7.5: Friction (Part 2)

Friction and the Inclined Plane

One situation where friction plays an obvious role is that of an object on a slope. It might be a crate being pushed up a ramp to a loading dock or a skateboarder coasting down a mountain, but the basic physics is the same. We usually generalize the sloping surface and call it an inclined plane but then pretend that the surface is flat. Let’s look at an example of analyzing motion on an inclined plane with friction.

Example 7.5.1: Downhill Skier

A skier with a mass of 62 kg is sliding down a snowy slope at a constant velocity. Find the coefficient of kinetic friction for the skier if friction is known to be 45.0 N.

Strategy

The magnitude of kinetic friction is given as 45.0 N. Kinetic friction is related to the normal force N by $f_k = \mu_k N$; thus, we can find the coefficient of kinetic friction if we can find the normal force on the skier. The normal force is always perpendicular to the surface, and since there is no motion perpendicular to the surface, the normal force should equal the component of the skier’s weight perpendicular to the slope. (See Figure 7.5.1, which repeats a figure from the chapter on Newton’s laws of motion.)

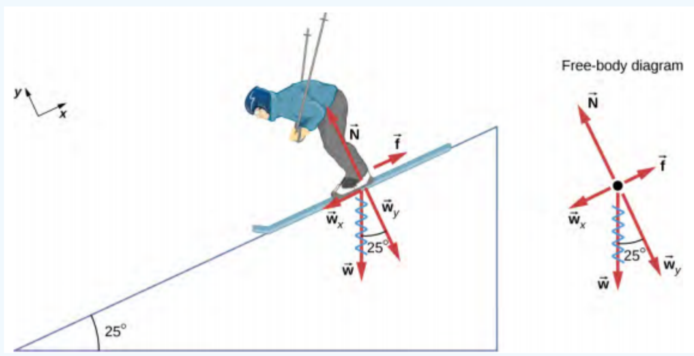

Figure 7.5.1: The motion of the skier and friction are parallel to the slope, so it is most convenient to project all forces onto a coordinate system where one axis is parallel to the slope and the other is perpendicular (axes shown to left of skier). The normal force $\vec{N}$ is perpendicular to the slope, and friction $\vec{f}$ is parallel to the slope, but the skier’s weight $\vec{w}$ has components along both axes, namely $\vec{w}_y$ and $\vec{w}_x$. The normal force $\vec{N}$ is equal in magnitude to $\vec{w}_y$, so there is no motion perpendicular to the slope. However, $\vec{f}$ is less than $\vec{w}_x$ in magnitude, so there is acceleration down the slope (along the x-axis).

We have

$$N = w_y = w\cos 25^o = mg\cos 25^o. \tag{7.5.1}$$

Substituting this into our expression for kinetic friction, we obtain

$$f_k = \mu_k mg\cos 25^o, \tag{7.5.2}$$

which can now be solved for the coefficient of kinetic friction μ_k.

Solution

Solving for μ_k gives

$$\mu_k = \frac{f_k}{N} = \frac{f_k}{w\cos 25^o} = \frac{f_k}{mg\cos 25^o}. \tag{7.5.3}$$

Substituting known values on the right-hand side of the equation,

$$\mu_k = \frac{45.0\ N}{(62\ kg)(9.80\ m/s^2)(0.906)} = 0.082. \tag{7.5.4}$$

Significance

This result is a little smaller than the coefficient listed in Table 6.1 for waxed wood on snow, but it is still reasonable since values of the coefficients of friction can vary greatly. In situations like this, where an object of mass m slides down a slope that makes an angle θ with the horizontal, friction is given by $f_k = \mu_k$ mg cos θ. All objects slide down a slope with constant acceleration under these circumstances.

We have discussed that when an object rests on a horizontal surface, the normal force supporting it is equal in magnitude to its weight. Furthermore, simple friction is always proportional to the normal force. When an object is not on a horizontal surface, as with the inclined plane, we must find the force acting on the object that is directed perpendicular to the surface; it is a component of the weight.

We now derive a useful relationship for calculating coefficient of friction on an inclined plane. Notice that the result applies only for situations in which the object slides at constant speed down the ramp.

An object slides down an inclined plane at a constant velocity if the net force on the object is zero. We can use this fact to measure the coefficient of kinetic friction between two objects. As shown in Example $7.5.1$, the kinetic friction on a slope is $f_k = \mu_k$ mg cos θ. The component of the weight down the slope is equal to mg sin θ (see the free-body diagram in Figure $7.5.1$). These forces act in opposite directions, so when they have equal magnitude, the acceleration is zero. Writing these out,

$$\mu_k mg\cos\theta = mg\sin\theta. \tag{7.5.5}$$

Solving for μ_k, we find that

$$\mu_k = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta. \tag{7.5.6}$$

Put a coin on a book and tilt it until the coin slides at a constant velocity down the book. You might need to tap the book lightly to get the coin to move. Measure the angle of tilt relative to the horizontal and find μ_k. Note that the coin does not start to slide at all until an angle greater than θ is attained, since the coefficient of static friction is larger than the coefficient of kinetic friction. Think about how this may affect the value for μ_k and its uncertainty.

