13.3: Simple Harmonic Motion

In the previous sections, we modeled the motion of a mass attached to a spring and found that its position, $x(t)$, was described by the following differential equation:

$$\frac{d^2x}{dt} = -\omega^2 x \tag{13.3.1}$$

A possible solution to that equation was given by:

$$x(t) = Acos(\omega t + \phi) \tag{13.3.2}$$

We then saw that the motion of a vertical spring-mass system, as well as that of a mass attached to two springs, could also be described by *Equation 13.3.1*. Any physical system that can described by *Equation 13.3.1* is said to undergo “simple harmonic motion”, or to be a “simple harmonic oscillator”. If we find that the physical model of a system leads to *Equation 13.3.1*, then we immediately know that the position of system can be described by *Equation 13.3.2*.

The key physical characteristic of a simple harmonic oscillator is that there is a “restoring force” whose magnitude is proportional to the displacement from the equilibrium position. A restoring force is a force that acts to place the system back in equilibrium, and is thus always in the direction that is opposite of the displacement relative to an equilibrium position. In the three systems that we considered so far, the net force on the mass was always such that it would restore the mass back to the equilibrium position, where the net force on the mass is zero.

Many systems in nature are well modeled as simple harmonic oscillators. Some examples are: the motion of a pendulum as it oscillates, the motion of a buoy bobbing up and down in the sea, the motion of electrons in a shorted capacitor, and the vibrations of atoms in a molecule.