10.8: Static Equilibrium and Elasticity (Summary)

Key Terms

breaking stress (ultimate stress)	value of stress at the fracture point
bulk modulus	elastic modulus for the bulk stress
bulk strain (or volume strain)	strain under the bulk stress, given as fractional change in volume
bulk stress (or volume stress)	stress caused by compressive forces, in all directions
center of gravity	point where the weight vector is attached
compressibility	reciprocal of the bulk modulus
compressive strain	strain that occurs when forces are contracting an object, causing its shortening
compressive stress	stress caused by compressive forces, only in one direction
elastic	object that comes back to its original size and shape when the load is no longer present
elastic limit	stress value beyond which material no longer behaves elastically and becomes permanently deformed
elastic modulus	proportionality constant in linear relation between stress and strain, in SI pascals
equilibrium	body is in equilibrium when its linear and angular accelerations are both zero relative to an inertial frame of reference
first equilibrium condition	expresses translational equilibrium; all external forces acting on the body balance out and their vector sum is zero
gravitational torque	torque on the body caused by its weight; it occurs when the center of gravity of the body is not located on the axis of rotation
linearity limit (proportionality limit)	largest stress value beyond which stress is no longer proportional to strain
normal pressure	pressure of one atmosphere, serves as a reference level for pressure
pascal (Pa)	SI unit of stress, SI unit of pressure
plastic behavior	material deforms irreversibly, does not go back to its original shape and size when load is removed and stress vanishes
pressure	force pressing in normal direction on a surface per the surface area, the bulk stress in fluids
second equilibrium condition	expresses rotational equilibrium; all torques due to external forces acting on the body balance out and their vector sum is zero
shear modulus	elastic modulus for shear stress
shear strain	strain caused by shear stress
shear stress	stress caused by shearing forces
static equilibrium	body is in static equilibrium when it is at rest in our selected inertial frame of reference
strain	dimensionless quantity that gives the amount of deformation of an object or medium under stress
stress	quantity that contains information about the magnitude of force causing deformation, defined as force per unit area
stress-strain diagram	graph showing the relationship between stress and strain, characteristic of a material

tensile strain	strain under tensile stress, given as fractional change in length, which occurs when forces are stretching an object, causing its elongation
tensile stress	stress caused by tensile forces, only in one direction, which occurs when forces are stretching an object, causing its elongation
Young's modulus	elastic modulus for tensile or compressive stress

Key Equations

First Equilibrium Condition	$$\sum_k \vec{F}_k = \vec{0} \tag{10.8.1}$$
Second Equilibrium Condition	$$\sum_k \vec{\tau}_k = \vec{0} \tag{10.8.2}$$
Linear relation between stress and strain	$$stress = (elastic\ modulus) \times strain \tag{10.8.3}$$
Young's modulus	$$Y = \frac{tensile\ stress}{tensile\ strain} = \frac{F_\perp}{A}\frac{L_0}{\Delta L} \tag{10.8.4}$$
Bulk modulus	$$B = \frac{bulk\ stress}{bulk\ strain} = -\Delta p \frac{V_0}{\Delta V} \tag{10.8.5}$$
Shear modulus	$$S = \frac{shear\ stress}{shear\ strain} = \frac{F_\parallel}{A}\frac{L_0}{\Delta x} \tag{10.8.6}$$

Summary

12.1 Conditions for Static Equilibrium

- A body is in equilibrium when it remains either in uniform motion (both translational and rotational) or at rest. When a body in a selected inertial frame of reference neither rotates nor moves in translational motion, we say the body is in static equilibrium in this frame of reference.
- Conditions for equilibrium require that the sum of all external forces acting on the body is zero (first condition of equilibrium), and the sum of all external torques from external forces is zero (second condition of equilibrium). These two conditions must be simultaneously satisfied in equilibrium. If one of them is not satisfied, the body is not in equilibrium.
- The free-body diagram for a body is a useful tool that allows us to count correctly all contributions from all external forces and torques acting on the body. Free-body diagrams for the equilibrium of an extended rigid body must indicate a pivot point and lever arms of acting forces with respect to the pivot.

12.2 Examples of Static Equilibrium

- A variety of engineering problems can be solved by applying equilibrium conditions for rigid bodies.
- In applications, identify all forces that act on a rigid body and note their lever arms in rotation about a chosen rotation axis. Construct a free-body diagram for the body. Net external forces and torques can be clearly identified from a correctly constructed free-body diagram. In this way, you can set up the first equilibrium condition for forces and the second equilibrium condition for torques.
- In setting up equilibrium conditions, we are free to adopt any inertial frame of reference and any position of the pivot point. All choices lead to one answer. However, some choices can make the process of finding the solution unduly complicated. We reach the same answer no matter what choices we make. The only way to master this skill is to practice.

12.3 Stress, Strain, and Elastic Modulus

- External forces on an object (or medium) cause its deformation, which is a change in its size and shape. The strength of the forces that cause deformation is expressed by stress, which in SI units is measured in the unit of pressure (pascal). The extent of

deformation under stress is expressed by strain, which is dimensionless.
- For a small stress, the relation between stress and strain is linear. The elastic modulus is the proportionality constant in this linear relation.
- Tensile (or compressive) strain is the response of an object or medium to tensile (or compressive) stress. Here, the elastic modulus is called Young's modulus. Tensile (or compressive) stress causes elongation (or shortening) of the object or medium and is due to an external forces acting along only one direction perpendicular to the cross-section.
- Bulk strain is the response of an object or medium to bulk stress. Here, the elastic modulus is called the bulk modulus. Bulk stress causes a change in the volume of the object or medium and is caused by forces acting on the body from all directions, perpendicular to its surface. Compressibility of an object or medium is the reciprocal of its bulk modulus.
- Shear strain is the deformation of an object or medium under shear stress. The shear modulus is the elastic modulus in this case. Shear stress is caused by forces acting along the object's two parallel surfaces.

12.4 Elasticity and Plasticity

- An object or material is elastic if it comes back to its original shape and size when the stress vanishes. In elastic deformations with stress values lower than the proportionality limit, stress is proportional to strain. When stress goes beyond the proportionality limit, the deformation is still elastic but nonlinear up to the elasticity limit.
- An object or material has plastic behavior when stress is larger than the elastic limit. In the plastic region, the object or material does not come back to its original size or shape when stress vanishes but acquires a permanent deformation. Plastic behavior ends at the breaking point.

Contributors and Attributions

-

  Samuel J. Ling (Truman State University), Jeff Sanny (Loyola Marymount University), and Bill Moebs with many contributing authors. This work is licensed by OpenStax University Physics under a Creative Commons Attribution License (by 4.0).
