8.6: Examples

You will work out a rather thorough example of projectile motion in the lab, and Section 8.3 above already has the problem of a block sliding down an inclined plane worked out for you. The following example will show you how to use the kinematic angular variables of section 8.4 to deal with motion in a circle, and to calculate the centripetal acceleration in a simple situation. The section on Advanced Topics deals with a few more challenging (but interesting) examples.

Example 8.6.1: The penny on the turntable

Suppose that you have a penny sitting on a turntable, a distance d = 10 cm from the axis of rotation. Assume the turntable starts moving, steadily spinning up from rest, in such a way that after 1.3 seconds it has reached its final rotation rate of 33.3 rpm (revolutions per minute). Answer the following questions:

a. What was the turntable's angular acceleration over the time interval from t = 0 to t = 1.3 s?
b. How many turns (complete and fractional) did the turntable make before reaching its final velocity?
c. Assuming the penny has not slipped, what is its centripetal acceleration once the turntable reaches its final velocity?
d. How large does the static friction coefficient between the penny and the turntable have to be for the penny not to slip throughout this process?

Solution

(a) We are told that the turntable spins up "steadily" from t = 0 to t = 1.3 s. The word "steadily" here is a keyword that means the (angular) acceleration is constant (that is, the angular velocity increases at a constant rate).

What is this rate? For constant α, we have, from Equation (8.4.10), $\alpha = \Delta\omega/\Delta t$. Here, the time interval Δt = 1.3 , so we just need to find $\Delta\omega$. By definition, $\Delta\omega = \omega_f - \omega_i$, and since we start from rest, ω_i = 0. So we just need ω_f. We are told that "the final rotation rate" is 33.3 rpm (revolutions per minute). What does this tell us about the angular velocity?

The angular velocity is the number of radians an object moving in a circle (such as the penny in this example) travels per second. A complete turn around the circle, or *revolution*, corresponds to 180°, or equivalently 2π radians. So, 33.3 revolutions, or turns, per minute means $33.3 \times 2\pi$ radians per 60 s, that is,

$$\omega_f = \frac{33.3 \times 2\pi \text{ rad}}{60 \text{ s}} = 3.49 \frac{\text{rad}}{\text{s}}. \tag{8.6.1}$$

The angular acceleration, therefore, is

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{\Delta t} = \frac{3.49 \text{ rad/s}}{1.3 \text{ s}} = 2.68 \frac{\text{rad}}{\text{s}^2}. \tag{8.6.2}$$

(b) The way to answer this question is to find out the total angular displacement, $\Delta\theta$, of the penny over the time interval considered (from t = 0 to t = 1.3 s), and then convert this to a number of turns, using the relationship 2π rad = 1 turn. To get $\Delta\theta$, we should use the equation (8.4.10) for motion with constant angular acceleration:

$$\Delta\theta = \omega_i \Delta t + \frac{1}{2}\alpha(\Delta t)^2. \tag{8.6.3}$$

We start from rest, so ω_i = 0, We know Δt = 1.3 s, and we just calculated α = 2.68 rad/s^2, so we have

$$\Delta\theta = \frac{1}{2} \times 2.68 \frac{\text{rad}}{\text{s}^2} \times (1.3 \text{ s})^2 = 2.26 \text{ rad}. \tag{8.6.4}$$

This is less than 2π radians, so it takes the turntable less than one complete revolution to reach its final angular velocity. To be precise, since 2π radians is one turn, 2.26 rad will be 2.26/(2π) turns, which is to say, 0.36 turns—a little more than 1/3 of a turn.

(c) For the questions above, the penny just served as a marker to keep track of the revolutions of the turntable. Now, we turn to the dynamics of the motion of the penny itself. First, to get its angular acceleration, we can just use Equation (8.4.18), in the form

$$a_c = R\omega^2 = 0.1 \text{ m} \times \left(3.49 \frac{\text{rad}}{\text{s}}\right)^2 = 1.22 \frac{\text{m}}{\text{s}^2} \tag{8.6.5}$$

noticing that R, the radius of the circle on which the penny moves, is just the distance d to the axis of rotation that we were given at the beginning of the problem, and ω, its angular velocity, is just the final angular velocity of the turntable (assuming, as we are told, that the penny has not slipped relative to the turntable).

(d) Finally, how about the force needed to keep the penny from slipping—that is to say, to keep it moving with the turntable? This is just the centripetal force needed "bend" the trajectory of the penny into a circle of radius R, so $F_c = ma_c$, where m is the mass of the penny and a_c is the centripetal acceleration we just calculated. Physically, we know that this force has to be provided by the *static* (as long as the penny does not slip!) friction force between the penny and the turntable. We know that $F^s \leq \mu_s F^n$, and we have for the normal force, in this simple situation, just $F^n = mg$. Therefore, setting $F^s = ma_c$ we have:

$$ma_c = F^s \leq \mu_s F^n = \mu_s mg. \tag{8.6.6}$$

This is equivalent to the single inequality $ma_c \leq \mu_s mg$, where we can cancel out the mass of the penny to conclude that we must have $a_c \leq \mu_s g$, and therefore

$$\mu_s \geq \frac{a_c}{g} = \frac{1.22 \text{ m/s}^2}{9.8 \text{ m/s}^2} = 0.124 \tag{8.6.7}$$