7.E: Linear Momentum and Collisions (Exercises)

Conceptual Questions

8.1: Linear Momentum and Force

1. An object that has a small mass and an object that has a large mass have the same momentum. Which object has the largest kinetic energy?

2. An object that has a small mass and an object that has a large mass have the same kinetic energy. Which mass has the largest momentum?

3. *Professional Application*

4. Football coaches advise players to block, hit, and tackle with their feet on the ground rather than by leaping through the air. Using the concepts of momentum, work, and energy, explain how a football player can be more effective with his feet on the ground.

5. How can a small force impart the same momentum to an object as a large force?

8.2: Impulse

6. *Professional Application*

> Explain in terms of impulse how padding reduces forces in a collision. State this in terms of a real example, such as the advantages of a carpeted vs. tile floor for a day care center.

7. While jumping on a trampoline, sometimes you land on your back and other times on your feet. In which case can you reach a greater height and why?

8. *Professional Application*

> Tennis racquets have “sweet spots.” If the ball hits a sweet spot then the player's arm is not jarred as much as it would be otherwise. Explain why this is the case.

8.3: Conservation of Momentum

9. *Professional Application*

> If you dive into water, you reach greater depths than if you do a belly flop. Explain this difference in depth using the concept of conservation of energy. Explain this difference in depth using what you have learned in this chapter.

10. Under what circumstances is momentum conserved?

11. Can momentum be conserved for a system if there are external forces acting on the system? If so, under what conditions? If not, why not?

12. Momentum for a system can be conserved in one direction while not being conserved in another. What is the angle between the directions? Give an example.

13. *Professional Application*

> Explain in terms of momentum and Newton’s laws how a car’s air resistance is due in part to the fact that it pushes air in its direction of motion.

14. Can objects in a system have momentum while the momentum of the system is zero? Explain your answer.

15. Must the total energy of a system be conserved whenever its momentum is conserved? Explain why or why not.

8.4: Elastic Collisions in One Dimension

16. What is an elastic collision?

8.5: Inelastic Collisions in One Dimension

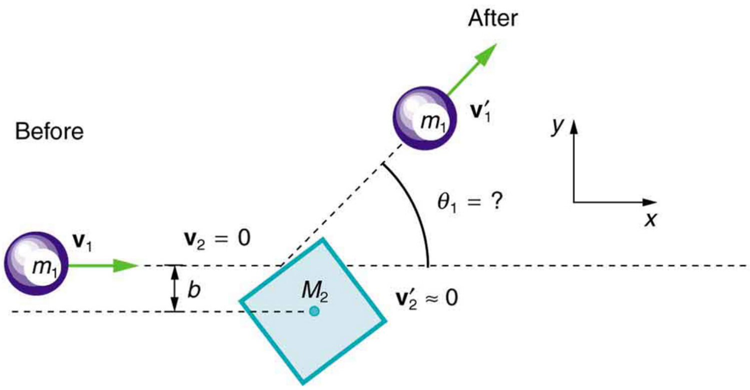
17. What is an inelastic collision? What is a perfectly inelastic collision?

18. Mixed-pair ice skaters performing in a show are standing motionless at arms length just before starting a routine. They reach out, clasp hands, and pull themselves together by only using their arms. Assuming there is no friction between the blades of their skates and the ice, what is their velocity after their bodies meet?

19. A small pickup truck that has a camper shell slowly coasts toward a red light with negligible friction. Two dogs in the back of the truck are moving and making various inelastic collisions with each other and the walls. What is the effect of the dogs on the motion of the center of mass of the system (truck plus entire load)? What is their effect on the motion of the truck?

8.6: Collisions of Point Masses in Two Dimensions

19. Figure shows a cube at rest and a small object heading toward it. (a) Describe the directions (angle θ_1) at which the small object can emerge after colliding elastically with the cube. How does θ_1 depend on b, the so-called impact parameter? Ignore any effects that might be due to rotation after the collision, and assume that the cube is much more massive than the small object. (b) Answer the same questions if the small object instead collides with a massive sphere.



A small object approaches a collision with a much more massive cube, after which its velocity has the direction θ_1. The angles at which the small object can be scattered are determined by the shape of the object it strikes and the impact parameter b.

8.7: Introduction to Rocket Propulsion

20. *Professional Application*

Suppose a fireworks shell explodes, breaking into three large pieces for which air resistance is negligible. How is the motion of the center of mass affected by the explosion? How would it be affected if the pieces experienced significantly more air resistance than the intact shell?