Atomic-Scale Explanations of Friction

The simpler aspects of friction dealt with so far are its macroscopic (large-scale) characteristics. Great strides have been made in the atomic-scale explanation of friction during the past several decades. Researchers are finding that the atomic nature of friction seems to have several fundamental characteristics. These characteristics not only explain some of the simpler aspects of friction—they also hold the potential for the development of nearly friction-free environments that could save hundreds of billions of dollars in energy which is currently being converted (unnecessarily) into heat.

Figure $7.5.2$ illustrates one macroscopic characteristic of friction that is explained by microscopic (small-scale) research. We have noted that friction is proportional to the normal force, but not to the amount of area in contact, a somewhat counterintuitive notion. When two rough surfaces are in contact, the actual contact area is a tiny fraction of the total area because only high spots touch. When a greater normal force is exerted, the actual contact area increases, and we find that the friction is proportional to this area.

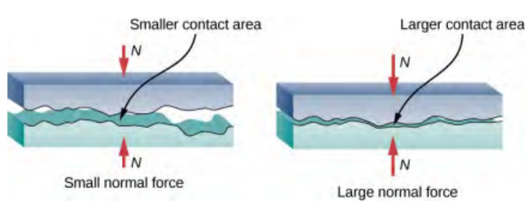

Figure $7.5.2$: Two rough surfaces in contact have a much smaller area of actual contact than their total area. When the normal force is larger as a result of a larger applied force, the area of actual contact increases, as does friction.

However, the atomic-scale view promises to explain far more than the simpler features of friction. The mechanism for how heat is generated is now being determined. In other words, why do surfaces get warmer when rubbed? Essentially, atoms are linked with one another to form lattices. When surfaces rub, the surface atoms adhere and cause atomic lattices to vibrate—essentially creating sound waves that penetrate the material. The sound waves diminish with distance, and their energy is converted into heat. Chemical reactions that are related to frictional wear can also occur between atoms and molecules on the surfaces. Figure $7.5.3$ shows how the tip of a probe drawn across another material is deformed by atomic-scale friction. The force needed to drag the tip can be measured and is found to be related to shear stress, which is discussed in Static Equilibrium and Elasticity. The variation in shear stress is remarkable (more than a factor of 1012) and difficult to predict theoretically, but shear stress is yielding a fundamental understanding of a large-scale phenomenon known since ancient times—friction.

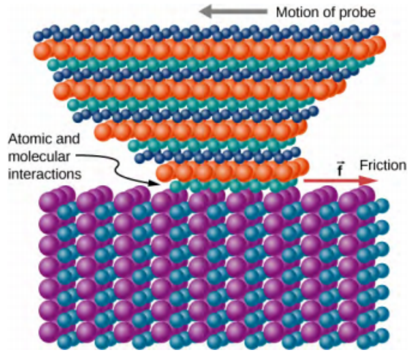

Figure 7.5.3: The tip of a probe is deformed sideways by frictional force as the probe is dragged across a surface. Measurements of how the force varies for different materials are yielding fundamental insights into the atomic nature of friction.

Simulation

Describe a model for friction on a molecular level. Describe matter in terms of molecular motion. The description should include diagrams to support the description; how the temperature affects the image; what are the differences and similarities between solid, liquid, and gas particle motion; and how the size and speed of gas molecules relate to everyday objects.

Example 7.5.2: Sliding Blocks

The two blocks of Figure 7.5.4 are attached to each other by a massless string that is wrapped around a frictionless pulley. When the bottom 4.00-kg block is pulled to the left by the constant force $\vec{P}$, the top 2.00-kg block slides across it to the right. Find the magnitude of the force necessary to move the blocks at constant speed. Assume that the coefficient of kinetic friction between all surfaces is 0.400.

