9.7: In Summary

(Note: this summary makes extensive use of cross products, but does not include a summary of cross product properties. Please refer to Section 9.3 for that!)

1. The angular velocity and acceleration of a particle moving in a circle can be treated as vectors perpendicular to the plane of the circle, $\vec{\omega}$ and $\vec{\alpha}$, respectively. The direction of $\vec{\omega}$ is such that the relation $\vec{v}=\vec{\omega}\times\vec{r}$ always holds, where $\vec{r}$ is the (instantaneous) position vector of the particle on the circle.
2. The particle's kinetic energy can be written as $K_{rot}=\frac{1}{2}I\omega^2$, where $I=mR^2$ is the *rotational inertia* or *moment of inertia*. For an extended object rotating about an axis, $K_{rot}=\frac{1}{2}I\omega^2$ also applies if I is defined as the sum of the quantities mr^2 for all the particles making up the object, where r is the particle's distance to the rotation axis.
3. For a rigid object that is rotating around an axis passing through its center of mass with angular velocity ω the total kinetic energy can be written as $K=K_{cm}+K_{rot}=\frac{1}{2}Mv_{cm}^2+\frac{1}{2}I\omega^2$. This would apply also to a non-rigid system, provided all the particles have the same angular velocity.
4. The *angular momentum*, $\vec{L}$, of a particle about a point O is defined as $\vec{L}=\vec{r}\times\vec{p}=m\vec{r}\times\vec{v}$, where $\vec{r}$ is the position vector of the particle relative to the origin O, and $\vec{v}$ and $\vec{p}$ its velocity and momentum vectors. For an extended object or system, $\vec{L}$ is defined as the sum of the quantities $m\vec{r}\times\vec{v}$ for all the particles making up the system.
5. For a solid object rotating around a symmetry axis, $\vec{L}=I\vec{\omega}$. This applies also to an essentially flat object rotating about a perpendicular axis, even if it is not an axis of symmetry.
6. The *torque*, $\vec{\tau}$, of a force about a point O is defined as $\tau=\vec{r}\times\vec{F}$, where $\vec{r}$ is the position vector of the point of application of the force relative to the origin O. It is a measure of the effectiveness of the force at causing a rotation around that point.
7. The rate of change of a system's angular momentum about a point O is equal to the sum of the torques, about that same point, of all the *external* forces acting on the system: $\sum\vec{\tau}_{ext}=d\vec{L}_{sys}/dt$. Hence, angular momentum is constant whenever all the external torques vanish (*conservation of angular momentum*).
8. For the cases considered in point 7 above, if the moment of inertia I is constant, the equation $\sum\vec{\tau}_{ext}=d\vec{L}_{sys}/dt$ can be written in the form $\sum\vec{\tau}_{ext}=I\vec{\alpha}$, which strongly resembles the familiar $\sum\vec{F}_{ext}=m\vec{a}$. Note, however, that deformable systems where I may change with time as a result of internal forces are relatively common, and for those systems this simpler equation would not apply.
9. For an object to be in *static equilibrium*, we require that both the sum of the external forces and of the external torques be equal to zero: $\sum\vec{F}_{ext}=0$ and $\sum\vec{\tau}_{ext}=0$. Note that if the first condition applies, it does not matter about which point we calculate the torque, so we are free to choose whichever is most convenient.
10. For a rigid object of radius R rolling without slipping on some surface, the relations $|v_{cm}|=R|\omega|$ and $|a_{cm}|=R|\alpha|$ hold. The relative signs of, for instance, v_{cm} and ω (understood here as the relevant components of their respective vectors) need to be chosen so as to be consistent with whatever convention one has adopted for the positive direction of motion and the positive direction of rotation (typically, a counterclockwise rotation is considered positive).