

## B8: Capacitors, Dielectrics, and Energy in Capacitors

Capacitance is a characteristic of a conducting object. Capacitance is also a characteristic of a pair of conducting objects.

Let's start with the capacitance of a single conducting object, isolated from its surroundings. Assume the object to be neutral. Now put some positive charge on the object. The electric potential of the object is no longer zero. Put some more charge on the object and the object will have a higher value of electric potential. What's interesting is, no matter how much, or how little charge you put on the object, the ratio of the amount of charge  $q$  on the object to the resulting electric potential  $\varphi$  of the object has one and the same value.

$$\frac{q}{\varphi} \text{ have the same value for any value of } q$$

You double the charge, and, the electric potential doubles. You reduce the amount of charge to one tenth of what it was, and, the electric potential becomes one tenth of what it was. The actual value of the unchanging ratio is called the capacitance  $C_{sc}$  of the object (where the subscript "sc" stands for "single conductor").

$$C_{sc} = \frac{q}{\varphi} \quad (\text{B8.1})$$

where:

- $C_{sc}$  is the capacitance of a single conductor, isolated (distant from) its surroundings,
- $q$  is the charge on the conductor, and,
- $\varphi$  is the electric potential of the conductor relative to the electric potential at infinity (the position defined for us to be our zero level of electric potential).

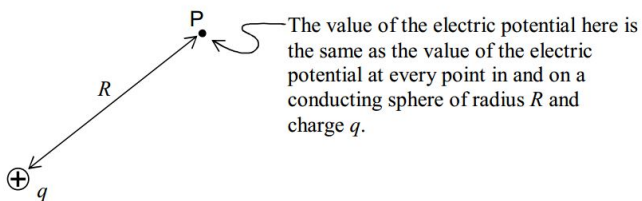
The capacitance of a conducting object is a property that an object has even if it has no charge at all. It depends on the size and shape of the object.

The more positive charge you need to add to an object to raise the potential of that object 1 volt, the greater the capacitance of the object. In fact, if you define  $q_1$  to be the amount of charge you must add to a particular conducting object to increase the electric potential of that object by one volt, then the capacitance of the object is  $\frac{q_1}{1 \text{ volt}}$ .

### The Capacitance of a Spherical Conductor

Consider a sphere (either an empty spherical shell or a solid sphere) of radius  $R$  made out of a perfectly-conducting material. Suppose that the sphere has a positive charge  $q$  and that it is isolated from its surroundings. We have already covered the fact that the electric field of the charged sphere, from an infinite distance away, all the way to the surface of the sphere, is indistinguishable from the electric field due to a point charge  $q$  at the position of the center of the sphere; and; everywhere inside the surface of the sphere, the electric field is zero. Thus, outside the sphere, the electric potential must be identical to the electric potential due to a point charge at the center of the sphere (instead of the sphere). Working your way in from infinity, however, as you pass the surface of the sphere, the electric potential no longer changes. Whatever the value of electric potential at the surface of the sphere is, that is the value of electric potential at every point inside the sphere.

This means that the electric potential of the sphere is equal to the electric potential that would be caused by a point charge (all by itself) at a point in space a distance  $R$  from the point charge (where  $R$  is the radius of the sphere).



Thus,  $\varphi = \frac{kq}{R}$  is the electric potential of a conducting sphere of radius  $R$  and charge  $q$ .

Solving this expression for  $\frac{q}{\varphi}$  yields:

$$\frac{q}{\varphi} = \frac{R}{k}$$

Since, by definition, the capacitance  $C_{sc} = \frac{q}{\varphi}$ , we have:

$$C_{sc} = \frac{R}{k} \quad (\text{B8.2})$$

The capacitance of a conducting sphere is directly proportional to the radius of the sphere. The bigger the sphere, the more charge you have to put on it to raise its potential one volt (in other words, the bigger the capacitance of the sphere). This is true of conducting objects in general. Since all the unbalanced charge on a conductor resides on the surface of the conductor, it really has to do with the amount of surface area of the object. The more surface area, the more room the charge has to spread out and, therefore, the more charge you have to put on the object to raise its potential one volt (in other words, the bigger the capacitance of the object).

Consider, for instance, a typical paper clip. It only takes an amount of charge on the order of a pC (picocoulomb,  $1 \times 10^{-12}$  coulombs) to raise the potential of a paper clip 10 volts.

