3.4: Lumped-Element Model

It is possible to ascertain the relevant behaviors of a transmission line using elementary circuit theory applied to a differential-length lumped-element model of the transmission line. The concept is illustrated in Figure $3.4.1$, which shows a generic transmission line aligned with its length along the z axis. The transmission line is divided into segments having small but finite length Δz. Each segment is modeled as an identical two-port having the equivalent circuit representation shown in Figure $3.4.2$. The equivalent circuit consists of 4 components as follows:

- The resistance $R'\Delta z$ represents the series-combined ohmic resistance of the two conductors. This should account for *both* conductors since the current in the actual transmission line must flow through both conductors. The prime notation reminds us that R' is resistance *per unit length*; i.e., Ω/m, and it is only after multiplying by length that we get a resistance in Ω.
- The conductance $G'\Delta z$ represents the leakage of current directly from one conductor to the other. When $G'\Delta z > 0$, the resistance between the conductors is less than infinite, and therefore, current may flow between the conductors. This amounts to a loss of power separate from the loss associated with R' above. G' has units of S/m. Further note that G' is *not* equal to $1/R'$ as defined above. G' and R' are describing entirely different physical mechanisms (and in principle *either* could be defined as either a resistance or a conductance).
- The capacitance $C'\Delta z$ represents the capacitance of the transmission line structure. Capacitance is the tendency to store energy in electric fields and depends on the cross-sectional geometry and the media separating the conductors. C' has units of F/m.
- The inductance $L'\Delta z$ represents the inductance of the transmission line structure. Inductance is the tendency to store energy in magnetic fields, and (like capacitance) depends on the cross-sectional geometry and the media separating the conductors. L' has units of H/m.

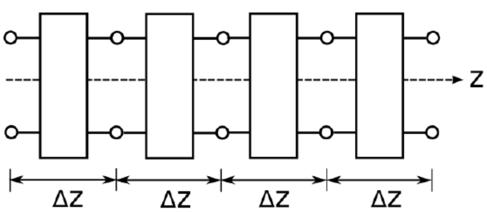

Figure $3.4.1$: Interpretation of a transmission line as a cascade of discrete series-connected two-ports.

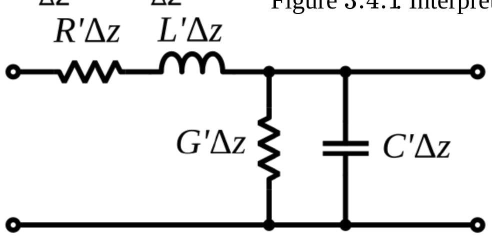

Figure $3.4.1$: Lumped-element equivalent circuit model for each of the two-ports in Figure $3.4.2$ (CC BY SA 3.0 Unported (modified))

In order to use the model, one must have values for R', G', C', and L'. Methods for computing these parameters are addressed elsewhere in this book.