5.1: Forces on Free Charges and Currents

5.1.1: Lorentz force equation and introduction to force

The Lorentz force equation (1.2.1) fully characterizes electromagnetic forces on stationary and moving charges. Despite the simplicity of this equation, it is highly accurate and essential to the understanding of all electrical phenomena because these phenomena are observable only as a result of forces on charges. Sometimes these forces drive motors or other actuators, and sometimes they drive electrons through materials that are heated, illuminated, or undergoing other physical or chemical changes. These forces also drive the currents essential to all electronic circuits and devices.

When the electromagnetic fields and the location and motion of free charges are known, the calculation of the forces acting on those charges is straightforward and is explained in Sections 5.1.2 and 5.1.3. When these charges and currents are confined within conductors instead of being isolated in vacuum, the approaches introduced in Section 5.2 can usually be used. Finally, when the charges and charge motion of interest are bound within stationary atoms or spinning charged particles, the Kelvin force density expressions developed in Section 5.3 must be added. The problem usually lies beyond the scope of this text when the force-producing electromagnetic fields are not given but are determined by those same charges on which the forces are acting (e.g., plasma physics), and when the velocities are relativistic.

The simplest case involves the forces arising from known electromagnetic fields acting on free charges in vacuum. This case can be treated using the *Lorentz force equation* (5.1.1) for the *force vector* $\overrightarrow{\mathrm{f}}$ acting on a charge q [Coulombs]:

$$\overrightarrow{\mathrm{f}}=\mathrm{q}\left(\overrightarrow{\mathrm{E}}+\overrightarrow{\mathrm{v}}\times\mu_{o}\overrightarrow{\mathrm{H}}\right)\quad[\text{Newtons}]\qquad\text{(Lorentz force equation)}\tag{5.1.1}$$

where $\overrightarrow{\mathrm{E}}$ and $\overrightarrow{\mathrm{H}}$ are the local electric and magnetic fields and $\overrightarrow{\mathrm{v}}$ is the charge velocity vector [m s^{-1}].

5.1.2: Electric Lorentz forces on free electrons

The *cathode-ray tube* (CRT) used for displays in older computers and television sets, as illustrated in Figure 5.1.1, provides a simple example of the Lorentz force law (5.1.1). Electrons thermally excited by a heated *cathode* at -V volts escape at low energy and are accelerated in vacuum at acceleration $\overrightarrow{\mathrm{a}}$ [m s^{-2}] toward the grounded *anode* by the electric field $\overrightarrow{\mathrm{E}}\cong-\hat{z}\mathrm{V}/\mathrm{s}$ between anode and cathode[13]; V and s are the voltage across the tube and the cathode-anode separation, respectively. In electronics the anode always has a more positive potential Φ than the cathode, by definition.

[13] The anode is grounded for safety reasons; it lies at the tube face where users may place their fingers on the other side of the glass faceplate. Also, the cathode and anode are sometimes shaped so that the electric field $\overrightarrow{\mathrm{E}}$, the force $\overrightarrow{\mathrm{f}}$, and the acceleration $\overrightarrow{\mathrm{a}}$ are functions of z instead of being constant; i.e., $\overrightarrow{\mathrm{E}}\neq-\hat{z}V/D$.

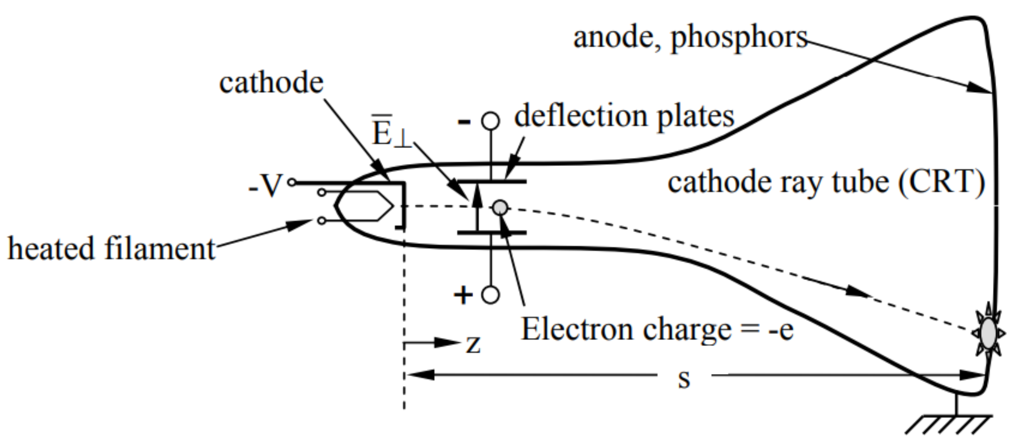

Figure 5.1.1: Cathode ray tube.

