1.1: Review of Foundations

1.1.1: Introduction

Electromagnetics involves the macroscopic behavior of electric charges in vacuum and matter. This behavior can be accurately characterized by the Lorentz force law and Maxwell's equations, which were derived from experiments showing how forces on charges depend on the relative locations and motions of other charges nearby. Additional relevant laws of physics include Newton's law, photon quantization, and the conservation relations for charge, energy, power, and momentum. Electromagnetic phenomena underlie most of the "electrical" in "electrical engineering" and are basic to a sound understanding of that discipline.

Electrical engineering has delivered four "miracles" — sets of phenomena that could each be considered true magic prior to their development. The first of these to impress humanity was the electrical phenomenon of lightning, often believed to be a tool of heaven, and the less powerful magnetic force that caused lodestones to point north. The explanation and application of these invisible forces during the eighteenth and nineteenth centuries vaulted electrical engineering to the forefront of commercial interest as motors, generators, electric lights, batteries, heaters, telephones, record players, and many other devices emerged.

The second set of miracles delivered the ability to communicate instantly without wires around the world, not only dots and dashes, but also voice, images, and data. Such capabilities had been commonplace in fairy tales, but were beyond human reach until Hertz demonstrated radiowave transmission in 1888, 15 years after Maxwell's predictions. Marconi extended the technique to intercontinental distances.

Third came electronics and photonics — the ability to electrically manipulate individual electrons and atoms in vacuum and in matter so as to generate, amplify, manipulate, and detect electromagnetic signals. During the twentieth century vacuum tubes, diodes, transistors, integrated circuits, lasers, and superconductors all vastly extended the capabilities and applications of electromagnetics.

The fourth set of electrical phenomena involves cybernetics and informatics — the manipulation of electrical signals so complex that entirely new classes of functionality are obtained, such as optimum signal processing, computers, robotics, and artificial intelligence. This text focuses on the electromagnetic nature of the first three sets of phenomena and explores many of their most important applications.

Chapter 1 of this text begins with a brief review of the underlying laws of physics, followed by the Lorentz force law and the nature of electric and magnetic fields. Chapter 2 introduces electrodynamics and Maxwell's equations, leading to uniform plane waves in space and media, and definitions of power, energy, boundary conditions, and uniqueness. The next four chapters address static and quasistatic systems beginning with Chapter 3, which explores electromagnetics in the context of RLC circuits and devices. Chapter 4 addresses the more general behavior of quasistatic electric and magnetic fields in homogeneous and inhomogeneous media. Chapter 5 introduces electromagnetic forces while Chapter 6 addresses their application to motors, generators, actuators, and sensors.

The second half of the text focuses on electrodynamics and waves, beginning with TEM transmission lines in Chapters 7 and 8, and waves in media and at boundaries in Chapter 9. Antennas and radiation are treated in Chapters 10 and 11, while optical and acoustic systems are addressed in Chapters 12 and 13, respectively. Acoustics is introduced on its own merits and as a useful way to review electromagnetic wave phenomena such as radiation and resonance in a more physical and familiar context. The appendices list natural constants and review some of the prerequisite mathematics.

The rationalized international system of units (rationalized SI units) is used, which largely avoids factors of 4π. SI units emphasize meters (m), kilograms (kg), seconds (s), Amperes (A), and Kelvins (K); most other units can be expressed in terms of these. The SI system also favors units in multiples of 10^3 ; for example, it favors meters and millimeters over centimeters. The algebraic convention used here is that operations within parentheses are performed before others. Within parentheses and exponents and elsewhere, exponentiation is performed first, and multiplication before division; all these operations are performed before addition and subtraction.

1.1.2: Review of basic physical concepts and definitions

The few basic concepts summarized below are central to electromagnetics. These concepts include conservation of energy, power, and charge, and the notion of a photon, which conveys one quantum of electromagnetic energy. In addition, Newton's laws characterize the kinematics of charged particles and objects influenced by electromagnetic fields. The conservation laws also follow from Maxwell's equations, which are presented in Section 2.1 and, together with the Lorentz force law, compress all macroscopic electromagnetic behavior into a few concise statements.

This text neglects relativistic issues introduced when mass approaches the velocity of light or is converted to or from energy, and therefore we have *conservation of mass*: the total mass m within a closed envelope remains constant.

Conservation of energy requires that the total energy w_T [Joules] remains constant within any system closed so that no power enters or leaves, even though the form of the internally stored energy may be changing. This total energy w_T may include electric energy w_e, magnetic energy w_m, thermal energy w_{Th}, mechanical kinetic energy w_k, mechanical potential energy w_p, and energy in chemical, atomic, or other forms w_{other}; w_{other} is neglected here. Conservation of energy means:

$$\mathrm{W_T = w_e + w_m + w_k + w_p + w_{Th} + w_{other}}\ [\,\mathrm{Joules}\,] = \mathrm{constant} \tag{1.1.1}$$

In this text we generally use lower case letters to indicate totals, and upper case letters to indicate densities. Thus we represent total energy by w_T [J] and total energy density by W_T [J m^{-3}]. Similarly, f [N] denotes the total force on an object and F [N m^{-3}] denotes the force density.

Unfortunately the number of electromagnetic variables is so large that many letters are used in multiple ways, and sometimes the meaning must be extracted from the context. For example, the symbol f is used to signify both force and frequency.

