

## 10.6: Mathematical Formulas - Vector Identities

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### Algebraic Identities

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B}) \quad (10.6.1)$$

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B}) \quad (10.6.2)$$

### Identities Involving Differential Operators

$$\nabla \cdot (\nabla \times \mathbf{A}) = 0 \quad (10.6.3)$$

$$\nabla \times (\nabla f) = 0 \quad (10.6.4)$$

$$\nabla \times (f\mathbf{A}) = f(\nabla \times \mathbf{A}) + (\nabla f) \times \mathbf{A} \quad (10.6.5)$$

$$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B}) \quad (10.6.6)$$

$$\nabla \cdot (\nabla f) = \nabla^2 f \quad (10.6.7)$$

$$\nabla \times \nabla \times \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} \quad (10.6.8)$$

$$\nabla^2 \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla \times (\nabla \times \mathbf{A}) \quad (10.6.9)$$

### Divergence Theorem

Given a closed surface  $\mathcal{S}$  enclosing a contiguous volume  $\mathcal{V}$ ,

$$\int_{\mathcal{V}} (\nabla \cdot \mathbf{A}) dv = \oint_{\mathcal{S}} \mathbf{A} \cdot d\mathbf{s}$$

where the surface normal  $d\mathbf{s}$  is pointing out of the volume.

### Stokes' Theorem

Given a closed curve  $\mathcal{C}$  bounding a contiguous surface  $\mathcal{S}$ ,

$$\int_{\mathcal{S}} (\nabla \times \mathbf{A}) \cdot d\mathbf{s} = \oint_{\mathcal{C}} \mathbf{A} \cdot d\mathbf{l}$$

where the direction of the surface normal  $d\mathbf{s}$  is related to the direction of integration along  $\mathcal{C}$  by the “right hand rule.”

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