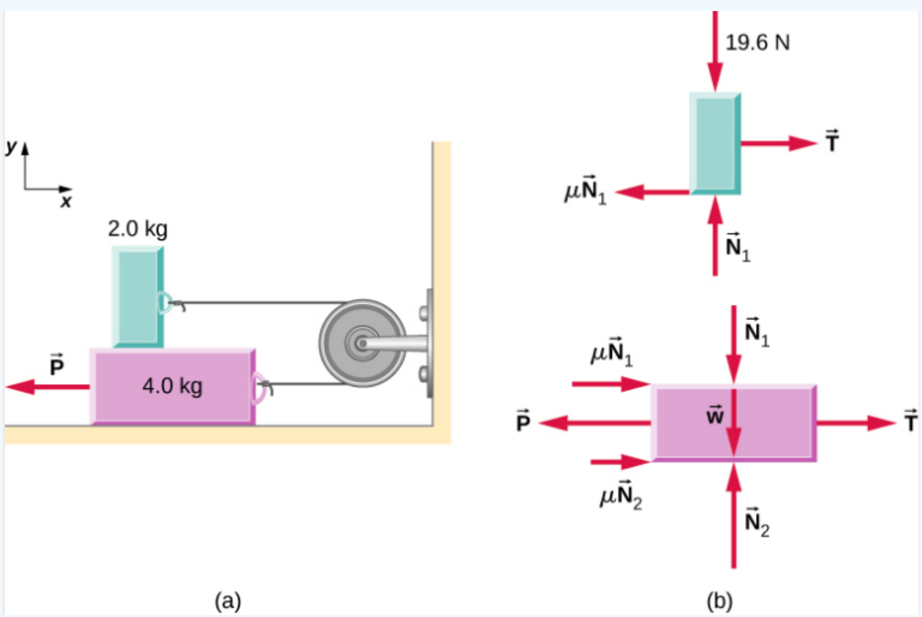

Figure 7.5.4: (a) Each block moves at constant velocity. (b) Free-body diagrams for the blocks.

Strategy

We analyze the motions of the two blocks separately. The top block is subjected to a contact force exerted by the bottom block. The components of this force are the normal force N_1 and the frictional force $-0.400\ N_1$. Other forces on the top block are the tension T in the string and the weight of the top block itself, 19.6 N. The bottom block is subjected to contact forces due to the top block and due to the floor. The first contact force has components $-N_1$ and $0.400\ N_1$, which are simply reaction forces to the contact forces that the bottom block exerts on the top block. The components of the contact force of the floor are N_2 and

0.400 N_2. Other forces on this block are −P, the tension T, and the weight –39.2 N. Solution Since the top block is moving horizontally to the right at constant velocity, its acceleration is zero in both the horizontal and the vertical directions. From Newton's second law,

$$\sum F_x = m_2 a_x \tag{7.5.7}$$

$$T - 0.400\ N_1 = 0 \tag{7.5.8}$$

$$\sum F_y = m_1 a_y \tag{7.5.9}$$

$$N_1 - 19.6\ N = 0. \tag{7.5.10}$$

Solving for the two unknowns, we obtain N_1 = 19.6 N and T = 0.40 N_1 = 7.84 N. The bottom block is also not accelerating, so the application of Newton's second law to this block gives

$$\sum F_x = m_2 a_x \tag{7.5.11}$$

$$T - P + 0.400\ N_1 + 0.400\ N_2 = 0 \tag{7.5.12}$$

$$\sum F_y = m_1 a_y \tag{7.5.13}$$

$$N_2 - 39.2\ N - N_1 = 0. \tag{7.5.14}$$

The values of N_1 and T were found with the first set of equations. When these values are substituted into the second set of equations, we can determine N_2 and P. They are

$$N_2 = 58.8\ N \ and\ P = 39.2\ N. \tag{7.5.15}$$

Significance

Understanding what direction in which to draw the friction force is often troublesome. Notice that each friction force labeled in Figure 7.5.4 acts in the direction opposite the motion of its corresponding block.

✔ Example 7.5.3: A Crate on an Accelerating Truck

A 50.0-kg crate rests on the bed of a truck as shown in Figure 7.5.5. The coefficients of friction between the surfaces are μ_k = 0.300 and μ_s = 0.400. Find the frictional force on the crate when the truck is accelerating forward relative to the ground at (a) 2.00 m/s^2, and (b) 5.00 m/s^2.

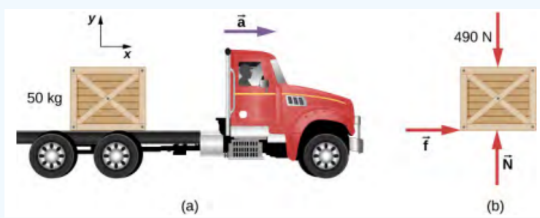

Figure 7.5.5: (a) A crate rests on the bed of the truck that is accelerating forward. (b) The free-body diagram of the crate.

Strategy

The forces on the crate are its weight and the normal and frictional forces due to contact with the truck bed. We start by assuming that the crate is not slipping. In this case, the static frictional force fs acts on the crate. Furthermore, the accelerations of the crate and the truck are equal.

