Tuskegee University

Algebra Based Physics I

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TABLE OF CONTENTS

Licensing

1: Nature of Physics

- 1.1: The Basics of Physics
- 1.2: Scientific Notation and Order of Magnitude
- 1.3: Units and Standards
- 1.4: Unit Conversion
- 1.5: Dimensional Analysis
- 1.6: Significant Figures
- 1.7: Summary
- 1.8: Exercises
- 1.9: Answers

2: One-Dimensional Kinematics

- 2.1: Prelude to One-Dimensional Kinematics
- 2.2: Displacement
- 2.3: Vectors, Scalars, and Coordinate Systems
- 2.4: Time, Velocity, and Speed
- 2.5: Acceleration
- 2.6: Motion Equations for Constant Acceleration in One Dimension
- 2.7: Problem-Solving Basics for One-Dimensional Kinematics
- 2.8: Falling Objects
- 2.9: Graphical Analysis of One-Dimensional Motion
- 2.E: Kinematics (Exercises)

3: Two-Dimensional Kinematics

- 3.1: Prelude to Two-Dimensional Kinematics
- 3.2: Kinematics in Two Dimensions - An Introduction
- 3.3: Vector Addition and Subtraction- Graphical Methods
- 3.4: Vector Addition and Subtraction- Analytical Methods
- 3.5: Projectile Motion
- 3.6: Addition of Velocities
- 3.E: Two-Dimensional Kinematics (Exercises)

4: Dynamics- Force and Newton's Laws of Motion

- 4.1: Prelude to Dynamics- Newton’s Laws of Motion
- 4.2: Development of Force Concept
- 4.3: Newton’s First Law of Motion- Inertia
- 4.4: Newton’s Second Law of Motion- Concept of a System
- 4.5: Newton’s Third Law of Motion- Symmetry in Forces
- 4.6: Normal, Tension, and Other Examples of Forces
- 4.7: Friction
- 4.8: Drag Forces
- 4.9: Problem-Solving Strategies
- 4.10: Further Applications of Newton’s Laws of Motion
- 4.11: Extended Topic- The Four Basic Forces—An Introduction

- 4.E: Dynamics- Force and Newton's Laws of Motion (Exercises)

5: Uniform Circular Motion and Gravitation

- 5.1: Prelude to Uniform Circular Motion and Gravitation
- 5.2: Rotation Angle and Angular Velocity
- 5.3: Centripetal Acceleration
- 5.4: Centripetal Force
- 5.5: Fictitious Forces and Non-inertial Frames - The Coriolis Force
- 5.6: Newton’s Universal Law of Gravitation
- 5.7: Satellites and Kepler’s Laws- An Argument for Simplicity
- 5.E: Uniform Circular Motion and Gravitation (Excercise)

6: Work, Energy, and Energy Resources

- 6.1: Prelude to Work, Energy, and Energy Resources
- 6.2: Work- The Scientific Definition
- 6.3: Kinetic Energy and the Work-Energy Theorem
- 6.4: Gravitational Potential Energy
- 6.5: Conservative Forces and Potential Energy
- 6.6: Nonconservative Forces
- 6.7: Conservation of Energy
- 6.8: Power
- 6.9: Work, Energy, and Power in Humans
- 6.10: World Energy Use
- 6.E: Work, Energy, and Energy Resources (Exercise)

7: Linear Momentum and Collisions

- 7.1: Prelude to Linear Momentum and Collisions
- 7.2: Linear Momentum and Force
- 7.3: Impulse
- 7.4: Conservation of Momentum
- 7.5: Elastic Collisions in One Dimension
- 7.6: Inelastic Collisions in One Dimension
- 7.7: Collisions of Point Masses in Two Dimensions
- 7.8: Introduction to Rocket Propulsion
- 7.E: Linear Momentum and Collisions (Exercises)

8: Heat and Heat Transfer Methods

- 8.1: Prelude to Heat and Heat Transfer Methods
- 8.2: Heat
- 8.3: Temperature Change and Heat Capacity
- 8.4: Phase Change and Latent Heat
- 8.5: Heat Transfer Methods
- 8.6: Conduction
- 8.7: Convection
- 8.8: Radiation
- 8.E: Heat and Heat Transfer Methods (Exercise)

Index

Glossary

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CHAPTER OVERVIEW

1: Nature of Physics

1.1: The Basics of Physics
1.2: Scientific Notation and Order of Magnitude
1.3: Units and Standards
1.4: Unit Conversion
1.5: Dimensional Analysis
1.6: Significant Figures
1.7: Summary
1.8: Exercises
1.9: Answers

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1.1: The Basics of Physics

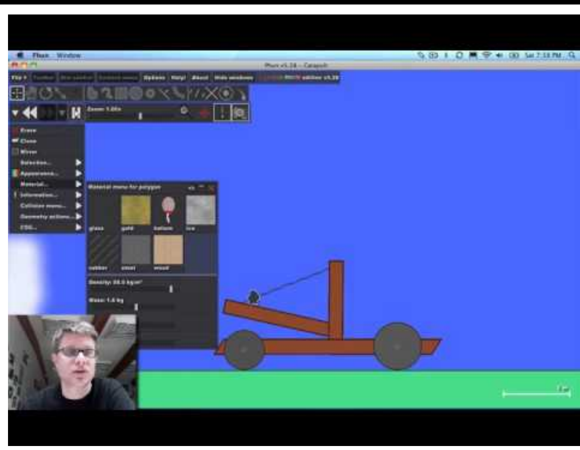
Introduction: Physics and Matter

Physics is a study of how the universe behaves.

learning objectives

- Apply physics to describe the function of daily life

Physics is a natural science that involves the study of matter and its motion through space and time, along with related concepts such as energy and force. More broadly, it is the study of nature in an attempt to understand how the universe behaves.

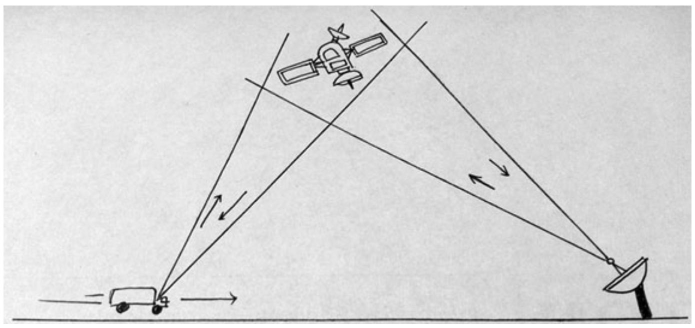


What is Physics?: Mr. Andersen explains the importance of physics as a science. History and virtual examples are used to give the discipline context.

Physics uses the scientific method to help uncover the basic principles governing light and matter, and to discover the implications of those laws. It assumes that there are rules by which the universe functions, and that those laws can be at least partially understood by humans. It is also commonly believed that those laws could be used to predict everything about the universe's future if complete information was available about the present state of all light and matter.

Matter is generally considered to be anything that has mass and volume. Many concepts integral to the study of classical physics involve theories and laws that explain matter and its motion. The law of conservation of mass, for example, states that mass cannot be created or destroyed. Further experiments and calculations in physics, therefore, take this law into account when formulating hypotheses to try to explain natural phenomena.

Physics aims to describe the function of everything around us, from the movement of tiny charged particles to the motion of people, cars, and spaceships. In fact, almost everything around you can be described quite accurately by the laws of physics. Consider a smart phone; physics describes how electricity interacts with the various circuits inside the device. This knowledge helps engineers select the appropriate materials and circuit layout when building the smart phone. Next, consider a GPS system; physics describes the relationship between the speed of an object, the distance over which it travels, and the time it takes to travel that distance. When you use a GPS device in a vehicle, it utilizes these physics equations to determine the travel time from one location to another. The study of physics is capable of making significant contributions through advances in new technologies that arise from theoretical breakthroughs.



Global Positioning System: GPS calculates the speed of an object, the distance over which it travels, and the time it takes to travel that distance using equations based on the laws of physics.

Physics and Other Fields

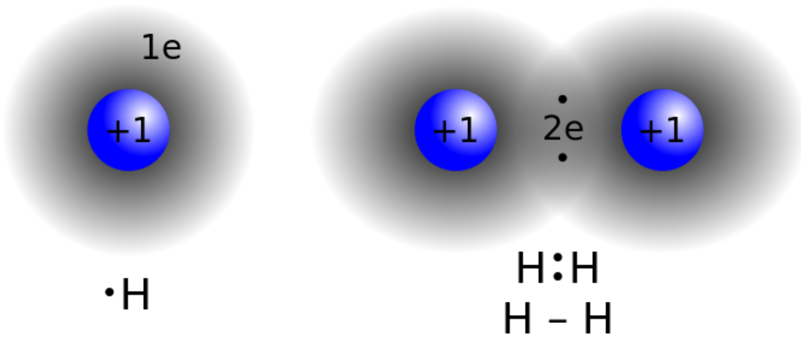
Physics is the foundation of many disciplines and contributes directly to chemistry, astronomy, engineering, and most scientific fields.

learning objectives

- Explain why the study of physics is integral to the study of other sciences

Physics and Other Disciplines

Physics is the foundation of many important disciplines and contributes directly to others. Chemistry deals with the interactions of atoms and molecules, so it is rooted in atomic and molecular physics. Most branches of engineering are applied physics. In architecture, physics is at the heart of structural stability and is involved in acoustics, heating, lighting, and the cooling of buildings. Parts of geology rely heavily on physics, such as the radioactive dating of rocks, earthquake analysis, and heat transfer in the Earth. Some disciplines, such as biophysics and geophysics, are hybrids of physics and other disciplines.



Physics in Chemistry: The study of matter and electricity in physics is fundamental towards the understanding of concepts in chemistry, such as the covalent bond.

Physics has many applications in the biological sciences. On the microscopic level, it helps describe the properties of cell walls and cell membranes. On the macroscopic level, it can explain the heat, work, and power associated with the human body. Physics is involved in medical diagnostics, such as X-rays, magnetic resonance imaging (MRI), and ultrasonic blood flow measurements. Medical therapy sometimes directly involves physics: cancer radiotherapy uses ionizing radiation, for instance. Physics can also explain sensory phenomena, such as how musical instruments make sound, how the eye detects color, and how lasers can transmit information.

The boundary between physics and the other sciences is not always clear. For instance, chemists study atoms and molecules, which are what matter is built from, and there are some scientists who would be equally willing to call themselves physical chemists or chemical physicists. It might seem that the distinction between physics and biology would be clearer, since physics seems to deal with inanimate objects. In fact, almost all physicists would agree that the basic laws of physics that apply to molecules in a test tube work equally well for the combination of molecules that constitutes a bacterium. What differentiates physics from biology is that many of the scientific theories that describe living things ultimately result from the fundamental laws of physics, but cannot be rigorously derived from physical principles.

It is not necessary to formally study all applications of physics. What is most useful is the knowledge of the basic laws of physics and skill in the analytical methods for applying them. The study of physics can also improve your problem-solving skills. Furthermore, physics has retained the most basic aspects of science, so it is used by all of the sciences. The study of physics makes other sciences easier to understand.

Models, Theories, and Laws

The terms *model*, *theory*, and *law* have exact meanings in relation to their usage in the study of physics.

learning objectives

- Define the terms model, theory, and law

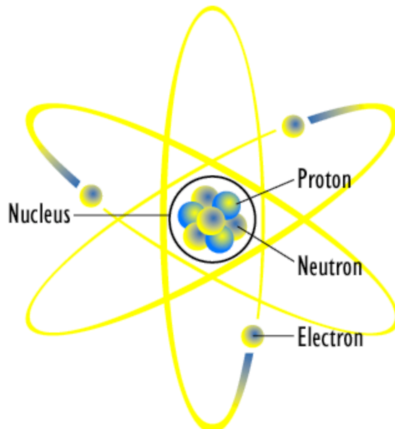
Definition of Terms: Model, Theory, Law

In colloquial usage, the terms *model*, *theory*, and *law* are often used interchangeably or have different interpretations than they do in the sciences. In relation to the study of physics, however, each term has its own specific meaning.

The *laws of nature* are concise descriptions of the universe around us. They are not explanations, but human statements of the underlying rules that all natural processes follow. They are intrinsic to the universe; humans did not create them and we cannot change them. We can only discover and understand them. The cornerstone of discovering natural laws is observation; science must describe the universe as it is, not as we may imagine it to be. Laws can never be known with absolute certainty, because it is impossible to perform experiments to establish and confirm a law in every possible scenario without exception. Physicists operate under the assumption that all scientific laws and theories are valid until a counterexample is observed. If a good-quality, verifiable experiment contradicts a well-established law, then the law must be modified or overthrown completely.

Models

A *model* is a representation of something that is often too difficult (or impossible) to display directly. While a model's design is justified using experimental information, it is only accurate under limited situations. An example is the commonly used "planetary model" of the atom, in which electrons are pictured as orbiting the nucleus, analogous to the way planets orbit the Sun. We cannot observe electron orbits directly, but the mental image helps explain the observations we can make, such as the emission of light from hot gases. Physicists use models for a variety of purposes. For example, models can help physicists analyze a scenario and perform a calculation, or they can be used to represent a situation in the form of a computer simulation.



Planetary Model of an Atom: The planetary model of the atom in which electrons are pictured as orbiting the nucleus, analogous to the way planets orbit the Sun

Theories

A *theory* is an explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers. *Some theories include models to help visualize phenomena, whereas others do not*. Newton's theory of gravity, for example, does not require a model or mental image, because we can observe the objects directly with our own senses. The kinetic theory of gases, on the other hand, makes use of a model in which a gas is viewed as being composed of atoms and molecules. Atoms and molecules are too small to be observed directly with our senses—thus, we picture them mentally to understand what our instruments tell us about the behavior of gases.

Laws

A law uses concise language to describe a generalized pattern in nature that is supported by scientific evidence and repeated experiments. Often, a law can be expressed in the form of a single mathematical equation. Laws and theories are similar in that they are both scientific statements that result from a tested hypothesis and are supported by scientific evidence. However, the designation law is reserved for a concise and very general statement that describes phenomena in nature, such as the law that energy is conserved during any process, or Newton's second law of motion, which relates force, mass, and acceleration by the simple equation $F = ma$. A theory, in contrast, is a less concise statement of observed phenomena. For example, the Theory of Evolution and the Theory of Relativity cannot be expressed concisely enough to be considered a law. The biggest difference between a law and a theory is that a law is much more complex and dynamic, and a theory is more explanatory. A law describes a single observable point of fact, whereas a theory explains an entire group of related phenomena. And, whereas a law is a postulate that forms the foundation of the scientific method, a theory is the end result of that process.

Key Points

- Physics is a natural science that involves the study of matter and its motion through space and time, along with related concepts such as energy and force.
- Matter is generally considered to be anything that has mass and volume.
- Scientific laws and theories express the general truths of nature and the body of knowledge they encompass. These laws of nature are rules that all natural processes appear to follow.
- Many scientific disciplines, such as biophysics, are hybrids of physics and other sciences.
- The study of physics encompasses all forms of matter and its motion in space and time.

- The application of physics is fundamental towards significant contributions in new technologies that arise from theoretical breakthroughs.
- Concepts in physics cannot be proven, they can only be supported or disproven through observation and experimentation.
- A model is an evidence-based representation of something that is either too difficult or impossible to display directly.
- A theory is an explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers.
- A law uses concise language, often expressed as a mathematical equation, to describe a generalized pattern in nature that is supported by scientific evidence and repeated experiments.

Key Terms

- **matter**: The basic structural component of the universe. Matter usually has mass and volume.
- **scientific method**: A method of discovering knowledge about the natural world based in making falsifiable predictions (hypotheses), testing them empirically, and developing peer-reviewed theories that best explain the known data.
- **application**: the act of putting something into operation
- **Model**: A representation of something difficult or impossible to display directly
- **Law**: A concise description, usually in the form of a mathematical equation, used to describe a pattern in nature
- **theory**: An explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers

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1.2: Scientific Notation and Order of Magnitude

Scientific Notation

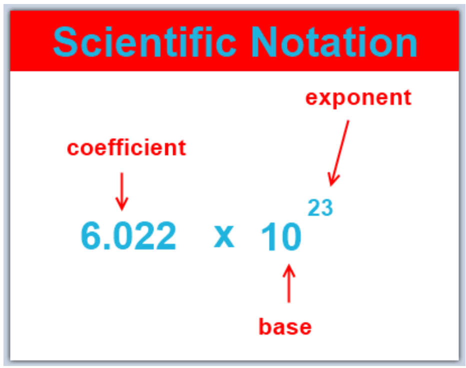
Scientific notation is a way of writing numbers that are too big or too small in a convenient and standard form.

learning objectives

- Convert properly between standard and scientific notation and identify appropriate situations to use it

Scientific Notation: A Matter of Convenience

Scientific notation is a way of writing numbers that are too big or too small in a convenient and standard form. Scientific notation has a number of useful properties and is commonly used in calculators and by scientists, mathematicians and engineers. In scientific notation all numbers are written in the form of $a \cdot 10^{b}$ (a multiplied by ten raised to the power of b), where the exponent b) is an integer, and the coefficient (a is any real number.



Scientific Notation: There are three parts to writing a number in scientific notation: the coefficient, the base, and the exponent.

Most of the interesting phenomena in our universe are not on the human scale. It would take about 1,000,000,000,000,000,000,000 bacteria to equal the mass of a human body. Thomas Young's discovery that light was a wave preceded the use of scientific notation, and he was obliged to write that the time required for one vibration of the wave was "$\frac{1}{500}$ of a millionth of a millionth of a second"; an inconvenient way of expressing the point. Scientific notation is a less awkward and wordy way to write very large and very small numbers such as these.

A Simple System

Scientific notation means writing a number in terms of a product of something from 1 to 10 and something else that is a power of ten.

For instance, $32 = 3.2 \cdot 10^{1}$

$320 = 3.2 \cdot 10^{2}$

$3200 = 3.2 \cdot 10^{3}$, and so forth…

Each number is ten times bigger than the previous one. Since 10^{1} is ten times smaller than 10^{2}, it makes sense to use the notation 10^{0} to stand for one, the number that is in turn ten times smaller than 10^{1}. Continuing on, we can write 10^{-1} to stand for 0.1, the number ten times smaller than 10^{0}. Negative exponents are used for small numbers:

$3.2 = 3.2 \cdot 10^{0}$

$0.32 = 3.2 \cdot 10^{-1}$

$0.032 = 3.2 \cdot 10^{-2}$

Scientific notation displayed calculators can take other shortened forms that mean the same thing. For example, $3.2 \cdot 10^{6}$(written notation) is the same as $3.2\mathrm{E}+6$ (notation on some calculators) and 3.2^{6} (notation on some other calculators).

Round-off Error

A round-off error is the difference between the calculated approximation of a number and its exact mathematical value.

learning objectives

- Explain the impact round-off errors may have on calculations, and how to reduce this impact

Round-off Error

A round-off error, also called a rounding error, is the difference between the calculated approximation of a number and its exact mathematical value. Numerical analysis specifically tries to estimate this error when using approximation equations, algorithms, or both, especially when using finitely many digits to represent real numbers. When a sequence of calculations subject to rounding errors is made, errors may accumulate, sometimes dominating the calculation.

Calculations rarely lead to whole numbers. As such, values are expressed in the form of a decimal with infinite digits. The more digits that are used, the more accurate the calculations will be upon completion. Using a slew of digits in multiple calculations, however, is often unfeasible if calculating by hand and can lead to much more human error when keeping track of so many digits. To make calculations much easier, the results are often 'rounded off' to the nearest few decimal places.

For example, the equation for finding the area of a circle is $\mathrm{A}=\pi \mathrm{r}^2$. The number π(pi) has infinitely many digits, but can be truncated to a rounded representation of as 3.14159265359. However, for the convenience of performing calculations by hand, this number is typically rounded even further, to the nearest two decimal places, giving just 3.14. Though this technically decreases the accuracy of the calculations, the value derived is typically 'close enough' for most estimation purposes.

However, when doing a series of calculations, numbers are rounded off at each subsequent step. This leads to an accumulation of errors, and if profound enough, can misrepresent calculated values and lead to miscalculations and mistakes.

The following is an example of round-off error:

$$\sqrt{4.58^2+3.28^2}=\sqrt{21.0+10.8}=5.64$$

Rounding these numbers off to one decimal place or to the nearest whole number would change the answer to 5.7 and 6, respectively. The more rounding off that is done, the more errors are introduced.

Order of Magnitude Calculations

An order of magnitude is the class of scale of any amount in which each class contains values of a fixed ratio to the class preceding it.

learning objectives

- Choose when it is appropriate to perform an order-of-magnitude calculation

Orders of Magnitude

An order of magnitude is the class of scale of any amount in which each class contains values of a fixed ratio to the class preceding it. In its most common usage, the amount scaled is 10, and the scale is the exponent applied to this amount (therefore, to be an order of magnitude greater is to be 10 times, or 10 to the power of 1, greater). Such differences in order of magnitude can be measured on the logarithmic scale in "decades," or factors of ten. It is common among scientists and technologists to say that a parameter whose value is not accurately known or is known only within a range is "on the order of" some value. The order of magnitude of a physical quantity is its magnitude in powers of ten when the physical quantity is expressed in powers of ten with one digit to the left of the decimal.

Orders of magnitude are generally used to make very approximate comparisons and reflect very large differences. If two numbers differ by one order of magnitude, one is about ten times larger than the other. If they differ by two orders of magnitude, they differ by a factor of about 100. Two numbers of the same order of magnitude have roughly the same scale — the larger value is less than ten times the smaller value.

It is important in the field of science that estimates be at least in the right ballpark. In many situations, it is often sufficient for an estimate to be within an order of magnitude of the value in question. Although making order-of-magnitude estimates seems simple and natural to experienced scientists, it may be completely unfamiliar to the less experienced.

Example 1.2.1:

Some of the mental steps of estimating in orders of magnitude are illustrated in answering the following example question: Roughly what percentage of the price of a tomato comes from the cost of transporting it in a truck?

Guessing the Number of Jelly Beans: Can you guess how many jelly beans are in the jar? If you try to guess directly, you will almost certainly underestimate. The right way to do it is to estimate the linear dimensions and then estimate the volume indirectly.

Incorrect solution: Let's say the trucker needs to make a profit on the trip. Taking into account her benifits, the cost of gas, and maintenance and payments on the truck, let's say the total cost is more like 2000. You might guess about 5000 tomatoes would fit in the back of the truck, so the extra cost per tomato is 40 cents. That means the cost of transporting one tomato is comparable to the cost of the tomato itself.

The problem here is that the human brain is not very good at estimating area or volume — it turns out the estimate of 5000 tomatoes fitting in the truck is way off. (This is why people have a hard time in volume-estimation contests, such as the one shown below.) When estimating area or volume, you are much better off estimating linear dimensions and computing the volume from there.

So, here's a better solution: As before, let's say the cost of the trip is $2000. The dimensions of the bin are probably 4m by 2m by 1m, for a volume of $8\ \mathrm{m}^3$. Since our goal is just an order-of-magnitude estimate, let's round that volume off to the nearest power of ten: $10\ \mathrm{m}^3$. The shape of a tomato doesn't follow linear dimensions, but since this is just an estimate, let's pretend that a tomato is an 0.1m by 0.1m by 0.1m cube, with a volume of $1 \cdot 10^{-3}\ \mathrm{m}^3$. We can find the total number of tomatoes by dividing the volume of the bin by the volume of one tomato: $\frac{10^3\ \mathrm{m}^3}{10^{-3}\ \mathrm{m}^3} = 10^6$ tomatoes. The transportation cost per tomato is $\frac{\$2000}{10^6\ \text{tomatoes}} = \$0.002$ per tomato. That means that transportation really doesn't contribute very much to the cost of a tomato. Approximating the shape of a tomato as a cube is an example of another general strategy for making order-of-magnitude estimates.

Key Points

- Scientific notation means writing a number in terms of a product of something from 1 to 10 and something else that is a power of 10.
- In scientific notation all numbers are written in the form of $a \cdot 10^b$ (a times ten raised to the power of b).
- Each consecutive exponent number is ten times bigger than the previous one; negative exponents are used for small numbers.
- When a sequence of calculations subject to rounding error is made, these errors can accumulate and lead to the misrepresentation of calculated values.
- Increasing the number of digits allowed in a representation reduces the magnitude of possible round-off errors, but may not always be feasible, especially when doing manual calculations.
- The degree to which numbers are rounded off is relative to the purpose of calculations and the actual value.
- Orders of magnitude are generally used to make very approximate comparisons and reflect very large differences.
- In the field of science, it is often sufficient for an estimate to be within an order of magnitude of the value in question.

- When estimating area or volume, you are much better off estimating linear dimensions and computing volume from those linear dimensions.

Key Terms

- **exponent**: The power to which a number, symbol or expression is to be raised. For example, the 3 in x^3.
- **Scientific notation**: A method of writing, or of displaying real numbers as a decimal number between 1 and 10 followed by an integer power of 10
- **approximation**: An imprecise solution or result that is adequate for a defined purpose.
- **Order of Magnitude**: The class of scale or magnitude of any amount, where each class contains values of a fixed ratio (most often 10) to the class preceding it. For example, something that is 2 orders of magnitude larger is 100 times larger; something that is 3 orders of magnitude larger is 1000 times larger; and something that is 6 orders of magnitude larger is one million times larger, because $10^2 = 100$, $10^3 = 1000$, and $10^6 =$ one million

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1.3: Units and Standards

Learning Objectives

- Describe how SI base units are defined.
- Describe how derived units are created from base units.
- Express quantities given in SI units using metric prefixes.

As we saw previously, the range of objects and phenomena studied in physics is immense. From the incredibly short lifetime of a nucleus to the age of Earth, from the tiny sizes of subnuclear particles to the vast distance to the edges of the known universe, from the force exerted by a jumping flea to the force between Earth and the Sun, there are enough factors of 10 to challenge the imagination of even the most experienced scientist. Giving numerical values for physical quantities and equations for physical principles allows us to understand nature much more deeply than qualitative descriptions alone. To comprehend these vast ranges, we must also have accepted units in which to express them. We shall find that even in the potentially mundane discussion of meters, kilograms, and seconds, a profound simplicity of nature appears: all physical quantities can be expressed as combinations of only seven base physical quantities.

We define a **physical quantity** either by specifying how it is measured or by stating how it is calculated from other measurements. For example, we might define distance and time by specifying methods for measuring them, such as using a meter stick and a stopwatch. Then, we could define average speed by stating that it is calculated as the total distance traveled divided by time of travel.

Measurements of physical quantities are expressed in terms of **units**, which are standardized values. For example, the length of a race, which is a physical quantity, can be expressed in units of meters (for sprinters) or kilometers (for distance runners). Without standardized units, it would be extremely difficult for scientists to express and compare measured values in a meaningful way (Figure 1.3.1).

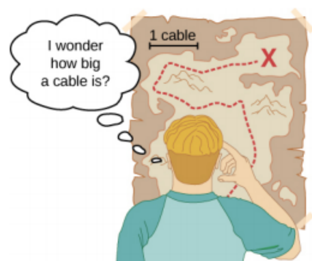

Figure 1.3.1: Distances given in unknown units are maddeningly useless.

Two major systems of units are used in the world: **SI units** (for the French **Système International d'Unités**), also known as the **metric system**, and **English units** (also known as the **customary** or **imperial system**). English units were historically used in nations once ruled by the British Empire and are still widely used in the United States. English units may also be referred to as the **foot–pound–second** (fps) system, as opposed to the **centimeter–gram–second** (cgs) system. You may also encounter the term **SAE units**, named after the Society of Automotive Engineers. Products such as fasteners and automotive tools (for example, wrenches) that are measured in inches rather than metric units are referred to as **SAE fasteners** or **SAE wrenches**.

Virtually every other country in the world (except the United States) now uses SI units as the standard. The metric system is also the standard system agreed on by scientists and mathematicians.

SI Units: Base and Derived Units

In any system of units, the units for some physical quantities must be defined through a measurement process. These are called the **base quantities** for that system and their units are the system's **base units**. All other physical quantities can then be expressed as algebraic combinations of the base quantities. Each of these physical quantities is then known as a **derived quantity** and each unit is called a **derived unit**. The choice of base quantities is somewhat arbitrary, as long as they are independent of each other and all other quantities can be derived from them. Typically, the goal is to choose physical quantities that can be measured accurately to a high precision as the base quantities. The reason for this is simple. Since the derived units can be expressed as algebraic combinations of the base units, they can only be as accurate and precise as the base units from which they are derived.

Based on such considerations, the International Standards Organization recommends using seven base quantities, which form the International System of Quantities (ISQ). These are the base quantities used to define the SI base units. Table 1.3.1 lists these seven ISQ base quantities and the corresponding SI base units.

Table 1.3.1: ISQ Base Quantities and Their SI Units

ISQ Base Quantity	SI Base Unit
Length	meter (m)
Mass	kilogram (kg)
Time	second (s)
Electrical Current	ampere (A)
Thermodynamic Temperature	kelvin (K)
Amount of Substance	mole (mol)
Luminous Intensity	candela (cd)

You are probably already familiar with some derived quantities that can be formed from the base quantities in Table 1.3.1. For example, the geometric concept of area is always calculated as the product of two lengths. Thus, area is a derived quantity that can be expressed in terms of SI base units using square meters ($m \times m = m^2$). Similarly, volume is a derived quantity that can be expressed in cubic meters (m^3). Speed is length per time; so in terms of SI base units, we could measure it in meters per second (m/s). Volume mass density (or just density) is mass per volume, which is expressed in terms of SI base units such as kilograms per cubic meter (kg/m^3). Angles can also be thought of as derived quantities because they can be defined as the ratio of the arc length subtended by two radii of a circle to the radius of the circle. This is how the radian is defined. Depending on your background and interests, you may be able to come up with other derived quantities, such as the mass flow rate (kg/s) or volume flow rate (m^3/s) of a fluid, electric charge (A • s), mass flux density [$kg/(m^2 \cdot s)$], and so on. We will see many more examples throughout this text. For now, the point is that every physical quantity can be derived from the seven base quantities in Table 1.3.1, and the units of every physical quantity can be derived from the seven SI base units.

For the most part, we use SI units in this text. Non-SI units are used in a few applications in which they are in very common use, such as the measurement of temperature in degrees Celsius (°C), the measurement of fluid volume in liters (L), and the measurement of energies of elementary particles in electron-volts (eV). Whenever non-SI units are discussed, they are tied to SI units through conversions. For example, 1 L is 10^{-3} m^3.

Check out a comprehensive source of information on SI units at the National Institute of Standards and Technology (NIST) Reference on Constants, Units, and Uncertainty.

Units of Time, Length, and Mass: The Second, Meter, and Kilogram

The initial chapters in this textmap are concerned with mechanics, fluids, and waves. In these subjects all pertinent physical quantities can be expressed in terms of the base units of length, mass, and time. Therefore, we now turn to a discussion of these three base units, leaving discussion of the others until they are needed later.

The Second

The SI unit for time, the **second** (abbreviated s), has a long history. For many years it was defined as 1/86,400 of a mean solar day. More recently, a new standard was adopted to gain greater accuracy and to define the second in terms of a nonvarying or constant physical phenomenon (because the solar day is getting longer as a result of the very gradual slowing of Earth's rotation). Cesium atoms can be made to vibrate in a very steady way, and these vibrations can be readily observed and counted. In 1967, the second was redefined as the time required for 9,192,631,770 of these vibrations to occur (Figure 1.3.2). Note that this may seem like more precision than you would ever need, but it isn't—GPSs rely on the precision of atomic clocks to be able to give you turn-by-turn directions on the surface of Earth, far from the satellites broadcasting their location.

Figure 1.3.2: An atomic clock such as this one uses the vibrations of cesium atoms to keep time to a precision of better than a microsecond per year. The fundamental unit of time, the second, is based on such clocks. This image looks down from the top of an atomic fountain nearly 30 feet tall. (credit: Steve Jurvetson)

The Meter

The SI unit for length is the **meter** (abbreviated m); its definition has also changed over time to become more precise. The meter was first defined in 1791 as 1/10,000,000 of the distance from the equator to the North Pole. This measurement was improved in 1889 by redefining the meter to be the distance between two engraved lines on a platinum–iridium bar now kept near Paris. By 1960, it had become possible to define the meter even more accurately in terms of the wavelength of light, so it was again redefined as 1,650,763.73 wavelengths of orange light emitted by krypton atoms. In 1983, the meter was given its current definition (in part for greater accuracy) as the distance light travels in a vacuum in 1/299,792,458 of a second (Figure 1.3.3). This change came after knowing the speed of light to be exactly 299,792,458 m/s. The length of the meter will change if the speed of light is someday measured with greater accuracy.

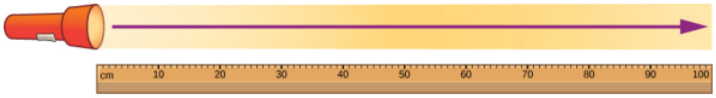

Figure 1.3.3: The meter is defined to be the distance light travels in 1/299,792,458 of a second in a vacuum. Distance traveled is speed multiplied by time.

The Kilogram

The SI unit for mass is the **kilogram** (abbreviated kg); From 1795–2018 it was defined to be the mass of a platinum–iridium cylinder kept with the old meter standard at the International Bureau of Weights and Measures near Paris. However, this cylinder has lost roughly 50 micrograms since it was created. Because this is the standard, this has shifted how we defined a kilogram. Therefore, a new definition was adopted in May 2019 based on the Planck constant and other constants which will never change in value. We will study Planck's constant in quantum mechanics, which is an area of physics that describes how the smallest pieces of the universe work. The kilogram is measured on a Kibble balance (see 1.3.4). When a weight is placed on a Kibble balance, an electrical current is produced that is proportional to Planck's constant. Since Planck's constant is defined, the exact current measurements in the balance define the kilogram.

Figure 1.3.4: Redefining the SI unit of mass. The U.S. National Institute of Standards and Technology's Kibble balance is a machine that balances the weight of a test mass with the resulting electrical current needed for a force to balance it.

Metric Prefixes

SI units are part of the **metric system**, which is convenient for scientific and engineering calculations because the units are categorized by factors of 10. Table 1.3.1 lists the metric prefixes and symbols used to denote various factors of 10 in SI units. For example, a centimeter is one-hundredth of a meter (in symbols, 1 cm = 10^{-2} m) and a kilometer is a thousand meters (1 km = 10^3 m). Similarly, a megagram is a million grams (1 Mg = 10^6 g), a nanosecond is a billionth of a second (1 ns = 10^{-9} s), and a terameter is a trillion meters (1 Tm = 10^{12} m).

Table 1.3.2: Metric Prefixes for Powers of 10 and Their Symbols

Prefix	Symbol	Meaning	Prefix	Symbol	Meaning
yotta-	Y	10^{24}	yocto-	Y	10^{-24}
zetta-	Z	10^{21}	zepto-	Z	10^{-21}
exa-	E	10^{18}	atto-	E	10^{-18}
peta-	P	10^{15}	femto-	P	10^{-15}
tera-	T	10^{12}	pico-	T	10^{-12}
giga-	G	10^{9}	nano-	G	10^{-9}
mega-	M	10^{6}	micro-	M	10^{-6}
kilo-	k	10^{3}	milli-	k	10^{-3}
hecto-	h	10^{2}	centi-	h	10^{-2}
deka-	da	10^{1}	deci-	da	10^{-1}

The only rule when using metric prefixes is that you cannot "double them up." For example, if you have measurements in petameters (1 Pm = 10^{15} m), it is not proper to talk about megagigameters, although 10^6 x 10^9 = 10^{15}. In practice, the only time this becomes a bit confusing is when discussing masses. As we have seen, the base SI unit of mass is the kilogram (kg), but metric prefixes need to be applied to the gram (g), because we are not allowed to "double-up" prefixes. Thus, a thousand kilograms (10^3 kg) is written as a megagram (1 Mg) since

$$10^3\ kg = 10^3 \times 10^3\ g = 10^6\ g = 1\ Mg. \tag{1.3.1}$$

Incidentally, 10^3 kg is also called a **metric ton**, abbreviated t. This is one of the units outside the SI system considered acceptable for use with SI units.

As we see in the next section, metric systems have the advantage that conversions of units involve only powers of 10. There are 100 cm in 1 m, 1000 m in 1 km, and so on. In nonmetric systems, such as the English system of units, the relationships are not as simple—there are 12 in in 1 ft, 5280 ft in 1 mi, and so on.

Another advantage of metric systems is that the same unit can be used over extremely large ranges of values simply by scaling it with an appropriate metric prefix. The prefix is chosen by the order of magnitude of physical quantities commonly found in the task at hand. For example, distances in meters are suitable in construction, whereas distances in kilometers are appropriate for air travel, and nanometers are convenient in optical design. With the metric system there is no need to invent new units for particular applications. Instead, we rescale the units with which we are already familiar.

Example 1.3.1: Using Metric Prefixes

Restate the mass 1.93 x 10^{13} kg using a metric prefix such that the resulting numerical value is bigger than one but less than 1000.

Strategy

Since we are not allowed to “double-up” prefixes, we first need to restate the mass in grams by replacing the prefix symbol k with a factor of 10^3 (Table $1.3.2$). Then, we should see which two prefixes in Table $1.3.2$ are closest to the resulting power of 10 when the number is written in scientific notation. We use whichever of these two prefixes gives us a number between one and 1000.

Solution

Replacing the k in kilogram with a factor of 10^3, we find that

$$1.93 \times 10^{13}\ kg = 1.93 \times 10^{13} \times 10^3\ g = 1.93 \times 10^{16}\ g.$$

From Table $1.3.2$, we see that 10^{16} is between “peta-” (10^{15}) and “exa-” (10^{18}). If we use the “peta-” prefix, then we find that 1.93×10^{16} g = 1.93×10^1 Pg, since 16 = 1 + 15. Alternatively, if we use the “exa-” prefix we find that 1.93 x 10^{16} g = 1.93 x 10^{-2} Eg, since 16 = −2 + 18. Because the problem asks for the numerical value between one and 1000, we use the “peta-” prefix and the answer is 19.3 Pg.

Significance

It is easy to make silly arithmetic errors when switching from one prefix to another, so it is always a good idea to check that our final answer matches the number we started with. An easy way to do this is to put both numbers in scientific notation and count powers of 10, including the ones hidden in prefixes. If we did not make a mistake, the powers of 10 should match up. In this problem, we started with 1.93 x 10^{13} kg, so we have 13 + 3 = 16 powers of 10. Our final answer in scientific notation is 1.93 x 10^1 Pg, so we have 1 + 15 = 16 powers of 10. So, everything checks out.

If this mass arose from a calculation, we would also want to check to determine whether a mass this large makes any sense in the context of the problem. For this, Figure 1.4 might be helpful.

Exercises 1.3.1

Restate 4.79 x 10^5 kg using a metric prefix such that the resulting number is bigger than one but less than 1000.

Answer

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1.4: Unit Conversion

Learning Objectives

- Use conversion factors to express the value of a given quantity in different units.

It is often necessary to convert from one unit to another. For example, if you are reading a European cookbook, some quantities may be expressed in units of liters and you need to convert them to cups. Or perhaps you are reading walking directions from one location to another and you are interested in how many miles you will be walking. In this case, you may need to convert units of feet or meters to miles.

Let's consider a simple example of how to convert units. Suppose we want to convert 80 m to kilometers. The first thing to do is to list the units you have and the units to which you want to convert. In this case, we have units in meters and we want to convert to kilometers. Next, we need to determine a conversion factor relating meters to kilometers. A **conversion factor** is a ratio that expresses how many of one unit are equal to another unit. For example, there are 12 in. in 1 ft, 1609 m in 1 mi, 100 cm in 1 m, 60 s in 1 min, and so on. Refer to Appendix B for a more complete list of conversion factors. In this case, we know that there are 1000 m in 1 km. Now we can set up our unit conversion. We write the units we have and then multiply them by the conversion factor so the units cancel out, as shown:

$$80\ \cancel{m} \times \frac{1\ km}{1000\ \cancel{m}} = 0.080\ km. \tag{1.4.1}$$

Note that the unwanted meter unit cancels, leaving only the desired kilometer unit. You can use this method to convert between any type of unit. Now, the conversion of 80 m to kilometers is simply the use of a metric prefix, as we saw in the preceding section, so we can get the same answer just as easily by noting that

$$80\ m = 8.0 \times 10^1\ m = 8.0 \times 10^{-2}\ km = 0.080\ km, \tag{1.4.2}$$

since "kilo-" means 10^3 and 1 = −2 + 3. However, using conversion factors is handy when converting between units that are not metric or when converting between derived units, as the following examples illustrate.

Example 1.4.1: Converting Nonmetric Units to Metric

The distance from the university to home is 10 mi and it usually takes 20 min to drive this distance. Calculate the average speed in meters per second (m/s). (**Note**: Average speed is distance traveled divided by time of travel.)

Strategy

First we calculate the average speed using the given units, then we can get the average speed into the desired units by picking the correct conversion factors and multiplying by them. The correct conversion factors are those that cancel the unwanted units and leave the desired units in their place. In this case, we want to convert miles to meters, so we need to know the fact that there are 1609 m in 1 mi. We also want to convert minutes to seconds, so we use the conversion of 60 s in 1 min.

Solution

1. Calculate average speed. Average speed is distance traveled divided by time of travel. (Take this definition as a given for now. Average speed and other motion concepts are covered in later chapters.) In equation form,

$$Average\ speed = \frac{Distance}{Time}.$$

2. Substitute the given values for distance and time:

$$Average\ speed = \frac{10\ mi}{20\ min} = 0.50\ \frac{mi}{min}.$$

3. Convert miles per minute to meters per second by multiplying by the conversion factor that cancels miles and leave meters, and also by the conversion factor that cancels minutes and leave seconds:

$$0.50\ \frac{\cancel{mile}}{\cancel{min}} \times \frac{1609\ m}{1\ \ \cancel{mile}} \times \frac{1\ \cancel{min}}{60\ s} = \frac{(0.50)(1609)}{60}\ m/s = 13\ m/s.$$

Significance

Check the answer in the following ways:

1. Be sure the units in the unit conversion cancel correctly. If the unit conversion factor was written upside down, the units do not cancel correctly in the equation. We see the "miles" in the numerator in 0.50 mi/min cancels the "mile" in the denominator in the first conversion factor. Also, the "min" in the denominator in 0.50 mi/min cancels the "min" in the numerator in the second conversion factor.
2. Check that the units of the final answer are the desired units. The problem asked us to solve for average speed in units of meters per second and, after the cancelations, the only units left are a meter (m) in the numerator and a second (s) in the denominator, so we have indeed obtained these units.

? Exercise 1.4.1

Light travels about 9 Pm in a year. Given that a year is about 3 x 10^7 s, what is the speed of light in meters per second?

Answer

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✓ Example 1.4.2: Converting between Metric Units

The density of iron is 7.86 g/cm^3 under standard conditions. Convert this to kg/m^3.

Strategy

We need to convert grams to kilograms and cubic centimeters to cubic meters. The conversion factors we need are 1 kg = 10^3 g and 1 cm = 10^{-2}m. However, we are dealing with cubic centimeters (cm^3 = cm x cm x cm), so we have to use the second conversion factor three times (that is, we need to cube it). The idea is still to multiply by the conversion factors in such a way that they cancel the units we want to get rid of and introduce the units we want to keep.

Solution

$$7.86\ \frac{\cancel{g}}{\cancel{cm^3}} \times \frac{kg}{10^3\ \ \cancel{g}} \times \left(\frac{\cancel{cm}}{10^{-2}\ \ m}\right)^3 = \frac{7.86}{(10^3)(10^{-6})}\ kg/m^3 = 7.86 \times 10^3\ kg/m^3$$

Significance

Remember, it's always important to check the answer.

1. Be sure to cancel the units in the unit conversion correctly. We see that the gram ("g") in the numerator in 7.86 g/cm^3 cancels the "g" in the denominator in the first conversion factor. Also, the three factors of "cm" in the denominator in 7.86 g/cm^3 cancel with the three factors of "cm" in the numerator that we get by cubing the second conversion factor.
2. Check that the units of the final answer are the desired units. The problem asked for us to convert to kilograms per cubic meter. After the cancelations just described, we see the only units we have left are "kg" in the numerator and three factors of "m" in the denominator (that is, one factor of "m" cubed, or "m^3"). Therefore, the units on the final answer are correct.

? Exercise 1.4.2

We know from Figure 1.4 that the diameter of Earth is on the order of 10^7 m, so the order of magnitude of its surface area is 10^{14} m^2. What is that in square kilometers (that is, km^2)? (Try doing this both by converting 10^7 m to km and then squaring it and then by converting 10^{14} m^2 directly to square kilometers. You should get the same answer both ways.)

Answer

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Unit conversions may not seem very interesting, but not doing them can be costly. One famous example of this situation was seen with the **Mars Climate Orbiter**. This probe was launched by NASA on December 11, 1998. On September 23, 1999, while attempting to guide the probe into its planned orbit around Mars, NASA lost contact with it. Subsequent investigations showed a piece of software called SM_FORCES (or "small forces") was recording thruster performance data in the English units of pound-seconds (lb • s). However, other pieces of software that used these values for course corrections expected them to be recorded in the SI units of newton-seconds (N • s), as dictated in the software interface protocols. This error caused the probe to follow a very different trajectory from what NASA thought it was following, which most likely caused the probe either to burn up in the Martian atmosphere or to shoot out into space. This failure to pay attention to unit conversions cost hundreds of millions of dollars, not to mention all the time invested by the scientists and engineers who worked on the project.

Exercise 1.4.3

Given that 1 lb (pound) is 4.45 N, were the numbers being output by SM_FORCES too big or too small?

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1.5: Dimensional Analysis

> **Learning Objectives**
>
> - Find the dimensions of a mathematical expression involving physical quantities.
> - Determine whether an equation involving physical quantities is dimensionally consistent.

The **dimension** of any physical quantity expresses its dependence on the base quantities as a product of symbols (or powers of symbols) representing the base quantities. Table 1.5.1 lists the base quantities and the symbols used for their dimension. For example, a measurement of length is said to have dimension L or L^1, a measurement of mass has dimension M or M^1, and a measurement of time has dimension T or T^1. Like units, dimensions obey the rules of algebra. Thus, area is the product of two lengths and so has dimension L^2, or length squared. Similarly, volume is the product of three lengths and has dimension L^3, or length cubed. Speed has dimension length over time, L/T or LT^{-1}. Volumetric mass density has dimension M/L^3 or ML^{-3}, or mass over length cubed. In general, the dimension of any physical quantity can be written as

$$L^aM^bT^cI^d\Theta^eN^fJ^g \tag{1.5.1}$$

for some powers a, b, c, d, e, f, and g. We can write the dimensions of a length in this form with a = 1 and the remaining six powers all set equal to zero:

$$L^1 = L^1M^0T^0I^0\Theta^0N^0J^0. \tag{1.5.2}$$

Any quantity with a dimension that can be written so that all seven powers are zero (that is, its dimension is $L^0M^0T^0I^0\Theta^0N^0J^0$) is called **dimensionless** (or sometimes "of dimension 1," because anything raised to the zero power is one). Physicists often call dimensionless quantities **pure numbers**.

Table 1.5.1: Base Quantities and Their Dimensions

Base Quantity	Symbol for Dimension
Length	L
Mass	M
Time	T
Current	I
Thermodynamic Temperature	Θ
Amount of Substance	N
Luminous Intensity	J

Physicists often use square brackets around the symbol for a physical quantity to represent the dimensions of that quantity. For example, if r is the radius of a cylinder and h is its height, then we write [r] = L and [h] = L to indicate the dimensions of the radius and height are both those of length, or L. Similarly, if we use the symbol A for the surface area of a cylinder and V for its volume, then [A] = L^2 and [V] = L^3. If we use the symbol m for the mass of the cylinder and ρ for the density of the material from which the cylinder is made, then [m] = M and [ρ] = ML^{-3}.

The importance of the concept of dimension arises from the fact that any mathematical equation relating physical quantities must be **dimensionally consistent**, which means the equation must obey the following rules:

- Every term in an expression must have the same dimensions; it does not make sense to add or subtract quantities of differing dimension (think of the old saying: "You can't add apples and oranges"). In particular, the expressions on each side of the equality in an equation must have the same dimensions.
- The arguments of any of the standard mathematical functions such as trigonometric functions (such as sine and cosine), logarithms, or exponential functions that appear in the equation must be dimensionless. These functions require pure numbers as inputs and give pure numbers as outputs.

If either of these rules is violated, an equation is not dimensionally consistent and cannot possibly be a correct statement of physical law. This simple fact can be used to check for typos or algebra mistakes, to help remember the various laws of physics, and even to suggest the form that new laws of physics might take. This last use of dimensions is beyond the scope of this text, but is something you will undoubtedly learn later in your academic career.

Example 1.5.1: Using Dimensions to Remember an Equation

Suppose we need the formula for the area of a circle for some computation. Like many people who learned geometry too long ago to recall with any certainty, two expressions may pop into our mind when we think of circles: πr^2 and $2\pi r$. One expression is the circumference of a circle of radius r and the other is its area. But which is which?

Strategy

One natural strategy is to look it up, but this could take time to find information from a reputable source. Besides, even if we think the source is reputable, we shouldn't trust everything we read. It is nice to have a way to double-check just by thinking about it. Also, we might be in a situation in which we cannot look things up (such as during a test). Thus, the strategy is to find the dimensions of both expressions by making use of the fact that dimensions follow the rules of algebra. If either expression does not have the same dimensions as area, then it cannot possibly be the correct equation for the area of a circle.

Solution

We know the dimension of area is L^2. Now, the dimension of the expression πr^2 is

$$[\pi r^2] = [\pi] \cdot [r]^2 = 1 \cdot L^2 = L^2, \tag{1.5.3}$$

since the constant π is a pure number and the radius r is a length. Therefore, πr^2 has the dimension of area. Similarly, the dimension of the expression $2\pi r$ is

$$[2\pi r] = [2] \cdot [\pi] \cdot [r] = 1 \cdot 1 \cdot L = L, \tag{1.5.4}$$

since the constants 2 and π are both dimensionless and the radius r is a length. We see that $2\pi r$ has the dimension of length, which means it cannot possibly be an area.

We rule out $2\pi r$ because it is not dimensionally consistent with being an area. We see that πr^2 is dimensionally consistent with being an area, so if we have to choose between these two expressions, πr^2 is the one to choose.

Significance

This may seem like kind of a silly example, but the ideas are very general. As long as we know the dimensions of the individual physical quantities that appear in an equation, we can check to see whether the equation is dimensionally consistent. On the other hand, knowing that true equations are dimensionally consistent, we can match expressions from our imperfect memories to the quantities for which they might be expressions. Doing this will not help us remember dimensionless factors that appear in the equations (for example, if you had accidentally conflated the two expressions from the example into $2\pi r^2$, then dimensional analysis is no help), but it does help us remember the correct basic form of equations.

Exercise 1.5.1

Suppose we want the formula for the volume of a sphere. The two expressions commonly mentioned in elementary discussions of spheres are $4\pi r^2$ and $\frac{4}{3}\pi r^3$. One is the volume of a sphere of radius r and the other is its surface area. Which one is the volume?

Answer

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Example 1.5.2: Checking Equations for Dimensional Consistency

Consider the physical quantities s, v, a, and t with dimensions [s] = L, [v] = LT^{-1}, [a] = LT^{-2}, and [t] = T. Determine whether each of the following equations is dimensionally consistent:

a. s = vt + 0.5at^2;
b. s = vt^2 + 0.5at; and
c. v = sin $\left(\frac{at^2}{s}\right)$.

Strategy

By the definition of dimensional consistency, we need to check that each term in a given equation has the same dimensions as the other terms in that equation and that the arguments of any standard mathematical functions are dimensionless.

Solution

a. There are no trigonometric, logarithmic, or exponential functions to worry about in this equation, so we need only look at the dimensions of each term appearing in the equation. There are three terms, one in the left expression and two in the expression on the right, so we look at each in turn:

$$[s] = L \tag{1.5.5}$$

$$[vt] = [v]\cdot[t] = LT^{-1}\cdot T = LT^0 = L \tag{1.5.6}$$

$$[0.5at^2] = [a]\cdot[t]^2 = LT^{-2}\cdot T^2 = LT^0 = L. \tag{1.5.7}$$

b. Again, there are no trigonometric, exponential, or logarithmic functions, so we only need to look at the dimensions of each of the three terms appearing in the equation:

$$[s] = L \tag{1.5.8}$$

$$[vt^2] = [v]\cdot[t]^2 = LT^{-1}\cdot T^2 = LT \tag{1.5.9}$$

$$[at] = [a]\cdot[t] = LT^{-2}\cdot T = LT^{-1}. \tag{1.5.10}$$

None of the three terms has the same dimension as any other, so this is about as far from being dimensionally consistent as you can get. The technical term for an equation like this is **nonsense**.

c. This equation has a trigonometric function in it, so first we should check that the argument of the sine function is dimensionless:

$$\left[\frac{at^2}{s}\right] = \frac{[a]\cdot[t]^2}{[s]} = \frac{LT^{-2}\cdot T^2}{L} = \frac{L}{L} = 1. \tag{1.5.11}$$

The argument is dimensionless. So far, so good. Now we need to check the dimensions of each of the two terms (that is, the left expression and the right expression) in the equation:

$$[v] = LT^{-1} \tag{1.5.12}$$

$$\left[sin\left(\frac{at^2}{s}\right)\right] = 1. \tag{1.5.13}$$

The two terms have different dimensions—meaning, the equation is not dimensionally consistent. This equation is another example of "nonsense."

Significance

If we are trusting people, these types of dimensional checks might seem unnecessary. But, rest assured, any textbook on a quantitative subject such as physics (including this one) almost certainly contains some equations with typos. Checking equations routinely by dimensional analysis save us the embarrassment of using an incorrect equation. Also, checking the dimensions of an equation we obtain through algebraic manipulation is a great way to make sure we did not make a mistake (or to spot a mistake, if we made one).

? Exercise 1.5.2

Is the equation v = at dimensionally consistent?

Answer

Add texts here. Do not delete this text first.

One further point that needs to be mentioned is the effect of the operations of calculus on dimensions. We have seen that dimensions obey the rules of algebra, just like units, but what happens when we take the derivative of one physical quantity with respect to another or integrate a physical quantity over another? The derivative of a function is just the slope of the line tangent to its graph and slopes are ratios, so for physical quantities v and t, we have that the dimension of the derivative of v with respect to t is just the ratio of the dimension of v over that of t:

$$\left[\frac{dv}{dt}\right] = \frac{[v]}{[t]}. \tag{1.5.14}$$

Similarly, since integrals are just sums of products, the dimension of the integral of v with respect to t is simply the dimension of v times the dimension of t:

$$\left[\int vdt\right] = [v]\cdot[t]. \tag{1.5.15}$$

By the same reasoning, analogous rules hold for the units of physical quantities derived from other quantities by integration or differentiation.

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1.6: Significant Figures

Learning Objectives

- Determine the correct number of significant figures for the result of a computation.
- Describe the relationship between the concepts of accuracy, precision, uncertainty, and discrepancy.
- Calculate the percent uncertainty of a measurement, given its value and its uncertainty.
- Determine the uncertainty of the result of a computation involving quantities with given uncertainties.

Figure 1.6.1 shows two instruments used to measure the mass of an object. The digital scale has mostly replaced the double-pan balance in physics labs because it gives more accurate and precise measurements. But what exactly do we mean by **accurate and precise**? Aren't they the same thing? In this section we examine in detail the process of making and reporting a measurement.

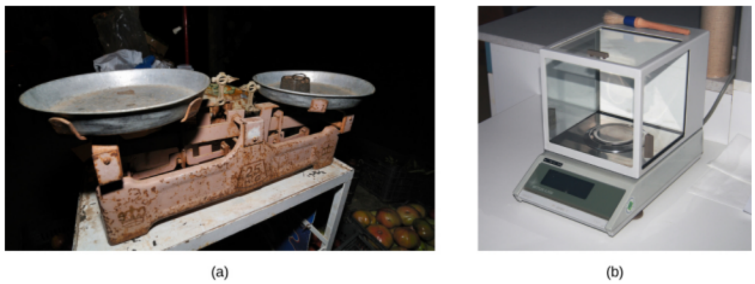

Figure 1.6.1: (a) A double-pan mechanical balance is used to compare different masses. Usually an object with unknown mass is placed in one pan and objects of known mass are placed in the other pan. When the bar that connects the two pans is horizontal, then the masses in both pans are equal. The "known masses" are typically metal cylinders of standard mass such as 1 g, 10 g, and 100 g. (b) Many mechanical balances, such as double-pan balances, have been replaced by digital scales, which can typically measure the mass of an object more precisely. A mechanical balance may read only the mass of an object to the nearest tenth of a gram, but many digital scales can measure the mass of an object up to the nearest thousandth of a gram. (credit a: modification of work by Serge Melki; credit b: modification of work by Karel Jakubec)

Accuracy and Precision of a Measurement

Science is based on observation and experiment—that is, on measurements. **Accuracy** is how close a measurement is to the accepted reference value for that measurement. For example, let's say we want to measure the length of standard printer paper. The packaging in which we purchased the paper states that it is 11.0 in. long. We then measure the length of the paper three times and obtain the following measurements: 11.1 in., 11.2 in., and 10.9 in. These measurements are quite accurate because they are very close to the reference value of 11.0 in. In contrast, if we had obtained a measurement of 12 in., our measurement would not be very accurate. Notice that the concept of accuracy requires that an accepted reference value be given.

The **precision** of measurements refers to how close the agreement is between repeated independent measurements (which are repeated under the same conditions). Consider the example of the paper measurements. The precision of the measurements refers to the spread of the measured values. One way to analyze the precision of the measurements is to determine the range, or difference, between the lowest and the highest measured values. In this case, the lowest value was 10.9 in. and the highest value was 11.2 in. Thus, the measured values deviated from each other by, at most, 0.3 in. These measurements were relatively precise because they did not vary too much in value. However, if the measured values had been 10.9 in., 11.1 in., and 11.9 in., then the measurements would not be very precise because there would be significant variation from one measurement to another. Notice that the concept of precision depends only on the actual measurements acquired and does not depend on an accepted reference value.

The measurements in the paper example are both accurate and precise, but in some cases, measurements are accurate but not precise, or they are precise but not accurate. Let's consider an example of a GPS attempting to locate the position of a restaurant in a city. Think of the restaurant location as existing at the center of a bull's-eye target and think of each GPS attempt to locate the restaurant as a black dot. In Figure $1.6.1a$, we see the GPS measurements are spread out far apart from each other, but they are all relatively close to the actual location of the restaurant at the center of the target. This indicates a low-precision, high-accuracy measuring system. However, in Figure $1.6.1b$, the GPS measurements are concentrated quite closely to one another, but they are far away from the target location. This indicates a high-precision, low-accuracy measuring system.

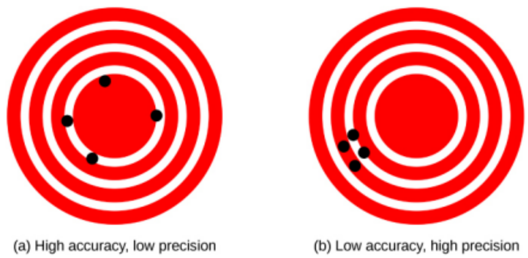

Figure $1.6.2$: A GPS attempts to locate a restaurant at the center of the bull's-eye. The black dots represent each attempt to pinpoint the location of the restaurant. (a) The dots are spread out quite far apart from one another, indicating low precision, but they are each rather close to the actual location of the restaurant, indicating high accuracy. (b) The dots are concentrated rather closely to one another, indicating high precision, but they are rather far away from the actual location of the restaurant, indicating low accuracy. (credit a and credit b: modification of works by Dark Evil)

Accuracy, Precision, Uncertainty, and Discrepancy

The precision of a measuring system is related to the **uncertainty** in the measurements whereas the accuracy is related to the **discrepancy** from the accepted reference value. Uncertainty is a quantitative measure of how much your measured values deviate from one another. There are many different methods of calculating uncertainty, each of which is appropriate to different situations. Some examples include taking the range (that is, the biggest less the smallest) or finding the standard deviation of the measurements. Discrepancy (or "measurement error") is the difference between the measured value and a given standard or expected value. If the measurements are not very precise, then the uncertainty of the values is high. If the measurements are not very accurate, then the discrepancy of the values is high.

Recall our example of measuring paper length; we obtained measurements of 11.1 in., 11.2 in., and 10.9 in., and the accepted value was 11.0 in. We might average the three measurements to say our best guess is 11.1 in.; in this case, our discrepancy is 11.1 – 11.0 = 0.1 in., which provides a quantitative measure of accuracy. We might calculate the uncertainty in our best guess by using the range of our measured values: 0.3 in. Then we would say the length of the paper is 11.1 in. plus or minus 0.3 in. The uncertainty in a measurement, A, is often denoted as δA (read "delta A"), so the measurement result would be recorded as A ± δA. Returning to our paper example, the measured length of the paper could be expressed as 11.1 ± 0.3 in. Since the discrepancy of 0.1 in. is less than the uncertainty of 0.3 in., we might say the measured value agrees with the accepted reference value to within experimental uncertainty.

Some factors that contribute to uncertainty in a measurement include the following:

- Limitations of the measuring device
- The skill of the person taking the measurement
- Irregularities in the object being measured
- Any other factors that affect the outcome (highly dependent on the situation)

In our example, such factors contributing to the uncertainty could be the smallest division on the ruler is 1/16 in., the person using the ruler has bad eyesight, the ruler is worn down on one end, or one side of the paper is slightly longer than the other. At any rate, the uncertainty in a measurement must be calculated to quantify its precision. If a reference value is known, it makes sense to calculate the discrepancy as well to quantify its accuracy.

Percent uncertainty

Another method of expressing uncertainty is as a percent of the measured value. If a measurement A is expressed with uncertainty δA, the percent uncertainty is defined as

$$Percent\ uncertainty = \frac{\delta A}{A} \times 100\% \tag{1.6.1}$$

Example 1.6.1: Calculating Percent Uncertainty: A Bag of Apples

A grocery store sells 5-lb bags of apples. Let's say we purchase four bags during the course of a month and weigh the bags each time. We obtain the following measurements:

- Week 1 weight: 4.8 lb
- Week 2 weight: 5.3 lb
- Week 3 weight: 4.9 lb
- Week 4 weight: 5.4 lb

We then determine the average weight of the 5-lb bag of apples is 5.1 ± 0.3 lb. What is the percent uncertainty of the bag's weight?

Strategy

First, observe that the average value of the bag's weight, A, is 5.1 lb. The uncertainty in this value, δA, is 0.3 lb. We can use the following equation to determine the percent uncertainty of the weight:

$$Percent\ uncertainty = \frac{\delta A}{A} \times 100\% \tag{1.6.2}$$

Solution

Substitute the values into the equation:

$$Percent\ uncertainty = \frac{\delta A}{A} \times 100\% = \frac{0.3\ lb}{5.1\ lb} \times 100\% = 5.9\% \approx 6\% \tag{1.6.3}$$

Significance

We can conclude the average weight of a bag of apples from this store is 5.1 lb ± 6%. Notice the percent uncertainty is dimensionless because the units of weight in δA = 0.3 lb canceled those inn A = 5.1 lb when we took the ratio.

Exercises 1.6.1

A high school track coach has just purchased a new stopwatch. The stopwatch manual states the stopwatch has an uncertainty of ±0.05 s. Runners on the track coach's team regularly clock 100-m sprints of 11.49 s to 15.01 s. At the school's last track meet, the first-place sprinter came in at 12.04 s and the second-place sprinter came in at 12.07 s. Will the coach's new stopwatch be helpful in timing the sprint team? Why or why not?

Uncertainties in Calculations

Uncertainty exists in anything calculated from measured quantities. For example, the area of a floor calculated from measurements of its length and width has an uncertainty because the length and width have uncertainties. How big is the uncertainty in something you calculate by multiplication or division? If the measurements going into the calculation have small uncertainties (a few percent or less), then the **method of adding percents** can be used for multiplication or division. This method states **the percent uncertainty in a quantity calculated by multiplication or division is the sum of the percent uncertainties in the items used to make the calculation**. For example, if a floor has a length of 4.00 m and a width of 3.00 m, with uncertainties of 2% and 1%, respectively, then the area of the floor is 12.0 m^2 and has an uncertainty of 3%. (Expressed as an area, this is 0.36 m^2 [12.0 m^2 x 0.03], which we round to 0.4 m^2 since the area of the floor is given to a tenth of a square meter.)

Precision of Measuring Tools and Significant Figures

An important factor in the precision of measurements involves the precision of the measuring tool. In general, a precise measuring tool is one that can measure values in very small increments. For example, a standard ruler can measure length to the nearest millimeter whereas a caliper can measure length to the nearest 0.01 mm. The caliper is a more precise measuring tool because it can measure extremely small differences in length. The more precise the measuring tool, the more precise the measurements.

When we express measured values, we can only list as many digits as we measured initially with our measuring tool. For example, if we use a standard ruler to measure the length of a stick, we may measure it to be 36.7 cm. We can't express this value as 36.71 cm because our measuring tool is not precise enough to measure a hundredth of a centimeter. It should be noted that the last digit in a measured value has been estimated in some way by the person performing the measurement. For example, the person measuring the length of a stick with a ruler notices the stick length seems to be somewhere in between 36.6 cm and 36.7 cm, and he or she must estimate the value of the last digit. Using the method of **significant figures**, the rule is that **the last digit written down in a measurement is the first digit with some uncertainty**. To determine the number of significant digits in a value, start with the first measured value at the left and count the number of digits through the last digit written on the right. For example, the measured value 36.7 cm has three digits, or three significant figures. Significant figures indicate the precision of the measuring tool used to measure a value.

Zeros

Special consideration is given to zeros when counting significant figures. The zeros in 0.053 are not significant because they are placeholders that locate the decimal point. There are two significant figures in 0.053. The zeros in 10.053 are not placeholders; they are significant. This number has five significant figures. The zeros in 1300 may or may not be significant, depending on the style of writing numbers. They could mean the number is known to the last digit or they could be placeholders. So 1300 could have two, three, or four significant figures. To avoid this ambiguity, we should write 1300 in scientific notation as 1.3 x 10^3, 1.30 x 10^3, or 1.300 x 10^3, depending on whether it has two, three, or four significant figures. **Zeros are significant except when they serve only as placeholders**.

Significant Figures in Calculations

When combining measurements with different degrees of precision, **the number of significant digits in the final answer can be no greater than the number of significant digits in the least-precise measured value**. There are two different rules, one for multiplication and division and the other for addition and subtraction.

1. **For multiplication and division, the result should have the same number of significant figures as the quantity with the least number of significant figures entering into the calculation.** For example, the area of a circle can be calculated from its radius using A = πr^2. Let's see how many significant figures the area has if the radius has only two—say, r = 1.2 m. Using a calculator with an eight-digit output, we would calculate
$$A=\pi r^2=(3.1415927...)\times(1.2\ m)^2=4.5238934\ m^2. \tag{1.6.4}$$
But because the radius has only two significant figures, it limits the calculated quantity to two significant figures, or
$$A=4.5\ m^2. \tag{1.6.5}$$
although π is good to at least eight digits.
2. **For addition and subtraction, the answer can contain no more decimal places than the least-precise measurement**. Suppose we buy 7.56 kg of potatoes in a grocery store as measured with a scale with precision 0.01 kg, then we drop off 6.052 kg of potatoes at your laboratory as measured by a scale with precision 0.001 kg. Then, we go home and add 13.7 kg of potatoes as measured by a bathroom scale with precision 0.1 kg. How many kilograms of potatoes do we now have and how many significant figures are appropriate in the answer? The mass is found by simple addition and subtraction:
$$\begin{array}{r} 7.56\ kg \\ -6.052\ kg \\ \underline{+13.7\ kg} \\ 15.208\ kg = 15.2\ kg. \end{array}$$
Next, we identify the least-precise measurement: 13.7 kg. This measurement is expressed to the 0.1 decimal place, so our final answer must also be expressed to the 0.1 decimal place. Thus, the answer is rounded to the tenths place, giving us 15.2 kg.

Significant Figures in This Text

In this text, most numbers are assumed to have three significant figures. Furthermore, consistent numbers of significant figures are used in all worked examples. An answer given to three digits is based on input good to at least three digits, for example. If the input has fewer significant figures, the answer will also have fewer significant figures. Care is also taken that the number of significant figures is reasonable for the situation posed. In some topics, particularly in optics, more accurate numbers are needed and we use more than three significant figures. Finally, if a number is exact, such as the two in the formula for the circumference of a circle, C = $2\pi r$, it does not affect the number of significant figures in a calculation. Likewise, conversion factors such as 100 cm/1 m are considered exact and do not affect the number of significant figures in a calculation.

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1.7: Summary

Key Terms

accuracy	the degree to which a measured value agrees with an accepted reference value for that measurement
base quantity	physical quantity chosen by convention and practical considerations such that all other physical quantities can be expressed as algebraic combinations of them
base unit	standard for expressing the measurement of a base quantity within a particular system of units; defined by a particular procedure used to measure the corresponding base quantity
conversion factor	a ratio that expresses how many of one unit are equal to another unit
derived quantity	physical quantity defined using algebraic combinations of base quantities
derived units	units that can be calculated using algebraic combinations of the fundamental units
dimension	expression of the dependence of a physical quantity on the base quantities as a product of powers of symbols representing the base quantities; in general, the dimension of a quantity has the form $L^a M^b T^c I^d \Theta^e N^f J^g$ for some powers a, b, c, d, e, f, and g
dimensionally consistent	equation in which every term has the same dimensions and the arguments of any mathematical functions appearing in the equation are dimensionless
dimensionless	quantity with a dimension of $L^0 M^0 T^0 I^0 \Theta^e N^0 J^0 = 1$; also called quantity of dimension 1 or a pure number
discrepancy	the difference between the measured value and a given standard or expected value
English units	system of measurement used in the United States; includes units of measure such as feet, gallons, and pounds
estimation	using prior experience and sound physical reasoning to arrive at a rough idea of a quantity’s value; sometimes called an “order-of-magnitude approximation,” a “guesstimate,” a “back-of-the-envelope calculation”, or a “Fermi calculation”
kilogram	SI unit for mass, abbreviated kg
law	description, using concise language or a mathematical formula, of a generalized pattern in nature supported by scientific evidence and repeated experiments
meter	SI unit for length, abbreviated m
method of adding percents	the percent uncertainty in a quantity calculated by multiplication or division is the sum of the percent uncertainties in the items used to make the calculation
metric system	system in which values can be calculated in factors of 10
model	representation of something often too difficult (or impossible) to display directly

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order of magnitude	the size of a quantity as it relates to a power of 10
percent uncertainty	the ratio of the uncertainty of a measurement to the measured value, expressed as a percentage
physical quantity	characteristic or property of an object that can be measured or calculated from other measurements
physics	science concerned with describing the interactions of energy, matter, space, and time; especially interested in what fundamental mechanisms underlie every phenomenon
precision	the degree to which repeated measurements agree with each other
second	the SI unit for time, abbreviated s
SI units	the international system of units that scientists in most countries have agreed to use; includes units such as meters, liters, and grams
significant figures	used to express the precision of a measuring tool used to measure a value
theory	testable explanation for patterns in nature supported by scientific evidence and verified multiple times by various groups of researchers
uncertainty	a quantitative measure of how much measured values deviate from one another
units	standards used for expressing and comparing measurements

Key Equations

Percent uncertainty	$$Percent\ uncertainty = \frac{\delta A}{A} \times 100\% \tag{1.7.1}$$

Summary

1.1 The Scope and Scale of Physics

- Physics is about trying to find the simple laws that describe all natural phenomena.
- Physics operates on a vast range of scales of length, mass, and time. Scientists use the concept of the order of magnitude of a number to track which phenomena occur on which scales. They also use orders of magnitude to compare the various scales.
- Scientists attempt to describe the world by formulating models, theories, and laws

1.2 Units and Standards

- Systems of units are built up from a small number of base units, which are defined by accurate and precise measurements of conventionally chosen base quantities. Other units are then derived as algebraic combinations of the base units.
- Two commonly used systems of units are English units and SI units. All scientists and most of the other people in the world use SI, whereas nonscientists in the United States still tend to use English units.
- The SI base units of length, mass, and time are the meter (m), kilogram (kg), and second (s), respectively.
- SI units are a metric system of units, meaning values can be calculated by factors of 10. Metric prefixes may be used with metric units to scale the base units to sizes appropriate for almost any application.

1.3 Unit Conversion

- To convert a quantity from one unit to another, multiply by conversion factors in such a way that you cancel the units you want to get rid of and introduce the units you want to end up with.
- Be careful with areas and volumes. Units obey the rules of algebra so, for example, if a unit is squared we need two factors to cancel it.

1.4 Dimensional Analysis

- The dimension of a physical quantity is just an expression of the base quantities from which it is derived.
- All equations expressing physical laws or principles must be dimensionally consistent. This fact can be used as an aid in remembering physical laws, as a way to check whether claimed relationships between physical quantities are possible, and even to derive new physical laws.

1.5 Estimates and Fermi Calculations

- An estimate is a rough educated guess at the value of a physical quantity based on prior experience and sound physical reasoning. Some strategies that may help when making an estimate are as follows:
  - Get big lengths from smaller lengths.
  - Get areas and volumes from lengths.
  - Get masses from volumes and densities.
  - If all else fails, bound it. One "sig. fig." is fine.
  - Ask yourself: Does this make any sense?

1.6 Significant Figures

- Accuracy of a measured value refers to how close a measurement is to an accepted reference value. The discrepancy in a measurement is the amount by which the measurement result differs from this value.
- Precision of measured values refers to how close the agreement is between repeated measurements. The uncertainty of a measurement is a quantification of this.
- The precision of a measuring tool is related to the size of its measurement increments. The smaller the measurement increment, the more precise the tool.
- Significant figures express the precision of a measuring tool.
- When multiplying or dividing measured values, the final answer can contain only as many significant figures as the least-precise value.
- When adding or subtracting measured values, the final answer cannot contain more decimal places than the least-precise value.

1.7 Solving Problems in Physics

The three stages of the process for solving physics problems used in this textmap are as follows:

- **Strategy**: Determine which physical principles are involved and develop a strategy for using them to solve the problem.
- **Solution**: Do the math necessary to obtain a numerical solution complete with units.
- **Significance**: Check the solution to make sure it makes sense (correct units, reasonable magnitude and sign) and assess its significance.

Contributors and Attributions

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1.8: Exercises

Conceptual Questions

1.1 The Scope and Scale of Physics

1. What is physics?
2. Some have described physics as a "search for simplicity." Explain why this might be an appropriate description.
3. If two different theories describe experimental observations equally well, can one be said to be more valid than the other (assuming both use accepted rules of logic)?
4. What determines the validity of a theory?
5. Certain criteria must be satisfied if a measurement or observation is to be believed. Will the criteria necessarily be as strict for an expected result as for an unexpected result?
6. Can the validity of a model be limited or must it be universally valid? How does this compare with the required validity of a theory or a law?

1.2 Units and Standards

7. Identify some advantages of metric units.
8. What are the SI base units of length, mass, and time?
9. What is the difference between a base unit and a derived unit? (b) What is the difference between a base quantity and a derived quantity? (c) What is the difference between a base quantity and a base unit?
10. For each of the following scenarios, refer to Figure 1.4 and Table 1.2 to determine which metric prefix on the meter is most appropriate for each of the following scenarios. (a) You want to tabulate the mean distance from the Sun for each planet in the solar system. (b) You want to compare the sizes of some common viruses to design a mechanical filter capable of blocking the pathogenic ones. (c) You want to list the diameters of all the elements on the periodic table. (d) You want to list the distances to all the stars that have now received any radio broadcasts sent from Earth 10 years ago.

1.6 Significant Figures

11. (a) What is the relationship between the precision and the uncertainty of a measurement? (b) What is the relationship between the accuracy and the discrepancy of a measurement?

1.7 Solving Problems in Physics

12. What information do you need to choose which equation or equations to use to solve a problem?
13. What should you do after obtaining a numerical answer when solving a problem?

Problems

1.1 The Scope and Scale of Physics

14. Find the order of magnitude of the following physical quantities.
    a. The mass of Earth's atmosphere: 5.1×10^{18} kg;
    b. The mass of the Moon's atmosphere: 25,000 kg;
    c. The mass of Earth's hydrosphere: 1.4×10^{21} kg;
    d. The mass of Earth: 5.97×10^{24} kg;
    e. The mass of the Moon: 7.34×10^{22} kg;
    f. The Earth–Moon distance (semi-major axis): 3.84×10^{8} m;
    g. The mean Earth–Sun distance: 1.5×10^{11} m;
    h. The equatorial radius of Earth: 6.38×10^{6} m;
    i. The mass of an electron: 9.11×10^{-31} kg;
    j. The mass of a proton: 1.67×10^{-27} kg;
    k. The mass of the Sun: 1.99×10^{30} kg.
15. Use the orders of magnitude you found in the previous problem to answer the following questions to within an order of magnitude.
    a. How many electrons would it take to equal the mass of a proton?
    b. How many Earths would it take to equal the mass of the Sun?

c. How many Earth–Moon distances would it take to cover the distance from Earth to the Sun?
d. How many Moon atmospheres would it take to equal the mass of Earth's atmosphere?
e. How many moons would it take to equal the mass of Earth?
f. How many protons would it take to equal the mass of the Sun?

For the remaining questions, you need to use Figure 1.4 to obtain the necessary orders of magnitude of lengths, masses, and times.

16. Roughly how many heartbeats are there in a lifetime?
17. A generation is about one-third of a lifetime. Approximately how many generations have passed since the year 0 AD?
18. Roughly how many times longer than the mean life of an extremely unstable atomic nucleus is the lifetime of a human?
19. Calculate the approximate number of atoms in a bacterium. Assume the average mass of an atom in the bacterium is 10 times the mass of a proton.
20. (a) Calculate the number of cells in a hummingbird assuming the mass of an average cell is 10 times the mass of a bacterium. (b) Making the same assumption, how many cells are there in a human?
21. Assuming one nerve impulse must end before another can begin, what is the maximum firing rate of a nerve in impulses per second?
22. About how many floating-point operations can a supercomputer perform each year?
23. Roughly how many floating-point operations can a supercomputer perform in a human lifetime?

1.2 Units and Standards

24. The following times are given using metric prefixes on the base SI unit of time: the second. Rewrite them in scientific notation without the prefix. For example, 47 Ts would be rewritten as 4.7×10^{13} s.
    a. 980 Ps;
    b. 980 fs;
    c. 17 ns;
    d. 577 μs.
25. The following times are given in seconds. Use metric prefixes to rewrite them so the numerical value is greater than one but less than 1000. For example, 7.9×10^{-2} s could be written as either 7.9 cs or 79 ms.
    a. 9.57×10^{5} s;
    b. 0.045 s;
    c. 5.5×10^{-7} s;
    d. 3.16×10^{7} s.
26. The following lengths are given using metric prefixes on the base SI unit of length: the meter. Rewrite them in scientific notation without the prefix. For example, 4.2 Pm would be rewritten as 4.2×10^{15} m.
    a. 89 Tm;
    b. 89 pm;
    c. 711 mm;
    d. 0.45 μm.
27. The following lengths are given in meters. Use metric prefixes to rewrite them so the numerical value is bigger than one but less than 1000. For example, 7.9×10^{-2} m could be written either as 7.9 cm or 79 mm.
    a. 7.59×10^{7} m;
    b. 0.0074 m;
    c. 8.8×10^{-11} m;
    d. 1.63×10^{13} m.
28. The following masses are written using metric prefixes on the gram. Rewrite them in scientific notation in terms of the SI base unit of mass: the kilogram. For example, 40 Mg would be written as 4×10^{4} kg.
    a. 23 mg;
    b. 320 Tg;
    c. 42 ng;
    d. 7 g;
    e. 9 Pg.

29. The following masses are given in kilograms. Use metric prefixes on the gram to rewrite them so the numerical value is bigger than one but less than 1000. For example, 7 × 10−4 kg could be written as 70 cg or 700 mg.
    a. 3.8 × 10−5 kg;
    b. 2.3 × 1017 kg;
    c. 2.4 × 10−11 kg;
    d. 8 × 1015 kg;
    e. 4.2 × 10−3 kg.

1.3 Unit Conversion

30. The volume of Earth is on the order of 10^{21} m^3. (a) What is this in cubic kilometers (km^3)? (b) What is it in cubic miles (mi^3)? (c) What is it in cubic centimeters (cm^3)?
31. The speed limit on some interstate highways is roughly 100 km/h. (a) What is this in meters per second? (b) How many miles per hour is this?
32. A car is traveling at a speed of 33 m/s. (a) What is its speed in kilometers per hour? (b) Is it exceeding the 90 km/ h speed limit?
33. In SI units, speeds are measured in meters per second (m/s). But, depending on where you live, you're probably more comfortable of thinking of speeds in terms of either kilometers per hour (km/h) or miles per hour (mi/h). In this problem, you will see that 1 m/s is roughly 4 km/h or 2 mi/h, which is handy to use when developing your physical intuition. More precisely, show that (a) 1.0 m/s = 3.6 km/h and (b) 1.0 m/s = 2.2 mi/h.
34. American football is played on a 100-yd-long field, excluding the end zones. How long is the field in meters? (Assume that 1 m = 3.281 ft.)
35. Soccer fields vary in size. A large soccer field is 115 m long and 85.0 m wide. What is its area in square feet? (Assume that 1 m = 3.281 ft.)
36. What is the height in meters of a person who is 6 ft 1.0 in. tall?
37. Mount Everest, at 29,028 ft, is the tallest mountain on Earth. What is its height in kilometers? (Assume that 1 m = 3.281 ft.)
38. The speed of sound is measured to be 342 m/s on a certain day. What is this measurement in kilometers per hour?
39. Tectonic plates are large segments of Earth's crust that move slowly. Suppose one such plate has an average speed of 4.0 cm/yr. (a) What distance does it move in 1.0 s at this speed? (b) What is its speed in kilometers per million years?
40. The average distance between Earth and the Sun is 1.5 × 10^{11} m. (a) Calculate the average speed of Earth in its orbit (assumed to be circular) in meters per second. (b) What is this speed in miles per hour?
41. The density of nuclear matter is about 10^{18} kg/m^3. Given that 1 mL is equal in volume to cm^3, what is the density of nuclear matter in megagrams per microliter (that is, Mg/μL)?
42. The density of aluminum is 2.7 g/cm^3. What is the density in kilograms per cubic meter?
43. A commonly used unit of mass in the English system is the pound-mass, abbreviated lbm, where 1 lbm = 0.454 kg. What is the density of water in pound-mass per cubic foot?
44. A furlong is 220 yd. A fortnight is 2 weeks. Convert a speed of one furlong per fortnight to millimeters per second.
45. It takes 2π radians (rad) to get around a circle, which is the same as 360°. How many radians are in 1°?
46. Light travels a distance of about 3 × 10^8 m/s. A light-minute is the distance light travels in 1 min. If the Sun is 1.5 × 10^{11} m from Earth, how far away is it in lightminutes?
47. A light-nanosecond is the distance light travels in 1 ns. Convert 1 ft to light-nanoseconds.
48. An electron has a mass of 9.11 × 10^{-31} kg. A proton has a mass of 1.67 × 10^{-27} kg. What is the mass of a proton in electron-masses?
49. A fluid ounce is about 30 mL. What is the volume of a 12 fl-oz can of soda pop in cubic meters?

1.4 Dimensional Analysis

50. A student is trying to remember some formulas from geometry. In what follows, assume A is area, V is volume, and all other variables are lengths. Determine which formulas are dimensionally consistent. (a) V = $\pi r^2 h$; (b) A = $2\pi r^2 + 2\pi r h$; (c) V = 0.5bh; (d) V = πd^2 ; (e) V = $\frac{\pi d^3}{6}$
51. Consider the physical quantities s, v, a, and t with dimensions [s] = L, [v] = LT^{-1}, [a] = LT^{-2}, and [t] = T. Determine whether each of the following equations is dimensionally consistent. (a) v^2 = 2as; (b) s = vt^2 + 0.5at^2; (c) v = s/t; (d) a = v/t.
52. Consider the physical quantities m, s, v, a, and t with dimensions [m] = M, [s] = L, [v] = LT^{-1}, [a] = LT^{-2}, and [t] = T. Assuming each of the following equations is dimensionally consistent, find the dimension of the quantity on the left-hand side of the equation: (a) F = ma; (b) K = 0.5mv^2 ; (c) p = mv; (d) W = mas; (e) L = mvr.

53. Suppose quantity s is a length and quantity t is a time. Suppose the quantities v and a are defined by v = ds/dt and a = dv/dt. (a) What is the dimension of v? (b) What is the dimension of the quantity a? What are the dimensions of (c) $\int$vdt, (d) $\int$adt, and (e) da/dt?
54. Suppose [V] = L3 , [ρ] = ML^{-3}, and [t] = T. (a) What is the dimension of $\int \rho dV$? (b) What is the dimension of dV/dt? (c) What is the dimension of ρ(dV/dt)?
55. The arc length formula says the length s of arc subtended by angle Θ in a circle of radius r is given by the equation s = rΘ. What are the dimensions of (a) s, (b) r, and (c) Θ?

1.5 Estimates and Fermi Calculations

56. Assuming the human body is made primarily of water, estimate the volume of a person.
57. Assuming the human body is primarily made of water, estimate the number of molecules in it. (Note that water has a molecular mass of 18 g/mol and there are roughly 10^{24} atoms in a mole.)
58. Estimate the mass of air in a classroom.
59. Estimate the number of molecules that make up Earth, assuming an average molecular mass of 30 g/mol. (Note there are on the order of 10^{24} objects per mole.)
60. Estimate the surface area of a person.
61. Roughly how many solar systems would it take to tile the disk of the Milky Way?
62. (a) Estimate the density of the Moon. (b) Estimate the diameter of the Moon. (c) Given that the Moon subtends at an angle of about half a degree in the sky, estimate its distance from Earth.
63. The average density of the Sun is on the order 10^3 kg/ m^3. (a) Estimate the diameter of the Sun. (b) Given that the Sun subtends at an angle of about half a degree in the sky, estimate its distance from Earth. 64. Estimate the mass of a virus.
64. A floating-point operation is a single arithmetic operation such as addition, subtraction, multiplication, or division. (a) Estimate the maximum number of floating-point operations a human being could possibly perform in a lifetime. (b) How long would it take a supercomputer to perform that many floating-point operations?

1.6 Significant Figures

66. Consider the equation 4000/400 = 10.0. Assuming the number of significant figures in the answer is correct, what can you say about the number of significant figures in 4000 and 400?
67. Suppose your bathroom scale reads your mass as 65 kg with a 3% uncertainty. What is the uncertainty in your mass (in kilograms)?
68. A good-quality measuring tape can be off by 0.50 cm over a distance of 20 m. What is its percent uncertainty?
69. An infant’s pulse rate is measured to be 130 ± 5 beats/ min. What is the percent uncertainty in this measurement?
70. (a) Suppose that a person has an average heart rate of 72.0 beats/min. How many beats does he or she have in 2.0 years? (b) In 2.00 years? (c) In 2.000 years?
71. A can contains 375 mL of soda. How much is left after 308 mL is removed?
72. State how many significant figures are proper in the results of the following calculations: (a) (106.7)(98.2) / (46.210)(1.01); (b) $(18.7)^2$; (c) $(1.60 \times 10^{-19})(3712)$
73. (a) How many significant figures are in the numbers 99 and 100.? (b) If the uncertainty in each number is 1, what is the percent uncertainty in each? (c) Which is a more meaningful way to express the accuracy of these two numbers: significant figures or percent uncertainties?
74. (a) If your speedometer has an uncertainty of 2.0 km/h at a speed of 90 km/h, what is the percent uncertainty? (b) If it has the same percent uncertainty when it reads 60 km/ h, what is the range of speeds you could be going?
75. (a) A person’s blood pressure is measured to be 120 ± 2 mm Hg. What is its percent uncertainty? (b) Assuming the same percent uncertainty, what is the uncertainty in a blood pressure measurement of 80 mm Hg?
76. A person measures his or her heart rate by counting the number of beats in 30 s. If 40 ± 1 beats are counted in 30.0 ± 0.5 s, what is the heart rate and its uncertainty in beats per minute?
77. What is the area of a circle 3.102 cm in diameter?
78. Determine the number of significant figures in the following measurements: (a) 0.0009, (b) 15,450.0, (c) 6×103 , (d) 87.990, and (e) 30.42.
79. Perform the following calculations and express your answer using the correct number of significant digits. (a) A woman has two bags weighing 13.5 lb and one bag with a weight of 10.2 lb. What is the total weight of the bags? (b) The force F on an object is equal to its mass m multiplied by its acceleration a. If a wagon with mass 55 kg accelerates at a rate of 0.0255 m/s^2, what is the force on the wagon? (The unit of force is called the newton and it is expressed with the symbol N.)

Additional Problems

80. Consider the equation y = mt +b, where the dimension of y is length and the dimension of t is time, and m and b are constants. What are the dimensions and SI units of (a) m and (b) b?
81. Consider the equation $s=s_0+v_0t+\frac{a_0t^2}{2}+\frac{j_0t^3}{6}+\frac{S_0t^4}{24}+\frac{ct^5}{120}$, where s is a length and t is a time. What are the dimensions and SI units of (a) s_0, (b) v_0, (c) a_0, (d) j_0, (e) S_0, and (f) c?
82. (a) A car speedometer has a 5% uncertainty. What is the range of possible speeds when it reads 90 km/h? (b) Convert this range to miles per hour. Note 1 km = 0.6214 mi.
83. A marathon runner completes a 42.188-km course in 2 h, 30 min, and 12 s. There is an uncertainty of 25 m in the distance traveled and an uncertainty of 1 s in the elapsed time. (a) Calculate the percent uncertainty in the distance. (b) Calculate the percent uncertainty in the elapsed time. (c) What is the average speed in meters per second? (d) What is the uncertainty in the average speed?
84. The sides of a small rectangular box are measured to be 1.80 ± 0.1 cm, 2.05 ± 0.02 cm, and 3.1 ± 0.1 cm long. Calculate its volume and uncertainty in cubic centimeters.
85. When nonmetric units were used in the United Kingdom, a unit of mass called the pound-mass (lbm) was used, where 1 lbm = 0.4539 kg. (a) If there is an uncertainty of 0.0001 kg in the pound-mass unit, what is its percent uncertainty? (b) Based on that percent uncertainty, what mass in pound-mass has an uncertainty of 1 kg when converted to kilograms?
86. The length and width of a rectangular room are measured to be 3.955 ± 0.005 m and 3.050 ± 0.005 m. Calculate the area of the room and its uncertainty in square meters.
87. A car engine moves a piston with a circular cross-section of 7.500 ± 0.002 cm in diameter a distance of 3.250 ± 0.001 cm to compress the gas in the cylinder. (a) By what amount is the gas decreased in volume in cubic centimeters? (b) Find the uncertainty in this volume.

Challenge Problems

88. The first atomic bomb was detonated on July 16, 1945, at the Trinity test site about 200 mi south of Los Alamos. In 1947, the U.S. government declassified a film reel of the explosion. From this film reel, British physicist G. I. Taylor was able to determine the rate at which the radius of the fireball from the blast grew. Using dimensional analysis, he was then able to deduce the amount of energy released in the explosion, which was a closely guarded secret at the time. Because of this, Taylor did not publish his results until 1950. This problem challenges you to recreate this famous calculation.
    a. Using keen physical insight developed from years of experience, Taylor decided the radius r of the fireball should depend only on time since the explosion, t, the density of the air, ρ, and the energy of the initial explosion, E. Thus, he made the educated guess that $r=kE^a\rho^bt^c$ for some dimensionless constant k and some unknown exponents a, b, and c. Given that [E] = ML^2T^{-2}, determine the values of the exponents necessary to make this equation dimensionally consistent. (Hint: Notice the equation implies that $k=rE^{-a}\rho^{-b}t^{-c}$ and that [k] = 1.)
    b. By analyzing data from high-energy conventional explosives, Taylor found the formula he derived seemed to be valid as long as the constant k had the value 1.03. From the film reel, he was able to determine many values of r and the corresponding values of t. For example, he found that after 25.0 ms, the fireball had a radius of 130.0 m. Use these values, along with an average air density of 1.25 kg/m^3, to calculate the initial energy release of the Trinity detonation in joules (J). (Hint: To get energy in joules, you need to make sure all the numbers you substitute in are expressed in terms of SI base units.) (c) The energy released in large explosions is often cited in units of “tons of TNT” (abbreviated “t TNT”), where 1 t TNT is about 4.2 GJ. Convert your answer to (b) into kilotons of TNT (that is, kt TNT). Compare your answer with the quick-and-dirty estimate of 10 kt TNT made by physicist Enrico Fermi shortly after witnessing the explosion from what was thought to be a safe distance. (Reportedly, Fermi made his estimate by dropping some shredded bits of paper right before the remnants of the shock wave hit him and looked to see how far they were carried by it.)
89. The purpose of this problem is to show the entire concept of dimensional consistency can be summarized by the old saying “You can’t add apples and oranges.” If you have studied power series expansions in a calculus course, you know the standard mathematical functions such as trigonometric functions, logarithms, and exponential functions can be expressed as infinite sums of the form $\sum_{n=0}^{\infty}a_nx^n=a_0+a_1x+a_2x^2+a^3x^3+\cdots$, where the a_n are dimensionless constants for all n = 0, 1, 2, ⋯ and x is the argument of the function. (If you have not studied power series in calculus yet, just trust us.) Use this fact to explain why the requirement that all terms in an equation have the same dimensions is sufficient as a definition of dimensional consistency. That is, it actually implies the arguments of standard mathematical functions must be dimensionless, so it is not

really necessary to make this latter condition a separate requirement of the definition of dimensional consistency as we have done in this section.

Contributors and Attributions

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1.9: Answers

Check your Understanding

1.1. 4.79×10^2 Mg

1.2. $3 \times 10^8 m/s$

1.3. $10^8 km^2$

1.4. The numbers were too small, by a factor of 4.45.

1.5. $4\pi r^3/3$

1.6. yes

1.7. $3 \times 10^4 m$ or 30 km. It is probably an underestimate because the density of the atmosphere decreases with altitude. (In fact, 30 km does not even get us out of the stratosphere.)

1.8. No, the coach's new stopwatch will not be helpful. The uncertainty in the stopwatch is too great to differentiate between the sprint times effectively.

Conceptual Questions

1. Physics is the science concerned with describing the interactions of energy, matter, space, and time to uncover the fundamental mechanisms that underlie every phenomenon.

3. No, neither of these two theories is more valid than the other. Experimentation is the ultimate decider. If experimental evidence does not suggest one theory over the other, then both are equally valid. A given physicist might prefer one theory over another on the grounds that one seems more simple, more natural, or more beautiful than the other, but that physicist would quickly acknowledge that he or she cannot say the other theory is invalid. Rather, he or she would be honest about the fact that more experimental evidence is needed to determine which theory is a better description of nature.

5. Probably not. As the saying goes, "Extraordinary claims require extraordinary evidence."

7. Conversions between units require factors of 10 only, which simplifies calculations. Also, the same basic units can be scaled up or down using metric prefixes to sizes appropriate for the problem at hand.

9. a. Base units are defined by a particular process of measuring a base quantity whereas derived units are defined as algebraic combinations of base units.

b. A base quantity is chosen by convention and practical considerations. Derived quantities are expressed as algebraic combinations of base quantities.

c. A base unit is a standard for expressing the measurement of a base quantity within a particular system of units. So, a measurement of a base quantity could be expressed in terms of a base unit in any system of units using the same base quantities. For example, length is a base quantity in both SI and the English system, but the meter is a base unit in the SI system only.

11. a. Uncertainty is a quantitative measure of precision. b. Discrepancy is a quantitative measure of accuracy.

13. Check to make sure it makes sense and assess its significance.

Problems

15. a. 10^3;

b. 10^5;

c. 10^2;

d. 10^{15};

e. 10^2;

f. 10^{57}

17. 10^2 generations

19. 10^{11} atoms

21. 10^3 nerve impulses/s

23. 10^{26} floating-point operations per human lifetime

25. a. 957 ks;

b. 4.5 cs or 45 ms;

c. 550 ns;

d. 31.6 Ms

27. a. 75.9 Mm;

b. 7.4 mm;

c. 88 pm;

d. 16.3 Tm

29. a. 3.8 cg or 38 mg;

b. 230 Eg;

c. 24 ng;

d. 8 Eg

e. 4.2 g

31. a. 27.8 m/s;

b. 62 mi/h

33. a. 3.6 km/h;

b. 2.2 mi/h

35. $1.05 \times 10^5 ft^2$

37. 8.847 km

39. a. $1.3 \times 10^{-9} m$;

b. 40 km/My

41. $10^6 Mg/\mu L$

43. 62.4 lbm/ft^3

45. 0.017 rad

47. 1 light-nanosecond

49. $3.6 \times 10^{-4} m^3$

51. a. Yes, both terms have dimension L^2T^{-2}

b. No.

c. Yes, both terms have dimension LT^{-1}

d. Yes, both terms have dimension LT^{-2}

53. a. $[v] = LT^{-1}$;

b. $[a] = LT^{-2}$;

c. $[\int v dt] = L$;

d. $[\int a dt] = LT^{-1}$;

e. $[\frac{da}{dt}] = LT^{-3}$

55. a. L;

b. L;

c. $L^0 = 1$ (that is, it is dimensionless)

57. 10^{28} atoms

59. 10^{51} molecules

61. 10^{16} solar systems

63. a. Volume = $10^{27}m^3$, diameter is 10^9 m.;

b. 10^{11} m

65. a. A reasonable estimate might be one operation per second for a total of $\boxed{10^\wedge}$ in a lifetime.;

b. about $(10^9)(10^{-17}s) = 10^{-8}s$, or about 10 ns

67. 2 kg

69. 4%

71. 67 mL

73. a. The number 99 has 2 significant figures; 100. has 3 significant figures.

b. 1.00%;

c. percent uncertainties

75. a. 2%;

b. 1 mm Hg

77. 7.557 cm^2

79. a. 37.2 lb; because the number of bags is an exact value, it is not considered in the significant figures;

b. 1.4 N; because the value 55 kg has only two significant figures, the final value must also contain two significant figures

Additional Problems

81. a. $[s_0] = L$ and units are meters (m);

b. $[v_0] = LT^{-1}$ and units are meters per second (m/s);

c. $[a_0] = LT^{-2}$ and units are meters per second squared (m/s^2);

d. $[j_0] = LT^{-3}$ and units are meters per second cubed (m/s^3);

e. $[S_0] = LT^{-4}$ and units are m/s^4;

f. $[c] = LT^{-5}$ and units are m/s^5.

83. a. 0.059%;

b. 0.01%;

c. 4.681 m/s;

d. 0.07%,

0.003 m/s

85. a. 0.02%;

b. 1×10^4 lbm

87. a. 143.6 cm^3;

b. 0.2 cm^3 or 0.14%

Challenge Problems

89. Since each term in the power series involves the argument raised to a different power, the only way that every term in the power series can have the same dimension is if the argument is dimensionless. To see this explicitly, suppose $[x] = L^a M^b T^c$. Then, $[x^n] = [x]^n = L^{an} M^{bn} T^{cn}$. If we want $[x] = [x^n]$, then an = a, bn = b, and cn = c for all n. The only way this can happen is if a = b = c = 0.

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CHAPTER OVERVIEW

2: One-Dimensional Kinematics

2.1: Prelude to One-Dimensional Kinematics
2.2: Displacement
2.3: Vectors, Scalars, and Coordinate Systems
2.4: Time, Velocity, and Speed
2.5: Acceleration
2.6: Motion Equations for Constant Acceleration in One Dimension
2.7: Problem-Solving Basics for One-Dimensional Kinematics
2.8: Falling Objects
2.9: Graphical Analysis of One-Dimensional Motion
2.E: Kinematics (Exercises)

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2.1: Prelude to One-Dimensional Kinematics

Objects are in motion everywhere we look. Everything from a tennis game to a space-probe flyby of the planet Neptune involves motion. When you are resting, your heart moves blood through your veins. And even in inanimate objects, there is continuous motion in the vibrations of atoms and molecules. Questions about motion are interesting in and of themselves: *How long will it take for a space probe to get to Mars? Where will a football land if it is thrown at a certain angle?* But an understanding of motion is also key to understanding other concepts in physics. An understanding of acceleration, for example, is crucial to the study of force.

Figure 2.1.1:The motion of an American kestrel through the air can be described by the bird's displacement, speed, velocity, and acceleration. When it flies in a straight line without any change in direction, its motion is said to be one dimensional. (credit: Vince Maidens, Wikimedia Commons)

Our formal study of physics begins with kinematics which is defined as the *study of motion without considering its causes*. The word "kinematics" comes from a Greek term meaning motion and is related to other English words such as "cinema" (movies) and "kinesiology" (the study of human motion). In one-dimensional kinematics and Two-Dimensional Kinematics we will study only the *motion* of a football, for example, without worrying about what forces cause or change its motion. Such considerations come in other chapters. In this chapter, we examine the simplest type of motion—namely, motion along a straight line, or one-dimensional motion. In Two-Dimensional Kinematics, we apply concepts developed here to study motion along curved paths (two- and three-dimensional motion); for example, that of a car rounding a curve.

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2.2: Displacement

Learning Objectives

- Define position, displacement, distance, and distance traveled.
- Explain the relationship between position and displacement.
- Distinguish between displacement and distance traveled.
- Calculate displacement and distance given initial position, final position, and the path between the two.

These cyclists in Vietnam can be described by their position relative to buildings and a canal. Their motion can be described by their change in position, or displacement, in the frame of reference.

