7.5: A Point Magnetic Dipole

Consider an oscillating magnetic dipole moment, mz, oriented along the z-axis and located at the origin of co-ordinates similar to the case of the oscillating electric dipole of Figure (7.4.3). If the dipole were static it would generate a vector potential having only a ϕ-component:

$$\mathrm{A}_{\phi}=\frac{\mu_{0}}{4 \pi} \frac{\mathrm{m}_{\mathrm{z}} \sin \theta}{\mathrm{R}^{2}}. \tag{7.5.1}$$

This follows from the general expression for the vector potential generated by a point dipole, Equation (4.3.4)

$$\vec{\mathrm{A}}=\frac{\mu_{0}}{4 \pi} \frac{(\vec{\mathrm{m}} \times \vec{\mathrm{R}})}{\mathrm{R}^{3}}.$$

However, it can be shown that due to the effects of time retardation the equation for the vector potential,(7.5.1) must be modified to read

$$\mathrm{A}_{\phi}=\frac{\mu_{0}}{4 \pi} \sin \theta\left[\frac{\mathrm{m}_{\mathrm{z}}}{\mathrm{R}^{2}}+\frac{\mathrm{m}_{\mathrm{z}}}{\mathrm{cR}}\right]. \tag{7.5.2}$$

The fields derived from this expression for the vector potential, $\vec{\mathrm{B}}=\operatorname{curl}(\vec{\mathrm{A}})$, are

$$\begin{gathered}
\mathrm{B}_{\mathrm{R}}=\frac{\mu_{0}}{4 \pi} 2 \cos \theta\left[\frac{\mathrm{m}_{\mathrm{z}}}{\mathrm{R}^{3}}+\frac{\dot{\mathrm{m}}_{\mathrm{z}}}{\mathrm{cR}^{2}}\right]_{\mathrm{t}_{\mathrm{R}}}, \\
\mathrm{B}_{\theta}=\frac{\mu_{0}}{4 \pi} \sin \theta\left[\frac{\mathrm{m}_{z}}{\mathrm{R}^{3}}+\frac{\dot{\mathrm{m}}_{z}}{\mathrm{cR}^{2}}+\frac{\ddot{\mathrm{m}}_{z}}{\mathrm{c}^{2} \mathrm{R}}\right]_{\mathrm{t}_{\mathrm{R}}}, \\
\mathrm{B}_{\phi}=0=\mathrm{E}_{\mathrm{R}}=\mathrm{E}_{\theta}, \\
\mathrm{E}_{\phi}=-\frac{\mu_{0}}{4 \pi} \sin \theta\left[\frac{\dot{\mathrm{m}}_{\mathrm{z}}}{\mathrm{R}^{2}}+\frac{\ddot{\mathrm{m}}_{\mathrm{z}}}{\mathrm{cR}}\right]_{\mathrm{t}_{\mathrm{R}}}.
\end{gathered} \tag{7.5.3}$$

where $\mathrm{t_R} = \mathrm{t} - \mathrm{R/c}$. Far from the dipole the radiation fields that decrease with distance like (1/R) are given by

$$\mathrm{B}_{\theta}=\frac{\mu_{0}}{4 \pi} \frac{\ddot{\mathrm{m}}_{z}}{\mathrm{c}^{2} \mathrm{R}} \sin \theta, \tag{7.5.4}$$

$$\mathrm{E}_{\phi}=-\frac{\mu_{0}}{4 \pi} \frac{\ddot{\mathrm{m}}_{z}}{\mathrm{cR}} \sin \theta=\mathrm{cB}_{\theta},$$

both evaluated at the retarded time $\mathrm{t_R}$. Just as for the electric dipole far fields $|\vec{\mathrm{E}}|=\mathrm{c}|\vec{\mathrm{B}}|$, and $\vec{E}$ and $\vec{B}$ are orthogonal to each other and to the line joining the position of the observer to the dipole.