1.1: Fundamental Postulates

The properties of ordinary matter are a consequence of the forces acting between charged particles. Extensive experimental investigations have established the following properties of electrical charges:

(1) There are two kinds of charges. These have been labeled positive charge and negative charge.

(2) Electrical charge is quantized. All particles so far observed carry charges which are integer multiples of the charge on an electron. In the MKS system of units, the charge on an electron is e = $-1.60\mathrm{x}10^{-19}$ Coulombs. By definition, the electron carries a negative charge and a proton carries a positive charge; the charge on a proton is $+1.60\mathrm{x}10^{-19}$ Coulombs. No one knows why charge comes in multiples of the electron charge.

(3) Equality of the positive and the negative charge quantum. The quantum of positive charge and the quantum of negative charge are equal to at least 1 part in 10^{20}. This has been determined from experiments designed to measure the net charge on neutral atoms.

(4) In any closed system charge is conserved. This means that the algebraic sum of all positive charges plus all negative charges does not change with time. This does not mean that individual charged particles are conserved. For example, a positron, which carries a positive charge of $1.60\mathrm{x}10^{-19}$ Coulombs, can interact with an electron, which carries a negative charge of $1.60\mathrm{x}10^{-19}$ Coulombs, in such a way that the electron and positron disappear and two neutral particles called photons are produced. The total charge before and after this transformation occurs remains exactly the same, namely zero. The individual charged particles have disappeared but the total charge has been conserved.

(5) Charges generate electric and magnetic fields. Charged particles set up a disturbance in space which can be described by two vector fields; an electric field, $\vec{E}$, and a magnetic field, $\vec{B}$. The units of the electric field are Volts/meter; the units of the magnetic field are Webers/m^2 . Since these are vector fields they are characterized by a direction and a magnitude. Each of these fields at any point in space can be described by its components along three mutually perpendicular axes. For example, with respect to a rectangular cartesian system of axes, xyz (see Figure (1.1.1)),

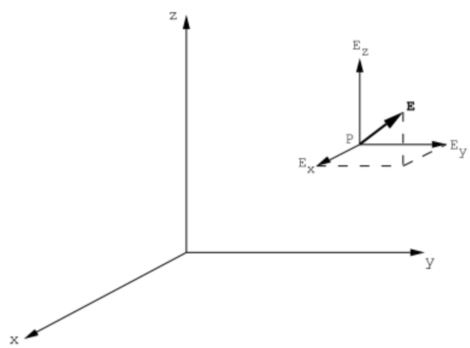

Figure 1.1.1: A cartesian co-ordinate system used to specify an electric vector field

the electric field can be resolved into the three components E_x(x,y,z,t), E_y(x,y,z,t), and E_z(x,y,z,t) where the magnitude of the electric field is given by $\mathrm{E}=\sqrt{\mathrm{E}_x^2+\mathrm{E}_y^2+\mathrm{E}_z^2}$. The components of these fields depend upon the orientation of the co-ordinate system used to describe them, however the magnitude of each field must be independent of the orientation of the co-ordinate system.

(6) The fields E and B are real physical objects. These fields can carry energy, momentum, and angular momentum from one place to another.

(7) The electromagnetic forces on a charged particle, q, can be obtained from a knowledge of the fields E, B generated at the position of q by all other charges. The force in Newtons is given by

$$\overrightarrow{\mathrm{F}}=q[\overrightarrow{\mathrm{E}}+(\overrightarrow{\mathrm{v}}\times\overrightarrow{\mathrm{B}})] \tag{1.1.1}$$

where $\vec{v}$ is the particle velocity in meters/sec. (Notice that $\vec{B}$ has the units of an electric field divided by a velocity). Formula (1.1.1) applies to a spinless particle. In actual fact the situation is more complicated because most particles carry an intrinsic magnetic moment associated with its intrinsic angular momentum (spin). In the rest system of the particle its magnetic moment is subject to a torque due to the presence of the field $\vec{B}$, and to a force due to spatial gradients of $\vec{B}$. These magnetic forces will be discussed later. For the present we shall discuss only spinless charged particles, and we shall ignore the fact that real charged particles are more complicated.

(8) Superposition. Electric and magnetic fields obey the rules of superposition. Given a system of charges which would by themselves produce the fields $\vec{E}_1$, $\vec{B}_1$; given a second system of charges which would by themselves produce the fields $\vec{E}_2$, $\vec{B}_2$; then together the two systems of charges produce the total fields

$$\overrightarrow{\mathrm{E}}=\overrightarrow{\mathrm{E}_1}+\overrightarrow{\mathrm{E}_2}, \quad \overrightarrow{\mathrm{B}}=\overrightarrow{\mathrm{B}_1}+\overrightarrow{\mathrm{B}_2}. \tag{1.1.2}$$

This rule enormously simplifies the calculation of electric and magnetic fields because it can be carried out particle by particle and the total field obtained as the vector sum of all the partial fields due to the individual charges.