Newton's law says that a one-Newton force f would cause an otherwise force-free kilogram mass to accelerate at one meter per second per second; this defines the *Newton*. One Newton is roughly the terrestrial gravitational force on a quarter-pound weight (e.g. the weight of the apple that allegedly fell on Newton's head, prompting him to conceive the law of gravity). Newton's law may be expressed as:

$$\mathrm{f = ma}\ [\mathrm{Newtons}] \tag{1.1.2}$$

where m is the mass of the object [kg] and a is the induced acceleration [ms^{-2}].

The unit of energy, the *Joule*, is the total energy w_T delivered to an object when a force f of one Newton is applied to it as it moves one meter in the direction z of the force. Therefore:

$$\mathrm{f} = \frac{\mathrm{dw_T}}{\mathrm{dz}} \tag{1.1.3}$$

The *kinetic energy* w_k of a mass m moving at velocity v is:

$$\mathrm{w_k} = \frac{1}{2}\mathrm{mv}^2\ [\mathrm{J}] \tag{1.1.4}$$

which, when added to its *potential energy* w_p, equals its total energy w_T relative to a motionless reference position; i.e.:

$$\mathrm{W_T = w_k + w_p} \tag{1.1.5}$$

It is easy to see that if w_p remains constant, then (1.1.3) and (1.1.4) are consistent with f = ma; that is, f = dw_T/dz = dw_k/dz = mv dv/dz = m(dz/dt)(dv/dz) = m dv/dt = ma.

Conservation of power means, for example, that the total power P_{in} [Js^{-1}] entering a closed volume must equal the rate of increase [Js^{-1}] of the total energy stored there; that is:

$$\mathrm{P_{in}[W]} = \frac{\mathrm{dw_T}}{\mathrm{dt}}\ \left[\mathrm{Js^{-1}}\right] \tag{1.1.6}$$

where dw_T/dt is the time derivative of w_T, and the units [Joules per second] are often replaced by their equivalent, Watts [W]. If dw_T/dt=0, then the power flowing into a closed volume must equal the power flowing out so that power is conserved. These laws also apply to electromagnetic power and energy, and their definition in terms of electromagnetic fields appears in Section 2.7.

In mechanical systems one watt is delivered to an object if it received one joule in one second. More generally the *mechanical power* P delivered to an object is $P = fv\,[W]$, where f is the only force [N] acting on the object, and v $\left[\mathrm{ms^{-1}}\right]$ is the object's velocity in the same direction as the force vector $overrightarrow$f. More generally,

$$P = \overrightarrow{f} \cdot \overrightarrow{v} \equiv fv\cos\theta\,[W] \tag{1.1.7}$$

where $\vec{v}$ is the velocity vector and θ is the angle between $\vec{f}$ and $\vec{v}$.

Conservation of momentum requires that the total momentum of a set of interacting masses mi remains constant if the set is free from external forces. The *momentum* of any object is mv [kg ms^{-1}], so in a force-free environment:

$$\mathrm{d}\left(\sum_i m_i v_i\right) \Big/ dt = 0 \tag{1.1.8}$$

The unit of charge, one *Coulomb*, is the charge conveyed by one Ampere flowing for one second, where the *Ampere* is the unit of electric current.

Photons carry the smallest unit of energy that can be conveyed by electromagnetic waves. The energy E of a single photon is:

$$\mathrm{e} = -1.6021 \times 10^{-19} \text{ Coulombs} \tag{1.1.9}$$

where h is Planck's constant (6.624×10^{-34} [J s]) and f is the photon frequency [Hz]. Sometimes it is more convenient to think of electromagnetic waves as continuous waves, and sometimes it is more convenient to think of them as consisting of particles (photons), each of energy E. The total power P conveyed by an electromagnetic wave at frequency f is therefore the number N of photons passing per second times the photon energy E:

$$\mathrm{E} = \mathrm{hf}\ [\mathrm{J}] \tag{1.1.10}$$

The frequency of a wave is simply related to its wavelength λ and the velocity of light c:

$$\mathrm{P} = \mathrm{N\ hf}\ \ [\mathrm{W}] \tag{1.1.11}$$

The frequency of a wave is simply related to its wavelength λ and the velocity of light c:

$$f = c/\lambda \tag{1.1.12}$$

Example 1.1.A

A typical fully charged 1-kilowatt-hour car battery can accelerate a perfectly efficient 1000-kg electric automobile to what maximum speed?

Solution

The battery energy w_e [J] equals 1000 watts times 3600 seconds (one kilowatt-hour). It also equals the maximum kinetic energy, wk = mv^2 /2, of the speeding automobile (mass = m = 1000, velocity = v) after the battery is totally drained. Therefore $w_k = 3.6\times10^6 \Rightarrow v = (2w_k/m)\ 0.5 = (7.2\times10^6 /1000)0.5 \cong 85$ m $s^{-1} \cong 190$ mph.

Example 1.1.B

A sunny day delivers ~1kw m^{-2}; to how many photons N per second per square meter does this correspond if we (incorrectly) assume they all have the same wavelength $\lambda = 5\times10^{-7}$ meters? (0.5 microns is in the visible band.)?

Solution

Power = Nhf = Nhc/λ = 1 kw, so $N = 10^3\ \lambda/hc \cong 10^3 \times 5\times10^{-7}/(6.6\times10^{-34} \times 3\times10^8) \cong 2.5\times10^{20}$ photons m^{-2} s^{-1}.