21. *Professional Application*

During a visit to the International Space Station, an astronaut was positioned motionless in the center of the station, out of reach of any solid object on which he could exert a force. Suggest a method by which he could move himself away from this position, and explain the physics involved.

22. *Professional Application*

It is possible for the velocity of a rocket to be greater than the exhaust velocity of the gases it ejects. When that is the case, the gas velocity and gas momentum are in the same direction as that of the rocket. How is the rocket still able to obtain thrust by ejecting the gases?

Problems & Exercises

8.1: Linear Momentum and Force

23. (a) Calculate the momentum of a 2000-kg elephant charging a hunter at a speed of 7.50 m/s size 12{7 "." "50"``"m/s"} {}.

(b) Compare the elephant's momentum with the momentum of a 0.0400-kg tranquilizer dart fired at a speed of 600 m/s size 12{"600"``"m/s"} {}.

(c) What is the momentum of the 90.0-kg hunter running at 7.40 m/s size 12{7 "." "40"``"m/s"} {} after missing the elephant?

Solution
(a) $1.50 \times 10^4 kg \cdot m/s$
(b) 625 to 1
(c) $6.66 \times 10^2 kg \cdot m/s$

24. (a) What is the mass of a large ship that has a momentum of $1.60 \times 10^9 kg \cdot m/s$, when the ship is moving at a speed of $48.0 km/h$?

(b) Compare the ship's momentum to the momentum of a 1100-kg artillery shell fired at a speed of $1200 m/s$.

25. (a) At what speed would a $2.00 \times 10^4 - kg$ airplane have to fly to have a momentum of $1.60 \times 10^9 kg \cdot m/s$ (the same as the ship's momentum in the problem above)?

(b) What is the plane's momentum when it is taking off at a speed of $60.0 m/s$?

(c) If the ship is an aircraft carrier that launches these airplanes with a catapult, discuss the implications of your answer to (b) as it relates to recoil effects of the catapult on the ship.

Solution
(a) $8.00 \times 10^4 m/s$
(b) $1.20 \times 10^6 kg \cdot m/s$
(c) Because the momentum of the airplane is 3 orders of magnitude smaller than of the ship, the ship will not recoil very much. The recoil would be $-0.0100 m/s$, which is probably not noticeable.

26. (a) What is the momentum of a garbage truck that is $1.20 \times 10^4 kg$ and is moving at $10.0 m/s$

(b) At what speed would an 8.00-kg trash can have the same momentum as the truck?

27. A runaway train car that has a mass of 15,000 kg travels at a speed of $5.4 m/s$ down a track. Compute the time required for a force of 1500 N to bring the car to rest.

Solution
54 s

28. The mass of Earth is $5.972 \times 10^{24} kg$ and its orbital radius is an average of $1.496 \times 10^{11} m$. Calculate its linear momentum.

8.2: Impulse

29. A bullet is accelerated down the barrel of a gun by hot gases produced in the combustion of gun powder. What is the average force exerted on a 0.0300-kg bullet to accelerate it to a speed of 600 m/s in a time of 2.00 ms (milliseconds)?

Solution
$9.00 \times 10^3 N$

30. *Professional Application*

A car moving at 10 m/s crashes into a tree and stops in 0.26 s. Calculate the force the seat belt exerts on a passenger in the car to bring him to a halt. The mass of the passenger is 70 kg.

31. A person slaps her leg with her hand, bringing her hand to rest in 2.50 milliseconds from an initial speed of 4.00 m/s.

(a) What is the average force exerted on the leg, taking the effective mass of the hand and forearm to be 1.50 kg?

(b) Would the force be any different if the woman clapped her hands together at the same speed and brought them to rest in the same time? Explain why or why not.

Solution
a) $2.40 \times 10^3 N$ toward the leg
b) The force on each hand would have the same magnitude as that found in part (a) (but in opposite directions by Newton's third law) because the change in momentum and the time interval are the same.

32. *Professional Application*

A professional boxer hits his opponent with a 1000-N horizontal blow that lasts for 0.150 s.

(a) Calculate the impulse imparted by this blow.

(b) What is the opponent's final velocity, if his mass is 105 kg and he is motionless in midair when struck near his center of mass?

(c) Calculate the recoil velocity of the opponent's 10.0-kg head if hit in this manner, assuming the head does not initially transfer significant momentum to the boxer's body.

(d) Discuss the implications of your answers for parts (b) and (c).

