

2.5: Electric and Magnetic Fields in Media

2.5.1: Maxwell's equations and media

The great success of Maxwell's equations lies partly in their simple prediction of electromagnetic waves and their simple characterization of materials in terms of conductivity σ [Siemens m^{-1}], permittivity ϵ [Farads m^{-1}], and permeability μ [Henries m^{-1}]. In vacuum we find $\sigma = 0$, $\epsilon = \epsilon_0$, and $\mu = \mu_0$, where $\epsilon_0 = 8.8542 \times 10^{-12}$ and $\mu_0 = 4\pi \times 10^{-7}$. For reference, Maxwell's equations are:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (2.5.1)$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad (2.5.2)$$

$$\nabla \cdot \vec{D} = \rho \quad (2.5.3)$$

$$\nabla \cdot \vec{B} = 0 \quad (2.5.4)$$

The electromagnetic properties of most media can be characterized by the *constitutive relations*:

$$\vec{D} = \epsilon \vec{E} \quad (2.5.5)$$

$$\vec{B} = \mu \vec{H} \quad (2.5.6)$$

$$\vec{J} = \sigma \vec{E} \quad (2.5.7)$$

In contrast, the nano-structure of media can be quite complex and requires quantum mechanics for its full explanation. Fortunately, simple classical approximations to atoms and molecules suffice to understand the origins of σ , ϵ , and μ , as discussed below in that sequence.

2.5.2: Conductivity

Conduction in metals and *n-type semiconductors*⁶ involves free electrons moving many atomic diameters before they lose momentum by interacting with atoms or other particles. Acceleration induced by the small applied electric field inside the conductor restores electron velocities to produce an equilibrium current. The total current density \vec{J} [A m^{-2}] is proportional to the product of the average electron velocity \vec{v} [m s^{-1}] and the number density n [m^{-3}] of free electrons. A related conduction process occurs in ionic liquids, where both negative and positive ions can carry charge long distances

In *metals* there is approximately one free electron per atom, and in warm *n-type semiconductors* there is approximately one free electron per donor atom, where the sparse donor atoms are easily ionized thermally. Since, for non-obvious reasons, the average electron velocity $\langle \vec{v} \rangle$ is proportional to \vec{E} , therefore $\vec{J} = -en_3 \langle \vec{v} \rangle = \sigma \vec{E}$, as stated in (2.5.7). As the conductivity σ approaches infinity the electric field inside a conductor approaches zero for any given current density \vec{J} .

Warm donor atoms in *n-type semiconductors* can be easily ionized and contribute electrons to the conduction band where they move freely. Only certain types of impurity atoms function as donors--those that are most easily ionized. As the density of donor atoms approaches zero and as temperature declines, the number of free electrons and the conductivity approach very low values that depend on temperature and any alternative ionization mechanisms that are present.

In *p-type semiconductors* the added impurity atoms readily trap nearby free electrons to produce a negative ion; this results in a corresponding number of positively ionized semiconductor atoms that act as "holes". As a result any free electrons typically move only short distances before they are trapped by one of these holes. Moreover, the threshold energy required to move an electron from a neutral atom to an adjacent positive ion is usually less than the available thermal energy, so such transfers occur rapidly, causing the hole to move quickly from place to place over long distances. Thus holes are the *dominant charge carriers* in *p-type semiconductors*, whereas electrons dominate in *n-type semiconductors*.

⁶ *n-type semiconductors* (e.g., silicon, germanium, gallium arsenide, indium phosphide, and others) are doped with a tiny percentage of donor atoms that ionize easily at temperatures of interest, releasing mobile electrons into the conduction band of the

semiconductor where they can travel freely across the material until they recombine with another ionized donor atom. The conduction band is not a place; it refers instead to an energy and wave state of electrons that enables them to move freely. The conductivity of semiconductors therefore increases with temperature; they become relatively insulating at low temperatures, depending on the ionization potentials of the impurity atoms.

More broadly, *semiconductors* have a *conduction band* in which free electrons can propagate long distances; this band is separated by an energy of one or a few electron volts from the *valence band* in which electrons cannot move. The conduction band is not a location, it is a family of possible electron wave states. When electrons are excited from the valence band to the conduction band by some energetic process, they become free to move in response to electric fields. Semiconductor conductivity is approximately proportional to the number of free electrons or holes produced by the scarce impurity atoms, and therefore to the doping density of those impurity atoms. Easily ionized impurity atoms are the principal mechanism by which electrons enter the conduction band, and impurities that readily trap adjacent electrons are the principal mechanism by which holes enter and move in the valence band. Semiconductors are discussed further in Section 8.2.4. The current leakage processes in insulators vaguely resemble electron and hole conduction in semiconductors, and can include weak surface currents as well as bulk conduction; microscopic flaws can also increase conductivity. The conductivities of typical materials are listed in Table 2.5.1.

Table 2.5.1: Nominal conductivities σ of common materials [Siemens m^{-1}].

paraffin	$10^{-14} - 10^{-16}$	sea water	3-5
glass	10^{-12}	iron	10^7
dry earth	$10^{-4} - 10^{-5}$	copper	5.8×10^7
distilled water	2×10^{-4}	silver	6.14×10^7

In some exotic materials the conductivity is a function of direction and can be represented by the 3×3 matrix $\vec{\sigma}$; such materials are not addressed here, but Section 2.5.3 addresses similar issues in the context of permittivity ϵ .

Some materials exhibit *superconductivity*, or infinite conductivity. In these materials pairs of electrons become loosely bound magnetically and move as a unit called a *Cooper pair*. Quantum mechanics prevents these pairs from colliding with the lattice and losing energy. Because the magnetic binding energy for these pairs involves electron spins, it is quite small. Normal conductivity returns above a threshold *critical temperature* at which the pairs are shaken apart, and it also returns above some threshold *critical magnetic field* at which the magnetic bonds coupling the electrons break as the electron spins all start to point in the same direction. Materials having critical temperatures above 77K (readily obtained in cryogenic refrigerators) are difficult to achieve. The finite number of such pairs at any temperature and magnetic field limits the current to some maximum value; moreover that current itself produces a magnetic field that can disrupt pairs. Even a few pairs can move so as to reduce electric fields to zero by short circuiting the normal electrons until the maximum current carrying capacity of those pairs is exceeded. If the applied fields have frequency $f > 0$, then the Cooper pairs behave much like collisionless electrons in a plasma and therefore the applied electric field can penetrate that plasma to its skin depth, as discussed in Section 9.8. Those penetrating electric fields interact with a small number of normal electrons to produce tiny losses in superconductors that increase with frequency.

2.5.3: Permittivity

The *permittivity* ϵ_0 of free space is 8.854×10^{-12} farads/meter, where $\vec{D} = \epsilon_0 \vec{E}$. The permittivity ϵ of any material deviates from ϵ_0 for free space if applied electric fields induce electric dipoles in the medium; such dipoles alter the applied electric field seen by neighboring atoms. Electric fields generally distort atoms because \vec{E} pulls on positively charged nuclei ($f = q\vec{E}$ [N]) and repels the surrounding negatively charged electron clouds. The resulting small offset \vec{d} of each atomic nucleus of charge $+q$ relative to the center of its associated electron cloud produces a tiny electric dipole in each atom, as suggested in Figure 2.5.1(a). In addition, most asymmetric molecules are permanently polarized, such as H_2O or NH_3 , and can rotate within fluids or gases to align with an applied field. Whether the dipole moments are induced, or permanent and free to rotate, the result is a complete or partial alignment of dipole moments as suggested in Figure 2.5.1(b).

These polarization charges generally cancel inside the medium, as suggested in Figure 2.5.1(b), but the immobile atomic dipoles on the outside surfaces of the medium are not fully canceled and therefore contribute the *surface polarization charge* ρ_{sp} .

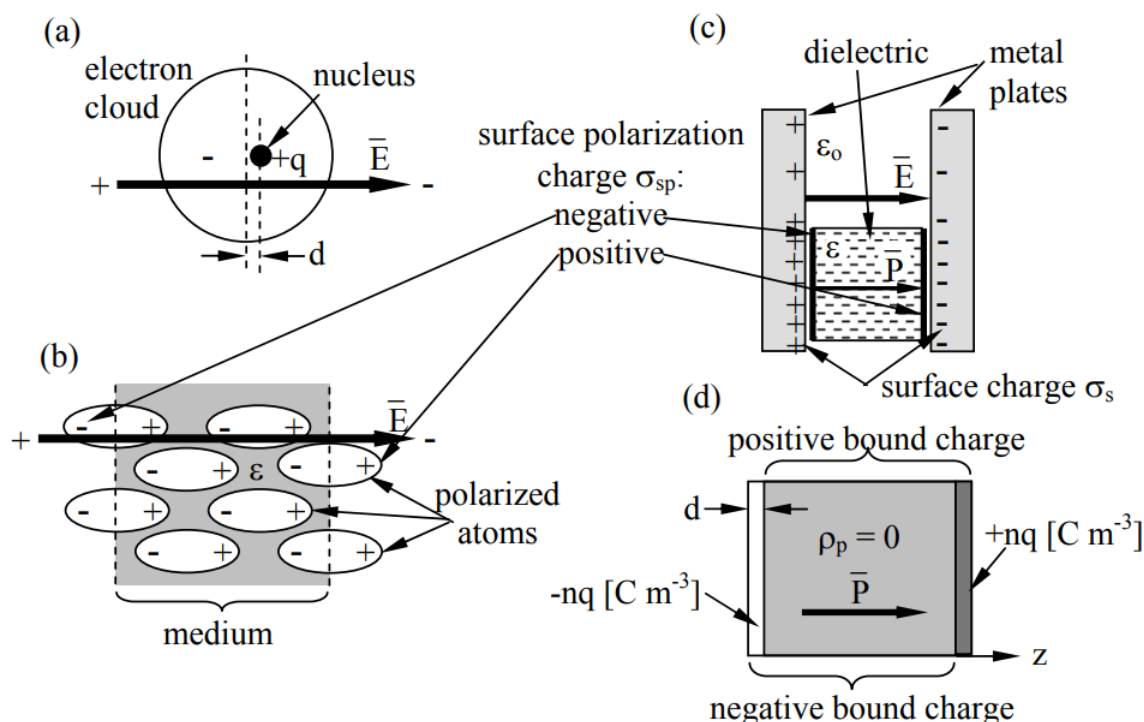


Figure 2.5.1: Polarized media.

Figure 2.5.1(c) suggests how two charged plates might provide an electric field \vec{E} that polarizes a dielectric slab having permittivity $\epsilon > \epsilon_0$. [The electric field \vec{E} is the same in vacuum as it is inside the dielectric (assuming no air gaps) because the path integral of $\vec{E} \cdot d\vec{s}$ from plate to plate equals their voltage difference V in both cases. The electric displacement overlinevector $\vec{D}_e = \epsilon \vec{E}$ and therefore differs.] We associate the difference between $\vec{D}_o = \epsilon_0 \vec{E}$ (vacuum) and $\vec{D}_\epsilon = \epsilon \vec{E}$ (dielectric) with the *electric polarization overlinevector* \vec{P} , where:

$$\vec{D} = \epsilon \vec{E} = \epsilon_0 \vec{E} + \vec{P} = \epsilon_0 \vec{E} (1 + \chi) \quad (2.5.8)$$

The *polarization overlinevector* \vec{P} is defined by (2.5.8) and is normally parallel to \vec{E} in the same direction, as shown in Figure 2.5.1(c); it points from the negative surface polarization charge to the positive surface polarization charge (unlike \vec{E} , which points from positive charges to negative ones). As suggested in (2.5.8), $\vec{P} = \vec{E} \epsilon_0 \chi$, where χ is defined as the dimensionless susceptibility of the dielectric. Because nuclei are bound rather tightly to their electron clouds, χ is generally less than 3 for most atoms, although some molecules and crystals, particularly in fluid form, can exhibit much higher values. It is shown later in (2.5.13) that \vec{P} simply equals the product of the number density n of these dipoles and the average overlinevector *electric dipole moment* of each atom or molecule, $\vec{p} = q \vec{d}$, where \vec{d} is the offset (meters) of the positive charge relative to the negative charge:

$$\vec{P} = nq \vec{d} \text{ [Cm}^{-2}\text{]} \quad (2.5.9)$$

where ρ_f is the free charge density [C m⁻³]

We can derive a relation similar to (2.5.10) for \vec{P} by treating materials as distributed bound positive and negative charges with vacuum between them; the net bound charge density is designated the *polarization charge density* ρ_p [C m⁻³]. Then in the vacuum between the charges $\vec{D} = \epsilon_0 \vec{E}$ and (2.5.10) becomes:

$$\epsilon_0 \nabla \cdot \vec{E} = \rho_f + \rho_p \quad (2.5.10)$$

From $\nabla \cdot (2.5.8)$, we obtain $\nabla \cdot \vec{D} = \epsilon_0 \nabla \cdot \vec{E} + \nabla \cdot \vec{P} = \rho_f$. Combining this with (2.5.11) yields:

$$\nabla \cdot \vec{P} = -\rho_p \quad (2.5.11)$$

The negative sign in (2.5.12) is consistent with the fact that \vec{P} , unlike \vec{E} , is directed from negative to positive polarization charge, as noted earlier.

Outside a polarized dielectric the polarization \vec{P} is zero, as suggested by Figure 2.5.1(d). Note that the net polarization charge density is $\pm nq$ for only an atomic-scale distance d at the boundaries of the dielectric, where we model the positive and negative charge distributions within the medium as continuous uniform rectilinear slabs of density $\pm nq$ [C m⁻³]. These two slabs are offset by the distance d . If \vec{P} is in the z direction and arises from n dipole moments $\vec{p} = q \vec{d}$ per cubic meter, where \vec{d} is the offset [m] of the positive charge relative to the negative charge, then (2.5.12) can be integrated over a volume V that encloses a unit area of the left-hand face of a polarized dielectric [see Figure 2.5.1(d)] to yield the polarization overlinevector \vec{P} inside the dielectric:

$$\vec{P} = \int_V \nabla \cdot \vec{P} dv = - \int_V \rho_p dv = nq \vec{d} \quad (2.5.12)$$

The first equality of (2.5.13) involving \vec{P} follows from Gauss's divergence theorem: $\int_V \nabla \cdot \vec{P} dv = \int_A \vec{P} \cdot \hat{z} da = P_z$ if $A = 1$. Therefore, $\vec{P} = nq \vec{d}$, proving (2.5.9).

When the electric displacement \vec{D} overlineineies with time it becomes displacement current, $\partial \vec{D} / \partial t$ [A/m²], analogous to \vec{J} , as suggested by Ampere's law: $\nabla \times \vec{H} = \vec{J} + \partial \vec{D} / \partial t$. For reference, Table 2.5.2 presents the *dielectric constants* ϵ/ϵ_0 for some common materials near 1 MHz, after Von Hippel (1954).

Table 2.5.2: Dielectric constants ϵ/ϵ_0 of common materials near 1 MHz.

vacuum	1.0	fused quartz	3.78
fir wood	1.8 – 2.0	ice	4.15
Teflon, petroleum	2.1	pyrex glass	5.1
vaseline	2.16	aluminum oxide	8.8
paper	2 – 3	ethyl alcohol	24.5
polystyrene	2.55	water	81.0
sandy soil	2.59	titaniumdioxide	100.0

Most dielectric materials are lossy because oscillatory electric fields dither the directions and magnitudes of the induced electric dipoles, and some of this motion heats the dielectric. This effect can be represented by a frequency-dependent complex permittivity $\underline{\epsilon}$, as discussed further in Section 9.5. Some dielectrics have direction-dependent permittivities that can be represented by $\underline{\underline{\epsilon}}$; such anisotropic materials are discussed in Section 9.6. Lossy anisotropic materials can be characterized by $\underline{\underline{\epsilon}}$.

Some special dielectric media are spontaneously polarized, even in the absence of an externally applied \vec{E} . This occurs for highly polarizable media where orientation of one electric dipole in the media can motivate its neighbors to orient themselves similarly, forming domains of atoms, molecules, or crystal unit cells that are all polarized the same. Such spontaneously polarized domains are illustrated for magnetic materials in Figure 2.5.2. As in the case of ferromagnetic domains, in the absence of externally applied fields, domain size is limited by the buildup of stored field energy external to the domain; adjacent domains are oriented so as to largely cancel each other. Such ferroelectric materials have large effective values of ϵ , although \vec{D} saturates if \vec{E} is sufficient to produce ~100% alignment of polarization. They can also exhibit hysteresis as do the ferromagnetic materials discussed in Section 2.5.4.

Example 2.5.A

What are the free and polarization charge densities ρ_f and ρ_p in a medium having conductivity $\sigma = \sigma_0(1+z)$, permittivity $\epsilon = 3\epsilon_0$, and current density $\vec{J} = \hat{z}J_0$?

Solution

$$\vec{J} = \sigma \vec{E}, \text{ so } \vec{E} = \hat{z}J_0(1+z)/\sigma_0 = \epsilon_0 \vec{E} + \vec{P}.$$

$$\text{From (2.5.10) } \rho_f = \nabla \cdot \vec{D} = (\partial/\partial z) [3\varepsilon_o J_o(1+z)/\sigma_o] = 3\varepsilon_o J_o/\sigma_o \text{ [Cm}^{-3}\text{]}$$

$$\begin{aligned} \text{From (2.5.12) } \rho_p &= -\nabla \cdot \vec{P} = -\nabla \cdot (\varepsilon - \varepsilon_o) \vec{E} = -(\partial/\partial z) 2\varepsilon_o J_o(1+z)/\sigma_o \\ &= 2\varepsilon_o J_o/\sigma_o \text{ [Cm}^{-3}\text{]} \end{aligned}$$

2.5.4: Permeability

The permeability μ_o of free space is $4\pi 10^{-7}$ o Henries/meter by definition, where $\vec{B} = \mu \vec{H}$. The permeability μ of matter includes the additional contributions to B from atomic magnetic dipoles aligning with the applied \vec{H} . These magnetic dipoles and their associated magnetic fields arise either from electrons orbiting about atomic nuclei or from spinning charged particles, where such moving charge is current. All electrons and protons have spin $\pm 1/2$ in addition to any orbital spin about the nucleus, and the net spin of an atom or nucleus can be non-zero. Their magnetic fields are linked to their equivalent currents by Ampere's law, $\nabla \times \vec{H} = \vec{J} + \partial \vec{D}/\partial t$. Quantum theory describes how these magnetic moments are quantized at discrete levels, and for some devices quantum descriptions are necessary. In this text we average over large numbers of atoms, so that $\vec{B} = \mu \vec{H}$ accurately characterizes media, and quantum effects can be ignored.

In any medium the cumulative contribution of induced magnetic dipoles to \vec{B} is characterized by the magnetization \vec{M} , which is defined by:

$$\vec{B} = \mu \vec{H} = \mu_o (\vec{H} + \vec{M}) = \mu_o \vec{H} (1 + \chi_m) \quad (2.5.13)$$

where χ_m is the *magnetic susceptibility* of the medium. Because of quantum effects χ_m for *diamagnetic* materials is slightly negative so that $\mu < \mu_o$; examples include silver, copper, and water, as listed in Table 2.5.3. The table also lists representative *paramagnetic* materials, which have slightly positive magnetic susceptibilities, and ferromagnetic materials, which have very large susceptibilities (e.g., cobalt, etc.).

Table 2.5.3: Approximate relative permeabilities μ/μ_o of common materials.

bismuth	0.99983	aluminum	1.00002
silver	0.99998	cobalt	250
lead	0.999983	nickel	600
copper	0.999991	mild steel	2000
water	0.999991	iron	5000
vacuum	1.000000	mumetal	100,000
air	1.0000004	supermalloy	1,000,000

Figure 2.5.2(c). Since domain walls cannot easily move across grain walls, the granular structure of the material can be engineered to control magnetic properties. If domain walls move easily, the magnetic susceptibility χ_m is large.

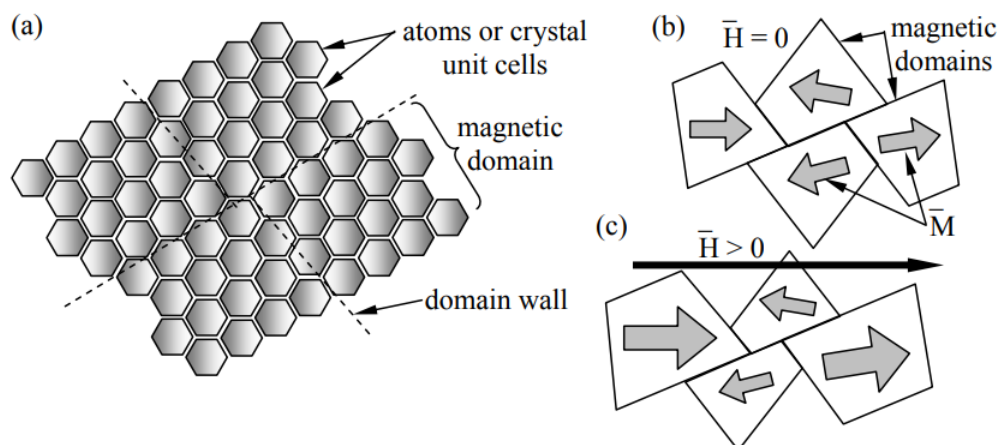


Figure 2.5.2: Magnetic domains in ferromagnetic materials.

At sufficiently high magnetic fields all domains will expand to their maximum size and/or rotate in the direction of \vec{H} . This corresponds to the maximum value of \vec{M} and *magnetic saturation*. The resulting typical non-linear behavior of the *magnetization curve* relating B and H for ferromagnetic materials is suggested in Figure 2.5.3(a). The slope of the B vs. H curve is $\sim\mu$ near the origin and $\sim\mu_0$ beyond saturation. If the domains resist change and dissipate energy when doing so, the *hysteresis curve* of Figure 2.5.3(b) results. It can be shown that the area enclosed by the figure is the energy dissipated per cycle as the applied \vec{H} oscillates.

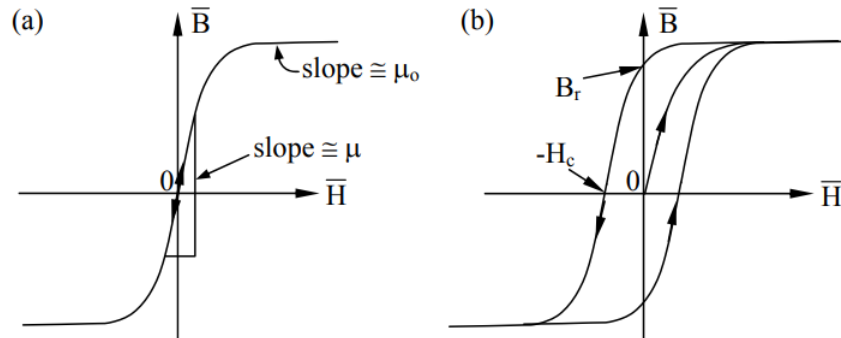


Figure 2.5.3: Magnetization curve and hysteresis loop for a ferromagnetic material.

Hard magnetic materials have large values of *residual flux density* B_r and magnetic coercive force or coercivity H_c , as illustrated. B_r corresponds to the magnetic strength B of this *permanent magnet* with no applied H. The magnetic energy density $W_m = \vec{B} \cdot \vec{H} / 2 \cong 0$ inside permanent magnets because $\vec{H} = 0$, while $W_m = \mu_0 H^2 / 2$ [Jm⁻³] outside. To *demagnetize* a permanent magnet we can apply a magnetic field H of magnitude H_c , which is the field strength necessary to drive B to zero.

If we represent the magnetic dipole moment of an atom by \vec{m} , where m for a current loop of magnitude I and area $\hat{n}A$ is $\hat{n}IA$ [Am]⁷, then it can be shown that the total magnetization \vec{M} of a medium is $n\vec{m}$ [Am⁻¹] via the same approach used to derive $\vec{P} = n\vec{p}$ (2.5.13) for the polarization of dielectrics; n is the number of dipoles per m³.

Example 2.5.B

Show how the power dissipated in a hysteretic magnetic material is related to the area circled in Figure 2.5.3(b) as H oscillates. For simplicity, approximate the loop in the figure by a rectangle bounded by $\pm H_0$ and $\pm B_0$.

Solution

We seek the energy dissipated in the material by one traverse of this loop as H goes from $+H_0$ to $-H_0$ and back to $+H_0$. The energy density $W_m = BH/2$ when $B = 0$ at $t = 0$ is $W_m = 0$; $W_m \rightarrow B_0 H_0 / 2$ [J m⁻³] as $B \rightarrow B_0$. As H returns to 0 while $B = B_0$, this energy is dissipated and cannot be recovered by an external circuit because any voltage induced in that circuit would be $\propto \partial B / \partial t = 0$. As $H \rightarrow -H_0$, $W_m \rightarrow B_0 H_0 / 2$; this energy can be recovered by an external circuit later as $B \rightarrow 0$ because $\partial B / \partial t \neq 0$. As $B \rightarrow -B_0$, $W_m \rightarrow B_0 H_0 / 2$, which is lost later as $H \rightarrow 0$ with $\partial B / \partial t = 0$. The energy stored as $H \rightarrow H_0$ with $B = -B_0$ is again recoverable as $B \rightarrow 0$ with $H = H_0$. Thus the minimum energy dissipated during one loop traverse is $v B_0 H_0$ [J], where v is material volume. If the drive circuits do not recapture the available energy but dissipate it, the total energy dissipated per cycle is doubled.

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