2.3: Free Fall

An important example of motion with (approximately) constant acceleration is provided by *free fall* near the surface of the Earth. We say that an object is in “free fall” when the only force acting on it is the force of gravity (the word “fall” here may be a bit misleading, since the object could actually be moving upwards some of the time, if it has been thrown straight up, for instance). The space station is in free fall, but because it is nowhere near the surface of the earth its direction of motion (and hence its acceleration, regarded as a two-dimensional vector) is constantly changing. Right next to the surface of the earth, on the other hand, the planet’s curvature is pretty much negligible and gravity provides an approximately constant, vertical acceleration, which, in the absence of other forces, turns out to be *the same for every object*, regardless of its size, shape, or weight.

The above result—that, in the absence of other forces, all objects should fall to the earth at the same rate, regardless of how big or heavy they are—is so contrary to our common experience that it took many centuries to discover it. The key, of course, as with the law of inertia, is to realize that, under normal circumstances, frictional forces are, in fact, acting all the time, so an object falling through the atmosphere is never *really* in “free” fall: there is always, at a minimum, and in addition to the force of gravity, an air drag force that opposes its motion. The magnitude of this force does depend on the object’s size and shape (basically, on how “aerodynamic” the object is); and thus a golf ball, for instance, falls much faster than a flat sheet of paper. Yet, if you crumple up the sheet of paper till it has the same size and shape as the golf ball, you can see for yourself that they do fall at approximately the same rate! The equality can never be exact, however, unless you get rid completely of air drag, either by doing the experiment in an evacuated tube, or (in a somewhat extreme way), by doing it on the surface of the moon, as the Apollo 15 astronauts did with a hammer and a feather back in 1971[2].

This still leaves us with something of a mystery, however: the force of gravity is the only force known to have the property that it imparts all objects the *same* acceleration, regardless of their mass or constitution. A way to put this technically is that the force of gravity on an object is proportional to that object’s *inertial mass*, a quantity that we will introduce properly in the next chapter. For the time being, we will simply record here that this acceleration, near the surface of the earth, has a magnitude of approximately 9.8 m/s^2, a quantity that is denoted by the symbol g. Thus, if we take the upwards direction as positive (as is usually done), we get for the acceleration of an object in free fall $a = -g$, and the equations of motion become

$$\Delta v = -g\Delta t \tag{2.3.1}$$

$$\Delta y = v_i \Delta t - \frac{1}{2} g(\Delta t)^2 \tag{2.3.2}$$

where I have used y instead of x for the position coordinate, since that is a more common choice for a vertical axis. Note that we could as well have chosen the downward direction as positive, and that may be a more natural choice in some problems. Regardless, the quantity g is always defined to be positive: g = 9.8 m/s^2. The acceleration, then, is g or $-g$, depending on which direction we take to be positive

In practice, the value of g changes a little from place to place around the earth, for various reasons (it is somewhat sensitive to the density of the ground below you, and it decreases as you climb higher away from the center of the earth). In a later chapter we will see how to calculate the value of g from the mass and radius of the earth, and also how to calculate the equivalent quantity for other planets.

In the meantime, we can use equations like (2.3.1) and (2.3.1) (as well as (2.2.10), with the appropriate substitutions) to answer a number of interesting questions about objects thrown or dropped straight up or down (always, of course, assuming that air drag is negligible). For instance, back at the beginning of this chapter I mentioned that if I dropped an object it might take about half a second to hit the ground. If you use Equation (2.3.2) with v_i = 0 (since I am dropping the object, not throwing it down, its initial velocity is zero), and substitute Δt = 0.5 s, you get Δy = 1.23 m (about 4 feet). This is a reasonable height from which to drop something.

On the other hand, you may note that half a second is not a very long time in which to make accurate observations (especially if you do not have modern electronic equipment), and as a result of that there was considerable confusion for many centuries as to the precise way in which objects fell. Some people believed that the speed did increase in some way as the object fell, while others appear to have believed that an object dropped would “instantaneously” (that is, at soon as it left your hand) acquire some speed and keep that unchanged all the way down. In reality, in the presence of air drag, what happens is a combination of both: initially the speed increases at an approximately constant rate (free, or nearly free fall), but the drag force increases with the speed as well, until eventually it balances out the force of gravity, and from that point on the speed does not increase anymore: we say that the object has reached "terminal velocity." Some objects reach terminal velocity almost instantly, whereas others (the more "aerodynamic" ones) may take a long time to do so. This accounts for the confusion that prevailed before Galileo's experiments in the early 1600's.

Galileo's main insight, on the theoretical side, was the realization that it was necessary to separate clearly the effect of gravity and the effect of the drag force. Experimentally, his big idea was to use an inclined plane to slow down the "fall" of an object, so as to make accurate measurements possible (and also, incidentally, reduce the air drag force!). These "inclined planes" were just basically ramps down which he sent small balls (like marbles) rolling. By changing the steepness of the ramp he could control how slowly the balls moved. He reasoned that, ultimately, the force that made the balls go down was essentially the same force of gravity, only not the whole force, but just a fraction of it. Today we know that, in fact, an object sliding (*not* rolling!) up or down on a *frictionless* incline will experience an acceleration directed downwards along the incline and with a magnitude equal to $g \sin \theta$, where \(\theta) is the angle that the slope makes with the horizontal:

$$a=g \sin \theta \quad (\text{inclined plane, taking } downwards \text{ to be positive}) \tag{2.3.3}$$

(for some reason, it seems more natural, when dealing with inclined planes, to take the downward direction as positive!). Equation (2.3.3) makes sense in the two extreme cases in which the plane is completely vertical ($\theta=90^{\circ}$, $a=g$) and completely horizontal ($\theta=0^{\circ}$, a = 0). For intermediate values, you will carry out experiments in the lab to verify this result.

We will show, in a later chapter, how Equation (2.3.3) comes about from a careful consideration of all the forces acting on the object; we will also see, later on, how it needs to be modified for the case of a rolling, rather than a sliding, object. This modification does not affect Galileo's main conclusion, which was, basically, that the natural falling motion in the absence of friction or drag forces is motion with *constant acceleration* (at least, near the surface of the earth, where g is constant to a very good approximation).

[2]The video of this is available online: https://www.youtube.com/watch?v=oYEgdZ3iEKA. It is, however, pretty low resolution and hard to see. A very impressive modern-day demonstration involving feathers and a bowling ball in a completely evacuated (airless) room is available here: https://www.youtube.com/watch?v=E43-CfukEgs.