27.5: Compressibility of a Fluid

When the pressure is uniform on all sides of an object in a fluid, the pressure will squeeze the object resulting in a smaller volume. When we increase the pressure by ΔP on a material of volume v_o, then the volume of the material will change by $\Delta V < 0$ and consequently the density of the material will also change. Define the bulk stress by the increase in pressure change

$$\sigma_B \equiv \Delta P$$

Define the bulk strain by the ratio

$$\varepsilon_B \equiv \frac{\Delta V}{V_0}$$

For many materials, for small pressure changes, the bulk stress is linearly proportional to the bulk strain,

$$\Delta P = -B\frac{\Delta V}{V_0} \tag{27.5.1}$$

where the constant of proportionality B is called the **bulk modulus**. The SI unit for bulk modulus is the pascal. If the bulk modulus of a material is very large, a large pressure change will result in only a small volume change. In that case the material is called *incompressible*. In Table 27.2, the bulk modulus is tabulated for various materials.

Table 27.2 Bulk Modulus for Various Materials

Material	Bulk Modulus, Y , (Pa)
Diamond	4.4×10^{11}
Iron	1.6×10^{11}
Nickel	1.7×10^{11}
Steel	1.6×10^{11}
Copper	1.4×10^{11}
Brass	6.0×10^{10}
Aluminum	7.5×10^{10}
Crown Glass	5.0×10^{10}
Lead	4.1×10^{10}
Water (value increases at higher pressure)	2.2×10^{9}
Air (adiabatic bulk modulus)	1.42×10^{5}
Air (isothermal bulk modulus)	1.01×10^{5}

Example 27.3: Compressibility of Water

Determine the percentage decrease in a fixed volume of water at a depth of 4 km where the pressure difference is 40 Mpa, with respect to sea level.

Solution

The bulk modulus of water is $2.2 \times 10^{9}\mathrm{Pa}$. From Equation 27.5.1,

$$\frac{\Delta V}{V_0} = -\frac{\Delta P}{B} = -\frac{40 \times 10^{6}\mathrm{Pa}}{2.2 \times 10^{9}\mathrm{Pa}} = -0.018.$$

There is only a 1.8% decrease in volume. Water is essentially incompressible even at great depths in ocean, justifying our assumption that the density of water is uniform in the ocean in Example 27.1.