

13.11: Chapter- 11

Problem (11.1).

A strip-line is constructed from a metal strip 1 mm wide ($W = 1$ mm) separated from a ground plane by an oxide layer whose thickness, D , is 20 μm . The relative dielectric constant of the oxide layer is $\epsilon_r = 8.00$, and its relative permeability is $\mu_r = 1.00$.

- What is the velocity of an electromagnetic wave on this line?
- What is the characteristic impedance of the strip-line?
- A pulse on the line is 10 meters long and corresponds to a constant potential difference of 10 Volts. How much energy is stored in the pulse?

Answer (11.1).

$$(a) v^2 = \frac{1}{\epsilon\mu} = \frac{c^2}{\epsilon_r} = \frac{c^2}{8} ; v = 1.06 \times 10^8 \text{ m/sec.}$$

(b) In the dielectric material one finds $\text{curl} \mathbf{E} = i\omega\mu_0 \mathbf{H}$ for a wave having a time dependence $e^{-i\omega t}$. Therefore

$$\frac{\partial E_x}{\partial z} = i\omega\mu_0 H_y,$$

and

$$E_x = \left(\frac{\omega\mu_0}{k} \right) H_y \quad \text{where } kv = \omega.$$

$$\text{Thus } \frac{E_x}{H_y} = v\mu_0 = 133.2 \text{ Ohms.}$$

In the strip line the potential is $V = E_x D$, and the current is given by $I = WH_y$. It follows that the characteristic impedance is given by

$$Z_0 = \frac{V}{I} = \frac{E_x D}{H_y W} = \left(\frac{D}{W} \right) (133.2) = 2.66 \text{ ohms.}$$

(c) The electric field in the insulator is $E_x = \frac{V}{D}$, so

$$E_x = \frac{10}{20 \times 10^{-6}} = 5 \times 10^5 \text{ Volts /m.}$$

$$H_y = \frac{E_x}{133.2} = 3.754 \times 10^3 \text{ Amps/m.}$$

The energy density stored in the electric field is given by

$$W_E = \frac{\epsilon E^2}{2} = \frac{\epsilon_r}{2} \epsilon_0 E_x^2 = 4\epsilon_0 E_x^2 \text{ Joules /m}^3.$$

The energy density stored in the magnetic field is given by

$$W_B = \frac{\mu H^2}{2} = \frac{\mu_0 H_y^2}{2} \text{ Joules /m}^3.$$

$$\text{But } \frac{E_x}{H_y} = \frac{C}{\sqrt{8}} \mu_0 \text{ or } H_y = \frac{\sqrt{8}}{c\mu_0} E_x$$

$$\text{so that } W_B = 4\epsilon_0 E_x^2 \text{ Joules /m}^3.$$

$$\text{The total energy density is } W = W_E + W_B = 8\epsilon_0 E_x^2.$$

$$\text{So } W = (8)(8.84 \times 10^{-12})(25 \times 10^{10}) = 17.86 \text{ Joules/m}^3$$

The volume which contains this energy density is given by

$$\text{Vol.} = (10) (10^{-3}) (20 \times 10^{-6}) = 2 \times 10^{-7} \text{ m}^3.$$

The total energy stored in the pulse is **3.54 x 10⁻⁶ Joules.**

Problem (11.2).

The space between the conductors in a co-axial cable is filled with polyethylene which has a relative dielectric constant $\epsilon_r = 2.25$. The characteristic impedance of the cable is 50 Ohms. A 10 meter length of cable is used to connect a pulse generator to a load of R Ohms. The incident pulse amplitude is V_0 .

- What is the amplitude of the reflected pulse if the cable is terminated by 50 Ohms?
- What is the amplitude of the reflected pulse if the cable is terminated by zero Ohms?
- What is the amplitude of the reflected pulse if the cable is terminated by an open circuit?
- What is the amplitude of the reflected pulse if the cable is terminated by 100 Ohms?
- What is the inductance per meter of cable?
- What is the capacitance per meter of cable?

Answer (11.2).

The velocity of a pulse on the cable is $v = \frac{c}{\sqrt{\epsilon_r}} = \frac{2c}{3} = 2.0 \times 10^8 \text{ m/sec}$, and the characteristic impedance is $Z_0 = 50$ Ohms. The reflection coefficient is given by $\frac{V_R}{V_0} = \frac{r-1}{r+1}$, where $r = \frac{R}{Z_0}$.

- $R = 50$ Ohms, therefore $r = 1$ and $V_R = 0$.
- $R = 0$ Ohms, therefore $r = 0$ and $V_R = -V_0$.
- $R = \infty$ Ohms, therefore $r = \infty$ and $V_R = +V_0$.
- $R = 100$ Ohms, therefore $r = 2$ and $\frac{V_R}{V_0} = \frac{1}{3}$.
- $V^2 = \frac{1}{LC}$ and $Z_0 = \sqrt{L/C}$ so that

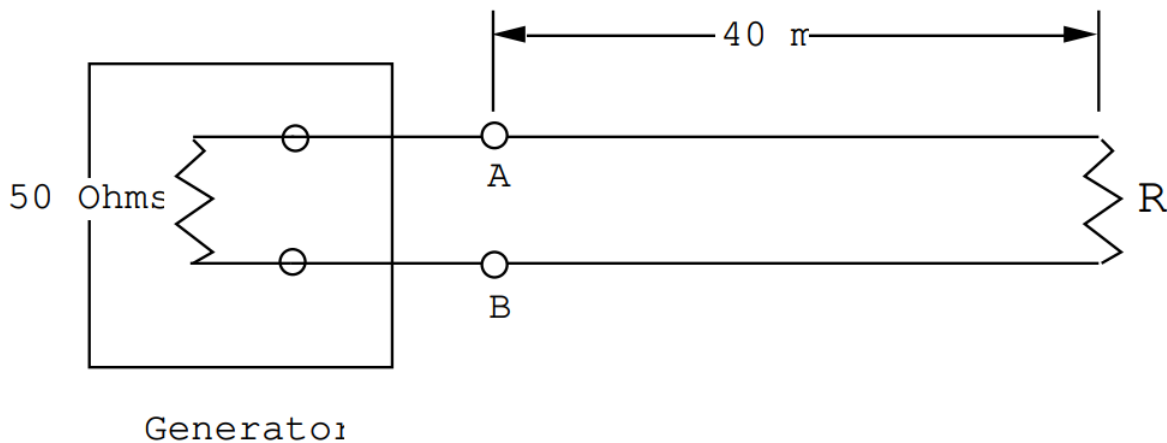
$$L/C = 2500 \quad \text{and} \quad LC = \frac{1}{4 \times 10^{16}}.$$

