9.1: Introduction Newton's Second Law and Circular Motion

I shall now recall to mind that the motion of the heavenly bodies is circular, since the motion appropriate to a sphere is rotation in a circle.

Nicholas Copernicus

We have already shown that when an object moves in a circular orbit of radius r with angular velocity $\overrightarrow{\boldsymbol{\omega}}$ it is most convenient to choose polar coordinates to describe the position, velocity and acceleration vectors. In particular, the acceleration vector is given by

$$\overrightarrow{\mathbf{a}}(t)=-r\left(\frac{d\theta}{dt}\right)^{2}\hat{\mathbf{r}}(t)+r\frac{d^{2}\theta}{dt^{2}}\hat{\boldsymbol{\theta}}(t)$$

Then Newton's Second Law, $\overrightarrow{\mathbf{F}}=m\overrightarrow{\mathbf{a}}$ can be decomposed into radial $(\hat{\mathbf{r}}-)$ and tangential $(\hat{\boldsymbol{\theta}}-)$ components

$$F_{r}=-mr\left(\frac{d\theta}{dt}\right)^{2}(\text{ circular motion })$$

$$F_{\theta}=mr\frac{d^{2}\theta}{dt^{2}}\quad(\text{ circular motion })$$

For the special case of uniform circular motion, $d^{2}\theta/dt^{2}=0$, and so the sum of the tangential components of the force acting on the object must therefore be zero,

$$F_{\theta}=0$$