Solution

a. Application of Newton's second law to the crate, using the reference frame attached to the ground, yields

$$\begin{aligned} \sum F_x &= ma_x \\ f_s &= (50.0\ kg)(2.00\ m/s^2) \\ &= 1.00 \times 10^2\ N \end{aligned}$$

$$\begin{aligned} \sum F_y &= ma_y \\ N - 4.90 \times 10^2\ N &= (50.0\ kg)(0) \\ N &= 4.90 \times 10^2\ N. \end{aligned}$$

We can now check the validity of our no-slip assumption. The maximum value of the force of static friction is

$$\mu_s N = (0.400)(4.90 \times 10^2\ N) = 196\ N, \tag{7.5.16}$$

whereas the **actual** force of static friction that acts when the truck accelerates forward at 2.00 m/s^2 is only 1.00 x 10^2 N. Thus, the assumption of no slipping is valid.

b. If the crate is to move with the truck when it accelerates at 5.0 m/s^2, the force of static friction must be

$$f_s = ma_x = (50.0\ kg)(5.00\ m/s^2) = 250\ N. \tag{7.5.17}$$

Since this exceeds the maximum of 196 N, the crate must slip. The frictional force is therefore kinetic and is

$$f_k = \mu_k N = (0.300)(4.90 \times 10^2\ N) = 147\ N. \tag{7.5.18}$$

The horizontal acceleration of the crate relative to the ground is now found from $$

$$\begin{aligned} \sum F_x &= ma_x \\ 147\ N &= (50.0\ kg)a_x, \\ so\ a_x &= 2.94\ m/s^2. \end{aligned}$$

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Significance

Relative to the ground, the truck is accelerating forward at 5.0 m/s^2 and the crate is accelerating forward at 2.94 m/s^2. Hence the crate is sliding backward relative to the bed of the truck with an acceleration 2.94 m/s^2 − 5.00 m/s^2 = −2.06 m/s^2.

Example 7.5.4: Snowboarding

Earlier, we analyzed the situation of a downhill skier moving at constant velocity to determine the coefficient of kinetic friction. Now let's do a similar analysis to determine acceleration. The snowboarder of Figure 7.5.6 glides down a slope that is inclined at θ = 13° to the horizontal. The coefficient of kinetic friction between the board and the snow is μ_k = 0.20. What is the acceleration of the snowboarder?

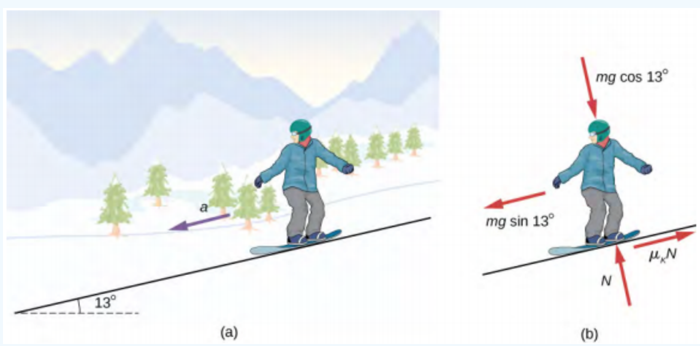

Figure 7.5.6: (a) A snowboarder glides down a slope inclined at 13° to the horizontal. (b) The free-body diagram of the snowboarder.

Strategy

The forces acting on the snowboarder are her weight and the contact force of the slope, which has a component normal to the incline and a component along the incline (force of kinetic friction). Because she moves along the slope, the most convenient reference frame for analyzing her motion is one with the x-axis along and the y-axis perpendicular to the incline. In this frame, both the normal and the frictional forces lie along coordinate axes, the components of the weight are mg sin θ along the slope and mg cos θ at right angles into the slope , and the only acceleration is along the x-axis ($a_y = 0$).

Solution

We can now apply Newton's second law to the snowboarder:

$\begin{aligned} \sum F_x &= ma_x \\ mg\sin\theta - \mu_k N &= ma_x \end{aligned}$	$\begin{aligned} \sum F_y &= ma_y \\ N - mg\cos\theta &= m(0). \end{aligned}$

From the second equation, N = mg cos θ. Upon substituting this into the first equation, we find

$$\begin{aligned} a_x &= g(\sin\theta - \mu_k\cos\theta) \\ &= g(\sin 13^o - 0.520\cos 13^o) = 0.29\ m/s^2. \end{aligned}$$

Significance

Notice from this equation that if θ is small enough or μ_k is large enough, a_x is negative, that is, the snowboarder slows down.

? Exercise 7.5.4

The snowboarder is now moving down a hill with incline 10.0°. What is the skier’s acceleration?