7.2: Buoyancy

Archimedes's Principle

In the previous chapter, we discussed the pressure difference between the top and bottom of a section of the fluid. What happens if we replace that same section with a solid object? As we have seen, the presence of the solid object doesn't affect the pressure difference at the two heights, since the fluid is continuous and static. But in the case of a solid object, the higher pressure at the bottom and the lower pressure at the top result in actual forces on the bottom and top surfaces of the object. The result of these two forces is a total force by the fluid upward, which is called the *buoyancy force*.

We concluded in the previous section that the pressures on top and bottom differed just enough to balance the weight of the section of fluid, so we know precisely what the resulting force of this unbalanced pressure is – it is the weight of the fluid that would be in that section if the solid object was not there. Put concisely, we have *Archimedes's principle*:

The buoyancy force on an object in a fluid equals the weight of the fluid displaced by that object.

As simple as this seems, it is very easy to get confused about this force. The main source of confusion tends to be distinguishing the buoyancy force from the net force on the object (which also experiences gravity). Here are some secrets to winding one's way through the daunting mazes commonly encountered regarding buoyancy:

- Keep in mind that buoyancy is *just* the force from the fluid on the object – it is completely independent from the gravity force on the object, and it is not the net force. Draw force diagrams whenever possible, with separate force vectors for gravity and buoyancy, and apply Archimedes's principle *only* to the buoyancy force vector.
- Apply Archimedes's principle *very strictly* – the volume of fluid displaced does not always equal the volume of the object (it has to be completely submerged for that).
- Be careful about drawing conclusions based on the density of the object only – whether an object sinks or floats, it experiences the buoyancy force described by Archimedes's principle.

Assuming the object in the fluid is completely submerged, then its full volume displaces fluid. This means it feels a buoyancy force equal to the weight of fluid that occupies that same volume. The net force on such an object is the buoyancy force up minus the gravity force down, so if the object weighs more than the displaced fluid, it sinks. Given that the object and the displaced fluid have equal volumes, we can just as easily compare their mass-to-volume ratios to determine which is heavier. This ratio for the fluid is simply its uniform density. For the object, it is its *average* density. The distinction is that the object may, for example, be hollow. This explains how an aircraft carrier, made of materials significantly denser than water, can float – the hollow parts of the vessel that contain only air reduce its average density greatly.

Examples

The full flower of the tricky topic of buoyancy only becomes apparent with examples, so here are a few...

Example 7.2.1

Two identical hollow metal cubes are in the Earth's atmosphere. One cube contains helium (at one atmosphere of pressure), and the other a vacuum. Which of these cubes experiences the greatest buoyancy force, and which registers the lowest weight on a scale?

Solution

The cubes are identical, so they displace the same volume of air, and the buoyancy force is the same on both. Because the buoyancy forces are equal, the cube that registers the higher weight on the scale will be the one that possesses the greater mass. Helium has more mass than a vacuum, so the cube filled with helium will register a larger weight on the scale. [Note: Helium does not "naturally rise" and pull things upward. It has mass, like all matter, and is therefore subject to gravity like everything else!]

Example 7.2.2

A tub of water sits on a spring scale. A toy boat is floated on the surface of the water, and the scale is read. Later, the boat is submerged until it takes on enough water to sink and settle on the bottom. Still later, the boat is removed from the water and placed on the scale beside the tub. Order the weighings from least to greatest.

Solution

The simple answer is that all of the forces between the water, the tub, and the boat are internal to the water + tub + boat system, and therefore all have Newton's third law pairs that cancel each other. The force external to the system, from the spring scale, is the same in all three cases.

While this makes sense, it may be a bit unsatisfying, so let's look very briefly at the details of the internal forces. Strange as it seems, buoyancy forces come in third-law pairs, just like every other force, which means that while the water pushes up on a floating boat, the floating boat pushes down equally on the body of water. The scale beneath the tub of water must be great enough to balance all the other forces on it, which includes the weight of the tub + water ***plus*** *the buoyancy force down on the water by the boat. Since the boat isn't accelerating up or down, the buoyancy force up on the boat must equal its weight, which means that the buoyancy force down on the water (and balanced by the scale) is exactly the weight of the boat.*

One might try to argue that the force down on the tub by the water must equal the pressure of the water at the bottom multiplied by the area of the bottom of the tub, and aren't these two things the same whether the boat is in the water or not? No! When the boat is floating, it is displacing water. Where does this displaced water go? Nowhere - it just gets deeper! The increased depth changes the pressure at the bottom just enough to contribute an additional force equal to the buoyancy force on/by the boat, and with the boat floating, this equals the weight of the boat.

Example $7.2.3$

A small balloon containing air (which we can treat as an ideal gas) has negligible mass, and is attached to a rock with a string. This combination is thrown into the middle of Lake Tahoe, and the hanging rock pulls down on the balloon enough to submerge one-eighth of its volume. A scuba diver grabs the balloon and submerges it, swimming downward. Find the depth to which the balloon must be submerged such that it will sink when it is released. Assume the temperature of the water doesn't change appreciably during the dive.

Solution

The gravity force on the rock equals the weight of water occupying one-eighth of the balloon's volume in the atmosphere. Therefore, when the balloon no longer displaces at least one eighth of its starting volume, the buoyancy force will be insufficient to keep it from sinking. But what would cause the balloon to stop displacing so much volume? The pressure on the outside of the balloon approximately balances the pressure inside the balloon (not counting the force from the balloon's elasticity), and as the balloon goes deeper, the outside pressure gets greater. With the temperature and the number of moles not changing, the increased pressure must be balanced by decreased volume.

The volume of the balloon must decrease by a factor of 8 to keep sinking, so the pressure of the water must equal 8 times atmospheric pressure. We found in the digression of the previous section that 1 atmosphere of pressure is added for approximately every 10 meters of depth in fresh water. The balloon starts at the surface with 1 atmosphere, so it needs 7 more atmospheres to reach the goal of one eighth of its original volume – the diver must push the balloon $70m$ *below the surface in order to pollute the bottom of Lake Tahoe with it.*

While this answer is in keeping with information we have been given to this point, it is not actually correct, and is off by a fairly large margin! The problem lies not in the logic of the solution, but in the assumption that the pressure increases by the amount of the atmosphere with every 10 meters of added depth. This is incorrect because that rule-of-thumb only applies to atmospheric pressure at sea level. The pressure of the atmosphere at the elevation of Lake Tahoe is only about 79% of what we refer to as "atmospheric pressure." This means that at the surface of the lake the pressure of the air in the balloon is 0.79 atmospheres, and in order to increase the pressure by that much, the balloon must be submerged only $7.9m$*. Following the logic of the original solution, in order to increase the pressure to eight times the pressure at the lake surface, the balloon must be submerged* $7 \times 7.9m \approx 55m$.