10.2: Short Dipole Antennas

10.2.1: Radiation from Hertzian dipoles

Since Maxwell's equations are linear, superposition applies and therefore the electromagnetic field produced by an arbitrary current distribution is simply the integral of the fields produced by each infinitesimal element. Thus the electromagnetic field response to an infinitesimal current element is analogous to the impulse response of a linear circuit, and comparably useful for calculating responses to arbitrary stimuli.

The simplest infinitesimal radiating element, called a **Hertzian dipole**, is a current element of length d carrying I(t) amperes. Conservation of charge requires charge reservoirs at each end of the current element containing ±q(t) coulombs, where I = dq/dt, as illustrated in Figure 10.2.1(a). The total charge is zero. If we align the z axis with the direction of the current and assume the cross-sectional area of the current element is A_c [m^2], then the current density within the element is:

$$\overrightarrow{\mathrm{J}}_{\mathrm{q}}(\mathrm{t})=\hat{\mathrm{z}} \mathrm{I}(\mathrm{t}) / \mathrm{A}_{\mathrm{c}}\ \left[\mathrm{Am}^{-2}\right] \tag{10.2.1}$$

Substituting this current density into the expression (10.1.39) for vector potential yields:

$$\overrightarrow{\mathrm{A}}_{\mathrm{p}}=\hat{z} \oiiint_{\mathrm{V}}\left[\mu_{\mathrm{o}} \mathrm{I}\left(\mathrm{t}-\frac{\mathrm{r}_{\mathrm{pq}}}{\mathrm{c}}\right) /\left(\mathrm{A}_{\mathrm{c}} 4 \pi \mathrm{r}_{\mathrm{pq}}\right)\right] \mathrm{dv}=\hat{z} \frac{\mu_{\mathrm{o}} \mathrm{d}}{4 \pi \mathrm{r}_{\mathrm{pq}}} \mathrm{I}\left(\mathrm{t}-\frac{\mathrm{r}_{\mathrm{pq}}}{\mathrm{c}}\right)\ [\mathrm{Vs} / \mathrm{m}] \tag{10.2.2}$$

where integration over the volume V of the current element yielded a factor of $A_c d$.

To obtain simple expressions for the radiated electric and magnetic fields we must now switch to: 1) time-harmonic representations because radiation is frequency dependent, and 2) polar coordinates because the symmetry of the radiation is polar, not Cartesian, as suggested in Figure 10.2.1(b). The time harmonic form of I(t - r_{pq}/c) is $\mathrm{Ie}^{-\mathrm{jkr}_{\mathrm{pq}}}$ and the polar form of $\hat{z}$ is $\hat{r} \cos \theta-\hat{\theta} \sin \theta$, so (10.2.2) becomes:

$$\underline{\overrightarrow{\mathrm{A}}}_{\mathrm{p}}=(\hat{r} \cos \theta-\hat{\theta} \sin \theta) \mu_{\mathrm{o}} \underline{\mathrm{I}} \mathrm{de}^{-\mathrm{jkr}_{\mathrm{pq}}} / 4 \pi \mathrm{r}_{\mathrm{pq}} \quad\left[\mathrm{Vsm}^{-1}\right] \tag{10.2.3}$$

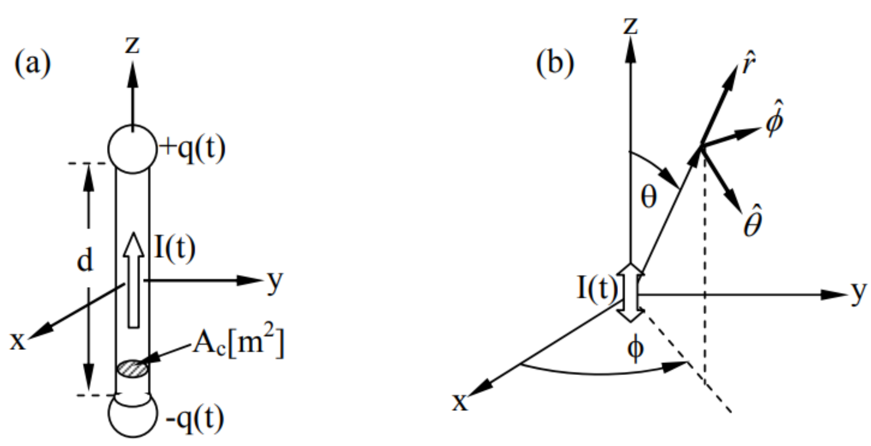

Figure 10.2.1: Hertzian dipole in spherical coordinates.

