B1: Charge & Coulomb's Law

Charge is a property of matter. There are two kinds of charge, positive "+" and negative "-". An object can have positive charge, negative charge, or no charge at all. A particle which has charge causes a force-per-charge-of-would-be-victim vector to exist at each point in the region of space around itself. The infinite set of force-per-charge-of-would be-victim vectors is called a vector field. Any charged particle that finds itself in the region of space where the force-per-charge-of-would-be-victim vector field exists will have a force exerted upon it by the force-per-charge-of-would-be-victim field. The force-per-charge-of-would-be-victim field is called the electric field. The charged particle causing the electric field to exist is called the source charge. (Regarding jargon: A charged particle is a particle that has charge. A charged particle is often referred to simply as "a charge.")

The source charge causes an electric field which exerts a force on the victim charge. The net effect is that the source charge causes a force to be exerted on the victim. While we have much to discuss about the electric field, for now, we focus on the net effect, which we state simply (neglecting the "middle man", the electric field) as, "A charged particle exerts a force on another charged particle." This statement is *Coulomb's Law* in its conceptual form. The force is called the *Coulomb force*, a.k.a. the *electrostatic force*.

Note that either charge can be viewed as the source charge and either can be viewed as the victim charge. Identifying one charge as the victim charge is equivalent to establishing a point of view, similar to identifying an object whose motion or equilibrium is under study for purposes of applying Newton's 2nd Law of motion. $\vec{a} = \frac{\sum \vec{F}}{m}$. In Coulomb's Law, the force exerted on one charged particle by another is directed along the line connecting the two particles, and, away from the other particle if both particles have the same kind of charge (both positive, or, both negative) but, toward the other particle if the kind of charge differs (one positive and the other negative). This fact is probably familiar to you as, "like charges repel and unlike attract."

The SI unit of charge is the coulomb, abbreviated C. One coulomb of charge is a lot of charge, so much that, two particles, each having a charge of +1 C and separated by a distance of 1 meter exert a force of $9 \times 10^9 N$, that is 9 billion newtons on each other.

This brings us to the equation form of Coulomb's Law which can be written to give the magnitude of the force exerted by one charged particle on another as:

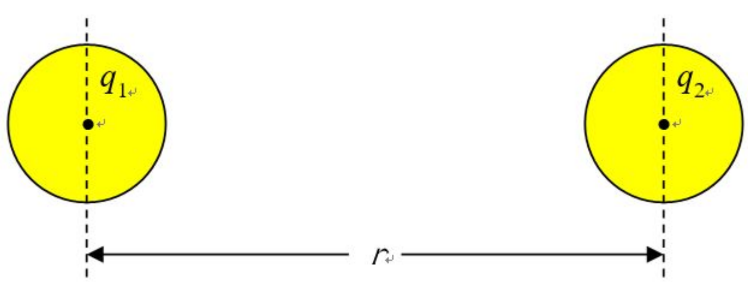
$$F = k\frac{|q_1||q_2|}{r^2} \tag{B1.1}$$

where:

- $k = 8.99 \times 10^9 \frac{N \cdot m^2}{C^2}$, a universal constant called the Coulomb constant,
- q_1 is the charge of particle 1,
- q_2 is the charge of particle 2, and
- r is the distance between the two particles.

The user of the equation (we are still talking about equation B1.1, $F = k\frac{|q_1||q_2|}{r^2}$) is expected to establish the direction of the force by means of "common sense" (the user's understanding of what it means for like charges to repel and unlike charges to attract each other).

While Coulomb's Law in equation form is designed to be exact for point particles, it is also exact for spherically symmetric charge distributions (such as uniform balls of charge) as long as one uses the center-to-center distance for \(*r*\).



Coulomb's Law is also a good approximation in the case of objects on which the charge is not spherically symmetric as long as the objects' dimensions are small compared to the separation of the objects (the truer this is, the better the approximation). Again, one uses the separation of the centers of the charge distributions in the Coulomb's Law equation.

Coulomb's Law can be written in vector form as:

$$\overrightarrow{F_{12}} = k\frac{q_1 q_2}{r^2}\hat{r}_{12} \tag{B1.2}$$

where:

- $\overrightarrow{F_{12}}$ is the force "of 1 on 2", that is, the force exerted by particle 1 on particle 2,
- $\hat{r}_{12}$ is a unit vector in the direction "from 1 to 2", and
- k, q_1 and q_2 are defined as before (the Coulomb constant, the charge on particle 1, and the charge on particle 2 respectively).

