3.E: Vectors (Exercises)

Conceptual Questions

2.1 Scalars and Vectors

1. A weather forecast states the temperature is predicted to be −5 °C the following day. Is this temperature a vector or a scalar quantity? Explain.
2. Which of the following is a vector: a person's height, the altitude on Mt. Everest, the velocity of a fly, the age of Earth, the boiling point of water, the cost of a book, Earth's population, or the acceleration of gravity?
3. Give a specific example of a vector, stating its magnitude, units, and direction.
4. What do vectors and scalars have in common? How do they differ?
5. Suppose you add two vectors $\vec{A}$ and $\vec{B}$. What relative direction between them produces the resultant with the greatest magnitude? What is the maximum magnitude? What relative direction between them produces the resultant with the smallest magnitude? What is the minimum magnitude?
6. Is it possible to add a scalar quantity to a vector quantity?
7. Is it possible for two vectors of different magnitudes to add to zero? Is it possible for three vectors of different magnitudes to add to zero? Explain.
8. Does the odometer in an automobile indicate a scalar or a vector quantity?
9. When a 10,000-m runner competing on a 400-m track crosses the finish line, what is the runner's net displacement? Can this displacement be zero? Explain.
10. A vector has zero magnitude. Is it necessary to specify its direction? Explain.
11. Can a magnitude of a vector be negative?
12. Can the magnitude of a particle's displacement be greater that the distance traveled?
13. If two vectors are equal, what can you say about their components? What can you say about their magnitudes? What can you say about their directions?
14. If three vectors sum up to zero, what geometric condition do they satisfy?

2.2 Coordinate Systems and Components of a Vector

15. Give an example of a nonzero vector that has a component of zero.
16. Explain why a vector cannot have a component greater than its own magnitude.
17. If two vectors are equal, what can you say about their components?
18. If vectors $\vec{A}$ and $\vec{B}$ are orthogonal, what is the component of $\vec{B}$ along the direction of $\vec{A}$? What is the component of $\vec{A}$ along the direction of $\vec{B}$?
19. If one of the two components of a vector is not zero, can the magnitude of the other vector component of this vector be zero?
20. If two vectors have the same magnitude, do their components have to be the same?

2.4 Products of Vectors

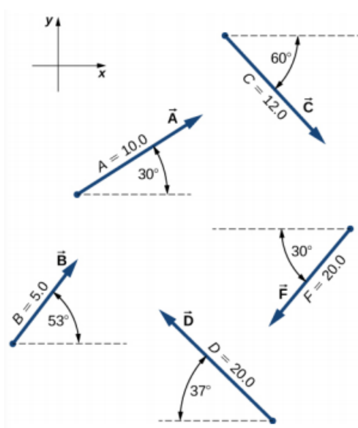
21. What is wrong with the following expressions? How can you correct them?
    a. $C = \vec{A}\vec{B}$,
    b. $\vec{C} = \vec{A}\vec{B}$,
    c. $C = \vec{A} \times \vec{B}$,
    d. $C = A\vec{B}$,
    e. $C + 2\vec{A} = B$,
    f. $\vec{C} = A \times \vec{B}$,
    g. $\vec{A} \cdot \vec{B} = \vec{A} \times \vec{B}$,
    h. $\vec{C} = 2\vec{A} \cdot \vec{B}$,
    i. $C = \vec{A}/\vec{B}$, and
    j. $C = \vec{A}/B$.
22. If the cross product of two vectors vanishes, what can you say about their directions?

23. If the dot product of two vectors vanishes, what can you say about their directions?
24. What is the dot product of a vector with the cross product that this vector has with another vector?

