9.3: Block Spin Transformation

Spin blocking refers to a process in which we replace a group of spins by a single spin whose direction is determined by 'majority rule'. That is, if most of the spins in the group are up, then the block spin is said to be up; if most spins are down, then the block spin is down. The block spins interact with a different set of couplings $\{K'_\alpha\}$.

Consider a $b\times b\times\cdots\times b$ block of spins, and define the *block spin projector*,

$$\mathcal{T}\big(\sigma'_a\,,\,\{\sigma_i\}\big)=\begin{cases}1 & \text{of}\ \sigma'_a=sgn\left(\sum_{i=1}^{b^d}\sigma_{a,i}\right)\\ 0 & \text{otherwise}\ .\end{cases}\tag{9.3.1}$$

Note that

$$\sum_{\sigma_a}\mathcal{T}\big(\sigma'_a\,,\,\{\sigma_{a,i}\}\big)=1\ .\tag{9.3.2}$$

Here a indexes the blocks, and $\sigma_{a,i}$ denotes the i^{th} spin within the a^{th} block. The block spin projector effects the 'majority rule' operation, assigning σ'_a to ±1 depending on whether the majority of the spins in the block a are up $(\sigma'_a=+1)$ or down $(\sigma'_a=-1)$. Note that such a procedure presumes an odd number of spins in each block. Then

$$\begin{aligned}Z&=\sum_{\{\sigma_i\}}e^{-\beta\hat{H}[\{\sigma_i\}]}\\ &=\sum_{\{\sigma'_a\}}\left\{\sum_{\{\sigma_i\}}e^{-\beta\hat{H}[\{\sigma_{a.i}\}]}\prod_a\mathcal{T}\big(\sigma'_a\,,\,\{\sigma_{a,i}\}\big)\right\}\\ &=\sum_{\{\sigma'_a\}}e^{-\beta\hat{H}'[\{\sigma'_a\}]}\ ,\end{aligned}$$

where

$$e^{-\beta\hat{H}'[\{\sigma'_a\}]}=\sum_{\{\sigma_i\}}e^{-\beta\hat{H}[\{\sigma_i\}]}\prod_a\mathcal{T}\big(\sigma'_a\,,\,\{\sigma_{a,i}\}\big)\ .\tag{9.3.3}$$