5.7: The Maxwell Stress Tensor

In analogy with the electrostatic case, the forces due to the magnetic field acting on the current distribution in a body can be obtained from a magnetic Maxwell stress tensor, see J.A.Stratton, Electromagnetic Theory, section 2.5, (McGraw-Hill, N.Y., 1941). If the magnetic materials in the system are linear so that $\vec{B}$ is proportional to $\vec{H}$, it can be shown that there exists a vector $\vec{T}_{\mathrm{M}}$ associated with the elements of the stress tensor such that the surface integral of $\vec{T}_{\mathrm{M}}$ over a closed surface S gives the net force acting on the material in the volume V enclosed by the surface S: it is assumed that the surface S is contained entirely within a fluid that can support no shearing stresses. The magnetic force acting on the material within the volume V can be calculated from

$$\overrightarrow{\mathrm{F}}_{M}=\int\int_{S} \overrightarrow{\mathrm{T}}_{M} \cdot \overrightarrow{\mathrm{dS}}, \tag{5.7.1}$$

where the magnitude of the Maxwell stress vector for a linear, isotropic material, is

$$\left|\overrightarrow{\mathrm{T}}_{M}\right|=\frac{\overrightarrow{\mathrm{B}} \cdot \overrightarrow{\mathrm{H}}}{2}, \tag{5.7.2}$$

and its direction is given by the construction shown in Figure (5.6.14). The stress vector $\vec{T}_{\mathrm{M}}$ is turned away from the surface normal through an angle that is twice the angle that the magnetic field $\vec{B}$ (or $\vec{H}$) makes with the surface normal. When $\vec{B}$ lies along the surface normal the magnetic force is a tension, but when the field $\vec{B}$ lies in the surface the magnetic force is a pressure.