

## 9.12: Linear Momentum and Collisions (Exercises)

### Conceptual Questions

#### 9.1 Linear Momentum

1. An object that has a small mass and an object that has a large mass have the same momentum. Which object has the largest kinetic energy?
2. An object that has a small mass and an object that has a large mass have the same kinetic energy. Which mass has the largest momentum?

#### 9.2 Impulse and Collisions

3. Is it possible for a small force to produce a larger impulse on a given object than a large force? Explain.
4. Why is a 10-m fall onto concrete far more dangerous than a 10-m fall onto water?
5. What external force is responsible for changing the momentum of a car moving along a horizontal road?
6. A piece of putty and a tennis ball with the same mass are thrown against a wall with the same velocity. Which object experience a greater force from the wall or are the forces equal? Explain.

#### 9.3 Conservation of Linear Momentum

7. Under what circumstances is momentum conserved?
8. Can momentum be conserved for a system if there are external forces acting on the system? If so, under what conditions? If not, why not?
9. Explain in terms of momentum and Newton's laws how a car's air resistance is due in part to the fact that it pushes air in its direction of motion.
10. Can objects in a system have momentum while the momentum of the system is zero? Explain your answer.
11. A sprinter accelerates out of the starting blocks. Can you consider him as a closed system? Explain.
12. A rocket in deep space (zero gravity) accelerates by firing hot gas out of its thrusters. Does the rocket constitute a closed system? Explain.

#### 9.4 Types of Collisions

13. Two objects of equal mass are moving with equal and opposite velocities when they collide. Can all the kinetic energy be lost in the collision?
14. Describe a system for which momentum is conserved but mechanical energy is not. Now the reverse: Describe a system for which kinetic energy is conserved but momentum is not.

#### 9.5 Collisions in Multiple Dimensions

15. Momentum for a system can be conserved in one direction while not being conserved in another. What is the angle between the directions? Give an example.

#### 9.6 Center of Mass

16. Suppose a fireworks shell explodes, breaking into three large pieces for which air resistance is negligible. How does the explosion affect the motion of the center of mass? How would it be affected if the pieces experienced significantly more air resistance than the intact shell?

#### 9.7 Rocket Propulsion

17. It is possible for the velocity of a rocket to be greater than the exhaust velocity of the gases it ejects. When that is the case, the gas velocity and gas momentum are in the same direction as that of the rocket. How is the rocket still able to obtain thrust by ejecting the gases?

### Problems

#### 9.1 Linear Momentum

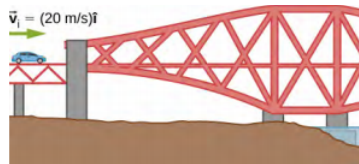
18. An elephant and a hunter are having a confrontation. A drawing of an elephant, on the left, and hunter, on the right. An xy-coordinate system has positive x to the right and positive y up. The elephant is labeled with  $m_E = 2000.0$  kg, and vector  $\mathbf{v}_E = 7.50$  m/s  $\hat{i}$ . An arrow above the  $\mathbf{v}_E$  vector points to the right. The hunter is labeled with  $m_{\text{hunter}} = 90.0$  kg, and

vector  $\vec{v}_{\text{hunter}} = 7.40 \text{ m/s } \hat{i}$ . An arrow above the  $\vec{v}_{\text{hunter}}$  vector points to the right. Between the hunter and elephant is a dart with a long arrow pointing to the left drawn near it and labeled vector  $\vec{v}_{\text{dart}} = -600 \text{ m/s } \hat{i}$ , and  $m_{\text{dart}} = 0.0400 \text{ kg}$ .

- Calculate the momentum of the 2000.0-kg elephant charging the hunter at a speed of 7.50 m/s.
  - Calculate the ratio of the elephant's momentum to the momentum of a 0.0400-kg tranquilizer dart fired at a speed of 600 m/s.
  - What is the momentum of the 90.0-kg hunter running at 7.40 m/s after missing the elephant?
- A skater of mass 40 kg is carrying a box of mass 5 kg. The skater has a speed of 5 m/s with respect to the floor and is gliding without any friction on a smooth surface. (a) Find the momentum of the box with respect to the floor. (b) Find the momentum of the box with respect to the floor after she puts the box down on the frictionless skating surface. (c) A car of mass 2000 kg is moving with a constant velocity of 10 m/s due east. What is the momentum of the car?
  - The mass of Earth is  $5.97 \times 10^{24} \text{ kg}$  and its orbital radius is an average of  $1.50 \times 10^{11} \text{ m}$ . Calculate the magnitude of its average linear momentum.
  - If a rainstorm drops 1 cm of rain over an area of  $10 \text{ km}^2$  in the period of 1 hour, what is the momentum of the rain that falls in one second? Assume the terminal velocity of a raindrop is 10 m/s.
  - What is the average momentum of an avalanche that moves a 40-cm-thick layer of snow over an area of 100 m by 500 m over a distance of 1 km down a hill in 5.5 s? Assume a density of  $350 \text{ kg/m}^3$  for the snow.
  - What is the average momentum of a 70.0-kg sprinter who runs the 100-m dash in 9.65 s?