Consequently, $L^2 = \frac{2500}{4 \times 10^{16}}$ and $L = \frac{1}{4} \mu\text{Henry/m}$.

$$C = \frac{L}{2500} = 100 \text{ pF/m}.$$

Problem (11.3).

A typical co-axial cable has a characteristic impedance of 50 Ohms ($Z_0 = 50$ Ohms). The dielectric material can be regarded as lossless and $\epsilon_r = 2.25$. The cable is connected to a 50 Ohm pulse generator and is terminated by a resistance R Ohms (see the figure).



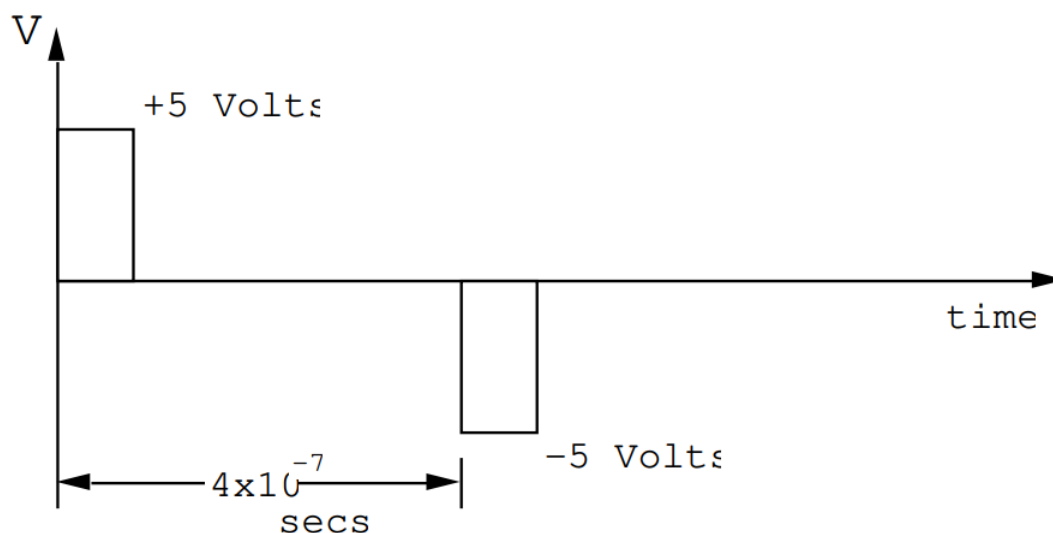
An oscilloscope is connected across AB: its impedance is effectively infinite so that it does not disturb the propagation of pulses on the line. The distance between AB and the end of the line is 40 meters. The generator emits a rectangular pulse whose amplitude is 5 Volts and whose length in time is 10^{-7} seconds.

- What is the velocity of pulses on this cable?
- Let $R = 0$. Make a sketch of the signal measured using the oscilloscope across AB.
- Let $R = 0$. Make a sketch of the signal measured using the oscilloscope connected across the resistor, R .
- Let $R = 50 \text{ Ohms}$. Make a sketch of the signal measured across AB.
- Let $R = 50 \text{ Ohms}$. Make a sketch of the signal measured across the resistor, R .
- Let $R \rightarrow \infty$ (an open circuit). Make a sketch of the signal measured across AB.
- Let $R \rightarrow \infty$. Make a sketch of the signal measured across the open end of the cable.

Answer (11.3).

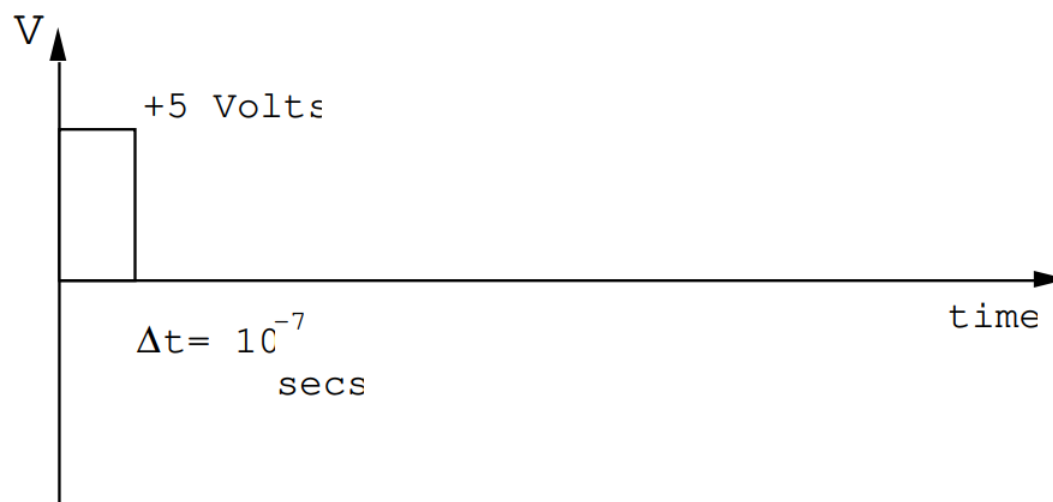
(a) For this cable $\epsilon_r = n^2 = 9/4$ therefore $n = 3/2$. The velocity of propagation $v = \frac{c}{n} = 2 \times 10^8 \text{ m/sec}$.

