

## 2.5: Examples

### Example 2.5.1: Motion with piecewise constant acceleration

Construct the position vs. time, velocity vs. time, and acceleration vs. time graphs for the motion described below. For each of the intervals (a)–(d) you'll need to figure out the position (height) and velocity of the rocket at the beginning and the end of the interval, and the acceleration for the interval. In addition, for interval (b) you need to figure out the maximum height reached by the rocket and the time at which it occurs. For interval (d) you need to figure out its duration, that is to say, the time at which the rocket hits the ground.

- A rocket is shot upwards, accelerating from rest to a final velocity of 20 m/s in 1 s as it burns its fuel. (Treat the acceleration as constant during this interval.)
- From  $t = 1$  s to  $t = 4$  s, with the fuel exhausted, the rocket flies under the influence of gravity alone. At some point during this time interval (you need to figure out when!) it stops climbing and starts falling.
- At  $t = 4$  s a parachute opens, suddenly causing an upwards acceleration (again, treat it as constant) lasting 1 s; at the end of this interval, the rocket's velocity is 5 m/s downwards.
- The last part of the motion, with the parachute deployed, is with constant velocity of 5 m/s downwards until the rocket hits the ground.

#### Solution

(a) For this first interval (for which I will use a subscript "1" throughout) we have

$$\Delta y_1 = \frac{1}{2} a_1 (\Delta t_1)^2 \quad (2.5.1)$$

using Equation (2.2.6) for motion with constant acceleration with zero initial velocity (I am using the variable  $y$ , instead of  $x$ , for the vertical coordinate; this is more or less customary, but, of course, I could have used  $x$  just as well).

Since the acceleration is constant, it is equal to its average value:

$$a_1 = \frac{\Delta v}{\Delta t} = 20 \frac{\text{m}}{\text{s}^2}.$$

Substituting this into (2.5.1) we get the height at  $t = 1$  s is 10 m. The velocity at that time, of course, is  $v_{f1} = 20$  m/s, as we were told in the statement of the problem.

(b) This part is free fall with initial velocity  $v_{i2} = 20$  m/s. To find how high the rocket climbs, use Equation (2.3.1) in the form  $v_{top} - v_{i2} = -g(t_{top} - t_{i2})$ , with  $v_{top} = 0$  (as the rocket climbs, its velocity decreases, and it stops climbing when its velocity is zero). This gives us  $t_{top} = 3.04$  s as the time at which the rocket reaches the top of its trajectory, and then starts coming down. The corresponding displacement is, by Equation (2.3.2),

$$\Delta y_{top} = v_{i2} (t_{top} - t_{i2}) - \frac{1}{2} g (t_{top} - t_{i2})^2 = 20.4 \text{ m}$$

so the maximum height it reaches is 30.4 m.

At the end of the full 3-second interval, the rocket's displacement is

$$\Delta y_2 = v_{i2} \Delta t_2 - \frac{1}{2} g (\Delta t_2)^2 = 15.9 \text{ m}$$

(so its height is 25.9 m above the ground), and the final velocity is

$$v_{f2} = v_{i2} - g \Delta t_2 = -9.43 \frac{\text{m}}{\text{s}}.$$

(c) The acceleration for this part is  $(v_{f3} - v_{i3}) / \Delta t_3 = (-5 + 9.43) / 1 = 4.43 \text{ m/s}^2$ . Note the positive sign. The displacement is

$$\Delta y_3 = -9.43 \times 1 + \frac{1}{2} \times 4.43 \times 1^2 = -7.22 \text{ m}$$

so the final height is  $25.9 - 7.21 = 18.7$  m.

(d) This is just motion with constant speed to cover 18.7 m at 5 m/s. The time it takes is 3.74 s. The graphs for this motion are shown earlier in the chapter, in [Figure 2.2.3](#).

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