5.5: Inductance Coefficients

Consider N circuits embedded in a linear, isotropic medium characterized by a permeability μ. The magnetic flux through a given circuit will depend upon the currents in all of the circuits. However, the magnetic field generated by the current in a particular circuit will be a linear function of the current in that circuit; if the current is doubled then the magnetic field due to that current will also be doubled because Maxwell's equations are linear in the current density. Since the magnetic field at any point is a linear function of the currents it follows that the flux through each circuit must be a linear function of the currents: i.e.

$$\begin{aligned}\Phi_1 &= \mathrm{L}_{11}\mathrm{I}_1 + \mathrm{L}_{12}\mathrm{I}_2 + \cdots + \mathrm{L}_{1\mathrm{N}}\mathrm{I}_\mathrm{N} \\ \Phi_2 &= \mathrm{L}_{21}\mathrm{I}_1 + \mathrm{L}_{22}\mathrm{I}_2 + \cdots + \mathrm{L}_{2\mathrm{N}}\mathrm{I}_\mathrm{N} \\ &\vdots \\ \Phi_N &= \mathrm{L}_{\mathrm{N}1}\mathrm{I}_1 + \mathrm{L}_{\mathrm{N}2}\mathrm{I}_2 + \cdots + \mathrm{L}_{\mathrm{NN}}\mathrm{I}_\mathrm{N}\end{aligned} .$$

The coefficients L_{MN} are called **coefficients of induction**. They have units of Henries.

The magnetostatic energy, Equation (5.4.9), can be written in terms of the current in each circuit and the induction coefficients. The magnetic energy must be independent of the order in which the circuit currents attains their final values. This condition requires that

$$\mathrm{L}_{\mathrm{MN}} = \mathrm{L}_{\mathrm{NM}}. \tag{5.5.1}$$

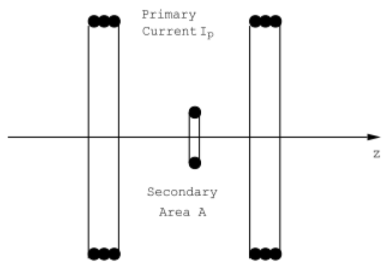

Figure 5.5.8: A system consisting of a primary coil carrying a current I_p and a secondary coil consisting of a single turn of wire enclosing an area A m^2.

There are, therefore, only N(N+1)/2 independent induction coefficients rather than N^2 of them. This symmetry property of the induction coefficients can be used to determine the flux produced in a coil system by a magnetized body. Consider a primary coil system carrying a current I_p. Let there be a small secondary coil as shown in Figure (5.5.8). The magnetic field produced by the primary coil at the position of the secondary coil is

$$\mathrm{B}_\mathrm{z} = \mathrm{KI}_\mathrm{p} \quad \text{Teslas} ,$$

K is just a constant that depends upon the geometry of the primary coil. If the area of the secondary coil is A, supposed to be very small, the flux through the secondary coil due to the primary current is given by

$$\Phi_s = \mathrm{B}_z\mathrm{A} = \mathrm{KAI}_\mathrm{p}, \tag{5.5.2}$$

and therefore the relevant inductance coefficient, L_{sp}, is

$$\mathrm{L}_{\mathrm{sp}} = \mathrm{KA}. \tag{5.5.3}$$

But this means that the flux through the primary coil system due to a current I_s in the secondary coil is given by

$$\Phi_p = \mathrm{L}_{\mathrm{pr}}\mathrm{I}_\mathrm{s} = \mathrm{L}_{\mathrm{xp}}\mathrm{I}_\mathrm{z} = \mathrm{KAI}_\mathrm{s}.$$

The small secondary coil carrying a current I_s constitutes a magnetic moment $m_z = I_s A \; Amp - m^2$. It follows that a magnetic dipole m_z produces a flux through a primary coil system given by

$$\Phi_p = \mathrm{Km}_z, \tag{5.5.4}$$

where the coil system produces the field $B_z = KI_p$ Teslas at the position of the magnetic moment. If the field produced by the coil system is uniform over a magnetic body it follows from the principle of superposition that the flux produced in the coil system by the body is proportional to its total magnetic moment. In particular, the flux through an infinite solenoid produced by a magnetic dipole, $\vec{m}$, oriented along the solenoid axis is given by

$$\Phi=\mu \mathrm{Nm}, \tag{5.5.5}$$

if the system is immersed in a medium whose permeability is μ. In most applications μ can be taken to be the permeability of free space, μ_o.