## 9.2 Impulse and Collisions

- A 75.0-kg person is riding in a car moving at 20.0 m/s when the car runs into a bridge abutment (see the following figure).
  - Calculate the average force on the person if he is stopped by a padded dashboard that compresses an average of 1.00 cm.
  - Calculate the average force on the person if he is stopped by an air bag that compresses an average of 15.0 cm.

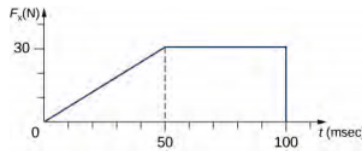


- One hazard of space travel is debris left by previous missions. There are several thousand objects orbiting Earth that are large enough to be detected by radar, but there are far greater numbers of very small objects, such as flakes of paint. Calculate the force exerted by a 0.100-mg chip of paint that strikes a spacecraft window at a relative speed of  $4.00 \times 10^3 \text{ m/s}$ , given the collision lasts  $6.00 \times 10^{-8} \text{ s}$ .
- A cruise ship with a mass of  $1.00 \times 10^7 \text{ kg}$  strikes a pier at a speed of 0.750 m/s. It comes to rest after traveling 6.00 m, damaging the ship, the pier, and the tugboat captain's finances. Calculate the average force exerted on the pier using the concept of impulse. (**Hint:** First calculate the time it took to bring the ship to rest, assuming a constant force.)

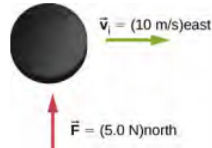


- Calculate the final speed of a 110-kg rugby player who is initially running at 8.00 m/s but collides head-on with a padded goalpost and experiences a backward force of  $1.76 \times 10^4 \text{ N}$  for  $5.50 \times 10^{-2} \text{ s}$ .
- Water from a fire hose is directed horizontally against a wall at a rate of 50.0 kg/s and a speed of 42.0 m/s. Calculate the force exerted on the wall, assuming the water's horizontal momentum is reduced to zero.
- A 0.450-kg hammer is moving horizontally at 7.00 m/s when it strikes a nail and comes to rest after driving the nail 1.00 cm into a board. Assume constant acceleration of the hammer-nail pair. (a) Calculate the duration of the impact. (b) What was the average force exerted on the nail?
- What is the momentum (as a function of time) of a 5.0-kg particle moving with a velocity  $\vec{v}(t) = (2.0 \hat{i} + 4.0t \hat{j}) \text{ m/s}$ ? What is the net force acting on this particle?
- The x-component of a force on a 46-g golf ball by a 7-iron versus time is plotted in the following figure:

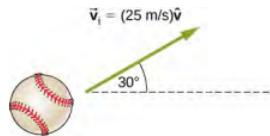
- a. Find the x-component of the impulse during the intervals (i) [0, 50 ms], and (ii) [50 ms, 100 ms].
- b. Find the change in the x-component of the momentum during the intervals (iii) [0, 50 ms], and (iv) [50 ms, 100 ms].



33. A hockey puck of mass 150 g is sliding due east on a frictionless table with a speed of 10 m/s. Suddenly, a constant force of magnitude 5 N and direction due north is applied to the puck for 1.5 s. Find the north and east components of the momentum at the end of the 1.5-s interval.

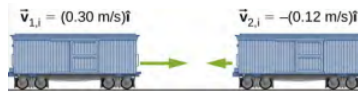


34. A ball of mass 250 g is thrown with an initial velocity of 25 m/s at an angle of  $30^\circ$  with the horizontal direction. Ignore air resistance. What is the momentum of the ball after 0.2 s? (Do this problem by finding the components of the momentum first, and then constructing the magnitude and direction of the momentum vector from the components.)

