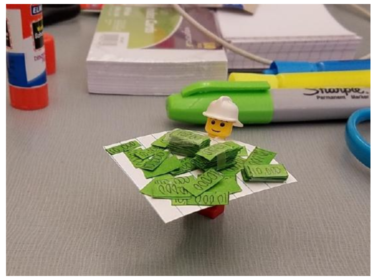
8.1: Million, Billion, Trillion- Big Numbers and Money

If you teach in the K-8 classroom, you may know that most of your students ***will not*** become scientists. Why then is it important to understand big numbers? Although most of us are not professional scientists, very large numbers touch our daily lives in many ways – money is the most common.



When we listen to legislators discuss state or national budgets on the news, or when we see a local bond measure to allocate millions – even billions of dollars to a project like a bridge, highway, or a railroad line, most people don't understand how much money we are talking about. Worse yet, they have no fundamental grasp of large numbers to help them understand the ideas being debated on their behalf.

This activity seeks to correct that deficit by giving students a physical model for the concepts of thousand, million, billion, and trillion – without too much tedious counting!

Academic Standards

Science and Engineering Practices

- Asking questions and defining problems.
- Developing and using models.
- Using mathematics.
- Obtain, evaluate, and communicate information.

Crosscutting Concepts

- Scale, proportion, and quantity.

Next Generation Science Standards

- Space systems (K-5, 6-8, 9-12).
- Structure and function (K-5, 6-8, 9-12).

For the Educator

Facts you need to know

1. We normally think of counting, multiplying in powers of 10 – One's place, Ten's place, Hundred's place, etc.
2. Large numbers: Thousand, Million, Billion, Trillion, are ***powers of 1000*** – each one of these numbers is ***one thousand times larger*** than its predecessor.
3. It is this change, from powers of 10 for everyday numbers to powers of 1000 for large numbers that confuses people.

Teaching and Pedagogy

Misconceptions about large numbers are some of the most persistent and troublesome bits of misinformation. Not just children, but a large fraction of adults in our society lack a good understanding of what large numbers are, how much larger a billion is than a million, and much more. Almost no one has a good visual concept of what a million of anything looks like. You can imagine a dozen doughnuts looks like and how large a box you need to put them in – but what does a million doughnuts look like?

When we reach the domain of millions, billions, trillions, and beyond, the names for our numbers now reflect ***powers of 1000***. Psychologists and anthropologists tell us that humans have an ability to easily mentally conceptualize groups of up to five or so, and with a bit of practice (and the help of our fingers!) we can get pretty good at conceptualizing groups of ten. In this way, visualizing 100 as ***ten groups of ten*** is well within almost anyone's grasp. In comparison to this, groups of 1000 are beyond anyone's cognitive grasp. Yes, we handle numbers like these mathematically and numerically; but conceptually, we get lost pretty quickly.

Want proof? Try this mental experiment: Think of ten marbles. Got the picture mentally? Good! Now think of ***one thousand*** marbles. How big is it? How much does it weigh? How large a container will you need to hold them all? If you needed 1000 marbles and you saw a group of 750 marbles, could you tell by sight that you don't have enough? I suppose if you had a job working with boxes of 1000 marbles all day long you might be able to tell, but generally, the answer to all these questions is cognitively beyond our grasp. As human beings, we just don't mentally image or handle large numbers very well without a great deal of practice.

Computers add to the confusion for most students (and adults!) A computer with 200 megabytes of memory (200 million bytes) looks just like the model with 3 gigabytes of memory (3 billion bytes). A USB memory stick with 32 megabytes looks just like a USB stick with 32 gigabytes even though the capacity is one thousand time greater! Newspapers and broadcasters reading the news on television are no better; announcers will jump from one story about a fire causing three million dollars damage to a story about a two trillion dollar spending bill in Congress with no attempt to explain the difference between them. Activities like this one, and a through discussion of the ideas they contain, are essential for your students.

As adults, we are expected to work with computers which routinely use terms like mega (million), giga (billion), and terra (trillion). As voters, we are expected to select candidates for office by listening to their economic and taxation plans involving millions, billions, and yes, trillions of dollars. How can we prepare the children of today to do any of these things successfully if they do not have a fundamental understanding of these concepts? This activity will finally put the ideas of million, billion, and trillion on a solid physical, and visual, foundation.