Figure 2.2.1: These cyclists in Vietnam can be described by their position relative to buildings and a canal. Their motion can be described by their change in position, or displacement, in the frame of reference. (credit: Suzan Black, Fotopedia)

Position

In order to describe the motion of an object, you must first be able to describe its position—where it is at any particular time. More precisely, you need to specify its position relative to a convenient reference frame. Earth is often used as a reference frame, and we often describe the position of an object as it relates to stationary objects in that reference frame. For example, a rocket launch would be described in terms of the position of the rocket with respect to the Earth as a whole, while a professor's position could be described in terms of where she is in relation to the nearby white board. In other cases, we use reference frames that are not stationary but are in motion relative to the Earth. To describe the position of a person in an airplane, for example, we use the airplane, not the Earth, as the reference frame.

Displacement

If an object moves relative to a reference frame (for example, if a professor moves to the right relative to a white board or a passenger moves toward the rear of an airplane), then the object's position changes. This change in position is known as displacement. The word "displacement" implies that an object has moved, or has been displaced.

Definition: Displacement

Displacement is the change in position of an object:

$$\Delta x = x_f - x_0,$$

where Δx is displacement, x_f is the final position, and x_0 is the initial position.

In this text the upper case Greek letter size Δ(delta) always means "change in" whatever quantity follows it; thus, size Δx means change in position. Always solve for displacement by subtracting initial position size x_0 from final position x_f.

Note that the SI unit for displacement is the meter (m) (see Section on Physical Quantities and Units), but sometimes kilometers, miles, feet, and other units of length are used. Keep in mind that when units other than the meter are used in a problem, you may need to convert them into meters to complete the calculation.

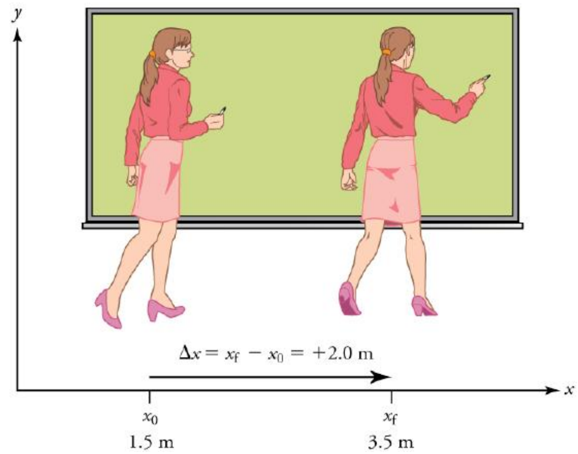


Figure $2.2.2$: A professor paces left and right while lecturing. Her position relative to Earth is given by x. The +2 m displacement of the professor relative to Earth is represented by an arrow pointing to the right.

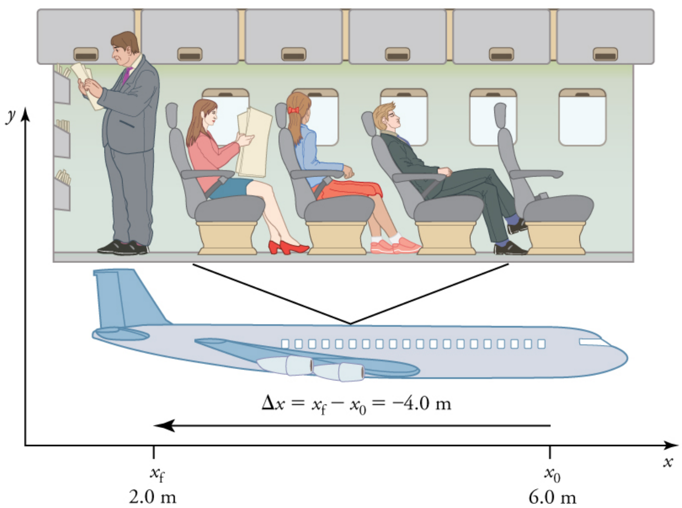


Figure $2.2.3$: A passenger moves from his seat to the back of the plane. His location relative to the airplane is given by x. The -4 m displacement of the passenger relative to the plane is represented by an arrow toward the rear of the plane. Notice that the arrow representing his displacement is twice as long as the arrow representing the displacement of the professor (he moves twice as far)

Note that displacement has a direction as well as a magnitude. The professor's displacement in Figure $2.2.2$ is 2.0 m to the right, and the airline passenger's displacement is 4.0 m toward the rear in Figure $2.2.3$. In one-dimensional motion, direction can be specified with a plus or minus sign. When you begin a problem, you should select which direction is positive (usually that will be to the right or up, but you are free to select positive as being any direction). The professor's initial position is size x_0 = 1.5 m and her final position is x_f = 3.5 m. Thus her displacement is

$$\begin{aligned} \Delta x &= x_f - x_0 \\ &= 3.5\,m - 1.5\,m \\ &= +2.0\,m. \end{aligned}$$

In this coordinate system, motion to the right is positive, whereas motion to the left is negative. Similarly, the airplane passenger's initial position is $x_0 = 6.0m$ and his final position is $x_f = 2.0m$, so his displacement is

$$\begin{aligned} \Delta x &= x_f - x_0 \\ &= 2.0\,m - 6.0\,m \\ &= -4.0\,m. \end{aligned}$$

His displacement is negative because his motion is toward the rear of the plane, or in the negative size 12{x} {} direction in our coordinate system.

Distance

Although displacement is described in terms of direction, distance is not. **Distance** is defined to be the *magnitude or size of displacement between two positions*. Note that the distance between two positions is not the same as the distance traveled between them. **Distance traveled** is *the total length of the path traveled between two positions.* Distance has no direction and, thus, no sign. For example, the distance the professor walks is 2.0 m. The distance the airplane passenger walks is 4.0 m.

Misconception Alert: Distance Traveled vs. Magnitude of Displacement

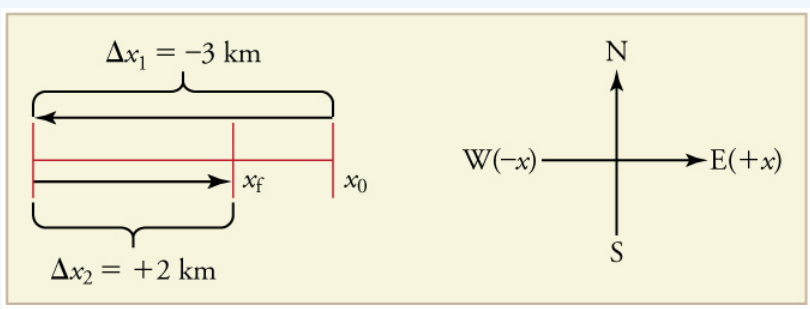
It is important to note that the distance traveled, however, can be greater than the magnitude of the displacement (by magnitude, we mean just the size of the displacement without regard to its direction; that is, just a number with a unit). For example, the professor could pace back and forth many times, perhaps walking a distance of 150 m during a lecture, yet still end up only 2.0 m to the right of her starting point. In this case her displacement would be +2.0 m, the magnitude of her displacement would be 2.0 m, but the distance she traveled would be 150 m. In kinematics we nearly always deal with displacement and magnitude of displacement, and almost never with distance traveled. One way to think about this is to assume you marked the start of the motion and the end of the motion. The displacement is simply the difference in the position of the two marks and is independent of the path taken in traveling between the two marks. The distance traveled, however, is the total length of the path taken between the two marks.

Exercise 2.2.1

A cyclist rides 3 km west and then turns around and rides 2 km east.

a. What is her displacement?
b. What distance does she ride?
c. What is the magnitude of her displacement?

Answer



Answer a

The rider's displacement is $\Delta_x = x_f - x_0 = -1 km$. (The displacement is negative because we take east to be positive and west to be negative.)

Answer b

The distance traveled is $3 km + 2 km = 5 km$.

Answer c

The magnitude of the displacement is $1 km$.

Summary

- Kinematics is the study of motion without considering its causes. In this chapter, it is limited to motion along a straight line, called one-dimensional motion.
- Displacement is the change in position of an object.
- In symbols, displacement Δx is defined to be

$$\Delta x = x_f - x_0 \ ,$$

where x_0 is the initial position and xf is the final position. In this text, the Greek letter Δ (delta) always means "change in" whatever quantity follows it. The SI unit for displacement is the meter (m). Displacement has a direction as well as a magnitude.

- When you start a problem, assign which direction will be positive.
- Distance is the magnitude of displacement between two positions.
- Distance traveled is the total length of the path traveled between two positions.

Glossary

kinematics
the study of motion without considering its causes

position
the location of an object at a particular time

displacement
the change in position of an object

distance
the magnitude of displacement between two positions

distance traveled
the total length of the path traveled between two positions

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2.3: Vectors, Scalars, and Coordinate Systems

Learning Objectives

By the end of this section, you will be able to:

- Define and distinguish between scalar and vector quantities.
- Assign a coordinate system for a scenario involving one-dimensional motion.

What is the difference between distance and displacement? Whereas displacement is defined by both direction and magnitude, distance is defined only by magnitude. Displacement is an example of a vector quantity. Distance is an example of a scalar quantity. A vector is any quantity with both *magnitude and direction*. Other examples of vectors include a velocity of 90 km/h east and a force of 500 newtons straight down.

Figure 2.3.1: The motion of this Eclipse Concept jet can be described in terms of the distance it has traveled (a scalar quantity) or its displacement in a specific direction (a vector quantity). In order to specify the direction of motion, its displacement must be described based on a coordinate system. In this case, it may be convenient to choose motion toward the left as positive motion (it is the forward direction for the plane), although in many cases, the x -coordinate runs from left to right, with motion to the right as positive and motion to the left as negative. (credit: Armchair Aviator, Flickr)

The direction of a vector in one-dimensional motion is given simply by a plus (+) or minus (−) sign. Vectors are represented graphically by arrows. An arrow used to represent a vector has a length proportional to the vector's magnitude (e.g., the larger the magnitude, the longer the length of the vector) and points in the same direction as the vector.

Some physical quantities, like distance, either have no direction or none is specified. A scalar is any quantity that has a magnitude, but no direction. For example, a 20ºC temperature, the 250 kilocalories (250 Calories) of energy in a candy bar, a 90 km/h speed limit, a person's 1.8 m height, and a distance of 2.0 m are all scalars—quantities with no specified direction. Note, however, that a scalar can be negative, such as a −20ºC temperature. In this case, the minus sign indicates a point on a scale rather than a direction. Scalars are never represented by arrows.

Coordinate Systems for One-Dimensional Motion

In order to describe the direction of a vector quantity, you must designate a coordinate system within the reference frame. For one-dimensional motion, this is a simple coordinate system consisting of a one-dimensional coordinate line. In general, when describing horizontal motion, motion to the right is usually considered positive, and motion to the left is considered negative. With vertical motion, motion up is usually positive and motion down is negative. In some cases, however, as with the jet in Figure \PageIndex{2}\), it can be more convenient to switch the positive and negative directions. For example, if you are analyzing the motion of falling objects, it can be useful to define downwards as the positive direction. If people in a race are running to the left, it is useful to define left as the positive direction. It does not matter as long as the system is clear and consistent. Once you assign a positive direction and start solving a problem, you cannot change it.

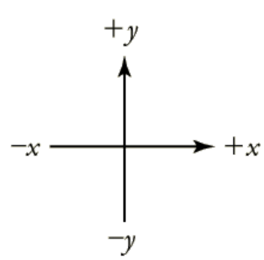

Figure 2.3.2: It is usually convenient to consider motion upward or to the right as positive (+) and motion downward or to the left as negative (−) .

Exercise 2.3.1

A person’s speed can stay the same as he or she rounds a corner and changes direction. Given this information, is speed a scalar or a vector quantity? Explain.

Answer

Speed is a scalar quantity. It does not change at all with direction changes; therefore, it has magnitude only. If it were a vector quantity, it would change as direction changes (even if its magnitude remained constant).

Summary

- A vector is any quantity that has magnitude and direction.
- A scalar is any quantity that has magnitude but no direction.
- Displacement and velocity are vectors, whereas distance and speed are scalars.
- In one-dimensional motion, direction is specified by a plus or minus sign to signify left or right, up or down, and the like.

Glossary

scalar

a quantity that is described by magnitude, but not direction

vector

a quantity that is described by both magnitude and direction

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2.4: Time, Velocity, and Speed

Learning Objectives

By the end of this section, you will be able to:

- Explain the relationships between instantaneous velocity, average velocity, instantaneous speed, average speed, displacement, and time.
- Calculate velocity and speed given initial position, initial time, final position, and final time.
- Derive a graph of velocity vs. time given a graph of position vs. time.
- Interpret a graph of velocity vs. time.

There is more to motion than distance and displacement. Questions such as, “How long does a foot race take?” and “What was the runner’s speed?” cannot be answered without an understanding of other concepts. In this section we add definitions of time, velocity, and speed to expand our description of motion.

Figure 2.4.1: The motion of these racing snails can be described by their speeds and their velocities. (credit: tobitasflickr, Flickr)

Time

As discussed in Physical Quantities and Units, the most fundamental physical quantities are defined by how they are measured. This is the case with time. Every measurement of time involves measuring a change in some physical quantity. It may be a number on a digital clock, a heartbeat, or the position of the Sun in the sky. In physics, the definition of **time** is simple—time is *change*, or the interval over which change occurs. It is impossible to know that time has passed unless something changes.

The amount of time or change is calibrated by comparison with a standard. The SI unit for time is the second, abbreviated s. We might, for example, observe that a certain pendulum makes one full swing every 0.75 s. We could then use the pendulum to measure time by counting its swings or, of course, by connecting the pendulum to a clock mechanism that registers time on a dial. This allows us to not only measure the amount of time, but also to determine a sequence of events.

How does time relate to motion? We are usually interested in elapsed time for a particular motion, such as how long it takes an airplane passenger to get from his seat to the back of the plane. To find elapsed time, we note the time at the beginning and end of the motion and subtract the two. For example, a lecture may start at 11:00 A.M. and end at 11:50 A.M., so that the elapsed time would be 50 min. **Elapsed time** Δt is the difference between the ending time and beginning time,

$$\Delta t = t_f - t_0 \,,$$

where Δt is the change in time or elapsed time, t_f is the time at the end of the motion, and t_0 is the time at the beginning of the motion. (As usual, the delta symbol, Δ, means the change in the quantity that follows it.)

Life is simpler if the beginning time t_0 is taken to be zero, as when we use a stopwatch. If we were using a stopwatch, it would simply read zero at the start of the lecture and 50 min at the end. If $t_0 = 0$, then

$$\Delta t = t_f \equiv t. \tag{2.4.1}$$

In this text, for simplicity’s sake,

- motion starts at time equal to zero ($t_0 = 0$)

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- the symbol t is used for elapsed time unless otherwise specified ($\Delta t = t_f \equiv t$)

Velocity

Your notion of velocity is probably the same as its scientific definition. You know that if you have a large displacement in a small amount of time you have a large velocity, and that velocity has units of distance divided by time, such as miles per hour or kilometers per hour.

Definition: AVERAGE VELOCITY

Average velocity is *displacement (change in position) divided by the time of travel,*

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_0}{t_f - t_0}. \tag{2.4.2}$$

where $\bar{v}$ is the average (indicated by the bar over the v) velocity, Δx is the change in position (or displacement), and x_f and x_0 are the final and beginning positions at times t_f and t_0, respectively. If the starting time t_0 is taken to be zero, then the average velocity is simply

$$\bar{v} = \frac{\Delta x}{t}. \tag{2.4.3}$$

Notice that this definition indicates that *velocity is a vector because displacement is a vector*. It has both magnitude and direction. The SI unit for velocity is meters per second or m/s, but many other units, such as km/h, mi/h (also written as mph), and cm/s, are in common use. Suppose, for example, an airplane passenger took 5 seconds to move −4 m (the negative sign indicates that displacement is toward the back of the plane). His average velocity would be

$$\bar{v} = \frac{\Delta x}{t} = \frac{-4\,m}{5\,s} = -0.8\,m/s. \tag{2.4.4}$$

The minus sign indicates the average velocity is also toward the rear of the plane.

The average velocity of an object does not tell us anything about what happens to it between the starting point and ending point, however. For example, we cannot tell from average velocity whether the airplane passenger stops momentarily or backs up before he goes to the back of the plane. To get more details, we must consider smaller segments of the trip over smaller time intervals.

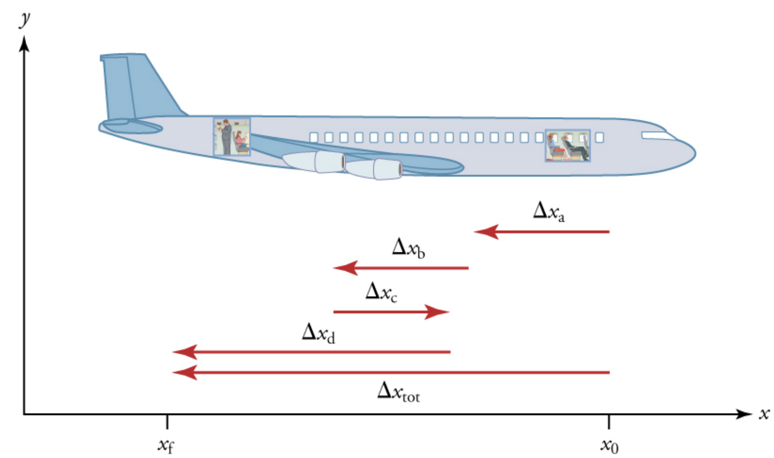

Figure 2.4.2: A more detailed record of an airplane passenger heading toward the back of the plane, showing smaller segments of his trip.

The smaller the time intervals considered in a motion, the more detailed the information. When we carry this process to its logical conclusion, we are left with an infinitesimally small interval. Over such an interval, the average velocity becomes the *instantaneous velocity* or the *velocity at a specific instant*. A car's speedometer, for example, shows the magnitude (but not the direction) of the instantaneous velocity of the car. (Police give tickets based on instantaneous velocity, but when calculating how long it will take to get from one place to another on a road trip, you need to use average velocity.) **Instantaneous velocity** v is the average velocity at a specific instant in time (or over an infinitesimally small time interval).

Mathematically, finding instantaneous velocity, v, at a precise instant t can involve taking a limit, a calculus operation beyond the scope of this text. However, under many circumstances, we can find precise values for instantaneous velocity without calculus.

Speed

In everyday language, most people use the terms “speed” and “velocity” interchangeably. In physics, however, they do not have the same meaning and they are distinct concepts. One major difference is that speed has no direction. Thus *speed is a scalar*. Just as we need to distinguish between instantaneous velocity and average velocity, we also need to distinguish between instantaneous speed and average speed.

Instantaneous speed is the magnitude of instantaneous velocity. For example, suppose the airplane passenger at one instant had an instantaneous velocity of −3.0 m/s (the minus meaning toward the rear of the plane). At that same time his instantaneous speed was 3.0 m/s. Or suppose that at one time during a shopping trip your instantaneous velocity is 40 km/h due north. Your instantaneous speed at that instant would be 40 km/h—the same magnitude but without a direction. Average speed, however, is very different from average velocity. **Average speed** is the distance traveled divided by elapsed time.

We have noted that distance traveled can be greater than displacement. So average speed can be greater than average velocity, which is displacement divided by time. For example, if you drive to a store and return home in half an hour, and your car’s odometer shows the total distance traveled was 6 km, then your average speed was 12 km/h. Your average velocity, however, was zero, because your displacement for the round trip is zero. (Displacement is change in position and, thus, is zero for a round trip.) Thus average speed is *not* simply the magnitude of average velocity.

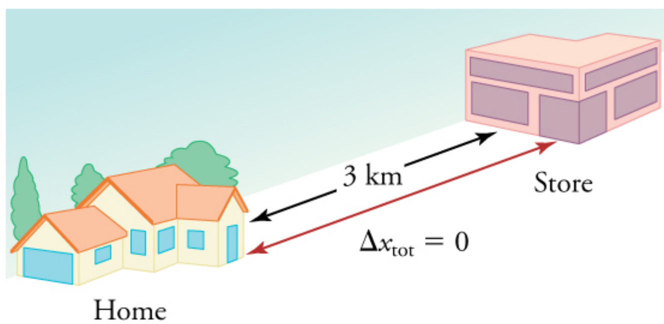

Figure $2.4.3$: During a 30-minute round trip to the store, the total distance traveled is 6 km. The average speed is 12 km/h. The displacement for the round trip is zero, since there was no net change in position. Thus the average velocity is zero.

Another way of visualizing the motion of an object is to use a graph. A plot of position or of velocity as a function of time can be very useful. For example, for this trip to the store, the position, velocity, and speed-vs.-time graphs are displayed in Figure $2.4.4$. (Note that these graphs depict a very simplified **model** of the trip. We are assuming that speed is constant during the trip, which is unrealistic given that we’ll probably stop at the store. But for simplicity’s sake, we will model it with no stops or changes in speed. We are also assuming that the route between the store and the house is a perfectly straight line.)

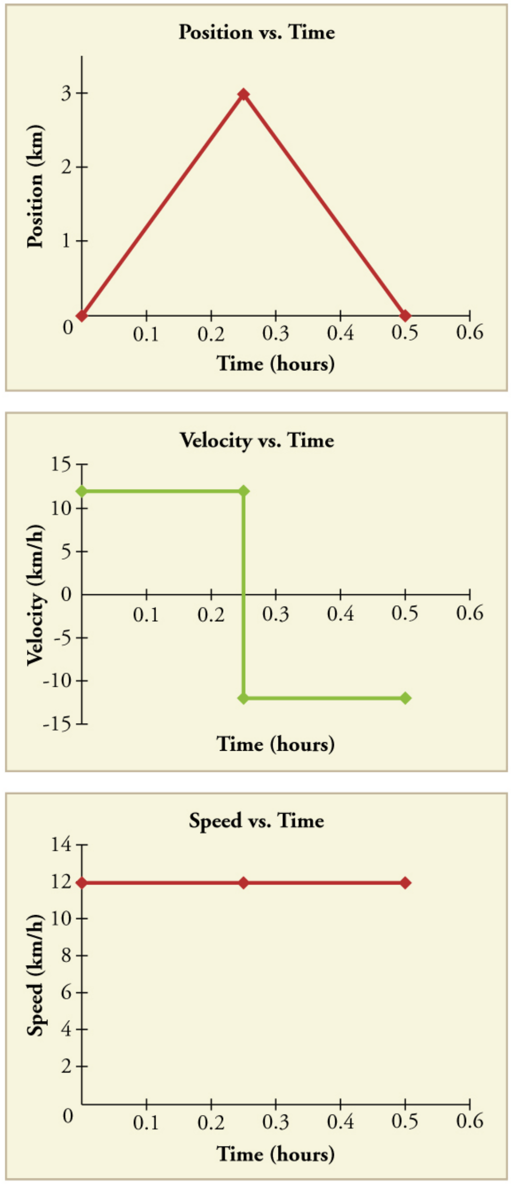

Figure 2.4.4: Position vs. time, velocity vs. time, and speed vs. time on a trip. Note that the velocity for the return trip is negative.

MAKING CONNECTIONS: TAKE-HOME INVESTIGATION - GETTING A SENSE OF SPEED

If you have spent much time driving, you probably have a good sense of speeds between about 10 and 70 miles per hour. But what are these in meters per second? What do we mean when we say that something is moving at 10 m/s? To get a better sense of what these values really mean, do some observations and calculations on your own:

- calculate typical car speeds in meters per second
- estimate jogging and walking speed by timing yourself; convert the measurements into both m/s and mi/h
- determine the speed of an ant, snail, or falling leaf

Exercise 2.4.1

A commuter train travels from Baltimore to Washington, DC, and back in 1 hour and 45 minutes. The distance between the two stations is approximately 40 miles. What is

a. the average velocity of the train, and
b. the average speed of the train in m/s?

Answer

(a) The average velocity of the train is zero because $x_f = x_0$; the train ends up at the same place it starts.

(b) The average speed of the train is calculated below. Note that the train travels 40 miles one way and 40 miles back, for a total distance of 80 miles.

$$\frac{distance}{time} = \frac{80 miles}{105 minutes}$$

$$\frac{80 miles}{105 minutes} \times \frac{5280 feet}{1 mile} \times \frac{1 meter}{3.28 feet} \times \frac{1 minute}{60 seconds} = 20 m/s$$

Summary

- Time is measured in terms of change, and its SI unit is the second (s). Elapsed time for an event is

$$\Delta t = t_f - t_0,$$

  where t_f is the final time and t_0 is the initial time. The initial time is often taken to be zero, as if measured with a stopwatch; the elapsed time is then just t.
- Average velocity $\bar{v}$ is defined as displacement divided by the travel time. In symbols, average velocity is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_0}{t_f - t_0}.$$

- The SI unit for velocity is m/s.
- Velocity is a vector and thus has a direction.
- Instantaneous velocity v is the velocity at a specific instant or the average velocity for an infinitesimal interval.
- Instantaneous speed is the magnitude of the instantaneous velocity.
- Instantaneous speed is a scalar quantity, as it has no direction specified.
- Average speed is the total distance traveled divided by the elapsed time. (Average speed is *not* the magnitude of the average velocity.) Speed is a scalar quantity; it has no direction associated with it.

Glossary

average speed
distance traveled divided by time during which motion occurs

average velocity
displacement divided by time over which displacement occurs

instantaneous velocity
velocity at a specific instant, or the average velocity over an infinitesimal time interval

instantaneous speed
magnitude of the instantaneous velocity

time
change, or the interval over which change occurs

model
simplified description that contains only those elements necessary to describe the physics of a physical situation

elapsed time
the difference between the ending time and beginning time

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2.5: Acceleration

Learning Objectives

By the end of this section, you will be able to:

- Define and distinguish between instantaneous acceleration, average acceleration, and deceleration.
- Calculate acceleration given initial time, initial velocity, final time, and final velocity.

In everyday conversation, to accelerate means to speed up. The accelerator in a car can in fact cause it to speed up. The greater the **acceleration**, the greater the change in velocity over a given time. The formal definition of acceleration is consistent with these notions, but more inclusive.

Figure 2.5.1: A plane decelerates, or slows down, as it comes in for landing in St. Maarten. Its acceleration is opposite in direction to its velocity. (credit: Steve Conry, Flickr)

Definition: Average Acceleration

The *average acceleration* is the rate at which velocity changes,

$$\bar{a}=\frac{\Delta v}{\Delta t}=\frac{v_f-v_0}{t_f-t_0} \tag{2.5.1}$$

where $\bar{a}$ is average acceleration, v is velocity, and t is time. (The bar over the a means *average*.)

Because acceleration is velocity in m/s divided by time in s, the SI units for acceleration are m/s^2, meters per second squared or meters per second per second, which literally means by how many meters per second the velocity changes every second.

Recall that velocity is a vector—it has both magnitude and direction. This means that a change in velocity can be a change in magnitude (or speed), but it can also be a change in direction. For example, if a car turns a corner at constant speed, it is accelerating because its direction is changing. The quicker you turn, the greater the acceleration. So there is an acceleration when velocity changes either in magnitude (an increase or decrease in speed) or in direction, or both.

ACCELERATION AS A VECTOR

Acceleration is a vector in the same direction as the change in velocity, Δv. Since velocity is a vector, it can change either in magnitude or in direction. Acceleration is therefore a change in either speed or direction, or both.

Keep in mind that although acceleration is in the direction of the *change* in velocity, it is not always in the direction of *motion*. When an object slows down, its acceleration is opposite to the direction of its motion. This is known as **deceleration.**

Figure 2.5.2: A subway train in Sao Paulo, Brazil, decelerates as it comes into a station. It is accelerating in a direction opposite to its direction of motion. (credit: Yusuke Kawasaki, Flickr)

MISCONCEPTION ALERT: DECELERATION VS. NEGATIVE ACCELERATION

Deceleration always refers to acceleration in the direction opposite to the direction of the velocity. Deceleration always reduces speed. Negative acceleration, however, is acceleration in the negative direction in the chosen coordinate system. Negative acceleration may or may not be deceleration, and deceleration may or may not be considered negative acceleration. If acceleration has the same sign as the velocity, the object is speeding up. If acceleration has the opposite sign as the velocity, the object is slowing down. For example, consider Figure $2.5.2$.

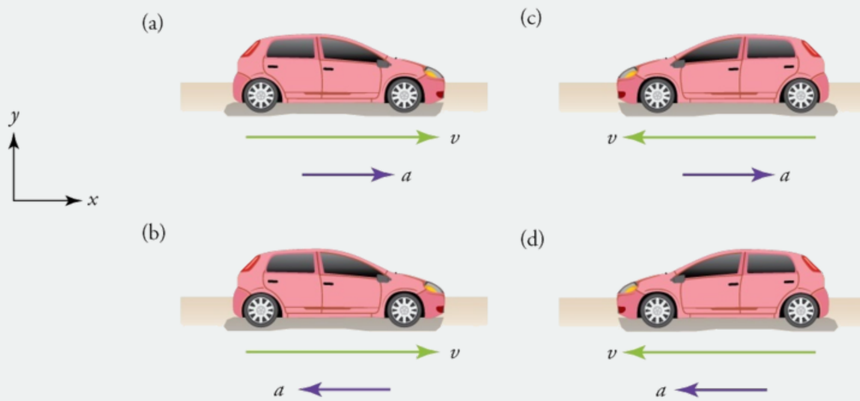

Figure $2.5.3$: (a) This car is speeding up as it moves toward the right. It therefore has positive acceleration in our coordinate system. (b) This car is slowing down as it moves toward the right. Therefore, it has negative acceleration in our coordinate system, because its acceleration is toward the left. The car is also decelerating: the direction of its acceleration is opposite to its direction of motion. (c) This car is moving toward the left, but slowing down over time. Therefore, its acceleration is positive in our coordinate system because it is toward the right. However, the car is decelerating because its acceleration is opposite to its motion. (d) This car is speeding up as it moves toward the left. It has negative acceleration because it is accelerating toward the left. However, because its acceleration is in the same direction as its motion, it is speeding up (not decelerating).

Example $2.5.1$: Calculating Acceleration: A Racehorse Leaves the Gate

A racehorse coming out of the gate accelerates from rest to a velocity of 15.0 m/s due west in 1.80 s. What is its average acceleration?

Figure 2.5.4: Two racehorses running toward the left. (credit: Jon Sullivan, PD Photo.org)

Strategy

First we draw a sketch and assign a coordinate system to the problem. This is a simple problem, but it always helps to visualize it. Notice that we assign east as positive and west as negative. Thus, in this case, we have negative velocity.

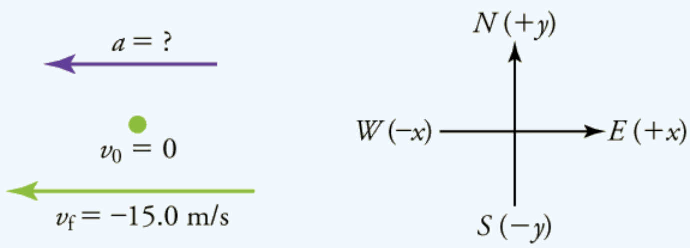

Figure 2.5.5.

We can solve this problem by identifying Δv and Δt from the given information and then calculating the average acceleration directly from the Equation ???:

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}.$$

Solution

1. Identify the knowns. $v_0 = 0, v_f = -15.0 m/s$ (the negative sign indicates direction toward the west), $\Delta t = 1.80 s$.
2. Find the change in velocity. Since the horse is going from zero to $-15.0 m/s$, its change in velocity equals its final velocity:

$$\Delta v = v_f = -15.0\, m/s.$$

3. Plug in the known values (Δv and Δt) and solve for the unknown $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-15.0 m/s}{1.80 s} = -8.33 m/s^2 .$$

Discussion

The negative sign for acceleration indicates that acceleration is toward the west. An acceleration of $8.33\, m/s^2$ due west means that the horse increases its velocity by 8.33 m/s due west each second, that is, 8.33 meters per second per second, which we write as $8.33\, m/s^2$. This is truly an average acceleration, because the ride is not smooth. We shall see later that an acceleration of this magnitude would require the rider to hang on with a force nearly equal to his weight.

Instantaneous Acceleration

Instantaneous acceleration a, or the acceleration at a specific instant in time, is obtained by the same process as discussed for instantaneous velocity in Time, Velocity, and Speed—that is, by considering an infinitesimally small interval of time. How do we find instantaneous acceleration using only algebra? The answer is that we choose an average acceleration that is representative of the motion. Figure $2.5.6$ shows graphs of instantaneous acceleration versus time for two very different motions. In Figure $2.5.6a$, the acceleration varies slightly and the average over the entire interval is nearly the same as the instantaneous acceleration at any time. In this case, we should treat this motion as if it had a constant acceleration equal to the average (in this case about $1.8 m/s^2$). In Figure $2.5.6b$, the acceleration varies drastically over time. In such situations it is best to consider smaller time intervals and choose an average acceleration for each. For example, we could consider motion over the time intervals from 0 to 1.0 s and from 1.0 to 3.0 s as separate motions with accelerations of $+3.0\, m/s^2$ and $-2.0\, m/s^2$, respectively.

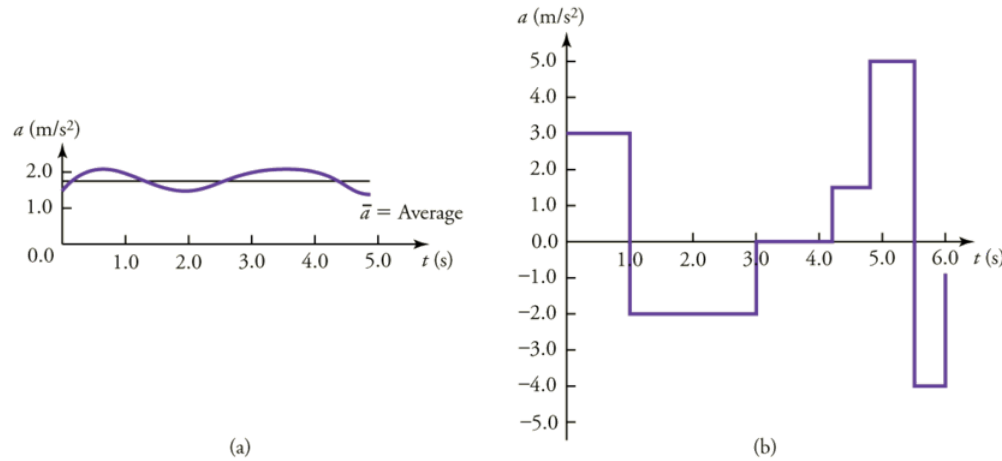

Figure $2.5.6$: Graphs of instantaneous acceleration versus time for two different one-dimensional motions. (a) Here acceleration varies only slightly and is always in the same direction, since it is positive. The average over the interval is nearly the same as the acceleration at any given time. (b) Here the acceleration varies greatly, perhaps representing a package on a post office conveyor belt that is accelerated forward and backward as it bumps along. It is necessary to consider small time intervals (such as from 0 to 1.0 s) with constant or nearly constant acceleration in such a situation.

The next several examples consider the motion of the subway train shown in Figure $2.5.7$. In Figure $2.5.7a$ the shuttle moves to the right, and in Figure $2.5.7b$ it moves to the left. The examples are designed to further illustrate aspects of motion and to illustrate some of the reasoning that goes into solving problems.

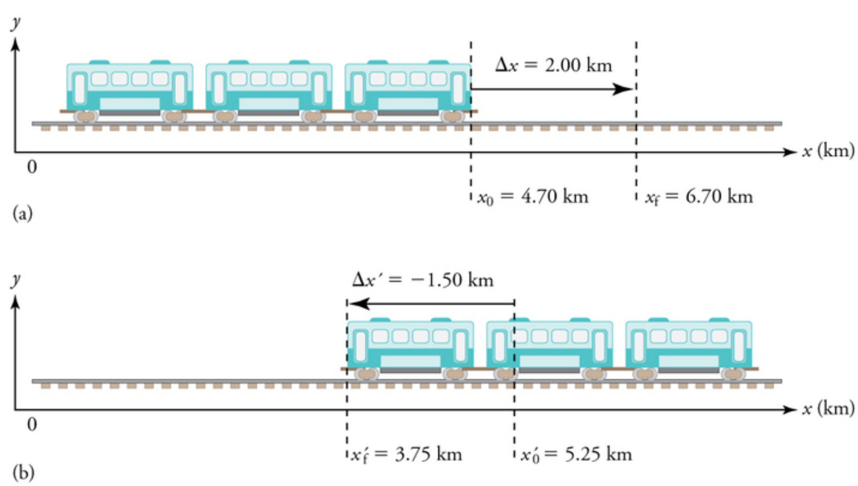

Figure $2.5.7$: One-dimensional motion of a subway train considered in Examples $2.5.2$ - $2.5.5$. Here we have chosen the x -axis so that + means to the right and − means to the left for displacements, velocities, and accelerations. (a) The subway train moves to the right from x_0 to x_f. Its displacement Δx is +2.0 km. (b) The train moves to the left from x_0 to x_f. Its displacement Δx is $-1.5 km$. (Note that the prime symbol (′) is used simply to distinguish between displacement in the two different situations. The distances of travel and the size of the cars are on different scales to fit everything into the diagram.)

Example $2.5.2$: Calculating Displacement - A Subway Train

What are the magnitude and sign of displacements for the motions of the subway train shown in parts (a) and (b) of Figure $2.5.7$?

Strategy

A drawing with a coordinate system is already provided, so we don't need to make a sketch, but we should analyze it to make sure we understand what it is showing. Pay particular attention to the coordinate system. To find displacement, we use the equation $\Delta x = x_f - x_0$. This is straightforward since the initial and final positions are given.

Solution

1. Identify the knowns. In the figure we see that $x_f = 6.70\,km$ and $x_0 = 4.70\,km$ for part (a), and $x'_f = 3.75\,km$ and $x'_0 = 5.25\,km$ for part (b).
2. Solve for displacement in part (a).

$$\begin{aligned}\Delta x &= x_f - x_0 \\ &= 6.70\,km - 4.70\,km \\ &= +2.00\,km\end{aligned}$$

3. Solve for displacement in part (b).

$$\begin{aligned}\Delta x' &= x'_f - x'_0 \\ &= 3.75\,km - 5.25\,km \\ &= -1.50\,km\end{aligned}$$

Discussion

The direction of the motion in (a) is to the right and therefore its displacement has a positive sign, whereas motion in (b) is to the left and thus has a negative sign.

Example 2.5.3: Comparing Distance Traveled with Displacement - A Subway Train

What are the distances traveled for the motions shown in parts (a) and (b) of the subway train in Figure $2.5.7$?

Strategy

To answer this question, think about the definitions of distance and distance traveled, and how they are related to displacement. Distance between two positions is defined to be the magnitude of displacement, which was found in Example $2.5.2$. Distance traveled is the total length of the path traveled between the two positions (see Section on Displacement). In the case of the subway train shown in Figure $2.5.7$, the distance traveled is the same as the distance between the initial and final positions of the train.

Solution

1. The displacement for part (a) was +2.00 km. Therefore, the distance between the initial and final positions was 2.00 km, and the distance traveled was 2.00 km.

2. The displacement for part (b) was $-1.5 km$. Therefore, the distance between the initial and final positions was 1.50 km, and the distance traveled was 1.50 km.

Discussion

Distance is a scalar. It has magnitude but no sign to indicate direction.

Example 2.5.4: Calculating Acceleration: A Subway Train Speeding Up

Suppose the train in Figure $2.5.7a$ accelerates from rest to 30.0 km/h in the first 20.0 s of its motion. What is its average acceleration during that time interval?

Strategy

It is worth it at this point to make a simple sketch:

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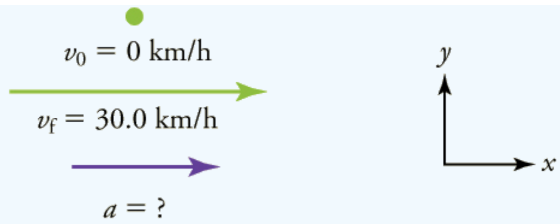

Figure 2.5.8: This problem involves three steps. First we must determine the change in velocity, then we must determine the change in time, and finally we use these values to calculate the acceleration.

Solution

1. Identify the knowns. $v_0 = 0$ (the trains starts at rest), $v_f = 30.0 km/h$, and $\Delta t = 20.0 s$.
2. Calculate Δv. Since the train starts from rest, its change in velocity is $\Delta v = +30.0 km/h$, where the plus sign means velocity to the right.
3. Plug in known values and solve for the unknown, $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{+30.0 km/h}{20.0 s}$$

4. Since the units are mixed (we have both hours and seconds for time), we need to convert everything into SI units of meters and seconds. (See Physical Quantities and Units for more guidance.)

$$\bar{a} = \left(\frac{+30 km/h}{20.0 s}\right)\left(\frac{10^3 m}{1 km}\right)\left(\frac{1 h}{3600 s}\right) = 0.417 m/s^2$$

Discussion

The plus sign means that acceleration is to the right. This is reasonable because the train starts from rest and ends up with a velocity to the right (also positive). So acceleration is in the same direction as the change in velocity, as is always the case.

Example 2.5.5: Calculate Acceleration

A Subway Train Slowing Down: Now suppose that at the end of its trip, the train in Figure $2.5.7a$ slows to a stop from a speed of 30.0 km/h in 8.00 s. What is its average acceleration while stopping?

Strategy

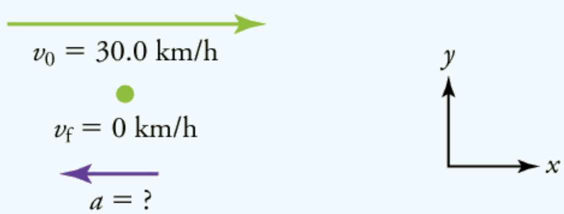

Figure 2.5.9:.

In this case, the train is decelerating and its acceleration is negative because it is toward the left. As in the previous example, we must find the change in velocity and the change in time and then solve for acceleration.

Solution

1. Identify the knowns. $v_0 = 30.0 km/h$, $v_f = 0 km/h$ (the train is stopped, so its velocity is 0), and $\Delta t = 8.00 s$.
2. Solve for the change in velocity, Δv.

$$\Delta v = v_f - v_0 = 0 - 30.0 km/h = -30.0 km/h$$

3. Plug in the knowns, Δv and Δt, and solve for $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-30.0 km/h}{8.00 s}$$

4. Convert the units to meters and seconds.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \left(\frac{-30.0km/h}{8.00s}\right)\left(\frac{10^3 m}{1km}\right)\left(\frac{1h}{3600s}\right) = -1.04m/s^2.$$

Discussion

The minus sign indicates that acceleration is to the left. This sign is reasonable because the train initially has a positive velocity in this problem, and a negative acceleration would oppose the motion. Again, acceleration is in the same direction as the change in velocity, which is negative here. This acceleration can be called a deceleration because it has a direction opposite to the velocity.

The graphs of position, velocity, and acceleration vs. time for the trains in Example $2.5.4$ and $2.5.5$ are displayed in Figure $2.5.10$. (We have taken the velocity to remain constant from 20 to 40 s, after which the train decelerates.)

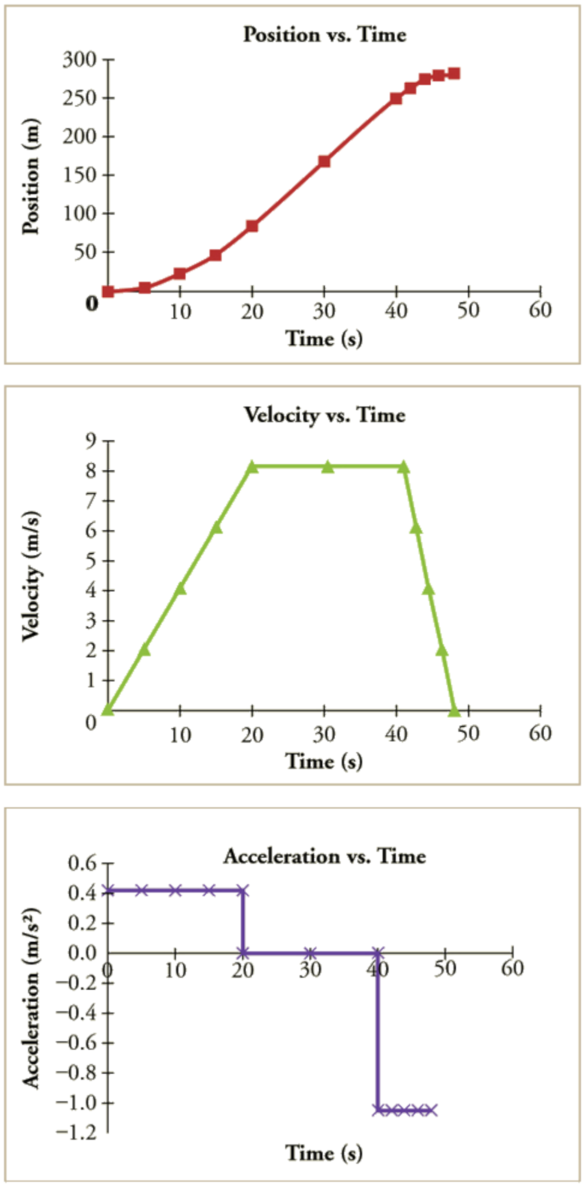

Figure $2.5.10$: (a) Position of the train over time. Notice that the train's position changes slowly at the beginning of the journey, then more and more quickly as it picks up speed. Its position then changes more slowly as it slows down at the end of the journey. In the middle of the journey, while the velocity remains constant, the position changes at a constant rate. (b) Velocity of the train over time. The train's velocity increases as it accelerates at the beginning of the journey. It remains the same in the middle of the journey (where there is no acceleration). It decreases as the train decelerates at the end of the journey. (c) The acceleration of the train over time. The train has positive acceleration as it speeds up at the beginning of the journey. It has no acceleration as it travels at constant velocity in the middle of the journey. Its acceleration is negative as it slows down at the end of the journey.

Example 2.5.6: Calculating Average Velocity: The Subway Train

What is the average velocity of the train in part b of Example $2.5.2$, and shown again below, if it takes 5.00 min to make its trip?

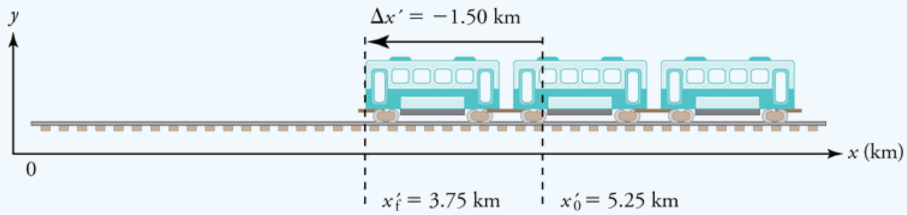

Figure 2.5.11

Strategy

Average velocity is displacement divided by time. It will be negative here, since the train moves to the left and has a negative displacement.

Solution

1. Identify the knowns.

$$x'_f = 3.75km,\ x'_0 = 5.25km,\ \Delta t = 5.00min.$$

2. Determine displacement, $\Delta x'$. We found $\Delta x'$ to be $-1.5km$ in Example $2.5.7$.
3. Solve for average velocity.

$$\bar{v} = \frac{\Delta x'}{\Delta t} = \frac{-1.50km}{5.00min}$$

4. Convert units.

$$\bar{v} = \frac{\Delta x'}{\Delta t} = (\frac{-1.50km}{5.00min})(\frac{60min}{1}h) = -18.0km/h$$

Discussion

The negative velocity indicates motion to the left.

Example 2.5.7: Calculating Deceleration: The Subway Train

Finally, suppose the train in Figure $2.5.7$ slows to a stop from a velocity of 20.0 km/h in 10.0 s. What is its average acceleration?

Strategy

Once again, let's draw a sketch:

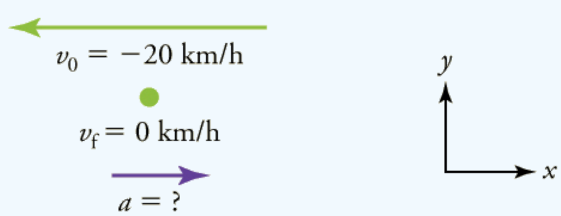

Figure 2.5.12: As before, we must find the change in velocity and the change in time to calculate average acceleration.

Solution

1. Identify the knowns. $v_0 = -20\ km/h$, $v_f = 0\ km/h$, $\Delta t = 10.0\ s$.
2. Calculate Δv. The change in velocity here is actually positive, since

$$\Delta v = v_f - v_0 = 0 - (-20\ km/h) = +20\ km/h.$$

3. Solve for $\bar{a}$.

$$\bar{a}=\frac{\Delta v}{\Delta t}=\frac{+20.0\ km/h}{10.0\ s}$$

4. Convert units.

$$\bar{a}=\left(\frac{+20.0\ km/h}{10.0\ s}\right)\left(\frac{10^3\ m}{1\ km}\right)\left(\frac{1\ h}{3600\ s}\right)=+0.556\ m/^s 2$$

Discussion

The plus sign means that acceleration is to the right. This is reasonable because the train initially has a negative velocity (to the left) in this problem and a positive acceleration opposes the motion (and so it is to the right). Again, acceleration is in the same direction as the change in velocity, which is positive here. As in Example $2.5.5$, this acceleration can be called a deceleration since it is in the direction opposite to the velocity.

Sign and Direction

Perhaps the most important thing to note about these examples is the signs of the answers. In our chosen coordinate system, plus means the quantity is to the right and minus means it is to the left. This is easy to imagine for displacement and velocity. But it is a little less obvious for acceleration. Most people interpret negative acceleration as the slowing of an object. This was not the case in Example $2.5.5$, where a positive acceleration slowed a negative velocity. The crucial distinction was that the acceleration was in the opposite direction from the velocity. In fact, a negative acceleration will increase a negative velocity. For example, the train moving to the left in Figure Figure $2.5.11$ is sped up by an acceleration to the left. In that case, both v and a are negative. The plus and minus signs give the directions of the accelerations. If acceleration has the same sign as the velocity, the object is speeding up. If acceleration has the opposite sign as the velocity, the object is slowing down.

Exercise 2.5.1

An airplane lands on a runway traveling east. Describe its acceleration.

Answer

If we take east to be positive, then the airplane has negative acceleration, as it is accelerating toward the west. It is also decelerating: its acceleration is opposite in direction to its velocity.

PHET EXPLORATIONS: MOVING MAN SIMULATION

Learn about position, velocity, and acceleration graphs with the PhET Moving Man simulation. Move the little man back and forth with the mouse and plot his motion. Set the position, velocity, or acceleration and let the simulation move the man for you.

Summary

- Acceleration is the rate at which velocity changes. In symbols, **average acceleration** $\bar{a}$ is $\bar{a}=\frac{\Delta v}{\Delta t}=\frac{v_f-v_0}{t_f-t_0}$.
- The SI unit for acceleration is 2 .
- Acceleration is a vector, and thus has a both a magnitude and direction.
- Acceleration can be caused by either a change in the magnitude or the direction of the velocity.
- Instantaneous acceleration is the acceleration at a specific instant in time.
- Deceleration is an acceleration with a direction opposite to that of the velocity.

Glossary

acceleration

the rate of change in velocity; the change in velocity over time

average acceleration

the change in velocity divided by the time over which it changes

instantaneous acceleration

acceleration at a specific point in time

deceleration

acceleration in the direction opposite to velocity; acceleration that results in a decrease in velocity

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2.6: Motion Equations for Constant Acceleration in One Dimension

Learning Objectives

By the end of this section, you will be able to:

- Calculate displacement of an object that is not accelerating, given initial position and velocity.
- Calculate final velocity of an accelerating object, given initial velocity, acceleration, and time.
- Calculate displacement and final position of an accelerating object, given initial position, initial velocity, time, and acceleration.

We might know that the greater the acceleration of, say, a car moving away from a stop sign, the greater the displacement in a given time. But we have not developed a specific equation that relates acceleration and displacement. In this section, we develop some convenient equations for kinematic relationships, starting from the definitions of displacement, velocity, and acceleration already covered.

Figure 2.6.1: Kinematic equations can help us describe and predict the motion of moving objects such as these kayaks racing in Newbury, England. (credit: Barry Skeates, Flickr)

Notation: *t*, *x*, *v*, *a*

First, let us make some simplifications in notation. Taking the initial time to be zero, as if time is measured with a stopwatch, is a great simplification. Since elapsed time is $\Delta t = t_f - t_0$, taking $t_0 = 0$ means that $\Delta t = t_f$, the final time on the stopwatch. When initial time is taken to be zero, we use the subscript 0 to denote initial values of position and velocity. That is, x_0 is the initial position and v_0 is *the initial velocity*. We put no subscripts on the final values. That is, t is *the final time*, x is *the final position*, and v is *the final velocity*. This gives a simpler expression for elapsed time—now, $\Delta t = t$. It also simplifies the expression for displacement, which is now $\Delta x = x - x_0$. Also, it simplifies the expression for change in velocity, which is now $\Delta v = v - v_0$. To summarize, using the simplified notation, with the initial time taken to be zero,

$$\Delta t = t$$
$$\Delta x = x - x_0$$
$$\Delta v = v - v_0$$

where *the subscript 0 denotes an initial value and the absence of a subscript denotes a final value* in whatever motion is under consideration.

We now make the important assumption that *acceleration is constant*. This assumption allows us to avoid using calculus to find instantaneous acceleration. Since acceleration is constant, the average and instantaneous accelerations are equal. That is,

$$\bar{a} = a = constant, \tag{2.6.1}$$

so we use the symbol a for acceleration at all times. Assuming acceleration to be constant does not seriously limit the situations we can study nor degrade the accuracy of our treatment. For one thing, acceleration is constant in a great number of situations. Furthermore, in many other situations we can accurately describe motion by assuming a constant acceleration equal to the average acceleration for that motion. Finally, in motions where acceleration changes drastically, such as a car accelerating to top speed and then braking to a stop, the motion can be considered in separate parts, each of which has its own constant acceleration.

SOLVING FOR DISPLACEMENT (Δx) AND FINAL POSITION (x) FROM AVERAGE VELOCITY WHEN ACCELERATION (a) IS CONSTANT

To get our first two new equations, we start with the definition of average velocity:

$$\bar{v} = \frac{\Delta x}{\Delta t} \tag{2.6.2}$$

Substituting the simplified notation for Δx and Δt yields

$$\bar{v} = \frac{x - x_0}{t}. \tag{2.6.3}$$

Solving for x yields

$$x = x_0 + \bar{v}t, \tag{2.6.4}$$

where the average velocity is

$$\bar{v} = \frac{v_0 + v}{2} \tag{2.6.5}$$

with constant a.

Equation 2.6.5 reflects the fact that, when acceleration is constant, v is just the simple average of the initial and final velocities. For example, if you steadily increase your velocity (that is, with constant acceleration) from 30 to 60 km/h, then your average velocity during this steady increase is 45 km/h. Using the equation $\bar{v} = \frac{v_0 + v}{2}$ to check this, we see that

$$\bar{v} = \frac{v_0 + v}{2} = \frac{30km/h + 60km/h}{2} = 45km/h,$$

which seems logical.

Example 2.6.1: Calculating Displacement - How Far does the Jogger Run?

A jogger runs down a straight stretch of road with an average velocity of 4.00 m/s for 2.00 min. What is his final position, taking his initial position to be zero?

Strategy

Draw a sketch.

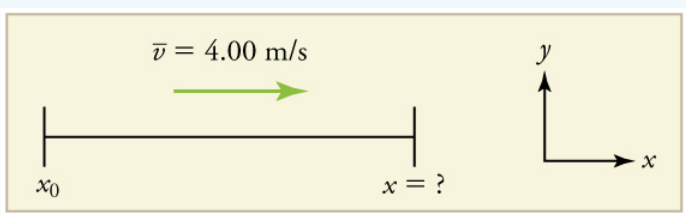

Figure 2.6.2

The final position is given by the equation

$x = x_0 + \bar{v}t$.

To find x, we identify the values of x_0, $\bar{v}$, and t from the statement of the problem and substitute them into the equation.

Solution

1. Identify the knowns. $\bar{v} = 4.00m/s$, $\Delta t = 2.00min$, and $x_0 = 0m$.
2. Enter the known values into the equation.

$$x = x_0 + \bar{v}t = 0 + (4.00m/s)(120s) = 480m$$

Discussion

Velocity and final displacement are both positive, which means they are in the same direction.

The equation $x = x_0 + \bar{v}t$ gives insight into the relationship between displacement, average velocity, and time. It shows, for example, that displacement is a linear function of average velocity. (By linear function, we mean that displacement depends on $\bar{v}$ rather than on $\bar{v}$ raised to some other power, such as $\bar{v}^2$. When graphed, linear functions look like straight lines with a constant slope.) On a car trip, for example, we will get twice as far in a given time if we average 90 km/h than if we average 45 km/h.

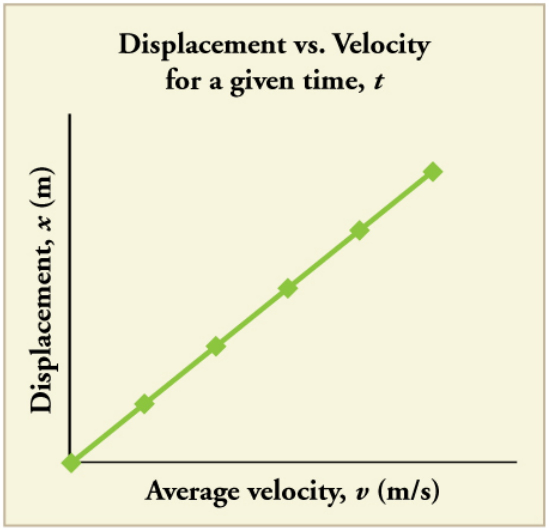

Figure 2.6.3: There is a linear relationship between displacement and average velocity. For a given time t, an object moving twice as fast as another object will move twice as far as the other object.

SOLVING FOR FINAL VELOCITY

We can derive another useful equation by manipulating the definition of acceleration.

$$a = \frac{\Delta v}{\Delta t}$$

Substituting the simplified notation for Δv and Δt gives us

$$a = \frac{v - v_0}{t} \tag{2.6.6}$$

(constant a).

Solving for v yields

$$v = v_0 + at \tag{2.6.7}$$

(constanta).

Example 2.6.2: Calculating Final Velocity: An Airplane Slowing Down after Landing

An airplane lands with an initial velocity of 70.0 m/s and then decelerates at $1.50 m/s^2$ for 40.0 s. What is its final velocity?

Strategy

Draw a sketch. We draw the acceleration vector in the direction opposite the velocity vector because the plane is decelerating.

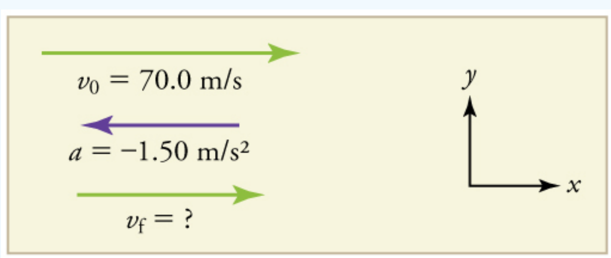

Figure 2.6.4

Solution

1. Identify the knowns. $v_0 = 70.0 m/s, a = -1.50 m/s^2, t = 40.0 s$.

2. Identify the unknown. In this case, it is final velocity, v_f.

3. Determine which equation to use. We can calculate the final velocity using the equation $v = v_0 + at$.

4. Plug in the known values and solve.

$$v = v_0 + at = 70.0 m/s + (-1.50 m/s^2)(40.0 s) = 10.0 m/s$$

Discussion

The final velocity is much less than the initial velocity, as desired when slowing down, but still positive. With jet engines, reverse thrust could be maintained long enough to stop the plane and start moving it backward. That would be indicated by a negative final velocity, which is not the case here.

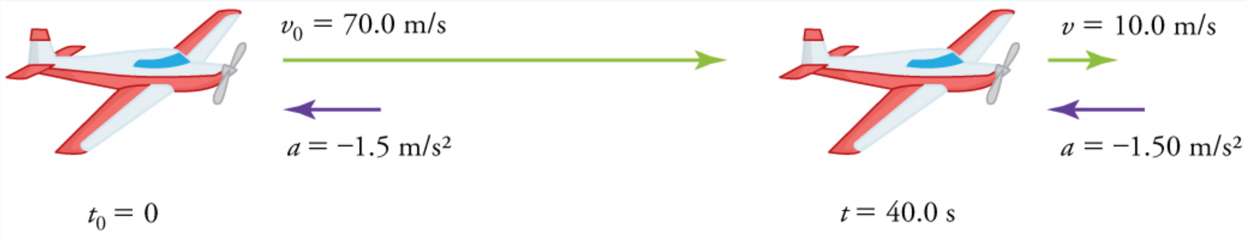

Figure 2.6.5: The airplane lands with an initial velocity of 70.0 m/s and slows to a final velocity of 10.0 m/s before heading for the terminal. Note that the acceleration is negative because its direction is opposite to its velocity, which is positive.

In addition to being useful in problem solving, the equation $v = v_0 + at$ gives us insight into the relationships among velocity, acceleration, and time. From it we can see, for example, that

- final velocity depends on how large the acceleration is and how long it lasts
- if the acceleration is zero, then the final velocity equals the initial velocity ($v = v_0$), as expected (i.e., velocity is constant)
- if a is negative, then the final velocity is less than the initial velocity

(All of these observations fit our intuition, and it is always useful to examine basic equations in light of our intuition and experiences to check that they do indeed describe nature accurately.)

MAKING CONNECTIONS: REAL-WORLD CONNECTION

Figure 2.6.6: The Space Shuttle Endeavor blasts off from the Kennedy Space Center in February 2010. (credit: Matthew Simantov, Flickr)

An intercontinental ballistic missile (ICBM) has a larger average acceleration than the Space Shuttle and achieves a greater velocity in the first minute or two of flight (actual ICBM burn times are classified—short-burn-time missiles are more difficult for an enemy to destroy). But the Space Shuttle obtains a greater final velocity, so that it can orbit the earth rather than come directly back down as an ICBM does. The Space Shuttle does this by accelerating for a longer time.

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SOLVING FOR FINAL POSITION WHEN VELOCITY IS NOT CONSTANT (a≠0)

We can combine the equations above to find a third equation that allows us to calculate the final position of an object experiencing constant acceleration. We start with

$$v=v_0+at.$$

Adding v_0 to each side of this equation and dividing by 2 gives

$$\frac{v_0+v}{2}=v_0+\frac{1}{2}at\ .$$

Since \\frac{(v_0+v}{2}=\bar{v}\) for constant acceleration, then

$$\bar{v}=v_0+\frac{1}{2}at\ .$$

Now we substitute this expression for $\bar{v}$ into the equation for displacement, $x=x_0+\bar{v}t$, yielding

$$x=x_0+v_0t+\frac{1}{2}at^2\ \text{(constant a)}.$$

Example 2.6.3: Calculating Displacement of an Accelerating Object - Dragsters

Dragsters can achieve average accelerations of $26.0 m/s^2$. Suppose such a dragster accelerates from rest at this rate for 5.56 s. How far does it travel in this time?

Figure 2.6.7: U.S. Army Top Fuel pilot Tony “The Sarge” Schumacher begins a race with a controlled burnout. (credit: Lt. Col. William Thurmond. Photo Courtesy of U.S. Army.)

Strategy

Draw a sketch.

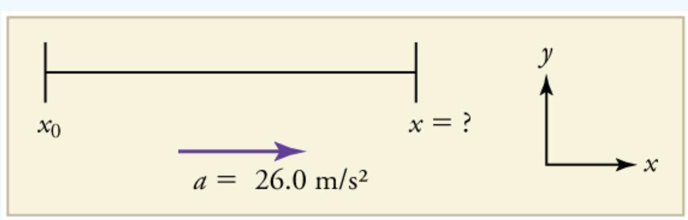

Figure 2.6.8

We are asked to find displacement, which is x if we take x_0 to be zero. (Think about it like the starting line of a race. It can be anywhere, but we call it 0 and measure all other positions relative to it.) We can use the equation $x=x_0+v_0t+\frac{1}{2}at^2$ once we identify v_0, a, and t from the statement of the problem.

Solution

1. Identify the knowns. Starting from rest means that $v_0=0$, a is given as $26.0 m/s^2$ and t is given as 5.56 s.

2. Plug the known values into the equation to solve for the unknown x:

$x = x_0 + v_0 t + \frac{1}{2} a t^2$.

Since the initial position and velocity are both zero, this simplifies to

$x = \frac{1}{2} a t^2$.

Substituting the identified values of a and t gives

$x = \frac{1}{2}(26.0 m/s^2)(5.56 s)^2$,

yielding

$x = 402 m$.

Discussion

If we convert 402 m to miles, we find that the distance covered is very close to one quarter of a mile, the standard distance for drag racing. So the answer is reasonable. This is an impressive displacement in only 5.56 s, but top-notch dragsters can do a quarter mile in even less time than this.

What else can we learn by examining the equation $x = x_0 + v_0 t + \frac{1}{2} a t^2$? We see that:

- displacement depends on the square of the elapsed time when acceleration is not zero. In Example, the dragster covers only one fourth of the total distance in the first half of the elapsed time
- if acceleration is zero, then the initial velocity equals average velocity ($v_0 = \bar{v}$) and $x = x_0 + v_0 t + \frac{1}{2} a t^2$ becomes $x = x_0 + v_0 t$

SOLVING FOR FINAL VELOCITY WHEN VELOCITY IS NOT CONSTANT (a≠0)

A fourth useful equation can be obtained from another algebraic manipulation of previous equations.

If we solve $v = v_0 + at$ for t, we get

$$t = \frac{v - v_0}{a}.$$

Substituting this and $\bar{v} = \frac{v_0 + v}{2}$ into $x = x_0 + \bar{v} t$, we get

$$v^2 = v_0^2 + 2a(x - x_0) \text{ (constant a).}$$

Example 2.6.4: Calculating Final Velocity: Dragsters

Calculate the final velocity of the dragster in Example 2.6.3 without using information about time.

Strategy

Draw a sketch.

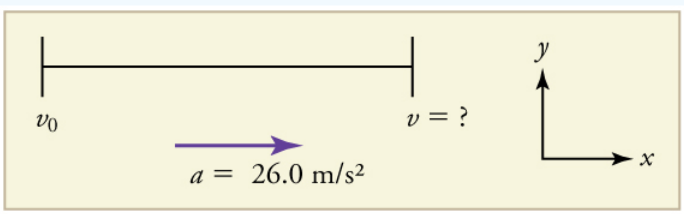

Figure 2.6.9

The equation $v^2 = v_0^2 + 2a(x - x_0)$ is ideally suited to this task because it relates velocities, acceleration, and displacement, and no time information is required.

Solution

1. Identify the known values. We know that $v_0 = 0$, since the dragster starts from rest. Then we note that $x - x_0 = 402m$ (this was the answer in Example). Finally, the average acceleration was given to be $a = 26.0m/s^2$.

2. Plug the knowns into the equation $v^2 = v_0^2 + 2a(x - x_0)$ and solve for v.

$$v^2 = 0 + 2(26.0m/s^2)(402m).$$

Thus

$$v^2 = 2.09 \times 10^4 m^2/s^2.$$

To get v, we take the square root:

$$v = \sqrt{2.09 \times 10^4 m^2/s^2} = 145m/s.$$

Discussion

145 m/s is about 522 km/h or about 324 mi/h, but even this breakneck speed is short of the record for the quarter mile. Also, note that a square root has two values; we took the positive value to indicate a velocity in the same direction as the acceleration.

An examination of the equation $v^2 = v_0^2 + 2a(x - x_0)$ can produce further insights into the general relationships among physical quantities:

- The final velocity depends on how large the acceleration is and the distance over which it acts
- For a fixed deceleration, a car that is going twice as fast doesn't simply stop in twice the distance—it takes much further to stop. (This is why we have reduced speed zones near schools.)

Putting Equations Together

In the following examples, we further explore one-dimensional motion, but in situations requiring slightly more algebraic manipulation. The examples also give insight into problem-solving techniques. The box below provides easy reference to the equations needed.

SUMMARY OF KINEMATIC EQUATIONS (CONSTANT a)

$$x = x_0 + \bar{v}t$$

$$\bar{v} = \frac{v_0 + v}{2}$$

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

Example 2.6.5: Calculating Displacement: How Far Does a Car Go When Coming to a Halt?

On dry concrete, a car can decelerate at a rate of $7.00m/s^2$, whereas on wet concrete it can decelerate at only $5.00m/s^2$. Find the distances necessary to stop a car moving at 30.0 m/s (about 110 km/h)

a. on dry concrete and
b. on wet concrete.
c. Repeat both calculations, finding the displacement from the point where the driver sees a traffic light turn red, taking into account his reaction time of 0.500 s to get his foot on the brake.

Strategy

Draw a sketch.

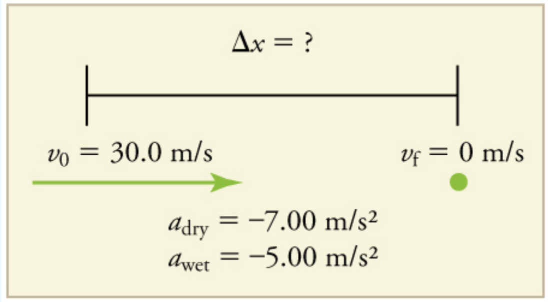

Figure 2.6.10

In order to determine which equations are best to use, we need to list all of the known values and identify exactly what we need to solve for. We shall do this explicitly in the next several examples, using tables to set them off.

Solution for (a)

1. Identify the knowns and what we want to solve for. We know that $v_0 = 30.0 m/s; v = 0; a = -7.00 m/s^2$ (a is negative because it is in a direction opposite to velocity). We take x_0 to be 0. We are looking for displacement Δx, or $x - x_0$.

2. Identify the equation that will help up solve the problem. The best equation to use is

$$v^2 = v_0^2 + 2a(x - x_0) \ .$$

This equation is best because it includes only one unknown, x. We know the values of all the other variables in this equation. (There are other equations that would allow us to solve for x, but they require us to know the stopping time, t, which we do not know. We could use them but it would entail additional calculations.)

3. Rearrange the equation to solve for x.

$$x - x_0 = \frac{v^2 - v_0^2}{2a}$$

4. Enter known values.

$$x - 0 = \frac{0^2 - (30.0 m/s)^2}{2(-7.00 m/s^2)}$$

Thus,

$$x = 64.3 m \text{ on dry concrete.}$$

Solution for (b)

This part can be solved in exactly the same manner as Part A. The only difference is that the deceleration is $-5.00 m/s^2$. The result is

$$x_{wet} = 90.0 m \text{ on wet concrete.}$$

Solution for (c)

Once the driver reacts, the stopping distance is the same as it is in Parts A and B for dry and wet concrete. So to answer this question, we need to calculate how far the car travels during the reaction time, and then add that to the stopping time. It is reasonable to assume that the velocity remains constant during the driver's reaction time.

1. Identify the knowns and what we want to solve for. We know that $\bar{v} = 30.0 m/s; t_{reaction} = 0.500 s; a_{reaction} = 0$. We take $x_{0-reaction}$ to be 0. We are looking for $x_{reaction}$.

2. Identify the best equation to use.

$x = x_0 + \bar{v}t$ works well because the only unknown value is x, which is what we want to solve for.

3. Plug in the knowns to solve the equation.

$$x = 0 + (30.0 m/s)(0.500 s) = 15.0 m.$$

This means the car travels 15.0 m while the driver reacts, making the total displacements in the two cases of dry and wet concrete 15.0 m greater than if he reacted instantly.

4. Add the displacement during the reaction time to the displacement when braking.

$$x_{braking} + x_{reaction} = x_{total}$$

a. 64.3 m + 15.0 m = 79.3 m when dry
b. 90.0 m + 15.0 m = 105 m when wet

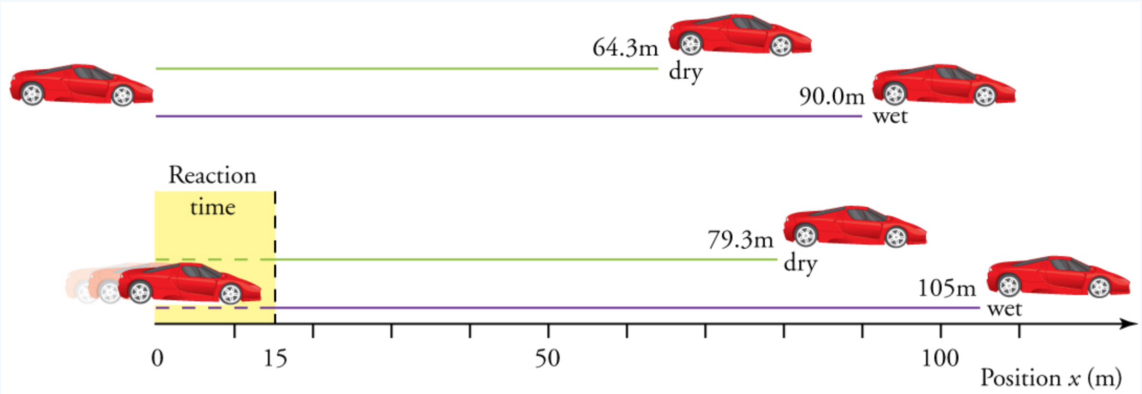

Figure 2.6.11: The distance necessary to stop a car varies greatly, depending on road conditions and driver reaction time. Shown here are the braking distances for dry and wet pavement, as calculated in this example, for a car initially traveling at 30.0 m/s. Also shown are the total distances traveled from the point where the driver first sees a light turn red, assuming a 0.500 s reaction time.

Discussion

The displacements found in this example seem reasonable for stopping a fast-moving car. It should take longer to stop a car on wet rather than dry pavement. It is interesting that reaction time adds significantly to the displacements. But more important is the general approach to solving problems. We identify the knowns and the quantities to be determined and then find an appropriate equation. There is often more than one way to solve a problem. The various parts of this example can in fact be solved by other methods, but the solutions presented above are the shortest.

Example 2.6.5: Calculating Time - A Car Merges into Traffic

Suppose a car merges into freeway traffic on a 200-m-long ramp. If its initial velocity is 10.0 m/s and it accelerates at \(\displaystyle .00 m/s^2\), how long does it take to travel the 200 m up the ramp? (Such information might be useful to a traffic engineer.)

Strategy

Draw a sketch.

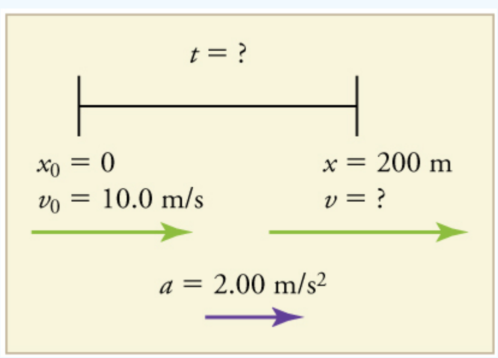

Figure 2.6.12: We are asked to solve for the time t. As before, we identify the known quantities in order to choose a convenient physical relationship (that is, an equation with one unknown, t).

Solution

1. Identify the knowns and what we want to solve for. We know that $v_0 = 10 m/s$; $a = 2.00 m/s^2$; and $x = 200 m$.

2. We need to solve for t. Choose the best equation. $x = x_0 + v_0 t + \frac{1}{2} a t^2$ works best because the only unknown in the equation is the variable t for which we need to solve.

3. We will need to rearrange the equation to solve for t. In this case, it will be easier to plug in the knowns first.

$$200m = 0m + (10.0m/s)t + \frac{1}{2}(2.00m/s^2)t^2$$

4. Simplify the equation. The units of meters (m) cancel because they are in each term. We can get the units of seconds (s) to cancel by taking $t = ts$, where t is the magnitude of time and s is the unit. Doing so leaves

$$200 = 10t + t^2.$$

5. Use the quadratic formula to solve for t.

(a) Rearrange the equation to get 0 on one side of the equation.

$$t^2 + 10t - 200 = 0$$

This is a quadratic equation of the form

$$at^2 + bt + c = 0,$$

where the constants are $a = 1.00$, $b = 10.0$,and $c = -200$.

(b) Its solutions are given by the quadratic formula:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

This yields two solutions for t, which are

$$t = 10.0 \text{ and } -20.0.$$

In this case, then, the time is $t = t$ in seconds, or

$$t = 10.0s \text{ and } -20.0s.$$

A negative value for time is unreasonable, since it would mean that the event happened 20 s before the motion began. We can discard that solution. Thus,

$$t = 10.0s.$$

Discussion

Whenever an equation contains an unknown squared, there will be two solutions. In some problems both solutions are meaningful, but in others, such as the above, only one solution is reasonable. The 10.0 s answer seems reasonable for a typical freeway on-ramp.

With the basics of kinematics established, we can go on to many other interesting examples and applications. In the process of developing kinematics, we have also glimpsed a general approach to problem solving that produces both correct answers and insights into physical relationships. Problem-Solving Basics discusses problem-solving basics and outlines an approach that will help you succeed in this invaluable task.

MAKING CONNECTIONS: TAKE-HOME EXPERIMENT—BREAKING NEWS

We have been using SI units of meters per second squared to describe some examples of acceleration or deceleration of cars, runners, and trains. To achieve a better feel for these numbers, one can measure the braking deceleration of a car doing a slow (and safe) stop. Recall that, for average acceleration, $\bar{a} = \Delta v/\Delta t$. While traveling in a car, slowly apply the brakes as you come up to a stop sign. Have a passenger note the initial speed in miles per hour and the time taken (in seconds) to stop. From this, calculate the deceleration in miles per hour per second. Convert this to meters per second squared and compare with other decelerations mentioned in this chapter. Calculate the distance traveled in braking.

Exercise 2.6.1

A manned rocket accelerates at a rate of $20 m/s^2$ during launch. How long does it take the rocket to reach a velocity of 400 m/s?

Answer

To answer this, choose an equation that allows you to solve for time t, given only a, v_0, and v.

$$v = v_0 + at$$

Rearrange to solve for t.

$$t = \frac{v - v_0}{a} = \frac{400 m/s - 0 m/s}{20 m/s^2} = 20 s$$

Summary

- To simplify calculations we take acceleration to be constant, so that $\bar{a} = a$ at all times.
- We also take initial time to be zero.
- Initial position and velocity are given a subscript 0; final values have no subscript. Thus,

$$\Delta t = t$$

$$\Delta x = x - x_0$$

$$\Delta v = v - v_0$$

- The following kinematic equations for motion with constant a are useful:

$$x = x_0 + \bar{v}t$$

$$\bar{v} = \frac{v_0 + v}{2}$$

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

- In vertical motion, y is substituted for x.

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2.7: Problem-Solving Basics for One-Dimensional Kinematics

Learning Objectives

By the end of this section, you will be able to:

- Apply problem-solving steps and strategies to solve problems of one-dimensional kinematics.
- Apply strategies to determine whether or not the result of a problem is reasonable, and if not, determine the cause.

Problem-Solving Basics for One-Dimensional Kinematics

Problem-solving skills are obviously essential to success in a quantitative course in physics. More importantly, the ability to apply broad physical principles, usually represented by equations, to specific situations is a very powerful form of knowledge. It is much more powerful than memorizing a list of facts. Analytical skills and problem-solving abilities can be applied to new situations, whereas a list of facts cannot be made long enough to contain every possible circumstance. Such analytical skills are useful both for solving problems in this text and for applying physics in everyday and professional life.

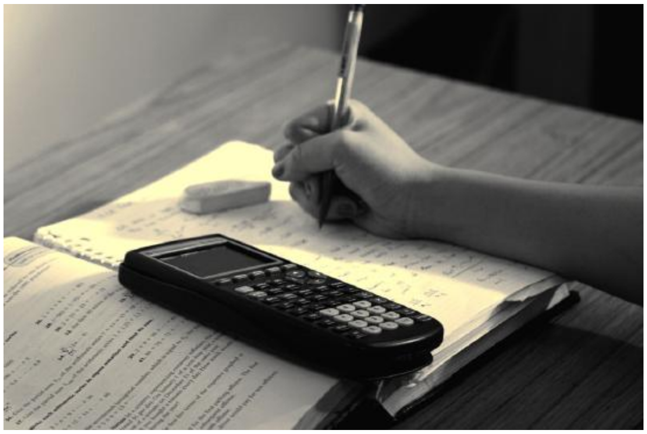

Figure 2.7.1: Problem-solving skills are essential to your success in Physics. (credit: scui3asteveo, Flickr)

Problem-Solving Steps

While there is no simple step-by-step method that works for every problem, the following general procedures facilitate problem solving and make it more meaningful. A certain amount of creativity and insight is required as well.

Step 1

Examine the situation to determine which physical principles are involved. It often helps to *draw a simple sketch* at the outset. You will also need to decide which direction is positive and note that on your sketch. Once you have identified the physical principles, it is much easier to find and apply the equations representing those principles. Although finding the correct equation is essential, keep in mind that equations represent physical principles, laws of nature, and relationships among physical quantities. Without a conceptual understanding of a problem, a numerical solution is meaningless.

Step 2

Make a list of what is given or can be inferred from the problem as stated (identify the knowns). Many problems are stated very succinctly and require some inspection to determine what is known. A sketch can also be very useful at this point. Formally identifying the knowns is of particular importance in applying physics to real-world situations. Remember, “stopped” means velocity is zero, and we often can take initial time and position as zero.

Step 3

Identify exactly what needs to be determined in the problem (identify the unknowns). In complex problems, especially, it is not always obvious what needs to be found or in what sequence. Making a list can help.

Step 4

Find an equation or set of equations that can help you solve the problem. Your list of knowns and unknowns can help here. It is easiest if you can find equations that contain only one unknown—that is, all of the other variables are known, so you can easily solve for the unknown. If the equation contains more than one unknown, then an additional equation is needed to solve the problem. In some problems, several unknowns must be determined to get at the one needed most. In such problems it is especially important to keep physical principles in mind to avoid going astray in a sea of equations. You may have to use two (or more) different equations to get the final answer.

Step 5

Substitute the knowns along with their units into the appropriate equation, and obtain numerical solutions complete with units. This step produces the numerical answer; it also provides a check on units that can help you find errors. If the units of the answer are incorrect, then an error has been made. However, be warned that correct units do not guarantee that the numerical part of the answer is also correct.

Step 6

Check the answer to see if it is reasonable: Does it make sense? This final step is extremely important—the goal of physics is to accurately describe nature. To see if the answer is reasonable, check both its magnitude and its sign, in addition to its units. Your judgment will improve as you solve more and more physics problems, and it will become possible for you to make finer and finer judgments regarding whether nature is adequately described by the answer to a problem. This step brings the problem back to its conceptual meaning. If you can judge whether the answer is reasonable, you have a deeper understanding of physics than just being able to mechanically solve a problem.

When solving problems, we often perform these steps in different order, and we also tend to do several steps simultaneously. There is no rigid procedure that will work every time. Creativity and insight grow with experience, and the basics of problem solving become almost automatic. One way to get practice is to work out the text's examples for yourself as you read. Another is to work as many end-of-section problems as possible, starting with the easiest to build confidence and progressing to the more difficult. Once you become involved in physics, you will see it all around you, and you can begin to apply it to situations you encounter outside the classroom, just as is done in many of the applications in this text.

Unreasonable Results

Physics must describe nature accurately. Some problems have results that are unreasonable because one premise is unreasonable or because certain premises are inconsistent with one another. The physical principle applied correctly then produces an unreasonable result. For example, if a person starting a foot race accelerates at $0.40 m/s^2$ for 100 s, his final speed will be $40 m/s$ (about 150 km/h)—clearly unreasonable because the time of 100 s is an unreasonable premise. The physics is correct in a sense, but there is more to describing nature than just manipulating equations correctly. Checking the result of a problem to see if it is reasonable does more than help uncover errors in problem solving—it also builds intuition in judging whether nature is being accurately described.

Use the following strategies to determine whether an answer is reasonable and, if it is not, to determine what is the cause.

Step 1

Solve the problem using strategies as outlined and in the format followed in the worked examples in the text. In the example given in the preceding paragraph, you would identify the givens as the acceleration and time and use the equation below to find the unknown final velocity. That is,

$$v = v0 + at = 0 + (0.40 m/s2)(100 s) = 40 m/s. \tag{2.7.1}$$

Step 2

Check to see if the answer is reasonable. Is it too large or too small, or does it have the wrong sign, improper units, …? In this case, you may need to convert meters per second into a more familiar unit, such as miles per hour.

$$(40 ms)(3.28 ftm)(1 mi5280 ft)(60 smin)(60 min1 h) = 89 mph \tag{2.7.2}$$

This velocity is about four times greater than a person can run—so it is too large.

Step 3

If the answer is unreasonable, look for what specifically could cause the identified difficulty. In the example of the runner, there are only two assumptions that are suspect. The acceleration could be too great or the time too long. First look at the acceleration and think about what the number means. If someone accelerates at 0.40 m/s2, their velocity is increasing by 0.4 m/s each second. Does this seem reasonable? If so, the time must be too long. It is not possible for someone to accelerate at a constant rate of 0.40 m/s2 for 100 s (almost two minutes).

Summary

- The six basic problem solving steps for physics are:

  Step 1. Examine the situation to determine which physical principles are involved.

  Step 2. Make a list of what is given or can be inferred from the problem as stated (identify the knowns).

  Step 3. Identify exactly what needs to be determined in the problem (identify the unknowns).

  Step 4. Find an equation or set of equations that can help you solve the problem.

  Step 5. Substitute the knowns along with their units into the appropriate equation, and obtain numerical solutions complete with units.

  Step 6. Check the answer to see if it is reasonable: Does it make sense?

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2.8: Falling Objects

> **Learning Objectives**
>
> By the end of this section, you will be able to:
>
> - Describe the effects of gravity on objects in motion.
> - Describe the motion of objects that are in free fall.
> - Calculate the position and velocity of objects in free fall.

Falling Objects

Falling objects form an interesting class of motion problems. For example, we can estimate the depth of a vertical mine shaft by dropping a rock into it and listening for the rock to hit the bottom. By applying the kinematics developed so far to falling objects, we can examine some interesting situations and learn much about gravity in the process.

Gravity

The most remarkable and unexpected fact about falling objects is that, if air resistance and friction are negligible, then in a given location all objects fall toward the center of Earth with the *same constant acceleration*, *independent of their mass*. This experimentally determined fact is unexpected, because we are so accustomed to the effects of air resistance and friction that we expect light objects to fall slower than heavy ones.

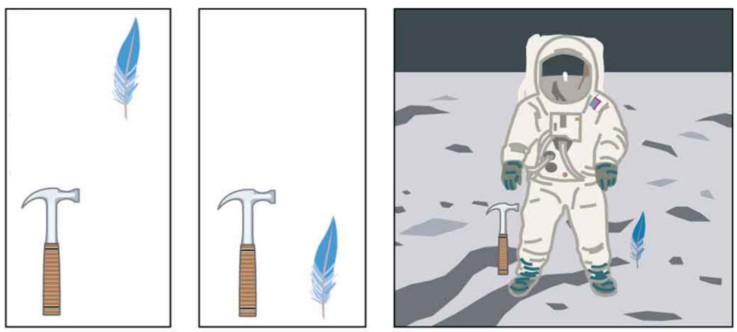


Figure 2.8.1: A hammer and a feather will fall with the same constant acceleration if air resistance is considered negligible. This is a general characteristic of gravity not unique to Earth, as astronaut David R. Scott demonstrated on the Moon in 1971, where the acceleration due to gravity is only .67 m/s2.

In the real world, air resistance can cause a lighter object to fall slower than a heavier object of the same size. A tennis ball will reach the ground after a hard baseball dropped at the same time. (It might be difficult to observe the difference if the height is not large.) Air resistance opposes the motion of an object through the air, while friction between objects—such as between clothes and a laundry chute or between a stone and a pool into which it is dropped—also opposes motion between them. For the ideal situations of these first few chapters, an object *falling without air resistance or friction* is defined to be in free-fall.

The force of gravity causes objects to fall toward the center of Earth. The acceleration of free-falling objects is therefore called the acceleration due to gravity. The acceleration due to gravity is *constant*, which means we can apply the kinematics equations to any falling object where air resistance and friction are negligible. This opens a broad class of interesting situations to us. The acceleration due to gravity is so important that its magnitude is given its own symbol, . It is constant at any given location on Earth and has the average value

$$g = 9.80 m/s^2. \tag{2.8.1}$$

Although varies from .78 m/s2 to 9.83 m/s2, depending on latitude, altitude, underlying geological formations, and local topography, the average value of 9.80 m/s2 will be used in this text unless otherwise specified. The direction of the acceleration due to gravity is downward (towards the center of Earth). In fact, its direction defines what we call vertical. Note that whether the acceleration a in the kinematic equations has the value +g or −g depends on how we define our coordinate system. If we define the upward direction as negative, then a=−g=−9.80 m/s2, and if we define the downward direction as positive, then =g=9.80 m/s2.

One-Dimensional Motion Involving Gravity

The best way to see the basic features of motion involving gravity is to start with the simplest situations and then progress toward more complex ones. So we start by considering straight up and down motion with no air resistance or friction. These assumptions mean that the velocity (if there is any) is vertical. If the object is dropped, we know the initial velocity is zero. Once the object has left contact with whatever held or threw it, the object is in free-fall. Under these circumstances, the motion is one-dimensional and has constant acceleration of magnitude g. We will also represent vertical displacement with the symbol y and use x for horizontal displacement.

KINEMATIC EQUATIONS FOR OBJECTS IN FREE-FALL WHERE ACCELERATION = -G

$$v=v_0-gt$$

$$y=y_0+v_0t-\frac{1}{2}gt^2$$

$$v^2=v_0^2-2g(y-y_0)$$

Example 2.8.1: Calculating Position and Velocity of a Falling Object: A Rock Thrown Upward

A person standing on the edge of a high cliff throws a rock straight up with an initial velocity of 13.0 m/s. The rock misses the edge of the cliff as it falls back to earth. Calculate the position and velocity of the rock 1.00 s, 2.00 s, and 3.00 s after it is thrown, neglecting the effects of air resistance.

Strategy

Draw a sketch.

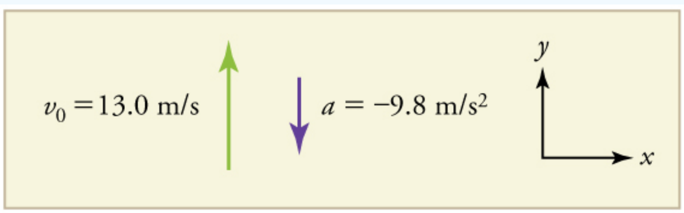


Figure 2.8.2

We are asked to determine the position y at various times. It is reasonable to take the initial position y_0 to be zero. This problem involves one-dimensional motion in the vertical direction. We use plus and minus signs to indicate direction, with up being positive and down negative. Since up is positive, and the rock is thrown upward, the initial velocity must be positive too. The acceleration due to gravity is downward, so a is negative. It is crucial that the initial velocity and the acceleration due to gravity have opposite signs. Opposite signs indicate that the acceleration due to gravity opposes the initial motion and will slow and eventually reverse it.

Since we are asked for values of position and velocity at three times, we will refer to these as y_1 and v_1; y_2 and v_2; and y_3 and v_3.

Solution for Position y_1

1. Identify the knowns. We know that $y_0=0$; $v_0=13.0 m/s$; $a=-g=-9.80 m/s^2$; and $t=1.00 s$.
2. Identify the best equation to use. We will use $y=y_0+v_0t+\frac{1}{2}at^2$ because it includes only one unknown, y (or y_1, here), which is the value we want to find.
3. Plug in the known values and solve for y_1.

$$y_1=0+(13.0 m/s)(1.00 s)+\frac{1}{2}(-9.80 m/s^2)(1.00 s)^2=8.10 m$$

Discussion

The rock is 8.10 m above its starting point at $t=1.00 s$, since $y_1>y_0$. It could be moving up or down; the only way to tell is to calculate v_1 and find out if it is positive or negative.

Solution for Velocity v_1

1. Identify the knowns. We know that $y_0 = 0; v_0 = 13.0 m/s; a = -g = -9.80 m/s^2$; and $t = 1.00 s$. We also know from the solution above that $y_1 = 8.10 m$.

2. Identify the best equation to use. The most straightforward is $v = v_0 - gt$ (from $v = v_0 + at$, where a=gravitational acceleration=−g).

3. Plug in the knowns and solve.

$$v_1 = v_0 - gt = 13.0 m/s - (9.80 m/s^2)(1.00 s) = 3.20 m/s$$

Discussion

The positive value for v_1 means that the rock is still heading upward at $t = 1.00 s$. However, it has slowed from its original 13.0 m/s, as expected.

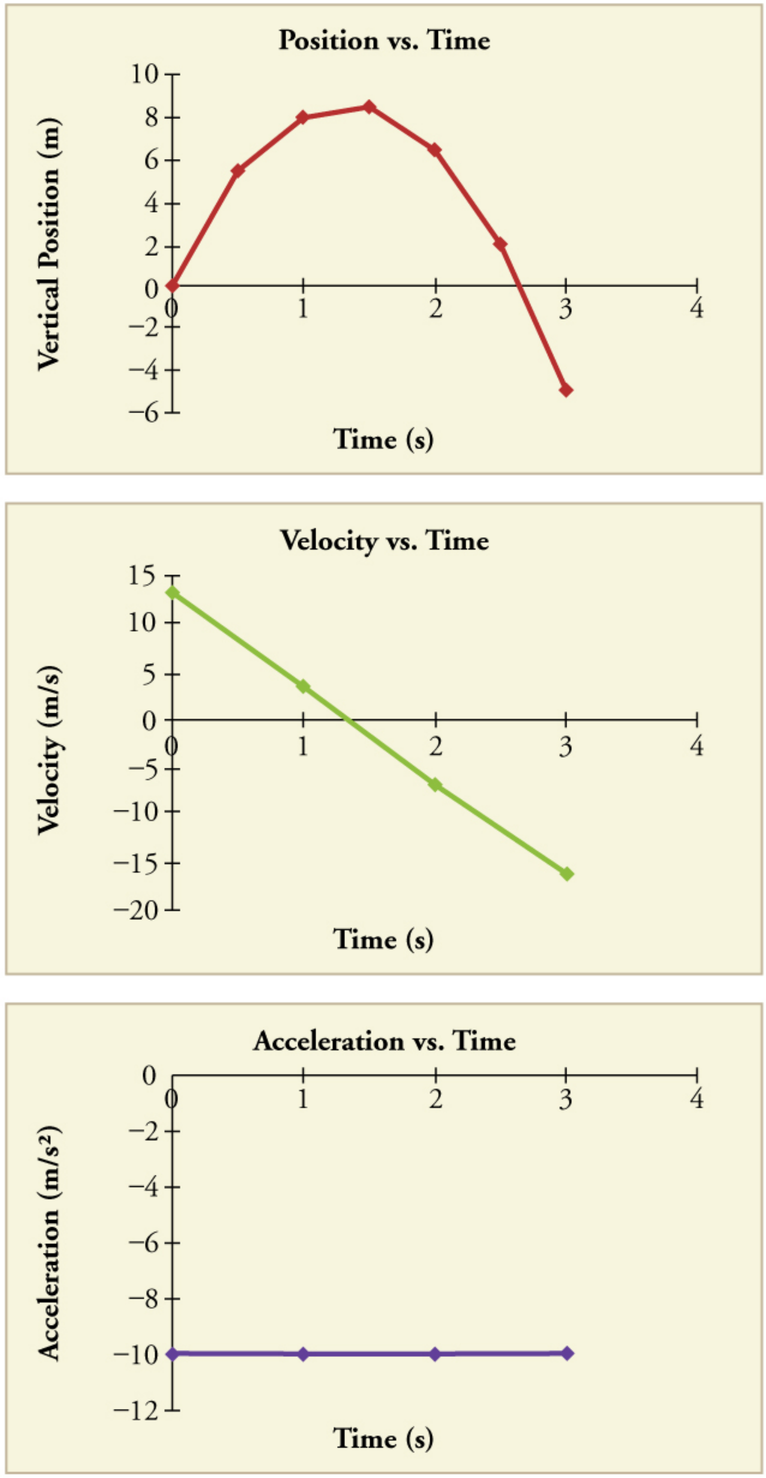
Solution for Remaining Times

The procedures for calculating the position and velocity at $t = 2.00 s$ and $3.00 s$ are the same as those above. The results are summarized in Table and illustrated in Figure.

Results

Time, *t*	Position, *y*	Velocity, *v*	Acceleration, *a*
.00 s	.10 m	.20 m/s\)	9.80 m/s^2\)
.00 s	.40 m	6.60 m/s\)	9.80 m/s^2\)
.00 s	5.10 m	16.4 m/s\)	9.80 m/s^2\)

Graphing the data helps us understand it more clearly.



PageIndex3: *Vertical position, vertical velocity, and vertical acceleration vs. time for a rock thrown vertically up at the edge of a cliff. Notice that velocity changes linearly with time and that acceleration is constant. Misconception Alert! Notice that the position vs. time graph shows vertical position only. It is easy to get the impression that the graph shows some horizontal motion—the shape of the graph looks like the path of a projectile. But this is not the case; the horizontal axis is time, not space. The actual path of the rock in space is straight up, and straight down.*

Discussion

The interpretation of these results is important. At 1.00 s the rock is above its starting point and heading upward, since y_1 and v_1 are both positive. At 2.00 s, the rock is still above its starting point, but the negative velocity means it is moving downward. At 3.00 s, both y_3 and v_3 are negative, meaning the rock is below its starting point and continuing to move downward. Notice that when the rock is at its highest point (at 1.5 s), its velocity is zero, but its acceleration is still $-9.80 m/s^2$. Its acceleration is $-9.80 m/s^2$ for the whole trip—while it is moving up and while it is moving down. Note that the values for y are the positions (or displacements) of the rock, not the total distances traveled. Finally, note that free-fall applies to upward motion as well as downward. Both have the same acceleration—the acceleration due to gravity, which remains constant the entire time. Astronauts training in the famous Vomit Comet, for example, experience free-fall while arcing up as well as down, as we will discuss in more detail later.The interpretation of these results is important. At 1.00 s the rock is above its starting point and heading upward, since y1 and v1 are both positive. At 2.00 s, the rock is still above its starting point, but the negative velocity means it is moving downward. At 3.00 s, both y3 and v3 are negative, meaning the rock is below its starting point and continuing to move downward. Notice that when the rock is at its highest point (at 1.5 s), its velocity is zero, but its acceleration is still −9.80 m/s2. Its acceleration is −9.80 m/s2 for the whole trip—while it is moving up and while it is moving down. Note that the values for y are the positions (or displacements) of the rock, not the total distances traveled. Finally, note that free-fall applies to upward motion as well as downward. Both have the same acceleration—the acceleration due to gravity, which remains constant the entire time. Astronauts training in the famous Vomit Comet, for example, experience free-fall while arcing up as well as down, as we will discuss in more detail later.

MAKING CONNECTIONS: TAKE-HOME EXPERIMENT—REACTION TIME

A simple experiment can be done to determine your reaction time. Have a friend hold a ruler between your thumb and index finger, separated by about 1 cm. Note the mark on the ruler that is right between your fingers. Have your friend drop the ruler unexpectedly, and try to catch it between your two fingers. Note the new reading on the ruler. Assuming acceleration is that due to gravity, calculate your reaction time. How far would you travel in a car (moving at 30 m/s) if the time it took your foot to go from the gas pedal to the brake was twice this reaction time?

Example 2.8.2: Calculating Velocity of a Falling Object: A Rock Thrown Down

What happens if the person on the cliff throws the rock straight down, instead of straight up? To explore this question, calculate the velocity of the rock when it is 5.10 m below the starting point, and has been thrown downward with an initial speed of 13.0 m/s.

Strategy

Draw a sketch.

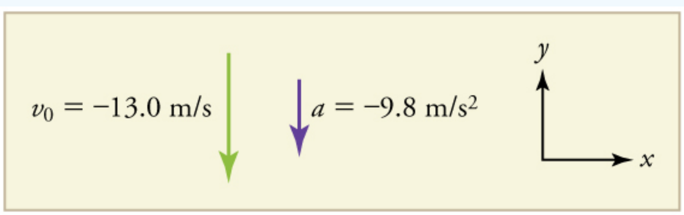


Figure 2.8.4

Since up is positive, the final position of the rock will be negative because it finishes below the starting point at $y_0 = 0$. Similarly, the initial velocity is downward and therefore negative, as is the acceleration due to gravity. We expect the final velocity to be negative since the rock will continue to move downward.

Solution

1. Identify the knowns. $y_0 = 0; y_1 = -5.10 m; v_0 = -13.0 m/s; a = -g = -9.80 m/s^2$.

2. Choose the kinematic equation that makes it easiest to solve the problem. The equation $v^2 = v_0^2 + 2a(y - y_0)$ works well because the only unknown in it is v. (We will plug y_1 in for y.)

3. Enter the known values

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$$v^2 = (-13.0m/s)^2 + 2(-9.80m/s^2)(-5.10m - 0m) = 268.96m^2/s^2,$$

where we have retained extra significant figures because this is an intermediate result.

Taking the square root, and noting that a square root can be positive or negative, gives

$$v = \pm 16.4m/s.$$

The negative root is chosen to indicate that the rock is still heading down. Thus,

$$v = -16.4m/s.$$

Discussion

Note that *this is exactly the same velocity the rock had at this position when it was thrown straight upward with the same initial speed.* (See Example and Figure(a).) This is not a coincidental result. Because we only consider the acceleration due to gravity in this problem, the speed of a falling object depends only on its initial speed and its vertical position relative to the starting point. For example, if the velocity of the rock is calculated at a height of 8.10 m above the starting point (using the method from Example) when the initial velocity is 13.0 m/s straight up, a result of $\pm 3.20m/s$ is obtained. Here both signs are meaningful; the positive value occurs when the rock is at 8.10 m and heading up, and the negative value occurs when the rock is at 8.10 m and heading back down. It has the same speed but the opposite direction.

