

## 14.3: Mathematical Identities

$$\begin{aligned}
 \vec{A} &= \hat{x}A_x + \hat{y}A_y + \hat{z}A_z \\
 \vec{A} \cdot \vec{B} &= A_x B_x + A_y B_y + A_z B_z = \hat{a} \times \hat{b} |\vec{A}| |\vec{B}| \cos \theta \\
 \vec{A} \times \vec{B} &= \det \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \\
 &= \hat{x}(A_y B_z - A_z B_y) + \hat{y}(A_z B_x - A_x B_z) + \hat{z}(A_x B_y - A_y B_x) \\
 &= \hat{a} \times \hat{b} |\vec{A}| |\vec{B}| \sin \theta \\
 \vec{A} \cdot (\vec{B} \times \vec{C}) &= \vec{B} \cdot (\vec{C} \times \vec{A}) = \vec{C} \cdot (\vec{A} \times \vec{B}) \\
 \vec{A} \times (\vec{B} \times \vec{C}) &= (\vec{A} \cdot \vec{C}) \vec{B} - (\vec{A} \cdot \vec{B}) \vec{C} \\
 (\vec{A} \times \vec{B}) \cdot (\vec{C} \times \vec{D}) &= (\vec{A} \cdot \vec{C})(\vec{B} \cdot \vec{D}) - (\vec{A} \cdot \vec{D})(\vec{B} \cdot \vec{C}) \\
 \nabla \times \nabla \Psi &= 0 \\
 \nabla \cdot (\nabla \times \vec{A}) &= 0 \\
 \nabla \times (\nabla \times \vec{A}) &= \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A} \\
 -\vec{A} \times (\nabla \times \vec{A}) &= (\vec{A} \cdot \nabla) \vec{A} - \frac{1}{2} \nabla(\vec{A} \cdot \vec{A}) \\
 \nabla(\Psi \Phi) &= \Psi \nabla \Phi + \Phi \nabla \Psi \\
 \nabla \cdot (\Psi \vec{A}) &= \vec{A} \cdot \nabla \Psi + \Psi \nabla \cdot \vec{A} \\
 \nabla \times (\Psi \vec{A}) &= \nabla \Psi \times \vec{A} + \Psi \nabla \times \vec{A} \\
 \nabla^2 \Psi &= \nabla \cdot \nabla \Psi \\
 \nabla(\vec{A} \cdot \vec{B}) &= (\vec{A} \cdot \nabla) \vec{B} + (\vec{B} \cdot \nabla) \vec{A} + \vec{A} \times (\nabla \times \vec{B}) + \vec{B} \times (\nabla \times \vec{A}) \\
 \nabla \cdot (\vec{A} \times \vec{B}) &= \vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\nabla \times \vec{B}) \\
 \nabla \times (\vec{A} \times \vec{B}) &= \vec{A}(\nabla \cdot \vec{B}) - \vec{B}(\nabla \cdot \vec{A}) + (\vec{B} \cdot \nabla) \vec{A} - (\vec{A} \cdot \nabla) \vec{B}
 \end{aligned}$$

### 14.3.1: Cartesian Coordinates (x,y,z):

$$\begin{aligned}
 \nabla \Psi &= \hat{x} \frac{\partial \Psi}{\partial x} + \hat{y} \frac{\partial \Psi}{\partial y} + \hat{z} \frac{\partial \Psi}{\partial z} \\
 \nabla \cdot \vec{A} &= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \\
 \nabla \times \vec{A} &= \hat{x} \left( \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \hat{y} \left( \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \hat{z} \left( \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \\
 \nabla^2 \Psi &= \frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2}
 \end{aligned}$$

### 14.3.2: Cylindrical coordinates (r,φ,z):

$$\begin{aligned}\nabla \Psi &= \hat{r} \frac{\partial \Psi}{\partial r} + \hat{\phi} \frac{1}{r} \frac{\partial \Psi}{\partial \phi} + \hat{z} \frac{\partial \Psi}{\partial z} \\ \nabla \cdot \vec{A} &= \frac{1}{r} \frac{\partial (r A_r)}{\partial r} + \frac{1}{r} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z} \\ \nabla \times \vec{A} &= \hat{r} \left( \frac{1}{r} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right) + \hat{\phi} \left( \frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right) + \hat{z} \frac{1}{r} \left( \frac{\partial (r A_\phi)}{\partial r} - \frac{\partial A_r}{\partial \phi} \right) = \frac{1}{r} \det \begin{vmatrix} \hat{r} / \partial r & \partial / \partial \phi & \partial / \partial z \\ A_r & r A_\phi & A_z \end{vmatrix} \\ \nabla^2 \Psi &= \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \Psi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \Psi}{\partial \phi^2} + \frac{\partial^2 \Psi}{\partial z^2}\end{aligned}$$

### 14.3.3: Spherical coordinates (r,θ,φ):

$$\begin{aligned}\nabla \Psi &= \hat{r} \frac{\partial \Psi}{\partial r} + \hat{\theta} \frac{1}{r} \frac{\partial \Psi}{\partial \theta} + \hat{\phi} \frac{1}{r \sin \theta} \frac{\partial \Psi}{\partial \phi} \\ \nabla \cdot \vec{A} &= \frac{1}{r^2} \frac{\partial (r^2 A_r)}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial (\sin \theta A_\theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi} \\ \nabla \times \vec{A} &= \hat{r} \frac{1}{r \sin \theta} \left( \frac{\partial (r \sin \theta A_\phi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi} \right) + \hat{\theta} \left( \frac{1}{r \sin \theta} \frac{\partial A_r}{\partial \phi} - \frac{1}{r} \frac{\partial (r A_\phi)}{\partial r} \right) + \hat{\phi} \frac{1}{r} \left( \frac{\partial (r A_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta} \right) \\ &= \frac{1}{r^2 \sin \theta} \det \begin{vmatrix} \hat{r} & r \hat{\theta} & r \sin \theta \hat{\phi} \\ \partial / \partial r & \partial / \partial \theta & \partial / \partial \phi \\ A_r & r A_\theta & r \sin \theta A_\phi \end{vmatrix} \\ \nabla^2 \Psi &= \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \Psi}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \Psi}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \Psi}{\partial \phi^2}\end{aligned}$$

### 14.3.4: Gauss' Divergence Theorem:

$$\int_V \nabla \cdot \vec{G} dv = \oint_A \vec{G} \cdot \hat{n} da$$

### 14.3.5: Stokes' Theorem:

$$\int_A (\nabla \times \vec{G}) \cdot \hat{n} da = \oint_C \vec{G} \cdot d\vec{\ell}$$

### 14.3.6: Fourier Transforms for pulse signals h(t):

$$\begin{aligned}\underline{H}(f) &= \int_{-\infty}^{\infty} h(t) e^{-j2\pi f t} dt \\ h(t) &= \int_{-\infty}^{\infty} \underline{H}(f) e^{+j2\pi f t} df\end{aligned}$$

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