5.E: Relativity (Exercises)

Conceptual Questions

5.1 Invariance of Physical Laws

1. Which of Einstein's postulates of special relativity includes a concept that does not fit with the ideas of classical physics? Explain.

2. Is Earth an inertial frame of reference? Is the sun? Justify your response.

3. When you are flying in a commercial jet, it may appear to you that the airplane is stationary and Earth is moving beneath you. Is this point of view valid? Discuss briefly.

5.3 Time Dilation

4. (a) Does motion affect the rate of a clock as measured by an observer moving with it?

(b) Does motion affect how an observer moving relative to a clock measures its rate?

5. To whom does the elapsed time for a process seem to be longer, an observer moving relative to the process or an observer moving with the process? Which observer measures the interval of proper time?

6. (a) How could you travel far into the future of Earth without aging significantly?

(b) Could this method also allow you to travel into the past?

5.4 Length Contraction

7. To whom does an object seem greater in length, an observer moving with the object or an observer moving relative to the object? Which observer measures the object's proper length?

8. Relativistic effects such as time dilation and length contraction are present for cars and airplanes. Why do these effects seem strange to us?

9. Suppose an astronaut is moving relative to Earth at a significant fraction of the speed of light.

(a) Does he observe the rate of his clocks to have slowed?

(b) What change in the rate of earthbound clocks does he see?

(c) Does his ship seem to him to shorten?

(d) What about the distance between two stars that lie in the direction of his motion? (e) Do he and an earthbound observer agree on his velocity relative to Earth?

5.7 Doppler Effect for Light

10. Explain the meaning of the terms "red shift" and "blue shift" as they relate to the relativistic Doppler effect.

11. What happens to the relativistic Doppler effect when relative velocity is zero? Is this the expected result?

12. Is the relativistic Doppler effect consistent with the classical Doppler effect in the respect that λ_{obs} is larger for motion away?

13. All galaxies farther away than about 50×10^6 **ly** exhibit a red shift in their emitted light that is proportional to distance, with those farther and farther away having progressively greater red shifts. What does this imply, assuming that the only source of red shift is relative motion?

5.8 Relativistic Momentum

14. How does modern relativity modify the law of conservation of momentum?

15. Is it possible for an external force to be acting on a system and relativistic momentum to be conserved? Explain.

5.9 Relativistic Energy

16. How are the classical laws of conservation of energy and conservation of mass modified by modern relativity?

17. What happens to the mass of water in a pot when it cools, assuming no molecules escape or are added? Is this observable in practice? Explain.

18. Consider a thought experiment. You place an expanded balloon of air on weighing scales outside in the early morning. The balloon stays on the scales and you are able to measure changes in its mass. Does the mass of the balloon change as the day progresses? Discuss the difficulties in carrying out this experiment.

19. The mass of the fuel in a nuclear reactor decreases by an observable amount as it puts out energy. Is the same true for the coal and oxygen combined in a conventional power plant? If so, is this observable in practice for the coal and oxygen? Explain.

20. We know that the velocity of an object with mass has an upper limit of c. Is there an upper limit on its momentum? Its energy? Explain.

21. Given the fact that light travels at c , can it have mass? Explain.

22. If you use an Earth-based telescope to project a laser beam onto the moon, you can move the spot across the moon's surface at a velocity greater than the speed of light. Does this violate modern relativity? (Note that light is being sent from the Earth to the moon, not across the surface of the moon.)

Problems

5.3 Time Dilation

23. (a) What is γ if $v = 0.250c$?

(b) If $v = 0.500c$?

24. (a) What is γ if $v = 0.100c$?

(b) If $v = 0.900c$?

25. Particles called π-mesons are produced by accelerator beams. If these particles travel at $2.70 \times 10^8 m/s$ and live $2.60 \times 10^{-8} s$ when at rest relative to an observer, how long do they live as viewed in the laboratory?

26. Suppose a particle called a kaon is created by cosmic radiation striking the atmosphere. It moves by you at $0.980c$, and it lives $1.24 \times 10^{-8} s$ when at rest relative to an observer. How long does it live as you observe it?

27. A neutral π-meson is a particle that can be created by accelerator beams. If one such particle lives $1.40 \times 10^{-16} s$ as measured in the laboratory, and $0.840 \times 10^{-16} s$ when at rest relative to an observer, what is its velocity relative to the laboratory?