#### Units: the Farad

The unit of capacitance is the coulomb-per-volt,  $\frac{C}{V}$ . That combination unit is given a name, the farad, abbreviated  $F$ .

$$1F = 1 \frac{C}{V}$$

## The Capacitance of a Pair of Conducting Objects

So far, we've been talking about the capacitance of a conducting object that is isolated from its surroundings. You put some charge on such an object, and, as a result, the object takes on a certain value of electric potential. The charge-to-potential ratio is called the capacitance of the object. But get this, if the conductor is near another conductor when you put the charge on it, the conductor takes on a different value of electric potential (compared to the value it takes on when it is far from all other conductors) for the exact same amount of charge. This means that just being in the vicinity of another conductor changes the effective capacitance of the conductor in question. In fact, if you put some charge on an isolated conductor, and then bring another conductor into the vicinity of the first conductor, the electric potential of the first conductor will change, meaning, its effective capacitance changes. Let's investigate a particular case to see how this comes about.

Consider a conducting sphere with a certain amount of charge,  $q$ , on it. Suppose that, initially, the sphere is far from its surroundings and, as a result of the charge on it, it is at a potential  $\varphi$ .

Let's take a moment to review what we mean when we say that the sphere is at a potential  $\varphi$ . Imagine that you take a test charge  $q_T$  from a great distance away from the sphere and take it to the surface of the sphere. Then you will have changed the potential energy of the test charge from zero to  $q_T\varphi$ . To do that, you have to do an amount of work  $q_T\varphi$  on the test charge. We're assuming that the test charge was initially at rest and is finally at rest. You have to push the charge onto the sphere. You apply a force over a distance to give that particle the potential energy  $q_T\varphi$ . You do positive work on it. The electric field of the sphere exerts a force on the test charge in the opposite direction to the direction in which you are moving the test charge. The electric field does a negative amount of work on the test charge such that the total work, the work done by you plus the work done by the electric field, is zero (as it must be since the kinetic energy of the test charge does not change). But I want you to focus your attention on the amount of work that you must do, pushing the test charge in the same direction in which it is going, to bring the test charge from infinity to the surface of the sphere. That amount of work is  $q_T\varphi$  because  $q_T\varphi$  is the amount by which you increase the potential energy of the charged particle. If you were to repeat the experiment under different circumstances and you found that you did not have to do as much work to bring the test charge from infinity to the surface of the sphere, then you would know that the sphere is at a lower potential than it was the first time.

Now, we are ready to explore the case that will illustrate that the charge-to-voltage ratio of the conducting object depends on whether or not there is another conductor in the vicinity. Let's bring an identical conducting sphere near one side of the first sphere. The first sphere still has the same amount of charge  $q$  on it that it always had, and, the second sphere is neutral. The question is, "Is the potential of the original sphere still the same as what it was when it was all alone?" Let's test it by bringing a charge in from an

infinite distance on the opposite side of the first sphere (as opposed to the side to which the second sphere now resides). Experimentally we find that it takes less work to bring the test charge to the original sphere than it did before, meaning that the original sphere now has a lower value of electric potential. How can that be? Well, when we brought the second sphere in close to the original sphere, the second sphere became polarized. (Despite the fact that it is neutral, it is a conductor so the balanced charge in it is free to move around.) The original sphere, having positive charge  $q$ , attracts the negative charge in the second sphere and repels the positive charge. The near side of the second sphere winds up with a negative charge and the far side, with the same amount of positive charge. (The second sphere remains neutral overall.) Now the negative charge on the near side of the second sphere attracts the (unbalanced) positive charge on the original sphere to it. So the charge on the original sphere, instead of being spread out uniformly over the surface as it was before the second sphere was introduced, is bunched up on the side of the original sphere that is closer to the second sphere. This leaves the other side of the original sphere, if not neutral, at least less charged than it was before. As a result, it takes less work to bring the positive test charge in from infinity to that side of the original sphere. As mentioned, this means that the electric potential of the original sphere must be lower than it was before the second sphere was brought into the picture. Since it still has the same charge that it always had, the new, lower potential, means that the original sphere has a greater charge-to-potential ratio, and hence a greater effective capacitance.

