7.17: Gravitational Potential Energy

Defining Graviational Potential Energy

Gravitational energy is the potential energy associated with gravitational force, such as elevating objects against the Earth's gravity.

> **learning objectives**
>
> - Express gravitational potential energy for two masses

Gravitational energy is the potential energy associated with gravitational force, as work is required to elevate objects against Earth's gravity. The potential energy due to elevated positions is called gravitational potential energy, and is evidenced by water in an elevated reservoir or kept behind a dam. If an object falls from one point to another point inside a gravitational field, the force of gravity will do positive work on the object, and the gravitational potential energy will decrease by the same amount.

Consider a book placed on top of a table. As the book is raised from the floor to the table, some external force works against the gravitational force. If the book falls back to the floor, the "falling" energy the book receives is provided by the gravitational force. Thus, if the book falls off the table, this potential energy goes to accelerate the mass of the book and is converted into kinetic energy. When the book hits the floor, this kinetic energy is converted into heat and sound by the impact.

The factors that affect an object's gravitational potential energy are its height relative to some reference point, its mass, and the strength of the gravitational field it is in. Thus, a book lying on a table has less gravitational potential energy than the same book on top of a taller cupboard, and less gravitational potential energy than a heavier book lying on the same table. An object at a certain height above the Moon's surface has less gravitational potential energy than at the same height above the Earth's surface because the Moon's gravity is weaker. Note that "height" in the common sense of the term cannot be used for gravitational potential energy calculations when gravity is not assumed to be a constant. The following sections provide more detail.

Local Approximation

The strength of a gravitational field varies with location. However, when the change of distance is small in relation to the distances from the center of the source of the gravitational field, this variation in field strength is negligible and we can assume that the force of gravity on a particular object is constant. Near the surface of the Earth, for example, we assume that the acceleration due to gravity is a constant $g = 9.8 m/s^2$ ("standard gravity"). In this case, a simple expression for gravitational potential energy can be derived using the $W = Fd$ equation for work. The upward force required while moving at a constant velocity is equal to the weight, mg, of an object, so the work done in lifting it through a height h is the product mgh. Thus, when accounting only for mass, gravity, and altitude, the equation is:

$$U = mgh \tag{7.17.1}$$

where U is the potential energy of the object relative to its being on the Earth's surface, m is the mass of the object, g is the acceleration due to gravity, and h is the altitude of the object. If m is expressed in kilograms, g in m/s^2 and h in meters then U will be calculated in joules. In most situations, the change in potential energy is the relevant quantity:

$$\Delta U = mg\Delta h \tag{7.17.2}$$

General Formula

However, over large variations in distance, the approximation that gg is constant is no longer valid, and we have to use calculus and the general mathematical definition of work to determine gravitational potential energy. For the computation of the potential energy, we can integrate the gravitational force, whose magnitude is given by Newton's law of gravitation, with respect to the distance r between the two bodies. Using that definition, the gravitational potential energy of a system of masses m_1 and M_2 at a distance r using gravitational constant G is

$$U = -G\frac{m_1 M_2}{r} + K \tag{7.17.3}$$

where K is the constant of integration. Choosing the convention that $K = 0$ makes calculations simpler, albeit at the cost of making U negative. Note that in this case the potential energy becomes zero when r is infinite, and approaches negative infinity as r goes to zero.

Trebuchet: A trebuchet uses the gravitational potential energy of the counterweight to throw projectiles over long distances.

Key Points

- If an object falls from one point to another point inside a gravitational field, the force of gravity will do positive work on the object, and the gravitational potential energy will decrease by the same amount.
- Near the surface of the Earth, the work done in lifting an object through a height h is the product mgh, so $U = mgh$.
- The gravitational potential energy, U, of a system of masses m_1 and M_2 at a distance r using gravitational constant G is $U = -G\frac{m_1 M_2}{r} + K$.

Key Terms

- **Newton's law of gravitation**: This law states that every point mass in the universe attracts every other point mass with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.
- **potential energy**: The energy an object has because of its position (in a gravitational or electric field) or its condition (as a stretched or compressed spring, as a chemical reactant, or by having rest mass)
- **gravity**: Resultant force on Earth's surface, of the attraction by the Earth's masses, and the centrifugal pseudo-force caused by the Earth's rotation.

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