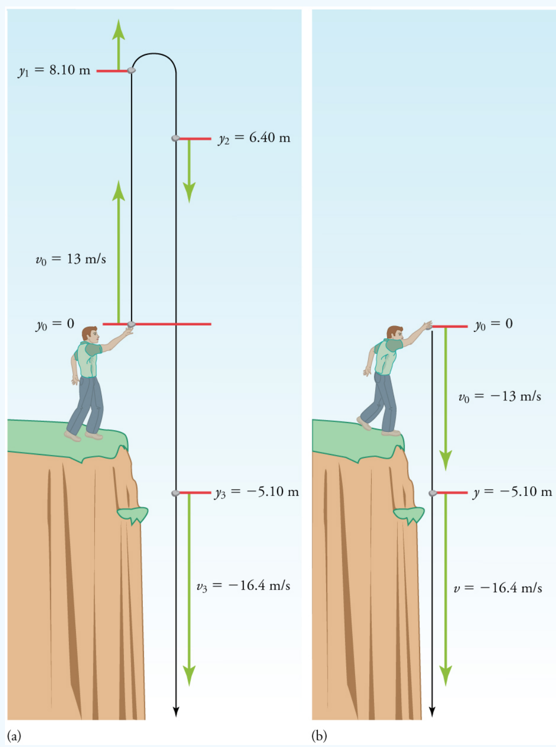
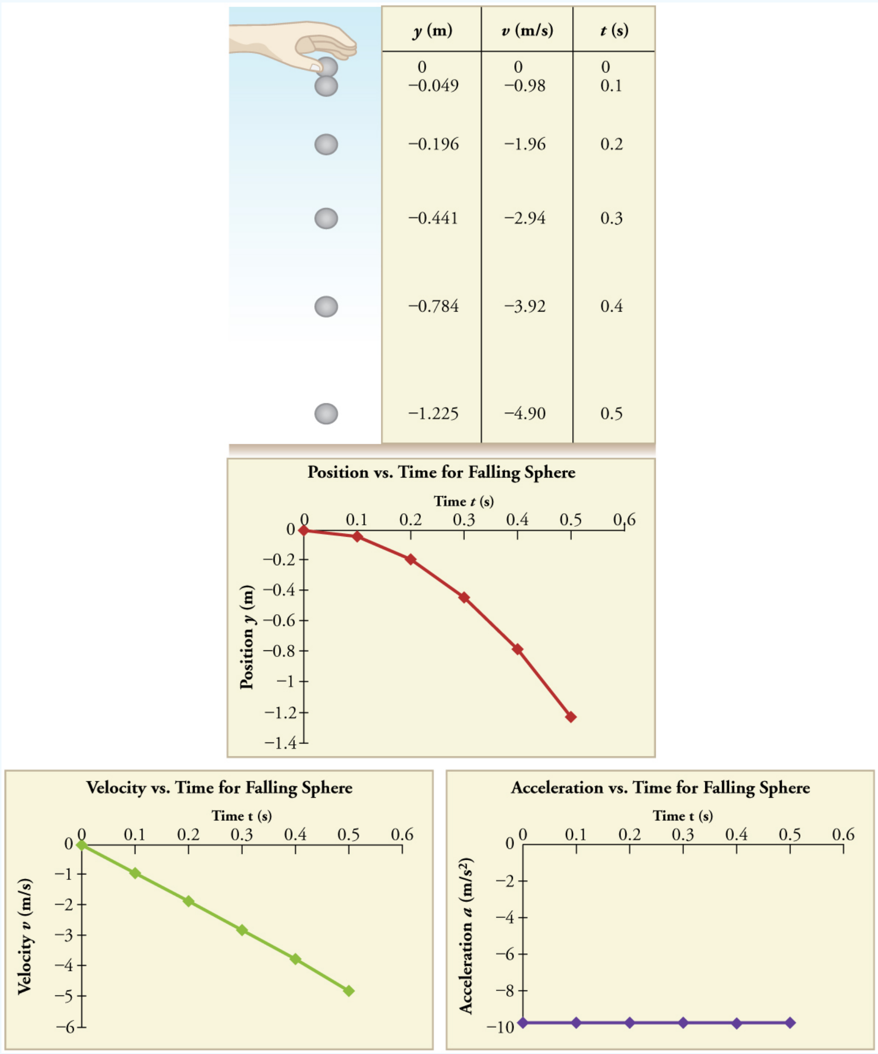


Figure $PageIndex5$: (a) A person throws a rock straight up, as explored in Example. The arrows are velocity vectors at 0, 1.00, 2.00, and 3.00 s. (b) A person throws a rock straight down from a cliff with the same initial speed as before, as in Example. Note that at the same distance below the point of release, the rock has the same velocity in both cases.

Another way to look at it is this: In Example, the rock is thrown up with an initial velocity of $13.0m/s$. It rises and then falls back down. When its position is $y = 0$ on its way back down, its velocity is $-13.0m/s$. That is, it has the same speed on its way down as on its way up. We would then expect its velocity at a position of $y = -5.10$ m to be the same whether we have thrown it upwards at $+13.0m/s$ or thrown it downwards at $-13.0m/s$. The velocity of the rock on its way down from $y = 0$ is the same whether we have thrown it up or down to start with, as long as the speed with which it was initially thrown is the same.

Example 2.8.3: Find g from Data on a Falling Object

The acceleration due to gravity on Earth differs slightly from place to place, depending on topography (e.g., whether you are on a hill or in a valley) and subsurface geology (whether there is dense rock like iron ore as opposed to light rock like salt beneath you.) The precise acceleration due to gravity can be calculated from data taken in an introductory physics laboratory course. An object, usually a metal ball for which air resistance is negligible, is dropped and the time it takes to fall a known distance is measured. See, for example, Figure. Very precise results can be produced with this method if sufficient care is taken in measuring the distance fallen and the elapsed time.



y (m)	v (m/s)	t (s)
0	0	0
−0.049	−0.98	0.1
−0.196	−1.96	0.2
−0.441	−2.94	0.3
−0.784	−3.92	0.4
−1.225	−4.90	0.5

Figure 2.8.6: Positions and velocities of a metal ball released from rest when air resistance is negligible. Velocity is seen to increase linearly with time while displacement increases with time squared. Acceleration is a constant and is equal to gravitational acceleration.

Suppose the ball falls 1.0000 m in 0.45173 s. Assuming the ball is not affected by air resistance, what is the precise acceleration due to gravity at this location?

Strategy

Draw a sketch.

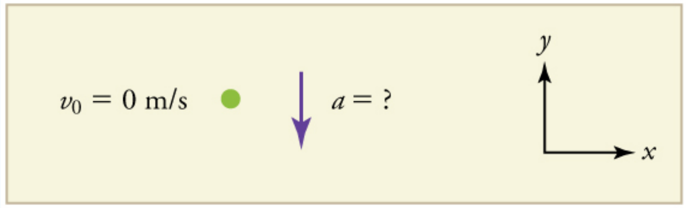


Figure 2.8.7:

We need to solve for acceleration a. Note that in this case, displacement is downward and therefore negative, as is acceleration.

Solution

1. Identify the knowns. $y_0 = 0; y = -1.0000m; t = 0.45173; v_0 = 0$.
2. Choose the equation that allows you to solve for a using the known values.

$$y = y_0 + v_0 t + \frac{1}{2}at^2$$

3. Substitute 0 for v_0 and rearrange the equation to solve for a. Substituting 0 for v_0 yields

$$y = y_0 + \frac{1}{2}at^2 \ .$$

Solving for a gives

$$a = \frac{2(y-y_0)}{t^2} \ .$$

4. Substitute known values yields

$$a = \frac{2(-1.0000m-0)}{(0.45173s)^2} = -9.8010m/s^2,$$

so, because $a = -g$ with the directions we have chosen,

$$g = 9.8010m/s^2.$$

Discussion

The negative value for a indicates that the gravitational acceleration is downward, as expected. We expect the value to be somewhere around the average value of $9.80m/s^2$, so $9.8010m/s^2$ makes sense. Since the data going into the calculation are relatively precise, this value for g is more precise than the average value of $9.80m/s$; it represents the local value for the acceleration due to gravity.

Exercise 2.8.1

A chunk of ice breaks off a glacier and falls 30.0 meters before it hits the water. Assuming it falls freely (there is no air resistance), how long does it take to hit the water?

Answer

We know that initial position $y_0 = 0$, final position $y = -30.0m$, and $a = -g = -9.80m/s^2$. We can then use the equation $y = y_0 + v_0 t + \frac{1}{2}at^2$ to solve for t. Inserting $a = -g$, we obtain

$$y = 0 + 0 - \frac{1}{2}gt^2$$

$$t^2 = \frac{2y}{-g}$$

$$t = \pm\sqrt{\frac{2y}{-g}} = \pm\sqrt{\frac{2(-30.0m)}{-9.80m/s^2}} = \pm\sqrt{6.12s^2} = 2.47s \approx 2.5s$$

where we take the positive value as the physically relevant answer. Thus, it takes about 2.5 seconds for the piece of ice to hit the water.

PHET EXPLORATIONS: EQUATION GRAPHER

Learn about graphing polynomials. The shape of the curve changes as the constants are adjusted. View the curves for the individual terms (e.g. $y = bx$) to see how they add to generate the polynomial curve.

PhET Interactive Simulation

Figure 2.8.8: Equation Grapher

Summary

- An object in free-fall experiences constant acceleration if air resistance is negligible.
- On Earth, all free-falling objects have an acceleration due to gravity g, which averages

$$g = 9.80 m/s^2.$$

- Whether the acceleration a should be taken as $+g$ or $-g$ is determined by your choice of coordinate system. If you choose the upward direction as positive, $a = -g = -9.80 m/s^2$ is negative. In the opposite case, $a = +g = 9.80 m/s^2$ is positive. Since acceleration is constant, the kinematic equations above can be applied with the appropriate $+g$ or $-g$ substituted for a.
- For objects in free-fall, up is normally taken as positive for displacement, velocity, and acceleration.

Glossary

free-fall

the state of movement that results from gravitational force only

acceleration due to gravity

acceleration of an object as a result of gravity

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2.9: Graphical Analysis of One-Dimensional Motion

Learning Objectives

By the end of this section, you will be able to:

- Describe a straight-line graph in terms of its slope and *y*-intercept.
- Determine average velocity or instantaneous velocity from a graph of position vs. time.
- Determine average or instantaneous acceleration from a graph of velocity vs. time.
- Derive a graph of velocity vs. time from a graph of position vs. time.
- Derive a graph of acceleration vs. time from a graph of velocity vs. time.

A graph, like a picture, is worth a thousand words. Graphs not only contain numerical information; they also reveal relationships between physical quantities. This section uses graphs of displacement, velocity, and acceleration versus time to illustrate one-dimensional kinematics.

Slopes and General Relationships

First note that graphs in this text have perpendicular axes, one horizontal and the other vertical. When two physical quantities are plotted against one another in such a graph, the horizontal axis is usually considered to be an independent variable and the vertical axis a dependent variable. If we call the horizontal axis the x-axis and the vertical axis the y-axis, as in Figure $2.9.1$, a straight-line graph has the general form

$$y=mx+b. \tag{2.9.1}$$

Here m is the slope, defined to be the rise divided by the run of the straight line (Figure $2.9.1$). The letter b is used for they-intercept, which is the point at which the line crosses the vertical axis.

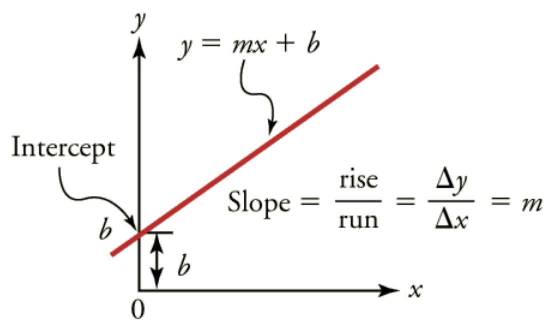

Figure $2.9.1$: A straight-line graph. The equation for a straight line is $y=mx+b$.

Graph of Displacement vs. Time (*a* = 0, so *v* is constant)

Time is usually an independent variable that other quantities, such as displacement, depend upon. A graph of displacement versus time would, thus, have on the vertical axis and on the horizontal axis. Figure $2.9.2$ is just such a straight-line graph. It shows a graph of displacement versus time for a jet-powered car on a very flat dry lake bed in Nevada.

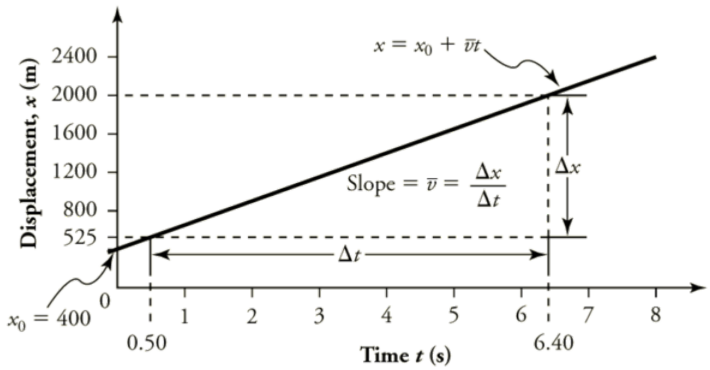

Figure 2.9.2: Graph of displacement versus time for a jet-powered car on the Bonneville Salt Flats.

Using the relationship between dependent and independent variables, we see that the slope in the graph above is average velocity $\bar{v}$ and the intercept is displacement at time zero—that is, x_0. Substituting these symbols into $y = mx + b$ gives

$$x = \bar{v}t + x_0 \tag{2.9.2}$$

or

$$x = x_0 + \bar{v}t. \tag{2.9.3}$$

Thus a graph of displacement versus time gives a general relationship among displacement, velocity, and time, as well as giving detailed numerical information about a specific situation.

THE SLOPE OF X VS. T

The slope of the graph of displacement x vs. time t is velocity v.

$$slope = \frac{\Delta x}{\Delta t} = v$$

Notice that this equation is the same as that derived algebraically from other motion equations in Motion Equations for Constant Acceleration in One Dimension.

From the figure we can see that the car has a displacement of 25 m at 0.50 s and 2000 m at 6.40 s. Its displacement at other times can be read from the graph; furthermore, information about its velocity and acceleration can also be obtained from the graph.

Example 2.9.1:Determining Average Velocity from a Graph of Displacement versus Time: Jet Car

Find the average velocity of the car whose position is graphed in Figure 2.9.2.

Strategy

The slope of a graph of x vs. t is average velocity, since slope equals rise over run. In this case, rise = change in position and run = change in time, so that

$$slope = \frac{\Delta x}{\Delta t} = \bar{v}.$$

Since the slope is constant here, any two points on the graph can be used to find the slope. (Generally speaking, it is most accurate to use two widely separated points on the straight line. This is because any error in reading data from the graph is proportionally smaller if the interval is larger.)

Solution

1. Choose two points on the line. In this case, we choose the points labeled on the graph: (6.4 s, 2000 m) and (0.50 s, 525 m). (Note, however, that you could choose any two points.)
2. Substitute the x and t values of the chosen points into the equation. Remember in calculating change (Δ) we always use final value minus initial value.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{2000m - 525m}{6.4s - 0.50s},$$

yielding

$$v- = 250m/s.$$

Discussion

This is an impressively large land speed (900 km/h, or about 560 mi/h): much greater than the typical highway speed limit of 60 mi/h (27 m/s or 96 km/h), but considerably shy of the record of 343 m/s (1234 km/h or 766 mi/h) set in 1997.

Graphs of Motion when is constant but ≠0

The graphs in Figure $2.9.3$ below represent the motion of the jet-powered car as it accelerates toward its top speed, but only during the time when its acceleration is constant. Time starts at zero for this motion (as if measured with a stopwatch), and the displacement and velocity are initially 200 m and 15 m/s, respectively.

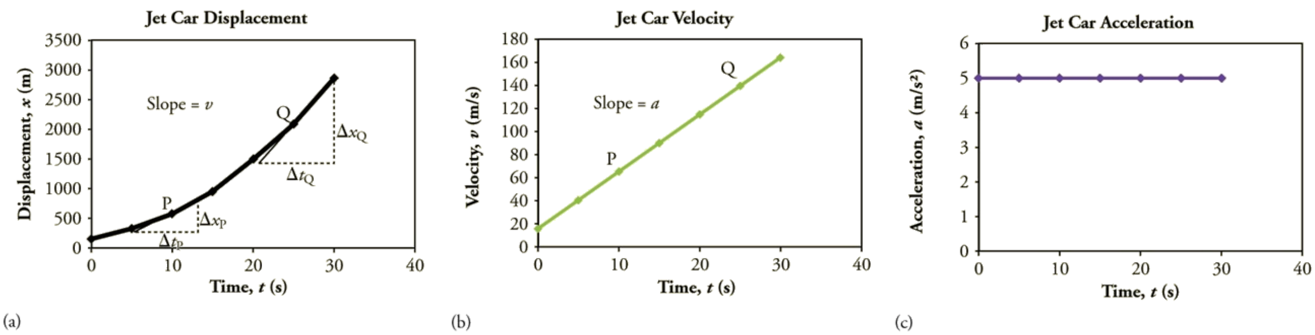

Figure 2.9.3: *Graphs of motion of a jet-powered car during the time span when its acceleration is constant. (a) The slope of an x vs. t graph is velocity. This is shown at two points, and the instantaneous velocities obtained are plotted in the next graph. Instantaneous velocity at any point is the slope of the tangent at that point. (b) The slope of the v vs. t graph is constant for this part of the motion, indicating constant acceleration. (c) Acceleration has the constant value of $5.0m/s^2$ over the time interval plotted.*

Figure $2.9.4$: A U.S. Air Force jet car speeds down a track. (credit: Matt Trostle, Flickr)

The graph of displacement versus time in Figure $2.9.3a$ is a curve rather than a straight line. The slope of the curve becomes steeper as time progresses, showing that the velocity is increasing over time. The slope at any point on a displacement-versus-time graph is the instantaneous velocity at that point. It is found by drawing a straight line tangent to the curve at the point of interest and taking the slope of this straight line. Tangent lines are shown for two points in Figure $2.9.3a$. If this is done at every point on the curve and the values are plotted against time, then the graph of velocity versus time shown in Figure $2.9.3b$ is obtained. Furthermore, the slope of the graph of velocity versus time is acceleration, which is shown in Figure $2.9.3c$.

Example 2.9.2:

Calculate the velocity of the jet car at a time of 25 s by finding the slope of the \(\displaystyle vs. \(\displaystyle graph in the graph below

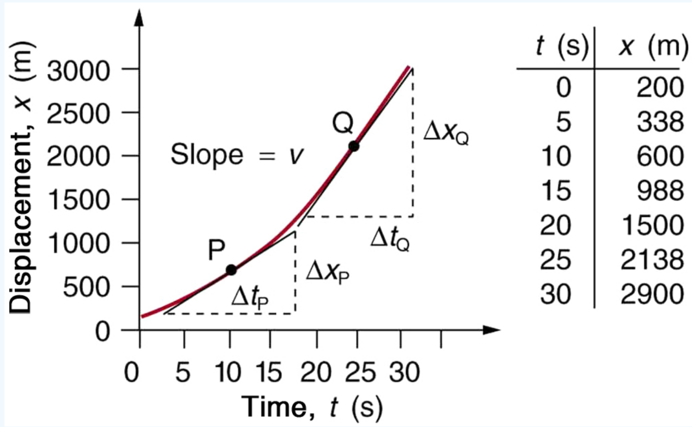

Figure 2.9.5: The slope of an \(vs. \(*graph is velocity. This is shown at two points. Instantaneous velocity at any point is the slope of the tangent at that point.*

Strategy

The slope of a curve at a point is equal to the slope of a straight line tangent to the curve at that point. This principle is illustrated in Figure, where Q is the point at $t = 25s$.

Solution

1. Find the tangent line to the curve at $t = 25s$.
2. Determine the endpoints of the tangent. These correspond to a position of 1300 m at time 19 s and a position of 3120 m at time 32 s.
3. Plug these endpoints into the equation to solve for the slope, .

$$slope = v_Q = \frac{\Delta x_Q}{\Delta t_Q} = \frac{(3120m - 1300m)}{(32s - 19s)}$$

Thus,

$$v_Q = \frac{1820m}{13s} = 140m/s.$$

Discussion

This is the value given in this figure's table for v at $t = 25s$. The value of 140 m/s for v_Q is plotted in Figure. The entire graph of v vs. t can be obtained in this fashion.

Carrying this one step further, we note that the slope of a velocity versus time graph is acceleration. Slope is rise divided by run; on a v vs. t graph, rise = change in velocity Δv and run = change in time Δt.

THE SLOPE OF V VS. T

The slope of a graph of velocity v vs. time t is acceleration a.

$$slope = \frac{\Delta v}{\Delta t} = a$$

Since the velocity versus time graph in Figure $2.9.3b$ is a straight line, its slope is the same everywhere, implying that acceleration is constant. Acceleration versus time is graphed in Figure(c).

Additional general information can be obtained from Figure and the expression for a straight line, $y = mx + b$.

In this case, the vertical axis y is V, the intercept b is v_0, the slope m is a, and the horizontal axis x is t. Substituting these symbols yields

$$v = v_0 + at.$$

A general relationship for velocity, acceleration, and time has again been obtained from a graph. Notice that this equation was also derived algebraically from other motion equations in Motion Equations for Constant Acceleration in One Dimension.

It is not accidental that the same equations are obtained by graphical analysis as by algebraic techniques. In fact, an important way to *discover* physical relationships is to measure various physical quantities and then make graphs of one quantity against another to see if they are correlated in any way. Correlations imply physical relationships and might be shown by smooth graphs such as those above. From such graphs, mathematical relationships can sometimes be postulated. Further experiments are then performed to determine the validity of the hypothesized relationships.

Graphs of Motion Where Acceleration is Not Constant

Now consider the motion of the jet car as it goes from 165 m/s to its top velocity of 250 m/s, graphed in Figure $2.9.6$. Time again starts at zero, and the initial position and velocity are 2900 m and 165 m/s, respectively. (These were the final position and velocity of the car in the motion graphed in Figure $2.9.4$) Acceleration gradually decreases from $5.0 m/s^2$ to zero when the car hits 250 m/s. The slope of the x vs. t graph increases until $t = 55 s$, after which time the slope is constant. Similarly, velocity increases until 55 s and then becomes constant, since acceleration decreases to zero at 55 s and remains zero afterward.

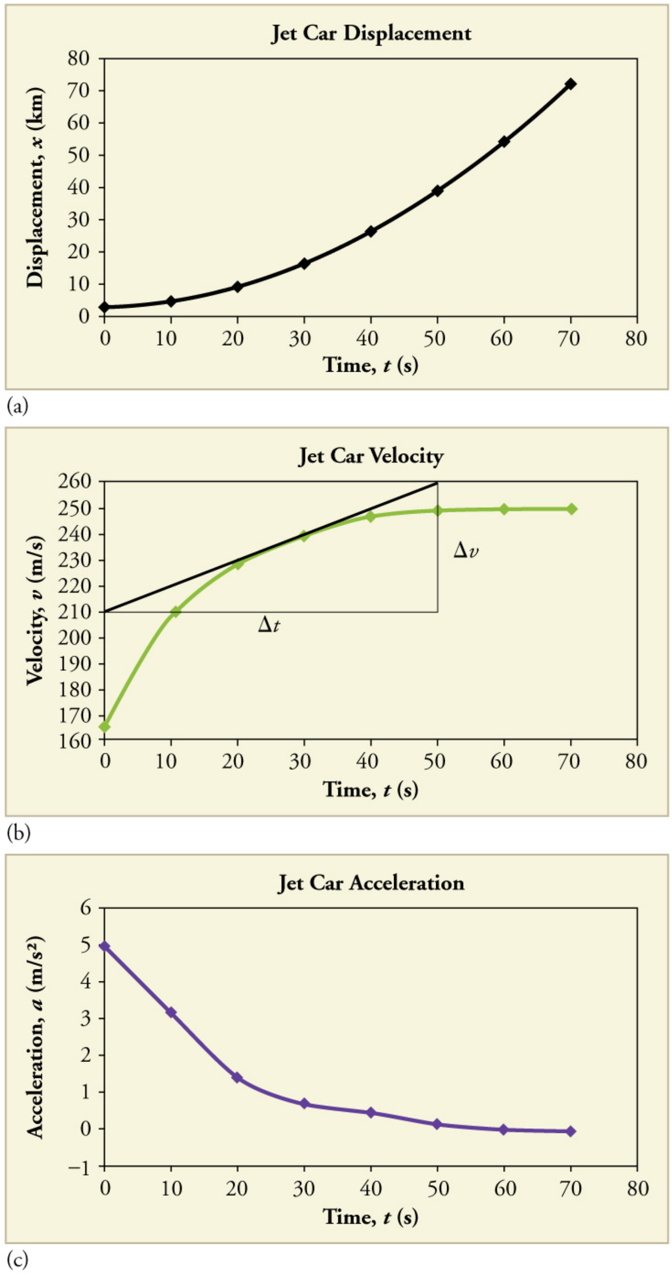

Figure 2.9.3 ends. (a) The slope of this graph is velocity; it is plotted in the next graph. (b) The velocity gradually approaches its top value. The slope of this graph is acceleration; it is plotted in the final graph. (c) Acceleration gradually declines to zero when velocity becomes constant.

Example 2.9.3:Calculating Acceleration from a Graph of Velocity versus Time

Calculate the acceleration of the jet car at a time of 25 s by finding the slope of the v vs. t graph in Figure 2.9.6*b*.

Strategy

The slope of the curve at $t = 25s$ is equal to the slope of the line tangent at that point, as illustrated in Figure 2.9.6*b*.

Solution

Determine endpoints of the tangent line from the figure, and then plug them into the equation to solve for slope, .

$$slope = \frac{\Delta v}{\Delta t} = \frac{(260m/s - 210m/s)}{(51s - 1.0s)}$$

$$a = \frac{50 m/s}{50 s} = 1.0 m/s^2.$$

Discussion

Note that this value for a is consistent with the value plotted in Figure(c) at $t = 25 s$.

A graph of displacement versus time can be used to generate a graph of velocity versus time, and a graph of velocity versus time can be used to generate a graph of acceleration versus time. We do this by finding the slope of the graphs at every point. If the graph is linear (i.e., a line with a constant slope), it is easy to find the slope at any point and you have the slope for every point. Graphical analysis of motion can be used to describe both specific and general characteristics of kinematics. Graphs can also be used for other topics in physics. An important aspect of exploring physical relationships is to graph them and look for underlying relationships.

Exercise 2.9.1:Check Your Understanding

A graph of velocity vs. time of a ship coming into a harbor is shown below.

a. Describe the motion of the ship based on the graph.
b. What would a graph of the ship's acceleration look like?

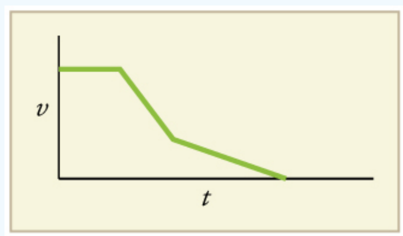

Figure 2.9.7

Answer a

(a) The ship moves at constant velocity and then begins to decelerate at a constant rate. At some point, its deceleration rate decreases. It maintains this lower deceleration rate until it stops moving.

Solution b

A graph of acceleration vs. time would show zero acceleration in the first leg, large and constant negative acceleration in the second leg, and constant negative acceleration.

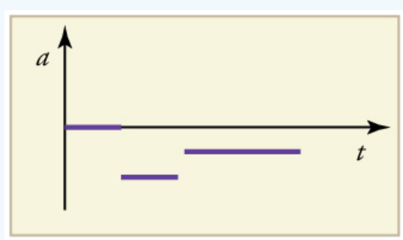

Figure 2.9.8

Summary

- Graphs of motion can be used to analyze motion.
- Graphical solutions yield identical solutions to mathematical methods for deriving motion equations.
- The slope of a graph of displacement x vs. time t is velocity v.
- The slope of a graph of velocity v vs. time t graph is acceleration a.
- Average velocity, instantaneous velocity, and acceleration can all be obtained by analyzing graphs.

Glossary

independent variable

the variable that the dependent variable is measured with respect to; usually plotted along the x-axis

dependent variable

the variable that is being measured; usually plotted along the y-axis

slope

the difference in y-value (the rise) divided by the difference in x-value (the run) of two points on a straight line

y-intercept

the y-value when x=0, or when the graph crosses the y-axis

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2.E: Kinematics (Exercises)

Conceptual Questions

2.1: Displacement

1. Give an example in which there are clear distinctions among distance traveled, displacement, and magnitude of displacement. Specifically identify each quantity in your example.

2. Under what circumstances does distance traveled equal magnitude of displacement? What is the only case in which magnitude of displacement and displacement are exactly the same?

3. Bacteria move back and forth by using their flagella (structures that look like little tails). Speeds of up to $50\mu m/s (50 \times 10^{-6} m/s)$ have been observed. The total distance traveled by a bacterium is large for its size, while its displacement is small. Why is this?

2.2: Vectors, Scalars, and Coordinate Systems

4. A student writes, "*A bird that is diving for prey has a speed of* $-10m/s$" What is wrong with the student's statement? What has the student actually described? Explain.

5. What is the speed of the bird in Exercise?

6. Acceleration is the change in velocity over time. Given this information, is acceleration a vector or a scalar quantity? Explain.

7. A weather forecast states that the temperature is predicted to be $-5°C$ the following day. Is this temperature a vector or a scalar quantity? Explain.

2.3: Time, Velocity, and Speed

8. Give an example (but not one from the text) of a device used to measure time and identify what change in that device indicates a change in time.

9. There is a distinction between average speed and the magnitude of average velocity. Give an example that illustrates the difference between these two quantities.

10. Does a car's odometer measure position or displacement? Does its speedometer measure speed or velocity?

11. If you divide the total distance traveled on a car trip (as determined by the odometer) by the time for the trip, are you calculating the average speed or the magnitude of the average velocity? Under what circumstances are these two quantities the same?

12. How are instantaneous velocity and instantaneous speed related to one another? How do they differ?

2.4: Acceleration

13. Is it possible for speed to be constant while acceleration is not zero? Give an example of such a situation.

14. Is it possible for velocity to be constant while acceleration is not zero? Explain.

15. Give an example in which velocity is zero yet acceleration is not.

16. If a subway train is moving to the left (has a negative velocity) and then comes to a stop, what is the direction of its acceleration? Is the acceleration positive or negative?

17. Plus and minus signs are used in one-dimensional motion to indicate direction. What is the sign of an acceleration that reduces the magnitude of a negative velocity? Of a positive velocity?

2.6: Problem-Solving Basics for One-Dimensional Kinematics

18. What information do you need in order to choose which equation or equations to use to solve a problem? Explain.

19. What is the last thing you should do when solving a problem? Explain.

2.7: Falling Objects

20. What is the acceleration of a rock thrown straight upward on the way up? At the top of its flight? On the way down?

21. An object that is thrown straight up falls back to Earth. This is one-dimensional motion.

(a) When is its velocity zero?

(b) Does its velocity change direction?

(c) Does the acceleration due to gravity have the same sign on the way up as on the way down?

22. Suppose you throw a rock nearly straight up at a coconut in a palm tree, and the rock misses on the way up but hits the coconut on the way down. Neglecting air resistance, how does the speed of the rock when it hits the coconut on the way down compare with what it would have been if it had hit the coconut on the way up? Is it more likely to dislodge the coconut on the way up or down? Explain.

23. If an object is thrown straight up and air resistance is negligible, then its speed when it returns to the starting point is the same as when it was released. If air resistance were not negligible, how would its speed upon return compare with its initial speed? How would the maximum height to which it rises be affected?

24. The severity of a fall depends on your speed when you strike the ground. All factors but the acceleration due to gravity being the same, how many times higher could a safe fall on the Moon be than on Earth (gravitational acceleration on the Moon is about 1/6 that of the Earth)?

25. How many times higher could an astronaut jump on the Moon than on Earth if his takeoff speed is the same in both locations (gravitational acceleration on the Moon is about 1/6 of g on Earth)?

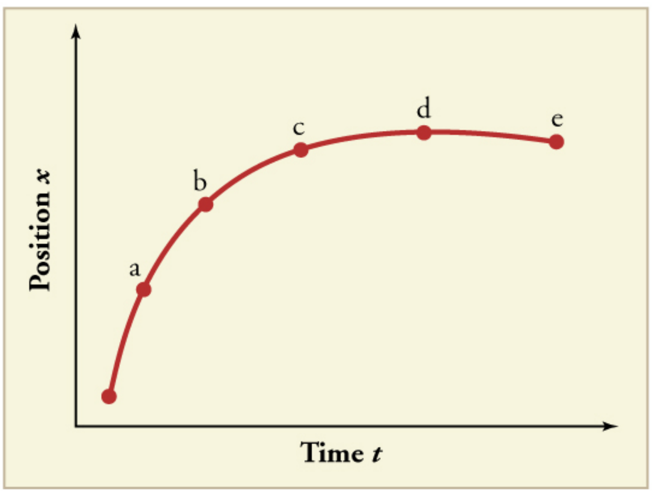
2.8: Graphical Analysis of One-Dimensional Motion

23. (a) Explain how you can use the graph of position versus time in Figure to describe the change in velocity over time. Identify

(b) the time (t_a, t_b, t_c, t_d, or t_e) at which the instantaneous velocity is greatest,

(c) the time at which it is zero, and

(d) the time at which it is negative.

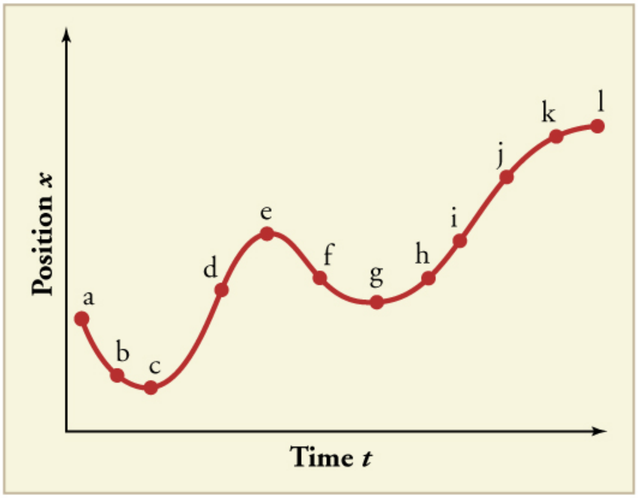


24. (a) Sketch a graph of velocity versus time corresponding to the graph of position versus time given in Figure.

(b) Identify the time or times (t_a, t_b, t_c, etc.) at which the instantaneous velocity is greatest.

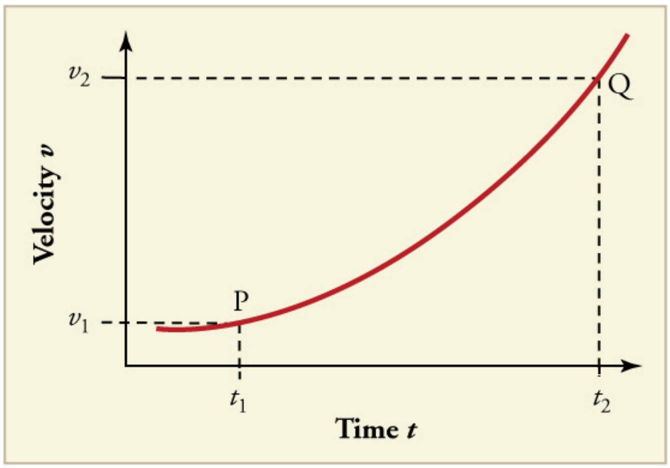
(c) At which times is it zero?

(d) At which times is it negative?



25. (a) Explain how you can determine the acceleration over time from a velocity versus time graph such as the one in Figure.

(b) Based on the graph, how does acceleration change over time?

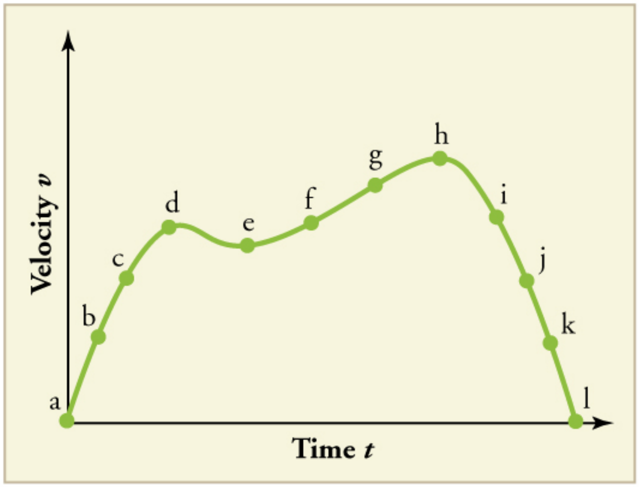


26. (a) Sketch a graph of acceleration versus time corresponding to the graph of velocity versus time given in Figure.

(b) Identify the time or times (t_a, t_b, t_c, etc.) at which the acceleration is greatest.

(c) At which times is it zero?

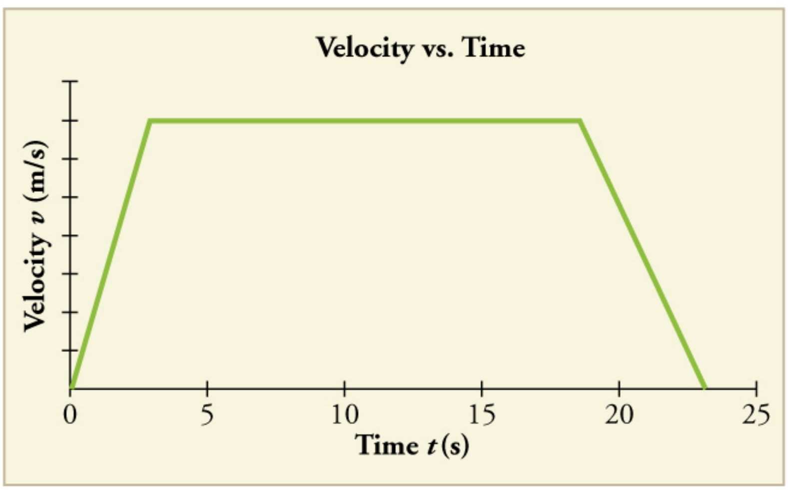
(d) At which times is it negative?



27. Consider the velocity vs. time graph of a person in an elevator shown in Figure. Suppose the elevator is initially at rest. It then accelerates for 3 seconds, maintains that velocity for 15 seconds, then decelerates for 5 seconds until it stops. The acceleration for the entire trip is not constant so we cannot use the equations of motion from Motion Equations for Constant Acceleration in One Dimension for the complete trip. (We could, however, use them in the three individual sections where acceleration is a constant.) Sketch graphs of

(a) position vs. time and

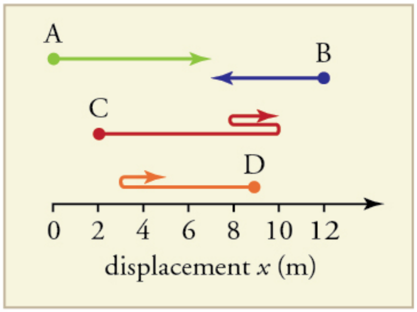
(b) acceleration vs. time for this trip.



28. A cylinder is given a push and then rolls up an inclined plane. If the origin is the starting point, sketch the position, velocity, and acceleration of the cylinder vs. time as it goes up and then down the plane.

Problems & Exercises

2.1: Displacement



29. Find the following for path A:

(a) The distance traveled.

(b) The magnitude of the displacement from start to finish.

(c) The displacement from start to finish.

Solution
(a) 7 m
(b) 7 m
(c) $+7m$

30. Find the following for path B:

(a) The distance traveled.

(b) The magnitude of the displacement from start to finish.

(c) The displacement from start to finish.

31. Find the following for path C:

(a) The distance traveled.

(b) The magnitude of the displacement from start to finish.

(c) The displacement from start to finish.

Solution
(a) 13 m
(b) 9 m
(c) $+9m$

32. Find the following for path D:

(a) The distance traveled.

(b) The magnitude of the displacement from start to finish

(c) The displacement from start to finish

2.3: Time, Velocity, and Speed

33. (a) Calculate Earth's average speed relative to the Sun.

(b) What is its average velocity over a period of one year?

Solution
(a) $3.0 \times 10^4 m/s$
(b) 0 m/s

34. A helicopter blade spins at exactly 100 revolutions per minute. Its tip is 5.00 m from the center of rotation.

(a) Calculate the average speed of the blade tip in the helicopter's frame of reference

(b) What is its average velocity over one revolution?

35. The North American and European continents are moving apart at a rate of about 3 cm/y. At this rate how long will it take them to drift 500 km farther apart than they are at present?

Solution
$2 \times 10^7 years$

36. Land west of the San Andreas fault in southern California is moving at an average velocity of about 6 cm/y northwest relative to land east of the fault. Los Angeles is west of the fault and may thus someday be at the same latitude as San Francisco, which is east of the fault. How far in the future will this occur if the displacement to be made is 590 km northwest, assuming the motion remains constant?

37. On May 26, 1934, a streamlined, stainless steel diesel train called the Zephyr set the world's nonstop long-distance speed record for trains. Its run from Denver to Chicago took 13 hours, 4 minutes, 58 seconds, and was witnessed by more than a million people along the route. The total distance traveled was 1633.8 km. What was its average speed in km/h and m/s?

Solution
$34.689 m/s = 124.88 km/h$

38. Tidal friction is slowing the rotation of the Earth. As a result, the orbit of the Moon is increasing in radius at a rate of approximately 4 cm/year. Assuming this to be a constant rate, how many years will pass before the radius of the Moon's orbit increases by $3.84 \times 10^6 m (1\%)$?

39. A student drove to the university from her home and noted that the odometer reading of her car increased by 12.0 km. The trip took 18.0 min.

(a) What was her average speed?

(b) If the straight-line distance from her home to the university is 10.3 km in a direction 25.0^o south of east, what was her average velocity?

(c) If she returned home by the same path 7 h 30 min after she left, what were her average speed and velocity for the entire trip?

Solution
(a) $40.0 km/h$
(b) 34.3 km/h, 25^o S of E.
(c) average speed = $3.20 km/h, \bar{v} = 0$.

40. The speed of propagation of the action potential (an electrical signal) in a nerve cell depends (inversely) on the diameter of the axon (nerve fiber). If the nerve cell connecting the spinal cord to your feet is 1.1 m long, and the nerve impulse speed is 18 m/s, how long does it take for the nerve signal to travel this distance?

41. Conversations
ith astronauts on the lunar surface were characterized by a kind of echo in which the earthbound person's voice was so loud in the astronaut's space helmet that it was picked up by the astronaut's microphone and transmitted back to Earth. It is reasonable to assume that the echo time equals the time necessary for the radio wave to travel from the Earth to the Moon and back (that is, neglecting any time delays in the electronic equipment). Calculate the distance from Earth to the Moon given that the echo time was 2.56 s and that radio waves travel at the speed of light ($3.00 \times 10^8 m/s$)

Solution
384,000 km

42. A football quarterback runs 15.0 m straight down the playing field in 2.50 s. He is then hit and pushed 3.00 m straight backward in 1.75 s. He breaks the tackle and runs straight forward another 21.0 m in 5.20 s. Calculate his average velocity

(a) for each of the three intervals and

(b) for the entire motion.

43. The planetary model of the atom pictures electrons orbiting the atomic nucleus much as planets orbit the Sun. In this model you can view hydrogen, the simplest atom, as having a single electron in a circular orbit $1.06 \times 10^{-10} m$ in diameter.

(a) If the average speed of the electron in this orbit is known to be $2.20 \times 10^6 m/s$, calculate the number of revolutions per second it makes about the nucleus.

(b) What is the electron's average velocity?

Solution
(a) $6.61 \times 10^{15} rev/s$
(b) 0 m/s

2.4: Acceleration

44. A cheetah can accelerate from rest to a speed of 30.0 m/s in 7.00 s. What is its acceleration?

Solution
$4.29 m/s^2$

45. *Professional Application*

Dr. John Paul Stapp was U.S. Air Force officer who studied the effects of extreme deceleration on the human body. On December 10, 1954, Stapp rode a rocket sled, accelerating from rest to a top speed of 282 m/s (1015 km/h) in 5.00 s, and was brought jarringly back to rest in only 1.40 s! Calculate his (a) acceleration and (b) deceleration. Express each in multiples of $g(9.80 m/s^2)$ by taking its ratio to the acceleration of gravity.

46. A commuter backs her car out of her garage with an acceleration of $1.40 m/s^2$

(a) How long does it take her to reach a speed of 2.00 m/s?

(b) If she then brakes to a stop in 0.800 s, what is her deceleration?

Solution
(a) $1.43 s$
(b) $-2.50 m/s^2$

47. Assume that an intercontinental ballistic missile goes from rest to a suborbital speed of 6.50 km/s in 60.0 s (the actual speed and time are classified). What is its average acceleration in m/s^2 and in multiples of $g(9.80 m/s^2)$?

2.5: Motion Equations for Constant Acceleration in One Dimension

48. An Olympic-class sprinter starts a race with an acceleration of $4.50 m/s^2$

(a) What is her speed 2.40 s later?

(b) Sketch a graph of her position vs. time for this period.

Solution
(a) $10.8 m/s$
(b)

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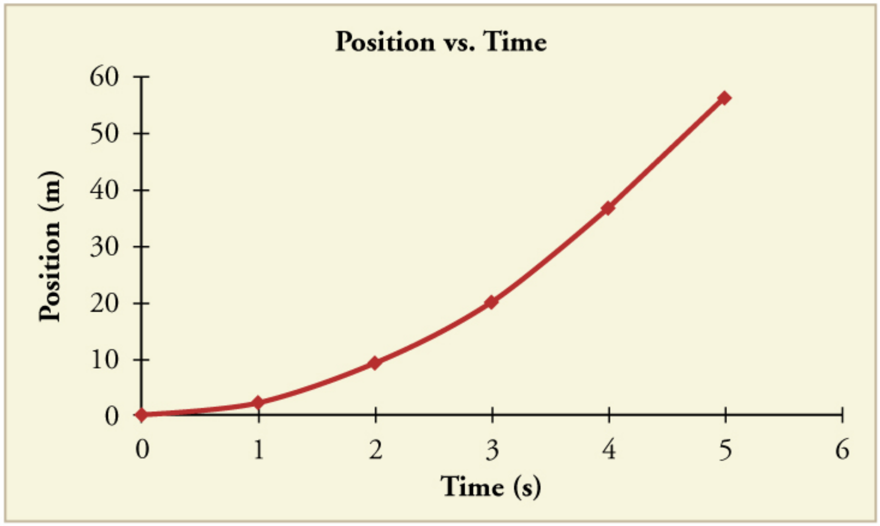


49. A well-thrown ball is caught in a well-padded mitt. If the deceleration of the ball is $2.10 \times 10^4 m/s^2$, and 1.85 ms $(1ms = 10^{-3}s)$ elapses from the time the ball first touches the mitt until it stops, what was the initial velocity of the ball?

Solution

38.9 m/s (about 87 miles per hour)

50. A bullet in a gun is accelerated from the firing chamber to the end of the barrel at an average rate of $6.20 \times 10^5 m/s^2$ for $8.10 \times 10^{-4} s$. What is its muzzle velocity (that is, its final velocity)?

51. (a) A light-rail commuter train accelerates at a rate of $1.35 m/s^2$. How long does it take to reach its top speed of 80.0 km/h, starting from rest?

(b) The same train ordinarily decelerates at a rate of $1.65 m/s^2$. How long does it take to come to a stop from its top speed?

(c) In emergencies the train can decelerate more rapidly, coming to rest from 80.0 km/h in 8.30 s. What is its emergency deceleration in m/s^2?

Solution

(a) $16.5s$

(b) $13.5s$

(c) $-2.68m/s^2$

52. While entering a freeway, a car accelerates from rest at a rate of $2.40m/s^2$ for 12.0 s.

(a) Draw a sketch of the situation.

(b) List the knowns in this problem.

(c) How far does the car travel in those 12.0 s? To solve this part, first identify the unknown, and then discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, check your units, and discuss whether the answer is reasonable.

(d) What is the car’s final velocity? Solve for this unknown in the same manner as in part (c), showing all steps explicitly.

53. At the end of a race, a runner decelerates from a velocity of 9.00 m/s at a rate of $2.00m/s^2$.

(a) How far does she travel in the next 5.00 s?

(b) What is her final velocity?

(c) Evaluate the result. Does it make sense?

Solution

(a) $20.0m$

(b) $-1.00 m/s$
(c) This result does not really make sense. If the runner starts at 9.00 m/s and decelerates at $2.00 m/s^2$ then she will have stopped after 4.50 s. If she continues to decelerate, she will be running backwards.

54. *Professional Application:*

Blood is accelerated from rest to 30.0 cm/s in a distance of 1.80 cm by the left ventricle of the heart.

(a) Make a sketch of the situation.

(b) List the knowns in this problem.

(c) How long does the acceleration take? To solve this part, first identify the unknown, and then discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, checking your units.

(d) Is the answer reasonable when compared with the time for a heartbeat?

55. In a slap shot, a hockey player accelerates the puck from a velocity of 8.00 m/s to 40.0 m/s in the same direction. If this shot takes $3.33 \times 10^{-2} s$, calculate the distance over which the puck accelerates.

Solution
0.799 m

56. A powerful motorcycle can accelerate from rest to 26.8 m/s (100 km/h) in only 3.90 s.

(a) What is its average acceleration?

(b) How far does it travel in that time?

57. Freight trains can produce only relatively small accelerations and decelerations.

(a) What is the final velocity of a freight train that accelerates at a rate of $0.0500 m/s^2$ for 8.00 min, starting with an initial velocity of 4.00 m/s?

(b) If the train can slow down at a rate of $0.550 m/s^2$, how long will it take to come to a stop from this velocity?

(c) How far will it travel in each case?

Solution
(a) $28.0 m/s$
(b) $50.9 s$
(c) 7.68 km to accelerate and 713 m to decelerate

58. A fireworks shell is accelerated from rest to a velocity of 65.0 m/s over a distance of 0.250 m.

(a) How long did the acceleration last?

(b) Calculate the acceleration.

59. A swan on a lake gets airborne by flapping its wings and running on top of the water.

(a) If the swan must reach a velocity of 6.00 m/s to take off and it accelerates from rest at an average rate of $0.350 m/s^2$, how far will it travel before becoming airborne?

(b) How long does this take?

Solution
(a) $51.4 m$
(b) $17.1 s$

60. *Professional Application:*

A woodpecker's brain is specially protected from large decelerations by tendon-like attachments inside the skull. While pecking on a tree, the woodpecker's head comes to a stop from an initial velocity of 0.600 m/s in a distance of only 2.00 mm.

(a) Find the acceleration in m/s^2 and in multiples of $g (g = 9.80 m/s^2)$.

(b) Calculate the stopping time.

(c) The tendons cradling the brain stretch, making its stopping distance 4.50 mm (greater than the head and, hence, less deceleration of the brain). What is the brain's deceleration, expressed in multiples of g?

61. An unwary football player collides with a padded goalpost while running at a velocity of 7.50 m/s and comes to a full stop after compressing the padding and his body 0.350 m.

(a) What is his deceleration?

(b) How long does the collision last?

Solution
(a) $-80.4 m/s^2$
(b) $9.33 \times 10^{-2} s$

62. In World War II, there were several reported cases of airmen who jumped from their flaming airplanes with no parachute to escape certain death. Some fell about 20,000 feet (6000 m), and some of them survived, with few life-threatening injuries. For these lucky pilots, the tree branches and snow drifts on the ground allowed their deceleration to be relatively small. If we assume that a pilot's speed upon impact was 123 mph (54 m/s), then what was his deceleration? Assume that the trees and snow stopped him over a distance of 3.0 m.

63. Consider a grey squirrel falling out of a tree to the ground.

(a) If we ignore air resistance in this case (only for the sake of this problem), determine a squirrel's velocity just before hitting the ground, assuming it fell from a height of 3.0 m.

(b) If the squirrel stops in a distance of 2.0 cm through bending its limbs, compare its deceleration with that of the airman in the previous problem.

Solution
(a) $7.7 m/s$
(b) $-15 \times 10^2 m/s^2$. This is about 3 times the deceleration of the pilots, who were falling from thousands of meters high!

64. An express train passes through a station. It enters with an initial velocity of 22.0 m/s and decelerates at a rate of $0.150 m/s^2$ as it goes through. The station is 210 m long.

(a) How long is the nose of the train in the station?

(b) How fast is it going when the nose leaves the station?

(c) If the train is 130 m long, when does the end of the train leave the station?

(d) What is the velocity of the end of the train as it leaves?

65. Dragsters can actually reach a top speed of 145 m/s in only 4.45 s—considerably less time than given in Example and Example.

(a) Calculate the average acceleration for such a dragster.

(b) Find the final velocity of this dragster starting from rest and accelerating at the rate found in (a) for 402 m (a quarter mile) without using any information on time.

(c) Why is the final velocity greater than that used to find the average acceleration?

Hint: Consider whether the assumption of constant acceleration is valid for a dragster. If not, discuss whether the acceleration would be greater at the beginning or end of the run and what effect that would have on the final velocity.

Solution
(a) $32.6 m/s^2$
(b) $162 m/s$
(c) $v > v_{max}$, because the assumption of constant acceleration is not valid for a dragster. A dragster changes gears, and would have a greater acceleration in first gear than second gear than third gear, etc. The acceleration would be greatest at the beginning, so it would not be accelerating at $32.6 m/s^2$ during the last few meters, but substantially less, and the final velocity would be less than 162 m/s.

66. A bicycle racer sprints at the end of a race to clinch a victory. The racer has an initial velocity of 11.5 m/s and accelerates at the rate of $0.500 m/s^2$ for 7.00 s.

(a) What is his final velocity?

(b) The racer continues at this velocity to the finish line. If he was 300 m from the finish line when he started to accelerate, how much time did he save?

(c) One other racer was 5.00 m ahead when the winner started to accelerate, but he was unable to accelerate, and traveled at 11.8 m/s until the finish line. How far ahead of him (in meters and in seconds) did the winner finish?

67. In 1967, New Zealander Burt Munro set the world record for an Indian motorcycle, on the Bonneville Salt Flats in Utah, with a maximum speed of 183.58 mi/h. The one-way course was 5.00 mi long. Acceleration rates are often described by the time it takes to reach 60.0 mi/h from rest. If this time was 4.00 s, and Burt accelerated at this rate until he reached his maximum speed, how long did it take Burt to complete the course?

Solution
104 s

68. (a) A world record was set for the men's 100-m dash in the 2008 Olympic Games in Beijing by Usain Bolt of Jamaica. Bolt "coasted" across the finish line with a time of 9.69 s. If we assume that Bolt accelerated for 3.00 s to reach his maximum speed, and maintained that speed for the rest of the race, calculate his maximum speed and his acceleration.

(b) During the same Olympics, Bolt also set the world record in the 200-m dash with a time of 19.30 s. Using the same assumptions as for the 100-m dash, what was his maximum speed for this race?

Solution
(a) $v = 12.2 m/s; a = 4.07 m/s^2$
(b) $v = 11.2 m/s$

2.7: Falling Objects

Assume air resistance is negligible unless otherwise stated.

69. Calculate the displacement and velocity at times of

(a) 0.500,

(b) 1.00,

(c) 1.50, and

(d) 2.00 s for a ball thrown straight up with an initial velocity of 15.0 m/s. Take the point of release to be $y_0 = 0$

Solution
(a) $y_1 = 6.28 m; v_1 = 10.1 m/s$
(b) $y_2 = 10.1 m; v_2 = 5.20 m/s$
(c) $y_3 = 11.5 m; v_3 = 0.300 m/s$
(d) $y_4 = 10.4 m; v_4 = -4.60 m/s$

70. Calculate the displacement and velocity at times of

(a) 0.500,

(b) 1.00,

(c) 1.50,

(d) 2.00, and

(e) 2.50 s for a rock thrown straight down with an initial velocity of 14.0 m/s from the Verrazano Narrows Bridge in New York City. The roadway of this bridge is 70.0 m above the water.

71. A basketball referee tosses the ball straight up for the starting tip-off. At what velocity must a basketball player leave the ground to rise 1.25 m above the floor in an attempt to get the ball?

Solution

$v_0 = 4.95 m/s$

72. A rescue helicopter is hovering over a person whose boat has sunk. One of the rescuers throws a life preserver straight down to the victim with an initial velocity of 1.40 m/s and observes that it takes 1.8 s to reach the water.

(a) List the knowns in this problem.

(b) How high above the water was the preserver released? Note that the downdraft of the helicopter reduces the effects of air resistance on the falling life preserver, so that an acceleration equal to that of gravity is reasonable.

73. A dolphin in an aquatic show jumps straight up out of the water at a velocity of 13.0 m/s.

(a) List the knowns in this problem.

(b) How high does his body rise above the water? To solve this part, first note that the final velocity is now a known and identify its value. Then identify the unknown, and discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, checking units, and discuss whether the answer is reasonable.

(c) How long is the dolphin in the air? Neglect any effects due to his size or orientation.

Solution

(a) $a = -9.80 m/s^2; v_0 = 13.0 m/s; y_0 = 0 m$

(b) $v = 0 m/s$. Unknown is distance y to top of trajectory, where velocity is zero. Use equation $v^2 = v_0^2 + 2a(y - y_0)$ because it contains all known values except for y, so we can solve for y. Solving for y gives

$$v^2 - v_0^2 = 2ay - y_0)$$

$$\frac{v^2 - v_0^2}{2a} = y - y_0$$

$$y = y0 + \frac{v^2 - v_0^2}{2a} = 0m + \frac{(0m/s)^2 - (13.0m/s)^2}{2(-9.80m/s^2)} = 8.62m$$

Dolphins measure about 2 meters long and can jump several times their length out of the water, so this is a reasonable result.

(c) $2.65 s$

74. A swimmer bounces straight up from a diving board and falls feet first into a pool. She starts with a velocity of 4.00 m/s, and her takeoff point is 1.80 m above the pool.

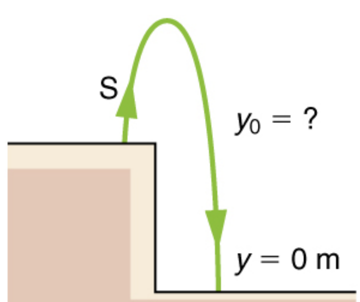
(a) How long are her feet in the air?

(b) What is her highest point above the board?

(c) What is her velocity when her feet hit the water?

75. (a) Calculate the height of a cliff if it takes 2.35 s for a rock to hit the ground when it is thrown straight up from the cliff with an initial velocity of 8.00 m/s.

(b) How long would it take to reach the ground if it is thrown straight down with the same speed?



Solution

(a) 8.26 m

(b) 0.717 s

76. A very strong, but inept, shot putter puts the shot straight up vertically with an initial velocity of 11.0 m/s. How long does he have to get out of the way if the shot was released at a height of 2.20 m, and he is 1.80 m tall?

77. You throw a ball straight up with an initial velocity of 15.0 m/s. It passes a tree branch on the way up at a height of 7.00 m. How much additional time will pass before the ball passes the tree branch on the way back down?

Solution
1.91 s

78. A kangaroo can jump over an object 2.50 m high.

(a) Calculate its vertical speed when it leaves the ground.

(b) How long is it in the air?

79. Standing at the base of one of the cliffs of Mt. Arapiles in Victoria, Australia, a hiker hears a rock break loose from a height of 105 m. He can't see the rock right away but then does, 1.50 s later.

(a) How far above the hiker is the rock when he can see it?

(b) How much time does he have to move before the rock hits his head?

Solution
(a) 94.0 m
(b) 3.13 s

80. An object is dropped from a height of 75.0 m above ground level.

(a) Determine the distance traveled during the first second.

(b) Determine the final velocity at which the object hits the ground. (c) Determine the distance traveled during the last second of motion before hitting the ground.

81. There is a 250-m-high cliff at Half Dome in Yosemite National Park in California. Suppose a boulder breaks loose from the top of this cliff.

(a) How fast will it be going when it strikes the ground?

(b) Assuming a reaction time of 0.300 s, how long will a tourist at the bottom have to get out of the way after hearing the sound of the rock breaking loose (neglecting the height of the tourist, which would become negligible anyway if hit)? The speed of sound is 335 m/s on this day.

Solution
(a) -70.0 m/s (downward)
(b) 6.10 s

82. A ball is thrown straight up. It passes a 2.00-m-high window 7.50 m off the ground on its path up and takes 0.312 s to go past the window. What was the ball's initial velocity? *Hint:* First consider only the distance along the window, and solve for the ball's velocity at the bottom of the window. Next, consider only the distance from the ground to the bottom of the window, and solve for the initial velocity using the velocity at the bottom of the window as the final velocity.

83. Suppose you drop a rock into a dark well and, using precision equipment, you measure the time for the sound of a splash to return.

(a) Neglecting the time required for sound to travel up the well, calculate the distance to the water if the sound returns in 2.0000 s.

(b) Now calculate the distance taking into account the time for sound to travel up the well. The speed of sound is 332.00 m/s in this well.

Solution
(a) $19.6m$
(b) $18.5m$

84. A steel ball is dropped onto a hard floor from a height of 1.50 m and rebounds to a height of 1.45 m.

(a) Calculate its velocity just before it strikes the floor.

(b) Calculate its velocity just after it leaves the floor on its way back up.

(c) Calculate its acceleration during contact with the floor if that contact lasts 0.0800 ms $(8.00 \times 10^{-5} s)$

(d) How much did the ball compress during its collision with the floor, assuming the floor is absolutely rigid?

85. A coin is dropped from a hot-air balloon that is 300 m above the ground and rising at 10.0 m/s upward. For the coin, find

(a) the maximum height reached,

(b) its position and velocity 4.00 s after being released, and

(c) the time before it hits the ground.

Solution
(a) 305 m
(b) 262 m, -29.2 m/s
(c) 8.91 s

86. A soft tennis ball is dropped onto a hard floor from a height of 1.50 m and rebounds to a height of 1.10 m.

(a) Calculate its velocity just before it strikes the floor.

(b) Calculate its velocity just after it leaves the floor on its way back up.

(c) Calculate its acceleration during contact with the floor if that contact lasts 3.50 ms $(3.50 \times 10^{-3} s)$

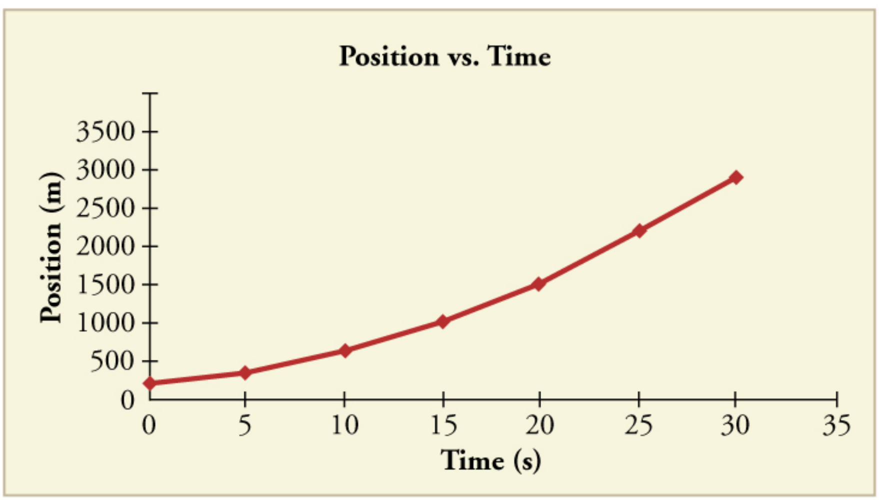
(d) How much did the ball compress during its collision with the floor, assuming the floor is absolutely rigid?

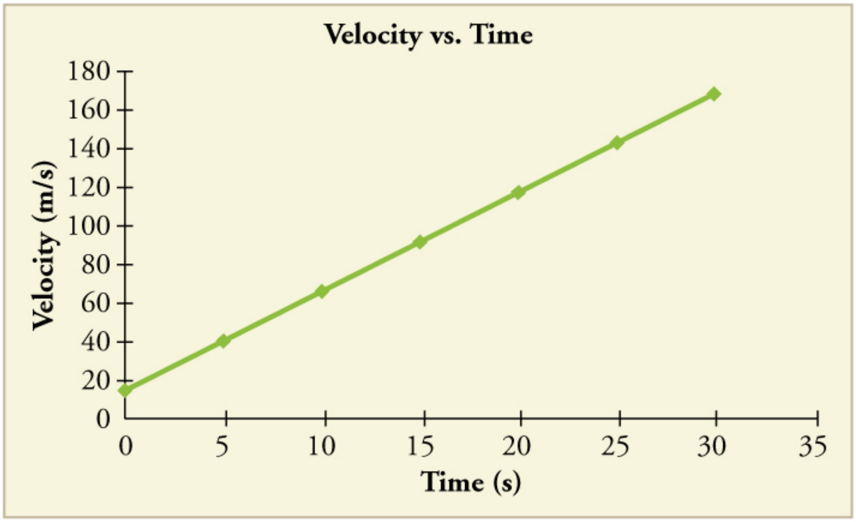
2.8: Graphical Analysis of One-Dimensional Motion

Note: There is always uncertainty in numbers taken from graphs. If your answers differ from expected values, examine them to see if they are within data extraction uncertainties estimated by you.

87. (a) By taking the slope of the curve in Figure, verify that the velocity of the jet car is 115 m/s at $t = 20$ s.

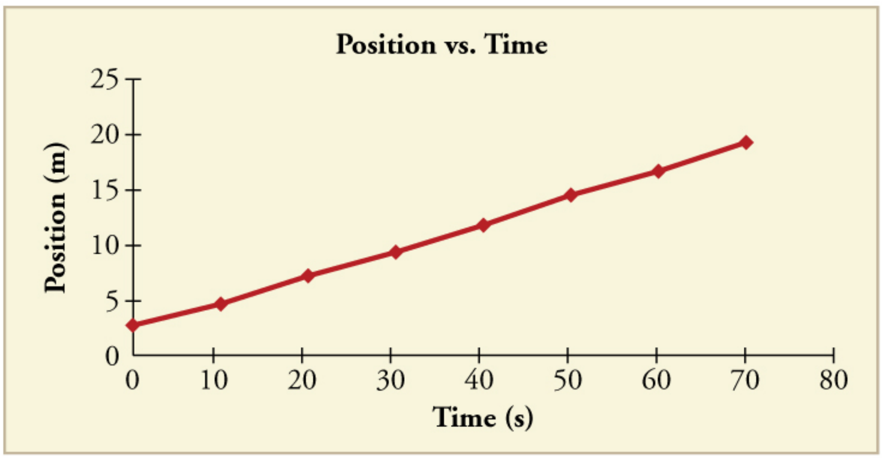
(b) By taking the slope of the curve at any point in Figure, verify that the jet car's acceleration is $5.0 m/s^2$





Solution
(a) $115 m/s$
(b) $5.0 m/s^2$

88. Using approximate values, calculate the slope of the curve in Figure to verify that the velocity at $t = 10.0 s$ is 0.208 m/s. Assume all values are known to 3 significant figures.



89. Using approximate values, calculate the slope of the curve in above Figure to verify that the velocity at $t = 30.0 s$ is approximately 0.24 m/s.

Solution
$$v = \frac{(11.7 - 6.95) \times 10^3 m}{(40.0 – 20.0) s} = 238 m/s$$

90. By taking the slope of the curve in Figure, verify that the acceleration is $3.2 m/s^2$ at $t = 10 s$

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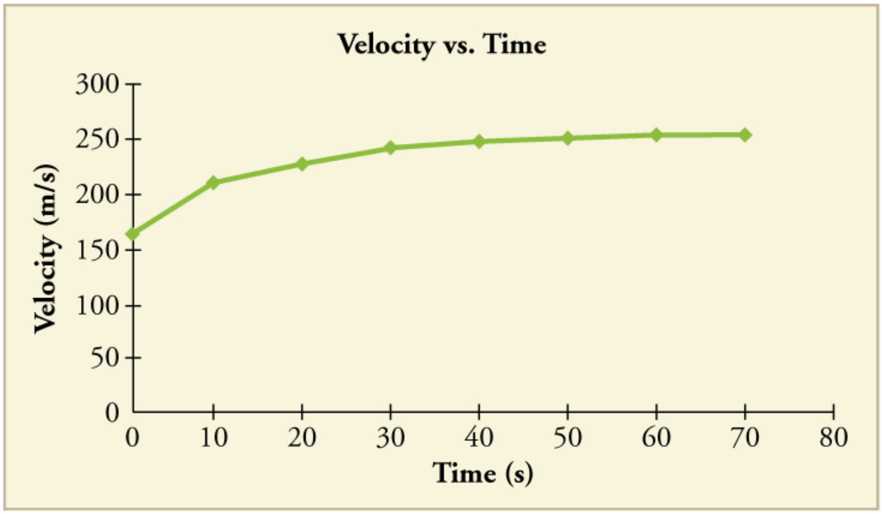
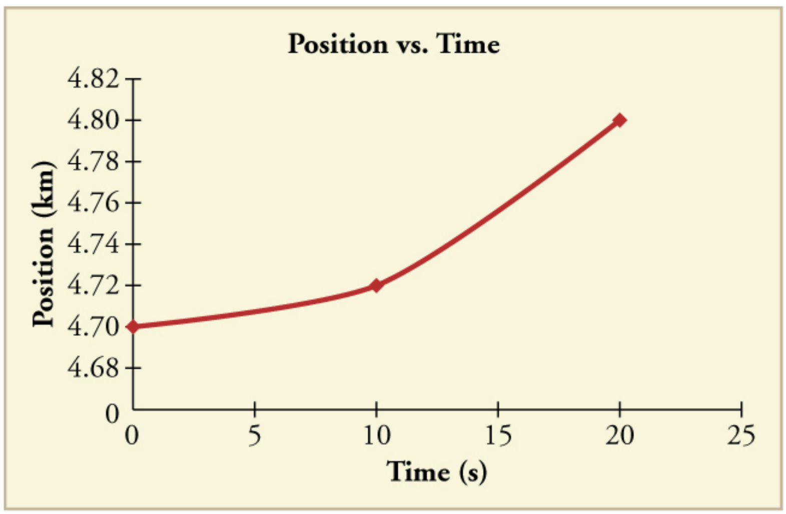


91. Construct the position graph for the subway shuttle train as shown in [link](a). Your graph should show the position of the train, in kilometers, from t = 0 to 20 s. You will need to use the information on acceleration and velocity given in the examples for this figure.



92. (a) Take the slope of the curve in Figure to find the jogger's velocity at $t = 2.5 s$.

(b) Repeat at 7.5 s. These values must be consistent with the graph in Figure.

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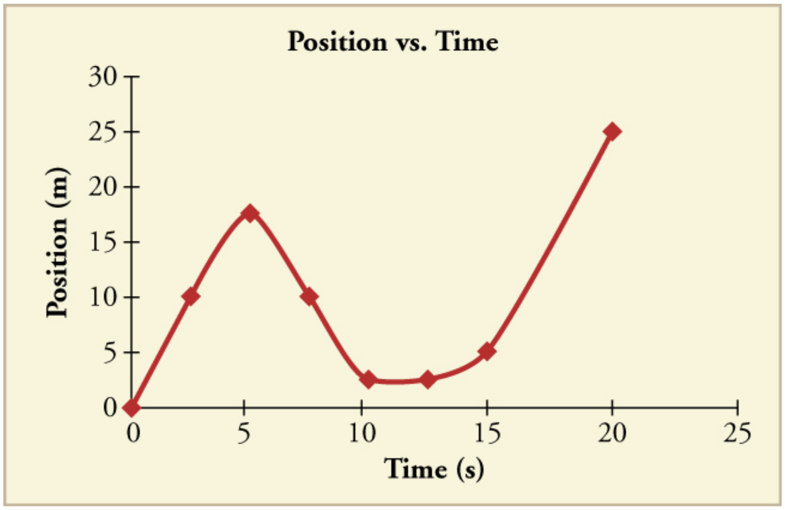
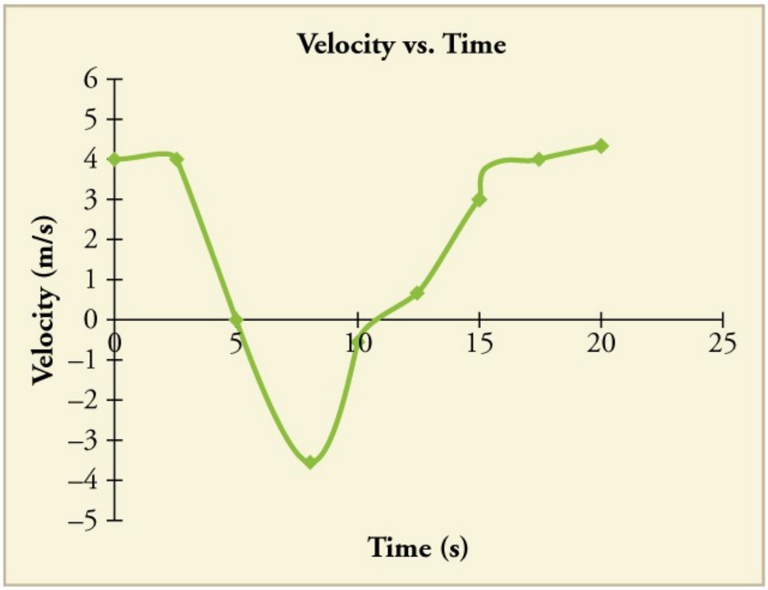
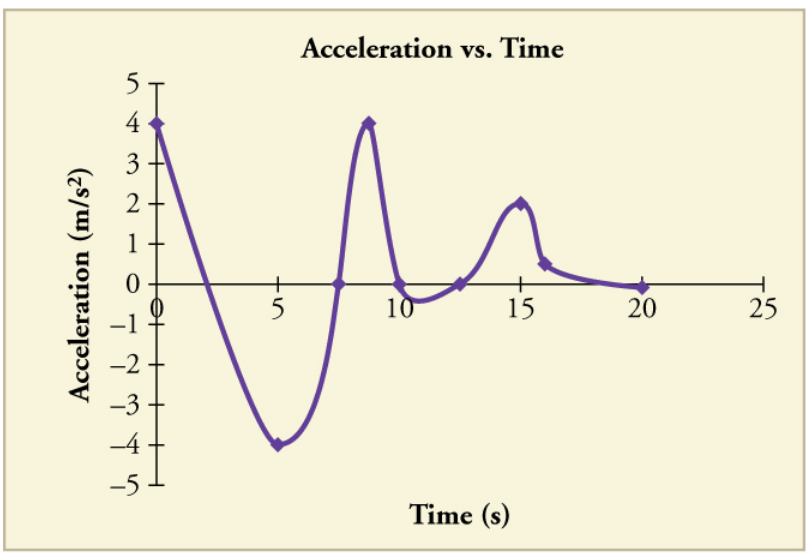






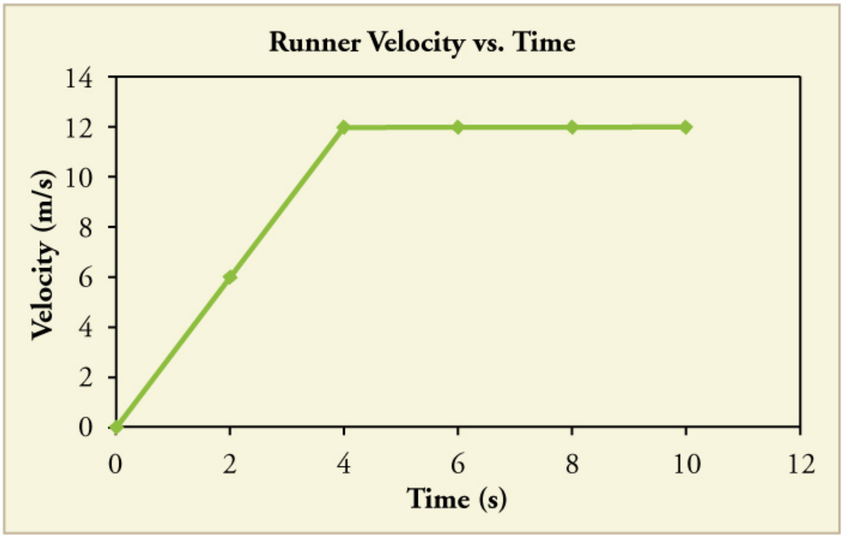
93. A graph of $v(t)$ is shown for a world-class track sprinter in a 100-m race. (See Figure).

(a) What is his average velocity for the first 4 s?

(b) What is his instantaneous velocity at $t = 5s$?

(c) What is his average acceleration between 0 and 4 s?

(d) What is his time for the race?



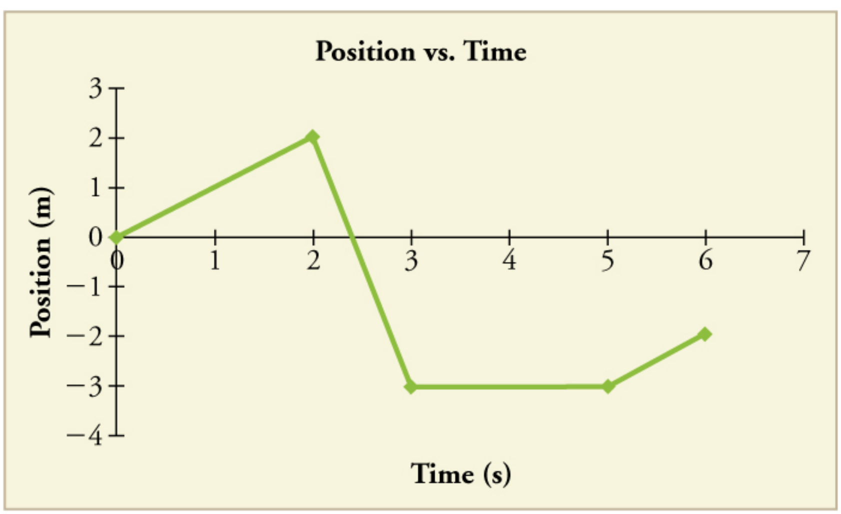
Solution
(a) $6m/s$
(b) $12m/s$
(c) $3m/s^2$
(d) $10s$

94. Figure shows the position graph for a particle for 6 s.

(a) Draw the corresponding Velocity vs. Time graph.

(b) What is the acceleration between 0 s and 2 s?

(c) What happens to the acceleration at exactly 2 s?



Contributors and Attributions

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CHAPTER OVERVIEW

3: Two-Dimensional Kinematics

The arc of a basketball, the orbit of a satellite, a bicycle rounding a curve, a swimmer diving into a pool, blood gushing out of a wound, and a puppy chasing its tail are but a few examples of motions along curved paths. In fact, most motions in nature follow curved paths rather than straight lines. Motion along a curved path on a flat surface or a plane (such as that of a ball on a pool table or a skater on an ice rink) is two-dimensional, and thus described by two-dimensional kinematics.

3.1: Prelude to Two-Dimensional Kinematics

3.2: Kinematics in Two Dimensions - An Introduction

3.3: Vector Addition and Subtraction- Graphical Methods

3.4: Vector Addition and Subtraction- Analytical Methods

3.5: Projectile Motion

3.6: Addition of Velocities

3.E: Two-Dimensional Kinematics (Exercises)

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3.1: Prelude to Two-Dimensional Kinematics

The arc of a basketball, the orbit of a satellite, a bicycle rounding a curve, a swimmer diving into a pool, blood gushing out of a wound, and a puppy chasing its tail are but a few examples of motions along curved paths. In fact, most motions in nature follow curved paths rather than straight lines. Motion along a curved path on a flat surface or a plane (such as that of a ball on a pool table or a skater on an ice rink) is two-dimensional, and thus described by two-dimensional kinematics. Motion not confined to a plane, such as a car following a winding mountain road, is described by three-dimensional kinematics. Both two- and three-dimensional kinematics are simple extensions of the one-dimensional kinematics developed for straight-line motion in the previous chapter. This simple extension will allow us to apply physics to many more situations, and it will also yield unexpected insights about nature.

Figure 3.1.1: Everyday motion that we experience is, thankfully, rarely as tortuous as a rollercoaster ride like this—the Dragon Khan in Spain's Universal Port Aventura Amusement Park. However, most motion is in curved, rather than straight-line, paths. Motion along a curved path is two- or three-dimensional motion, and can be described in a similar fashion to one-dimensional motion. (credit: Boris23/Wikimedia Commons)

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3.2: Kinematics in Two Dimensions - An Introduction

Learning Objectives

By the end of this section, you will be able to:

- Observe that motion in two dimensions consists of horizontal and vertical components.
- Understand the independence of horizontal and vertical vectors in two-dimensional motion.

Figure 3.2.1: Walkers and drivers in a city like New York are rarely able to travel in straight lines to reach their destinations. Instead, they must follow roads and sidewalks, making two-dimensional, zigzagged paths. (credit: Margaret W. Carruthers)

Two-Dimensional Motion: Walking in a City

Suppose you want to walk from one point to another in a city with uniform square blocks, as pictured in Figure 3.2.2.

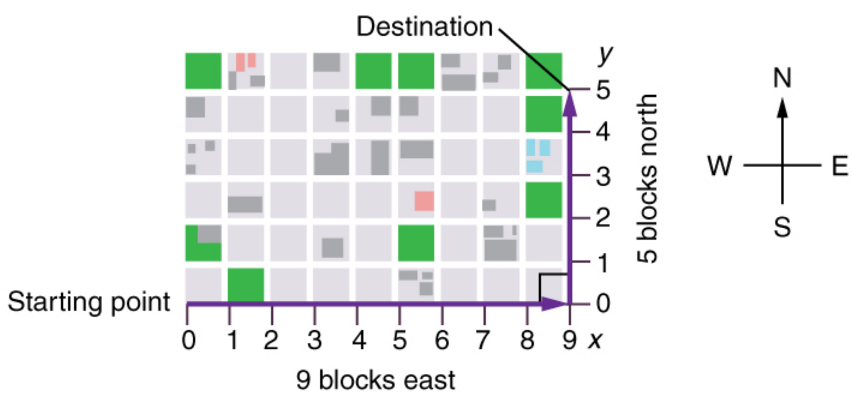

Figure 3.2.2: A pedestrian walks a two-dimensional path between two points in a city. In this scene, all blocks are square and are the same size.

The straight-line path that a helicopter might fly is blocked to you as a pedestrian, and so you are forced to take a two-dimensional path, such as the one shown. You walk 14 blocks in all, 9 east followed by 5 north. What is the straight-line distance?

An old adage states that the shortest distance between two points is a straight line. The two legs of the trip and the straight-line path form a right triangle, and so the Pythagorean theorem,

$$a^2 + b^2 = c^2 \tag{3.2.1}$$

can be used to find the straight-line distance.

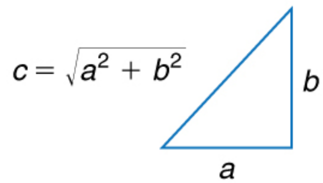

Figure 3.2.3: The Pythagorean theorem relates the length of the legs of a right triangle, labeled a and b, with the hypotenuse, labeled c. The relationship is given by: $a^2 + b^2 = c^2$. This can be rewritten, solving for c : $c = \sqrt{a^2 + b^2}$.

The hypotenuse of the triangle is the straight-line path, and so in this case its length in units of city blocks is $\sqrt{(9 blocks)^2 + (5 blocks)^2} = 10.3 blocks$, considerably shorter than the 14 blocks you walked. (Note that we are using three significant figures in the answer. Although it appears that "9" and "5" have only one significant digit, they are discrete numbers. In this case "9 blocks" is the same as "9.0 or 9.00 blocks." We have decided to use three significant figures in the answer in order to show the result more precisely.)

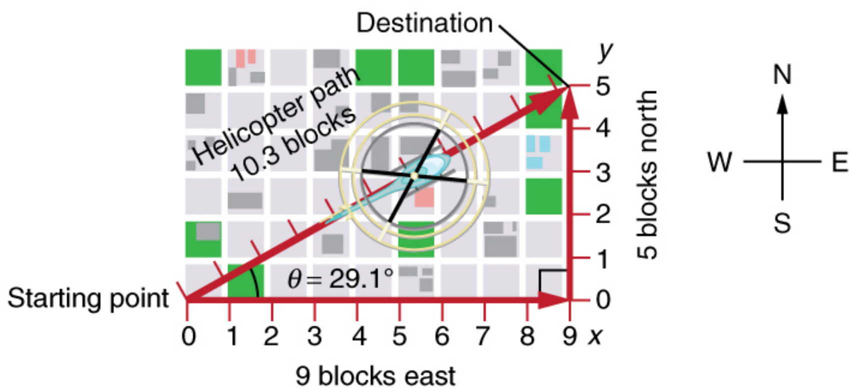

Figure 3.2.4: The straight-line path followed by a helicopter between the two points is shorter than the 14 blocks walked by the pedestrian. All blocks are square and the same size.

The fact that the straight-line distance (10.3 blocks) in Figure is less than the total distance walked (14 blocks) is one example of a general characteristic of vectors. (Recall that vectors are quantities that have both magnitude and direction.)

As for one-dimensional kinematics, we use arrows to represent vectors. The length of the arrow is proportional to the vector's magnitude. The arrow's length is indicated by hash marks in Figure and Figure. The arrow points in the same direction as the vector. For two-dimensional motion, the path of an object can be represented with three vectors: one vector shows the straight-line path between the initial and final points of the motion, one vector shows the horizontal component of the motion, and one vector shows the vertical component of the motion. The horizontal and vertical components of the motion add together to give the straight-line path. For example, observe the three vectors in Figure. The first represents a 9-block displacement east. The second represents a 5-block displacement north. These vectors are added to give the third vector, with a 10.3-block total displacement. The third vector is the straight-line path between the two points. Note that in this example, the vectors that we are adding are perpendicular to each other and thus form a right triangle. This means that we can use the Pythagorean theorem to calculate the magnitude of the total displacement. (Note that we cannot use the Pythagorean theorem to add vectors that are not perpendicular. We will develop techniques for adding vectors having any direction, not just those perpendicular to one another, in Vector Addition and Subtraction: Graphical Methods and Vector Addition and Subtraction: Analytical Methods.)

The Independence of Perpendicular Motions

The person taking the path shown in Figure walks east and then north (two perpendicular directions). How far he or she walks east is only affected by his or her motion eastward. Similarly, how far he or she walks north is only affected by his or her motion northward.

INDEPENDENCE OF MOTION

The horizontal and vertical components of two-dimensional motion are independent of each other. Any motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.

This is true in a simple scenario like that of walking in one direction first, followed by another. It is also true of more complicated motion involving movement in two directions at once. For example, let's compare the motions of two baseballs. One baseball is dropped from rest. At the same instant, another is thrown horizontally from the same height and follows a curved path. A stroboscope has captured the positions of the balls at fixed time intervals as they fall.

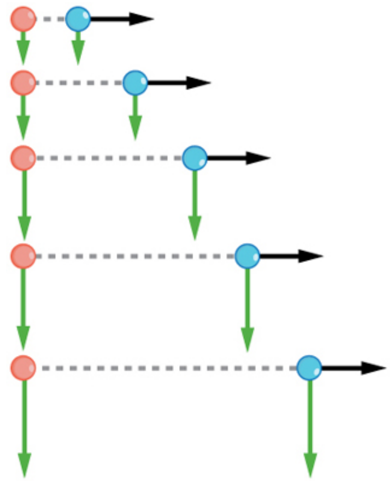

Figure 3.2.5: This shows the motions of two identical balls—one falls from rest, the other has an initial horizontal velocity. Each subsequent position is an equal time interval. Arrows represent horizontal and vertical velocities at each position. The ball on the right has an initial horizontal velocity, while the ball on the left has no horizontal velocity. Despite the difference in horizontal velocities, the vertical velocities and positions are identical for both balls. This shows that the vertical and horizontal motions are independent.

It is remarkable that for each flash of the strobe, the vertical positions of the two balls are the same. This similarity implies that the vertical motion is independent of whether or not the ball is moving horizontally. (Assuming no air resistance, the vertical motion of a falling object is influenced by gravity only, and not by any horizontal forces.) Careful examination of the ball thrown horizontally shows that it travels the same horizontal distance between flashes. This is due to the fact that there are no additional forces on the ball in the horizontal direction after it is thrown. This result means that the horizontal velocity is constant, and affected neither by vertical motion nor by gravity (which is vertical). Note that this case is true only for ideal conditions. In the real world, air resistance will affect the speed of the balls in both directions.

The two-dimensional curved path of the horizontally thrown ball is composed of two independent one-dimensional motions (horizontal and vertical). The key to analyzing such motion, called *projectile motion*, is to *resolve* (break) it into motions along perpendicular directions. Resolving two-dimensional motion into perpendicular components is possible because the components are independent. We shall see how to resolve vectors in Vector Addition and Subtraction: Graphical Methods and Vector Addition and Subtraction: Analytical Methods. We will find such techniques to be useful in many areas of physics.

PHET EXPLORATIONS: LADYBUG MOTION 2D

Learn about position, velocity and acceleration vectors. Move the ladybug by setting the position, velocity or acceleration, and see how the vectors change. Choose linear, circular or elliptical motion, and record and playback the motion to analyze the behavior.

Figure 3.2.5: Ladybug Motion 2D

Summary

- The shortest path between any two points is a straight line. In two dimensions, this path can be represented by a vector with horizontal and vertical components.
- The horizontal and vertical components of a vector are independent of one another. Motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.

Glossary

vector
: a quantity that has both magnitude and direction; an arrow used to represent quantities with both magnitude and direction

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3.3: Vector Addition and Subtraction- Graphical Methods

Learning Objectives

By the end of this section, you will be able to:

- Understand the rules of vector addition, subtraction, and multiplication.
- Apply graphical methods of vector addition and subtraction to determine the displacement of moving objects.

Vector Addition and Subtraction: Graphical Methods

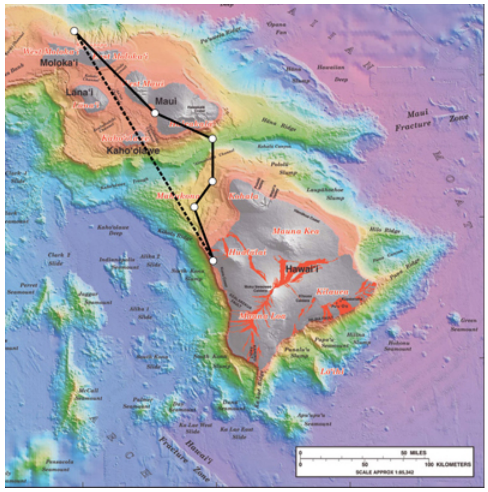

Figure 3.3.1: Displacement can be determined graphically using a scale map, such as this one of the Hawaiian Islands. A journey from Hawai'i to Moloka'i has a number of legs, or journey segments. These segments can be added graphically with a ruler to determine the total two-dimensional displacement of the journey. (credit: US Geological Survey)

Vectors in Two Dimensions

A vector is a quantity that has magnitude and direction. Displacement, velocity, acceleration, and force, for example, are all vectors. In one-dimensional, or straight-line, motion, the direction of a vector can be given simply by a plus or minus sign. In two dimensions (2-d), however, we specify the direction of a vector relative to some reference frame (i.e., coordinate system), using an arrow having length proportional to the vector's magnitude and pointing in the direction of the vector.

Figure shows such a *graphical representation of a vector*, using as an example the total displacement for the person walking in a city considered in Kinematics in Two Dimensions: An Introduction. We shall use the notation that a boldface symbol, such as D, stands for a vector. Its magnitude is represented by the symbol in italics, D, and its direction by θ.

VECTORS IN THIS TEXT

In this text, we will represent a vector with a boldface variable. For example, we will represent the quantity force with the vector F, which has both magnitude and direction. The magnitude of the vector will be represented by a variable in italics, such as F, and the direction of the variable will be given by an angle θ.

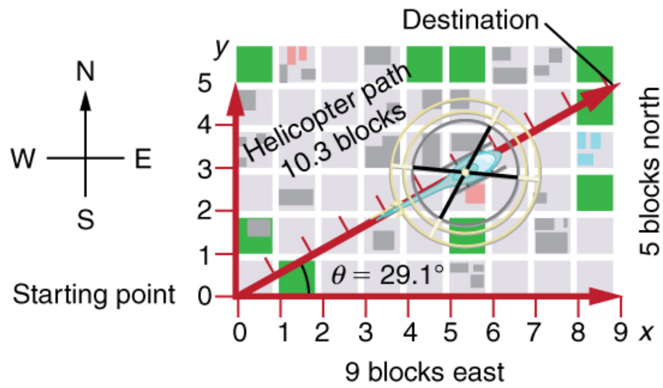

***Figure 3.3.2**: A person walks 9 blocks east and 5 blocks north. The displacement is 10.3 blocks at an angle .1° north of east.*

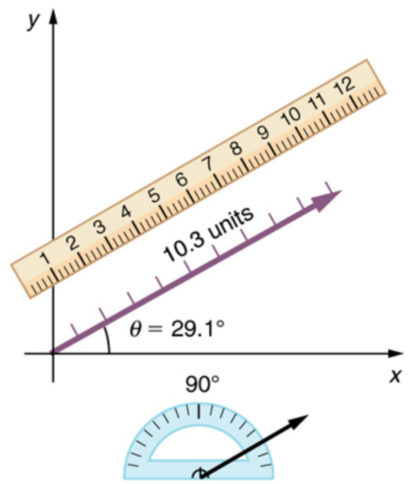

Figure graphically, draw an arrow to represent the total displacement vector D. Using a protractor, draw a line at an angle θ relative to the east-west axis. The length D of the arrow is proportional to the vector's magnitude and is measured along the line with a ruler. In this example, the magnitude D of the vector is 10.3 units, and the direction θ is 29.1° north of east.

Vector Addition: Head-to-Tail Method

The **head-to-tail method** is a graphical way to add vectors, described in Figure below and in the steps following. The **tail** of the vector is the starting point of the vector, and the **head** (or tip) of a vector is the final, pointed end of the arrow.

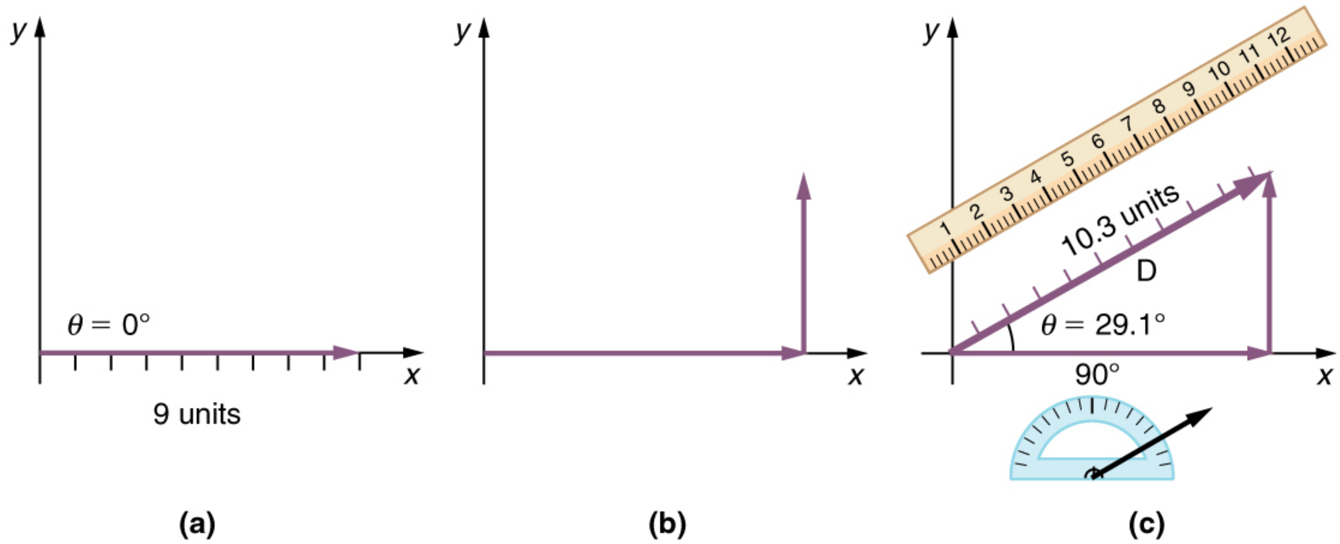

Figure. (a) Draw a vector representing the displacement to the east. (b) Draw a vector representing the displacement to the north. The tail of this vector should originate from the head of the first, east-pointing vector. (c) Draw a line from the tail of the east-pointing vector to the head of the north-pointing vector to form the sum or resultant vector D. The length of the arrow D is proportional to the vector's magnitude and is measured to be 10.3 units . Its direction, described as the angle with respect to the east (or horizontal axis) θ is measured with a protractor to be 29.1^{o}.

Step 1. *Draw an arrow to represent the first vector (9 blocks to the east) using a ruler and protractor*.

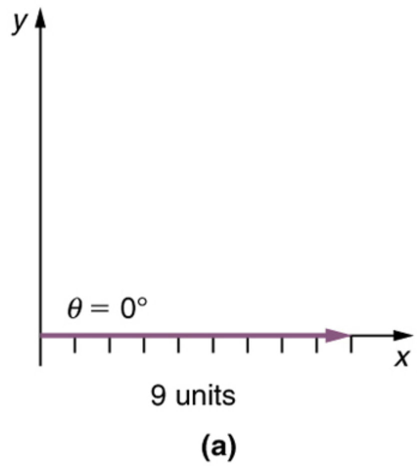

Figure 3.3.5

Step 2. *Now draw an arrow to represent the second vector (5 blocks to the north). Place the tail of the second vector at the head of the first vector.*

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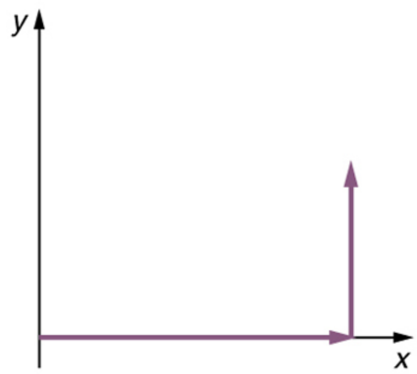
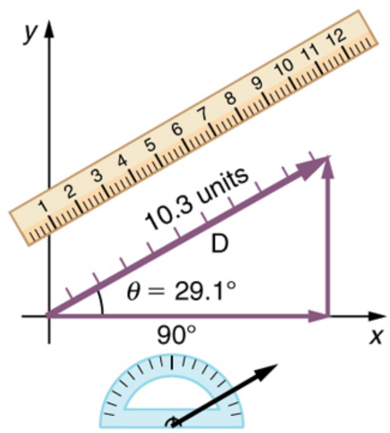


(b)

Figure 3.3.6

Step 3. *If there are more than two vectors, continue this process for each vector to be added. Note that in our example, we have only two vectors, so we have finished placing arrows tip to tail.*

Step 4. *Draw an arrow from the tail of the first vector to the head of the last vector*. This is the **resultant**, or the sum, of the other vectors.



(c)

Figure 3.3.7

Step 5. To get the **magnitude** of the resultant, *measure its length with a ruler. (Note that in most calculations, we will use the Pythagorean theorem to determine this length.)*

Step 6. To get the **direction** of the resultant, *measure the angle it makes with the reference frame using a protractor. (Note that in most calculations, we will use trigonometric relationships to determine this angle.)*

The graphical addition of vectors is limited in accuracy only by the precision with which the drawings can be made and the precision of the measuring tools. It is valid for any number of vectors.

Example 3.3.1:Adding Vectors Graphically Using the Head-to-Tail Method: A Woman Takes a Walk

Use the graphical technique for adding vectors to find the total displacement of a person who walks the following three paths (displacements) on a flat field. First, she walks 25.0 m in a direction north of east. Then, she walks 23.0 m heading north of east. Finally, she turns and walks 32.0 m in a direction **68.0°** south of east.

Strategy

Represent each displacement vector graphically with an arrow, labeling the first , the second , and the third , making the lengths proportional to the distance and the directions as specified relative to an east-west line. The head-to-tail method outlined above will give a way to determine the magnitude and direction of the resultant displacement, denoted .

Solution

(1) Draw the three displacement vectors.

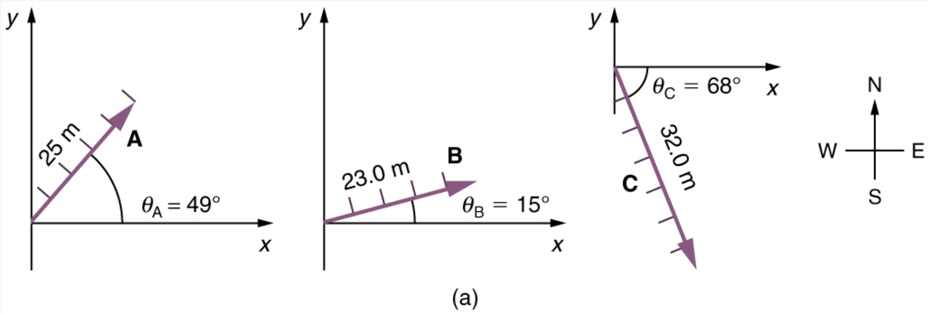


Figure 3.3.8

(2) Place the vectors head to tail retaining both their initial magnitude and direction.

