13.5: Summary

Key Takeaways

The equation of motion for the position, $x(t)$, of the mass in a one-dimensional spring-mass system with no friction can be written:

$$\frac{d^2x}{dt^2} = -\sqrt{\frac{k}{m}}x = -\omega^2 x$$

and has a solution:

$$x(t) = A\cos(\omega t + \phi)$$

where A is the amplitude of the motion, ϕ is the phase, which depends on our choice of initial conditions (when we choose time $t = 0$), and ω:

$$\omega = \sqrt{\frac{k}{m}}$$

is the angular frequency of the motion. The mass will oscillate about an equilibrium position with a period, T, and frequency, f, given by:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$
$$f = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

The velocity and acceleration of the mass are found by taking the time derivatives of the position $x(t)$:

$$\begin{aligned}
x(t) &= A\cos(\omega t + \phi)\\
v(t) &= \frac{d}{dt}x(t) = -A\omega\sin(\omega t + \phi)\\
a(t) &= \frac{d^2}{dt^2}x(t) = \frac{d}{dt}(-A\omega\sin(\omega t + \phi)) = -A\omega^2\cos(\omega t + \phi)
\end{aligned}$$

The total mechanical energy of the mass, at some position x, is given by:

$$E = U + K = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kA^2$$

and is conserved.

Any system that can be described by the equation of motion:

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

is said to be a simple harmonic oscillator, and its position will be described by:

$$x(t) = A\cos(\omega t + \phi)$$

A simple harmonic oscillator will always oscillate about an equilibrium position, where the net force on the oscillator is zero. The net force on a simple harmonic oscillator is always directed towards the equilibrium position, and has a magnitude proportional to the distance of the oscillator from its equilibrium position. The force is called a restoring force. A vertical spring-mass system, and a mass attached to two springs will both undergo simple harmonic motion about their respective equilibrium position.

A simple pendulum will undergo simple harmonic oscillations, if the amplitude of the oscillations is small. The angular frequency for the oscillations of a simple pendulum only depends on the length of the pendulum:

$$\omega = \sqrt{\frac{g}{L}}$$

This is valid in the small angle approximation, where:

$$\sin\theta \approx \theta$$

A physical pendulum of mass m which oscillates about an axis through the object will also undergo simple harmonic oscillation in the small angle approximation. The angular frequency of the oscillations for a physical pendulum is given by:

$$\omega = \sqrt{\frac{mgh}{I}}$$

where h is the distance between the center of mass and the axis of rotation, and I is the moment of inertia of the object about the rotation axis.

Important Equations

Position, velocity, and acceleration for SHM:

$$\begin{aligned} x(t) &= A\cos(\omega t + \phi) \\ v(t) &= \frac{d}{dt}x(t) = -A\omega\sin(\omega t + \phi) \\ a(t) &= \frac{d^2}{dt^2}x(t) = -A\omega^2\cos(\omega t + \phi) \end{aligned}$$

Period and frequency:

$$\begin{aligned} \omega &= \sqrt{\frac{k}{m}} \\ T &= \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \\ f &= \frac{1}{T} = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \end{aligned}$$

Mechanical energy:

$$E = U + K = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kA^2$$

Simple pendulum (small angles):

$$\omega = \sqrt{\frac{g}{L}}$$

Physical pendulum (small angles):

$$\omega = \sqrt{\frac{mgh}{I}}$$

Important Definitions

Definition

Angular frequency: is related to a usual frequency by a factor of 2π. For an object rotating around a circle at constant speed, the angular frequency of the rotation is the same as the angular speed (the rate of change of a position angle). SI units: $[\mathrm{rad/s}]$. Common variable(s): ω.

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