28. A neutron lives 900 s when at rest relative to an observer. How fast is the neutron moving relative to an observer who measures its life span to be 2065 s?

29. If relativistic effects are to be less than 1%, then γ must be less than 1.01. At what relative velocity is $\gamma = 1.01$?

30. If relativistic effects are to be less than 3%, then γ must be less than 1.03. At what relative velocity is $\gamma = 1.03$?

5.4 Length Contraction

31. A spaceship, 200 m long as seen on board, moves by the Earth at 0.970c. What is its length as measured by an earthbound observer?

32. How fast would a 6.0 m-long sports car have to be going past you in order for it to appear only 5.5 m long?

33. (a) How far does the muon in Example 5.1 travel according to the earthbound observer?

(b) How far does it travel as viewed by an observer moving with it? Base your calculation on its velocity relative to the Earth and the time it lives (proper time).

(c) Verify that these two distances are related through length contraction $\gamma = 3.20$.

34. (a) How long would the muon in Example 5.1 have lived as observed on Earth if its velocity was $0.0500c$?

(b) How far would it have traveled as observed on Earth?

(c) What distance is this in the muon's frame?

35. Unreasonable Results A spaceship is heading directly toward Earth at a velocity of 0.800c. The astronaut on board claims that he can send a canister toward the Earth at 1.20c relative to Earth.

(a) Calculate the velocity the canister must have relative to the spaceship.

(b) What is unreasonable about this result?

(c) Which assumptions are unreasonable or inconsistent?

5.5 The Lorentz Transformation

36. Describe the following physical occurrences as events, that is, in the form **(x, y, z, t)**:

(a) A postman rings a doorbell of a house precisely at noon.

(b) At the same time as the doorbell is rung, a slice of bread pops out of a toaster that is located 10 m from the door in the east direction from the door.

(c) Ten seconds later, an airplane arrives at the airport, which is 10 km from the door in the east direction and 2 km to the south.

37. Describe what happens to the angle $\alpha = tan(v/c)$, and therefore to the transformed axes in Figure 5.17, as the relative velocity **v** of the S and S' frames of reference approaches **c**.

38. Describe the shape of the world line on a space-time diagram of

(a) an object that remains at rest at a specific position along the **x**-axis;

(b) an object that moves at constant velocity **u** in the **x**-direction;

(c) an object that begins at rest and accelerates at a constant rate of in the positive **x**-direction.

39. A man standing still at a train station watches two boys throwing a baseball in a moving train. Suppose the train is moving east with a constant speed of 20 m/s and one of the boys throws the ball with a speed of 5 m/s with respect to himself toward the other boy, who is 5 m west from him. What is the velocity of the ball as observed by the man on the station?

40. When observed from the sun at a particular instant, Earth and Mars appear to move in opposite directions with speeds 108,000 km/h and 86,871 km/h, respectively. What is the speed of Mars at this instant when observed from Earth?

41. A man is running on a straight road perpendicular to a train track and away from the track at a speed of 12 m/s. The train is moving with a speed of 30 m/s with respect to the track. What is the speed of the man with respect to a passenger sitting at rest in the train?

42. A man is running on a straight road that makes **30°** with the train track. The man is running in the direction on the road that is away from the track at a speed of 12 m/s. The train is moving with a speed of 30 m/s with respect to the track. What is the speed of the man with respect to a passenger sitting at rest in the train?

43. In a frame at rest with respect to the billiard table, a billiard ball of mass m moving with speed v strikes another billiard ball of mass m at rest. The first ball comes to rest after the collision while the second ball takes off with speed v in the original direction of the motion of the first ball. This shows that momentum is conserved in this frame.

(a) Now, describe the same collision from the perspective of a frame that is moving with speed v in the direction of the motion of the first ball.

(b) Is the momentum conserved in this frame?

44. In a frame at rest with respect to the billiard table, two billiard balls of same mass **m** are moving toward each other with the same speed **v**. After the collision, the two balls come to rest.

(a) Show that momentum is conserved in this frame.

(b) Now, describe the same collision from the perspective of a frame that is moving with speed **v** in the direction of the motion of the first ball.

(c) Is the momentum conserved in this frame?