33. *Professional Application*

Suppose a child drives a bumper car head on into the side rail, which exerts a force of 4000 N on the car for 0.200 s.

(a) What impulse is imparted by this force?

(b) Find the final velocity of the bumper car if its initial velocity was 2.80 m/s and the car plus driver have a mass of 200 kg. You may neglect friction between the car and floor.

Solution

a) $800 kg \cdot m/s$ away from the wall
b) $1.20 m/s$ away from the wall

34. *Professional Application*

One hazard of space travel is debris left by previous missions. There are several thousand objects orbiting Earth that are large enough to be detected by radar, but there are far greater numbers of very small objects, such as flakes of paint. Calculate the force exerted by a 0.100-mg chip of paint that strikes a spacecraft window at a relative speed of $4.00 \times 10^3 m/s$, given the collision lasts $6.00 \times 10^{-8} s$.

35. Professional Application

A 75.0-kg person is riding in a car moving at 20.0 m/s when the car runs into a bridge abutment.

(a) Calculate the average force on the person if he is stopped by a padded dashboard that compresses an average of 1.00 cm.

(b) Calculate the average force on the person if he is stopped by an air bag that compresses an average of 15.0 cm.

Solution

(a) $1.50 \times 10^6 N$ away from the dashboard
(b) $1.00 \times 10^5 N$ away from the dashboard

36. Professional Application

Military rifles have a mechanism for reducing the recoil forces of the gun on the person firing it. An internal part recoils over a relatively large distance and is stopped by damping mechanisms in the gun. The larger distance reduces the average force needed to stop the internal part.

(a) Calculate the recoil velocity of a 1.00-kg plunger that directly interacts with a 0.0200-kg bullet fired at 600 m/s from the gun.

(b) If this part is stopped over a distance of 20.0 cm, what average force is exerted upon it by the gun?

(c) Compare this to the force exerted on the gun if the bullet is accelerated to its velocity in 10.0 ms (milliseconds).

37. A cruise ship with a mass of $1.00 \times 10^7 kg$ strikes a pier at a speed of 0.750 m/s. It comes to rest 6.00 m later, damaging the ship, the pier, and the tugboat captain's finances. Calculate the average force exerted on the pier using the concept of impulse. (Hint: First calculate the time it took to bring the ship to rest.)

Solution

$4.69 \times 10^5 N$ in the boat's original direction of motion

38. Calculate the final speed of a 110-kg rugby player who is initially running at 8.00 m/s but collides head-on with a padded goalpost and experiences a backward force of $1.76 \times 10^4 N$ for $5.50 \times 10^{-2} s$.

39. Water from a fire hose is directed horizontally against a wall at a rate of 50.0 kg/s and a speed of 42.0 m/s. Calculate the magnitude of the force exerted on the wall, assuming the water's horizontal momentum is reduced to zero.

Solution
$2.10 \times 10^3 N$ away from the wall

40. A 0.450-kg hammer is moving horizontally at 7.00 m/s when it strikes a nail and comes to rest after driving the nail 1.00 cm into a board.

(a) Calculate the duration of the impact.

(b) What was the average force exerted on the nail?

41. Starting with the definitions of momentum and kinetic energy, derive an equation for the kinetic energy of a particle expressed as a function of its momentum.

Solution

$$p = mv \Rightarrow p^2 = m^2v^2 \Rightarrow \frac{p^2}{m} = mv^2 \Rightarrow \frac{p^2}{2m} = \frac{1}{2}mv^2 = KE$$

$$KE = \frac{p^2}{2m}$$

42. A ball with an initial velocity of 10 m/s moves at an angle 60° above the $+x$-direction. The ball hits a vertical wall and bounces off so that it is moving 60° above the $-x$-direction with the same speed. What is the impulse delivered by the wall?

43. When serving a tennis ball, a player hits the ball when its velocity is zero (at the highest point of a vertical toss). The racquet exerts a force of 540 N on the ball for 5.00 ms, giving it a final velocity of 45.0 m/s. Using these data, find the mass of the ball.

Solution
60.0 g

44. A punter drops a ball from rest vertically 1 meter down onto his foot. The ball leaves the foot with a speed of 18 m/s at an angle 55º size 12{"55"°} {} above the horizontal. What is the impulse delivered by the foot (magnitude and direction)?