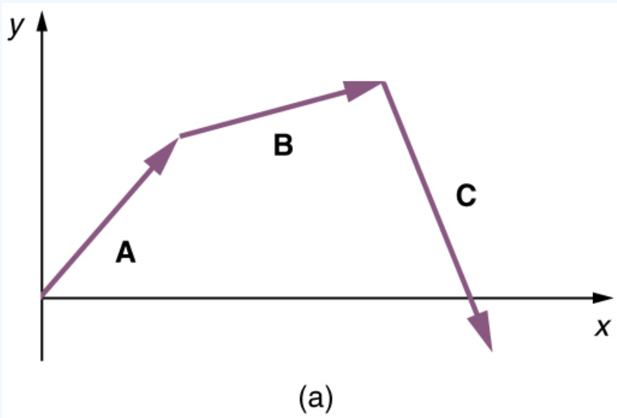


Figure 3.3.9

(3) Draw the resultant vector, .

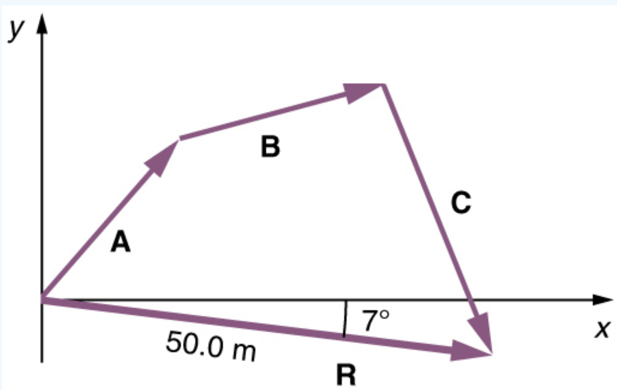


Figure 3.3.10

(4) Use a ruler to measure the magnitude of , and a protractor to measure the direction of . While the direction of the vector can be specified in many ways, the easiest way is to measure the angle between the vector and the nearest horizontal or vertical axis. Since the resultant vector is south of the eastward pointing axis, we flip the protractor upside down and measure the angle between the eastward axis and the vector.

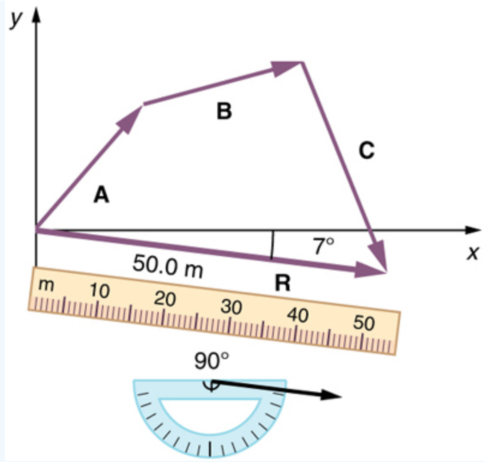


Figure 3.3.11

In this case, the total displacement is seen to have a magnitude of 50.0 m and to lie in a direction south of east. By using its magnitude and direction, this vector can be expressed as **=50.0 m** and =7.0º south of east.

Discussion

The head-to-tail graphical method of vector addition works for any number of vectors. It is also important to note that the resultant is independent of the order in which the vectors are added. Therefore, we could add the vectors in any order as illustrated in Figure and we will still get the same solution.

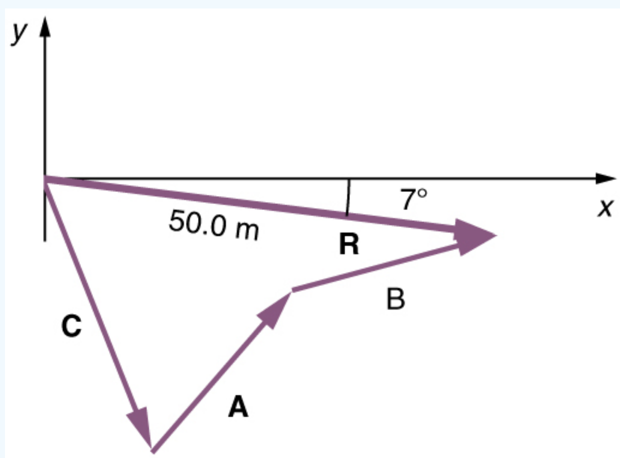


Figure 3.3.12

Here, we see that when the same vectors are added in a different order, the result is the same. This characteristic is true in every case and is an important characteristic of vectors. Vector addition is **commutative**. Vectors can be added in any order.

$$A+B=B+A.$$

(This is true for the addition of ordinary numbers as well—you get the same result whether you add **+3** or +2, for example).

Vector Subtraction

Vector subtraction is a straightforward extension of vector addition. To define subtraction (say we want to subtract from , written **– B** , we must first define what we mean by subtraction. The *negative* of a vector is defined to be ; that is, graphically *the negative of any vector has the same magnitude but the opposite direction*, as shown in Figure. In other words, has the same length as , but points in the opposite direction. Essentially, we just flip the vector so it points in the opposite direction.

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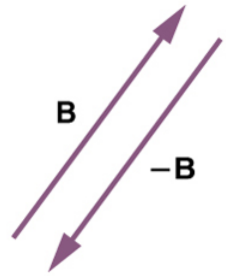

Figure 3.3.13: The negative of a vector is just another vector of the same magnitude but pointing in the opposite direction. So is the negative of ; it has the same length but opposite direction.

The *subtraction* of vector from vector is then simply defined to be the addition of to . Note that vector subtraction is the addition of a negative vector. The order of subtraction does not affect the results.

A – B = A + (–B).

This is analogous to the subtraction of scalars (where, for example, (–2)**)**. Again, the result is independent of the order in which the subtraction is made. When vectors are subtracted graphically, the techniques outlined above are used, as the following example illustrates.

Example 3.3.1:Subtracting Vectors Graphically: A Woman Sailing a Boat

A woman sailing a boat at night is following directions to a dock. The instructions read to first sail 27.5 m in a direction north of east from her current location, and then travel 30.0 m in a direction north of east (or west of north). If the woman makes a mistake and travels in the *opposite* direction for the second leg of the trip, where will she end up? Compare this location with the location of the dock.

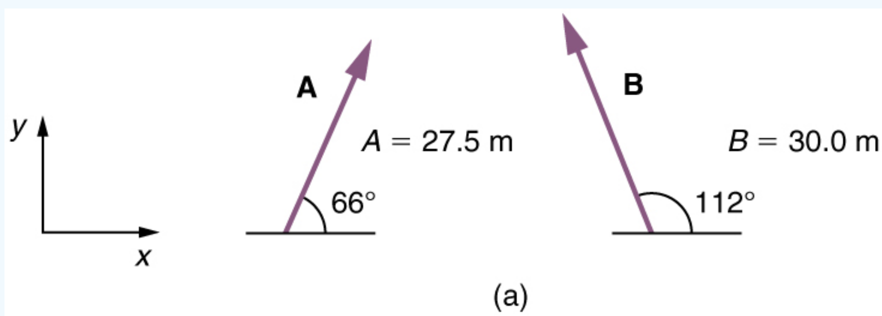

Figure 3.3.14

Strategy

We can represent the first leg of the trip with a vector , and the second leg of the trip with a vector . The dock is located at a location +B. If the woman mistakenly travels in the *opposite* direction for the second leg of the journey, she will travel a distance (30.0 m) in the direction **–112º=68º** south of east. We represent this as , as shown below. The vector has the same magnitude as but is in the opposite direction. Thus, she will end up at a location +(–B), or **–B**.

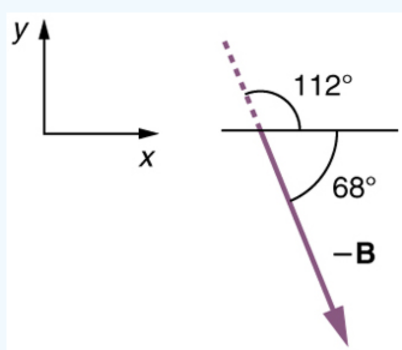

Figure 3.3.15

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We will perform vector addition to compare the location of the dock, + B, with the location at which the woman mistakenly arrives, **+ (–B)**.

Solution

(1) To determine the location at which the woman arrives by accident, draw vectors and .

(2) Place the vectors head to tail.

(3) Draw the resultant vector .

(4) Use a ruler and protractor to measure the magnitude and direction of .

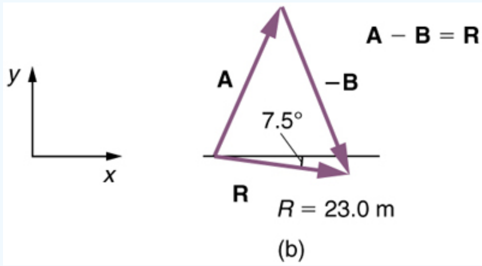

Figure 3.3.16

In this case, **=23.0 m** and **=7.5º** south of east.

(5) To determine the location of the dock, we repeat this method to add vectors and . We obtain the resultant vector **'**:

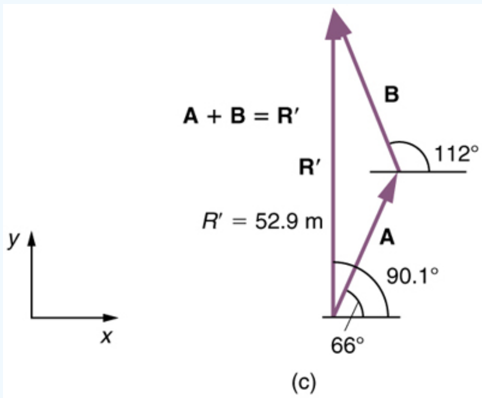

Figure 3.3.17

In this case **= 52.9 m** and **=90.1º** north of east.

We can see that the woman will end up a significant distance from the dock if she travels in the opposite direction for the second leg of the trip.

Discussion

Because subtraction of a vector is the same as addition of a vector with the opposite direction, the graphical method of subtracting vectors works the same as for addition.

Multiplication of Vectors and Scalars

If we decided to walk three times as far on the first leg of the trip considered in the preceding example, then we would walk **× 27.5 m**, or 82.5 m, in a direction **.0º** north of east. This is an example of multiplying a vector by a positive **scalar**. Notice that the magnitude changes, but the direction stays the same.

If the scalar is negative, then multiplying a vector by it changes the vector's magnitude and gives the new vector the *opposite*direction. For example, if you multiply by –2, the magnitude doubles but the direction changes. We can summarize these rules in the following way: When vector is multiplied by a scalar ,

- the magnitude of the vector becomes the absolute value of ,
- if is positive, the direction of the vector does not change,

- if is negative, the direction is reversed.

In our case, =3 and =27.5 m. Vectors are multiplied by scalars in many situations. Note that division is the inverse of multiplication. For example, dividing by 2 is the same as multiplying by the value (1/2). The rules for multiplication of vectors by scalars are the same for division; simply treat the divisor as a scalar between 0 and 1.

Resolving a Vector into Components

In the examples above, we have been adding vectors to determine the resultant vector. In many cases, however, we will need to do the opposite. We will need to take a single vector and find what other vectors added together produce it. In most cases, this involves determining the perpendicular **components** of a single vector, for example the *x- and y*-components, or the north-south and east-west components.

For example, we may know that the total displacement of a person walking in a city is 10.3 blocks in a direction **.0º** north of east and want to find out how many blocks east and north had to be walked. This method is called *finding the components (or parts)* of the displacement in the east and north directions, and it is the inverse of the process followed to find the total displacement. It is one example of finding the components of a vector. There are many applications in physics where this is a useful thing to do. We will see this soon in Projectile Motion, and much more when we cover **forces** in Dynamics: Newton's Laws of Motion. Most of these involve finding components along perpendicular axes (such as north and east), so that right triangles are involved. The analytical techniques presented in Vector Addition and Subtraction: Analytical Methods are ideal for finding vector components.

PHET EXPLORATIONS: MAZE GAME

Learn about position, velocity, and acceleration in the "Arena of Pain". Use the green arrow to move the ball. Add more walls to the arena to make the game more difficult. Try to make a goal as fast as you can.

Figure 3.3.18: Maze Game

Summary

- The **graphical method of adding vectors** and involves drawing vectors on a graph and adding them using the head-to-tail method. The resultant vector is defined such that +B=R**.** The magnitude and direction of are then determined with a ruler and protractor, respectively.
- The **graphical method of subtracting vector** from involves adding the opposite of vector , which is defined as **B**. In this case, **–B=A+(–B)=R**. Then, the head-to-tail method of addition is followed in the usual way to obtain the resultant vector .
- Addition of vectors is **commutative** such that **+B=B+A** .
- The **head-to-tail method** of adding vectors involves drawing the first vector on a graph and then placing the tail of each subsequent vector at the head of the previous vector. The resultant vector is then drawn from the tail of the first vector to the head of the final vector.
- If a vector is multiplied by a scalar quantity , the magnitude of the product is given by . If is positive, the direction of the product points in the same direction as ; if is negative, the direction of the product points in the opposite direction as .

Glossary

component (of a 2-d vector)

a piece of a vector that points in either the vertical or the horizontal direction; every 2-d vector can be expressed as a sum of two vertical and horizontal vector components

commutative

refers to the interchangeability of order in a function; vector addition is commutative because the order in which vectors are added together does not affect the final sum

direction (of a vector)
: the orientation of a vector in space

head (of a vector)
: the end point of a vector; the location of the tip of the vector's arrowhead; also referred to as the "tip"

head-to-tail method
: a method of adding vectors in which the tail of each vector is placed at the head of the previous vector

magnitude (of a vector)
: the length or size of a vector; magnitude is a scalar quantity

resultant
: the sum of two or more vectors

resultant vector
: the vector sum of two or more vectors

scalar
: a quantity with magnitude but no direction

tail
: the start point of a vector; opposite to the head or tip of the arrow

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3.4: Vector Addition and Subtraction- Analytical Methods

Learning Objectives

By the end of this section, you will be able to:

- Understand the rules of vector addition and subtraction using analytical methods.
- Apply analytical methods to determine vertical and horizontal component vectors.
- Apply analytical methods to determine the magnitude and direction of a resultant vector.

Analytical methods of vector addition and subtraction employ geometry and simple trigonometry rather than the ruler and protractor of graphical methods. Part of the graphical technique is retained, because vectors are still represented by arrows for easy visualization. However, analytical methods are more concise, accurate, and precise than graphical methods, which are limited by the accuracy with which a drawing can be made. Analytical methods are limited only by the accuracy and precision with which physical quantities are known.

Resolving a Vector into Perpendicular Components

Analytical techniques and right triangles go hand-in-hand in physics because (among other things) motions along perpendicular directions are independent. We very often need to separate a vector into perpendicular components. For example, given a vector like A in Figure $3.4.1$, we may wish to find which two perpendicular vectors, A_x and A_y, add to produce it.

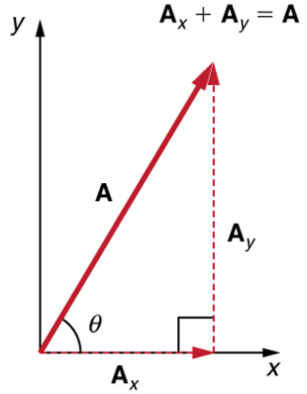


Figure 3.4.1:The vector A, with its tail at the origin of an x, y-coordinate system, is shown together with its x- and y-components, A_x and A_y. These vectors form a right triangle. The analytical relationships among these vectors are summarized below.

A_x and A_y are defined to be the components of A along the x- and y-axes. The three vectors A, A_x, and A_y form a right triangle:

$$A_x + A_y = A. \tag{3.4.1}$$

Note that this relationship between vector components and the resultant vector holds only for vector quantities (which include both magnitude and direction). The relationship does not apply for the magnitudes alone. For example, if $A_x = 3m$ east, $A_y = 4m$ north, and $A = 5m$north-east, then it is true that the vectors $A_x + A_y = A$. However, it is not true that the sum of the magnitudes of the vectors is also equal. That is,

$$3m + 4m \neq 5m \tag{3.4.2}$$

Thus,

$$A_x + A_y \neq A \tag{3.4.3}$$

If the vector A is known, then its magnitude A (its length) and its angle θ (its direction) are known. To find A_x and A_y, its x- and y-components, we use the following relationships for a right triangle.

$$A_x = Acos\theta \tag{3.4.4}$$

and

$$A_y = Asin\theta. \tag{3.4.5}$$

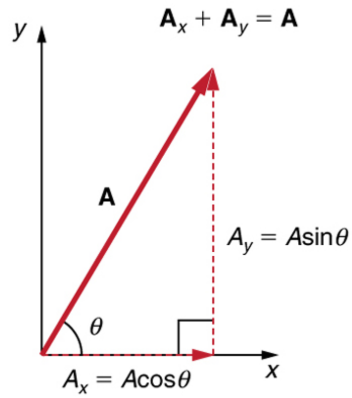


Figure 3.4.2: The magnitudes of the vector components A_x and A_y can be related to the resultant vector A and the angle θ with trigonometric identities. Here we see that $A_x = Acos\theta$ and $A_y = Asin\theta$.

Suppose, for example, that **A** is the vector representing the total displacement of the person walking in a city considered in Kinematics in Two Dimensions: An Introduction and Vector Addition and Subtraction: Graphical Methods.

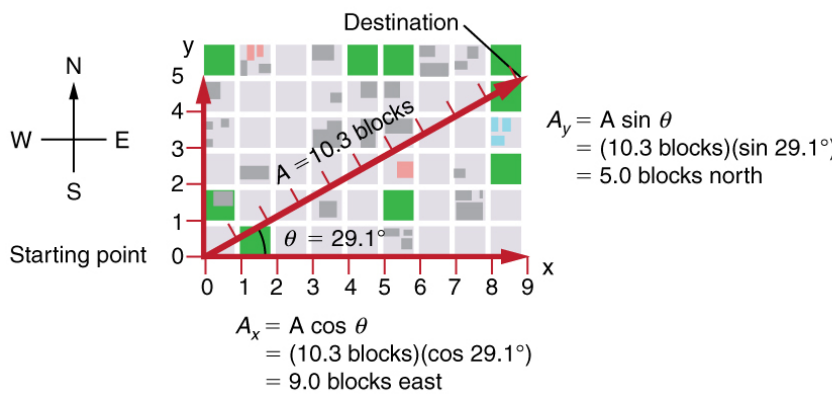


Figure 3.4.3: We can use the relationships $A_x = Acos\theta$ and $A_y = Asin\theta$ to determine the magnitude of the horizontal and vertical component vectors in this example.

Then $A = 10.3$ blocks and $\theta = 29.1^{\circ}$, so that

$$A_x = A\cos\theta = (10.3 blocks)(\cos 29.1^{\circ}) = 9.0 \quad blocks \tag{3.4.6}$$

$$A_y = A\sin\theta = (10.3 blocks)(\sin 29.1^{\circ}) = 5.0 \quad blocks \tag{3.4.7}$$

Calculating a Resultant Vector

If the perpendicular components A_x and A_y of a vector A are known, then A can also be found analytically. To find the magnitude A and direction θ of a vector from its perpendicular components A_x and A_y, we use the following relationships:

$$A = \sqrt{A_{x^2} + A_{y^2}} \tag{3.4.8}$$

$$\theta = tan^{-1}(A_y/A_x) \tag{3.4.9}$$

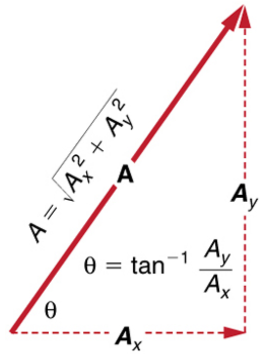


Figure 3.4.4: The magnitude and direction of the resultant vector can be determined once the horizontal and vertical components A_x and A_y have been determined.

Note that the equation $A = \sqrt{A_x^2 + A_y^2}$ is just the Pythagorean theorem relating the legs of a right triangle to the length of the hypotenuse. For example, if A_x and A_y are 9 and 5 blocks, respectively, then $A = \sqrt{9^2 + 5^2} = 10.3$ blocks, again consistent with the example of the person walking in a city. Finally, the direction is $\theta = tan^{-1}(5/9) = 29.1°$, as before.

DETERMINING VECTORS AND VECTOR COMPONENTS WITH ANALYTICAL METHODS

Equations $A_x = Acos\theta$ and $A_y = Asin\theta$ are used to find the perpendicular components of a vector—that is, to go from A and θ to A_x and A_y. Equations $A = \sqrt{A_x^2 + A_y^2}$ and $\theta = tan^{-1}(A_y/A_x)$ are used to find a vector from its perpendicular components—that is, to go from A_x and A_y to A and θ. Both processes are crucial to analytical methods of vector addition and subtraction.

Adding Vectors Using Analytical Methods

To see how to add vectors using perpendicular components, consider Figure $3.4.5$, in which the vectors A and B are added to produce the resultant R.

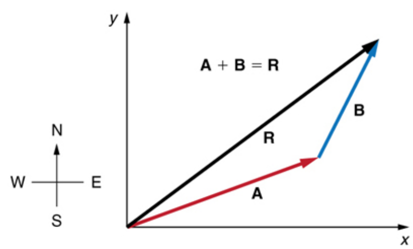


Figure 3.4.5: Vectors A and$B$ are two legs of a walk, and R is the resultant or total displacement. You can use analytical methods to determine the magnitude and direction of R.

If A and B represent two legs of a walk (two displacements), then R is the total displacement. The person taking the walk ends up at the tip of R. There are many ways to arrive at the same point. In particular, the person could have walked first in the x-direction and then in the y-direction. Those paths are the x- and y-components of the resultant, R_x and R_y. If we know R_x and R_y, we can find R and θ using the equations $A = \sqrt{A_x^2 + A_y^2}$ and $\theta = tan^{-1}(A_y/A_x)$. When you use the analytical method of vector addition, you can determine the components or the magnitude and direction of a vector.

Step 1. *Identify the x- and y-axes that will be used in the problem. Then, find the components of each vector to be added along the chosen perpendicular axes.* Use the equations $A_x = Acos\theta$ and $A_y = Asin\theta$ to find the components. In Figure, these components are A_x, A_y, B_x, and B_y. The angles that vectors A and B make with the x-axis are θ_A and θ_B, respectively.

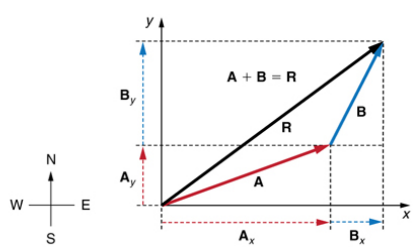


Figure 3.4.6: To add vectors A and B, first determine the horizontal and vertical components of each vector. These are the dotted vectors A_x, A_y, B_x and $B - y$ shown in the image.

Step 2. *Find the components of the resultant along each axis by adding the components of the individual vectors along that axis. That is, as shown in Figure* \(\PageIndex{7}\,

$$R_x = A_x + B_x \tag{3.4.10}$$

and

$$R_y = A_y + B_y. \tag{3.4.11}$$

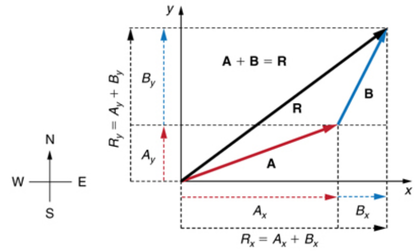


Figure 3.4.7: The magnitude of the vectors A_x and B_x add to give the magnitude R_x of the resultant vector in the horizontal direction. Similarly, the magnitudes of the vectors A_y and B_y add to give the magnitude R_y of the resultant vector in the vertical direction.

Components along the same axis, say the *x*-axis, are vectors along the same line and, thus, can be added to one another like ordinary numbers. The same is true for components along the *y*-axis. (For example, a 9-block eastward walk could be taken in two legs, the first 3 blocks east and the second 6 blocks east, for a total of 9, because they are along the same direction.) So resolving vectors into components along common axes makes it easier to add them. Now that the components of *R* are known, its magnitude and direction can be found.

Step 3. *To get the magnitude R of the resultant, use the Pythagorean theorem:*

$$R = \sqrt{R_x^2 + R_y^2} \tag{3.4.12}$$

Step 4. *To get the direction of the resultant:*

$$\theta = tan^{-1}(R_y/R_x) \tag{3.4.13}$$

The following example illustrates this technique for adding vectors using perpendicular components.

Example 3.4.1: Adding Vectors Using Analytical Methods

Add the vector A to the vector B shown in Figure, using perpendicular components along the *x*- and *y*-axes. The *x*- and *y*-axes are along the east–west and north–south directions, respectively. Vector A represents the first leg of a walk in which a person walks $53.0 m$ in a direction 20.0^{o} north of east. Vector B represents the second leg, a displacement of $34.0 m$ in a direction 63.0^{o} north of east.

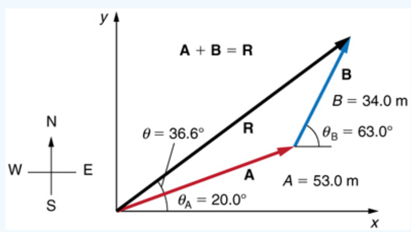


Figure 3.4.8: Vector A has magnitude $53.0 m$ and direction 20.0^{o} north of the *x*-axis. Vector B has magnitude $34.0 m$ and direction 63.0^{o} north of the x-axis. You can use analytical methods to determine the magnitude and direction of R.

Strategy

The components of A and B along the *x*- and *y*-axes represent walking due east and due north to get to the same ending point. Once found, they are combined to produce the resultant.

Solution

Following the method outlined above, we first find the components of A and B along the *x*- and *y*-axes. Note that $A = 53.0 m, \theta_A = 20.0^{o}, B = 34.0 m$, and $\theta_B = 63.0^{o}$. We find the *x*-components by using $A_x = Acos\theta$, which gives

$$A_x = Acos\theta_A = (53.0 m)(cos 20.0^{o})(53.0 m)(0.940) = 49.8 m \tag{3.4.14}$$

and

$$B_x = Bcos\theta_B = (34.0 m)(cos 63.0^{o})(34.0 m)(0.454) = 15.4 m. \tag{3.4.15}$$

Similarly, the *y*-components are found using $A_y = Asin\theta_A$:

$$A_y = Asin\theta_A = (53.0 m)(sin 20.0^{o})(53.0 m)(0.342) = 18.1 m \tag{3.4.16}$$

and

$$B_y = B sin\theta_B = (34.0m)(sin 63.0^\circ)(34.0m)(0.891) = 30.3m. \tag{3.4.17}$$

The *x*- and *y*-components of the resultant are thus

$$R_x = A_x + B_x = 49.8m + 15.4m = 65.2m \tag{3.4.18}$$

and

$$R_y = A_y + B_y = 18.1m + 30.3m = 48.4m. \tag{3.4.19}$$

Now we can find the magnitude of the resultant by using the Pythagorean theorem:

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(65.2)^2 + (48.4)^2 m} \tag{3.4.20}$$

so that

$$R = 81.2m. \tag{3.4.21}$$

Finally, we find the direction of the resultant:

$$\theta = tan^{-1}(R_y/R_x) = +tan^{-1}(48.4/65.2). \tag{3.4.22}$$

Thus,

$$\theta = tan^{-1}(0.742) = 36.6^\circ. \tag{3.4.23}$$

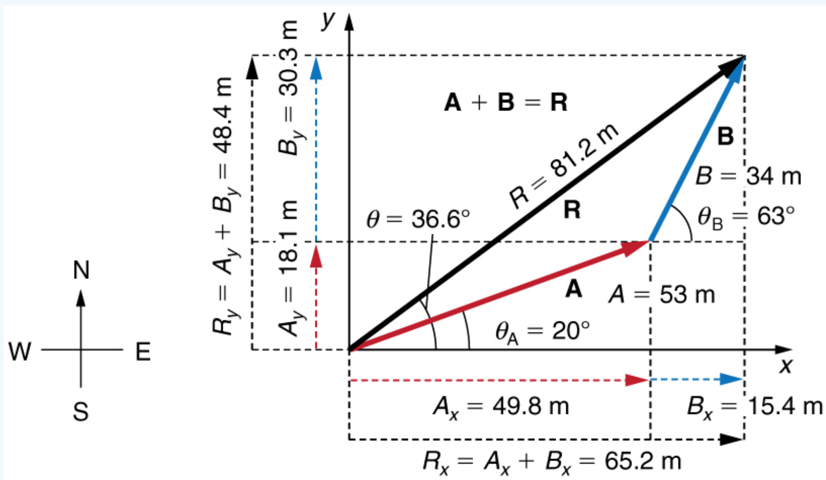


Figure 3.4.9: Using analytical methods, we see that the magnitude of R is $81.2m$ and its direction is 36.6° north of east.

Discussion

This example illustrates the addition of vectors using perpendicular components. Vector subtraction using perpendicular components is very similar—it is just the addition of a negative vector.

Subtraction of vectors is accomplished by the addition of a negative vector. That is, $A - B \equiv A + (-B)$. Thus, *the method for the subtraction of vectors using perpendicular components is identical to that for addition.* The components of $-B$ are the negatives of the components of B. The *x*- and *y*-components of the resultant $A - B = R$ are thus

$$R_x = A_x + (-B_x) \tag{3.4.24}$$

and

$$R_y = A_y + (-B_y) \tag{3.4.25}$$

and the rest of the method outlined above is identical to that for addition. (See Figure 3.4.10.)

Analyzing vectors using perpendicular components is very useful in many areas of physics, because perpendicular quantities are often independent of one another. The next module, Projectile Motion, is one of many in which using perpendicular components helps make the picture clear and simplifies the physics.

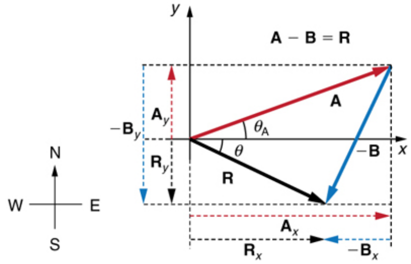
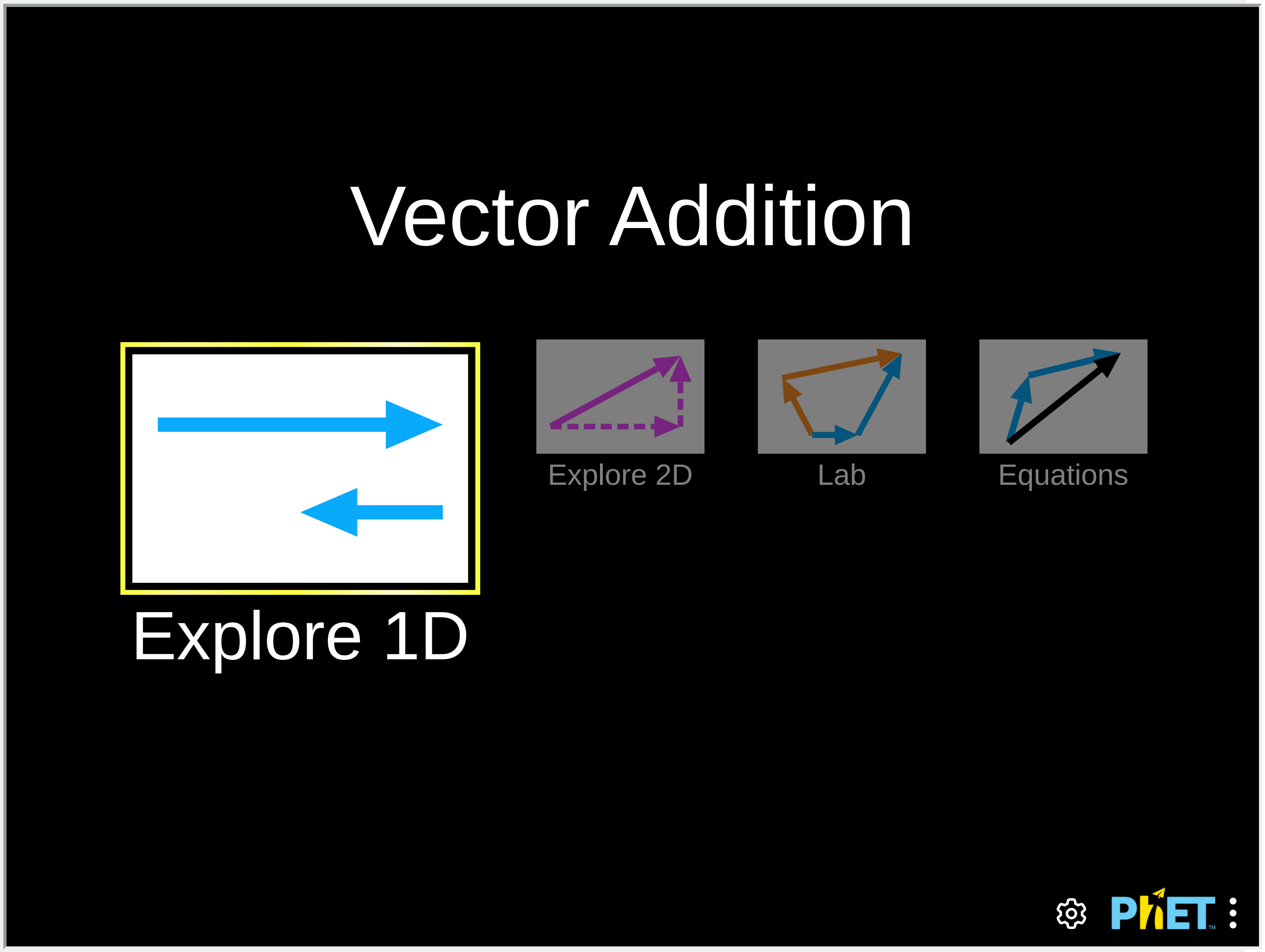


Figure 3.4.10. The components of $-B$ are the negatives of the components of B. The method of subtraction is the same as that for addition.

PHET EXPLORATIONS: VECTOR ADDITION

Learn how to add vectors. Drag vectors onto a graph, change their length and angle, and sum them together. The magnitude, angle, and components of each vector can be displayed in several formats.



Summary

- The analytical method of vector addition and subtraction involves using the Pythagorean theorem and trigonometric identities to determine the magnitude and direction of a resultant vector.
- The steps to add vectors A and B using the analytical method are as follows:

  Step 1: Determine the coordinate system for the vectors. Then, determine the horizontal and vertical components of each vector using the equations

$$A_x = A\cos\theta$$
$$B_x = B\cos\theta$$

  and

$$A_y = Asin\theta$$
$$B_y = Bsin\theta.$$

Step 2: Add the horizontal and vertical components of each vector to determine the components Rx and Ry of the resultant vector, R:

$$R_x = A_x + B_x$$

and

$$R_y = A_y + B_y\ .$$

Step 3: Use the Pythagorean theorem to determine the magnitude, R, of the resultant vector R:

$$R = \sqrt{R_x^2 + R_y^2}\ .$$

Step 4: Use a trigonometric identity to determine the direction, θ, of **R**:

$$\theta = tan^{-1}(R_y/R_x)\,.$$

Glossary

analytical method

the method of determining the magnitude and direction of a resultant vector using the Pythagorean theorem and trigonometric identities

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3.5: Projectile Motion

Learning Objectives

By the end of this section, you will be able to:

- Identify and explain the properties of a projectile, such as acceleration due to gravity, range, maximum height, and trajectory.
- Determine the location and velocity of a projectile at different points in its trajectory.
- Apply the principle of independence of motion to solve projectile motion problems.

Projectile motion is the **motion** of an object thrown or projected into the air, subject to only the acceleration of gravity. The object is called a **projectile**, and its path is called its **trajectory.** The motion of falling objects, as covered in Problem-Solving Basics for One-Dimensional Kinematics, is a simple one-dimensional type of projectile motion in which there is no horizontal movement. In this section, we consider two-dimensional projectile motion, such as that of a football or other object for which **air resistance** *is negligible*.

The most important fact to remember here is that motions along perpendicular axes are independent and thus can be analyzed separately. This fact was discussed in Kinematics in Two Dimensions: An Introduction, where vertical and horizontal motions were seen to be independent. The key to analyzing two-dimensional projectile motion is to break it into two motions, one along the horizontal axis and the other along the vertical. (This choice of axes is the most sensible, because acceleration due to gravity is vertical—thus, there will be no acceleration along the horizontal axis when air resistance is negligible.) As is customary, we call the horizontal axis the *x*-axis and the vertical axis the *y*-axis. Figure illustrates the notation for displacement, where s is defined to be the total displacement and x and y are its components along the horizontal and vertical axes, respectively. The magnitudes of these vectors are s, x, and y. (Note that in the last section we used the notation A to represent a vector with components A_x and A_y. If we continued this format, we would call displacement s with components s_x and s_y. However, to simplify the notation, we will simply represent the component vectors as x and y.)

Of course, to describe motion we must deal with velocity and acceleration, as well as with displacement. We must find their components along the x- and *y*-axes, too. We will assume all forces except gravity (such as air resistance and friction, for example) are negligible. The components of acceleration are then very simple: $a_y =-g =-9.80 m/s^2$(. (Note that this definition assumes that the upwards direction is defined as the positive direction. If you arrange the coordinate system instead such that the downwards direction is positive, then acceleration due to gravity takes a positive value.) Because gravity is vertical, $a_x =0$. Both accelerations are constant, so the kinematic equations can be used.

REVIEW OF KINEMATIC EQUATIONS (CONSTANT)

$$x=x_0+\bar{v}t$$
$$\bar{v}=\frac{v_0+v}{2}$$
$$v=v_0+at$$
$$x=x_0+v_0t+\frac{1}{2}at^2$$
$$v^2=v_0^2+2a(x-x_0)\ .$$

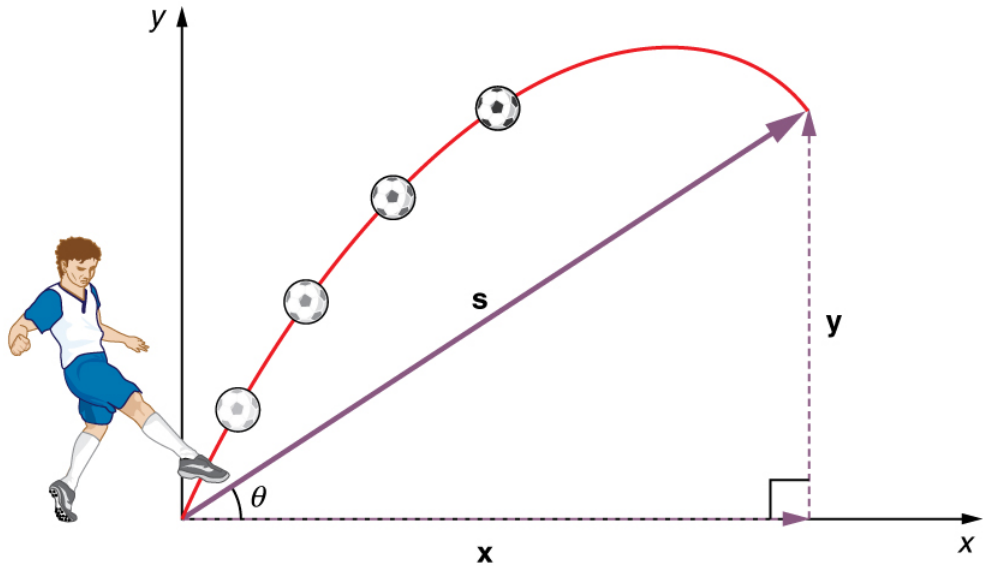

Figure 3.5.1: The total displacement s of a soccer ball at a point along its path. The vector s has components x and y along the horizontal and vertical axes. Its magnitude is s, and it makes an angle θ with the horizontal.

Given these assumptions, the following steps are then used to analyze projectile motion:

Step 1. *Resolve or break the motion into horizontal and vertical components along the x- and y-axes.* These axes are perpendicular, so $A_x = Acos\theta$ and $A_y = Asin\theta$ are used. The magnitude of the components of displacement s along these axes are x and y. The magnitudes of the components of the velocity v are $v_x = vcos\theta$ and $v_y = vsin\theta$, where v is the magnitude of the velocity and θ is its direction, as shown in Figure. Initial values are denoted with a subscript 0, as usual.

Step 2. *Treat the motion as two independent one-dimensional motions, one horizontal and the other vertical.* The kinematic equations for horizontal and vertical motion take the following forms:

$$\text{Horizontal Motion}(a_x = 0)$$
$$x = x_0 + v_x t$$
$$v_x = v_{0x} = v_x = \text{velocity is a constant} .$$
$$\text{Vertical Motion}(\text{assuming positive is up} a_y = -g = -9.80 m/s^2)$$
$$y = y_0 + \frac{1}{2}(v_{0y} + v_y)t$$
$$v_y = v_{0y} - gt$$
$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$
$$v_y^2 = v_{0y}^2 - 2g(y - y_0) .$$

Step 3. *Solve for the unknowns in the two separate motions—one horizontal and one vertical.* Note that the only common variable between the motions is time *t*. The problem solving procedures here are the same as for one-dimensional **kinematics** and are illustrated in the solved examples below.

Step 4. *Recombine the two motions to find the total displacement* s *and velocity* v. Because the *x* - and *y* -motions are perpendicular, we determine these vectors by using the techniques outlined in the Vector Addition and Subtraction: Analytical Methods and employing $A = \sqrt{A_x^2 + A_y^2}$ and $\theta = tan^{-1}(A_y/A_x)$ in the following form, where θ is the direction of the displacement s and θ_v is the direction of the velocity v:

Total displacement and velocity

$$s = \sqrt{x^2 + y^2}$$
$$\theta = tan^{-1}(y/x)$$
$$v = \sqrt{v_x^2 + v_y^2}$$
$$\theta_v = tan^{-1}(v_y/v_x) .$$

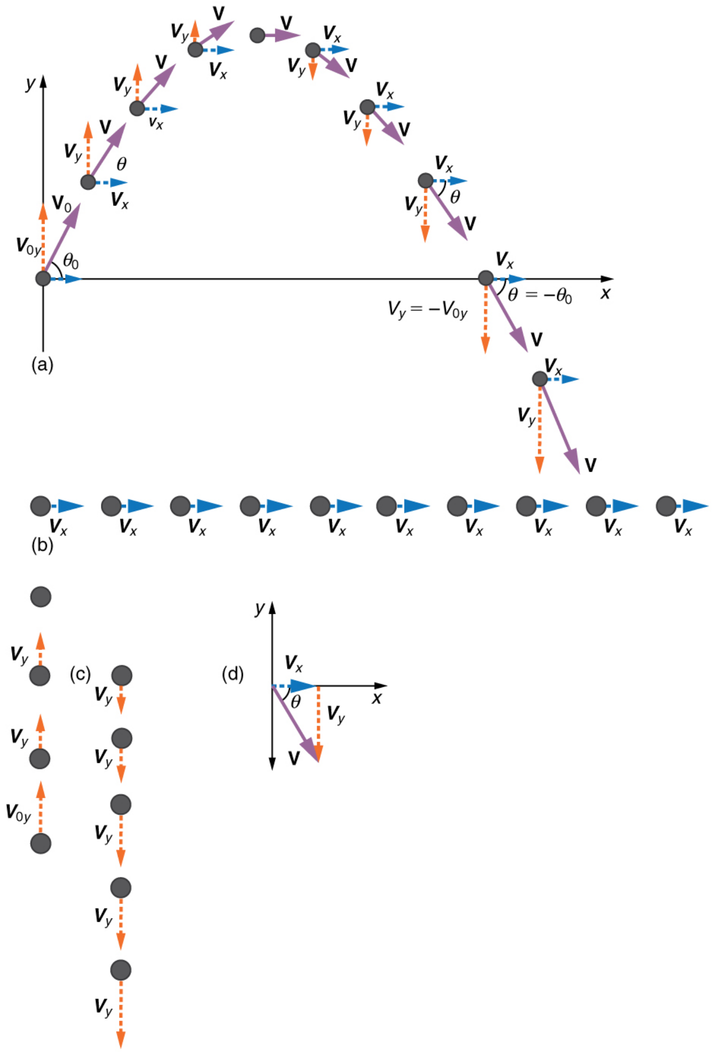

Figure 3.5.2: (a) We analyze two-dimensional projectile motion by breaking it into two independent one-dimensional motions along the vertical and horizontal axes. (b) The horizontal motion is simple, because $a_x = 0$ and v_x is thus constant. (c) The velocity in the vertical direction begins to decrease as the object rises; at its highest point, the vertical velocity is zero. As the object falls towards the Earth again, the vertical velocity increases again in magnitude but points in the opposite direction to the initial vertical velocity. (d) The x - and y -motions are recombined to give the total velocity at any given point on the trajectory.

Example 3.5.1: A Fireworks Projectile Explodes High and Away

During a fireworks display, a shell is shot into the air with an initial speed of 70.0 m/s at an angle of **75.0°** above the horizontal, as illustrated in Figure. The fuse is timed to ignite the shell just as it reaches its highest point above the ground.

a. Calculate the height at which the shell explodes.
b. How much time passed between the launch of the shell and the explosion?
c. What is the horizontal displacement of the shell when it explodes?

Strategy

Because air resistance is negligible for the unexploded shell, the analysis method outlined above can be used. The motion can be broken into horizontal and vertical motions in which $a_x = 0$ and $a_y = -g$. We can then define x_0 and y_0 to be zero and solve for the desired quantities.

Solution for (a)

By “height” we mean the altitude or vertical position y above the starting point. The highest point in any trajectory, called the apex, is reached when $v_y = 0$. Since we know the initial and final velocities as well as the initial position, we use the following equation to find y:

$$v_y^2 = v_{0y}^2 - 2g(y - y_0)\ .$$

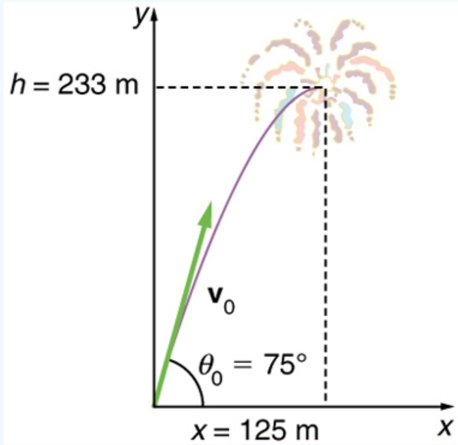

Figure 3.5.3: The trajectory of a fireworks shell. The fuse is set to explode the shell at the highest point in its trajectory, which is found to be at a height of 233 m and 125 m away horizontally.

Because y_0 and v_y are both zero, the equation simplifies to

$$0 = v_{0y}^2 - 2gy.$$

Solving for y gives

$$y = \frac{v_{0y}^2}{2g}.$$

Now we must find v_{0y}, the component of the initial velocity in the *y*-direction. It is given by $v_{0y} = v_0 sin\theta$, where v_{0y} is the initial velocity of 70.0 m/s, and $\theta_0 = 75.0^o$ is the initial angle. Thus,

$$v_{0y} = v_0 sin\theta_0 = (70.0m/s)(sin75^o) = 67.6m/s.$$

and y is

$$y = \frac{(67.6m/s)^2}{2(9.80m/s^2)},$$

so that

$$y = 233m.$$

Discussion for (a)

Note that because up is positive, the initial velocity is positive, as is the maximum height, but the acceleration due to gravity is negative. Note also that the maximum height depends only on the vertical component of the initial velocity, so that any projectile with a 67.6 m/s initial vertical component of velocity will reach a maximum height of 233 m (neglecting air resistance). The numbers in this example are reasonable for large fireworks displays, the shells of which do reach such heights before exploding. In practice, air resistance is not completely negligible, and so the initial velocity would have to be somewhat larger than that given to reach the same height.

Solution for (b)

As in many physics problems, there is more than one way to solve for the time to the highest point. In this case, the easiest method is to use $y = y_0 + \frac{1}{2}(v_{0y} + v_y)t$. Because y0 is zero, this equation reduces to simply

$$y = \frac{1}{2}(v_{0y} + v_y)t\ .$$

Note that the final vertical velocity, v_y, at the highest point is zero. Thus,

$$t = \frac{2y}{(v_{0y} + v_y)} = \frac{2(233m)}{(67.6m/s)} = 6.90s.$$

Discussion for (b)

This time is also reasonable for large fireworks. When you are able to see the launch of fireworks, you will notice several seconds pass before the shell explodes. (Another way of finding the time is by using $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$, and solving the quadratic equation for t.)

Solution for (c)

Because air resistance is negligible, $a_x = 0$ and the horizontal velocity is constant, as discussed above. The horizontal displacement is horizontal velocity multiplied by time as given by $x = x_0 + v_x t$, where x_0 is equal to zero:

$$x = v_x t,$$

where v_x is the *x*-component of the velocity, which is given by $v_x = v_0 cos\theta_0$. Now,

$$v_x = v_0 cos\theta_0 = (70.0 m/s)(cos 75.0^o) = 18.1 m/s.$$

The time t for both motions is the same, and so x is

$$x = (18.1 m/s)(6.90 s) = 125 m.$$

Discussion for (c)

The horizontal motion is a constant velocity in the absence of air resistance. The horizontal displacement found here could be useful in keeping the fireworks fragments from falling on spectators. Once the shell explodes, air resistance has a major effect, and many fragments will land directly below.

In solving part (a) of the preceding example, the expression we found for y is valid for any projectile motion where air resistance is negligible. Call the maximum height $y = h$; then,

$$h = \frac{v_{0y}^2}{2g}.$$

This equation defines the *maximum height* of a projectile and depends only on the vertical component of the initial velocity.

DEFINING A COORDINATE SYSTEM

It is important to set up a coordinate system when analyzing projectile motion. One part of defining the coordinate system is to define an origin for the x and y positions. Often, it is convenient to choose the initial position of the object as the origin such that $x_0 = 0$ and $y_0 = 0$. It is also important to define the positive and negative directions in the x and y directions. Typically, we define the positive vertical direction as upwards, and the positive horizontal direction is usually the direction of the object's motion. When this is the case, the vertical acceleration, g, takes a negative value (since it is directed downwards towards the Earth). However, it is occasionally useful to define the coordinates differently. For example, if you are analyzing the motion of a ball thrown downwards from the top of a cliff, it may make sense to define the positive direction downwards since the motion of the ball is solely in the downwards direction. If this is the case, g takes a positive value.

Example 3.5.2: Calculating Projectile Motion: Hot Rock Projectile

Kilauea in Hawaii is the world's most continuously active volcano. Very active volcanoes characteristically eject red-hot rocks and lava rather than smoke and ash. Suppose a large rock is ejected from the volcano with a speed of 25.0 m/s and at an angle above the horizontal, as shown in Figure. The rock strikes the side of the volcano at an altitude 20.0 m lower than its starting point. (a) Calculate the time it takes the rock to follow this path. (b) What are the magnitude and direction of the rock's velocity at impact?

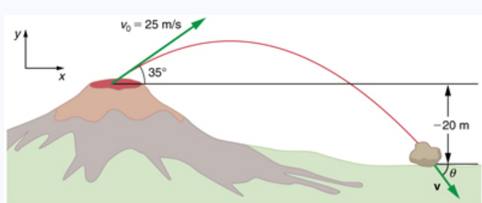

Figure 3.5.4: The trajectory of a rock ejected from the Kilauea volcano.

Strategy

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Again, resolving this two-dimensional motion into two independent one-dimensional motions will allow us to solve for the desired quantities. The time a projectile is in the air is governed by its vertical motion alone. We will solve for t first. While the rock is rising and falling vertically, the horizontal motion continues at a constant velocity. This example asks for the final velocity. Thus, the vertical and horizontal results will be recombined to obtain v and θ_v at the final time t determined in the first part of the example.

Solution for (a)

While the rock is in the air, it rises and then falls to a final position 20.0 m lower than its starting altitude. We can find the time for this by using

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2 \ .$$

If we take the initial position y_0 to be zero, then the final position is $y = -20.0m$. Now the initial vertical velocity is the vertical component of the initial velocity, found from $v_{0y} = v_0 sin\theta_0 = (25.0m/s)(sin35.0^{\circ}) = 14.3m/s$. Substituting known values yields

$$-20.0m = (14.3m/s)t - (4.90m/s^2)t^2.$$

Rearranging terms gives a quadratic equation in t:

$$(4.90m/s^2)t^2 - (14.3m/s)t - (20.0m) = 0.$$

This expression is a quadratic equation of the form $at^2 + bt + c = 0$, where the constants are $a = 4.90, b =- 14.3$, and $c =- 20.0.$ Its solutions are given by the quadratic formula:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \ .$$

This equation yields two solutions: $t = 3.96$ and $t =- 1.03$. (It is left as an exercise for the reader to verify these solutions.) The time is $t = 3.96s$ or $-1.03s$. The negative value of time implies an event before the start of motion, and so we discard it. Thus,

$$t = 3.96s.$$

Discussion for (a)

The time for projectile motion is completely determined by the vertical motion. So any projectile that has an initial vertical velocity of 14.3 m/s and lands 20.0 m below its starting altitude will spend 3.96 s in the air.

Solution for (b)

From the information now in hand, we can find the final horizontal and vertical velocities v_x and v_y and combine them to find the total velocity v and the angle θ_0 it makes with the horizontal. Of course, v_x is constant so we can solve for it at any horizontal location. In this case, we chose the starting point since we know both the initial velocity and initial angle. Therefore:

$$v_x = v_0 cos\theta_0 = (25.0m/s)(cos35^{\circ}) = 20.5m/s.$$

The final vertical velocity is given by the following equation:

$$v_y = v_{0y} - gt \ ,$$

where v_{0y} was found in part (a) to be **14.3 m/s**. Thus,

$$v_y = 14.3m/s - (9.80m/s^2)(3.96s)$$

so that

$$v_y = -24.5m/s.$$

To find the magnitude of the final velocity v we combine its perpendicular components, using the following equation:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(20.5m/s)^2 + (-24.5m/s)^2} \ ,$$

which gives

$$v = 31.9 m/s.$$

The direction θv is found from the equation:

$$\theta_v = tan^{-1}(v_y/v_x)$$

so that

$$\theta_v = tan^{-1}(-24.5/20.5) = tan^{-1}(-1.19).$$

Thus,

$$\theta_v = -50.1^o.$$

Discussion for (b)

The negative angle means that the velocity is **50.1°** below the horizontal. This result is consistent with the fact that the final vertical velocity is negative and hence downward—as you would expect because the final altitude is 20.0 m lower than the initial altitude. (See Figure.)

One of the most important things illustrated by projectile motion is that vertical and horizontal motions are independent of each other. Galileo was the first person to fully comprehend this characteristic. He used it to predict the range of a projectile. On level ground, we define **range** to be the horizontal distance traveled by a projectile. Galileo and many others were interested in the range of projectiles primarily for military purposes—such as aiming cannons. However, investigating the range of projectiles can shed light on other interesting phenomena, such as the orbits of satellites around the Earth. Let us consider projectile range further.

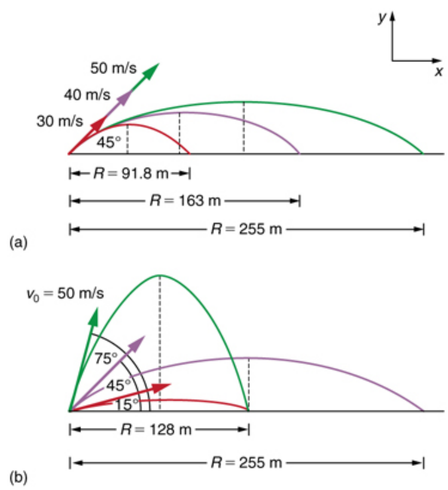

Figure 3.5.5: Trajectories of projectiles on level ground. (a) The greater the initial speed v_0, the greater the range for a given initial angle. (b) The effect of initial angle θ_0 on the range of a projectile with a given initial speed. Note that the range is the same for 15° and 75°, although the maximum heights of those paths are different.

How does the initial velocity of a projectile affect its range? Obviously, the greater the initial speed v_0, the greater the range, as shown in Figure(a). The initial angle θ_0 also has a dramatic effect on the range, as illustrated in Figure(b). For a fixed initial speed, such as might be produced by a cannon, the maximum range is obtained with $\theta_0 = 45^o$. This is true only for conditions neglecting air resistance. If air resistance is considered, the maximum angle is approximately 38^o. Interestingly, for every initial angle except 45^o, there are two angles that give the same range—the sum of those angles is 90^o. The range also depends on the value of the acceleration of gravity g. The lunar astronaut Alan Shepherd was able to drive a golf ball a great distance on the Moon because gravity is weaker there. The range R of a projectile on level ground for which air resistance is negligible is given by

$$R = \frac{v_0^2 sin 2\theta_0}{g},$$

where v_0 is the initial speed and θ_0 is the initial angle relative to the horizontal. The proof of this equation is left as an end-of-chapter problem (hints are given), but it does fit the major features of projectile range as described.

When we speak of the range of a projectile on level ground, we assume that R is very small compared with the circumference of the Earth. If, however, the range is large, the Earth curves away below the projectile and acceleration of gravity changes direction along the path. The range is larger than predicted by the range equation given above because the projectile has farther to fall than it would on level ground. (See Figure.) If the initial speed is great enough, the projectile goes into orbit. This is called exit velocity. This possibility was recognized centuries before it could be accomplished. When an object is in orbit, the Earth curves away from underneath the object at the same rate as it falls. The object thus falls continuously but never hits the surface. These and other aspects of orbital motion, such as the rotation of the Earth, will be covered analytically and in greater depth later in this text.

Once again we see that thinking about one topic, such as the range of a projectile, can lead us to others, such as the Earth orbits. In Addition of Velocities, we will examine the addition of velocities, which is another important aspect of two-dimensional kinematics and will also yield insights beyond the immediate topic.

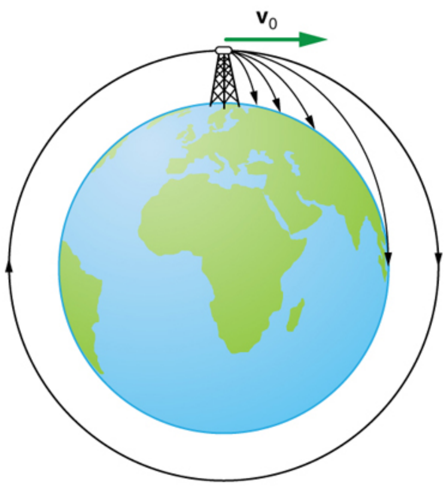

Figure 3.5.6: Projectile to satellite. In each case shown here, a projectile is launched from a very high tower to avoid air resistance. With increasing initial speed, the range increases and becomes longer than it would be on level ground because the Earth curves away underneath its path. With a large enough initial speed, orbit is achieved.

PHET EXPLORATIONS: PROJECTILE MOTION

Blast a Buick out of a cannon! Learn about projectile motion by firing various objects. Set the angle, initial speed, and mass. Add air resistance. Make a game out of this simulation by trying to hit a target.

Figure 3.5.7: Projectile Motion

Summary

- Projectile motion is the motion of an object through the air that is subject only to the acceleration of gravity.
- To solve projectile motion problems, perform the following steps:

1. Determine a coordinate system. Then, resolve the position and/or velocity of the object in the horizontal and vertical components. The components of position s are given by the quantities x and y, and the components of the velocity v are given by vx=vcosθ and vy=vsinθ, where v is the magnitude of the velocity and θ is its direction.

2. Analyze the motion of the projectile in the horizontal direction using the following equations:

$$\text{Horizontal motion}(a_x = 0)$$

$$x = x_0 + v_x t$$

$$v_x = v_{0x} = v_x = \text{velocity is a constant} \ .$$

3. Analyze the motion of the projectile in the vertical direction using the following equations:

$$\text{Vertical motion}(\text{Assuming positive direction is up}; a_y = -g = -9.80 m/s^2)$$

$$y = y_0 + \frac{1}{2}(v_{0y} + v_y)t$$

$$v_y = v_{0y} - gt$$

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

$$v_y^2 = v_{0y}^2 - 2g(y - y_0) \ .$$

4. Recombine the horizontal and vertical components of location and/or velocity using the following equations:

$$s = \sqrt{x^2 + y^2}$$

$$\theta = tan^{-1}(y/x)$$

$$v = \sqrt{v_x^2 + v_y^2}$$

$$\theta_v = tan^{-1}(v_y/v_x) \, .$$

- The maximum height h of a projectile launched with initial vertical velocity v_{0y} is given by

$$h = \frac{v_{0y}^2}{2g} \, .$$

- The maximum horizontal distance traveled by a projectile is called the **range**. The range R of a projectile on level ground launched at an angle θ_0 above the horizontal with initial speed v_0 is given by

$$R = \frac{v_0^2 sin 2\theta_0}{g} \, .$$

Glossary

air resistance
: a frictional force that slows the motion of objects as they travel through the air; when solving basic physics problems, air resistance is assumed to be zero

kinematics
: the study of motion without regard to mass or force

motion
: displacement of an object as a function of time

projectile
: an object that travels through the air and experiences only acceleration due to gravity

projectile motion
: the motion of an object that is subject only to the acceleration of gravity

range
: the maximum horizontal distance that a projectile travels

trajectory
: the path of a projectile through the air

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3.6: Addition of Velocities

Learning Objectives

By the end of this section, you will be able to:

- Apply principles of vector addition to determine relative velocity.
- Explain the significance of the observer in the measurement of velocity.

Relative Velocity

If a person rows a boat across a rapidly flowing river and tries to head directly for the other shore, the boat instead moves diagonallyrelative to the shore, as in Figure $3.6.1$. The boat does not move in the direction in which it is pointed. The reason, of course, is that the river carries the boat downstream. Similarly, if a small airplane flies overhead in a strong crosswind, you can sometimes see that the plane is not moving in the direction in which it is pointed, as illustrated in Figure $3.6.2$. The plane is moving straight ahead relative to the air, but the movement of the air mass relative to the ground carries it sideways.

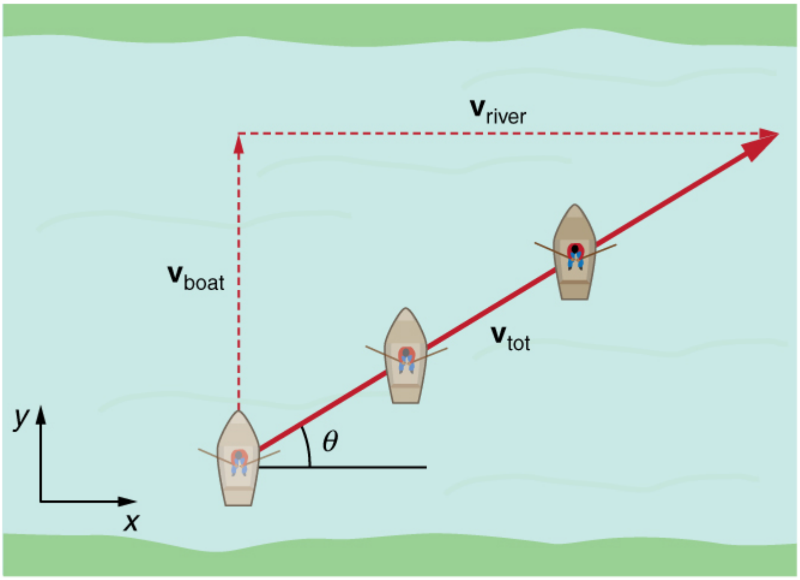

Figure $3.6.1$: A boat trying to head straight across a river will actually move diagonally relative to the shore as shown. Its total velocity (solid arrow) relative to the shore is the sum of its velocity relative to the river plus the velocity of the river relative to the shore.

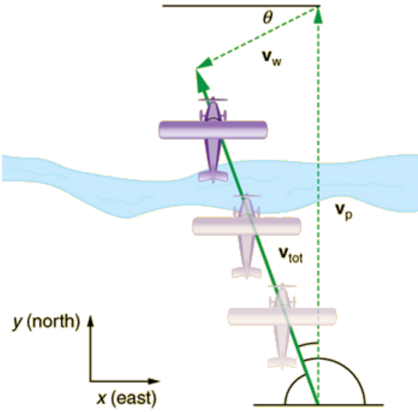

Figure $3.6.2$: An airplane heading straight north is instead carried to the west and slowed down by wind. The plane does not move relative to the ground in the direction it points; rather, it moves in the direction of its total velocity (solid arrow).

In each of these situations, an object has a velocity relative to a medium (such as a river) and that medium has a velocity relative to an observer on solid ground. The velocity of the object relative to the observer is the sum of these velocity vectors, as indicated in

Figures 3.6.1 and 3.6.2. These situations are only two of many in which it is useful to add velocities. In this module, we first re-examine how to add velocities and then consider certain aspects of what relative velocity means.

How do we add velocities? Velocity is a vector (it has both magnitude and direction); the rules of vector addition discussed inVector Addition and Subtraction: Graphical Methods and Vector Addition and Subtraction: Analytical Methods apply to the addition of velocities, just as they do for any other vectors. In one-dimensional motion, the addition of velocities is simple—they add like ordinary numbers. For example, if a field hockey player is moving at 5 m/s straight toward the goal and drives the ball in the same direction with a velocity of 30 m/s relative to her body, then the velocity of the ball is 35 m/s relative to the stationary, profusely sweating goalkeeper standing in front of the goal.

In two-dimensional motion, either graphical or analytical techniques can be used to add velocities. We will concentrate on analytical techniques. The following equations give the relationships between the magnitude and direction of velocity (v and θ) and its components (v_x and v_y) along the x- and y-axes of an appropriately chosen coordinate system:

$$v_x = v\cos\theta \tag{3.6.1}$$

$$v_y = v\sin\theta \tag{3.6.2}$$

$$v = \sqrt{v^2x + v^2} \tag{3.6.3}$$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right). \tag{3.6.4}$$

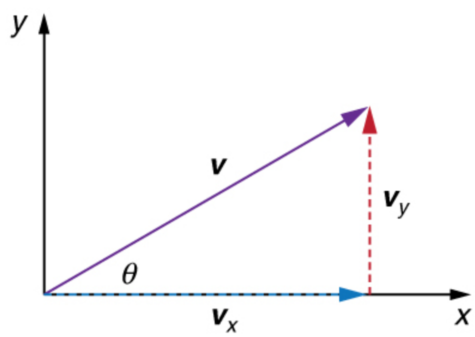

Figure 3.6.3: The velocity, v, of an object traveling at an angle θ to the horizontal axis is the sum of component vectors v_x and v_y.

These equations are valid for any vectors and are adapted specifically for velocity. The first two equations are used to find the components of a velocity when its magnitude and direction are known. The last two are used to find the magnitude and direction of velocity when its components are known.

TAKE-HOME EXPERIMENT: RELATIVE VELOCITY OF A BOAT

Fill a bathtub half-full of water. Take a toy boat or some other object that floats in water. Unplug the drain so water starts to drain. Try pushing the boat from one side of the tub to the other and perpendicular to the flow of water. Which way do you need to push the boat so that it ends up immediately opposite? Compare the directions of the flow of water, heading of the boat, and actual velocity of the boat.

Example 3.6.1: Adding Velocities - A Boat on a River

Refer to Figure 3.6.4, which shows a boat trying to go straight across the river. Let us calculate the magnitude and direction of the boat's velocity relative to an observer on the shore, v_{tot}. The velocity of the boat, vboat , is 0.75 m/s in the y -direction relative to the river and the velocity of the river, v_{river}, is 1.20 m/s to the right.

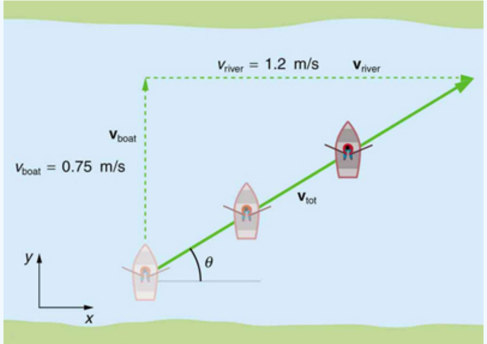

Figure 3.6.4: A boat attempts to travel straight across a river at a speed 0.75 m/s. The current in the river, however, flows at a speed of 1.20 m/s to the right. What is the total displacement of the boat relative to the shore?

Strategy

We start by choosing a coordinate system with its x-axis parallel to the velocity of the river, as shown in Figure. Because the boat is directed straight toward the other shore, its velocity relative to the water is parallel to the y-axis and perpendicular to the velocity of the river. Thus, we can add the two velocities by using the equations $v_{tot} = \sqrt{v_x^2 + v_y^2}$ and $\theta = tan^{-1}(v_y/v_x)$ directly.

Solution

The magnitude of the total velocity is

$$v_{tot} = \sqrt{v_x^2 + v_y^2},$$

where

$$v_x = v_{river} = 1.20 m/s$$

and

$$v_y = v_{boat} = 0.750 m/s.$$

Thus,

$$v_{tot} = \sqrt{(1.20 m/s)^2 + (0.750 m/s)^2}$$

yielding

$$v_{tot} = 1.42 m/s.$$

The direction of the total velocity θ is given by:

$$\theta = tan^{-1}(v_y/v_x) = tan^{-1}(0.750/1.20).$$

This equation gives

$$\theta = 32.0^{\circ}.$$

Discussion

Both the magnitude v and the direction θ of the total velocity are consistent with Figure. Note that because the velocity of the river is large compared with the velocity of the boat, it is swept rapidly downstream. This result is evidenced by the small angle (only **32.0º**) the total velocity has relative to the riverbank.

Example 3.6.2: Calculating Velocity - Wind Velocity Causes an Airplane to Drift

Calculate the wind velocity for the situation shown in Figure 3.6.5. The plane is known to be moving at 45.0 m/s due north relative to the air mass, while its velocity relative to the ground (its total velocity) is 38.0 m/s in a direction **20.0º** west of north.

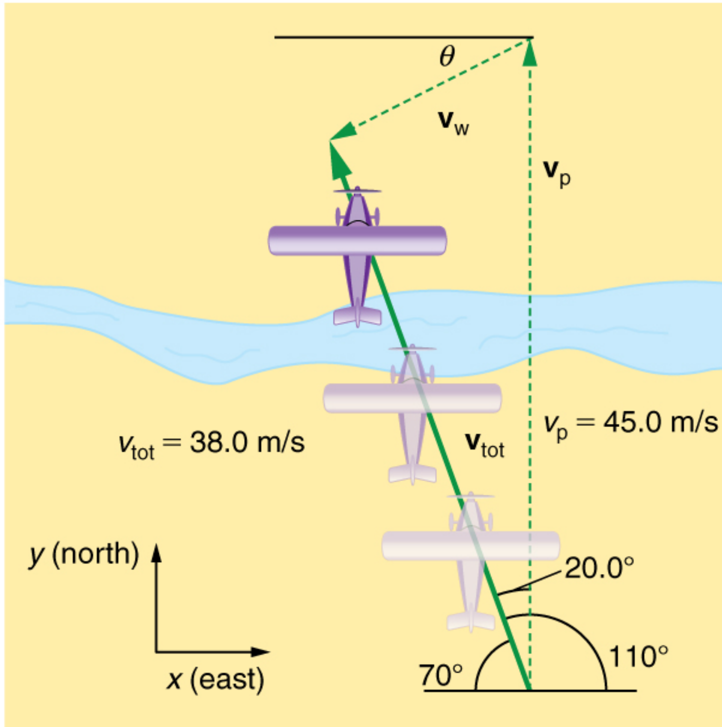

Figure 3.6.5: An airplane is known to be heading north at 45.0 m/s, though its velocity relative to the ground is 38.0 m/s at an angle west of north. What is the speed and direction of the wind?

Strategy

In this problem, somewhat different from the previous example, we know the total velocity v_{to}t and that it is the sum of two other velocities, v_w (the wind) and v_p (the plane relative to the air mass). The quantity v_p is known, and we are asked to find vw. None of the velocities are perpendicular, but it is possible to find their components along a common set of perpendicular axes. If we can find the components of v_w, then we can combine them to solve for its magnitude and direction. As shown in Figure, we choose a coordinate system with its *x*-axis due east and its *y*-axis due north (parallel to v_p). (You may wish to look back at the discussion of the addition of vectors using perpendicular components in Vector Addition and Subtraction: Analytical Methods.)

Solution

Because v_{tot} is the vector sum of the v_w and v_p, its *x*- and *y*-components are the sums of the *x*- and *y*-components of the wind and plane velocities. Note that the plane only has vertical component of velocity so $v_{px} = 0$ and $v_{py} = v_p$. That is,

$$v_{tot}x = v_{wx}$$

and

$$v_{toty} = v_{wy} + v_p \ .$$

We can use the first of these two equations to find v_{wx}:

$$v_{wx} = v_{totx} = v_{tot}cos110^o.$$

Because $v_{tot} = 38.0m/s$ and $cos110^o = -0.342$ we have

$$v_{wx} = (38.0m/s)(-0.342) = -13m/s.$$

The minus sign indicates motion west which is consistent with the diagram.

Now, to find v_{wy} we note that

$$v_{toty} = v_{wy} + v_p$$

Here $v_{toty} = v_{tot}sin110^o$; thus,

$$v_{wy} = (38.0m/s)(0.940) - 45.0m/s = -9.29m/s.$$

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This minus sign indicates motion south which is consistent with the diagram.

Now that the perpendicular components of the wind velocity v_{wx} and v_{wy} are known, we can find the magnitude and direction of v_w. First, the magnitude is

$$v_w = \sqrt{v_{wx}^2 + v_{wy}^2} = \sqrt{(-13.0 m/s)^2 + (-9.29 m/s)^2}$$

so that

$$v_w = 16.0 m/s.$$

The direction is:

$$\theta = tan^{-1}(v_{wy}/v_{wx}) = tan^{-1}(-9.29/-13.0)$$

giving

$$\theta = 35.6^o.$$

Discussion

The wind's speed and direction are consistent with the significant effect the wind has on the total velocity of the plane, as seen in Figure. Because the plane is fighting a strong combination of crosswind and head-wind, it ends up with a total velocity significantly less than its velocity relative to the air mass as well as heading in a different direction.

Note that in both of the last two examples, we were able to make the mathematics easier by choosing a coordinate system with one axis parallel to one of the velocities. We will repeatedly find that choosing an appropriate coordinate system makes problem solving easier. For example, in projectile motion we always use a coordinate system with one axis parallel to gravity.

Relative Velocities and Classical Relativity

When adding velocities, we have been careful to specify that the *velocity is relative to some reference frame*. These velocities are called **relative velocities**. For example, the velocity of an airplane relative to an air mass is different from its velocity relative to the ground. Both are quite different from the velocity of an airplane relative to its passengers (which should be close to zero). Relative velocities are one aspect of **relativity**, which is defined to be the study of how different observers moving relative to each other measure the same phenomenon.

Nearly everyone has heard of relativity and immediately associates it with Albert Einstein (1879–1955), the greatest physicist of the 20th century. Einstein revolutionized our view of nature with his *modern* theory of relativity, which we shall study in later chapters. The relative velocities in this section are actually aspects of classical relativity, first discussed correctly by Galileo and Isaac Newton.**Classical relativity** is limited to situations where speeds are less than about 1% of the speed of light—that is, less than . Most things we encounter in daily life move slower than this speed.

Let us consider an example of what two different observers see in a situation analyzed long ago by Galileo. Suppose a sailor at the top of a mast on a moving ship drops his binoculars. Where will it hit the deck? Will it hit at the base of the mast, or will it hit behind the mast because the ship is moving forward? The answer is that if air resistance is negligible, the binoculars will hit at the base of the mast at a point directly below its point of release. Now let us consider what two different observers see when the binoculars drop. One observer is on the ship and the other on shore. The binoculars have no horizontal velocity relative to the observer on the ship, and so he sees them fall straight down the mast. (Figure $3.6.1$; blue curve) To the observer on shore, the binoculars and the ship have the *same* horizontal velocity, so both move the same distance forward while the binoculars are falling. This observer sees the red curved path shown in Figure $3.6.6$. Although the paths look different to the different observers, each sees the same result—the binoculars hit at the base of the mast and not behind it. To get the correct description, it is crucial to correctly specify the velocities relative to the observer.

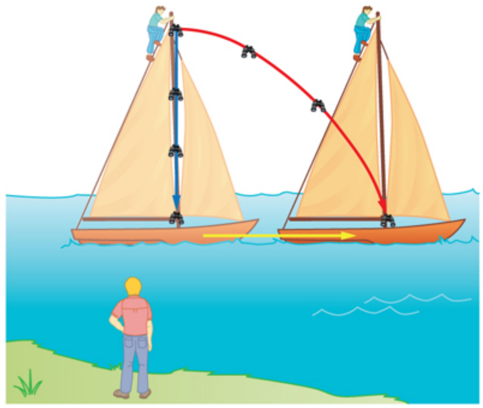

Figure 3.6.6: Classical relativity. The same motion as viewed by two different observers. An observer on the moving ship sees the binoculars dropped from the top of its mast fall straight down. An observer on shore sees the binoculars take the curved path, moving forward with the ship. Both observers see the binoculars strike the deck at the base of the mast. The initial horizontal velocity is different relative to the two observers. (The ship is shown moving rather fast to emphasize the effect.)

Example 3.6.3: Calculating Relative Velocity: An Airline Passenger Drops a Coin

An airline passenger drops a coin while the plane is moving at 260 m/s. What is the velocity of the coin when it strikes the floor 1.50 m below its point of release: (a) Measured relative to the plane? (b) Measured relative to the Earth?

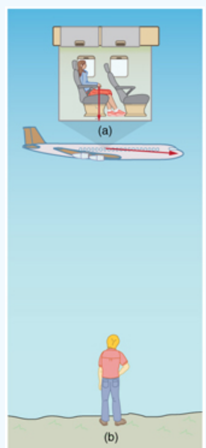

Figure 3.6.7: The motion of a coin dropped inside an airplane as viewed by two different observers. (a) An observer in the plane sees the coin fall straight down. (b) An observer on the ground sees the coin move almost horizontally.

Strategy

Both problems can be solved with the techniques for falling objects and projectiles. In part (a), the initial velocity of the coin is zero relative to the plane, so the motion is that of a falling object (one-dimensional). In part (b), the initial velocity is 260 m/s horizontal relative to the Earth and gravity is vertical, so this motion is a projectile motion. In both parts, it is best to use a coordinate system with vertical and horizontal axes.

Solution for (a)

Using the given information, we note that the initial velocity and position are zero, and the final position is 1.50 m. The final velocity can be found using the equation:

$$v_y^2 = v_{0y}^2 - 2g(y - y_0).$$

Substituting known values into the equation, we get

$$v_y^2 = 0^2 - 2(9.80 m/s^2)(-1.50 m - 0 m) = 29.4 m^2/s^2$$

yielding

$$v_y = -5.42 m/s.$$

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We know that the square root of 29.4 has two roots: 5.42 and -5.42. We choose the negative root because we know that the velocity is directed downwards, and we have defined the positive direction to be upwards. There is no initial horizontal velocity relative to the plane and no horizontal acceleration, and so the motion is straight down relative to the plane.

Solution for (b)

Because the initial vertical velocity is zero relative to the ground and vertical motion is independent of horizontal motion, the final vertical velocity for the coin relative to the ground is $v_y = -5.42m/s$, the same as found in part (a). In contrast to part (a), there now is a horizontal component of the velocity. However, since there is no horizontal acceleration, the initial and final horizontal velocities are the same and $v_x = 260m/s$. The *x*- and *y*-components of velocity can be combined to find the magnitude of the final velocity:

$$v = \sqrt{v_x^2 + v_y^2}\ .$$

Thus,

$$v = \sqrt{(260m/s)^2 + (-5.42m/s)^2}$$

yielding

$$v = 260.06m/s.$$

The direction is given by:

$$\theta = tan^{-1}(v_y/v_x) = tan^{-1}(-5.42/260)$$

so that

$$\theta = tan^{-1}(-0.0208) = -1.19^{\circ}.$$

Discussion

In part (a), the final velocity relative to the plane is the same as it would be if the coin were dropped from rest on the Earth and fell 1.50 m. This result fits our experience; objects in a plane fall the same way when the plane is flying horizontally as when it is at rest on the ground. This result is also true in moving cars. In part (b), an observer on the ground sees a much different motion for the coin. The plane is moving so fast horizontally to begin with that its final velocity is barely greater than the initial velocity. Once again, we see that in two dimensions, vectors do not add like ordinary numbers—the final velocity v in part (b) is not $(260 - 5.42)m/s$; rather, it is $260.06m/s$. The velocity's magnitude had to be calculated to five digits to see any difference from that of the airplane. The motions as seen by different observers (one in the plane and one on the ground) in this example are analogous to those discussed for the binoculars dropped from the mast of a moving ship, except that the velocity of the plane is much larger, so that the two observers see very different paths. (See Figure.) In addition, both observers see the coin fall 1.50 m vertically, but the one on the ground also sees it move forward 144 m (this calculation is left for the reader). Thus, one observer sees a vertical path, the other a nearly horizontal path.

MAKING CONNECTIONS: RELATIVITY AND EINSTEIN

Because Einstein was able to clearly define how measurements are made (some involve light) and because the speed of light is the same for all observers, the outcomes are spectacularly unexpected. Time varies with observer, energy is stored as increased mass, and more surprises await.

PHET EXPLORATIONS: MOTION IN 2D

Try the new "Ladybug Motion 2D" simulation for the latest updated version. Learn about position, velocity, and acceleration vectors. Move the ball with the mouse or let the simulation move the ball in four types of motion (2 types of linear, simple harmonic, circle).

Figure 3.6.1: Motion in 2D

Summary

- Velocities in two dimensions are added using the same analytical vector techniques, which are rewritten as

$$v_x = vcos\theta$$
$$v_y = vsin\theta$$
$$v = \sqrt{v_x^2 + v^2 + y}$$
$$\theta = tan^{-1}(v_y/v_x).$$

- Relative velocity is the velocity of an object as observed from a particular reference frame, and it varies dramatically with reference frame.
- **Relativity** is the study of how different observers measure the same phenomenon, particularly when the observers move relative to one another. **Classical relativity** is limited to situations where speed is less than about 1% of the speed of light (3000 km/s).

Glossary

classical relativity

the study of relative velocities in situations where speeds are less than about 1% of the speed of light—that is, less than 3000 km/s

relative velocity

the velocity of an object as observed from a particular reference frame

relativity

the study of how different observers moving relative to each other measure the same phenomenon

velocity

speed in a given direction

vector addition

the rules that apply to adding vectors together

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3.E: Two-Dimensional Kinematics (Exercises)

Conceptual Questions

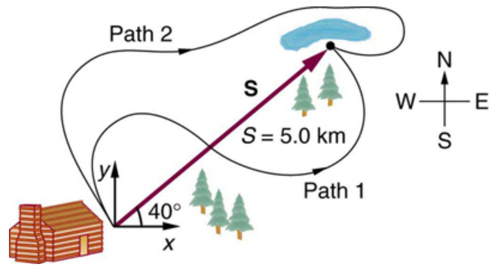
3.2: Vector Addition and Subtraction: Graphical Methods

1. Which of the following is a vector: a person's height, the altitude on Mt. Everest, the age of the Earth, the boiling point of water, the cost of this book, the Earth's population, the acceleration of gravity?

2. Give a specific example of a vector, stating its magnitude, units, and direction.

3. What do vectors and scalars have in common? How do they differ?

4. Two campers in a national park hike from their cabin to the same spot on a lake, each taking a different path, as illustrated below. The total distance traveled along Path 1 is 7.5 km, and that along Path 2 is 8.2 km. What is the final displacement of each camper?



5. If an airplane pilot is told to fly 123 km in a straight line to get from San Francisco to Sacramento, explain why he could end up anywhere on the circle shown in Figure. What other information would he need to get to Sacramento?

6. Suppose you take two steps A and **B** (that is, two nonzero displacements). Under what circumstances can you end up at your starting point? More generally, under what circumstances can two nonzero vectors add to give zero? Is the maximum distance you can end up from the starting point **A+B** the sum of the lengths of the two steps?

7. Explain why it is not possible to add a scalar to a vector

8. If you take two steps of different sizes, can you end up at your starting point? More generally, can two vectors with different magnitudes ever add to zero? Can three or more?

3.3: Vector Addition and Subtraction: Analytical Methods

9. Suppose you add two vectors **A** and **B**. What relative direction between them produces the resultant with the greatest magnitude? What is the maximum magnitude? What relative direction between them produces the resultant with the smallest magnitude? What is the minimum magnitude?

10. Give an example of a nonzero vector that has a component of zero.

11. Explain why a vector cannot have a component greater than its own magnitude.

12. If the vectors **A** and **B** are perpendicular, what is the component of **A** along the direction of **B**? What is the component of **B** along the direction of **A**?

3.4: Projectile Motion

13. Answer the following questions for projectile motion on level ground assuming negligible air resistance (the initial angle being neither **0º** nor **90º**):

(a) Is the velocity ever zero?

(b) When is the velocity a minimum? A maximum?

(c) Can the velocity ever be the same as the initial velocity at a time other than at $t = 0$?

(d) Can the speed ever be the same as the initial speed at a time other than at $t = 0$?

14. Answer the following questions for projectile motion on level ground assuming negligible air resistance (the initial angle being neither 0º nor 90º):

(a) Is the acceleration ever zero?

(b) Is the acceleration ever in the same direction as a component of velocity?

(c) Is the acceleration ever opposite in direction to a component of velocity?

15. For a fixed initial speed, the range of a projectile is determined by the angle at which it is fired. For all but the maximum, there are two angles that give the same range. Considering factors that might affect the ability of an archer to hit a target, such as wind, explain why the smaller angle (closer to the horizontal) is preferable. When would it be necessary for the archer to use the larger angle? Why does the punter in a football game use the higher trajectory?

16. During a lecture demonstration, a professor places two coins on the edge of a table. She then flicks one of the coins horizontally off the table, simultaneously nudging the other over the edge. Describe the subsequent motion of the two coins, in particular discussing whether they hit the floor at the same time.

3.5: Addition of Velocities

17. What frame or frames of reference do you instinctively use when driving a car? When flying in a commercial jet airplane?

18. A basketball player dribbling down the court usually keeps his eyes fixed on the players around him. He is moving fast. Why doesn’t he need to keep his eyes on the ball?

19. If someone is riding in the back of a pickup truck and throws a softball straight backward, is it possible for the ball to fall straight down as viewed by a person standing at the side of the road? Under what condition would this occur? How would the motion of the ball appear to the person who threw it?

20. The hat of a jogger running at constant velocity falls off the back of his head. Draw a sketch showing the path of the hat in the jogger’s frame of reference. Draw its path as viewed by a stationary observer.

21. A clod of dirt falls from the bed of a moving truck. It strikes the ground directly below the end of the truck. What is the direction of its velocity relative to the truck just before it hits? Is this the same as the direction of its velocity relative to ground just before it hits? Explain your answers.

Problems & Exercises

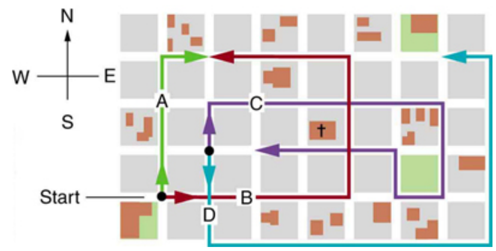
3.2: Vector Addition and Subtraction: Graphical Methods

Use graphical methods to solve these problems. You may assume data taken from graphs is accurate to three digits.

22. Find the following for path A in Figure:

(a) the total distance traveled, and

(b) the magnitude and direction of the displacement from start to finish.



The various lines represent paths taken by different people walking in a city. All blocks are 120 m on a side.

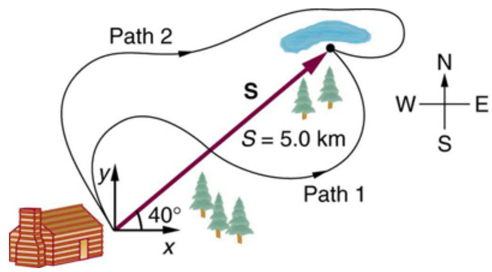
solution:
(a) **480 m**
(b) **379 m**, **18.4º** east of north

23. Find the following for path B in Figure:

(a) the total distance traveled, and

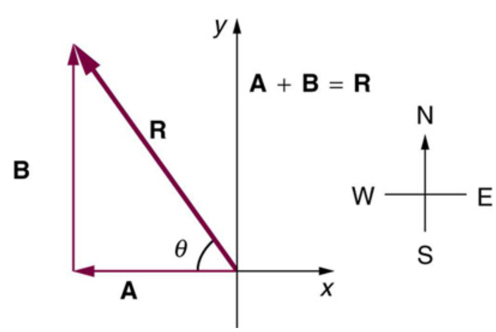
(b) the magnitude and direction of the displacement from start to finish.

24. Find the north and east components of the displacement for the hikers shown in Figure.



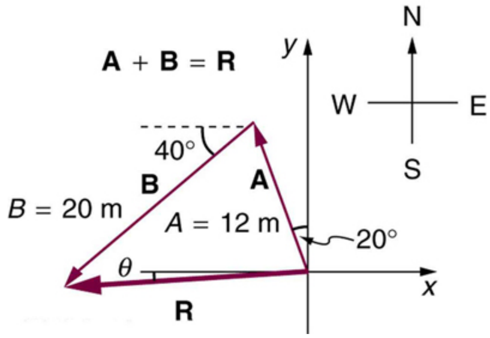
Solution
north component 3.21 km, east component 3.83 km

25. Suppose you walk 18.0 m straight west and then 25.0 m straight north. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements **A** and **B** , as in Figure, then this problem asks you to find their sum **R=A+B**.)



*The two displacements **A** and **B** add to give a total displacement **R** having magnitude R and direction θ.*

26. Suppose you first walk 12.0 m in a direction **20º** west of north and then 20.0 m in a direction **40.0º** south of west. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements **A** and **B**, as in Figure, then this problem finds their sum **R = A + B**.)



Solution
$19.5m, 4.65^{\circ}$south of west

27. Repeat the problem above, but reverse the order of the two legs of the walk; show that you get the same final result. That is, you first walk leg **B**, which is 20.0 m in a direction exactly **40º** south of west, and then leg **A** size 12{A} {}, which is 12.0 m in a direction exactly **20º** west of north. (This problem shows that **A+B=B+A**.)

28. (a) Repeat the problem two problems prior, but for the second leg you walk 20.0 m in a direction **40.0º** north of east (which is equivalent to subtracting **B** from **A** —that is, to finding **R'=A−B**).

(b) Repeat the problem two problems prior, but now you first walk 20.0 m in a direction **40.0º** south of west and then 12.0 m in a direction **20.0º** east of south (which is equivalent to subtracting **A** from **B** —that is, to finding **R''=B−A=−R'**). Show that this is the case.

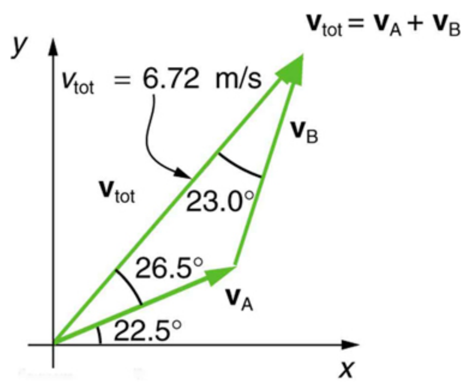
Solution
(a) $26.6m, 65.1^{\circ}$north of east
(b) $26.6m, 65.1^{\circ}$south of west

29. Show that the order of addition of three vectors does not affect their sum. Show this property by choosing any three vectors **A**, **B** and **C**, all having different lengths and directions. Find the sum **A + B + C** then find their sum when added in a different order and show the result is the same. (There are five other orders in which **A**, **B**, and **C** can be added; choose only one.)

30. Show that the sum of the vectors discussed in Example gives the result shown in Figure.

Solution
$52.9m, 90.1^{\circ}$with respect to the x-axis.

31. Find the magnitudes of velocities v_A and v_B in Figure



The two velocities v_A and v_B add to give a total v_{tot}.

32. Find the components of v_{tot} along the x- and y-axes in Figure.

Solution
x-component 4.41 m/s
y-component 5.07 m/s

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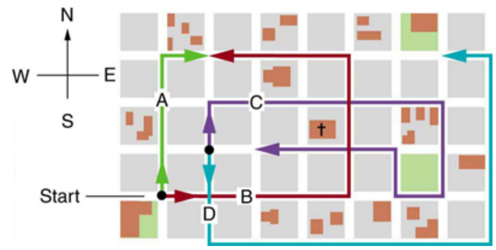
33. Find the components of v_{tot} along a set of perpendicular axes rotated 30^{o} counterclockwise relative to those in Figure.

3.3: Vector Addition and Subtraction: Analytical Methods

34. Find the following for path C in Figure:

(a) the total distance traveled and

(b) the magnitude and direction of the displacement from start to finish. In this part of the problem, explicitly show how you follow the steps of the analytical method of vector addition.



The various lines represent paths taken by different people walking in a city. All blocks are 120 m on a side.

Solution

(a) 1.56 km

(b) 120 m east

35. Find the following for path D in Figure:

(a) the total distance traveled and

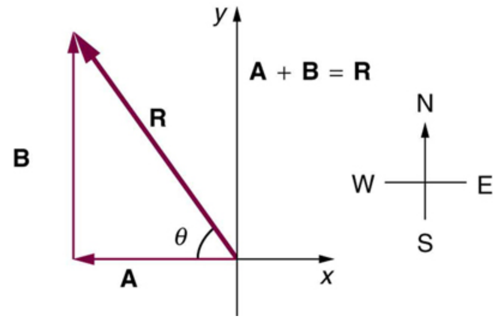
(b) the magnitude and direction of the displacement from start to finish. In this part of the problem, explicitly show how you follow the steps of the analytical method of vector addition.

36. Find the north and east components of the displacement from San Francisco to Sacramento shown in Figure.

Solution

North-component 87.0 km, east-component 87.0 km

37. Solve the following problem using analytical techniques: Suppose you walk 18.0 m straight west and then 25.0 m straight north. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements **A** and **B**, as in Figure, then this problem asks you to find their sum **R=A+B**.)



*The two displacements **A** and **B** add to give a total displacement **R** having magnitude R and direction θ.*

Note that you can also solve this graphically. Discuss why the analytical technique for solving this problem is potentially more accurate than the graphical technique.

38. Repeat Exercise using analytical techniques, but reverse the order of the two legs of the walk and show that you get the same final result. (This problem shows that adding them in reverse order gives the same result—that is, **B + A = A + B**.) Discuss how taking another path to reach the same point might help to overcome an obstacle blocking you other path.

Solution
30.8 m, 35.8 west of north

You drive **7.50 km** in a straight line in a direction **15º**.

(a) Find the distances you would have to drive straight east and then straight north to arrive at the same point. (This determination is equivalent to find the components of the displacement along the east and north directions.)

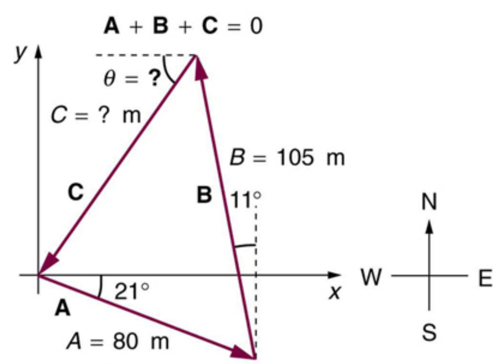
(b) Show that you still arrive at the same point if the east and north legs are reversed in order.

39. a) Do Exercise again using analytical techniques and change the second leg of the walk to **25.0 m** straight south. (This is equivalent to subtracting **B** from **A** —that is, finding **R'=A – B**)

(b) Repeat again, but now you first walk 25.0 m size 12{"25" "." "0 m"} {} north and then **18.0 m** size 12{"18" "." "0 m"} {} east. (This is equivalent to subtract **A** size 12{A} {} from **B** size 12{B} {} —that is, to find **A=B+C** size 12{A=B+C} {}. Is that consistent with your result?)

Solution
(a) $30.8m, 54.2^{\circ}$south of west
(b) $30.8m, 54.2^{\circ}$north of east

40. A new landowner has a triangular piece of flat land she wishes to fence. Starting at the west corner, she measures the first side to be 80.0 m long and the next to be 105 m. These sides are represented as displacement vectors **A** from **B** in Figure. She then correctly calculates the length and orientation of the third side C. What is her result?



41. You fly **32.0 km** size 12{"32" "." "0 km"} {} in a straight line in still air in the direction **35.0º** size 12{"35"°} {} south of west.

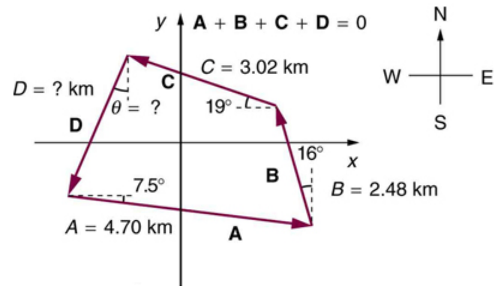
(a) Find the distances you would have to fly straight south and then straight west to arrive at the same point. (This determination is equivalent to finding the components of the displacement along the south and west directions.)

(b) Find the distances you would have to fly first in a direction **45.0º** south of west and then in a direction **45.0º** west of north. These are the components of the displacement along a different set of axes—one rotated **45º**.

Solution

18.4 km south, then 26.2 km west(b) 31.5 km at **45.0º** south of west, then 5.56 km at **45.0º** west of north

42. A farmer wants to fence off his four-sided plot of flat land. He measures the first three sides, shown as **A**, **B**, and **C** in Figure, and then correctly calculates the length and orientation of the fourth side D. What is his result?

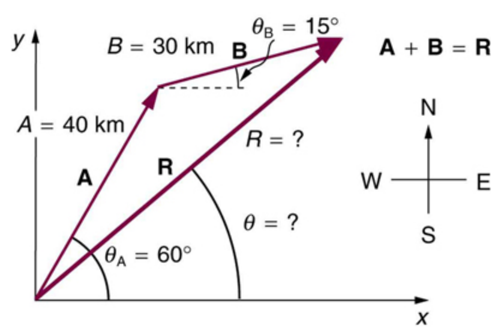


43. In an attempt to escape his island, Gilligan builds a raft and sets to sea. The wind shifts a great deal during the day, and he is blown along the following straight lines: **2.50 km 45.0º** north of west; then **4.70 km 60.0º** south of east; then **1.30km 25.0º** south of west; then **5.10 km** straight east; then **1.70km 5.00º** east of north; then **7.20 km 55.0º** south of west; and finally **2.80 km 10.0º** north of east. What is his final position relative to the island?

Solution

$7.34 km, 63.5^{o}$south of east

44. Suppose a pilot flies **40.0 km** in a direction **60º** north of east and then flies **30.0 km** in a direction **15º** north of east as shown in Figure. Find her total distance R from the starting point and the direction θ of the straight-line path to the final position. Discuss qualitatively how this flight would be altered by a wind from the north and how the effect of the wind would depend on both wind speed and the speed of the plane relative to the air mass.



3.4: Projectile Motion

45. A projectile is launched at ground level with an initial speed of 50.0 m/s at an angle of **30.0º** above the horizontal. It strikes a target above the ground 3.00 seconds later. What are the x and y distances from where the projectile was launched to where it lands?

Solution

$x = 1.30m \times 10$

$y = 30.9m.$

46. A ball is kicked with an initial velocity of 16 m/s in the horizontal direction and 12 m/s in the vertical direction.

(a) At what speed does the ball hit the ground?

(b) For how long does the ball remain in the air?

(c)What maximum height is attained by the ball?

47. A ball is thrown horizontally from the top of a 60.0-m building and lands 100.0 m from the base of the building. Ignore air resistance.

(a) How long is the ball in the air?

(b) What must have been the initial horizontal component of the velocity?

(c) What is the vertical component of the velocity just before the ball hits the ground? (d) What is the velocity (including both the horizontal and vertical components) of the ball just before it hits the ground?

Solution
(a) 3.50 s
(b) 28.6 m/s
(c) 34.3 m/s
(d) 44.7 m/s, **50.2º** below horizontal

48. (a) A daredevil is attempting to jump his motorcycle over a line of buses parked end to end by driving up a **32º** ramp at a speed of **40.0 m/s** (**144 km/h**). How many buses can he clear if the top of the takeoff ramp is at the same height as the bus tops and the buses are 20.0 m long?

(b) Discuss what your answer implies about the margin of error in this act—that is, consider how much greater the range is than the horizontal distance he must travel to miss the end of the last bus. (Neglect air resistance.)

49. An archer shoots an arrow at a 75.0 m distant target; the bull's-eye of the target is at same height as the release height of the arrow.

(a) At what angle must the arrow be released to hit the bull's-eye if its initial speed is 35.0 m/s? In this part of the problem, explicitly show how you follow the steps involved in solving projectile motion problems.

(b) There is a large tree halfway between the archer and the target with an overhanging horizontal branch 3.50 m above the release height of the arrow. Will the arrow go over or under the branch?

Solution
(a) **18.4º**
(b) The arrow will go over the branch.

50. A rugby player passes the ball 7.00 m across the field, where it is caught at the same height as it left his hand.

(a) At what angle was the ball thrown if its initial speed was 12.0 m/s, assuming that the smaller of the two possible angles was used?

(b) What other angle gives the same range, and why would it not be used?

(c) How long did this pass take?

51. Verify the ranges for the projectiles in Figure(a) for $\theta = 45^{\circ}$ and the given initial velocities.

Solution
$R = \frac{v_0^2}{sin 2\theta_0 g}$
For $\theta = 45^{\circ}$, $R = \frac{v_0^2}{g}$
$R = 91.8m$ for $v_0 = 30m/s$; $R = 163m$ for $v_0 = 40m/s$; $R = 255m$ for $v_{=}50m/s$.

52. Verify the ranges shown for the projectiles in Figure(b) for an initial velocity of 50 m/s at the given initial angles.

53. The cannon on a battleship can fire a shell a maximum distance of 32.0 km.

(a) Calculate the initial velocity of the shell.

(b) What maximum height does it reach? (At its highest, the shell is above 60% of the atmosphere—but air resistance is not really negligible as assumed to make this problem easier.)

(c) The ocean is not flat, because the Earth is curved. Assume that the radius of the Earth is $6.37 \times 10^3 km$. How many meters lower will its surface be 32.0 km from the ship along a horizontal line parallel to the surface at the ship? Does your answer imply that error introduced by the assumption of a flat Earth in projectile motion is significant here?