45. In a frame S, two events are observed: event 1: a pion is created at rest at the origin and event 2: the pion disintegrates after time τ. Another observer in a frame S' is moving in the positive direction along the positive x-axis with a constant speed v and observes the same two events in his frame. The origins of the two frames coincide at $t = t' = 0$.

(a) Find the positions and timings of these two events in the frame S' (a) according to the Galilean transformation, and

(b) according to the Lorentz transformation.

5.6 Relativistic Velocity Transformation

46. If two spaceships are heading directly toward each other at 0.800**c**, at what speed must a canister be shot from the first ship to approach the other at 0.999**c** as seen by the second ship?

47. Two planets are on a collision course, heading directly toward each other at 0.250**c**. A spaceship sent from one planet approaches the second at 0.750**c** as seen by the second planet. What is the velocity of the ship relative to the first planet?

48. When a missile is shot from one spaceship toward another, it leaves the first at 0.950**c** and approaches the other at 0.750**c**. What is the relative velocity of the two ships?

49. What is the relative velocity of two spaceships if one fires a missile at the other at 0.750**c** and the other observes it to approach at 0.950**c**?

50. Prove that for any relative velocity **v** between two observers, a beam of light sent from one to the other will approach at speed **c** (provided that **v** is less than **c**, of course).

51. Show that for any relative velocity **v** between two observers, a beam of light projected by one directly away from the other will move away at the speed of light (provided that **v** is less than **c**, of course).

5.7 Doppler Effect for Light

52. A highway patrol officer uses a device that measures the speed of vehicles by bouncing radar off them and measuring the Doppler shift. The outgoing radar has a frequency of 100 GHz and the returning echo has a frequency 15.0 kHz higher. What is the velocity of the vehicle? Note that there are two Doppler shifts in echoes. Be certain not to round off until the end of the problem, because the effect is small.

5.8 Relativistic Momentum

53. Find the momentum of a helium nucleus having a mass of $6.68 \times 10^{-27} kg$ that is moving at 0.200**c**.

54. What is the momentum of an electron traveling at 0.980**c**?

55. (a) Find the momentum of a $1.00 \times 10^9 - kg$ asteroid heading towards Earth at 30.0 km/s.

(b) Find the ratio of this momentum to the classical momentum. (Hint: Use the approximation that $\gamma = 1 + (1/2)v^2/c^2$ at low velocities.)

56. (a) What is the momentum of a 2000-kg satellite orbiting at 4.00 km/s? (b) Find the ratio of this momentum to the classical momentum. (Hint: Use the approximation that $\gamma = 1 + (1/2)v^2/c^2$ at low velocities.)

57. What is the velocity of an electron that has a momentum of $3.04 \times 10^{-21} kg \cdot m/s$? Note that you must calculate the velocity to at least four digits to see the difference from **c**.

58. Find the velocity of a proton that has a momentum of $4.48 \times 10^{-19} kg \cdot m/s$.

5.9 Relativistic Energy

59. What is the rest energy of an electron, given its mass is $9.11 \times 10^{-31} kg$? Give your answer in joules and MeV.

60. Find the rest energy in joules and MeV of a proton, given its mass is $1.67 \times 10^{-27} kg$.

61. If the rest energies of a proton and a neutron (the two constituents of nuclei) are 938.3 and 939.6 MeV, respectively, what is the difference in their mass in kilograms?

62. The Big Bang that began the universe is estimated to have released $10^{68} J$ of energy. How many stars could half this energy create, assuming the average star's mass is $4.00 \times 10^{30} kg$?

63. A supernova explosion of a $2.00 \times 10^{31} kg$ star produces $1.00 \times 10^{44} J$ of energy.

(a) How many kilograms of mass are converted to energy in the explosion?

(b) What is the ratio $\Delta m/m$ of mass destroyed to the original mass of the star?

64. (a) Using data from Potential Energy of a System, calculate the mass converted to energy by the fission of 1.00 kg of uranium.

(b) What is the ratio of mass destroyed to the original mass, $\Delta m/m$?

65. (a) Using data from Potential Energy of a System, calculate the amount of mass converted to energy by the fusion of 1.00 kg of hydrogen.

(b) What is the ratio of mass destroyed to the original mass, $\Delta m/m$?

(c) How does this compare with $\Delta m/m$ for the fission of 1.00 kg of uranium?