8.3: Conservation of Momentum

45. *Professional Application*

Train cars are coupled together by being bumped into one another. Suppose two loaded train cars are moving toward one another, the first having a mass of 150,000 kg and a velocity of 0.300 m/s, and the second having a mass of 110,000 kg and a velocity of $-0.120 m/s$. (The minus indicates direction of motion.) What is their final velocity?

Solution
0.122 m/s

46. Suppose a clay model of a koala bear has a mass of 0.200 kg and slides on ice at a speed of 0.750 m/s. It runs into another clay model, which is initially motionless and has a mass of 0.350 kg. Both being soft clay, they naturally stick together. What is their final velocity?

47. *Professional Application*

Consider the following question: *A car moving at 10 m/s crashes into a tree and stops in 0.26 s. Calculate the force the seatbelt exerts on a passenger in the car to bring him to a halt.* The mass of the passenger is 70 kg. Would the answer to this question be different if the car with the 70-kg passenger had collided with a car that has a mass equal to and is traveling in the opposite direction and at the same speed? Explain your answer.

Solution
In a collision with an identical car, momentum is conserved. Afterwards $v_f = 0$ for both cars. The change in momentum will be the same as in the crash with the tree. However, the force on the body is not determined since the time is not known. A padded stop will reduce injurious force on body.

48. What is the velocity of a 900-kg car initially moving at 30.0 m/s, just after it hits a 150-kg deer initially running at 12.0 m/s in the same direction? Assume the deer remains on the car.

49. A 1.80-kg falcon catches a 0.650-kg dove from behind in midair. What is their velocity after impact if the falcon's velocity is initially 28.0 m/s and the dove's velocity is 7.00 m/s in the same direction?

Solution
22.4 m/s in the same direction as the original motion

8.4: Elastic Collisions in One Dimension

50. Two identical objects (such as billiard balls) have a one-dimensional collision in which one is initially motionless. After the collision, the moving object is stationary and the other moves with the same speed as the other originally had. Show that both momentum and kinetic energy are conserved.

51. *Professional Application*

Two manned satellites approach one another at a relative speed of 0.250 m/s, intending to dock. The first has a mass of $4.00 \times 10^3 kg$, and the second a mass of $7.50 \times 10^3 kg$. If the two satellites collide elastically rather than dock, what is their final relative velocity?

Solution
0.250 m/s

52. A 70.0-kg ice hockey goalie, originally at rest, catches a 0.150-kg hockey puck slapped at him at a velocity of 35.0 m/s. Suppose the goalie and the ice puck have an elastic collision and the puck is reflected back in the direction from which it came. What would their final velocities be in this case?

8.5: Inelastic Collisions in One Dimension

53. A 0.240-kg billiard ball that is moving at 3.00 m/s strikes the bumper of a pool table and bounces straight back at 2.40 m/s (80% of its original speed). The collision lasts 0.0150 s.

(a) Calculate the average force exerted on the ball by the bumper.

(b) How much kinetic energy in joules is lost during the collision?

(c) What percent of the original energy is left?

Solution
(a) 86.4 N perpendicularly away from the bumper
(b) 0.389 J
(c) 64.0%

54. During an ice show, a 60.0-kg skater leaps into the air and is caught by an initially stationary 75.0-kg skater.

(a) What is their final velocity assuming negligible friction and that the 60.0-kg skater's original horizontal velocity is 4.00 m/s?

(b) How much kinetic energy is lost?

55. *Professional Application*

Using mass and speed data from [link] and assuming that the football player catches the ball with his feet off the ground with both of them moving horizontally, calculate:

(a) the final velocity if the ball and player are going in the same direction and

(b) the loss of kinetic energy in this case.

(c) Repeat parts (a) and (b) for the situation in which the ball and the player are going in opposite directions. Might the loss of kinetic energy be related to how much it hurts to catch the pass?

Solution
(a) 8.06 m/s

(b) -56.0 J
(c)(i) 7.88 m/s; (ii) -223 J

56. A battleship that is $6.00 \times 10^7 kg$ and is originally at rest fires a 1100-kg artillery shell horizontally with a velocity of 575 m/s.

(a) If the shell is fired straight aft (toward the rear of the ship), there will be negligible friction opposing the ship's recoil. Calculate its recoil velocity.

(b) Calculate the increase in internal kinetic energy (that is, for the ship and the shell). This energy is less than the energy released by the gun powder—significant heat transfer occurs.

57. *Professional Application*

Two manned satellites approaching one another, at a relative speed of 0.250 m/s, intending to dock. The first has a mass of $4.00 \times 10^3 kg$, and the second a mass of $7.50 \times 10^3 kg$.

