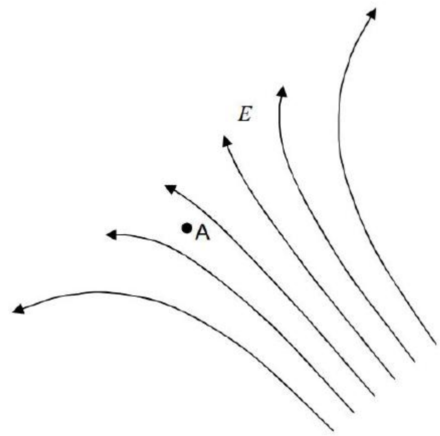
B7: Equipotential Surfaces, Conductors, and Voltage

Consider a region of space in which there exists an electric field. Focus your attention on a specific point in that electric field, call it point A.



Imagine placing a positive test charge at point A. (Assume that, by means not specified, you can move the test charge anywhere you want to.) Please think about the answer to the following question before reading on: Is it possible for you to move the test charge around in the electric field in such a manner that the electric field does no work on the test charge?

If we move the positive test charge in the "downfield" direction (toward the upper left corner of the diagram), there will be a positive amount of work (force-along-the-path times the length of the path) done on the test charge. And, if we move the positive test charge in the "upfield" direction there will be a negative amount of work done on it. But, if we move the positive test charge at right angles to the electric field, no work is done on it. That is, if we choose a path for the positive test charge such that every infinitesimal displacement of the particle is normal to the electric field at the location of the particle when it (the particle) undergoes said infinitesimal displacement, then the work done on the test charge, by the electric field, is zero. The set of all points that can be reached by such paths makes up an infinitesimally thin shell, a surface, which is everywhere perpendicular to the electric field. In moving a test charge along the surface from one point (call it point A) to another point (call it point B) on the surface, the work done is zero because the electric field is perpendicular to the path at all points along the path. Let's (momentarily) call the kind of surface we have been discussing a "zero-work surface." We have constructed the surface by means of force-along-the-path times the length-of-the-path work considerations. But the work done by the electric field when a test charge is moved from point A on the surface to point B on the surface must also turn out to be zero if we calculate it as the negative of the change in the potential energy of the test charge. Let's do that and see where it leads us. We know that the work $W=0$.

Also

$$W = -\Delta U \tag{B7.1}$$

$$= -(U_B - U_A) \tag{B7.2}$$

In terms of the electric potential φ, $U = q\varphi$ so the work can be expressed as

$$W = -(q\varphi_B - q\varphi_A) \tag{B7.3}$$

$$= -q(\varphi_B - \varphi_A) \tag{B7.4}$$

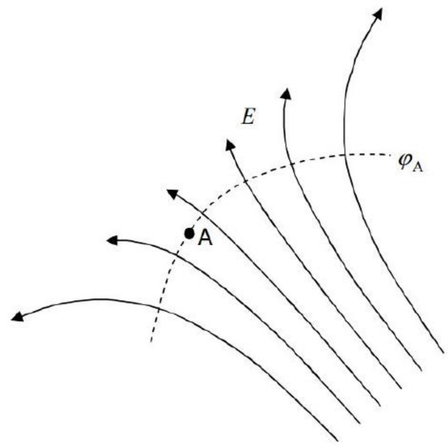
Given that $W=0$, this means that

$$0 = -q(\varphi_B - \varphi_A)$$

$$\varphi_B - \varphi_A = 0$$

$$\varphi_B = \varphi_A$$

This is true for any point B on the entire "zero-work" surface. This means that every point on the entire surface is at the same value of electric potential. Thus a "zero-work" surface is also an equipotential surface. Indeed, this is the name (equipotential surface) that physicists use for such a surface. An equipotential surface is typically labeled with the corresponding potential value (φ_A in the case at hand). In the following diagram, the dashed curve represents the equipotential surface viewed edge on.



Summarizing:

- An equipotential surface is an imaginary surface on which every point has one and the same value of electric potential.
- An equipotential surface is everywhere perpendicular to the electric field that it characterizes.
- The work done by the electric field on a particle when it is moved from one point on an equipotential surface to another point on the same equipotential surface is always zero.

Perfect Conductors and the Electric Potential

Please recall what you know about perfect conductors and the electric field. Namely, that everywhere inside and on a perfect conductor, the electric field is zero. This goes for solid conductors as well as hollow, empty shells of perfectly conducting material. This means that the work done by the electric field on a test charge that is moved from one point in or on a perfect conductor (consider this to be a thought experiment), to another point in or on the same conductor, is zero. This means that the difference in the electric potential between any two points in or on a perfect conductor must be zero. This means that the electric potential at every point in and on a perfect conductor must have one and the same value. Note that the value is not, in general, zero.

