8.2: Fundamental Laws of Nature

Force laws are mathematical models of physical processes. They arise from observation and experimentation, and they have limited ranges of applicability. Does the linear force law for the spring hold for all springs? Each spring will most likely have a different range of linear behavior. So the model for stretching springs still lacks a universal character. As such, there should be some hesitation to generalize this observation to all springs unless some property of the spring, universal to all springs, is responsible for the force law.

Perhaps springs are made up of very small components, which when pulled apart tend to contract back together. This would suggest that there is some type of force that contracts spring molecules when they are pulled apart. What holds molecules together? Can we find some fundamental property of the interaction between atoms that will suffice to explain the macroscopic force law? This search for fundamental forces is a central task of physics.

In the case of springs, this could lead into an investigation of the composition and structural properties of the atoms that compose the steel in the spring. We would investigate the geometric properties of the lattice of atoms and determine whether there is some fundamental property of the atoms that create this lattice. Then we ask how stable is this lattice under deformations. This may lead to an investigation into the electron configurations associated with each atom and how they overlap to form bonds between atoms. These particles carry charges, which obey Coulomb's Law, but also the Laws of Quantum Mechanics. So in order to arrive at a satisfactory explanation of the elastic restoring properties of the spring, we need models that describe the fundamental physics that underline Hooke's Law.

Universal Law of Gravitation

At points significantly far away from the surface of Earth, the gravitational force is no longer constant with respect to the distance to the center of Earth. Newton's Universal Law of Gravitation describes the gravitational force between two objects with masses, m_1 and m_2. This force points along the line connecting the objects, is attractive, and its magnitude is proportional to the inverse square of the distance, $r_{1,2}$ between the two point-like objects (Figure 8.4a). The force on object 2 due to the gravitational interaction between the two objects is given by

$$\overrightarrow{\mathbf{F}}^{G}_{1,2} = -G\frac{m_1 m_2}{r^2_{1,2}}\hat{\mathbf{r}}_{1,2}$$

where $\overrightarrow{\mathbf{r}}_{1,2} = \overrightarrow{\mathbf{r}}_2 - \overrightarrow{\mathbf{r}}_1$ is a vector directed from object 1 to object 2, $r_{1,2} = \left|\overrightarrow{\mathbf{r}}_{1,2}\right|$, and $\hat{\mathbf{r}}_{1,2} = \overrightarrow{\mathbf{r}}_{1,2}/\left|\overrightarrow{\mathbf{r}}_{1,2}\right|$ is a unit vector directed from object 1 to object 2 (Figure 8.4b). The constant of proportionality in SI units is $G = 6.67\times 10^{-11}\mathrm{N}\cdot\mathrm{m}^2\cdot\mathrm{kg}^{-2}$.

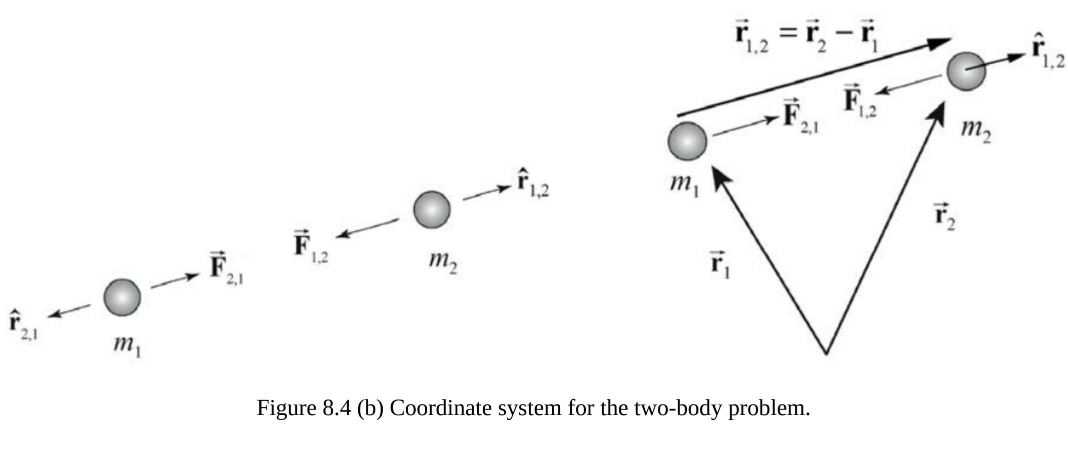

Figure 8.4 (b) Coordinate system for the two-body problem.

Principle of Equivalence:

The Principle of Equivalence states that the mass that appears in the Universal Law of Gravity is identical to the inertial mass that is determined with respect to the standard kilogram. From this point on, the equivalence of inertial and gravitational mass will be assumed and the mass will be denoted by the symbol m.

Gravitational Force near the Surface of the Earth

Near the surface of Earth, the gravitational interaction between an object and Earth is mutually attractive and has a magnitude of

$$\left|\overrightarrow{\mathbf{F}}^{G}_{\text{earth,object}}\right| = mg$$

where g is a positive constant.

