

7.1: Force and Quantity of Matter

I have not as yet been able to discover the reason for these properties of gravity from phenomena, and I do not feign hypotheses. For whatever is not deduced from the phenomena must be called a hypothesis; and hypotheses, whether metaphysical or physical, or based on occult qualities, or mechanical, have no place in experimental philosophy. In this philosophy particular propositions are inferred from the phenomena, and afterwards rendered general by induction.

~Isaac Newton

In our daily experience, we can cause a body to move by either pushing or pulling that body. Ordinary language use describes this action as the effect of a person's strength or force. However, bodies placed on inclined planes, or when released at rest and undergo free fall, will move without any push or pull. Galileo referred to a force acting on these bodies, a description of which he published in *Mechanics* in 1623. In 1687, Isaac Newton published his three laws of motion in the *Philosophiae Naturalis Principia Mathematica* ("Mathematical Principles of Natural Philosophy"), which extended Galileo's observations. The First Law expresses the idea that when no force acts on a body, it will remain at rest or maintain uniform motion; when a force is applied to a body, it will change its state of motion.

Many scientists, especially Galileo, recognized the idea that force produces motion before Newton but Newton extended the concept of force to any circumstance that produces acceleration. When a body is initially at rest, the direction of our push or pull corresponds to the direction of motion of the body. If the body is moving, the direction of the applied force may change both the direction of motion of the body and how fast it is moving. Newton defined the force acting on an object as proportional to the acceleration of the object.

An impressed force is an action exerted upon a body, in order to change its state, either of rest, or of uniform motion in a right line.

In order to define the magnitude of the force, he introduced a constant of proportionality, the inertial mass, which Newton called "quantity of matter".

The quantity of matter is the measure of the same, arising from its density and bulk conjointly. Thus air of double density, in a double space, is quadruple in quantity; in a triple space, sextuple in quantity. The same thing is to be understood of snow, and fine dust or powders, that are condensed by compression or liquefaction, and of all bodies that are by any causes whatever differently condensed. I have no regard in this place to a medium, if any such there is, that freely pervades the interstices between the parts of bodies. It is this quantity that I mean hereafter everywhere under the name of body or mass. And the same is known by the weight of each body, for it is proportional to the weight, as I have found by experiment on pendulums, very accurately made, which shall be shown hereafter.

Suppose we apply a force to a body that is an identical copy of the standard mass, (we shall refer to this body as a standard body). The force will induce the standard body to accelerate with magnitude $|\vec{a}|$ that can be measured by an accelerometer (any device that measures acceleration). The magnitude of the force $|\vec{F}|$ acting on the standard body is defined to be the product of the standard mass m_s with the magnitude of the acceleration $|\vec{a}|$. Force is a vector quantity. The direction of the force on the standard body is defined to be the direction of the acceleration of the body. Thus

$$\vec{F} \equiv m_s \vec{a}$$

In order to justify the statement that force is a vector quantity, we need to apply two forces \vec{F}_1 and \vec{F}_2 simultaneously to our standard body and show that the resultant force \vec{F}^T is the vector sum of the two forces when the forces are applied one at a time.

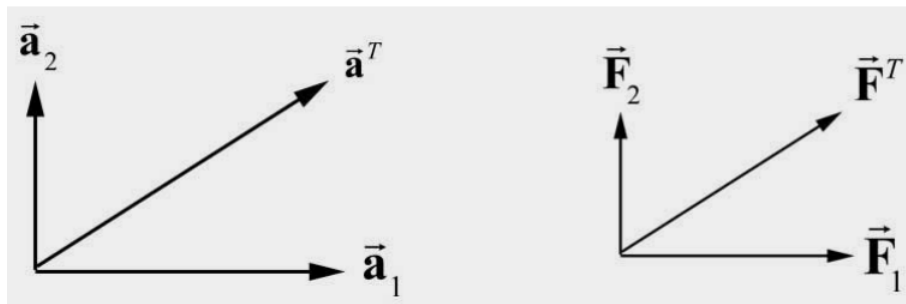


Figure 7.2 Force adds as vectors.

We apply each force separately and measure the accelerations \vec{a}_1 and \vec{a}_2 , noting that

$$\vec{F}_1 = m_s \vec{a}_1$$

$$\vec{F}_2 = m_s \vec{a}_2$$

When we apply the two forces simultaneously, we measure the acceleration \vec{a} . The force by definition is now

$$\vec{F}^T \equiv m_s \vec{a}$$

We then compare the accelerations. The results of these three measurements, and for that matter any similar experiment, confirms that the accelerations add as vectors (Figure 7.1)

$$\vec{a} = \vec{a}_1 + \vec{a}_2$$

Therefore the forces add as vectors as well (Figure 7.2),

$$\vec{F}^T = \vec{F}_1 + \vec{F}_2$$

This last statement is not a definition but a consequence of the experimental result described by Equation (7.1.5) and our definition of force.

Example 7.1 Vector Decomposition Solution

Two horizontal ropes are attached to a post that is stuck in the ground. The ropes pull the post producing the vector forces $\vec{F}_1 = 70\text{N}\hat{i} + 20\text{N}\hat{j}$ and $\vec{F}_2 = -30\text{N}\hat{i} + 40\text{N}\hat{j}$ as shown in Figure 7.3. Find the direction and magnitude of the horizontal component of a contact force of the ground on the post.