(9) A Stationary Charged Particle. In the co-ordinate system in which a charged particle is stationary with respect to the observer the electric and magnetic fields which it generates are very simple:

$$\overrightarrow{\mathrm{E}}=\frac{q}{4\pi\epsilon_0}\left(\frac{\overrightarrow{\mathrm{R}}}{\mathrm{R}^3}\right) \quad \overrightarrow{\mathrm{B}}=0 \tag{1.1.3}$$

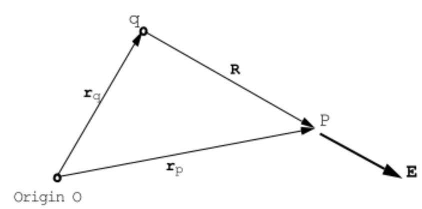

Figure 1.1.2: The fields generated by a point charge of q Coulombs at $\vec{r}_q$ that is stationary with respect to the observer located at $\vec{r}_P$. $\overrightarrow{\mathrm{R}}=\overrightarrow{\mathrm{r}_p}-\overrightarrow{r_q}$.

Equation (1.1.3) is called Coulomb's law. See Figure (1.1.2)

The electric field strength is measured in Volts/meter. The amplitude of the electric field decreases with distance from the charge like the square of the distance ie. $\sim\frac{1}{R^2}$ where the exponent is equal to two within 1 part in 10^{10}. The MKS system of units has been used to write eqn(1.1.3) in which the charge is measured in Coulombs. A current of 1 Amp`ere at some point in a circuit consists of an amount of charge equal to 1 Coulomb passing that point each second. Distances in (1.1.3) are measured in meters. The factor of proportionality is

$$\frac{1}{4\pi\epsilon_0}=10^{-7}\times c^2=8.987\times 10^9 \quad \text{meters / farad} \tag{1.1.4}$$

where c = 2.9979×10^8 m/sec is the velocity of light in vacuum. The size of

$$\frac{1}{4\pi\epsilon_0}$$

is purely the consequence of the historical definitions of the Volt and the Amp`ere. A second system of units which is very commonly used in the current magnetism literature is the CGS system in which distances are measured in centimeters, mass is measured in grams, and time is measured in seconds. In the CGS system the unit of charge, the statcoulomb, has been chosen to make Coulomb's law very simple. In the CGS system the field due to a stationary point charge is given by

$$\overrightarrow{\mathrm{E}}=q\left(\frac{\overrightarrow{\mathrm{R}}}{\mathrm{R}^3}\right) \quad \text{statvolts /cm} \quad \text{and} \quad \overrightarrow{\mathrm{B}}=0 \quad \text{Gauss.} \tag{1.1.5}$$

The price that is paid for the simplicity of Equation (1.1.5) is that the conventional engineering units for the current and potential, Amp`eres and Volts, cannot be used. The scaling factors between MKS and CGS electrical units involve the numerical value of the velocity of light, c. For example, in the CGS system the charge on a proton is $e_p = 4.803\times10^{-10}$ esu whereas in the MKS system it is $e_p = 1.602\times10^{-19}$ Coulombs. The ratio of these two numbers is

$$\frac{e_p|_{\text{esu}}}{e_p|_{MKS}} = 2.9979\times 10^9. \tag{1.1.6}$$

(10) The Fields generated by a Moving Charged Particle. Consider a co-ordinate system in which a spinless charged particle moves with respect to the observer with a constant velocity v which is much smaller than the speed of light in vacuum, c : ie. (v/c) << 1. The electric and magnetic fields generated by such a slowly moving charge are given by

$$\overrightarrow{\mathrm{E}}(\overrightarrow{\mathrm{R}},t) = \frac{q}{4\pi\epsilon_0}\left(\frac{\overrightarrow{\mathrm{R}}}{\mathrm{R}^3}\right)\quad V/m\quad \overrightarrow{\mathrm{B}}(\overrightarrow{\mathrm{R}},t) = \frac{1}{c^2}(\vec{v}\times\overrightarrow{\mathrm{E}})\quad \text{Webers}\ /m^2. \tag{1.1.7}$$

