26.2: Stress and Strain in Tension and Compression

Consider a rod with cross sectional area A and length l_0 Two forces of the same magnitude $F_\perp$ are applied perpendicularly at the two ends of the section stretching the rod to a length l (Figure $26.2.1$), where the beam has been stretched by a positive amount $\delta l = l - l_0$.

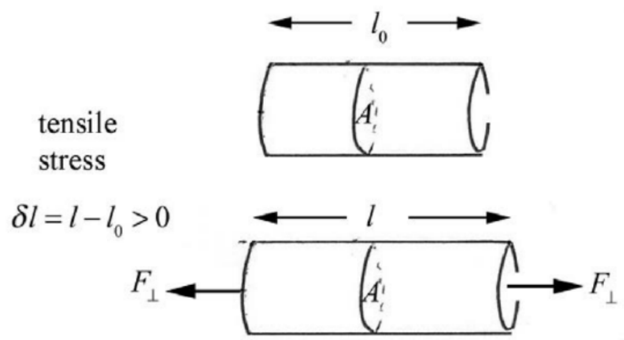

Figure $26.2.1$: Tensile stress on a rod

The ratio of the applied perpendicular force to the cross-sectional area is called the **tensile stress**,

$$\sigma_T = \frac{F_\perp}{A} \tag{26.2.1}$$

The ratio of the amount the section has stretched to the original length is called the **tensile strain**,

$$\varepsilon_T = \frac{\delta l}{l_0} \tag{26.2.2}$$

Experimentally, for sufficiently small stresses, for many materials the stress and strain are linearly proportional,

$$\frac{F_\perp}{A} = Y \frac{\delta l}{l_0} \quad (\text{ Hooke's Law }) \tag{26.2.3}$$

where the constant of proportionality Y is called **Young's modulus**. The SI unit for Young's Modulus is the **pascal** where $1 \mathrm{Pa} \equiv 1 \mathrm{N} \cdot \mathrm{m}^{-2}$. Note the following conversion factors between SI and English units: $1 \mathrm{bar} \equiv 10^5 \mathrm{Pa}, \quad 1 \mathrm{psi} \equiv 6.9 \times 10^{-2} \mathrm{bar}$, and $1 \mathrm{bar} = 14.5 \mathrm{psi}$ In Table 26.1, Young's Modulus is tabulated for various materials. Figure $26.2.2$ shows a plot of the stress-strain relationship for various human bones. For stresses greater than approximately $70 \mathrm{N} \cdot \mathrm{mm}^{-2}$, the material is no longer elastic. At a certain point for each bone, the stress-strain relationship stops, representing the fracture point.

Material	Young's Modulus, Y (Pa)
Iron	21×10^{10}
Nickel	21×10^{10}
Steel	20×10^{10}
Copper	11×10^{10}
Brass	9.0×10^{10}
Aluminum	7.0×10^{10}
Crown Glass	6.0×10^{10}
Cortical Bone	$7 \times 10^9 - 30 \times 10^9$
Lead	1.6×10^{10}
Tendon	2×10^7
Rubber	$7 \times 10^5 - 40 \times 10^5$
Blood vessels	2×10^5

Table 26.1: Young's Modulus for various materials

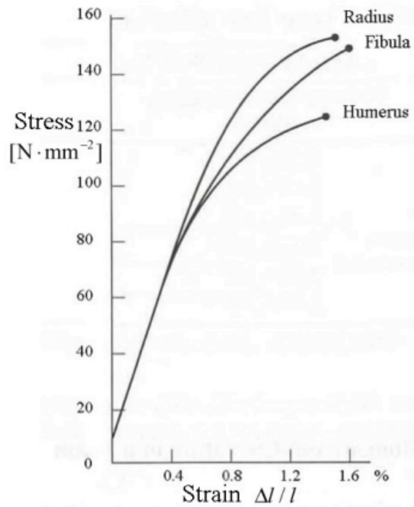

Figure 26.2.2: Stress-strain relation for various human bones (figure from H. Yamada, Strength of Biological Materials)

When the material is under compression, the forces on the ends are directed towards each other producing a compressive stress resulting in a compressive strain (Figure 26.2.2). For compressive strains, if we define $\delta l = l_0 - l > 0$ then Equation 26.2.3 holds for compressive stresses provided the compressive stress is not too large. For many materials, Young's Modulus is the same when the material is under tension and compression. There are some important exceptions. Concrete and stone can undergo compressive stresses but fail when the same tensile stress is applied. When building with these materials, it is important to design the structure so that the stone or concrete is never under tensile stresses. Arches are used as an architectural structural element primarily for this reason.

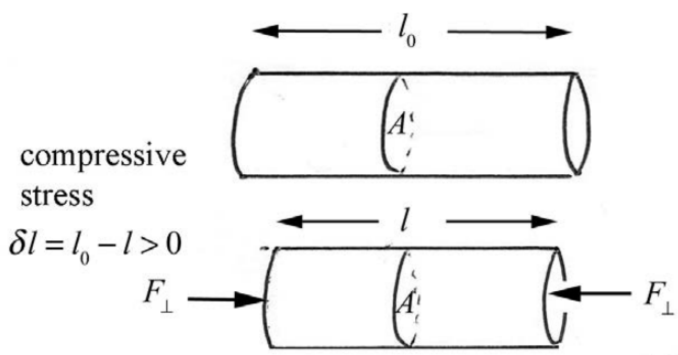

Figure 26.2.3: Compressive Stress