(a) Calculate the final velocity (after docking) by using the frame of reference in which the first satellite was originally at rest.

(b) What is the loss of kinetic energy in this inelastic collision?

(c) Repeat both parts by using the frame of reference in which the second satellite was originally at rest. Explain why the change in velocity is different in the two frames, whereas the change in kinetic energy is the same in both.

Solution
(a) 0.163 m/s in the direction of motion of the more massive satellite
(b) 81.6 J
(c) $8.70 \times 10^{-2} m/s$ in the direction of motion of the less massive satellite, 81.5 J. Because there are no external forces, the velocity of the center of mass of the two-satellite system is unchanged by the collision. The two velocities calculated above are the velocity of the center of mass in each of the two different individual reference frames. The loss in KE is the same in both reference frames because the KE lost to internal forces (heat, friction, etc.) is the same regardless of the coordinate system chosen.

58. *Professional Application*

A 30,000-kg freight car is coasting at 0.850 m/s with negligible friction under a hopper that dumps 110,000 kg of scrap metal into it.

(a) What is the final velocity of the loaded freight car?

(b) How much kinetic energy is lost?

59. *Professional Application*

Space probes may be separated from their launchers by exploding bolts. (They bolt away from one another.) Suppose a 4800-kg satellite uses this method to separate from the 1500-kg remains of its launcher, and that 5000 J of kinetic energy is supplied to the two parts. What are their subsequent velocities using the frame of reference in which they were at rest before separation?

Solution
0.704 m/s
–2.25 m/s

60. A 0.0250-kg bullet is accelerated from rest to a speed of 550 m/s in a 3.00-kg rifle. The pain of the rifle's kick is much worse if you hold the gun loosely a few centimeters from your shoulder rather than holding it tightly against your shoulder.

(a) Calculate the recoil velocity of the rifle if it is held loosely away from the shoulder.

(b) How much kinetic energy does the rifle gain?

(c) What is the recoil velocity if the rifle is held tightly against the shoulder, making the effective mass 28.0 kg?

(d) How much kinetic energy is transferred to the rifle-shoulder combination? The pain is related to the amount of kinetic energy, which is significantly less in this latter situation.

(e) Calculate the momentum of a 110-kg football player running at 8.00 m/s. Compare the player's momentum with the momentum of a hard-thrown 0.410-kg football that has a speed of 25.0 m/s. Discuss its relationship to this problem.

Solution
(a) 4.58 m/s away from the bullet
(b) 31.5 J
(c) –0.491 m/s
(d) 3.38 J

61. *Professional Application*

One of the waste products of a nuclear reactor is plutonium-239 (^{239}Pu). This nucleus is radioactive and decays by splitting into a helium-4 nucleus and a uranium-235 nucleus ($^{4}He + ^{235}U$), the latter of which is also radioactive and will itself decay some time later. The energy emitted in the plutonium decay is (8.40×10^{–13}J\) and is entirely converted to kinetic energy of the helium and uranium nuclei. The mass of the helium nucleus is $6.68 \times 10^{-27} kg$, while that of the uranium is $3.92 \times 10^{-25} kg$ (note that the ratio of the masses is 4 to 235).

(a) Calculate the velocities of the two nuclei, assuming the plutonium nucleus is originally at rest.

(b) How much kinetic energy does each nucleus carry away? Note that the data given here are accurate to three digits only.

62. *Professional Application*

The Moon's craters are remnants of meteorite collisions. Suppose a fairly large asteroid that has a mass of \(\displaystyle 5.00×10^{12}kg\(\displaystyle about a kilometer across) strikes the Moon at a speed of 15.0 km/s.

(a) At what speed does the Moon recoil after the perfectly inelastic collision (the mass of the Moon is $7.36 \times 10^{22} kg$) ?

(b) How much kinetic energy is lost in the collision? Such an event may have been observed by medieval English monks who reported observing a red glow and subsequent haze about the Moon.

(c) In October 2009, NASA crashed a rocket into the Moon, and analyzed the plume produced by the impact. (Significant amounts of water were detected.) Answer part (a) and (b) for this real-life experiment. The mass of the rocket was 2000 kg and its speed upon impact was 9000 km/h. How does the plume produced alter these results?