The acceleration $\overrightarrow{\mathrm{a}}$ is governed by *Newton's law*:

$$\overrightarrow{\mathrm{f}} = \mathrm{m}\overrightarrow{a} \qquad \text{(Newton's law)} \tag{5.1.2}$$

where m is the mass of the unconstrained accelerating particle. Therefore the acceleration a of the electron charge q = -e in an electric field E = V/s is:

$$\mathrm{a} = \mathrm{f}/\mathrm{m} = \mathrm{qE}/\mathrm{m} \cong \mathrm{eV}/\mathrm{ms} \quad \left[\mathrm{ms}^{-2}\right] \tag{5.1.3}$$

The subsequent velocity $\overrightarrow{\mathrm{v}}$ and position z of the particle can be found by integration of the acceleration $\hat{z}a$:

$$\overrightarrow{\mathrm{v}} = \int_0^{\mathrm{t}} \overrightarrow{\mathrm{a}}(\mathrm{t})\mathrm{dt} = \overrightarrow{\mathrm{v}}_{\mathrm{o}} + \hat{z}\mathrm{at} \quad \left[\mathrm{ms}^{-1}\right] \tag{5.1.4}$$

$$z = z_0 + \hat{z} \bullet \int_0^t \overrightarrow{v}(t)dt = z_0 + \hat{z} \bullet \bar{v}_0 t + at^2/2\ [\mathrm{m}] \tag{5.1.5}$$

where we have defined the initial electron position and velocity at t = 0 as z_o and $\overrightarrow{\mathrm{v}}_{\mathrm{o}}$, respectively.

The increase w_k in the kinetic energy of the electron equals the accumulated work done on it by the electric field $\overrightarrow{\mathrm{E}}$. That is, the increase in the kinetic energy of the electron is the product of the constant force f acting on it and the distance s the electron moved in the direction of $\overrightarrow{\mathrm{f}}$ while experiencing that force. If s is the separation between anode and cathode, then:

$$\mathrm{w_k} = \mathrm{fs} = (\mathrm{eV}/\mathrm{s})\mathrm{s} = \mathrm{eV}\ [\mathrm{J}] \tag{5.1.6}$$

Thus the kinetic energy acquired by the electron in moving through the potential difference V is eV Joules. If V = 1 volt, then wk is one "*electron volt*", or "e" Joules, where e ≅ 1.6 × 10^{-19} Coulombs. The kinetic energy increase equals eV even when $\overrightarrow{\mathrm{E}}$ is a function of z because:

$$\mathrm{w_k} = \int_0^{\mathrm{D}} \mathrm{eE_z dz} = \mathrm{eV} \tag{5.1.7}$$

Typical values for V in television CRT's are generally less than 50 kV so as to minimize dangerous x-rays produced when the electrons impact the phosphors on the CRT faceplate, which is often made of x-ray-absorbing leaded glass.

Figure 5.1.1 also illustrates how time-varying lateral electric fields $\overrightarrow{\mathrm{E}}_{\perp}(\mathrm{t})$ can be applied by deflection plates so as to scan the electron beam across the CRT faceplate and "paint" the image to be displayed. At higher tube voltages V the electrons move so quickly that the lateral electric forces have no time to act, and magnetic deflection is used instead because lateral magnetic forces increase with electron velocity v.

5.1.3: Magnetic Lorentz forces on free charges

An alternate method for laterally scanning the electron beam in a CRT utilizes magnetic deflection applied by coils that produce a magnetic field perpendicular to the electron beam, as illustrated in Figure 5.1.2. The magnetic Lorentz force on the charge q = -e (1.6021×10^{-19} Coulombs) is easily found from (5.1.1) to be:

$$\overrightarrow{\mathrm{f}} = -\mathrm{e}\overrightarrow{\mathrm{v}} \times \mu_{\mathrm{o}}\overrightarrow{\mathrm{H}}\ [\mathrm{N}] \tag{5.1.8}$$

Thus the illustrated CRT electron beam would be deflected upwards, where the magnetic field $\overrightarrow{\mathrm{H}}$ produced by the coil is directed out of the paper; the magnitude of the force on each electron is $ev\mu_oH$ [N].