Note the absence of the absolute value signs around q_1 and q_2. A particle which has a certain amount, say, 5 coulombs of the negative kind of charge is said to have a charge of -5 coulombs and one with 5 coulombs of the positive kind of charge is said to have a charge of +5 coulombs) and indeed the plus and minus signs designating the kind of charge have the usual arithmetic meaning when the charges enter into equations. For instance, if you create a composite object by combining an object that has a charge of $q_1 = +3C$ with an object that has a charge of $q_2 = -5C$, then the composite object has a charge of

$$q = q_1 + q_2$$

$$q = +3C + (-5C)$$

$$q = -2C$$

Note that the arithmetic interpretation of the kind of charge in the vector form of Coulomb's Law causes that equation to give the correct direction of the force for any combination of kinds of charge. For instance, if one of the particles has positive charge and the other negative, then the value of the product \(q_1q_2\) in equation B1.2

$$\overrightarrow{F_{12}} = k\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

has a negative sign which we can associate with the unit vector. Now $-\hat{r}_{12}$ is in the direction opposite "from 1 to 2" meaning it is in the direction "from 2 to 1." This means that $\overrightarrow{F_{12}}$, the force of 1 on 2, is directed toward particle 1. This is consistent with our understanding that opposites attract. Similarly, if q_1 and q_2 2 are both positive, or both negative in $\overrightarrow{F_{12}} = k\frac{q_1 q_2}{r^2}\hat{r}_{12}$ then the value of the product \(q_1q_2\) is positive meaning that the direction of the force of 1 on 2 is $\hat{r}_{12}$ (from 1 to 2), that is, away from 1, consistent with the fact that like charges repel. We've been talking about the force of 1 on 2. Particle 2 exerts a force on particle 1 as well. It is given by $\overrightarrow{F_{21}} = k\frac{q_1 q_2}{r^2}\hat{r}_{21}$. The unit vector $\hat{r}_{21}$, pointing from 2 to 1, is just the negative of the unit vector pointing from 1 to 2:

$$\hat{r_{21}} = -\hat{r}_{12}$$

If we make this substitution into our expression for the force exerted by particle 2 on particle 1, we obtain:

$$\overrightarrow{F_{21}} = k\frac{q_1 q_2}{r^2}(-\hat{r}_{12})$$

$$\overrightarrow{F_{21}} = -k\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

Comparing the right side with our expression for the force of 1 on 2 (namely, $\overrightarrow{F_{12}} = k\frac{q_1 q_2}{r^2}\hat{r}_{12}$), we see that

$$\overrightarrow{F_{21}} = -\overrightarrow{F_{12}}$$

So, according to Coulomb's Law, if particle 1 is exerting a force $\overrightarrow{F_{12}}$ on particle 2, then particle 2 is, at the same time, exerting an equal but opposite force $-\overrightarrow{F_{12}}$ back on particle 2, which, as we know, by Newton's 3$^{\text{rd}}$ Law, it must.

In our macroscopic world we find that charge is not an inherent fixed property of an object but, rather, something that we can change. Rub a neutral rubber rod with animal fur, for instance, and you'll find that afterwards, the rod has some charge and the fur has the opposite kind of charge. Ben Franklin defined the kind of charge that appears on the rubber rod to be negative charge and the other kind to be positive charge. To provide some understanding of how the rod comes to have negative charge, we delve briefly into the atomic world and even the subatomic world.

The stable matter with which we are familiar consists of protons, neutrons, and electrons. Neutrons are neutral, protons have a fixed amount of positive charge, and electrons have the same fixed amount of negative charge. Unlike the rubber rod of our macroscopic world, you cannot give charge to the neutron and you can neither add charge to, nor remove charge from, either the proton or the electron. Every proton has the same fixed amount of charge, namely $1.60 \times 10^{-19} C$. Scientists have never been able to isolate any smaller amount of charge. That amount of charge is given a name. It is called the e, abbreviated e and pronounced "ee". The e is a non-SI unit of charge. As stated $1e = 1.60 \times 10^{-19} C$. In units of e, the charge of a proton is $1e$ (exactly) and the charge of an electron is $-1e$. For some reason, there is a tendency among humans to interpret the fact that the unit the e is equivalent to $1.60 \times 10^{-19} C$ to mean that $1e$ equals $-1.60 \times 10^{-19} C$. This is wrong! Rather,