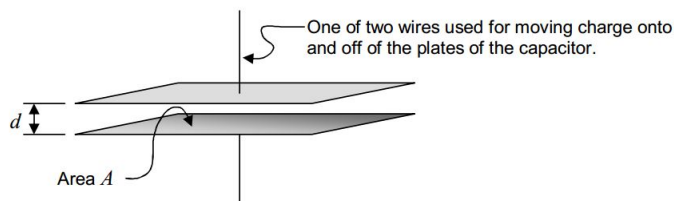
In practice, rather than call the charge-to-potential ratio of a conductor that is near another conductor, the “effective capacitance” of the first conductor, we define a capacitance for the pair of conductors. Consider a pair of conductors, separated by vacuum or insulating material, with a given position relative to each other. We call such a configuration a capacitor. Start with both conductors being neutral. Take some charge from one conductor and put it on the other. The amount of charge moved from one conductor to the other is called the charge of the capacitor. (Contrast this with the actual total charge of the device which is still zero.) As a result of the repositioning of the charge, there is a potential difference between the two conductors. This potential difference  $\Delta\varphi$  is called the voltage of the capacitor or, more often, the voltage across the capacitor. We use the symbol  $V$  to represent the voltage across the capacitor. In other words,  $V \equiv \Delta\varphi$ . The ratio of the amount of charge moved from one conductor to the other, to, the resulting potential difference of the capacitor, is the capacitance of the capacitor (the pair of conductors separated by vacuum or insulator).

$$C = \frac{q}{V} \quad (\text{B8.3})$$

where:

- $C$  is the capacitance of a capacitor, a pair of conductors separated by vacuum or an insulating material,
- $q$  is the “charge on the capacitor,” the amount of charge that has been moved from one initially neutral conductor to the other. One conductor of the capacitor actually has an amount of charge  $q$  on it and the other actually has an amount of charge  $-q$  on it.
- $V$  is the electric potential difference  $\Delta\varphi$  between the conductors. It is known as the voltage of the capacitor. It is also known as the voltage across the capacitor.

A two-conductor capacitor plays an important role as a component in electric circuits. The simplest kind of capacitor is the parallel-plate capacitor. It consists of two identical sheets of conducting material (called plates), arranged such that the two sheets are parallel to each other. In the simplest version of the parallel-plate capacitor, the two plates are separated by vacuum.



The capacitance of such a capacitor is given by

$$C = \epsilon_o \frac{A}{d}$$

where:

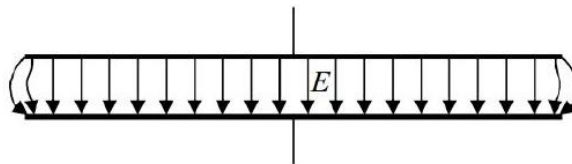
- $C$  is the capacitance of the parallel-plate capacitor whose plates are separated by vacuum,
- $d$  is the distance between the plates,
- $A$  is the area of one face of one of the plates,

- $\epsilon_o$  is a universal constant called the permittivity of free space.  $\epsilon_o$  is closely related to the Coulomb constant  $k$ . In fact,  

$$k = \frac{1}{4\pi\epsilon_o}.$$

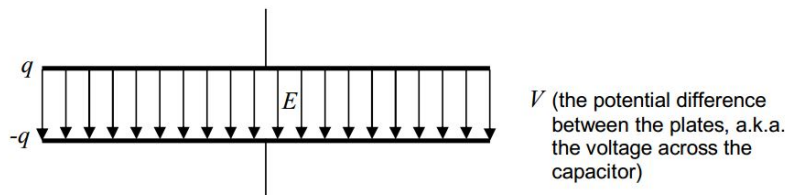
Thus,  $\epsilon = 8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}$ . Our equation for the capacitance can be expressed in terms of the Coulomb constant  $k$  as  $C = \frac{1}{4\pi k} \frac{A}{d}$ , but, it is more conventional to express the capacitance in terms of  $\epsilon_o$ .

This equation for the capacitance is an approximate formula. It is a good approximation as long as the plate separation  $d$  is small compared to a representative plate dimension (the diameter in the case of circular plates, the smaller edge length in the case of rectangular plates). The derivation of the formula is based on the assumption that the electric field, in the region between the plates is uniform, and the electric field outside that region is zero. In fact, the electric field is not uniform in the vicinity of the edges of the plates. As long as the region in which the electric field is not well-approximated by a uniform electric field is small compared to the region in which it is, our formula for the capacitance is good.