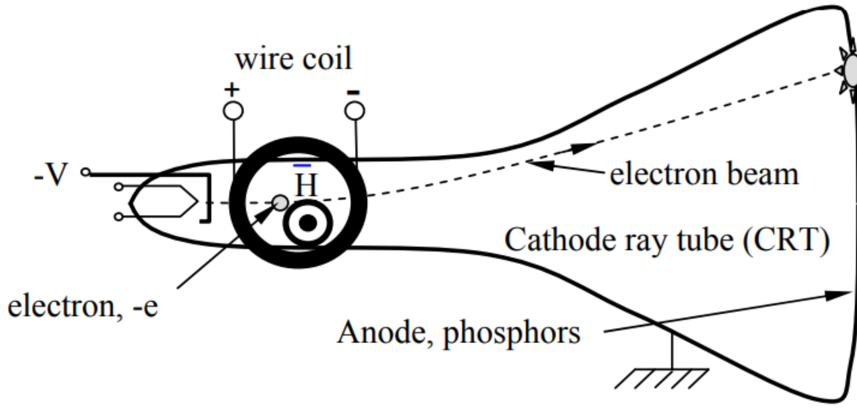

Figure 5.1.2: Magnetic deflection of electrons in a cathode ray tube.

The lateral force on the electrons $ev\mu_oH$ can be related to the CRT voltage V. Electrons accelerated from rest through a potential difference of V volts have kinetic energy eV [J], where:

$$\mathrm{eV} = \mathrm{mv}^2/2 \tag{5.1.9}$$

Therefore the electron velocity $v = (2eV/m)^{0.5}$, where m is the electron mass (9.107×10^{-31} kg), and the lateral deflection increases with tube voltage V, whereas it decreases if electrostatic deflection is used instead.

Another case of magnetic deflection is illustrated in Figure 5.1.3 where a free electron moving perpendicular to a magnetic field $\overrightarrow{\mathrm{B}}$ experiences a force $\overrightarrow{\mathrm{f}}$ orthogonal to its velocity vector $\overrightarrow{\mathrm{v}}$, since $\overrightarrow{\mathrm{f}} = \mathrm{q}\overrightarrow{\mathrm{v}} \times \mu_o \overrightarrow{\mathrm{H}}$.

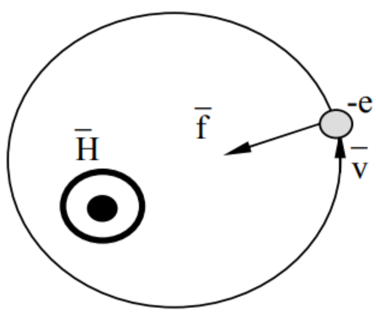

Figure 5.1.3: Cyclotron motion of an electron.

This force $|\overrightarrow{\mathrm{f}}|$ is always orthogonal to $\overrightarrow{\mathrm{v}}$ and therefore the trajectory of the electron will be circular with radius R at angular frequency ω_e [radians s^{-1}]:

$$|\mathrm{f}| = \mathrm{ev}\,\mu_0 H = m_e a = m_e \omega_e^2 R = m_e v \omega_e \tag{5.1.10}$$

where $v = \omega_e R$. We can solve (5.1.9) for this "electron *cyclotron frequency*" ω_e:

$$\omega_{\mathrm{e}} = \mathrm{e}\mu_o \mathrm{H}/\mathrm{m_e} \qquad \text{(electron cyclotron frequency)} \tag{5.1.11}$$

which is independent of v and the electron energy, provided the electron is not relativistic. Thus the magnitudes of magnetic fields can be measured by observing the radiation frequency ω_e of free electrons in the region of interest.

Example 5.1.B: Cyclotron Motion

What is the radius r_e of cyclotron motion for a 100 e.v. free electron in the terrestrial magnetosphere[14] where $B \cong 10^{-6}$ Tesla? What is the radius r_p for a free proton with the same energy? The masses of electrons and protons are $\sim 9.1\times10^{-31}$ and 1.7×10^{-27} kg, respectively.

Solution

The magnetic force on a charged particle is $qv\mu_oH = ma = mv^2/r$, where the velocity v follows from (5.1.9): $eV = mv^2/2 \Rightarrow v = (2eV/m)^{0.5}$. Solving for r_e yields

$$\begin{aligned} r_e &= m_{eV}/e\mu_o H \\ &= (2Vm/e)^{0.5}/\mu_o H \\ &\cong \left(2 \times 100 \times 9.1 \times 10^{-31}/1.6 \times 10^{-19}\right)^{0.5}/10^{-6} \\ &\cong 34 \text{ m} \end{aligned}$$

for electrons and ~2.5 km for protons.

[14] The magnetosphere extends from the ionosphere to several planetary radii; particle collisions are rare compared to the cyclotron frequency.