

## 13.11: Work-Kinetic Energy Theorem in Three Dimensions

Recall our mathematical result that for one-dimensional motion

$$m \int_i^f a_x dx = m \int_i^f \frac{dv_x}{dt} dx = m \int_i^f dv_x \frac{dx}{dt} = m \int_i^f v_x dv_x = \frac{1}{2} m v_{x,f}^2 - \frac{1}{2} m v_{x,i}^2$$

Using Newton's Second Law in the form  $F_x = ma_x$ , we concluded that

$$\int_i^f F_x dx = \frac{1}{2} m v_{x,f}^2 - \frac{1}{2} m v_{x,i}^2$$

Equation (13.11.2) generalizes to the y - and z -directions:

$$\int_i^f F_y dy = \frac{1}{2} m v_{y,f}^2 - \frac{1}{2} m v_{y,i}^2$$

$$\int_i^f F_z dz = \frac{1}{2} m v_{z,f}^2 - \frac{1}{2} m v_{z,i}^2$$

Adding Equations (13.11.2), (13.11.3), and (13.11.4) yields

$$\int_i^f (F_x dx + F_y dy + F_z dz) = \frac{1}{2} m (v_{x,f}^2 + v_{y,f}^2 + v_{z,f}^2) - \frac{1}{2} m (v_{x,i}^2 + v_{y,i}^2 + v_{z,i}^2)$$

Recall (Equation (13.8.24)) that the left hand side of Equation (13.11.5) is the work done by the force  $\vec{\mathbf{F}}$  on the object

$$W = \int_i^f dW = \int_i^f (F_x dx + F_y dy + F_z dz) = \int_i^f \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$$

The right hand side of Equation (13.11.5) is the change in kinetic energy of the object

$$\Delta K \equiv K_f - K_i = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = \frac{1}{2} m (v_{x,f}^2 + v_{y,f}^2 + v_{z,f}^2) - \frac{1}{2} m (v_{x,i}^2 + v_{y,i}^2 + v_{z,i}^2)$$

Therefore Equation (13.11.5) is the three dimensional generalization of the work-kinetic energy theorem

$$\int_i^f \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = K_f - K_i$$

When the work done on an object is positive, the object will increase its speed, and negative work done on an object causes a decrease in speed. When the work done is zero, the object will maintain a constant speed.

### Instantaneous Power Applied by a Non-Constant Force for Three Dimensional Motion

Recall that for one-dimensional motion, the instantaneous power at time t is defined to be the limit of the average power as the time interval  $[t, t + \Delta t]$  approaches zero,

$$P(t) = F_x^a(t) v_x(t)$$

A more general result for the instantaneous power is found by using the expression for dW as given in Equation (13.8.23),

$$P = \frac{dW}{dt} = \frac{\vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}}{dt} = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$$

The time rate of change of the kinetic energy for a body of mass m is equal to the power,

$$\frac{dK}{dt} = \frac{1}{2} m \frac{d}{dt} (\vec{\mathbf{v}} \cdot \vec{\mathbf{v}}) = m \frac{d\vec{\mathbf{v}}}{dt} \cdot \vec{\mathbf{v}} = m \vec{\mathbf{a}} \cdot \vec{\mathbf{v}} = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}} = P$$

where the we used Equation (13.8.9), Newton's Second Law and Equation (13.11.10).

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