3.3.2: Q-Factor Simulation

This simulation expands on what was shown in the previous simulation. Now there are fifty-one independent, damped, driven harmonic oscillators instead of five. Each oscillator is independent of the others and has a different mass which means their natural frequencies are all slightly different. The ratio of the driving frequency, f, to the natural frequency, f_o for each oscillator is given on the horizontal axis. The masses are heaviest on the left (blue) and lightest on the right (red). Each mass is driven by the same sinusoidal driver with the same amplitude and the same driving frequency. A red arrow shows the driving force being applied to each of the masses. The amplitude of the driving force (red arrow) is the same for all damping but will appear different because the scale on the left changes to match the amplitude of the center mass.

In the previous simulations we examined the effects of damping, driving amplitude and driving frequency on a damped, driven harmonic oscillator. In this simulation, the only adjustable parameter is the amount of damping, b which is the same for each oscillator. The grey dots show the maximum displacement of each of the masses for a given amount of damping but note that the scale on the left changes depending on amplitude. Keep in mind the masses are independent and do not interact with each other.

Simulation Questions

1. On which side of the center is the driving frequency higher than the natural frequency? On which side is the driving frequency lower than the natural frequency?
2. Damped driven harmonic oscillators settle into a steady state after a certain time as you saw in the previous two simulations. How long does it take for the steady state behavior (when the center mass reaches its maximum shown by the grey circles) to emerge if $b = 0.1 \text{ Ns/m}$? If $b = 0.2 \text{ Ns/m}$?
3. The time to reach steady state is also an indication of how long the oscillations take to relax to zero if the driver is turned off. Based on your previous answer, which amount of damping will let the system 'ring' the longest?
4. How does the damping coefficient affect the maximum displacement of the center mass (look at the scale on the left for different values of damping)?
5. Describe the shape of the maximum amplitudes of all the masses when damping is small compared to when damping is large. In which case are more oscillators moving, low damping or high damping?
6. Based on what you have seen, what is the relationship between damping and Q-factor?

Chapter Four Summary

Oscillators will vibrate at their natural frequency which is determined by the physical properties of the system (mass, stiffness, etc.). Most real oscillators have some damping (friction) so that they will gradually stop unless external forces are acting. If an oscillator is pushed (driven) with a periodic force it may have several different amplitudes of vibration, depending on the frequency of the driving force. The largest amplitude occurs when the driving frequency equals the natural frequency. This condition is called resonance.

Resonances may occur at a broad range of frequencies (low Q) or a very sharp, single frequency (high Q) in which case the vibrations will die away more slowly once the driving force is turned off.