2.1: Orders of magnitude

Although you should try to fight intuition when building a model to describe a particular phenomenon, you should not abandon critical thinking and should always ask if a prediction from your model makes sense. One of the most straightforward ways to estimate if a model makes sense is to ask whether it predicts the correct order of magnitude for a quantity. Usually, the order of magnitude for a quantity can be determined by making a very simple model, ideally one that you can work through in your head. When we say that a prediction gives the right "order of magnitude", we usually mean that the prediction is within a factor of "a few" (up to a factor of 10) of the correct answer. For example, if a measurement gives a value of 2000, then we would consider that a model prediction of 8,000 gave the right order of magnitude (it differs from the correct answer by a factor of 4, whereas a prediction of 24,000 would not (it differs by a factor of 12).

Example 2.1.1

How many ping pong balls can you fit into a school bus? Is it of order $10,000$ or $100,000$ or more?

Solution

Our strategy is to estimate the volumes of a school bus and of a ping pong ball, and then calculate how many times the volume of the ping pong ball fits into the volume of the school bus.

We can model a school bus as a box, say $20\ \text{m} \times 2\ \text{m} \times 2\ \text{m}$, with a volume of $80\ \text{m}^3 \sim 100\ \text{m}^3$.

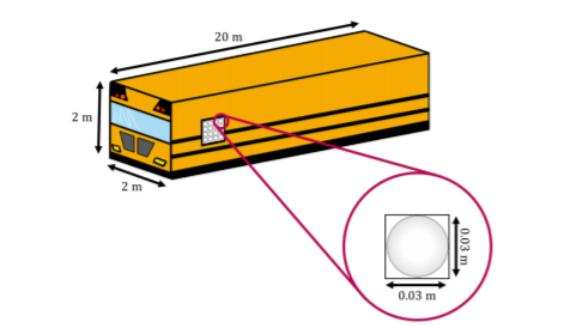

Figure 2.1.1: A school bus and ping pong balls modeled as boxes.

We can model a ping pong ball as a sphere with a diameter of $0.03\ \text{m}$ $(3\ \text{cm})$. When stacking the ping pong balls, we can model them as little cubes with a side given by their diameter, so the volume of a ping pong ball, for stacking, is $\sim 0.00003\ \text{m}^3 = 3 \times 10^{-5}\ \text{m}^3$. If we divide $100\ \text{m}^3$ by $3 \times 10^{-5}\ m^3$, using scientific notation:

$$\frac{100\ \text{m}^3}{3 \times\ 10^{-5}\text{m}^3} = \frac{1 \times 10^2}{3 \times 10^{-5}} = \frac{1}{3} \times 10^7 \sim 3 \times 10^6 \tag{2.1.1}$$

Thus, we expect to be able to fit about three million ping pong balls in a school bus.

Exercise 2.1.1

Fill in the following table, giving the order of magnitude (in meters) of the sizes of different physical objects. Feel free to look these up on the internet!

Table 2.1.1

Object	Order of Magnitude
Proton	
Nucleus of atom	
Hydrogen atom	
Virus	
Human skin cell	

Object	Order of Magnitude
Width of human hair	
Human	1 m
Height of Mt. Everset	
Radius of the Earth	
Radius of the Sun	
Radius of the Milky Way	

Answer