14.1.3: Temperament Simulation

The simulation shows an octave of notes on a keyboard starting with C which has a modern day frequency of $261.63\ \mathrm{Hz}$. You can click on each key to hear the frequency corresponding to note shown on the key. The buttons across the top allow you to select the tunings of four different temperaments; choosing a different temperament will change the frequency of the notes between C and C an octave higher to match the chosen temperament. The notes sound mechanical because they are pure sine wave with no overtones and so have no timbre. The scale button plays the notes on the scale and the $5\mathrm{th}$ button plays a perfect fifth; C and G for this scale. Clicking on the overtone checkbox will add the second harmonic (twice the fundamental) at half the volume of the fundamental and the third harmonic with a volume one third the fundamental to any note being played. These overtones are louder than would be normal for a stringed instrument in order to better demonstrate beating and dissonance in the different chord combinations (Go back to chapter 10 to review beats and dissonance if you have forgotten). Three chords used in popular music can also be played. The C Major chord plays the notes C, E and G; the D major chord plays the notes D, $\mathrm{F}^{\#}$ (F sharp) and A; the G major chord plays the notes G, B, and D.

Note

Press Reset if sound does not play when the simulation first loads.

Simulation Quesions:

1. Play a few notes by clicking on the keyboard and then change temperaments and try the same notes. Can you hear the difference?
2. Because the Just Noticeable Different (JND) for most people is a few hertz, you probably were not able to hear the difference between the same note in two different temperaments if they are played separately. Push the scale button and listen to the scale in different temperaments. Can you detect the difference for a string of notes?
3. Again, since the difference between the same note in different temperaments is less than three Hertz you probably can't tell from individual notes played one after the other which temperament you are playing. Now try the fifth in each of the temperaments. What do you notice? Which temperaments have the least beating for the fifth?
4. You can hear the difference in temperaments by listening to a $5\mathrm{th}$ or chords of three notes. Try each of the three chords in different temperaments. You may hear some beating for some temperaments that you don't hear in others. Describe what you hear for each temperament.
5. Now click the overtone check box and play a few individual notes. What is different?
6. Only the first two overtones are added and they are much louder than would occur in a real instrument so the timbre still isn't pleasant like a real instrument would be. Now play the fifth and the chords in different temperaments with overtones. Which temperaments have less beating and dissonance?
7. You may have noticed less beating and dissonance for the Pythagorean and Just temperaments with overtones. Recall that strings with length ratios of $3/2$ and $4/3$ are thought to sound good together because the frequencies in these two scales have exact ratios. Harmonics of these frequencies also have the same ratio so they also sound good together; the harmonics of one note are also harmonics of other notes. This will not be the case for an equal temperament scale because the 12 notes of the scale are equally spaced in the octave which results in the $5\mathrm{th}$ note not being an exact $3/2$ ratio of the tonic (it is in fact a ratio of 1.4953between C and G for the equal temperament instead of $3/2=1.5$ as is the case for the Pythagorean temperament). How does the choice of temperament change how musical scales sound?

Video/audio examples:

- Web site on how to read music.
- Brief explanation of music theory from a musicians' frame of reference.
- Wikipedia on musical tuning systems (towards the bottom) has samples as well.
- The choice of mode is actually a bit more complicated than simply choosing a starting note. See The Physics of Musical Scales by Durfee and Colton.
- A more in-depth discussion of Harmonic Series as it relates to musical scales.
- Wikipedia diagram of harmonic series intervals.
- A more detailed explanation of musical intervals and scales.
- The circle of fifths is a way to see how different musical keys are related. Here is a circle of fifths simulation from Wolfram (you may need to download their plug in to play with this demonstration).

- Pierre Lewis gives a detailed explanation of temperaments.
- Dallin Durfee's Temperament Studio , a Java applet that plays scales and chords in different temperaments.
- Pierre Lewis's Java Tuner, an applet that plays scales in different temperaments. If security blocks the Java you can still download it and use it locally (scroll to the bottom of the page).
- Wolfram's demonstration of musical scales and temperaments (you may need to download their plug in to play with this demonstration).
- Music by Bill Sethares using alternate tunings (click on mp3s for listening).
- Article in *American Scientist* by Cook and Hayashi about The Psychoacoustics of Harmony Perception.
- Human voices are not restricted to sounding a specific note like most musical instruments; the voice is very flexible in the choice of tones it can make. The so called barbershop seventh is an example. The notes in a typical chord have the ratios $4:5:6:7$ which are notes in the Just scale. These frequencies also cause a missing fundamental frequency so that four singers will create five perceived notes. The chord makes use of a tuning scale that cannot be reproduced on a piano tuned to an equal tempered scale. Here is an example from The Music Man (skip to 40 seconds to hear the first chord).

Summary

Musicians even before recorded history have made music and musical instruments that sounded good to them. The oldest flute found is about $35,000$ years old and is made of the bone of a giant vulture. Ancient Greek mathematicians realized that the notes on a string that sounded pleasant together have specific length ratios. For a long time after that mathematicians thought they could codify the scales used by musicians into set ratios of frequencies. With the advent of modern music which uses more than one octave, much more harmony and instruments that have fixed scales that are not easily changed, compromises in the 'perfect' ratios had to be made. The slight differences between Pythagorean and equal tempered scales can be heard by a trained ear but are not enough to change the overall feel of a musical piece.

Questions on Scales:

1. What is the Pythagorean scale and how was it developed?
2. What is an octave?
3. If you hold a guitar string down in the middle each side of the string will be a ________ lower (or half) than the original sound of the full length.
4. In what way is the Pythagorean scale limited? What problems arise from its limitation?
5. Define a perfect fifth and a perfect fourth.
6. What is the ratio of a string’s length that produces a perfect 5th? A perfect 4th?
7. Why do notes with a ratio of $2/3$ (a perfect fifth) and other ratios sound good together?
8. Explain the contribution Ptolemy made to the Pythagorean scale.
9. Explain how the Pythagorean formula eventually calculates notes which are not on the desired scale.
10. Why is it that most people cannot hear the small frequency differences between notes on the Polemic scale and the Pythagorean scale (Hint: Review the chapter on perception.)?
11. What is a mode?
12. A particular choice of starting note and system of generating a note scale is called a __________?
13. What is a temperament?
14. What is a semitone?
15. How many semitones are in an octave? How many cents are in a semitone?
16. Why did the question of music scales become more complicated beginning with the 14th century?
17. How are modern musical scales in Western culture different from older classical music?
18. Why and when was the equal tempered scale developed? What problems did it solve?
19. Who popularized the equal tempered scale?
20. On the equal temperament scale _______ cents is about 0.3 Hz.
21. What is the current scale that most modern composers use? What is the compromise of using this scale?
22. When tuning a piano, how are the higher notes tuned as compared to the lower notes?
23. What is a Railsback curve?
24. Why are the overtones in a piano not exactly harmonic? What is the result of this mismatch?
25. Barbershop quartets, generally, have a very rich sound and use a specific type of scale. What scale do they use? Why is it easier for the quartet to use this scale compared to a piano and what happens when their frequencies combine?

26. The note A gradually changed in frequency over time from $415\ Hz$ to $440\ Hz$. What made this possible?