13.6: Examples

Example 13.6.1: Calorimetry

The specific heat of aluminum is 900 J/kg·K, and that of water is 4186 J·K. Suppose you drop a block of aluminum of mass 1 kg at a temperature of 80°C in a liter of water (which also has a mass of 1 kg) at a temperature of 20°C. What is the final temperature of the system, assuming no exchange of heat with the environment takes place? How much energy does the aluminum lose/the water gain?

Solution

Let us call T_{Al} the initial temperature of the aluminum, T_{water} the initial temperature of the water, and T_f their final common temperature. The thermal energy given off by the aluminum equals $\Delta E_{Al} = C_{Al}(T_f - T_{Al})$ (this follows from the definition (13.2.1) of heat capacity; we could equally well call this quantity "the heat given off by the aluminum"). In the same way, the thermal energy change of the water (heat absorbed by the water) equals $\Delta E_{water} = C_{water}(T_f - T_{water})$. If the total system is closed, the sum of these two quantities, each with its appropriate sign, must be zero:

$$0 = \Delta E_{\text{Al}} + \Delta E_{\text{water}} = C_{\text{Al}}\left(T_f - T_{\text{Al}}\right) + C_{\text{water}}\left(T_f - T_{\text{water}}\right). \tag{13.6.1}$$

This equation for T_f has the solution

$$T_f = \frac{C_{\text{Al}} T_{\text{Al}} + C_{\text{water}} T_{\text{water}}}{C_{\text{Al}} + C_{\text{water}}}. \tag{13.6.2}$$

As you can see, the result is a weighted average of the two starting temperatures, with the corresponding heat capacities as the weighting factors.

The heat capacities C are equal to the given specific heats multiplied by the respective masses. In this case, the mass of aluminum and the mass of the water are the same, so they will cancel in the final result. Also, we can use the temperatures in degrees Celsius, instead of Kelvin. This is not immediately obvious from the final expression (13.6.2), but if you look at (13.6.1) you'll see it involves only temperature differences, and those have the same value in the Kelvin and Celsius scales.

Substituting the given values in (13.6.2), then, we get

$$T_f = \frac{900 \times 80 + 4186 \times 20}{900 + 4186} = 30.6^\circ\text{C}. \tag{13.6.3}$$

This is much closer to the initial temperature of the water, as expected, since it has the greater heat capacity. The amount of heat exchanged is

$$C_{\text{water}}\left(T_f - T_{\text{water}}\right) = 4186 \times (30.6 - 20) = 44,440 \text{ J} = 44.4 \text{ kJ}. \tag{13.6.4}$$

So, 1 kg of aluminum gives off 44.4 kJ of thermal energy and its temperature drops almost 50°C, from 80°C to 30.6°C, whereas 1 kg of water takes in the same amount of thermal energy and its temperature only rises about 10.6°C.

Example 13.6.2: Equipartition of energy

Estimate the speed of an oxygen molecule in air at room temperature (about 300 K).

Solution

Recall that in Section 13.2 I mentioned that the average translational kinetic energy of a molecule in a system at a temperature T is $\frac{3}{2}k_B T$ (Equation (13.2.7), where k_B, Boltzmann's constant, is equal to 1.38 × 10^{-23} J/K. So, at T = 300 K, a molecule of oxygen (or of anything else, for that matter) should have, on average, a kinetic energy of

$$\langle K_{\text{trans}} \rangle = \frac{3}{2} k_B T = \frac{3}{2} \times 1.38 \times 10^{-23} \times 300 \text{ J} = 6.21 \times 10^{-21} \text{ J}. \tag{13.6.5}$$

Since $K = \frac{1}{2}mv^2$, we can figure out the average value of v^2 if we know the mass of an oxygen molecule. This is something you can look up, or derive like this: One mole of oxygen atoms has a mass of 16 grams (16 is the atomic mass number of oxygen) and contains Avogadro’s number of atoms, 6.02×10^{23}. So a single atom has a mass of 0.016 kg/6.02×10^{23} = 2.66×10^{-26} kg. A molecule of oxygen contains two atoms, so it has twice the mass, m = 5.32×10^{-26} kg. Then,

$$\left\langle v^2\right\rangle=\frac{2\left\langle K_{\text{trans}}\right\rangle}{m}=\frac{2\times 6.21\times 10^{-21}\text{ J}}{5.32\times 10^{-26}\text{ kg}}=2.33\times 10^5\ \frac{\text{m}^2}{\text{s}^2}. \tag{13.6.6}$$

The square root of this will give us what is called the “root mean square” velocity, or v_{rms}:

$$v_{rms}=\sqrt{2.33\times 10^5\ \frac{\text{m}^2}{\text{s}^2}}=483\ \frac{\text{m}}{\text{s}}. \tag{13.6.7}$$

This is of the same order of magnitude as (but larger than) the speed of sound in air at room temperature (about 340 m/s, as you may recall from Chapter 12).