(b) Shorted Cable. At AB one sees the original pulse followed by the reflected pulse after a time delay of $80/v = 4 \times 10^{-7}$ seconds (40 m out and 40 m back). The reflected pulse is inverted.



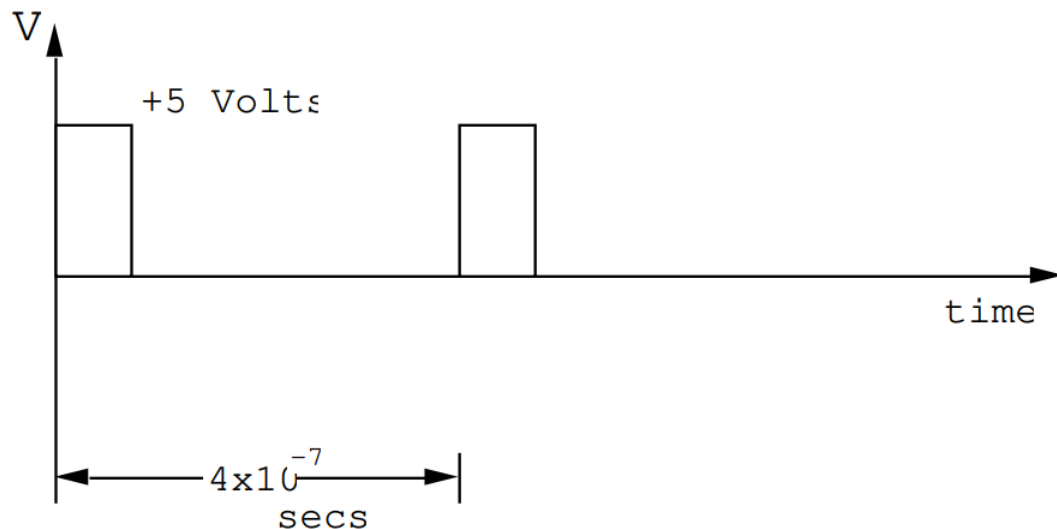
The reflected pulse is absorbed in the generator because the generator impedance is $Z_0 = 50 \text{ Ohms}$.

- Shorted Cable. Nothing will be seen across the short at the end of the cable ($R = 0$).
- Cable terminated by $Z_0 = 50 \text{ Ohms}$. One will measure only the initial pulse. There is no reflected pulse.



(e) Cable terminated by 50 Ohms. The voltage across the 50 Ohms will just look like the incident pulse but delayed by $40/v = 2 \times 10^{-7}$ secs.

(f) Open circuit. At AB one will see the original pulse followed $80/(2 \times 10^8) = 4 \times 10^{-7}$ secs. later by a similar pulse. The reflected pulse will then be absorbed in the generator.



This is a standard technique for generating a delayed pulse.

(g) At the open end of the cable one will measure a single pulse whose amplitude is twice that of the original pulse. (One measures $V_o + V_R$). There will be a time delay of 2×10^{-7} secs.

Problem (11.4).

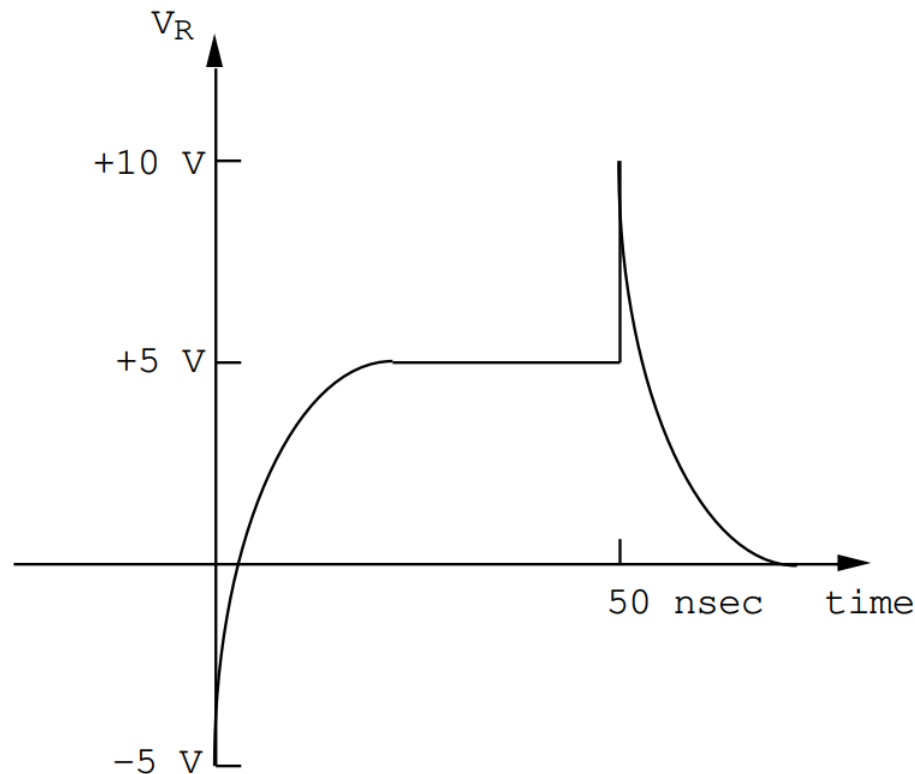
A certain co-axial cable is characterized by a velocity of $v = 2.00 \times 10^8$ meters/sec., and it has a characteristic impedance of 50 Ohms. The cable is terminated by a capacitor $C = 100$ pF. A 10 Meter long rectangular pulse whose amplitude is 5 Volts is launched along the cable. Make a sketch of the reflected pulse. Carefully indicate the voltage and time scales; let the reflected pulse reach the observer at $t=0$. What is the maximum voltage in the reflected pulse?

Answer (11.4).

