4.4: Examples

Example 4.4.1: Collision Graph revisited

Look again at the collision graph from Example 3.5.1 from the point of view of the kinetic energy of the two carts.

a. What is the initial kinetic energy of the system?
b. How much of this is in the center of mass motion, and how much of is convertible?
c. Does the convertible kinetic energy go to zero at some point during the collision? If so, when? Is it fully recovered after the collision is over?
d. What kind of collision is this? (Elastic, inelastic, etc.) What is the coefficient of restitution?

Solution

(a) From the solution to Example 3.5.1 we know that

$$\begin{aligned} v_{1i} &= -1 \ \frac{\mathrm{m}}{\mathrm{s}} \qquad & v_{2i} &= 0.5 \ \frac{\mathrm{m}}{\mathrm{s}} \\ v_{1f} &= 1 \ \frac{\mathrm{m}}{\mathrm{s}} \qquad & v_{2f} &= -0.5 \ \frac{\mathrm{m}}{\mathrm{s}} \end{aligned}$$

and m_1 = 1 kg and m_2 = 2 kg. So the initial kinetic energy is

$$K_{sys,i} = \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = 0.5 \text{ J} + 0.25 \text{ J} = 0.75 \text{ J} \tag{4.4.1}$$

(b) To calculate $K_{cm} = \frac{1}{2}(m_1 + m_2) v_{cm}^2$, we need v_{cm}, which in this case is equal to

$$v_{cm} = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = \frac{-1 + 2 \times 0.5}{3} = 0$$

so K_{cm} = 0, which means all the kinetic energy is convertible. We can also calculate that directly:

$$K_{\mathrm{conv},i} = \frac{1}{2} \mu v_{12,i}^2 = \frac{1}{2} \left(\frac{1 \times 2}{1+2} \text{ kg} \right) \times \left(0.5 \ \frac{\mathrm{m}}{\mathrm{s}} - (-1) \ \frac{\mathrm{m}}{\mathrm{s}} \right)^2 = \frac{1.5^2}{3} \text{ J} = 0.75 \text{ J} \tag{4.4.2}$$

(c) If we look at figure 3.5.1, we can see that the carts do not pass through each other, so their relative velocity must be zero at some point, and with that, the convertible energy. In fact, the figure makes it quite clear that *both* v_1 and v_2 are zero at t = 5 s, so at that point also v_{12} = 0, and the convertible energy K_{conv} = 0. (And so is the total K_{sys} = 0 at that time, since K_{cm} = 0 throughout.)

On the other hand, it is also clear that K_{conv} is fully recovered after the collision is over, since the relative velocity just changes sign:

$$\begin{aligned} v_{12,i} &= v_{2i} - v_{1i} = 0.5 \ \tfrac{\mathrm{m}}{\mathrm{s}} - (-1) \ \tfrac{\mathrm{m}}{\mathrm{s}} = 1.5 \ \tfrac{\mathrm{m}}{\mathrm{s}} \\ v_{12,f} &= v_{2f} - v_{1f} = -0.5 \ \tfrac{\mathrm{m}}{\mathrm{s}} - 1 \ \tfrac{\mathrm{m}}{\mathrm{s}} = -1.5 \ \tfrac{\mathrm{m}}{\mathrm{s}} \end{aligned} \tag{4.4.3}$$

Therefore

$$K_{\mathrm{conv},f} = \frac{1}{2} \mu v_{12,f}^2 = \frac{1}{2} \mu v_{12,i}^2 = K_{\mathrm{conv},i}$$

(d) Since the total kinetic energy (which in this case is only convertible energy) is fully recovered when the collision is over, the collision is elastic. Using equation (4.4.3), we can see that the coefficient of restitution is

$$e = -\frac{v_{12,f}}{v_{12,i}} = -\frac{-1.5}{1.5} = 1$$

as it should be.

Example 4.4.2: Inelastic collision and explosive separation

Analyze Example 3.5.2 from the point of view of the system's kinetic energy. In particular, answer the following questions:

a. What is the total kinetic energy of the system (*i*) before the players collide, (*ii*) right after the collision, when they are holding to one another, and (*iii*) after they separate. How much of this energy is translational (that is, center-of-mass kinetic energy), and how much is convertible?
b. Answer the same questions from the point of view of the player who is skating at a constant 1.5 m/s to the right (player 3) (To avoid needless repetition, you may use already established results, such as conservation of momentum.)

