7.7: Examples

Example 7.7.1: Braking

Suppose you are riding your bicycle and hit the brakes to come to a stop. Assuming no slippage between the tire and the road:

a. Which force is responsible for removing your momentum? (By “you” I mean throughout “you and the bicycle.”)
b. Which force is responsible for removing your kinetic energy?

Solution

(a) According to what we saw in previous chapters, for example, Equation (6.1.10)

$$\frac{\Delta p_{\text{sys}}}{\Delta t} = F_{\text{ext,net}} \tag{7.7.1}$$

the total momentum of the system can only be changed by the action of an external force, and the only available external force is the force of static friction between the tire and the road (static, because we assume no slippage). So it is this force that removes the forward momentum from the system. The stopping distance, Δx_{cm}, and the force, can be related using Equation (7.3.1):

$$F^s_{r,t} \Delta x_{cm} = \Delta K_{cm}. \tag{7.7.2}$$

(b) Now, here is an interesting fact: the force of static friction, although fully responsible for stopping your center of mass motion *does no work in this case*. That is because the point where it is applied—the point of the tire that is momentarily in contact with the road—is also momentarily at rest relative to the road: it is, precisely, *not slipping*, so Δx in the equation $W = F\Delta x$ is zero. By the time that bit of the tire has moved on, so you actually have a nonzero Δx, you no longer have an F: the force of static friction is no longer acting on that bit of the tire, it is acting on a different bit—on which it will, again, do no work, for the same reason.

So, as you bring your bicycle to a halt the work $W_{ext,sys}$ = 0, and it follows from Equation (7.4.8) that the total energy of your system is, in fact, conserved: *all* your initial kinetic energy is converted to thermal energy by the brake pad rubbing on the wheel, and the internal force responsible for that conversion is the force of *kinetic* friction between the pad and the wheel.

Example 7.7.2: Work, energy and the choice of system- dissipative case

Consider again the situation shown in Figure 7.4.3. Let m_1 = 1 kg, m_2 = 2 kg, and μ_k = 0.3. Use the solutions provided in Section 6.3 to calculate the work done by all the forces, and the changes in all energies, when the system undergoes a displacement of 0.5 m, and represent the changes graphically using bar diagrams like the ones in Figure 7.4.1 (for system A and B separately)

Solution

From Equation (6.3.11), we have

$$\begin{aligned} a &= \frac{m_2 - \mu_k m_1}{m_1 + m_2} g = 5.55\ \frac{\text{m}}{\text{s}^2} \\ F^t &= \frac{m_1 m_2 (1+\mu_k)}{m_1+m_2} g = 8.49\ \text{N} \end{aligned} \tag{7.7.3}$$

We can use the acceleration to calculate the change in kinetic energy, since we have Equation (2.2.10) for motion with constant acceleration:

$$v_f^2 - v_i^2 = 2a\Delta x = 2 \times \left(5.55\ \frac{\text{m}}{\text{s}^2}\right) \times 0.5\ \text{m} = 5.55\ \frac{\text{m}^2}{\text{s}^2} \tag{7.7.4}$$

so the change in kinetic energy of the two blocks is

$$\begin{aligned} \Delta K_1 &= \frac{1}{2} m_1 \left(v_f^2 - v_i^2\right) = 2.78\ \text{J} \\ \Delta K_2 &= \frac{1}{2} m_2 \left(v_f^2 - v_i^2\right) = 5.55\ \text{J}. \end{aligned} \tag{7.7.5}$$

We can also use the tension to calculate the work done by the external force on each system:

$$\begin{aligned} W_{ext,A} &= F^t_{r,1}\Delta x = (8.49\text{ N}) \times (0.5\text{ m}) = 4.25\text{ J} \\ W_{ext,B} &= F^t_{r,2}\Delta y = (8.49\text{ N}) \times (-0.5\text{ m}) = -4.25\text{ J}. \end{aligned} \tag{7.7.6}$$

Lastly, we need the change in the gravitational potential energy of system B:

$$\Delta U^G_B = m_2 g \Delta y = (2\text{ kg}) \times \left(9.8\ \frac{\text{m}}{\text{s}^2}\right) \times (-0.5\text{ m}) = -9.8\text{ J} \tag{7.7.7}$$