Problems

2.1 Scalars and Vectors

25. A scuba diver makes a slow descent into the depths of the ocean. His vertical position with respect to a boat on the surface changes several times. He makes the first stop 9.0 m from the boat but has a problem with equalizing the pressure, so he ascends 3.0 m and then continues descending for another 12.0 m to the second stop. From there, he ascends 4 m and then descends for 18.0 m, ascends again for 7 m and descends again for 24.0 m, where he makes a stop, waiting for his buddy. Assuming the positive direction up to the surface, express his net vertical displacement vector in terms of the unit vector. What is his distance to the boat?
26. In a tug-of-war game on one campus, 15 students pull on a rope at both ends in an effort to displace the central knot to one side or the other. Two students pull with force 196 N each to the right, four students pull with force 98 N each to the left, five students pull with force 62 N each to the left, three students pull with force 150 N each to the right, and one student pulls with force 250 N to the left. Assuming the positive direction to the right, express the net pull on the knot in terms of the unit vector. How big is the net pull on the knot? In what direction?
27. Suppose you walk 18.0 m straight west and then 25.0 m straight north. How far are you from your starting point and what is the compass direction of a line connecting your starting point to your final position? Use a graphical method.
28. For the vectors given in the following figure, use a graphical method to find the following resultants:
    a. $\vec{A}+\vec{B}$,
    b. $\vec{C}+\vec{B}$,
    c. $\vec{D}+\vec{F}$,
    d. $\vec{A}-\vec{B}$,
    e. $\vec{D}-\vec{F}$,
    f. $\vec{A}+2\vec{F}$,
    g. $\vec{A}-4\vec{D}+2\vec{F}$.



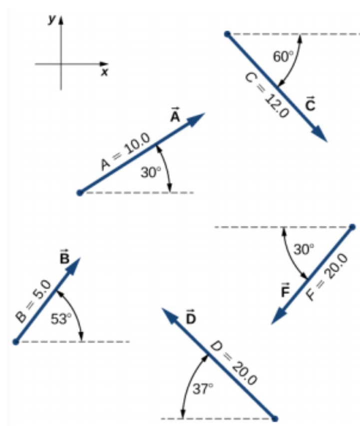
29. A delivery man starts at the post office, drives 40 km north, then 20 km west, then 60 km northeast, and finally 50 km north to stop for lunch. Use a graphical method to find his net displacement vector.
30. An adventurous dog strays from home, runs three blocks east, two blocks north, one block east, one block north, and two blocks west. Assuming that each block is about 100 m, how far from home and in what direction is the dog? Use a graphical method.
31. In an attempt to escape a desert island, a castaway builds a raft and sets out to sea. The wind shifts a great deal during the day and he is blown along the following directions: 2.50 km and 45.0° north of west, then 4.70 km and 60.0° south of east, then 1.30 km and 25.0° south of west, then 5.10 km straight east, then 1.70 km and 5.00° east of north, then 7.20 km and 55.0° south of west, and finally 2.80 km and 10.0° north of east. Use a graphical method to find the castaway's final position relative to the island.

32. A small plane flies 40.0 km in a direction 60° north of east and then flies 30.0 km in a direction 15° north of east. Use a graphical method to find the total distance the plane covers from the starting point and the direction of the path to the final position.
33. A trapper walks a 5.0-km straight-line distance from his cabin to the lake, as shown in the following figure. Use a graphical method (the parallelogram rule) to determine the trapper's displacement directly to the east and displacement directly to the north that sum up to his resultant displacement vector. If the trapper walked only in directions east and north, zigzagging his way to the lake, how many kilometers would he have to walk to get to the lake?

34. A surveyor measures the distance across a river that flows straight north by the following method. Starting directly across from a tree on the opposite bank, the surveyor walks 100 m along the river to establish a baseline. She then sights across to the tree and reads that the angle from the baseline to the tree is 35°. How wide is the river?
35. A pedestrian walks 6.0 km east and then 13.0 km north. Use a graphical method to find the pedestrian's resultant displacement and geographic direction.
36. The magnitudes of two displacement vectors are A = 20 m and B = 6 m. What are the largest and the smallest values of the magnitude of the resultant $\vec{R} = \vec{A} + \vec{B}$?