Student Outcomes

What will the student discover?

1. The fact that large numbers increase in powers of 1000 instead of the usual power of 10 makes them harder to visualize easily.
2. Learning to visualize and comprehend the powers of 1000 in thousand, million, billion, trillion takes practice! We have to try imagining large numbers of various things to become familiar with these scales.

What will your students learn about science?

1. These large numbers are of particular interest in astronomy, where distances and sizes vary so greatly. Our use of powers of 1000 makes it easier to discuss, and understand large sizes and distances.
2. More advanced classes in science and mathematics (high school and college) use something called ***scientific notation*** to help handle these very large and very small numbers. Scientific notation is beyond the scope of this book, but these methods of writing and calculating with very large and small numbers helps us handle everything from the distance between stars to the tremendously small size of the atom.

Conducting the Activity

Materials

1. A trillion dollars. No? Okay, how about a package of index cards or several manila file folders that we can cut up and color to represent money?
2. A couple LEGO® figures or similar 2-inch toy action figure.
3. Two, 3-foot square pieces of cardboard.
4. Paints and markers (green, of course!), highlighters work well for this.
5. Glue sticks and hot glue.

Building the Big Numbers Model

1. Cut up index cards into ¼-inch wide strips. Cut each strip into ½-inch pieces. You are going to need a lot of these, so if each student does one card, you should have enough.
2. Label five pieces with "$100" and glue them together so that they are fanned out like a hand of cards. When dry, color this stack over with green highlighter. Glue this fanned stack of cash into the hand of one of your small action figures to represent $10,000 – be sure to paint a smile on the little fellow's face! Remind your students that it takes ***one hundred*** $100 bills to make $10,000 dollars.

3. Now use glue sticks to glue stacks of these cut pieces of index card together – four pieces per stack. Once they are dry, label each stack “$10,000” in pen or black marker and color it over with green highlighter or pale green marker. Each stack now represents a pile of 100, one hundred dollar bills — $10,000 cash each!
4. Make 100 of these piles. Yeah, each student in your class of 30 is going to have to produce 3-4 of these for you to have enough! Once you have one hundred of these piles – each representing $10,000 – then you have ‘printed’ ***one million dollars***… and you now know how those folks at the U.S. Mint feel! Making money is a lot of work!
5. Make a little cafeteria tray out of a piece of cardboard or plastic from a milk container. Glue this into the second action figure’s hands and stack the $1,000,000 dollars on it. You’ll have to stack neatly, it makes quite a tidy pile of cash, doesn’t it? This is a pretty good model for the physical size of one million dollars cash in $100 bills! (***BIG*** smile, little guy!)

Exploring the Big Numbers Model

1. Now it’s time to go big… it’s time to make a ***billion dollars!*** No, we aren’t going to need a lot more index cards, a single manila folder and some white glue and markers will do just fine. One billion dollars is a stack of $100 bills that is ***one thousand times larger*** than our million dollar stack. This is a neatly stacked cube of $100 bills that is ***eight feet long on each side.*** Assuming your 2-inch tall action figure is 6-feet tall, let’s plan on a pile of play money that is a 2½-inch cube. Take your manila folder and cut out 2.5 inch cube as shown in the diagram below, glue it together and decorate it.

   To give you some idea, our billion dollar pile contains ***ten million $100 bills***. This is like having a solid cube of dense wood 8-feet on a side – it would weigh ten tonnes (10,000 kg), and only the largest industrial forklifts could move it.

