19.7: Summary

Key Takeaways

Electric current, I, is defined as the rate at which charges cross some plane (for example a plane perpendicular to a wire) per unit time. That is, if an amount of charge, ΔQ, enters a wire during an amount of time, Δt, the current, I, in that wire is defined to be:

$$I = \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

where a derivative is taken if the rate at which charges are moving is not constant with time.

Electric current is a macroscopic quantity that can be measured. Conventional current is defined to be positive in the direction in which positive charges flow. In most situations, it is electrons that move inside a conductor, so the current is defined to be positive in the *opposite direction* of the actual motion of the (negative) electrons.

The current density, $\vec j$, is defined to be the current per unit area at some point in a conductor, and is a vector in the direction of the electric field, $\hat E$, at that point:

$$\vec j = \frac{I}{A}\hat E$$

The current density can be related to the microscopic motion of charges within the conductor. If the current density, $\vec j$, is known, the corresponding current, I, that crosses a surface with area, A, and normal vector, $\hat n$, is given by:

$$I = A\vec j \cdot \hat n = \int \vec j \cdot d\vec A$$

where the integral must be taken if the current density is not constant over the surface.

A conducting material through which current is flowing is called a resistor. When a potential difference is applied across a resistor, the resulting electric field will drive the flow of electrons through the resistor. The electrons will flow with an average "drift velocity", $\vec v_d$, which is much lower than the actual (Fermi) speed of the electrons in the material. Inside the resistor, electrons are constantly accelerated before they collide with atoms in the material losing their kinetic energy, and then accelerating again. Thus, the potential energy that is available to the electrons is "used" to heat the resistor, and the electrons, on average, drift quite slowly through the resistor.

The current density in a resistor can be related to the drift velocity of the electrons and the "density of free electrons" in the material, n:

$$\vec j = -en\vec v_d$$

where, e, is the magnitude of the charge of the electrons and the minus sign indicates that the current density is in the opposite direction of the velocity of the (negative) electrons.

Ohm's Law states that the current density, $\vec j$, at some point in the conductor is proportional to the electric field, $\vec E$, at that point:

$$\vec j = \sigma \vec E = \frac{1}{\rho}\vec E$$

where the constant of proportionality, σ, is called the "conductivity" of the material (and is a property of the material through which current is flowing). The resistivity, ρ, is a material property that is simply the inverse of the conductivity. Both of these properties are a measure of how large a current (or current density) will exist in a material given a certain electric field. For example, the conductivity of an insulating material is close to zero (and its resistivity close to infinity).

For most materials, resistivity usually increases linearly with temperature:

$$\rho(T) = \rho_0[1 + \alpha(T - T_0)]$$

where ρ_0 is the resistivity as measured at some reference temperature, T_0 (usually $10^\circ\mathrm{C}$), and α, is the "temperature coefficient" for that material. Note that this model of resistivity does not hold for extreme temperatures (very cold or very hot), and for some materials, resistivity decreases with temperature (α is negative).

If we apply Ohm's Law to a resistor of length, L, cross-sectional area, A, made of a material with resistivity, ρ, we find that the potential difference applied across the resistor, ΔV, is proportional to the current flowing through the resistor:

$$\Delta V=\rho\frac{L}{A}I$$

The constant of proportionality depends on the material with which the resistor is made (through the resistivity) and on the dimensions of the resistor (through the length and cross-sectional area). The constant of proportionality is called the "resistance" of the resistor, R:

$$R=\rho\frac{L}{A}$$

Ohm's Law is often written for a resistor as the relationship between the current through the resistor, I, and the potential difference across the resistor, ΔV:

$$\Delta V=RI$$

although, technically, Ohm's Law is the relation between current density and electric field.

Resistors can be combined in series, in which case, the effective resistance of the combination is found by adding the resistances of the individual resistors:

$$R_{eff}=R_1+R_2+R_2+\dots\quad\text{(Series resistors)}$$

When combined in parallel, the inverse of the effective resistance is given by the inverse of the sum of the inverse of the resistances of the individual resistors:

$$R_{eff}=\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\dots}\quad\text{(Parallel resistors)}$$

As charges move through a resistor of resistance, R, under a potential difference, ΔV, and current, I, they transfer their kinetic energy into heating up the resistor. The rate at which they transfer the energy, also called the "power dissipated in the resistor", is given by:

$$P=I\Delta V=\frac{(\Delta V)^2}{R}=I^2R$$

where the various combinations can be obtained by applying the macroscopic version of Ohm's Law.