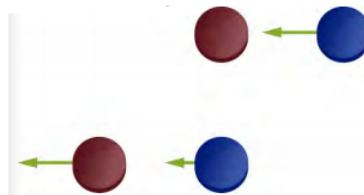


### 9.3 Conservation of Linear Momentum

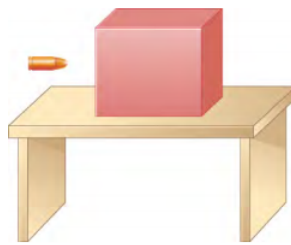
35. Train cars are coupled together by being bumped into one another. Suppose two loaded train cars are moving toward one another, the first having a mass of  $1.50 \times 10^5$  kg and a velocity of  $(0.30 \text{ m/s}) \hat{i}$ , and the second having a mass of  $1.10 \times 10^5$  kg and a velocity of  $-(0.12 \text{ m/s}) \hat{i}$ . What is their final velocity?



36. Two identical pucks collide elastically on an air hockey table. Puck 1 was originally at rest; puck 2 has an incoming speed of 6.00 m/s and scatters at an angle of  $30^\circ$  with respect to its incoming direction. What is the velocity (magnitude and direction) of puck 1 after the collision?



37. The figure below shows a bullet of mass 200 g traveling horizontally towards the east with speed 400 m/s, which strikes a block of mass 1.5 kg that is initially at rest on a frictionless table. After striking the block, the bullet is embedded in the block and the block and the bullet move together as one unit. (a) What is the magnitude and direction of the velocity of the block/bullet combination immediately after the impact? (b) What is the magnitude and direction of the impulse by the block on the bullet? (c) What is the magnitude and direction of the impulse from the bullet on the block? (d) If it took 3 ms for the bullet to change the speed from 400 m/s to the final speed after impact, what is the average force between the block and the bullet during this time?



38. A 20-kg child is coasting at 3.3 m/s over flat ground in a 4.0-kg wagon. The child drops a 1.0-kg ball out the back of the wagon. What is the final speed of the child and wagon?
39. A 5000-kg paving truck coasts over a road at 2.5 m/s and quickly dumps 1000 kg of gravel on the road. What is the speed of the truck after dumping the gravel?
40. Explain why a cannon recoils when it fires a shell.
41. Two figure skaters are coasting in the same direction, with the leading skater moving at 5.5 m/s and the trailing skater moving at 6.2 m/s. When the trailing skater catches up with the leading skater, he picks her up without applying any horizontal forces on his skates. If the trailing skater is 50% heavier than the 50-kg leading skater, what is their speed after he picks her up?
42. A 2000-kg railway freight car coasts at 4.4 m/s underneath a grain terminal, which dumps grain directly down into the freight car. If the speed of the loaded freight car must not go below 3.0 m/s, what is the maximum mass of grain that it can accept?

#### 9.4 Types of Collisions

43. A 5.50-kg bowling ball moving at 9.00 m/s collides with a 0.850-kg bowling pin, which is scattered at an angle of  $15.8^\circ$  to the initial direction of the bowling ball and with a speed of 15.0 m/s. (a) Calculate the final velocity (magnitude and direction) of the bowling ball. (b) Is the collision elastic?
44. Ernest Rutherford (the first New Zealander to be awarded the Nobel Prize in Chemistry) demonstrated that nuclei were very small and dense by scattering helium-4 nuclei from gold-197 nuclei. The energy of the incoming helium nucleus was  $8.00 \times 10^{-13}$  J, and the masses of the helium and gold nuclei were  $6.68 \times 10^{-27}$  kg and  $3.29 \times 10^{-25}$  kg, respectively (note that their mass ratio is 4 to 197). (a) If a helium nucleus scatters to an angle of  $120^\circ$  during an elastic collision with a gold nucleus, calculate the helium nucleus's final speed and the final velocity (magnitude and direction) of the gold nucleus. (b) What is the final kinetic energy of the helium nucleus?



