

## 7.3: Solving Problems with Newton's Laws (Part 2)

### Newton's Laws of Motion and Kinematics

Physics is most interesting and most powerful when applied to general situations that involve more than a narrow set of physical principles. Newton's laws of motion can also be integrated with other concepts that have been discussed previously in this text to solve problems of motion. For example, forces produce accelerations, a topic of kinematics, and hence the relevance of earlier chapters.

When approaching problems that involve various types of forces, acceleration, velocity, and/or position, listing the givens and the quantities to be calculated will allow you to identify the principles involved. Then, you can refer to the chapters that deal with a particular topic and solve the problem using strategies outlined in the text. The following worked example illustrates how the problem-solving strategy given earlier in this chapter, as well as strategies presented in other chapters, is applied to an integrated concept problem.

#### ✓ Example 6.6: What Force Must a Soccer Player Exert to Reach Top Speed?

A soccer player starts at rest and accelerates forward, reaching a velocity of 8.00 m/s in 2.50 s. (a) What is her average acceleration? (b) What average force does the ground exert forward on the runner so that she achieves this acceleration? The player's mass is 70.0 kg, and air resistance is negligible.

##### Strategy

To find the answers to this problem, we use the problem-solving strategy given earlier in this chapter. The solutions to each part of the example illustrate how to apply specific problem-solving steps. In this case, we do not need to use all of the steps. We simply identify the physical principles, and thus the knowns and unknowns; apply Newton's second law; and check to see whether the answer is reasonable.

##### Solution

- a. We are given the initial and final velocities (zero and 8.00 m/s forward); thus, the change in velocity is  $\Delta v = 8.00 \text{ m/s}$ . We are given the elapsed time, so  $\Delta t = 2.50 \text{ s}$ . The unknown is acceleration, which can be found from its definition:

$$a = \frac{\Delta v}{\Delta t}. \quad (7.3.1)$$

Substituting the known values yields

$$a = \frac{8.00 \text{ m/s}}{2.50 \text{ s}} = 3.20 \text{ m/s}^2. \quad (7.3.2)$$

- b. Here we are asked to find the average force the ground exerts on the runner to produce this acceleration. (Remember that we are dealing with the force or forces acting on the object of interest.) This is the reaction force to that exerted by the player backward against the ground, by Newton's third law. Neglecting air resistance, this would be equal in magnitude to the net external force on the player, since this force causes her acceleration. Since we now know the player's acceleration and are given her mass, we can use Newton's second law to find the force exerted. That is,

$$F_{\text{net}} = ma. \quad (7.3.3)$$

Substituting the known values of  $m$  and  $a$  gives

$$F_{\text{net}} = (70.0 \text{ kg})(3.20 \text{ m/s}^2) = 224 \text{ N}. \quad (7.3.4)$$

This is a reasonable result: The acceleration is attainable for an athlete in good condition. The force is about 50 pounds, a reasonable average force.

##### Significance

This example illustrates how to apply problem-solving strategies to situations that include topics from different chapters. The first step is to identify the physical principles, the knowns, and the unknowns involved in the problem. The second step is to solve for the unknown, in this case using Newton's second law. Finally, we check our answer to ensure it is reasonable. These

techniques for integrated concept problems will be useful in applications of physics outside of a physics course, such as in your profession, in other science disciplines, and in everyday life.

### ? Exercise 6.4

The soccer player stops after completing the play described above, but now notices that the ball is in position to be stolen. If she now experiences a force of 126 N to attempt to steal the ball, which is 2.00 m away from her, how long will it take her to get to the ball?

### ✓ Example 6.7: What Force Acts on a Model Helicopter?

A 1.50-kg model helicopter has a velocity of  $5.00 \hat{j}$  m/s at  $t = 0$ . It is accelerated at a constant rate for two seconds (2.00 s) after which it has a velocity of  $(6.00 \hat{i} + 12.00 \hat{j})$  m/s. What is the magnitude of the resultant force acting on the helicopter during this time interval?