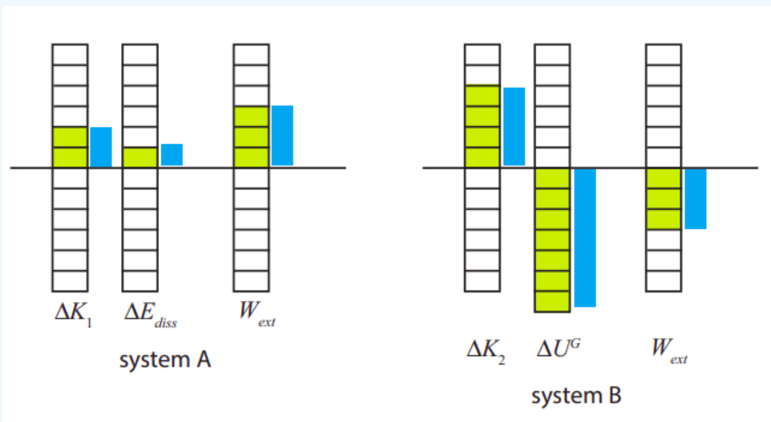
and the increase in dissipated energy in system A, which we can get from Equation (7.4.16):

$$\Delta E_{\text{diss}} = -F^k_{s,1}\Delta x = \mu_k F^n_{s,1}\Delta x = \mu_k m_1 g \Delta x = 0.3 \times (1\text{ kg}) \times \left(9.8\ \frac{\text{m}}{\text{s}^2}\right) \times (0.5\text{ m}) = 1.47\text{ J}. \tag{7.7.8}$$

We can now put all this together to show that Equation (7.4.8) indeed holds:

$$\begin{aligned} W_{ext,A} &= \Delta E_A = \Delta K_1 + \Delta E_{diss} = 2.78\text{ J} + 1.47\text{ J} = 4.25\text{ J} \\ W_{ext,B} &= \Delta E_B = \Delta K_2 + \Delta U^G_B = 5.55\text{ J} - 9.8\text{ J} = -4.25\text{ J}. \end{aligned} \tag{7.7.9}$$

To plot all this as energy bars, if you do not have access to a very precise drawing program, you typically have to make some approximations. In this case, we see that $\Delta K_2 = 2\Delta K_1$ (exactly), whereas $\Delta K_1 \simeq 2\Delta E_{diss}$, so we can use one box to represent E_{diss}, two boxes for ΔK_1, three for $W_{ext,A}$, four for ΔK_2, and so on. The result is shown in green in the picture below; the blue bars have been drawn more exactly to scale, and are shown for your information only.



Example 7.7.3: WOrk, energy and the choice of system- non-dissipative case

Suppose you hang a spring from the ceiling, then attach a block to the end of the spring and let go. The block starts swinging up and down on the spring. Consider the initial time just before you let go, and the final time when the block momentarily stops at the bottom of the swing. For each of the choices of a system listed below, find the net energy change of the system in this process, and relate it explicitly to the work done on the system by an external force (or forces)

a. System is the block and the spring.
b. System is the block alone.
c. System is the block and the earth.

Solution

(a) The block alone has kinetic energy, and the spring alone has (elastic) potential energy, so the total energy of this system is the sum of these two. For the interval considered, the change in kinetic energy is zero, because the block starts and ends (momentarily) at rest, so only the spring energy changes. This has to be equal to the work done by gravity, which is the only external force.

So, if the spring stretches a distance d, its potential energy goes from zero to $\frac{1}{2}kd^2$, and the block falls the same distance, so gravity does an amount of work equal to mgd, and we have

$$W_{grav} = mgd = \Delta E_{sys} = \Delta K + \Delta U^{spr} = 0 + \frac{1}{2}kd^2. \tag{7.7.10}$$

(b) If the system is the block alone, the only energy it has is kinetic energy, which, as stated above, does not see a net change in this process. This means the net work done on the block by the external forces must be zero. The external forces in this case are the spring force and gravity, so we have

$$W_{spr} + W_{grav} = \Delta K = 0. \tag{7.7.11}$$

We have calculated W_{grav} above, so from this we get that the work done by the spring on the block, as it stretches, is $-mgd$, or (by Equation (7.7.10)) $-\frac{1}{2}kd^2$. Note that the force exerted by the spring is *not* constant as it stretches (or compresses) so we cannot just use Equation (7.2.1) to calculate it; rather, we need to calculate it as an integral, as in Equation (7.2.6), or derive it in some indirect way as we have just done here.