45. A 90.0-kg ice hockey player hits a 0.150-kg puck, giving the puck a velocity of 45.0 m/s. If both are initially at rest and if the ice is frictionless, how far does the player recoil in the time it takes the puck to reach the goal 15.0 m away?
46. A 100-g firecracker is launched vertically into the air and explodes into two pieces at the peak of its trajectory. If a 72-g piece is projected horizontally to the left at 20 m/s, what is the speed and direction of the other piece?
47. In an elastic collision, a 400-kg bumper car collides directly from behind with a second, identical bumper car that is traveling in the same direction. The initial speed of the leading bumper car is 5.60 m/s and that of the trailing car is 6.00 m/s. Assuming that the mass of the drivers is much, much less than that of the bumper cars, what are their final speeds?
48. Repeat the preceding problem if the mass of the leading bumper car is 30.0% greater than that of the trailing bumper car.
49. An alpha particle ( $^4\text{He}$ ) undergoes an elastic collision with a stationary uranium nucleus ( $^{235}\text{U}$ ). What percent of the kinetic energy of the alpha particle is transferred to the uranium nucleus? Assume the collision is one-dimensional.
50. You are standing on a very slippery icy surface and throw a 1-kg football horizontally at a speed of 6.7 m/s. What is your velocity when you release the football? Assume your mass is 65 kg.
51. A 35-kg child sleds down a hill and then coasts along the flat section at the bottom, where a second 35-kg child jumps on the sled as it passes by her. If the speed of the sled is 3.5 m/s before the second child jumps on, what is its speed after she jumps on?
52. A boy sleds down a hill and onto a frictionless ice-covered lake at 10.0 m/s. In the middle of the lake is a 1000-kg boulder. When the sled crashes into the boulder, he is propelled over the boulder and continues sliding over the ice. If the

boy's mass is 40.0 kg and the sled's mass is 2.50 kg, what is the speed of the sled and the boulder after the collision?

### 9.5 Collisions in Multiple Dimensions

53. A 0.90-kg falcon is diving at 28.0 m/s at a downward angle of  $35^\circ$ . It catches a 0.325-kg pigeon from behind in midair. What is their combined velocity after impact if the pigeon's initial velocity was 7.00 m/s directed horizontally? Note that  $\vec{v}_{1,i}$  is a unit vector pointing in the direction in which the falcon is initially flying.

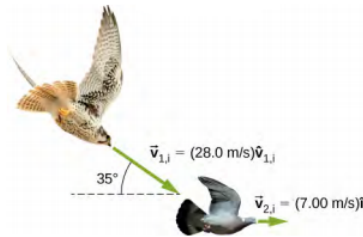
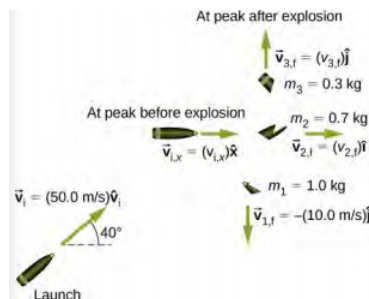


Figure 9.12.1 - (credit "hawk": modification of work by "USFWS Mountain-Prairie"/Flickr; credit "dove": modification of work by Jacob Spinks)

54. A billiard ball, labeled 1, moving horizontally strikes another billiard ball, labeled 2, at rest. Before impact, ball 1 was moving at a speed of 3.00 m/s, and after impact it is moving at 0.50 m/s at  $50^\circ$  from the original direction. If the two balls have equal masses of 300 g, what is the velocity of the ball 2 after the impact?
55. A projectile of mass 2.0 kg is fired in the air at an angle of  $40.0^\circ$  to the horizon at a speed of 50.0 m/s. At the highest point in its flight, the projectile breaks into three parts of mass 1.0 kg, 0.7 kg, and 0.3 kg. The 1.0-kg part falls straight down after breakup with an initial speed of 10.0 m/s, the 0.7-kg part moves in the original forward direction, and the 0.3-kg part goes straight up. (a) Find the speeds of the 0.3-kg and 0.7-kg pieces immediately after the break-up. (b) How high from the break-up point does the 0.3-kg piece go before coming to rest? (c) Where does the 0.7-kg piece land relative to where it was fired from?