The International Committee on Weights and Measures has adopted as a standard value for the acceleration of an object freely falling in a vacuum $g = 9.80665 \mathrm{m} \cdot \mathrm{s}^{-2}$. The actual value of g varies as a function of elevation and latitude. If ϕ is the latitude and h the elevation in meters then the acceleration of gravity in SI units is

$$g = \left(9.80616 - 0.025928 \cos(2\phi) + 0.000069 \cos^{2}(2\phi) - 3.086 \times 10^{-4} h\right) \mathrm{m} \cdot \mathrm{s}^{-2}$$

This is known as Helmert's equation. The strength of the gravitational force on the standard kilogram at 42° latitude is $9.80345 \mathrm{N} \cdot \mathrm{kg}^{-1}$ and the acceleration due to gravity at sea level is therefore $g = 9.80345 \mathrm{m} \cdot \mathrm{s}^{-2}$ for all objects. At the equator, $g = 9.78 \mathrm{m} \cdot \mathrm{s}^{-2}$ and at the poles $g = 9.83 \mathrm{m} \cdot \mathrm{s}^{-2}$ This difference is primarily due to the earth's rotation, which introduces an apparent (fictitious) repulsive force that affects the determination of g as given in Equation (8.2.2) and also flattens the spherical shape of Earth (the distance from the center of Earth is larger at the equator than it is at the poles by about 26.5 km). Both the magnitude and the direction of the gravitational force also show variations that depend on local features to an extent that's useful in prospecting for oil, investigating the water table, navigating submerged submarines, and as well as many other practical uses. Such variations in g can be measured with a sensitive spring balance. Local variations have been much studied over the past two decades in attempts to discover a proposed "fifth force" which would fall off faster than the gravitational force that falls off as the inverse square of the distance between the objects.

Electric Charge and Coulomb's Law

Matter has properties other than mass. Matter can also carry one of two types of observed electric charge, positive and negative. Like charges repel, and opposite charges attract each other. The unit of charge in the SI system of units is called the coulomb [C].

The smallest unit of "free" charge known in nature is the charge of an electron or proton, which has a magnitude of

$$e = 1.602 \times 10^{-19} \mathrm{C}$$

It has been shown experimentally that charge carried by ordinary objects is quantized in integral multiples of the magnitude of this free charge. The electron carries one unit of negative charge $(q_e = -e)$ and the proton carries one unit of positive charge $(q_p = +e)$. In an isolated system, the charge stays constant; in a closed system, an amount of unbalanced charge can neither be created nor destroyed. Charge can only be transferred from one object to another.

Consider two point-like objects with charges q_1 and q_2 separated by a distance $r_{1,2}$ in vacuum. By experimental observation, the two objects repel each other if they are both positively or negatively charged (Figure 8.4a). They attract each other if they are oppositely charged (Figure 8.5b). The force exerted on object 2 due to the interaction between objects 1 and 2 is given by Coulomb's Law,

$$\overrightarrow{\mathbf{F}}^{E}_{1,2} = k_e \frac{q_1 q_2}{r_{1,2}^{2}} \hat{\mathbf{r}}_{1,2}$$

where $\hat{\mathbf{r}}_{1,2} = \overrightarrow{\mathbf{r}}_{1,2} / \left|\overrightarrow{\mathbf{r}}_{1,2}\right|$ is a unit vector directed from object 1 to object 2, and in SI units, $k_e = 8.9875 \times 10^{9} \mathrm{N} \cdot \mathrm{m}^{2} \cdot \mathrm{C}^{-2}$ as illustrated in the Figure 8.5a. This law was derived empirically by Charles Augustin de Coulomb in the late 18th century.

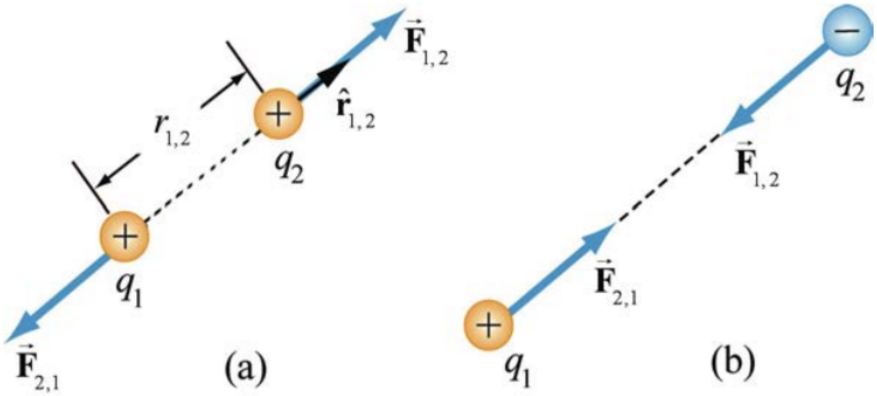

Figure 8.5 (a) and 8.5 (b) Coulomb interaction between two charges

Example 8.2.1: Coulomb's Law and the Universal Law of Gravitation

Show that both Coulomb's Law and the Universal Law of Gravitation satisfy Newton's Third Law.

Solution

To see this, interchange 1 and 2 in the Universal Law of Gravitation to find the force on object 1 due to the interaction between the objects. The only quantity to change sign is the unit vector

$$\hat{\mathbf{r}}_{2,1} = -\hat{\mathbf{r}}_{1,2}$$

Then

$$\overrightarrow{\mathbf{F}}^{G}_{2,1} = -G\frac{m_2 m_1}{r^2_{2,1}}\hat{\mathbf{r}}_{2,1} = G\frac{m_1 m_2}{r^2_{1,2}}\hat{\mathbf{r}}_{1,2} = -\overrightarrow{\mathbf{F}}^{G}_{1,2}$$

Coulomb's Law also satisfies Newton's Third Law since the only quantity to change sign is the unit vector, just as in the case of the Universal Law of Gravitation.