

The Kinetic Theory of Gases (Answer)

Check Your Understanding

2.1. We first need to calculate the molar mass (the mass of one mole) of niacin. To do this, we must multiply the number of atoms of each element in the molecule by the element's molar mass.

$$(6\text{mol of carbon})(12.0\text{g/mol}) + (5\text{mol hydrogen})(1.0\text{g/mol}) + (1\text{mol nitrogen})(14\text{g/mol}) \\ + (2\text{mol oxygen})(16.0\text{g/mol}) = 123\text{g/mol}$$

Then we need to calculate the number of moles in 14 mg.

$$\left(\frac{14\text{mg}}{123\text{g/mol}}\right)\left(\frac{1\text{g}}{1000\text{mg}}\right) = 1.14 \times 10^{-4}\text{mol}.$$

Then, we use Avogadro's number to calculate the number of molecules:

$$N = nN_A = (1.14 \times 10^{-4}\text{mol})(6.02 \times 10^{23}\text{molecules/mol}) = 6.85 \times 10^{19}\text{molecules}.$$

2.2. The density of a gas is equal to a constant, the average molecular mass, times the number density N/V . From the ideal gas law, $pV = Nk_B T$, we see that $N/V = p/k_B T$. Therefore, at constant temperature, if the density and, consequently, the number density are reduced by half, the pressure must also be reduced by half, and $p_f = 0.500\text{atm}$.

2.3. Density is mass per unit volume, and volume is proportional to the size of a body (such as the radius of a sphere) cubed. So if the distance between molecules increases by a factor of 10, then the volume occupied increases by a factor of 1000, and the density decreases by a factor of 1000. Since we assume molecules are in contact in liquids and solids, the distance between their centers is on the order of their typical size, so the distance in gases is on the order of 10 times as great.

2.4. Yes. Such fluctuations actually occur for a body of any size in a gas, but since the numbers of molecules are immense for macroscopic bodies, the fluctuations are a tiny percentage of the number of collisions, and the averages spoken of in this section vary imperceptibly. Roughly speaking, the fluctuations are inversely proportional to the square root of the number of collisions, so for small bodies, they can become significant. This was actually observed in the nineteenth century for pollen grains in water and is known as Brownian motion.

2.5. In a liquid, the molecules are very close together, constantly colliding with one another. For a gas to be nearly ideal, as air is under ordinary conditions, the molecules must be very far apart. Therefore the mean free path is much longer in the air.

2.6. As the number of moles is equal and we know the molar heat capacities of the two gases are equal, the temperature is halfway between the initial temperatures, 300 K.

Conceptual Questions

1. 2 moles, as that will contain twice as many molecules as the 1 mole of oxygen

3. pressure

5. The flame contains hot gas (heated by combustion). The pressure is still atmospheric pressure, in mechanical equilibrium with the air around it (or roughly so). The density of the hot gas is proportional to its number density N/V (neglecting the difference in composition between the gas in the flame and the surrounding air). At higher temperature than the surrounding air, the ideal gas law says that $N/V = p/k_B T$ is less than that of the surrounding air. Therefore the hot air has lower density than the surrounding air and is lifted by the buoyant force.

7. The mean free path is inversely proportional to the square of the radius, so it decreases by a factor of 4. The mean free time is proportional to the mean free path and inversely proportional to the rms speed, which in turn is inversely proportional to the square root of the mass. That gives a factor of $\sqrt{8}$ in the numerator, so the mean free time decreases by a factor of $\sqrt{2}$.

9. Since they're more massive, their gravity is stronger, so the escape velocity from them is higher. Since they're farther from the Sun, they're colder, so the speeds of atmospheric molecules including hydrogen and helium are lower. The combination of those facts means that relatively few hydrogen and helium molecules have escaped from the outer planets.

11. One where nitrogen is stored, as excess CO_2 will cause a feeling of suffocating, but excess nitrogen and insufficient oxygen will not.

13. Less, because at lower temperatures their heat capacity was only $3RT/2$.

15. a. false; b. true; c. true; d. true

17. 1200 K

Problems

19. a. 0.137 atm;

b. $p_g = (1 \text{ atm}) \frac{T_2 V_1}{T_1 V_2} - 1 \text{ atm}$. Because of the expansion of the glass, $V_2 = 0.99973$. Multiplying by that factor does not make any significant difference.

