

4.3: Ideal Gas Model

Ideal Gas Law

The *Ideal Gas Law* is a good approximation for the behavior of most gases. The word "ideal" refers to the following assumption about the gas:

- the gas is made-up of a large number of particles whose motion is random.
- the gas particles are negligibly small compared to the the volume the gas occupies.
- there is no intermolecular forces between the particles.
- the there is no energy loss due to particle collisions.

Given the assumption described above, there is an important relationship between the pressure, temperature, volume, and number of particles in an ideal gas:

$$PV = Nk_B T \quad (4.3.1)$$

or

$$PV = nRT \quad (4.3.2)$$

where n is the number of moles, while N is the actual number of gas molecules. The number of moles is defined as:

$$n = \frac{N}{N_A} \quad (4.3.3)$$

where N_A is Avogadro's constant. The Boltzmann's constant is k_B is:

$$k_B = \frac{R}{N_A} = 1.381 \times 10^{-23} \frac{J}{K} \quad (4.3.4)$$

an the gas constant R is:

$$R = 8.314 \frac{J}{mol \cdot K} \quad (4.3.5)$$

The *Ideal Gas Law* states that pressure and volume are not uniquely related, but depend on the temperature. Therefore, the work done when compressing a gas will depend on how temperature changes as the pressure and volume change. That is, it takes different amounts of work to compress a gas, depending on how the temperature varies during the compression, as heat is or is not allowed to enter the system. Pressure vs. Volume plots (PV diagrams) introduced in the next section and a new way of stating conservation of energy will allow us to determine the contributions of work and heat to changes in systems.

The Ideal Gas connects the *microscopic* way of thinking about gases to the *macroscopic* way of thinking about gases. The temperature term forms a bridge between the particle model of thermal energy and macroscopic thermodynamics. Temperature connects directly to the meaning of thermal energy through the fundamental relationship of temperature as a measure of the random thermal motion in any energy mode at thermal equilibrium, while at the same time, temperature is seen to be directly proportional to the pressure of a gas.

Heat Capacity for an Ideal Gas

In Section 3.4 we defined heat capacity at a constant volume. Most chemical reactions or biological processes occur in open containers, where the gas is allowed to expand or compress, keeping the pressure constant. The assumption for heat capacity at constant volume was that no work was being done on or by the system. Based on the definition in the previous section, $W = -\int P dV$, we can now see that when the volume is not changing $dV = 0$, there is no work being done. In an experiment where the container is open, we can no longer make that assumption.

Using energy conservation we know that, $\Delta E_{tot} = Q + W$. There are no intermolecular interaction in an ideal gas, thus, there are no changes in bond energy and only thermal energy can change during an interaction, $\Delta E_{tot} = \Delta E_{th}$. Going back to the definition of heat capacity, $C = Q/\Delta T$, and plugging in for Q , heat capacity can be written as:

$$C = \frac{\Delta E_{th} - W}{\Delta T} \quad (4.3.6)$$

In Equation 3.4.10 we defined heat capacity at constant volume at $C_V = \Delta E_{th} / \Delta T$. Thus, we can write the above equation as:

$$C = C_V - \frac{W}{\Delta T} \quad (4.3.7)$$

For constant pressure the integral equation for work simplifies to $W = -P\Delta V$, and using the Ideal Gas Law Equation 4.3.2 the above equation simplifies to:

$$C_P = C_V + nR \quad (4.3.8)$$

where the subscript "P" refers to heat capacity at constant pressure.

For one mole the expression for the molar heat capacity at constant pressure for an ideal gas becomes:

$$c_{pm} = c_{vm} + R \quad (4.3.9)$$

For an ideal gas, the molar heat capacity at constant pressure is larger than at constant volume by exactly the value R . This is true for any ideal gas, whether monatomic, diatomic, or polyatomic, because the Ideal Gas Law does not depend on *intramolecular* motions and interactions. Looking back at Table 3.2.1 of experimentally determined heat capacities, we see indeed that the molar heat capacity measured at constant pressure is larger than the constant volume heat capacity by R . Our model of matter does indeed work pretty well for gases!

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