9.13: Linear Momentum and Collisions (Summary)

Key Terms

center of mass	weighted average position of the mass
closed system	system for which the mass is constant and the net external force on the system is zero
elastic	collision that conserves kinetic energy
explosion	single object breaks up into multiple objects; kinetic energy is not conserved in explosions
external force	force applied to an extended object that changes the momentum of the extended object as a whole
impulse	effect of applying a force on a system for a time interval; this time interval is usually small, but does not have to be
impulse-momentum theorem	change of momentum of a system is equal to the impulse applied to the system
inelastic	collision that does not conserve kinetic energy
internal force	force that the simple particles that make up an extended object exert on each other. Internal forces can be attractive or repulsive
Law of Conservation of Momentum	total momentum of a closed system cannot change
linear mass density	λ, expressed as the number of kilograms of material per meter
momentum	measure of the quantity of motion that an object has; it takes into account both how fast the object is moving, and its mass; specifically, it is the product of mass and velocity; it is a vector quantity
perfectly inelastic	collision after which all objects are motionless, the final kinetic energy is zero, and the loss of kinetic energy is a maximum
rocket equation	derived by the Soviet physicist Konstantin Tsiolkovsky in 1897, it gives us the change of velocity that the rocket obtains from burning a mass of fuel that decreases the total rocket mass from m_i down to m
system	object or collection of objects whose motion is currently under investigation; however, your system is defined at the start of the problem, you must keep that definition for the entire problem

Key Equations

Definition of momentum	$$\vec{p} = m\vec{v} \tag{9.13.1}$$
Impulse	$$\vec{J} \equiv \int_{t_i}^{t_f} \vec{F}(t)dt \; or \; \vec{J} = \vec{F}_{ave}\Delta t \tag{9.13.2}$$
Impulse-momentum theorem	$$\vec{J} = \Delta\vec{p} \tag{9.13.3}$$
Average force from momentum	$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t} \tag{9.13.4}$$

Instantaneous force from momentum (Newton's second law)	$$\vec{F}(t)=\frac{d\vec{p}}{dt} \tag{9.13.5}$$
Conservation of momentum	$$\frac{d\vec{p}_1}{dt}+\frac{d\vec{p}_2}{dt}=0\ or\ \vec{p}_1+\vec{p}_2=constant \tag{9.13.6}$$
Generalized conservation of momentum	$$\sum_{j=1}^{N}\vec{p}_j=constant \tag{9.13.7}$$
Conservation of momentum in two dimensions	$$p_{f,x}=p_{1,i,x}+p_{2,i,x} \tag{9.13.8}$$ $$p_{f,y}=p_{1,i,y}+p_{2,i,y} \tag{9.13.9}$$
External forces	$$\vec{F}_{ext}=\sum_{j=1}^{N}\frac{d\vec{p}_j}{dt} \tag{9.13.10}$$
Newton's second law for an extended object	$$\vec{F}=\frac{d\vec{p}_{CM}}{dt} \tag{9.13.11}$$
Acceleration of the center of mass	$$\vec{a}_{CM}=\frac{d^2}{dt^2}\left(\frac{1}{M}\sum_{j=1}^{N}m_j\vec{r}_j\right)=\frac{1}{M}\sum_{j=1}^{N}m_j\vec{a}_j \tag{9.13.12}$$
Position of the center of mass for a system of particles	$$\vec{r}_{CM}\equiv\sum_{j=1}^{N}m_j\vec{r}_j \tag{9.13.13}$$
Velocity of the center of mass	$$\vec{v}_{CM}=\frac{d}{dt}\left(\frac{1}{M}\sum_{j=1}^{N}m_j\vec{r}_j\right)=\frac{1}{M}\sum_{j=1}^{N}m_j\vec{v}_j \tag{9.13.14}$$
Position of the center of mass of a continuous object	$$\vec{r}_{CM}\equiv\frac{1}{M}\int\vec{r}dm \tag{9.13.15}$$
Rocket equation	$$\Delta v=u\ln\left(\frac{m_i}{m}\right) \tag{9.13.16}$$

Summary

9.1 Linear Momentum

- The motion of an object depends on its mass as well as its velocity. Momentum is a concept that describes this. It is a useful and powerful concept, both computationally and theoretically. The SI unit for momentum is kg • m/s.

9.2 Impulse and Collisions

- When a force is applied on an object for some amount of time, the object experiences an impulse.
- This impulse is equal to the object's change of momentum.
- Newton's second law in terms of momentum states that the net force applied to a system equals the rate of change of the momentum that the force causes.

9.3 Conservation of Linear Momentum

- The law of conservation of momentum says that the momentum of a closed system is constant in time (conserved).
- A closed (or isolated) system is defined to be one for which the mass remains constant, and the net external force is zero.

- The total momentum of a system is conserved only when the system is closed.

9.4 Types of Collisions

- An elastic collision is one that conserves kinetic energy.
- An inelastic collision does not conserve kinetic energy.
- Momentum is conserved regardless of whether or not kinetic energy is conserved.
- Analysis of kinetic energy changes and conservation of momentum together allow the final velocities to be calculated in terms of initial velocities and masses in one-dimensional, two-body collisions.

9.5 Collisions in Multiple Dimensions

- The approach to two-dimensional collisions is to choose a convenient coordinate system and break the motion into components along perpendicular axes.
- Momentum is conserved in both directions simultaneously and independently.
- The Pythagorean theorem gives the magnitude of the momentum vector using the x- and y-components, calculated using conservation of momentum in each direction.

9.6 Center of Mass

- An extended object (made up of many objects) has a defined position vector called the center of mass.
- The center of mass can be thought of, loosely, as the average location of the total mass of the object.
- The center of mass of an object traces out the trajectory dictated by Newton’s second law, due to the net external force.
- The internal forces within an extended object cannot alter the momentum of the extended object as a whole.

9.7 Rocket Propulsion

- A rocket is an example of conservation of momentum where the mass of the system is not constant, since the rocket ejects fuel to provide thrust.
- The rocket equation gives us the change of velocity that the rocket obtains from burning a mass of fuel that decreases the total rocket mass.
