18.6: Summary

Key Takeaways

The electric force is conservative, so we can define a potential energy function, $U(\vec{r})$. The potential energy function for a point charge, q, at position, $\vec{r}$, relative to a point charge, Q, is given by:

$$U(\vec{r})=\frac{kQ}{r}q+C \tag{18.6.1}$$

where, C, is an arbitrary constant, since only difference in potential energy are physically meaningful (as they correspond to work). Note that the sign of the electrical potential energy will depend on the relative sign of q and Q.

If a collection of charges are held together, the total electrical potential energy that is stored is called "electrostatic potential energy".

In a similar way as the electric field, $\vec{E}(\vec{r})$, corresponds to electric force per unit charge, "electric potential", $V(\vec{r})$, corresponds to electric potential energy per unit charge. The electric potential at a position, $\vec{r}$, relative to a point charge, Q, is given by:

$$V(\vec{r})=\frac{U(\vec{r})}{q}=\frac{kQ}{r}+C'$$

and also depends on an arbitrary constant, C', since only differences in electric potential will lead to differences in potential energy. The value of the electric potential, V, at some position in space, $\vec{r}$, allows us to determine the electric potential energy, U, at that position for any charge, q:

$$U=qV$$

This is analogous to determining the force on a charge q when we know the electric field at some point in space:

$$\vec{F}=q\vec{E}$$

Differences in electric potential are called "voltages", and the S.I. unit of potential is called the "volt" (V). In S.I. units, the electric field is often expressed in units of volts per meter (V/m).

When a particle with charge, q, changes position such that the corresponding change in electric potential is ΔV, the particle's potential energy will change by:

$$\Delta U=q\Delta V$$

In particular, a negative charge will experience a decrease in potential energy when the electric potential increases, whereas a positive charge will experience an increase in potential energy when the electric potential increases. This reflects the fact that the electric force associated with the electric potential will act in opposite directions on a positive and a negative charge.

In order to describe the energies of particles interacting with electric forces, it is more convenient to use the "electron volt" instead of the Joule. An electron volt is defined as the energy that is gained by a charge with a magnitude e (the magnitude of the charge of the electron) when accelerated through a potential difference of $\Delta V=1\text{V}$:

$$1\text{eV}=(e)(1\text{V})=1.6\times 10^{-19}\text{J}$$

The electric potential function can be determined in two different ways:

1. By modelling the charge distribution as the sum of infinitesimal point charges, dq, and adding together the electric potentials, dV, from all charges, dq. This requires that one choose 0V to be located at infinity, so that the dV are all relative to the same point.
2. By calculating the electric field (either as a integral or from Gauss' Law), and using:

$$\Delta V = V(\vec{r}_B) - V(\vec{r}_A) = -\int_A^B \vec{E}\cdot d\vec{r}$$

It is worth noting that one needs to be very careful with the signs when using the above integral. In particular note that one takes the negative of the integral, from A to B, to determine the potential at B minus the potential at A.

Similarly, one can determine the value of the electric field, $\vec{E}(\vec{r}) = \vec{E}(x,y,z)$, from the electric potential, $V(\vec{r}) = V(x,y,z)$:

$$\vec{E}(x,y,z) = -\nabla V = -\frac{\partial V}{\partial x}\hat{x} - \frac{\partial V}{\partial y}\hat{y} - \frac{\partial V}{\partial z}\hat{z}$$

where, ∇V, is the gradient of the electric potential.

The electric potential can be visualized in a number of ways. The most common is to draw contours of constant electric potential, akin to the contours on geographical maps that are used to show regions of constant altitude (i.e. constant gravitational potential energy).

Regions of constant electric potential are called “equipotentials”, and can be lines, surfaces or volumes. Equipotentials are always perpendicular to the electric field. In electrostatics (when charges are not moving), the electric field in a conductor must be zero, so that a conductor always forms an equipotential, and the electric field at the surface of a conductor is always perpendicular to the surface.

When charges are placed on a conductor, they will spread out along the outer surface of the conductor. The surface density of charges will be the highest where the conductor has the smallest radius of curvature (e.g. at a sharp point). Consequently, the electric field at the surface of a charged conductor is highest near sharp points.

Capacitors are devices that are used to store charge. They are usually made using two conducting plates (“terminals” or “electrodes”) that hold equal and opposite charge, Q, at a fixed potential difference, ΔV, between the electrodes. The amount of charge that is stored on the capacitor is observed to be proportional to the potential difference between the electrodes:

$$Q = C\Delta V$$

where the constant of proportionality, C, is called the “capacitance” of the capacitor. The S.I. unit of capacitance is the “Farad” (F). The capacitance of a capacitor depends on its geometry (e.g. its size) and the materials that it is placed between the electrodes.

Usually, one places a dielectric material between the two electrodes in order to increase the capacitance, and to reduce the risk of breakdown. If that material has a “dielectric constant”, K, then the capacitance is given by:

$$C = KC_0$$

where, C_0, corresponds to the capacitance if there were vacuum between the electrodes. The dielectric constant of air is very close to 1, so that a capacitor in air is very similar to a capacitor in vacuum. A dielectric material is one that is made of molecules that can be polarized under the presence of an electric field; that is, the molecules have an electric dipole moment. When the molecules in a material are polarized, this reduces the total electric field in the material, which increases the capacitance of the capacitor. Inside a dielectric material, we can define the “permittivity”, ϵ, as:

$$\epsilon = K\epsilon_0$$

where ϵ_0 is the permittivity of free space. Within a dielectric material, Gauss’ Law is modified to:

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q^{enc}}{\epsilon}$$

Since charges are held at a fixed potential difference on a capacitor, capacitors are a way of storing electric potential energy. The amount of electric potential energy stored in a capacitor with capacitance, C, when the capacitor has a potential difference, ΔV, across its electrodes, is given by:

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}Q\Delta V$$

Important Equations

Electric potential energy from a point charge

$$U(r)=\frac{kQq}{r}+C$$

Electric potential

$$V=\frac{U}{q}$$

Electric potential:

$$\Delta V=V(\vec{r}_B)-V(\vec{r}_A)$$

$$\Delta V=-\int_A^B \vec{E}\cdot d\vec{r}$$

Electric field:

$$\vec{E}=-\nabla V=-\frac{\partial V}{\partial x}\hat{x}-\frac{\partial V}{\partial y}\hat{y}-\frac{\partial V}{\partial z}\hat{z}$$

Electric potential from a point charge

$$V(r)=\frac{kQ}{r}+C$$

Electric potential between two parallel plates

$$\Delta V=EL$$

Charge stored in a capacitor:

$$Q=C\Delta V$$

Energy stored in a capacitor

$$U=\frac{1}{2}\frac{Q^2}{C}=\frac{1}{2}C(\Delta V)^2=\frac{1}{2}Q\Delta V$$

Important Definitions

> **Definition**
>
> **Electric Potential:** Electric potential energy per unit charge. SI units: $[\mathrm{V}]$. Common variable(s): V, often appearing as ΔV (potential difference).

> **Definition**
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> **Capacitance:** How much charge a capacitor can hold given the potential difference between the terminals of the capacitor. SI units: $[\mathrm{F}]$. Common variable(s): C.

> **Definition**
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> **Dielectric constant:** A constant which is defined as the (dimensionless) ratio of the dielectric permittivity of a substance and the dielectric permittivity of a vacuum. Common variable(s): K.