4.6: Totally Inelastic Collisions

For the case of two particles colliding totally inelastically, conservation of momentum gives:

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_f \tag{4.6.1}$$

If the masses and initial velocities of the particles are known, calculating the final velocity of the composite particle is thus straightforward.

4.6.1. Worked Example: Bike Crash

You're late for class and it's raining to boot, so you cycle as fast as you possibly can, without paying attention in which direction you're going. A classmate, similarly late, comes towards you from a side street that makes an angle ϕ with yours. When your streets cross, you crash into each other, moving together in a big clutch of people and bikes, see Figure 4.6.1. Suppose you're about equally heavy, but you're the faster biker, with initially twice the speed of your classmate. Find the velocity (i.e., magnitude and direction) you and your classmate have immediately after the collision.

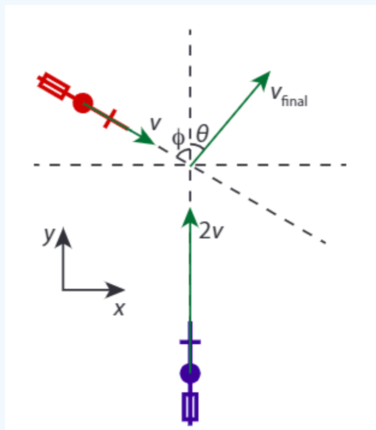

Figure 4.6.1: Two colliding cyclists.

Solution

Let's call your (and your classmate's) mass m, your initial speed 2v (so your classmate's speed is v), and your combined final speed v_f, with an angle θ with your initial direction. After the collision you move as one object, so the collision is completely inelastic. During the collision we have conservation of momentum in both the x and y directions, which gives:

$$0 + v \sin\phi = v_f \sin\theta \tag{4.6.2}$$

$$2v - v\cos\phi = v_f \cos\theta \tag{4.6.3}$$

We need to solve for both v_f and θ. To eliminate θ, we square both (4.6.2) and (4.6.3) and add them, which gives

$$v_f^2 = v^2 \sin^2\phi + v^2(2-\cos\phi)^2 = v^2(5 - 4\cos\phi) \tag{4.6.4}$$

or $v_f = v\sqrt{5 - 4\cos\phi}$. To get θ, we divide (4.6.2) by (4.6.3), which gives

We can easily check that these answers make sense when $\phi = 0$, which gives $v_f = v$ and $\theta = 0$, as it should.