66. There is approximately $10^{34} J$ of energy available from fusion of hydrogen in the world's oceans.

(a) If $10^{33} J$ of this energy were utilized, what would be the decrease in mass of the oceans?

(b) How great a volume of water does this correspond to?

(c) Comment on whether this is a significant fraction of the total mass of the oceans.

67. A muon has a rest mass energy of 105.7 MeV, and it decays into an electron and a massless particle.

(a) If all the lost mass is converted into the electron's kinetic energy, find γ for the electron.

(b) What is the electron's velocity?

68. A π-meson is a particle that decays into a muon and a massless particle. The π-meson has a rest mass energy of 139.6 MeV, and the muon has a rest mass energy of 105.7 MeV. Suppose the π-meson is at rest and all of the missing mass goes into the muon's kinetic energy. How fast will the muon move?

69. (a) Calculate the relativistic kinetic energy of a 1000-kg car moving at 30.0 m/s if the speed of light were only 45.0 m/s.

(b) Find the ratio of the relativistic kinetic energy to classical.

70. Alpha decay is nuclear decay in which a helium nucleus is emitted. If the helium nucleus has a mass of $6.80 \times 10^{-27} kg$ and is given 5.00 MeV of kinetic energy, what is its velocity?

71. (a) Beta decay is nuclear decay in which an electron is emitted. If the electron is given 0.750 MeV of kinetic energy, what is its velocity?

(b) Comment on how the high velocity is consistent with the kinetic energy as it compares to the rest mass energy of the electron.

Additional Problems

72. (a) At what relative velocity is $\gamma = 1.50$?

(b) At what relative velocity is $\gamma = 100$?γ=100?

73. (a) At what relative velocity is $\gamma = 2.00$?

(b) At what relative velocity is $\gamma = 10.0$?

74. Unreasonable Results (a) Find the value of γ required for the following situation. An earthbound observer measures 23.9 h to have passed while signals from a high-velocity space probe indicate that 24.0 h have passed on board.

(b) What is unreasonable about this result?

(c) Which assumptions are unreasonable or inconsistent?

75. (a) How long does it take the astronaut in Example 5.5 to travel 4.30 ly at $0.99944c$(as measured by the earthbound observer)?

(b) How long does it take according to the astronaut?

(c) Verify that these two times are related through time dilation with $\gamma = 30.00$ as given.

76. (a) How fast would an athlete need to be running for a 100-m race to look 100 yd long?

(b) Is the answer consistent with the fact that relativistic effects are difficult to observe in ordinary circumstances? Explain.

77. (a) Find the value of γ for the following situation. An astronaut measures the length of his spaceship to be 100 m, while an earthbound observer measures it to be 25.0 m.

(b) What is the speed of the spaceship relative to Earth?

78. A clock in a spaceship runs one-tenth the rate at which an identical clock on Earth runs. What is the speed of the spaceship?

79. An astronaut has a heartbeat rate of 66 beats per minute as measured during his physical exam on Earth. The heartbeat rate of the astronaut is measured when he is in a spaceship traveling at 0.5c with respect to Earth by an observer (A) in the ship and by an observer (B) on Earth.

(a) Describe an experimental method by which observer B on Earth will be able to determine the heartbeat rate of the astronaut when the astronaut is in the spaceship.

(b) What will be the heartbeat rate(s) of the astronaut reported by observers A and B?

80. A spaceship (A) is moving at speed **c/2** with respect to another spaceship (B). Observers in A and B set their clocks so that the event at **(x, y, z, t)** of turning on a laser in spaceship B has coordinates (0, 0, 0, 0) in A and also (0, 0, 0, 0) in B. An observer at the origin of B turns on the laser at $t = 0$ and turns it off at $t = \tau$ in his time. What is the time duration between on and off as seen by an observer in A?

81. Same two observers as in the preceding exercise, but now we look at two events occurring in spaceship A. A photon arrives at the origin of A at its time $t = 0$ and another photon arrives at $(x = 1.00m, 0, 0)$at $t = 0$ in the frame of ship A.

(a) Find the coordinates and times of the two events as seen by an observer in frame B.

(b) In which frame are the two events simultaneous and in which frame are they are not simultaneous?

82. Same two observers as in the preceding exercises. A rod of length 1 m is laid out on the **x**-axis in the frame of B from origin to $(x = 1.00m, 0, 0)$. What is the length of the rod observed by an observer in the frame of spaceship A?

