5.1: Introduction- Sources in a Uniform Permeable Material

The equations of magnetostatics are given by Equation (4.1.2)

$$\operatorname{div}(\overrightarrow{\mathrm{B}})=0,$$

and Equation (4.1.3)

$$\operatorname{curl}(\overrightarrow{\mathrm{B}})=\mu_0\left(\overrightarrow{\mathrm{J}}_f+\operatorname{curl}(\overrightarrow{\mathrm{M}})\right).$$

(refer to section(4.1)). For a linear, isotropic, magnetic medium $\vec{B}$ is proportional to $\vec{H}$ where the factor of proportionality is called the permeability.

$$\overrightarrow{\mathrm{B}}=\mu\overrightarrow{\mathrm{H}}=\mu_0(\overrightarrow{\mathrm{H}}+\overrightarrow{\mathrm{M}}),$$

so that

$$\overrightarrow{\mathrm{M}}=\left(\frac{\mu}{\mu_0}-1\right)\overrightarrow{\mathrm{H}},$$

or

$$\overrightarrow{\mathrm{M}}=(\mu_r-1)\overrightarrow{\mathrm{H}}.\tag{5.1.1}$$

In Equation (5.1.1) $\mu r = \mu/\mu 0$ is the relative permeability. The second of the above Maxwell's equations can be re-written in the form

$$\operatorname{curl}(\overrightarrow{\mathrm{H}})=\overrightarrow{\mathrm{J}}_f,$$

or

$$\operatorname{curl}(\overrightarrow{\mathrm{B}})=\mu\overrightarrow{\mathrm{J}}_f.\tag{5.1.2}$$

The substitution $\vec{B}$ = curl($\vec{A}$) ensures that Equation (4.1.2) will be satisfied since the divergence of any curl is zero. Using this substitution in Equation (5.1.2) gives

$$\operatorname{curlcurl}(\overrightarrow{\mathrm{A}})=\mu\overrightarrow{\mathrm{J}}_f.\tag{5.1.3}$$

If in addition one chooses

$$\operatorname{div}(\overrightarrow{\mathrm{A}})=0,\tag{5.1.4}$$

then

$$\nabla^2\overrightarrow{\mathrm{A}}=-\mu\overrightarrow{\mathrm{J}}_f,\tag{5.1.5}$$

and this equation has the particular solution

$$\overrightarrow{\mathrm{A}}(\overrightarrow{\mathrm{R}})=\frac{\mu}{4\pi}iiint_{Space}\mathrm{d}\tau\frac{\overrightarrow{\mathrm{J}}_f(\overrightarrow{\mathrm{r}})}{|\overrightarrow{\mathrm{R}}-\overrightarrow{\mathrm{r}}|},\tag{5.1.6}$$

where $\mathrm{d}\tau$ is an element of volume. This development exactly follows the procedure described in Chpt.(4); the only difference is that the integration in Equation (5.1.6) is carried out over the free current density distribution, and the fields due to the effective current density curl($\vec{M}$) are taken into account through the permeability μ that multiplies the integral. It should be noted that this procedure only works if μ does not depend upon position in space. If there are regions characterized by different values of μ the problem of calculating the magnetic field distribution becomes much more difficult. This is because at the boundaries between regions having different permeabilities there are discontinuities in the normal and tangential components of $\vec{M}$ that act as field sources.

In the usual situation the current density is zero except within a finite number of thin wires. For a current of I Amps carried in a wire of negligible cross-section Equation (5.1.6) becomes

$$\overrightarrow{\mathrm{A}}_P=\frac{\mu \mathrm{I}}{4 \pi} \int_{Wire} \frac{|\mathrm{d} \overrightarrow{\mathrm{L}}}{|\overrightarrow{\mathrm{r}}|}, \tag{5.1.7}$$

where $\vec{r}$ is the vector from the element of length d$\vec{L}$ to the point P where the vector potential $\vec{A}$ is to be calculated. From $\vec{B}$ = curl($\vec{A}$) one obtains

$$\overrightarrow{\mathrm{B}}(\overrightarrow{\mathrm{r}})=\frac{\mu \mathrm{I}}{4 \pi} \int_{Wire} \frac{\mathrm{d} \overrightarrow{\mathrm{L}} \times \overrightarrow{\mathrm{r}}}{|\overrightarrow{\mathrm{r}}|^3}. \tag{5.1.8}$$

These formulae are very similar to Equations (4.2.1) and (4.17) of Chpt.(4). The fields corresponding to the standard problems of a long straight wire, the field along the axis of a circular loop, and along the axis of a finite solenoid are given by Equations (4.3.3), (4.3.4), and (4.3.5) where the permeability of free space, μ_0, is replaced by the permeability μ. In particular, the field of an infinite solenoid that is filled with a magnetic material is given by

$$\overrightarrow{\mathrm{B}}=\mu \mathrm{NI}, \tag{5.1.9}$$

where N is the number of turns per meter.