1.1.3: Density and Pressure

The ***density*** of something is its mass divided by its volume (m/V) and is measured in *kilograms per cubic meter*, $\mathrm{kg/m^3}$ (or sometimes grams per cubic centimeter; $\mathrm{g/cm^3}$). So a kilogram of feathers and a kilogram of iron have the same mass and weigh the same but since the kilogram of feathers takes up more space (larger volume) it is less dense. One other version of density we will use is called the ***linear density*** which is the mass per length in $\mathrm{kg/m}$. Bass strings on guitars and in pianos have a larger linear density than the strings used for the higher notes. We will see why later on.

Pressure is defined to be a force acting over an area; $P = F/A$. There are several units of pressure; we will use the *pascal*, Pa, which is a newton per meter squared, $\mathrm{N/m^2}$. Other units are the bar; *atmospheres*; millimeters of mercury, mmHg; inches of water; torr, etc. A larger force over the same area increases pressure but the same force over a larger area decreases pressure. A dull knife does not apply the same pressure as a sharp knife because the area of contact of the dull blade is larger than the area of contact of the sharp blade. Being stepped on by the heel of a high heeled shoe hurts a lot more than if the same person steps on you with a low heel because the same force (the persons weight) applied with a high heel acts over a smaller area so the pressure is much higher. As we will see, the loudness of a sound wave is related to pressure; high volume sound exerts more pressure on average, and therefore more force on the surface of your eardrum.

For gasses in a closed container, pressure and volume are inversely proportional (pressure increases as volume decreases). Pressure and volume are also each directly proportional to temperature (either pressure or volume or both will increase if the temperature increases). These properties are sometimes summarized as the ***ideal gas law*** which can be written as $PV = nRT$. Here P is pressure, V is volume, T is temperature in kelvin, K. The variable n indicates how much gas there is (in moles where a mole is 6.0×10^{23} atoms or molecules) and R is a constant equal to $3.14\ \mathrm{J/mol\ K}$

Pressure in a liquid or gas is the weight of the liquid pushing downward on an area at a given depth (and is measured in the same units as pressure) or $P = \rho g h$ where h is the depth, ρ (Greek letter rho) is the density in kilograms per meter cubed, and g is gravitational acceleration. We sit at the bottom of a sea of air that pushes down on us. This pressure is called atmospheric pressure and it varies a little bit from day today because the air above us is moving and also because of changes in temperature and humidity (and so its density changes). When you use a straw you are decreasing the pressure inside the straw and atmospheric pressure outside the straw pushes the liquid up into the straw. This is why a straw would not work in a vacuum. If you are under water, the water above you pushes down on you in addition to the air above the water which pushes down on the water. Since water is much more dense than air, pressure changes a lot faster as you go deeper under water than it does if you change altitude in the air.

Bernoulli's principle says that if the speed of a fluid (liquid or gas) increases, the internal pressure in the fluid decreases. Take a strip of paper one inch wide and 12 inches long. Hold the short end up to your lips and blow. You'll notice that the strip pulls upward to meet the flowing air. This is because the moving air above has a slightly lower pressure than the stationary air below. A similar effect causes baseballs to change direction (curve balls) and airplane wings to have lift. Some wind instruments and the human voice operate in part because of forces due to the Bernoulli effect, as we shall see.

Video/audio examples:

- Pressure and a bed of nails: with a balloon, with a person. What would happen to the pressure if the number of nails is reduced? What would this do to the person lying on them?
- Bernoulli's principle: Hair dryer. Why does the ping pong ball stay suspended? Soccer ball. Why does the ball not follow Newton's first law and travel in a straight line? Several Examples. Why do the balloon's come together?

Questions on Density and Pressure:

Density

1. Does the mass of a car change if it is crushed into a cube? Explain.
2. Does the density of a car change if it is crushed into a cube? Explain.
3. Does a dieting person lose mass? Weight? Density? Explain your answers.
4. How does the density of water change when it freezes into ice?
5. Which is more dense, a kilogram of feathers or a kilogram of iron?
6. Which weighs more on the earth, a kilogram of feathers or a kilogram of iron?
7. What is the difference between density and linear density?

8. Given what you know about Newton's second law ($F = ma$), why would you expect a denser guitar string to vibrate more slowly when plucked with the same force?

Pressure

1. Why does a sharp knife cut better than a dull knife (even when you apply the same force)?
2. An old time magic trick (that originally came from India) was to lie down on a bed of nails (hundreds of nails sticking up through a board) without getting hurt. Using the definition of pressure, explain how this is possible.
3. Does a bathroom scale measure pressure or force? Explain. (Try this at home: Stand with both feet on the scale, look at the reading and then stand on one foot on the scale and check. Are the readings different?)
4. Which exerts more pressure on the ground, the foot of an elephant or a person in high-heeled shoes? State your reasoning.
5. You may notice that an unopened bag of chips is soft while on the ground but is puffs out to be firm when at cruising altitude in an airplane. Explain why.
6. Why would you want the bottom of a dam to be stronger than the top?
7. Why would it be slightly more difficult to suck soda through a straw on top of a high mountain as compared to sea level?
8. You decide you want to use a piece of garden hose with one end above water to go to the bottom of a pool $3\ \mathrm{m}$ deep and be able to breath. What is wrong with this plan?
9. A siphon is a tube that transfers liquid from a higher level to a lower level. How does it work?
10. What causes the 'lift' on an airplane wing?
11. Why is it easier on your heart when you are lying down compared to when you are standing?
12. Why does a lightweight shower curtain move in towards you when the shower is running?
13. How does an airplane wing provide lift?
14. The pressure variation in a sound wave from a jet engine is around $200\ \mathrm{Pa}$ (Pascal). What is this in $\mathrm{N/m^2}$?
15. Suppose a marimba mallet makes contact with a wooden marimba bar and applies a strike force of $600\ \mathrm{N}$. You measure the contact area of the mallet to be 5 square millimeters ($5.0 \times 10^{-6}\ \mathrm{m^2}$). What pressure (in $\mathrm{N/m^2}$) was exerted on the marimba bar? Convert this to atmospheres (1 atmosphere $= 101325\ \mathrm{N/m^2}$). Do you think this could do damage to a marimba bar?
16. For sound waves from a normal conversation the pressure at the listener's ear fluctuates by around $0.2\ \mathrm{N/m^2}$. How much does the force change on the eardrum if the area is $1\ \mathrm{cm^2}$ ($1.0 \times 10^{-4}\ \mathrm{m^2}$)?
17. Suppose the pressure of a sound wave reaching a microphone fluctuates by 0.002 atmospheres. What is the force on the microphone if it has an area of $2\ \mathrm{cm^2}$ ($2.0 \times 10^{-4}\ \mathrm{m^2}$)?
