19.1: Introduction

The situation, in brief, is that newtonian physics is incapable of predicting conservation of angular momentum, but no isolated system has yet been encountered experimentally for which angular momentum is not conserved. We conclude that conservation of angular momentum is an independent physical law, and until a contradiction is observed, our physical understanding must be guided by it

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When we consider a system of objects, we have shown that the external force, acting at the center of mass of the system, is equal to the time derivative of the total momentum of the system,

$$\overrightarrow{\mathbf{F}}^{\text{ext}}=\frac{d\overrightarrow{\mathbf{p}}_{\text{sys}}}{dt}$$

We now introduce the rotational analog of Equation (19.1.1). We will first introduce the concept of angular momentum for a point-like particle of mass m with linear momentum $\overrightarrow{\mathbf{p}}$ about a point S, defined by the equation

$$\overrightarrow{\mathbf{L}}_S=\overrightarrow{\mathbf{r}}_S\times\overrightarrow{\mathbf{p}}$$

where $\overrightarrow{\mathbf{r}}_S$ is the vector from the point S to the particle. We will show in this chapter that the torque about the point S acting on the particle is equal to the rate of change of the angular momentum about the point S of the particle,

$$\vec{\tau}_S=\frac{d\overrightarrow{\mathbf{L}}_S}{dt}$$

Equation (19.1.3) generalizes to any body undergoing rotation.

We shall concern ourselves first with the special case of rigid body undergoing fixed axis rotation about the z-axis with angular velocity $\overrightarrow{\boldsymbol{\omega}}=\omega_z\hat{\mathbf{k}}$ We divide up the rigid body into N elements labeled by the index $i, i=1,2,\ldots N$, the i^{th} element having mass m_i i and position vector $\overrightarrow{\mathbf{r}}_{S,i}$. The rigid body has a moment of inertia I_S about some point S on the fixed axis, (often taken to be the z-axis, but not always) which rotates with angular velocity $\overrightarrow{\boldsymbol{\omega}}$ about this axis. The angular momentum is then the vector sum of the individual angular momenta,

$$\overrightarrow{\mathbf{L}}_S=\sum_{i=1}^{i=N}\overrightarrow{\mathbf{L}}_{S,i}=\sum_{i=1}^{i=N}\overrightarrow{\mathbf{r}}_{S,i}\times\overrightarrow{\mathbf{p}}_i$$

When the rotation axis is the z-axis the z-component of the angular momentum, $L_{S,z}$, about the point S is then given by

$$L_{S,z}=I_S\omega_z$$

We shall show that the z-component of the torque about the point $S, \tau_{S,z}$ is then the time derivative of the z-component of angular momentum about the point S,

$$\tau_{S,z}=\frac{dL_{S,z}}{dt}=I_S\frac{d\omega_z}{dt}=I_S\alpha_z$$