# 12.4: Energy Losses in the Waveguide Walls

When a metal is exposed to a time-varying magnetic field eddy currents are induced which flow so as to shield the interior of the metal from the magnetic field. Let the strength of the magnetic field at the metal surface be $H_0$, and let the field be oriented along the y-direction. The magnetic field decays towards the interior of the metal, Chapter(10), section(10.4), according to the formula

$$\mathrm{H_y}(\xi)=\mathrm{H_0}\exp(i[\mathrm{k}\xi-\omega\mathrm{t}]),$$

where ξ measures distance into the metal along the normal to the surface; the metal surface is assumed to lie in the y-z plane. The wave-vector, k, is given by

$$\mathrm{k}=\sqrt{\frac{\omega\sigma\mu_0}{2}}(1+i).$$

The electric field that generates the shielding currents in the metal is orthogonal to the magnetic field and parallel with the metal surface:

$$\mathrm{E_z}(\xi)=\left(\frac{i\mathrm{k}}{\sigma}\right)\mathrm{H_0}\exp(i[\mathrm{k}\xi-\omega\mathrm{t}]).$$

The Poynting vector at the metal surface is directed into the metal; its time average is given by

$$<\mathrm{S}_\xi>=\frac{|\mathrm{H_0}|^2}{2}\sqrt{\frac{\omega\mu_0}{2\sigma}}\quad Watts/m^2. \tag{12.4.1}$$

This energy is converted into heat in the metal wall. This Joule heat must, of course, be supplied by the microwaves propagating along the guide, and results in a gradual decrease in signal strength. The resistivity of brass is typically $\rho = 8\times10^{-8}$ Ohm-meters corresponding to $\sigma = 1/\rho = 1.25\times10^7$ per Ohm-m at room temperature. The rate of energy loss to a brass waveguide wall at room temperature and for a frequency of 10 GHz is

$$<\mathrm{S}_\xi>=0.028|\mathrm{H_0}|^2\quad \mathrm{Watts}\ /m^2. \tag{12.4.2}$$

Since energy is lost to the waveguide walls the average energy moving down the guide must decrease with distance, and therefore the amplitude of the wave must decrease. For the $TE_{10}$ mode one has (using the co-ordinate system of Figure (12.2.4))

$$\begin{aligned}
\mathrm{H_z}&=\mathrm{H_0}\cos\left(\frac{\pi\mathrm{x}}{\mathrm{a}}\right)\exp(i[\mathrm{kgz}-\omega\mathrm{t}]), \qquad (12.4.3)\\
\mathrm{H_x}&=-i\left(\frac{\mathrm{k_g}}{\pi/\mathrm{a}}\right)\mathrm{H_0}\sin\left(\frac{\pi\mathrm{x}}{\mathrm{a}}\right)\exp(i[\mathrm{kgz}-\omega\mathrm{t}]),\\
\mathrm{E_y}&=i\left(\frac{\omega\mu_0}{\pi/\mathrm{a}}\right)\mathrm{H_0}\sin\left(\frac{\pi\mathrm{x}}{\mathrm{a}}\right)\exp(i[\mathrm{kgz}-\omega\mathrm{t}]).
\end{aligned}$$

The average rate at which energy is transported past a waveguide crosssection can be calculated from $<\mathrm{S_z}>=(1/2)\,\mathrm{Real}(-\mathrm{E_y}\mathrm{H_x^*})$ and this must be integrated over the area of the guide:

$$P_z=\frac{dE}{dt}=\left(\frac{ab}{4}\right)\frac{\omega\mu_0k_g}{(\pi/a)^2}H_0^2\quad Watts. \tag{12.4.4}$$

Let us apply these ideas to calculate the rate at which a microwave signal propagating in a brass X-band waveguide decays with distance. The energy loss per second per unit length of guide due to eddy current losses in the narrow sides of the guide is given by

$$\frac{\mathrm{d}<\mathrm{P_1}>}{\mathrm{dz}}=(2\mathrm{b})\mathrm{H_0^2}(0.028)\quad \mathrm{Watts}\,; \tag{12.4.5}$$

these losses are due to the component $H_z$ whose amplitude at the walls is $H_0$. The factor two arises because there are contributions from two walls; the factor 0.028 comes from Equation (12.4.2). The contribution from the energy losses at the broad sides of the waveguide are more complicated since there are two magnetic field components $H_x$ and $H_z$, and both components must be averaged over the x spatial dependence:

$$\frac{d < P_2 >}{dz} = (2a)\left(\frac{H_0^2}{2}\right)\left[1 + \left(\frac{k_g}{\pi/a}\right)^2\right](0.028)\text{ Watts. .} \tag{12.4.6}$$

Thus the power passing through the guide cross-section must decrease in the distance dz by an amount that is given by the sum of Equations (12.4.5) and (12.4.6):

$$dP = (0.028)H_0^2\left[2b + a\left(1 + \left(\frac{k_g}{\pi/a}\right)^2\right)\right]dz. \tag{12.4.7}$$

By differentiating Equation (12.4.4) one obtains

$$dP = \left(\frac{ab}{2}\right)\left(\frac{\omega\mu_0 k_g}{(\pi/a)^2}\right)H_0\left(\frac{dH_0}{dz}\right)dz. \tag{12.4.8}$$

Equating Equations (12.4.7) and (12.4.8) gives an equation for the rate of change of the wave amplitude, $H_0$, with distance

$$\frac{dH_0}{dz} = -H_0\left(\frac{0.056(\pi/a)^2}{\omega\mu_0 k_g}\right)\left[\frac{2}{a} + \frac{1}{b}\left(1 + \left(\frac{k_g}{\pi/a}\right)^2\right)\right].$$

This can be re-written in the form

$$\frac{dH_0}{dz} = -\gamma H_0, \tag{12.4.9}$$

where for X-band waveguide (a=2.29 cm, b=1.02 cm) and for a frequency of 10 GHz kg = 158 $m^{-1}$ and $\gamma = 2.67 \times 10^{-2}$ per meter. Equation (12.4.9) implies that the amplitude of a wave propagating along a waveguide falls off exponentially

$$H_0 = A\exp(-\gamma z); \tag{12.4.10}$$

the amplitude decreases by 1/e after having travelled a distance of z= 1/γ meters. This distance is 37.5 meters for X-band waveguide at 10 GHz.