Solution

(a) Before the players collide, we have

$$K_{sys,i} = \frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}(80\text{ kg}) \times \left(3\ \frac{\text{m}}{\text{s}}\right)^2 + \frac{1}{2}(90\text{ kg}) \times \left(-2\ \frac{\text{m}}{\text{s}}\right)^2 = 540\text{ J}. \tag{4.4.4}$$

While they are still holding to each other, we know from the solution to Example 3.5.2 that their joint velocity is 0.353 , and that this has to be also the velocity of their center of mass, which is unchanged by the collision. So, we have

$$K_{cm} = \frac{1}{2}(m_1 + m_2)\, v_{cm}^2 = \frac{1}{2}(170\text{ kg})\left(0.353\ \frac{\text{m}}{\text{s}}\right)^2 = 10.6\text{ J}. \tag{4.4.5}$$

This is K_{cm} throughout, as well as K_{sys} right after the collision, since the collision is totally inelastic and that means that K_{conv} drops to zero. Also, subtracting this from (4.4.4) will give us the initial value of the convertible energy, without the need for a separate calculation, so

$$K_{conv,i} = K_{sys,i} - K_{cm} = 540\text{ J} - 10.6\text{ J} = 529.4\text{ J} \simeq 529\text{ J}. \tag{4.4.6}$$

After the separation, the new total kinetic energy (for which I will use the subscript *f*) is

$$K_{sys,i} = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2 = \frac{1}{2}(80\text{ kg}) \times \left(-0.176\ \frac{\text{m}}{\text{s}}\right)^2 + \frac{1}{2}(90\text{ kg}) \times \left(0.824\ \frac{\text{m}}{\text{s}}\right)^2 = 31.8\text{ J} \tag{4.4.7}$$

where I have gotten the values for v_{1f} and v_{2f} from the solution to part (d) of Example 3.5.2. Subtracting K_{cm} from this will give us the final value of the convertible energy:

$$K_{\text{conv},f} = K_{\text{sys,f}} - K_{cm} = 31.8\text{ J} - 10.6\text{ J} = 21.2\text{ J} \tag{4.4.8}$$

To summarize, then, we have:

- Before the collision:

$$K_{sys,i} = 540\text{ J}, \quad K_{cm} = 10.6\text{ J}, \quad K_{conv,i} = 529.4\text{ J}$$

- Right after the collision (players still holding to each other):

$$K_{sys} = K_{cm} = 10.6\text{ J}, \quad K_{conv} = 0$$

- After the (explosive) separation:

$$K_{sys,f} = 31.8\text{ J}, \quad K_{cm} = 10.6\text{ J}, \quad K_{conv,i} = 21.2\text{ J}$$

So, in the collision, approximately 529 J of kinetic energy "disappeared" from the system (or, we could say, were "converted into some form of internal energy"), whereas the players' pushing on each other managed to put about 21 J of kinetic energy back into the system; we will explore these kinds of processes in more detail in the following chapter!

(b) We need to repeat all the above calculations with all the velocities shifted down by 1.5 m/s, to bring them to the reference frame of player 3. Instead of putting a subscript "3" on all the quantities, since we already have tons of subscripts to worry about, I'm going to follow an alternative convention and use a "prime" superscript (′) to denote all the quantities in this frame of reference. In brief, we have

$$K'_{sys,i} = \frac{1}{2}m_1 (v'_{1i})^2 + \frac{1}{2}m_2 (v'_{2i})^2 = \frac{1}{2}(80\text{ kg}) \times \left(1.5\frac{\text{m}}{\text{s}}\right)^2 + \frac{1}{2}(90\text{ kg}) \times \left(-3.5\ \frac{\text{m}}{\text{s}}\right)^2 = 641.3\text{ J} \tag{4.4.9}$$

$$K'_{cm} = \frac{1}{2}(m_1 + m_2)\,(v'_{cm})^2 = \frac{1}{2}(170\text{ kg})\left(0.353\ \frac{\text{m}}{\text{s}} - 1.5\ \frac{\text{m}}{\text{s}}\right)^2 = 111.8\text{J} \tag{4.4.10}$$

$$K'_{\text{conv,i}} = K'_{sys,i} - K'_{cm} = 641.3\text{ J} - 111.8\text{ J} = 529.5\text{ J} \simeq 529\text{ J} \tag{4.4.11}$$

This shows explicitly that the convertible energy, as I pointed out earlier in this chapter, is the same in every reference frame! (The equality is exact, if you keep enough decimals in the calculation.)

Knowing this, we can simplify the calculation of the final kinetic energy, after the explosive separation: the convertible energy, $K'_{conv,f}$, will be the same as in the earth reference frame, that is to say, 21.2 J, and the total kinetic energy will be $K'_{sys,f} = K'_c m + K'_{conv,f}$ = 111.8 J +21.2 J = 133 J.

So, in this frame of reference, we have (to three significant figures):

$$
\begin{aligned}
K'_{sys,i} = 641 \text{ J}, \quad K'_{cm} = 112 \text{ J}, \quad K'_{\text{conv,i}} = 529 \text{ J} \qquad & \text{(before the collision)} \\
K'_{sys} = K'_{cm} = 112 \text{ J}, \quad K'_{conv} = 0 \qquad & \text{(right after the collision)} \\
K'_{sys,f} = 133 \text{ J}, \quad K'_{cm} = 112 \text{ J}, \quad K'_{conv,i} = 21.2 \text{ J} \qquad & \text{(after the separation)}
\end{aligned}
$$

So, even though the total kinetic energy is different in the two reference frames, all the (inertial) observers will agree as to the amount of kinetic energy “lost” in the collision, as well as the amount of kinetic energy put back into the system by the players’ pushing on each other.