Solution
(a) $1.02 \times 10^{-6} m/s$
(b) $5.63 \times 10^{20} J$ (almost all KE lost)
(c) Recoil speed is $6.79 \times 10^{-17} m/s$, energy lost is $6.25 \times 10^{9} J$. The plume will not affect the momentum result because the plume is still part of the Moon system. The plume may affect the kinetic energy result because a significant part of the initial kinetic energy may be transferred to the kinetic energy of the plume particles.

63. *Professional Application*

Two football players collide head-on in midair while trying to catch a thrown football. The first player is 95.0 kg and has an initial velocity of 6.00 m/s, while the second player is 115 kg and has an initial velocity of –3.50 m/s. What is their velocity just after impact if they cling together?

64. What is the speed of a garbage truck that is $1.20 \times 10^{4} kg$ and is initially moving at 25.0 m/s just after it hits and adheres to a trash can that is 80.0 kg and is initially at rest?

Solution
24.8 m/s

65. During a circus act, an elderly performer thrills the crowd by catching a cannon ball shot at him. The cannon ball has a mass of 10.0 kg and the horizontal component of its velocity is 8.00 m/s when the 65.0-kg performer catches it. If the performer is on nearly frictionless roller skates, what is his recoil velocity?

66. (a) During an ice skating performance, an initially motionless 80.0-kg clown throws a fake barbell away. The clown's ice skates allow her to recoil frictionlessly. If the clown recoils with a velocity of 0.500 m/s and the barbell is thrown with a velocity of 10.0 m/s, what is the mass of the barbell?

(b) How much kinetic energy is gained by this maneuver?

(c) Where does the kinetic energy come from?

Solution
(a) 4.00 kg
(b) 210 J
(c) The clown does work to throw the barbell, so the kinetic energy comes from the muscles of the clown. The muscles convert the chemical potential energy of ATP into kinetic energy.

8.6: Collisions of Point Masses in Two Dimensions

67. Two identical pucks collide on an air hockey table. One puck was originally at rest.

(a) If the incoming puck has a speed of 6.00 m/s and scatters to an angle of 30.0^{o},what is the velocity (magnitude and direction) of the second puck? (You may use the result that $\theta_1 - \theta_2 = 90^{o}$ for elastic collisions of objects that have identical masses.)

(b) Confirm that the collision is elastic.

Solution
(a) 3.00 m/s, 60^{o} below x-axis

(b) Find speed of first puck after collision: $0 = mv'_1 sin30^{o} - mv'_2 sin60^{o} \Rightarrow v'_1 = v'_2 \frac{sin60^{o}}{sin30^{o} =} 5.196 m/s$

Verify that ratio of initial to final KE equals one: $KE = \frac{1}{2}mv_1^2 = 18mJ$

$KE = \frac{1}{2}mv_1^{\prime 2} + \frac{1}{2}mv_2^{\prime 2} = 18mJ \quad \frac{KE}{KE'} = 1.00$

68. Confirm that the results of the example Example do conserve momentum in both the x- and y-directions.

69. A 3000-kg cannon is mounted so that it can recoil only in the horizontal direction.

(a) Calculate its recoil velocity when it fires a 15.0-kg shell at 480 m/s at an angle of 20.0^{o} above the horizontal.

(b) What is the kinetic energy of the cannon? This energy is dissipated as heat transfer in shock absorbers that stop its recoil.

(c) What happens to the vertical component of momentum that is imparted to the cannon when it is fired?

Solution
(a) $-2.26 m/s$
(b) $7.63 \times 10^3 J$
(c) The ground will exert a normal force to oppose recoil of the cannon in the vertical direction. The momentum in the vertical direction is transferred to the earth. The energy is transferred into the ground, making a dent where the cannon is. After long barrages, cannon have erratic aim because the ground is full of divots.

70. *Professional Application*

A 5.50-kg bowling ball moving at 9.00 m/s collides with a 0.850-kg bowling pin, which is scattered at an angle of 85.0^{o} to the initial direction of the bowling ball and with a speed of 15.0 m/s.

(a) Calculate the final velocity (magnitude and direction) of the bowling ball.

(b) Is the collision elastic?

(c) Linear kinetic energy is greater after the collision. Discuss how spin on the ball might be converted to linear kinetic energy in the collision.

71. *Professional Application*

Ernest Rutherford (the first New Zealander to be awarded the Nobel Prize in Chemistry) demonstrated that nuclei were very small and dense by scattering helium-4 nuclei (^{4}He) from gold-197 nuclei (^{197}Au). The energy of the incoming helium nucleus was $8.00 \times 10^{-13} J$, and the masses of the helium and gold nuclei were $6.68 \times 10^{-27} kg$ and $3.29 \times 10^{-25} kg$, respectively (note that their mass ratio is 4 to 197).

