6.1: Introduction to Circular Motion

We shall now investigate a special class of motions, motion in a plane about a central point, a motion we shall refer to as central motion, the most outstanding case of which is circular motion. Special cases often dominate our study of physics, and circular motion about a central point is certainly no exception. There are many instances of central motion about a point; a bicycle rider on a circular track, a ball spun around by a string, and the rotation of a spinning wheel are just a few examples. Various planetary models described the motion of planets in circles before any understanding of gravitation. The motion of the moon around the earth is nearly circular. The motions of the planets around the sun are nearly circular. Our sun moves in nearly a circular orbit about the center of our galaxy, 50,000 light years from a massive black hole at the center of the galaxy. When Newton solved the two-body under a gravitational central force, he discovered that the orbits can be circular, elliptical, parabolic or hyperbolic. All of these orbits still display central force motion about the center of mass of the two-body system. Another example of central force motion is the scattering of particles by a Coulombic central force, for example Rutherford scattering of an alpha particle (two protons and two neutrons bound together into a particle identical to a helium nucleus) against an atomic nucleus such as a gold nucleus.

We shall begin by describing the kinematics of circular motion, the position, velocity, and acceleration, as a special case of two-dimensional motion. We will see that unlike linear motion, where velocity and acceleration are directed along the line of motion, in circular motion the direction of velocity is always tangent to the circle. This means that as the object moves in a circle, the direction of the velocity is always changing. When we examine this motion, we shall see that the direction of the change of the velocity is towards the center of the circle. This means that there is a non-zero component of the acceleration directed radially inward, which is called the centripetal acceleration. If our object is increasing its speed or slowing down, there is also a non-zero tangential acceleration in the direction of motion. But when the object is moving at a constant speed in a circle then only the centripetal acceleration is non-zero.

In 1666, twenty years before Newton published his Principia, he realized that the moon is always “falling” towards the center of the earth; otherwise, by the First Law, it would continue in some linear trajectory rather than follow a circular orbit. Therefore there must be a centripetal force, a radial force pointing inward, producing this centripetal acceleration.

In all of these instances, when an object is constrained to move in a circle, there must exist a force $\overrightarrow{\mathbf{F}}$ acting on the object directed towards the center. Because Newton’s Second Law is a vector equality, the radial component of the Second Law is

$$F_r = ma_r$$