19.5: Angular Impulse and Change in Angular Momentum

If there is a total applied torque $\vec{\tau}_S$ about a point S over an interval of time $\Delta t = t_f - t_i$, then the torque applies an angular impulse about a point S, given by

$$\overrightarrow{\mathbf{J}}_S = \int_{t_i}^{t_f} \vec{\tau}_S dt$$

Because $\vec{\tau}_S = d\overrightarrow{\mathbf{L}}_S^{\text{total}} / dt$ the angular impulse about S is equal to the change in angular momentum about S,

$$\overrightarrow{\mathbf{J}}_S = \int_{t_i}^{t_f} \vec{\tau}_S dt = \int_{t_i}^{t_f} \frac{d\overrightarrow{\mathbf{L}}_S}{dt} dt = \Delta \overrightarrow{\mathbf{L}}_S = \overrightarrow{\mathbf{L}}_{S,f} - \overrightarrow{\mathbf{L}}_{S,i}$$

This result is the rotational analog to linear impulse, which is equal to the change in momentum,

$$\overrightarrow{\mathbf{I}} = \int_{t_i}^{t_f} \overrightarrow{\mathbf{F}} dt = \int_{t_i}^{t_f} \frac{d\overrightarrow{\mathbf{p}}}{dt} dt = \Delta \overrightarrow{\mathbf{p}} = \overrightarrow{\mathbf{p}}_f - \overrightarrow{\mathbf{p}}_i$$