10.4: Impulse and Collisions (Part 2)

Effect of Impulse

Since an impulse is a force acting for some amount of time, it causes an object's motion to change. Recall

$$\vec{J} = m\Delta\vec{v}. \tag{10.4.1}$$

Because m$\vec{v}$ is the momentum of a system, m$\Delta\vec{v}$ is the change of momentum $\Delta\vec{p}$. This gives us the following relation, called the **impulse-momentum theorem** (or relation).

Impulse-Momentum Theorem

An impulse applied to a system changes the system's momentum, and that change of momentum is exactly equal to the impulse that was applied:

$$\vec{J} = \Delta\vec{p}. \tag{10.4.2}$$

The impulse-momentum theorem is depicted graphically in Figure 10.4.1.

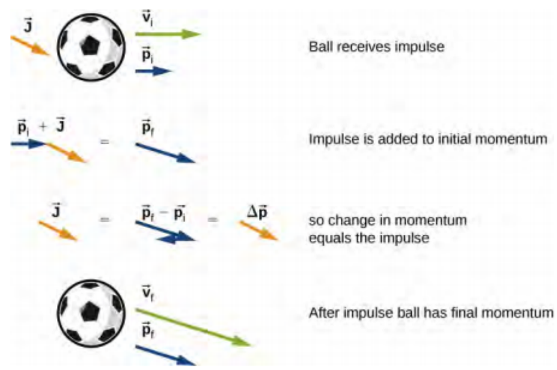

Figure 10.4.1: Illustration of impulse-momentum theorem. (a) A ball with initial velocity $\vec{v}_0$ and momentum $\vec{p}_0$ receives an impulse $\vec{J}$. (b) This impulse is added vectorially to the initial momentum. (c) Thus, the impulse equals the change in momentum, $\vec{J} = \Delta\vec{p}$. (d) After the impulse, the ball moves off with its new momentum $\vec{p}_f$.

There are two crucial concepts in the impulse-momentum theorem:

1. Impulse is a vector quantity; an impulse of, say, −(10 N • s) $\hat{i}$ is very different from an impulse of +(10 N • s) $\hat{i}$; they cause completely opposite changes of momentum.
2. An impulse does not cause momentum; rather, it causes a change in the momentum of an object. Thus, you must subtract the final momentum from the initial momentum, and—since momentum is also a vector quantity—you must take careful account of the signs of the momentum vectors.

The most common questions asked in relation to impulse are to calculate the applied force, or the change of velocity that occurs as a result of applying an impulse. The general approach is the same.

Problem-Solving Strategy: Impulse-Momentum Theorem

1. Express the impulse as force times the relevant time interval.
2. Express the impulse as the change of momentum, usually mΔv.
3. Equate these and solve for the desired quantity.

Enterprise

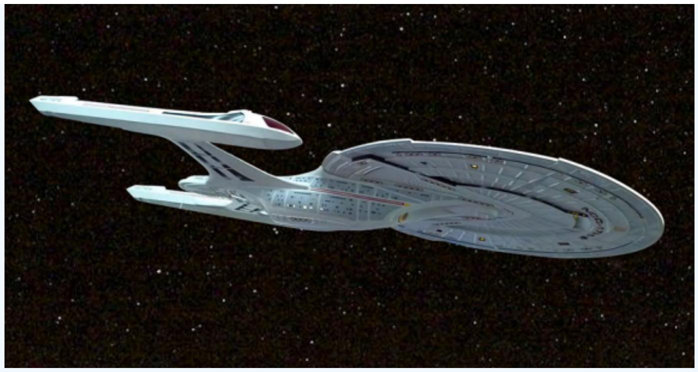

Figure 10.4.2: The fictional starship Enterprise from the Star Trek adventures operated on so-called “impulse engines” that combined matter with antimatter to produce energy.

“Mister Sulu, take us out; ahead one-quarter impulse.” With this command, Captain Kirk of the starship **Enterprise** (Figure 10.4.2) has his ship start from rest to a final speed of $v_f = \frac{1}{4}(3.0 \times 10^8$ m/s). Assuming this maneuver is completed in 60 s, what average force did the impulse engines apply to the ship?

