# 4.5: Summary

## Key Takeaways

When the motion of an object is in more than one dimension, we describe the position of the object using a vector, $\vec{r}$.

$$\vec{r}(t)=\begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix}=x(t)\hat{x}+y(t)\hat{y}+z(t)\hat{z}$$

where $x(t)$, $y(t)$, and $z(t)$, are the position coordinates of the object. We treat the motion in each dimension as independent.

The instantaneous velocity vector and the acceleration vector are given by:

$$\vec{v}(t)=\frac{d}{dt}\vec{r}(t)$$
$$\vec{a}(t)=\frac{d}{dt}\vec{v}(t)$$

If the acceleration vector is constant (in magnitude and direction), then the position and velocity of the object are described by:

$$\begin{aligned}\vec{r}(t) &= \vec{r}_0+\vec{v}_0 t+\frac{1}{2}\vec{a}t^2 \\ \vec{v}(t) &= \vec{v}_0+\vec{a}t\end{aligned}$$

where each of these vector equations represents 3 independent equations, one for each of the $x$, $y$, and $z$ component of the vectors.

If an object has position $\vec{r}^A$ as measured in a frame of reference $xy$ that is moving at constant speed $\vec{v}'^B$ as measured in a second frame of reference $x'y'$, then in the $x'y'$ reference frame:

$$\begin{aligned}\vec{r}'^A(t) &= \vec{v}'^B t+\vec{r}^A(t) \\ \vec{v}'^A(t) &= \vec{v}'^B+\vec{v}^A(t) \\ \vec{a}'^A(t) &= \vec{a}^A(t)\end{aligned}$$

An acceleration can change the magnitude and/or the direction of the velocity vector.

1. The component of the acceleration vector that is parallel to the velocity vector changes the magnitude of the velocity.
2. The component of the acceleration vector that is perpendicular to the velocity vector changes the direction of the velocity.

The acceleration vector for motion in two dimensions can be written as the sum of vectors that are parallel ($\vec{a}_{\parallel}$) and perpendicular ($\vec{a}_{\perp}$) to the velocity vector:

$$\vec{a}=\frac{dv}{dt}\hat{v}(t)+v(t)\frac{d\hat{v}}{dt}=\vec{a}_{\parallel}+\vec{a}_{\perp}$$

If the position of an object moving in a circle of radius $R$ is described by its position along the curved axis $s$, then its position along the circle can be described using an angle, $\theta$, in radians:

$$\theta(t)=\frac{s(t)}{R}$$

For an object moving along a circle, we can write its position vector, $\vec{r}(t)$, as:

$$\vec{r}(t)=\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}=R\begin{pmatrix} \cos(\theta(t)) \\ \sin(\theta(t)) \end{pmatrix}$$

The angular velocity, $\omega$, is the rate of change of the angle. The angular acceleration, $\alpha$, is the rate of change of the angular velocity:

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt}$$

The linear kinematic quantities can be found from the angular quantities:

$$s = R\theta$$

$$v_s = R\omega$$

$$a_s = R\alpha$$

For circular motion, the velocity vector is tangent to the circle and the perpendicular component of the acceleration is called the centripetal acceleration. The centripetal acceleration points towards the center of the circle and has a magnitude of:

$$a_c(t) = \omega^2(t)R = \frac{v^2(t)}{R}$$

The centripetal acceleration vector can be written as:

$$\vec{a}_\perp(t) = \omega^2 R[-\cos(\theta)\hat{x} - \sin(\theta)\hat{y}]$$

Uniform circular is the motion of an object around a circle with a constant speed. The period, $T$, is the time that it takes for the object to complete one revolution. The frequency, $f$, is the inverse of the period, and can be thought of as the number of revolutions completed per second:

$$T = \frac{2\pi}{\omega}$$

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

## Important Equations

### Motion in 2D:

$$\vec{r}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = x(t)\hat{x} + y(t)\hat{y}$$

$$\vec{v}(t) = \frac{d}{dt}\vec{r}(t)$$

$$\vec{a}(t) = \frac{d}{dt}\vec{v}(t)$$

### Relative Motion 2D:

$$\vec{r}'^A(t) = \vec{v}'^B t + \vec{r}^A(t)$$

$$\vec{v}'^A(t) = \vec{v}'^B + \vec{v}^A(t)$$

$$\vec{a}'^A(t) = \vec{a}^A(t)$$

### Acceleration Vector 2D:

$$\vec{a} = \frac{dv}{dt}\hat{v}(t) + v(t)\frac{d\hat{v}}{dt}$$

$$(\text{constant speed:}) \quad \vec{a} = \frac{dv_x}{dt}\left(\hat{x} - \frac{v_x(t)}{v_y(t)}\hat{y}\right)$$

### Circular Motion:

$$\vec{r}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = R \begin{pmatrix} \cos(\theta(t)) \\ \sin(\theta(t)) \end{pmatrix}$$

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt}$$

$$s = R\theta$$

$$v_s = R\omega$$

$$a_s = R\alpha$$

$$a_c(t) = \omega^2(t)R = \frac{v^2(t)}{R}$$

$$\vec{a}_\perp(t) = \omega^2 R[-\cos(\theta)\hat{x} - \sin(\theta)\hat{y}]$$

$$T = \frac{2\pi}{\omega}$$

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

## Important Definitions

**Definition**

**Position vector:** A vector, usually labelled, $\vec{r}$, to describe the position of an object relative to the origin of a coordinate system. In Cartesian coordinates, the position vector is simply given by the $x$, $y$, and $z$ coordinates of the object, $\vec{r} = x\hat{x} = y\hat{y} + z\hat{z}$ .

**Definition**

**Velocity vector:** A vector, usually labelled, $\vec{v}$, which corresponds to the time-rate of change (the derivative with respect to time) of the position vector.

**Definition**

**Acceleration vector:** A vector, usually labelled, $\vec{a}$, which corresponds to the time-rate of change (the derivative with respect to time) of the velocity vector.

**Definition**

**Angular position:** The angle that the position vector makes with either the $x$ or $z$ axis. SI units: none. Common variable(s): $\theta$ (angle with the $z$ axis), $\phi$ (angle with the $x$ axis).

**Definition**

**Angular velocity:** The rate at which an angle changes with respect to time. SI units: [ s^-1]. Common variable(s): $\vec{\omega}$. The angular velocity can be represented by a vector, using the right-hand rule for axial vectors.

**Definition**

**Angular acceleration:** The rate at which angular velocity changes with respect to time. SI units: [ s^-2]. Common variable(s): $\vec{\alpha}$. The angular acceleration can be represented by a vector, using the right-hand rule for axial vectors.

Definition

**Uniform circular motion**: The motion of an object with constant speed around a circle.

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