$$1e = 1.60 \times 10^{-19} C$$

A typical neutral atom consists of a nucleus made up of neutrons and protons surrounded by orbiting electrons such that the number of electrons in orbit about the nucleus is equal to the number of protons in the nucleus. Let's see what this means in terms of an everyday object such as a polystyrene cup. A typical polystyrene cup has a mass of about 2 grams. It consists of roughly: 6×10^{23} neutrons, 6×10^{23} protons, and, when neutral, 6×10^{23} electrons. Thus, when neutral it has about $1 \times 10^{5} C$ of positive charge and $1 \times 10^{5} C$ of negative charge, for a total of 0 charge. Now if you rub a polystyrene cup with animal fur you can give it a noticeable charge. If you rub it all over with the fur on a dry day and then experimentally determine the charge on the cup, you will find it to be about $-5 \times 10^{-8} C$. This represents an increase of about 0.00000000005%in the number of electrons on the cup. They were transferred from the fur to the cup. We are talking about 3×10^{10} electrons, which sure would be a lot of marbles but represents a minuscule fraction of the total number of electrons in the material of the cup.

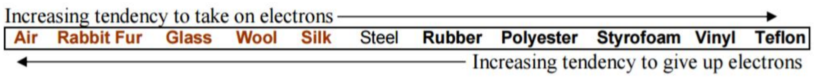
The main points of the preceding discussion are:

- A typical neutral macroscopic object consists of incredibly huge amounts of both kinds of charge (about 50 million coulombs of each for every kilogram of matter), the same amount of each kind.
- When we charge an object, we transfer a relatively minuscule amount of charge to or from that object.
- A typical everyday amount of charge (such as the amount of charge on a clingy sock just out of the dryer) is 10^{-7} coulombs.
- When we transfer charge from one object to another, we are actually moving charged particles, typically electrons, from one object to the other.

One point that we did not make in the discussion above is that charge is conserved. For instance, if, by rubbing a rubber rod with fur, we transfer a certain amount of negative charge to the rubber rod, then, the originally-neutral fur is left with the exact same amount of positive charge. Recalling the exact balance between the incredibly huge amount of negative charge and the incredibly huge amount of positive charge in any macroscopic object, we recognize that, in charging the rubber rod, the fur becomes positively charged not because it somehow gains positive charge, but, because it loses negative charge, meaning that the original incredibly huge amount of positive charge now (slightly) exceeds the (still incredibly huge) amount of negative charge remaining on and in the fur.

Charging by Rubbing

One might well wonder why rubbing a rubber rod with animal fur would cause electrons to be transferred from the fur to the rod. If one could imagine some way that even one electron might, by chance, find its way from the fur to the rod, it would seem that, then, the rod would be negatively charged and the fur positively charged so that any electron that got free from the fur would be attracted back to the fur by the positive charge on it and repelled by the negative charge on the rod. So why would any more charge ever be transferred from the fur to the rod? The answer comes under the heading of "distance matters." In rubbing the rod with the fur you bring lots of fur molecules very close to rubber molecules. In some cases, the outer electrons in the atoms of the fur come so close to nuclei of the atoms on the surface of the rubber that the force of attraction of these positive nuclei is greater than the force of attraction of the nucleus of the atom of which they are a part. The net force is then toward the rod, the electrons in question experience acceleration toward the rod that changes the velocity such that the electrons move to the rod. Charging by rubbing depends strongly on the molecular structure of the materials in question. One interesting aspect of the process is that the rubbing only causes lots of molecules in the fur to come very close to molecules in the rubber. It is not as if the energy associated with the rubbing motion is somehow given to the electrons causing them to jump from the fur to the rubber. It should be noted that fur is not the only material that has a tendency to give up electrons and rubber is not the only material with a tendency to acquire them. The phenomenon of charging by rubbing is called triboelectrification. The following ordered list of the tendency of (a limited number of) materials to give up or accept electrons is called a triboelectric sequence:



The presence and position of air on the list suggests that it is easier to maintain a negative charge on objects in air than it is to maintain a positive charge on them.

