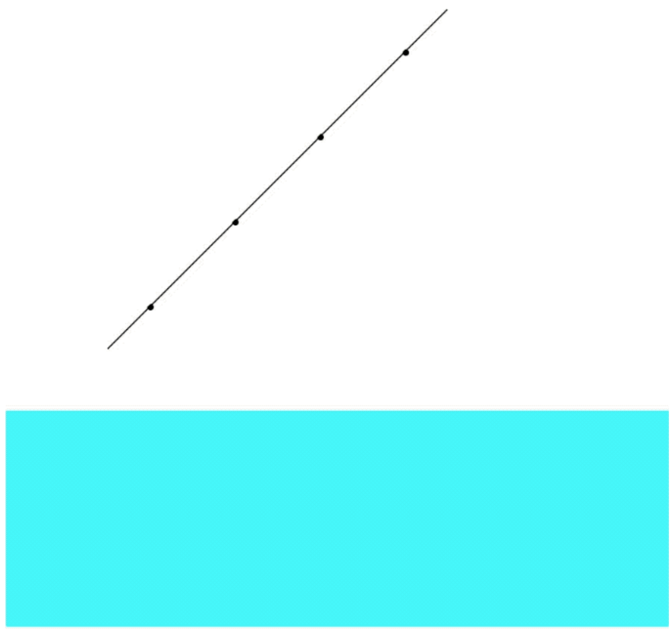
10.4: Refraction

Refraction

In the previous two sections we analyzed light rays changing direction through reflection, a process that occurs when a ray meets another medium. We now consider another way that direction change can occur, instead of reflecting from the medium, the ray moves into the new medium it encounters. This process, called *refraction*. To get to the essence of this phenomenon from Huygens's principle, we don't have a symmetry trick like we did for reflection, so rather than use a point source of the light, we can look at the effect that changing the medium has on a plane wave.

We saw in Figure 8.7.1 how a plane wave propagates according to Huygens's Principle. We can't sketch every one wavelets emerging from the infinite number of points on the wavefront, but we can sketch a few representative wavelets, and if those wavelets have propagated for equal periods of time, then a line tangent to all the wavelets will represent the next wavefront. It's clear that following this procedure for a plane wave will continue the plane wave in the same direction. But now let's imagine that such a plane wave approaches a new medium from an angle, as shown in the figure below. As each point on the wave front comes in contact with the new medium, it becomes a source for a new Huygens wavelet *within the medium*. These wavelets will travel at a different rate than they traveled in the previous medium (in the figure, the light wave is slowing down in the new medium). This means that the distance the wave in medium #1 travels is farther than it travels in medium #2 during the same time. The effect is a bending of the direction of the plane wave in medium #2 relative to medium #1.

Figure 10.4.1: Huygens's Principle Refracts a Plane Wave



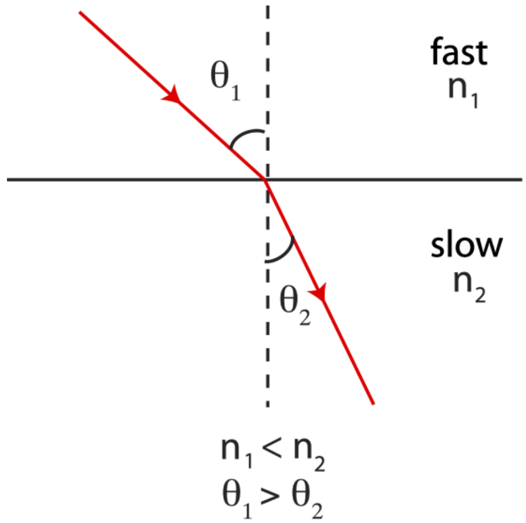
The amount that the direction of the light ray changes when the wave enters a new medium depends upon how much the wave slows down or speeds up upon changing media. The property of the medium which determines the speed of light is known as its *index of refraction*, n, and is defined as:

$$n=\frac{c}{v} \tag{10.4.1}$$

where c is the speed of light in a vacuum and v is a speed of light in the medium. When the light is traveling in a vacuum, $v=c$, the resulting index of refraction is one, $n=1$. Since $v<c$ in any other medium except for the vacuum, the index of refraction will be greater than one, $n>1$. In air light slows down by very slightly such that the index of refraction of air is, $n_{\text{air}}=1.00029$. Since we will often analyze problems where light enters a new medium from air, for simplicity we assume that the index of refraction in air is approximately one. For other materials such as glass the index of refraction becomes significant, $n_{\text{glass}}=1.52$.

