10.3: In Summary

1. In Newton's theory of gravity, two particles of inertial masses m_1 and m_2, separated by a distance r_{12}, exert a gravitational force on each other which is attractive, along the line joining the two particles, and has magnitude $F_{12}^G = F_{21}^G = Gm_1m_2/r_{12}^2$, with G = 6.674 × 10^{-11} $m^3kg^{-1}s^{-2}$.
2. The gravitational force between two extended objects is found by adding (vectorially) the forces between all the pairs of particles that make up the objects. For objects with spherical symmetry, the result has the same form as above, with r_{12} being now the distance between the centers of the spheres.
3. The gravitational potential energy of a system of two particles of masses m_1 and m_2 is $U^G = -Gm_1m_2/r_{12}$. For systems of more particles, one should just add the corresponding energies for all the possible pairs. For a pair of spheres, one may use the same result as for two particles, as long as one is not interested in the spheres' gravitational self-energy.
4. The expressions given above for F^G and U^G reduce, respectively, to mg and $mgy + C$ (where C is an unimportant constant) near the surface of the earth, to a good approximation, provided the distance y to the surface is much smaller than the radius of the earth, R_E. If M_E is the mass of the earth, one has $g = GM_E/R_E^2$.
5. A good first approximation to many astronomical problems is obtained by considering the motion of a particle (or sphere) of mass m under the gravitational pull of an object (also treated as a particle or sphere) of much larger mass M, which is assumed to not move at all. This is sometimes called the *Kepler problem*.
6. The solutions to the Kepler problem are of two types, depending on the system's total energy E: bound, elliptical orbits (including circles as a special case), if E < 0; and unbound hyperbolic trajectories, if E > 0. The special trajectory obtained when E = 0 is a parabola.
7. For the elliptical orbits, one has $E = -GMm/2a$, where a is the ellipse's semimajor axes. The large mass object is not at the center, but at one of the foci of the ellipse. The distance from the focus to the center is equal to ea, where e is called the *eccentricity* of the ellipse.
8. The *escape speed* of an object bound gravitationally to a mass M, a distance r_i away from that mass's center, is obtained by setting the total energy of the system equal to zero. It is the speed the object needs in order to be able to just escape to "infinity" and "stop there" (mathematically, $v \to 0$ as $r \to \infty$, which makes $E = K + U^G = 0$).
9. The angular momentum of a particle in a Kepler trajectory (circle, ellipse, parabola or hyperbola), relative to the point where the large mass M is located, is constant. For a given energy, orbits with less angular momentum are more eccentric.
10. A consequence of conservation of angular momentum is Kepler's second law, or "law of areas": The orbiting object's position vector (with the origin at the location of the large mass), sweeps equal areas in equal times.
11. The square of the orbital period of any object in a Kepler elliptical orbit is proportional to the cube of the semimajor axis of the ellipse. This is Kepler's third law. Specifically, one finds, from Newton's theory, $T^2 = (4\pi^2/GM)a^3$.
12. According to Einstein's principle of equivalence, a constant acceleration a of a reference frame is experienced by every object in that reference frame as an "extra weight," or gravitational force, equal to $-ma$ (that is, of magnitude ma and in the direction opposite the acceleration).