A 10 m pulse has a time duration of 5×10^{-8} seconds. The time constant associated with the capacitor is $CZ_0 = 5 \times 10^{-9}$ secs., therefore the capacitor will become fully charged during the time that the pulse is applied to it.

(i) Initially the capacitor behaves like a short circuit; the reflected pulse will have an amplitude of -5 Volts. This amplitude decays to +5 Volts as the capacitor becomes fully charged and looks like an open circuit. Note that when fully charged the potential across the capacitor is $V_o + V_R = 10$ Volts.

(ii) At the end of the incident pulse the capacitor, which has been charged to +10 Volts, deposits its charge back into the line at a rate determined by C and the characteristic impedance, Z_0 .



Problem (11.5).

A certain co-axial cable is characterized by a velocity of $V = 2.00 \times 10^8$ meters/sec., and it has a characteristic impedance of 50 Ohms. The cable is terminated by an inductor $L = 0.25 \mu\text{H}$. A 10 Meter long rectangular pulse whose amplitude is 5 Volts is launched along the cable. Make a sketch of the reflected pulse. Carefully indicate the voltage and time scales; let the reflected pulse reach the observer at $t=0$. What is the maximum voltage in the reflected pulse?

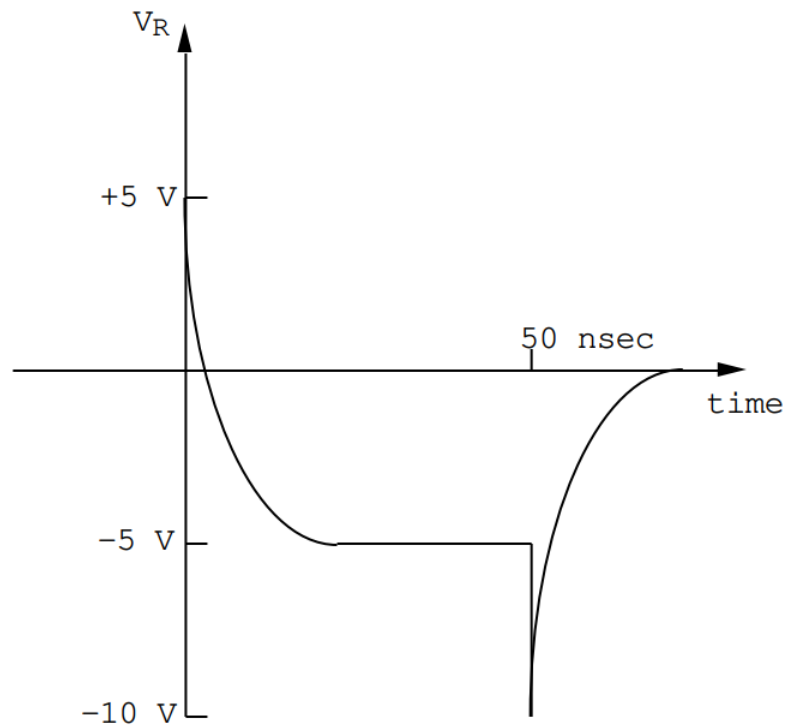
Answer (11.5).

The time duration of the pulse is 5×10^{-8} secs. = 50 nsecs., whereas the time constant associated with the inductor is $\tau = \frac{L}{Z_0} = 0.5 \times 10^{-8}$ secs. = 5 nsecs ; thus the inductor will become fully charged with magnetic energy during the course of the pulse.

(i) At $t=0$ the inductor looks like an open circuit because it resists a change in the current flowing through it. The reflected pulse will therefore have an amplitude of +5 Volts, equal to the amplitude of the incident pulse. The reflected amplitude will decay with a time constant $\tau = L/Z_0$ as the current through the inductor reaches a steady state value. When the current has become constant, the inductor looks like a short circuit and the reflected pulse amplitude is -5 Volts.

(ii) The steady state value of the current through the inductor is just twice the current in the incident pulse, i.e. $I_0 = \frac{2V_0}{Z_0}$ Amps, corresponding to a short circuit. Upon termination of the pulse, this current collapses to give an initial voltage

$$V = L \frac{dI}{dt} = -\frac{L}{\tau} I_0 = -Z_0 \left(\frac{2V_0}{Z_0} \right) = -2V_0.$$



Problem (11.6).

A certain co-axial cable is characterized by a velocity of $V = 2.00 \times 10^8$ meters/sec., and it has a characteristic impedance of 50 Ohms. A piece of this cable 21 m long is used to connect a 250 MHz oscillator to a load impedance Z_L .

- What load impedance will be presented to the generator if Z_L is a 50 Ohm resistor?
- What load impedance will be presented to the generator if Z_L is a 1.00 μH inductor?
- What load impedance will be presented to the generator if Z_L is a 100 pF capacitor?
- What impedance will be presented to the generator in the above three cases if the co-axial cable has a length of 20.0 meters?

Answer (11.6).

At 250 MHz and for $v = 2.00 \times 10^8$ m/sec. the wavelength on the cable is $\lambda = \frac{2 \times 10^8}{2.5 \times 10^8} = \frac{4}{5}$ meters .

(a) Terminated by the characteristic impedance. The generator looks into 50 Ohms.

(b) At 250 MHz. the impedance of a 1.0 μH inductor is given by $Z_L = iL\omega = 1571i$ Ohms, since $\omega = 1.57 \times 10^9$ radians/sec.
 $\frac{Z_L}{Z_0} = i31.42$

$$2ikl = i \frac{4\pi(21)}{\lambda} = 105i\pi$$

which is equivalent to a phase shift of π . Since the impedance seen by the generator is

$$\frac{Z_G}{Z_0} = \frac{1 + b/a}{1 - b/a},$$

where

$$\frac{b}{a} = \left(\frac{z - 1}{z + 1} \right) e^{-2ikl},$$

and

$$z = \frac{Z_L}{Z_0},$$

one finds $\frac{b}{a} = \frac{1-i31.42}{1+i31.42} = -0.998 - i0.0636$.

$$\frac{Z_G}{Z_0} = \frac{1+b/a}{1-b/a} = -i0.0319,$$

or $Z_G = -1.59i$ Ohms. The load appears to the generator like a capacitor with $C = 400$ pF!