#### Strategy

We can easily set up a coordinate system in which the x-axis ( $\hat{i}$  direction) is horizontal, and the y-axis ( $\hat{j}$  direction) is vertical. We know that  $\Delta t = 2.00$  s and  $\Delta \mathbf{v} = (6.00 \hat{i} + 12.00 \hat{j})$  m/s  $- (5.00 \hat{j})$  m/s. From this, we can calculate the acceleration by the definition; we can then apply Newton's second law.

#### Solution

We have

$$\begin{aligned} \mathbf{a} = \frac{\Delta \mathbf{v}}{\Delta t} &= \frac{(6.00 \hat{i} + 12.00 \hat{j}) \text{ m/s} - (5.00 \hat{j}) \text{ m/s}}{2.00 \text{ s}} = 3.00 \hat{i} + 3.50 \hat{j} \text{ m/s}^2 \\ &= (1.50 \text{ kg})(3.00 \hat{i} + 3.50 \hat{j} \text{ m/s}^2) = 4.50 \hat{i} + 5.25 \hat{j} \text{ N}. \end{aligned} \quad (7.3.5)$$

The magnitude of the force is now easily found:

$$F = \sqrt{(4.50 \text{ N})^2 + (5.25 \text{ N})^2} = 6.91 \text{ N}. \quad (7.3.6)$$

#### Significance

The original problem was stated in terms of  $\hat{i} - \hat{j}$  vector components, so we used vector methods. Compare this example with the previous example.

### ? Exercise 6.5

Find the direction of the resultant for the 1.50-kg model helicopter.

### ✓ Example 6.8: Baggage Tractor

Figure 7.3.7(a) shows a baggage tractor pulling luggage carts from an airplane. The tractor has mass 650.0 kg, while cart A has mass 250.0 kg and cart B has mass 150.0 kg. The driving force acting for a brief period of time accelerates the system from rest and acts for 3.00 s. (a) If this driving force is given by  $F = (820.0t)$  N, find the speed after 3.00 seconds. (b) What is the horizontal force acting on the connecting cable between the tractor and cart A at this instant?

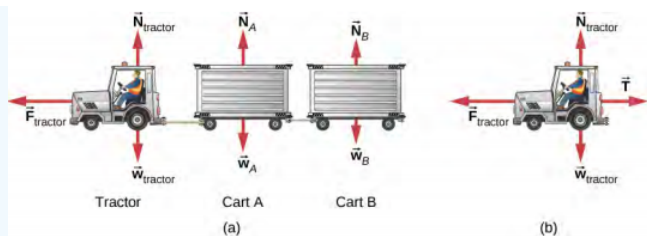


Figure 7.3.7: (a) A free-body diagram is shown, which indicates all the external forces on the system consisting of the tractor and baggage carts for carrying airline luggage. (b) A free-body diagram of the tractor only is shown isolated in order to calculate the tension in the cable to the carts.

### Strategy

A free-body diagram shows the driving force of the tractor, which gives the system its acceleration. We only need to consider motion in the horizontal direction. The vertical forces balance each other and it is not necessary to consider them. For part b, we make use of a free-body diagram of the tractor alone to determine the force between it and cart A. This exposes the coupling force  $\vec{T}$ , which is our objective.

### Solution

a. 
$$\sum F_x = m_{\text{system}} a_x \text{ and } \sum F_x = 820.0t, \quad (7.3.7)$$

so

$$820.0t = (650.0 + 250.0 + 150.0)a \quad (7.3.8)$$

$$a = 0.7809t. \quad (7.3.9)$$

Since acceleration is a function of time, we can determine the velocity of the tractor by using  $a = \frac{dv}{dt}$  with the initial condition that  $v_0 = 0$  at  $t = 0$ . We integrate from  $t = 0$  to  $t = 3$ :

$$\begin{aligned} dv &= a dt \\ \int_0^3 dv &= \int_0^{3.00} a dt = \int_0^{3.00} 0.7809t dt \\ v &= 0.3905t^2 \Big|_0^{3.00} = 3.51 \text{ m/s}. \end{aligned}$$

b. Refer to the free-body diagram in Figure 7.3.7(b)

$$\begin{aligned} \sum F_x &= m_{\text{tractor}} a_x \\ 820.0t - T &= m_{\text{tractor}} (0.7805)t \\ (820.0)(3.00) - T &= (650.0)(0.7805)(3.00) \\ T &= 938 \text{ N}. \end{aligned}$$

### Significance

Since the force varies with time, we must use calculus to solve this problem. Notice how the total mass of the system was important in solving Figure 7.3.7(a), whereas only the mass of the truck (since it supplied the force) was of use in Figure 7.3.7(b).

