7.4: Summary

Key Takeaways

The work, W, done on an object by a force, $\vec{F}$, while the object has moved through a displacement, $\vec{d}$, is defined as the scalar product:

$$\begin{aligned} W = \vec{F}\cdot\vec{d} &= Fd\cos\theta \\ &= F_x d_x + F_y d_y + F_z d_z \end{aligned}$$

If the force changes with position and/or the object moves along an arbitrary path in space, the work done by that force over the path is given by:

$$W = \int_A^B \vec{F}(\vec{r})\cdot d\vec{l}$$

Work allows us to quantify how a force acting on an object changes the speed of that object.

If multiple forces are exerted on an object, then one can calculate the net force on the object (the vector sum of the forces), and the net work done on the object will be equal to the work done by the net force:

$$W^{net} = \int_A^B \vec{F}^{net}(\vec{r})\cdot d\vec{l}$$

If the net work done on an object is zero, that object does not accelerate. We can define the kinetic energy, $K(v)$ of an object of mass m that has speed v as:

$$K(v) = \frac{1}{2}mv^2$$

The Work-Energy Theorem states that the net work done on an object in going from position A to position B is equal to the object's change in kinetic energy:

$$W^{net} = \Delta K = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2$$

where v_A and v_B are the speed of the object at positions A and B, respectively.

The rate at which work is being done is called power and is defined as:

$$P = \frac{dW}{dt}$$

If a constant force $\vec{F}$ is exerted on an object that has a constant velocity $\vec{v}$, then the power that corresponds to the work being done by that force is:

$$\begin{aligned} P &= \frac{d}{dt}W = \frac{d}{dt}(\vec{F}\cdot\vec{d}) \\ &= \vec{F}\cdot\frac{d}{dt}\vec{d} = \vec{F}\cdot\vec{v} \end{aligned}$$

Important Equations

Work:

$$\begin{aligned} W &= \vec{F} \cdot \vec{d} = F d \cos\theta \\ W &= F_x d_x + F_y d_y + F_z d_z \\ W &= \int_A^B \vec{F}(\vec{r}) \cdot d\vec{l} \\ W^{net} &= \int_A^B \vec{F}^{net}(\vec{r}) \cdot d\vec{l} \end{aligned}$$

Kinetic Energy:

$$K(v) = \frac{1}{2}mv^2$$

Work-Energy Theorem:

$$W^{net} = \Delta K = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2$$

Power:

$$\begin{aligned} P &= \frac{dW}{dt} \\ P &= \vec{F} \cdot \vec{v} \end{aligned}$$

Important Definitions

> Definition
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> **Work**: A scalar quantity to quantity the amount of energy that a force can input into a system when it is exerted over a given distance. SI units: $[\mathrm{J}]$. Common variable(s): W.

> Definition
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> **Kinetic energy**: A form of energy that an object with a mass has by virtue of having a non-zero speed. SI units: $[\mathrm{J}]$. Common variable(s): K.

> Definition
>
> **Power**: The rate at which energy is converted with respect to time. SI units: $[\mathrm{W}]$. Common variable(s): P.