(c) The load impedance is a 100 pF capacitor.

For the capacitor $Z_c = \frac{-i}{\omega C} = -i6.366$ Ohms .

$$z = \frac{z_L}{z_0} = -i0.1273$$

As before $e^{-2ikl} = -1$ so that

$$b/a = \frac{1-z}{1+z} = 0.9681 + i0.2505.$$

$$\frac{Z_G}{Z_0} = \frac{1+b/a}{1-b/a} = i7.855,$$

from which $Z_G = i392.8$ Ohms. .

The load appears to the generator like a 0.25 μ H inductor.

(d) A 20 m cable contains an integer number of wavelengths, therefore the generator will look into the load impedance exactly as if the cable had zero length.

(a) $Z_G = 50$ Ohms.

(b) $Z_G = i1571$ Ohms (Inductive).

(c) $Z_G = -i6.37$ Ohms.

Problem (11.7).

A co-axial cable is characterized by a characteristic impedance of $Z_0 = 50$ Ohms and a velocity of propagation of 2×10^8 m/sec. It is used to connect a 10 Ohm load to a generator. Calculate the impedance as seen from the generator for a cable having the following lengths, L:

(a) $L = \lambda/8$

(b) $L = \lambda/4$

(c) $L = 3\lambda/8$

(d) $L = \lambda/2$

(e) Calculate the Voltage Standing Wave Ratio, VSWR.

Answer (11.7).

For a load of $Z_L = 10$ one has a normalized impedance $Z_L = \frac{10}{50} = 0.20$

$$\therefore \Gamma = \frac{z_L - 1}{z_L + 1} = -\frac{0.80}{1.2} = -\frac{2}{3} = \frac{2}{3}e^{i\pi}$$

$$(a) L = \frac{\lambda}{8} \quad \therefore e^{-2ikL} = e^{-i4\pi L/\lambda} = e^{-i\pi/2} = -i$$

$$\therefore \Gamma e^{-2ikL} = \frac{2i}{3}$$

$$\therefore z_G = \frac{1 + \Gamma e^{-2ikL}}{1 - \Gamma e^{-2ikL}} = \frac{1 + 2i/3}{1 - 2i/3} = \frac{3 + 2i}{3 - 2i} = \frac{(3 + 2i)(3 + 2i)}{9 + 4}$$

$$\therefore Z_G = \frac{5 + 12i}{13} = 0.385 + 0.923i$$

$$\therefore Z_G = 50 Z_G = \mathbf{19.23 + 46.15i \text{ Ohms}}$$

i.e. a large inductive component

$$(b) L = \lambda/4 \quad e^{-2ikL} = e^{-i4\pi L/\lambda} = e^{-i\pi} = -1$$

$$\therefore \Gamma e^{-2ikL} = 2/3 \text{ (real)}$$

$$\therefore Z_G = \frac{1+2/3}{1-2/3} = \frac{5/3}{1/3} = 5$$

$\therefore Z_G = \mathbf{250 \text{ Ohms}}$ (purely real and relatively large!).

$$(c) L = \frac{3\lambda}{8} \quad e^{-2ikL} = e^{-i4\pi L/\lambda} = e^{i\pi/2} = +i$$

$$\therefore \Gamma e^{-2ikL} = -\frac{2i}{3}$$

$$Z_G = \frac{1 - 2i/3}{1 + 2i/3} = \frac{3 - 2i}{3 + 2i} = \frac{(3 - 2i)(3 - 2i)}{9 + 4} = \frac{5 - 12i}{13}$$

$$\therefore Z_G = \mathbf{19.23 - 46.15i \text{ Ohms}}$$

i.e. there is a large capacitive component.

$$(d) L = \frac{\lambda}{2} \quad e^{-2ikL} = e^{-i4\pi L/\lambda} = e^{-i2\pi} = +1$$

$$\therefore \Gamma e^{-2ikL} = \Gamma = -2/3$$

$$Z_G = \frac{1-2/3}{1+2/3} = \frac{1}{5} = 0.2$$

$$\therefore Z_G = \mathbf{10 \text{ Ohms.}}$$

i.e. The generator looks directly into the load.

(e) The standing wave ratio is given by

$$\text{VSWR} = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 2/3}{1 - 2/3} = 5$$

In a slotted line there would be no change in the position of $|V_{\min}|$ when the load was exchanged for a short.

Problem (11.8).

Given a co-axial cable for which $Z_0 = 50 \text{ Ohms}$ and $v = 2 \times 10^8 \text{ m/sec}$. A piece of this cable of length L meters is used to connect a load impedance Z_L to the generator: $Z_L = (10 + 20i) \text{ Ohms}$.

Calculate the impedance seen by the generator for

$$(a) L = \frac{\lambda}{16}$$

$$(b) L = \frac{3\lambda}{16}$$

$$(c) L = \frac{5\lambda}{16}$$

$$(d) L = \frac{\lambda}{2}$$

(e) Calculate the Voltage Standing Wave Ratio, VSWR.

