11.4: In Summary

1. Most stable physical systems will oscillate when displaced from their equilibrium position. The oscillations are due to a restoring force (or a restoring torque) and the system's own inertia.
2. The period T and frequency f of an oscillatory motion are related by $f = 1/T$. The units of frequency are s^{-1} or hertz (abbreviated Hz).
3. A special (but very common) kind of oscillation is *simple harmonic motion*. This happens whenever the restoring force is a linear function of (that is, it is proportional to) the system's displacement from the equilibrium position.
4. The *angular frequency*, ω, of a simple harmonic oscillator is related to the regular frequency by $\omega = 2\pi f$. One of the properties of simple harmonic motion is that its frequency does not depend on the initial conditions, that is, on the velocity or displacement with which the motion is started.
5. If the equilibrium position is chosen to correspond to x = 0, the most general form of the position function for a simple harmonic oscillator is $x(t) = A\cos(\omega t + \phi)$, where The amplitude A and phase angle ϕ are determined by the initial conditions. The velocity function is then $v(t) = -\omega A \sin(\omega t + \phi)$, and the acceleration $a(t) = -\omega^2 A \cos(\omega t + \phi)$.
6. The total energy (potential plus kinetic) in a simple harmonic oscillator is equal to $E_{sys} = \frac{1}{2} m\omega^2 A^2$. The kinetic and potential energies oscillate (in opposition, that is to say, each being maximum when the other is minimum) between this value and zero.
7. A mass attached to an ideal (massless) spring is an example of a simple harmonic oscillator. If the spring constant is k, the angular frequency of this system is $\omega = \sqrt{k/m}$.
8. An external, constant force acting on a harmonic oscillator does not change its period (or frequency), only its equilibrium position. For the mass-on-a-spring system, a force F will cause a displacement of the equilibrium position equal to F/k, in the direction of the force.
9. A simple pendulum (a point particle of mass m suspended from a massless, inextensible string of length l) will perform harmonic oscillations around the vertical provided the small angle, approximation, $\sin\theta \simeq \theta$, holds. The angular frequency of these oscillations is $\omega = \sqrt{g/l}$.
10. A rigid object of mass m pivoted around some point and performing oscillations under the influence of gravity is sometimes called a *physical pendulum*. Just as for the simple pendulum, the oscillations will be harmonic if the small-angle approximation holds. The angular frequency will then be $\sqrt{mgd/I}$, where I is the moment if inertia around the pivot point, and d the distance from the pivot point to the center of mass.