Recall that  $v = \frac{ds}{dt}$  and  $a = \frac{dv}{dt}$ . If acceleration is a function of time, we can use the calculus forms developed in [Motion Along a Straight Line](#), as shown in this example. However, sometimes acceleration is a function of displacement. In this case, we can derive an important result from these calculus relations. Solving for  $dt$  in each, we have  $dt = \frac{ds}{v}$  and  $dt = \frac{dv}{a}$ . Now, equating these expressions, we have  $\frac{ds}{v} = \frac{dv}{a}$ . We can rearrange this to obtain  $a ds = v dv$ .

### ✓ Example 6.9: Motion of a Projectile Fired Vertically

A 10.0-kg mortar shell is fired vertically upward from the ground, with an initial velocity of 50.0 m/s (see Figure 7.3.8). Determine the maximum height it will travel if atmospheric resistance is measured as  $F_D = (0.0100 v^2) \text{ N}$ , where  $v$  is the speed at any instant.



Figure 7.3.8: (a) The mortar fires a shell straight up; we consider the friction force provided by the air. (b) A free-body diagram is shown which indicates all the forces on the mortar shell.

### Strategy

The known force on the mortar shell can be related to its acceleration using the equations of motion. Kinematics can then be used to relate the mortar shell's acceleration to its position.

### Solution

Initially,  $y_0 = 0$  and  $v_0 = 50.0$  m/s. At the maximum height  $y = h$ ,  $v = 0$ . The free-body diagram shows  $F_D$  to act downward, because it slows the upward motion of the mortar shell. Thus, we can write

$$\begin{aligned}\sum F_y &= ma_y \\ -F_D - w &= ma_y \\ -0.0100v^2 - 98.0 &= 10.0a \\ a &= -0.00100v^2 - 9.80.\end{aligned}$$

The acceleration depends on  $v$  and is therefore variable. Since  $a = f(v)$ , we can relate  $a$  to  $v$  using the rearrangement described above,

$$ads = vdv. \quad (7.3.10)$$

We replace  $ds$  with  $dy$  because we are dealing with the vertical direction,

$$\begin{aligned}ady &= vdv \\ (-0.00100v^2 - 9.80)dy &= vdv.\end{aligned}$$

We now separate the variables ( $v$ 's and  $dv$ 's on one side;  $dy$  on the other):

$$\begin{aligned}\int_0^h dy &= \int_{50.0}^0 \frac{v dv}{(-0.00100v^2 - 9.80)} \\ &= - \int_{50.0}^0 \frac{v dv}{(-0.00100v^2 + 9.80)} \\ &= (-5 \times 10^3) \ln(0.00100v^2 + 9.80) \Big|_{50.0}^0.\end{aligned}$$

Thus,  $h = 114$  m.

### Significance

Notice the need to apply calculus since the force is not constant, which also means that acceleration is not constant. To make matters worse, the force depends on  $v$  (not  $t$ ), and so we must use the trick explained prior to the example. The answer for the height indicates a lower elevation if there were air resistance. We will deal with the effects of air resistance and other drag forces in greater detail in [Drag Force and Terminal Speed](#).

### ? Exercise 6.6

If atmospheric resistance is neglected, find the maximum height for the mortar shell. Is calculus required for this solution?

### 📌 Simulation

Explore the forces at work in [this simulation](#) when you try to push a filing cabinet. Create an applied force and see the resulting frictional force and total force acting on the cabinet. Charts show the forces, position, velocity, and acceleration vs. time. View a free-body diagram of all the forces (including gravitational and normal forces).

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