23.1: Introduction to Periodic Motion

> …Indeed it is not in the nature of a simple pendulum to provide equal and reliable measurements of time, since the wide lateral excursions often made may be observed to be slower than more narrow ones; however, we have been led in a different direction by geometry, from which we have found a means of suspending the pendulum, with which we were previously unacquainted, and by giving close attention to a line with a certain curvature, the time of the swing can be chosen equal to some calculated value and is seen clearly in practice to be in wonderful agreement with that ratio. As we have checked the lapses of time measured by these clocks after making repeated land and sea trials, the effects of motion are seen to have been avoided, so sure and reliable are the measurements; now it can be seen that both astronomical studies and the art of navigation will be greatly helped by them…
>
> ~Christian Huygens

There are two basic ways to measure time: by duration or periodic motion. Early clocks measured duration by calibrating the burning of incense or wax, or the flow of water or sand from a container. Our calendar consists of years determined by the motion of the sun; months determined by the motion of the moon; days by the rotation of the earth; hours by the motion of cyclic motion of gear trains; and seconds by the oscillations of springs or pendulums. In modern times a second is defined by a specific number of vibrations of radiation, corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom.

Sundials calibrate the motion of the sun through the sky, including seasonal corrections. A clock escapement is a device that can transform continuous movement into discrete movements of a gear train. The early escapements used oscillatory motion to stop and start the turning of a weight-driven rotating drum. Soon, complicated escapements were regulated by pendulums, the theory of which was first developed by the physicist Christian Huygens in the mid 17th century. The accuracy of clocks was increased and the size reduced by the discovery of the oscillatory properties of springs by Robert Hooke. By the middle of the 18th century, the technology of timekeeping advanced to the point that William Harrison developed timekeeping devices that were accurate to one second in a century.

Simple Harmonic Motion: Quantitative

One of the most important examples of periodic motion is simple harmonic motion (SHM), in which some physical quantity varies sinusoidally. Suppose a function of time has the form of a sine wave function,

$$y(t) = A \sin(2\pi t/T)$$

where $A > 0$ is the amplitude (maximum value). The function y(t) varies between A and − A, because a sine function varies between +1 and −1. A plot of y(t) vs. time is shown in Figure 23.1.

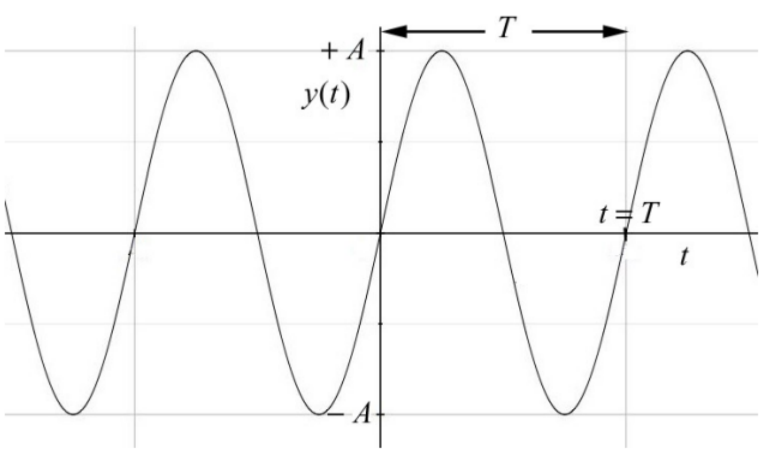

Figure 23.1 Sinusoidal function of time

The sine function is periodic in time. This means that the value of the function at time t will be exactly the same at a later time $t' = t + T$, where T is the period. That the sine function satisfies the periodic condition can be seen from

$$y(t+T) = A \sin\left[\frac{2\pi}{T}(t+T)\right] = A \sin\left[\frac{2\pi}{T}t + 2\pi\right] = A \sin\left[\frac{2\pi}{T}t\right] = y(t)$$

The frequency, f , is defined to be

$$f \equiv 1/T$$

The SI unit of frequency is inverse seconds, $\left[\mathrm{s}^{-1}\right]$, or hertz [Hz]. The angular frequency of oscillation is defined to be

$$\omega_0 \equiv 2\pi/T = 2\pi f$$

and is measured in radians per second. (The angular frequency of oscillation is denoted by ω_0 to distinguish from the angular speed $\omega = |d\theta/dt|$.) One oscillation per second, 1Hz , corresponds to an angular frequency of $2\pi \text{ rad} \cdot \mathrm{s}^{-1}$. (Unfortunately, the same symbol ω is used for angular speed in circular motion. For uniform circular motion the angular speed is equal to the angular frequency but for non-uniform motion the angular speed is not constant. The angular frequency for simple harmonic motion is a constant by definition.) We therefore have several different mathematical representations for sinusoidal motion

$$y(t) = A\sin(2\pi t/T) = A\sin(2\pi f t) = A\sin(\omega_0 t)$$