Answer (11.8).

$$Z_L = (10 + 20i) \text{ Ohms}$$

$$Z_L = \left(\frac{1+2i}{5} \right)$$

$$\Gamma = \frac{z_L - 1}{z_L + 1} = \frac{1 + 2i - 5}{1 + 2i + 5} = \frac{-4 + 2i}{6 + 2i} = \frac{-2 + i}{3 + i}$$

$$\therefore \Gamma = \frac{(-2+i)(3-i)}{9+1} = \frac{-5+5i}{10} = \frac{-1+i}{2}$$

$$\therefore \Gamma = \frac{1}{\sqrt{2}} e^{3\pi i/4}$$

Now $e^{-2ikL} = e^{-i4\pi L/\lambda} \therefore$ (a) $\frac{L}{\lambda} = \frac{1}{16} \quad e^{-2ikL} = e^{-\pi i/4}$

(b) $\frac{L}{\lambda} = \frac{3}{16} \quad e^{-2ikL} = e^{-3\pi i/4}$

(c) $\frac{L}{\lambda} = \frac{5}{16} \quad e^{-2ikL} = e^{-5\pi i/4}$

(d) $\frac{L}{\lambda} = \frac{1}{2} \quad e^{-2ikL} = e^{-2\pi i} \equiv +1$

So

(a) $\Gamma e^{-2ikL} = \frac{1}{\sqrt{2}} e^{i\pi/2} = \frac{i}{\sqrt{2}}$

$$z_G = \frac{1 + \Gamma e^{-2ikL}}{1 - \Gamma e^{-2ikL}} = \frac{1 + i/\sqrt{2}}{1 - i/\sqrt{2}} = \frac{\sqrt{2} + i}{\sqrt{2} - i}$$

$$\therefore z_G = \frac{(\sqrt{2} + i)(\sqrt{2} + i)}{3} = \frac{1 + i2\sqrt{2}}{3}$$

$$\therefore Z_G = 50 z_G = (16.67 + 47.14 i) \text{ Ohms.}$$

(b) $\Gamma e^{-2ikL} = \frac{1}{\sqrt{2}} \quad (\text{purely real})$

$$\therefore z_G = \frac{1 + 1/\sqrt{2}}{1 - 1/\sqrt{2}} = 5.83$$

$$\therefore Z_G = (5.83)(50) = \mathbf{291.4 \text{ Ohms}} \text{ (Purely real!).}$$

(c) $\Gamma e^{-2ikL} = \frac{1}{\sqrt{2}} e^{-i\pi/2} = -\frac{i}{\sqrt{2}}$

$$z_G = \frac{1 - i/\sqrt{2}}{1 + i/\sqrt{2}} = \frac{\sqrt{2} - i}{\sqrt{2} + i}$$

(The reciprocal of case (a))

$$\therefore z_G = \frac{1 - i2\sqrt{2}}{3}, \text{ and } Z_G = \mathbf{(16.67 - 47.14 i) \text{ Ohms}},$$

and now the generator load has a capacitive component.

(d) $\Gamma e^{-2ikL} = \Gamma \quad z_G = \frac{1+\Gamma}{1-\Gamma} \equiv z_L$

$$\therefore Z_G \equiv Z_L = \mathbf{(10 + 20i) \text{ Ohms.}}$$

(e) $\text{VSWR} = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+1/\sqrt{2}}{1-1/\sqrt{2}} = \mathbf{5.83}$

Problem (11.9).

A 50 Ohm piece of co-axial cable of length L meters is used to connect a load to a generator. The load impedance is given by

$$Z_L = (10 + 100 i) \text{ Ohms}$$

Calculate the impedance seen by the generator for

(a) $L/\lambda = 0.0732$

(b) $L/\lambda = 0.250$

(c) $L/\lambda = 0.3232$

(d) $L/\lambda = 0.5000$

(e) Calculate the Voltage Standing Wave Ratio, VSWR.

Answer (11.9).

$$z_L = \frac{1}{5} + 2i = \frac{1+10i}{5}$$

$$\therefore \Gamma = \frac{z_L - 1}{z_L + 1} = \frac{-4/5 + 2i}{6/5 + 2i} = \frac{-2+5i}{3+5i} = \mathbf{0.924e^{+0.921i}}$$

$$(a) e^{-2ikL} = e^{-0.92i} \therefore \Gamma e^{-2ikL} = 0.923 \text{ purely real}$$

$$\therefore z_G = \frac{1 + \Gamma e^{-2ikL}}{1 - \Gamma e^{-2ikL}} = \frac{1 + .9235 + i0.00095}{1 - .9235 - i0.00095} = (25.16 - i0.34) \text{ Ohms.}$$

$Z_G = (1257.8 + i 16.9) \text{ Ohms.}$ Almost resistive!

$$(b) e^{-2ikL} = e^{-i\pi} = -1 \therefore \Gamma e^{-2ikL} = \frac{2-5i}{3+5i} = -0.559 - 0.735i$$

$$\therefore z_G = \frac{(1 - 0.559) - 0.735i}{[1.559 + 0.735i]} = \frac{(.441 - .735i)(1.559 - .735i)}{2.971}$$

$$\therefore z_G = \frac{0.148 - 1.47i}{2.971} \text{ and } Z_G = \mathbf{2.48 - 24.75i \text{ Ohms}}$$

Capacitive Loading.

$$(c) \frac{L}{\lambda} = 0.3232 e^{-2ikL} = e^{-i4\pi L/\lambda} = e^{-4.06i}$$

$$\therefore \Gamma e^{-2ikL} = 0.923e^{-3.141i} = 0.923e^{-i\pi} = -0.923 \text{ (real)}$$

$$\therefore z_G = \frac{1 - .9235 - i0.00099}{1 + .9235 + i0.00099}$$

and $Z_G = (1.987 - i 0.027) \text{ Ohms.}$

A small, nearly purely real, load.

(d) When $L/\lambda = \frac{1}{2}$ one gets the same effect as connecting the load directly across the generator.

$$\therefore Z_G = Z_L = \mathbf{(10 + 100i) \text{ Ohms.}}$$

$$(e) \text{ VSWR} = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+.923}{1-.923} = \mathbf{25.16}$$

Problem (11.10).

A certain co-axial cable is characterized by a velocity of $V = 2.00 \times 10^8$ meters/sec., and it has a characteristic impedance of 50 Ohms. The attenuation parameter for the cable is $\alpha = 0.02$ per meter. A piece of this cable 21 m long is used to connect a 250 MHz oscillator to a load consisting of 100 pF shunted by a resistance of 5.0 Ohms. Calculate the load on the generator.