(a) If a helium nucleus scatters to an angle of 120° during an elastic collision with a gold nucleus, calculate the helium nucleus's final speed and the final velocity (magnitude and direction) of the gold nucleus.

(b) What is the final kinetic energy of the helium nucleus?

Solution
(a) $5.36 \times 10^{5} m/s$ at -29.5°
(b) $7.52 \times 10^{-13} J$

72. *Professional Application*

Two cars collide at an icy intersection and stick together afterward. The first car has a mass of 1200 kg and is approaching at $8.00 m/s$ due south. The second car has a mass of 850 kg and is approaching at $17.0 m/s$ due west.

(a) Calculate the final velocity (magnitude and direction) of the cars.

(b) How much kinetic energy is lost in the collision? (This energy goes into deformation of the cars.) Note that because both cars have an initial velocity, you cannot use the equations for conservation of momentum along the x-axis and y-axis; instead, you must look for other simplifying aspects.

73. Starting with equations $m_1 v_1 = m_1 v_1' cos\theta_1 + m_2 v_2' cos\theta_2$ and $0 = m_1 v_1' sin\theta_1 + m_2 v_2' sin\theta_2$ for conservation of momentum in the $x-$ and $y-$directions and assuming that one object is originally stationary, prove that for an elastic collision of two objects of equal masses,

$$\frac{1}{2} m v_1^2 = \frac{1}{2} m v_1'^2 + \frac{1}{2} m v_2'^2 + m v_1' v_2' cos(\theta_1 - \theta_2)$$

as discussed in the text.

Solution
We are given that $m_1 = m_2 \equiv m$. The given equations then become:

$$v_1 = v_1 cos\theta_1 + v_2 cos\theta_2$$

and

$$0 = v_1' sin\theta_1 + v_2' sin\theta_2.$$

Square each equation to get

$$\begin{aligned} v_1^2 &= v_1'^2 cos^2\theta_1 + v_2'^2 cos^2\theta_2 + 2 v_1' v_2' cos\theta_1 cos\theta_2 \\ 0 &= v_1'^2 sin^2\theta_1 + v_2'^2 sin^2\theta_2 + 2 v_1' v_2' sin\theta_1 sin\theta_2. \end{aligned}$$

Add these two equations and simplify:

$$\begin{aligned} v_1^2 &= v_1'^2 + v_2'^2 + 2 v_1' v_2' (cos\theta_1 cos\theta_2 + sin\theta_1 sin\theta_2) \\ &= v_1'^2 + v_2'^2 + 2 v_1' v_2' [\frac{1}{2} cos(\theta_1 - \theta_2) + \frac{1}{2} cos(\theta_1 + \theta_2) + \frac{1}{2} cos(\theta_1 - \theta_2) - \frac{1}{2} cos(\theta_1 + \theta_2)] \\ &= v_1'^2 + v_2'^2 + 2 v_1' v_2' cos(\theta_1 - \theta_2). \end{aligned}$$

Multiply the entire equation by $\frac{1}{2} m$ to recover the kinetic energy:

$$\frac{1}{2} m v_1^2 = \frac{1}{2} m v_1'^2 + \frac{1}{2} m v_2'^2 + m v_1' v_2' cos(\theta_1 - \theta_2)$$

74. *Integrated Concepts*

A 90.0-kg ice hockey player hits a 0.150-kg puck, giving the puck a velocity of 45.0 m/s. If both are initially at rest and if the ice is frictionless, how far does the player recoil in the time it takes the puck to reach the goal 15.0 m away?

75. *Professional Application*

Antiballistic missiles (ABMs) are designed to have very large accelerations so that they may intercept fast-moving incoming missiles in the short time available. What is the takeoff acceleration of a 10,000-kg ABM that expels 196 kg of gas per second at an exhaust velocity of $2.50 \times 10^3 m/s$?

Solution
$39.2 m/s^2$

76. *Professional Application*

What is the acceleration of a 5000-kg rocket taking off from the Moon, where the acceleration due to gravity is only $1.6 m/s^2$, if the rocket expels 8.00 kg of gas per second at an exhaust velocity of $2.20 \times 10^3 m/s$?

