# 5.9: Independence of Path

In Section 5.8, we found that the potential difference (“voltage”) associated with a path $\mathcal{C}$ in an electric field intensity $\mathbf{E}$ is

$$V_{21} = -\int_{\mathcal{C}} \mathbf{E} \cdot d\mathbf{l}$$

where the curve begins at point 1 and ends at point 2. Let these points be identified using the position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$, respectively (see Section 4.1). Then:

$$V_{21} = -\int_{\mathbf{r}_1,\ \text{along}\ \mathcal{C}}^{\mathbf{r}_2} \mathbf{E} \cdot d\mathbf{l}$$

The associated work done by a particle bearing charge $q$ is

$$W_{21} = qV_{21}$$

This work represents the change in potential energy of the system consisting of the electric field and the charged particle. So, it must also be true that

$$W_{21} = W_2 - W_1$$

where $W_2$ and $W_1$ are the potential energies when the particle is at $\mathbf{r}_2$ and $\mathbf{r}_1$, respectively. It is clear from the above equation that $W_{21}$ does not depend on $\mathcal{C}$; it depends only on the positions of the start and end points and not on any of the intermediate points along $\mathcal{C}$. That is,

$$\boxed{V_{21} = -\int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{E} \cdot d\mathbf{l} \quad \text{, independent of } \mathcal{C}} \tag{5.9.1}$$

Since the result of the integration in Equation 5.9.1 is independent of the path of integration, *any* path that begins at $\mathbf{r}_1$ and ends at $\mathbf{r}_2$ yields the same value of $W_{21}$ and $V_{21}$. We refer to this concept as *independence of path*.

> The integral of the electric field over a path between two points depends only on the locations of the start and end points and is independent of the path taken between those points.

A practical application of this concept is that some paths may be easier to use than others, so there may be an advantage in computing the integral in Equation 5.9.1 using some path other than the path actually traversed.