

5.13: Electric Potential Field due to a Continuous Distribution of Charge

The electrostatic potential field at \mathbf{r} associated with N charged particles is

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon} \sum_{n=1}^N \frac{q_n}{|\mathbf{r} - \mathbf{r}_n|} \quad (5.13.1)$$

where q_n and \mathbf{r}_n are the charge and position of the n^{th} particle. However, it is more common to have a continuous distribution of charge as opposed to a countable number of charged particles. We now consider how to compute $V(\mathbf{r})$ three types of these commonly-encountered distributions. Before beginning, it's worth noting that the methods will be essentially the same, from a mathematical viewpoint, as those developed in Section 5.4; therefore, a review of that section may be helpful before attempting this section.

Continuous Distribution of Charge Along a Curve

Consider a continuous distribution of charge along a curve \mathcal{C} . The curve can be divided into short segments of length Δl . Then, the charge associated with the n^{th} segment, located at \mathbf{r}_n , is

$$q_n = \rho_l(\mathbf{r}_n) \Delta l$$

where ρ_l is the line charge density (units of C/m) at \mathbf{r}_n . Substituting this expression into Equation 5.13.1, we obtain

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon} \sum_{n=1}^N \frac{\rho_l(\mathbf{r}_n)}{|\mathbf{r} - \mathbf{r}_n|} \Delta l$$

Taking the limit as $\Delta l \rightarrow 0$ yields:

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon} \int_{\mathcal{C}} \frac{\rho_l(l)}{|\mathbf{r} - \mathbf{r}'|} dl \quad (5.13.2)$$

where \mathbf{r}' represents the varying position along \mathcal{C} with integration along the length l .

Continuous Distribution of Charge Over a Surface

Consider a continuous distribution of charge over a surface \mathcal{S} . The surface can be divided into small patches having area Δs . Then, the charge associated with the n^{th} patch, located at \mathbf{r}_n , is

$$q_n = \rho_s(\mathbf{r}_n) \Delta s$$

where ρ_s is surface charge density (units of C/m²) at \mathbf{r}_n . Substituting this expression into Equation 5.13.1, we obtain

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon} \sum_{n=1}^N \frac{\rho_s(\mathbf{r}_n)}{|\mathbf{r} - \mathbf{r}_n|} \Delta s$$

Taking the limit as $\Delta s \rightarrow 0$ yields:

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon} \int_{\mathcal{S}} \frac{\rho_s(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} ds \quad (5.13.3)$$

where \mathbf{r}' represents the varying position over \mathcal{S} with integration.

Continuous Distribution of Charge in a Volume

Consider a continuous distribution of charge within a volume \mathcal{V} . The volume can be divided into small cells (volume elements) having area Δv . Then, the charge associated with the n^{th} cell, located at \mathbf{r}_n , is

$$q_n = \rho_v(\mathbf{r}_n) \Delta v$$

where ρ_v is the volume charge density (units of C/m³) at \mathbf{r}_n . Substituting this expression into Equation 5.13.1, we obtain

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon} \sum_{n=1}^N \frac{\rho_v(\mathbf{r}_n)}{|\mathbf{r} - \mathbf{r}_n|} \Delta v$$

Taking the limit as $\Delta v \rightarrow 0$ yields:

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon} \int_{\mathcal{V}} \frac{\rho_v(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} dv$$

where \mathbf{r}' represents the varying position over \mathcal{V} with integration.

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