2.2 Coordinate Systems and Components of a Vector

37. Assuming the +x-axis is horizontal and points to the right, resolve the vectors given in the following figure to their scalar components and express them in vector component form.



38. Suppose you walk 18.0 m straight west and then 25.0 m straight north. How far are you from your starting point? What is your displacement vector? What is the direction of your displacement? Assume the +x-axis is to the east.
39. You drive 7.50 km in a straight line in a direction 15° east of north. (a) Find the distances you would have to drive straight east and then straight north to arrive at the same point. (b) Show that you still arrive at the same point if the east and north legs are reversed in order. Assume the +xaxis is to the east.
40. A sledge is being pulled by two horses on a flat terrain. The net force on the sledge can be expressed in the Cartesian coordinate system as vector $\vec{F}$= (−2980.0 $\hat{i}$ + 8200.0 $\hat{j}$)N , where $\hat{i}$ and $\hat{j}$ denote directions to the east and north, respectively. Find the magnitude and direction of the pull.
41. A trapper walks a 5.0-km straight-line distance from her cabin to the lake, as shown in the following figure. Determine the east and north components of her displacement vector. How many more kilometers would she have to walk if she walked along the component displacements? What is her displacement vector?

42. The polar coordinates of a point are $\frac{4\pi}{3}$ and 5.50 m. What are its Cartesian coordinates?
43. Two points in a plane have polar coordinates P_1 (2.500 m, $\frac{\pi}{6}$) and P_2 (3.800 m, $\frac{2\pi}{3}$). Determine their Cartesian coordinates and the distance between them in the Cartesian coordinate system. Round the distance to a nearest centimeter.
44. A chameleon is resting quietly on a lanai screen, waiting for an insect to come by. Assume the origin of a Cartesian coordinate system at the lower left-hand corner of the screen and the horizontal direction to the right as the +x-direction. If its coordinates are (2.000 m, 1.000 m), (a) how far is it from the corner of the screen? (b) What is its location in polar coordinates?
45. Two points in the Cartesian plane are A(2.00 m, −4.00 m) and B(−3.00 m, 3.00 m). Find the distance between them and their polar coordinates.
46. A fly enters through an open window and zooms around the room. In a Cartesian coordinate system with three axes along three edges of the room, the fly changes its position from point b(4.0 m, 1.5 m, 2.5 m) to point e(1.0 m, 4.5 m, 0.5 m). Find the scalar components of the fly's displacement vector and express its displacement vector in vector component form. What is its magnitude?

