11.7: Exercises

Exercise 11.7.1

A block of mass m is sliding on a frictionless, horizontal surface, with a velocity v_i. It hits an ideal spring, of spring constant k, which is attached to the wall. The spring compresses until the block momentarily stops, and then starts expanding again, so the block ultimately bounces off (see Example 5.6.2).

a. Write down an equation of motion (a function $x(t)$) for the block, which is valid for as long as it is in contact with the spring. For simplicity, assume the block is initially moving to the right, take the time when it first makes contact with the spring to be $t = 0$, and let the position of the block at that time to be $x = 0$. Make sure that you express any constants in your equation (such as A or ω) in terms of the given data, namely, m, v_i, and k.
b. Sketch the function $x(t)$ for the relevant time interval.

Exercise 11.7.2

For this problem, imagine that you are on a ship that is oscillating up and down on a rough sea. Assume for simplicity that this is simple harmonic motion (in the vertical direction) with amplitude 5 cm and frequency 2 Hz. There is a box on the floor with mass $m = 1$ kg.

a. Assuming the box remains in contact with the floor throughout, find the maximum and minimum values of the normal force exerted on it by the floor over an oscillation cycle.
b. How large would the amplitude of the oscillations have to become for the box to lose contact with the floor, assuming the frequency remains constant? (Hint: what is the value of the normal force at the moment the box loses contact with the floor?)

Exercise 11.7.3

Imagine a simple pendulum swinging in an elevator. If the cable holding the elevator up was to snap, allowing the elevator to go into free fall, what would happen to the frequency of oscillation of the pendulum? Justify your answer.

Exercise 11.7.4

Consider a block of mass m attached to *two* springs, one on the left with spring constant k_1 and one on the right with spring constant k_2. Each spring is attached on the other side to a wall, and the block slides without friction on a horizontal surface. When the block is sitting at $x = 0$, both springs are relaxed.

Write Newton's second law, $F = ma$, as a differential equation for an arbitrary position x of the block. What is the period of oscillation of this system?

Exercise 11.7.5

Consider the block hanging from a spring shown in Figure 11.2.5. Suppose the mass of the block is 1.5 kg and the system is at rest when the spring has been stretched 2 cm from its original length (that is, with reference to the figure, $y_0 - y_0' = 0.02$ m).

a. What is the value of the spring constant k?
b. If you stretch the spring by an additional 2 cm downward from this equilibrium position, and release it, what will be the frequency of the oscillations?
c. Now consider the system formed by the spring, the block, and the earth. Take the "zero" of gravitational potential energy to be at the height y_0' (the equilibrium point; you may also use this as the origin for the vertical coordinate!), and calculate all the energies in the system (kinetic, spring/elastic, and gravitational) at the highest point in the oscillation, the equilibrium point, and the lowest point. Verify that the sum is indeed constant.