10.6: Exercises

Exercise 10.6.1

Suppose you fire a projectile straight up from the Earth's North Pole with a speed of 10.5 km/s. Ignore air resistance.

a. How far from the center of the Earth does the projectile rise? How high above the surface of the Earth is that? (The radius of the Earth is R_E = 6.37 × 10^6 m, and the mass of the Earth is M = 5.97 × 10^{24} kg.)
b. How different is the result you got in part (a) above from what you would have obtained if you had treated the Earth's gravitational force as a constant (independent of height), as we did in previous chapters?
c. Using the correct expression for the gravitational potential energy, what is the total energy of the projectile-Earth system, if the projectile's mass is 1, 000 kg?

   Now assume the projectile is fired *horizontally* instead, with the same speed. This time, it actually goes into orbit! (Well, it would, if you could neglect things like air resistance, and mountains and stuff like that. Assume it does, anyway, and answer the following questions:)
d. What is the projectile's angular momentum around the center of the Earth?
e. How far from the center of the Earth does it make it this time? (You will need to use conservation of energy and angular momentum to answer this one, unless you can think of a shortcut...)
f. Draw a sketch of the Earth and the projectile's trajectory.

Exercise 10.6.2

You want to put a satellite in a *geosynchronous* orbit around the earth. (This means the asteroid takes 1 day to complete a turn around the earth.)

a. At what height above the surface do you need to put it?
b. How fast is it moving?
c. How does the answer to (b) compare to the escape speed from the earth, for an object *at this height*?

Exercise 10.6.3

Suppose that one day astronomers discover a new asteroid that moves on a very elliptical orbit around the sun. At the point of closest approach (perihelion), the asteroid is 1.61 × 10^8 km away from the (center of the) sun, and its speed is 38.9 km/s.

a. What is the escape velocity from the sun at this distance? The mass of the sun is 2 × 10^{30} kg.
b. The astronomers estimate the mass of the asteroid as 10^{12} kg. What is its kinetic energy at perihelion?
c. What is the gravitational potential energy of the sun-asteroid system at perihelion?
d. What is the total energy of the sun-asteroid system? Is it positive or negative? Is this consistent with the assumption that the orbit is an ellipse? What would a positive total energy mean?
e. At perihelion, the asteroid's velocity vector is perpendicular to its position vector (as drawn from the sun). What is then its angular momentum?
f. Draw a sketch of an elliptical orbit. On your sketch, indicate (1) the semimajor axis, and (2) qualitatively, where the sun might be.
g. The point in its orbit where the asteroid is farthest away from the sun is called aphelion. Use conservation of energy and angular momentum to figure out the asteroid's distance to the sun at aphelion. (Hint: if solving simultaneous equations does not appeal to you, there is a formula in this chapter which you can use to answer this question fairly quickly, based on something you have calculated already.)
h. How fast is the asteroid moving at aphelion?

Exercise 10.6.4

The mass of the moon is 7.34 × 10^{22} kg, and its radius is about 1.74 × 10^6 m.

a. What is the value of "g_{moon}", that is, the acceleration of gravity for a falling object near the surface of the moon?

b. What is the escape speed (from the moon) for an object on the surface of the moon?
c. What is the escape speed *from the earth* for an object that is as far from the earth as the orbit of the moon?
d. At some point between the earth and the moon, an object would be pulled with equal strength towards both bodies. How far from the earth is that point?

Exercise 10.6.5

On August 17, 2017, the LIGO observatory reported the detection of gravitational waves from the merger of two neutron stars. Neutron stars are extremely dense ("a teaspoon of neutron star material has a mass of about a billion tons") and very small—only about 10 or 20 km in diameter. The stars were estimated to have been separated by about 300 km when the merger signal became detectable.

Let us start a little before that. Suppose the stars have the same mass, $M = 2.6 \times 10^{30}$ kg (approximately 1.3 times the mass of the sun), and are separated (center-to-center distance) by 1000 km. They pull on each other gravitationally, and as a result each one moves in a circular orbit around their common center of mass. What is then (a) their period of revolution, and (b) their speed? (Hint: what is the centripetal force in this case?)

Exercise 10.6.6

a. Consider two possible circular orbits for a satellite around a planet, with radii R_1 and R_2. If $R_1 < R_2$, which of the two orbits has (i) the largest total energy, and (ii) the largest total angular momentum? Explain.
b. For an object in a circular orbit around a planet, how does the orbital velocity compare to the escape velocity from the same orbit?

Exercise 10.6.7

Jupiter's distance to the sun is 5.2 astronomical units. How long does it take for Jupiter to complete an orbit around the sun, in earth years? (Do not look it up! You need to show how you can calculate it using what you have learned in this chapter.)