2.4: Summary

Key Takeaways

Measurable quantities have dimensions and units. A physical quantity should always be reported with units, preferably SI units.

When you build a model to predict a physical quantity, you should always ask if the prediction makes sense (Does it have a reasonable order of magnitude? Does it have the right dimensions?).

Any quantity that you measure will have an uncertainty. Almost any quantity that you determine from a model or theory will also have an uncertainty.

The best way to determine an uncertainty is to repeat the measurement and use the mean and standard deviation of the measurements as the central value and uncertainty. If we have N measurements of some quantity t, $\{t_1, t_2, t_3, \ldots t_N\}$, then the mean, $\bar{t}$, and standard deviation, σ_t, are defined as:

$$\bar{t} = \frac{1}{n}\sum_{i=1}^{i=N} t_i = \frac{t_1 + t_2 + t_3 + \ldots t_N}{N}$$

$$\sigma_t^2 = \frac{1}{N-1}\sum_{i=1}^{i=N}(t_i - \bar{t})^2 = \frac{(t_1-\bar{t})^2 + (t_2-\bar{t})^2 + (t_3-\bar{t})^2 + \ldots + (t_N-\bar{t})^2}{N-1}$$

$$\sigma_t = \sqrt{\sigma_t^2}$$

You have to pay special attention to systematic uncertainties, which are difficult to determine. You should always think of ways that your measured values could be wrong, even after repeated measurements. Relative uncertainties tell you whether your measurement is precise.

There are multiple ways to propagate uncertainties. You can estimate the uncertainty using relative uncertainties or use the Min-Max method, which tends to overestimate the uncertainties. The preferred way to propagate uncertainties is with the derivative method, which you can use so long as the relative uncertainties on the measurements are small. If we have a function, $F(x,y)$ that depends on multiple variables with uncertainties (e.g. $x \pm \sigma_x, y \pm \sigma_y$) , then the central value and uncertainty in $F(x,y)$ are given by:

$$\overline{F} = F(\overline{x}, \overline{y})$$

$$\sigma_F = \sqrt{\left(\frac{\partial F}{\partial x}\sigma_x\right)^2 + \left(\frac{\partial F}{\partial y}\sigma_y\right)^2}$$

This can be easily calculated using a computer.

If you expect two measured quantities to be linearly related (one is proportional to the other), plot them to find out! Use a computer to do so!

Important Equations

Central value and uncertainty:

$$\bar{t} = \frac{1}{n}\sum_{i=1}^{i=N} t_1 = \frac{t_1 + t_2 + t_3 + \ldots + t_N}{N} \tag{2.4.1}$$

$$\sigma_t^2 = \frac{1}{N-1}\sum_{i=1}^{i=N}(t_i - \bar{t})^2 = \frac{(t_1-\bar{t})^2 + (t_2-\bar{t})^2 + (t_3-\bar{t})^2 + \ldots + (t_N-\bar{t})^2}{N-1} \tag{2.4.2}$$

$$\sigma_t = \sqrt{\sigma_t^2} \tag{2.4.3}$$

Derivative method:

$$\overline{F} = F(\overline{x}, \overline{y}) \tag{2.4.4}$$

$$\sigma_F = \sqrt{\left(\frac{\partial F}{\partial x}\sigma_x\right)^2 + \left(\frac{\partial F}{\partial y}\sigma_y\right)^2} \tag{2.4.5}$$

Operation to get z	Uncertainty in z
$z = x + y$ (addition)	$\sigma_z = \sqrt{\sigma_x^2 + \sigma_y^2}$
$z = x - y$ (subtraction)	$\sigma_z = \sqrt{\sigma_x^2 + \sigma_y^2}$
$z = xy$ (multiplication)	$\sigma_z = xy\sqrt{\left(\frac{\sigma_x}{x}\right)^2 + \left(\frac{\sigma_y}{y}\right)^2}$
$z = \frac{x}{y}$ (division)	$\sigma_z = \frac{x}{y}\sqrt{\left(\frac{\sigma_x}{x}\right)^2 + \left(\frac{\sigma_y}{y}\right)^2}$
$z = f(x)$ (a function of 1 variable)	$\sigma_z = \left\lvert\frac{df}{dx}\sigma_x\right\rvert$

Table 2.4.1: *How to propagate uncertainties from measured values $x \pm \sigma_x$ and $y \pm \sigma_y$ to a quantity $z(x, y)$ for common operations.*