11.24: Angular Momentum (Summary)

Key Terms

angular momentum	rotational analog of linear momentum, found by taking the product of moment of inertia a
law of conservation of angular momentum	angular momentum is conserved, that is, the initial angular momentum is equal to the final angular momentum whe
precession	circular motion of the pole of the axis of a spinning object around another axis due
rolling motion	combination of rotational and translational motion with or without slippi

Key Equations

Velocity of center of mass of rolling object	$$v_{CM} = R\omega \tag{11.24.1}$$
Acceleration of center of mass of rolling object	$$a_{CM} = R\alpha \tag{11.24.2}$$
Displacement of center of mass of rolling object	$$d_{CM} = R\theta \tag{11.24.3}$$
Acceleration of an object rolling without slipping	$$a_{CM} = \frac{mg\sin\theta}{m + \left(\frac{I_{CM}}{r^2}\right)} \tag{11.24.4}$$
Angular momentum	$$\vec{l} = \vec{r} \times \vec{p} \tag{11.24.5}$$
Derivative of angular momentum equals torque	$$\frac{d\vec{l}}{dt} = \sum \vec{\tau} \tag{11.24.6}$$
Angular momentum of a system of particles	$$\vec{L} = \vec{l}_1 + \vec{l}_2 + \cdots + \vec{l}_N \tag{11.24.7}$$
For a system of particles, derivative of angular momentum equals torque	$$\frac{d\vec{L}}{dt} = \sum \vec{\tau} \tag{11.24.8}$$
Angular momentum of a rotating rigid body	$$L = I\omega \tag{11.24.9}$$
Conservation of angular momentum	$$\frac{d\vec{L}}{dt} = 0 \tag{11.24.10}$$
Conservation of angular momentum	$$\vec{L} = \vec{l}_1 + \vec{l}_2 + \cdots + \vec{l}_N = constant \tag{11.24.11}$$
Precessional angular velocity	$$\omega_P = \frac{rMg}{I\omega} \tag{11.24.12}$$

Summary

11.1 Rolling Motion

- In rolling motion without slipping, a static friction force is present between the rolling object and the surface. The relations $v_{CM} = R\omega$, $a_{CM} = R\alpha$, and $d_{CM} = R\theta$ all apply, such that the linear velocity, acceleration, and distance of the center of mass are the angular variables multiplied by the radius of the object.
- In rolling motion with slipping, a kinetic friction force arises between the rolling object and the surface. In this case, $v_{CM} \neq R\omega$, $a_{CM} \neq R\alpha$, and $d_{CM} \neq R\theta$.
- Energy conservation can be used to analyze rolling motion. Energy is conserved in rolling motion without slipping. Energy is not conserved in rolling motion with slipping due to the heat generated by kinetic friction.

11.2 Angular Momentum

- The angular momentum $\vec{l} = \vec{r} \times \vec{p}$ of a single particle about a designated origin is the vector product of the position vector in the given coordinate system and the particle's linear momentum.
- The angular momentum $\vec{l} = \sum_i \vec{l}_i$ of a system of particles about a designated origin is the vector sum of the individual momenta of the particles that make up the system.
- The net torque on a system about a given origin is the time derivative of the angular momentum about that origin: $\frac{d\vec{L}}{dt} = \sum \vec{\tau}$
- A rigid rotating body has angular momentum L = Iω directed along the axis of rotation. The time derivative of the angular momentum $\frac{dL}{dt} = \sum \tau$ gives the net torque on a rigid body and is directed along the axis of rotation.

11.3 Conservation of Angular Momentum

- In the absence of external torques, a system's total angular momentum is conserved. This is the rotational counterpart to linear momentum being conserved when the external force on a system is zero.
- For a rigid body that changes its angular momentum in the absence of a net external torque, conservation of angular momentum gives $I_f \omega_f = I_i \omega_i$. This equation says that the angular velocity is inversely proportional to the moment of inertia. Thus, if the moment of inertia decreases, the angular velocity must increase to conserve angular momentum.
- Systems containing both point particles and rigid bodies can be analyzed using conservation of angular momentum. The angular momentum of all bodies in the system must be taken about a common axis.

11.4 Precession of a Gyroscope

- When a gyroscope is set on a pivot near the surface of Earth, it precesses around a vertical axis, since the torque is always horizontal and perpendicular to $\vec{L}$. If the gyroscope is not spinning, it acquires angular momentum in the direction of the torque, and it rotates about a horizontal axis, falling over just as we would expect.
- The precessional angular velocity is given by $\omega_P = \frac{rMg}{I\omega}$, where r is the distance from the pivot to the center of mass of the gyroscope, I is the moment of inertia of the gyroscope's spinning disk, M is its mass, and ω is the angular frequency of the gyroscope disk.

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