18.1: Introduction Static Equilibrium

The proof of the correctness of a new rule can be attained by the repeated application of it, the frequent comparison with experience, the putting of it to the test under the most diverse circumstances. This process, would in the natural course of events, be carried out in time. The discoverer, however hastens to reach his goal more quickly. He compares the results that flow from his rule with all the experiences with which he is familiar, with all older rules, repeatedly tested in times gone by, and watches to see if he does not light on contradictions. In this procedure, the greatest credit is, as it should be, conceded to the oldest and most familiar experiences, the most thoroughly tested rules. Our instinctive experiences, those generalizations that are made involuntarily, by the irresistible force of the innumerable facts that press upon us, enjoy a peculiar authority; and this is perfectly warranted by the consideration that it is precisely the elimination of subjective caprice and of individual error that is the object aimed at.

Ernst Mach

When the vector sum of the forces acting on a point-like object is zero then the object will continue in its state of rest, or of uniform motion in a straight line. If the object is in uniform motion we can always change reference frames so that the object will be at rest. We showed that for a collection of point-like objects the sum of the external forces may be regarded as acting at the center of mass. So if that sum is zero the center of mass will continue in its state of rest, or of uniform motion in a straight line. We introduced the idea of a rigid body, and again showed that in addition to the fact that the sum of the external forces may be regarded as acting at the center of mass, forces like the gravitational force that acts at every point in the body may be treated as acting at the center of mass. However for an extended rigid body it matters where the force is applied because even though the sum of the forces on the body may be zero, a non-zero sum of torques on the body may still produce angular acceleration. In particular for fixed axis rotation, the torque along the axis of rotation on the object is proportional to the angular acceleration. It is possible that sum of the torques may be zero on a body that is not constrained to rotate about a fixed axis and the body may still undergo rotation. We will restrict ourselves to the special case in which in an inertial reference frame both the center of mass of the body is at rest and the body does not undergo any rotation, a condition that is called **static equilibrium of an extended object.**

The two sufficient and necessary conditions for a rigid body to be in static equilibrium are:

(1) The sum of the forces acting on the rigid body is zero,

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \cdots = \vec{\mathbf{0}}$$

(2) The vector sum of the torques about any point S in a rigid body is zero,

$$\vec{\tau}_S = \vec{\tau}_{S,1} + \vec{\tau}_{S,2} + \cdots = \vec{0}$$