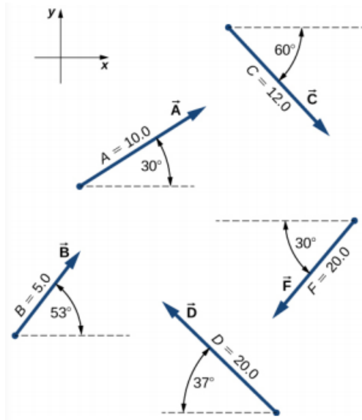
2.3 Algebra of Vectors

47. For vectors $\vec{B} = -\hat{i} - 4\hat{j}$ and $\vec{A} = -3\hat{i} - 2\hat{j}$, calculate (a) $\vec{A} + \vec{B}$ and its magnitude and direction angle, and (b) $\vec{A} - \vec{B}$ and its magnitude and direction angle.
48. A particle undergoes three consecutive displacements given by vectors $\vec{D}_1$ = (3.0 $\hat{i}$ − 4.0 $\hat{j}$ − 2.0 $\hat{k}$)mm, $\vec{D}_2$ = (1.0 $\hat{i}$ − 7.0 $\hat{j}$ + 4.0 $\hat{i}$)mm, and $\vec{D}_3$ = (−7.0 $\hat{i}$ + 4.0 $\hat{j}$ + 1.0 $\hat{k}$)mm. (a) Find the resultant displacement vector of the particle. (b) What is the magnitude of the resultant displacement? (c) If all displacements were along one line, how far would the particle travel?
49. Given two displacement vectors $\vec{A}$ = (3.00 $\hat{i}$ − 4.00 $\hat{j}$ + 4.00 $\hat{k}$)m and $\vec{B}$ = (2.00 $\hat{i}$ + 3.00 $\hat{j}$ − 7.00 $\hat{k}$)m, find the displacements and their magnitudes for (a) $\vec{C} = \vec{A} + \vec{B}$ and (b) $\vec{D} = 2\vec{A} - \vec{B}$.
50. A small plane flies 40.0 km in a direction 60° north of east and then flies 30.0 km in a direction 15° north of east. Use the analytical method to find the total distance the plane covers from the starting point, and the geographic direction of its displacement vector. What is its displacement vector?
51. . In an attempt to escape a desert island, a castaway builds a raft and sets out to sea. The wind shifts a great deal during the day, and she is blown along the following straight lines: 2.50 km and 45.0° north of west, then 4.70 km and 60.0° south of east, then 1.30 km and 25.0° south of west, then 5.10 km due east, then 1.70 km and 5.00° east of north, then 7.20 km and 55.0° south of west, and finally 2.80 km and 10.0° north of east. Use the analytical method to find the resultant vector of all her displacement vectors. What is its magnitude and direction?
52. Assuming the +x-axis is horizontal to the right for the vectors given in the following figure, use the analytical method to find the following resultants:
    a. $\vec{A} + \vec{B}$,
    b. $\vec{C} + \vec{B}$,
    c. $\vec{D} + \vec{F}$,
    d. $\vec{A} - \vec{B}$,
    e. $\vec{D} - \vec{F}$,
    f. $\vec{A} + 2\vec{F}$,
    g. $\vec{C} - 2\vec{B} + 3\vec{F}$, and
    h. $\vec{A} - 4\vec{D} + 2\vec{F}$.

