5.6: Photonic Forces

Photonic forces arise whenever electromagnetic waves are absorbed or reflected by objects, and can be found using either wave or photon paradigms. Section 5.2.2 derived the magnetic pressure $\overrightarrow{\mathrm{P}}_{\mathrm{m}}$ (5.2.13) applied by a surface magnetic field $H_s(t)$ that is parallel to a flat perfect conductor in the x-y plane:

$$\overrightarrow{\mathrm{P}}_{\mathrm{m}}=\hat{z} \mu_{\mathrm{o}} \mathrm{H}_{\mathrm{s}}^{2} / 2\ \left[\mathrm{Nm}^{-2}\right] \qquad \text{(magnetic pressure on perfect conductor)} \tag{5.6.1}$$

Thus this instantaneous magnetic pressure perpendicular to the conductor surface equals the adjacent magnetic energy density ([N m^{-2}] = [J m^{-3}]).

In the sinusoidal steady state the time average pressure is half the peak instantaneous value given by (5.6.1), where $H_s(t) = H_s \cos \omega t$. This average pressure on a perfectly reflecting conductor can also be expressed in terms of the time-average Poynting vector $\langle\overrightarrow{\mathrm{S}}(\mathrm{t})\rangle$ of an incident wave characterized by $H_+ \cos \omega t$:

$$\left\langle\overrightarrow{\mathrm{P}}_{\mathrm{m}}(\mathrm{t})\right\rangle=\hat{z} \mu_{\mathrm{o}}\left\langle\mathrm{H}_{\mathrm{s}}^{2}(\mathrm{t})\right\rangle / 2=2\langle\overrightarrow{\mathrm{S}}(\mathrm{t})\rangle / \mathrm{c} \quad\left[\mathrm{Nm}^{-2}\right] \tag{5.6.2}$$

where $\mathrm{H}_{\mathrm{s}}=2 \mathrm{H}_{+}$ and $\langle\mathrm{S}(\mathrm{t})\rangle=\eta_{0} \mathrm{H}_{+}^{2} / 2$; the impedance of free space $\eta_{\mathrm{o}}=\mu_{\mathrm{o}} / \mathrm{c}$.

It is now easy to relate $\langle\mathrm{S}(\mathrm{t})\rangle$ to the photon momentum flux, which also yields pressure. We recall[17] that:

$$\text{photon momentum } \mathrm{M}=\frac{\mathrm{hf}}{\mathrm{c}} \quad\left[\mathrm{Nms}^{-1}\right] \tag{5.6.3}$$

The momentum transferred to a mirror upon perfect reflection of a single photon at normal incidence is therefore $2hf/c$.

We recall from mechanics that the force f required to change momentum mv is:

$$f=\frac{d(m v)}{d t}\ [\mathrm{N}] \tag{5.6.4}$$

so that the total *radiation pressure* on a perfect mirror reflecting directly backwards n photons [$s^{-1}m^{-2}$] is:

consistent with (5.6.2). Thus we have shown that both the Lorentz force method and the photonic force method yield the same pressure on perfectly reflecting mirrors; $P_m = P^r$. The factor of two in (5.6.5) arises because photon momentum is not zeroed but reversed by a mirror. If these photons were absorbed rather than reflected, the rate of momentum transfer to the absorber would be halved. In general if the incident and normally reflected power densities are $\langle S_1\rangle$ and $\langle S_2\rangle$, respectively, then the average radiation pressure on the mirror is:

$$\langle P\rangle=\frac{\langle S_1+S_2\rangle}{c} \tag{5.6.5}$$

If the photons are incident at an angle, the momentum transfer is reduced by the cosine of the angle of incidence and reflection. And if the mirror is partially transparent, the momentum transfer is reduced by that fraction of the photon momentum that passes through unaltered.

[17] A crude plausibility argument for (5.6.3) is the following. The energy of a photon is hf [J], half being magnetic and half being electric. We have seen in (5.2.1) and (5.2.13) that only the magnetic fields contribute to the Lorentz force on a normal reflecting conductor for which both $E_\perp$ and $H_\perp$ = 0, so we might notionally associate hf/2 with the "kinetic energy of a photon", where kinetic energy is linked to momentum. If photons had mass m, this notional kinetic energy hf/2 would equal $mc^2/2$, and the notional associated momentum mc of a photon would then equal hf/c, its actual value.

Consider the simple example of a reflective *solar sail* blown by radiation pressure across the solar system, sailing from planet to planet. At earth the *solar radiation* intensity is ~1400 W/m2 , so (5.6.6) yields, for example, the total force f on a sail of projected area A intercepting one square kilometer of radiation:

$$\mathrm{f}=\mathrm{A}\langle\mathrm{P}\rangle=\mathrm{A} 2\langle\mathrm{S}(\mathrm{t})\rangle / \mathrm{c} \leq 10^{6} \times 2 \times 1400 /\left(3 \times 10^{8}\right) \cong 9\ [\mathrm{N}] \tag{5.6.6}$$

A sail this size one micron thick and having the density of water would have a mass m of 1000 kg. Since the sail velocity v = at = (f/m)t, where a is acceleration and t is time, it follows that after one year the accumulated velocity of a sail facing such constant pressure in vacuum could be as much as $(9/1000)3\times10^{7} \cong 3\times10^{5}\ ms^{-1} = c/1000$. Of course the solar photon pressure declines as the square of the solar distance, and solar gravity would also act on such sails.

Example $5.6.A$

What force F [N] is exerted on a 3-watt flashlight ($\lambda \cong 0.5$ microns) as a result of the exiting photons?

Solution

E = hf and power P = Nhf = 3 watts, where N is the number of photons per second. The force F = Nhf/c, where hf/c is the momentum of a single photon, and N = 3/hf here. So F = 3/c = 10^{-8} Newtons. A Newton approximates the gravitational force on the quarter-pound package of fig newtons. This force pushes the flashlight in the direction opposite to that of the light beam.