11.7: The Slotted Line

A slotted line is a section of a co-axial cable that uses air as a dielectric medium and in which a narrow slot has been cut along the length of the outer conductor; the slot is sufficiently narrow so that its presence does not appreciably affect the electric field distribution between the conducting cylinders which form the walls of the cable. A thin pin is inserted through the slot and is used to pick up a signal that is a measure of the electric field strength in the co-axial cable. This pin is mounted on a carriage whose position along the slotted line can be accurately measured. A picture and a sketch of a slotted line can be found in Ginzton's book (see his Figures (5.11) and (5.12)). The signal picked up by the probe pin is usually rectified by means of a high frequency diode and it is the resulting dc signal that is measured. The dc signal provides a measure of the amplitude of the potential difference at any point along the slotted line. If the signal picked up is very small, less than ∼1 mV say, the dc signal provides a measure of the time averaged square of the potential difference between the outer and inner conductors; in many instances the signal picked up is greater than 10 mV, and in such cases the high frequency diodes commonly used for such measurements produce a dc signal that is proportional to the root mean square of the potential difference between the outer and inner conductors. If the slotted line is used to connect a generator operating at a fixed frequency with a load that is different from the characteristic impedance of the line, the rectified probe signal will be found to exhibit a sinusoidal variation between a maximum signal and a minimum signal as the probe is moved along the line. The ratio of the maximum signal to the minimum signal as the probe is moved along the slotted line provides a measure of the ratio of the forward wave amplitude to the reflected wave amplitude, i.e. the amplitudes a and b of Equation (11.6.2) that describes the position dependence of the voltage along the cable.

From Equation (11.6.2)

$$\mathrm{V(z,t) = a\exp(-\mathit{i}kz)\left(1+\frac{b}{a}\exp(2\mathit{i}kz)\right)\exp(\mathit{i}\omega t)},$$

where, from Equations (11.6.4)

$$\left(\frac{\mathrm{b}}{\mathrm{a}}\right)=\left(\frac{\frac{\mathrm{Z_L}}{\mathrm{Z_0}}-1}{\frac{\mathrm{Z_L}}{\mathrm{Z_0}}+1}\right)\exp(-2\mathit{i}\mathrm{kL}).$$

But by definition, Equation (11.6.5),

$$\Gamma=|\Gamma|\exp(i\theta)=\left(\frac{\frac{\mathrm{Z_L}}{\mathrm{Z_0}}-1}{\frac{\mathrm{Z_L}}{\mathrm{Z_0}}+1}\right),$$

or, in terms of the normalized impedance

$$\mathrm{z_L}=\frac{\mathrm{Z_L}}{\mathrm{Z_0}}$$

one has

$$\Gamma=\left(\frac{\mathrm{z_L}-1}{\mathrm{z_L}+1}\right)=|\Gamma|\exp(i\theta). \tag{11.7.1}$$

The ratio (b/a) can be written

$$\left(\frac{b}{a}\right)=|\Gamma|\exp(-i[2kL-\theta]),$$

from which one obtains

$$\mathrm{V(z,t)=a\exp(-\mathit{i}kz)(1+|\Gamma|\exp(\mathit{i}[2k(z-L)+\theta]))\exp(\mathit{i}\omega t)},$$

and

$$\mathrm{V^*(z,t)=a^*\exp(+\mathit{i}kz)(1+|\Gamma|\exp(-\mathit{i}[2k(z-L)+\theta]))\exp(-\mathit{i}\omega t)},$$

where V* (z,t) is the complex conjugate of the potential function V(z,t). The time averaged value of the square of the voltage is given by

$$<\mathrm{V}^2>=\frac{1}{2}\mathrm{Real}(\mathrm{VV}^*),$$

or

$$<\mathrm{V}^2>=\frac{|\mathrm{a}|^2}{2}\left(1+|\Gamma|^2+2|\Gamma|\cos(2\mathrm{k}[\mathrm{z}-\mathrm{L}]+\theta)\right). \tag{11.7.2}$$

The maximum value of $<\mathrm{V}^2>$, or of $\sqrt{<V^2>}$, occurs at those positions z such that (2k[z − L] + θ) = 2πn where n is an integer. These maxima are spaced a half-wavelength apart; the variation of the pick-up signal as the probe is moved along the slotted line provides a direct measure of the wavelength of the radiation. The maximum value of the root mean square potential difference is

$$(\sqrt{<\mathrm{V}^2>})_{\max}=\frac{|\mathrm{a}|}{\sqrt{2}}(1+|\Gamma|).$$

The minimum value of the pick-up signal occurs at those positions such that (2k[z − L]+θ) = πm where m is an odd integer: these minima are also spaced one half-wavelength apart. At a minimum the root mean square voltage is

$$(\sqrt{<\mathrm{V}^2>})_{\min}=\frac{|\mathrm{a}|}{\sqrt{2}}(1-|\Gamma|).$$

The ratio of these two voltages is called the ”Voltage Standing Wave Ratio” and is usually designated by VSWR:

$$\mathrm{VSWR}=\frac{1+|\Gamma|}{1-|\Gamma|}. \tag{11.7.3}$$

This expression can be inverted to give

$$|\Gamma|=\left(\frac{\mathrm{VSWR}-1}{\mathrm{VSWR}+1}\right). \tag{11.7.4}$$

The Voltage Standing Wave Ratio, which can be easily measured by means of a slotted line, provides information about the load impedance through the absolute value of the parameter Γ = (z_L − 1)/(z_L +1). In order to determine the phase of the load impedance it is also necessary to measure the phase of the parameter Γ: from the definition of Γ

$$\mathrm{z}_{\mathrm{L}}=\frac{\mathrm{Z}_{\mathrm{L}}}{\mathrm{Z}_0}=\left(\frac{1+\Gamma}{1-\Gamma}\right). \tag{11.7.5}$$