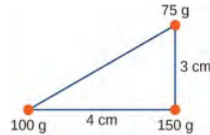
56. Two asteroids collide and stick together. The first asteroid has mass of  $15 \times 10^3$  kg and is initially moving at 770 m/s. The second asteroid has mass of  $20 \times 10^3$  kg and is moving at 1020 m/s. Their initial velocities made an angle of  $20^\circ$  with respect to each other. What is the final speed and direction with respect to the velocity of the first asteroid?
57. A 200-kg rocket in deep space moves with a velocity of  $(121 \text{ m/s})\hat{i} + (38.0 \text{ m/s})\hat{j}$ . Suddenly, it explodes into three pieces, with the first (78 kg) moving at  $(-321 \text{ m/s})\hat{i} + (228 \text{ m/s})\hat{j}$  and the second (56 kg) moving at  $(16.0 \text{ m/s})\hat{i} - (88.0 \text{ m/s})\hat{j}$ . Find the velocity of the third piece.
58. A proton traveling at  $3.0 \times 10^6$  m/s scatters elastically from an initially stationary alpha particle and is deflected at an angle of  $85^\circ$  with respect to its initial velocity. Given that the alpha particle has four times the mass of the proton, what percent of its initial kinetic energy does the proton retain after the collision?
59. Three 70-kg deer are standing on a flat 200-kg rock that is on an ice-covered pond. A gunshot goes off and the deer scatter, with deer A running at  $(15 \text{ m/s})\hat{i} + (5.0 \text{ m/s})\hat{j}$ , deer B running at  $(-12 \text{ m/s})\hat{i} + (8.0 \text{ m/s})\hat{j}$ , and deer C running at  $(1.2 \text{ m/s})\hat{i} - (18.0 \text{ m/s})\hat{j}$ . What is the velocity of the rock on which they were standing?
60. A family is skating. The father (75 kg) skates at 8.2 m/s and collides and sticks to the mother (50 kg), who was initially moving at 3.3 m/s and at  $45^\circ$  with respect to the father's velocity. The pair then collides with their daughter (30 kg), who was stationary, and the three slide off together. What is their final velocity?
61. An oxygen atom (mass 16 u) moving at 733 m/s at  $15.0^\circ$  with respect to the  $\hat{i}$  direction collides and sticks to an oxygen molecule (mass 32 u) moving at 528 m/s at  $128^\circ$  with respect to the  $\hat{i}$  direction. The two stick together to form ozone.

What is the final velocity of the ozone molecule?

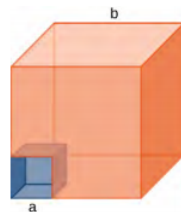
62. Two cars approach an extremely icy four-way perpendicular intersection. Car A travels northward at 30 m/s and car B is travelling eastward. They collide and stick together, traveling at  $28^\circ$  north of east. What was the initial velocity of car B?

### 9.6 Center of Mass

63. Three point masses are placed at the corners of a triangle as shown in the figure below. Find the center of mass of the three-mass system.



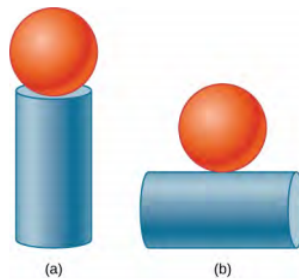
64. Two particles of masses  $m_1$  and  $m_2$  separated by a horizontal distance  $D$  are released from the same height  $h$  at the same time. Find the vertical position of the center of mass of these two particles at a time before the two particles strike the ground. Assume no air resistance.
65. Two particles of masses  $m_1$  and  $m_2$  separated by a horizontal distance  $D$  are let go from the same height  $h$  at different times. Particle 1 starts at  $t = 0$ , and particle 2 is let go at  $t = T$ . Find the vertical position of the center of mass at a time before the first particle strikes the ground. Assume no air resistance.
66. Two particles of masses  $m_1$  and  $m_2$  move uniformly in different circles of radii  $R_1$  and  $R_2$  about origin in the  $x,y$ -plane. The  $x$ - and  $y$ -coordinates of the center of mass and that of particle 1 are given as follows (where length is in meters and  $t$  in seconds):  $x_1(t) = 4\cos(2t)$ ,  $y_1(t) = 4\sin(2t)$  and:  $x_{CM}(t) = 3\cos(2t)$ ,  $y_{CM}(t) = 3\sin(2t)$ . (a) Find the radius of the circle in which particle 1 moves. (b) Find the  $x$ - and  $y$ -coordinates of particle 2 and the radius of the circle this particle moves.
67. Two particles of masses  $m_1$  and  $m_2$  move uniformly in different circles of radii  $R_1$  and  $R_2$  about the origin in the  $x, y$ -plane. The coordinates of the two particles in meters are given as follows ( $z = 0$  for both). Here  $t$  is in seconds:  $x_1(t) = 4\cos(2t)$ ,  $y_1(t) = 4\sin(2t)$ ,  $x_2(t) = 2\cos(3t - \frac{\pi}{2})$ ,  $y_2(t) = 2\sin(3t - \frac{\pi}{2})$  (a) Find the radii of the circles of motion of both particles. (b) Find the  $x$ - and  $y$ -coordinates of the center of mass. (c) Decide if the center of mass moves in a circle by plotting its trajectory.
68. Find the center of mass of a one-meter long rod, made of 50 cm of iron (density  $8 \text{ g/cm}^3$ ) and 50 cm of aluminum (density  $2.7 \text{ g/cm}^3$ ).
69. Find the center of mass of a rod of length  $L$  whose mass density changes from one end to the other quadratically. That is, if the rod is laid out along the  $x$ -axis with one end at the origin and the other end at  $x = L$ , the density is given by  $\rho(x) = \rho_0 + (\rho_1 - \rho_0) \left(\frac{x}{L}\right)^2$ , where  $\rho_0$  and  $\rho_1$  are constant values.
70. Find the center of mass of a rectangular block of length  $a$  and width  $b$  that has a nonuniform density such that when the rectangle is placed in the  $x,y$ -plane with one corner at the origin and the block placed in the first quadrant with the two edges along the  $x$ - and  $y$ -axes, the density is given by  $\rho(x, y) = \rho_0 x$ , where  $\rho_0$  is a constant.
71. Find the center of mass of a rectangular material of length  $a$  and width  $b$  made up of a material of nonuniform density. The density is such that when the rectangle is placed in the  $xy$ -plane, the density is given by  $\rho(x, y) = \rho_0 xy$ .
72. A cube of side  $a$  is cut out of another cube of side  $b$  as shown in the figure below. Find the location of the center of mass of the structure. (**Hint:** Think of the missing part as a negative mass overlapping a positive mass.)