Using ray representation refraction can be depicted as in the figure below, which depicts the same situation as in Figure 10.4.1, where light travels from a fast to a slow medium, or one with higher index of refraction to one with a lower one. As for reflection, we will measure angles relative to the normal of the surface separating two types of media. The incident ray makes an angle marked θ_1 with the normal, and the refracted ray makes an angle marked θ_2 with the normal. As demonstrated by Huygen's principle above, the rays bend down or toward the normal as they refract from a faster to a slower medium. Thus, in this example $\theta_2 < \theta_1$.

Figure 10.4.2: Ray Refraction Going from Fast to Slow Medium

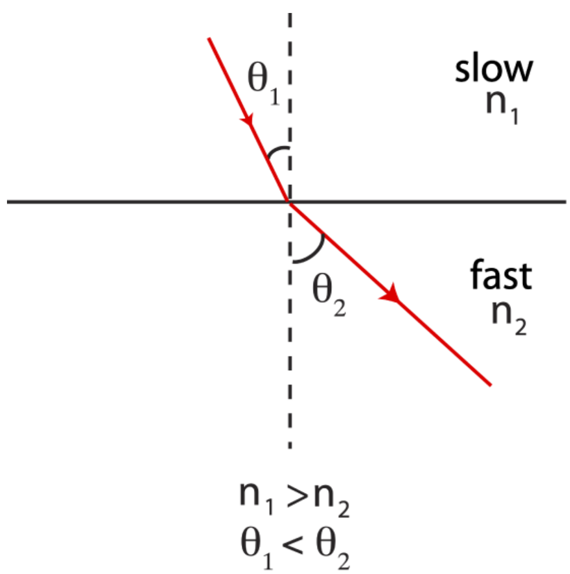


The relationship between the two angles and the two indices of refraction is described by *Snell's Law* (you can see the derivation of this equation in the digression below using Fermat's principle):

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{10.4.2}$$

Snell's Law is perfectly symmetric, which means that the same rules govern refraction when light moves from a slower to a faster medium. In other words, you can reverse the direction of rays shown in Figure 10.4.2, and you would obtain the situation depicted below. In this case the ray will bend away from the normal.

Figure 10.4.3: Ray Refraction Going from Slow to Fast Medium

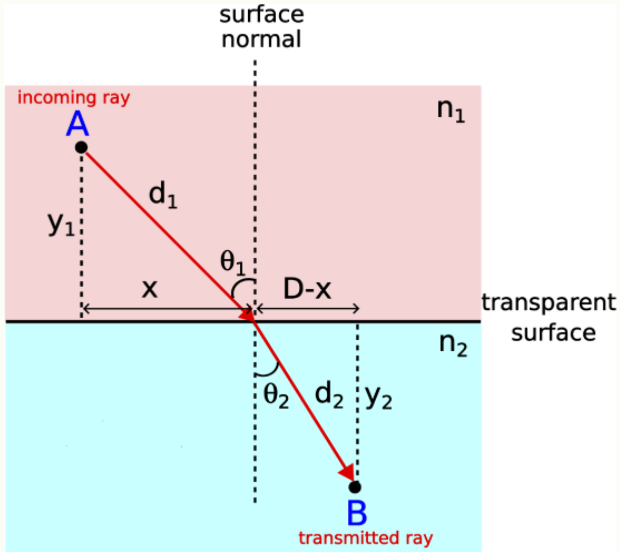


To summarize the general rules for refraction:

A ray will bend toward the normal when it moves from a faster to a slower medium, and it will bend away from the normal when it moves from a slower to a faster medium.

Digression: Snell's Law from Fermat's Principle

Fermat's principle states that light travels between two points such that it takes the shortest amount of time. Consider the diagram below. Here a incoming ray which originates at point A in a material with index of refraction n_1 is transmitted to another material with index of refraction n_2. We want to determine the path which will minimize the time of the ray's travel from point A to B.



Our goal is to find the the distance that the ray travels, $d_1 + d_2$, such that the time is minimized. As we did when we derived the Law of Reflection in Section 11.2, we will solve for the distance x, marked in the figure, such that it insures the shortest path between A and B with refraction.