83. An observer at origin of inertial frame S sees a flashbulb go off at $x = 150km, y = 15.0km$, and $z = 1.00km$ at time $t = 4.5 \times 10^{-4} s$. At what time and position in the S' system did the flash occur, if S' is moving along shared **x**-direction with S at a velocity $v = 0.6c$?

84. An observer sees two events $1.5 \times 10^{-8} s$ apart at a separation of 800 m. How fast must a second observer be moving relative to the first to see the two events occur simultaneously?

85. An observer standing by the railroad tracks sees two bolts of lightning strike the ends of a 500-m-long train simultaneously at the instant the middle of the train passes him at 50 m/s. Use the Lorentz transformation to find the time between the lightning strikes as measured by a passenger seated in the middle of the train.

86. Two astronomical events are observed from Earth to occur at a time of 1 s apart and a distance separation of $1.5 \times 10^9 m$ from each other.

(a) Determine whether separation of the two events is space like or time like.

(b) State what this implies about whether it is consistent with special relativity for one event to have caused the other?

87. Two astronomical events are observed from Earth to occur at a time of 0.30 s apart and a distance separation of $2.0 \times 10^9 m$ from each other. How fast must a spacecraft travel from the site of one event toward the other to make the events occur at the same time when measured in the frame of reference of the spacecraft?

88. A spacecraft starts from being at rest at the origin and accelerates at a constant rate **g**, as seen from Earth, taken to be an inertial frame, until it reaches a speed of **c/2**.

(a) Show that the increment of proper time is related to the elapsed time in Earth's frame by:

$$d\tau = \sqrt{1 - v2/c^2}\, dt$$

.

(b) Find an expression for the elapsed time to reach speed **c/2** as seen in Earth's frame.

(c) Use the relationship in (a) to obtain a similar expression for the elapsed proper time to reach c/2 as seen in the spacecraft, and determine the ratio of the time seen from Earth with that on the spacecraft to reach the final speed.

89. (a) All but the closest galaxies are receding from our own Milky Way Galaxy. If a galaxy $12.0 \times 10^9 ly$ away is receding from us at 0.900c, at what velocity relative to us must we send an exploratory probe to approach the other galaxy at 0.990c as measured from that galaxy?

(b) How long will it take the probe to reach the other galaxy as measured from Earth? You may assume that the velocity of the other galaxy remains constant.

(c) How long will it then take for a radio signal to be beamed back? (All of this is possible in principle, but not practical.)

90. Suppose a spaceship heading straight toward the Earth at 0.750**c** can shoot a canister at 0.500**c** relative to the ship.

(a) What is the velocity of the canister relative to Earth, if it is shot directly at Earth?

(b) If it is shot directly away from Earth?

91. Repeat the preceding problem with the ship heading directly away from Earth.

92. If a spaceship is approaching the Earth at 0.100**c** and a message capsule is sent toward it at 0.100**c** relative to Earth, what is the speed of the capsule relative to the ship?

93. (a) Suppose the speed of light were only 3000 m/s. A jet fighter moving toward a target on the ground at 800 m/s shoots bullets, each having a muzzle velocity of 1000 m/s. What are the bullets' velocity relative to the target?

(b) If the speed of light was this small, would you observe relativistic effects in everyday life? Discuss.

94. If a galaxy moving away from the Earth has a speed of 1000 km/s and emits 656 nm light characteristic of hydrogen (the most common element in the universe).

(a) What wavelength would we observe on Earth?

(b) What type of electromagnetic radiation is this? (c) Why is the speed of Earth in its orbit negligible here?

95. A space probe speeding towards the nearest star moves at $0.250c$and sends radio information at a broadcast frequency of 1.00 GHz. What frequency is received on Earth?

96. Near the center of our galaxy, hydrogen gas is moving directly away from us in its orbit about a black hole. We receive 1900 nm electromagnetic radiation and know that it was 1875 nm when emitted by the hydrogen gas. What is the speed of the gas?

97. (a) Calculate the speed of a $1.00 - \mu g$ particle of dust that has the same momentum as a proton moving at 0.999**c**.

(b) What does the small speed tell us about the mass of a proton compared to even a tiny amount of macroscopic matter?

98. (a) Calculate γ for a proton that has a momentum of $1.00 kg \cdot m/s$.