73. Find the center of mass of cone of uniform density that has a radius  $R$  at the base, height  $h$ , and mass  $M$ . Let the origin be at the center of the base of the cone and have  $+z$  going through the cone vertex.
74. Find the center of mass of a thin wire of mass  $m$  and length  $L$  bent in a semicircular shape. Let the origin be at the center of the semicircle and have the wire arc from the  $+x$  axis, cross the  $+y$  axis, and terminate at the  $-x$  axis.
75. Find the center of mass of a uniform thin semicircular plate of radius  $R$ . Let the origin be at the center of the semicircle, the plate arc from the  $+x$  axis to the  $-x$  axis, and the  $z$  axis be perpendicular to the plate.



76. Find the center of mass of a sphere of mass  $M$  and radius  $R$  and a cylinder of mass  $m$ , radius  $r$ , and height  $h$  arranged as shown below. Express your answers in a coordinate system that has the origin at the center of the cylinder.



## 9.7 Rocket Propulsion

77. A 5.00-kg squid initially at rest ejects 0.250 kg of fluid with a velocity of 10.0 m/s. (a) What is the recoil velocity of the squid if the ejection is done in 0.100 s and there is a 5.00-N frictional force opposing the squid's movement? (b) How much energy is lost to work done against friction?
78. A rocket takes off from Earth and reaches a speed of 100 m/s in 10.0 s. If the exhaust speed is 1500 m/s and the mass of fuel burned is 100 kg, what was the initial mass of the rocket?
79. Repeat the preceding problem but for a rocket that takes off from a space station, where there is no gravity other than the negligible gravity due to the space station. 8
80. How much fuel would be needed for a 1000-kg rocket (this is its mass with no fuel) to take off from Earth and reach 1000 m/s in 30 s? The exhaust speed is 1000 m/s.
81. What exhaust speed is required to accelerate a rocket in deep space from 800 m/s to 1000 m/s in 5.0 s if the total rocket mass is 1200 kg and the rocket only has 50 kg of fuel left?
82. **Unreasonable Results** Squids have been reported to jump from the ocean and travel 30.0 m (measured horizontally) before re-entering the water. (a) Calculate the initial speed of the squid if it leaves the water at an angle of  $20.0^\circ$ , assuming negligible lift from the air and negligible air resistance. (b) The squid propels itself by squirting water. What fraction of its mass would it have to eject in order to achieve the speed found in the previous part? The water is ejected at 12.0 m/s; gravitational force and friction are neglected. (c) What is unreasonable about the results? (d) Which premise is unreasonable, or which premises are inconsistent?