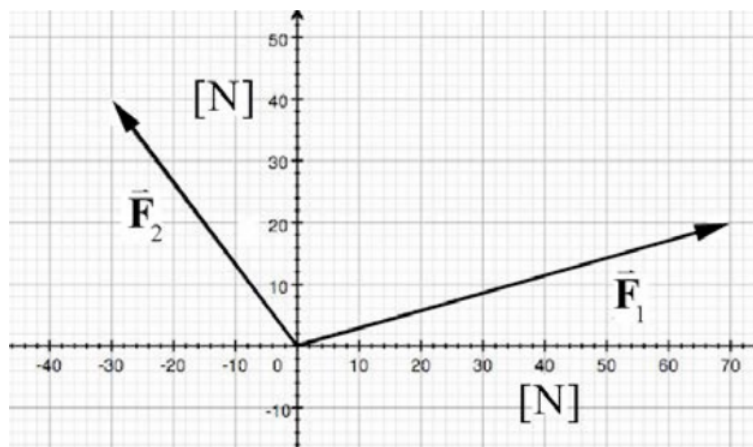


Figure 7.3 Example 7.1

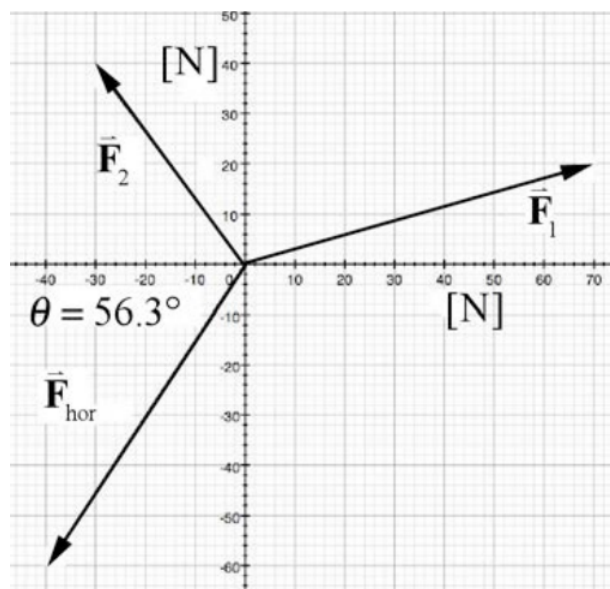


Figure 7.4 Vector sum of the horizontal forces

Solution: Because the ropes are pulling the post horizontally, the contact force must have a horizontal component that is equal to the negative of the sum of the two horizontal forces exerted by the rope on the post (Figure 7.4). There is an additional vertical component of the contact force that balances the gravitational force exerted on the post by the earth. We restrict our attention to the horizontal component of the contact force. Let $\vec{\mathbf{F}}_3$ denote the sum of the forces due to the ropes. Then we can write the vector $\vec{\mathbf{F}}_3$ as

$$\begin{aligned}\vec{\mathbf{F}}_3 &= (F_{1x} + F_{2x})\hat{\mathbf{i}} + (F_{1y} + F_{2y})\hat{\mathbf{j}} = (70\text{N} + -30\text{N})\hat{\mathbf{i}} + (20\text{N} + 40\text{N})\hat{\mathbf{j}} \\ &= (40\text{N})\hat{\mathbf{i}} + (60\text{N})\hat{\mathbf{j}}\end{aligned}$$

Therefore the horizontal component of the contact force satisfies the condition that

$$\vec{\mathbf{F}}_{hor} = -\vec{\mathbf{F}}_3 = -\left(\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2\right) = (-40\text{N})\hat{\mathbf{i}} + (-60\text{N})\hat{\mathbf{j}}$$

The magnitude is $\left|\vec{\mathbf{F}}_{hor}\right| = \sqrt{(-40\text{N})^2 + (-60\text{N})^2} = 72\text{N}$. The horizontal component of the contact force makes an angle

$$\theta = \tan^{-1}\left[\frac{60\text{N}}{40\text{N}}\right] = 56.3^\circ$$

as shown in the figure above.

Mass Calibration

So far, we have only used the standard body to measure force. Instead of performing experiments on the standard body, we can calibrate the masses of all other bodies in terms of the standard mass by the following experimental procedure. We shall refer to the mass measured in this way as the inertial mass and denote it by m_{in} .

We apply a force of magnitude F to the standard body and measure the magnitude of the acceleration a_s . Then we apply the same force to a second body of unknown mass m_{in} and measure the magnitude of the acceleration a_{in} . Because the same force is applied to both bodies,

$$F = m_{in}a_{in} = m_s a_s$$

the ratio of the inertial mass to the standard mass is equal to the inverse ratio of the magnitudes of the accelerations,

$$\frac{m_{in}}{m_s} = \frac{a_s}{a_{in}}$$

Therefore the second body has inertial mass equal to

$$m_{in} = m_s \frac{a_s}{a_{in}}$$

This method is justified by the fact that we can repeat the experiment using a different force and still find that the ratios of the acceleration are the same. For simplicity we shall denote the inertial mass by m .

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