

## 5.3: Charge Distributions

In principle, the smallest unit of electric charge that can be isolated is the charge of a single electron, which is  $\cong -1.60 \times 10^{-19}$  C. This is very small, and we rarely deal with electrons one at a time, so it is usually more convenient to describe charge as a quantity that is continuous over some region of space. In particular, it is convenient to describe charge as being distributed in one of three ways: along a curve, over a surface, or within a volume.

### Line Charge Distribution

Imagine that charge is distributed along a curve  $\mathcal{C}$  through space. Let  $\Delta q$  be the total charge along a short segment of the curve, and let  $\Delta l$  be the length of this segment. The *line charge density*  $\rho_l$  at any point along the curve is defined as

$$\rho_l \triangleq \lim_{\Delta l \rightarrow 0} \frac{\Delta q}{\Delta l} = \frac{dq}{dl}$$

which has units of C/m. We may then define  $\rho_l$  to be a function of position along the curve, parameterized by  $l$ ; e.g.,  $\rho_l(l)$ . Then, the total charge  $Q$  along the curve is

$$Q = \int_{\mathcal{C}} \rho_l(l) dl$$

which has units of C. In other words, line charge density integrated over length yields total charge.

### Surface Charge Distribution

Imagine that charge is distributed over a surface. Let  $\Delta q$  be the total charge on a small patch on this surface, and let  $\Delta s$  be the area of this patch. The *surface charge density*  $\rho_s$  at any point on the surface is defined as

$$\rho_s \triangleq \lim_{\Delta s \rightarrow 0} \frac{\Delta q}{\Delta s} = \frac{dq}{ds}$$

which has units of C/m<sup>2</sup>. Let us define  $\rho_s$  to be a function of position on this surface. Then the total charge over a surface  $\mathcal{S}$  is

$$Q = \int_{\mathcal{S}} \rho_s ds$$

In other words, surface charge density integrated over a surface yields total charge.

### Volume Charge Distribution

Imagine that charge is distributed over a volume. Let  $\Delta q$  be the total charge in a small cell within this volume, and let  $\Delta v$  be the volume of this cell. The *volume charge density*  $\rho_v$  at any point in the volume is defined as

$$\rho_v \triangleq \lim_{\Delta v \rightarrow 0} \frac{\Delta q}{\Delta v} = \frac{dq}{dv}$$

which has units of C/m<sup>3</sup>. Since  $\rho_v$  is a function of position within this volume, the total charge within a volume  $\mathcal{V}$  is

$$Q = \int_{\mathcal{V}} \rho_v dv$$

In other words, volume charge density integrated over a volume yields total charge.

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