Strategy

We are asked for a force; we know the initial and final speeds (and hence the change in speed), and we know the time interval over which this all happened. In particular, we know the amount of time that the force acted. This suggests using the impulse-momentum relation. To use that, though, we need the mass of the **Enterprise**. An internet search gives a best estimate of the mass of the **Enterprise** (in the 2009 movie) as 2 x 10^9 kg.

Solution

Because this problem involves only one direction (i.e., the direction of the force applied by the engines), we only need the scalar form of the impulse-momentum theorem Equation 10.4.2, which is

$$\Delta p = J \tag{10.4.3}$$

with

$$\Delta p = m\Delta v \tag{10.4.4}$$

and

$$J = F\Delta t. \tag{10.4.5}$$

Equating these expressions gives

$$F\Delta t = m\Delta v. \tag{10.4.6}$$

Solving for the magnitude of the force and inserting the given values leads to

$$F = \frac{m\Delta v}{\Delta t} = \frac{(2 \times 10^9\ kg)(7.35 \times 10^7\ m/s)}{60\ s} = 2.5 \times 10^{15}\ N. \tag{10.4.7}$$

Significance

This is an unimaginably huge force. It goes almost without saying that such a force would kill everyone on board instantly, as well as destroying every piece of equipment. Fortunately, the **Enterprise** has “inertial dampeners.” It is left as an exercise for the reader’s imagination to determine how these work.

? EXercise 10.4.1

The U.S. Air Force uses “10gs” (an acceleration equal to 10 x 9.8 m/s^2) as the maximum acceleration a human can withstand (but only for several seconds) and survive. How much time must the **Enterprise** spend accelerating if the humans on board are to experience an average of at most 10gs of acceleration? (Assume the inertial dampeners are offline.)

Example 10.4.2: The iPhone Drop

Apple released its iPhone 6 Plus in November 2014. According to many reports, it was originally supposed to have a screen made from sapphire, but that was changed at the last minute for a hardened glass screen. Reportedly, this was because the sapphire screen cracked when the phone was dropped. What force did the iPhone 6 Plus experience as a result of being dropped?

Strategy

The force the phone experiences is due to the impulse applied to it by the floor when the phone collides with the floor. Our strategy then is to use the impulse-momentum relationship. We calculate the impulse, estimate the impact time, and use this to calculate the force. We need to make a couple of reasonable estimates, as well as find technical data on the phone itself. First, let's suppose that the phone is most often dropped from about chest height on an average-height person. Second, assume that it is dropped from rest, that is, with an initial vertical velocity of zero. Finally, we assume that the phone bounces very little—the height of its bounce is assumed to be negligible.

Solution

Define upward to be the +y-direction. A typical height is approximately h = 1.5 m and, as stated, $\vec{v}_i$ = (0 m/s) $\hat{i}$. The average force on the phone is related to the impulse the floor applies on it during the collision:

$$\vec{F}_{ave} = \frac{\vec{J}}{\Delta t}. \tag{10.4.8}$$

The impulse $\vec{J}$ equals the change in momentum,

$$\vec{J} = \Delta \vec{p} \tag{10.4.9}$$

so

$$\vec{F}_{ave} = \frac{\Delta \vec{p}}{\Delta t}. \tag{10.4.10}$$

Next, the change of momentum is

$$\Delta \vec{p} = m \Delta \vec{v}. \tag{10.4.11}$$

We need to be careful with the velocities here; this is the change of velocity due to the collision with the floor. But the phone also has an initial drop velocity [$\vec{v}_i$ = (0 m/s) $\hat{j}$], so we label our velocities. Let:

- $\vec{v}_i$ = the initial velocity with which the phone was dropped (zero, in this example)
- $\vec{v}_1$ = the velocity the phone had the instant just before it hit the floor
- $\vec{v}_2$ = the final velocity of the phone as a result of hitting the floor

Figure 10.4.3 shows the velocities at each of these points in the phone's trajectory.

