2.4: The Tangential Components of E

It follows from the first Maxwell equation, Equation (2.1.1) curl($\vec{E}$) = 0, that **the tangential components of the electric field vector must be continuous across any surface**. Consider a loop dL long and dw wide that spans a surface SS: the loop has one side in region (1) and the other side in region (2) as shown in Figure (2.4.5); the sides dw are chosen to be perpendicular to the surface SS. E_{t1} and E_{t2} are the electric field components parallel with the surface SS - the tangential electric field components. From Stokes' theorem, Section (1.3.4), one has

$$\iint_{Loop} dS(\hat{\mathbf{n}}\cdot \operatorname{curl}(\overrightarrow{\mathrm{E}})) = \oint_{Loop} \overrightarrow{\mathrm{E}}\cdot d\overrightarrow{\mathrm{L}}.$$

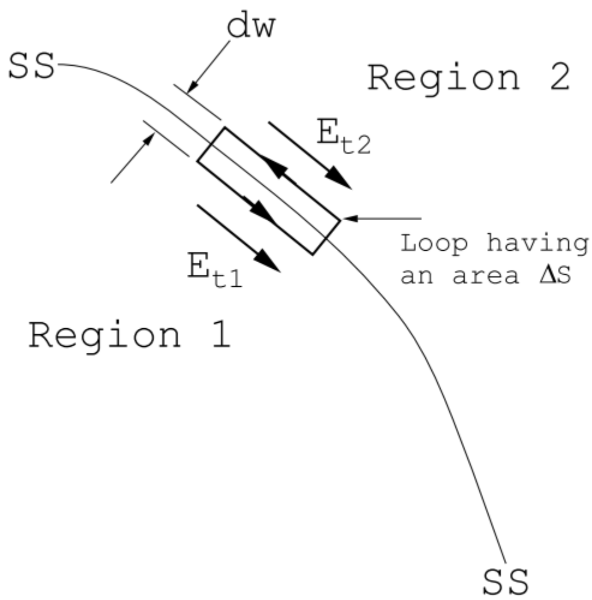

Figure 2.4.5: A rectangular loop having sides dL long and dw wide used for the application of Stokes' Theorem.

But curl($\vec{E}$) = 0, therefore the line integral must vanish:

$$\oint_{\mathrm{Loop}} \overrightarrow{\mathrm{E}}\cdot \overrightarrow{\mathrm{d}} = 0.$$

In calculating the line integral one can take the limit as dw becomes very small so that contributions from the electric field components parallel with dw and therefore normal to the surface can be made negligibly small. In this limit the line integral becomes

$$\oint_{Loop} \overrightarrow{\mathrm{E}}\cdot d\overrightarrow{\mathrm{L}} = E_{t1}dL - E_{t2}dL.$$

The negative sign arises because in Region(2) the loop is traversed in the direction opposite to the direction of E_{t2}. It follows from the fact that the line integral must vanish that

$$E_{t2} = E_{t1}, \tag{2.4.1}$$

or in other words the tangential components of $\vec{E}$ must be continuous across the surface SS. Since SS is an arbitrary surface it follows that the tangential components of the electric field must be continuous across **any surface**.