13.2: Kinetic Energy

The first form of energy that we will study is an energy associated with the coherent motion of molecules that constitute a body of mass m; this energy is called the kinetic energy (from the Greek word kinetikos which translates as moving). Let us consider a car moving along a straight road (along which we will place the x -axis). For an observer at rest with respect to the ground, the car has velocity $\overrightarrow{\mathbf{v}}=v_x \hat{\mathbf{i}}$ The speed of the car is the magnitude of the velocity, $v \equiv |v_x|$

The kinetic energy K of a non-rotating body of mass m moving with speed v is defined to be the positive scalar quantity.

$$K \equiv \frac{1}{2} m v^2$$

The kinetic energy is proportional to the square of the speed. The SI units for kinetic energy are $\left[\mathrm{kg} \cdot \mathrm{m}^2 \cdot \mathrm{s}^{-2}\right]$. This combination of units is defined to be a joule and is denoted by [J], thus $\mathrm{IJ} \equiv 1 \mathrm{kg} \cdot \mathrm{m}^2 \cdot \mathrm{s}^{-2}$ (The SI unit of energy is named for James Prescott Joule.) The above definition of kinetic energy does not refer to any direction of motion, just the speed of the body.

Let's consider a case in which our car changes velocity. For our initial state, the car moves with an initial velocity $\overrightarrow{\mathbf{v}}_i=v_{x,i} \hat{\mathbf{i}}$ along the x -axis. For the final state (at some later time), the car has changed its velocity and now moves with a final velocity $\overrightarrow{\mathbf{v}}_f=v_{x,f} \hat{\mathbf{i}}$. Therefore the change in the kinetic energy is

Example 13.1 Change in Kinetic Energy of a Car

Suppose car A increases its speed from 10 to 20 mph and car B increases its speed from 50 to 60 mph. Both cars have the same mass m. (a) What is the ratio of the change of kinetic energy of car B to the change of kinetic energy of car A? In particular, which car has a greater change in kinetic energy? (b) What is the ratio of the change in kinetic energy of car B to car A as seen by an observer moving with the initial velocity of car A?

Solution: (a) The ratio of the change in kinetic energy of car B to car A is

$$\begin{aligned}
\frac{\Delta K_B}{\Delta K_A} &= \frac{\frac{1}{2} m\left(v_{B,f}\right)^2-\frac{1}{2} m\left(v_{B,i}\right)^2}{\frac{1}{2} m\left(v_{A,f}\right)^2-\frac{1}{2} m\left(v_{A,i}\right)^2}=\frac{\left(v_{B,f}\right)^2-\left(v_{B,i}\right)^2}{\left(v_{A,f}\right)^2-\left(v_{A,i}\right)^2} \\
&= \frac{(60 \mathrm{mph})^2-(50 \mathrm{mph})^2}{(20 \mathrm{mph})^2-(10 \mathrm{mph})^2}=11/3
\end{aligned}$$

Thus car B has a much greater increase in its kinetic energy than car A.

(b) In a reference moving with the speed of car A , car A increases its speed from rest to 10 mph and car B increases its speed from 40 to 50 mph. The ratio is now

$$\begin{aligned}
\frac{\Delta K_B}{\Delta K_A} &= \frac{\frac{1}{2} m\left(v_{B,f}\right)^2-\frac{1}{2} m\left(v_{B,0}\right)^2}{\frac{1}{2} m\left(v_{A,f}\right)^2-\frac{1}{2} m\left(v_{A,0}\right)^2}=\frac{\left(v_{B,f}\right)^2-\left(v_{B,0}\right)^2}{\left(v_{A,f}\right)^2-\left(v_{A,0}\right)^2} \\
&= \frac{(50 \mathrm{mph})^2-(40 \mathrm{mph})^2}{(10 \mathrm{mph})^2}=9
\end{aligned}$$

The ratio is greater than that found in part a). Note that from the new reference frame both car A and car B have smaller increases in kinetic energy.