5.6: Examples

Example 5.6.1: Inelastic Collision in the middle of a swing

Tarzan swings on a vine to rescue a helpless explorer (as usual) from some attacking animal or another. He begins his swing from a branch a height of 15 m above the ground, grabs the explorer at the bottom of his swing, and continues the swing, upwards this time, until they both land safely on another branch. Suppose that Tarzan weighs 90 kg and the explorer weighs 70, and that Tarzan doesn't just drop from the branch, but pushes himself off so that he starts the swing with a speed of 5 m/s. How high a branch can he and the explorer reach?

Solution

Let us break this down into parts. The first part of the swing involves the conversion of some amount of initial gravitational potential energy into kinetic energy. Then comes the collision with the explorer, which is completely inelastic and we can analyze using conservation of momentum (assuming Tarzan and the explorer form an isolated system for the brief time the collision lasts). After that, the second half of their swing involves the complete conversion of their kinetic energy into gravitational potential energy.

Let m_1 be Tarzan's mass, m_2 the explorer's mass, h_i the initial height, and h_f the final height. We also have three velocities to worry about (or, more properly in this case, speeds, since their direction is of no concern, as long as they all point the way they are supposed to): Tarzan's initial velocity at the beginning of the swing, which we may call v_{top}; his velocity at the bottom of the swing, just before he grabs the explorer, which we may call v_{bot1}, and his velocity just after he grabs the explorer, which we may call v_{bot2}. (If you find those subscripts confusing, I am sorry, they are the best I could do; please feel free to make up your own.)

- *First part: the downswing*. We apply conservation of energy, in the form Equation (5.4.1), to the first part of the swing. The system we consider consists of Tarzan and the earth, and it has kinetic energy as well as gravitational potential energy. We ignore the source energy and the dissipated energy terms, and consider the system closed despite the fact that Tarzan is holding onto a vine (as we shall see in a couple of chapters, the vine does no "work" on Tarzan—meaning, it does not change his energy, only his direction of motion—because the force it exerts on Tarzan is always perpendicular to his displacement):

$$K_{top}+U^G_{top}=K_{bot1}+U^G_{bot}. \tag{5.6.1}$$

  In terms of the quantities I introduced above, this equation becomes:

$$\frac{1}{2}m_1v^2_{top}+m_1gh_i=\frac{1}{2}m_1v^2_{bot1}+0$$

  which can be solved to give

$$v^2_{bot1}=v^2_{top}+2gh_i \tag{5.6.2}$$

  (note that this is just the familiar result (2.2.10) for free fall! This is because, as I pointed out above, the vine does no work on the system.). Substituting, we get

$$v_{bot1}=\sqrt{\left(5\ \frac{\mathrm{m}}{\mathrm{s}}\right)^2+2\left(9.8\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)\times(15\ \mathrm{m})}=17.9\ \frac{\mathrm{m}}{\mathrm{s}}$$

- *Second part: the completely inelastic collision*. The explorer is initially at rest (we assume he has not seen the wild beast ready to pounce yet, or he has seen it and he is paralyzed by fear!). After Tarzan grabs him they are moving together with a speed \(v_{bot2}. Conservation of momentum gives

$$m_1v_{bot1}=(m_1+m_2)\,v_{bot2} \tag{5.6.3}$$

  which we can solve to get

$$v_{bot2}=\frac{m_1 v_{bot1}}{m_1+m_2}=\frac{(90\text{ kg})\times(17.9\text{ m/s})}{160\text{ kg}}=10\ \frac{\text{m}}{\text{s}}$$

- *Third part: the upswing*. Here we use again conservation of energy in the form

$$K_{bot2}+U^G_{bot}=K_f+U^G_f \tag{5.6.4}$$

where the subscript *f* refers to the very end of the swing, when they both safely reach their new branch, and all their kinetic energy has been converted to gravitational potential energy, so K_f = 0 (which means that is as high as they can go, unless they start climbing the vine!). This equation can be rewritten as

$$\frac{1}{2}(m_1+m_2)\,v^2_{bot2}+0=0+(m_1+m_2)\,gh_f$$

and solving for h_f we get

$$h_f=\frac{v^2_{bot2}}{2g}=\frac{(10\text{ m/s})^2}{2\times 9.8\text{ m/s}^2}=5.15\text{ m}$$

Example 5.6.2: kinetic energy to spring potential energy- block collides with spring

A block of mass m is sliding on a frictionless, horizontal surface, with a velocity v_i. It hits an ideal spring, of spring constant k, which is attached to the wall. The spring compresses until the block momentarily stops, and then starts expanding again, so the block ultimately bounces off.

a. In the absence of dissipation, what is the block's final speed?
b. By how much is the spring compressed?

Solution

This is a simpler version of the problem considered in Section 5.1, and in the next example. The problem involves the conversion of kinetic energy into elastic potential energy, and back. In the absence of dissipation, Equation (5.4.1), specialized to this system (the spring and the block) reads:

$$K+U^{spr}=\text{constant} \tag{5.6.5}$$

For part (a), we consider the whole process where the spring starts relaxed and ends relaxed, so $U^{spr}_i=U^{spr}_f$ = 0. Therefore, we must also have $K_f=K_i$, which means the block's final speed is the same as its initial speed. As explained in the chapter, this is characteristic of a conservative interaction.

For part (b), we take the final state to be the instant where the spring is maximally compressed and the block is momentarily at rest, so all the energy in the system is spring (which is to say, elastic) potential energy. If the spring is compressed a distance d (that is, $x-x_0=-d$ in Equation (5.1.5)), this potential energy is $\frac{1}{2}kd^2$, so setting that equal to the system's initial energy we get:

$$K_i+0=0+\frac{1}{2}kd^2 \tag{5.6.6}$$

or

$$\frac{1}{2}mv_i^2=\frac{1}{2}kd^2$$

which can be solved to get

$$d=\sqrt{\frac{m}{k}}v_i.$$