25.1: Introduction- The Kepler Problem

...and if you want the exact moment in time, it was conceived mentally on 8th March in this year one thousand six hundred and eighteen, but submitted to calculation in an unlucky way, and therefore rejected as false, and finally returning on the 15th of May and adopting a new line of attack, stormed the darkness of my mind. So strong was the support from the combination of my labour of seventeen years on the observations of Brahe and the present study, which conspired together, that at first I believed I was dreaming, and assuming my conclusion among my basic premises. But it is absolutely certain and exact that "the proportion between the periodic times of any two planets is precisely the sesquialterate proportion of their mean distances ..."

~ Johannes Kepler

Johannes Kepler first formulated the laws that describe planetary motion,

I. Each planet moves in an ellipse with the sun at one focus.

II. The radius vector from the sun to a planet sweeps out equal areas in equal time.

III. The period of revolution T of a planet about the sun is related to the semi-major axis a of the ellipse by T 2 = k a 3 where k is the same for all planets.

The third law was published in 1619, and efforts to discover and solve the equation of motion of the planets generated two hundred years of mathematical and scientific discovery. In his honor, this problem has been named the Kepler Problem.

When there are more than two bodies, the problem becomes impossible to solve exactly. The most important “three-body problem” in the 17th and 18th centuries involved finding the motion of the moon, due to gravitational interaction with both the sun and the earth. Newton realized that if the exact position of the moon were known, the longitude of any observer on the earth could be determined by measuring the moon’s position with respect to the stars.

In the eighteenth century, Leonhard Euler and other mathematicians spent many years trying to solve the three-body problem, and they raised a deeper question. Do the small contributions from the gravitational interactions of all the planets make the planetary system unstable over long periods of time? At the end of 18th century, Pierre Simon Laplace and others found a series solution to this stability question, but it was unknown whether or not the series solution converged after a long period of time. Henri Poincaré proved that the series actually diverged. Poincaré went on to invent new mathematical methods that produced the modern fields of differential geometry and topology in order to answer the stability question using geometric arguments, rather than analytic methods. Poincaré and others did manage to show that the three-body problem was indeed stable, due to the existence of periodic solutions. Just as in the time of Newton and Leibniz and the invention of calculus, unsolved problems in celestial mechanics became the experimental laboratory for the discovery of new mathematics.