To find $\underline{\overrightarrow{\mathrm{H}}}$ and $\underline{\overrightarrow{\mathrm{E}}}$ radiated by this current element, we need to compute the curl of $\underline{\overrightarrow{\mathrm{A}}}$ in spherical coordinates:

$$\underline{\vec{\mathrm{H}}}=(\nabla \times \underline{\overrightarrow{\mathrm{A}}}) / \mu_{\mathrm{o}}=\left(\mu_{\mathrm{o}} \mathrm{r}^{2} \sin \theta\right)^{-1} \operatorname{det}\left|\begin{array}{ccc}\hat{r} & r \hat{\theta} & r \sin \theta \hat{\phi} \\ \partial / \partial r & \partial / \partial \theta & \partial / \partial \phi \\ A_{r} & r A_{\theta} & r \sin \theta A_{\phi}\end{array}\right| \tag{10.2.4}$$

Since $\underline{\overrightarrow{\mathrm{A}}}$ is independent of position ϕ (so $\partial/\partial\phi$ = 0) and has no ϕ component, (10.2.4) becomes:

$$\underline{\overrightarrow{\mathrm{H}}}=\hat{\phi}(\mathrm{jk} \underline{\mathrm{I}} \mathrm{d} / 4 \pi \mathrm{r}) \mathrm{e}^{-\mathrm{jkr}}\left[1+(\mathrm{jkr})^{-1}\right] \sin \theta \tag{10.2.5}$$

After some computation the radiated electric field can be found from (10.2.5) using Ampere's law (2.3.17):

$$\begin{aligned}\overrightarrow{\underline{E}} &= (\nabla \times \overrightarrow{\underline{H}})/j\omega\varepsilon_o \\ &= j\frac{k\underline{I}d\eta_o}{4\pi r}e^{-jkr}\left\{\hat{r}\left[\frac{1}{jkr}+\frac{1}{(jkr)^2}\right]2\cos\theta+\hat{\theta}\left[1+\frac{1}{jkr}+\frac{1}{(jkr)^2}\right]\sin\theta\right\}\end{aligned} \tag{10.2.6}$$

These solutions (10.2.5–6) for the Hertzian dipole are fundamental because they permit us to calculate easily the radiation from arbitrary current sources. It suffices to know the source current distribution because it uniquely determines the charge distribution via conservation of charge (2.1.19), and therefore the charge does not radiate independently.

These solutions for $\overrightarrow{E}$ and $\overrightarrow{H}$ are polynomials in 1/jkr, so they have two asymptotes—one for large values of kr and one for small values. When kr is very large the lowest order terms dominate, so kr = 2πr/λ >> 1, or:

$$r >> \lambda/2\pi \qquad \text{(far field)} \tag{10.2.7}$$

which we call the *far field* of the dipole.

In the far field the expressions (10.2.5) and (10.2.6) for $\overrightarrow{H}$ and $\overrightarrow{E}$ simplify to:

$$\overrightarrow{\underline{E}} = \hat{\theta}\frac{jk\underline{I}d\eta_o}{4\pi r}e^{-jkr}\sin\theta \qquad \text{(far-field electric field)} \tag{10.2.8}$$

$$\overrightarrow{\underline{H}} = \hat{\phi}\left(jk\underline{I}de^{-jkr}\sin\theta\right)/4\pi r \qquad \text{(far-field magnetic field)} \tag{10.2.9}$$

These expressions are identical, except that $\overrightarrow{E}$ points in the θ direction while $\overrightarrow{H}$ points in the orthogonal ϕ direction; the radial components are negligible in the far field. Also:

$$|\overrightarrow{\underline{E}}| = |\overrightarrow{\underline{H}}|\eta_o \tag{10.2.10}$$

where the impedance of free space $\eta_o = \sqrt{\mu_o/\varepsilon_o} \cong 377$ ohms. We found similar orthogonality and proportionality for uniform plane waves in Sections 2.3.2 and 2.3.3.

We can calculate the radiated intensity in the far field using (2.7.41) and the field expressions (10.2.8) and (10.2.9):

$$\langle\overrightarrow{S}(t)\rangle = 0.5R_e[\overrightarrow{\underline{S}}] = 0.5R_e\left[\overrightarrow{\underline{E}}\times\overrightarrow{\underline{H}}^*\right] \tag{10.2.11}$$

$$\langle\overrightarrow{S}(t)\rangle = \hat{r}\left|\overrightarrow{\underline{E}}_o\right|^2/2\eta_o = \hat{r}\,(\eta_o/2)\,|kId/4\pi r|^2\sin^2\theta\ \left[Wm^{-2}\right] \tag{10.2.12}$$

The *radiation pattern* $\langle S(t,\theta)\rangle$ for a Hertzian dipole is therefore a donut-shaped figure of revolution about its z axis, as suggested in Figure 10.2.2(b).

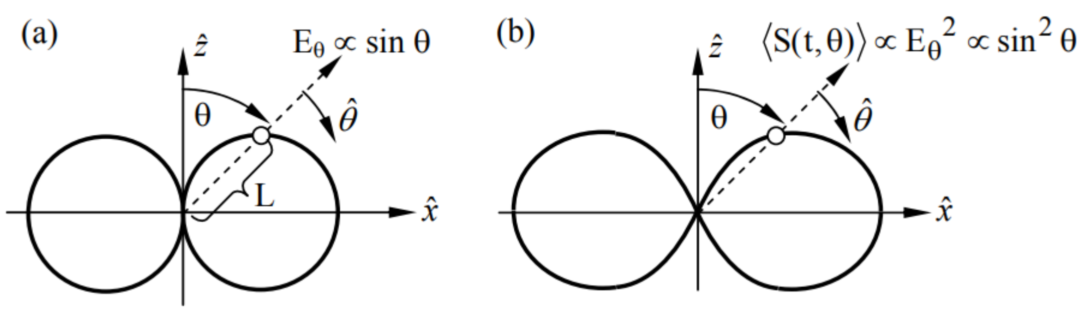

Figure 10.2.2: Electric field strength E_θ and power $\langle S(t,\theta)\rangle$radiated by a Hertzian dipole.

Hertzian dipoles preferentially radiate laterally, with zero radiation along their axis. The electric field strength E_θ varies as sin θ, which yields a circle in a polar plot, as illustrated in Figure 10.2.2(a). The distance L in the plot corresponds to $|\underline{E}_\theta|$. Since $\sin^2 45^o = 0.5$, the width of the beam between half-power points in the θ direction is 90º .

The total power P_R radiated by a Hertzian dipole can be calculated by integrating the radial component $\langle S_r\rangle$ of $\langle \vec{S}(t,\theta)\rangle$ over all directions:

$$\begin{aligned} P_R &= \int_0^{2\pi} d\phi \int_0^{\pi} \langle S_r\rangle\, r^2 \sin\theta d\theta = \pi\eta_o|k\underline{I}d/4\pi|^2 \int_0^{\pi} \sin^3\theta d\theta \\ &= (\eta_o/12\pi)\,|k\underline{I}d|^2 \cong 395(Id/\lambda)^2\ [W] \end{aligned} \tag{10.2.13}$$

Thus the radiated fields increase linearly with $\underline{I}d/\lambda$ and the total radiated power increases as the square of this factor, i.e. as $|\underline{I}d/\lambda|^2$. Since the electric and magnetic fields are in phase with each other in the far field, the imaginary power $I_m\{\underline{\vec{S}}\} = 0$ there.

Example 10.2.A

Equation (10.2.13) says the current I input to a Hertzian dipole radiates P_R watts. What value resistor R_r would dissipate the same power for the same I?

Solution

$P_R = (\eta_0/12\pi)\,|k\underline{I}d^2 = |\,\underline{I}|^2 R_r/2 \Rightarrow R_r = (2\pi\eta_0/3)\,(d/\lambda)^2$ ohms; this quantity is often called the **radiation resistance** of the radiator.

10.2.2: Near fields of a Hertzian dipole

If we examine the near fields radiated by a Hertzian dipole close to the origin where kr << 1, then the $(kr)^{-2}$ terms in the expression (10.2.7) for $\underline{\vec{E}}$ dominate all other terms for both $\underline{\vec{E}}$ (10.2.6) and $\underline{\vec{H}}$ (10.2.5), so we are left primarily with $\underline{\vec{E}}$:

$$\underline{\vec{E}} \cong \left(\underline{I}d/j\omega 4\pi\varepsilon_o r^3\right)(\hat{r}2\cos\theta + \hat{\theta}\sin\theta) \tag{10.2.14}$$

This is simply the electric field produced by a static *electric dipole* of length d with a *dipole moment* of $\underline{p} = \underline{q}d$, where the charges at the ends of the dipole are ± q and $\underline{I} = j\omega\underline{q}$ to conserve charge.

Substituting this definition of $\underline{p}$ into (10.2.14) yields:

$$\underline{\vec{E}} \cong \left(\underline{p}/4\pi\varepsilon_o r^3\right)(\hat{r}2\cos\theta + \hat{\theta}\sin\theta) \qquad \text{(near-field electric field)} \tag{10.2.15}$$

The dominant term for $\underline{\vec{H}}$ in the near field is:

$$\underline{\vec{H}} \cong \hat{\phi}\left(j\omega\underline{p}/4\pi r^2\right)\sin\theta \qquad \text{(near-field magnetic field)} \tag{10.2.16}$$

Because $\underline{\vec{S}} = \underline{\vec{E}} \times \underline{\vec{H}}^*$ is purely negative imaginary in the near fields, these fields correspond to reactive power and stored electric energy. Integrating the exact expressions for $\underline{\vec{S}}$ over 4π steradians yields a real part that is independent of r; that is, the total power radiated is the same (10.2.13) regardless of the radius r at which we integrate, even in the near field.

A simple expression for $\vec{H}$ in the near field of the source is called the *Biot-Savart law*; it easily follows from the expression (10.2.5) for magnetic fields close to a current element $\underline{I}d\hat{z}$ when kr << 1:

$$\underline{\vec{H}} = \hat{\phi}(\underline{I}d\sin\theta)/4\pi r^2 \tag{10.2.17}$$

The Biot-Savart law relates arbitrary current distributions $\vec{J}\left(\vec{r}',t\right)$ and $\vec{H}(\vec{r},t)$ when the distance $r = \left|\vec{r}' - \vec{r}\right| << \lambda/2\pi$; it therefore applies to static current distributions as well. But $\underline{I}d$ here is simply $\int_V \underline{J}\,dx\,dy\,dz$, where the current $\underline{I}$ and current density $\underline{J}$ are in the z direction and V is the volume of the sources of $\vec{r}'$ ' With these substitutions (10.2.17) becomes:

$$\underline{\vec{H}}(\vec{r}) = \iiint_V \frac{\underline{J}\hat{\phi}\sin\theta}{4\pi\left|\vec{r} - \vec{r}'\right|^2} dx\,dy\,dz \tag{10.2.18}$$

We can also use the definition of vector cross product to replace $\hat{\phi}\underline{\mathrm{J}}\sin\theta$:

$$\hat{\phi}\underline{\mathrm{J}}\sin\theta=\frac{\overrightarrow{\mathrm{J}}\times\left(\overrightarrow{\mathrm{r}}-\overrightarrow{\mathrm{r}}^{\prime}\right)}{\left|\overrightarrow{\mathrm{r}}-\overrightarrow{\mathrm{r}}^{\prime}\right|} \tag{10.2.19}$$

Substituting (10.2.19) into (10.2.18) in their time-domain forms yields the Biot-Savart law:

$$\overrightarrow{\mathrm{H}}(\overrightarrow{\mathrm{r}},t)=\oiiint_{\mathrm{V}^{\prime}}\frac{\overrightarrow{\mathrm{J}}\times\left(\overrightarrow{\mathrm{r}}-\overrightarrow{\mathrm{r}}^{\prime}\right)}{4\pi\left|\overrightarrow{\mathrm{r}}-\overrightarrow{\mathrm{r}}^{\prime}\right|^{3}}\mathrm{dx\,dy\,dz} \tag{10.2.20}$$

Equation (10.2.20) has been generalized to $\overrightarrow{\mathrm{H}}(\overrightarrow{\mathrm{r}},\mathrm{t})$ because $\overrightarrow{\mathrm{H}}$ is independent of frequency if $|\mathrm{r}-\mathrm{r}^{\prime}|<<\lambda/2\pi$.

10.2.3: Short dipole antennas

Antennas transform freely propagating electromagnetic waves into circuit voltages for reception, and also transform such voltages into free-space waves for transmission. They are used for wireless communications, power transmission, or surveillance at wavelengths ranging from micrometers (infrared and visible wavelengths) to hundreds of kilometers. Their sophistication and performance continue to increase as improved computational and fabrication methods are developed, although simple structures still dominate today.

Determining the fields and currents associated with a given antenna can be difficult using traditional approaches to boundary value problems because many waves must usually be superimposed in order to match boundary conditions, even when well chosen orthogonal wave expansions other than plane-waves are used. Fortunately modern computer software tools can handle most antenna problems. Here we take a traditional alternative approach to antenna analysis that yields acceptable solutions for most common configurations by making one key assumption—that the current distribution on the antenna is known. Determination of antenna current distributions is discussed in Sections 11.1–2.

Arbitrary antenna current distributions can be approximated by superimposing infinitesimal Hertzian dipole radiators that have constant current $\overrightarrow{\mathrm{I}}$ over an infinitesimal length d. The electric far fields each dipole radiates are given by (10.2.8), and the total radiated field $\overrightarrow{\mathrm{E}}$ is simply the sum of these differential contributions. Superposition of these fields is valid because Maxwell's equations are linear. $\overrightarrow{\mathrm{H}}$ can then be readily found using Faraday's law or direct integration. This is the approach taken here; the far fields of the short dipole antenna are found by integrating the contributions from each infinitesimal element of that dipole. From these fields the antenna gain, effective area, and circuit properties can then be found, as discussed in Section 10.3. Many practical antennas, such as those used in many cars for the ~1-MHz Amplitude-Modulated (AM) band, are approximately short-dipole antennas with lengths less than a few percent of the associated wavelength λ. Their simple behavior provides an easy introduction to antenna analysis.

Consider the *short-dipole antenna* illustrated in Figure 10.2.3. It has length d << λ and is driven by a current source with $\underline{\mathrm{I}}_0$ amperes at angular frequency ω [radians/second]. Complex notation is used here for simplicity because antenna characteristics are frequency-dependent. As discussed later, the wires, or "feed-lines", providing current to the dipole do not alter the dipole's radiated fields because those wires are perpendicular to the antenna fields and also do not radiate.

The electric far field $\overrightarrow{\underline{\mathrm{E}}}^{\prime}_{\mathrm{ff}}$ radiated by an infinitesimal current element $\underline{\mathrm{I}}$ of length δ is given by (10.2.8):

$$\overrightarrow{\underline{\mathrm{E}}}^{\prime}_{\mathrm{ff}}=\hat{\theta}\frac{\mathrm{jk}\underline{\mathrm{I}}\hat{\mathrm{n}}\eta_{\mathrm{o}}}{4\pi\mathrm{r}}\mathrm{e}^{-\mathrm{jkr}}\sin\theta \qquad \text{(far-field electric field)} \tag{10.2.21}$$

This expression can be integrated over the contributions from all infinitesimal elements of the current distribution on the short dipole to find the total radiated electric far field $\overrightarrow{\underline{\mathrm{E}}}^{\prime}_{\mathrm{ff}}$:

$$\overrightarrow{\underline{\mathrm{E}}}_{\mathrm{ff}}=\int_{-\mathrm{d}/2}^{\mathrm{d}/2}\overrightarrow{\underline{\mathrm{E}}}^{\prime}_{\mathrm{ff}}(\mathrm{r}^{\prime},\theta)\,dz=\frac{\mathrm{jk}\eta_{\mathrm{o}}}{4\pi}\int_{-\mathrm{d}/2}^{\mathrm{d}/2}\hat{\theta}^{\prime}\left[\underline{\mathrm{I}}(\mathrm{z})\mathrm{e}^{-\mathrm{jkr}^{\prime}}\frac{\sin\theta^{\prime}}{\mathrm{r}^{\prime}}\right]\mathrm{dz} \tag{10.2.22}$$

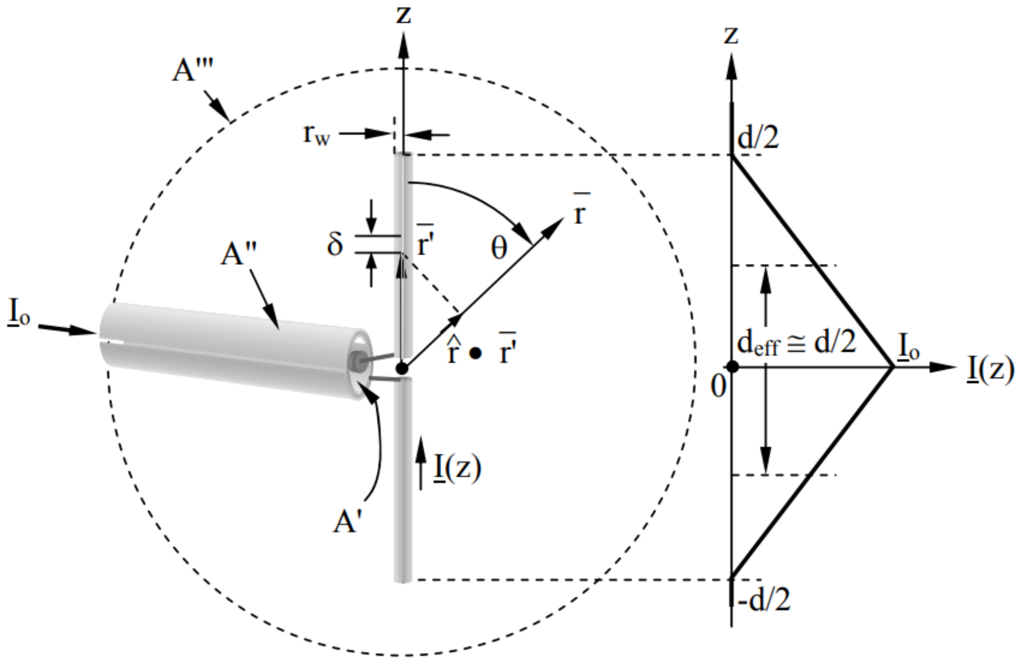

Figure 10.2.3: Short-dipole antenna.

This integral can be simplified if the observer is far from the antenna relative to its length d so that $\theta' = \theta$), \($\hat{\theta}' \cong \hat{\theta}$, ad $r'^{-1} \cong r^{-1}$. In addition, if d << λ/2π for all z, then:

$$e^{-jkr'} \cong e^{-jkr} \tag{10.2.23}$$

$$\begin{aligned} \overrightarrow{\underline{E}}_{ff} &\cong \hat{\theta}\,\hat{j}\,\frac{k\eta_o}{4\pi s}\sin\theta e^{-jkr}\int_{-d/2}^{d/2}\underline{I}(z)dz \\ &= \hat{\theta}\,\hat{j}\,\frac{k\eta_0\underline{I}_0 d_{eff}}{4\pi r}\sin\theta e^{-jkr} \qquad \text{(far-field radiation)} \end{aligned} \tag{10.2.24}$$

where the *effective length* of the dipole d_{eff} is illustrated in Figure 10.2.3 and is defined as:

$$d_{eff} \equiv \underline{I}_0^{-1}\int_{-d/2}^{d/2}\underline{I}(z)dz \qquad \text{(effective length of short dipole)} \tag{10.2.25}$$

Because both these far fields $\overrightarrow{\underline{E}}_{ff}$ and the near fields are perpendicular to the x-y plane where the feed line is located, they are consistent with the boundary conditions associated with a sufficiently small conducting feed line located in that plane, and no reflected waves are produced. Moreover, if the feed line is a coaxial cable with a conducting sheath, Poynting's vector on its outer surface is zero so it radiates no power.

The far fields radiated by a short dipole antenna are thus radially propagating θ-polarized plane waves with ϕ-directed magnetic fields $\overrightarrow{\underline{H}}$ of magnitude $|\overrightarrow{\underline{E}}|/\eta_o$. The time-average intensity $\overrightarrow{P}$ of these radial waves is given by Poynting's vector:

$$\overrightarrow{P} = \frac{1}{2}R_e\{\overrightarrow{\underline{S}}\} = \frac{1}{2}R_e\left\{\overrightarrow{\underline{E}} \times \overrightarrow{\underline{H}}^*\right\} = \hat{r}\frac{|\overrightarrow{\underline{E}}_{ff}|^2}{2\eta_o}\;[\mathrm{Wm^{-2}}] \tag{10.2.26}$$

$$\overrightarrow{P} = \hat{r}\frac{\eta_0}{2}\left(\frac{k|\underline{I}_o d_{eff}|}{4\pi r}\right)^2\sin^2\theta = \hat{r}\frac{\eta_o}{2}\left|\frac{\underline{I}_o d_{eff}}{\lambda 2r}\right|^2\sin^2\theta \tag{10.2.27}$$

This angular distribution of radiated power is illustrated in Figure 10.2.4.

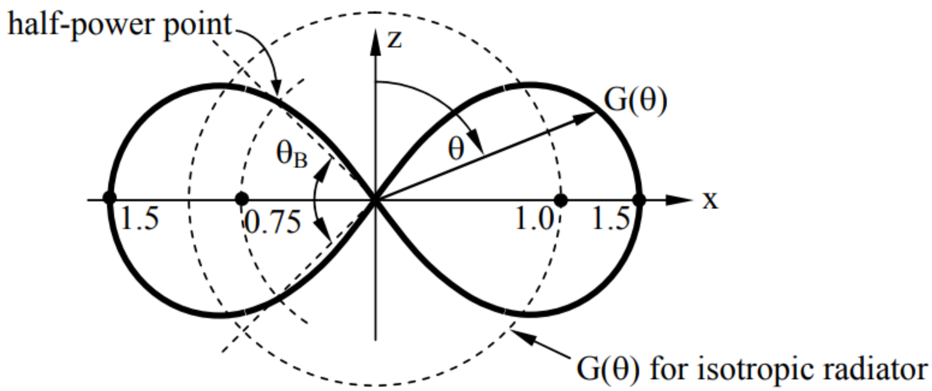

Figure 10.2.4: Antenna gain G(θ) for a short dipole or Hertzian antenna.

The total power radiated is the integral of this intensity over 4π steradians[52]:

$$\mathrm{P_T}=\int_{4\pi}\left[\overrightarrow{\mathrm{P}}(\mathrm{r},\theta)\bullet\hat{r}\right]\mathrm{r}^2\sin\theta d\theta d\phi=\frac{\eta_0\pi}{3}\left|\frac{\underline{\mathrm{I}}_0\mathrm{d_{eff}}}{\lambda}\right|^2[\mathrm{W}]\qquad\text{(radiated power)}\qquad(10.2.28)$$

[52] Recall $\int_0^{\pi/2}\sin^n x dx=[2\cdot4\cdot6\ldots(n-1)]/[1\cdot3\cdot5\ldots(n)]$ for n odd; $(\pi/2)[1\cdot3\cdot5\ldots(\mathrm{n}-1)]/[2\cdot4\cdot6\ldots(\mathrm{n})]$ for n even.