{\mathrm{d}^2 x'}{\mathrm{d}(t')^2} = \frac{\mathrm{d}^2(x - ut)}{\mathrm{d}t^2} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}x}{\mathrm{d}t} - u\right) = \frac{\mathrm{d}^2 x}{\mathrm{d}t^2} \tag{11.1.2}$$