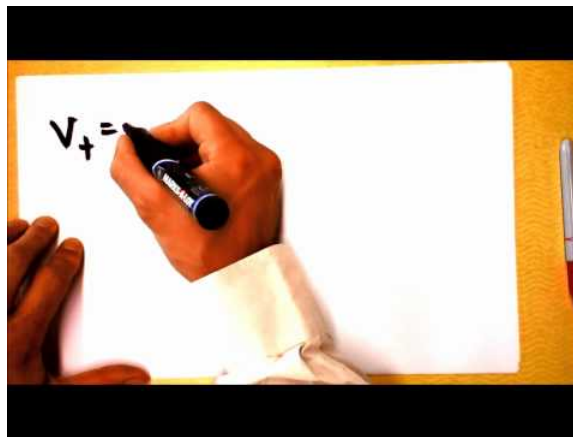
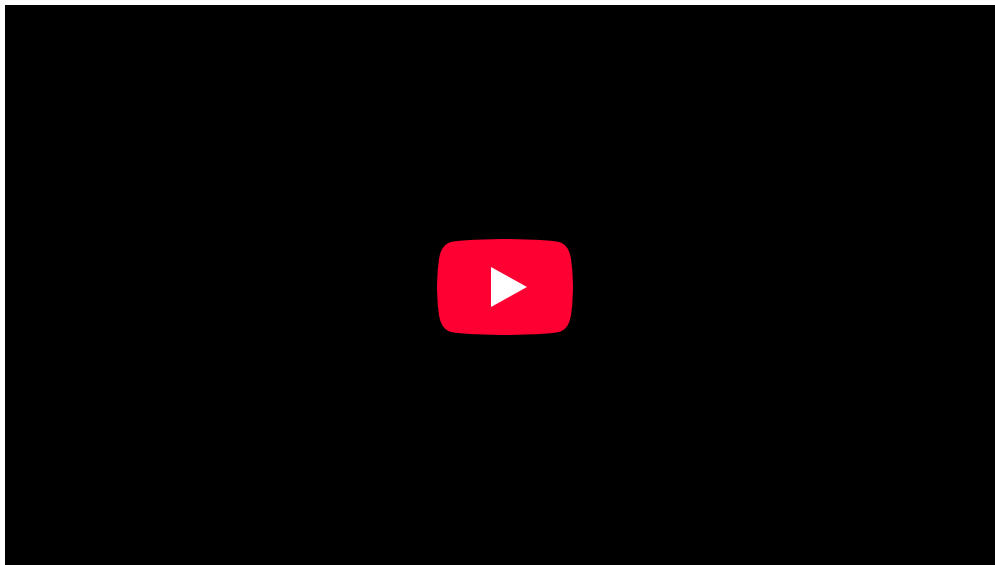


## 11.15: Rotational Kinematics

### learning objectives

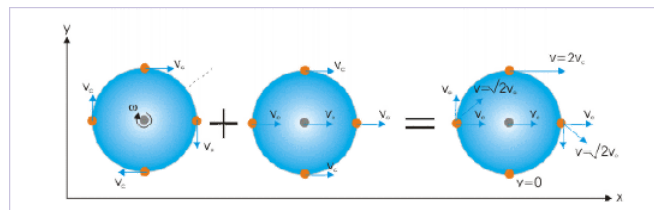
- Distinguish the two different motions in which rolling without slipping is broken down

Rolling without slipping generally occurs when an object rolls without skidding. To relate this to a real world phenomenon, we can consider the example of a wheel rolling on a flat, horizontal surface.



**Connecting Linear and Rotational Motion! Rolling without Slipping!** How fast does the axle of a bike wheel move? How fast does the BOTTOM of a wheel move?

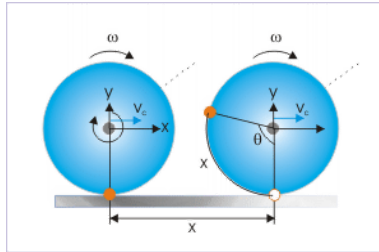
Rolling without slipping can be better understood by breaking it down into two different motions: 1) Motion of the center of mass, with linear velocity  $v$  (translational motion); and 2) rotational motion around its center, with angular velocity  $\omega$ .



**Rolling Motion:** Rolling motion is a combination of rotational and translational motion.

When an object is rolling on a plane without slipping, the point of contact of the object with the plane does not move. If we imagine a wheel moving forward by rolling on a plane at speed  $v$ , it must also be rotating about its axis at an angular speed  $\omega$  since it is rolling.

The object's angular velocity  $\omega$  is directly proportional to its velocity, because as we know, the faster we are driving in a car, the faster the wheels have to turn. So, to determine the exact relationship between linear velocity and angular velocity, we can imagine the scenario in which the wheel travels a distance of  $x$  while rotating through an angle  $\theta$ .



**Rolling Without Slipping:** A body rolling a distance of  $x$  on a plane without slipping.

In mathematical terms, the length of the arc is equal to the angle of the segment multiplied by the object's radius,  $R$ . Therefore, we can say that the length of the arc of the wheel that has rotated an angle  $\theta$ , is equal to  $R\theta$ . Furthermore, since the wheel is in constant contact with the ground, the length of the arc correlating to the angle is also equal to  $x$ . Therefore,

$$x = R\theta \quad (11.15.1)$$

Since  $x$  and  $\theta$  depend on time, we can take the derivative of both sides to obtain:

$$\frac{dx}{dt} = R \frac{d\theta}{dt} \quad (11.15.2)$$

where  $\frac{dx}{dt}$  is equal to the linear velocity  $v$ , and  $\frac{d\theta}{dt}$  is equal to the angular velocity  $\omega$ . We can then simplify this equation to:

$$v = \omega R \quad (11.15.3)$$

## Key Points

- Rolling without slipping can be better understood by breaking it down into translational motion and rotational motion.
- When an object is rolling on a plane without slipping, the point of contact of the object with the plane does not move.
- A rolling object's velocity  $v$  is directly related to its angular velocity  $\omega$ , and is mathematically expressed as  $v = \omega R$ , where  $R$  is the object's radius and  $v$  is its linear velocity.

## Key Terms

- angular velocity:** A vector quantity describing the motion of an object in circular motion; its magnitude is equal to the angular speed ( $\omega$ ) of the particle, and the direction is perpendicular to the plane of its circular motion.
- linear velocity:** A vector quantity that denotes the rate of change of position with respect to time of the object's center of mass.

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