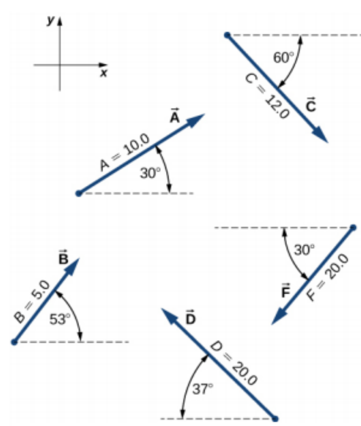
53. Given the vectors in the preceding figure, find vector $\vec{R}$ that solves equations (a) $\vec{D}+\vec{R}=\vec{F}$ and (b) $\vec{C}-2\vec{D}+5\vec{R}=3\vec{F}$. Assume the +x-axis is horizontal to the right.
54. A delivery man starts at the post office, drives 40 km north, then 20 km west, then 60 km northeast, and finally 50 km north to stop for lunch. Use the analytical method to determine the following: (a) Find his net displacement vector. (b) How far is the restaurant from the post office? (c) If he returns directly from the restaurant to the post office, what is his displacement vector on the return trip? (d) What is his compass heading on the return trip? Assume the +x-axis is to the east.
55. An adventurous dog strays from home, runs three blocks east, two blocks north, and one block east, one block north, and two blocks west. Assuming that each block is about a 100 yd, use the analytical method to find the dog's net displacement vector, its magnitude, and its direction. Assume the +x-axis is to the east. How would your answer be affected if each block was about 100 m?
56. If $\vec{D}$ = (6.00 $\hat{i}$ − 8.00 $\hat{j}$m, $\vec{B}$ = (−8.00 $\hat{i}$ + 3.00 $\hat{j}$)m , and $\vec{A}$ = (26.0 $\hat{i}$ + 19.0 $\hat{j}$)m, find the unknown constants a and b such that a $\vec{D}+b\vec{B}+\vec{A}=\vec{0}$.
57. Given the displacement vector $\vec{D}$ = (3 $\hat{i}$ − 4 $\hat{j}$)m, find the displacement vector $\vec{R}$ so that $\vec{D}$ + $\vec{R}$ = −4D $\hat{j}$.
58. Find the unit vector of direction for the following vector quantities: (a) Force $\vec{F}$ = (3.0 $\hat{i}$ − 2.0 $\hat{j}$)N, (b) displacement $\vec{D}$ = (−3.0 $\hat{i}$ − 4.0 $\hat{j}$)m, and (c) velocity $\vec{v}$ = (−5.00 $\hat{i}$ + 4.00 $\hat{j}$)m/s.
59. At one point in space, the direction of the electric field vector is given in the Cartesian system by the unit vector $\hat{E}=\frac{1}{\sqrt{5}}\hat{i}-\frac{2}{\sqrt{5}}\hat{j}$. If the magnitude of the electric field vector is E = 400.0 V/m, what are the scalar components E_x, E_y, and E_z of the electric field vector $\vec{E}$ at this point? What is the direction angle θ_E of the electric field vector at this point?
60. A barge is pulled by the two tugboats shown in the following figure. One tugboat pulls on the barge with a force of magnitude 4000 units of force at 15° above the line AB (see the figure and the other tugboat pulls on the barge with a force of magnitude 5000 units of force at 12° below the line AB. Resolve the pulling forces to their scalar components and find the components of the resultant force pulling on the barge. What is the magnitude of the resultant pull? What is its direction relative to the line AB?
61. In the control tower at a regional airport, an air traffic controller monitors two aircraft as their positions change with respect to the control tower. One plane is a cargo carrier Boeing 747 and the other plane is a Douglas DC-3. The Boeing is at an altitude of 2500 m, climbing at 10° above the horizontal, and moving 30° north of west. The DC-3 is at an altitude of 3000 m, climbing at 5° above the horizontal, and cruising directly west. (a) Find the position vectors of the planes relative to the control tower. (b) What is the distance between the planes at the moment the air traffic controller makes a note about their positions?

2.4 Products of Vectors

62. Assuming the +x-axis is horizontal to the right for the vectors in the following figure, find the following scalar products:
    a. $\vec{A}\cdot\vec{C}$,
    b. $\vec{A}\cdot\vec{F}$,
    c. $\vec{D}\cdot\vec{C}$,
    d. $\vec{A}\cdot(\vec{F}+2\vec{C})$,
    e. $\hat{i}\cdot\vec{B}$,
    f. $\hat{j}\cdot\vec{B}$,
    g. $(3\hat{i}-\hat{j})\cdot\vec{B}$ and
    h. $\hat{B}\cdot\vec{B}$.