Solution
(a) 560 m/s
(b) $8.00 \times 10^3 m$
(c) 80.0 m. This error is not significant because it is only 1% of the answer in part (b).

54. An arrow is shot from a height of 1.5 m toward a cliff of height H size 12{H} {}. It is shot with a velocity of 30 m/s at an angle of **60º** above the horizontal. It lands on the top edge of the cliff 4.0 s later.

(a) What is the height of the cliff?

(b) What is the maximum height reached by the arrow along its trajectory?

(c) What is the arrow's impact speed just before hitting the cliff?

55. In the standing broad jump, one squats and then pushes off with the legs to see how far one can jump. Suppose the extension of the legs from the crouch position is 0.600 m and the acceleration achieved from this position is 1.25 times the acceleration due to gravity, g size 12{g} {}. How far can they jump? State your assumptions. (Increased range can be achieved by swinging the arms in the direction of the jump.)

Solution
1.50 m, assuming launch angle of **45º**

56. The world long jump record is 8.95 m (Mike Powell, USA, 1991). Treated as a projectile, what is the maximum range obtainable by a person if he has a take-off speed of 9.5 m/s? State your assumptions.

57. Serving at a speed of 170 km/h, a tennis player hits the ball at a height of 2.5 m and an angle θ below the horizontal. The base line is 11.9 m from the net, which is 0.91 m high. What is the angle θ such that the ball just crosses the net? Will the ball land in the service box, whose service line is 6.40 m from the net?

Solution
$\theta = 6.1^{\circ}$
yes, the ball lands at 5.3 m from the net

58. A football quarterback is moving straight backward at a speed of 2.00 m/s when he throws a pass to a player 18.0 m straight downfield.

(a) If the ball is thrown at an angle of **25º** relative to the ground and is caught at the same height as it is released, what is its initial speed relative to the ground?

(b) How long does it take to get to the receiver?

(c) What is its maximum height above its point of release?

59. Gun sights are adjusted to aim high to compensate for the effect of gravity, effectively making the gun accurate only for a specific range.

(a) If a gun is sighted to hit targets that are at the same height as the gun and 100.0 m away, how low will the bullet hit if aimed directly at a target 150.0 m away? The muzzle velocity of the bullet is 275 m/s.

(b) Discuss qualitatively how a larger muzzle velocity would affect this problem and what would be the effect of air resistance.

Solution
(a) −0.486 m
(b) The larger the muzzle velocity, the smaller the deviation in the vertical direction, because the time of flight would be smaller. Air resistance would have the effect of decreasing the time of flight, therefore increasing the vertical deviation.

60. An eagle is flying horizontally at a speed of 3.00 m/s when the fish in her talons wiggles loose and falls into the lake 5.00 m below. Calculate the velocity of the fish relative to the water when it hits the water.

61. An owl is carrying a mouse to the chicks in its nest. Its position at that time is 4.00 m west and 12.0 m above the center of the 30.0 cm diameter nest. The owl is flying east at 3.50 m/s at an angle 30.0º size 12{"30º"} below the horizontal when it accidentally drops the mouse. Is the owl lucky enough to have the mouse hit the nest? To answer this question, calculate the horizontal position of the mouse when it has fallen 12.0 m.

Solution
4.23 m. No, the owl is not lucky; he misses the nest.

62. Suppose a soccer player kicks the ball from a distance 30 m toward the goal. Find the initial speed of the ball if it just passes over the goal, 2.4 m above the ground, given the initial direction to be **40º** above the horizontal.

63. Can a goalkeeper at her/ his goal kick a soccer ball into the opponent's goal without the ball touching the ground? The distance will be about 95 m. A goalkeeper can give the ball a speed of 30 m/s.

Solution
No, the maximum range (neglecting air resistance) is about 92 m.

64. The free throw line in basketball is 4.57 m (15 ft) from the basket, which is 3.05 m (10 ft) above the floor. A player standing on the free throw line throws the ball with an initial speed of 8.15 m/s, releasing it at a height of 2.44 m (8 ft) above the floor. At what angle above the horizontal must the ball be thrown to exactly hit the basket? Note that most players will use a large initial angle rather than a flat shot because it allows for a larger margin of error. Explicitly show how you follow the steps involved in solving projectile motion problems.

65. In 2007, Michael Carter (U.S.) set a world record in the shot put with a throw of 24.77 m. What was the initial speed of the shot if he released it at a height of 2.10 m and threw it at an angle of **38.0º** above the horizontal? (Although the maximum distance for a projectile on level ground is achieved at **45º** when air resistance is neglected, the actual angle to achieve maximum range is smaller; thus, **38º** will give a longer range than **45º** in the shot put.)

Solution
15.0 m/s

66. A basketball player is running at **5.00 m/s** directly toward the basket when he jumps into the air to dunk the ball. He maintains his horizontal velocity.

(a) What vertical velocity does he need to rise 0.750 m above the floor?

(b) How far from the basket (measured in the horizontal direction) must he start his jump to reach his maximum height at the same time as he reaches the basket?

67. A football player punts the ball at a **45.0º** angle. Without an effect from the wind, the ball would travel 60.0 m horizontally.

(a) What is the initial speed of the ball?

(b) When the ball is near its maximum height it experiences a brief gust of wind that reduces its horizontal velocity by 1.50 m/s. What distance does the ball travel horizontally?

Solution
(a) 24.2 m/s
(b) The ball travels a total of 57.4 m with the brief gust of wind.

68. Prove that the trajectory of a projectile is parabolic, having the form $y=ax+bx^2$. To obtain this expression, solve the equation $x=v_{0x}t$ for t and substitute it into the expression for $y=v_{0y}t–(1/2)gt^2$ (These equations describe the x and y positions of a projectile that starts at the origin.) You should obtain an equation of the form $y=ax+bx^2$ where a and b are constants.

69. Derive $R=\frac{v_0^2 sin2\theta_0}{g}$ for the range of a projectile on level ground by finding the time t at which y becomes zero and substituting this value of t into the expression for $x-x_0$, noting that $R=x-x_0$

Solution
$y-y_0=0=v_{0y}t-\frac{1}{2}gt^2=(v_0sin\theta)t-\frac{1}{2}gt^2$
so that $t=\frac{2(v0sin\theta)}{g}$
$x-x_0=v_0xt=(v_0cos\theta)t=R,$ and substituting for t gives:
$R=v_0cos\theta(\frac{2v_0sin\theta}{g})=\frac{2v_0^2sin\theta cos\theta}{g}$

since $2sin\theta cos\theta = sin2\theta$, the range is:
$R = \frac{v_0^2 sin2\theta}{g}$

70. *Unreasonable Results*

(a) Find the maximum range of a super cannon that has a muzzle velocity of 4.0 km/s.

(b) What is unreasonable about the range you found?

(c) Is the premise unreasonable or is the available equation inapplicable? Explain your answer.

(d) If such a muzzle velocity could be obtained, discuss the effects of air resistance, thinning air with altitude, and the curvature of the Earth on the range of the super cannon.

71. *Construct Your Own Problem*

Consider a ball tossed over a fence. Construct a problem in which you calculate the ball's needed initial velocity to just clear the fence. Among the things to determine are; the height of the fence, the distance to the fence from the point of release of the ball, and the height at which the ball is released. You should also consider whether it is possible to choose the initial speed for the ball and just calculate the angle at which it is thrown. Also examine the possibility of multiple solutions given the distances and heights you have chosen.

3.5: Addition of Velocities

72. Bryan Allen pedaled a human-powered aircraft across the English Channel from the cliffs of Dover to Cap Gris-Nez on June 12, 1979.

(a) He flew for 169 min at an average velocity of 3.53 m/s in a direction 45^o south of east. What was his total displacement?

(b) Allen encountered a headwind averaging 2.00 m/s almost precisely in the opposite direction of his motion relative to the Earth. What was his average velocity relative to the air?

(c) What was his total displacement relative to the air mass?

Solution
(a) $35.8km, 45^o$south of east
(b) $5.53m/s, 45^o$south of east
(c) $56.1km, 45^o$south of east

73. A seagull flies at a velocity of 9.00 m/s straight into the wind.

(a) If it takes the bird 20.0 min to travel 6.00 km relative to the Earth, what is the velocity of the wind?

(b) If the bird turns around and flies with the wind, how long will he take to return 6.00 km?

(c) Discuss how the wind affects the total round-trip time compared to what it would be with no wind.

74. Near the end of a marathon race, the first two runners are separated by a distance of 45.0 m. The front runner has a velocity of 3.50 m/s, and the second a velocity of 4.20 m/s.

(a) What is the velocity of the second runner relative to the first?

(b) If the front runner is 250 m from the finish line, who will win the race, assuming they run at constant velocity?

(c) What distance ahead will the winner be when she crosses the finish line?

Solution
(a) 0.70 m/s faster
(b) Second runner wins
(c) 4.17 m

75. Verify that the coin dropped by the airline passenger in the Example travels 144 m horizontally while falling 1.50 m in the frame of reference of the Earth.

76. A football quarterback is moving straight backward at a speed of 2.00 m/s when he throws a pass to a player 18.0 m straight downfield. The ball is thrown at an angle of 25.0^o relative to the ground and is caught at the same height as it is released. What is the initial velocity of the ball *relative to the quarterback* ?

Solution
$17.0 m/s, 22.1^{\circ}$

77. A ship sets sail from Rotterdam, The Netherlands, heading due north at 7.00 m/s relative to the water. The local ocean current is 1.50 m/s in a direction 40.0° north of east. What is the velocity of the ship relative to the Earth?

78. (a) A jet airplane flying from Darwin, Australia, has an air speed of 260 m/s in a direction 5.0° south of west. It is in the jet stream, which is blowing at 35.0 m/s in a direction 15° south of east. What is the velocity of the airplane relative to the Earth?

(b) Discuss whether your answers are consistent with your expectations for the effect of the wind on the plane's path.

Solution
(a) $230 m/s, 8.0^{\circ}$south of west
(b) The wind should make the plane travel slower and more to the south, which is what was calculated.

79. (a) In what direction would the ship in Exercise have to travel in order to have a velocity straight north relative to the Earth, assuming its speed relative to the water remains $7.00 m/s$?

(b) What would its speed be relative to the Earth?

80. (a) Another airplane is flying in a jet stream that is blowing at 45.0 m/s in a direction 20º size 12{"20º"} {} south of east (as in Exercise). Its direction of motion relative to the Earth is 45.0º size 12{"45.0º"} {} south of west, while its direction of travel relative to the air is 5.00º size 12{5.00º} {} south of west. What is the airplane's speed relative to the air mass? (b) What is the airplane's speed relative to the Earth?

Solution
(a) 63.5 m/s
(b) 29.6 m/s

81. A sandal is dropped from the top of a 15.0-m-high mast on a ship moving at 1.75 m/s due south. Calculate the velocity of the sandal when it hits the deck of the ship:

(a) relative to the ship and

(b) relative to a stationary observer on shore.

(c) Discuss how the answers give a consistent result for the position at which the sandal hits the deck.

82. The velocity of the wind relative to the water is crucial to sailboats. Suppose a sailboat is in an ocean current that has a velocity of 2.20 m/s in a direction 30.0° east of north relative to the Earth. It encounters a wind that has a velocity of 4.50 m/s in a direction of 50.0° south of west relative to the Earth. What is the velocity of the wind relative to the water?

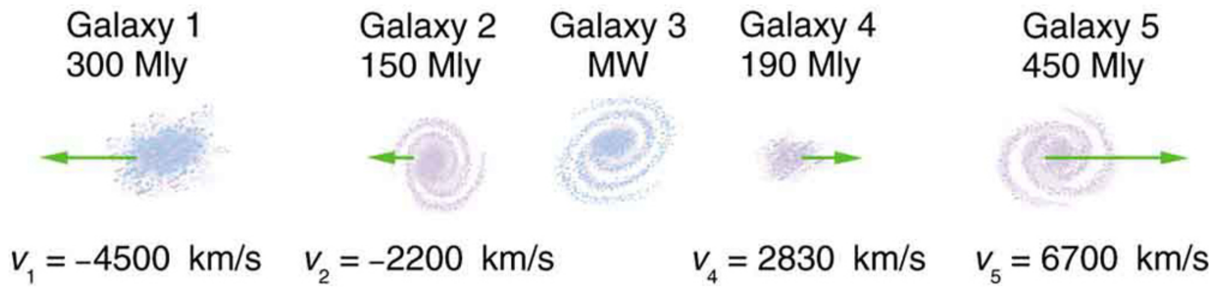
Solution
$6.68 m/s, 53.3^{\circ}$south of west

83. The great astronomer Edwin Hubble discovered that all distant galaxies are receding from our Milky Way Galaxy with velocities proportional to their distances. It appears to an observer on the Earth that we are at the center of an expanding universe. Figure illustrates this for five galaxies lying along a straight line, with the Milky Way Galaxy at the center. Using the data from the figure, calculate the velocities:

(a) relative to galaxy 2 and

(b) relative to galaxy 5.

The results mean that observers on all galaxies will see themselves at the center of the expanding universe, and they would likely be aware of relative velocities, concluding that it is not possible to locate the center of expansion with the given information.



Five galaxies on a straight line, showing their distances and velocities relative to the Milky Way (MW) Galaxy. The distances are in millions of light years (Mly), where a light year is the distance light travels in one year. The velocities are nearly proportional to the distances. The sizes of the galaxies are greatly exaggerated; an average galaxy is about 0.1 Mly across.

84. (a) Use the distance and velocity data in Figure to find the rate of expansion as a function of distance.

(b) If you extrapolate back in time, how long ago would all of the galaxies have been at approximately the same position? The two parts of this problem give you some idea of how the Hubble constant for universal expansion and the time back to the Big Bang are determined, respectively.

Solution

(a) $H_{average} = 14.9 \frac{km/s}{Mly}$

(b) 20.2 billion years

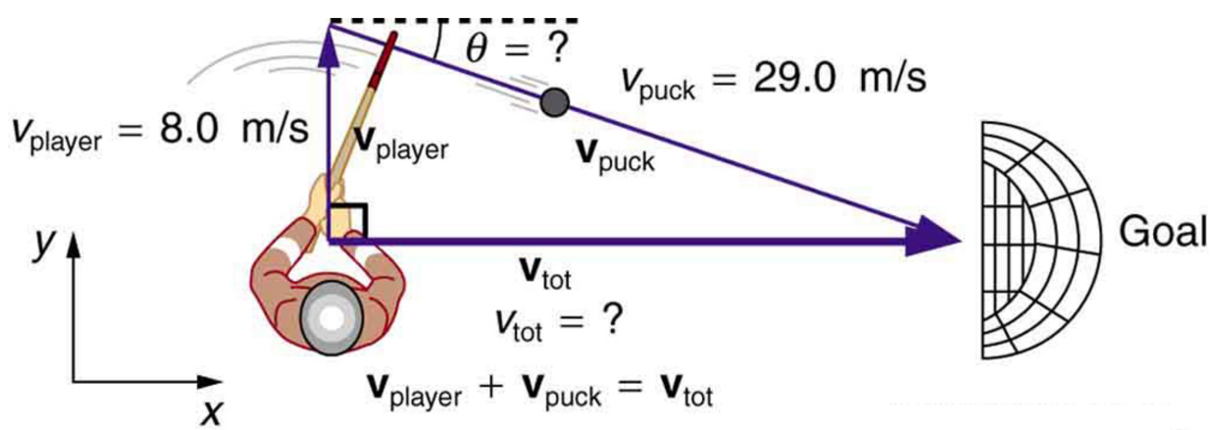
85. An athlete crosses a 25-m-wide river by swimming perpendicular to the water current at a speed of 0.5 m/s relative to the water. He reaches the opposite side at a distance 40 m downstream from his starting point. How fast is the water in the river flowing with respect to the ground? What is the speed of the swimmer with respect to a friend at rest on the ground?

86. A ship sailing in the Gulf Stream is heading 25.0° west of north at a speed of 4.00 m/s relative to the water. Its velocity relative to the Earth is $4.80 m/s 5.00^{\circ}$ west of north. What is the velocity of the Gulf Stream? (The velocity obtained is typical for the Gulf Stream a few hundred kilometers off the east coast of the United States.)

Solution

$1.72 m/s, 42.3^{\circ}$ north of east

87. An ice hockey player is moving at 8.00 m/s when he hits the puck toward the goal. The speed of the puck relative to the player is 29.0 m/s. The line between the center of the goal and the player makes a 90.0° angle relative to his path as shown in Figure. What angle must the puck's velocity make relative to the player (in his frame of reference) to hit the center of the goal?



An ice hockey player moving across the rink must shoot backward to give the puck a velocity toward the goal.

88. *Unreasonable Results*

Suppose you wish to shoot supplies straight up to astronauts in an orbit 36,000 km above the surface of the Earth.

(a) At what velocity must the supplies be launched?

(b) What is unreasonable about this velocity?

(c) Is there a problem with the relative velocity between the supplies and the astronauts when the supplies reach their maximum height?

(d) Is the premise unreasonable or is the available equation inapplicable? Explain your answer.

89. *Unreasonable Results*

A commercial airplane has an air speed of $280 m/s$ due east and flies with a strong tailwind. It travels 3000 km in a direction 5^{o} south of east in 1.50 h.

(a) What was the velocity of the plane relative to the ground?

(b) Calculate the magnitude and direction of the tailwind's velocity.

(c) What is unreasonable about both of these velocities?

(d) Which premise is unreasonable?

90. *Construct Your Own Problem*

Consider an airplane headed for a runway in a cross wind. Construct a problem in which you calculate the angle the airplane must fly relative to the air mass in order to have a velocity parallel to the runway. Among the things to consider are the direction of the runway, the wind speed and direction (its velocity) and the speed of the plane relative to the air mass. Also calculate the speed of the airplane relative to the ground. Discuss any last minute maneuvers the pilot might have to perform in order for the plane to land with its wheels pointing straight down the runway.

Contributors and Attributions

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CHAPTER OVERVIEW

4: Dynamics- Force and Newton's Laws of Motion

4.1: Prelude to Dynamics- Newton’s Laws of Motion
4.2: Development of Force Concept
4.3: Newton’s First Law of Motion- Inertia
4.4: Newton’s Second Law of Motion- Concept of a System
4.5: Newton’s Third Law of Motion- Symmetry in Forces
4.6: Normal, Tension, and Other Examples of Forces
4.7: Friction
4.8: Drag Forces
4.9: Problem-Solving Strategies
4.10: Further Applications of Newton’s Laws of Motion
4.11: Extended Topic- The Four Basic Forces—An Introduction
4.E: Dynamics- Force and Newton's Laws of Motion (Exercises)

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4.1: Prelude to Dynamics- Newton’s Laws of Motion

Motion draws our attention. Motion itself can be beautiful, causing us to marvel at the forces needed to achieve spectacular motion, such as that of a dolphin jumping out of the water, or a pole vaulter, or the flight of a bird, or the orbit of a satellite. The study of motion is kinematics, but kinematics only *describes* the way objects move—their velocity and their acceleration. **Dynamics** considers the forces that affect the motion of moving objects and systems. Newton’s laws of motion are the foundation of dynamics. These laws provide an example of the breadth and simplicity of principles under which nature functions. They are also universal laws in that they apply to similar situations on Earth as well as in space.

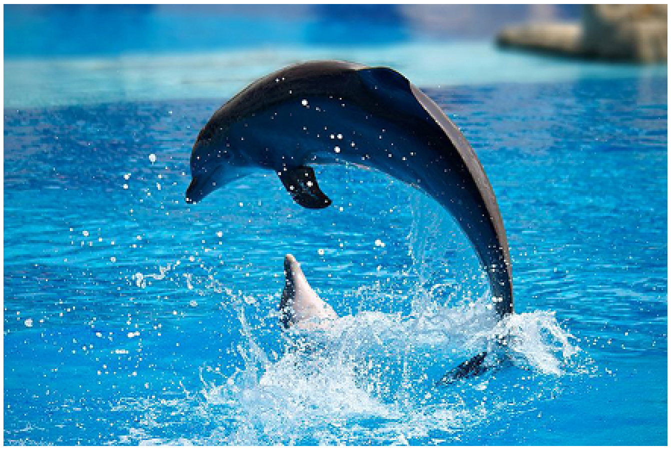

Figure 4.1.1. Newton’s laws of motion describe the motion of the dolphin’s path. (credit: Jin Jang)

Isaac Newton’s (1642–1727) laws of motion were just one part of the monumental work that has made him legendary. The development of Newton’s laws marks the transition from the Renaissance into the modern era. This transition was characterized by a revolutionary change in the way people thought about the physical universe. For many centuries natural philosophers had debated the nature of the universe based largely on certain rules of logic with great weight given to the thoughts of earlier classical philosophers such as Aristotle (384–322 BC). Among the many great thinkers who contributed to this change were Newton and Galileo.

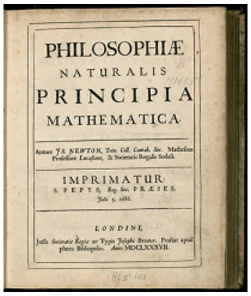

Figure 4.1.2. Issac Newton’s monumental work, Philosophiae Naturalis Principia Mathematica, was published in 1687. It proposed scientific laws that are still used today to describe the motion of objects. (credit: Service commun de la documentation de l'Université de Strasbourg)

Galileo was instrumental in establishing *observation* as the absolute determinant of truth, rather than “logical” argument. Galileo’s use of the telescope was his most notable achievement in demonstrating the importance of observation. He discovered moons orbiting Jupiter and made other observations that were inconsistent with certain ancient ideas and religious dogma. For this reason, and because of the manner in which he dealt with those in authority, Galileo was tried by the Inquisition and punished. He spent the final years of his life under a form of house arrest. Because others before Galileo had also made discoveries by *observing* the nature of the universe, and because repeated observations verified those of Galileo, his work could not be suppressed or denied. After his death, his work was verified by others, and his ideas were eventually accepted by the church and scientific communities.

Galileo also contributed to the formation of what is now called Newton’s first law of motion. Newton made use of the work of his predecessors, which enabled him to develop laws of motion, discover the law of gravity, invent calculus, and make great contributions to the theories of light and color. It is amazing that many of these developments were made with Newton working alone, without the benefit of the usual interactions that take place among scientists today.

It was not until the advent of modern physics early in the 20th century that it was discovered that Newton's laws of motion produce a good approximation to motion only when the objects are moving at speeds much, much less than the speed of light and when those objects are larger than the size of most molecules (about m in diameter). These constraints define the realm of classical mechanics, as discussed in Introduction to the Nature of Science and Physics. At the beginning of the 20th century, Albert Einstein (1879–1955) developed the theory of relativity and, along with many other scientists, developed quantum theory. This theory does not have the constraints present in classical physics. All of the situations we consider in this chapter, and all those preceding the introduction of relativity in Special Relativity, are in the realm of classical physics.

MAKING CONNECTIONS: PAST AND PRESENT PHILOSOPHY

The importance of observation and the concept of *cause and effect* were not always so entrenched in human thinking. This realization was a part of the evolution of modern physics from natural philosophy. The achievements of Galileo, Newton, Einstein, and others were key milestones in the history of scientific thought. Most of the scientific theories that are described in this book descended from the work of these scientists.*The importance of observation* and the concept of *cause and effect* were not always so entrenched in human thinking. This realization was a part of the evolution of modern physics from natural philosophy. The achievements of Galileo, Newton, Einstein, and others were key milestones in the history of scientific thought. Most of the scientific theories that are described in this book descended from the work of these scientists.

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4.2: Development of Force Concept

Learning Objectives

By the end of this section, you will be able to:

- Understand the definition of force.

Dynamics is the study of the forces that cause objects and systems to move. To understand this, we need a working definition of force. Our intuitive definition of **force**—that is, a push or a pull—is a good place to start. We know that a push or pull has both magnitude and direction (therefore, it is a vector quantity) and can vary considerably in each regard. For example, a cannon exerts a strong force on a cannonball that is launched into the air. In contrast, Earth exerts only a tiny downward pull on a flea. Our everyday experiences also give us a good idea of how multiple forces add. If two people push in different directions on a third person, as illustrated in Figure $4.2.1$, we might expect the total force to be in the direction shown. Since force is a vector, it adds just like other vectors, as illustrated in Figure $4.2.1a$ for two ice skaters. Forces, like other vectors, are represented by arrows and can be added using the familiar head-to-tail method or by trigonometric methods. These ideas were developed in Two-Dimensional Kinematics.

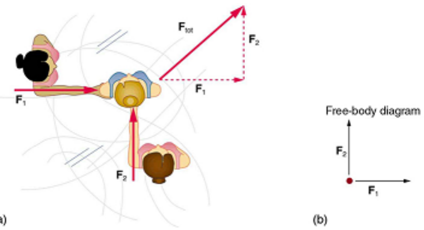

Figure $4.2.1$: Part (a) shows an overhead view of two ice skaters pushing on a third. Forces are vectors and add like other vectors, so the total force on the third skater is in the direction shown. In part (b), we see a free-body diagram representing the forces acting on the third skater.

Figure $4.2.1b$ is our first example of a free-body diagram, which is a technique used to illustrate all the external forces acting on a body. The body is represented by a single isolated point (or free body), and only those forces acting *on* the body from the outside (external forces) are shown. (These forces are the only ones shown, because only external forces acting on the body affect its motion. We can ignore any internal forces within the body.) Free-body diagrams are very useful in analyzing forces acting on a system and are employed extensively in the study and application of Newton's laws of motion.

A more quantitative definition of force can be based on some standard force, just as distance is measured in units relative to a standard distance. One possibility is to stretch a spring a certain fixed distance, as illustrated in Figure $4.2.2$, and use the force it exerts to pull itself back to its relaxed shape—called a *restoring force*—as a standard. The magnitude of all other forces can be stated as multiples of this standard unit of force. Many other possibilities exist for standard forces. (One that we will encounter in Magnetism is the magnetic force between two wires carrying electric current.) Some alternative definitions of force will be given later in this chapter.

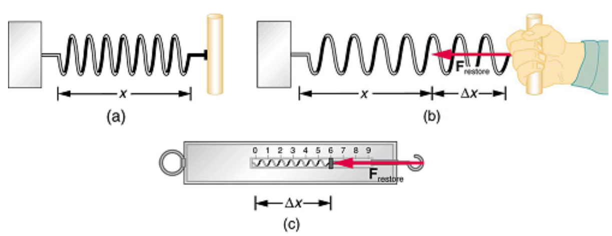

Figure $4.2.2$:The force exerted by a stretched spring can be used as a standard unit of force. (a) This spring has a length when undistorted. (b) When stretched a distance Δx the spring exerts a restoring force, $F_{restore}$ *which is reproducible. (c) A spring scale is one device that uses a spring to measure force. The force* $F_{restore}$ *is exerted on whatever is attached to the hook. Here* $F_{restore}$ *has a magnitude of 6 units in the force standard being employed.*

TAKE HOME EXPERIMENT: FORCE STANDARDS

To investigate force standards and cause and effect, get two identical rubber bands. Hang one rubber band vertically on a hook. Find a small household item that could be attached to the rubber band using a paper clip, and use this item as a weight to investigate the stretch of the rubber band. Measure the amount of stretch produced in the rubber band with one, two, and four of these (identical) items suspended from the rubber band. What is the relationship between the number of items and the amount of stretch? How large a stretch would you expect for the same number of items suspended from two rubber bands? What happens to the amount of stretch of the rubber band (with the weights attached) if the weights are also pushed to the side with a pencil?

Summary

- **Dynamics** is the study of how forces affect the motion of objects.
- **Force** is a push or pull that can be defined in terms of various standards, and it is a vector having both magnitude and direction.
- **External forces** are any outside forces that act on a body. A **free-body diagram** is a drawing of all external forces acting on a body.

Glossary

dynamics
: the study of how forces affect the motion of objects and systems

external force
: a force acting on an object or system that originates outside of the object or system

free-body diagram
: a sketch showing all of the external forces acting on an object or system; the system is represented by a dot, and the forces are represented by vectors extending outward from the dot

force
: a push or pull on an object with a specific magnitude and direction; can be represented by vectors; can be expressed as a multiple of a standard force

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4.3: Newton’s First Law of Motion- Inertia

Learning Objectives

- Define mass and inertia.
- Understand Newton's first law of motion.

Experience suggests that an object at rest will remain at rest if left alone, and that an object in motion tends to slow down and stop unless some effort is made to keep it moving. What **Newton’s first law of motion** states, however, is the following:

Newton’s First Law of Motion

A body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force.

Note the repeated use of the verb “remains.” We can think of this law as preserving the status quo of motion.

Rather than contradicting our experience, **Newton’s first law of motion** states that there must be a *cause* (which is a net external force) *for there to be any change in velocity (either a change in magnitude or direction)*. We will define *net external force* in the next section. An object sliding across a table or floor slows down due to the net force of friction acting on the object. If friction disappeared, would the object still slow down?

The idea of cause and effect is crucial in accurately describing what happens in various situations. For example, consider what happens to an object sliding along a rough horizontal surface. The object quickly grinds to a halt. If we spray the surface with talcum powder to make the surface smoother, the object slides farther. If we make the surface even smoother by rubbing lubricating oil on it, the object slides farther yet. Extrapolating to a frictionless surface, we can imagine the object sliding in a straight line indefinitely. Friction is thus the *cause* of the slowing (consistent with Newton’s first law). The object would not slow down at all if friction were completely eliminated. Consider an air hockey table. When the air is turned off, the puck slides only a short distance before friction slows it to a stop. However, when the air is turned on, it creates a nearly frictionless surface, and the puck glides long distances without slowing down. Additionally, if we know enough about the friction, we can accurately predict how quickly the object will slow down. Friction is an external force.

Newton’s first law is completely general and can be applied to anything from an object sliding on a table to a satellite in orbit to blood pumped from the heart. Experiments have thoroughly verified that any change in velocity (speed or direction) must be caused by an external force. The idea of *generally applicable or universal laws* is important not only here—it is a basic feature of all laws of physics. Identifying these laws is like recognizing patterns in nature from which further patterns can be discovered. The genius of Galileo, who first developed the idea for the first law, and Newton, who clarified it, was to ask the fundamental question, “What is the cause?” Thinking in terms of cause and effect is a worldview fundamentally different from the typical ancient Greek approach when questions such as “Why does a tiger have stripes?” would have been answered in Aristotelian fashion, “That is the nature of the beast.” True perhaps, but not a useful insight.

Mass

The property of a body to remain at rest or to remain in motion with constant velocity is called **inertia**. Newton’s first law is often called the law of inertia. As we know from experience, some objects have more inertia than others. It is obviously more difficult to change the motion of a large boulder than that of a basketball, for example. The inertia of an object is measured by its **mass**. Roughly speaking, mass is a measure of the amount of “stuff” (or matter) in something. The quantity or amount of matter in an object is determined by the numbers of atoms and molecules of various types it contains. Unlike weight, mass does not vary with location. The mass of an object is the same on Earth, in orbit, or on the surface of the Moon. In practice, it is very difficult to count and identify all of the atoms and molecules in an object, so masses are not often determined in this manner. Operationally, the masses of objects are determined by comparison with the standard kilogram.

Exercise 4.3.1

Which has more mass: a kilogram of cotton balls or a kilogram of gold?

Answer

They are equal. A kilogram of one substance is equal in mass to a kilogram of another substance. The quantities that might differ between them are volume and density.

Summary

- **Newton's first law of motion** states that a body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force. This is also known as the **law of inertia**.
- **Inertia** is the tendency of an object to remain at rest or remain in motion. Inertia is related to an object's mass.
- **Mass** is the quantity of matter in a substance.

Glossary

inertia

the tendency of an object to remain at rest or remain in motion

law of inertia

see Newton's first law of motion

mass

the quantity of matter in a substance; measured in kilograms

Newton's first law of motion

a body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force; also known as the law of inertia

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4.4: Newton’s Second Law of Motion- Concept of a System

Learning Objectives

By the end of this section, you will be able to:

- Define net force, external force, and system.
- Understand Newton’s second law of motion.
- Apply Newton’s second law to determine the weight of an object.

Newton’s second law of motion is closely related to Newton’s first law of motion. It mathematically states the cause and effect relationship between force and changes in motion. Newton’s second law of motion is more quantitative and is used extensively to calculate what happens in situations involving a force. Before we can write down Newton’s second law as a simple equation giving the exact relationship of force, mass, and acceleration, we need to sharpen some ideas that have already been mentioned.

First, what do we mean by a change in motion? The answer is that a change in motion is equivalent to a change in velocity. A change in velocity means, by definition, that there is an **acceleration**. Newton’s first law says that a net external force causes a change in motion; thus, we see that a *net external force causes acceleration*.

Another question immediately arises. What do we mean by an external force? An intuitive notion of external is correct — an **external force** acts from outside the **system** of interest. For example, in Figure $4.4.1a$ the system of interest is the wagon plus the child in it. The two forces exerted by the other children are external forces. An internal force acts between elements of the system. Again looking at Figure $4.4.1a$, the force the child in the wagon exerts to hang onto the wagon is an internal force between elements of the system of interest. Only external forces affect the motion of a system, according to Newton’s first law. (The internal forces actually cancel, as we shall see in the next section.) *You must define the boundaries of the system before you can determine which forces are external*. Sometimes the system is obvious, whereas other times identifying the boundaries of a system is more subtle. The concept of a system is fundamental to many areas of physics, as is the correct application of Newton’s laws. This concept will be revisited many times on our journey through physics.

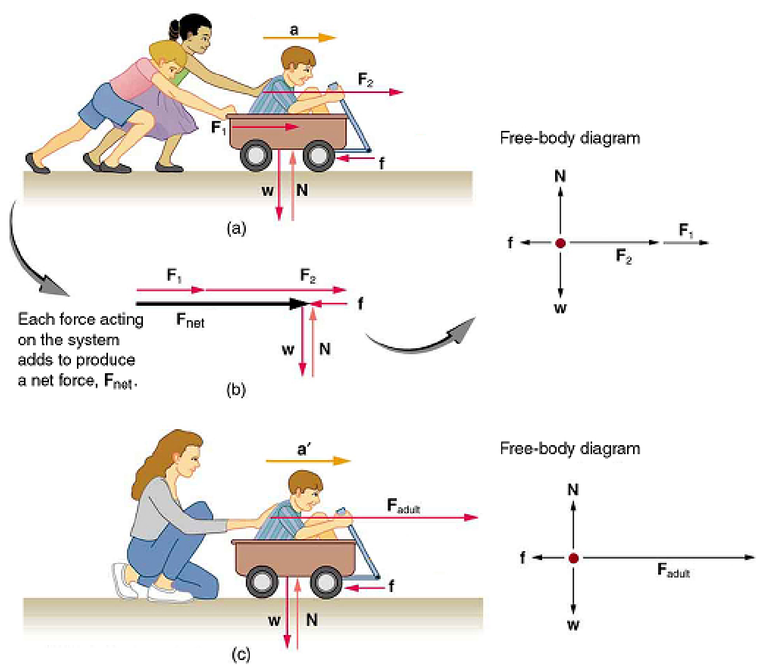

Figure $4.4.1$: Different forces exerted on the same mass produce different accelerations. (a) Two children push a wagon with a child in it. Arrows representing all external forces are shown. The system of interest is the wagon and its rider. The weight w of the system and the support of the ground N are also shown for completeness and are assumed to cancel. The vector f represents the friction acting on the wagon, and it acts to the left, opposing the motion of the wagon. (b) All of the external forces acting on the system add together to produce a net force, F_{net}. The free-body diagram shows all of the forces acting on the system of interest. The dot represents the center of mass of the system. Each force vector extends from this dot. Because there are two forces acting to the right, we draw the vectors collinearly. (c) A larger net external force produces a larger acceleration $(al > a)$ when an adult pushes the child.

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Now, it seems reasonable that acceleration should be directly proportional to and in the same direction as the net (total) external force acting on a system. This assumption has been verified experimentally and is illustrated in Figure. In part (a), a smaller force causes a smaller acceleration than the larger force illustrated in part (c). For completeness, the vertical forces are also shown; they are assumed to cancel since there is no acceleration in the vertical direction. The vertical forces are the weight w and the support of the ground N, and the horizontal force f represents the force of friction. These will be discussed in more detail in later sections. For now, we will define **friction** as a force that opposes the motion past each other of objects that are touching. Figure $4.4.1b$ shows how vectors representing the external forces add together to produce a net force, F_{net}.

To obtain an equation for Newton's second law, we first write the relationship of acceleration and net external force as the proportionality

$$a \propto F_{net} \tag{4.4.1}$$

where the symbol $\propto$ means "proportional to," and F_{net} is the **net external force**. (The net external force is the vector sum of all external forces and can be determined graphically, using the head-to-tail method, or analytically, using components. The techniques are the same as for the addition of other vectors, and are covered in the chapter section on Two-Dimensional Kinematics.) This proportionality states what we have said in words—*acceleration is directly proportional to the net external force*. Once the system of interest is chosen, it is important to identify the external forces and ignore the internal ones. It is a tremendous simplification not to have to consider the numerous internal forces acting between objects within the system, such as muscular forces within the child's body, let alone the myriad of forces between atoms in the objects, but by doing so, we can easily solve some very complex problems with only minimal error due to our simplification

Now, it also seems reasonable that acceleration should be inversely proportional to the mass of the system. In other words, the larger the mass (the inertia), the smaller the acceleration produced by a given force. And indeed, as illustrated in Figure, the same net external force applied to a car produces a much smaller acceleration than when applied to a basketball. The proportionality is written as

$$a \propto \frac{1}{m}, \tag{4.4.2}$$

where m is the mass of the system. Experiments have shown that acceleration is exactly inversely proportional to mass, just as it is exactly linearly proportional to the net external force.

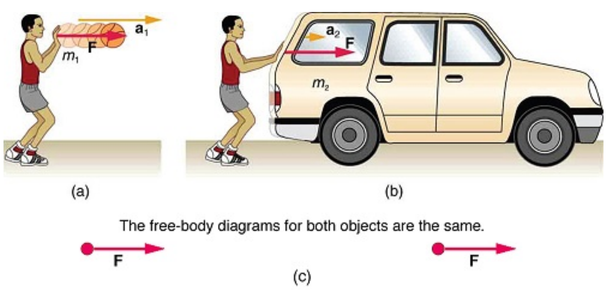

Figure 4.4.2:The same force exerted on systems of different masses produces different accelerations. (a) A basketball player pushes on a basketball to make a pass. (The effect of gravity on the ball is ignored.) (b) The same player exerts an identical force on a stalled SUV and produces a far smaller acceleration (even if friction is negligible). (c) The free-body diagrams are identical, permitting direct comparison of the two situations. A series of patterns for the free-body diagram will emerge as you do more problems.

It has been found that the acceleration of an object depends *only* on the net external force and the mass of the object. Combining the two proportionalities just given yields Newton's second law of motion.

Newton's Second Law of Motion

The acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system, and inversely proportional to its mass. In equation form, Newton's second law of motion is

$$a = \frac{F_{net}}{m} \tag{4.4.3}$$

This is often written in the more familiar form

$$F_{net} = ma. \tag{4.4.4}$$

When only the magnitude of force and acceleration are considered, this equation is simply

$$F_{net} = ma. \tag{4.4.5}$$

Although these last two equations are really the same, the first gives more insight into what Newton's second law means. The law is a *cause and effect relationship* among three quantities that is not simply based on their definitions. The validity of the second law is completely based on experimental verification.

Units of Force

$F_{net} = ma$ is used to define the units of force in terms of the three basic units for mass, length, and time. The SI unit of force is called the newton (abbreviated N) and is the force needed to accelerate a 1-kg system at the rate of $1m/s^2$ That is, since $F_{net} = ma,$

$$1N = 1kg \cdot s^2 \tag{4.4.6}$$

While almost the entire world uses the newton for the unit of force, in the United States the most familiar unit of force is the pound (lb), where 1 N = 0.225 lb.

Weight and the Gravitational Force

When an object is dropped, it accelerates toward the center of Earth. Newton's second law states that a net force on an object is responsible for its acceleration. If air resistance is negligible, the net force on a falling object is the gravitational force, commonly called its **weight** w. Weight can be denoted as a vector w because it has a direction; down is, by definition, the direction of gravity, and hence weight is a downward force. The magnitude of weight is denoted as w. Galileo was instrumental in showing that, in the absence of air resistance, all objects fall with the same acceleration w. Using Galileo's result and Newton's second law, we can derive an equation for weight.

Consider an object with mass m falling downward toward Earth. It experiences only the downward force of gravity, which has magnitude w. Newton's second law states that the magnitude of the net external force on an object is $F_{net} = ma$.

Since the object experiences only the downward force of gravity, $F_{net} = w$. We know that the acceleration of an object due to gravity is g, or $a = g$. Substituting these into Newton's second law gives

WEIGHT

This is the equation for weight - the gravitational force on mass m:

$$w = mg \tag{4.4.7}$$

Since weight $g = 9.80m/s^2$ on Earth, the weight of a 1.0 kg object on Earth is 9.8 N, as we see:

$$w = mg = (1.0kg)(9.8m/s^2) = 9.8N. \tag{4.4.8}$$

Recall that g can take a positive or negative value, depending on the positive direction in the coordinate system. Be sure to take this into consideration when solving problems with weight.

When the net external force on an object is its weight, we say that it is in **free-fall**. That is, the only force acting on the object is the force of gravity. In the real world, when objects fall downward toward Earth, they are never truly in free-fall because there is always some upward force from the air acting on the object.

The acceleration due to gravity g varies slightly over the surface of Earth, so that the weight of an object depends on location and is not an intrinsic property of the object. Weight varies dramatically if one leaves Earth's surface. On the Moon, for example, the acceleration due to gravity is only $1.67m/s^2$. A 1.0-kg mass thus has a weight of 9.8 N on Earth and only about 1.7 N on the Moon.

The broadest definition of weight in this sense is that *the weight of an object is the gravitational force on it from the nearest large body*, such as Earth, the Moon, the Sun, and so on. This is the most common and useful definition of weight in physics. It differs dramatically, however, from the definition of weight used by NASA and the popular media in relation to space travel and exploration. When they speak of “weightlessness” and “microgravity,” they are really referring to the phenomenon we call “free-fall” in physics. We shall use the above definition of weight, and we will make careful distinctions between free-fall and actual weightlessness.

It is important to be aware that weight and mass are very different physical quantities, although they are closely related. Mass is the quantity of matter (how much “stuff”) and does not vary in classical physics, whereas weight is the gravitational force and does vary depending on gravity. It is tempting to equate the two, since most of our examples take place on Earth, where the weight of an object only varies a little with the location of the object. Furthermore, the terms *mass* and *weight* are used interchangeably in everyday language; for example, our medical records often show our “weight” in kilograms, but never in the correct units of newtons.

COMMON MISCONCEPTIONS: MASS VS. WEIGHT

Mass and weight are often used interchangeably in everyday language. However, in science, these terms are distinctly different from one another. Mass is a measure of how much matter is in an object. The typical measure of mass is the kilogram (or the “slug” in English units). Weight, on the other hand, is a measure of the force of gravity acting on an object. Weight is equal to the mass of an object (m) multiplied by the acceleration due to gravity (g). Like any other force, weight is measured in terms of newtons (or pounds in English units).

Assuming the mass of an object is kept intact, it will remain the same, regardless of its location. However, because weight depends on the acceleration due to gravity, the weight of an object can change when the object enters into a region with stronger or weaker gravity. For example, the acceleration due to gravity on the Moon is $1.67 m/s^2$(which is much less than the acceleration due to gravity on Earth, $9.80 m/s^2$). If you measured your weight on Earth and then measured your weight on the Moon, you would find that you “weigh” much less, even though you do not look any skinnier. This is because the force of gravity is weaker on the Moon. In fact, when people say that they are “losing weight,” they really mean that they are losing “mass” (which in turn causes them to weigh less)

TAKE-HOME EXPERIMENT: MASS AND WEIGHT

What do bathroom scales measure? When you stand on a bathroom scale, what happens to the scale? It depresses slightly. The scale contains springs that compress in proportion to your weight—similar to rubber bands expanding when pulled. The springs provide a measure of your weight (for an object which is not accelerating). This is a force in newtons (or pounds). In most countries, the measurement is divided by 9.80 to give a reading in mass units of kilograms. The scale measures weight but is calibrated to provide information about mass. While standing on a bathroom scale, push down on a table next to you. What happens to the reading? Why? Would your scale measure the same “mass” on Earth as on the Moon?

Example 4.4.1:What Acceleration Can a Person Produce when pushing a Lawn Mower?

Suppose that the net external force (push minus friction) exerted on a lawn mower is 51 N (about 11 lb) parallel to the ground. The mass of the mower is 24 kg. What is its acceleration?

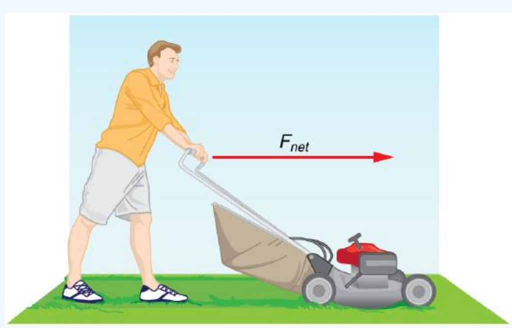

Figure 4.4.3: The net force on a lawn mower is 51 N to the right. At what rate does the lawn mower accelerate to the right?

Strategy

Since F_{net} and m are given, the acceleration can be calculated directly from Newton’s second law as stated in $F_{net} = ma$.

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Solution

The magnitude of the acceleration a is $a = \frac{F_{net}}{m}$. Entering known values gives

$$a = \frac{51\ N}{24\ kg} \tag{4.4.9}$$

Substituting the units $kg \cdot m/s^2$ for N yields

$$a = \frac{51\ kg/s^2}{24\ kg} = 2.1\ m/s^2 \tag{4.4.10}$$

Discussion

The direction of the acceleration is the same direction as that of the net force, which is parallel to the ground. There is no information given in this example about the individual external forces acting on the system, but we can say something about their relative magnitudes. For example, the force exerted by the person pushing the mower must be greater than the friction opposing the motion (since we know the mower moves forward), and the vertical forces must cancel if there is to be no acceleration in the vertical direction (the mower is moving only horizontally). The acceleration found is small enough to be reasonable for a person pushing a mower. Such an effort would not last too long because the person's top speed would soon be reached.

Example 4.4.2:What Rocket Thrust Accelerates This Sled?

Prior to manned space flights, rocket sleds were used to test aircraft, missile equipment, and physiological effects on human subjects at high speeds. They consisted of a platform that was mounted on one or two rails and propelled by several rockets. Calculate the magnitude of force exerted by each rocket, called its thrust T for the four-rocket propulsion system shown in Figure. The sled's initial acceleration is $49m/s^2$ the mass of the system is 2100 kg, and the force of friction opposing the motion is known to be 650 N.

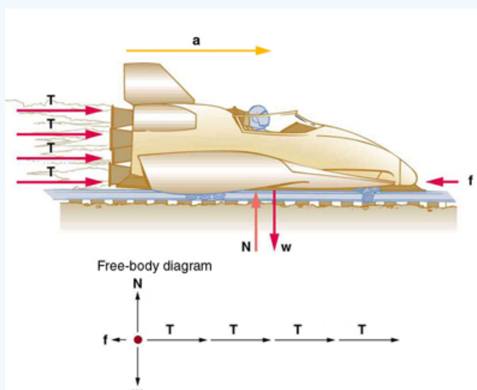

Figure 4.4.4. A sled experiences a rocket thrust that accelerates it to the right. Each rocket creates an identical thrust T. As in other situations where there is only horizontal acceleration, the vertical forces cancel. The ground exerts an upward force N on the system that is equal in magnitude and opposite in direction to its weight, w. The system here is the sled, its rockets, and rider, so none of the forces between these objects are considered. The arrow representing friction (f) is drawn larger than scale.

Strategy

Although there are forces acting vertically and horizontally, we assume the vertical forces cancel since there is no vertical acceleration. This leaves us with only horizontal forces and a simpler one-dimensional problem. Directions are indicated with plus or minus signs, with right taken as the positive direction. See the free-body diagram in the figure.

Solution

Since acceleration, mass, and the force of friction are given, we start with Newton's second law and look for ways to find the thrust of the engines. Since we have defined the direction of the force and acceleration as acting "to the right," we need to consider only the magnitudes of these quantities in the calculations. Hence we begin with

$$F_{net} = ma. \tag{4.4.11}$$

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, where F_{net} is the net force along the horizontal direction. We can see from Figure that the engine thrusts add, while friction opposes the thrust. In equation form, the net external force is

$$F_{net} = 4T - f. \tag{4.4.12}$$

Substituting this into Newton's second law gives

$$F_{net} = ma = 4T - f. \tag{4.4.13}$$

Using a little algebra, we solve for the total thrust 4T:

$$4T = ma + f. \tag{4.4.14}$$

Substituting known values yields

$$4T = ma + f = (2100\,kg)(49\,m/s^2) + 650\,N \tag{4.4.15}$$

So the total thrust is

$$1 \times 10^5 N, \tag{4.4.16}$$

and the individual thrusts are

$$T = \frac{1 \times 10^5}{4} = 2.6 \times 10^4\,N \tag{4.4.17}$$

Discussion

The numbers are quite large, so the result might surprise you. Experiments such as this were performed in the early 1960s to test the limits of human endurance and the setup designed to protect human subjects in jet fighter emergency ejections. Speeds of 1000 km/h were obtained, with accelerations of 45 g-s. (Recall that g, the acceleration due gravity is $9.80\,m/s^2$. When we say that an acceleration is 45 g-s, it is $45 \times 9.80 m/s^2$, which is approximately $440 m/s^2$). While living subjects are not used any more, land speeds of 10,000 km/h have been obtained with rocket sleds. In this example, as in the preceding one, the system of interest is obvious. We will see in later examples that choosing the system of interest is crucial—and the choice is not always obvious.

Newton's second law of motion is more than a definition; it is a relationship among acceleration, force, and mass. It can help us make predictions. Each of those physical quantities can be defined independently, so the second law tells us something basic and universal about nature. The next section introduces the third and final law of motion.

Summary

- Acceleration, a, is defined as a change in velocity, meaning a change in its magnitude or direction, or both.
- An external force is one acting on a system from outside the system, as opposed to internal forces, which act between components within the system.
- Newton's second law of motion states that the acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system, and inversely proportional to its mass.
- In equation form, Newton's second law of motion is $a = \frac{F_{net}}{m}$
- This is often written in the more familiar form: $F_{net} = ma$.
- The weight w of an object is defined as the force of gravity acting on an object of mass m. The object experiences an acceleration due to gravity g:

$$w = mg. \tag{4.4.18}$$

- If the only force acting on an object is due to gravity, the object is in free fall.
- Friction is a force that opposes the motion past each other of objects that are touching.

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4.5: Newton’s Third Law of Motion- Symmetry in Forces

Learning Objectives

By the end of this section, you will be able to:

- Understand Newton’s third law of motion.
- Apply Newton’s third law to define systems and solve problems of motion.

There is a passage in the musical *Man of la Mancha* that relates to Newton’s third law of motion. Sancho, in describing a fight with his wife to Don Quixote, says, “Of course I hit her back, Your Grace, but she’s a lot harder than me and you know what they say, ‘Whether the stone hits the pitcher or the pitcher hits the stone, it’s going to be bad for the pitcher.’” This is exactly what happens whenever one body exerts a force on another—the first also experiences a force (equal in magnitude and opposite in direction). Numerous common experiences, such as stubbing a toe or throwing a ball, confirm this. It is precisely stated in **Newton’s third law of motion.**

NEWTON’S THIRD LAW OF MOTION

Whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that it exerts.

This law represents a certain *symmetry in nature*: Forces always occur in pairs, and one body cannot exert a force on another without experiencing a force itself. We sometimes refer to this law loosely as “action-reaction,” where the force exerted is the action and the force experienced as a consequence is the reaction. Newton’s third law has practical uses in analyzing the origin of forces and understanding which forces are external to a system.

We can readily see Newton’s third law at work by taking a look at how people move about. Consider a swimmer pushing off from the side of a pool, as illustrated in Figure. She pushes against the pool wall with her feet and accelerates in the direction *opposite* to that of her push. The wall has exerted an equal and opposite force back on the swimmer. You might think that two equal and opposite forces would cancel, but they do not *because they act on different systems*. In this case, there are two systems that we could investigate: the swimmer or the wall. If we select the swimmer to be the system of interest, as in the figure, then $F_{wall\ on\ feet}$ is an external force on this system and affects its motion. The swimmer moves in the direction of $F_{wall\ on\ feet}$. In contrast, the force $F_{feet\ on\ wall}$ acts on the wall and not on our system of interest. Thus $F_{feet\ on\ wall}$ does not directly affect the motion of the system and does not cancel $F_{wall\ on\ feet}$. Note that the swimmer pushes in the direction opposite to that in which she wishes to move. The reaction to her push is thus in the desired direction.

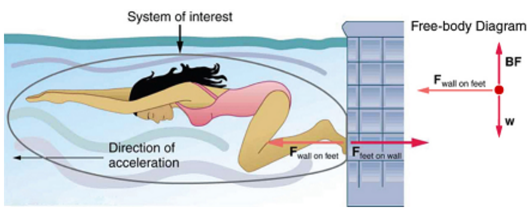

Figure 4.5.1: When the swimmer exerts a force $F_{feet\ on\ wall}$ on the wall, she accelerates in the direction opposite to that of her push. This means the net external force on her is in the direction opposite to $F_{feet\ on\ wall}$ This opposition occurs because, in accordance with Newton’s third law of motion, the wall exerts a force $F_{wall\ on\ feet}$ on her, equal in magnitude but in the direction opposite to the one she exerts on it. The line around the swimmer indicates the system of interest. Note that $F_{feet\ on\ wall}$ does not act on this system (the swimmer) and, thus, does not cancel $F_{wall\ on\ feet}$. Thus the free-body diagram shows only $F_{wall\ on\ feet}$, w, the gravitational force, and BF, the buoyant force of the water supporting the swimmer’s weight. The vertical forces w and BF cancel since there is no vertical motion.

Other examples of Newton’s third law are easy to find. As a professor paces in front of a whiteboard, she exerts a force backward on the floor. The floor exerts a reaction force forward on the professor that causes her to accelerate forward. Similarly, a car accelerates because the ground pushes forward on the drive wheels in reaction to the drive wheels pushing backward on the ground. You can see evidence of the wheels pushing backward when tires spin on a gravel road and throw rocks backward. In another example, rockets move forward by expelling gas backward at high velocity. This means the rocket exerts a large backward force on the gas in the rocket combustion chamber, and the gas therefore exerts a large reaction force forward on the rocket. This reaction force is called thrust. It is a common misconception that rockets propel themselves by pushing on the ground or on the air behind them. They actually work better in a vacuum, where they can more readily expel the exhaust gases. Helicopters similarly create lift by pushing air down, thereby experiencing an upward reaction force. Birds and airplanes also fly by exerting force on air in a direction opposite to that of whatever force they need. For example, the wings of a bird force air downward and backward in order to get lift and move forward. An octopus propels itself in the water by ejecting water through a funnel from its body, similar to a jet ski. In a situation similar to Sancho's, professional cage fighters experience reaction forces when they punch, sometimes breaking their hand by hitting an opponent's body.

Exercise 4.5.1: Getting up to speed: Choosing the Correct System

A physics professor pushes a cart of demonstration equipment to a lecture hall, as seen in Figure. Her mass is 65.0 kg, the cart's is 12.0 kg, and the equipment's is 7.0 kg. Calculate the acceleration produced when the professor exerts a backward force of 150 N on the floor. All forces opposing the motion, such as friction on the cart's wheels and air resistance, total 24.0 N.

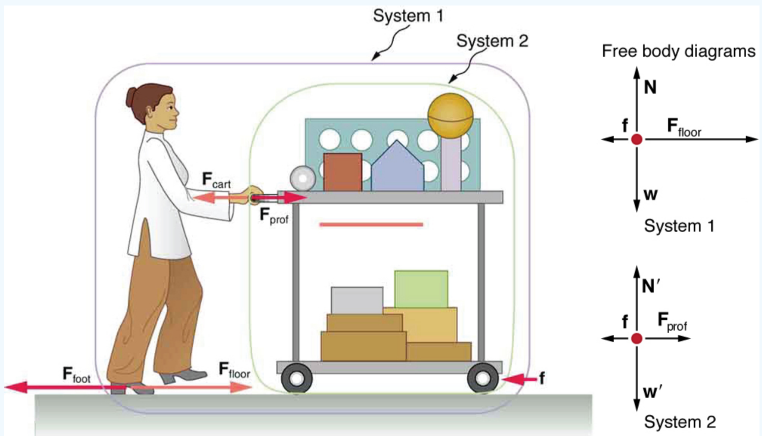

Figure 4.5.2: A professor pushes a cart of demonstration equipment. The lengths of the arrows are proportional to the magnitudes of the forces (except for f, since it is too small to draw to scale). Different questions are asked in each example; thus, the system of interest must be defined differently for each. System 1 is appropriate for Example, since it asks for the acceleration of the entire group of objects. Only F_{floor} and f are external forces acting on System 1 along the line of motion. All other forces either cancel or act on the outside world. System 2 is chosen for this example so that F_{prof} will be an external force and enter into Newton's second law. Note that the free-body diagrams, which allow us to apply Newton's second law, vary with the system chosen.

Strategy

Since they accelerate as a unit, we define the system to be the professor, cart, and equipment. This is System 1 in Figure. The professor pushes backward with a force F_{foot} of 150 N. According to Newton's third law, the floor exerts a forward reaction force F_{floor} of 150 N on System 1. Because all motion is horizontal, we can assume there is no net force in the vertical direction. The problem is therefore one-dimensional along the horizontal direction. As noted, f opposes the motion and is thus in the opposite direction of F_{floor}. Note that we do not include the forces F_{prof} or F_{cart} because these are internal forces, and we do not include F_{foot} because it acts on the floor, not on the system. There are no other significant forces acting on System 1. If the net external force can be found from all this information, we can use Newton's second law to find the acceleration as requested. See the free-body diagram in the figure.

Solution

Newton's second law is given by

$$a = \frac{F_{net}}{m}.$$

The net external force on System 1 is deduced from Figure and the discussion above to be

$$F_{net} = F_{floor} - f = 150\,N - 24.0\,N = 126\,N.$$

The mass of System 1 is

$$m = (65.0 + 12.0 + 7.0) = 84\,kg.$$

These values of F_{net} and m produce an acceleration of

$$a=\frac{F_{net}}{m}$$

$$a=\frac{126\,N}{84\,kg}=1.5\,m/s^2.$$

Discussion

None of the forces between components of System 1, such as between the professor's hands and the cart, contribute to the net external force because they are internal to System 1. Another way to look at this is to note that forces between components of a system cancel because they are equal in magnitude and opposite in direction. For example, the force exerted by the professor on the cart results in an equal and opposite force back on her. In this case both forces act on the same system and, therefore, cancel. Thus internal forces (between components of a system) cancel. Choosing System 1 was crucial to solving this problem.

Example 4.5.2: Force on the Cart: Choosing a New System

Calculate the force the professor exerts on the cart in Figure using data from the previous example if needed.

Strategy

If we now define the system of interest to be the cart plus equipment (System 2 in Figure), then the net external force on System 2 is the force the professor exerts on the cart minus friction. The force she exerts on the cart, F_{prof} is an external force acting on System 2. F_{prof} was internal to System 1, but it is external to System 2 and will enter Newton's second law for System 2.

Solution

Newton's second law can be used to find F_{prof} Starting with

$$a=\frac{F_{net}}{m}$$

and noting that the magnitude of the net external force on System 2 is

$$F_{net}=F_{prof}-f,$$

we solve for F_{prof}, the desired quantity

$$F_{net}+f.$$

The value of f is given, so we must calculate net F_{net}. That can be done since both the acceleration and mass of System 2 are known. Using Newton's second law we see that

$$F_{net}=ma,$$

where the mass of System 2 is 19.0 kg (m = 12.0 kg + 7.0 kg) and its acceleration was found to be $a=1.5\,m/s^2$ in the previous example. Thus,

$$F_{net}=ma$$

$$F_{net}=(19.0\,kg)(1.5\,m/s^2)=29\,N.$$

Now we can find the desired force:

$$F_{prof}=F_{net}+f,$$

$$F_{prof}=29\,N+24.0\,N=53\,N.$$

Discussion

It is interesting that this force is significantly less than the 150-N force the professor exerted backward on the floor. Not all of that 150-N force is transmitted to the cart; some of it accelerates the professor.

The choice of a system is an important analytical step both in solving problems and in thoroughly understanding the physics of the situation (which is not necessarily the same thing).

PHET EXPLORATIONS: GRAVITY FORCE LAB

Visualize the gravitational force that two objects exert on each other. Change properties of the objects in order to see how it changes the gravity force.

Section Summary

- **Newton's third law of motion** represents a basic symmetry in nature. It states: Whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that the first body exerts.
- A **thrust** is a reaction force that pushes a body forward in response to a backward force. Rockets, airplanes, and cars are pushed forward by a thrust reaction force.

Glossary

Newton's third law of motion

whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that the first body exerts

thrust

a reaction force that pushes a body forward in response to a backward force; rockets, airplanes, and cars are pushed forward by a thrust reaction force

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4.6: Normal, Tension, and Other Examples of Forces

Learning Objectives

By the end of this section, you will be able to:

- Define normal and tension forces.
- Apply Newton's laws of motion to solve problems involving a variety of forces.
- Use trigonometric identities to resolve weight into components.

Normal Force

Weight (also called force of gravity) is a pervasive force that acts at all times and must be counteracted to keep an object from falling. You definitely notice that you must support the weight of a heavy object by pushing up on it when you hold it stationary, as illustrated in Figure(a). But how do inanimate objects like a table support the weight of a mass placed on them, such as shown in Figure(b)? When the bag of dog food is placed on the table, the table actually sags slightly under the load. This would be noticeable if the load were placed on a card table, but even rigid objects deform when a force is applied to them. Unless the object is deformed beyond its limit, it will exert a restoring force much like a deformed spring (or trampoline or diving board). The greater the deformation, the greater the restoring force. So when the load is placed on the table, the table sags until the restoring force becomes as large as the weight of the load. At this point the net external force on the load is zero. That is the situation when the load is stationary on the table. The table sags quickly, and the sag is slight so we do not notice it. But it is similar to the sagging of a trampoline when you climb onto it.

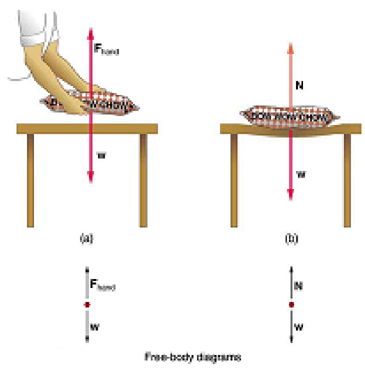


Figure 4.6.1: (a) The person holding the bag of dog food must supply an upward force F_{hand} equal in magnitude and opposite in direction to the weight of the food w. (b) The card table sags when the dog food is placed on it, much like a stiff trampoline. Elastic restoring forces in the table grow as it sags until they supply a force N equal in magnitude and opposite in direction to the weight of the load.

We must conclude that whatever supports a load, be it animate or not, must supply an upward force equal to the weight of the load, as we assumed in a few of the previous examples. If the force supporting a load is perpendicular to the surface of contact between the load and its support, this force is defined to be a **normal force** and here is given the symbol N. (This is not the unit for force N.) The word *normal* means perpendicular to a surface. The normal force can be less than the object's weight if the object is on an incline, as you will see in the next example.

COMMON MISCONCEPTION: NORMAL FORCE (N) VS. NEWTON (N)

In this section we have introduced the quantity normal force, which is represented by the variable N This should not be confused with the symbol for the newton, which is also represented by the letter N. These symbols are particularly important to distinguish because the units of a normal force N happen to be newtons (N). For example, the normal force N hat the floor exerts on a chair might be $N = 100\ N$ One important difference is that normal force is a vector, while the newton is simply a unit. Be careful not to confuse these letters in your calculations! You will encounter more similarities among variables and units as you proceed in physics. Another example of this is the quantity work W and the unit watts (W).

Example 4.6.1: Weight on an incline, a Two-Dimensional problem

Consider the skier on a slope shown in Figure. Her mass including equipment is 60.0 kg. (a) What is her acceleration if friction is negligible? (b) What is her acceleration if friction is known to be 45.0 N?

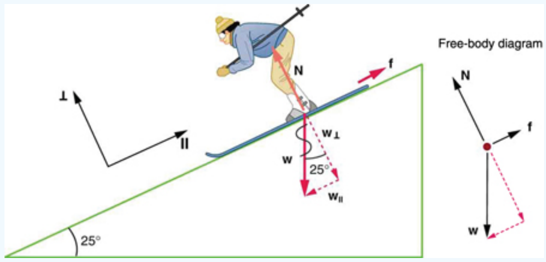


Figure 4.6.2: Since motion and friction are parallel to the slope, it is most convenient to project all forces onto a coordinate system where one axis is parallel to the slope and the other is perpendicular (axes shown to left of skier). N is perpendicular to the slope and f is parallel to the slope, but w has components along both axes, namely $w_{\perp}$ and $w_{\|}$ N is equal in magnitude to $w_{\perp}$, so that there is no motion perpendicular to the slope, but f is less than $w_{\|}$, so that there is a downslope acceleration (along the parallel axis).

Strategy

This is a two-dimensional problem, since the forces on the skier (the system of interest) are not parallel. The approach we have used in two-dimensional kinematics also works very well here. Choose a convenient coordinate system and project the vectors onto its axes, creating two connected one-dimensional problems to solve. The most convenient coordinate system for motion on an incline is one that has one coordinate parallel to the slope and one perpendicular to the slope. (Remember that motions along mutually perpendicular axes are independent.) We use the symbols $\perp$ and $\|$ to represent perpendicular and parallel, respectively. This choice of axes simplifies this type of problem, because there is no motion perpendicular to the slope and because friction is always parallel to the surface between two objects. The only external forces acting on the system are the skier's weight, friction, and the support of the slope, respectively labeled w, f and N in Figure. N is always perpendicular to the slope, and f is parallel to it. But w is not in the direction of either axis, and so the first step we take is to project it into components along the chosen axes, defining $w_{\|}$ to be the component of weight parallel to the slope and $w_{\perp}$ the component of weight perpendicular to the slope. Once this is done, we can consider the two separate problems of forces parallel to the slope and forces perpendicular to the slope.

Solution

The magnitude of the component of the weight parallel to the slope is $w_{\|} = w\, sin\,(25^o)$ = mg \, sin \, (25^o), \) and the magnitude of the component of the weight perpendicular to the slope is $w_{\perp} = w\, cos\,(25^o) = mg\, cos\,(25^o)$.

(a) Neglecting friction. Since the acceleration is parallel to the slope, we need only consider forces parallel to the slope. (Forces perpendicular to the slope add to zero, since there is no acceleration in that direction.) The forces parallel to the slope are the amount of the skier's weight parallel to the slope $w_{\|}$ and friction f. Using Newton's second law, with subscripts to denote quantities parallel to the slope,

$$a_{\|} = \frac{F_{net\|}}{m} \tag{4.6.1}$$

where $F_{net\|} = w_{\|} = mg\, sin\,(25^o)$, assuming no friction for this part, so that

$$a_{\|} = \frac{F_{net\|}}{m} = \frac{mg\, sin(25^o)}{m} = g\, sin(25^o) \tag{4.6.2}$$

$$(9.80\, m/s^2)(0.4226) = 4.14\, m/s^2 \tag{4.6.3}$$

is the acceleration.

(b) Including friction. We now have a given value for friction, and we know its direction is parallel to the slope and it opposes motion between surfaces in contact. So the net external force is now

$$F_{net\|} = w_{\|} - f, \tag{4.6.4}$$

and substituting this into Newton's second law, $a_{\parallel} = \frac{F_{net\parallel}}{m}$, gives

$$a_{\parallel} = \frac{F_{net\parallel}}{m} = \frac{w_{\parallel} - f}{m} = \frac{mg\,sin(25^o) - f}{m}. \tag{4.6.5}$$

We substitute known values to obtain

$$a_{\parallel} = \frac{(60.0\,kg)(9.80\,m/s^2)(0.4226) - 45.0\,N}{60.0\,kg}, \tag{4.6.6}$$

which yields

$$a_{\parallel} = 3.39\,m/s^2, \tag{4.6.7}$$

which is the acceleration parallel to the incline when there is 45.0 N of opposing friction.

Discussion

Since friction always opposes motion between surfaces, the acceleration is smaller when there is friction than when there is none. In fact, it is a general result that if friction on an incline is negligible, then the acceleration down the incline is $a = g\,sin\,\theta$, regardless of mass. This is related to the previously discussed fact that all objects fall with the same acceleration in the absence of air resistance. Similarly, all objects, regardless of mass, slide down a frictionless incline with the same acceleration (if the angle is the same).

RESOLVING WEIGHT INTO COMPONENTS

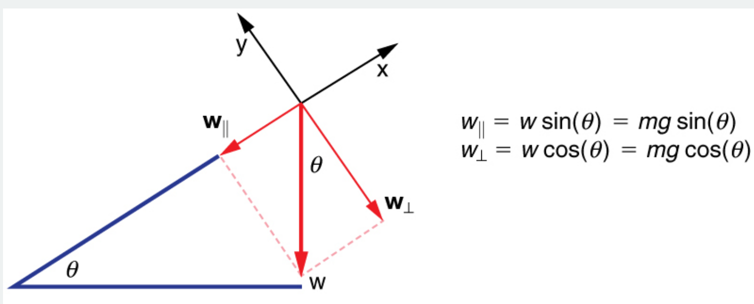


Figure 4.6.3: An object rests on an incline that makes an angle θ with the horizontal.

When an object rests on an incline that makes an angle θ with the horizontal, the force of gravity acting on the object is divided into two components: a force acting perpendicular to the plane, $w_{\perp}$ and a force acting parallel to the plane, $w_{\parallel}$. The perpendicular force of weight, $w_{\perp}$ is typically equal in magnitude and opposite in direction to the normal force, N. The force acting parallel to the plane, $w_{\parallel}$ causes the object to accelerate down the incline. The force of friction, f opposes the motion of the object, so it acts upward along the plane.

It is important to be careful when resolving the weight of the object into components. If the angle of the incline is at an angle θ to the horizontal, then the magnitudes of the weight components are

$$w_{\parallel} = w\ sin\ (\theta) = mg\,sin\,(\theta) \tag{4.6.8}$$

and

$$w_{\perp} = w\ sin\ (\theta) = mg\,sin\,(\theta) \tag{4.6.9}$$

Instead of memorizing these equations, it is helpful to be able to determine them from reason. To do this, draw the right triangle formed by the three weight vectors. Notice that the angle θ of the incline is the same as the angle formed between $w$and $w_{\perp}$. Knowing this property, you can use trigonometry to determine the magnitude of the weight components:

$$cos\ (\theta) = \frac{w_{\perp}}{w} \tag{4.6.10}$$

$$w_{\perp} = w\ cos\,(\theta) = mg\,cos\,(\theta) \tag{4.6.11}$$

$$sin\,(\theta)=\frac{w_{\parallel}}{w} \tag{4.6.12}$$

$$w_{\parallel}=w\;sin\,(\theta)=mg\,sin\,(\theta) \tag{4.6.13}$$

TAKE-HOME EXPERIMENT: FORCE PARALLEL

To investigate how a force parallel to an inclined plane changes, find a rubber band, some objects to hang from the end of the rubber band, and a board you can position at different angles. How much does the rubber band stretch when you hang the object from the end of the board? Now place the board at an angle so that the object slides off when placed on the board. How much does the rubber band extend if it is lined up parallel to the board and used to hold the object stationary on the board? Try two more angles. What does this show?

Tension

A **tension** is a force along the length of a medium, especially a force carried by a flexible medium, such as a rope or cable. The word "tension" comes from a Latin word meaning "to stretch." Not coincidentally, the flexible cords that carry muscle forces to other parts of the body are called *tendons*. Any flexible connector, such as a string, rope, chain, wire, or cable, can exert pulls only parallel to its length; thus, a force carried by a flexible connector is a tension with direction parallel to the connector. It is important to understand that tension is a pull in a connector. In contrast, consider the phrase: "You can't push a rope." The tension force pulls outward along the two ends of a rope.

Consider a person holding a mass on a rope as shown in Figure.

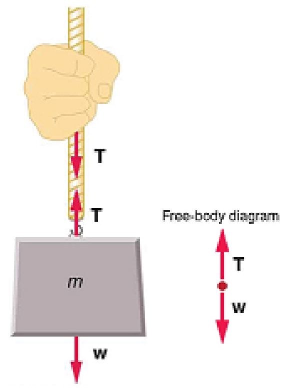


Figure 4.6.4:When a perfectly flexible connector (one requiring no force to bend it) such as this rope transmits a force T that force must be parallel to the length of the rope, as shown. The pull such a flexible connector exerts is a tension. Note that the rope pulls with equal force but in opposite directions on the hand and the supported mass (neglecting the weight of the rope). This is an example of Newton's third law. The rope is the medium that carries the equal and opposite forces between the two objects. The tension anywhere in the rope between the hand and the mass is equal. Once you have determined the tension in one location, you have determined the tension at all locations along the rope.

Tension in the rope must equal the weight of the supported mass, as we can prove using Newton's second law. If the 5.00-kg mass in the figure is stationary, then its acceleration is zero, and thus $F_{net}=0$. The only external forces acting on the mass are its weight w and the tension T supplied by the rope. Thus,

$$F_{net}=T-w=0, \tag{4.6.14}$$

where T and w are the magnitudes of the tension and weight and their signs indicate direction, with up being positive here. Thus, just as you would expect, the tension equals the weight of the supported mass:

$$T=w=mg. \tag{4.6.15}$$

For a 5.00-kg mass, then (neglecting the mass of the rope) we see that

$$T=mg=(5.00\;kg)(9.80\;m/s^2)=49.0\;N \tag{4.6.16}$$

If we cut the rope and insert a spring, the spring would extend a length corresponding to a force of 49.0 N, providing a direct observation and measure of the tension force in the rope.

Flexible connectors are often used to transmit forces around corners, such as in a hospital traction system, a finger joint, or a bicycle brake cable. If there is no friction, the tension is transmitted undiminished. Only its direction changes, and it is always parallel to the flexible connector. This is illustrated in Figure (a) and (b).

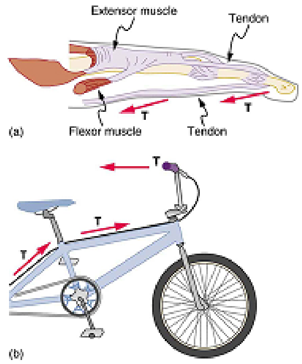


Figure $4.6.5$: (a) Tendons in the finger carry force T from the muscles to other parts of the finger, usually changing the force's direction, but not its magnitude (the tendons are relatively friction free). (b) The brake cable on a bicycle carries the tension T from the handlebars to the brake mechanism. Again, the direction but not the magnitude of T is changed.

Example $4.6.1$: What Is the Tension in a Tightrope?

Calculate the tension in the wire supporting the 70.0-kg tightrope walker shown in Figure.

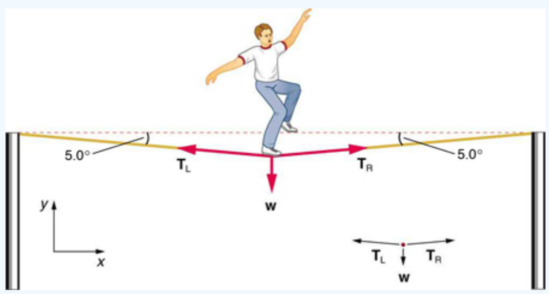


Figure $4.6.5$: The weight of a tightrope walker causes a wire to sag by 5.0 degrees. The system of interest here is the point in the wire at which the tightrope walker is standing.

Strategy

As you can see in the figure, the wire is not perfectly horizontal (it cannot be!), but is bent under the person's weight. Thus, the tension on either side of the person has an upward component that can support his weight. As usual, forces are vectors represented pictorially by arrows having the same directions as the forces and lengths proportional to their magnitudes. The system is the tightrope walker, and the only external forces acting on him are his weight w and the two tensions T_L (left tension) and T_R (right tension), as illustrated. It is reasonable to neglect the weight of the wire itself. The net external force is zero since the system is stationary. A little trigonometry can now be used to find the tensions. One conclusion is possible at the outset—we can see from part (b) of the figure that the magnitudes of the tensions T_L and T_R must be equal. This is because there is no horizontal acceleration in the rope, and the only forces acting to the left and right are T_L and T_R. Thus, the magnitude of those forces must be equal so that they cancel each other out.

Whenever we have two-dimensional vector problems in which no two vectors are parallel, the easiest method of solution is to pick a convenient coordinate system and project the vectors onto its axes. In this case the best coordinate system has one axis horizontal and the other vertical. We call the horizontal the x- axis and the vertical the y-axis.

Solution

First, we need to resolve the tension vectors into their horizontal and vertical components. It helps to draw a new free-body diagram showing all of the horizontal and vertical components of each force acting on the system.

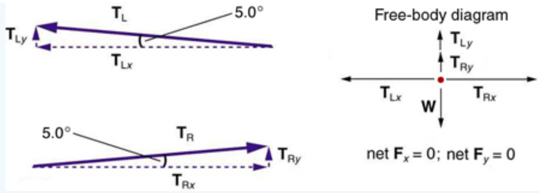


Figure 4.6.7: When the vectors are projected onto vertical and horizontal axes, their components along those axes must add to zero, since the tightrope walker is stationary. The small angle results in T being much greater than w.

Consider the horizontal components of the forces (denoted with a subscript x):

$$F_{net\,x} = T_{Lx} - T_{Rx}. \tag{4.6.17}$$

The net external horizontal force $F_{net\,x} = 0$, since the person is stationary. Thus,

$$F_{net\,x} = 0 = T_{Lx} - T_{Rx} \tag{4.6.18}$$

$$T_{Lx} = T_{Rx}. \tag{4.6.19}$$

Now, observe Figure. You can use trigonometry to determine the magnitude of T_L and T_R Notice that:

$$cos(5.0^o) = \frac{T_{Lx}}{T_L} \tag{4.6.20}$$

$$T_{Lx} = T_L\,cos(5.0^o) \tag{4.6.21}$$

$$cos(5.0^o) = \frac{T_{Lx}}{T_L} \tag{4.6.22}$$

$$T_{Rx} = T_R\,cos(5.0^o) \tag{4.6.23}$$

Equating T_{Lx} and T_{Rx}:

$$T_L\,cos(5.0^o) = T_R\,cos(5.0^o) \tag{4.6.24}$$

$$T_L = T_R = T \tag{4.6.25}$$

as predicted. Now, considering the vertical components (denoted by a subscript y), we can solve for T. Again, since the person is stationary, Newton's second law implies that net $F_y = 0$. Thus, as illustrated in the free-body diagram in Figure,

$$F_{net\,y} = T_{Ly} + T_{Ry} - w = 0 \tag{4.6.26}$$

Observing Figure, we can use trigonometry to determine the relationship between T_{Ly}, T_{Ry} and T. As we determined from the analysis in the horizontal direction, $T_L = T_R = T$.

$$sin(5.0^o) = \frac{T_{Ly}}{T_L} \tag{4.6.27}$$

$$T_{Ly} = T_L\,sin(5.0^o) = T\,sin(5.0^o) \tag{4.6.28}$$

$$sin(5.0^o) = \frac{T_{Ry}}{T_R} \tag{4.6.29}$$

$$T_{Ry} = T_R\,sin(5.0^o) = T\,sin(5.0^o) \tag{4.6.30}$$

Now, we can substitute the values for T_{Ly} and T_{Ry}, into the net force equation in the vertical direction:

$$F_{net\,y} = T_{Ly} + T_{Ry} - w = 0 \tag{4.6.31}$$

$$F_{net\,y} = T\,sin(5.0^o) + T\,sin(5.0^o) - w = 0 \tag{4.6.32}$$

$$2T\,sin(5.0^o) - w = 0 \tag{4.6.33}$$

$$2T\,sin(5.0^o) = w \tag{4.6.34}$$

and

$$T=\frac{w}{2\,sin(5.0^o)}=\frac{mg}{2\,sin(5.0^o)} \tag{4.6.35}$$

so that

$$T=\frac{(70.0\,kg)(9.80\,m/s^2)}{2(0.0872)}, \tag{4.6.36}$$

$$T=3900\,N. \tag{4.6.37}$$

Discussion

Note that the vertical tension in the wire acts as a normal force that supports the weight of the tightrope walker. The tension is almost six times the 686-N weight of the tightrope walker. Since the wire is nearly horizontal, the vertical component of its tension is only a small fraction of the tension in the wire. The large horizontal components are in opposite directions and cancel, and so most of the tension in the wire is not used to support the weight of the tightrope walker.

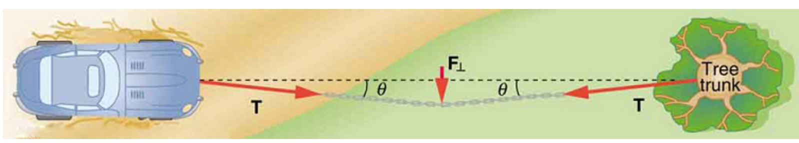
If we wish to *create* a very large tension, all we have to do is exert a force perpendicular to a flexible connector, as illustrated in Figure. As we saw in the last example, the weight of the tightrope walker acted as a force perpendicular to the rope. We saw that the tension in the roped related to the weight of the tightrope walker in the following way:

$$T=\frac{w}{2\,sin\,(\theta)} \tag{4.6.38}$$

We can extend this expression to describe the tension T created when a perpendicular force $(F_\perp)$ is exerted at the middle of a flexible connector:

$$T=\frac{F_\perp}{2\,sin\,(\theta)}. \tag{4.6.39}$$

Note that θ is the angle between the horizontal and the bent connector. In this case, T becomes very large as θ approaches zero. Even the relatively small weight of any flexible connector will cause it to sag, since an infinite tension would result if it were horizontal (i.e., $\theta=0$ and $sin\,\theta=0$). (See Figure.)



Figure, except that the tensions shown here are those transmitted to the car and the tree rather than those acting at the point where $F_\perp$ is applied.

Figure 4.6.9:Unless an infinite tension is exerted, any flexible connector—such as the chain at the bottom of the picture—will sag under its own weight, giving a characteristic curve when the weight is evenly distributed along the length. Suspension bridges—such as the Golden Gate Bridge shown in this image—are essentially very heavy flexible connectors. The weight of the bridge is evenly distributed along the length of flexible connectors, usually cables, which take on the characteristic shape. (credit: Leaflet, Wikimedia Commons)

Extended Topic: Real Forces and Inertial Frames

There is another distinction among forces in addition to the types already mentioned. Some forces are real, whereas others are not. *Real forces* are those that have some physical origin, such as the gravitational pull. Contrastingly, *fictitious forces* are those that arise simply because an observer is in an accelerating frame of reference, such as one that rotates (like a merry-go-round) or undergoes linear acceleration (like a car slowing down). For example, if a satellite is heading due north above Earth's northern hemisphere, then to an observer on Earth it will appear to experience a force to the west that has no physical origin. Of course, what is happening here is that Earth is rotating toward the east and moves east under the satellite. In Earth's frame this looks like a westward force on the satellite, or it can be interpreted as a violation of Newton's first law (the law of inertia). An **inertial frame of reference** is one in which all forces are real and, equivalently, one in which Newton's laws have the simple forms given in this chapter.

Earth's rotation is slow enough that Earth is nearly an inertial frame. You ordinarily must perform precise experiments to observe fictitious forces and the slight departures from Newton's laws, such as the effect just described. On the large scale, such as for the rotation of weather systems and ocean currents, the effects can be easily observed.

The crucial factor in determining whether a frame of reference is inertial is whether it accelerates or rotates relative to a known inertial frame. Unless stated otherwise, all phenomena discussed in this text are considered in inertial frames.

All the forces discussed in this section are real forces, but there are a number of other real forces, such as lift and thrust, that are not discussed in this section. They are more specialized, and it is not necessary to discuss every type of force. It is natural, however, to ask where the basic simplicity we seek to find in physics is in the long list of forces. Are some more basic than others? Are some different manifestations of the same underlying force? The answer to both questions is yes, as will be seen in the next (extended) section and in the treatment of modern physics later in the text.

PHET EXPLORATIONS: FORCES IN 1 DIMENSION

Explore the forces at work when you try to push a filing cabinet. Create an applied force and see the resulting friction force and total force acting on the cabinet. Charts show the forces, position, velocity, and acceleration vs. time. View a free-body diagram of all the forces (including gravitational and normal forces).

Figure 4.6.10: Forces in 1 Dimension

Summary

- When objects rest on a surface, the surface applies a force to the object that supports the weight of the object. This supporting force acts perpendicular to and away from the surface. It is called a normal force, T
- When objects rest on a non-accelerating horizontal surface, the magnitude of the normal force is equal to the weight of the object:

$$N = mg \tag{4.6.40}$$

- When objects rest on an inclined plane that makes an angle θ with the horizontal surface, the weight of the object can be resolved into components that act perpendicular $(w_{\perp})$ and parallel $w_{\parallel})$ to the surface of the plane. These components can be calculated using:

$$w_{\parallel} = w\,sin\,(\theta) = mg\,sin\,(\theta) \tag{4.6.41}$$

$$w_{\perp} = w\,cos\,(\theta) = mg\,cos\,(\theta) \tag{4.6.42}$$

- The pulling force that acts along a stretched flexible connector, such as a rope or cable, is called tension, \(T\.) When a rope supports the weight of an object that is at rest, the tension in the rope is equal to the weight of the object:

$$T = mg. \tag{4.6.43}$$

- In any inertial frame of reference (one that is not accelerated or rotated), Newton's laws have the simple forms given in this chapter and all forces are real forces having a physical origin.

Glossary

inertial frame of reference

a coordinate system that is not accelerating; all forces acting in an inertial frame of reference are real forces, as opposed to fictitious forces that are observed due to an accelerating frame of reference

normal force

the force that a surface applies to an object to support the weight of the object; acts perpendicular to the surface on which the object rests

tension

the pulling force that acts along a medium, especially a stretched flexible connector, such as a rope or cable; when a rope supports the weight of an object, the force on the object due to the rope is called a tension force

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4.7: Friction

Learning Objectives

By the end of this section, you will be able to:

- Discuss the general characteristics of friction.
- Describe the various types of friction.
- Calculate the magnitude of static and kinetic friction.

Friction is a force that is around us all the time that opposes relative motion between systems in contact but also allows us to move (which you have discovered if you have ever tried to walk on ice). While a common force, the behavior of friction is actually very complicated and is still not completely understood. We have to rely heavily on observations for whatever understandings we can gain. However, we can still deal with its more elementary general characteristics and understand the circumstances in which it behaves.

Friction

Friction is a force that opposes relative motion between systems in contact.

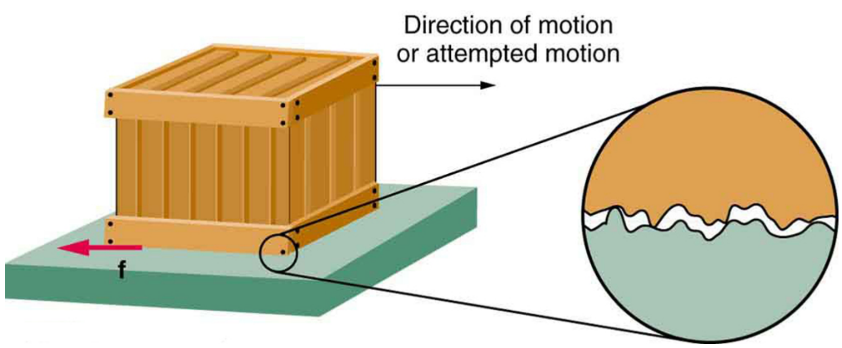
One of the simpler characteristics of friction is that it is parallel to the contact surface between systems and always in a direction that opposes motion or attempted motion of the systems relative to each other. If two systems are in contact and moving relative to one another, then the friction between them is called kinetic friction. For example, friction slows a hockey puck sliding on ice. But when objects are stationary**, static friction** can act between them; the static friction is usually greater than the kinetic friction between the objects.

Kinetic Friction

If two systems are in contact and moving relative to one another, then the friction between them is called kinetic friction.

Imagine, for example, trying to slide a heavy crate across a concrete floor—you may push harder and harder on the crate and not move it at all. This means that the static friction responds to what you do—it increases to be equal to and in the opposite direction of your push. But if you finally push hard enough, the crate seems to slip suddenly and starts to move. Once in motion it is easier to keep it in motion than it was to get it started, indicating that the kinetic friction force is less than the static friction force. If you add mass to the crate, say by placing a box on top of it, you need to push even harder to get it started and also to keep it moving. Furthermore, if you oiled the concrete you would find it to be easier to get the crate started and keep it going (as you might expect).

Figure is a crude pictorial representation of how friction occurs at the interface between two objects. Close-up inspection of these surfaces shows them to be rough. So when you push to get an object moving (in this case, a crate), you must raise the object until it can skip along with just the tips of the surface hitting, break off the points, or do both. A considerable force can be resisted by friction with no apparent motion. The harder the surfaces are pushed together (such as if another box is placed on the crate), the more force is nearly independent of speed.



4.7.1*: Frictional forces, such as f, always oppose motion or attempted motion between objects in contact. Friction arises in part because of the roughness of the surfaces in contact, as seen in the expanded view. In order for the object to move, it must rise to where the peaks can skip along the bottom surface. Thus a force is required just to set the object in motion. Some of the peaks will* *be broken off, also requiring a force to maintain motion. Much of the friction is actually due to attractive forces between molecules making up the two objects, so that even perfectly smooth surfaces are not friction-free. Such adhesive forces also depend on the substances the surfaces are made of, explaining, for example, why rubber-soled shoes slip less than those with leather soles.*

The magnitude of the frictional force has two forms: one for static situations (static friction), the other for when there is motion (kinetic friction).

When there is no motion between the objects, the **magnitude of static friction** f_s is

$$f_s \leq \mu_s N,$$

where μ_s is the coefficient of static friction and N is the magnitude of the normal force (the force perpendicular to the surface).

MAGNITUDE OF STATIC FRICTION

Magnitude of static friction f_s is

$$f_s \leq \mu_s N,$$

where μ_s is the coefficient of static friction and N is the magnitude of the normal force.

The symbol $\leq$ means *less than or equal to*, implying that static friction can have a minimum and a maximum value of μsN. Static friction is a responsive force that increases to be equal and opposite to whatever force is exerted, up to its maximum limit. Once the applied force exceeds fs(max), the object will move. Thus

$$f_{s(max)} = \mu_s N.$$

Once an object is moving, the **magnitude of kinetic friction** f_k is given by

$$f_k = \mu_k N,$$

where μ_k is the coefficient of kinetic friction. A system in which $f_k = \mu_k N$ is described as a system in which *friction behaves simply.*

MAGNITUDE OF KINETIC FRICTION

The magnitude of kinetic friction f_k is given by

$$f_k = \mu_k N,$$

where μ_k is the coefficient of kinetic friction.

As seen in Table, the coefficients of kinetic friction are less than their static counterparts. That values of μ in Table are stated to only one or, at most, two digits is an indication of the approximate description of friction given by the above two equations.

	Static Friction	**Kinetic Friction**
System	μ_s	μ_k
Rubber on dry concrete	1.0	0.7
Rubber on wet concrete	0.7	0.5
Wood on wood	0.5	0.3
Waxed wood on wet snow	0.14	0.1
Metal on wood	0.5	0.3
Steel on steel (dry)	0.6	0.3
Steel on steel (oiled)	0.05	0.063
Teflon on steel	0.04	0.04
Bone lubricated by synovial fluid	0.016	0.015

Shoes on wood	0.9	0.7
Shoes on ice	0.1	0.05
Ice on ice	0.1	0.03
Steel on ice	0.04	0.02

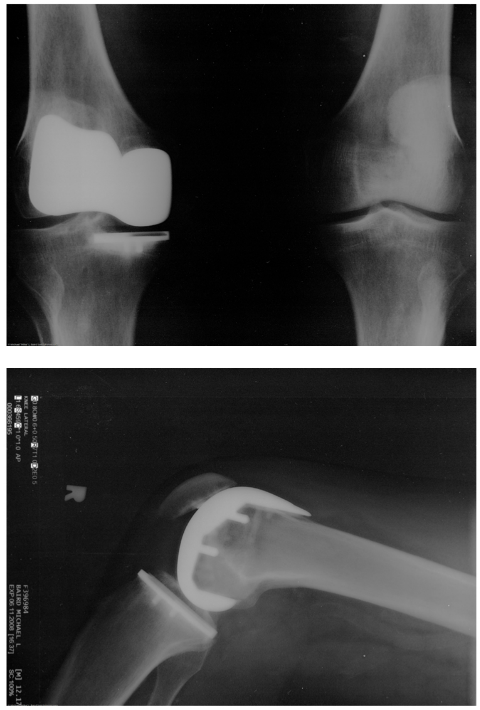
Coefficients of Static and Kinetic Friction

The equations given earlier include the dependence of friction on materials and the normal force. The direction of friction is always opposite that of motion, parallel to the surface between objects, and perpendicular to the normal force. For example, if the crate you try to push (with a force parallel to the floor) has a mass of 100 kg, then the normal force would be equal to its weight, $W = mg = (100kg)(9.80m/s^2) = 980N$, perpendicular to the floor. If the coefficient of static friction is 0.45, you would have to exert a force parallel to the floor greater than $f_{s(max)} = \mu_s N = (0.45)(980N) = 440N$ to move the crate. Once there is motion, friction is less and the coefficient of kinetic friction might be 0.30, so that a force of only 290 N $(f_k = \mu_k N = (0.30)(980N) = 290N)$ would keep it moving at a constant speed. If the floor is lubricated, both coefficients are considerably less than they would be without lubrication. Coefficient of friction is a unit less quantity with a magnitude usually between 0 and 1.0. The coefficient of the friction depends on the two surfaces that are in contact.

TAKE-HOME EXPERIMENT

Find a small plastic object (such as a food container) and slide it on a kitchen table by giving it a gentle tap. Now spray water on the table, simulating a light shower of rain. What happens now when you give the object the same-sized tap? Now add a few drops of (vegetable or olive) oil on the surface of the water and give the same tap. What happens now? This latter situation is particularly important for drivers to note, especially after a light rain shower. Why?

Many people have experienced the slipperiness of walking on ice. However, many parts of the body, especially the joints, have much smaller coefficients of friction—often three or four times less than ice. A joint is formed by the ends of two bones, which are connected by thick tissues. The knee joint is formed by the lower leg bone (the tibia) and the thighbone (the femur). The hip is a ball (at the end of the femur) and socket (part of the pelvis) joint. The ends of the bones in the joint are covered by cartilage, which provides a smooth, almost glassy surface. The joints also produce a fluid (synovial fluid) that reduces friction and wear. A damaged or arthritic joint can be replaced by an artificial joint (Figure). These replacements can be made of metals (stainless steel or titanium) or plastic (polyethylene), also with very small coefficients of friction.



4.7.2: *Artificial knee replacement is a procedure that has been performed for more than 20 years. In this figure, we see the post-op x rays of the right knee joint replacement. (credit: Mike Baird, Flickr)*

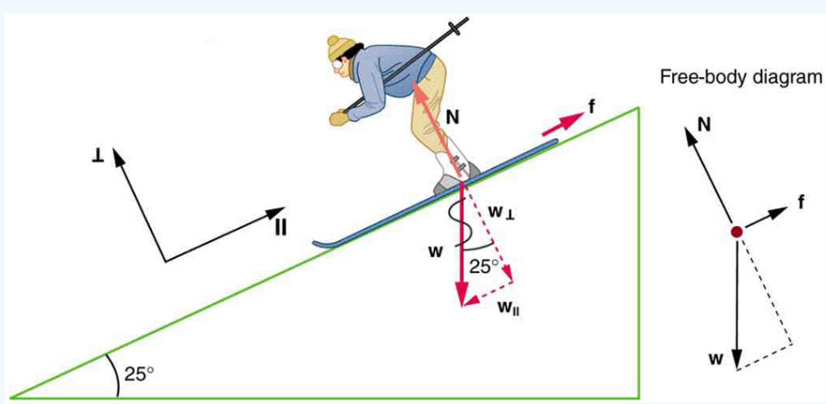
Other natural lubricants include saliva produced in our mouths to aid in the swallowing process, and the slippery mucus found between organs in the body, allowing them to move freely past each other during heartbeats, during breathing, and when a person moves. Artificial lubricants are also common in hospitals and doctor's clinics. For example, when ultrasonic imaging is carried out, the gel that couples the transducer to the skin also serves to lubricate the surface between the transducer and the skin—thereby reducing the coefficient of friction between the two surfaces. This allows the transducer to move freely over the skin.

Exercise 4.7.1: Skiing Exercise

A skier with a mass of 62 kg is sliding down a snowy slope. Find the coefficient of kinetic friction for the skier if friction is known to be 45.0 N.

Strategy

The magnitude of kinetic friction was given in to be 45.0 N. Kinetic friction is related to the normal force N as $f_k = \mu_k N$; thus, the coefficient of kinetic friction can be found if we can find the normal force of the skier on a slope. The normal force is always perpendicular to the surface, and since there is no motion perpendicular to the surface, the normal force should equal the component of the skier's weight perpendicular to the slope. (See the skier and free-body diagram in Figure.)



4.7.3: *The motion of the skier and friction are parallel to the slope and so it is most convenient to project all forces onto a coordinate system where one axis is parallel to the slope and the other is perpendicular (axes shown to left of skier). N (the normal force) is perpendicular to the slope, and f (the friction) is parallel to the slope, but w (the skier's weight) has components along both axes, namely $w_{\perp}$ and $W_{//}$. N is equal in magnitude to $w_{\perp}$, so there is no motion perpendicular to the slope. However, f is less than $W_{//}$ in magnitude, so there is acceleration down the slope (along the x-axis).*

That is,

$$N = w_{\perp} = wcos25^{o} = mgcos25^{o}.$$

Substituting this into our expression for kinetic friction, we get

$$f_k = \mu_k mgcos25^{o},$$

which can now be solved for the coefficient of kinetic friction μ_k.

Solution

Solving for μ_k gives

$$\mu_k = \frac{f_k}{N} = \frac{f_k}{wcos25^{o}} = \frac{f_k}{mgcos25^{o}}$$

Substituting known values on the right-hand side of the equation,

$$\mu_k = \frac{45.0N}{(62kg)(9.80m/s^2)(0.906)} = 0.082.$$

Discussion

This result is a little smaller than the coefficient listed in Table for waxed wood on snow, but it is still reasonable since values of the coefficients of friction can vary greatly. In situations like this, where an object of mass m slides down a slope that makes an angle θ with the horizontal, friction is given by $f_k = \mu_k mgcos\theta$. All objects will slide down a slope with constant acceleration under these circumstances. Proof of this is left for this chapter's Problems and Exercises.

TAKE-HOME EXPERIMENT

An object will slide down an inclined plane at a constant velocity if the net force on the object is zero. We can use this fact to measure the coefficient of kinetic friction between two objects. As shown in Example, the kinetic friction on a slope $f_k = \mu_k mgcos\theta$. The component of the weight down the slope is equal to $mgsin\theta$ (see the free-body diagram in Figure). These forces act in opposite directions, so when they have equal magnitude, the acceleration is zero. Writing these out:

$$f_k = Fg_x$$

$$\mu_k mgcos\theta = mgsin\theta \ .$$

Solving for μ_k, we find that

$$\mu_k = \frac{mgsin\theta}{mgcos\theta} = tan\theta \ .$$

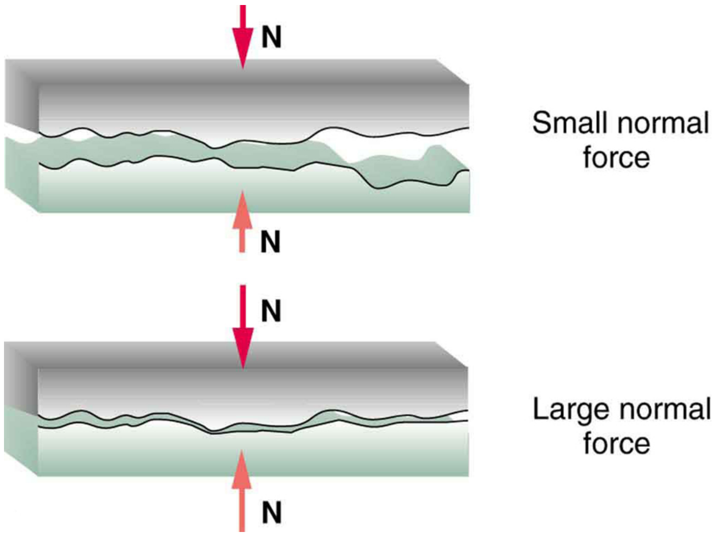
Put a coin on a book and tilt it until the coin slides at a constant velocity down the book. You might need to tap the book lightly to get the coin to move. Measure the angle of tilt relative to the horizontal and find μ_k. Note that the coin will not start to slide at all until an angle greater than θ is attained, since the coefficient of static friction is larger than the coefficient of kinetic friction. Discuss how this may affect the value for μ_k and its uncertainty.

We have discussed that when an object rests on a horizontal surface, there is a normal force supporting it equal in magnitude to its weight. Furthermore, simple friction is always proportional to the normal force.