(c) If the system is the block and the earth, it has kinetic energy and gravitational potential energy. The force exerted by the spring is an external force now, so we have:

$$W_{spr} = \Delta E_{sys} = \Delta K + \Delta U^G = 0 - mgd \tag{7.7.12}$$

so we end up again with the result that $W_{spr} = -mgd = -\frac{1}{2}kd^2$. Note that both the work done by the spring and the work done by gravity are equal to the negative of the changes in their respective potential energies, as they should be.

Example 7.7.4: Jumping

For a standing jump, you start standing straight (A) so your body's center of mass is at a height h_1 above the ground. You then bend your knees so your center of mass is now at a (lower) height h_2 (B). Finally, you straighten your legs, pushing hard on the ground, and take off, so your center of mass ends up achieving a maximum height, h_3, above the ground (C). Answer the following questions in as much detail as you can.

a. Consider the system to be your body only. In going from (A) to (B), which external forces are acting on it? How do their magnitudes compare, as a function of time?
b. In going from (A) to (B), does any of the forces you identified in part (a) do work on your body? If so, which one, and by how much? Does your body's energy increase or decrease as a result of this? Into what kind of energy do you think this work is primarily converted?
c. In going from (B) to (C), which external forces are acting on you? (Not all of them need to be acting all the time.) How do their magnitudes compare, as a function of time?
d. In going from (B) to (C), does any of the forces you identified do work on your body? If so, which one, and by how much? Does your body's kinetic energy see a net change from (B) to (C)? What other energy change needs to take place in order for Equation (7.4.8) (always with your body as the system) to be valid for this process?

Solution

(a) The external forces on your body are gravity, pointing down, and the normal force from the floor, pointing up. Initially, as you start lowering your center of mass, the normal force has to be slightly smaller than gravity, since your center of mass acquires a small downward acceleration. However, eventually F^n would have to exceed F^G in order to stop the downward motion.

(b) The normal force does no work, because its point of application (the soles of your feet) does not move, so Δx in the expression $W = F\Delta x$ (Equation (7.2.1)) is zero.

Gravity, on the other hand, does positive work, since you may always treat the center of mass as the point of application of gravity (see Section 7.3, footnote). We have $F_y^G = -mg$, and $\Delta y = h_2 - h_1$, so

$$W_{grav} = F_y^G \Delta y = -mg(h_2 - h_1) = mg(h_1 - h_2).$$

Since this is the net work done by all the external forces on my body, and it is positive, the total energy in my body must have increased (by the theorem ((7.4.8)): $W_{ext,sys} = \Delta E_{sys}$). In this case, it is clear that the main change has to be an increase in my body's *elastic potential energy*, as my muscles tense for the jump. (An increase in thermal energy is always possible too.)

(c) During the jump, the external forces acting on me are again gravity and the normal force, which together determine the acceleration of my center of mass. At the beginning of the jump, the normal force has to be much stronger than gravity, to give me a large upwards acceleration. Since the normal force is a reaction force, I accomplish this by pushing very hard with my feet on the ground, as I extend my leg's muscles: by Newton's third law, the ground responds with an equal and opposite force upwards.

As my legs continue to stretch, and move upwards, the force they exert on the ground decreases, and so does F^n, which eventually becomes less than F^G. At that point (probably even before my feet leave the ground) the acceleration of my center of mass becomes negative (that is, pointing down). This ultimately causes my upwards motion to stop, and my body to come down.

(d) The only force that does work on my body during the process described in (c) is gravity, since, again, the point of application of F^n is the point of contact between my feet and the ground, and that point does not move up or down—it is always level with the ground. So $W_{ext,sys} = W_{grav}$, which in this case is actually *negative*: $W_{grav} = -mg(h_3 - h_2)$.

In going from (B) to (C), there is no change in your kinetic energy, since you start at rest and end (momentarily) with zero velocity at the top of the jump. So the fact that there is a net negative work done on you means that the energy inside your body must have gone down. Clearly, some of this is just a decrease in elastic potential energy. However, since h_3 (the final height of your center of mass) is greater than h_1 (its initial height at (A), before crouching), there is a *net* loss of energy in your body as a result of the whole process. The most obvious place to look for this loss is in chemical energy: you "burned" some calories in the process, primarily when pushing hard against the ground.