14.4: The speed of a wave

In the previous section we found that the speed of a transverse wave in a rope is related to the ratio of the tension in the rope to the linear mass density of the rope:

$$v = \sqrt{\frac{F_T}{\mu}}$$

The speed of a wave in any medium is usually given by a ratio, where the numerator is a measure of how easy it is to deform the medium, and the denominator is measure of the inertia of the medium. For a rope, the tension is a measure of how stiff the rope is. A higher tension makes it more difficult to disturb the rope from equilibrium and it will "snap back" faster when disturbed, so the pulse will propagate faster. The heavier the rope, the harder it will be for the disturbance to propagate as the rope has more inertia, which will slow down the pulse.

The only way that a wave can propagate through a medium is if that medium can be deformed and the particles in the medium can be displaced from their equilibrium position, much like simple harmonic oscillators. The wave will propagate faster if those oscillators have a stiff spring constant and there is a strong force trying to restore them to equilibrium. However, if those oscillators have a large inertia, even with a large restoring force, they will accelerate back to their equilibrium with a smaller acceleration.

In general, the speed of a wave is given by:

$$v = \sqrt{\frac{\text{Stiffness of medium}}{\text{Inertia of medium}}}$$

For example, the speed of longitudinal pressure waves in a solid is given by:

$$v = \sqrt{\frac{E}{\rho}}$$

where E is the "elastic (or Young's) modulus" for the material, and ρ is the density of the material. The elastic modulus of a solid is a measure of the material's resistance to being deformed when a force (or pressure) is exerted on it. The more easily it is deformed, the lower its elastic modulus will be.

For the propagation of longitudinal pressure waves through a fluid, the speed is given by:

$$v = \sqrt{\frac{B}{\rho}}$$

where B is the bulk modulus of the liquid, and ρ its density.

? Exercise 14.4.1

A wave will propagate faster through...

A. ice.
B. water.

Answer

A.