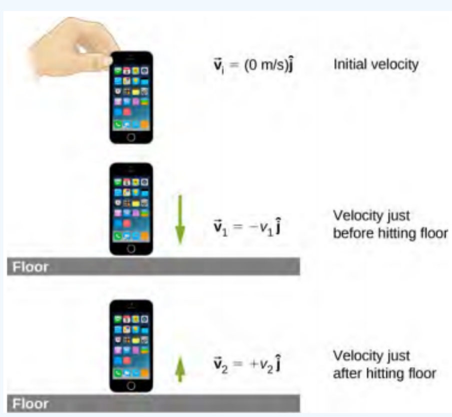

Figure 10.4.3: (a) The initial velocity of the phone is zero, just after the person drops it. (b) Just before the phone hits the floor, its velocity is $\vec{v}_1$, which is unknown at the moment, except for its direction, which is downward ($-\hat{j}$). (c) After bouncing off the floor, the phone has a velocity $\vec{v}_2$, which is also unknown, except for its direction, which is upward ($+\hat{j}$).

With these definitions, the change of momentum of the phone during the collision with the floor is

$$m\Delta\vec{v} = m(\vec{v}_2 - \vec{v}_1). \tag{10.4.12}$$

Since we assume the phone doesn't bounce at all when it hits the floor (or at least, the bounce height is negligible), then $\vec{v}_2$ is zero, so

$$m\Delta\vec{v} = m\left[0 - (-v_1\ \hat{j})\right] \tag{10.4.13}$$

$$m\Delta\vec{v} = +mv_1\ \hat{j}. \tag{10.4.14}$$

We can get the speed of the phone just before it hits the floor using either kinematics or conservation of energy. We'll use conservation of energy here; you should re-do this part of the problem using kinematics and prove that you get the same answer.

First, define the zero of potential energy to be located at the floor. Conservation of energy then gives us:

$$\begin{aligned} E_i &= E_1 \\ K_i + U_i &= K_1 + U_1 \\ \frac{1}{2}mv_i^2 + mgh_{drop} &= \frac{1}{2}mv_1^2 + mgh_{floor}. \end{aligned}$$

Defining $h_{floor} = 0$ and using $\vec{v}_i = (0\text{ m/s})\ \hat{j}$ gives

$$\begin{aligned} \frac{1}{2}mv_1^2 &= mgh_{drop} \\ v_1 &= \pm\sqrt{2gh_{drop}}. \end{aligned}$$

Because v_1 is a vector magnitude, it must be positive. Thus, $m\Delta v = mv_1 = m\sqrt{2gh_{drop}}$. Inserting this result into the expression for force gives

$$\begin{aligned} \vec{F} &= \frac{\Delta\vec{p}}{\Delta t} \\ &= \frac{m\Delta\vec{v}}{\Delta t} \\ &= \frac{+mv_1\ \hat{j}}{\Delta t} \\ &= \frac{m\sqrt{2gh}}{\Delta t}\ \hat{j}. \end{aligned}$$

Finally, we need to estimate the collision time. One common way to estimate a collision time is to calculate how long the object would take to travel its own length. The phone is moving at 5.4 m/s just before it hits the floor, and it is 0.14 m long, giving an estimated collision time of 0.026 s. Inserting the given numbers, we obtain

$$\vec{F} = \frac{(0.172\ kg)\sqrt{2(9.8\ m/s^2)(1.5\ m)}}{0.026\ s}\ \hat{j} = (36\ N)\hat{j}. \tag{10.4.15}$$

Significance

The iPhone itself weighs just (0.172 kg)(9.81 m/s^2) = 1.68 N; the force the floor applies to it is therefore over 20 times its weight.

? Exercise 10.4.2

What if we had assumed the phone **did** bounce on impact? Would this have increased the force on the iPhone, decreased it, or made no difference?

Momentum and Force

In Example 10.4.1, we obtained an important relationship:

$$\vec{F}_{ave} = \frac{\Delta \vec{p}}{\Delta t}. \tag{10.4.16}$$

In words, the average force applied to an object is equal to the change of the momentum that the force causes, divided by the time interval over which this change of momentum occurs. This relationship is very useful in situations where the collision time Δt is small, but measurable; typical values would be 1/10th of a second, or even one thousandth of a second. Car crashes, punting a football, or collisions of subatomic particles would meet this criterion.

