

19.3: Torque and the Time Derivative of Angular Momentum about a Point for a Particle

We will now show that the torque about a point S is equal to the time derivative of the angular momentum about S ,

$$\vec{\tau}_S = \frac{d\vec{\mathbf{L}}_S}{dt}$$

Take the time derivative of the angular momentum about S ,

$$\frac{d\vec{\mathbf{L}}_S}{dt} = \frac{d}{dt}(\vec{\mathbf{r}}_S \times \vec{\mathbf{p}})$$

In this equation we are taking the time derivative of a vector product of two vectors. There are two important facts that will help us simplify this expression. First, the time derivative of the vector product of two vectors satisfies the [product rule](#),

$$\frac{d}{dt}(\vec{\mathbf{r}}_S \times \vec{\mathbf{p}}) = \left(\left(\frac{d\vec{\mathbf{r}}_S}{dt} \right) \times \vec{\mathbf{p}} \right) + \left(\vec{\mathbf{r}}_S \times \left(\frac{d\vec{\mathbf{p}}}{dt} \right) \right)$$

Second, the first term on the right hand side vanishes,

$$\frac{d\vec{\mathbf{r}}_S}{dt} \times \vec{\mathbf{p}} = \vec{\mathbf{v}} \times m\vec{\mathbf{v}} = \vec{\mathbf{0}}$$

The rate of angular momentum change about the point S is then

$$\frac{d\vec{\mathbf{L}}_S}{dt} = \vec{\mathbf{r}}_S \times \frac{d\vec{\mathbf{p}}}{dt}$$

From Newton's Second Law, the force on the particle is equal to the derivative of the linear momentum,

$$\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt}$$

Therefore the rate of change in time of angular momentum about the point S is

$$\frac{d\vec{\mathbf{L}}_S}{dt} = \vec{\mathbf{r}}_S \times \vec{\mathbf{F}} \quad (19.3.1)$$

Recall that the torque about the point S due to the force $\vec{\mathbf{F}}$ acting on the particle is

$$\vec{\tau}_S = \vec{\mathbf{r}}_S \times \vec{\mathbf{F}} \quad (19.3.2)$$

Combining the expressions in Equation [19.3.1](#) and [19.3.2](#), it is readily seen that the torque about the point S is equal to the rate of change of angular momentum about the point S ,

$$\vec{\tau}_S = \frac{d\vec{\mathbf{L}}_S}{dt}$$

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