7.3: Momentum, Newton's Second Law and Third Law

Newton began his analysis of the cause of motion by introducing the quantity of motion:

Definition: Quantity of Motion

The quantity of motion is the measure of the same, arising from the velocity and quantity of matter conjointly.

The motion of the whole is the sum of the motion of all its parts; and therefore in a body double in quantity, with equal velocity, the motion is double, with twice the velocity, it is quadruple.

Our modern term for quantity of motion is momentum and it is a vector quantity

$$\overrightarrow{\mathbf{p}}=m\overrightarrow{\mathbf{v}}$$

where m is the inertial mass and $\overrightarrow{\mathbf{V}}$ is the velocity of the body. Newton's Second Law states that

Law II: The change of motion is proportional to the motive force impressed, and is made in the direction of the right line in which that force is impressed.

If any force generates a motion, a double force will generate double the motion, a triple force triple the motion, whether that force is impressed altogether and at once or gradually and successively. And this motion (being always directed the same way with the generating force), if the body moved before, is added or subtracted from the former motion, according as they directly conspire with or are directly contrary to each other; or obliquely joined, when they are oblique, so as to produce a new motion compounded from the determination of both.

Suppose that a force is applied to a body for a time interval Δt. The impressed force or impulse (a vector quantity $\overrightarrow{\mathbf{I}}$) produces a change in the momentum of the body,

$$\overrightarrow{\mathbf{I}}=\overrightarrow{\mathbf{F}}\Delta t=\Delta\overrightarrow{\mathbf{p}}$$

From the commentary to the second law, Newton also considered forces that were applied continually to a body instead of impulsively. The instantaneous action of the total force acting on a body at a time t is defined by taking the mathematical limit as the time interval Δt becomes smaller and smaller,

$$\overrightarrow{\mathbf{F}}=\lim_{\Delta t\to 0}\frac{\Delta\overrightarrow{\mathbf{p}}}{\Delta t}\equiv\frac{d\overrightarrow{\mathbf{p}}}{dt}$$

When the mass remains constant in time, the Second Law can be recast in its more familiar form,

$$\overrightarrow{\mathbf{F}}=m\frac{d\overrightarrow{\mathbf{v}}}{dt}$$

Because the derivative of velocity is the acceleration, the force is the product of mass and acceleration,

$$\overrightarrow{\mathbf{F}}=m\overrightarrow{\mathbf{a}}$$

Because we defined force in terms of change in motion, the Second Law appears to be a restatement of this definition, and devoid of predictive power since force is only determined by measuring acceleration. What transforms the Second Law from just a definition is the additional input that comes from force laws that are based on experimental observations on the interactions between bodies. Throughout this book, we shall investigate these force laws and learn to use them in order to determine the forces and accelerations acting on a body (left-hand-side of Newton's Second Law). When a physical body is constrained to move along a surface, or inside a container (for example gas molecules in a container), there are constraint forces that are not determined beforehand by any force law but are only determined by their effect on the motion of the body. For any given constrained motion, these constraint forces are unknown and must be determined by the particular motion of the body that we are studying, for example the contact force of the surface on the body, or the force of the wall on the gas particles.

The right-hand-side of Newton's Second Law is the product of mass with acceleration. Acceleration is a mathematical description of how the velocity of a body changes. Knowledge of all the forces acting on the body enables us to predict the acceleration. Equation (7.3.5) is known as the *equation of motion*. Once we know this equation we may be able to determine the velocity and position of that body at all future times by integration techniques, or computational techniques. For constrained motion, if we know the acceleration of the body, we can also determine the constraint forces acting on the body.