21. a. $1.79 \times 10^{-3} \text{ mol}$;

b. 0.227 mol;

c. 1.08×10^{21} molecules for the nitrogen, 1.37×10^{23} molecules for the carbon dioxide

23. $7.84 \times 10^{-2} \text{ mol}$

25. 1.87×10^3

27. 2.47×10^7 molecules

29. $6.95 \times 10^5 \text{ Pa}$; 6.86 atm

31. a. $9.14 \times 10^6 \text{ Pa}$;

b. $8.22 \times 10^6 \text{ Pa}$;

c. 2.15 K;

d. no

33. 40.7 km

35. a. 0.61 N;

b. 0.20 Pa

37. a. 5.88 m/s;

b. 5.89 m/s

39. 177 m/s

41. 4.54×10^3

43. a. 0.0352 mol;

b. $5.65 \times 10^{-21} \text{ J}$;

c. 139 J

45. 21.1 kPa

47. 458 K

49. $3.22 \times 10^3 \text{ K}$

51. a. 1.004;

b. 764 K;

c. This temperature is equivalent to **915°F**, which is high but not impossible to achieve. Thus, this process is feasible. At this temperature, however, there may be other considerations that make the process difficult. (In general, uranium enrichment by gaseous diffusion is indeed difficult and requires many passes.)

53. 65 mol

55. a. 0.76 atm;

- b. 0.29 atm;
 c. The pressure there is barely above the quickly fatal level.
57. $4.92 \times 10^5 K$; Yes, that's an impractically high temperature.
59. polyatomic
61. $3.08 \times 10^3 J$
63. $29.2^\circ C$
65. $-1.6^\circ C$
67. 0.00157
69. About 0.072. Answers may vary slightly. A more accurate answer is 0.074.
71. a. 419 m/s;
 b. 472 m/s;
 c. 513 m/s
73. 541 K
75. 2400 K for all three parts

Additional Problems

77. a. $1.20 kg/m^3$;
 b. $65.9 kg/m^3$
79. 7.9 m
81. a. supercritical fluid;
 b. $3.00 \times 10^7 Pa$
83. 40.18%
85. a. $2.21 \times 10^{27} molecules/m^3$;
 b. $3.67 \times 10^3 mol/m^3$
87. 8.2 mm
89. a. $1080 J/kg^\circ C$;
 b. 12%
91. $2\sqrt{e}/3$ about 1.10
93. a. 411 m/s;
 b. According to Table 2.3, the C_V of H_2 is significantly different from the theoretical value, so the ideal gas model does not describe it very well at room temperature and pressure, and the Maxwell-Boltzmann speed distribution for ideal gases may not hold very well, even less well at a lower temperature.

Challenge Problems

95. 29.5 N/m

97. Substituting $v = \sqrt{\frac{2k_B T}{m}} u$ and $dv = \sqrt{\frac{2k_B T}{m}} du$ gives

$$\int_0^\infty \frac{4}{\sqrt{\pi}} \left(\frac{m}{2k_B T}\right)^{3/2} v^2 e^{-mv^2/2k_B T} dv = \int_0^\infty \frac{4}{\sqrt{\pi}} \left(\frac{m}{2k_B T}\right)^{3/2} \left(\frac{2k_B T}{m}\right) u^2 e^{-u^2} \sqrt{\frac{2k_B T}{m}} du = \int_0^\infty \frac{4}{\sqrt{\pi}} u^2 e^{-u^2} du$$

$$= \frac{4}{\sqrt{\pi}} \frac{\sqrt{\pi}}{4} = 1$$

99. Making the scaling transformation as in the previous problems, we find that $\bar{v}^2 = \int_0^\infty \frac{4}{\sqrt{\pi}} \left(\frac{m}{2k_B T}\right)^{3/2} v^2 e^{-mv^2/2k_B T} dv = \int_0^\infty \frac{4}{\sqrt{\pi}} \frac{2k_B T}{m} u^4 e^{-u^2} du$. As in the previous problem, we integrate by parts: $\int_0^\infty u^4 e^{-u^2} du = \left[-\frac{1}{2} u^3 e^{-u^2}\right]_0^\infty + \frac{3}{2} \int_0^\infty u^2 e^{-u^2} du$. Again, the first term is 0, and we were given in an earlier problem that the integral in the second term equals $\frac{\sqrt{\pi}}{4}$. We now have $\bar{v}^2 = \frac{4}{\sqrt{\pi}} \frac{2k_B T}{m} \frac{3}{2} \frac{\sqrt{\pi}}{4} = \frac{3k_B T}{m}$. Taking the square root of both sides gives the desired result: $v_{rms} = \sqrt{\frac{3k_B T}{m}}$.

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