2. Okay, big is a relative thing. What about ***one trillion dollars?*** Well, let’s consider the standard school soccer field… yes, really. If we took an American school soccer field of 100 yards x 60 yards, we get an area of 6,000 square yards. On the other hand, if we have one thousand cubes of one billion dollars, that make 6,250 yards. That means our stack of ***ten billion $100 bills*** would take up an entire soccer field, plus about 8 feet extra on either end to have room for the goals, and piled it eight feet deep in neatly stacked $100 bills… Yeah, a trillion dollars is a ***LOT*** of money.
3. To keep in scale, we will need an area 10’4” x 4’2” and 2.5” tall. This may be a little much for your class to tackle, but if you want to build it in sections and put them next to each other – it makes a powerful display on the gymnasium floor… especially if you make little soccer goals and use a little gumball for a soccer ball and have several action figures running around on it!
4. If modeling something this big is a little much, consider taking a 3-ft square of cardboard and measuring off and building a cube as shown below. Make it 30-inches long, 18-inches wide, and 1½-inches high. At this scale, your figure is just about ¼-inch tall. I’ve never seen an action figure or toy this small, I suppose you would just have to draw one on paper and glue it to your soccer field of money (don’t forget that big smile!)
5. Oh, and if you want to model the national debt on the scale of our action figure? That’s a stack of billion dollar blocks, 10’4” long, 4’2” wide, and 4’ tall. That’s ***Nineteen Trillion Dollars***… and it’s collecting interest!

Discussion Questions

1. You have won the lottery! They offer you your choice: One billion dollars today… or one million dollars a day for a year! Which should you choose and why?
   - **Answer** A billion is 1000x larger than a million. The year is only 365 days long – you would end up with only 1/3 of the money if you took the million a day!
2. A new highway building project will cost 1.3 billion dollars. How much money does the .3 represent?
   - **Answer** 300 million dollars!

Supplemental Materials

Going Deeper

Talking about money is fun, but students often don’t have a good grasp of money, especially when they are younger. The million-billion-trillion problem can be fun in lots of ways, let’s make it about something most every student knows and loves – doughnuts.

You get 12 doughnuts in a 9x9x4 inch box. Now ask your students to figure out the size and space for a larger amount of doughnuts.

1. How many cubic inches in a doughnut box? (Length x width x height)

      a. 324 in3

2. How many boxes to hold 1000 doughnuts? How many cubic inches is this?
    a. 83.33 boxes, 27,000 in3 total
    b. This is a 30-inch cube of doughnuts!
3. If a 30-inch cube holds 1000 doughnuts – how big will ***one million*** doughnuts be?
    a. One million doughnuts is 1000 times bigger – our cube must be 10x bigger on each side: 300 inches or 25 feet wide, long, and tall!
    b. This is three 25×25 foot classrooms with 8 ½ foot ceilings – completely full floor to ceiling with doughnuts!
4. How big are one billion doughnuts?
    a. One thousand times bigger! Three thousand classrooms full of doughnuts!
5. How big are one trillion doughnuts?
    a. One thousand times bigger! Three million classrooms full of doughnuts!
    b. Three million classrooms with 25 students each would be 75 million students. This is about the number of students in the United States, Mexico, and Canada combined!

Being an Astronomer

What does million, billion, and trillion mean to an astronomer? Let's consider the size and scale of the solar system to get an idea. One of the most important measurements that astronomers make is distance – how far away in space are planets, moons, and stars from one another?

One thousand kilometers. This is a good scale to measure moons circling planets. Few moons are larger than 1000 kilometers wide; most are between 100 and 1000 kilometers in diameter. Our own Moon is about 3,500 kilometers wide and is one of the largest satellites in our solar system!

One million kilometers. This is a good scale to measure the orbits of moons circling around planets. If you look at all the moons in our solar system (there are hundreds of them!), almost all are within 1,000,000 km of the planet they orbit. Our own Moon circles the Earth at an average distance of about 385,000 km – about one third of a million kilometers away!

One billion kilometers. This is a good scale to measure the distance from a star out to its planets. Our own solar system's outermost major planets are about three billion kilometers from the Sun. The Sun's influence extends only about 20 billion kilometers – farther away than this, the Sun's gravity and magnetic field have no influence at all. Astronomers call this ***interstellar space***.

Only the two Voyager spacecraft have made it this far away from Earth. They were launched 40 years ago in 1977, both probes are now about 20 billion km from Earth and moving away from us at about 60,000 kph. At this rate, they will reach the one trillion kilometer mark in their journey in about two thousand years!

One trillion kilometers. This is a good scale to measure the distances between stars. It takes a beam of light one year to travel six trillion kilometers – this is called a ***light year.*** The nearest star to us is just over four light years away, or about 25 trillion kilometers away!

Following Up

Look for examples in the news that use the terms million, billion, or trillion. It will help if you look for examples that talk about money such as national or state budgets. Science websites that have space news like www.space.com have many stories that deal with large numbers.