4.E: Momentum (Exercises)

4.1 **Celestial centers of mass** We say that the Moon orbits the Earth, because the Earth's gravity pulls on the Moon, causing it to orbit. However, by Newton's third law, the Moon exerts a force back on the Earth.Therefore, the Earth should move in response to the Moon. Thus a more accurate statement would be that the Moon and the Earth both orbit the center of mass of the Earth-Moon system. Useful values: $M_E = 5.97 \cdot 10^{24} kg, R_E = 6.37 \cdot 10^6 m, R_{E,orbit} = 1.50 \cdot 10^{11} m, M_M = 7.35 \cdot 10^{22} kg, R_M = 1.74 \cdot 10^6 m, R_{M,orbit} = 3.84 \cdot 10^8 m, M_S = 1.99 \cdot 10^{30} kg, R_S = 6.96 \cdot 10^8 m$

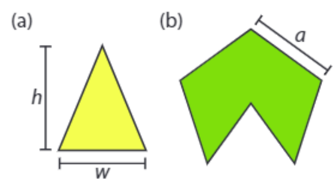
a. Find the center of mass of the Earth-Moon system (in an appropriately chosen, and clearly defined,coordinate system). Does this center of mass lie inside the Earth?
b. Find the location of the center of mass of the Earth-Moon-Sun system during a full Moon.
c. Find the location of the center of mass of the Earth-Moon-Sun system when the Moon is in its first quarter.

4.2 A shell is shot with an initial velocity of 20 m/s, at an angle of $60^{\circ}$with the horizontal. At the top of the trajectory, the shell explodes into two fragments of equal mass. One fragment, whose speed immediately after the explosion is zero, drops to the ground vertically. How far from the gun does the other fragment land (assuming no air drag and level terrain)?

4.3 Two cannonballs with masses m_1 and m_2 are simultaneously fired from two cannons situated a distance L apart.

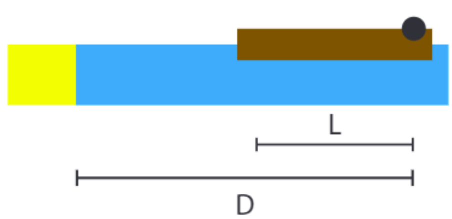
a. Find the equations of motion for the horizontal and vertical components of the vector describing the center of mass of the cannonballs.
b. Show that the motion of the center of mass is a parabola through space.

4.4 Center of mass of some solid objects



a. Find the center of mass of an isosceles triangle with a base width w and height h(see figure a).
b. Find the center of mass of a pentagon with five equal sides of length a, but with one triangle missing(see figure b). Hint: use your result from (a).
c. Find the position of the center of mass of a semicylinder (half of a solid cylinder, i.e., a solid cylinder sliced in two along a plane containing its symmetry axis). *Hint:* first calculate the center of mass of half a solid disk.

4.5 A dog (black dot in the sketch below) of mass m stands at the end of a boat of mass M and length L at an initial distance D from the shore. The dog then walks to the other end of the boat and stops there. Assuming no friction between the boat and the water, how far is the dog then from the shore? (Hint:what is conserved?).



4.6 [For optional Section 4.4] Every point in a tennis match stars with one of the players serving. The most commonly used service involves the player tossing the ball in the air and hitting it with their racket. To get the ball to move as fast as possible, players commonly swing the racket to give it a large momentum,and deliver a maximal impulse to the ball.

Figure 4.E.1 shows Serena Williams serving during the 2008 Wimbledon championships. Williams is widely regarded as one of the best women tennis players and holds the record of most aces (scoring a point from a serve without the opponent touching the ball)

by a female player at a Grand Slam tournament.

Williams is 175 cm tall. As you can see in the figure, at the top of its trajectory, the ball is about twice Williams' height above the ground. Also, as the span of people's arms is about the same as their height, and the shoulders of an adult are at about 5/6 of their height, we can estimate Williams' arms to be about 75 cm long and her shoulders to be at 145 cm above the ground. The distance between the point where a player holds the racket and where they hit the ball is typically about 40 cm.

Figure $4.E.1$: Serena Williams serving at the 2008 Wimbledon championships [18]

a. Find the speed of the ball coming down at the moment Williams hits it, assuming that she hits it with a fully upward stretched arm.
b. Williams's personal record for serve speed (speed of the ball after it was hit by the racket) is 207 km/h. Determine the impulse she delivered with her racket on the 58.0 g tennis ball as she hit it.
c. Assuming a typical racket weight of 360 g, calculate the change in speed of the racket from just before to just after Williams hit the ball.
d. Calculate the magnitude of the force on Williams' hand at the moment she hits the ball with her racket.

4.7 A 2.75 g bullet embeds itself in a 1.50 kg block, which is attached to a spring of force constant 850 N/m. The maximum compression of the spring is 4.30 cm.

a. Determine the initial speed of the bullet.
b. Find the time it takes the bullet-block system to come to rest.

4.8 **Head-on collision between two balls** A ball of mass m has velocity v when it makes a head-on collision with another ball of mass M that is originally at rest. After the collision the ball of mass m rebounds straight back along its path with 2/3 of its initial kinetic energy. We assume that the collision is totally elastic.

a. Sketch the situation before and after the collision, indicating directions of velocity, and values (if known, give symbols otherwise).
b. Write down all applicable conservation laws for this case.
c. From the conservation laws, solve for the mass M of the ball that is initially at rest.
d. Also solve for the velocity of that ball after the collision.

4.9 A small ball of mass m is aligned above a larger ball of mass M with a slight separation, and the two are dropped simultaneously from a height h. Assume the radii of the two balls and the initial separation are negligible compared to h.

a. If the larger ball rebounds elastically from the floor and then the small ball rebounds elastically from the larger ball, what value of m (as a fraction of M) results in the larger ball stopping when it collides with the small ball?
b. What height does the small ball then reach?

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