Answer (11.10).

The impedance of the capacitor is $z_c = \frac{-i}{\omega} = -i6.366 \text{ Ohms.}$ The load impedance is Z_C in parallel with a 5 Ohm resistor;

$$\frac{1}{Z_L} = \frac{1}{5} + \frac{1}{Z_C} = 0.20 + i \frac{\pi}{20},$$

so that $Z_L = 3.092 - i 2.429 \text{ Ohms, and}$

$$z = \frac{z_L}{z_0} = 0.0618 - i0.0486 = 0.0786 \angle -38.15^\circ.$$

We have

$$z = \frac{Z_L}{Z_0} = \frac{1 + \frac{b}{a} e^{2\alpha l} e^{2ikl}}{1 - \frac{b}{a} e^{2\alpha l} e^{2ikl}},$$

where $e^{2ikl} = -1$ and where $e^{2\alpha l} = e^{2(1.04)} = 2.316$.

$$\text{Let } \Gamma = \frac{z-1}{z+1} = (-0.880 - i0.086)$$

and

$$b/a = \Gamma \exp(-2\alpha l - 2ikl) = (0.380 + i0.037)$$

The impedance seen by the generator is $\frac{Z_G}{Z_0} = \frac{1+b/a}{1-b/a}$.

$$\frac{Z_G}{Z_0} = (2.213 + i0.192),$$

therefore $Z_G = (110.7 + i 9.62)$ Ohms.

This can be compared with an impedance $Z_G = 500 + i 393$ Ohms for the same length of lossless cable. In the limit of a very long cable the impedance seen by the generator must, of course, approach the characteristic impedance of 50 Ohms.

Problem (11.11).

A slotted line is terminated by a load impedance $Z_L = (10 + 10i)$ Ohms. The characteristic impedance is $Z_0 = 50$ Ohms. The position of the voltage minimum is found to be at z_1 . The load is then replaced by a short and the voltage minimum is found to be at z_2 .

(a) How large is the shift $\frac{(z_1 - z_2)}{\lambda}$?

Is this shift positive (i.e. $z_1 > z_2$) corresponding to the shorted line minimum closer to the generator, or is it negative (i.e. $z_2 > z_1$) corresponding to the shorted line minimum closer to the load?

(b) Calculate the Voltage Standing Wave Ratio, VSWR.

Answer (11.11).

(a) $Z_0 = 50$ Ohms $Z_L = (10 + 10i)$ Ohms

$$\therefore z_L = \frac{Z_L}{Z_0} = 0.2(1 + i)$$

$$\Gamma = \frac{z_L - 1}{z_L + 1} = \frac{-0.8 + 0.2i}{1.2 + 0.2i} = \frac{-0.92 + 0.40i}{(1.2)^2 + 0.04}$$

$$= -0.622 + 0.27i$$

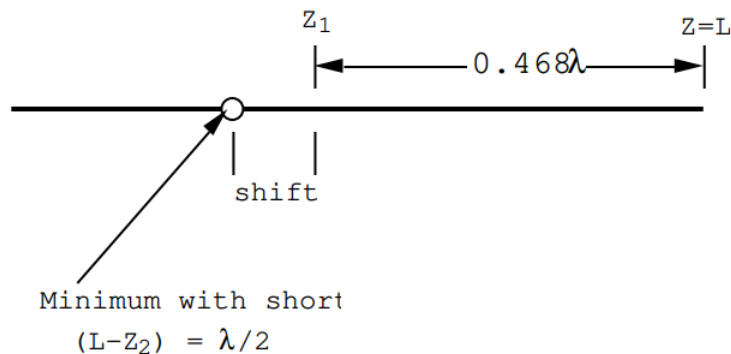
$$\therefore \Gamma = 0.678e^{2.731i} = 0.678e^{i(0.869)\pi}$$

We have at a voltage minimum

$$\cos\left[\frac{4\pi}{\lambda}(L - z_1) - \theta\right] = -1$$

$$\therefore \frac{4\pi}{\lambda}(L - z_1) - 0.87\pi = \pi$$

$$\therefore L - z_1 = \frac{1.87\lambda}{4} = .468\lambda$$



So upon shorting the line, the minimum moves 0.0327λ towards the generator.

$$(b) \text{VSWR} = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+.678}{1-.678} = \mathbf{5.21}$$

Problem (11.12).

A slotted line is terminated by a load impedance $Z_L = (10 - 10i)$ Ohms. The characteristic impedance of the slotted line is 50 Ohms. The voltage minimum is found to be at z_1 on the line. When the load is replaced by a short the voltage minimum moves to z_2 .

(a) Calculate the shift $\frac{(z_1 - z_2)}{\lambda}$. When the line is shorted does the minimum move towards the generator or towards the load?

(b) Calculate the Voltage Standing Wave Ratio, VSWR.

Answer (11.12).

(a) $Z_L = 10(1 - i)$ Ohms

$$\frac{Z_L}{Z_0} = \frac{1}{5} - \frac{i}{5} = z_L$$

$$\Gamma = \frac{z_L - 1}{z_L + 1} = \frac{-4/5 - i/5}{6/5 - i/5} = \frac{-4 - i}{6 - i} = \frac{-23 - 10i}{37}$$

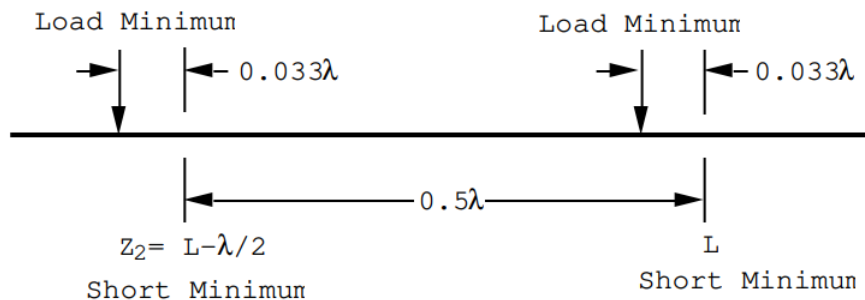
$$\therefore \Gamma = -0.622 - 0.27i = 0.678e^{i1.13\pi}$$

$$\theta = 203.5^\circ = 1.131\pi \text{ radians.}$$

Minimum when $\cos[2k(L - z) - \theta] = -1$

$$\text{or } \frac{4\pi}{\lambda}(L - z) - 1.13\pi = \pi$$

$$\therefore L - z = 0.533\lambda$$



When the load is replaced by a short, the minimum moves $.0327 \lambda$ towards the load.

$$(b) \text{VSWR} = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1.678}{1-.678} = \mathbf{5.21}$$

The same VSWR as for the impedance of problem (9.11).

Problem (11.13).

The Voltage Standing Wave Ratio is found to be $S = 2.0$ on a lossless 300 Ohm transmission line terminated by an unknown load impedance, Z_L . The nearest voltage minimum is $\frac{3\lambda}{10}$ from the load i.e. $z_1 = (L - 0.3\lambda)$.

(a) When the above line is shorted where will the voltage minimum be located which is nearest the load, but not right at the load?

(b) Calculate the real and imaginary parts of the unknown load impedance, Z_L .

Answer (11.13).

(a) When the load is replaced by a short the minimum will be located $\frac{\lambda}{2}$ from the short.

$$\therefore z_2 = L - \frac{\lambda}{2}$$

$$(b) S = \text{VSWR} = 2.0 = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$\therefore |\Gamma| = 1/3 \quad \Gamma = |\Gamma|e^{i\theta}$$

Minimum at z_1 where

$$\cos [2k(L-z) - \theta] = -1$$

$$k = \frac{2\pi}{\lambda} \text{ and } \frac{4\pi}{\lambda}(L - z_1) - \theta = \pi$$

$$\text{So } \pi + \theta = \left(\frac{4\pi}{\lambda}\right)\left(\frac{3\lambda}{10}\right) = 1.2\pi$$

$$\therefore \theta = 0.2\pi$$

$$\text{So } \Gamma = \frac{1}{3}e^{0.2\pi i}$$

$$\Gamma = 0.270 + .196i$$

$$\text{But } z_L = \frac{Z_L}{Z_0} = \frac{1+\Gamma}{1-\Gamma}$$

$$\therefore z_L = 1.555 + .685i$$

$$\therefore Z_L = (z_L)(300) = 466.5 + 205.6i \text{ Ohms}$$

Problem (11.14). A slotted line is characterized by a velocity $V = 3.00 \times 10^8$ m/sec, and by a characteristic impedance of 50 Ohms. The slotted line is connected to an oscillator on one end and to an unknown load on the other end. The voltage standing wave ratio is found to be $VSWR = 2.0$. Moreover, when the load is replaced by a short circuit the position of the voltage minimum shifts 5 cm towards the load. The position of the first minimum from the shorted end occurs 40.0 cm from the short. Calculate the impedance of the load.

Answer (11.14).

The voltage minimum on a shorted line occurs at a distance $\lambda/2$ from the short; therefore for this problem the generator frequency corresponds to a wavelength of $\lambda = 80$ cm = 0.80 meters. The velocity on the slotted line is $c = 3 \times 10^8$ m/sec, so that the frequency is $f = c/\lambda = 375$ MHz. The corresponding circular frequency is $\omega = 2\pi f = 2.356 \times 10^9$ radians/sec. The wavevector on the line is $k = 2\pi/\lambda = 7.854$ m⁻¹. Let the load be at $z=L$, with the generator somewhere to the left (at $z=0$). For a time dependence $e^{i\omega t}$

$$V = ae^{-ikz} + be^{ikz}$$

and

$$z_0 I = ae^{-ikz} - be^{ikz}.$$

$$\text{At } z=L \quad V = ae^{-ikL} + be^{ikL}$$

and

$$z_0 I = ae^{-ikL} - be^{ikL}$$

Thus

$$\frac{z_L}{z_0} = \frac{1 + (b/a)e^{2ikL}}{1 - (b/a)e^{2ikL}} = \frac{1 + \Gamma e^{i\theta}}{1 - \Gamma e^{i\theta}},$$

where

$$\left(\frac{b}{a}\right)e^{2ikL} = \Gamma e^{i\theta} = \frac{(Z_L/Z_0) - 1}{(Z_L/Z_0) + 1}.$$

One can write

$$V(z) = ae^{-ikz} \left(1 + \Gamma e^{i\theta} e^{2ik(z-L)}\right);$$

$$\text{clearly } |V_{\max}| = |a|(1 + \Gamma),$$

$$\text{whereas } |V_{\min}| = |a|(1 - \Gamma),$$

$$\text{so that } \frac{|V_{\max}|}{|V_{\min}|} = \frac{1+\Gamma}{1-\Gamma} = 2.0$$

Therefore $\Gamma = 1/3$. With the load connected the minimum occurs at z_1 . At the minimum $e^{i(2k(z_1-L)+\theta)} = -1$,

or $2k(z_1 - L) + \theta = \pm\pi$.

When the line is shorted the minimum occurs 40 cm from the load. With the load in place the minimum shifts 5 cm towards the generator. That means that z_1 is such that $L - z_1 = 45 \text{ cm} = 0.5625\lambda$. Thus

$$\theta = \pm\pi + 2k(L - z_1),$$

or $\theta = \pm\pi + 7.0686$. The appropriate value is less than 2π so that $\theta = 3.9270$ radians

$$\frac{Z_L}{Z_0} = \frac{1 + \frac{1}{3}e^{3.927i}}{1 - \frac{1}{3}e^{3.927i}} = 0.562 - i0.298$$

and the load impedance is **$Z_L = 28.08 - i14.89 \text{ Ohms}$** . This is equivalent to a resistance of 28.08 Ohms in series with a 28.5×10^{-12} Farad capacitor.

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