

25.10: Appendix 25C Analytic Geometric Properties of Ellipses

Consider Equation (25.3.20), and for now take $\varepsilon < 1$, so that the equation is that of an ellipse. We shall now show that we can write it as

$$\frac{(x - x_0)^2}{a^2} + \frac{y^2}{b^2} = 1$$

where the ellipse has axes parallel to the x - and y -coordinate axes, center at $(x_0, 0)$, semi-major axis a and semi-minor axis b. We begin by rewriting Equation (25.3.20) as

$$x^2 - \frac{2\varepsilon r_0}{1 - \varepsilon^2} x + \frac{y^2}{1 - \varepsilon^2} = \frac{r_0^2}{1 - \varepsilon^2}$$

We next complete the square,

$$\begin{aligned} x^2 - \frac{2\varepsilon r_0}{1 - \varepsilon^2} x + \frac{\varepsilon^2 r_0^2}{(1 - \varepsilon^2)^2} + \frac{y^2}{1 - \varepsilon^2} &= \frac{r_0^2}{1 - \varepsilon^2} + \frac{\varepsilon^2 r_0^2}{(1 - \varepsilon^2)^2} \Rightarrow \\ \left(x - \frac{\varepsilon r_0}{1 - \varepsilon^2}\right)^2 + \frac{y^2}{1 - \varepsilon^2} &= \frac{r_0^2}{(1 - \varepsilon^2)^2} \Rightarrow \\ \frac{\left(x - \frac{\varepsilon r_0}{1 - \varepsilon^2}\right)^2}{\left(r_0 / (1 - \varepsilon^2)\right)^2} + \frac{y^2}{\left(r_0 / \sqrt{1 - \varepsilon^2}\right)^2} &= 1 \end{aligned}$$

The last expression in (25.C.3) is the equation of an ellipse with semi-major axis

$$a = \frac{r_0}{1 - \varepsilon^2}$$

semi-minor axis

$$b = \frac{r_0}{\sqrt{1 - \varepsilon^2}} = a \sqrt{1 - \varepsilon^2}$$

and center at

$$x_0 = \frac{\varepsilon r_0}{(1 - \varepsilon^2)} = \varepsilon a$$

as found in Equation (25.B.10).

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