For a **continuously** changing momentum—due to a continuously changing force—this becomes a powerful conceptual tool. In the limit Δt → dt, Equation 9.3.1 becomes

$$\vec{F} = \frac{d\vec{p}}{dt}. \tag{10.4.17}$$

This says that the rate of change of the system's momentum (implying that momentum is a function of time) is exactly equal to the net applied force (also, in general, a function of time). This is, in fact, Newton's second law, written in terms of momentum rather than acceleration. This is the relationship Newton himself presented in his **Principia Mathematica** (although he called it "quantity of motion" rather than "momentum").

If the mass of the system remains constant, Equation 9.3.3 reduces to the more familiar form of Newton's second law. We can see this by substituting the definition of momentum:

$$\vec{F} = \frac{d(m\vec{v})}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a}. \tag{10.4.18}$$

The assumption of constant mass allowed us to pull m out of the derivative. If the mass is not constant, we cannot use this form of the second law, but instead must start from Equation 9.3.3. Thus, one advantage to expressing force in terms of changing momentum is that it allows for the mass of the system to change, as well as the velocity; this is a concept we'll explore when we study the motion of rockets.

Newton's Second Law of Motion in Terms of Momentum

The net external force on a system is equal to the rate of change of the momentum of that system caused by the force:

$$\vec{F} = \frac{d\vec{p}}{dt}. \tag{10.4.19}$$

Although Equation 9.3.3 allows for changing mass, as we will see in Rocket Propulsion, the relationship between momentum and force remains useful when the mass of the system is constant, as in the following example.

Example 10.4.3: Calculating Force: Venus Williams' Tennis Serve

During the 2007 French Open, Venus Williams hit the fastest recorded serve in a premier women's match, reaching a speed of 58 m/s (209 km/h). What is the average force exerted on the 0.057-kg tennis ball by Venus Williams' racquet? Assume that the ball's speed just after impact is 58 m/s, as shown in Figure $10.4.4$, that the initial horizontal component of the velocity before impact is negligible, and that the ball remained in contact with the racquet for 5.0 ms.

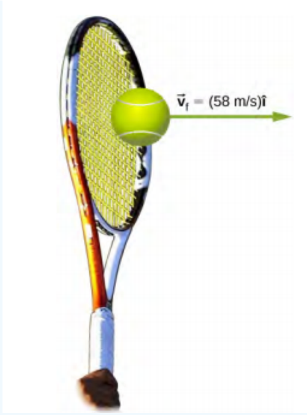

Figure 10.4.4: The final velocity of the tennis ball is $\vec{v}_f$ = (58 m/s) $\hat{i}$.

Strategy

This problem involves only one dimension because the ball starts from having no horizontal velocity component before impact. Newton's second law stated in terms of momentum is then written as

$$\vec{F} = \frac{d\vec{p}}{dt}. \tag{10.4.20}$$

As noted above, when mass is constant, the change in momentum is given by

$$\Delta p = m\Delta v = m(v_f - v_i) \tag{10.4.21}$$

where we have used scalars because this problem involves only one dimension. In this example, the velocity just after impact and the time interval are given; thus, once Δp is calculated, we can use F = $\frac{\Delta p}{\Delta t}$ to find the force.

Solution

To determine the change in momentum, insert the values for the initial and final velocities into the equation above:

$$\begin{aligned} \Delta p &= m(v_f - v_i) \\ &= (0.057\ kg)(58\ m/s - 0\ m/s) \\ &= 3.3\ kg \cdot m/s. \end{aligned}$$

Now the magnitude of the net external force can be determined by using

$$F = \frac{\Delta p}{\Delta t} = \frac{3.3\ kg \cdot m/s}{5.0 \times 10^{-3}\ s} = 6.6 \times 10^2\ N. \tag{10.4.22}$$

where we have retained only two significant figures in the final step.

Significance

This quantity was the average force exerted by Venus Williams' racquet on the tennis ball during its brief impact (note that the ball also experienced the 0.57-N force of gravity, but that force was not due to the racquet). This problem could also be solved by first finding the acceleration and then using F = ma, but one additional step would be required compared with the strategy used in this example.