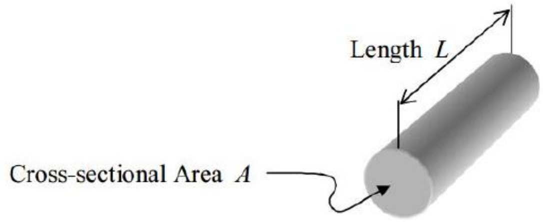
B11: Resistivity and Power

In chapter 9 we discussed resistors that conform to Ohm's Law. From the discussion, one could deduce that the resistance of such a resistor depends on the nature of the material of which the resistor is made and on the size and shape of the resistor. In fact, for resistors made out of a single kind of material, in the shape of a wire with a terminal at each end,



the resistance is given by:

$$R = \varrho \frac{L}{A} \tag{B11.1}$$

where:

- R is the resistance of the resistor as measured between the ends,
- ϱ is the resistivity of the substance of which the resistor is made,
- A is the cross-sectional area of the wire-shaped resistor, and
- L is the length of the resistor.

The values of resistivity for several common materials are provided in the following table:

Material	Resisitivity ϱ
Silver	$1.6 \times 10^{-8}\ \Omega \cdot m$
Copper	$1.7 \times 10^{-8}\ \Omega \cdot m$
Gold	$2.4 \times 10^{-8}\ \Omega \cdot m$
Aluminum	$3 \times 10^{-8}\ \Omega \cdot m$
Tungsten	$5.6 \times 10^{-8}\ \Omega \cdot m$
Nichrome	$1.0 \times 10^{-6}\ \Omega \cdot m$
Seawater	$0.25\ \Omega \cdot m$
Rubber	$1 \times 10^{13}\ \Omega \cdot m$
Glass	1×10^{10} to $1 \times 10^{14}\ \Omega \cdot m$
Quartz	5×10^{15} to $7.5 \times 10^{17}\ \Omega \cdot m$

In the expression $R = \varrho \frac{L}{A}$, the resistivity ϱ depends on the charge carrier density, that is, the number-of-charge-carriers-per-volume. The more charge carriers per volume, the smaller the resistance since, for a given velocity of the charge carriers, more of them will be passing any point along the length of the resistor every second for a given voltage across the resistor. The resistivity also depends on the retarding force factor. We said that the retarding force on each charge carrier is proportional to the velocity of that charge carrier.

$$\text{Retarding Force} = \text{-(factor) times (charge carrier velocity)}$$

(The minus sign is there because the retarding force is in the direction opposite that of the charge-carrier velocity.) The bigger the retarding force factor, the greater the resistivity of the material for which the factor applies.

The charge carrier density and the retarding force factor determine the value of ϱ. The effect of ϱ on the resistance is evident in the expression $R=\varrho\frac{L}{A}$. The bigger ϱ is, the greater the resistance is.

Why the factor of L in $R=\varrho\frac{L}{A}$? It's saying that the greater the length of the single-substance resistor in the shape of a wire, the greater the resistance of the resistor, all other things being equal (same substance, same cross-sectional area). It means, for instance, that if you have two resistors, identical in all respects except that one is twice as long as the other, and you put the same voltage across each of the resistors, you'll get half the current in the longer resistor. Why is that?

To get at the answer, we need to consider the electric field inside the wire-shaped resistor when we have a voltage V across the resistor. The thing is, the electric field inside the resistor is directed along the length of the resistor, and, it has the same magnitude everywhere along the length of the resistor. Evidence for this can be obtained by means of simple voltage measurements. Use a voltmeter to measure the potential difference $\Delta\varphi$ between two points on the resistor that are separated by a certain distance Δx, say 2 mm (measured along the length of the resistor) for instance. It turns out that no matter where along the length you pick the pair of points (separated from each other by the Δx), you always get the same voltage reading. Imagine (this part is a thought experiment) moving a positive test charge q_T that distanceΔx along the resistor from high potential toward low potential. No matter where along the length of the resistor you do that, the work done (by the electric field characterized by the potential) $q_T\Delta\varphi$ (calculated as the negative of the change of the potential energy of the test charge) is the same. The work, calculated as force times distance, is $q_T E\Delta x$. For that to be the same at every point along the length of the resistor, the electric field E has to have the same value everywhere along the length of the resistor. Furthermore, setting the two expressions for the work equal to each other yields:

$$q_T E\Delta x=q_T\Delta\varphi$$

$$E=\frac{\Delta\varphi}{\Delta x}$$

E being constant thus means that $\frac{\Delta\varphi}{\Delta x}$ is constant which means that a graph of φ vs. x is a straight line with slope $\frac{\Delta V}{\Delta x}$. But, in calculating that slope, since it is a straight line, we don't have to use a tiny little Δx. We can use the entire length of the resistor and the corresponding potential difference, which is the voltage V across the resistor. Thus,

$$E=\frac{V}{L}$$

where:

- E is the magnitude of the electric field everywhere in the single-substance wire-shaped resistor,
- V is the voltage across the resistor, and
- L is the length of the resistor.

This result ($E=\frac{V}{L}$) is profound in and of itself, but, if you recall, we were working on answering the question about why the resistance R, of a single-substance wire-shaped resistor, is proportional to the length of the resistor. We are almost there. The resistance is the ratio of the voltage across the resistor to the current in it. According to $E=\frac{V}{L}$, the longer the resistor, the weaker the electric field in the resistor is for a given voltage across it. A weaker E results in a smaller terminal velocity for the charge carriers in the resistor, which results in a smaller current. Thus, the longer the resistor, the smaller the current is; and; the smaller the current, the greater the voltage-to-current ratio is; meaning, the greater the resistance.

The next resistance-affecting characteristic in $R=\varrho\frac{L}{A}$ that I want to discuss is the area A. Why should that affect the resistance the way it does? Its presence in the denominator means that the bigger the cross-sectional area of the wire-shaped resistor, the smaller the resistance. Why is that?

If we compare two different resistors made of the same material and having the same length (but different cross-sectional areas) both having the same voltage across them, they will have the same electric field $E=\frac{V}{L}$ in them. As a result, the charge carriers will have the same velocity v. In an amount of time Δt,

$$L=v\Delta t$$

$$\Delta t=\frac{L}{v}$$

all the free-to-move charge carriers in either resistor will flow out the lower potential end of the resistor (while the same amount of charge flows in the higher potential end). This time Δt is the same for the two different resistors because both resistors have the same length, and the charge carriers in them have the same v. The number of charge carriers in either resistor is proportional to the volume of the resistor. Since the volume is given by volume = LA, the number of charge carriers in either resistor is proportional to the cross-sectional area A of the resistor. Since the number of charge carriers in either resistor, divided by the time Δt is the current in that resistor, this means that the current is proportional to the area.

If the current is proportional to the area, then the resistance, being the ratio of the voltage to the current, must be inversely proportional to the area. And so ends our explanation regarding the presence of the A in the denominator in the expression

$$R=\varrho \frac{L}{A}$$

Power

You were introduced to power in Volume I of this book. It is the rate at which work is done. It is the rate at which energy is transferred. And, it is the rate at which energy is transformed from one form of energy into another form of energy. The unit of power is the watt, W.

$$1 W=1 \frac{\mathrm{J}}{\mathrm{s}}$$

In a case in which the power is the rate that energy is transformed from one form to another, the amount of energy that is transformed from time 0 to time t:

- if the power is constant, is simply the power times the duration of the time interval:

$$\text{Energy}=\mathrm{Pt}$$

- if the power is a function of time, letting t′ be the time variable that changes from 0 to t, is:

$$\text{Energy}=\int_0^t P\left(t^{\prime}\right) d t^{\prime}$$

The Power of a Resistor

In a resistor across which there is a voltage V, energy is transformed from electric potential energy into thermal energy. A particle of charge q, passing through the resistor, loses an amount of potential energy qV but it does not gain any kinetic energy. As it passes through the resistor, the electric field in the resistor does an amount of work qV on the charged particle, but, at a same time, the retarding force exerted on the charged particle by the background material of the resistor, does the negative of that same amount of work. The retarding force, like friction, is a non-conservative force. It is exerted on the charge carrier when the charge carrier collides with impurities and ions (especially at sites of defects and imperfections in the structure of the material). During those collisions, the charge carriers impart energy to the ions with which they collide. This gives the ions vibrational energy which manifests itself, on a macroscopic scale, (early in the process) as an increase in temperature. Some of the thermal energy is continually transferred to the surroundings. Under steady state conditions, arrived at after the resistor has warmed up, thermal energy is transferred to the surroundings at the same rate that it is being transformed from electrical potential energy in the resistor.

The rate at which electric potential energy is converted to thermal energy in the resistor is the power of the resistor (a.k.a. the power dissipated by the resistor). It is the rate at which the energy is being delivered to the resistor. The energy conversion that occurs in the resistor is sometimes referred to as the dissipation of energy. One says that the resistor power is the rate at which energy is dissipated in the resistor. It's pretty easy to arrive at an expression for the power of a resistor in terms of circuit quantities. Each time a coulomb of charge passes through a resistor that has a voltage V across it, an amount of energy equal to one coulomb times V is converted to thermal energy. The current I is the number of coulombs-per-second passing through the resistor. Hence V times I is the number of joules-per-second converted to thermal energy. That's the power of the resistor. In short,

$$P=IV \tag{B11.2}$$

where:

- P is the power of the resistor. It is the rate at which the resistor is converting electrical potential energy into thermal energy. The unit of power is the watt. $1 W=1 \frac{J}{s}$.
- I is the current in the resistor. It is the rate at which charge is flowing through the resistor. The unit of current is the ampere. $1 A=1 \frac{C}{s}$.

- V is the voltage across the resistor. It is the amount by which the value of electric potential (the electric potential energy per charge) at one terminal of the resistor exceeds that at the other terminal. The unit of voltage is the volt. $1 \text{ volt} = 1\frac{J}{C}$.

The Power of a Seat of EMF

In a typical circuit, a seat of EMF causes positive charge carriers (in our positive-charge-carrier model) to go from a lower-potential conductor, through itself, to a higher-potential conductor. The electric field of the conductors exerts a force on the charge carriers inside the seat of EMF in the direction opposite to the direction in which the charge carriers are going. The charged particles gain electric potential energy in moving from the lower-potential terminal of the seat of EMF to the higher-potential terminal. Where does that energy come from?

In the case of a battery, the energy comes from chemical potential energy stored in the battery and released in chemical reactions that occur as the battery moves charge from one terminal to the other. In the case of a power supply, the power supply, when plugged into a wall outlet and turned on, becomes part of a huge circuit including transmission wires extending all the way back to a power plant. At the power plant, depending on the kind of power plant, kinetic energy of moving water, or thermal energy used to make steam to turn turbines, or chemical potential energy stored in wood, coal, or oil; is converted to electric potential energy. Whether it is part of a battery, or a part of a power supply, the seat of EMF converts energy into electric potential energy. It keeps one of its terminals at a potential ε higher than the other terminal. Each time it moves a coulomb of charge from the lower potential terminal to the higher potential terminal, it increases the potential energy of that charge by one coulomb times ε. Since the current I is the number of coulombs per second that the seat of EMF moves from one terminal to the other, the power, the rate at which the seat of EMF delivers energy to the circuit, is given by:

$$P = I\varepsilon$$

Recall that it is common to use the symbol V (as well as ε) to represent the voltage across a seat of EMF. If you use V, then the power of the seat of EMF is given by:

$$P = IV$$

where:

- P is the rate at which a seat of EMF delivers energy to a circuit,
- I is the current in the seat of EMF (the rate at which charge flows through the seat of EMF), and
- V is the voltage across the seat of EMF.

This is the same expression as the expression for the power of a resistor (Equation B11.2).