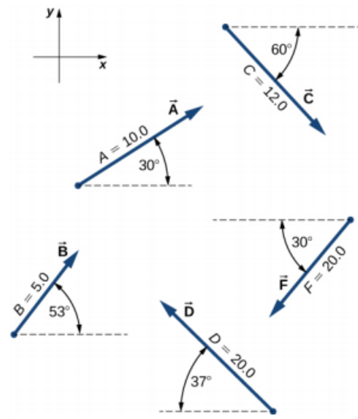


63. Assuming the +x-axis is horizontal to the right for the vectors in the preceding figure, find (a) the component of vector $\vec{A}$ along vector $\vec{C}$, (b) the component of vector $\vec{C}$ along vector $\vec{A}$, (c) the component of vector $\hat{i}$ along vector $\vec{F}$, and (d) the component of vector $\vec{F}$ along vector $\hat{i}$.
64. Find the angle between vectors for
    a. $\vec{D} = (-3.0\,\hat{i} - 4.0\,\hat{j})$m and $\vec{A} = (-3.0\,\hat{i} + 4.0\,\hat{j})$m and
    b. $\vec{D} = (2.0\,\hat{i} - 4.0\,\hat{j} + \hat{k})$m and $\vec{B} = (-2.0\,\hat{i} + 3.0\,\hat{j} + 2.0\,\hat{k})$m.
65. Find the angles that vector $\vec{D} = (2.0\,\hat{i} - 4.0\,\hat{j} + \hat{k})$m makes with the x-, y-, and z-axes.
66. Show that the force vector $\vec{D} = (2.0\,\hat{i} - 4.0\,\hat{j} + \hat{k})$N is orthogonal to the force vector $\vec{G} = (3.0\,\hat{i} + 4.0\,\hat{j} + 10.0\,\hat{k})$N.
67. Assuming the +x-axis is horizontal to the right for the vectors in the following figure, find the following vector products:
    a. $\vec{A} \times \vec{C}$,
    b. $\vec{A} \times \vec{F}$,
    c. $\vec{D} \times \vec{C}$
    d. $\vec{A} \times (\vec{F} + 2\vec{C})$,
    e. $\hat{i} \times \vec{B}$,
    f. $\hat{j} \times \vec{B}$,
    g. $(3\hat{i} - \hat{j}) \times \vec{B}$ and
    h. $\hat{B} \times \vec{B}$.



68. Find the cross product $\vec{A} \times \vec{C}$ for
    a. $\vec{A} = 2.0\,\hat{i} - 4.0\,\hat{j} + \hat{k}$ and $\vec{C} = 3.0\,\hat{i} + 4.0\,\hat{j} + 10.0\hat{k}$,
    b. $\vec{A} = 3.0\,\hat{i} + 4.0\,\hat{j} + 10.0\,\hat{k}$ and $\vec{C} = 2.0\,\hat{i} - 4.0\,\hat{j} + \hat{k}$,
    c. $\vec{A} = -3.0\,\hat{i} - 4.0\,\hat{j}$ and $\vec{C} = -3.0\,\hat{i} + 4.0\,\hat{j}$, and
    d. $\vec{C} = -2.0\,\hat{i} + 3.0\,\hat{j} + 2.0\,\hat{k}$ and $\vec{A} = -9.0\,\hat{j}$.

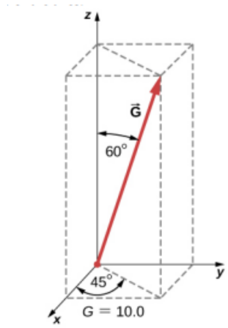
69. For the vectors in the following figure, find (a) $(\vec{A} \times \vec{F} \cdot \vec{D})$, (b) $(\vec{A} \times \vec{F}) \cdot (\vec{A} \times \vec{C})$, and (c) $(\vec{A} \cdot \vec{F})(\vec{D} \times \vec{B})$.

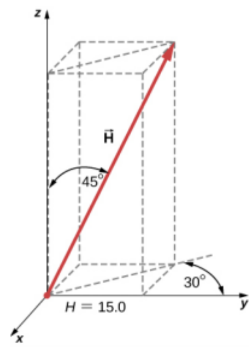


70. (a) If $\vec{A} \times \vec{F} = \vec{B} \times \vec{F}$, can we conclude $\vec{A} = \vec{B}$? (b) If $\vec{A} \cdot \vec{F} = \vec{B} \cdot \vec{F}$, can we conclude $\vec{A} = \vec{B}$? (c) If $F\vec{A} = \vec{B}F$, can we conclude $\vec{A} = \vec{B}$? Why or why not?

