21.4: Translation and Rotation of a Rigid Body Undergoing Fixed Axis Rotation

For the special case of rigid body of mass m, we showed that with respect to a reference frame in which the center of mass of the rigid body is moving with velocity $\overrightarrow{\mathbf{V}}_{cm}$ all elements of the rigid body are rotating about the center of mass with the same angular velocity $\overrightarrow{\boldsymbol{\omega}}_{\mathrm{cm}}$ For the rigid body of mass m and momentum $\overrightarrow{\mathbf{p}} = m\overrightarrow{\mathbf{V}}_{cm}$ the translational equation of motion is still given by Equation (21.2.1), which we repeat in the form

$$\overrightarrow{\mathbf{F}}^{\mathrm{ext}} = m\overrightarrow{\mathbf{A}}_{\mathrm{cm}}$$

For fixed axis rotation, choose the z -axis as the axis of rotation that passes through the center of mass of the rigid body. We have already seen in our discussion of angular momentum of a rigid body that the angular momentum does not necessary point in the same direction as the angular velocity. However we can take the z -component of Equation (21.3.28)

$$\tau_{cm,z}^{\mathrm{ext}} = \frac{dL_{cm,z}^{\mathrm{spin}}}{dt}$$

For a rigid body rotating about the center of mass with $\vec{\omega}_{\mathrm{cm}} = \omega_{\mathrm{cm},z}\hat{\mathbf{k}}$ the z -component of angular momentum about the center of mass is

$$L_{cm,z}^{\mathrm{spin}} = I_{\mathrm{cm}}\omega_{\mathrm{cm},z}$$

The z -component of the rotational equation of motion about the center of mass is

$$\tau_{cm,z}^{\mathrm{ext}} = I_{\mathrm{cm}}\frac{d\omega_{\mathrm{cm},z}}{dt} = I_{\mathrm{cm}}\alpha_{\mathrm{cm},z}$$