10.2: Boundary Conditions

10.2.1 The Tangential Components of the Electric Field.

Apply Stokes' theorem to the Maxwell equation

$$\operatorname{curl}(\vec{E}) = -\frac{\partial \vec{B}}{\partial t}$$

and the small loop whose sides are L long and δ long as shown in Figure (10.1.2):

$$\oint \vec{E} \cdot d\overrightarrow{L} = -\frac{\partial}{\partial t} \int\int_{\text{Area}} \vec{B} \cdot d\vec{A}.$$

One then takes the limit as the sides δ shrink to zero. The line integral of the electric field gives

$$\oint \vec{E} \cdot d\vec{L} = (E_{t1} - E_{t2})\, L,$$

where E_{t1} is the field component parallel with L in material number 1 (vacuum in this case) and E_{t2} is the electric field component parallel with L in material number 2. The flux of the magnetic field through the loop goes to zero as δ goes to zero, therefore

$$(E_{t1} - E_{t2}) = 0$$

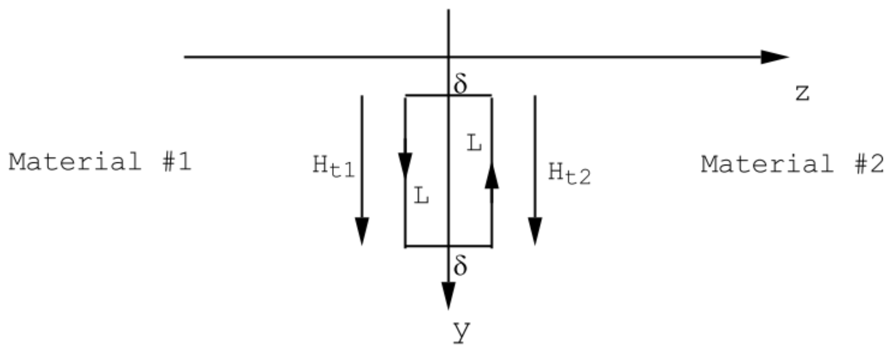

Figure 10.2.3: The Maxwell equation $\operatorname{curl}(\vec{H}) = -\partial\vec{D}/\partial t$ requires the tangential components of $\vec{H}$ to be continuous across any interface. See the text.

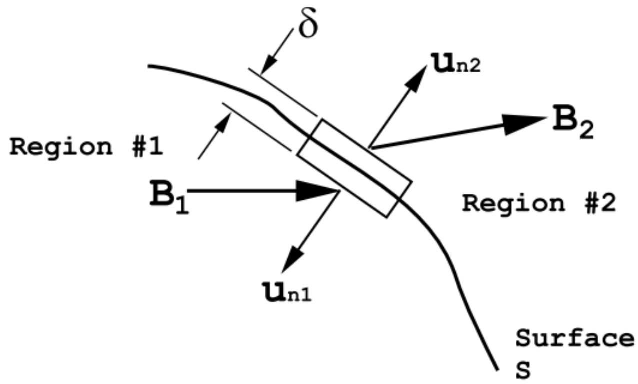

Figure 10.2.4: The Maxwell equation $\mathrm{div}(\vec{\mathrm{B}})=0$ requires the normal component of $\vec{B}$ to be continuous across any interface. See the text.

or

$$\mathrm{E_{t1}}=\mathrm{E_{t2}}. \tag{10.2.1}$$

At the boundary between two materials the transverse components of $\vec{E}$ must be continuous.

10.2.2 The Tangential Components of the Magnetic Field.

Apply Stokes' theorem to a small loop as shown in fig(10.2.3):

$$\mathrm{curl}(\vec{\mathrm{H}})=\frac{\partial\vec{\mathrm{D}}}{\partial \mathrm{t}},$$

where it has been assumed that there are no free currents in either material, and no surface free current density on the interface between material number(1) and material number(2). Therefore

$$\oint_C \vec{\mathrm{H}}\cdot d\vec{\mathrm{L}}=\frac{\partial}{\partial \mathrm{t}}\int\int_{Area}\vec{\mathrm{D}}\cdot d\overrightarrow{\mathrm{S}}.$$

Upon taking the limit as δ shrinks to zero the surface integral over $\vec{D}$ gives nothing and

$$(\mathrm{H_{t1}}-\mathrm{H_{t2}})\,\mathrm{L}=0,$$

that is

$$\mathrm{H_{t1}}=\mathrm{H_{t2}}. \tag{10.2.2}$$

The transverse components of the magnetic field $\vec{H}$ must be continuous across the boundary between two materials.

10.2.3 The Normal Component of the Field B.

The normal component of the magnetic field $\vec{B}$ must be continuous across **any** interface as a consequence of the Maxwell equation $div(\vec{\mathrm{B}})=0$; see Figure (10.2.4). In Figure (10.2.4) Gauss' theorem is applied to a small pill-box that spans an arbitrary surface. The height of the pill-box, δ, is taken to be so small that any contributions to the surface integral from the sides of the box can be neglected. The continuity of the normal component of $\vec{B}$ is then forced by the requirement that the surface integral of $\vec{B}$ over the pill-box be zero:

$$\mathrm{B_{n1}}=\mathrm{B_{n2}}. \tag{10.2.3}$$