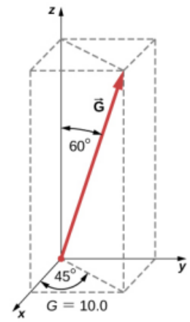
Additional Problems

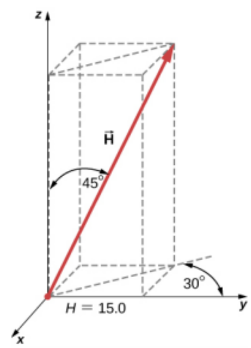
71. You fly 32.0 km in a straight line in still air in the direction 35.0° south of west. (a) Find the distances you would have to fly due south and then due west to arrive at the same point. (b) Find the distances you would have to fly first in a direction 45.0° south of west and then in a direction 45.0° west of north. Note these are the components of the displacement along a different set of axes—namely, the one rotated by 45° with respect to the axes in (a).
72. Rectangular coordinates of a point are given by (2, y) and its polar coordinates are given by (r, $\frac{\pi}{6}$). Find y and r.
73. If the polar coordinates of a point are (r,φ) and its rectangular coordinates are (x, y), determine the polar coordinates of the following points: (a) (−x, y), (b) (−2x, −2y), and (c) (3x, −3y).
74. Vectors $\vec{A}$ and $\vec{B}$ have identical magnitudes of 5.0 units. Find the angle between them if $\vec{A} + \vec{B} = 5\ 2\ \hat{j}$.
75. Starting at the island of Moi in an unknown archipelago, a fishing boat makes a round trip with two stops at the islands of Noi and Poi. It sails from Moi for 4.76 nautical miles (nmi) in a direction 37° north of east to Noi. From Noi, it sails 69° west of north to Poi. On its return leg from Poi, it sails 28° east of south. What distance does the boat sail between Noi and Poi? What distance does it sail between Moi and Poi? Express your answer both in nautical miles and in kilometers. Note: 1 nmi = 1852 m.
76. An air traffic controller notices two signals from two planes on the radar monitor. One plane is at altitude 800 m and in a 19.2-km horizontal distance to the tower in a direction 25° south of west. The second plane is at altitude 1100 m and its horizontal distance is 17.6 km and 20° south of west. What is the distance between these planes?
77. Show that when $\vec{A} + \vec{B} = \vec{C}$, then $C^2 = A^2 + B^2 + 2AB \cos \varphi$, where φ is the angle between vectors $\vec{A}$ and $\vec{B}$.
78. Four force vectors each have the same magnitude f. What is the largest magnitude the resultant force vector may have when these forces are added? What is the smallest magnitude of the resultant? Make a graph of both situations.
79. A skater glides along a circular path of radius 5.00 m in clockwise direction. When he coasts around one-half of the circle, starting from the west point, find (a) the magnitude of his displacement vector and (b) how far he actually skated. (c) What is the magnitude of his displacement vector when he skates all the way around the circle and comes back to the west point?
80. A stubborn dog is being walked on a leash by its owner. At one point, the dog encounters an interesting scent at some spot on the ground and wants to explore it in detail, but the owner gets impatient and pulls on the leash with force $\vec{F}$ = (98.0 $\hat{i}$ + 132.0 $\hat{j}$ + 32.0 $\hat{j}$)N along the leash. (a) What is the magnitude of the pulling force? (b) What angle does the leash make with the vertical?
81. If the velocity vector of a polar bear is $\vec{u}$ = (−18.0 $\hat{i}$ − 13.0 $\hat{j}$)km/h , how fast and in what geographic direction is it heading? Here, $\hat{i}$ and $\hat{j}$ are directions to geographic east and north, respectively.
82. Find the scalar components of three-dimensional vectors $\vec{G}$ and $\vec{H}$ in the following figure and write the vectors in vector component form in terms of the unit vectors of the axes.