MAKING CONNECTIONS: SUBMICROSCOPIC EXPLANATIONS OF FRICTION

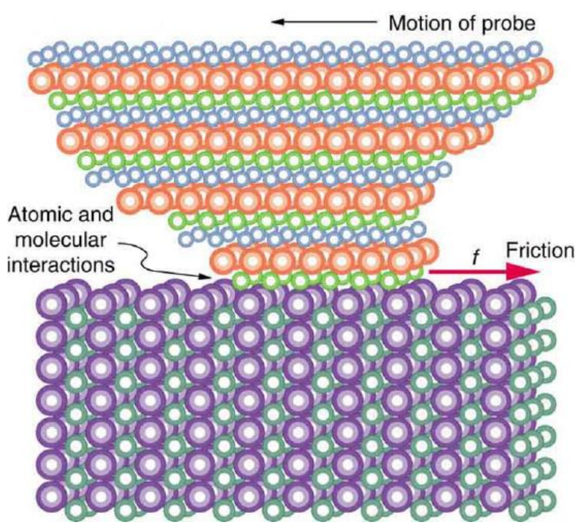
The simpler aspects of friction dealt with so far are its macroscopic (large-scale) characteristics. Great strides have been made in the atomic-scale explanation of friction during the past several decades. Researchers are finding that the atomic nature of friction seems to have several fundamental characteristics. These characteristics not only explain some of the simpler aspects of friction—they also hold the potential for the development of nearly friction-free environments that could save hundreds of billions of dollars in energy which is currently being converted (unnecessarily) to heat.

Figure illustrates one macroscopic characteristic of friction that is explained by microscopic (small-scale) research. We have noted that friction is proportional to the normal force, but not to the area in contact, a somewhat counterintuitive notion. When two rough surfaces are in contact, the actual contact area is a tiny fraction of the total area since only high spots touch. When a greater normal force is exerted, the actual contact area increases, and it is found that the friction is proportional to this area.



4.7.4*: Two rough surfaces in contact have a much smaller area of actual contact than their total area. When there is a greater normal force as a result of a greater applied force, the area of actual contact increases as does friction.*

But the atomic-scale view promises to explain far more than the simpler features of friction. The mechanism for how heat is generated is now being determined. In other words, why do surfaces get warmer when rubbed? Essentially, atoms are linked with one another to form lattices. When surfaces rub, the surface atoms adhere and cause atomic lattices to vibrate—essentially creating sound waves that penetrate the material. The sound waves diminish with distance and their energy is converted into heat. Chemical reactions that are related to frictional wear can also occur between atoms and molecules on the surfaces. Figure shows how the tip of a probe drawn across another material is deformed by atomic-scale friction. The force needed to drag the tip can be measured and is found to be related to shear stress, which will be discussed later in this chapter. The variation in shear stress is remarkable (more than a factor of 10^{12}) and difficult to predict theoretically, but shear stress is yielding a fundamental understanding of a large-scale phenomenon known since ancient times—friction.



4.7.5*:* The tip of a probe is deformed sideways by frictional force as the probe is dragged across a surface. Measurements of how the force varies for different materials are yielding fundamental insights into the atomic nature of friction.

PHET EXPLORATIONS: FORCES AND MOTION

Explore the forces at work when you try to push a filing cabinet. Create an applied force and see the resulting friction force and total force acting on the cabinet. Charts show the forces, position, velocity, and acceleration vs. time. Draw a free-body diagram of all the forces (including gravitational and normal forces).

PhET Interactive Simulation

4.7.6: *Forces and Motion*

Summary

- Friction is a contact force between systems that opposes the motion or attempted motion between them. Simple friction is proportional to the normal force N pushing the systems together. (A normal force is always perpendicular to the contact surface between systems.) Friction depends on both of the materials involved. The magnitude of static friction f_s between systems stationary relative to one another is given by

$$f_s \leq \mu_s N,$$

where μ_s is the coefficient of static friction, which depends on both of the materials.

- The kinetic friction force f_k between systems moving relative to one another is given by

$$f_k = \mu_k N,$$

where μ_k is the coefficient of kinetic friction, which also depends on both materials.

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4.8: Drag Forces

Learning Objectives

By the end of this section, you will be able to:

- Express mathematically the drag force.
- Discuss the applications of drag force.
- Define terminal velocity.
- Determine the terminal velocity given mass.

Another interesting force in everyday life is the force of drag on an object when it is moving in a fluid (either a gas or a liquid). You feel the drag force when you move your hand through water. You might also feel it if you move your hand during a strong wind. The faster you move your hand, the harder it is to move. You feel a smaller drag force when you tilt your hand so only the side goes through the air—you have decreased the area of your hand that faces the direction of motion. Like friction, the drag force always opposes the motion of an object. Unlike simple friction, the drag force is proportional to some function of the velocity of the object in that fluid. This functionality is complicated and depends upon the shape of the object, its size, its velocity, and the fluid it is in. For most large objects such as bicyclists, cars, and baseballs not moving too slowly, the magnitude of the drag force F_D is found to be proportional to the square of the speed of the object. We can write this relationship mathematically as $F_D \propto v^2$. When taking into account other factors, this relationship becomes

$$F_D = \frac{1}{2} C_\rho A v^2, \tag{4.8.1}$$

where C is the drag coefficient, A is the area of the object facing the fluid, and ρ is the density of the fluid. (Recall that density is mass per unit volume.) This equation can also be written in a more generalized fashion as $F_D = bv^2$, where b is a constant equivalent to $0.5C\rho A$. We have set the exponent for these equations as 2 because, when an object is moving at high velocity through air, the magnitude of the drag force is proportional to the square of the speed. As we shall see in a few pages on fluid dynamics, for small particles moving at low speeds in a fluid, the exponent is equal to 1.

Definition: DRAG FORCE

Drag Force F_D is found to be proportional to the square of the speed of the object. Mathematically

$$F_D \propto v^2 \tag{4.8.2}$$

$$F_D = \frac{1}{2} C_\rho A v^2, \tag{4.8.3}$$

where C is the drag coefficient A is the area of the object facing the fluid, and ρ is the density of the fluid.

Athletes as well as car designers seek to reduce the drag force to lower their race times. (See Figure). “Aerodynamic” shaping of an automobile can reduce the drag force and so increase a car’s gas mileage.

Figure 4.8.1: From racing cars to bobsled racers, aerodynamic shaping is crucial to achieving top speeds. Bobsleds are designed for speed. They are shaped like a bullet with tapered fins. (credit: U.S. Army, via Wikimedia Commons)

The value of the drag coefficient, C is determined empirically, usually with the use of a wind tunnel. (See Figure).

Figure 4.8.2: NASA researchers test a model plane in a wind tunnel. (credit: NASA/Ames)

The drag coefficient can depend upon velocity, but we will assume that it is a constant here. Table lists some typical drag coefficients for a variety of objects. Notice that the drag coefficient is a dimensionless quantity. At highway speeds, over 50% of the power of a car is used to overcome air drag. The most fuel-efficient cruising speed is about 70–80 km/h (about 45–50 mi/h). For this reason, during the 1970s oil crisis in the United States, maximum speeds on highways were set at about 90 km/h (55 mi/h).

Object	C
Airfoil	0.05
Toyota Camry	0.28
Ford Focus	0.32
Honda Civic	0.36
Ferrari Testarossa	0.37
Dodge Ram pickup	0.43
Sphere	0.45
Hummer H2 SUV	0.64
Skydiver (feet first)	0.70
Bicycle	0.90
Skydiver (horizontal)	1.0
Circular flat plate	1.12

Substantial research is under way in the sporting world to minimize drag. The dimples on golf balls are being redesigned as are the clothes that athletes wear. Bicycle racers and some swimmers and runners wear full bodysuits. Australian Cathy Freeman wore a full body suit in the 2000 Sydney Olympics, and won the gold medal for the 400 m race. Many swimmers in the 2008 Beijing Olympics wore (Speedo) body suits; it might have made a difference in breaking many world records (See Figure). Most elite swimmers (and cyclists) shave their body hair. Such innovations can have the effect of slicing away milliseconds in a race, sometimes making the difference between a gold and a silver medal. One consequence is that careful and precise guidelines must be continuously developed to maintain the integrity of the sport.

Figure 4.8.3: Body suits, such as this LZR Racer Suit, have been credited with many world records after their release in 2008. Smoother “skin” and more compression forces on a swimmer’s body provide at least 10% less drag. (credit: NASA/Kathy Barnstorff)

Some interesting situations connected to Newton’s second law occur when considering the effects of drag forces upon a moving object. For instance, consider a skydiver falling through air under the influence of gravity. The two forces acting on him are the force of gravity and the drag force (ignoring the buoyant force). The downward force of gravity remains constant regardless of the velocity at which the person is moving. However, as the person’s velocity increases, the magnitude of the drag force increases until the magnitude of the drag force is equal to the gravitational force, thus producing a net force of zero. A zero net force means that there is no acceleration, as given by Newton’s second law. At this point, the person’s velocity remains constant and we say that the person has reached his *terminal velocity* (v_t). Since F_D is proportional to the speed, a heavier skydiver must go faster for F_D to equal his weight. Let’s see how this works out more quantitatively.

At the terminal velocity,

$$F_{net} = mg - F_D = ma = 0 \tag{4.8.4}$$

Thus,

$$mg = F_D. \tag{4.8.5}$$

Using the equation for drag force, we have

$$mg = \frac{1}{2}\rho C A v^2. \tag{4.8.6}$$

Solving for the velocity, we obtain

$$v = \sqrt{\frac{2mg}{\rho C A}} \tag{4.8.7}$$

Assume the density of air is $\rho = 1.21\,kg/m^3$. A 75-kg skydiver descending head first will have an area approximately $A = 0.18\,m^2$ and a drag coefficient of approximately $C = 0.70$. We find that

$$v = \sqrt{\frac{2(75\,kg)(9.80\,m/s^2)}{(1.21\,kg/m^3)(0.70)(0.18\,m^2)}} \tag{4.8.8}$$

$$= 98\,m/s \tag{4.8.9}$$

$$= 350\,km/h. \tag{4.8.10}$$

This means a skydiver with a mass of 75 kg achieves a maximum terminal velocity of about 350 km/h while traveling in a pike (head first) position, minimizing the area and his drag. In a spread-eagle position, that terminal velocity may decrease to about 200 km/h as the area increases. This terminal velocity becomes much smaller after the parachute opens.

TAKE-HOME EXPERIMENT

This interesting activity examines the effect of weight upon terminal velocity. Gather together some nested coffee filters. Leaving them in their original shape, measure the time it takes for one, two, three, four, and five nested filters to fall to the floor from the same height (roughly 2 m). (Note that, due to the way the filters are nested, drag is constant and only mass varies.) They obtain terminal velocity quite quickly, so find this velocity as a function of mass. Plot the terminal velocity v versus mass. Also plot v^2 versus mass. Which of these relationships is more linear? What can you conclude from these graphs?

Example $4.8.1$: The terminal Velocity

Find the terminal velocity of an 85-kg skydiver falling in a spread-eagle position.

Strategy

At terminal velocity, $F_{net} = 0$ Thus the drag force on the skydiver must equal the force of gravity (the person's weight). Using the equation of drag force, we find $mg = \frac{1}{2}\rho CAv^2$.

Thus the terminal velocity v_t can be written as

$$v_t = \sqrt{\frac{2mg}{\rho CA}} \tag{4.8.11}$$

Solution

All quantities are known except the person's projected area. This is an adult (85 kg) falling spread eagle. We can estimate the frontal area as

$$A = (2\,m)(0.35\,m) = 0.70\,m^2 \tag{4.8.12}$$

Using our equation for v_t, we find that

$$v_t = \sqrt{\frac{2(85\,kg)(9.80\,m/s^2)}{91.21\,kg/m^3)(1.0)(0.70\,m^2)}} \tag{4.8.13}$$

$$= 44\,m/s \tag{4.8.14}$$

Discussion

This result is consistent with the value for v_t mentioned earlier. The 75-kg skydiver going feet first had a $v = 98\,m/s$. He weighed less but had a smaller frontal area and so a smaller drag due to the air.

The size of the object that is falling through air presents another interesting application of air drag. If you fall from a 5-m high branch of a tree, you will likely get hurt—possibly fracturing a bone. However, a small squirrel does this all the time, without getting hurt. You don't reach a terminal velocity in such a short distance, but the squirrel does.

The following interesting quote on animal size and terminal velocity is from a 1928 essay by a British biologist, J.B.S. Haldane, titled "On Being the Right Size."

To the mouse and any smaller animal, [gravity] presents practically no dangers. You can drop a mouse down a thousand-yard mine shaft; and, on arriving at the bottom, it gets a slight shock and walks away, provided that the ground is fairly soft. A rat is killed, a man is broken, and a horse splashes. For the resistance presented to movement by the air is proportional to the surface of the moving object. Divide an animal's length, breadth, and height each by ten; its weight is reduced to a thousandth, but its surface only to a hundredth. So the resistance to falling in the case of the small animal is relatively ten times greater than the driving force.

The above quadratic dependence of air drag upon velocity does not hold if the object is very small, is going very slow, or is in a denser medium than air. Then we find that the drag force is proportional just to the velocity. This relationship is given by Stokes' law, which states that

$$F_S = 6\pi r\eta v \tag{4.8.15}$$

where r is the radius of the object, η is the viscosity of the fluid and v is the object's velocity.

Definition: STOKES LAW

$$F_S = 6\pi r\eta v \tag{4.8.16}$$

where r is the radius of the object, η is the viscosity of the fluid and v is the object's velocity.

Good examples of this law are provided by microorganisms, pollen, and dust particles. Because each of these objects is so small, we find that many of these objects travel unaided only at a constant (terminal) velocity. Terminal velocities for bacteria (size about $1\ \mu m$) can be about $2\ m/s$. To move at a greater speed, many bacteria swim using flagella (organelles shaped like little tails) that are powered by little motors embedded in the cell. Sediment in a lake can move at a greater terminal velocity (about $5\ \mu m/s$,) so it can take days to reach the bottom of the lake after being deposited on the surface.

If we compare animals living on land with those in water, you can see how drag has influenced evolution. Fishes, dolphins, and even massive whales are streamlined in shape to reduce drag forces. Birds are streamlined and migratory species that fly large distances often have particular features such as long necks. Flocks of birds fly in the shape of a spear head as the flock forms a streamlined pattern (see Figure). In humans, one important example of streamlining is the shape of sperm, which need to be efficient in their use of energy.

Figure 4.8.4:Geese fly in a V formation during their long migratory travels. This shape reduces drag and energy consumption for individual birds, and also allows them a better way to communicate. (credit: Julo, Wikimedia Commons)

GALILEO'S EXPERIMENT

Galileo is said to have dropped two objects of different masses from the Tower of Pisa. He measured how long it took each to reach the ground. Since stopwatches weren't readily available, how do you think he measured their fall time? If the objects were the same size, but with different masses, what do you think he should have observed? Would this result be different if done on the Moon?

Summary

- Drag forces acting on an object moving in a fluid oppose the motion. For larger objects (such as a baseball) moving at a velocity v in air, the drag force is given by

$$F_D = \frac{1}{2} C \rho A v^2, \tag{4.8.17}$$

where C is the drag coefficient (typical values are given in Table), A is the area of the object facing the fluid, and ρ is the fluid density.
- For small objects (such as a bacterium) moving in a denser medium (such as water), the drag force is given by Stokes' law,

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4.9: Problem-Solving Strategies

Learning Objectives

By the end of this section, you will be able to:

- Understand and apply a problem-solving procedure to solve problems using Newton's laws of motion.

Success in problem solving is obviously necessary to understand and apply physical principles, not to mention the more immediate need of passing exams. The basics of problem solving, presented earlier in this text, are followed here, but specific strategies useful in applying Newton's laws of motion are emphasized. These techniques also reinforce concepts that are useful in many other areas of physics. Many problem-solving strategies are stated outright in the worked examples, and so the following techniques should reinforce skills you have already begun to develop.

Problem-Solving Strategy for Newton's Laws of Motion

Step 1. As usual, it is first necessary to identify the physical principles involved. *Once it is determined that Newton's laws of motion are involved (if the problem involves forces), it is particularly important to draw a careful sketch of the situation*. Such a sketch is shown in Figure(a). Then, as in Figure(b), use arrows to represent all forces, label them carefully, and make their lengths and directions correspond to the forces they represent (whenever sufficient information exists).

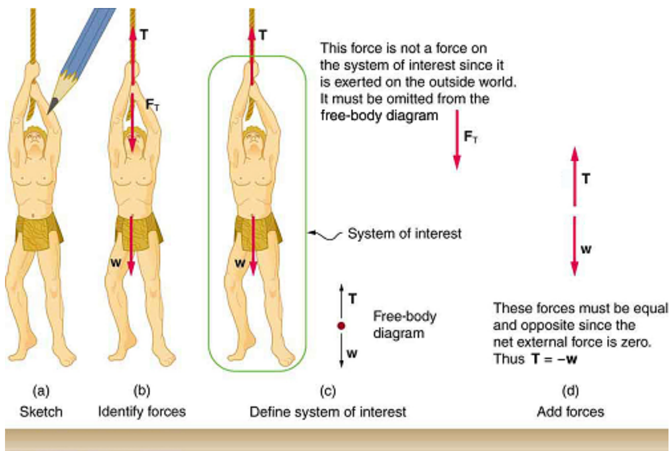

Figure 4.9.1: (a) A sketch of Tarzan hanging from a vine. (b) Arrows are used to represent all forces. T is the tension in the vine above Tarzan F_T is the force he exerts on the vine, and w is his weight. All other forces, such as the nudge of a breeze, are assumed negligible. (c) Suppose we are given the ape man's mass and asked to find the tension in the vine. We then define the system of interest as shown and draw a free-body diagram. F_T is no longer shown, because it is not a force acting on the system of interest; rather, F_T acts on the outside world. (d) Showing only the arrows, the head-to-tail method of addition is used. It is apparent that $T = -w$ if Tarzan is stationary.

Step 2. Identify what needs to be determined and what is known or can be inferred from the problem as stated. That is, make a list of knowns and unknowns. *Then carefully determine the system of interest*. This decision is a crucial step, since Newton's second law involves only external forces. Once the system of interest has been identified, it becomes possible to determine which forces are external and which are internal, a necessary step to employ Newton's second law. (See Figure(c).) Newton's third law may be used to identify whether forces are exerted between components of a system (internal) or between the system and something outside (external). As illustrated earlier in this chapter, the system of interest depends on what question we need to answer. This choice becomes easier with practice, eventually developing into an almost unconscious process. Skill in clearly defining systems will be beneficial in later chapters as well.

A diagram showing the system of interest and all of the external forces is called a free-body diagram. Only forces are shown on free-body diagrams, not acceleration or velocity. We have drawn several of these in worked examples. Figure(c) shows a free-body diagram for the system of interest. Note that no internal forces are shown in a free-body diagram.

Step 3. Once a free-body diagram is drawn, *Newton's second law can be applied to solve the problem*. This is done in Figure(d) for a particular situation. In general, once external forces are clearly identified in free-body diagrams, it should be a straightforward task to put them into equation form and solve for the unknown, as done in all previous examples. If the problem is one-dimensional

—that is, if all forces are parallel—then they add like scalars. If the problem is two-dimensional, then it must be broken down into a pair of one-dimensional problems. This is done by projecting the force vectors onto a set of axes chosen for convenience. As seen in previous examples, the choice of axes can simplify the problem. For example, when an incline is involved, a set of axes with one axis parallel to the incline and one perpendicular to it is most convenient. It is almost always convenient to make one axis parallel to the direction of motion, if this is known.

APPLYING NEWTON'S SECOND LAW

Before you write net force equations, it is critical to determine whether the system is accelerating in a particular direction. If the acceleration is zero in a particular direction, then the net force is zero in that direction. Similarly, if the acceleration is nonzero in a particular direction, then the net force is described by the equation: $F_{net} = ma$

For example, if the system is accelerating in the horizontal direction, but it is not accelerating in the vertical direction, then you will have the following conclusions:

$$F_{net\,x} = ma \tag{4.9.1}$$

$$F_{net\,y} = 0 \tag{4.9.2}$$

You will need this information in order to determine unknown forces acting in a system.

Step 4. As always, *check the solution to see whether it is reasonable*. In some cases, this is obvious. For example, it is reasonable to find that friction causes an object to slide down an incline more slowly than when no friction exists. In practice, intuition develops gradually through problem solving, and with experience it becomes progressively easier to judge whether an answer is reasonable. Another way to check your solution is to check the units. If you are solving for force and end up with units of m/s, then you have made a mistake.

Summary

- To solve problems involving Newton's laws of motion, follow the procedure described:
  1. Draw a sketch of the problem.
  2. Identify known and unknown quantities, and identify the system of interest. Draw a free-body diagram, which is a sketch showing all of the forces acting on an object. The object is represented by a dot, and the forces are represented by vectors extending in different directions from the dot. If vectors act in directions that are not horizontal or vertical, resolve the vectors into horizontal and vertical components and draw them on the free-body diagram.
  3. Write Newton's second law in the horizontal and vertical directions and add the forces acting on the object. If the object does not accelerate in a particular direction (for example, the x-direction) then $F_{net\,x} = 0$. If the object does accelerate in that direction, $F_{net\,x} = ma$.
  4. Check your answer. Is the answer reasonable? Are the units correct?

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4.10: Further Applications of Newton's Laws of Motion

Learning Objectives

By the end of this section, you will be able to:

- Apply problem-solving techniques to solve for quantities in more complex systems of forces.
- Integrate concepts from kinematics to solve problems using Newton's laws of motion.

There are many interesting applications of Newton's laws of motion, a few more of which are presented in this section. These serve also to illustrate some further subtleties of physics and to help build problem-solving skills.

Example 4.10.1: Drag Force on a Barge

Suppose two tugboats push on a barge at different angles, as shown in Figure. The first tugboat exerts a force of $2.7 \times 10^5\ N$ in the x-direction, and the second tugboat exerts a force of $3.6 \times 10^5\ N$ in the y-direction.

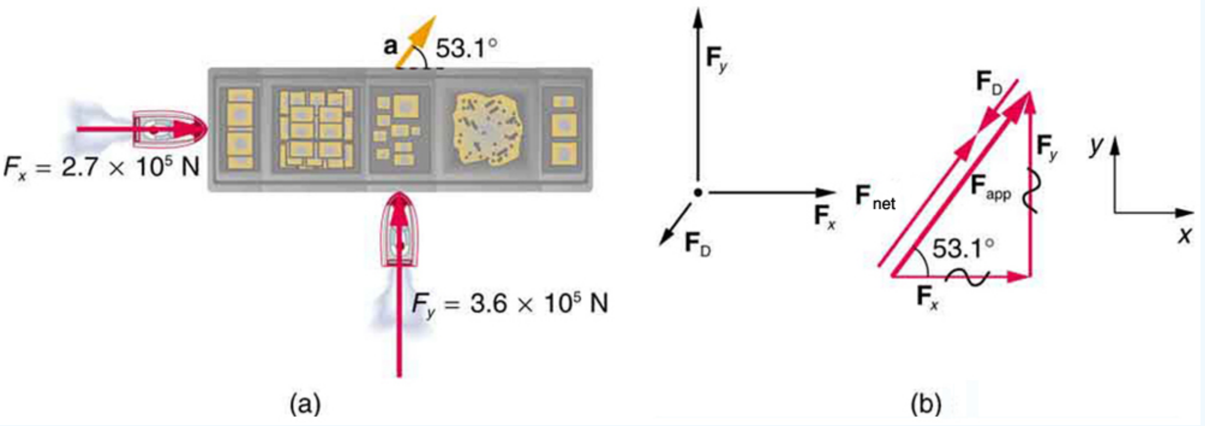

Figure 4.10.1: (a) A view from above of two tugboats pushing on a barge. (b) The free-body diagram for the ship contains only forces acting in the plane of the water. It omits the two vertical forces—the weight of the barge and the buoyant force of the water supporting it cancel and are not shown. Since the applied forces are perpendicular, the x- and y-axes are in the same direction as F_x and F_y The problem quickly becomes a one-dimensional problem along the direction of F_{app}, since friction is in the direction opposite to F_{app}.

If the mass of the barge is $5.0 \times 10^6\ kg$ and its acceleration is observed to be $7.5 \times 10^{-2}\ m/s^2$ in the direction shown, what is the drag force of the water on the barge resisting the motion? (Note: drag force is a frictional force exerted by fluids, such as air or water. The drag force opposes the motion of the object.)

Strategy

The directions and magnitudes of acceleration and the applied forces are given in Figure (a). We will define the total force of the tugboats on the barge as F_{app} so that:

$$F_{app} = F_x + F_y$$

Since the barge is flat bottomed, the drag of the water F_D will be in the direction opposite to F_{app} as shown in the free-body diagram in Figure (b). The system of interest here is the barge, since the forces on it are given as well as its acceleration. Our strategy is to find the magnitude and direction of the net applied force F_{app}, and then apply Newton's second law to solve for the drag force F_D.

Solution

Since F_x and F_y are perpendicular, the magnitude and direction of F_{app} are easily found. First, the resultant magnitude is given by the Pythagorean theorem:

$$F_{app} = \sqrt{F_x^2 + F_y^2}$$

$$F_{app} = \sqrt{(2.7 \times 10^5\ N)^2 + (3.6 \times 10^5\ N)^2} = 4.5 \times 10^5\ N.$$

The angle is given by

$$\theta = tan^{-1}\left(\frac{F_y}{F_x}\right)$$

$$\theta = tan^{-1}\left(\frac{3.6\times 10^5\ N}{2.7\times 10^5\ N}\right) = 53^o,$$

which we know, because of Newton's first law, is the same direction as the acceleration. F_D is in the opposite direction of F_{app}, since it acts to slow down the acceleration. Therefore, the net external force is in the same direction as F_{app}, but its magnitude is slightly less than F_{app}. The problem is now one-dimensional. From Figure (b) we can see that

$$F_{net} = F_{app} - F_D.$$

But Newton's second law states that

$$F_{net} = ma$$

Thus,

$$F_{app} - F_D = ma$$

This can be solved for the magnitude of the drag force of the water F_D in terms of known quantities:

$$F_D = F_{app} - ma$$

Substituting known values gives

$$F_D = (4.5\times 10^5\ N) - (5.0\times 10^6\ kg)(7.5\times 10^{-2}\ m/s^2) = 7.5\times 10^4\ N$$

The direction of F_D has already been determined to be in the direction opposite to F_{app} or at an angle of 53^o south of west.

Discussion

The numbers used in this example are reasonable for a moderately large barge. It is certainly difficult to obtain larger accelerations with tugboats, and small speeds are desirable to avoid running the barge into the docks. Drag is relatively small for a well-designed hull at low speeds, consistent with the answer to this example, where F_D is less than 1/600th of the weight of the ship.

In the earlier example of a tightrope walker we noted that the tensions in wires supporting a mass were equal only because the angles on either side were equal. Consider the following example, where the angles are not equal; slightly more trigonometry is involved.

Example 4.10.2: Different Tensions at Different Angles

Consider the traffic light (mass 15.0 kg) suspended from two wires as shown in Figure. Find the tension in each wire, neglecting the masses of the wires.

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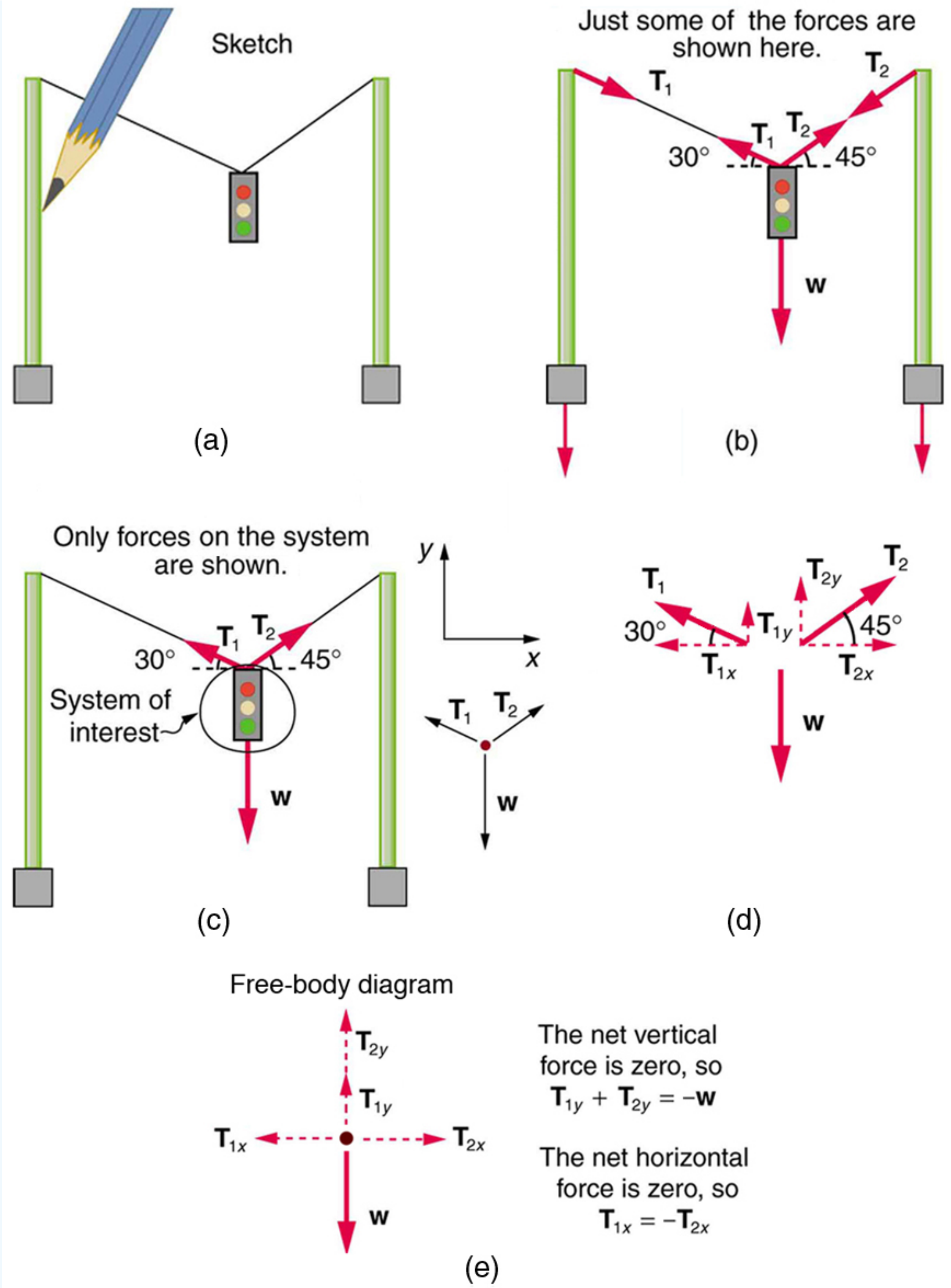

Figure 4.10.2: A traffic light is suspended from two wires. (b) Some of the forces involved. (c) Only forces acting on the system are shown here. The free-body diagram for the traffic light is also shown. (d) The forces projected onto vertical (y) and horizontal (x) axes. The horizontal components of the tensions must cancel, and the sum of the vertical components of the tensions must equal the weight of the traffic light. (e) The free-body diagram shows the vertical and horizontal forces acting on the traffic light.

Strategy

The system of interest is the traffic light, and its free-body diagram is shown in Figure(c). The three forces involved are not parallel, and so they must be projected onto a coordinate system. The most convenient coordinate system has one axis vertical and one horizontal, and the vector projections on it are shown in part (d) of the figure. There are two unknowns in this problem (T_1 and T_2\)), so two equations are needed to find them. These two equations come from applying Newton's second law along the vertical and horizontal axes, noting that the net external force is zero along each axis because acceleration is zero.

Solution

First consider the horizontal or x-axis:

$$F_{net\,x} = T_{2x} - T_{1x} = 0.$$

Thus, as you might expect,

$$T_{1x} = T2x$$

This gives us the following relationship between T_1 and T_2:

$$T_1\, cos\, 30^o = T_2\, cos\, 45^o$$

Thus,

$$T_2 = (1.225)T_1.$$

Note that T_1 and T_2 are not equal in this case, because the angles on either side are not equal. It is reasonable that T_2 ends up being greater than T_1, because it is exerted more vertically than T_1.

Now consider the force components along the vertical or y-axis:

$$F_{net\,y} = T_{1y} + T_{2y} - w = 0$$

This implies

$$T_{1y} + T_{2y} = w$$

Substituting the expressions for the vertical components gives

$$T_1\, sin\,(30^o) + T_2\, sin\,(45^o) = w.$$

There are two unknowns in this equation, but substituting the expression for T_2 in terms of T_1 reduces this to one equation with one unknown:

$$T_1(0.500) + (1.225T_1)(0.707) = w = mg$$

which yields

$$(1.366)T_1 = (15.0\, kg)(9.80\, m/s^2).$$

Solving this last equation gives the magnitude of T_1 to be

$$T_1 = 108\, N.$$

Finally, the magnitude of T_2 is determined using the relationship between them, $T_2 = 1.225T_1$ found above. Thus we obtain

$$T_2 = 132\, N.$$

Discussion

Both tensions would be larger if both wires were more horizontal, and they will be equal if and only if the angles on either side are the same (as they were in the earlier example of a tightrope walker).

The bathroom scale is an excellent example of a normal force acting on a body. It provides a quantitative reading of how much it must push upward to support the weight of an object. But can you predict what you would see on the dial of a bathroom scale if you stood on it during an elevator ride? Will you see a value greater than your weight when the elevator starts up? What about when the elevator moves upward at a constant speed: will the scale still read more than your weight at rest? Consider the following example.

Example 4.10.3: What does the Bathroom Scale Read in an Elevator?

Figure shows a 75.0-kg man (weight of about 165 lb) standing on a bathroom scale in an elevator. Calculate the scale reading: (a) if the elevator accelerates upward at a rate of $1.20 m/s^2$ and (b) if the elevator moves upward at a constant speed of 1 m/s.

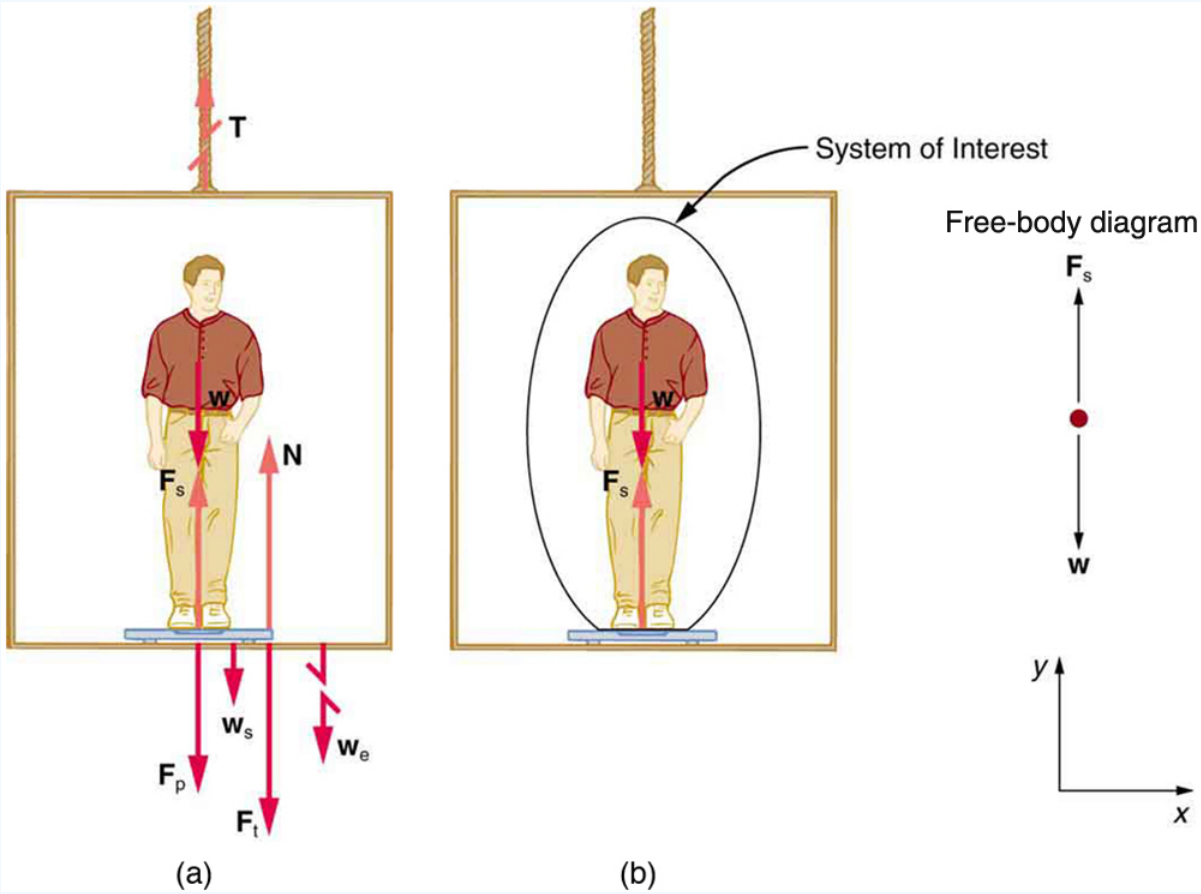

Figure 4.10.3:(a) The various forces acting when a person stands on a bathroom scale in an elevator. The arrows are approximately correct for when the elevator is accelerating upward—broken arrows represent forces too large to be drawn to scale. T is the tension in the supporting cable, w is the weight of the person, w_s is the weight of the scale, w_e is the weight of the elevator, F_s is the force of the scale on the person, F_p is the force of the person on the scale, F_t is the force of the scale on the floor of the elevator, and N is the force of the floor upward on the scale. (b) The free-body diagram shows only the external forces acting on the designated system of interest—the person.

Strategy

If the scale is accurate, its reading will equal F_p the magnitude of the force the person exerts downward on it. Figure (a) shows the numerous forces acting on the elevator, scale, and person. It makes this one-dimensional problem look much more formidable than if the person is chosen to be the system of interest and a free-body diagram is drawn as in Figure (b). Analysis of the free-body diagram using Newton's laws can produce answers to both parts (a) and (b) of this example, as well as some other questions that might arise. The only forces acting on the person are his weight w and the upward force of the scale F_s. According to Newton's third law F_p and F_s are equal in magnitude and opposite in direction, so that we need to find F_s in order to find what the scale reads. We can do this, as usual, by applying Newton's second law,

$$F_{net} = ma$$

From the free-body diagram we see that $F_{net} = F_s - w,$ so that

$$F_s - w = ma.$$

Solving for F_s gives an equation with only one unknown:

$$F_s = ma + w,$$

or, because $w = mg$, simply

$$F_s = ma + mg.$$

No assumptions were made about the acceleration, and so this solution should be valid for a variety of accelerations in addition to the ones in this exercise.

Solution for (a)

In this part of the problem, $a = 1.20 m/s^2$, so that

$$F_s = (75.0\,kg)(1.20\,m/s^2) + (75.0\,kg)(9.80\,m/s^2),$$

yielding

$$F_s = 825\,N.$$

Discussion for (a)

This is about 185 lb. What would the scale have read if he were stationary? Since his acceleration would be zero, the force of the scale would be equal to his weight:

$$F_{net} = ma = 0 = F_s - w$$

$$F_s = w = mg$$

$$F_s = (75.0\,kg)(9.80\,m/s^2)$$

$$F_s = 735\,N.$$

So, the scale reading in the elevator is greater than his 735-N (165 lb) weight. This means that the scale is pushing up on the person with a force greater than his weight, as it must in order to accelerate him upward. Clearly, the greater the acceleration of the elevator, the greater the scale reading, consistent with what you feel in rapidly accelerating versus slowly accelerating elevators.

Solution for (b)

Now, what happens when the elevator reaches a constant upward velocity? Will the scale still read more than his weight? For any constant velocity—up, down, or stationary—acceleration is zero because $a = \frac{\Delta v}{\Delta t}$ and $\Delta v = 0$.

Thus,

$$F_s = ma + mg = 0 + mg.$$

Now

$$F_s = (75.0\,kg)(9.80\,m/s^2),$$

which gives

$$F_s = 735\,N.$$

Discussion for (b)

The scale reading is 735 N, which equals the person's weight. This will be the case whenever the elevator has a constant velocity—moving up, moving down, or stationary.

The solution to the previous example also applies to an elevator accelerating downward, as mentioned. When an elevator accelerates downward, a is negative, and the scale reading is *less* than the weight of the person, until a constant downward velocity is reached, at which time the scale reading again becomes equal to the person's weight. If the elevator is in free-fall and accelerating downward at g, then the scale reading will be zero and the person will *appear* to be weightless.

Integrating Concepts: Newton’s Laws of Motion and Kinematics

Physics is most interesting and most powerful when applied to general situations that involve more than a narrow set of physical principles. Newton’s laws of motion can also be integrated with other concepts that have been discussed previously in this text to solve problems of motion. For example, forces produce accelerations, a topic of kinematics, and hence the relevance of earlier chapters. When approaching problems that involve various types of forces, acceleration, velocity, and/or position, use the following steps to approach the problem:

Problem-Solving Strategy

Step 1. *Identify which physical principles are involved*. Listing the givens and the quantities to be calculated will allow you to identify the principles involved.

Step 2. *Solve the problem using strategies outlined in the text*. If these are available for the specific topic, you should refer to them. You should also refer to the sections of the text that deal with a particular topic. The following worked example illustrates how these strategies are applied to an integrated concept problem.

Example 4.10.4: What Force Must a Soccer Player Exert to Reach Top Speed?

A soccer player starts from rest and accelerates forward, reaching a velocity of 8.00 m/s in 2.50 s. (a) What was his average acceleration? (b) What average force did he exert backward on the ground to achieve this acceleration? The player’s mass is 70.0 kg, and air resistance is negligible.

Strategy

1. To solve an integrated concept problem, we must first identify the physical principles involved and identify the chapters in which they are found. Part (a) of this example considers acceleration along a straight line. This is a topic of kinematics. Part (b) deals with force, a topic of dynamics found in this chapter.
2. The following solutions to each part of the example illustrate how the specific problem-solving strategies are applied. These involve identifying knowns and unknowns, checking to see if the answer is reasonable, and so forth.

Solution for (a)

We are given the initial and final velocities (zero and 8.00 m/s forward); thus, the change in velocity is $\Delta v=8.00\, m/s$. We are given the elapsed time, and so $\Delta t=2.50\, s$. The unknown is acceleration, which can be found from its definition:

$$a=\frac{\Delta v}{\Delta t}.$$

Substituting the known values yields

$$a=\frac{8.00\, m/s}{2.50\, s}=3.20\, m/s^2$$

Discussion for (a)

This is an attainable acceleration for an athlete in good condition.

Solution for (b)

Here we are asked to find the average force the player exerts backward to achieve this forward acceleration. Neglecting air resistance, this would be equal in magnitude to the net external force on the player, since this force causes his acceleration. Since we now know the player’s acceleration and are given his mass, we can use Newton’s second law to find the force exerted. That is,

$$F_{net}=ma$$

Substituting the known values of m and a gives

$$F_{net}=(70\, kg)(3.2\, m/s^2)=224\, N.$$

Discussion for (b)

This is about 50 pounds, a reasonable average force.

This worked example illustrates how to apply problem-solving strategies to situations that include topics from different chapters. The first step is to identify the physical principles involved in the problem. The second step is to solve for the unknown using familiar problem-solving strategies. These strategies are found throughout the text, and many worked examples show how to use them for single topics. You will find these techniques for integrated concept problems useful in applications of physics outside of a physics course, such as in your profession, in other science disciplines, and in everyday life. The following problems will build your skills in the broad application of physical principles.

Summary

- Newton's laws of motion can be applied in numerous situations to solve problems of motion.
- Some problems will contain multiple force vectors acting in different directions on an object. Be sure to draw diagrams, resolve all force vectors into horizontal and vertical components, and draw a free-body diagram. Always analyze the direction in which an object accelerates so that you can determine whether $F_{net} = ma$ or $F_{net} = 0$.
- The normal force on an object is not always equal in magnitude to the weight of the object. If an object is accelerating, the normal force will be less than or greater than the weight of the object. Also, if the object is on an inclined plane, the normal force will always be less than the full weight of the object.
- Some problems will contain various physical quantities, such as forces, acceleration, velocity, or position. You can apply concepts from kinematics and dynamics in order to solve these problems of motion.

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4.11: Extended Topic- The Four Basic Forces—An Introduction

Learning Objectives

By the end of this section, you will be able to:

- Understand the four basic forces that underlie the processes in nature.

One of the most remarkable simplifications in physics is that only four distinct forces account for all known phenomena. In fact, nearly all of the forces we experience directly are due to only one basic force, called the electromagnetic force. (The gravitational force is the only force we experience directly that is not electromagnetic.) This is a tremendous simplification of the myriad of *apparently* different forces we can list, only a few of which were discussed in the previous section. As we will see, the basic forces are all thought to act through the exchange of microscopic carrier particles, and the characteristics of the basic forces are determined by the types of particles exchanged. Action at a distance, such as the gravitational force of Earth on the Moon, is explained by the existence of a force field rather than by "physical contact."

The *four basic forces* are the gravitational force, the electromagnetic force, the weak nuclear force, and the strong nuclear force. Their properties are summarized in Table. Since the weak and strong nuclear forces act over an extremely short range, the size of a nucleus or less, we do not experience them directly, although they are crucial to the very structure of matter. These forces determine which nuclei are stable and which decay, and they are the basis of the release of energy in certain nuclear reactions. Nuclear forces determine not only the stability of nuclei, but also the relative abundance of elements in nature. The properties of the nucleus of an atom determine the number of electrons it has and, thus, indirectly determine the chemistry of the atom. More will be said of all of these topics in later chapters.

CONCEPT CONNECTIONS: THE FOUR BASIC FORCES

The four basic forces will be encountered in more detail as you progress through the text. The gravitational force is defined in Uniform Circular Motion and Gravitation, electric force in Electric Charge and Electric Field, magnetic force in Magnetism, and nuclear forces in Radioactivity and Nuclear Physics. On a macroscopic scale, electromagnetism and gravity are the basis for all forces. The nuclear forces are vital to the substructure of matter, but they are not directly experienced on the macroscopic scale.

Force	Approximate relative strengths	Range	Attraction/Repulsion	Carrier Particle
Gravitational	10^{-38}	∞	attractive only	Graviton
Electromagnetic	10^{-2}	∞	attractive and repulsive	Photon
Weak Nuclear	10^{-13}	$< 10^{-18}\ m$	attractive and repulsive	$W^+,\ W^-,\ Z^0$
Strong Nuclear	1	$< 10^{-15}\ m$	attractive and repulsive	gluons

The gravitational force is surprisingly weak—it is only because gravity is always attractive that we notice it at all. Our weight is the gravitational force due to the *entire* Earth acting on us. On the very large scale, as in astronomical systems, the gravitational force is the dominant force determining the motions of moons, planets, stars, and galaxies. The gravitational force also affects the nature of space and time. As we shall see later in the study of general relativity, space is curved in the vicinity of very massive bodies, such as the Sun, and time actually slows down near massive bodies.

Electromagnetic forces can be either attractive or repulsive. They are long-range forces, which act over extremely large distances, and they nearly cancel for macroscopic objects. (Remember that it is the *net* external force that is important.) If they did not cancel, electromagnetic forces would completely overwhelm the gravitational force. The electromagnetic force is a combination of electrical forces (such as those that cause static electricity) and magnetic forces (such as those that affect a compass needle). These two forces were thought to be quite distinct until early in the 19th century, when scientists began to discover that they are different manifestations of the same force. This discovery is a classical case of the *unification of forces*. Similarly, friction, tension, and all of the other classes of forces we experience directly (except gravity, of course) are due to electromagnetic interactions of atoms and molecules. It is still convenient to consider these forces separately in specific applications, however, because of the ways they manifest themselves.

CONCEPT CONNECTIONS: UNIFYING FORCES

Attempts to unify the four basic forces are discussed in relation to elementary particles later in this text. By "unify" we mean finding connections between the forces that show that they are different manifestations of a single force. Even if such unification is achieved, the forces will retain their separate characteristics on the macroscopic scale and may be identical only under extreme conditions such as those existing in the early universe.

Physicists are now exploring whether the four basic forces are in some way related. Attempts to unify all forces into one come under the rubric of Grand Unified Theories (GUTs), with which there has been some success in recent years. It is now known that under conditions of extremely high density and temperature, such as existed in the early universe, the electromagnetic and weak nuclear forces are indistinguishable. They can now be considered to be different manifestations of one force, called the *electroweak* force. So the list of four has been reduced in a sense to only three. Further progress in unifying all forces is proving difficult—especially the inclusion of the gravitational force, which has the special characteristics of affecting the space and time in which the other forces exist.

While the unification of forces will not affect how we discuss forces in this text, it is fascinating that such underlying simplicity exists in the face of the overt complexity of the universe. There is no reason that nature must be simple—it simply is.

Action at a Distance: Concept of a Field

All forces act at a distance. This is obvious for the gravitational force. Earth and the Moon, for example, interact without coming into contact. It is also true for all other forces. Friction, for example, is an electromagnetic force between atoms that may not actually touch. What is it that carries forces between objects? One way to answer this question is to imagine that a **force field** surrounds whatever object creates the force. A second object (often called a *test object*) placed in this field will experience a force that is a function of location and other variables. The field itself is the "thing" that carries the force from one object to another. The field is defined so as to be a characteristic of the object creating it; the field does not depend on the test object placed in it. Earth's gravitational field, for example, is a function of the mass of Earth and the distance from its center, independent of the presence of other masses. The concept of a field is useful because equations can be written for force fields surrounding objects (for gravity, this yields $w = mg$ at Earth's surface), and motions can be calculated from these equations. (See Figure.)

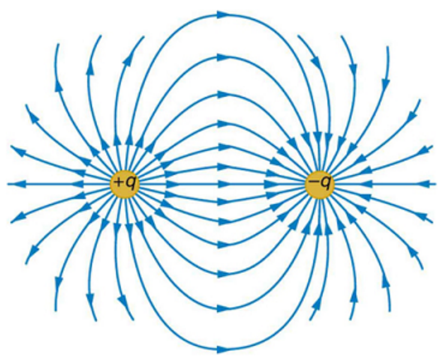

Figure 4.11.1: The electric force field between a positively charged particle and a negatively charged particle. When a positive test charge is placed in the field, the charge will experience a force in the direction of the force field lines.

CONCEPT CONNECTIONS: FORCE FIELDS

The concept of a *force field* is also used in connection with electric charge and is presented in Electric Charge and Electric Field. It is also a useful idea for all the basic forces, as will be seen in Particle Physics. Fields help us to visualize forces and how they are transmitted, as well as to describe them with precision and to link forces with subatomic carrier particles.

The field concept has been applied very successfully; we can calculate motions and describe nature to high precision using field equations. As useful as the field concept is, however, it leaves unanswered the question of what carries the force. It has been proposed in recent decades, starting in 1935 with Hideki Yukawa's (1907–1981) work on the strong nuclear force, that all forces are transmitted by the exchange of elementary particles. We can visualize particle exchange as analogous to macroscopic phenomena such as two people passing a basketball back and forth, thereby exerting a repulsive force without touching one another. (See Figure.)

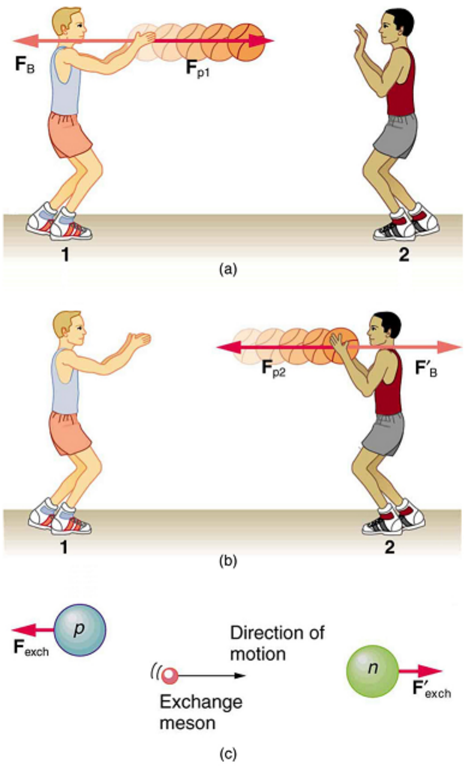

Figure 4.11.2:The exchange of masses resulting in repulsive forces. (a) The person throwing the basketball exerts a force F_{p1} on it toward the other person and feels a reaction force F_B away from the second person. (b) The person catching the basketball exerts a forceF_{p2} on it to stop the ball and feels a reaction force F'_B away from the first person. (c) The analogous exchange of a meson between a proton and a neutron carries the strong nuclear forces F_{exch} and F'_{exch} between them. An attractive force can also be exerted by the exchange of a mass—if person 2 pulled the basketball away from the first person as he tried to retain it, then the force between them would be attractive

This idea of particle exchange deepens rather than contradicts field concepts. It is more satisfying philosophically to think of something physical actually moving between objects acting at a distance. Table lists the exchange or **carrier particles,** both observed and proposed, that carry the four forces. But the real fruit of the particle-exchange proposal is that searches for Yukawa's proposed particle found it *and* a number of others that were completely unexpected, stimulating yet more research. All of this research eventually led to the proposal of quarks as the underlying substructure of matter, which is a basic tenet of GUTs. If successful, these theories will explain not only forces, but also the structure of matter itself. Yet physics is an experimental science, so the test of these theories must lie in the domain of the real world. As of this writing, scientists at the CERN laboratory in Switzerland are starting to test these theories using the world's largest particle accelerator: the Large Hadron Collider. This accelerator (27 km in circumference) allows two high-energy proton beams, traveling in opposite directions, to collide. An energy of 14 trillion electron volts will be available. It is anticipated that some new particles, possibly force carrier particles, will be found. (See Figure.) One of the force carriers of high interest that researchers hope to detect is the Higgs boson. The observation of its properties might tell us why different particles have different masses.

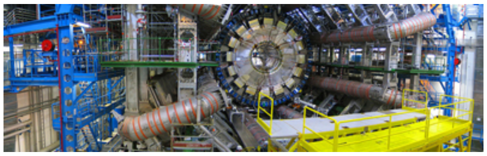

Figure 4.11.3:The world's largest particle accelerator spans the border between Switzerland and France. Two beams, traveling in opposite directions close to the speed of light, collide in a tube similar to the central tube shown here. External magnets determine the beam's path. Special detectors will analyze particles created in these collisions. Questions as broad as what is the origin of mass and what was matter like the first few seconds of our universe will be explored. This accelerator began preliminary operation in 2008. (credit: Frank Hommes)

Tiny particles also have wave-like behavior, something we will explore more in a later chapter. To better understand force-carrier particles from another perspective, let us consider gravity. The search for gravitational waves has been going on for a number of years. Almost 100 years ago, Einstein predicted the existence of these waves as part of his general theory of relativity. Gravitational waves are created during the collision of massive stars, in black holes, or in supernova explosions—like shock waves. These gravitational waves will travel through space from such sites much like a pebble dropped into a pond sends out ripples—except these waves move at the speed of light. A detector apparatus has been built in the U.S., consisting of two large installations nearly 3000 km apart—one in Washington state and one in Louisiana! The facility is called the Laser Interferometer Gravitational-Wave Observatory (LIGO). Each installation is designed to use optical lasers to examine any slight shift in the relative positions of two masses due to the effect of gravity waves. The two sites allow simultaneous measurements of these small effects to be separated from other natural phenomena, such as earthquakes. Initial operation of the detectors began in 2002, and work is proceeding on increasing their sensitivity. Similar installations have been built in Italy (VIRGO), Germany (GEO600), and Japan (TAMA300) to provide a worldwide network of gravitational wave detectors.

International collaboration in this area is moving into space with the joint EU/US project LISA (Laser Interferometer Space Antenna). Earthquakes and other Earthly noises will be no problem for these monitoring spacecraft. LISA will complement LIGO by looking at much more massive black holes through the observation of gravitational-wave sources emitting much larger wavelengths. Three satellites will be placed in space above Earth in an equilateral triangle (with 5,000,000-km sides) (Figure). The system will measure the relative positions of each satellite to detect passing gravitational waves. Accuracy to within 10% of the size of an atom will be needed to detect any waves. The launch of this project might be as early as 2018.

"I'm sure LIGO will tell us something about the universe that we didn't know before. The history of science tells us that any time you go where you haven't been before, you usually find something that really shakes the scientific paradigms of the day. Whether gravitational wave astrophysics will do that, only time will tell." —David Reitze, LIGO Input Optics Manager, University of Florida.

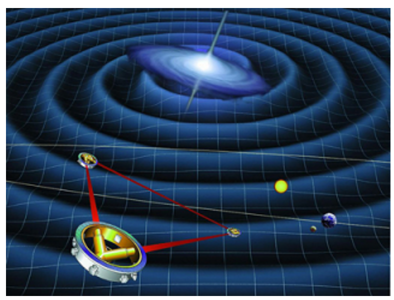

Figure 4.11.4:Space-based future experiments for the measurement of gravitational waves. Shown here is a drawing of LISA's orbit. Each satellite of LISA will consist of a laser source and a mass. The lasers will transmit a signal to measure the distance between each satellite's test mass. The relative motion of these masses will provide information about passing gravitational waves. (credit: NASA)

The ideas presented in this section are but a glimpse into topics of modern physics that will be covered in much greater depth in later chapters.

Summary

- The various types of forces that are categorized for use in many applications are all manifestations of the *four basic forces* in nature.
- The properties of these forces are summarized in Table.
- Everything we experience directly without sensitive instruments is due to either electromagnetic forces or gravitational forces. The nuclear forces are responsible for the submicroscopic structure of matter, but they are not directly sensed because of their short ranges. Attempts are being made to show all four forces are different manifestations of a single unified force.
- A force field surrounds an object creating a force and is the carrier of that force.

Footnotes

The graviton is a proposed particle, though it has not yet been observed by scientists. See the discussion of gravitational waves later in this section. The particles W^+, W^-, and Z^0 are called vector bosons; these were predicted by theory and first observed in 1983. There are eight types of gluons proposed by scientists, and their existence is indicated by meson exchange in the nuclei of atoms.

Glossary

carrier particle

a fundamental particle of nature that is surrounded by a characteristic force field; photons are carrier particles of the electromagnetic force

force field

a region in which a test particle will experience a force

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4.E: Dynamics- Force and Newton's Laws of Motion (Exercises)

Conceptual Questions

4.1: Development of Force Concept

1. Propose a force standard different from the example of a stretched spring discussed in the text. Your standard must be capable of producing the same force repeatedly.

2. What properties do forces have that allow us to classify them as vectors?

4.2: Newton's First Law of Motion: Inertia

3. How are inertia and mass related?

4. What is the relationship between weight and mass? Which is an intrinsic, unchanging property of a body?

4.3: Newton's Second Law of Motion: Concept of a System

5. Which statement is correct? Explain your answer and give an example.

(a) Net force causes motion.

(b) Net force causes change in motion.

6. Why can we neglect forces such as those holding a body together when we apply Newton's second law of motion?

7. Explain how the choice of the "system of interest" affects which forces must be considered when applying Newton's second law of motion.

8. Describe a situation in which the net external force on a system is not zero, yet its speed remains constant.

9. system can have a nonzero velocity while the net external force on it is zero. Describe such a situation.

10. A rock is thrown straight up. What is the net external force acting on the rock when it is at the top of its trajectory?

11. (a) Give an example of different net external forces acting on the same system to produce different accelerations.

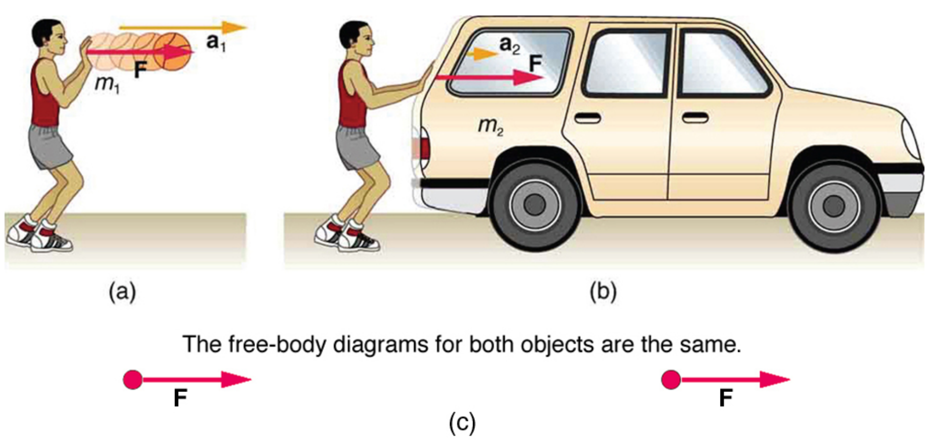
(b) Give an example of the same net external force acting on systems of different masses, producing different accelerations.

(c) What law accurately describes both effects? State it in words and as an equation.

12. If the acceleration of a system is zero, are no external forces acting on it? What about internal forces? Explain your answers.

13. If a constant, nonzero force is applied to an object, what can you say about the velocity and acceleration of the object?

14. The gravitational force on the basketball in Figure is ignored. When gravity is taken into account, what is the direction of the net external force on the basketball—above horizontal, below horizontal, or still horizontal?



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4.4: Newton’s Third Law of Motion: Symmetry in Forces

15. When you take off in a jet aircraft, there is a sensation of being pushed back into the seat. Explain why you move backward in the seat—is there really a force backward on you? (The same reasoning explains whiplash injuries, in which the head is apparently thrown backward.)

16. A device used since the 1940s to measure the kick or recoil of the body due to heart beats is the “ballistocardiograph.” What physics principle(s) are involved here to measure the force of cardiac contraction? How might we construct such a device?

17. Describe a situation in which one system exerts a force on another and, as a consequence, experiences a force that is equal in magnitude and opposite in direction. Which of Newton’s laws of motion apply?

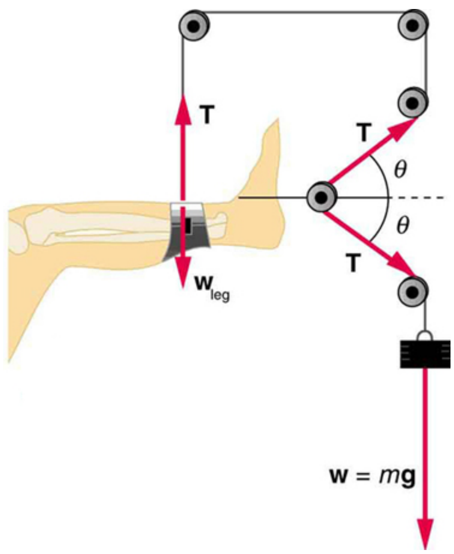
18. Why does an ordinary rifle recoil (kick backward) when fired? The barrel of a recoilless rifle is open at both ends. Describe how Newton’s third law applies when one is fired. Can you safely stand close behind one when it is fired?

19. An American football lineman reasons that it is senseless to try to out-push the opposing player, since no matter how hard he pushes he will experience an equal and opposite force from the other player. Use Newton’s laws and draw a free-body diagram of an appropriate system to explain how he can still out-push the opposition if he is strong enough.

20. Newton’s third law of motion tells us that forces always occur in pairs of equal and opposite magnitude. Explain how the choice of the “system of interest” affects whether one such pair of forces cancels.

4.5: Normal, Tension, and Other Examples of Forces

21. If a leg is suspended by a traction setup as shown in Figure, what is the tension in the rope?



A leg is suspended by a traction system in which wires are used to transmit forces. Frictionless pulleys change the direction of the force T without changing its magnitude.

22. In a traction setup for a broken bone, with pulleys and rope available, how might we be able to increase the force along the tibia using the same weight? (See Figure.) (Note that the tibia is the shin bone shown in this image.)

4.7: Further Applications of Newton’s Laws of Motion

23. To simulate the apparent weightlessness of space orbit, astronauts are trained in the hold of a cargo aircraft that is accelerating downward at g. Why will they appear to be weightless, as measured by standing on a bathroom scale, in this accelerated frame of reference? Is there any difference between their apparent weightlessness in orbit and in the aircraft?

24. A cartoon shows the toupee coming off the head of an elevator passenger when the elevator rapidly stops during an upward ride. Can this really happen without the person being tied to the floor of the elevator? Explain your answer.

4.8: Extended Topic: The Four Basic Forces—An Introduction

25. Explain, in terms of the properties of the four basic forces, why people notice the gravitational force acting on their bodies if it is such a comparatively weak force.

26. What is the dominant force between astronomical objects? Why are the other three basic forces less significant over these very large distances?

27. Give a detailed example of how the exchange of a particle can result in an attractive force. (For example, consider one child pulling a toy out of the hands of another.)

Problems & Exercises

4.3: Newton's Second Law of Motion: Concept of a System

You may assume data taken from illustrations is accurate to three digits.

28. A 63.0-kg sprinter starts a race with an acceleration of $4.20 m/s^2$. What is the net external force on him?

Solution
265 N

29. If the sprinter from the previous problem accelerates at that rate for 20 m, and then maintains that velocity for the remainder of the 100-m dash, what will be his time for the race?

30. A cleaner pushes a 4.50-kg laundry cart in such a way that the net external force on it is 60.0 N. Calculate the magnitude of its acceleration.

Solution
$13.3 m/s^2$

31. Since astronauts in orbit are apparently weightless, a clever method of measuring their masses is needed to monitor their mass gains or losses to adjust diets. One way to do this is to exert a known force on an astronaut and measure the acceleration produced. Suppose a net external force of 50.0 N is exerted and the astronaut's acceleration is measured to be $0.893 m/s^2$.

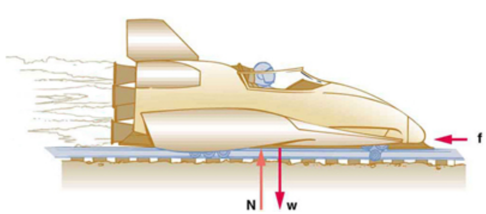
(a) Calculate her mass.

(b) By exerting a force on the astronaut, the vehicle in which they orbit experiences an equal and opposite force. Discuss how this would affect the measurement of the astronaut's acceleration. Propose a method in which recoil of the vehicle is avoided.

32. In Figure 4.4.3, the net external force on the 24-kg mower is stated to be 51 N. If the force of friction opposing the motion is 24 N, what force F (in newtons) is the person exerting on the mower? Suppose the mower is moving at 1.5 m/s when the force F is removed. How far will the mower go before stopping?

Solution
1.1 m

33. The same rocket sled drawn in Figure is decelerated at a rate of $196 m/s^2$. What force is necessary to produce this deceleration? Assume that the rockets are off. The mass of the system is 2100 kg.



34. (a) If the rocket sled shown in Figure starts with only one rocket burning, what is the magnitude of its acceleration? Assume that the mass of the system is 2100 kg, the thrust T is 2.4×10^4 N, and the force of friction opposing the motion is known to be 650 N.

(b) Why is the acceleration not one-fourth of what it is with all rockets burning?

Solution
(a) $11 m/s^2$
(b) The acceleration is not one-fourth of what it was with all rockets burning because the frictional force is still as large as it was with all rockets burning.

35. What is the deceleration of the rocket sled if it comes to rest in 1.1 s from a speed of 1000 km/h? (Such deceleration caused one test subject to black out and have temporary blindness.)

36. Suppose two children push horizontally, but in exactly opposite directions, on a third child in a wagon. The first child exerts a force of 75.0 N, the second a force of 90.0 N, friction is 12.0 N, and the mass of the third child plus wagon is 23.0 kg.

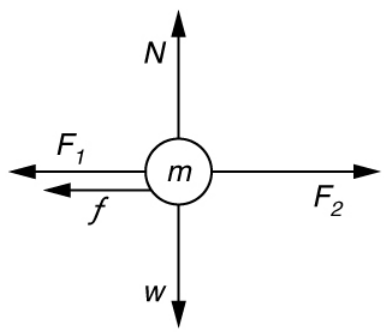
(a) What is the system of interest if the acceleration of the child in the wagon is to be calculated?

(b) Draw a free-body diagram, including all forces acting on the system.

(c) Calculate the acceleration.

(d) What would the acceleration be if friction were 15.0 N?

Solution
(a) The system is the child in the wagon plus the wagon.
(b)



(c) $a = 0.130 m/s^2$ in the direction of the second child's push.
(d) $a = 0.00 m/s^2$

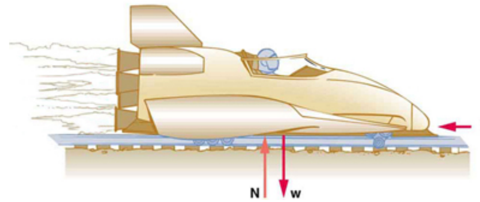
37. A powerful motorcycle can produce an acceleration of $3.50 m/s^2$ while traveling at 90.0 km/h. At that speed the forces resisting motion, including friction and air resistance, total 400 N. (Air resistance is analogous to air friction. It always opposes the motion of an object.) What is the magnitude of the force the motorcycle exerts backward on the ground to produce its acceleration if the mass of the motorcycle with rider is 245 kg?

38. The rocket sled shown in Figure accelerates at a rate of $49.0 m/s^2$. Its passenger has a mass of 75.0 kg.

(a) Calculate the horizontal component of the force the seat exerts against his body. Compare this with his weight by using a ratio.

(b) Calculate the direction and magnitude of the total force the seat exerts against his body.

Solution
(a) $3.68 \times 10^3 N$. This force is 5.00 times greater than his weight.
(b) $3750 N; 11.3^{o} above horizontal$



39. Repeat the previous problem for the situation in which the rocket sled decelerates at a rate of $201 m/s^2$. In this problem, the forces are exerted by the seat and restraining belts.

40. The weight of an astronaut plus his space suit on the Moon is only 250 N. How much do they weigh on Earth? What is the mass on the Moon? On Earth?

Solution
$1.5 \times 10^3 N, 150 kg, 150 kg$

41. Suppose the mass of a fully loaded module in which astronauts take off from the Moon is 10,000 kg. The thrust of its engines is 30,000 N. (a) Calculate its the magnitude of acceleration in a vertical takeoff from the Moon. (b) Could it lift off from Earth? If not, why not? If it could, calculate the magnitude of its acceleration.

4.4: Newton's Third Law of Motion: Symmetry in Forces

42. What net external force is exerted on a 1100-kg artillery shell fired from a battleship if the shell is accelerated at $2.40 \times 10^4 m/s^2$? What is the magnitude of the force exerted on the ship by the artillery shell?

Solution
Force on shell: $2.64 \times 10^7 N$
Force exerted on ship = $-2.64 \times 10^7 N$, by Newton's third law

43. A brave but inadequate rugby player is being pushed backward by an opposing player who is exerting a force of 800 N on him. The mass of the losing player plus equipment is 90.0 kg, and he is accelerating at 1.20m/s2 size 12{1 "." "20"" m/s" rSup { size 8{2} } } {} backward. (a) What is the force of friction between the losing player's feet and the grass? (b) What force does the winning player exert on the ground to move forward if his mass plus equipment is 110 kg? (c) Draw a sketch of the situation showing the system of interest used to solve each part. For this situation, draw a free-body diagram and write the net force equation.

4.5: Normal, Tension, and Other Examples of Forces

44. Two teams of nine members each engage in a tug of war. Each of the first team's members has an average mass of 68 kg and exerts an average force of 1350 N horizontally. Each of the second team's members has an average mass of 73 kg and exerts an average force of 1365 N horizontally.

(a) What is magnitude of the acceleration of the two teams?

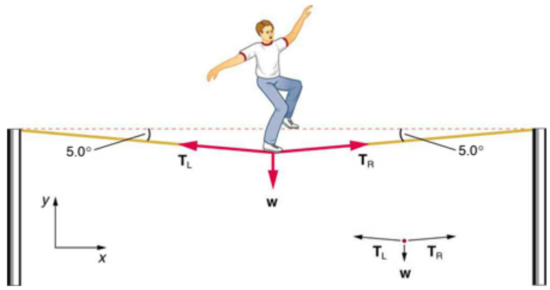
(b) What is the tension in the section of rope between the teams?

Solution
a. $0.11 m/s^2$
b. $1.2 \times 10^4 N$

45. What force does a trampoline have to apply to a 45.0-kg gymnast to accelerate her straight up at $7.50 m/s^2$? Note that the answer is independent of the velocity of the gymnast—she can be moving either up or down, or be stationary.

46. (a) Calculate the tension in a vertical strand of spider web if a spider of mass $8.00 \times 10^{-5} kg$ hangs motionless on it.

(b) Calculate the tension in a horizontal strand of spider web if the same spider sits motionless in the middle of it much like the tightrope walker in Figure. The strand sags at an angle of 12° below the horizontal. Compare this with the tension in the vertical strand (find their ratio).



Solution
(a) $7.84 \times 10^{-4} N$
(b) $1.89 \times 10^{-3} N$. This is 2.41 times the tension in the vertical strand.

47. Suppose a 60.0-kg gymnast climbs a rope.

(a) What is the tension in the rope if he climbs at a constant speed?

(b) What is the tension in the rope if he accelerates upward at a rate of $1.50 m/s^2$?

48. Show that, as stated in the text, a force $F_\perp$ exerted on a flexible medium at its center and perpendicular to its length (such as on the tightrope wire in Figure) gives rise to a tension of magnitude $T = \frac{F_\perp}{2sin(\theta)}$.

Solution
Newton's second law applied in vertical direction gives
$F_y = F - 2T sin\theta = 0$
$F = 2T sin\theta$
$T = \frac{F}{2sin\theta}$.

49. Consider the baby being weighed in Figure.

(a) What is the mass of the child and basket if a scale reading of 55 N is observed?

(b) What is the tension T_1 in the cord attaching the baby to the scale?

(c) What is the tension T_2 in the cord attaching the scale to the ceiling, if the scale has a mass of 0.500 kg?

(d) Draw a sketch of the situation indicating the system of interest used to solve each part. The masses of the cords are negligible.

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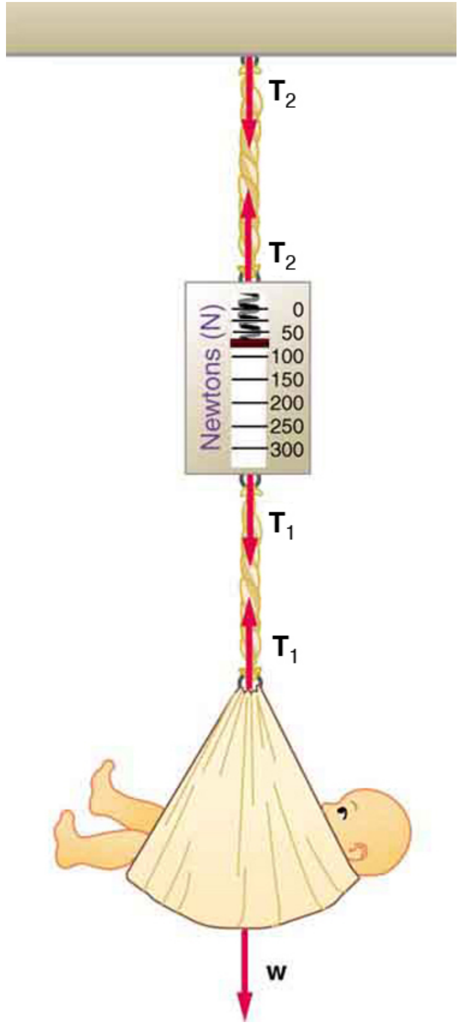
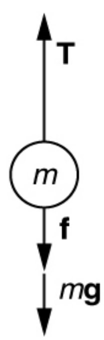


A baby is weighed using a spring scale.

4.6: Problem-Solving Strategies

50. A $5.00 \times 10^5 - kg$ rocket is accelerating straight up. Its engines produce $1.250 \times 10^7 N$ of thrust, and air resistance is $4.50 \times 10^6 N$. What is the rocket's acceleration? Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion.

Solution



Using the free-body diagram:

$F_{net} = T - f - mg = ma$

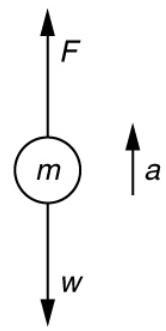
so that

$$a = \frac{T - f - mg}{m} = \frac{1.250 \times 10^7 N - 4.50 \times 10^6 N - (5.00 \times 10^5 kg)(9.80 m/s^2)}{5.00 \times 10^5 kg} = 6.20 m/s^2$$

51. The wheels of a midsize car exert a force of 2100 N backward on the road to accelerate the car in the forward direction. If the force of friction including air resistance is 250 N and the acceleration of the car is $1.80 m/s^2$, what is the mass of the car plus its occupants? Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion. For this situation, draw a free-body diagram and write the net force equation.

52. Calculate the force a 70.0-kg high jumper must exert on the ground to produce an upward acceleration 4.00 times the acceleration due to gravity. Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion.

Solution
1.Use Newton's laws of motion.



2. Given : $a = 4.00g = (4.00)(9.80 m/s^2) = 39.2 m/s^2; m = 70.0 kg.$

Find: F
3. $\sum F = +F - w = ma$, so that $F = ma + w = ma + mg = m(a + g)$
$F = (70.0 kg)[(39.2 m/s^2) + (9.80 m/s^2)] = 3.43 \times 10^3 N$ The force exerted by the high-jumper is actually down on the ground, but F size 12{F} is up from the ground and makes him jump.
4. This result is reasonable, since it is quite possible for a person to exert a force of the magnitude of $10^3 N$

53. When landing after a spectacular somersault, a 40.0-kg gymnast decelerates by pushing straight down on the mat. Calculate the force she must exert if her deceleration is 7.00 times the acceleration due to gravity. Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion.

54. A freight train consists of two $8.00 \times 10^4 - kg$ engines and 45 cars with average masses of $5.50 \times 10^4 kg$.

(a) What force must each engine exert backward on the track to accelerate the train at a rate of $5.00 \times 10^{-2} m/s^2$ if the force of friction is $7.50 \times 10^5 N$, assuming the engines exert identical forces? This is not a large frictional force for such a massive system. Rolling friction for trains is small, and consequently trains are very energy-efficient transportation systems.

(b) What is the force in the coupling between the 37th and 38th cars (this is the force each exerts on the other), assuming all cars have the same mass and that friction is evenly distributed among all of the cars and engines?

Solution
(a) $4.41 \times 10^5 N$
(b) $1.50 \times 10^5 N$

55. Commercial airplanes are sometimes pushed out of the passenger loading area by a tractor.

(a) An 1800-kg tractor exerts a force of $1.75 \times 10^4 N$ backward on the pavement, and the system experiences forces resisting motion that total 2400 N. If the acceleration is $0.150 m/s^2$, what is the mass of the airplane?

(b) Calculate the force exerted by the tractor on the airplane, assuming 2200 N of the friction is experienced by the airplane.

(c) Draw two sketches showing the systems of interest used to solve each part, including the free-body diagrams for each.

56. A 1100-kg car pulls a boat on a trailer.

(a) What total force resists the motion of the car, boat, and trailer, if the car exerts a 1900-N force on the road and produces an acceleration of $0.550 m/s^2$? The mass of the boat plus trailer is 700 kg.

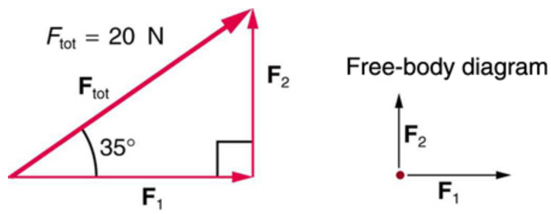
(b) What is the force in the hitch between the car and the trailer if 80% of the resisting forces are experienced by the boat and trailer?

Solution
(a) $910 N$
(b) $1.11 \times 10^3 N$

57. (a) Find the magnitudes of the forces F_1 and F_2 that add to give the total force F_{tot} shown in Figure. This may be done either graphically or by using trigonometry.

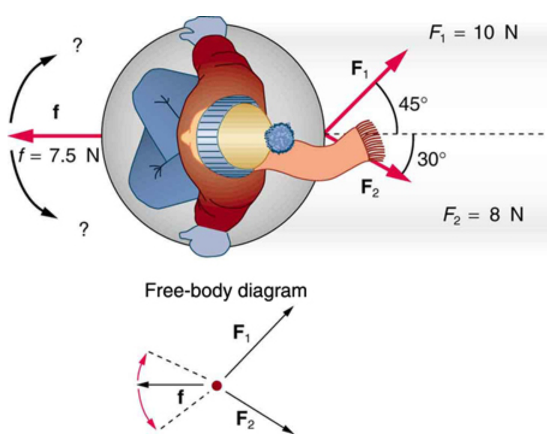
(b) Show graphically that the same total force is obtained independent of the order of addition of F_1 and F_2

(c) Find the direction and magnitude of some other pair of vectors that add to give F_{tot}. Draw these to scale on the same drawing used in part (b) or a similar picture.



58. Two children pull a third child on a snow saucer sled exerting forces F_1 and F_2 as shown from above in Figure. Find the acceleration of the 49.00-kg sled and child system. Note that the direction of the frictional force is unspecified; it will be in the opposite direction of the sum of F_1 and F_2.

Solution
$a = 0.139 m/s, \theta = 12.4^\circ$ north of east

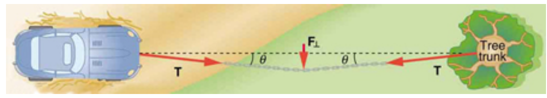


An overhead view of a child sitting on a snow saucer sled.

59. Suppose your car was mired deeply in the mud and you wanted to use the method illustrated in Figure to pull it out.

(a) What force would you have to exert perpendicular to the center of the rope to produce a force of 12,000 N on the car if the angle is 2.00°? In this part, explicitly show how you follow the steps in the Problem-Solving Strategy for Newton’s laws of motion.

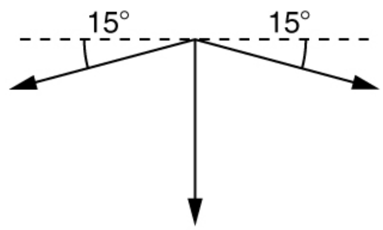
(b) Real ropes stretch under such forces. What force would be exerted on the car if the angle increases to 7.00° and you still apply the force found in part (a) to its center?



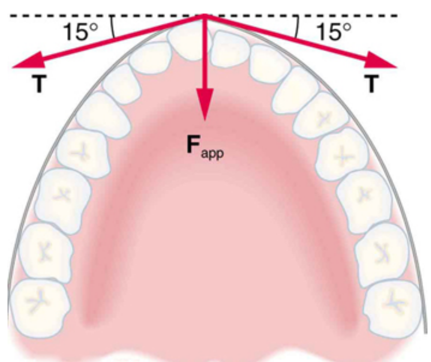
60. What force is exerted on the tooth in Figure if the tension in the wire is 25.0 N? Note that the force applied to the tooth is smaller than the tension in the wire, but this is necessitated by practical considerations of how force can be applied in the mouth. Explicitly show how you follow steps in the Problem-Solving Strategy for Newton's laws of motion.

Solution

1. Use Newton's laws since we are looking for forces.
2. Draw a free-body diagram:



3. The tension is given as $T=25.0N$. Find F_{app}. Using Newton's laws gives: $\Sigma F_y=0$, so that applied force is due to the *y*-components of the two tensions: $F_{app}=2T sin\theta=2(25.0N)sin(15^{\circ})=12.9N$
The *x*-components of the tension cancel. $\sum F_x=0$.
4. This seems reasonable, since the applied tensions should be greater than the force applied to the tooth.



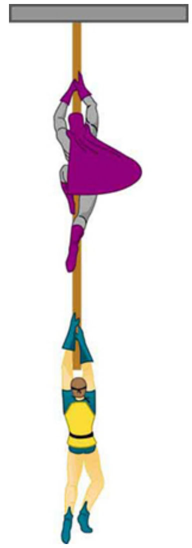
Braces are used to apply forces to teeth to realign them. Shown in this figure are the tensions applied by the wire to the protruding tooth. The total force applied to the tooth by the wire, F_{app}, points straight toward the back of the mouth.

61. Figure shows Superhero and Trusty Sidekick hanging motionless from a rope. Superhero's mass is 90.0 kg, while Trusty Sidekick's is 55.0 kg, and the mass of the rope is negligible.

(a) Draw a free-body diagram of the situation showing all forces acting on Superhero, Trusty Sidekick, and the rope.

(b) Find the tension in the rope above Superhero.

(c) Find the tension in the rope between Superhero and Trusty Sidekick. Indicate on your free-body diagram the system of interest used to solve each part.



Superhero and Trusty Sidekick hang motionless on a rope as they try to figure out what to do next. Will the tension be the same everywhere in the rope?

62. A nurse pushes a cart by exerting a force on the handle at a downward angle 35.0^{o} below the horizontal. The loaded cart has a mass of 28.0 kg, and the force of friction is 60.0 N.

(a) Draw a free-body diagram for the system of interest.

(b) What force must the nurse exert to move at a constant velocity?

63. *Construct Your Own Problem*

Consider the tension in an elevator cable during the time the elevator starts from rest and accelerates its load upward to some cruising velocity. Taking the elevator and its load to be the system of interest, draw a free-body diagram. Then calculate the tension in the cable. Among the things to consider are the mass of the elevator and its load, the final velocity, and the time taken to reach that velocity.

64. *Construct Your Own Problem*

Consider two people pushing a toboggan with four children on it up a snow-covered slope. Construct a problem in which you calculate the acceleration of the toboggan and its load. Include a free-body diagram of the appropriate system of interest as the basis for your analysis. Show vector forces and their components and explain the choice of coordinates. Among the things to be considered are the forces exerted by those pushing, the angle of the slope, and the masses of the toboggan and children.

65. *Unreasonable Results*

(a) Repeat Exercise, but assume an acceleration of $1.20 m/s^2$ is produced.

(b) What is unreasonable about the result?

(c) Which premise is unreasonable, and why is it unreasonable?

66. *Unreasonable Results*

(a) What is the initial acceleration of a rocket that has a mass of $1.50 \times 10^6 kg$ at takeoff, the engines of which produce a thrust of $2.00 \times 10^6 N$? Do not neglect gravity.

(b) What is unreasonable about the result? (This result has been unintentionally achieved by several real rockets.)

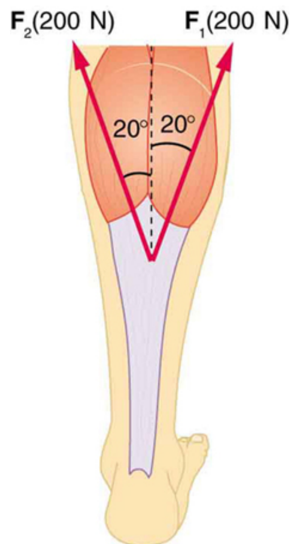
(c) Which premise is unreasonable, or which premises are inconsistent? (You may find it useful to compare this problem to the rocket problem earlier in this section.)

67. A flea jumps by exerting a force of $1.20 \times 10^{-5} N$ straight down on the ground. A breeze blowing on the flea parallel to the ground exerts a force of $0.500 \times 10^{-6} N$ on the flea. Find the direction and magnitude of the acceleration of the flea if its mass is $6.00 \times 10^{-7} kg$. Do not neglect the gravitational force.

Solution

$10.2 m/s^2, 4.67^{\circ}$ from vertical

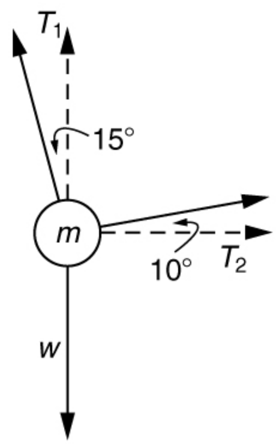
68. Two muscles in the back of the leg pull upward on the Achilles tendon, as shown in Figure. (These muscles are called the medial and lateral heads of the gastrocnemius muscle.) Find the magnitude and direction of the total force on the Achilles tendon. What type of movement could be caused by this force?



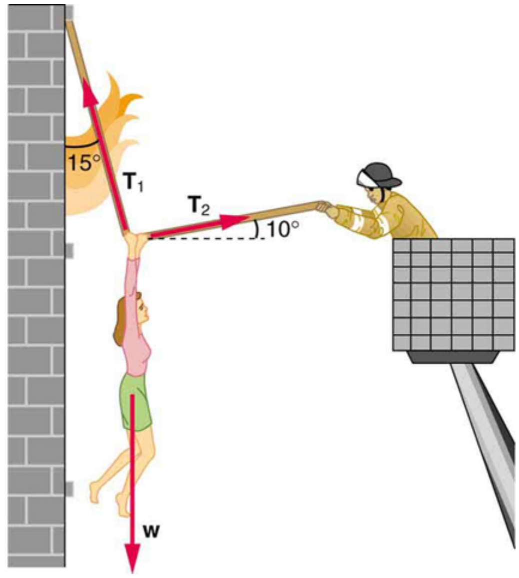
Achilles tendon

69. A 76.0-kg person is being pulled away from a burning building as shown in Figure. Calculate the tension in the two ropes if the person is momentarily motionless. Include a free-body diagram in your solution.

Solution



$T_1 = 736 N$
$T_2 = 194 N$



The force T_2 needed to hold steady the person being rescued from the fire is less than her weight and less than the force T_1 size 12{T rSub { size 8{1} } } {} in the other rope, since the more vertical rope supports a greater part of her weight (a vertical force).

70. *Integrated Concepts*

A 35.0-kg dolphin decelerates from 12.0 to 7.50 m/s in 2.30 s to join another dolphin in play. What average force was exerted to slow him if he was moving horizontally? (The gravitational force is balanced by the buoyant force of the water.)

71. *Integrated Concepts*

When starting a foot race, a 70.0-kg sprinter exerts an average force of 650 N backward on the ground for 0.800 s.

(a) What is his final speed?

(b) How far does he travel?

Solution

(a) $7.43m/s$

(b) $2.97m$

72. *Integrated Concepts*

A large rocket has a mass of $2.00 \times 10^6 kg$ at takeoff, and its engines produce a thrust of $3.50 \times 10^7 N$.

(a) Find its initial acceleration if it takes off vertically.

(b) How long does it take to reach a velocity of 120 km/h straight up, assuming constant mass and thrust?

(c) In reality, the mass of a rocket decreases significantly as its fuel is consumed. Describe qualitatively how this affects the acceleration and time for this motion.

73. *Integrated Concepts*

A basketball player jumps straight up for a ball. To do this, he lowers his body 0.300 m and then accelerates through this distance by forcefully straightening his legs. This player leaves the floor with a vertical velocity sufficient to carry him 0.900 m above the floor.

(a) Calculate his velocity when he leaves the floor.

(b) Calculate his acceleration while he is straightening his legs. He goes from zero to the velocity found in part (a) in a distance of 0.300 m.

(c) Calculate the force he exerts on the floor to do this, given that his mass is 110 kg.

Solution
(a) $4.20 m/s$
(b) $29.4 m/s^2$
(c) $4.31 \times 10^3 N$

74. *Integrated Concepts*

A 2.50-kg fireworks shell is fired straight up from a mortar and reaches a height of 110 m.

(a) Neglecting air resistance (a poor assumption, but we will make it for this example), calculate the shell's velocity when it leaves the mortar.

(b) The mortar itself is a tube 0.450 m long. Calculate the average acceleration of the shell in the tube as it goes from zero to the velocity found in (a).

(c) What is the average force on the shell in the mortar? Express your answer in newtons and as a ratio to the weight of the shell.

75. Integrated Concepts Repeat Exercise for a shell fired at an angle 10.0^o from the vertical.

Solution
(a) $47.1 m/s$
(b) $2.47 \times 10^3 m/s^2$
(c) $6.18 \times 10^3 N$.

76. *Integrated Concepts*

An elevator filled with passengers has a mass of 1700 kg.

(a) The elevator accelerates upward from rest at a rate of $1.20 m/s^2$. Calculate the tension in the cable supporting the elevator.

(b) The elevator continues upward at constant velocity for 8.50 s. What is the tension in the cable during this time?

(c) The elevator decelerates at a rate of $0.600 m/s^2$ for 3.00 s. What is the tension in the cable during deceleration?

(d) How high has the elevator moved above its original starting point, and what is its final velocity?

77. *Unreasonable Results*

(a) What is the final velocity of a car originally traveling at 50.0 km/h that decelerates at a rate of $0.400 m/s^2$ for 50.0 s?

(b) What is unreasonable about the result?

(c) Which premise is unreasonable, or which premises are inconsistent?

78. *Unreasonable Results*

A 75.0-kg man stands on a bathroom scale in an elevator that accelerates from rest to 30.0 m/s in 2.00 s.

(a) Calculate the scale reading in newtons and compare it with his weight. (The scale exerts an upward force on him equal to its reading.)

(b) What is unreasonable about the result?

(c) Which premise is unreasonable, or which premises are inconsistent?

4.8: Extended Topic: The Four Basic Forces—An Introduction

79. (a) What is the strength of the weak nuclear force relative to the strong nuclear force?

(b) What is the strength of the weak nuclear force relative to the electromagnetic force? Since the weak nuclear force acts at only very short distances, such as inside nuclei, where the strong and electromagnetic forces also act, it might seem surprising that we have any knowledge of it at all. We have such knowledge because the weak nuclear force is responsible for beta decay, a type of nuclear decay not explained by other forces.

Solution
(a) 1×10^{-13}
(b) 1×10^{-11}

80. (a) What is the ratio of the strength of the gravitational force to that of the strong nuclear force?

(b) What is the ratio of the strength of the gravitational force to that of the weak nuclear force?

(c) What is the ratio of the strength of the gravitational force to that of the electromagnetic force? What do your answers imply about the influence of the gravitational force on atomic nuclei?

81. What is the ratio of the strength of the strong nuclear force to that of the electromagnetic force? Based on this ratio, you might expect that the strong force dominates the nucleus, which is true for small nuclei. Large nuclei, however, have sizes greater than the range of the strong nuclear force. At these sizes, the electromagnetic force begins to affect nuclear stability. These facts will be used to explain nuclear fusion and fission later in this text.

Solution
10^2

Contributors and Attributions

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CHAPTER OVERVIEW

5: Uniform Circular Motion and Gravitation

This chapter deals with the simplest form of curved motion, **uniform circular motion**, motion in a circular path at constant speed. Studying this topic illustrates most concepts associated with rotational motion and leads to the study of many new topics we group under the name *rotation*. Pure *rotational motion* occurs when points in an object move in circular paths centered on one point. Pure *translational motion* is motion with no rotation. Some motion combines both types, such as a rotating hockey puck moving along ice.

5.1: Prelude to Uniform Circular Motion and Gravitation
5.2: Rotation Angle and Angular Velocity
5.3: Centripetal Acceleration
5.4: Centripetal Force
5.5: Fictitious Forces and Non-inertial Frames - The Coriolis Force
5.6: Newton's Universal Law of Gravitation
5.7: Satellites and Kepler's Laws- An Argument for Simplicity
5.E: Uniform Circular Motion and Gravitation (Excercise)

Thumbnail: Two bodies of different mass orbiting a common barycenter. The relative sizes and type of orbit are similar to the Pluto–Charon system. (public domain; Zhatt).

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5.1: Prelude to Uniform Circular Motion and Gravitation

Many motions, such as the arc of a bird's flight or Earth's path around the Sun, are curved. Recall that Newton's first law tells us that motion is along a straight line at constant speed unless there is a net external force. We will therefore study not only motion along curves, but also the forces that cause it, including gravitational forces. In some ways, this chapter is a continuation of Dynamics: Newton's Laws of Motion as we study more applications of Newton's laws of motion.

Figure 5.1.1: This Australian Grand Prix Formula 1 race car moves in a circular path as it makes the turn. Its wheels also spin rapidly—the latter completing many revolutions, the former only part of one (a circular arc). The same physical principles are involved in each. (credit: Richard Munckton)

This chapter deals with the simplest form of curved motion, **uniform circular motion**, motion in a circular path at constant speed. Studying this topic illustrates most concepts associated with rotational motion and leads to the study of many new topics we group under the name *rotation*. Pure *rotational motion* occurs when points in an object move in circular paths centered on one point. Pure *translational motion* is motion with no rotation. Some motion combines both types, such as a rotating hockey puck moving along ice.

Glossary

uniform circular motion

the motion of an object in a circular path at constant speed

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5.2: Rotation Angle and Angular Velocity

Learning Objectives

By the end of this section, you will be able to:

- Define arc length, rotation angle, radius of curvature and angular velocity.
- Calculate the angular velocity of a car wheel spin.

In Kinematics, we studied motion along a straight line and introduced such concepts as displacement, velocity, and acceleration. Two-Dimensional Kinematics dealt with motion in two dimensions. Projectile motion is a special case of two-dimensional kinematics in which the object is projected into the air, while being subject to the gravitational force, and lands a distance away. In this chapter, we consider situations where the object does not land but moves in a curve. We begin the study of uniform circular motion by defining two angular quantities needed to describe rotational motion.

Rotation Angle

When objects rotate about some axis—for example, when the CD (compact disc) in Figure rotates about its center—each point in the object follows a circular arc. Consider a line from the center of the CD to its edge. Each pit used to record sound along this line moves through the same angle in the same amount of time. The rotation angle is the amount of rotation and is analogous to linear distance. We define the rotation angle $\Delta\theta$ to be the ratio of the arc length to the radius of curvature:

$$\Delta\theta = \frac{\Delta s}{r}. \tag{5.2.1}$$

Figure 5.2.1: All points on a CD travel in circular arcs. The pits along a line from the center to the edge all move through the same angle $\Delta\theta$ in a time Δt.

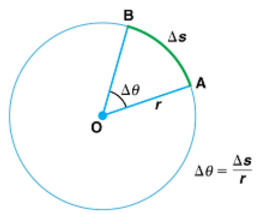

Figure 5.2.2: The radius of a circle is rotated through an angle $\Delta\theta$. The arc length δs is described on the circumference.

The arc length Δs is the distance traveled along a circular path as shown in Figure. Note that is the radius of curvature of the circular path.We know that for one complete revolution, the arc length is the circumference of a circle of radius r. The circumference of a circle is $2\pi r$.

Thus for one complete revolution the rotation angle is

$$\Delta\theta = \frac{2\pi r}{r} = 2\pi. \tag{5.2.2}$$

This result is the basis for defining the units used to measure rotation angles, $\Delta\theta$ to be **radians** (rad), defined so that

$$2\pi\, radians = 1\, revolution. \tag{5.2.3}$$

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A comparison of some useful angles expressed in both degrees and radians is shown in Table 5.2.1.

Table 5.2.1: Comparison in Angular Units

Degree Measure	Radian Measure
30^o	$\frac{\pi}{6}$
60^o	$\frac{\pi}{3}$
90^o	$\frac{\pi}{2}$
120^o	$\frac{2\pi}{3}$
135^o	$\frac{3\pi}{4}$
180^o	π

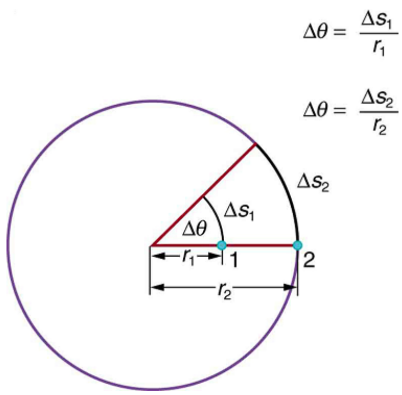

Figure 5.2.3: Points 1 and 2 rotate through the same angle $(\Delta\theta)$, but point 2 moves through a greater arc length (Δs) because it is at a greater distance from the center of rotation (r).

If $\Delta\theta = 2\pi\, rad$, then the CD has made one complete revolution, and every point on the CD is back at its original position. Because there are 360^o in a circle or one revolution, the relationship between radians and degrees is thus

$$2\pi\, rad = 360^o \tag{5.2.4}$$

so that

$$1\, rad = \frac{360^o}{2\pi} \approx 57.3^o \tag{5.2.5}$$

Angular Velocity

How fast is an object rotating? We define angular velocity ω as the rate of change of an angle. In symbols, this is

$$\omega = \frac{\Delta\theta}{\Delta t}, \tag{5.2.6}$$

where an angular rotation $\Delta\theta$ takes place in a time Δt. The greater the rotation angle in a given amount of time, the greater the angular velocity. The units for angular velocity are radians per second (rad/s).Angular velocity ω is analogous to linear velocity v. To get the precise relationship between angular and linear velocity, we again consider a pit on the rotating CD. This pit moves an arc length Δs in a time Δt, and so it has a linear velocity

$$v = \frac{\Delta s}{\Delta t}. \tag{5.2.7}$$

From $\Delta\theta = \frac{\Delta s}{r}$ we see that $\Delta s = r\Delta\theta$. Substituting this into the expression for v gives

$$v = \frac{r\Delta\theta}{\Delta t} = r\omega. \tag{5.2.8}$$

We write this relationship in two different ways and gain two different insights:

$$v = r\omega, \; or \; \omega = \frac{v}{r}. \tag{5.2.9}$$

The first relationship in $v = r\omega, \; or \; \omega = \frac{v}{r}$ states that the linear velocity v is proportional to the distance from the center of rotation, thus, it is largest for a point on the rim (largest r), as you might expect. We can also call this linear speed v of a point on the rim the ***tangential speed***. The second relationship in $v = r\omega, \; or \; \omega = \frac{v}{r}$ can be illustrated by considering the tire of a moving car. Note that the speed of a point on the rim of the tire is the same as the speed v of the car. See Figure So the faster the car moves, the faster the tire spins—large v means a large ω, because $v = r\omega$. Similarly, a larger-radius tire rotating at the same angular velocity (ω) will produce a greater linear speed (v) for the car.

