5.4: Measuring the Earth with Eratosthenes

An ancient Greek astronomer named ***Eratosthenes*** was the first man to measure the size of the Earth accurately. His method was very simple: he measured the angle made by a shadow cast from a vertical stick in two different cities on the same day and time. With the help of another teacher, you can recreate Eratosthenes' experiment and your students can measure the size of the Earth for themselves! All you will need is two yardsticks, a protractor, a magnetic compass, and a bit of string.

Academic Standards

Science and Engineering Practices

- Asking questions and defining problems.
- Planning and carrying out investigations.
- Analyzing and interpreting data.
- Using mathematics.
- Constructing explanations.
- Argument from evidence.
- Obtain, evaluate, and communicate information.

Crosscutting Concepts

- Scale, proportion, and quantity.
- Systems and system models.

Next Generation Science Standards

- Engineering and design (K-5, 6-8, 9-12).
- The Earth-Moon system (6-8, 9-12).

For the Educator

Facts you need to know

1. The Earth's circumference was first accurately measured more than 2,200 years ago by a Greek astronomer named Eratosthenes.
2. Eratosthenes method was very simple; he measured the length of a shadow from a vertical stick of a known height in two cities on the same day. The ratio between the north-south distance between the two cities and the angles measured gave a ratio which allowed Eratosthenes to calculate the size of the Earth.

Teaching and Pedagogy

This is a wonderful example of practical geometry and a powerful introduction into ancient cultures; the activity is not just STEM, but cross-curricular as well. It is a common misconception that just because cultures were ancient, they must have been primitive or simplistic. We often confuse technological sophistication for learning and knowledge. The activity where students actually work together with children from another school is living proof that this is not so.

This activity is also another example of the practical application of mathematics. Math needn't be complex or totally divorced from reality; children actually respond and learn better when mathematics are presented in a real-world concept. I can think of no more dramatic answer to the perennial question: "What are we gonna use this math junk for anyway?" than to say: "We're going to measure the size of the Earth today!"

Student Outcomes

What will the student discover?

1. This is a lovely project for many reasons; as with Activity #10 and #11, students are able to use simple methods to do amazing things, in this case to measure the entire Earth.
2. Eratosthenes measured the Earth to within 2% of the modern measured value. Using a stick, protractor, and a piece of string you students can easily do as well.

What will your students learn about science?

1. Science is a cooperative venture. Without the help of student scientists at another school, this activity is not possible. Even though the activity itself is extremely simple (measure one angle at a specific time of day,) without cooperation nothing is gained.

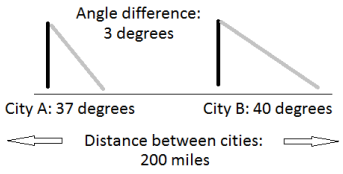
Conducting the Activity

Materials

1. A meter stick
2. String or twine
3. An accurate protractor

Measuring the Earth with Eratosthenes

1. The first step is to contact another teacher at your same grade level who lives at least 100 miles directly north or south of you – farther apart is better for this experiment. A direct north-south line between the cities is also important for this, you will need to know as exactly as possible how many miles north or south of you the other school is as opposed to the direct mileage between the cities. Look a map and select a likely city, research their schools on the internet and reach out to someone by email and send them an invitation to join your class in this exciting project. It may take one or two tries, but I bet you can find a partner without too much difficulty!
2. When the big day arrives, send an email in the morning to be sure you have sunny weather in both cities. A few minutes before noon, set up the yard sticks in the playground area. One stick should be held vertically, (use a small carpenter's level for this). Use the compass to lay out the second yardstick flat on the ground so that it points directly north. You have now made a simple sundial! Watch as the shadow moves clockwise; when the shadow lies directly along the flat yardstick, measure and record the position where the tip of the shadow falls. Depending on your location and the time of year, the shadow may extend past the end of the flat yardstick – that's okay, just mark its position with some sidewalk chalk.
3. Now that you've marked the tip of the shadow, stretch a piece of string from the top of the vertical yardstick down to where the tip of the shadow touched the ground. Measure the angle between the vertical stick and the string with a protractor as accurately as you can and record it. Email this information to each other – it will be the ***difference between the angles*** that will be important for this activity!
4. Eratosthenes believed that the Earth was round, and so the angle of the Sun in the sky would be different depending on how far north you were from the equator – and he was right! By setting up a simple ratio and proportion between the difference in the two angles and the distance between the cities, he was able to accurately measure the circumference of the Earth for the first time about 2,300 years ago. Eratosthenes' calculation for the size of the Earth was accurate to within about 2% of our modern value, how close can your students get? Set up your calculation as shown below!