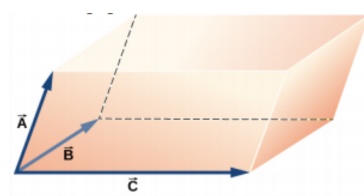


83. A diver explores a shallow reef off the coast of Belize. She initially swims 90.0 m north, makes a turn to the east and continues for 200.0 m, then follows a big grouper for 80.0 m in the direction 30° north of east. In the meantime, a local current displaces her by 150.0 m south. Assuming the current is no longer present, in what direction and how far should she now swim to come back to the point where she started?
84. A force vector $\vec{A}$ has x- and y-components, respectively, of −8.80 units of force and 15.00 units of force. The x- and y-components of force vector $\vec{B}$ are, respectively, 13.20 units of force and −6.60 units of force. Find the components of force vector $\vec{C}$ that satisfies the vector equation $\vec{A} - \vec{B} + 3\vec{C} = 0$.
85. Vectors $\vec{A}$ and $\vec{B}$ are two orthogonal vectors in the xy-plane and they have identical magnitudes. If $\vec{A} = 3.0\,\hat{i} + 4.0\,\hat{j}$, find $\vec{B}$.
86. For the three-dimensional vectors in the following figure, find (a) $\vec{G} \times \vec{H}$, (b) $|\vec{G} \times \vec{H}|$, and (c) $\vec{G} \cdot \vec{H}$.



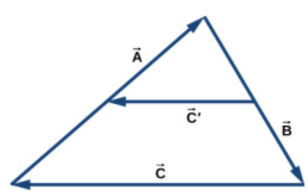


87. Show that $(\vec{B} \times \vec{C}) \cdot \vec{A}$ is the volume of the parallelepiped, with edges formed by the three vectors in the following figure.



Challenge Problems

88. Vector $\vec{B}$ is 5.0 cm long and vector $\vec{A}$ is 4.0 cm long. Find the angle between these two vectors when $|\vec{A}+\vec{B}|$ = 3.0 cm and $|\vec{A} - \vec{B}|$ = 3.0 cm.
89. What is the component of the force vector $\vec{G}$ = (3.0 $\hat{i}$ + 4.0 $\hat{j}$ + 10.0 $\hat{k}$)N along the force vector $\vec{H}$ = (1.0 $\hat{i}$ + 4.0 $\hat{j}$)N?
90. The following figure shows a triangle formed by the three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$. If vector $\vec{C}\,'$ is drawn between the midpoints of vectors $\vec{A}$ and $\vec{B}$, show that $\vec{C}\,' = \frac{\vec{C}}{2}$.



91. Distances between points in a plane do not change when a coordinate system is rotated. In other words, the magnitude of a vector is **invariant** under rotations of the coordinate system. Suppose a coordinate system S is rotated about its origin by angle φ to become a new coordinate system S′, as shown in the following figure. A point in a plane has coordinates (x, y) in S and coordinates (x′, y′) in S′.
    a. Show that, during the transformation of rotation, the coordinates in S′ are expressed in terms of the coordinates in S by the following relations:

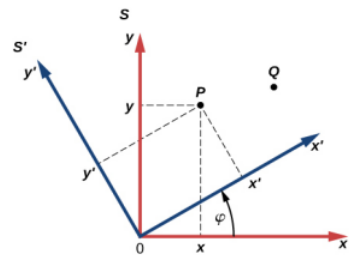
$$\begin{cases} x' = x\cos\varphi + y\sin\varphi \\ y' = -x\sin\varphi + y\cos\varphi \end{cases}. \tag{3.E.1}$$

b. Show that the distance of point P to the origin is invariant under rotations of the coordinate system. Here, you have to show that

$$\sqrt{x^2+y^2} = \sqrt{x'^2+y'^2}. \tag{3.E.2}$$

c. Show that the distance between points P and Q is invariant under rotations of the coordinate system. Here, you have to show that

$$\sqrt{(x_P-x_Q)^2+(y_P-y_Q)^2} = \sqrt{(x'_P-x'_Q)^2+(y'_P-y'_Q)^2}. \tag{3.E.3}$$



Contributors

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