19.9: Sample problems and solutions

Exercise 19.9.1

A cylindrical wire as a current density that increases with radius as $j(r) = ar$, where r, is the radial distance from the center of the wire, and a, is a constant. If the wire has a radius of $R = 1.5\text{cm}$, what is the total current in the wire?

Answer

To determine the current through the entire cross section of the wire, we first divide the cross-section of the wire into infinitesimally small concentric rings of radius, r, and width, dr. The cross-sectional area of one ring is given by:

$$dA = 2\pi r dr$$

so that the current through one ring is given by:

$$dI = j(r)dA = 2\pi ar^2 dr$$

The current through the whole wire is then found by summing the currents through each ring:

$$I = \int dI = \int_0^R 2\pi ar^2 dr = \frac{2}{3}\pi aR^3$$

Exercise 19.9.2

A resistor is measured to have a resistance of $R_1 = 103.4\Omega$ at a temperature of $T_1 = 30^{\circ}\text{C}$, and a resistance of $R_2 = 106.8\Omega$ at a temperature of $T_1 = 40^{\circ}\text{C}$. Using the values in *Table 19.3.1*, determine the material from which the resistor is made.

Answer

To determine the material of the resistor, we can find the temperature coefficient, α, since we are given measurements of resistance, R_1 and R_2, at two different temperatures, T_1, and T_2, respectively. The reference temperature is set to be $T_0 = 20^{\circ}\text{Celsius}$, so that we can compare with *Table 19.3.1*.

We know that the resistance will vary with temperature, since the resistivity is temperature-dependent. The temperature dependence of resistivity is given by:

$$\rho(T) = \rho_0[1 + \alpha(T - T_0)]$$

If the resistor has length, L, and cross-sectional area, A, it will have resistance, R, given by:

$$R(T) = \rho(T)\frac{L}{A} = \frac{\rho_0 L}{A}[1 + \alpha(T - T_0)] = R_0[1 + \alpha(T - T_0)]$$

where R_0 is the resistance at the reference temperature, T_0. Since we are given the resistance at two different temperatures, we can determine both α and R_0, for a choice of $T_0 = 20^{\circ}\text{C}$:

$$
\begin{aligned}
R_1 &= R_0[1+\alpha(T_1-T_0)] \\
R_2 &= R_0[1+\alpha(T_2-T_0)] \\
\therefore \frac{R_1}{R_2} &= \frac{1+\alpha(T_1-T_0)}{1+\alpha(T_2-T_0)} \\
R_1[1+\alpha(T_2-T_0)] &= R_2[1+\alpha(T_1-T_0)] \\
\alpha\left(R_1(T_2-T_0)-R_2(T_2-T_0)\right) &= R_2-R_1 \\
\therefore \alpha &= \frac{R_2-R_1}{R_1(T_2-T_0)-R_2(T_1-T_0)} \\
&= \frac{(106.8\Omega)-(103.4\Omega)}{(103.4\Omega)((40^\circ\text{Celsius})-(20^\circ\text{Celsius}))-(106.8\Omega)((30^\circ\text{Celsius})-(20^\circ\text{Celsius}))} \\
&= 0.0034
\end{aligned}
$$

Referring to *Table 19.3.1*, the material could likely be gold.