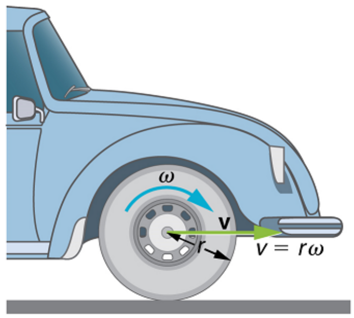

Figure 5.2.4: A car moving at a velocity v to the right has a tire rotating with an angular velocity ω. The speed of the tread of the tire relative to the axle is v, the same as if the car were jacked up. Thus the car moves forward at linear velocity $v = r\omega$, where r is the tire radius. A larger angular velocity for the tire means a greater velocity for the car.

Example 5.2.1: How Fast Does the Car Tire Spin?

Calculate the angular velocity of a 0.300 m radius car tire when the car travels at $15.0 m/s$ (about $54 \; km/h$). See Figure.

Strategy

Because the linear speed of the tire rim is the same as the speed of the car, we have $v = 15.0 m/s$. The radius of the tire is given to be $r = 0.300 \, m$. Knowing v and r, we can use the second relationship in $v = \omega r$, $\omega = \frac{v}{r}$ to calculate the angular velocity.

Solution

To calculate the angular velocity, we will use the following relationship:

$$\omega = \frac{v}{r}. \tag{5.2.10}$$

Substituting the knowns,

$$\omega = \frac{15.0 \, m/s}{0.300 \, m} = 50.0 \, rad/s. \tag{5.2.11}$$

Discussion

When we cancel units in the above calculation, we get 50.0/s. But the angular velocity must have units of rad/s. Because radians are actually unitless (radians are defined as a ratio of distance), we can simply insert them into the answer for the angular velocity. Also note that if an earth mover with much larger tires, say 1.20 m in radius, were moving at the same speed of 15.0 m/s, its tires would rotate more slowly. They would have an angular velocity

$$\omega = (15.0 \, m/s)/(1.20 \, m) = 12.5 \, rad/s. \tag{5.2.12}$$

Both ω and v have directions (hence they are angular and linear *velocities*, respectively). Angular velocity has only two directions with respect to the axis of rotation—it is either clockwise or counterclockwise. Linear velocity is tangent to the path, as illustrated in Figure.

TAKE-HOME EXPERIMENT

Tie an object to the end of a string and swing it around in a horizontal circle above your head (swing at your wrist). Maintain uniform speed as the object swings and measure the angular velocity of the motion. What is the approximate speed of the object? Identify a point close to your hand and take appropriate measurements to calculate the linear speed at this point. Identify other circular motions and measure their angular velocities.

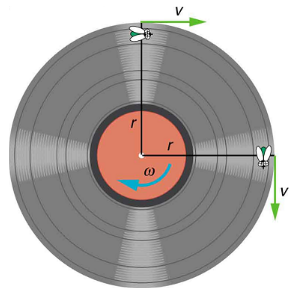

Figure 5.2.5: As an object moves in a circle, here a fly on the edge of an old-fashioned vinyl record, its instantaneous velocity is always tangent to the circle. The direction of the angular velocity is clockwise in this case.

PHET EXPLORATIONS: LADYBUG REVOLUTION

Join the ladybug in an exploration of rotational motion. Rotate the merry-go-round to change its angle, or choose a constant angular velocity or angular acceleration. Explore how circular motion relates to the bug's x,y position, velocity, and acceleration using vectors or graphs.

Section Summary

- Uniform circular motion is motion in a circle at constant speed. The rotation angle $\delta\theta$ is defined as the ratio of the arc length to the radius of curvature:

$$\Delta\theta = \frac{\Delta s}{r} \tag{5.2.13}$$

where arc length δs is distance traveled along a circular path and r is the radius of curvature of the circular path. The quantity $\Delta\theta$ is measured in units of radians (rad), for which

$$2\pi\, rad = 360^o = 1\, revolution. \tag{5.2.14}$$

- The conversion between radians and degrees is

$$1\, rad = 57.3^o. \tag{5.2.15}$$

- Angular velocity ω is the rate of change of an angle,

$$\omega = \frac{\Delta\theta}{\Delta t}, \tag{5.2.16}$$

where a rotation $\Delta\theta$ takes place in a time Δt. The units of angular velocity are radians per second (rad/s). Linear velocity v and angular velocity ω are related by

$$v = r\omega,\ or\, \omega = \frac{v}{r}. \tag{5.2.17}$$

Glossary

arc length

Δs, the distance traveled by an object along a circular path

pit

a tiny indentation on the spiral track moulded into the top of the polycarbonate layer of CD

rotation angle

the ratio of the arc length to the radius of curvature on a circular path: \(Δθ=\frac{Δs}{r}\)

radius of curvature

radius of a circular path

radians

a unit of angle measurement

angular velocity

ω, the rate of change of the angle with which an object moves on a circular path

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5.3: Centripetal Acceleration

Learning Objectives

By the end of this section, you will be able to:

- Establish the expression for centripetal acceleration.
- Explain the centrifuge.

We know from kinematics that acceleration is a change in velocity, either in its magnitude or in its direction, or both. In uniform circular motion, the direction of the velocity changes constantly, so there is always an associated acceleration, even though the magnitude of the velocity might be constant. You experience this acceleration yourself when you turn a corner in your car. (If you hold the wheel steady during a turn and move at constant speed, you are in uniform circular motion.) What you notice is a sideways acceleration because you and the car are changing direction. The sharper the curve and the greater your speed, the more noticeable this acceleration will become. In this section we examine the direction and magnitude of that acceleration.

Figure $5.3.1$ shows an object moving in a circular path at constant speed. The direction of the instantaneous velocity is shown at two points along the path. Acceleration is in the direction of the change in velocity, which points directly toward the center of rotation (the center of the circular path). This pointing is shown with the vector diagram in the figure. We call the acceleration of an object moving in uniform circular motion (resulting from a net external force) the **centripetal acceleration** a_c; centripetal means "toward the center" or "center seeking."

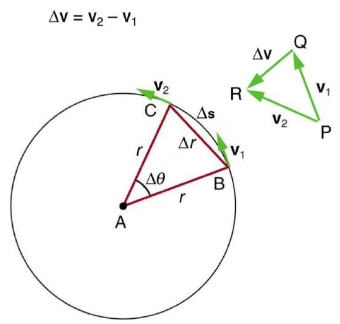

Figure $5.3.1$:The directions of the velocity of an object at two different points are shown, and the change in velocity Δv is seen to point directly toward the center of curvature. (See small inset.) Because $a_c = \Delta v/\Delta t$, the acceleration is also toward the center; a_c is called centripetal acceleration. (Because $\delta\theta$ is very small, the arc length Δs is equal to the chord length Δr for small time differences.)

The direction of centripetal acceleration is toward the center of curvature, but what is its magnitude? Note that the triangle formed by the velocity vectors and the one formed by the radii r and Δs are similar. Both the triangles ABC and PQR are isosceles triangles (two equal sides). The two equal sides of the velocity vector triangle are the speeds $v_1 = v_2 = v$ Using the properties of two similar triangles, we obtain

$$\frac{\Delta v}{v} = \frac{\Delta s}{r}. \tag{5.3.1}$$

Acceleration is $\Delta v/\Delta t$ and so we first solve this expression for δv:

$$\delta v = \frac{v}{r}\Delta s. \tag{5.3.2}$$

Then we divide this by Δt, yielding

$$\frac{\Delta v}{\Delta t} = \frac{v}{r} \times \frac{\Delta s}{\Delta t}. \tag{5.3.3}$$

Finally, noting that $\Delta v/\Delta t = a_c$ and that $\delta s/\Delta t = v$ the linear or tangential speed, we see that the magnitude of the centripetal acceleration is

$$a_c = \frac{v^2}{r}, \tag{5.3.4}$$

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which is the acceleration of an object in a circle of radius r at a speed v.

So, centripetal acceleration is greater at high speeds and in sharp curves (smaller radius), as you have noticed when driving a car. But it is a bit surprising that a_c is proportional to speed squared, implying, for example, that it is four times as hard to take a curve at 100 km/h than at 50 km/h. A sharp corner has a small radius, so that a_c is greater for tighter turns, as you have probably noticed.It is also useful to express a_c in terms of angular velocity. Substituting $v = r\omega$ into the above expression, we find $a_c = (r\omega^2)/r = r\omega^2$. We can express the magnitude of centripetal acceleration using either of two equations:

$$a_c = \frac{v^2}{r};\ a_c = r\omega^2 \tag{5.3.5}$$

Recall that the direction of a_c is toward the center. You may use whichever expression is more convenient, as illustrated in examples below.

A **centrifuge** (Figure $5.3.2b$) is a rotating device used to separate specimens of different densities. High centripetal acceleration significantly decreases the time it takes for separation to occur, and makes separation possible with small samples. Centrifuges are used in a variety of applications in science and medicine, including the separation of single cell suspensions such as bacteria, viruses, and blood cells from a liquid medium and the separation of macromolecules, such as DNA and protein, from a solution. Centrifuges are often rated in terms of their centripetal acceleration relative to acceleration due to gravity (g) maximum centripetal acceleration of several hundred thousand g is possible in a vacuum. Human centrifuges, extremely large centrifuges, have been used to test the tolerance of astronauts to the effects of accelerations larger than that of Earth's gravity.

Example $5.3.1$: Centripetal Acceleration vs. Gravity?

What is the magnitude of the centripetal acceleration of a car following a curve of radius 500 m at a speed of 25.0 m/s (about 90 km/h)? Compare the acceleration with that due to gravity for this fairly gentle curve taken at highway speed. See Figure $5.3.2a$.

Strategy

Because v and r are given, the first expression in $a_c = \frac{v^2}{r}$: $a_c = r\omega^2$ is the most convenient to use.

Solution

Entering the given values of $v = 25.0\, m/s and r = 500\, m$ into the first expression for a_c gives

$$a_c = \frac{v^2}{r} = \frac{(25.0 m/s)^2}{500\, m} = 1.25\, m/s^2.$$

Discussion

To compare this with the acceleration due to gravity $(g = 9.80\, m/s^2)$, we take the ratio of $a_c/g = (1.25\, m/s^2)/(9.80\, m/s^2) = 0.128$.Thus, $a_c = 0.128g$ and is noticeable especially if you were not wearing a seat belt.

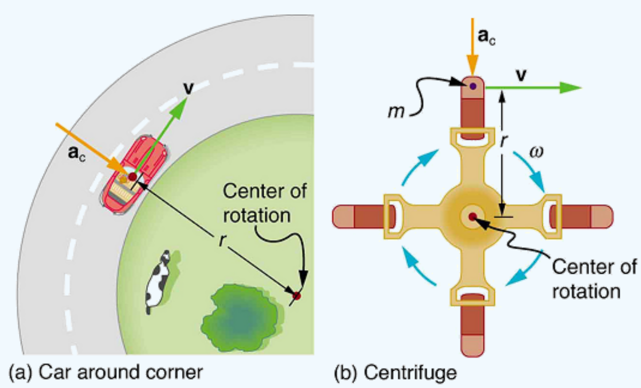

Figure $5.3.2$: (a) The car following a circular path at constant speed is accelerated perpendicular to its velocity, as shown. The magnitude of this centripetal acceleration is found in Example. (b) A particle of mass in a centrifuge is rotating at constant angular velocity . It must be accelerated perpendicular to its velocity or it would continue in a straight line. The magnitude of the necessary acceleration is found in Example.

Example 5.3.2: How Big is the Centripetal Acceleration in an Ultracentrifuge?

Calculate the centripetal acceleration of a point 7.50 cm from the axis of an **ultracentrifuge** spinning at $7.4\times10^7\,rev/min$. Determine the ratio of this acceleration to that due to gravity. See Figure Figure $5.3.2b$.

Strategy

The term rev/min stands for revolutions per minute. By converting this to radians per second, we obtain the angular velocity ω. Because r is given, we can use the second expression in the equation $a_c=\frac{v^2}{r};\, a_c=r\omega^2$ to calculate the centripetal acceleration.

Solution

To convert $7.40\times10^4\,rev/min$ to radians per second, we use the facts that one revolution is $2\pi\,rad$ and one minute is 60.0 s. Thus,

$$\omega=7.40\times10^4\,\frac{rev}{min}\times\frac{2\pi\,rad}{1\,rev}\times\frac{1\,min}{60\,sec}=7745\,rad/sec.$$

Now the centripetal acceleration is given by the second expression in $a_c=\frac{v^2}{r};\, a_c=r\omega^2$ as

$$a_c=r\omega^2.$$

Converting 7.50 cm to meters and substituting known values gives

$$a_c=(0.0750\,m)(7854\,rad/sec)^2=4.50\times10^6\,m/s^2.$$

Note that the unitless radians are discarded in order to get the correct units for centripetal acceleration. Taking the ratio of a_c to g yields

$$\frac{a_c}{g}=\frac{4.63\times10^6}{9.80}=4.59\times10^5.$$

Discussion

This last result means that the centripetal acceleration is 472,000 times as strong as g. It is no wonder that such high ω centrifuges are called ultracentrifuges. The extremely large accelerations involved greatly decrease the time needed to cause the sedimentation of blood cells or other materials.

Of course, a net external force is needed to cause any acceleration, just as Newton proposed in his second law of motion. So a net external force is needed to cause a centripetal acceleration. In the section on Centripetal Force, we will consider the forces involved in circular motion.

PHET EXPLORATIONS: LADYBUG MOTION 2D

Learn about position, velocity and acceleration vectors. Move the ladybug by setting the position, velocity or acceleration, and see how the vectors change. Choose linear, circular or elliptical motion, and record and playback the motion to analyze the behavior.

Summary

- Centripetal acceleration a_c is the acceleration experienced while in uniform circular motion. It always points toward the center of rotation. It is perpendicular to the linear velocity v and has the magnitude

$$a_c=\frac{v^2}{r};\, a_c=r\omega^2.$$

- The unit of centripetal acceleration is m/s^2.

Glossary

centripetal acceleration

the acceleration of an object moving in a circle, directed toward the center

ultracentrifuge

a centrifuge optimized for spinning a rotor at very high speeds

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5.4: Centripetal Force

Learning Objectives

By the end of this section, you will be able to:

- Calculate coefficient of friction on a car tire.
- Calculate ideal speed and angle of a car on a turn.

Any force or combination of forces can cause a centripetal or radial acceleration. Just a few examples are the tension in the rope on a tether ball, the force of Earth's gravity on the Moon, friction between roller skates and a rink floor, a banked roadway's force on a car, and forces on the tube of a spinning centrifuge.

Any net force causing uniform circular motion is called a **centripetal force**. The direction of a centripetal force is toward the center of curvature, the same as the direction of centripetal acceleration. According to Newton's second law of motion, net force is mass times acceleration: net $F = ma$. For uniform circular motion, the acceleration is the centripetal acceleration - $a = a_c$. Thus, the magnitude of centripetal force F_c is

$$F_c = ma_c. \tag{5.4.1}$$

By using the expressions for centripetal acceleration a_c from $a_c = \frac{v^2}{r};\ a_c = r\omega^2$, we get two expressions for the centripetal force F_c in terms of mass, velocity, angular velocity, and radius of curvature:

$$F_c = m\frac{v^2}{r};\ F_c = mr\omega^2. \tag{5.4.2}$$

You may use whichever expression for centripetal force is more convenient. Centripetal force F_c is always perpendicular to the path and pointing to the center of curvature, because a_c is perpendicular to the velocity and pointing to the center of curvature.Note that if you solve the first expression for r, you get

$$r = \frac{mv^2}{F_c}. \tag{5.4.3}$$

This implies that for a given mass and velocity, a large centripetal force causes a small radius of curvature—that is, a tight curve.

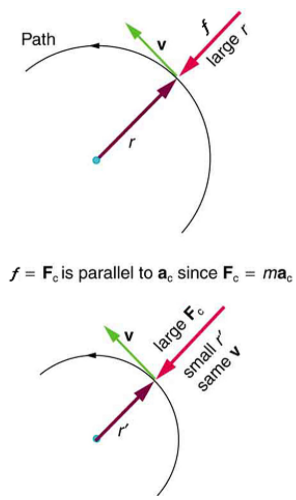

Figure 5.4.1:The frictional force supplies the centripetal force and is numerically equal to it. Centripetal force is perpendicular to velocity and causes uniform circular motion. The larger the F_c, the smaller the radius of curvature r and the sharper the curve. The second curve has the same v, but a larger F_c produces a smaller r'.

Example 5.4.1: What Coefficient of Friction Do Car Tires Need on a Flat Curve?

A. Calculate the centripetal force exerted on a 900 kg car that negotiates a 500 m radius curve at 25.0 m/s.
B. Assuming an unbanked curve, find the minimum static coefficient of friction, between the tires and the road, static friction being the reason that keeps the car from slipping (see Figure).

Strategy and Solution for (a)

We know that $F_c = \frac{mv^2}{r}$. Thus,

$$F_c = \frac{mv^2}{r} = \frac{(900\,kg)(25.0\,m/s)^2}{500\,m)} = 1125\,N.$$

Strategy for (b)

Figure shows the forces acting on the car on an unbanked (level ground) curve.

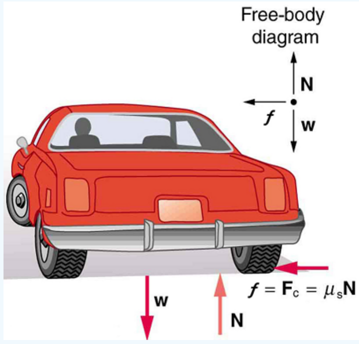

Figure 5.4.2: This car on level ground is moving away and turning to the left. The centripetal force causing the car to turn in a circular path is due to friction between the tires and the road. A minimum coefficient of friction is needed, or the car will move in a larger-radius curve and leave the roadway.

Friction is to the left, keeping the car from slipping, and because it is the only horizontal force acting on the car, the friction is the centripetal force in this case. We know that the maximum static friction (at which the tires roll but do not slip) is $\mu_s N$, where μ_s is the static coefficient of friction and N is the normal force. The normal force equals the car's weight on level ground,)so that $N = mg$. Thus the centripetal force in this situation is

$$F_c = f = \mu_s N = \mu_s mg.$$

Now we have a relationship between centripetal force and the coefficient of friction. Using the first expression for F_c from the equation

$$F_c = m\frac{v^2}{r}$$

$$= mr\omega^2$$

$$m\frac{v^2}{r} = \mu_s mg.$$

We solve this for μ_s, noting that mass cancels, and obtain

$$\mu_s = \frac{v^2}{rg}.$$

Solution for (b)

Substituting the knowns,

$$\mu_s = \frac{(25.0\,m/s)^2}{(500\,m)(9.80\,m/s^2)} = 0.13.$$

(Because coefficients of friction are approximate, the answer is given to only two digits.)

Discussion

We could also solve part (a) using the first expression in

$$F_c = m\frac{v^2}{r}$$
$$= mr\omega^2$$

because m, v and r are given. The coefficient of friction found in part (b) is much smaller than is typically found between tires and roads. The car will still negotiate the curve if the coefficient is greater than 0.13, because static friction is a responsive force, being able to assume a value less than but no more than $\mu_s N$. A higher coefficient would also allow the car to negotiate the curve at a higher speed, but if the coefficient of friction is less, the safe speed would be less than 25 m/s. Note that mass cancels, implying that in this example, it does not matter how heavily loaded the car is to negotiate the turn. Mass cancels because friction is assumed proportional to the normal force, which in turn is proportional to mass. If the surface of the road were banked, the normal force would be less as will be discussed below.

Let us now consider banked curves, where the slope of the road helps you negotiate the curve. See Figure. The greater the angle θ, the faster you can take the curve. Race tracks for bikes as well as cars, for example, often have steeply banked curves. In an "ideally banked curve," the angle θ is such that you can negotiate the curve at a certain speed without the aid of friction between the tires and the road. We will derive an expression for θ for an ideally banked curve and consider an example related to it.

For ideal banking, the net external force equals the horizontal centripetal force in the absence of friction. The components of the normal force N in the horizontal and vertical directions must equal the centripetal force and the weight of the car, respectively. In cases in which forces are not parallel, it is most convenient to consider components along perpendicular axes—in this case, the vertical and horizontal directions.

Figure shows a free body diagram for a car on a frictionless banked curve. If the angle θ is ideal for the speed and radius, then the net external force will equal the necessary centripetal force. The only two external forces acting on the car are its weight w and the normal force of the road N. (A frictionless surface can only exert a force perpendicular to the surface—that is, a normal force.) These two forces must add to give a net external force that is horizontal toward the center of curvature and has magnitude mv^2/r. Because this is the crucial force and it is horizontal, we use a coordinate system with vertical and horizontal axes. Only the normal force has a horizontal component, and so this must equal the centripetal force—that is,

$$N \sin\theta = \frac{mv^2}{r}. \tag{5.4.4}$$

Because the car does not leave the surface of the road, the net vertical force must be zero, meaning that the vertical components of the two external forces must be equal in magnitude and opposite in direction. From the figure, we see that the vertical component of the normal force is $N\, cos\,\theta$, and the only other vertical force is the car's weight. These must be equal in magnitude; thus,

$$N \cos\theta = mg. \tag{5.4.5}$$

Now we can combine the last two equations to eliminate N and get an expression for θ, as desired. Solving the second equation for $N = mg/(cos\,\theta)$, and substituting this into the first yields

$$mg\frac{\sin\theta}{\cos\theta} = \frac{mv^2}{r} \tag{5.4.6}$$

$$mg\,tan\,\theta = \frac{mv^2}{r} \tag{5.4.7}$$

$$\tan\theta = \frac{v^2}{rg} \tag{5.4.8}$$

Taking the inverse tangent gives

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)\,(ideally\,banked\,curve,\;no\,friction). \tag{5.4.9}$$

This expression can be understood by considering how θ depends on v and r. A large θ will be obtained for a large v and a small r. That is, roads must be steeply banked for high speeds and sharp curves. Friction helps, because it allows you to take the curve at greater or lower speed than if the curve is frictionless. Note that θ does not depend on the mass of the vehicle.

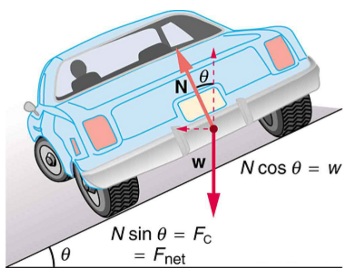

Figure 5.4.3: The car on this banked curve is moving away and turning to the left.

Example 5.4.2: What is the Ideal Speed to Take a Steeply Banked Tight Curve?

Curves on some test tracks and race courses, such as the Daytona International Speedway in Florida, are very steeply banked. This banking, with the aid of tire friction and very stable car configurations, allows the curves to be taken at very high speed. To illustrate, calculate the speed at which a 100 m radius curve banked at 65.0° should be driven if the road is frictionless.

Strategy

We first note that all terms in the expression for the ideal angle of a banked curve except for speed are known; thus, we need only rearrange it so that speed appears on the left-hand side and then substitute known quantities.

Solution

Starting with

$$\tan\theta = \frac{v^2}{rg}$$

we get

$$v = (rg tan\,\theta)^{\frac{1}{2}}.$$

Noting that tan 65.0º = 2.14, we obtain

$$\begin{aligned} v &= [(100\,m)(9.80\,m/s^2)(2.14)]^{\frac{1}{2}} \\ &= 45.8\,m/s \end{aligned}$$

Discussion

This is just about 165 km/h, consistent with a very steeply banked and rather sharp curve. Tire friction enables a vehicle to take the curve at significantly higher speeds.

Calculations similar to those in the preceding examples can be performed for a host of interesting situations in which centripetal force is involved—a number of these are presented in this chapter’s Problems and Exercises.

TAKE-HOME EXPERIMENT

Ask a friend or relative to swing a golf club or a tennis racquet. Take appropriate measurements to estimate the centripetal acceleration of the end of the club or racquet. You may choose to do this in slow motion.

PHET EXPLORATIONS: GRAVITY AND ORBITS

Move the sun, earth, moon and space station to see how it affects their gravitational forces and orbital paths. Visualize the sizes and distances between different heavenly bodies, and turn off gravity to see what would happen without it!

Summary

- Centripetal force F_c is any force causing uniform circular motion. It is a “center-seeking” force that always points toward the center of rotation. It is perpendicular to linear velocity v and has magnitude

$$F_c = ma_c$$

which can also be expressed as

$$F_c = \frac{v^2}{r}$$

or

$$F_c = mr\omega^2$$

Glossary

centripetal force
any net force causing uniform circular motion

ideal banking
the sloping of a curve in a road, where the angle of the slope allows the vehicle to negotiate the curve at a certain speed without the aid of friction between the tires and the road; the net external force on the vehicle equals the horizontal centripetal force in the absence of friction

ideal speed
the maximum safe speed at which a vehicle can turn on a curve without the aid of friction between the tire and the road

ideal angle
the angle at which a car can turn safely on a steep curve, which is in proportion to the ideal speed

banked curve
the curve in a road that is sloping in a manner that helps a vehicle negotiate the curve

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5.5: Fictitious Forces and Non-inertial Frames - The Coriolis Force

Learning Objectives

By the end of this section, you will be able to:

- Discuss the inertial frame of reference.
- Discuss the non-inertial frame of reference.
- Describe the effects of the Coriolis force.

What do taking off in a jet airplane, turning a corner in a car, riding a merry-go-round, and the circular motion of a tropical cyclone have in common? Each exhibits fictitious forces—unreal forces that arise from motion and may *seem* real, because the observer's frame of reference is accelerating or rotating.

When taking off in a jet, most people would agree it feels as if you are being pushed back into the seat as the airplane accelerates down the runway. Yet a physicist would say that *you* tend to remain stationary while the *seat* pushes forward on you, and there is no real force backward on you. An even more common experience occurs when you make a tight curve in your car—say, to the right. You feel as if you are thrown (that is, *forced*) toward the left relative to the car. Again, a physicist would say that *you* are going in a straight line but the *car* moves to the right, and there is no real force on you to the left. Recall Newton's first law.

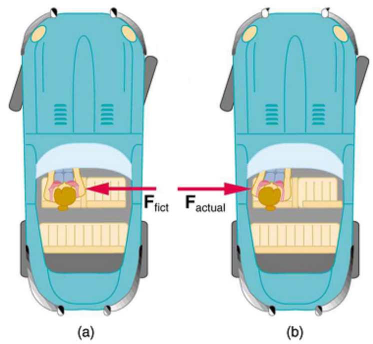

Figure 5.5.1: (a) The car driver feels herself forced to the left relative to the car when she makes a right turn. This is a fictitious force arising from the use of the car as a frame of reference. (b) In the Earth's frame of reference, the driver moves in a straight line, obeying Newton's first law, and the car moves to the right. There is no real force to the left on the driver relative to Earth. There is a real force to the right on the car to make it turn.

We can reconcile these points of view by examining the frames of reference used. Let us concentrate on people in a car. Passengers instinctively use the car as a frame of reference, while a physicist uses Earth. The physicist chooses Earth because it is very nearly an inertial frame of reference—one in which all forces are real (that is, in which all forces have an identifiable physical origin). In such a frame of reference, Newton's laws of motion take the form given in Dynamics: Newton's Laws of Motion. The car is a **non-inertial frame of reference** because it is accelerated to the side. The force to the left sensed by car passengers is a **fictitious** force having no physical origin. There is nothing real pushing them left—the car, as well as the driver, is actually accelerating to the right.

Let us now take a mental ride on a merry-go-round—specifically, a rapidly rotating playground merry-go-round. You take the merry-go-round to be your frame of reference because you rotate together. In that non-inertial frame, you feel a fictitious force, named **centrifugal force (**not to be confused with centripetal force**)**, trying to throw you off. You must hang on tightly to counteract the centrifugal force. In Earth's frame of reference, there is no force trying to throw you off. Rather you must hang on to make yourself go in a circle because otherwise you would go in a straight line, right off the merry-go-round.

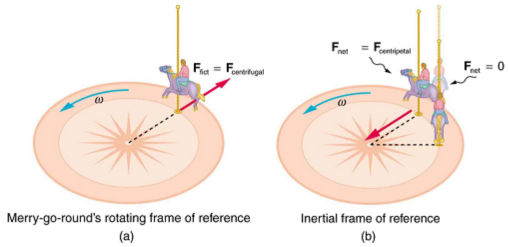

Figure 5.5.2: (a) A rider on a merry-go-round feels as if he is being thrown off. This fictitious force is called the centrifugal force—it explains the rider's motion in the rotating frame of reference. (b) In an inertial frame of reference and according to Newton's laws, it is his inertia that carries him off and not a real force (the unshaded rider has $F_{net} = 0$ and heads in a straight line). A real force, $F_{centripetal}$, is needed to cause a circular path.

This inertial effect, carrying you away from the center of rotation if there is no centripetal force to cause circular motion, is put to good use in centrifuges (see Figure). A centrifuge spins a sample very rapidly, as mentioned earlier in this chapter. Viewed from the rotating frame of reference, the fictitious centrifugal force throws particles outward, hastening their sedimentation. The greater the angular velocity, the greater the centrifugal force. But what really happens is that the inertia of the particles carries them along a line tangent to the circle while the test tube is forced in a circular path by a centripetal force.

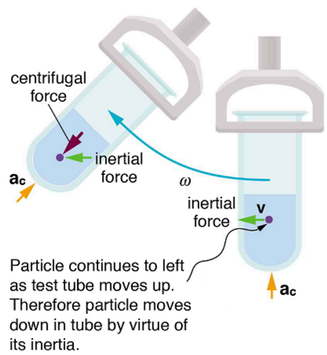

Figure 5.5.3: Centrifuges use inertia to perform their task. Particles in the fluid sediment come out because their inertia carries them away from the center of rotation. The large angular velocity of the centrifuge quickens the sedimentation. Ultimately, the particles will come into contact with the test tube walls, which will then supply the centripetal force needed to make them move in a circle of constant radius.

Let us now consider what happens if something moves in a frame of reference that rotates. For example, what if you slide a ball directly away from the center of the merry-go-round, as shown in Figure? The ball follows a straight path relative to Earth (assuming negligible friction) and a path curved to the right on the merry-go-round's surface. A person standing next to the merry-go-round sees the ball moving straight and the merry-go-round rotating underneath it. In the merry-go-round's frame of reference, we explain the apparent curve to the right by using a fictitious force, called the **Coriolis** force, that causes the ball to curve to the right. The fictitious Coriolis force can be used by anyone in that frame of reference to explain why objects follow curved paths and allows us to apply Newton's Laws in non-inertial frames of reference.

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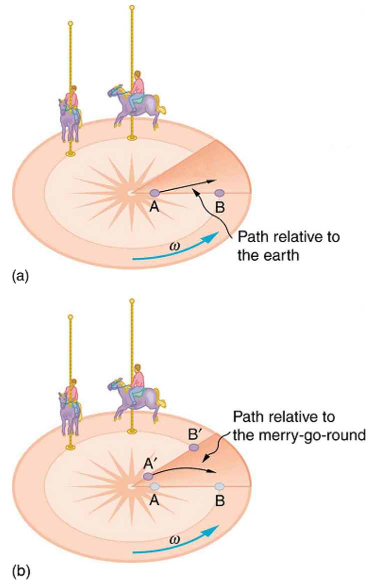

Figure 5.5.4:. Looking down on the counterclockwise rotation of a merry-go-round, we see that a ball slid straight toward the edge follows a path curved to the right. The person slides the ball toward point B, starting at point A. Both points rotate to the shaded positions (A’ and B’) shown in the time that the ball follows the curved path in the rotating frame and a straight path in Earth’s frame.

Up until now, we have considered Earth to be an inertial frame of reference with little or no worry about effects due to its rotation. Yet such effects *do* exist—in the rotation of weather systems, for example. Most consequences of Earth’s rotation can be qualitatively understood by analogy with the merry-go-round. Viewed from above the North Pole, Earth rotates counterclockwise, as does the merry-go-round in Figure. As on the merry-go-round, any motion in Earth’s northern hemisphere experiences a Coriolis force to the right. Just the opposite occurs in the southern hemisphere; there, the force is to the left. Because Earth’s angular velocity is small, the Coriolis force is usually negligible, but for large-scale motions, such as wind patterns, it has substantial effects.

The Coriolis force causes hurricanes in the northern hemisphere to rotate in the counterclockwise direction, while the tropical cyclones (what hurricanes are called below the equator) in the southern hemisphere rotate in the clockwise direction. The terms hurricane, typhoon, and tropical storm are regionally-specific names for tropical cyclones, storm systems characterized by low pressure centers, strong winds, and heavy rains. Figure helps show how these rotations take place. Air flows toward any region of low pressure, and tropical cyclones contain particularly low pressures. Thus winds flow toward the center of a tropical cyclone or a low-pressure weather system at the surface. In the northern hemisphere, these inward winds are deflected to the right, as shown in the figure, producing a counterclockwise circulation at the surface for low-pressure zones of any type. Low pressure at the surface is associated with rising air, which also produces cooling and cloud formation, making low-pressure patterns quite visible from space. Conversely, wind circulation around high-pressure zones is clockwise in the northern hemisphere but is less visible because high pressure is associated with sinking air, producing clear skies.

The rotation of tropical cyclones and the path of a ball on a merry-go-round can just as well be explained by inertia and the rotation of the system underneath. When non-inertial frames are used, fictitious forces, such as the Coriolis force, must be invented to explain the curved path. There is no identifiable physical source for these fictitious forces. In an inertial frame, inertia explains the path, and no force is found to be without an identifiable source. Either view allows us to describe nature, but a view in an inertial frame is the simplest and truest, in the sense that all forces have real origins and explanations.

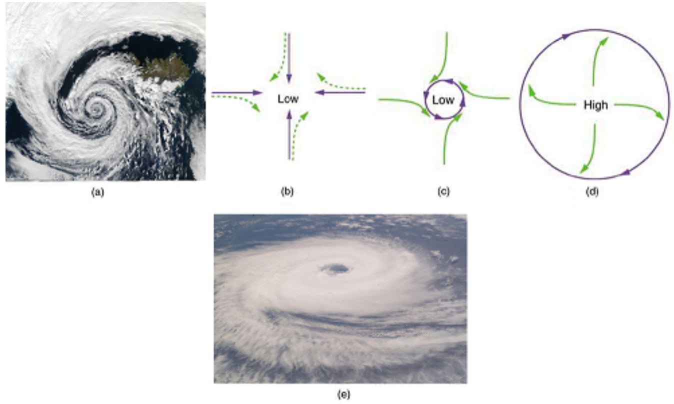

Figure \(\PageIndex{5}\): (a) The counterclockwise rotation of this northern hemisphere hurricane is a major consequence of the Coriolis force. (credit: NASA) (b) Without the Coriolis force, air would flow straight into a low-pressure zone, such as that found in tropical cyclones. (c) The Coriolis force deflects the winds to the right, producing a counterclockwise rotation. (d) Wind flowing away from a high-pressure zone is also deflected to the right, producing a clockwise rotation. (e) The opposite direction of rotation is produced by the Coriolis force in the southern hemisphere, leading to tropical cyclones. (credit: NASA)

Section Summary

- Rotating and accelerated frames of reference are non-inertial.
- Fictitious forces, such as the Coriolis force, are needed to explain motion in such frames.

Glossary

fictitious force
: a force having no physical origin

centrifugal force
: a fictitious force that tends to throw an object off when the object is rotating in a non-inertial frame of reference

Coriolis force
: the fictitious force causing the apparent deflection of moving objects when viewed in a rotating frame of reference

non-inertial frame of reference
: an accelerated frame of reference

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5.6: Newton’s Universal Law of Gravitation

Learning Objectives

By the end of this section, you will be able to:

- Explain Earth’s gravitational force.
- Describe the gravitational effect of the Moon on Earth.
- Discuss weightlessness in space.
- Examine the Cavendish experiment

What do aching feet, a falling apple, and the orbit of the Moon have in common? Each is caused by the gravitational force. Our feet are strained by supporting our weight—the force of Earth’s gravity on us. An apple falls from a tree because of the same force acting a few meters above Earth’s surface. And the Moon orbits Earth because gravity is able to supply the necessary centripetal force at a distance of hundreds of millions of meters. In fact, the same force causes planets to orbit the Sun, stars to orbit the center of the galaxy, and galaxies to cluster together. Gravity is another example of underlying simplicity in nature. It is the weakest of the four basic forces found in nature, and in some ways the least understood. It is a force that acts at a distance, without physical contact, and is expressed by a formula that is valid everywhere in the universe, for masses and distances that vary from the tiny to the immense.

Sir Isaac Newton was the first scientist to precisely define the gravitational force, and to show that it could explain both falling bodies and astronomical motions. See Figure. But Newton was not the first to suspect that the same force caused both our weight and the motion of planets. His forerunner Galileo Galilei had contended that falling bodies and planetary motions had the same cause. Some of Newton’s contemporaries, such as Robert Hooke, Christopher Wren, and Edmund Halley, had also made some progress toward understanding gravitation. But Newton was the first to propose an exact mathematical form and to use that form to show that the motion of heavenly bodies should be conic sections—circles, ellipses, parabolas, and hyperbolas. This theoretical prediction was a major triumph—it had been known for some time that moons, planets, and comets follow such paths, but no one had been able to propose a mechanism that caused them to follow these paths and not others.

Figure 5.6.1: According to early accounts, Newton was inspired to make the connection between falling bodies and astronomical motions when he saw an apple fall from a tree and realized that if the gravitational force could extend above the ground to a tree, it might also reach the Sun. The inspiration of Newton’s apple is a part of worldwide folklore and may even be based in fact. Great importance is attached to it because Newton’s universal law of gravitation and his laws of motion answered very old questions about nature and gave tremendous support to the notion of underlying simplicity and unity in nature. Scientists still expect underlying simplicity to emerge from their ongoing inquiries into nature.

The gravitational force is relatively simple. It is always attractive, and it depends only on the masses involved and the distance between them. Stated in modern language, Newton’s universal law of gravitation states that every particle in the universe attracts every other particle with a force along a line joining them. The force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

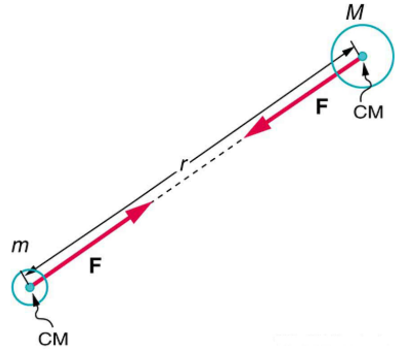

Figure 5.6.2: Gravitational attraction is along a line joining the centers of mass of these two bodies. The magnitude of the force is the same on each, consistent with Newton's third law.

MISCONCEPT ALERT

The magnitude of the force on each object (one has larger mass than the other) is the same, consistent with Newton's third law.

The bodies we are dealing with tend to be large. To simplify the situation we assume that the body acts as if its entire mass is concentrated at one specific point called the **center of mass** (CM), which will be further explored in Linear Momentum and Collisions. For two bodies having masses m and M with a distance r between their centers of mass, the equation for Newton's universal law of gravitation is

$$F=G\frac{mM}{r^2}, \tag{5.6.1}$$

where F is the magnitude of the gravitational force and G is a proportionality factor called the gravitational constant. G is a universal gravitational constant—that is, it is thought to be the same everywhere in the universe. It has been measured experimentally to be

$$G=6.673\times10^{-11}\frac{N\cdot m^2}{kg^2} \tag{5.6.2}$$

in SI units. Note that the units of G are such that a force in newtons is obtained from $F=G\frac{mM}{r^2}$, when considering masses in kilograms and distance in meters. For example, two 1.000 kg masses separated by 1.000 m will experience a gravitational attraction of $6.673\times10^{-11}\ N$.

This is an extraordinarily small force. The small magnitude of the gravitational force is consistent with everyday experience. We are unaware that even large objects like mountains exert gravitational forces on us. In fact, our body weight is the force of attraction of the *entire Earth* on us with a mass of $6\times10^{24}\ kg$.

Recall that the acceleration due to gravity *g* is about $9.80\ m/s^2$ on Earth. We can now determine why this is so. The weight of an object *mg* is the gravitational force between it and Earth. Substituting *mg* for F in Newton's universal law of gravitation gives

$$mg=G\frac{mM}{r^2}, \tag{5.6.3}$$

where m is the mass of the object, M is the mass of Earth, and r is the distance to the center of Earth (the distance between the centers of mass of the object and Earth). See Figure. The mass m of the object cancels, leaving an equation for *g*:

$$g=G\frac{M}{r^2}. \tag{5.6.4}$$

Substituting known values for Earth's mass and radius (to three significant figures),

$$g=\left(6.673\times10^{-11}\frac{N\cdot m^2}{kg^2}\right)\times\frac{5.98\times10^{24}\ kg}{(6.38\times10^6\ m)^2}, \tag{5.6.5}$$

and we obtain a value for the acceleration of a falling body:

$$g = 9.80\,m/s^2. \tag{5.6.6}$$

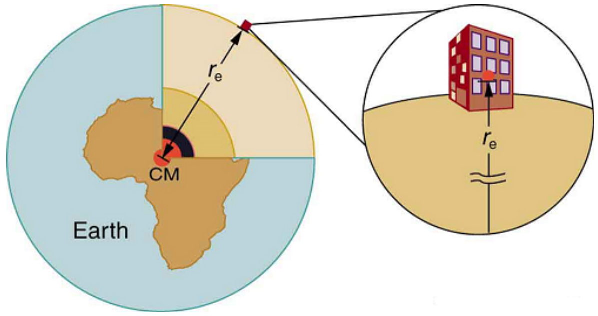

Figure 5.6.3: The distance between the centers of mass of Earth and an object on its surface is very nearly the same as the radius of Earth, because Earth is so much larger than the object.

This is the expected value *and is independent of the body's mass*. Newton's law of gravitation takes Galileo's observation that all masses fall with the same acceleration a step further, explaining the observation in terms of a force that causes objects to fall—in fact, in terms of a universally existing force of attraction between masses.

TAKE HOME EXPERIMENT

Take a marble, a ball, and a spoon and drop them from the same height. Do they hit the floor at the same time? If you drop a piece of paper as well, does it behave like the other objects? Explain your observations.

MAKING CONNECTIONS

Attempts are still being made to understand the gravitational force. As we shall see in Particle Physics, modern physics is exploring the connections of gravity to other forces, space, and time. General relativity alters our view of gravitation, leading us to think of gravitation as bending space and time.

In the following example, we make a comparison similar to one made by Newton himself. He noted that if the gravitational force caused the Moon to orbit Earth, then the acceleration due to gravity should equal the centripetal acceleration of the Moon in its orbit. Newton found that the two accelerations agreed "pretty nearly."

Example 5.6.1: Earth's Gravitational Force Is the Centripetal Force Making the Moon Move in a Curved Path

a. Find the acceleration due to Earth's gravity at the distance of the Moon.
b. Calculate the centripetal acceleration needed to keep the Moon in its orbit (assuming a circular orbit about a fixed Earth), and compare it with the value of the acceleration due to Earth's gravity that you have just found.

Strategy for (a)

This calculation is the same as the one finding the acceleration due to gravity at Earth's surface, except that r is the distance from the center of Earth to the center of the Moon. The radius of the Moon's nearly circular orbit is $3.84 \times 10^8\ m$.

Solution for (a)

Substituting known values into the expression for g found above, remembering that M is the mass of Earth not the Moon, yields

$$g = G\frac{M}{r^2} = \left(6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}\right) \times \frac{5.98 \times 10^{24}\,kg}{3.84 \times 10^8\ m)^2} \tag{5.6.7}$$

$$= 2.70 \times 10^{-3}\ m/s^2. \tag{5.6.8}$$

Strategy for (b)

Centripetal acceleration can be calculated using either form of

$$a_c = \frac{v^2}{r} \tag{5.6.9}$$

$$a_c = r\omega^2 \tag{5.6.10}$$

We choose to use the second form:

$$a_c = r\omega^2, \tag{5.6.11}$$

where ω is the angular velocity of the Moon about Earth.

Solution for (b)

Given that the period (the time it takes to make one complete rotation) of the Moon's orbit is 27.3 days, (d) and using

$$1\,d \times 24\frac{hr}{d} \times 60\frac{min}{hr} \times 60\frac{s}{min} = 86,400\,s \tag{5.6.12}$$

we see that

$$a_c = r\omega^2 = (3.84 \times 10^8\,m)(2.66 \times 10^{-6}\,rad/s^2) \tag{5.6.13}$$

$$= 2.72 \times 10^{-3}\,m/s^2. \tag{5.6.14}$$

The direction of the acceleration is toward the center of the Earth.

Discussion

The centripetal acceleration of the Moon found in (b) differs by less than 1% from the acceleration due to Earth's gravity found in (a). This agreement is approximate because the Moon's orbit is slightly elliptical, and Earth is not stationary (rather the Earth-Moon system rotates about its center of mass, which is located some 1700 km below Earth's surface). The clear implication is that Earth's gravitational force causes the Moon to orbit Earth.

Why does Earth not remain stationary as the Moon orbits it? This is because, as expected from Newton's third law, if Earth exerts a force on the Moon, then the Moon should exert an equal and opposite force on Earth (see Figure). We do not sense the Moon's effect on Earth's motion, because the Moon's gravity moves our bodies right along with Earth but there are other signs on Earth that clearly show the effect of the Moon's gravitational force as discussed in Satellites and Kepler's Laws: An Argument for Simplicity.

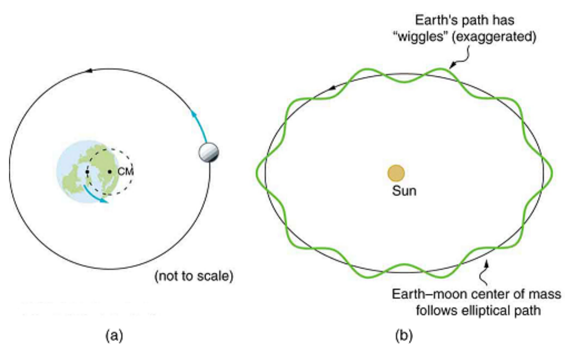

Figure 5.6.4: (a) Earth and the Moon rotate approximately once a month around their common center of mass. (b) Their center of mass orbits the Sun in an elliptical orbit, but Earth's path around the Sun has "wiggles" in it. Similar wiggles in the paths of stars have been observed and are considered direct evidence of planets orbiting those stars. This is important because the planets' reflected light is often too dim to be observed.

Tides

Ocean tides are one very observable result of the Moon's gravity acting on Earth. Figure is a simplified drawing of the Moon's position relative to the tides. Because water easily flows on Earth's surface, a high tide is created on the side of Earth nearest to the Moon, where the Moon's gravitational pull is strongest. Why is there also a high tide on the opposite side of Earth? The answer is that Earth is pulled toward the Moon more than the water on the far side, because Earth is closer to the Moon. So the water on the side of Earth closest to the Moon is pulled away from Earth, and Earth is pulled away from water on the far side. As Earth rotates, the tidal bulge (an effect of the tidal forces between an orbiting natural satellite and the primary planet that it orbits) keeps its orientation with the Moon. Thus there are two tides per day (the actual tidal period is about 12 hours and 25.2 minutes), because the Moon moves in its orbit each day as well).

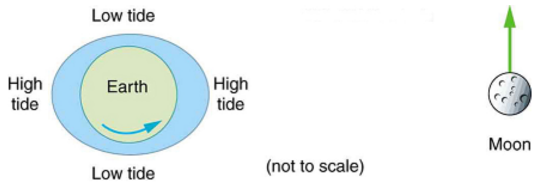

Figure 5.6.5: The Moon causes ocean tides by attracting the water on the near side more than Earth, and by attracting Earth more than the water on the far side. The distances and sizes are not to scale. For this simplified representation of the Earth-Moon system, there are two high and two low tides per day at any location, because Earth rotates under the tidal bulge.

The Sun also affects tides, although it has about half the effect of the Moon. However, the largest tides, called spring tides, occur when Earth, the Moon, and the Sun are aligned. The smallest tides, called neap tides, occur when the Sun is at a 90^o angle to the Earth-Moon alignment.

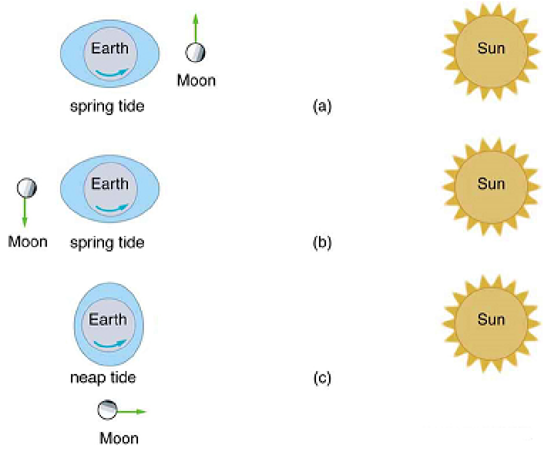

Figure 5.6.6: (a, b) Spring tides: The highest tides occur when Earth, the Moon, and the Sun are aligned. (c) Neap tide: The lowest tides occur when the Sun lies at 90^o to the Earth-Moon alignment. Note that this figure is not drawn to scale.

Tides are not unique to Earth but occur in many astronomical systems. The most extreme tides occur where the gravitational force is the strongest and varies most rapidly, such as near black holes (see Figure). A few likely candidates for black holes have been observed in our galaxy. These have masses greater than the Sun but have diameters only a few kilometers across. The tidal forces near them are so great that they can actually tear matter from a companion star.

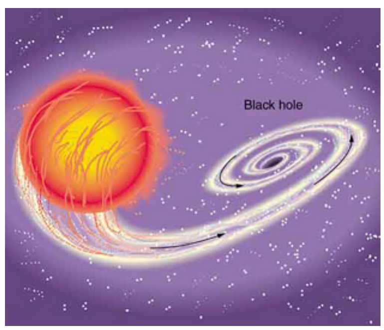

Figure 5.6.7: A black hole is an object with such strong gravity that not even light can escape it. This black hole was created by the supernova of one star in a two-star system. The tidal forces created by the black hole are so great that it tears matter from the companion star. This matter is compressed and heated as it is sucked into the black hole, creating light and X-rays observable from Earth.

"Weightlessness" and Microgravity

In contrast to the tremendous gravitational force near black holes is the apparent gravitational field experienced by astronauts orbiting Earth. What is the effect of "weightlessness" upon an astronaut who is in orbit for months? Or what about the effect of weightlessness upon plant growth? Weightlessness doesn't mean that an astronaut is not being acted upon by the gravitational force. There is no "zero gravity" in an astronaut's orbit. The term just means that the astronaut is in free-fall, accelerating with the acceleration due to gravity. If an elevator cable breaks, the passengers inside will be in free fall and will experience weightlessness. You can experience short periods of weightlessness in some rides in amusement parks.

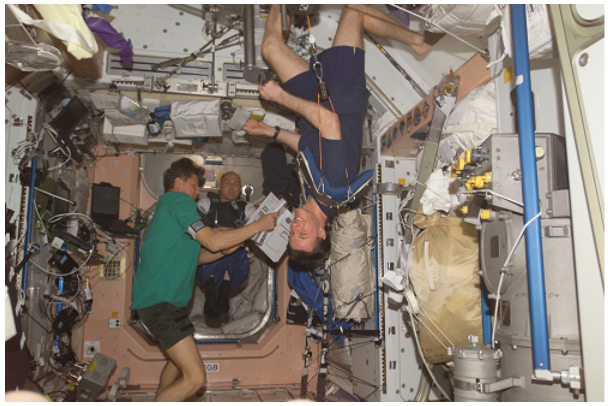

Figure 5.6.8: Astronauts experiencing weightlessness on board the International Space Station. (credit: NASA).

Microgravity refers to an environment in which the apparent net acceleration of a body is small compared with that produced by Earth at its surface. Many interesting biology and physics topics have been studied over the past three decades in the presence of microgravity. Of immediate concern is the effect on astronauts of extended times in outer space, such as at the International Space Station. Researchers have observed that muscles will atrophy (waste away) in this environment. There is also a corresponding loss of bone mass. Study continues on cardiovascular adaptation to space flight. On Earth, blood pressure is usually higher in the feet than in the head, because the higher column of blood exerts a downward force on it, due to gravity. When standing, 70% of your blood is below the level of the heart, while in a horizontal position, just the opposite occurs. What difference does the absence of this pressure differential have upon the heart?

Some findings in human physiology in space can be clinically important to the management of diseases back on Earth. On a somewhat negative note, spaceflight is known to affect the human immune system, possibly making the crew members more vulnerable to infectious diseases. Experiments flown in space also have shown that some bacteria grow faster in microgravity than they do on Earth. However, on a positive note, studies indicate that microbial antibiotic production can increase by a factor of two in space-grown cultures. One hopes to be able to understand these mechanisms so that similar successes can be achieved on the ground. In another area of physics space research, inorganic crystals and protein crystals have been grown in outer space that have much higher quality than any grown on Earth, so crystallography studies on their structure can yield much better results.

Plants have evolved with the stimulus of gravity and with gravity sensors. Roots grow downward and shoots grow upward. Plants might be able to provide a life support system for long duration space missions by regenerating the atmosphere, purifying water, and producing food. Some studies have indicated that plant growth and development are not affected by gravity, but there is still uncertainty about structural changes in plants grown in a microgravity environment.

The Cavendish Experiment: Then and Now

As previously noted, the universal gravitational constant G is determined experimentally. This definition was first done accurately by Henry Cavendish (1731–1810), an English scientist, in 1798, more than 100 years after Newton published his universal law of gravitation. The measurement of G is very basic and important because it determines the strength of one of the four forces in nature. Cavendish's experiment was very difficult because he measured the tiny gravitational attraction between two ordinary-sized masses (tens of kilograms at most), using apparatus like that in Figure. Remarkably, his value for G differs by less than 1% from the best modern value.One important consequence of knowing G was that an accurate value for Earth's mass could finally be obtained. This was done by measuring the acceleration due to gravity as accurately as possible and then calculating the mass of Earth M from the relationship Newton's universal law of gravitation gives

$$mg = G\frac{mM}{r^2}, \tag{5.6.15}$$

where m is the mass of the object, M is the mass of Earth, and r is the distance to the center of Earth (the distance between the centers of mass of the object and Earth). See Figure. The mass m of the object cancels, leaving an equation for g:

$$g = G\frac{M}{r^2}. \tag{5.6.16}$$

Rearranging to solve for M yields

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$$M = \frac{gr^2}{G}. \tag{5.6.17}$$

so M can be calculated because all quantities on the right, including the radius of Earth r, are known from direct measurements. We shall see in Satellites and Kepler's Laws: An Argument for Simplicity that knowing G also allows for the determination of astronomical masses. Interestingly, of all the fundamental constants in physics, G is by far the least well determined.

The Cavendish experiment is also used to explore other aspects of gravity. One of the most interesting questions is whether the gravitational force depends on substance as well as mass—for example, whether one kilogram of lead exerts the same gravitational pull as one kilogram of water. A Hungarian scientist named Roland von Eötvös pioneered this inquiry early in the 20th century. He found, with an accuracy of five parts per billion, that the gravitational force does not depend on the substance. Such experiments continue today, and have improved upon Eötvös' measurements. Cavendish-type experiments such as those of Eric Adelberger and others at the University of Washington, have also put severe limits on the possibility of a fifth force and have verified a major prediction of general relativity—that gravitational energy contributes to rest mass. Ongoing measurements there use a torsion balance and a parallel plate (not spheres, as Cavendish used) to examine how Newton's law of gravitation works over sub-millimeter distances. On this small-scale, do gravitational effects depart from the inverse square law? So far, no deviation has been observed.

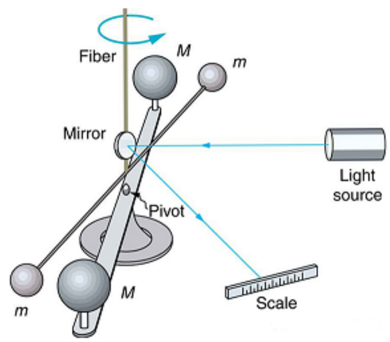

Figure 5.6.9: Cavendish used an apparatus like this to measure the gravitational attraction between the two suspended spheres (m) and the two on the stand (M) by observing the amount of torsion (twisting) created in the fiber. Distance between the masses can be varied to check the dependence of the force on distance. Modern experiments of this type continue to explore gravity.

Summary

- Newton's universal law of gravitation: Every particle in the universe attracts every other particle with a force along a line joining them. The force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. In equation form, this is

$$F = G\frac{mM}{r^2} \tag{5.6.18}$$

  where F is the magnitude of the gravitational force. G is the gravitational constant, given by $G = 6.63 \times 10^{-11}\ N \cdot m^2/kg^2$.

- Newton's law of gravitation applies universally.

Glossary

gravitational constant, *G*

a proportionality factor used in the equation for Newton's universal law of gravitation; it is a universal constant—that is, it is thought to be the same everywhere in the universe

center of mass

the point where the entire mass of an object can be thought to be concentrated

microgravity

an environment in which the apparent net acceleration of a body is small compared with that produced by Earth at its surface

Newton's universal law of gravitation

every particle in the universe attracts every other particle with a force along a line joining them; the force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them

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5.7: Satellites and Kepler's Laws- An Argument for Simplicity

Learning Objectives

By the end of this section, you will be able to:

- State Kepler's laws of planetary motion.
- Derive the third Kepler's law for circular orbits.
- Discuss the Ptolemaic model of the universe.

Examples of gravitational orbits abound. Hundreds of artificial satellites orbit Earth together with thousands of pieces of debris. The Moon's orbit about Earth has intrigued humans from time immemorial. The orbits of planets, asteroids, meteors, and comets about the Sun are no less interesting. If we look further, we see almost unimaginable numbers of stars, galaxies, and other celestial objects orbiting one another and interacting through gravity.

All these motions are governed by gravitational force, and it is possible to describe them to various degrees of precision. Precise descriptions of complex systems must be made with large computers. However, we can describe an important class of orbits without the use of computers, and we shall find it instructive to study them. These orbits have the following characteristics:

1. *A small mass $m$orbits a much larger mass M*. This allows us to view the motion as if M were stationary—in fact, as if from an inertial frame of reference placed on M —without significant error. Mass m is the satellite of M, if the orbit is gravitationally bound.
2. *The system is isolated from other masses*. This allows us to neglect any small effects due to outside masses.

The conditions are satisfied, to good approximation, by Earth's satellites (including the Moon), by objects orbiting the Sun, and by the satellites of other planets. Historically, planets were studied first, and there is a classical set of three laws, called Kepler's laws of planetary motion, that describe the orbits of all bodies satisfying the two previous conditions (not just planets in our solar system). These descriptive laws are named for the German astronomer Johannes Kepler (1571–1630), who devised them after careful study (over some 20 years) of a large amount of meticulously recorded observations of planetary motion done by Tycho Brahe (1546–1601). Such careful collection and detailed recording of methods and data are hallmarks of good science. Data constitute the evidence from which new interpretations and meanings can be constructed.

Kepler's Laws of Planetary Motion

Kepler's First Law

The orbit of each planet about the Sun is an ellipse with the Sun at one focus.

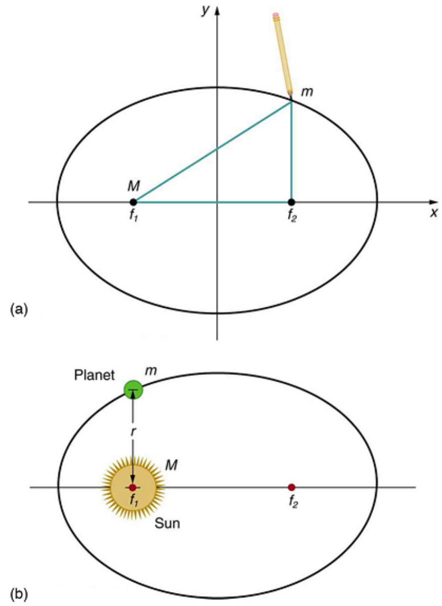

Figure 5.7.1: (a) An ellipse is a closed curve such that the sum of the distances from a point on the curve to the two foci $(f_1 and f_2)$ is a constant. You can draw an ellipse as shown by putting a pin at each focus, and then placing a string around a pencil and the pins and tracing a line on paper. A circle is a special case of an ellipse in which the two foci coincide (thus any point on the circle is the same distance from the center). (b) For any closed gravitational orbit, m follows an elliptical path with M at one focus. Kepler's first law states this fact for planets orbiting the Sun.

Kepler's Second Law

Each planet moves so that an imaginary line drawn from the Sun to the planet sweeps out equal areas in equal times (see Figure).

Kepler's Third Law

The ratio of the squares of the periods of any two planets about the Sun is equal to the ratio of the cubes of their average distances from the Sun. In equation form, this is

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3} \tag{5.7.1}$$

where T is the period (time for one orbit) and r is the average radius. This equation is valid only for comparing two small masses orbiting the same large one. Most importantly, this is a descriptive equation only, giving no information as to the cause of the equality.

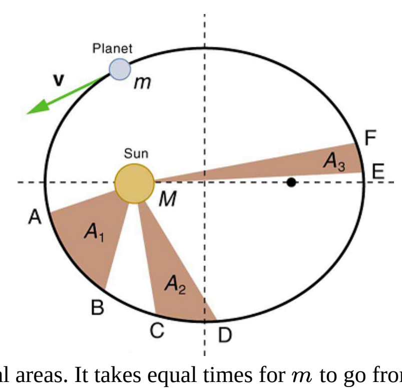

Figure 5.7.2: The shaded regions have equal areas. It takes equal times for m to go from A to B, from C to D, and from E to F. The mass m moves fastest when it is closest to M. Kepler's second law was originally devised for planets orbiting the Sun, but it has broader validity.

Note again that while, for historical reasons, Kepler's laws are stated for planets orbiting the Sun, they are actually valid for all bodies satisfying the two previously stated conditions.

Example 5.7.1: Find the Time for One Orbit of an Earth Satellite

Given that the Moon orbits Earth each 27.3 d and that it is an average distance of $3.84 \times 10^8\, m$ from the center of Earth, calculate the period of an artificial satellite orbiting at an average altitude of 1500 km above Earth's surface.

Strategy

The period, or time for one orbit, is related to the radius of the orbit by Kepler's third law, given in mathematical form in $\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$. Let us use the subscript 1 for the Moon and the subscript 2 for the satellite. We are asked to find T_2. The given information tells us that the orbital radius of the Moon is $r_1 = 3.84 \times 10^8\, m$, and that the period of the Moon is $T_1 = 27.3\, d$.

The height of the artificial satellite above Earth's surface is given, and so we must add the radius of Earth (6380 km) to get $r_2 = (1500 + 6380) km = 7880\, km$. Now all quantities are known, and so T_2 can be found.

Solution

Kepler's third law is

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}. \tag{5.7.2}$$

To solve for T_2, we cross-multiply and take the square root, yielding

$$T_2^2 = T_1^2 \left(\frac{r_2}{r_1}\right)^3 \tag{5.7.3}$$

$$T_2 = T_1 \left(\frac{r_2}{r_1}\right)^{\frac{3}{2}}. \tag{5.7.4}$$

Substituting known values yields

$$T_2 = 27.3\, d \times \frac{24\, h}{d} \times \left(\frac{7880\, km}{3.84 \times 10^5 km}\right)^{\frac{3}{2}} \tag{5.7.5}$$

$$= 1.93\, h. \tag{5.7.6}$$

Discussion This is a reasonable period for a satellite in a fairly low orbit. It is interesting that any satellite at this altitude will orbit in the same amount of time. This fact is related to the condition that the satellite's mass is small compared with that of Earth.

People immediately search for deeper meaning when broadly applicable laws, like Kepler's, are discovered. It was Newton who took the next giant step when he proposed the law of universal gravitation. While Kepler was able to discover *what* was happening, Newton discovered that gravitational force was the cause.

Derivation of Kepler's Third Law for Circular Orbits

We shall derive Kepler's third law, starting with Newton's laws of motion and his universal law of gravitation. The point is to demonstrate that the force of gravity is the cause for Kepler's laws (although we will only derive the third one).

Let us consider a circular orbit of a small mass m around a large mass M, satisfying the two conditions stated at the beginning of this section. Gravity supplies the centripetal force to mass m. Starting with Newton's second law applied to circular motion,

$$F_{net} = ma_c = m\frac{v^2}{r}. \tag{5.7.7}$$

The net external force on mass m is gravity, and so we substitute the force of gravity for F_{net}:

$$G\frac{mM}{r^2} = m\frac{v^2}{r}. \tag{5.7.8}$$

The mass m cancels, yielding

$$G\frac{M}{r}=v^{2}. \tag{5.7.9}$$

The fact that m cancels out is another aspect of the oft-noted fact that at a given location all masses fall with the same acceleration. Here we see that at a given orbital radius r, all masses orbit at the same speed. (This was implied by the result of the preceding worked example.) Now, to get at Kepler's third law, we must get the period T into the equation. By definition, period T is the time for one complete orbit. Now the average speed v is the circumference divided by the period—that is,

$$v=\frac{2\pi r}{T}. \tag{5.7.10}$$

Substituting this into the previous equation gives

$$G\frac{M}{r}=\frac{4\pi^{2}r^{2}}{T^{2}}. \tag{5.7.11}$$

Solving for T^2 yields

$$T^{2}=\frac{4\pi^{2}}{GM}r^{3}. \tag{5.7.12}$$

Using subscripts 1 and 2 to denote two different satellites, and taking the ratio of the last equation for satellite 1 to satellite 2 yields

$$\frac{T_{1}^{2}}{T_{2}^{2}}=\frac{r_{1}^{3}}{r_{2}^{3}}. \tag{5.7.13}$$

This is Kepler's third law. Note that Kepler's third law is valid only for comparing satellites of the same parent body, because only then does the mass of the parent body M cancel.

Now consider what we get if we solve $T^2=\frac{4\pi^2}{GM}r^3$ for the ratio r^3/T^2. We obtain a relationship that can be used to determine the mass M of a parent body from the orbits of its satellites:

$$\frac{r^{3}}{T^{2}}=\frac{G}{4\pi^{2}}M. \tag{5.7.14}$$

If r and T are known for a satellite, then the mass M of the parent can be calculated. This principle has been used extensively to find the masses of heavenly bodies that have satellites. Furthermore, the ratio r^3/T^2 should be a constant for all satellites of the same parent body (because $r^3/T^2=GM/4\pi$. (See Table).

It is clear from Table that the ratio of r^3/T^2 is constant, at least to the third digit, for all listed satellites of the Sun, and for those of Jupiter. Small variations in that ratio have two causes—uncertainties in the r and T data, and perturbations of the orbits due to other bodies. Interestingly, those perturbations can be—and have been—used to predict the location of new planets and moons. This is another verification of Newton's universal law of gravitation.

MAKING CONNECTIONS

Newton's universal law of gravitation is modified by Einstein's general theory of relativity, as we shall see in Particle Physics. Newton's gravity is not seriously in error—it was and still is an extremely good approximation for most situations. Einstein's modification is most noticeable in extremely large gravitational fields, such as near black holes. However, general relativity also explains such phenomena as small but long-known deviations of the orbit of the planet Mercury from classical predictions.

The Case for Simplicity

The development of the universal law of gravitation by Newton played a pivotal role in the history of ideas. While it is beyond the scope of this text to cover that history in any detail, we note some important points. The definition of planet set in 2006 by the International Astronomical Union (IAU) states that in the solar system, a planet is a celestial body that:

1. is in orbit around the Sun,
2. has sufficient mass to assume hydrostatic equilibrium and
3. has cleared the neighborhood around its orbit.

A non-satellite body fulfilling only the first two of the above criteria is classified as “dwarf planet.”

In 2006, Pluto was demoted to a ‘dwarf planet’ after scientists revised their definition of what constitutes a “true” planet.

Parent	Satellite	Average orbital radius r(km)	Period $T(y)$	$r^3/T^2(km^3/y^2)$
Earth	Moon	3.84×10^5	0.07481	1.01×10^{19}
Sun	Mercury	5.79×10^7	0.2409	$3,34 \times 10^{24}$
	Venus	1.082×10^8	0.6150	3.35×10^{24}
	Earth	1.496×10^8	1.000	3.35×10^{24}
	Mars	2.279×10^8	1.881	3.35×10^{24}
	Jupiter	7.783×10^8	11.86	3.35×10^{24}
	Saturn	1.427×10^9	29.46	3.35×10^{24}
	Neptune	4.497×10^9	164.8	3.35×10^{24}
	Pluto	5.90×10^9	248.3	3.33×10^{24}
Jupiter	Io	4.22×10^5	0.00485 (1.77 d)	3.19×10^{21}
	Europa	6.71×10^5	0.00972 (3.55 d)	3.20×10^{21}
	Ganymede	1.07×10^6	0.0196 (7.16 d)	3.19×10^{21}
	Callisto	1.88×10^6	0.0457 (16.19 d)	3.20×10^{21}

The universal law of gravitation is a good example of a physical principle that is very broadly applicable. That single equation for the gravitational force describes all situations in which gravity acts. It gives a cause for a vast number of effects, such as the orbits of the planets and moons in the solar system. It epitomizes the underlying unity and simplicity of physics.

Before the discoveries of Kepler, Copernicus, Galileo, Newton, and others, the solar system was thought to revolve around Earth as shown in Figure (a). This is called the Ptolemaic view, for the Greek philosopher who lived in the second century AD. This model is characterized by a list of facts for the motions of planets with no cause and effect explanation. There tended to be a different rule for each heavenly body and a general lack of simplicity.

Figure (b) represents the modern or Copernican model. In this model, a small set of rules and a single underlying force explain not only all motions in the solar system, but all other situations involving gravity. The breadth and simplicity of the laws of physics are compelling. As our knowledge of nature has grown, the basic simplicity of its laws has become ever more evident.

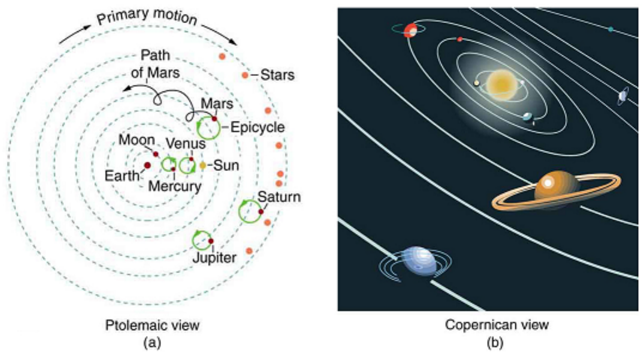

Figure $5.7.3$: (a) The Ptolemaic model of the universe has Earth at the center with the Moon, the planets, the Sun, and the stars revolving about it in complex superpositions of circular paths. This geocentric model, which can be made progressively more accurate by adding more circles, is purely descriptive, containing no hints as to what are the causes of these motions. (b) The Copernican model has the Sun at the center of the solar system. It is fully explained by a small number of laws of physics, including Newton's universal law of gravitation.

Summary

- Kepler's laws are stated for a small mass m orbiting a larger mass M in near-isolation. Kepler's laws of planetary motion are then as follows:

  Kepler's first law

  The orbit of each planet about the Sun is an ellipse with the Sun at one focus.

  Kepler's second law

  Each planet moves so that an imaginary line drawn from the Sun to the planet sweeps out equal areas in equal times.

  Kepler's third law

  The ratio of the squares of the periods of any two planets about the Sun is equal to the ratio of the cubes of their average distances from the Sun:

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}, \tag{5.7.15}$$

  where T is the period (time for one orbit) and r is the average radius of the orbit.

- The period and radius of a satellite's orbit about a larger body M are related by

$$T^2 = \frac{4\pi^2}{GM} r^3 \tag{5.7.16}$$

or

$$\frac{r^3}{T^2} = \frac{G}{4\pi^2} M. \tag{5.7.17}$$

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5.E: Uniform Circular Motion and Gravitation (Excercise)

Conceptual Questions

6.1: Rotation Angle and Angular Velocity

1. There is an analogy between rotational and linear physical quantities. What rotational quantities are analogous to distance and velocity?

6.2: Centripetal Acceleration

2. Can centripetal acceleration change the speed of circular motion? Explain.

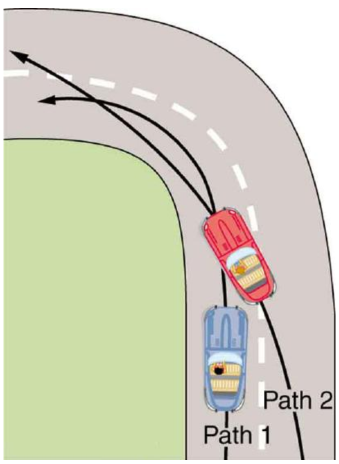
6.3: Centripetal Force

3. If you wish to reduce the stress (which is related to centripetal force) on high-speed tires, would you use large- or small-diameter tires? Explain.

4. Define centripetal force. Can any type of force (for example, tension, gravitational force, friction, and so on) be a centripetal force? Can any combination of forces be a centripetal force?

5. If centripetal force is directed toward the center, why do you feel that you are 'thrown' away from the center as a car goes around a curve? Explain.

6. Race car drivers routinely cut corners as shown in Figure. Explain how this allows the curve to be taken at the greatest speed.



Two paths around a race track curve are shown. Race car drivers will take the inside path (called cutting the corner) whenever possible because it allows them to take the curve at the highest speed.

7. A number of amusement parks have rides that make vertical loops like the one shown in Figure. For safety, the cars are attached to the rails in such a way that they cannot fall off. If the car goes over the top at just the right speed, gravity alone will supply the centripetal force. What other force acts and what is its direction if:

(a) The car goes over the top at faster than this speed?

(b)The car goes over the top at slower than this speed?

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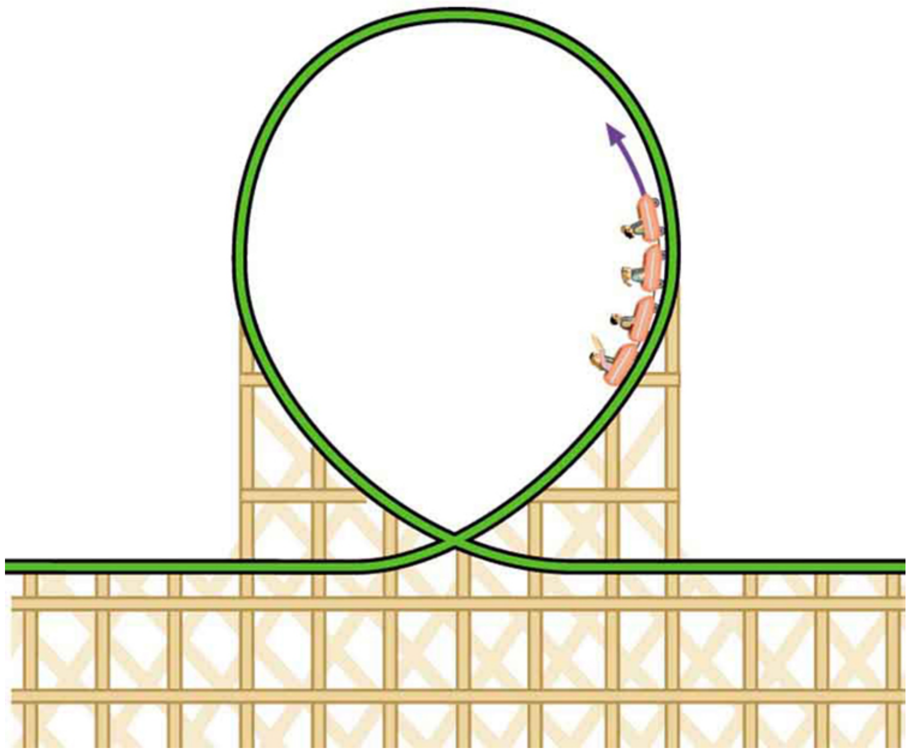


Amusement rides with a vertical loop are an example of a form of curved motion.

8. What is the direction of the force exerted by the car on the passenger as the car goes over the top of the amusement ride pictured in Figure under the following circumstances:

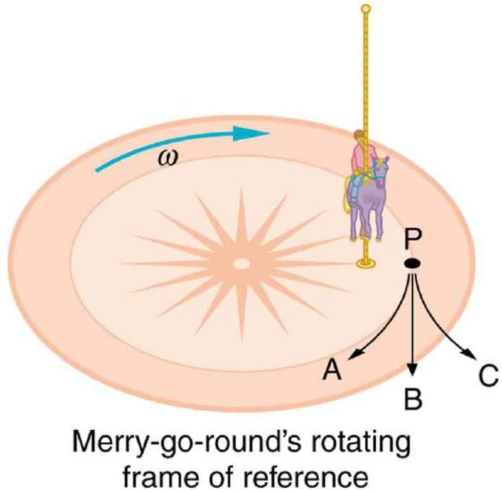
(a) The car goes over the top at such a speed that the gravitational force is the only force acting?

(b) The car goes over the top faster than this speed?

(c) The car goes over the top slower than this speed?

9. As a skater forms a circle, what force is responsible for making her turn? Use a free body diagram in your answer.

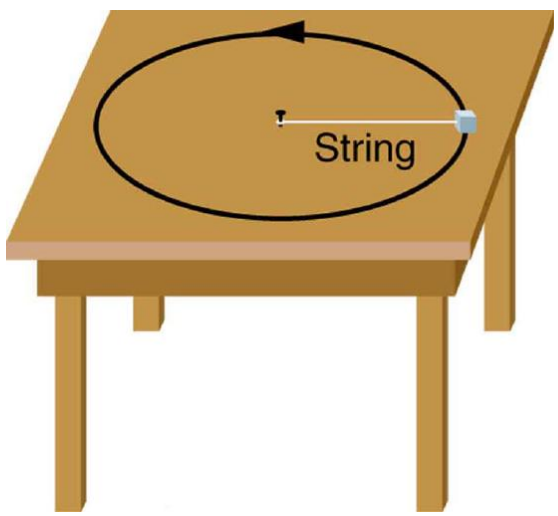
10. Suppose a child is riding on a merry-go-round at a distance about halfway between its center and edge. She has a lunch box resting on wax paper, so that there is very little friction between it and the merry-go-round. Which path shown in Figure will the lunch box take when she lets go? The lunch box leaves a trail in the dust on the merry-go-round. Is that trail straight, curved to the left, or curved to the right? Explain your answer.



A child riding on a merry-go-round releases her lunch box at point P. This is a view from above the clockwise rotation. Assuming it slides with negligible friction, will it follow path A, B, or C, as viewed from Earth's frame of reference? What will be the shape of the path it leaves in the dust on the merry-go-round?

11. Do you feel yourself thrown to either side when you negotiate a curve that is ideally banked for your car's speed? What is the direction of the force exerted on you by the car seat?

12. Suppose a mass is moving in a circular path on a frictionless table as shown in figure. In the Earth's frame of reference, there is no centrifugal force pulling the mass away from the centre of rotation, yet there is a very real force stretching the string attaching the mass to the nail. Using concepts related to centripetal force and Newton's third law, explain what force stretches the string, identifying its physical origin.



A mass attached to a nail on a frictionless table moves in a circular path. The force stretching the string is real and not fictional. What is the physical origin of the force on the string?

6.4: Fictitious Forces and Non-inertial Frames: The Coriolis Force

13. When a toilet is flushed or a sink is drained, the water (and other material) begins to rotate about the drain on the way down. Assuming no initial rotation and a flow initially directly straight toward the drain, explain what causes the rotation and which direction it has in the northern hemisphere. (Note that this is a small effect and in most toilets the rotation is caused by directional water jets.) Would the direction of rotation reverse if water were forced up the drain?

14. Is there a real force that throws water from clothes during the spin cycle of a washing machine? Explain how the water is removed.

15. In one amusement park ride, riders enter a large vertical barrel and stand against the wall on its horizontal floor. The barrel is spun up and the floor drops away. Riders feel as if they are pinned to the wall by a force something like the gravitational force. This is a fictitious force sensed and used by the riders to explain events in the rotating frame of reference of the barrel. Explain in an inertial frame of reference (Earth is nearly one) what pins the riders to the wall, and identify all of the real forces acting on them.

16. Action at a distance, such as is the case for gravity, was once thought to be illogical and therefore untrue. What is the ultimate determinant of the truth in physics, and why was this action ultimately accepted?

17. Two friends are having a conversation. Anna says a satellite in orbit is in freefall because the satellite keeps falling toward Earth. Tom says a satellite in orbit is not in freefall because the acceleration due to gravity is not 9.80 m/s^2. Who do you agree with and why?

18. A non-rotating frame of reference placed at the center of the Sun is very nearly an inertial one. Why is it not exactly an inertial frame?

6.5: Newton's Universal Law of Gravitation

19. Action at a distance, such as is the case for gravity, was once thought to be illogical and therefore untrue. What is the ultimate determinant of the truth in physics, and why was this action ultimately accepted?

20. Two friends are having a conversation. Anna says a satellite in orbit is in freefall because the satellite keeps falling toward Earth. Tom says a satellite in orbit is not in freefall because the acceleration due to gravity is not $9.80 m/s^2$. Who do you agree with and why?

21. Draw a free body diagram for a satellite in an elliptical orbit showing why its speed increases as it approaches its parent body and decreases as it moves away.

22. Newton's laws of motion and gravity were among the first to convincingly demonstrate the underlying simplicity and unity in nature. Many other examples have since been discovered, and we now expect to find such underlying order in complex situations. Is there proof that such order will always be found in new explorations?

6.6: Satellites and Kepler's Laws: An Argument for Simplicity

23. In what frame(s) of reference are Kepler's laws valid? Are Kepler's laws purely descriptive, or do they contain causal information?

Problem Exercises

6.1: Rotation Angle and Angular Velocity

24. Semi-trailer trucks have an odometer on one hub of a trailer wheel. The hub is weighted so that it does not rotate, but it contains gears to count the number of wheel revolutions—it then calculates the distance traveled. If the wheel has a 1.15 m diameter and goes through 200,000 rotations, how many kilometers should the odometer read?

Solution
723 km

25. Microwave ovens rotate at a rate of about 6 rev/min. What is this in revolutions per second? What is the angular velocity in radians per second?

26. An automobile with 0.260 m radius tires travels 80,000 km before wearing them out. How many revolutions do the tires make, neglecting any backing up and any change in radius due to wear?

Solution
5×10^7 rotations

27. (a) What is the period of rotation of Earth in seconds?

(b) What is the angular velocity of Earth?

(c) Given that Earth has a radius of $6.4 \times 10^6 m$ at its equator, what is the linear velocity at Earth's surface?

28. A baseball pitcher brings his arm forward during a pitch, rotating the forearm about the elbow. If the velocity of the ball in the pitcher's hand is 35.0 m/s and the ball is 0.300 m from the elbow joint, what is the angular velocity of the forearm?

Solution
117 rad/s

29. In lacrosse, a ball is thrown from a net on the end of a stick by rotating the stick and forearm about the elbow. If the angular velocity of the ball about the elbow joint is 30.0 rad/s and the ball is 1.30 m from the elbow joint, what is the velocity of the ball?

30. A truck with 0.420-m-radius tires travels at 32.0 m/s. What is the angular velocity of the rotating tires in radians per second? What is this in rev/min?

Solution
76.2 rad/s
728 rpm

31. *Integrated Concepts*

When kicking a football, the kicker rotates his leg about the hip joint.

(a) If the velocity of the tip of the kicker's shoe is 35.0 m/s and the hip joint is 1.05 m from the tip of the shoe, what is the shoe tip's angular velocity?

(b) The shoe is in contact with the initially stationary 0.500 kg football for 20.0 ms. What average force is exerted on the football to give it a velocity of 20.0 m/s?

(c) Find the maximum range of the football, neglecting air resistance.

Solution
(a) 33.3 rad/s
(b) 500 N
(c) 40.8 m

32. *Construct Your Own Problem*

Consider an amusement park ride in which participants are rotated about a vertical axis in a cylinder with vertical walls. Once the angular velocity reaches its full value, the floor drops away and friction between the walls and the riders prevents them from sliding down. Construct a problem in which you calculate the necessary angular velocity that assures the riders will not slide down the wall. Include a free body diagram of a single rider. Among the variables to consider are the radius of the cylinder and the coefficients of friction between the riders' clothing and the wall.

6.2: Centripetal Acceleration

33. A fairground ride spins its occupants inside a flying saucer-shaped container. If the horizontal circular path the riders follow has an 8.00 m radius, at how many revolutions per minute will the riders be subjected to a centripetal acceleration whose magnitude is 1.50 times that due to gravity?

Solution
12.9 rev/min

34. A runner taking part in the 200 m dash must run around the end of a track that has a circular arc with a radius of curvature of 30 m. If he completes the 200 m dash in 23.2 s and runs at constant speed throughout the race, what is the magnitude of his centripetal acceleration as he runs the curved portion of the track?

35. Taking the age of Earth to be about 4×10^9 years and assuming its orbital radius of 1.5×10^{11} m has not changed and is circular, calculate the approximate total distance Earth has traveled since its birth (in a frame of reference stationary with respect to the Sun).

Solution
$4 \times 10^{21} m$

36. The propeller of a World War II fighter plane is 2.30 m in diameter.

(a) What is its angular velocity in radians per second if it spins at 1200 rev/min?

(b) What is the linear speed of its tip at this angular velocity if the plane is stationary on the tarmac?

(c) What is the centripetal acceleration of the propeller tip under these conditions? Calculate it in meters per second squared and convert to multiples of g.

37. An ordinary workshop grindstone has a radius of 7.50 cm and rotates at 6500 rev/min.

(a) Calculate the magnitude of the centripetal acceleration at its edge in meters per second squared and convert it to multiples of g.

(b) What is the linear speed of a point on its edge?

Solution
a) $3.47 \times 10^4 m/s^2, 3.55 \times 10^3 g$
b) $51.1 m/s$

38. Helicopter blades withstand tremendous stresses. In addition to supporting the weight of a helicopter, they are spun at rapid rates and experience large centripetal accelerations, especially at the tip.

(a) Calculate the magnitude of the centripetal acceleration at the tip of a 4.00 m long helicopter blade that rotates at 300 rev/min.

(b) Compare the linear speed of the tip with the speed of sound (taken to be 340 m/s).

39. Olympic ice skaters are able to spin at about 5 rev/s.

(a) What is their angular velocity in radians per second?

(b) What is the centripetal acceleration of the skater's nose if it is 0.120 m from the axis of rotation?

(c) An exceptional skater named Dick Button was able to spin much faster in the 1950s than anyone since—at about 9 rev/s. What was the centripetal acceleration of the tip of his nose, assuming it is at 0.120 m radius?

(d) Comment on the magnitudes of the accelerations found. It is reputed that Button ruptured small blood vessels during his spins.

Solution
a) 31.4 rad/s
b) 118 m/s
c) 384 m/s
d)The centripetal acceleration felt by Olympic skaters is 12 times larger than the acceleration due to gravity. That's quite a lot of acceleration in itself. The centripetal acceleration felt by Button's nose was 39.2 times larger than the acceleration due to gravity. It is no wonder that he ruptured small blood vessels in his spins.

40. What percentage of the acceleration at Earth's surface is the acceleration due to gravity at the position of a satellite located 300 km above Earth?

41. Verify that the linear speed of an ultracentrifuge is about 0.50 km/s, and Earth in its orbit is about 30 km/s by calculating:

(a) The linear speed of a point on an ultracentrifuge 0.100 m from its center, rotating at 50,000 rev/min.

(b) The linear speed of Earth in its orbit about the Sun (use data from the text on the radius of Earth's orbit and approximate it as being circular).

Solution
a) 0.524 km/s
b) 29.7 km/s

42. A rotating space station is said to create "artificial gravity"—a loosely-defined term used for an acceleration that would be crudely similar to gravity. The outer wall of the rotating space station would become a floor for the astronauts, and centripetal acceleration supplied by the floor would allow astronauts to exercise and maintain muscle and bone strength more naturally than in non-rotating space environments. If the space station is 200 m in diameter, what angular velocity would produce an "artificial gravity" of $9.80 m/s^2$ at the rim?

43. At takeoff, a commercial jet has a 60.0 m/s speed. Its tires have a diameter of 0.850 m.

(a) At how many rev/min are the tires rotating?

(b) What is the centripetal acceleration at the edge of the tire?

(c) With what force must a determined $1.00 \times 10^{-15} kg$ bacterium cling to the rim?

(d) Take the ratio of this force to the bacterium's weight.

Solution
(a) $1.35 \times 10^{3} rpm$
(b) $8.47 \times 10^{3} m/s^{2}$
(c) $8.47 \times 10^{-12} N$
(d) 865

44. *Integrated Concepts*

Riders in an amusement park ride shaped like a Viking ship hung from a large pivot are rotated back and forth like a rigid pendulum. Sometime near the middle of the ride, the ship is momentarily motionless at the top of its circular arc. The ship then swings down under the influence of gravity.

(a) Assuming negligible friction, find the speed of the riders at the bottom of its arc, given the system's center of mass travels in an arc having a radius of 14.0 m and the riders are near the center of mass.

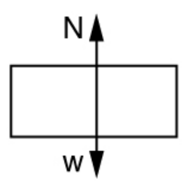
(b) What is the centripetal acceleration at the bottom of the arc?

(c) Draw a free body diagram of the forces acting on a rider at the bottom of the arc.

(d) Find the force exerted by the ride on a 60.0 kg rider and compare it to her weight.

(e) Discuss whether the answer seems reasonable.

Solution
(a) $16.6 m/s$
(b) $19.6 m/s^{2}$
(c)



(d) $1.76 \times 10^{3} N$ or $3.00 w$, that is, the normal force (upward) is three times her weight.
(e) This answer seems reasonable, since she feels like she's being forced into the chair MUCH stronger than just by gravity.

45. *Unreasonable Results*

A mother pushes her child on a swing so that his speed is 9.00 m/s at the lowest point of his path. The swing is suspended 2.00 m above the child's center of mass.

(a) What is the magnitude of the centripetal acceleration of the child at the low point?

(b) What is the magnitude of the force the child exerts on the seat if his mass is 18.0 kg?

(c) What is unreasonable about these results?

(d) Which premises are unreasonable or inconsistent?

Solution
a) $40.5 m/s^{2}$
b) 905 N
c) The force in part (b) is very large. The acceleration in part (a) is too much, about 4 g.
d) The speed of the swing is too large. At the given velocity at the bottom of the swing, there is enough kinetic energy to send the child all the way over the top, ignoring friction.

46. (a) A 22.0 kg child is riding a playground merry-go-round that is rotating at 40.0 rev/min. What centripetal force must she exert to stay on if she is 1.25 m from its center?

(b) What centripetal force does she need to stay on an amusement park merry-go-round that rotates at 3.00 rev/min if she is 8.00 m from its center?

(c) Compare each force with her weight.

Solution
a) 483 N
b) 17.4 N
c) 2.24 times her weight, 0.0807 times her weight

47. Calculate the centripetal force on the end of a 100 m (radius) wind turbine blade that is rotating at 0.5 rev/s. Assume the mass is 4 kg.

48. What is the ideal banking angle for a gentle turn of 1.20 km radius on a highway with a 105 km/h speed limit (about 65 mi/h), assuming everyone travels at the limit?

Solution
4.14°

48. What is the ideal speed to take a 100 m radius curve banked at a 20.0° angle?

49. (a) What is the radius of a bobsled turn banked at 75.0° and taken at 30.0 m/s, assuming it is ideally banked?

(b) Calculate the centripetal acceleration.

(c) Does this acceleration seem large to you?

Solution
a) 24.6 m
b) $36.6 m/s^2$
c) $a_c = 3.73g$. This does not seem too large, but it is clear that bobsledders feel a lot of force on them going through sharply banked turns.

50. Part of riding a bicycle involves leaning at the correct angle when making a turn, as seen in Figure. To be stable, the force exerted by the ground must be on a line going through the center of gravity. The force on the bicycle wheel can be resolved into two perpendicular components—friction parallel to the road (this must supply the centripetal force), and the vertical normal force (which must equal the system's weight).

(a) Show that θ (as defined in the figure) is related to the speed v size 12{v} {} and radius of curvature r size 12{r} {} of the turn in the same way as for an ideally banked roadway—that is, $\theta = tan^{-1} v^2/rg$

(b) Calculate θ for a 12.0 m/s turn of radius 30.0 m (as in a race).

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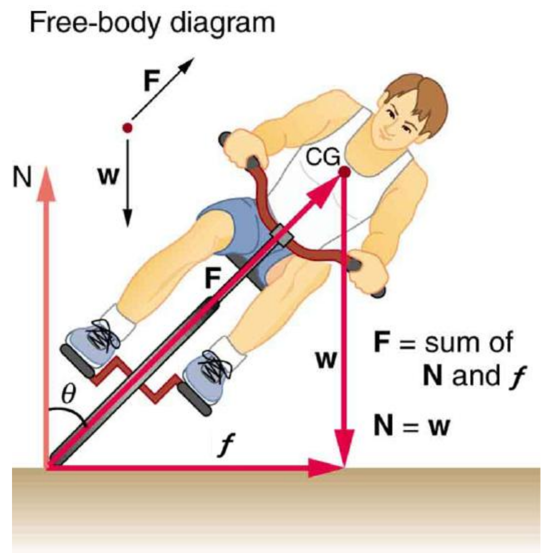


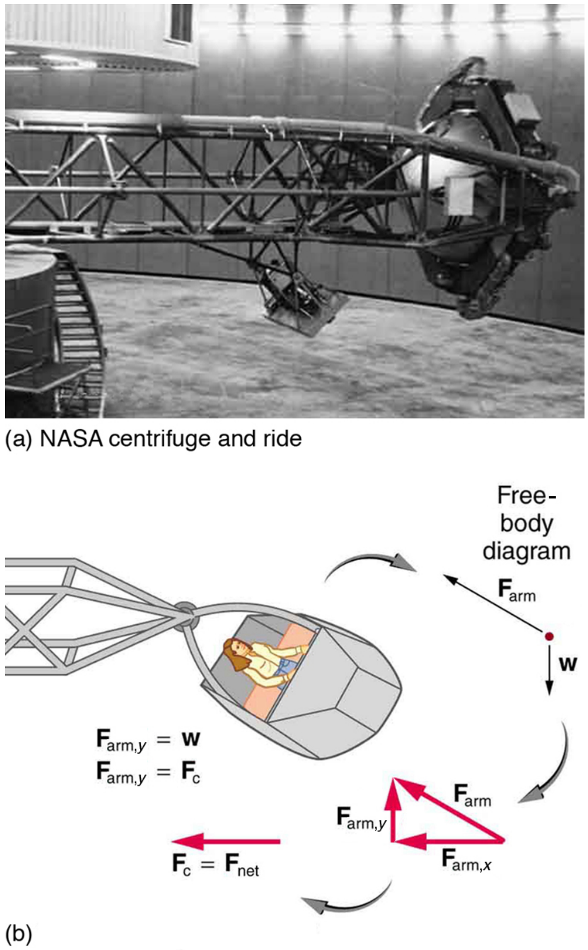
A bicyclist negotiating a turn on level ground must lean at the correct angle—the ability to do this becomes instinctive. The force of the ground on the wheel needs to be on a line through the center of gravity. The net external force on the system is the centripetal force. The vertical component of the force on the wheel cancels the weight of the system while its horizontal component must supply the centripetal force. This process produces a relationship among the angle θ, the speed v, and the radius of curvature r of the turn similar to that for the ideal banking of roadways.

51. A large centrifuge, like the one shown in Figure(a), is used to expose aspiring astronauts to accelerations similar to those experienced in rocket launches and atmospheric reentries.

(a) At what angular velocity is the centripetal acceleration $10g$ if the rider is 15.0 m from the center of rotation?

(b) The rider's cage hangs on a pivot at the end of the arm, allowing it to swing outward during rotation as shown in Figure(b). At what angle θ below the horizontal will the cage hang when the centripetal acceleration is $10g$?

(Hint: The arm supplies centripetal force and supports the weight of the cage. Draw a free body diagram of the forces to see what the angle θ size 12{θ} {} should be.)



(a) NASA centrifuge used to subject trainees to accelerations similar to those experienced in rocket launches and reentries. (credit: NASA) (b) Rider in cage showing how the cage pivots outward during rotation. This allows the total force exerted on the rider by the cage to be along its axis at all times.

Solution
a) 2.56 rad/s
b) 5.71º

bIntegrated Concepts

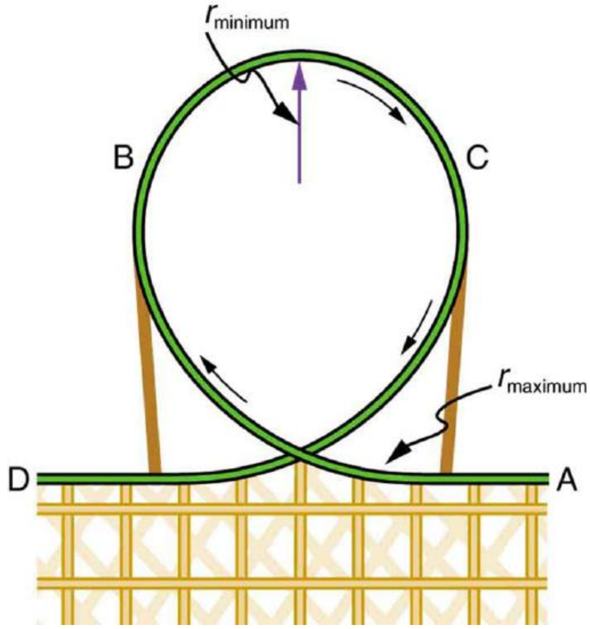
If a car takes a banked curve at less than the ideal speed, friction is needed to keep it from sliding toward the inside of the curve (a real problem on icy mountain roads).

(a) Calculate the ideal speed to take a 100 m radius curve banked at 15.0º.

(b) What is the minimum coefficient of friction needed for a frightened driver to take the same curve at 20.0 km/h?

Solution
a) 16.2 m/s
b) 0.234

53. Modern roller coasters have vertical loops like the one shown in Figure. The radius of curvature is smaller at the top than on the sides so that the downward centripetal acceleration at the top will be greater than the acceleration due to gravity, keeping the passengers pressed firmly into their seats. What is the speed of the roller coaster at the top of the loop if the radius of curvature there is 15.0 m and the downward acceleration of the car is 1.50 g?



Teardrop-shaped loops are used in the latest roller coasters so that the radius of curvature gradually decreases to a minimum at the top. This means that the centripetal acceleration builds from zero to a maximum at the top and gradually decreases again. A circular loop would cause a jolting change in acceleration at entry, a disadvantage discovered long ago in railroad curve design. With a small radius of curvature at the top, the centripetal acceleration can more easily be kept greater than g so that the passengers do not lose contact with their seats nor do they need seat belts to keep them in place.

54. *Unreasonable Results*

(a) Calculate the minimum coefficient of friction needed for a car to negotiate an unbanked 50.0 m radius curve at 30.0 m/s.

(b) What is unreasonable about the result?

(c) Which premises are unreasonable or inconsistent?

Solution

a) 1.84
b) A coefficient of friction this much greater than 1 is unreasonable .
c) The assumed speed is too great for the tight curve.

6.5: Newton's Universal Law of Gravitation

55. (a) Calculate Earth's mass given the acceleration due to gravity at the North Pole is $9.830 m/s^2$ and the radius of the Earth is 6371 km from center to pole.

(b) Compare this with the accepted value of $5.979 \times 10^{24} kg$.

Solution

a) $5.979 \times 10^{24} kg$
b) This is identical to the best value to three significant figures.

56. (a) Calculate the magnitude of the acceleration due to gravity on the surface of Earth due to the Moon.

(b) Calculate the magnitude of the acceleration due to gravity at Earth due to the Sun.

(c) Take the ratio of the Moon's acceleration to the Sun's and comment on why the tides are predominantly due to the Moon in spite of this number.

57. (a) What is the acceleration due to gravity on the surface of the Moon?

(b) On the surface of Mars? The mass of Mars is $6.418 \times 10^{23} kg$ and its radius is $3.38 \times 10^{6} m$

Solution
a) $1.62 m/s^2$
b) $3.75 m/s^2$

58. (a) Calculate the acceleration due to gravity on the surface of the Sun.

(b) By what factor would your weight increase if you could stand on the Sun? (Never mind that you cannot.)

59. The Moon and Earth rotate about their common center of mass, which is located about 4700 km from the center of Earth. (This is 1690 km below the surface.)

(a) Calculate the magnitude of the acceleration due to the Moon's gravity at that point.

(b) Calculate the magnitude of the centripetal acceleration of the center of Earth as it rotates about that point once each lunar month (about 27.3 d) and compare it with the acceleration found in part (a). Comment on whether or not they are equal and why they should or should not be.

Solution
a) $3.42 \times 10^{-5} m/s^2$
b) $3.34 \times 10^{-5} m/s^2$
The values are nearly identical. One would expect the gravitational force to be the same as the centripetal force at the core of the system.

60. Solve part (b) of Example using $a_c = v^2/r$.

61. Astrology, that unlikely and vague pseudo science, makes much of the position of the planets at the moment of one's birth. The only known force a planet exerts on Earth is gravitational.

(a) Calculate the magnitude of the gravitational force exerted on a 4.20 kg baby by a 100 kg father 0.200 m away at birth (he is assisting, so he is close to the child).

(b) Calculate the magnitude of the force on the baby due to Jupiter if it is at its closest distance to Earth, some $6.29 \times 10^{11} m$ away. How does the force of Jupiter on the baby compare to the force of the father on the baby? Other objects in the room and the hospital building also exert similar gravitational forces. (Of course, there could be an unknown force acting, but scientists first need to be convinced that there is even an effect, much less that an unknown force causes it.)

Solution
a) $7.01 \times 10^{-7} N$
b) $1.35 \times 10^{-6} N$

62. The existence of the dwarf planet Pluto was proposed based on irregularities in Neptune's orbit. Pluto was subsequently discovered near its predicted position. But it now appears that the discovery was fortuitous, because Pluto is small and the irregularities in Neptune's orbit were not well known. To illustrate that Pluto has a minor effect on the orbit of Neptune compared with the closest planet to Neptune:

(a) Calculate the acceleration due to gravity at Neptune due to Pluto when they are $4.50 \times 10^{12} m$ apart, as they are at present. The mass of Pluto is $1.4 \times 10^{22} kg$.