The electrical outlets in our daily lives provide an "alternating" voltage, $\Delta V(t)$, which oscillates sinusoidally:

$$\Delta V(t)=\Delta V_0\sin(\omega t)$$

with a maximum amplitude, ΔV_0, and an angular frequency, $\omega=2\pi f$. When this potential difference is applied across a resistor, an alternating current is formed, in which the electrons move back and forth, with no net displacement:

$$I(t)=\frac{\Delta V_0}{R}=I_0\sin(\omega t)$$

Even though there is not net displacement, the electrons will still transfer energy into the resistor in the form of heat. The average rate at which power is dissipated in the resistor is given by:

$$\bar{P}=\frac{1}{2}RI_0^2$$

We introduce the "root mean square" current (voltage), I_{rms} (ΔV_{rms}), as an average effective current (voltage):

$$\begin{aligned}I_{rms}&=\frac{1}{\sqrt{2}}I_0\\\Delta V_{rms}&=\frac{1}{\sqrt{2}}\Delta V_0\end{aligned}$$

such that the power can be expressed using a similar formula as in the direct current case, using the root mean square values:

$$\bar{P} = I_{rms}^2 R = I_{rms} \Delta V_{rms} = \frac{(\Delta V_{Rms})^2}{R}$$

There are two main types of hazards associated with the use of electricity: fire and electrocution. Electrical fires can arise when a large current goes through a wire, since this will dissipate a large amount of heat into the wire (which could set fire to its insulation or other nearby flammable items). Electrocution occurs when a current traverses the human body. If a current above $\sim 80\text{mA}$ crosses the upper body, this can result in ventricular fibrillation, whereby the heart beats very irregularly. Of course, one can also be burned by a large current. The amount of current through the body is what will ultimately determine the severity of injuries, and is why one often hears that "it's current that kills". A large voltage may not lead to a large current if the resistance of the person is large or if the power supply cannot provide a large current at that large voltage.

Important Equations

Current:

$$I = \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

Current density:

$$\vec{j} = \frac{I}{a}\hat{E}$$

$$I = \int \vec{j} \cdot d\vec{A}$$

Microscopic model of current:

$$\vec{j} = -en\vec{v}_d$$

Ohm's Law:

$$\vec{j} = \sigma \vec{E}$$

Resistivity:

$$\rho = \frac{1}{\sigma}$$

$$\rho(T) = \rho_0[1 + \alpha(T - T_0)]$$

Resistance:

$$R = \rho \frac{L}{A}$$

Ohm's Law (macroscopic):

$$\Delta V = RI$$

Combining resistors:

$$R_{eff} = R_1 + R_2 + R_2 + \dots \quad \text{(Series)}$$

$$R_{eff} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots} \quad \text{(Parallel)}$$

Power dissipated in a resistor:

$$P = I\Delta V = \frac{(\Delta V)^2}{R} = I^2 R$$

RMS voltage and current:

$$I_{rms} = \frac{1}{\sqrt{2}} I_0$$

$$\Delta V_{rms} = \frac{1}{\sqrt{2}} \Delta V_0$$

Important Definitions

Definition

Current: The rate at which charges flow across a given surface. SI units: $[\mathrm{A}]$. Common variable(s): I.

Definition

Current density: A measure of current per unit area, in the direction of the local electric field. SI units: $[\mathrm{Am}^{-1}]$. Common variable(s): $\vec{j}$.

Definition

Resistance: A measure of a specific object's opposition to the flow of charge. SI units: $[\Omega]$. Common variable(s): R.

Definition

Resistivity: A measure of a material's opposition to the flow of charge. SI units: $[\Omega\mathrm{m}]$. Common variable(s): ρ.

Definition

Conductivity: The inverse of resistivity. SI units: $[\Omega^{-1}\mathrm{m}^{-1}]$. Common variable(s): σ.

Definition

Drift velocity: The average velocity of an electron drifting in a conductor under the influence of an electric field. SI units: $[\mathrm{ms}^{-1}]$. Common variable(s): $\vec{v}_d$.