4.3: In Summary

1. The *kinetic energy* of a particle of mass m moving with velocity v is defined as $K=\frac{1}{2}mv^2$. It is a scalar quantity, and it is always positive. For a system of particles or an extended object, we define K_{sys} as the sum of the kinetic energies of all the particles making up the system.
2. For any system, the total kinetic energy can be written as the sum of the *translational* (or *center of mass*) kinetic energy, K_{cm}, and another term that involves the motion of the parts of the system relative to each other. (See Equation (4.2.2) above.) The translational kinetic energy is constant for an isolated system, and is always given by $K_{cm}=\frac{1}{2}Mv_{cm}^2$.
3. The kinetic energy of relative motion (which, in the context of collisions, is called the *convertible energy*) is given, for the special case of a system consisting of two particles (or two non-rotating extended objects), by $K_{conv}=\frac{1}{2}\mu v_{12}^2$, where $\mu=m_1m_2/(m_1+m_2)$ is the reduced mass, and $v_{12}=v_2-v_1$ is the relative velocity of the two objects.
4. In a one-dimensional collision between two objects that do not pass through each other, the convertible energy always drops to zero at some point, as a result of the interaction; that is, it is converted entirely into some other form of energy. At the end of the interaction, all the convertible energy may be recovered (elastic collision), or only part of it (inelastic collision), or none of it (completely inelastic collision).
5. In terms of the *coefficient of restitution* e, defined as $e=-v_{12,f}/v_{12,i}$, elastic collisions have e = 1, totally inelastic collisions have e = 0, and inelastic collisions 0 < e < 1. The total change in kinetic energy in the collision can be written as $\Delta K_{sys}=\Delta K_{conv}=(e^2-1)K_{conv,i}$.
6. Another way to say the above is that in an elastic collision in one dimension, the two objects move apart after the collision at the same rate (relative speed) at which they approached each other initially. In a totally inelastic collision, conversely, the two objects do not move apart at all after the collision—they become "stuck together."
7. Besides the cases considered above, one may have collisions where the objects pass through each other, giving e < 0, and "explosive collisions," where e > 1. In these latter collisions some internal source of energy is converted into additional kinetic energy when the objects interact. The extreme case of this is an *explosive separation*, which is the reverse of a totally inelastic collision—two objects initially moving together fly apart, with a net increase in the system's kinetic energy.
8. The translational kinetic energy of a system will, in general, have different values for observers moving with different velocities. The convertible kinetic energy, on the other hand, is seen by all observers to have the same value, regardless of their relative state of motion.