## Additional Problems

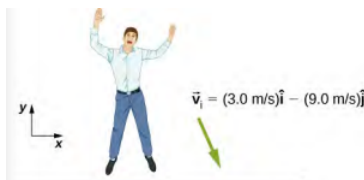
83. Two 70-kg canoers paddle in a single, 50-kg canoe. Their paddling moves the canoe at 1.2 m/s with respect to the water, and the river they're in flows at 4 m/s with respect to the land. What is their momentum with respect to the land?
84. Which has a larger magnitude of momentum: a 3000-kg elephant moving at 40 km/h or a 60-kg cheetah moving at 112 km/h?
85. A driver applies the brakes and reduces the speed of her car by 20%, without changing the direction in which the car is moving. By how much does the car's momentum change?
86. You friend claims that momentum is mass multiplied by velocity, so things with more mass have more momentum. Do you agree? Explain.
87. Dropping a glass on a cement floor is more likely to break the glass than if it is dropped from the same height on a grass lawn. Explain in terms of the impulse.
88. Your 1500-kg sports car accelerates from 0 to 30 m/s in 10 s. What average force is exerted on it during this acceleration?
89. A ball of mass  $m$  is dropped. What is the formula for the impulse exerted on the ball from the instant it is dropped to an arbitrary time  $\tau$  later? Ignore air resistance.
90. Repeat the preceding problem, but including a drag force due to air of  $f_{\text{drag}} = -b\vec{v}$ .
91. A 5.0-g egg falls from a 90-cm-high counter onto the floor and breaks. What impulse is exerted by the floor on the egg?
92. A car crashes into a large tree that does not move. The car goes from 30 m/s to 0 in 1.3 m. (a) What impulse is applied to the driver by the seatbelt, assuming he follows the same motion as the car? (b) What is the average force applied to the driver by the seatbelt?
93. Two hockey players approach each other head on, each traveling at the same speed  $v_i$ . They collide and get tangled together, falling down and moving off at a speed  $\frac{v_i}{5}$ . What is the ratio of their masses?

94. You are coasting on your 10-kg bicycle at 15 m/s and a 5.0-g bug splatters on your helmet. The bug was initially moving at 2.0 m/s in the same direction as you. If your mass is 60 kg, (a) what is the initial momentum of you plus your bicycle? (b) What is the initial momentum of the bug? (c) What is your change in velocity due to the collision with the bug? (d) What would the change in velocity have been if the bug were traveling in the opposite direction?
95. A load of gravel is dumped straight down into a 30 000-kg freight car coasting at 2.2 m/s on a straight section of a railroad. If the freight car's speed after receiving the gravel is 1.5 m/s, what mass of gravel did it receive?
96. Two carts on a straight track collide head on. The first cart was moving at 3.6 m/s in the positive x direction and the second was moving at 2.4 m/s in the opposite direction. After the collision, the second car continues moving in its initial direction of motion at 0.24 m/s. If the mass of the second car is 5.0 times that of the first, what is the final velocity of the first car?
97. A 100-kg astronaut finds himself separated from his spaceship by 10 m and moving away from the spaceship at 0.1 m/s. To get back to the spaceship, he throws a 10-kg tool bag away from the spaceship at 5.0 m/s. How long will he take to return to the spaceship?
98. Derive the equations giving the final speeds for two objects that collide elastically, with the mass of the objects being  $m_1$  and  $m_2$  and the initial speeds being  $v_{1,i}$  and  $v_{2,i} = 0$  (i.e., second object is initially stationary).
99. Repeat the preceding problem for the case when the initial speed of the second object is nonzero.
100. A child sleds down a hill and collides at 5.6 m/s into a stationary sled that is identical to his. The child is launched forward at the same speed, leaving behind the two sleds that lock together and slide forward more slowly. What is the speed of the two sleds after this collision?
101. For the preceding problem, find the final speed of each sled for the case of an elastic collision.
102. A 90-kg football player jumps vertically into the air to catch a 0.50-kg football that is thrown essentially horizontally at him at 17 m/s. What is his horizontal speed after catching the ball?
103. Three skydivers are plummeting earthward. They are initially holding onto each other, but then push apart. Two skydivers of mass 70 and 80 kg gain horizontal velocities of 1.2 m/s north and 1.4 m/s southeast, respectively. What is the horizontal velocity of the third skydiver, whose mass is 55 kg?
104. Two billiard balls are at rest and touching each other on a pool table. The cue ball travels at 3.8 m/s along the line of symmetry between these balls and strikes them simultaneously. If the collision is elastic, what is the velocity of the three balls after the collision?
105. A billiard ball traveling at  $(2.2 \text{ m/s}) \hat{i} - (0.4 \text{ m/s}) \hat{j}$  collides with a wall that is aligned in the  $\hat{j}$  direction. Assuming the collision is elastic, what is the final velocity of the ball?
106. Two identical billiard balls collide. The first one is initially traveling at  $(2.2 \text{ m/s}) \hat{i} - (0.4 \text{ m/s}) \hat{j}$  and the second one at  $-(1.4 \text{ m/s}) \hat{i} + (2.4 \text{ m/s}) \hat{j}$ . Suppose they collide when the center of ball 1 is at the origin and the center of ball 2 is at the point  $(2R, 0)$  where  $R$  is the radius of the balls. What is the final velocity of each ball?
107. Repeat the preceding problem if the balls collide when the center of ball 1 is at the origin and the center of ball 2 is at the point  $(0, 2R)$ .
108. Repeat the preceding problem if the balls collide when the center of ball 1 is at the origin and the center of ball 2 is at the point  $\left(\frac{\sqrt{3}R}{2}, \frac{R}{2}\right)$ .
109. Where is the center of mass of a semicircular wire of radius  $R$  that is centered on the origin, begins and ends on the x axis, and lies in the x,y plane?
110. Where is the center of mass of a slice of pizza that was cut into eight equal slices? Assume the origin is at the apex of the slice and measure angles with respect to an edge of the slice. The radius of the pizza is  $R$ .
111. If the entire population of Earth were transferred to the Moon, how far would the center of mass of the Earth-Moon-population system move? Assume the population is 7 billion, the average human has a mass of 65 kg, and that the population is evenly distributed over both the Earth and the Moon. The mass of the Earth is  $5.97 \times 10^{24}$  kg and that of the Moon is  $7.34 \times 10^{22}$  kg. The radius of the Moon's orbit is about  $3.84 \times 10^5$  m.
112. You friend wonders how a rocket continues to climb into the sky once it is sufficiently high above the surface of Earth so that its expelled gasses no longer push on the surface. How do you respond?
113. To increase the acceleration of a rocket, should you throw rocks out of the front window of the rocket or out of the back window?