### The Effect of Insulating Material Between the Plates of a Capacitor

To get at the effect of insulating material, rather than vacuum, between the plates of a capacitor, I need to at least outline the derivation of the formula  $C = \epsilon_o \frac{A}{d}$ . Keep in mind that the capacitance is the charge-per-voltage of the capacitor. Suppose that we move charge  $q$  from one initially-neutral plate to the other. We assume that the electric field is uniform between the plates of the capacitor and zero elsewhere.



By means that you will learn about later in this book we establish that the value of the electric field (valid everywhere between the plates) is given by:

$$E = \frac{q}{A\epsilon_o} \quad (\text{B8.4})$$

Also, we know that the work done on a test charge  $q_T$  by the electric field when the test charge is moved from the higher-potential plate to the lower-potential plate is the same whether we calculate it as force-along the path times the length of the path, or, as the negative of the change in the potential energy. This results in a relation between the electric field and the electric potential as follows:

W calculated as force times distance = W calculated as minus change in potential energy

$$F\Delta x = -\Delta U$$

$$q_T E d = -q_T \Delta \varphi$$

$$E d = -(-V)$$

$$V = E d$$

Using Equation B8.4 ( $E = \frac{q}{A\epsilon_o}$ ) to replace the  $E$  in  $V = E d$  with  $\frac{q}{A\epsilon_o}$  gives us:

$$V = \frac{q}{A\epsilon_o}d$$

Solving this for  $q/V$  yields

$$\frac{q}{V} = \epsilon_o \frac{A}{d}$$

for the charge-to-voltage ratio. Since the capacitance is the charge-to-voltage ratio, this means

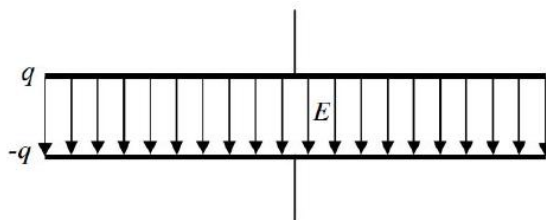
$$C = \epsilon_o \frac{A}{d}$$

which is what we set out to derive.

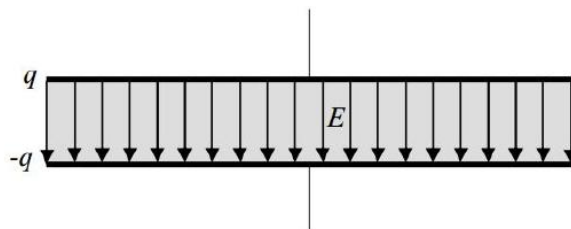
Okay now, here's the deal on having an insulator between the plates: Consider a capacitor that is identical in all respects to the one we just dealt with, except that there is an insulating material between the plates, rather than vacuum. Further suppose that the capacitor has the same amount of charge  $q$  on it as the vacuum-between-the-plates capacitor had on it. The presence of the insulator between the plates results in a weaker electric field between the plates. This means that a test charge moved from one plate to another would have less work done on it by the electric field, meaning that it would experience a smaller change in potential energy, meaning the electric potential difference between the plates is smaller. So, with the same charge, but a smaller potential difference, the charge-to-voltage ratio (that is, the capacitance of the capacitor) must be bigger.

The presence of the insulating material makes the capacitance bigger. The part of the preceding argument that still needs explaining is that part about the insulating material weakening the electric field. Why does the insulating material make the field weaker? Here's the answer:

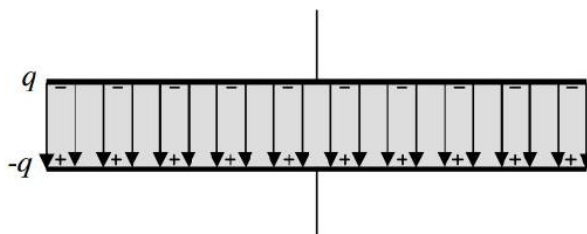
Starting with vacuum between the plates,



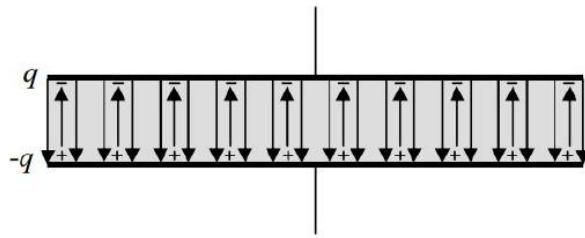
we insert some insulating material:



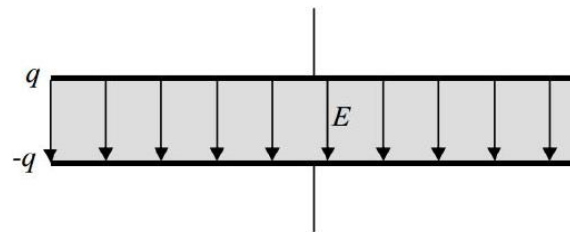
The original electric field polarizes the insulating material:



The displaced charge creates an electric field of its own, in the direction opposite that of the original electric field:



The net electric field, being at each point in space, the vector sum of the two contributions to it, is in the same direction as the original electric field, but weaker than the original electric field:



This is what we wanted to show. The presence of the insulating material makes for a weaker electric field (for the same charge on the capacitor), meaning a smaller potential difference, meaning a bigger charge-to-voltage ratio, meaning a bigger capacitance. How much bigger depends on how much the insulator is polarized which depends on what kind of material the insulator consists of. An insulating material, when placed between the plates of a capacitor is called a dielectric. The net effect of using a dielectric instead of vacuum between the plates is to multiply the capacitance by a factor known as the dielectric constant. Each dielectric is characterized by a unitless dielectric constant specific to the material of which the dielectric is made. The capacitance of a parallel-plate capacitor which has a dielectric in between the plates, rather than vacuum, is just the dielectric constant  $\kappa$  times the capacitance of the same capacitor with vacuum in between the plates.

$$C = \kappa \epsilon_o \frac{A}{d} \quad (\text{B8.5})$$

where:

- $C$  is the capacitance of the parallel-plate capacitor whose plates are separated by an insulating material,
- $\kappa$  is the dielectric constant characterizing the insulating material between the plates,
- $d$  is the distance between the plates,
- $A$  is the area of one face of one of the plates, and
- $\epsilon_o$  is a universal constant called the permittivity of free space.

Calling the dielectric constant for vacuum 1 (exactly one), we can consider this equation to apply to all parallel-plate capacitors. Some dielectric constants of materials used in manufactured capacitors are provided in the following table:

Substance	Dielectric Constant
Air	1.00
Aluminium Oxide (a corrosion product found in many electrolytic capacitors)	7
Mica	3-8
Titanium Dioxide	114
Vacuum	1 (exactly)
Waxed Paper	2.5-3.5

## Energy Stored in a Capacitor

Moving charge from one initially-neutral capacitor plate to the other is called **charging the capacitor**. When you charge a capacitor, you are storing energy in that capacitor. Providing a conducting path for the charge to go back to the plate it came from is called discharging the capacitor. If you discharge the capacitor through an electric motor, you can definitely have that charge do some work on the surroundings. So, how much energy is stored in a charged capacitor? Imagine the charging process. You use some force to move some charge over a distance from one plate to another. At first, it doesn't take much force because both plates are neutral. But the more charge that you have already relocated, the harder it is to move more charge. Think about it. If you are moving positive charge, you are pulling positive charge from a negatively charged plate and pushing it onto a positively charged plate. The total amount of work you do in moving the charge is the amount of energy you store in the capacitor. Let's calculate that amount of work.

In this derivation, a lower case  $q$  represents the variable amount of charge on the capacitor plate (it increases as we charge the capacitor), and an upper case  $Q$  represents the final amount of charge. Similarly, a lower case  $v$  represents the variable amount of voltage across the capacitor (it too increases as we charge the capacitor), and the upper case  $V$  represents the final voltage across the capacitor. Let  $U$  represent the energy stored in the capacitor:

$$dU = vdp$$

but the voltage across the capacitor is related to the charge of the capacitor by  $C = q/v$  (Equation B8.3), which, solved for  $v$  is  $v = q/C$ , so:

$$dU = \frac{q}{C} dq \quad (\text{B8.6})$$

$$\int dU = \frac{1}{C} \int_0^Q q dq \quad (\text{B8.7})$$

$$U = \frac{1}{C} \frac{q^2}{2} \Big|_0^Q \quad (\text{B8.8})$$

$$U = \frac{1}{C} \left( \frac{Q^2}{2} - \frac{0^2}{2} \right) \quad (\text{B8.9})$$

$$U = \frac{1}{2} \frac{1}{C} Q^2 \quad (\text{B8.10})$$

Using  $C = Q/V$ , we can also express the energy stored in the capacitor as  $U = \frac{1}{2} QV$ , or

$$U = \frac{1}{2} CV^2 \quad (\text{B8.11})$$

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