Thus a knowledge of the amplitude and phase of Γ serves to determine the amplitude and phase of the load impedance, Z_L. The phase of Γ can be obtained from the position of the voltage maximum or minimum on the slotted line; it is preferable to use the position of the minimum because the position of a minimum signal can be measured much more accurately than the position of a maximum signal.

The structure of the relationship between the complex number Γ and the complex load impedance, z_L = Z_L/Z_0, is such that for a load impedance

having an inductive component the phase angle θ must lie between 0 and π radians, whereas for a load impedance having a capacitive component the phase angle θ must lie between 0 and $-\pi$ radians. As an example consider a purely inductive load such that z_L = +iβ. For this case

$$\Gamma=\frac{-1+i\beta}{1+i\beta}.$$

The numerator of Γ may be written

$$N=\sqrt{1+\beta^2}\exp(i[\pi-\phi]),$$

where tan ϕ = β. The denominator of Γ may be written

$$D=\sqrt{1+\beta^2}\exp(i\phi),$$

where, as above, tan ϕ = β. Thus for this example

$$\Gamma=\exp(i[\pi-2\phi]),$$

where for β → 0 θ → ϕ, and for β very large θ → 0. This case is a special one but using complex algebra it can be shown that θ must lie between 0 and ϕ radians for any load having an inductive component. Consider the position of a minimum in the slotted line voltage corresponding to an inductive load. From Equation (11.7.2) one of the minima occurs at position z when

$$2\mathrm{k}(\mathrm{z}-\mathrm{L})+\theta=\pi,$$

or since k = 2ϕ/λ, the minimum occurs at

$$\mathrm{z}=\mathrm{L}+\frac{(\pi-\theta)}{4\pi}\lambda.$$

The position of this particular minimum ranges from z=L for θ = π to z = L + (λ/4) for θ = 0. Clearly one cannot measure the position of this minimum because it lies outside the slotted line (z>L). However, the pattern described by Equation (11.7.2) repeats itself every half wavelength along the slotted line. Therefore one has only to measure the position of the voltage minimum relative to a position, z_2, located exactly one half wavelength from the end of the slotted line. This position, z_2, can be found simply by locating the position of the appropriate minimum signal when the load impedance is replaced by a short circuit. The condition for the position of a minimum voltage with the load in place can then be written

$$2\mathrm{k}\left(\mathrm{z}-\mathrm{z}_{2}\right)+\theta=\pi$$

and therefore

$$\theta=\pi-\frac{4\pi}{\lambda}\left(\mathrm{z}-\mathrm{z}_{2}\right), \tag{11.7.6}$$

where z > z_2 for a load having an inductive component.

For a load having a capacitive component consider the minimum corresponding to

$$2\mathrm{k}(\mathrm{z}-\mathrm{L})+\theta=-\pi.$$

This minimum in the standing wave voltage occurs at

$$\mathrm{z}=\mathrm{L}-\frac{(\pi+\theta)}{4\pi}\lambda.$$

Since θ lies between 0 and −π radians for this case the position of the minimum varies from z = L − λ/4 to z=L. In other words the position of the minimum is shifted towards the generator. This minimum is accessible on the slotted line but it is more convenient to measure the position of that particular minimum that is located near z_2, the position that is a half wavelength removed from the end of the slotted line. The condition that describes the position of the minimum that lies within λ/4 of z_2 is given by

$$2\mathrm{k}\left(\mathrm{z}-\mathrm{z}_{2}\right)+\theta=-\pi,$$

and therefore

$$\theta=-\pi+\frac{4\pi}{\lambda}\left(\mathrm{z}_{2}-\mathrm{z}\right), \tag{11.7.7}$$

where for an impedance having a capacitive component z_2 > z.

Recapitulation

In order to determine an unknown impedance using a slotted line one must obtain the amplitude of the parameter Γ from the voltage standing wave ratio, Equation (11.7.4), as well as the phase of Γ from the position of a voltage minimum on the slotted line. The phase of Γ, θ, can be determined from the position of the minimum, z, relative to a position, z_2, located exactly λ/2 from the end of the slotted line. The position of z_2 is determined by the position of the appropriate minimum when the slotted line is terminated with a short circuit. With the slotted line terminated by the unknown impedance one looks for a voltage minimum located within λ/4 of the shorted position z_2. If the position of this minimum is displaced from z_2 towards the **load** then that impedance has an inductive component and the phase angle θ is to be calculated using Equation (11.7.6). If the position of the minimum is displaced from z_2 towards the generator then the load has a capacitive component and the phase angle θ is to be calculated using Equation (11.7.7).