Conductors and Insulators

Suppose you charge a rubber rod and then touch it to a neutral object. Some charge, repelled by the negative charge on the rod, will be transferred to the originally-neutral object. What happens to that charge then depends on the material of which the originally-neutral object consists. In the case of some materials, the charge will stay on the spot where the originally neutral object is touched by the charged rod. Such materials are referred to as insulators, materials through which charge cannot move, or, through which the movement of charge is very limited. Examples of good insulators are quartz, glass, and air. In the case of other materials, the charge, almost instantly spreads out all over the material in question, in response to the force of repulsion (recalling that force causes acceleration which leads to the movement) that each elementary particle of the charge exerts on every other elementary particle of charge. Materials in which the charge is free to move about are referred to as conductors. Examples of good conductors are metals and saltwater.

When you put some charge on a conductor, it immediately spreads out all over the conductor. The larger the conductor, the more it spreads out. In the case of a very large object, the charge can spread out so much that any chunk of the object has a negligible amount of charge and hence, behaves as if were neutral. Near the surface of the earth, the earth itself is large enough to play such a role. If we bury a good conductor such as a long copper rod or pipe, in the earth, and connect to it another good conductor such as a copper wire, which we might connect to another metal object, such as a cover plate for an electrical socket, above but near the surface of the earth, we can take advantage of the earth's nature as a huge object made largely of conducting material. If we touch a charged rubber rod to the metal cover plate just mentioned, and then withdraw the rod, the charge that is transferred to the metal plate spreads out over the earth to the extent that the cover plate is neutral. We use the expression "the charge that was transferred to the cover plate has flowed into the earth." A conductor that is connected to the earth in the manner that the cover plate just discussed is connected is called "ground." The act of touching a charged object to ground is referred to as grounding the object. If the object itself is a conductor, grounding it (in the absence of other charged objects) causes it to become neutral.

Charging by Induction

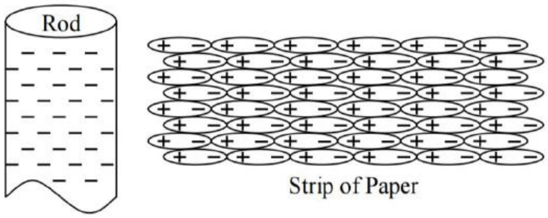
If you hold one side of a conductor in contact with ground and bring a charged object very near the other side of the conductor, and then, keeping the charged object close to the conductor without touching it, break the contact of the conductor with ground, you will find that the conductor is charged with the opposite kind of the charge that was originally on the charged object. Here's why. When you bring the charged object near the conductor, it repels charge in the conductor right out of the conductor and into the earth. Then, with those charges gone, if you break the path to ground, the conductor is stuck with the absence of those charged particles that were repelled into the ground. Since the original charged object repels the same kind of charge that it has, the conductor is left with the opposite kind of charge.

Polarization

Let's rub that rubber rod with fur again and bring the rubber rod near one end of a small strip of neutral aluminum foil. We find that the foil is attracted to the rubber rod, even though the foil remains neutral. Here's why:

The negatively charged rubber rod repels the free-to-move negative charge in the strip to the other end of the strip. As a result, the near end of the aluminum strip is positively charged and the far end is negatively charged. So, the rubber rod attracts the near end of the rod and repels the far end. But, because the near end is nearer, the force of attraction is greater than the force of repulsion and the net force is toward the rod. The separation of charge that occurs in the neutral strip of aluminum is called polarization, and, when the neutral aluminum strip is positive on one end and negative on the other, we say that it is polarized.

Polarization takes place in the case of insulators as well, despite the fact that charge is not free to move about within an insulator. Let's bring a negatively-charged rod near one end of a piece of paper. Every molecule in the paper has a positive part and a negative part. The positive part is attracted to the rod and the negative part is repelled. The effect is that each molecule in the paper is polarized and stretched. Now, if every bit of positive charge gets pulled just a little bit closer to the rod and every bit of negative charge gets pushed a little farther away, the net effect in the bulk of the paper is to leave it neutral, but, at the ends there is a net charge. On the near end, the repelled negative charge leaves the attracted positive charge all by itself, and, on the far end, the attracted positive charge leaves the repelled negative charge all by itself.



As in the case of the aluminum strip, the negative rubber rod attracts the near, positive, end and repels the far, negative, end, but, the near end is closer so the attractive force is greater, meaning that the net force on the strip of paper is attractive. Again, the separation of the charge in the paper is called polarization and the fact that one end of the neutral strip of paper is negative and the other is positive means that the strip of paper is polarized.