24.1: Introduction to Physical Pendulums

We have already used Newton's Second Law or Conservation of Energy to analyze systems like the spring-object system that oscillate. We shall now use torque and the rotational equation of motion to study oscillating systems like pendulums and torsional springs.

Simple Pendulum: Torque Approach

Recall the simple pendulum from Chapter 23.3.1.The coordinate system and force diagram for the simple pendulum is shown in Figure 24.1.

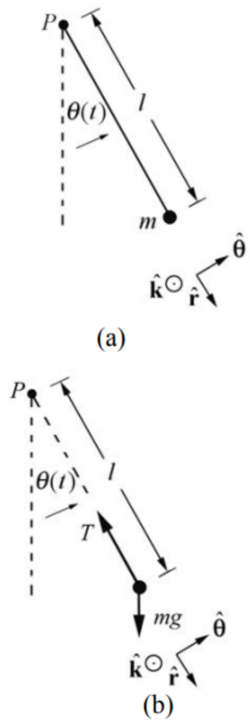

Figure 24.1 (a) Coordinate system and (b) torque diagram for simple pendulum

The torque about the pivot point P is given by

$$\overrightarrow{\boldsymbol{\tau}}_P=\overrightarrow{\mathbf{r}}_{P,m}\times m\overrightarrow{\mathbf{g}}=l\hat{\mathbf{r}}\times mg(\cos\theta\hat{\mathbf{r}}-\sin\theta\hat{\boldsymbol{\theta}})=-\mathrm{lm}\,g\sin\theta\hat{\mathbf{k}}$$

The z -component of the torque about point P

$$(\tau_P)_z=-mgl\sin\theta$$

When $\theta>0$, $(\tau_P)_z<0$ and the torque about P is directed in the negative $\hat{\mathbf{k}}$-direction (into the plane of Figure 24.1b) when $\theta<0$, $(\tau_P)_z>0$ and the torque about P is directed in the positive $\hat{\mathbf{k}}$-direction (out of the plane of Figure 24.1b). The moment of inertia of a point mass about the pivot point P is $I_P=ml^2$. The rotational equation of motion is then

$$(\tau_P)_z=I_P\alpha_z\equiv I_P\frac{d^2\theta}{dt^2}$$
$$-mgl\sin\theta=ml^2\frac{d^2\theta}{dt^2}$$

Thus we have

$$\frac{d^2\theta}{dt^2}=-\frac{g}{l}\sin\theta$$

agreeing with Equation 23. 3.14. When the angle of oscillation is small, we may use the small angle approximation

$$\sin\theta\cong\theta$$

and Equation (24.1.4) reduces to the simple harmonic oscillator equation

$$\frac{d^2\theta}{dt^2} \cong -\frac{g}{l}\theta$$

We have already studied the solutions to this equation in Chapter 23.3. A procedure for determining the period when the small angle approximation does not hold is given in Appendix 24A.