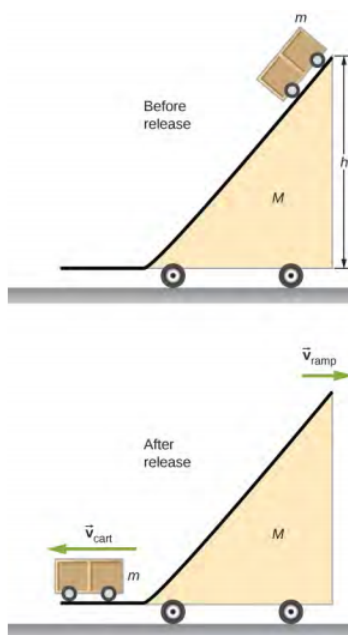


## Challenge Problems

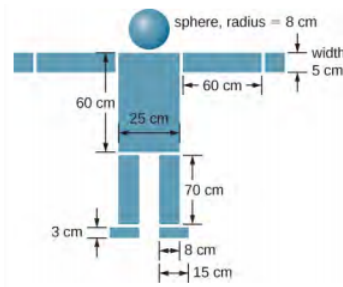
114. A 65-kg person jumps from the first floor window of a burning building and lands almost vertically on the ground with a horizontal velocity of 3 m/s and vertical velocity of  $-9$  m/s. Upon impact with the ground he is brought to rest in a short time. The force experienced by his feet depends on whether he keeps his knees stiff or bends them. Find the force on his feet in each case. (a) First find the impulse on the person from the impact on the ground. Calculate both its magnitude and direction. (b) Find the average force on the feet if the person keeps his leg stiff and straight and his center of mass drops by only 1 cm vertically and 1 cm horizontally during the impact. (c) Find the average force on the feet if the person bends his legs throughout the impact so that his center of mass drops by 50 cm vertically and 5 cm horizontally during the impact. (d) Compare the results of part (b) and (c), and draw conclusions about which way is better. You will need to find the time the impact lasts by making reasonable assumptions about the deceleration. Although the force is not constant during the impact, working with constant average force for this problem is acceptable.



115. Two projectiles of mass  $m_1$  and  $m_2$  are fired at the same speed but in opposite directions from two launch sites separated by a distance  $D$ . They both reach the same spot in their highest point and strike there. As a result of the impact they stick together and move as a single body afterwards. Find the place they will land.
116. Two identical objects (such as billiard balls) have a one-dimensional collision in which one is initially motionless. After the collision, the moving object is stationary and the other moves with the same speed as the other originally had. Show that both momentum and kinetic energy are conserved.
117. A ramp of mass  $M$  is at rest on a horizontal surface. A small cart of mass  $m$  is placed at the top of the ramp and released. What are the velocities of the ramp and the cart relative to the ground at the instant the cart leaves the ramp?



118. Find the center of mass of the structure given in the figure below. Assume a uniform thickness of 20 cm, and a uniform density of  $1 \text{ g/cm}^3$ .



## Contributors and Attributions

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