8.4: A Non-Sinusoidal Time Dependence

Nothing in the calculation of the radiation fields required the time variation of the dipole moment to be sinusoidal. If a charge undergoes an acceleration ~a at the retarded time $t_R = t - R/c$ then the Poynting vector at time t on a surface of radius R will have the radial component

$$S_r = \frac{1}{c\mu_0}\left(\frac{qa\sin\theta}{4\pi\epsilon_0 c^2 R}\right)^2,$$

(see Equations (7.4.5)). This expression can be written

$$S_r = \frac{c}{4\pi}\frac{1}{4\pi\epsilon_0}\left(\frac{q^2a^2\sin^2\theta}{c^4R^2}\right)_{tr}, \tag{8.4.1}$$

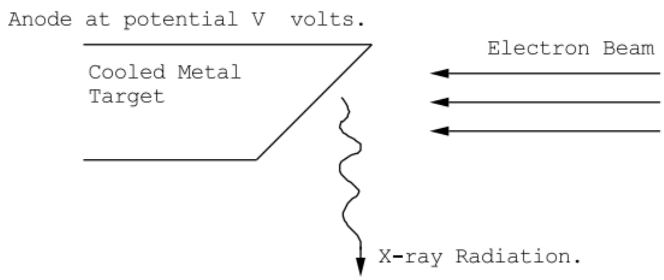

Figure 8.4.4: A schematic diagram of an X-ray tube illustrating the production of the white X-ray spectrum. The electrons undergo a de-acceleration upon striking the metal anode. This de-acceleration is of order a= 1024 m/sec^2 for a typical 20 keV potential drop between the anode and the cathode: this assumes an electron stopping distance of 35×10^{-10} m. During a brief period, $\sim 10^{-16}$ seconds, the electron radiates at the rate of $\sim 5.7\times10^{-6}$ Watts, therefore each electron emits a pulse of radiation containing $\sim 5.7\times10^{-22}$ Joules. The number of electrons that impinge on the anode per second for a beam of 1 mAmp is 6.25×10^{15}. The average power in the X-ray beam will be $(6.25\times10^{15})(5.7\times10^{-22}) = 3.6\times10^{-6}$ Watts. This energy is distributed over a range of frequencies from zero to 4.8×10^{18} Hz ($h\nu_{max} = |e|V$). This calculation does not include the energy contained in the characteristic X-ray spectrum emitted from the target.

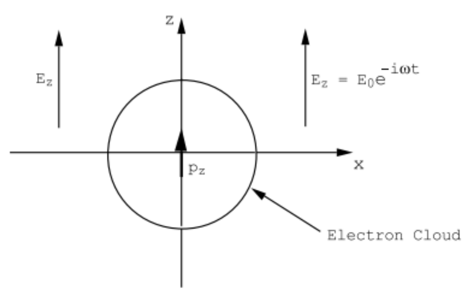

Figure 8.4.5: Schematic diagram of an atom in a time varying electric field. The atom develops a time varying dipole moment that scatters the incident radiation.

and the power integrated over a sphere of radius R is given by

$$P_q = \frac{2}{3}\frac{1}{4\pi\epsilon_0}\left(\frac{q^2a^2}{c^3}\right)_{t_R} \quad \text{Watts}, \tag{8.4.2}$$

where $a(t_R)$ means that the acceleration is measured at the retarded time $(t - R/c)$ if the power is measured at the time t. Eqn.(8.4.2) can be used to understand the production of the continuous X-ray spectrum, refer to Figure (8.4.4). The conversion efficiency for X-ray production is rather small; approximately 10^{-7} of the incident power is converted to continuous spectrum X-ray energy.