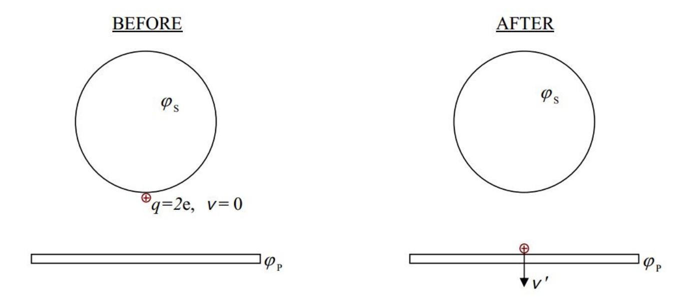
Some Electric Potential Jargon

When we talk about the electric potential in the context of a perfect conductor (or an object that approximates a perfect conductor), because every point in and on the conductor has the same value of electric potential, we typically call that value the electric potential of the conductor. We also use expressions such as, "the conductor is at a potential of 25 volts," meaning that the value of electric potential at every point in and on the conductor is 25 volts with respect to infinity (meaning that the zero of electric potential is at an infinite distance from the conductor) and/or with respect to "ground" (meaning that the potential of the earth is the zero of electric potential).

Electric Potential Difference, a.k.a. Voltage

In general, what is at issue when one talks about conductors and electric potential is not the value of the electric potential of a conductor, but rather, the electric potential difference between one conductor and another.

A hollow metal sphere is at a potential that is 472 volts higher than that of a nearby metal plate. A particle of charge $2e$ is released from rest at the surface of the sphere. It subsequently strikes the plate. With what kinetic energy does the charged particle strike the plate? (Assume that the only force acting on the particle is that due to the electric field corresponding to the given information.)



(Given $\varphi_S - \varphi_P = \Delta\varphi = 472$ volts)

$$\text{Energy Before} = \text{Energy After}$$

$$0 + U = K' + U'$$

$$q\varphi_S = K' + q\varphi_P$$

$$K' = q\varphi_S - q\varphi_P$$

$$K' = q(\varphi_S - \varphi_P)$$

$$K' = q\Delta\varphi$$

$$K' = 2e(472\text{volts})$$

$$K' = 944eV$$

Note that in the solution to the example problem, we never needed to know the value of the electric potential of either the sphere or the plate, only the difference between the two potentials. There is a device which can be used to measure the potential difference between two points in space. The device is called a voltmeter. A typical voltmeter consists of a box with two wires extending from it. On the end of each wire is a short metal wand called a probe. Each wire and each probe, except for the tip of the probe, is covered with insulating material. The box displays, either by means of a digital readout or the position of a needle, the potential difference between the two wires. In typical use, one presses the metal tip of one probe against a conductor of interest and holds the tip there. That causes that probe and wire to be at the same potential as the conductor. One presses the tip of the other probe against another conductor. This causes that probe and wire to be at the potential of the second conductor. With each probe in contact with a conductor, the voltmeter continually displays the potential difference between the two conductors.

Based on the SI units of measurement, the electric potential difference between two points in space goes by another name, namely, voltage. Voltage means electric potential difference which means, the difference between the electric-potential-energy-per-charge-of-would-be-victim at one point in space and the electric-potential-energy-per-charge-of-would-be-victim at another point in space. While voltage literally means potential difference, the word is also, quite often used to mean electric potential itself, where, one particular conductor or point in space is defined to be the zero of potential. If no conductor or point in space has been defined to be the zero, then it is understood that “infinity” is considered to be at the zero of electric potential. So, if you read that a metal object is at a potential of 230 volts (when no conductor or point in space has been identified as the zero of electric potential), you can interpret the statement to mean the same thing as a statement that the electric potential of the metal object is 230 volts higher than the electric potential at any point that is an infinite distance away from the object.

As you move on in your study of physics, onward to your study and work with electric circuits, it is important to keep in mind that voltage, in a circuit, is the difference in the value of a characteristic (the electric potential) of one conductor, and the value of the same characteristic (electric potential) of another conductor.

Analogy Between Voltage and Altitude

One can draw a pretty good analogy between voltage (electric potential) and altitude. Consider a particular altitude above the surface of the earth (measured, for instance, from sea level). The value of the altitude characterizes a point in space or a set of points in space. In fact, the set of all points in space that are at the same altitude above the surface of the earth forms an “equi-altitude” surface. On a local scale, we can think of that “equi-altitude” surface as a plane. On a global scale, looking at the big picture, we recognize it to be a spheroidal shell. Flocks of birds can be at that altitude and when they are, we attribute the altitude to the flock of birds. We say that the flock of birds has such and such an altitude. But, whether or not the flock of birds is there, the altitude exists. Regarding a particular altitude, we can have birds and air and clouds moving or flowing through space at that altitude, but the altitude itself just exists—it doesn't flow or go anywhere. This is like the voltage in a circuit. The voltage in a circuit exists. The voltage characterizes a conductor in a circuit. Charged particles can move and flow in and through a conductor that is at that voltage, but, the voltage doesn't flow or go anywhere, any more than altitude flows or goes anywhere.