The time it takes for the ray to travel from point A to point B is the time it takes to travel in medium 1 plus the time it travels in medium 2, $t_{\text{tot}} = t_1 + t_2$. The time in each medium is determined by the distance divided by speed, $v = c/n$:

$$t = \frac{d_1}{v_1} + \frac{d_2}{v_2} = d_1\left(\frac{n_1}{c}\right) + d_2\left(\frac{n_2}{c}\right)$$

Using Pythagorean theorem for each right triangle formed in the figure, the above equation can be written in terms of the horizontal distance x:

$$t = \sqrt{x^2 + y_1^2}\left(\frac{n_1}{c}\right) + \sqrt{(D-x)^2 + y_2^2}\left(\frac{n_2}{c}\right)$$

The vertical distances y_1 and y_2 and the horizontal distance D are fixed for given points A and B. To find the distance x which minimizes time, we need to take the derivative of time with the respect to x and set it to zero:

$$\frac{dt}{dx} = 0 = \left(\frac{x}{\sqrt{x^2 + y_1^2}}\right)\left(\frac{n_1}{c}\right) - \left(\frac{(D-x)}{\sqrt{(D-x)^2 + y_2^2}}\right)\left(\frac{n_2}{c}\right)$$

Expressing the above equation in terms of the angles θ_1 and θ_2 we get:

$$0 = \frac{n_1}{c}\sin\theta_1 - \frac{n_2}{c}\sin\theta_2$$

Rearranging, we arrive at Snell's Law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Total Internal Reflection

It was noted above that light which passes from a slower medium to a faster one bends away from the perpendicular. What happens then if the incoming angle is made larger and larger (obviously it can't be more than 90^o)? For example, suppose we have $n_1 = 2.0$, $\theta_1 = 45^o$, and $n_2 = 1.0$. Plugging these values into Snell's law gives:

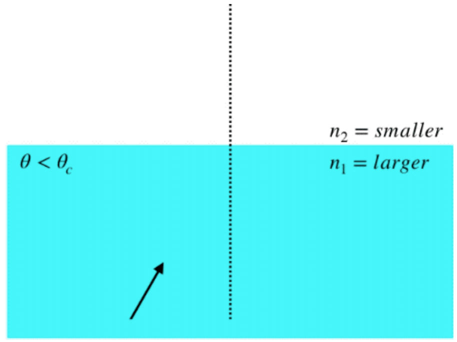
$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1 = 2.0 \cdot \sin 45^o = 1.4 \tag{10.4.3}$$

The sine function can never exceed 1, so there is no solution to this. This means that the *light incident at this angle cannot be transmitted into the new medium*. Every time light strikes a new medium some can be transmitted, and some reflected, so this result tells us that all of it must be reflected back into the medium in which it started. This phenomenon is called *total internal reflection*. The angle at which all of this first blows up is the one where the outgoing angle equals 90^o (the outgoing light refracts parallel to the surface between the two media). This angle is called the *critical angle*, and is computed by choosing the outgoing angle to be 90^o:

$$n_1 \sin\theta_c = n_2 \sin 90^o \quad \Rightarrow \quad \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) \tag{10.4.4}$$

The process of increasing the incoming angle until total internal reflection is achieved is illustrated below.

Figure 10.4.4: Partial and Total Internal Reflections By Incident Angle



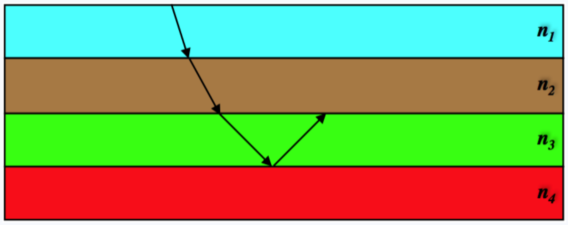
Note that there is at least partial reflection (obeying the law of reflection) every time the light hits the surface, but all of the light along that ray is only reflected when the ray's angle exceeds the critical angle.

Alert

Note that when light is coming from one medium to another, unless that light is a plane wave, it will be moving in many directions at once. Only the portions of the light wave with rays that equal or exceed the critical angle are not transmitted into the new medium. So the word "total" in "total internal reflection" to express the fraction of light at a specific angle that is reflected back, not necessarily the fraction of all the light that is reflected back.

Example 10.4.1

The diagram below shows the path of a ray of monochromatic light as it hits the surfaces between four different media (only the primary ray is considered – partial reflections are ignored). Order the four media according to the magnitudes of their indices of refraction.



Solution

We know from Snell's Law that when light passes from a higher index to a lower one, it bends away from the perpendicular, so we immediately have $n_1 > n_2 > n_3$. For the ray to reflect back from the fourth medium, it has to be a total internal reflection (we are only considering primary rays, so this is not a partial reflection), which can only occur when light is going from a higher index of refraction to a lower one, so $n_3 > n_4$.