(b) What is its speed? Such protons form a rare component of cosmic radiation with uncertain origins.

99. Show that the relativistic form of Newton's second law is

(a) $F = m\dfrac{du}{dt}\dfrac{1}{(1-u^2/c^2)^{3/2}}$;

(b) Find the force needed to accelerate a mass of 1 kg by 1 m/s^2 when it is traveling at a velocity of **c/2**.

100. A positron is an antimatter version of the electron, having exactly the same mass. When a positron and an electron meet, they annihilate, converting all of their mass into energy.

(a) Find the energy released, assuming negligible kinetic energy before the annihilation.

(b) If this energy is given to a proton in the form of kinetic energy, what is its velocity?

(c) If this energy is given to another electron in the form of kinetic energy, what is its velocity?

101. What is the kinetic energy in MeV of a π-meson that lives $1.40 \times 10^{-16} s$ as measured in the laboratory, and $0.840 \times 10^{-16} s$ when at rest relative to an observer, given that its rest energy is 135 MeV?

102. Find the kinetic energy in MeV of a neutron with a measured life span of 2065 s, given its rest energy is 939.6 MeV, and rest life span is 900s.

103. (a) Show that $(pc)^2/(mc^2)^2 = \gamma^2 - 1$. This means that at large velocities $pc >> mc^2$.

(b) Is $E \approx pc$ when $\gamma = 30.0$, as for the astronaut discussed in the twin paradox?

104. One cosmic ray neutron has a velocity of $0.250c$relative to the Earth.

(a) What is the neutron's total energy in MeV?

(b) Find its momentum.

(c) Is $E \approx pc$ in this situation? Discuss in terms of the equation given in part (a) of the previous problem.

105. What is γ for a proton having a mass energy of 938.3 MeV accelerated through an effective potential of 1.0 TV (teravolt)?

106. (a) What is the effective accelerating potential for electrons at the Stanford Linear Accelerator, if $\gamma = 1.00 \times 10^5$ for them?

(b) What is their total energy (nearly the same as kinetic in this case) in GeV?

107. (a) Using data from Potential Energy of a System, find the mass destroyed when the energy in a barrel of crude oil is released.

(b) Given these barrels contain 200 liters and assuming the density of crude oil is $750 kg/m^3$, what is the ratio of mass destroyed to original mass, $\Delta m/m$?

108. (a) Calculate the energy released by the destruction of 1.00 kg of mass.

(b) How many kilograms could be lifted to a 10.0 km height by this amount of energy?

109. A Van de Graaff accelerator utilizes a 50.0 MV potential difference to accelerate charged particles such as protons.

(a) What is the velocity of a proton accelerated by such a potential?

(b) An electron?

110. Suppose you use an average of $500 kW \cdot h$ of electric energy per month in your home.

(a) How long would 1.00 g of mass converted to electric energy with an efficiency of 38.0% last you?

(b) How many homes could be supplied at the $500 kW \cdot h$ per month rate for one year by the energy from the described mass conversion?

111. (a) A nuclear power plant converts energy from nuclear fission into electricity with an efficiency of 35.0%. How much mass is destroyed in one year to produce a continuous 1000 MW of electric power?

(b) Do you think it would be possible to observe this mass loss if the total mass of the fuel is $10^4 kg$?

112. Nuclear-powered rockets were researched for some years before safety concerns became paramount.

(a) What fraction of a rocket's mass would have to be destroyed to get it into a low Earth orbit, neglecting the decrease in gravity? (Assume an orbital altitude of 250 km, and calculate both the kinetic energy (classical) and the gravitational potential energy needed.)

(b) If the ship has a mass of $1.00 \times 10^5 kg$ (100 tons), what total yield nuclear explosion in tons of TNT is needed?

113. The sun produces energy at a rate of 3.85×10^{26} W by the fusion of hydrogen. About **0.7%** of each kilogram of hydrogen goes into the energy generated by the Sun.

(a) How many kilograms of hydrogen undergo fusion each second?

(b) If the sun is 90.0% hydrogen and half of this can undergo fusion before the sun changes character, how long could it produce energy at its current rate?

(c) How many kilograms of mass is the sun losing per second?

(d) What fraction of its mass will it have lost in the time found in part (b)?

114. Show that $E^2 - p^2c^2$ for a particle is invariant under Lorentz transformations.