77. *Professional Application*

Calculate the increase in velocity of a 4000-kg space probe that expels 3500 kg of its mass at an exhaust velocity of $2.00 \times 10^3 m/s$. You may assume the gravitational force is negligible at the probe's location.

Solution
$4.16 \times 10^3 m/s$

78. *Professional Application*

Ion-propulsion rockets have been proposed for use in space. They employ atomic ionization techniques and nuclear energy sources to produce extremely high exhaust velocities, perhaps as great as $8.00 \times 10^6 m/s$. These techniques allow a much more favorable payload-to-fuel ratio. To illustrate this fact:

(a) Calculate the increase in velocity of a 20,000-kg space probe that expels only 40.0-kg of its mass at the given exhaust velocity.

(b) These engines are usually designed to produce a very small thrust for a very long time—the type of engine that might be useful on a trip to the outer planets, for example. Calculate the acceleration of such an engine if it expels $4.50 \times 10^{-6} kg/s$ at the given velocity, assuming the acceleration due to gravity is negligible.

79. Derive the equation for the vertical acceleration of a rocket.

Solution
The force needed to give a small mass Δman acceleration $a_{\Delta m}$ is $F = \Delta m a_{\Delta m}$. To accelerate this mass in the small time interval Δt at a speed $v_e = a_{\Delta m} \Delta t$, so $F = v_e \frac{\Delta m}{\Delta t}$. By Newton's third law, this force is equal in magnitude to the thrust force acting on the rocket, so $F_{thrust} = v_e \frac{\Delta m}{\Delta t}$, where all quantities are positive. Applying Newton's second law to the rocket gives $F_{thrust} - mg = ma \Rightarrow a = \frac{v_e}{m} \frac{\Delta m}{\Delta t} - g$, where m is the mass of the rocket and unburnt fuel.

80. *Professional Application*

(a) Calculate the maximum rate at which a rocket can expel gases if its acceleration cannot exceed seven times that of gravity. The mass of the rocket just as it runs out of fuel is 75,000-kg, and its exhaust velocity is $2.40 \times 10^3 m/s$. Assume that the acceleration of gravity is the same as on Earth's surface $(9.80 m/s^2)$.

(b) Why might it be necessary to limit the acceleration of a rocket?

Solution
Given the following data for a fire extinguisher-toy wagon rocket experiment, calculate the average exhaust velocity of the gases expelled from the extinguisher. Starting from rest, the final velocity is 10.0 m/s. The total mass is initially 75.0 kg and is 70.0 kg after the extinguisher is fired.

81. How much of a single-stage rocket that is 100,000 kg can be anything but fuel if the rocket is to have a final speed of $8.00 km/s$, given that it expels gases at an exhaust velocity of $2.20 \times 10^3 m/s$?

Solution
$2.63 \times 10^3 kg$

82. *Professional Application*

(a) A 5.00-kg squid initially at rest ejects 0.250-kg of fluid with a velocity of 10.0 m/s. What is the recoil velocity of the squid if the ejection is done in 0.100 s and there is a 5.00-N frictional force opposing the squid's movement.

(b) How much energy is lost to work done against friction?

Solution
(a) 0.421 m/s away from the ejected fluid.
(b) $0.237 J$

83. *Unreasonable Results*

Squids have been reported to jump from the ocean and travel $30.0 m$ (measured horizontally) before re-entering the water.

(a) Calculate the initial speed of the squid if it leaves the water at an angle of 20.0°, assuming negligible lift from the air and negligible air resistance.

(b) The squid propels itself by squirting water. What fraction of its mass would it have to eject in order to achieve the speed found in the previous part? The water is ejected at $12.0 m/s$; gravitational force and friction are neglected.

(c) What is unreasonable about the results?

(d) Which premise is unreasonable, or which premises are inconsistent?

84. *Construct Your Own Problem*

Consider an astronaut in deep space cut free from her space ship and needing to get back to it. The astronaut has a few packages that she can throw away to move herself toward the ship. Construct a problem in which you calculate the time it takes her to get back by throwing all the packages at one time compared to throwing them one at a time. Among the things to be considered are the masses involved, the force she can exert on the packages through some distance, and the distance to the ship.

85. *Construct Your Own Problem*

Consider an artillery projectile striking armor plating. Construct a problem in which you find the force exerted by the projectile on the plate. Among the things to be considered are the mass and speed of the projectile and the distance over which its speed is reduced. Your instructor may also wish for you to consider the relative merits of depleted uranium versus lead projectiles based on the greater density of uranium.

Contributors and Attributions

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