Eratosthenes' experiment to measure the Earth!

$$\frac{360 \text{ degrees}}{\text{Angle difference}} = \frac{\text{Size of Earth}}{\text{Distance between cities}}$$

$$\text{Size of Earth} = \frac{360 \text{ degrees} \times \text{Distance between cities}}{\text{Angle difference}}$$

$$\text{Size of Earth} = 24{,}000 \text{ miles}$$

5. The actual circumference of the Earth is 24,900 miles. The example above was done by my own students several years ago and shows a value within 4% of the true size of the Earth – pretty good for kids using some string and a protractor! How close will your students get!

Discussion Questions

1. Eratosthenes obviously didn't have a telephone or the internet, how do you think he managed to do this activity in ancient Egypt? (Egypt was then part of the Greek/Macedonian empire.)
    - **Answer:** Eratosthenes did not take both measurements on the same day! The astronomer took a measure of the solar angle in the town of Syene in southern Egypt on the summer solstice. He then walked to the town of Alexandria in northern Egypt and carefully measured the distance along the way and measured the solar angle again on the summer solstice in the following year.
2. We sometimes think of ancient peoples as 'primitive' or even 'ignorant'. What do you think of the ancient Greek culture of Eratosthenes now that you know that people in this era were able to measure the size of the Earth and Moon, and even measure the distance between them accurately?
    - **Answer:** The ancient cultures were not all ignorant or primitive! Many cultures have had 'dark ages' where learning was not advanced, but ancient cultures were in many ways remarkably advanced!

Supplemental Materials

Going Deeper

Understanding what is happening when we measure the solar angle at two different locations, and how this helps us measure the Earth, is a masterpiece of scientific thinking. Sometimes the power of a simple experiment or argument are difficult to grasp.

One of the ways to comprehend the thinking of Eratosthenes is to draw the Earth and Sun, showing the angles between the Earth's core and the lines representing the rays of the Sun. See if you can understand Eratosthenes ideas this way!

There are many drawings of Eratosthenes ideas on the internet to help you!

Being an Astronomer

Measuring the solar angle with a stick, string, and protractor is another exercise that can show how the sky changes through the seasons. If your students can measure the solar angle once a week and keep a running record of the results, you will find that the solar angle changes measurably through the seasons.

Can you find a relation between the solar angle and the season?

Being a Scientist:

Climatic change is a hot topic in research and political debate these days, but climate doesn't just change slowly over centuries. The climatic change of the seasonal weather caused by the change in the solar angle is both powerful and measureable.

If your students keep a running record of both the solar angle and the average high temperature for each week, and interesting relationship will be revealed.

Create two graphs, one showing the solar angle over time, the other showing the weekly average high temperature over time. Compare the two graphs; what do you find?

The Sun is the most powerful factor in our climatic change. By comparing solar angle to temperature fluctuations, we can find a powerful link between how much sunlight we receive and our local temperatures.

Following Up

Ancient scientists like Eratosthenes, Pythagoras, Aristotle, and many others contributed to our modern scientific knowledge. Look into some of the ideas and discoveries of these ancient masters and see what you can find!
