13.2: Lorentz Transformation Matrix and Metric Tensor

In this section, we've joined space and time in a single four-vector and defined a new inner product on the space of those four-vectors. In Chapter 11 we defined the Lorentz transformations of the space and time coordinates, which are linear transformations. Linear transformations can, of course, be represented by matrices, and for our four-vectors, we can write down the appropriate Lorentz transformation matrix, rewriting equation (11.12) as a vector equation:

$$\overline{\boldsymbol{x}}^{\prime}=L \overline{\boldsymbol{x}} \tag{13.2.1}$$

Here L is a 4×4 matrix:

$$L=\left(\begin{array}{cccc}
\gamma(u) & -\gamma(u) \frac{u}{c} & 0 & 0 \\
-\gamma(u) \frac{u}{c} & \gamma(u) & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right) \tag{13.2.2}$$

Likewith the four-vectors, we start labeling the rows and columns of L with index 0. To indicate the difference with matrices in regular space, it is conventional to indicate indices of regular-space vectors and matrices with Roman letters (like $\boldsymbol{v}_i$ for the ith component of vector v, and A_{ij} for the ith row, jth column of matrix A), and those of Minkowski-space vectors and matrices with Greek letters - so we write x_μ for the μth component of the four-vector $\overline{\boldsymbol{x}}$, where μ can be 0, 1, 2, or 3.

We can also write Equation ??? in index form: