4.1: Introduction to Static and Quasistatic Fields

Static electric and magnetic fields are governed by the static forms of Maxwell's equations in differential and integral form for which ∂/∂t → 0:

$$\nabla \times \overrightarrow{\mathrm{E}} = 0 \quad \oint_{\mathrm{C}} \overrightarrow{\mathrm{E}} \cdot \mathrm{d}\overrightarrow{\mathrm{s}} = 0 \tag{4.1.1}$$

$$\nabla \times \overrightarrow{\mathrm{H}} = \overrightarrow{\mathrm{J}} \quad \oint_{\mathrm{C}} \overrightarrow{\mathrm{H}} \bullet \mathrm{d}\overrightarrow{\mathrm{s}} = \iint_{\mathrm{A}} \mathrm{J} \bullet \hat{n} \mathrm{da} \tag{4.1.2}$$

$$\nabla \bullet \overrightarrow{\mathrm{D}} = \rho \quad \oiint_{\mathrm{A}} (\overrightarrow{\mathrm{D}} \bullet \hat{n}) \mathrm{da} = \int\int\int_{\mathrm{V}} \rho \mathrm{dv} = \mathrm{Q} \tag{4.1.3}$$

$$\nabla \bullet \overrightarrow{\mathrm{B}} = 0 \quad \oiint_{\mathrm{A}} (\overrightarrow{\mathrm{B}} \bullet \hat{n}) \mathrm{da} = 0 \tag{4.1.4}$$

As shown in (1.3.5), Gauss's law (Equation 4.1.3) leads to the result that a single point charge Q at the origin in vacuum yields produces an electric field at radius r of:

$$\overrightarrow{\mathrm{E}}(\mathrm{r}) = \hat{\mathrm{r}} \mathrm{Q} / 4 \pi \varepsilon_{\mathrm{o}} \mathrm{r}^{2} \tag{4.1.5}$$

Superposition of such contributions to E(r) from a charge distribution ρ(r ') located within the volume V' yields:

$$\mathrm{E}(\mathrm{T}) = \hat{\mathrm{r}} \int\int\int\int_{\mathrm{V}'} \frac{\rho(\overrightarrow{\mathrm{r}})}{4 \pi \varepsilon_{\mathrm{o}} |\mathrm{r} - \overrightarrow{\mathrm{r}}|^{2}} \mathrm{dv}' \tag{4.1.6}$$

where r̂ is outside the integral because $r \gg 3V'$. A more complex derivation given in Section 10.1 yields the corresponding equation for static magnetic fields:

$$\mathrm{H}(\mathrm{r}, \mathrm{t}) = \int\int\int_{\mathrm{V}'} \frac{\overrightarrow{\mathrm{J}} \times (\overrightarrow{\mathrm{r}} - \overrightarrow{\mathrm{r}})}{4 \pi \left| \overrightarrow{\mathrm{r}} - \overrightarrow{\mathrm{r}}' \right|^{3}} \mathrm{dv}' \tag{4.1.7}$$

Any static electric field can be related to an electric potential distribution Φ[volts] because $\nabla \times \overrightarrow{E} = 0$ implies $\overrightarrow{E} = -\nabla\Phi$, where the voltage difference between two points (1.3.12) is:

$$\Phi_{1} - \Phi_{2} = \int_{1}^{2} \overrightarrow{\mathrm{E}} \cdot \mathrm{d}\overrightarrow{\mathrm{s}} \tag{4.1.8}$$

Similarly, in current-free regions of space $\nabla \times \overrightarrow{\mathrm{H}} = 0$ implies $\overrightarrow{\mathrm{H}} = -\nabla \Psi$ [Amperes], where Ψ is magnetic potential. Therefore the magnetic potential difference between two points is:

$$\Psi_{1} - \Psi_{2} = \int_{1}^{2} \bar{H} \cdot d\bar{s} \tag{4.1.9}$$

This definition of magnetic potential is useful in understanding the magnetic circuits discussed in Section 4.4.3.

Often not all source charges and currents are given because some reside on given equipotential surfaces and assume an unknown distribution consistent with that constraint. To address this case, Maxwell's equations can be simply manipulated to form Laplace's equation, which can sometimes be solved by separation of variables, as discussed in Section 4.5, or usually by numerical methods. Section 4.6 then discusses the utility of flux tubes and field mapping for understanding static field distributions.

Quasistatics assumes that the field strengths change so slowly that the electric and magnetic fields induced by those changes (the contributions to $\overrightarrow{\mathrm{E}}$ and $\overrightarrow{\mathrm{H}}$ from the ∂/∂t terms in Faraday's and Ampere's laws) are sufficiently small that their own induced fields ($\propto(\partial/\partial t)^2$) can be neglected; only the original and first-order induced fields are therefore of interest. Quasistatic examples were discussed in Chapter 3 in the context of resistors, capacitors, and inductors. The mirror image technique described in Section 4.2 is used for static, quasistatic, and dynamic problems and incidentally in the discussion in Section 4.3 concerning exponential relaxation of field strengths in conducting media and skin depth.