These expressions are correct to order $(v/c)^2$. $\vec{R}$ is the vector drawn from the position of the charged particle at the time of observation to the position of the observer. Note that the moving charge generates both an electric and a magnetic field. The above fields can be used to calculate the force on a particle q_2 located at $\vec{R}$:

$$\overrightarrow{\mathrm{F}_2}(\overrightarrow{\mathrm{R}},t) = q_2[\overrightarrow{\mathrm{E}}(\overrightarrow{\mathrm{R}},t) + (\overrightarrow{\mathrm{v}_2}(\overrightarrow{\mathrm{R}},t)\times\overrightarrow{\mathrm{B}}(\overrightarrow{\mathrm{R}},t))]\quad \text{Newtons.} \tag{1.1.8}$$

The particle q of Equations (1.1.7) generates the fields that exert forces on the particle q_2. Equations (1.1.7) are simplified versions of the general expressions for the electric and magnetic fields generated by a spinless point charge moving in an arbitrary fashion: see "The Feynman Lectures on Physics", Volume II, page 21-1 (R.P.Feynman, R.B.Leighton, and M.Sands, Addison-Wesley, Reading, Mass., 1964). These general expressions are

$$\overrightarrow{\mathrm{E}}(\overrightarrow{\mathrm{R}},t) = \frac{q}{4\pi\epsilon_0}\left[\left(\frac{\overrightarrow{\mathrm{R}}}{\mathrm{R}^3}\right) + \left(\frac{\mathrm{R}}{c}\right)\frac{d}{dt}\left(\frac{\overrightarrow{\mathrm{R}}}{\mathrm{R}^3}\right) + \frac{1}{\mathrm{c}^2}\frac{d^2}{dt^2}\left(\frac{\overrightarrow{\mathrm{R}}}{\mathrm{R}}\right)\right]\Bigg|_{Retarded} \tag{1.1.9}$$

$$c\overrightarrow{\mathrm{B}}(\overrightarrow{\mathrm{R}},t) = \frac{\overrightarrow{\mathrm{R}}}{\mathrm{R}}\Bigg|_{Retarded}\times\overrightarrow{\mathrm{E}}(\overrightarrow{\mathrm{R}},t). \tag{1.1.10}$$

The label "Retarded" refers to the retarded time $t_R = t - \frac{R}{c}$. The distances that appear in Equation (1.1.9) and Equation (1.1.10) are not evaluated at the time of observation, t, but at the earlier time, the retarded time, in order to take into account the finite speed of light. Any change in position requires the minimum time R/c to reach the observer, where c is the speed of light in vacuum. This corresponds to the requirement that changes in the motion of the particle can not be communicated to the observer faster than is permitted by the speed of light in vacuum, see Figure (1.1.3).

For a slowly moving particle, the first two terms of Equation (1.1.9) add together to give Coulomb's law in which the distance R is evaluated at the time of observation rather than at the retarded time; in other words, one can ignore time retardation if v/c is small. The last term in Equation (1.1.9) gives a field that is proportional to the component of acceleration perpendicular to the position vector $\vec{R}$ in the limit (v/c) << 1. This field decreases with distance like 1/R as opposed to the $1/R^2$ decrease of Coulomb's law. It is called the radiation field and is given by the expression

$$\overrightarrow{\mathrm{E}}_{rad} = \frac{q}{4\pi\epsilon_0}\frac{[\overrightarrow{\mathrm{a}}\times\overrightarrow{\mathrm{R}}]\times\overrightarrow{\mathrm{R}}}{\mathrm{c}^2R^3}\Bigg|_{t-\frac{R}{c}} \tag{1.1.11}$$

$$c\overrightarrow{\mathrm{B}} = \frac{\overrightarrow{\mathrm{R}}}{\mathrm{R}}\Bigg|_{t-\frac{R}{\epsilon}}\times\overrightarrow{\mathrm{E}}_{rad}, \tag{1.1.12}$$

where $\vec{a}$ is the acceleration of the charge.

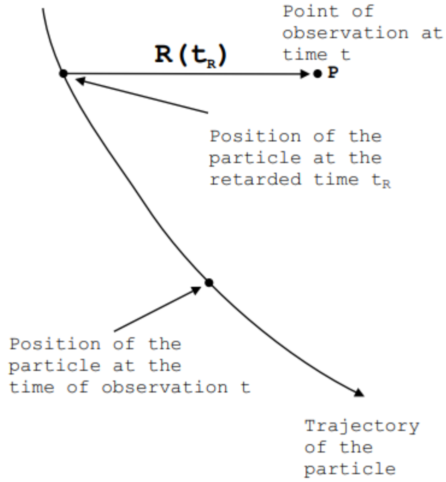

Figure 1.1.3: The electric and magnetic fields generated at the point of observation P at the time t depend upon the position, the velocity, and the acceleration of the charged particle at the retarded time $t_R = \left(t - \frac{R}{c}\right)$.