(b) Calculate the acceleration due to gravity at Neptune due to Uranus, presently about $2.50 \times 10^{12} m$ apart, and compare it with that due to Pluto. The mass of Uranus is $8.62 \times 10^{25} kg$.

63. (a) The Sun orbits the Milky Way galaxy once each $2.60x10^8 y$ with a roughly circular orbit averaging $3.00x10^4$ light years in radius. (A light year is the distance traveled by light in 1 y.) Calculate the centripetal acceleration of the Sun in its galactic orbit. Does your result support the contention that a nearly inertial frame of reference can be located at the Sun?

(b) Calculate the average speed of the Sun in its galactic orbit. Does the answer surprise you?

Solution
a) $1.66 \times 10^{-10} m/s^2$
b) $2.17 \times 10^5 m/s$

64. *Unreasonable Result*

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A mountain 10.0 km from a person exerts a gravitational force on him equal to 2.00% of his weight.

(a) Calculate the mass of the mountain.

(b) Compare the mountain's mass with that of Earth.

(c) What is unreasonable about these results?

(d) Which premises are unreasonable or inconsistent? (Note that accurate gravitational measurements can easily detect the effect of nearby mountains and variations in local geology.)

Solution
a) $2.937 \times 10^{17} kg$
b) 4.91×10^{-8} of the Earth's mass.
c) The mass of the mountain and its fraction of the Earth's mass are too great.
d) The gravitational force assumed to be exerted by the mountain is too great.

6.6: Satellites and Kepler's Laws: An Argument for Simplicity

65. A geosynchronous Earth satellite is one that has an orbital period of precisely 1 day. Such orbits are useful for communication and weather observation because the satellite remains above the same point on Earth (provided it orbits in the equatorial plane in the same direction as Earth's rotation). Calculate the radius of such an orbit based on the data for the moon in Table.

66. Calculate the mass of the Sun based on data for Earth's orbit and compare the value obtained with the Sun's actual mass.

Solution
$1.98 \times 10^{30} kg$

67. Find the mass of Jupiter based on data for the orbit of one of its moons, and compare your result with its actual mass.

68. Find the ratio of the mass of Jupiter to that of Earth based on data in Table.

Solution
$$\frac{M_J}{M_E} = 316$$

69. Astronomical observations of our Milky Way galaxy indicate that it has a mass of about 8.0×10^{11} solar masses. A star orbiting on the galaxy's periphery is about 6.0×10^4 light years from its center. (a) What should the orbital period of that star be? (b) If its period is 6.0×10^7 instead, what is the mass of the galaxy? Such calculations are used to imply the existence of "dark matter" in the universe and have indicated, for example, the existence of very massive black holes at the centers of some galaxies.

70. *Integrated Concepts*

Space debris left from old satellites and their launchers is becoming a hazard to other satellites.

(a) Calculate the speed of a satellite in an orbit 900 km above Earth's surface.

(b) Suppose a loose rivet is in an orbit of the same radius that intersects the satellite's orbit at an angle of 90° relative to Earth. What is the velocity of the rivet relative to the satellite just before striking it?

(c) Given the rivet is 3.00 mm in size, how long will its collision with the satellite last?

(d) If its mass is 0.500 g, what is the average force it exerts on the satellite?

(e) How much energy in joules is generated by the collision? (The satellite's velocity does not change appreciably, because its mass is much greater than the rivet's.)

Solution
a) $7.4 \times 10^3 m/s$
b) $1.05 \times 10^3 m/s$
c) $2.86 \times 10^{-7} s$
d) $1.84 \times 10^7 N$
e) $2.76 \times 10^4 J$

71. *Unreasonable Results*

(a) Based on Kepler's laws and information on the orbital characteristics of the Moon, calculate the orbital radius for an Earth satellite having a period of 1.00 h.

(b) What is unreasonable about this result?

(c) What is unreasonable or inconsistent about the premise of a 1.00 h orbit?

Solution
a) $5.08 \times 10^3 km$
b) This radius is unreasonable because it is less than the radius of earth.
c) The premise of a one-hour orbit is inconsistent with the known radius of the earth.

72. *Construct Your Own Problem*

On February 14, 2000, the NEAR spacecraft was successfully inserted into orbit around Eros, becoming the first artificial satellite of an asteroid. Construct a problem in which you determine the orbital speed for a satellite near Eros. You will need to find the mass of the asteroid and consider such things as a safe distance for the orbit. Although Eros is not spherical, calculate the acceleration due to gravity on its surface at a point an average distance from its center of mass. Your instructor may also wish to have you calculate the escape velocity from this point on Eros.

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CHAPTER OVERVIEW

6: Work, Energy, and Energy Resources

There is no simple, yet accurate, scientific definition for energy. Energy is characterized by its many forms and the fact that it is conserved. We can loosely define energy as the ability to do work, admitting that in some circumstances not all **energy** is available to do work. Because of the association of energy with work, we begin the chapter with a discussion of work. Work is intimately related to energy and how energy moves from one system to another or changes form.

6.1: Prelude to Work, Energy, and Energy Resources

6.2: Work- The Scientific Definition

6.3: Kinetic Energy and the Work-Energy Theorem

6.4: Gravitational Potential Energy

6.5: Conservative Forces and Potential Energy

6.6: Nonconservative Forces

6.7: Conservation of Energy

6.8: Power

6.9: Work, Energy, and Power in Humans

6.10: World Energy Use

6.E: Work, Energy, and Energy Resources (Exercise)

Thumbnail: One form of energy is mechanical work, the energy required to move an object of mass m a distance d when opposed by a force F, such as gravity. Image use with permission (CC-SA-BY-NC -3.0; anonymous).

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6.1: Prelude to Work, Energy, and Energy Resources

Energy plays an essential role both in everyday events and in scientific phenomena. You can no doubt name many forms of energy, from that provided by our foods, to the energy we use to run our cars, to the sunlight that warms us on the beach. You can also cite examples of what people call energy that may not be scientific, such as someone having an energetic personality. Not only does energy have many interesting forms, it is involved in almost all phenomena, and is one of the most important concepts of physics. What makes it even more important is that the total amount of energy in the universe is constant. Energy can change forms, but it cannot appear from nothing or disappear without a trace. Energy is thus one of a handful of physical quantities that we say is *conserved*.

Figure 6.1.1: How many forms of energy can you identify in this photograph of a wind farm in Iowa? (credit: Jürgen from Sandesneben, Germany, Wikimedia Commons)

Conservation of energy (as physicists like to call the principle that energy can neither be created nor destroyed) is based on experiment. Even as scientists discovered new forms of energy, conservation of energy has always been found to apply. Perhaps the most dramatic example of this was supplied by Einstein when he suggested that mass is equivalent to energy (his famous equation $E = mc^2$.

From a societal viewpoint, energy is one of the major building blocks of modern civilization. Energy resources are key limiting factors to economic growth. The world use of energy resources, especially oil, continues to grow, with ominous consequences economically, socially, politically, and environmentally. We will briefly examine the world's energy use patterns at the end of this chapter.

There is no simple, yet accurate, scientific definition for energy. Energy is characterized by its many forms and the fact that it is conserved. We can loosely define energy as the ability to do work, admitting that in some circumstances not all **energy** is available to do work. Because of the association of energy with work, we begin the chapter with a discussion of work. Work is intimately related to energy and how energy moves from one system to another or changes form.

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6.2: Work- The Scientific Definition

Learning Objectives

By the end of this section, you will be able to:

- Explain how an object must be displaced for a force on it to do work.
- Explain how relative directions of force and displacement determine whether the work done is positive, negative, or zero.

What It Means to Do Work

The scientific definition of work differs in some ways from its everyday meaning. Certain things we think of as hard work, such as writing an exam or carrying a heavy load on level ground, are not work as defined by a scientist. The scientific definition of work reveals its relationship to energy—whenever work is done, energy is transferred. For work, in the scientific sense, to be done, a force must be exerted and there must be motion or displacement in the direction of the force.

Formally, the work done on a system by a constant force is defined to be *the product of the component of the force in the direction of motion times the distance through which the force acts*. For one-way motion in one dimension, this is expressed in equation form as

$$W=|\vec{F}|\cos\theta|\vec{d}| \tag{6.2.1}$$

where W is work, d is the displacement of the system, and θ is the angle between the force vector $\vec{F}$ and the displacement vector $\vec{d}$, as in Figure $6.2.1$. We can also write Equation $6.2.1$ as

$$W=Fd\cos\theta \tag{6.2.2}$$

To find the work done on a system that undergoes motion that is not one-way or that is in two or three dimensions, we divide the motion into one-way one-dimensional segments and add up the work done over each segment.

What is Work?

The work done on a system by a constant force is *the product of the component of the force in the direction of motion times the distance through which the force acts*. For one-way motion in one dimension, this is expressed in equation form as

$$W=Fd\cos\theta \tag{6.2.3}$$

where W is work, F is the magnitude of the force on the system, d is the magnitude of the displacement of the system, and θ is the angle between the force vector F d the displacement vector d.

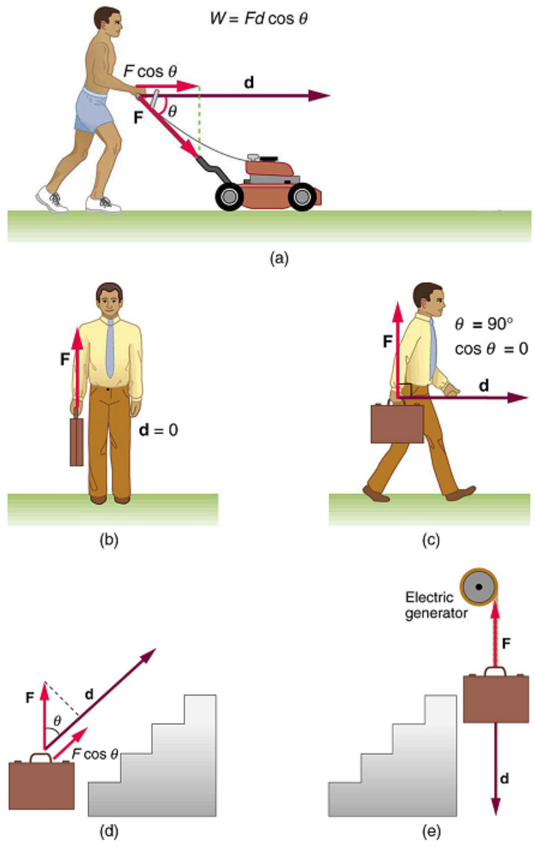

Figure 6.2.1: Examples of work. (a) The work done by the force F on this lawn mower is $Fd\,cos\,\theta$. Note that $F\,cos\,\theta$ is the component of the force in the direction of motion. (b) A person holding a briefcase does no work on it, because there is no motion. No energy is transferred to or from the briefcase. (c) The person moving the briefcase horizontally at a constant speed does no work on it, and transfers no energy to it. (d) Work is done on the briefcase by carrying it upstairs at constant speed, because there is necessarily a component of force F in the direction of the motion. Energy is transferred to the briefcase and could in turn be used to do work. (e) When the briefcase is lowered, energy is transferred out of the briefcase and into an electric generator. Here the work done on the briefcase by the generator is negative, removing energy from the briefcase, because F and d are in opposite directions.

To examine what the definition of work means, let us consider the other situations shown in Figure. The person holding the briefcase in Figure $6.2.1b$ does no work, for example. Here $d=0$, so $W=0$. Why is it you get tired just holding a load? The answer is that your muscles are doing work against one another, *but they are doing no work on the system of interest* (the "briefcase-Earth system" - see Gravitational Potential Energy for more details). There must be motion for work to be done, and there must be a component of the force in the direction of the motion. For example, the person carrying the briefcase on level ground in Figure $6.2.1c$ does no work on it, because the force is perpendicular to the motion. That is, cos $90^o=0$, so $W=0$.

In contrast, when a force exerted on the system has a component in the direction of motion, such as in Figure $6.2.1d$, work *is* done—energy is transferred to the briefcase. Finally, in Figure $6.2.1e$, energy is transferred from the briefcase to a generator. There are two good ways to interpret this energy transfer. One interpretation is that the briefcase's weight does work on the generator, giving it energy. The other interpretation is that the generator does negative work on the briefcase, thus removing energy from it. The drawing shows the latter, with the force from the generator upward on the briefcase, and the displacement downward. This makes $\theta=180^o$, and cos $180^o=-1$, therefore W is negative.

Calculating Work

Work and energy have the same units. From the definition of work, we see that those units are force times distance. Thus, in SI units, work and energy are measured in newton-meters. A newton-meter is given the special name joule (J), and $1\,J=1\,N\cdot m=1\,kgm^2/s^2$. One joule is not a large amount of energy; it would lift a small 100-gram apple a distance of about 1 meter.

Example 6.2.1: Calculating the Work You Do to Push a Lawn Mower Across a Large Lawn

How much work is done on the lawn mower by the person in Figure (a) if he exerts a constant force of 75.0 N at an angle 35^o below the horizontal and pushes the mower 25 m. on level ground? Convert the amount of work from joules to kilocalories and compare it with this person's average daily intake of **10,000 kJ** (about **2400 kcal**) of food energy. One *calorie* (1 cal) of heat is the amount required to warm 1 g of water by 1^oC and is equivalent to **4,184 J**, while one *food calorie* (**1 kcal**) is equivalent to **4,184 J**.

Strategy

We can solve this problem by substituting the given values into the definition of work done on a system, stated in the equation $W = Fd\,cos\,\theta$. The force, angle, and displacement are given, so that only the work W is unknown.

Solution

The equation for the work is (Equation 6.2.2):

$$W = Fd\ \cos\ \theta$$

Substituting the known values gives

$$\begin{aligned} W &= (75\ N)(25.0\ m)(cos\,35^o) \\ &= 1536\ J \\ &= 1.54 \times 10^3\ J \end{aligned}$$

Converting the work in joules to kilocalories yields $W = (1536\ J)(1\ kcal/4184\ J) = 0.367 kcal$.The ratio of the work done to the daily consumption is

$$\frac{W}{2400\ kcal} = 1.53 \times 10^{-4}.$$

Discussion

This ratio is a tiny fraction of what the person consumes, but it is typical. Very little of the energy released in the consumption of food is used to do work. Even when we "work" all day long, less than 10% of our food energy intake is used to do work and more than 90% is converted to thermal energy or stored as chemical energy in fat.

Summary

- Work is the transfer of energy by a force acting on an object as it is displaced.
- The work W that a force F does on an object is the product of the magnitude F of the force, times the magnitude d of the displacement, times the cosine of the angle θ between them. In symbols,

$$W = Fd\ \cos\ \theta. \tag{6.2.4}$$

- The SI unit for work and energy is the joule (J), where $1\ J = 1\ N \cdot m = 1\ kg m^2/s^2$.
- The work done by a force is zero if the displacement is either zero or perpendicular to the force.
- The work done is positive if the force and displacement have the same direction, and negative if they have opposite direction.

Glossary

energy

the ability to do work

work

the transfer of energy by a force that causes an object to be displaced; the product of the component of the force in the direction of the displacement and the magnitude of the displacement

joule

SI unit of work and energy, equal to one newton-meter

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6.3: Kinetic Energy and the Work-Energy Theorem

Learning Objectives

By the end of this section, you will be able to:

- Explain work as a transfer of energy and net work as the work done by the net force.
- Explain and apply the work-energy theorem.

Work Transfers Energy

What happens to the work done on a system? Energy is transferred into the system, but in what form? Does it remain in the system or move on? The answers depend on the situation. For example, if the lawn mower in [link](a) is pushed just hard enough to keep it going at a constant speed, then energy put into the mower by the person is removed continuously by friction, and eventually leaves the system in the form of heat transfer. In contrast, work done on the briefcase by the person carrying it up stairs in [link](d) is stored in the briefcase-Earth system and can be recovered at any time, as shown in [link](e). In fact, the building of the pyramids in ancient Egypt is an example of storing energy in a system by doing work on the system. Some of the energy imparted to the stone blocks in lifting them during construction of the pyramids remains in the stone-Earth system and has the potential to do work.

In this section we begin the study of various types of work and forms of energy. We will find that some types of work leave the energy of a system constant, for example, whereas others change the system in some way, such as making it move. We will also develop definitions of important forms of energy, such as the energy of motion.

Net Work and the Work-Energy Theorem

We know from the study of Newton's laws in Dynamics: Force and Newton's Laws of Motion that net force causes acceleration. We will see in this section that work done by the net force gives a system energy of motion, and in the process we will also find an expression for the energy of motion.

Let us start by considering the total, or net, work done on a system. **Net work** is defined to be the sum of work done by all external forces—that is, net work is the work done by the net external force F_{net}. In equation form, this is $W_{net} = F_{net} d \, cos \, \theta$, where θ is the angle between the force vector and the displacement vector. Figure (a) shows a graph of force versus displacement for the component of the force in the direction of the displacement—that is, an $F \, cos \, \theta$ vs. d graph. In this case, $F \, cos \, \theta$ is constant. You can see that the area under the graph is $F \, cos \, \theta$, or the work done. Figure (b) shows a more general process where the force varies. The area under the curve is divided into strips, each having an average force $(F \, cos \, \theta)_{i(ave)}$. The work done is e $(F \, cos \, \theta)_{i(ave)} d_i$ for each strip, and the total work done is the sum of the W_i. Thus the total work done is the total area under the curve, a useful property to which we shall refer later.

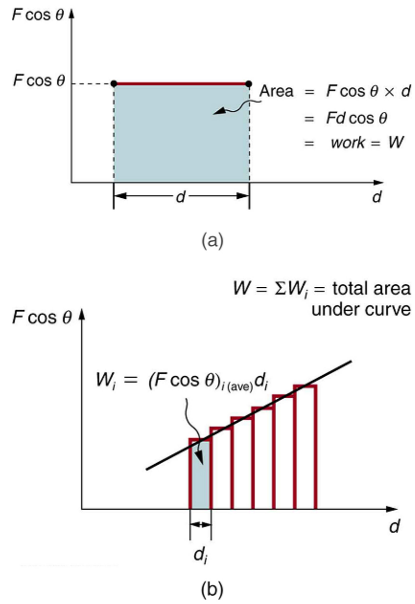

Figure 6.3.1: (a) A graph of $F\,cos\,\theta$ vs. d when $F\,cos\,\theta$ is constant. The area under the curve represents the work done by the force. (b) A graph of $F\,cos\,\theta$ vs. d in which the force varies. The work done for each interval is the area of each strip; thus, the total area under the curve equals the total work done.

Net work will be simpler to examine if we consider a one-dimensional situation where a force is used to accelerate an object in a direction parallel to its initial velocity. Such a situation occurs for the package on the roller belt conveyor system shown in Figure.

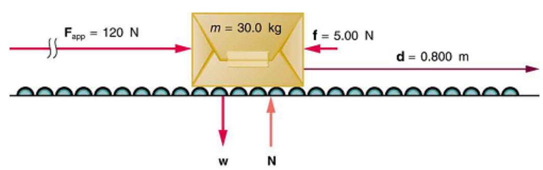

Figure 6.3.2: A package on a roller belt is pushed horizontally through a distance d.

The force of gravity and the normal force acting on the package are perpendicular to the displacement and do no work. Moreover, they are also equal in magnitude and opposite in direction so they cancel in calculating the net force. The net force arises solely from the horizontal applied force F_{app} and the horizontal friction force f. Thus, as expected, the net force is parallel to the displacement, so that $\theta = 0$ and $cos\,\theta = 1$, and the net work is given by

$$W_{net} = F_{net} d. \tag{6.3.1}$$

The effect of the net force F_{net} is to accelerate the package from v_0 to v The kinetic energy of the package increases, indicating that the net work done on the system is positive. (See Example.) By using Newton's second law, and doing some algebra, we can reach an interesting conclusion. Substituting $F = ma$ from Newton's second law gives

$$W_{net} = mad. \tag{6.3.2}$$

To get a relationship between net work and the speed given to a system by the net force acting on it, we take $d = x - x_0$ and use the equation studied in Motion Equations for Constant Acceleration in One Dimension for the change in speed over a distance d if the acceleration has the constant value a, namely $v^2 = v_0^2 + 2ad$. (note that a appears in the expression for the net work). Solving for acceleration gives $a = \frac{v^2 - v_0^2}{2d}$. When a is substituted into the preceding expression for W_{net} we obtain

$$W_{net} = m\left(\frac{v^2 - v_0^2}{2d}\right) d. \tag{6.3.3}$$

The d cancels, and we rearrange this to obtain

$$W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2. \tag{6.3.4}$$

This expression is called the **work-energy theorem,** and it actually applies *in general* (even for forces that vary in direction and magnitude), although we have derived it for the special case of a constant force parallel to the displacement. The theorem implies that the net work on a system equals the change in the quantity $\frac{1}{2}mv^2$. This quantity is our first example of a form of energy.

Work-Energy Theorem

The net work on a system equals the change in the quantity $\frac{1}{2}mv^2$.

$$W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2. \tag{6.3.5}$$

The quantity $\frac{1}{2}mv^2$ in the work-energy theorem is defined to be the translational **kinetic energy** (KE) of a mass m moving at a speed v. (*Translational* kinetic energy is distinct from *rotational* kinetic energy, which is considered later.) In equation form, the translational kinetic energy,

$$KE = \frac{1}{2}mv^2, \tag{6.3.6}$$

is the energy associated with translational motion. Kinetic energy is a form of energy associated with the motion of a particle, single body, or system of objects moving together.

We are aware that it takes energy to get an object, like a car or the package in Figure, up to speed, but it may be a bit surprising that kinetic energy is proportional to speed squared. This proportionality means, for example, that a car traveling at 100 km/h has four times the kinetic energy it has at 50 km/h, helping to explain why high-speed collisions are so devastating. We will now consider a series of examples to illustrate various aspects of work and energy.

Example 6.3.1: Calculating the Kinetic Energy of a Package

Suppose a 30.0-kg package on the roller belt conveyor system in Figure 7.03.2 is moving at 0.500 m/s. What is its kinetic energy?

Strategy

Because the mass m and the speed v are given, the kinetic energy can be calculated from its definition as given in the equation $KE = \frac{1}{2}mv^2$.

Solution

The kinetic energy is given by

$$KE = \frac{1}{2}mv^2. \tag{6.3.7}$$

Entering known values gives

$$KE = 0.5(30.0\,kg)(0.500\,m/s)^2, \tag{6.3.8}$$

which yields

$$KE = 3.75\,kg \cdot m^2/s^2 = 3.75\,J \tag{6.3.9}$$

Discussion

Note that the unit of kinetic energy is the joule, the same as the unit of work, as mentioned when work was first defined. It is also interesting that, although this is a fairly massive package, its kinetic energy is not large at this relatively low speed. This fact is consistent with the observation that people can move packages like this without exhausting themselves.

Example 6.3.2: Determining the Work to Accelerate a Package

Suppose that you push on the 30.0-kg package in Figure 7.03.2. with a constant force of 120 N through a distance of 0.800 m, and that the opposing friction force averages 5.00 N.

(a) Calculate the net work done on the package. (b) Solve the same problem as in part (a), this time by finding the work done by each force that contributes to the net force.

Strategy and Concept for (a)

This is a motion in one dimension problem, because the downward force (from the weight of the package) and the normal force have equal magnitude and opposite direction, so that they cancel in calculating the net force, while the applied force, friction, and the displacement are all horizontal. (See Figure 7.03.2.) As expected, the net work is the net force times distance.

Solution for (a)

The net force is the push force minus friction, or $F_{net} = 120\,N - 5.00\,N = 115\,N$. Thus the net work is

$$W_{net} = F_{net}d = (115\,N)(0.800\,m) \tag{6.3.10}$$

$$= 92.0\,N \cdot m = 92.0\,J \tag{6.3.11}$$

Discussion for (a)

This value is the net work done on the package. The person actually does more work than this, because friction opposes the motion. Friction does negative work and removes some of the energy the person expends and converts it to thermal energy. The net work equals the sum of the work done by each individual force.

Strategy and Concept for (b)

The forces acting on the package are gravity, the normal force, the force of friction, and the applied force. The normal force and force of gravity are each perpendicular to the displacement, and therefore do no work.

Solution for (b)

The applied force does work.

$$W_{app} = F_{app}d\,cos\,(0^o) = F_{app}d \tag{6.3.12}$$

$$= (120\,N)(0.800\,m) \tag{6.3.13}$$

$$= 96.0\,J \tag{6.3.14}$$

The friction force and displacement are in opposite directions, so that $\theta = 180^o$, and the work done by friction is

$$W_{fr} = F_{fr}d\,cos\,(180^o) \tag{6.3.15}$$

$$= -(5.00\,N)(0.800\,m) \tag{6.3.16}$$

$$= -4.00\,J \tag{6.3.17}$$

So the amounts of work done by gravity, by the normal force, by the applied force, and by friction are, respectively,

$$W_{gr} = 0, \tag{6.3.18}$$

$$W_N = 0, \tag{6.3.19}$$

$$W_{app} = 96.0\,J, \tag{6.3.20}$$

$$W_{fr} = -4\,J. \tag{6.3.21}$$

The total work done as the sum of the work done by each force is then seen to be

$$W_{total} = W_{gr} + W_N + W_{app} + W_{fr} = 92.0\,J. \tag{6.3.22}$$

Discussion for (b)

The calculated total work W_{total} as the sum of the work by each force agrees, as expected, with the work W_{net} done by the net force. The work done by a collection of forces acting on an object can be calculated by either approach.

Example 6.3.3: Determining Speed from Work and Energy

Find the speed of the package in Figure 7.03.2. at the end of the push, using work and energy concepts.

Strategy

Here the work-energy theorem can be used, because we have just calculated the net work W_{net} and the initial kinetic energy, $\frac{1}{2}mv_0^2$ These calculations allow us to find the final kinetic energy, $\frac{1}{2}mv^2$ and thus the final speed v.

Solution

The work-energy theorem in equation form is

$$W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2. \tag{6.3.23}$$

Solving for $\frac{1}{2}mv^2$ gives

$$\frac{1}{2}mv^2 = W_{net} + \frac{1}{2}mv_0^2 \tag{6.3.24}$$

Thus,

$$\frac{1}{2}mv^2 = 92.0\,J + 3.75\,J = 95.75\,J. \tag{6.3.25}$$

Solving for the final speed as requested and entering known values gives

$$v = \sqrt{\frac{2(95.75\,J)}{m}} = \sqrt{\frac{191.5\,kg\cdot m^2/s^2}{30.0\,kg}} \tag{6.3.26}$$

$$= 2.53\,m/s \tag{6.3.27}$$

Discussion

Using work and energy, we not only arrive at an answer, we see that the final kinetic energy is the sum of the initial kinetic energy and the net work done on the package. This means that the work indeed adds to the energy of the package.

Example 6.3.4: Work and Energy Can Reveal Distance, Too

How far does the package in Figure 7.03.2. coast after the push, assuming friction remains constant? Use work and energy considerations.

Strategy

We know that once the person stops pushing, friction will bring the package to rest. In terms of energy, friction does negative work until it has removed all of the package’s kinetic energy. The work done by friction is the force of friction times the distance traveled times the cosine of the angle between the friction force and displacement; hence, this gives us a way of finding the distance traveled after the person stops pushing.

Solution

The normal force and force of gravity cancel in calculating the net force. The horizontal friction force is then the net force, and it acts opposite to the displacement, so $\theta = 180^o$. To reduce the kinetic energy of the package to zero, the work W_{fr} by friction must be minus the kinetic energy that the package started with plus what the package accumulated due to the pushing. Thus $W_{fr} = -95.75\,J$. Furthermore, $W_{fr} = df'\,cos\,\theta = -Fd'$, where d' is the distance it takes to stop. Thus,

$$d' = -\frac{W_{fr}}{f} = \frac{-95.75\,J}{5.00\,N}, \tag{6.3.28}$$

and so

$$d' = 19.2\,m \tag{6.3.29}$$

Discussion

This is a reasonable distance for a package to coast on a relatively friction-free conveyor system. Note that the work done by friction is negative (the force is in the opposite direction of motion), so it removes the kinetic energy.

Some of the examples in this section can be solved without considering energy, but at the expense of missing out on gaining insights about what work and energy are doing in this situation. On the whole, solutions involving energy are generally shorter and easier than those using kinematics and dynamics alone.

Summary

- The net work W_{net} is the work done by the net force acting on an object.
- Work done on an object transfers energy to the object.
- The translational kinetic energy of an object of mass m moving at speed v is $KE = \frac{1}{2}mv^2$.
- The work-energy theorem states that the net work W_{net} on a system changes its kinetic energy, $W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$.

Glossary

net work

work done by the net force, or vector sum of all the forces, acting on an object

work-energy theorem

the result, based on Newton's laws, that the net work done on an object is equal to its change in kinetic energy

kinetic energy

the energy an object has by reason of its motion, equal to $\frac{1}{2}mv^2$ for the translational (i.e., non-rotational) motion of an object of mass m moving at speed v

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6.4: Gravitational Potential Energy

Learning Objectives

By the end of this section, you will be able to:

- Explain gravitational potential energy in terms of work done against gravity.
- Show that the gravitational potential energy of an object of mass *m* at height *h* on Earth is given by $PE_g = mgh$
- Show how knowledge of the potential energy as a function of position can be used to simplify calculations and explain physical phenomena.

Work Done Against Gravity

Climbing stairs and lifting objects is work in both the scientific and everyday sense—it is work done against the gravitational force. When there is work, there is a transformation of energy. The work done against the gravitational force goes into an important form of stored energy that we will explore in this section.

Let us calculate the work done in lifting an object of mass m through a height h such as in Figure. If the object is lifted straight up at constant speed, then the force needed to lift it is equal to its weight mg. The work done on the mass is then $W = Fd = mgh$. We define this to be the **gravitational potential energy** (PE_g) put into (or gained by) the object-Earth system. This energy is associated with the state of separation between two objects that attract each other by the gravitational force. For convenience, we refer to this as the PE_g gained by the object, recognizing that this is energy stored in the gravitational field of Earth. Why do we use the word "system"? Potential energy is a property of a system rather than of a single object—due to its physical position. An object's gravitational potential is due to its position relative to the surroundings within the Earth-object system. The force applied to the object is an external force, from outside the system. When it does positive work it increases the gravitational potential energy of the system. Because gravitational potential energy depends on relative position, we need a reference level at which to set the potential energy equal to 0. We usually choose this point to be Earth's surface, but this point is arbitrary; what is important is the *difference* in gravitational potential energy, because this difference is what relates to the work done. The difference in gravitational potential energy of an object (in the Earth-object system) between two rungs of a ladder will be the same for the first two rungs as for the last two rungs.

Converting Between Potential Energy and Kinetic Energy

Gravitational potential energy may be converted to other forms of energy, such as kinetic energy. If we release the mass, gravitational force will do an amount of work equal to mgh on it, thereby increasing its kinetic energy by that same amount (by the work-energy theorem). We will find it more useful to consider just the conversion of PE_g to KE without explicitly considering the intermediate step of work. (See Example $6.4.2$.) This shortcut makes it is easier to solve problems using energy (if possible) rather than explicitly using forces.

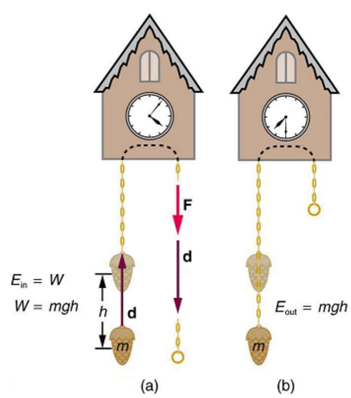

Figure $6.4.1$: (a) The work done to lift the weight is stored in the mass-Earth system as gravitational potential energy. (b) As the weight moves downward, this gravitational potential energy is transferred to the cuckoo clock.

More precisely, we define the *change* in gravitational potential energy ΔPE_g to be

$$\Delta PE_g = mgh, \tag{6.4.1}$$

where, for simplicity, we denote the change in height by h rather than the usual Δh. Note that h is positive when the final height is greater than the initial height, and vice versa. For example, if a 0.500-kg mass hung from a cuckoo clock is raised 1.00 m, then its change in gravitational potential energy is

$$mgh = (0.500\,kg)(9.80\,m/s^2)(1.00\,m) \tag{6.4.2}$$

$$= 4.90\,kg \cdot m^2/s^2 = 4.90\,J. \tag{6.4.3}$$

Note that the units of gravitational potential energy turn out to be joules, the same as for work and other forms of energy. As the clock runs, the mass is lowered. We can think of the mass as gradually giving up its 4.90 J of gravitational potential energy, *without directly considering the force of gravity that does the work*.

Using Potential Energy to Simplify Calculations

The equation $\Delta PE_g = mgh$ applies for any path that has a change in height of h, not just when the mass is lifted straight up. (See Figure.) It is much easier to calculate mgh (a simple multiplication) than it is to calculate the work done along a complicated path. The idea of gravitational potential energy has the double advantage that it is very broadly applicable and it makes calculations easier. From now on, we will consider that any change in vertical position h of a mass m is accompanied by a change in gravitational potential energy mgh, and we will avoid the equivalent but more difficult task of calculating work done by or against the gravitational force.

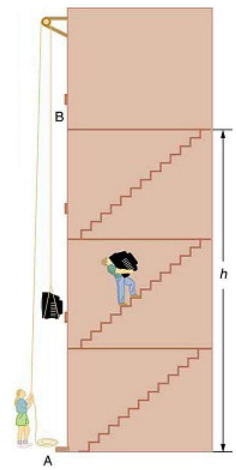

Figure 6.4.2: The change in gravitational potential energy (ΔPE_g) between points A and B is independent of the path $\Delta PE_g = mgh$ for any path between the two points. Gravity is one of a small class of forces where the work done by or against the force depends only on the starting and ending points, not on the path between them.

Example 6.4.1: The Force to Stop Falling

A 60.0-kg person jumps onto the floor from a height of 3.00 m. If he lands stiffly (with his knee joints compressing by 0.500 cm), calculate the force on the knee joints.

Strategy

This person's energy is brought to zero in this situation by the work done on him by the floor as he stops. The initial PE_g is transformed into KE as he falls. The work done by the floor reduces this kinetic energy to zero.

Solution

The work done on the person by the floor as he stops is given by

$$W = Fd\,cos\,\theta = -Fd \tag{6.4.4}$$

with a minus sign because the displacement while stopping and the force from floor are in opposite directions $(cos\,\theta = cos\,180^o = -1.)$ The floor removes energy from the system, so it does negative work.

The kinetic energy the person has upon reaching the floor is the amount of potential energy lost by falling through height h:

$$KE = -\Delta PE_g = -mgh \tag{6.4.5}$$

The distance d that the person's knees bend is much smaller than the height h of the fall, so the additional change in gravitational potential energy during the knee bend is ignored. The work W done by the floor on the person stops the person and brings the person's kinetic energy to zero:

$$W = -KE = mgh \tag{6.4.6}$$

Combining this equation with the expression for W gives

$$-Fd = mgh. \tag{6.4.7}$$

Recalling that h is negative because the person fell *down*, the force on the knee joints is given by

$$F = -\frac{mgh}{d} = -\frac{(60.0\,kg)(9.80\,m/s^2)(-3.00\,m)}{5.00\times 10^{-3}\,m} = 3.53\times 10^5\,N. \tag{6.4.8}$$

Discussion

Such a large force (500 times more than the person's weight) over the short impact time is enough to break bones. A much better way to cushion the shock is by bending the legs or rolling on the ground, increasing the time over which the force acts. A bending motion of 0.5 m this way yields a force 100 times smaller than in the example. A kangaroo's hopping shows this method in action. The kangaroo is the only large animal to use hopping for locomotion, but the shock in hopping is cushioned by the bending of its hind legs in each jump.(See Figure.)

Figure 6.4.3: The work done by the ground upon the kangaroo reduces its kinetic energy to zero as it lands. However, by applying the force of the ground on the hind legs over a longer distance, the impact on the bones is reduced. (credit: Chris Samuel, Flickr)

Example 6.4.2: Finding the Speed of a Roller Coaster from its Height

(a) What is the final speed of the roller coaster shown in Figure, if it starts from rest at the top of the 20.0 m hill and work done by frictional forces is negligible? (b) What is its final speed (again assuming negligible friction) if its initial speed is 5.00 m/s?

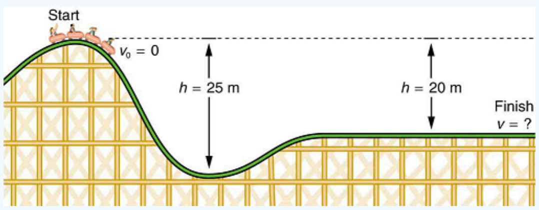

Figure 6.4.4: The speed of a roller coaster increases as gravity pulls it downhill and is greatest at its lowest point. Viewed in terms of energy, the roller-coaster-Earth system's gravitational potential energy is converted to kinetic energy. If work done by friction is negligible, all ΔPE_g is converted to KE.

Strategy

The roller coaster loses potential energy as it goes downhill. We neglect friction, so that the remaining force exerted by the track is the normal force, which is perpendicular to the direction of motion and does no work. The net work on the roller coaster is then done by gravity alone. The *loss* of gravitational potential energy from moving *downward* through a distance h

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equals the *gain* in kinetic energy. This can be written in equation form as $-\Delta PE = \Delta KE$. Using the equations for PE_g and KE we can solve for the final speed v, which is the desired quantity.

Solution for (a)

Here the initial kinetic energy is zero, so that $\Delta KE = \frac{1}{2}mv^2$. The equation for change in potential energy states that $\Delta PE_g = mgh$. Since h is negative in this case, we will rewrite this as $\Delta PE_g = -mg|h|$ to show the minus sign clearly. Thus,

$$-\Delta PE_g = \Delta KE \tag{6.4.9}$$

becomes

$$mg|h| = \frac{1}{2}mv^2 \tag{6.4.10}$$

Solving for v we find that mass cancels and that

$$v = \sqrt{2g|h|}. \tag{6.4.11}$$

Substituting known values,

$$v = \sqrt{2(9.80\,m/s^2)(20.0\,m)} \tag{6.4.12}$$

$$= 19.8\,m/s \tag{6.4.13}$$

Solution for (b)

Again $-\Delta PE_g = \Delta KE$. In this case there is initial kinetic energy, so $\Delta KE = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$. Thus,

$$mgh = \frac{1}{2}mv^2 - \frac{1}{2}m_0^2. \tag{6.4.14}$$

Rearranging gives

$$\frac{1}{2}mv^2 = mg|h| + \frac{1}{2}mv_0^2. \tag{6.4.15}$$

This means that the final kinetic energy is the sum of the initial kinetic energy and the gravitational potential energy. Mass again cancels, and

$$v = \sqrt{2g|h| + v_0^2}. \tag{6.4.16}$$

This equation is very similar to the kinematics equation $v = \sqrt{v_0^2 + 2ad}$, but it is more general—the kinematics equation is valid only for constant acceleration, whereas our equation above is valid for any path regardless of whether the object moves with a constant acceleration. Now, substituting known values gives

$$v = \sqrt{2(9.80\,m/s^2)(20.0\,m) + (5.00)^2} \tag{6.4.17}$$

$$= 20.4\,m/s. \tag{6.4.18}$$

Discussion and Implications

First, note that mass cancels. This is quite consistent with observations made in Falling Objects that all objects fall at the same rate if friction is negligible. Second, only the speed of the roller coaster is considered; there is no information about its direction at any point. This reveals another general truth. When friction is negligible, the speed of a falling body depends only on its initial speed and height, and not on its mass or the path taken. For example, the roller coaster will have the same final speed whether it falls 20.0 m straight down or takes a more complicated path like the one in the figure. Third, and perhaps unexpectedly, the final speed in part (b) is greater than in part (a), but by far less than 5.00 m/s. Finally, note that speed can be found at *any* height along the way by simply using the appropriate value of h at the point of interest.

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We have seen that work done by or against the gravitational force depends only on the starting and ending points, and not on the path between, allowing us to define the simplifying concept of gravitational potential energy. We can do the same thing for a few other forces, and we will see that this leads to a formal definition of the law of conservation of energy.

Making Connections: Take-Home Investigation—Converting Potential to Kinetic Energy

One can study the conversion of gravitational potential energy into kinetic energy in this experiment. On a smooth, level surface, use a ruler of the kind that has a groove running along its length and a book to make an incline (see Figure). Place a marble at the 10-cm position on the ruler and let it roll down the ruler. When it hits the level surface, measure the time it takes to roll one meter. Now place the marble at the 20-cm and the 30-cm positions and again measure the times it takes to roll 1 m on the level surface. Find the velocity of the marble on the level surface for all three positions. Plot velocity squared versus the distance traveled by the marble. What is the shape of each plot? If the shape is a straight line, the plot shows that the marble's kinetic energy at the bottom is proportional to its potential energy at the release point.

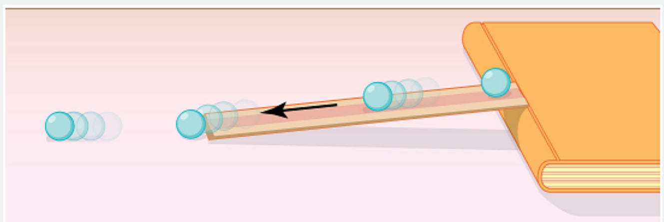

Figure 6.4.5: A marble rolls down a ruler, and its speed on the level surface is measured.

Summary

- Work done against gravity in lifting an object becomes potential energy of the object-Earth system.
- The change in gravitational potential energy ΔPE_g, is $\Delta PE_g = mgh$, with h being the increase in height and g the acceleration due to gravity.
- The gravitational potential energy of an object near Earth's surface is due to its position in the mass-Earth system. Only differences in gravitational potential energy, ΔPE_g, have physical significance.
- As an object descends without friction, its gravitational potential energy changes into kinetic energy corresponding to increasing speed, so that $\Delta KE = -\Delta PE_g$.

Glossary

gravitational potential energy

the energy an object has due to its position in a gravitational field

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6.5: Conservative Forces and Potential Energy

Learning Objectives

By the end of this section, you will be able to:

- Define conservative force, potential energy, and mechanical energy.
- Explain the potential energy of a spring in terms of its compression when Hooke's law applies.
- Use the work-energy theorem to show how having only conservative forces implies conservation of mechanical energy.

Work is done by a force, and some forces, such as weight, have special characteristics. A *conservative force* is one, like the gravitational force, for which work done by or against it depends only on the starting and ending points of a motion and not on the path taken. We can define a *potential energy (PE)* for any conservative force, just as we did for the gravitational force. For example, when you wind up a toy, an egg timer, or an old-fashioned watch, you do work against its spring and store energy in it. (We treat these springs as ideal, in that we assume there is no friction and no production of thermal energy.) This stored energy is recoverable as work, and it is useful to think of it as potential energy contained in the spring. Indeed, the reason that the spring has this characteristic is that its force is *conservative*. That is, a conservative force results in stored or potential energy. Gravitational potential energy is one example, as is the energy stored in a spring. We will also see how conservative forces are related to the conservation of energy.

Potential Energy and Conservative Forces

Potential energy is the energy a system has due to position, shape, or configuration. It is stored energy that is completely recoverable.

A conservative force is one for which work done by or against it depends only on the starting and ending points of a motion and not on the path taken.

We can define a potential energy (PE) for any conservative force. The work done against a conservative force to reach a final configuration depends on the configuration, not the path followed, and is the potential energy added.

Potential Energy of a Spring

First, let us obtain an expression for the potential energy stored in a spring $(PE_s$. We calculate the work done to stretch or compress a spring that obeys Hooke's law. (Hooke's law was examined in Elasticity: Stress and Strain, and states that the magnitude of force F on the spring and the resulting deformation ΔL are proportional, $F = k\Delta L$). (See Figure.) For our spring, we will replace ΔL (the amount of deformation produced by a force F) by the distance x that the spring is stretched or compressed along its length. So the force needed to stretch the spring has magnitude $F = kx$, where k is the spring's force constant. The force increases linearly from 0 at the start to kx in the fully stretched position. The average force is $kx/2$. Thus the work done in stretching or compressing the spring is

$$W_s = Fd = (\frac{kx}{2})x = \frac{1}{2}kx^2. \tag{6.5.1}$$

Alternatively, we noted in Kinetic Energy and the Work-Energy Theorem that the area under a graph of F vs. x is the work done by the force. In Figure (c) we see that this area is also $\frac{1}{2}kx^2$. We therefore define the **potential energy of a spring**, \(PE_s|) to be

$$PE_s = \frac{1}{2}kx^2, \tag{6.5.2}$$

where k is the spring's force constant and x is the displacement from its undeformed position. The potential energy represents the work done *on* the spring and the energy stored in it as a result of stretching or compressing it a distance x. The potential energy of the spring PE_s does not depend on the path taken; it depends only on the stretch or squeeze x in the final configuration.

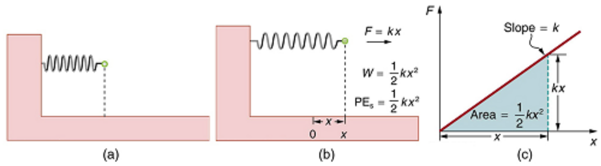

Figure 6.5.1: (a) An undeformed spring has no PE_s stored in it. (b) The force needed to stretch (or compress) the spring a distance x has a magnitude $F = kx$, and the work done to stretch (or compress) it is $\frac{1}{2}kx^2$. Because the force is conservative, this work is stored as potential energy (PE_s) in the spring, and it can be fully recovered. (c) A graph of F vs x has a slope of k, and the area under the graph is $\frac{1}{2}kx^2$. Thus the work done or potential energy stored is $\frac{1}{2}kx^2$.

The equation $PE_s = \frac{1}{2}kx^2$ has general validity beyond the special case for which it was derived. Potential energy can be stored in any elastic medium by deforming it. Indeed, the general definition of potential energy is energy due to position, shape, or configuration. For shape or position deformations, stored energy is $PE_s = \frac{1}{2}kx^2$, where k is the force constant of the particular system and x is its deformation. Another example is seen in Figure for a guitar string.

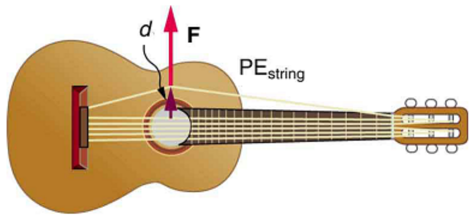

Figure 6.5.2: Work is done to deform the guitar string, giving it potential energy. When released, the potential energy is converted to kinetic energy and back to potential as the string oscillates back and forth. A very small fraction is dissipated as sound energy, slowly removing energy from the string.

Conservation of Mechanical Energy

Let us now consider what form the work-energy theorem takes when only conservative forces are involved. This will lead us to the conservation of energy principle. The work-energy theorem states that the net work done by all forces acting on a system equals its change in kinetic energy. In equation form, this is

$$W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = \Delta KE. \tag{6.5.3}$$

If only conservative forces act, then

$$W_{net} = W_c \tag{6.5.4}$$

where W_c is the total work done by all conservative forces. Thus,

$$W_c = \Delta KE. \tag{6.5.5}$$

Now, if the conservative force, such as the gravitational force or a spring force, does work, the system loses potential energy. That is,

$$W_c = -\Delta PE \tag{6.5.6}$$

Therefore,

$$-\Delta PE = KE \tag{6.5.7}$$

or

$$\Delta PE + \Delta KE = 0. \tag{6.5.8}$$

This equation means that the total kinetic and potential energy is constant for any process involving only conservative forces. That is,

$$KE + PE = constant \tag{6.5.9}$$

or

$$KE_i + PE_i = KE_f + PE_f \tag{6.5.10}$$

$$(conservative\,forces\,only), \tag{6.5.11}$$

where i and f denote initial and final values. This equation is a form of the work-energy theorem for conservative forces; it is known as the **conservation of mechanical energy principle**. Remember that this applies to the extent that all the forces are conservative, so that friction is negligible. The total kinetic plus potential energy of a system is defined to be its **mechanical energy**, $(KE+PE)$. In a system that experiences only conservative forces, there is a potential energy associated with each force, and the energy only changes form between KE and the various types of PE, with the total energy remaining constant.

Example 6.5.1: Using Conservation of Mechanical Energy to Calculate the Speed of a Toy Car

A 0.100-kg toy car is propelled by a compressed spring, as shown in Figure. The car follows a track that rises 0.180 m above the starting point. The spring is compressed 4.00 cm and has a force constant of 250.0 N/m. Assuming work done by friction to be negligible, find (a) how fast the car is going before it starts up the slope and (b) how fast it is going at the top of the slope.

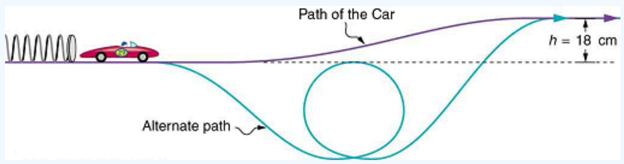

Figure 6.5.3: A toy car is pushed by a compressed spring and coasts up a slope. Assuming negligible friction, the potential energy in the spring is first completely converted to kinetic energy, and then to a combination of kinetic and gravitational potential energy as the car rises. The details of the path are unimportant because all forces are conservative—the car would have the same final speed if it took the alternate path shown.

Strategy

The spring force and the gravitational force are conservative forces, so conservation of mechanical energy can be used. Thus,

$$KE_i + PE_i = KE_f + PE_f \tag{6.5.12}$$

or

$$\frac{1}{2}mv_i^2 + mgh_i + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + mgh_f + \frac{1}{2}kx_f^2$$

where h is the height (vertical position) and x is the compression of the spring. This general statement looks complex but becomes much simpler when we start considering specific situations. First, we must identify the initial and final conditions in a problem; then, we enter them into the last equation to solve for an unknown.

Solution for (a)

This part of the problem is limited to conditions just before the car is released and just after it leaves the spring. Take the initial height to be zero, so that both h_i and h_f are zero. Furthermore, the initial speed v_i is zero and the final compression of the spring x_f is zero, and so several terms in the conservation of mechanical energy equation are zero and it simplifies to

$$\frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2.$$

In other words, the initial potential energy in the spring is converted completely to kinetic energy in the absence of friction. Solving for the final speed and entering known values yields

$$\begin{aligned} v_f &= \sqrt{\frac{k}{m}}x_i \\ &= \sqrt{\frac{250N/m}{0.100kg}}(0.0400\,m) \\ &= 2.00m/s \end{aligned}$$

Solution for (b)

One method of finding the speed at the top of the slope is to consider conditions just before the car is released and just after it reaches the top of the slope, completely ignoring everything in between. Doing the same type of analysis to find which terms are zero, the conservation of mechanical energy becomes

$$\frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + mgh_f.$$

This form of the equation means that the spring's initial potential energy is converted partly to gravitational potential energy and partly to kinetic energy. The final speed at the top of the slope will be less than at the bottom. Solving for v_f and substituting known values gives

$$v_f = \sqrt{\frac{kx^2}{m} - 2gh_f}$$

$$= \sqrt{\left(\frac{250N/m}{0.100kg}\right)(0.0400m)^2 - 2(9.80m/s^2)(0.180m)}$$

$$= 0.687\,m/s$$

Discussion

Another way to solve this problem is to realize that the car's kinetic energy before it goes up the slope is converted partly to potential energy—that is, to take the final conditions in part (a) to be the initial conditions in part (b).

Note that, for conservative forces, we do not directly calculate the work they do; rather, we consider their effects through their corresponding potential energies, just as we did in Example. Note also that we do not consider details of the path taken—only the starting and ending points are important (as long as the path is not impossible). This assumption is usually a tremendous simplification, because the path may be complicated and forces may vary along the way.

PHET Explorations: Energy Skate Park

Learn about conservation of energy with a skater dude! Build tracks, ramps and jumps for the skater and view the kinetic energy, potential energy and friction as he moves. You can also take the skater to different planets or even space!

Summary

- A conservative force is one for which work depends only on the starting and ending points of a motion, not on the path taken.
- We can define potential energy $(PE$ for any conservative force, just as we defined PE_g for the gravitational force.
- The potential energy of a spring is $PE_s = \frac{1}{2}kx^2$, where k is the spring's force constant and |(x\) is the displacement from its undeformed position.
- Mechanical energy is defined to be $KE = PE$ for conservative force.
- When only conservative forces act on and within a system, the total mechanical energy is constant. In equation form,

$$KE + PE = constant \tag{6.5.13}$$

or

$$KE_i + PE_i = KE_f + PE_f \tag{6.5.14}$$

where i and f denote initial and final values. This is known as the conservation of mechanical energy.

Glossary

conservative force

a force that does the same work for any given initial and final configuration, regardless of the path followed

potential energy

energy due to position, shape, or configuration

potential energy of a spring

the stored energy of a spring as a function of its displacement; when Hooke's law applies, it is given by the expression $\frac{1}{2}kx^2$ where x is the distance the spring is compressed or extended and k is the spring constant

conservation of mechanical energy

the rule that the sum of the kinetic energies and potential energies remains constant if only conservative forces act on and within a system

mechanical energy

the sum of kinetic energy and potential energy

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6.6: Nonconservative Forces

Learning Objectives

By the end of this section, you will be able to:

- Define nonconservative forces and explain how they affect mechanical energy.
- Show how the principle of conservation of energy can be applied by treating the conservative forces in terms of their potential energies and any nonconservative forces in terms of the work they do.

Nonconservative Forces and Friction

Forces are either conservative or nonconservative. A **nonconservative** force is one for which work depends on the path taken. Friction is a good example of a nonconservative force. As illustrated in Figure $6.6.1$, work done against friction depends on the length of the path between the starting and ending points. Because of this dependence on path, there is no potential energy associated with nonconservative forces. An important characteristic is that the work done by a nonconservative force *adds or removes mechanical energy from a system*. **Friction**, for example, creates **thermal energy** that dissipates, removing energy from the system. Furthermore, even if the thermal energy is retained or captured, it cannot be fully converted back to work, so it is lost or not recoverable in that sense as well.

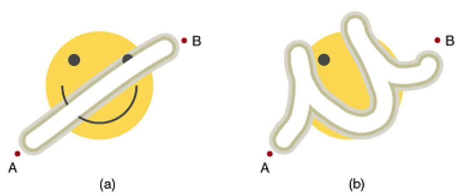

Figure $6.6.1$: The amount of the happy face erased depends on the path taken by the eraser between points A and B, as does the work done against friction. Less work is done and less of the face is erased for the path in (a) than for the path in (b). The force here is friction, and most of the work goes into thermal energy that subsequently leaves the system (the happy face plus the eraser). The energy expended cannot be fully recovered.

How Nonconservative Forces Affect Mechanical Energy

Mechanical energy *may* not be conserved when nonconservative forces act. For example, when a car is brought to a stop by friction on level ground, it loses kinetic energy, which is dissipated as thermal energy, reducing its mechanical energy. Figure compares the effects of conservative and nonconservative forces. We often choose to understand simpler systems such as that described in Figure $6.6.2a$ first before studying more complicated systems as in Figure $6.6.2b$.

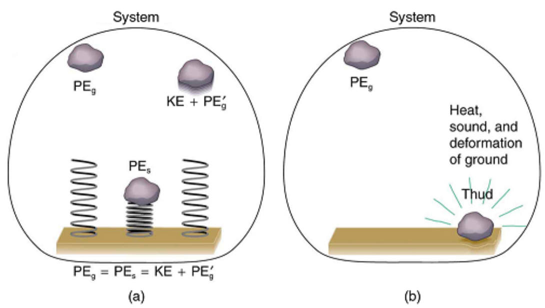

Figure $6.6.2$: Comparison of the effects of conservative and nonconservative forces on the mechanical energy of a system. (a) A system with only conservative forces. When a rock is dropped onto a spring, its mechanical energy remains constant (neglecting air resistance) because the force in the spring is conservative. The spring can propel the rock back to its original height, where it once again has only potential energy due to gravity. (b) A system with nonconservative forces. When the same rock is dropped onto the ground, it is stopped by nonconservative forces that dissipate its mechanical energy as thermal energy, sound, and surface distortion. The rock has lost mechanical energy.

How the Work-Energy Theorem Applies

Now let us consider what form the work-energy theorem takes when both conservative and nonconservative forces act. We will see that the work done by nonconservative forces equals the change in the mechanical energy of a system. As noted in Kinetic Energy and the Work-Energy Theorem, the work-energy theorem states that the net work on a system equals the change in its kinetic energy, or $W_{net} = \Delta KE$. The net work is the sum of the work by nonconservative forces plus the work by conservative forces. That is,

$$W_{net} = W_{nc} + W_c, \tag{6.6.1}$$

so that

$$W_{nc} + W_c = \Delta KE, \tag{6.6.2}$$

where W_{nc} is the total work done by all nonconservative forces and W_c is the total work done by all conservative forces.

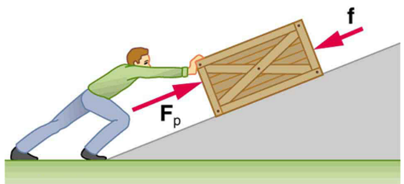

Figure 6.6.3: A person pushes a crate up a ramp, doing work on the crate. Friction and gravitational force (not shown) also do work on the crate; both forces oppose the person's push. As the crate is pushed up the ramp, it gains mechanical energy, implying that the work done by the person is greater than the work done by friction.

Consider Figure 6.6.3, in which a person pushes a crate up a ramp and is opposed by friction. As in the previous section, we note that work done by a conservative force comes from a loss of gravitational potential energy, so that $W_c = -\Delta PE$. Substituting this equation into the previous one and solving for W_{nc} gives

$$W_{nc} = \Delta KE + \Delta PE. \tag{6.6.3}$$

This equation means that the total mechanical energy $(KE + PE)$ changes by exactly the amount of work done by nonconservative forces. In Figure 6.6.3, this is the work done by the person minus the work done by friction. So even if energy is not conserved for the system of interest (such as the crate), we know that an equal amount of work was done to cause the change in total mechanical energy. We rearrange Equation 6.6.4 to obtain

$$KE_i + PE_i + W_{nc} = KE_f + PE_f \tag{6.6.4}$$

This means that the amount of work done by nonconservative forces adds to the mechanical energy of a system. If W_{nc} is positive, then mechanical energy is increased, such as when the person pushes the crate up the ramp in Figure. If W_{nc} is negative, then mechanical energy is decreased, such as when the rock hits the ground in Figure 6.6.2b. If W_{nc} is negative, then mechanical energy is decreased, such as when the rock hits the ground in Figure 6.6.2b. If W_{nc} is zero, then mechanical energy is conserved, and nonconservative forces are balanced. For example, when you push a lawn mower at constant speed on level ground, your work done is removed by the work of friction, and the mower has a constant energy.

Applying Energy Conservation with Nonconservative Forces

When no change in potential energy occurs, applying Equation 6.6.4 amounts to applying the work-energy theorem by setting the change in kinetic energy to be equal to the net work done on the system, which in the most general case includes both conservative and nonconservative forces. But when seeking instead to find a change in total mechanical energy in situations that involve changes in both potential and kinetic energy, the previous equation $KE_i + PE_i + W_{nc} = KE_f + PE_f$ says that you can start by finding the change in mechanical energy that would have resulted from just the conservative forces, including the potential energy changes, and add to it the work done, with the proper sign, by any nonconservative forces involved.

Example 6.6.1: Calculating Distance Traveled: How Far a Baseball Player Slides

Consider the situation shown in Figure 6.6.4, where a baseball player slides to a stop on level ground. Using energy considerations, calculate the distance the 65.0-kg baseball player slides, given that his initial speed is 6.00 m/s and the force of friction against him is a constant 450 N.

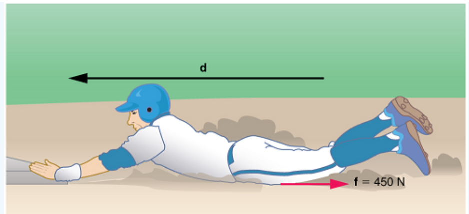

Figure 6.6.4: The baseball player slides to a stop in a distance d. In the process, friction removes the player's kinetic energy by doing an amount of work fd equal to the initial kinetic energy.

Strategy

Friction stops the player by converting his kinetic energy into other forms, including thermal energy. In terms of the work-energy theorem, the work done by friction, which is negative, is added to the initial kinetic energy to reduce it to zero. The work done by friction is negative, because f is in the opposite direction of the motion (that is, $\theta = 180^o$, and so $cos\, \theta = -1$). Thus, $W_{nc} = -fd$. The equation simplifies to

$$\frac{1}{2}mv_i^2 - bfd = 0$$

or

$$fd = \frac{1}{2}mv_i^2.$$

This equation can now be solved for the distance d.

Solution

Solving the previous equation for d and substituting known values yields

$$\begin{aligned} d &= \frac{mv_i^2}{2f} \\ &= \frac{65.0\, kg)(6.00\, m/s)^2}{(2)(450\, N)} \\ &= 2.60\, m. \end{aligned}$$

Discussion

The most important point of this example is that the amount of nonconservative work equals the change in mechanical energy. For example, you must work harder to stop a truck, with its large mechanical energy, than to stop a mosquito.

Example 6.6.2: Calculating Distance Traveled: Sliding Up an Incline

Suppose that the player from Example $6.6.1$ is running up a hill having a 5.00^o incline upward with a surface similar to that in the baseball stadium. The player slides with the same initial speed, and the frictional force is still 450 N. Determine how far he slides.

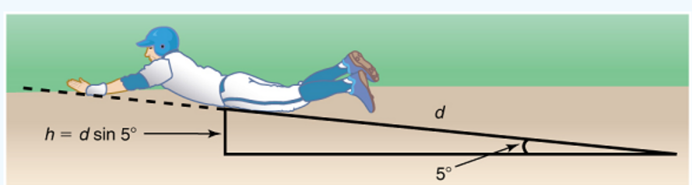

Figure 6.6.5: The same baseball player slides to a stop on a 5.00^o slope.

Strategy

In this case, the work done by the nonconservative friction force on the player reduces the mechanical energy he has from his kinetic energy at zero height, to the final mechanical energy he has by moving through distance d to reach height h along the hill, with $h = d\, \sin\, 5.00^o$. This is expressed by Equation 6.6.4

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$$KE_i + PE_i + W_{nc} = KE_f + PE_f.$$

Solution

The work done by friction is again $W_{nc} = -fd$; initially the potential energy is $PE_i = mg \cdot 0 = 0$ and the kinetic energy is $KE_i = \frac{1}{2}mv^2$; the final energy contributions are $KE_f = 0$ for the kinetic energy and $PE_f = mgh = mgd\,sin\,\theta$ for the potential energy.

Substituting these values gives

$$\frac{1}{2}mv_i^2 + 0 + (-fd) = 0 + mgd\ \sin\ \theta.$$

Solve this for d to obtain

$$\begin{aligned} d &= \frac{\frac{1}{2}mv_i^2}{f + mg\ \sin\ \theta} \\ &= \frac{(0.5)(65.0\,kg)(6.00\,m/s)^2}{450\,N + (65.0\,kg)(9.80\,m/s^2)sin\,(5.00^o)} \\ &= 2.31\,m. \end{aligned}$$

Discussion

As might have been expected, the player slides a shorter distance by sliding uphill. Note that the problem could also have been solved in terms of the forces directly and the work energy theorem, instead of using the potential energy. This method would have required combining the normal force and force of gravity vectors, which no longer cancel each other because they point in different directions, and friction, to find the net force. You could then use the net force and the net work to find the distance d that reduces the kinetic energy to zero. By applying conservation of energy and using the potential energy instead, we need only consider the gravitational potential energy mgh, without combining and resolving force vectors. This simplifies the solution considerably.

Making Connections: Take-Home Investigation - Determining Friction from the Stopping Distance

This experiment involves the conversion of gravitational potential energy into thermal energy. Use the ruler, book, and marble from Take-Home Investigation—Converting Potential to Kinetic Energy. In addition, you will need a foam cup with a small hole in the side, as shown in Figure 6.6.6. From the 10-cm position on the ruler, let the marble roll into the cup positioned at the bottom of the ruler. Measure the distance d the cup moves before stopping. What forces caused it to stop? What happened to the kinetic energy of the marble at the bottom of the ruler? Next, place the marble at the 20-cm and the 30-cm positions and again measure the distance the cup moves after the marble enters it. Plot the distance the cup moves versus the initial marble position on the ruler. Is this relationship linear? With some simple assumptions, you can use these data to find the coefficient of kinetic friction μ_k of the cup on the table. The force of friction f on the cup is $\mu_k N$, where the normal force N is just the weight of the cup plus the marble. The normal force and force of gravity do no work because they are perpendicular to the displacement of the cup, which moves horizontally. The work done by friction is fd. You will need the mass of the marble as well to calculate its initial kinetic energy.

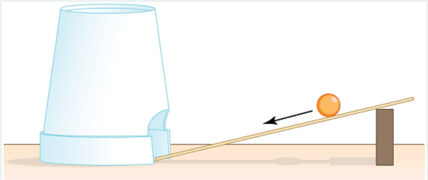

Figure 6.6.6: Rolling a marble down a ruler into a foam cup.

It is interesting to do the above experiment also with a steel marble (or ball bearing). Releasing it from the same positions on the ruler as you did with the glass marble, is the velocity of this steel marble the same as the velocity of the marble at the bottom of the ruler? Is the distance the cup moves proportional to the mass of the steel and glass marbles?

Phet Explorations: The Ramp

Explore forces, energy and work as you push household objects up and down a ramp. Lower and raise the ramp to see how the angle of inclination affects the parallel forces acting on the file cabinet. Graphs show forces, energy and work.

Figure 6.6.7: The Ramp

Summary

- A nonconservative force is one for which work depends on the path.
- Friction is an example of a nonconservative force that changes mechanical energy into thermal energy.
- Work W_{nc} done by a nonconservative force changes the mechanical energy of a system. In equation form, $W_{nc} = \Delta KE + \Delta PE$ or, equivalently, $KE_i + PE_i + W_{nc} = KE_f + PE_f$.
- When both conservative and nonconservative forces act, energy conservation can be applied and used to calculate motion in terms of the known potential energies of the conservative forces and the work done by nonconservative forces, instead of finding the net work from the net force, or having to directly apply Newton's laws

Glossary

nonconservative force

a force whose work depends on the path followed between the given initial and final configurations

friction

the force between surfaces that opposes one sliding on the other; friction changes mechanical energy into thermal energy

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6.7: Conservation of Energy

Learning Objectives

By the end of this section, you will be able to:

- Explain the law of the conservation of energy.
- Describe some of the many forms of energy.
- Define efficiency of an energy conversion process as the fraction left as useful energy or work, rather than being transformed, for example, into thermal energy.

Energy, as we have noted, is conserved, making it one of the most important physical quantities in nature. The law of conservation of energy can be stated as follows:

We have explored some forms of energy and some ways it can be transferred from one system to another. This exploration led to the definition of two major types of energy—mechanical energy $(KE+PE)$ and energy transferred via work done by nonconservative forces (W_{nc}) But energy takes *many* other forms, manifesting itself in *many* different ways, and we need to be able to deal with all of these before we can write an equation for the above general statement of the conservation of energy.

Other Forms of Energy than Mechanical Energy

At this point, we deal with all other forms of energy by lumping them into a single group called other energy (OE). Then we can state the conservation of energy in equation form as

$$KE_i+PE_i+W_{nc}+OE_i=KE_f+PE_f+OE_f. \tag{6.7.1}$$

All types of energy and work can be included in this very general statement of conservation of energy. Kinetic energy is KE, work done by a conservative force is represented by PE, work done by nonconservative forces is W_{nc} and all other energies are included as OE. This equation applies to all previous examples; in those situations OE was constant, and so it subtracted out and was not directly considered.

Usefulness of the Energy Conservation Principle

The fact that energy is conserved and has many forms makes it very important. You will find that energy is discussed in many contexts, because it is involved in all processes. It will also become apparent that many situations are best understood in terms of energy and that problems are often most easily conceptualized and solved by considering energy.

When does OE play a role? One example occurs when a person eats. Food is oxidized with the release of carbon dioxide, water, and energy. Some of this chemical energy is converted to kinetic energy when the person moves, to potential energy when the person changes altitude, and to thermal energy (another form of OE).

Some of the Many Forms of Energy

What are some other forms of energy? You can probably name a number of forms of energy not yet discussed. Many of these will be covered in later chapters, but let us detail a few here. **Electrical energy** is a common form that is converted to many other forms and does work in a wide range of practical situations. Fuels, such as gasoline and food, carry **chemical energy** that can be transferred to a system through oxidation. Chemical fuel can also produce electrical energy, such as in batteries. Batteries can in turn produce light, which is a very pure form of energy. Most energy sources on Earth are in fact stored energy from the energy we receive from the Sun. We sometimes refer to this as **radiant energy**, or electromagnetic radiation, which includes visible light, infrared, and ultraviolet radiation. **Nuclear energy** comes from processes that convert measurable amounts of mass into energy. Nuclear energy is transformed into the energy of sunlight, into electrical energy in power plants, and into the energy of the heat transfer and blast in weapons. Atoms and molecules inside all objects are in random motion. This internal mechanical energy from the random motions is called **thermal energy**, because it is related to the temperature of the object. These and all other forms of energy can be converted into one another and can do work.

Table gives the amount of energy stored, used, or released from various objects and in various phenomena. The range of energies and the variety of types and situations is impressive.

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Problem-Solving Strategies for Energy

You will find the following problem-solving strategies useful whenever you deal with energy. The strategies help in organizing and reinforcing energy concepts. In fact, they are used in the examples presented in this chapter. The familiar general problem-solving strategies presented earlier—involving identifying physical principles, knowns, and unknowns, checking units, and so on —continue to be relevant here.

Step 1. Determine the system of interest and identify what information is given and what quantity is to be calculated. A sketch will help.

Step 2. Examine all the forces involved and determine whether you know or are given the potential energy from the work done by the forces. Then use step 3 or step 4.

Step 3. If you know the potential energies for the forces that enter into the problem, then forces are all conservative, and you can apply conservation of mechanical energy simply in terms of potential and kinetic energy. The equation expressing conservation of energy is

$$KE_i + PE_i = KE_f + PE_f. \tag{6.7.2}$$

Step 4. If you know the potential energy for only some of the forces, possibly because some of them are nonconservative and do not have a potential energy, or if there are other energies that are not easily treated in terms of force and work, then the conservation of energy law in its most general form must be used.

$$KE_i + PE_i + W_{nc} + OE_i = KE_f + PE_f + OE_f. \tag{6.7.3}$$

In most problems, one or more of the terms is zero, simplifying its solution. Do not calculate W_c, the work done by conservative forces; it is already incorporated in the PE terms.

Step 5. You have already identified the types of work and energy involved (in step 2). Before solving for the unknown, *eliminate terms wherever possible* to simplify the algebra. For example, choose $h=0$ at either the initial or final point, so that PE_g is zero there. Then solve for the unknown in the customary manner.

Step 6. *Check the answer to see if it is reasonable*. Once you have solved a problem, reexamine the forms of work and energy to see if you have set up the conservation of energy equation correctly. For example, work done against friction should be negative, potential energy at the bottom of a hill should be less than that at the top, and so on. Also check to see that the numerical value obtained is reasonable. For example, the final speed of a skateboarder who coasts down a 3-m-high ramp could reasonably be 20 km/h, but *not* 80 km/h.

Transformation of Energy

The transformation of energy from one form into others is happening all the time. The chemical energy in food is converted into thermal energy through metabolism; light energy is converted into chemical energy through photosynthesis. In a larger example, the chemical energy contained in coal is converted into thermal energy as it burns to turn water into steam in a boiler. This thermal energy in the steam in turn is converted to mechanical energy as it spins a turbine, which is connected to a generator to produce electrical energy. (In all of these examples, not all of the initial energy is converted into the forms mentioned. This important point is discussed later in this section.)

Another example of energy conversion occurs in a solar cell. Sunlight impinging on a solar cell (Figure 7.7.1) produces electricity, which in turn can be used to run an electric motor. Energy is converted from the primary source of solar energy into electrical energy and then into mechanical energy.

Figure 6.7.1: Solar energy is converted into electrical energy by solar cells, which is used to run a motor in this solar-power aircraft. (credit: NASA)

Object/phenomenon	Energy in joules
Big Bang	10^{68}
Energy released in a supernova	10^{44}
Fusion of all the hydrogen in Earth’s oceans	10^{34}
Annual world energy use	4×10^{20}
Large fusion bomb (9 megaton)	3.8×10^{16}
1 kg hydrogen (fusion to helium)	6.4×10^{14}
1 kg uranium (nuclear fission)	8.0×10^{13}
Hiroshima-size fission bomb (10 kiloton)	4.2×10^{13}
90,000-ton aircraft carrier at 30 knots	1.1×10^{10}
1 barrel crude oil	5.9×10^{9}
1 ton TNT	4.2×10^{9}
1 gallon of gasoline	1.2×10^{8}
Daily home electricity use (developed countries)	$7n \times 10^{7}$
Daily adult food intake (recommended)	1.2×10^{7}
1000-kg car at 90 km/h	3.1×10^{5}
1 g fat (9.3 kcal)	3.9×10^{4}
ATP hydrolysis reaction	3.2×10^{4}
1 g carbohydrate (4.1 kcal)	1.7×10^{4}
1 g protein (4.1 kcal)	1.7×10^{4}
Tennis ball at 100 km/h	22
Mosquito $^{-2}$ g at $0.5\ \mathrm{m/s})$	1.3×10^{-6}
Single electron in a TV tube beam	4.0×10^{-15}
Energy to break one DNA strand	4.0×10^{-19}

Efficiency

Even though energy is conserved in an energy conversion process, the output of *useful energy* or work will be less than the energy input. The efficiency E_{ff} of an energy conversion process is defined as

$$Efficiency\,(E_{ff}) = \frac{useful\,energy\,or\,work\,output}{total\,energy\,input} = \frac{W_{out}}{E_{in}}. \tag{6.7.4}$$

Table lists some efficiencies of mechanical devices and human activities. In a coal-fired power plant, for example, about 40% of the chemical energy in the coal becomes useful electrical energy. The other 60% transforms into other (perhaps less useful) energy forms, such as thermal energy, which is then released to the environment through combustion gases and cooling towers.

Activity/device	Efficiency (%)
Cycling and climbing	20
Swimming, surface	2
Swimming, submerged	4
Shoveling	3
Weightlifting	9
Steam engine	17
Gasoline engine	30
Diesel engine	35
Nuclear power plant	35
Coal power plant	42
Electric motor	98
Compact fluorescent light	20
Gas heater (residential)	90
Solar cell	10

Efficiency of the Human Body and Mechanical Devices

PhET Explorations: Masses and Springs

A realistic mass and spring laboratory. Hang masses from springs and adjust the spring stiffness and damping. You can even slow time. Transport the lab to different planets. A chart shows the kinetic, potential, and thermal energies for each spring.

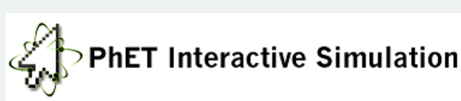

Figure 6.7.2: Masses and Springs

Summary

- The law of conservation of energy states that the total energy is constant in any process. Energy may change in form or be transferred from one system to another, but the total remains the same.
- When all forms of energy are considered, conservation of energy is written in equation form as

$$KE_i + PE_i + W_{nc} + OE_i = KE_f + PE_f + OE_f, \tag{6.7.5}$$

  where OE is all **other forms of energy** besides mechanical energy.
- Commonly encountered forms of energy include electric energy, chemical energy, radiant energy, nuclear energy, and thermal energy.

- Energy is often utilized to do work, but it is not possible to convert all the energy of a system to work.

The efficiency E_{ff} of a machine or human is defined to be $E_{ff}=\frac{W_{out}}{E_{in}}$, where W_{out} is useful work output and E_{in} s the energy consumed.

Glossary

law of conservation of energy
: the general law that total energy is constant in any process; energy may change in form or be transferred from one system to another, but the total remains the same

electrical energy
: the energy carried by a flow of charge

chemical energy
: the energy in a substance stored in the bonds between atoms and molecules that can be released in a chemical reaction

radiant energy
: the energy carried by electromagnetic waves

nuclear energy
: energy released by changes within atomic nuclei, such as the fusion of two light nuclei or the fission of a heavy nucleus

thermal energy
: the energy within an object due to the random motion of its atoms and molecules that accounts for the object's temperature

efficiency
: a measure of the effectiveness of the input of energy to do work; useful energy or work divided by the total input of energy

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6.8: Power

Learning Objectives

By the end of this section, you will be able to:

- Calculate power by calculating changes in energy over time.
- Examine power consumption and calculations of the cost of energy consumed.

What is Power?

Power—the word conjures up many images: a professional football player muscling aside his opponent, a dragster roaring away from the starting line, a volcano blowing its lava into the atmosphere, or a rocket blasting off, as in Figure.

Figure 6.8.1: This powerful rocket on the Space Shuttle Endeavor did work and consumed energy at a very high rate. (credit: NASA)

These images of power have in common the rapid performance of work, consistent with the scientific definition of power P as the rate at which work is done.

Power

Power is the rate at which work is done.

$$\boldsymbol{P=\frac{W}{t}} \tag{6.8.1}$$

The SI unit for power is the watt W, where 1 watt equals 1 joule/second $(1\,W=1\,J/s)$.

Because work is energy transfer, power is also the rate at which energy is expended. A 60-W light bulb, for example, expends 60 J of energy per second. Great power means a large amount of work or energy developed in a short time. For example, when a powerful car accelerates rapidly, it does a large amount of work and consumes a large amount of fuel in a short time.

Calculating Power from Energy

Example $6.8.1$: Calculating the Power to Climb Stairs

What is the power output for a 60.0-kg woman who runs up a 3.00 m high flight of stairs in 3.50 s, starting from rest but having a final speed of 2.00 m/s? (See Figure.)

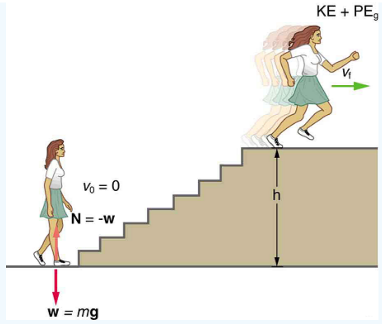

Figure 6.8.2:When this woman runs upstairs starting from rest, she converts the chemical energy originally from food into kinetic energy and gravitational potential energy. Her power output depends on how fast she does this.

Strategy and Concept

The work going into mechanical energy is $W = KE + PE$. At the bottom of the stairs, we take both KE and PE as initially zero; thus $W = KE_f + PE_g = \frac{1}{2}mv_f^2 + mgh,$ where h is the vertical height of the stairs. Because all terms are given, we can calculate W and then divide it by time to get power.

Solution

Substituting the expression for W into the definition of power given in the previous equation, $P = W/t$ yields

$$W = \frac{W}{t} = \frac{\frac{1}{2}mv_f^2 + mgh}{t}. \tag{6.8.2}$$

Entering known values yields

$$P = \frac{0.5(60\,kg)(2.00 m/s^2) + (60.0\,kg)(9.80 m/s^2)(3.00\,m)}{3.50\,s} \tag{6.8.3}$$

$$= \frac{120\,J + 1764\,J}{3.50\,s} \tag{6.8.4}$$

$$= 538\,W. \tag{6.8.5}$$

Discussion

The woman does 1764 J of work to move up the stairs compared with only 120 J to increase her kinetic energy; thus, most of her power output is required for climbing rather than accelerating.

It is impressive that this woman's useful power output is slightly less than 1 horsepower $(1\,hp = 746\,W)$. People can generate more than a horsepower with their leg muscles for short periods of time by rapidly converting available blood sugar and oxygen into work output. (A horse can put out 1 hp for hours on end.) Once oxygen is depleted, power output decreases and the person begins to breathe rapidly to obtain oxygen to metabolize more food—this is known as the *aerobic* stage of exercise. If the woman climbed the stairs slowly, then her power output would be much less, although the amount of work done would be the same.

Making Connections: Take-Home Investigation —Measure Your Power Rating

- Determine your own power rating by measuring the time it takes you to climb a flight of stairs. We will ignore the gain in kinetic energy, as the above example showed that it was a small portion of the energy gain. Don't expect that your output will be more than about 0.5 hp.

Examples of Power

Examples of power are limited only by the imagination, because there are as many types as there are forms of work and energy. (See Table for some examples.) Sunlight reaching Earth's surface carries a maximum power of about 1.3 kilowatts per square meter kW/m^2 A tiny fraction of this is retained by Earth over the long term. Our consumption rate of fossil fuels is far greater than the rate at which they are stored, so it is inevitable that they will be depleted. Power implies that energy is transferred, perhaps changing form. It is never possible to change one form completely into another without losing some of it as thermal energy. For example, a 60-W incandescent bulb converts only 5 W of electrical power to light, with 55 W dissipating into thermal energy. Furthermore, the typical electric power plant converts only 35 to 40% of its fuel into electricity. The remainder becomes a huge amount of thermal energy that must be dispersed as heat transfer, as rapidly as it is created. A coal-fired power plant may produce 1000 megawatts; 1 megawatt (MW) is 10^6 of electric power. But the power plant consumes chemical energy at a rate of about 2500 MW, creating heat transfer to the surroundings at a rate of 1500 MW. (See Figure.)

Figure 6.8.3: Tremendous amounts of electric power are generated by coal-fired power plants such as this one in China, but an even larger amount of power goes into heat transfer to the surroundings. The large cooling towers here are needed to transfer heat as rapidly as it is produced. The transfer of heat is not unique to coal plants but is an unavoidable consequence of generating electric power from any fuel—nuclear, coal, oil, natural gas, or the like. (credit: Kleinolive, Wikimedia Commons)

Object or Phenomenon	Power in Watts
Supernova (at peak)	5×10^{37}
Milky Way galaxy	10^{37}
Crab Nebula pulsar	10^{28}
The Sun	4×10^{26}
Volcanic eruption (maximum)	4×10^{15}
Lightning bolt	2×10^{12}
Nuclear power plant (total electric and heat transfer)	3×10^{9}
Aircraft carrier (total useful and heat transfer)	10^{8}
Dragster (total useful and heat transfer)	2×10^{6}
Car (total useful and heat transfer)	8×10^{4}
Football player (total useful and heat transfer)	5×10^{3}
Clothes dryer	4×10^{3}
Person at rest (all heat transfer)	100
Typical incandescent light bulb (total useful and heat transfer)	60
Heart, person at rest (total useful and heat transfer)	8
Electric clock	3
Pocket calculator	10^{-3}

Power and Energy Consumption

We usually have to pay for the energy we use. It is interesting and easy to estimate the cost of energy for an electrical appliance if its power consumption rate and time used are known. The higher the power consumption rate and the longer the appliance is used, the greater the cost of that appliance. The power consumption rate is $P = \frac{W}{t} = \frac{E}{t}$, where E is the energy supplied by the electricity company. So the energy consumed over a time t is

$$E = Pt. \tag{6.8.6}$$

Electricity bills state the energy used in units of kilowatt-hours $(kW \cdot h)$, which is the product of power in kilowatts and time in hours. This unit is convenient because electrical power consumption at the kilowatt level for hours at a time is typical.

Example 6.8.2: Calculating Energy Costs

What is the cost of running a 0.200-kW computer 6.00 h per day for 30.0 d if the cost of electricity is \$0.120 per $kW \cdot h$?

Strategy

Cost is based on energy consumed; thus, we must find E from $E = Pt$ and then calculate the cost. Because electrical energy is expressed in $kW \cdot h$ at the start of a problem such as this it is convenient to convert the units into kW and hours.

Solution

The energy consumed in $kW \cdot h$ is

$$E = Pt = (0.200\,kW)(6.00\,h/d)(30.0\,d) \tag{6.8.7}$$

$$= 36.0\,kW \cdot h, \tag{6.8.8}$$

and the cost is simply given by

$$cost = (36.0\,kW \cdot h)(\$0.120\,per\,kW \cdot h) = \$4.32\,per\,month. \tag{6.8.9}$$

Discussion

The cost of using the computer in this example is neither exorbitant nor negligible. It is clear that the cost is a combination of power and time. When both are high, such as for an air conditioner in the summer, the cost is high.

The motivation to save energy has become more compelling with its ever-increasing price. Armed with the knowledge that energy consumed is the product of power and time, you can estimate costs for yourself and make the necessary value judgments about where to save energy. Either power or time must be reduced. It is most cost-effective to limit the use of high-power devices that normally operate for long periods of time, such as water heaters and air conditioners. This would not include relatively high power devices like toasters, because they are on only a few minutes per day. It would also not include electric clocks, in spite of their 24-hour-per-day usage, because they are very low power devices. It is sometimes possible to use devices that have greater efficiencies—that is, devices that consume less power to accomplish the same task. One example is the compact fluorescent light bulb, which produces over four times more light per watt of power consumed than its incandescent cousin.

Modern civilization depends on energy, but current levels of energy consumption and production are not sustainable. The likelihood of a link between global warming and fossil fuel use (with its concomitant production of carbon dioxide), has made reduction in energy use as well as a shift to non-fossil fuels of the utmost importance. Even though energy in an isolated system is a conserved quantity, the final result of most energy transformations is waste heat transfer to the environment, which is no longer useful for doing work. As we will discuss in more detail in Thermodynamics, the potential for energy to produce useful work has been "degraded" in the energy transformation.

Summary

- Power is the rate at which work is done, or in equation form, for the average power P for work W done over a time t, $P = W/t$.
- The SI unit for power is the watt (W), where $1\,W = 1\,J/s$.
- The power of many devices such as electric motors is also often expressed in horsepower (hp), where $1\,hp = 746\,W$.

Glossary

power

the rate at which work is done

watt

(W) SI unit of power, with $1W = 1J/s$

horsepower

an older non-SI unit of power, with $1 hp = 746 W$

kilowatt-hour

$(kW \cdot h)$ unit used primarily for electrical energy provided by electric utility companies

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6.9: Work, Energy, and Power in Humans

Learning Objectives

By the end of this section, you will be able to:

- Explain the human body's consumption of energy when at rest vs. when engaged in activities that do useful work.
- Calculate the conversion of chemical energy in food into useful work.

Energy Conversion in Humans

Our own bodies, like all living organisms, are energy conversion machines. Conservation of energy implies that the chemical energy stored in food is converted into work, thermal energy, and/or stored as chemical energy in fatty tissue. (Figure 7.09.1.) The fraction going into each form depends both on how much we eat and on our level of physical activity. If we eat more than is needed to do work and stay warm, the remainder goes into body fat.

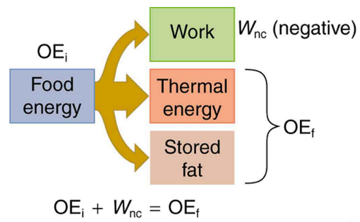

Figure 6.9.1: Energy consumed by humans is converted to work, thermal energy, and stored fat. By far the largest fraction goes to thermal energy, although the fraction varies depending on the type of physical activity.

Power Consumed at Rest

The *rate* at which the body uses food energy to sustain life and to do different activities is called the metabolic rate. The total energy conversion rate of a person *at rest* is called the basal metabolic rate (BMR) and is divided among various systems in the body, as shown in Table. The largest fraction goes to the liver and spleen, with the brain coming next. Of course, during vigorous exercise, the energy consumption of the skeletal muscles and heart increase markedly. About 75% of the calories burned in a day go into these basic functions. The BMR is a function of age, gender, total body weight, and amount of muscle mass (which burns more calories than body fat). Athletes have a greater BMR due to this last factor.

Basal Metabolic Rates (BMR):

Organ	Power consumed at rest (W)	Oxygen consumption (mL/min)	Percent of BMR
Liver & spleen	23	67	27
Brain	16	47	19
Skeletal muscle	15	45	18
Kidney	9	26	10
Heart	6	17	7
Other	16	48	19
Totals	85 W	250 mL/min	100%

Energy consumption is directly proportional to oxygen consumption because the digestive process is basically one of oxidizing food. We can measure the energy people use during various activities by measuring their oxygen use. (See Figure7.09.1.) Approximately 20 kJ of energy are produced for each liter of oxygen consumed, independent of the type of food. Table shows energy and oxygen consumption rates (power expended) for a variety of activities.

Power of Doing Useful Work

Work done by a person is sometimes called useful work, which is *work done on the outside world*, such as lifting weights. Useful work requires a force exerted through a distance on the outside world, and so it excludes internal work, such as that done by the heart when pumping blood. Useful work does include that done in climbing stairs or accelerating to a full run, because these are accomplished by exerting forces on the outside world. Forces exerted by the body are non-conservative, so that they can change the mechanical energy $(KE + PE)$ of the system worked upon, and this is often the goal. A baseball player throwing a ball, for example, increases both the ball's kinetic and potential energy.

If a person needs more energy than they consume, such as when doing vigorous work, the body must draw upon the chemical energy stored in fat. So exercise can be helpful in losing fat. However, the amount of exercise needed to produce a loss in fat, or to burn off extra calories consumed that day, can be large, as Example 7.09.1 illustrates.

Example 6.9.1: Calculating Weight Loss from Exercising

If a person who normally requires an average of 12,000 kJ (3000 kcal) of food energy per day consumes 13,000 kJ per day, he will steadily gain weight. How much bicycling per day is required to work off this extra 1000 kJ?

Solution

Table states that 400 W are used when cycling at a moderate speed. The time required to work off 1000 kJ at this rate is then

$$Time = \frac{energy}{\left(\frac{energy}{time}\right)} = \frac{1000\,kJ}{400\,W} = 2500\,s = 42\,min. \tag{6.9.1}$$

Discussion

If this person uses more energy than he or she consumes, the person's body will obtain the needed energy by metabolizing body fat. If the person uses 13,000 kJ but consumes only 12,000 kJ, then the amount of fat loss will be

$$Fat\,loss = (1000\,kJ)\left(\frac{1\,g\,fat}{30\,kJ}\right) = 26\,g, \tag{6.9.2}$$

assuming the energy content of fat to be 39 kJ/g.

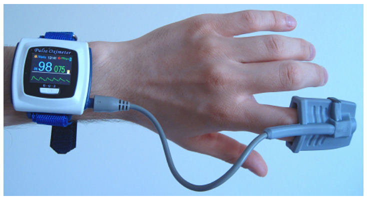

Figure 6.9.2: A pulse oxymeter is an apparatus that measures the amount of oxygen in blood. A knowledge of oxygen and carbon dioxide levels indicates a person's metabolic rate, which is the rate at which food energy is converted to another form. a person's metabolic rate, which is the rate at which food energy is converted to another form. Such measurements can indicate the level of athletic conditioning as well as certain medical problems. (credit: UusiAjaja, Wikimedia Commons)

Energy and Oxygen Consumption Rates:

Activity	Energy consumption in watts	Oxygen consumption in liters O_2/min
Sleeping	83	0.24
Sitting at rest	120	0.34
Standing relaxed	125	0.36
Sitting in class	210	0.60
Walking (5 km/h)	280	0.80
Cycling (13–18 km/h)	400	1.14
Shivering	425	1.21
Playing tennis	440	1.26
Swimming breaststroke	475	1.36

Activity	Energy consumption in watts	Oxygen consumption in liters O_2/min
Ice skating (14.5 km/h)	545	1.56
Climbing stairs (116/min)	685	1.96
Cycling (21 km/h)	700	2.00
Running cross-country	740	2.12
Playing basketball	800	2.28
Cycling, professional racer	1855	5.30
Sprinting	2415	6.90

All bodily functions, from thinking to lifting weights, require energy. (See Figure 7.09.3.) The many small muscle actions accompanying all quiet activity, from sleeping to head scratching, ultimately become thermal energy, as do less visible muscle actions by the heart, lungs, and digestive tract. Shivering, in fact, is an involuntary response to low body temperature that pits muscles against one another to produce thermal energy in the body (and do no work). The kidneys and liver consume a surprising amount of energy, but the biggest surprise of all it that a full 25% of all energy consumed by the body is used to maintain electrical potentials in all living cells. (Nerve cells use this electrical potential in nerve impulses.) This bioelectrical energy ultimately becomes mostly thermal energy, but some is utilized to power chemical processes such as in the kidneys and liver, and in fat production.

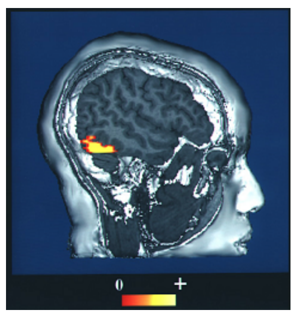

Figure 6.9.3: This MRI scan shows an increased level of energy consumption in the vision center of the brain. Here, the patient was being asked to recognize faces. (credit: NIH via Wikimedia Commons)

Summary

- The human body converts energy stored in food into work, thermal energy, and/or chemical energy that is stored in fatty tissue.
- The *rate* at which the body uses food energy to sustain life and to do different activities is called the metabolic rate, and the corresponding rate when at rest is called the basal metabolic rate (BMR)
- The energy included in the basal metabolic rate is divided among various systems in the body, with the largest fraction going to the liver and spleen, and the brain coming next.
- About 75% of food calories are used to sustain basic body functions included in the basal metabolic rate.
- The energy consumption of people during various activities can be determined by measuring their oxygen use, because the digestive process is basically one of oxidizing food.

Glossary

metabolic rate

the rate at which the body uses food energy to sustain life and to do different activities

basal metabolic rate

the total energy conversion rate of a person at rest

useful work

work done on an external system

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6.10: World Energy Use

Learning Objectives

By the end of this section, you will be able to:

- Describe the distinction between renewable and nonrenewable energy sources.
- Explain why the inevitable conversion of energy to less useful forms makes it necessary to conserve energy resources.

Energy is an important ingredient in all phases of society. We live in a very interdependent world, and access to adequate and reliable energy resources is crucial for economic growth and for maintaining the quality of our lives. But current levels of energy consumption and production are not sustainable. About 40% of the world's energy comes from oil, and much of that goes to transportation uses. Oil prices are dependent as much upon new (or foreseen) discoveries as they are upon political events and situations around the world. The U.S., with 4.5% of the world's population, consumes 24% of the world's oil production per year; 66% of that oil is imported!

Renewable and Nonrenewable Energy Sources

The principal energy resources used in the world are shown in Figure 7.10.1. The fuel mix has changed over the years but now is dominated by oil, although natural gas and solar contributions are increasing. Renewable forms of energy are those sources that cannot be used up, such as water, wind, solar, and biomass. About 85% of our energy comes from nonrenewable fossil fuels—oil, natural gas, coal. The likelihood of a link between global warming and fossil fuel use, with its production of carbon dioxide through combustion, has made, in the eyes of many scientists, a shift to non-fossil fuels of utmost importance—but it will not be easy.

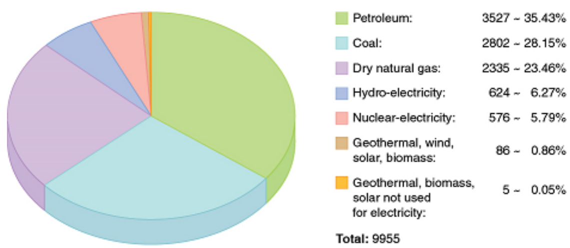

Figure 6.10.1: World energy consumption by source, in billions of kilowatt-hours: 2006. (credit: KVDP)

The World's Growing Energy Needs

World energy consumption continues to rise, especially in the developing countries. (See Figure 7.10.1.) Global demand for energy has tripled in the past 50 years and might triple again in the next 30 years. While much of this growth will come from the rapidly booming economies of China and India, many of the developed countries, especially those in Europe, are hoping to meet their energy needs by expanding the use of renewable sources. Although presently only a small percentage, renewable energy is growing very fast, especially wind energy. For example, Germany plans to meet 20% of its electricity and 10% of its overall energy needs with renewable resources by the year 2020. (See 7.10.2.) Energy is a key constraint in the rapid economic growth of China and India. In 2003, China surpassed Japan as the world's second largest consumer of oil. However, over 1/3 of this is imported. Unlike most Western countries, coal dominates the commercial energy resources of China, accounting for 2/3 of its energy consumption. In 2009 China surpassed the United States as the largest generator of CO_2 In India, the main energy resources are biomass (wood and dung) and coal. Half of India's oil is imported. About 70% of India's electricity is generated by highly polluting coal. Yet there are sizeable strides being made in renewable energy. India has a rapidly growing wind energy base, and it has the largest solar cooking program in the world.

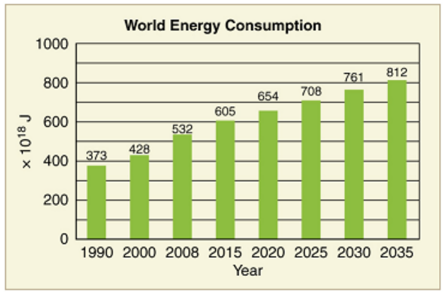

Figure 6.10.2: Past and projected world energy use (source: Based on data from U.S. Energy Information Administration, 2011)

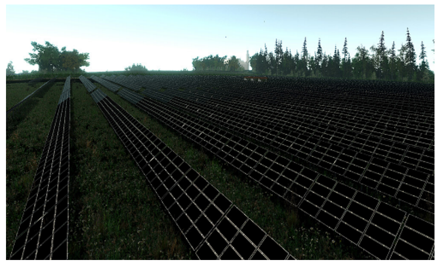

Figure 6.10.3: Solar cell arrays at a power plant in Steindorf, Germany (credit: Michael Betke, Flickr)

Table displays the 2006 commercial energy mix by country for some of the prime energy users in the world. While non-renewable sources dominate, some countries get a sizeable percentage of their electricity from renewable resources. For example, about 67% of New Zealand's electricity demand is met by hydroelectric. Only 10% of the U.S. electricity is generated by renewable resources, primarily hydroelectric. It is difficult to determine total contributihttp://physwiki.ucdavis.ed..._Energy_Useons of renewable energy in some countries with a large rural population, so these percentages in this table are left blank.

Energy Consumption—Selected Countries (2006):

Country	Consumption, in EJ (10^{18} J)	Oil	Natural Gas	Coal	Nuclear	Hydro	Other Renewables	Electricity Use per capita (kWh/yr)	Energy Use per capita (GJ/yr)
Australia	5.4	34%	17%	44%	0%	3%	1%	10000	260
Brazil	9.6	48%	7%	5%	1%	35%	2%	2000	50
China	63	22%	3%	69%	1%	6%		1500	35
Egypt	2.4	50%	41%	1%	0%	6%		990	32
Germany	16	37%	24%	24%	11%	1%	3%	6400	173
India	15	34%	7%	52%	1%	5%		470	13
Indonesia	4.9	51%	26%	16%	0%	2%	3%	420	22
Japan	24	48%	14%	21%	12%	4%	1%	7100	176
New Zealand	0.44	32%	26%	6%	0%	11%	19%	8500	102
Russia	31	19%	53%	16%	5%	6%		5700	202
U.S.	105	40%	23%	22%	8%	3%	1%	12500	340
World	**432**	**39%**	**23%**	**24%**	**6%**	**6%**	**2%**	**2600**	**71**

Energy and Economic Well-being

The last two columns in this table examine the energy and electricity use per capita. Economic well-being is dependent upon energy use, and in most countries higher standards of living, as measured by GDP (gross domestic product) per capita, are matched by higher levels of energy consumption per capita. This is borne out in Figure 7.10.4. Increased efficiency of energy use will change this dependency. A global problem is balancing energy resource development against the harmful effects upon the environment in its extraction and use.

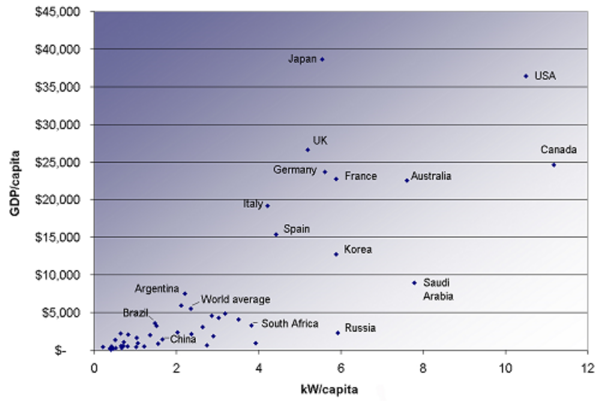

Figure 6.10.4: Power consumption per capita versus GDP per capita for various countries. Note the increase in energy usage with increasing GDP. (2007, credit: Frank van Mierlo, Wikimedia Commons)

Conserving Energy

As we finish this chapter on energy and work, it is relevant to draw some distinctions between two sometimes misunderstood terms in the area of energy use. As has been mentioned elsewhere, the “law of the conservation of energy” is a very useful principle in analyzing physical processes. It is a statement that cannot be proven from basic principles, but is a very good bookkeeping device, and no exceptions have ever been found. It states that the total amount of energy in an isolated system will always remain constant. Related to this principle, but remarkably different from it, is the important philosophy of energy conservation. This concept has to do with seeking to decrease the amount of energy used by an individual or group through (1) reduced activities (e.g., turning down thermostats, driving fewer kilometers) and/or (2) increasing conversion efficiencies in the performance of a particular task—such as developing and using more efficient room heaters, cars that have greater miles-per-gallon ratings, energy-efficient compact fluorescent lights, etc.

Since energy in an isolated system is not destroyed or created or generated, one might wonder why we need to be concerned about our energy resources, since energy is a conserved quantity. The problem is that the final result of most energy transformations is waste heat transfer to the environment and conversion to energy forms no longer useful for doing work. To state it in another way, the potential for energy to produce useful work has been “degraded” in the energy transformation. (This will be discussed in more detail in Thermodynamics.)

Summary

- The relative use of different fuels to provide energy has changed over the years, but fuel use is currently dominated by oil, although natural gas and solar contributions are increasing.
- Although non-renewable sources dominate, some countries meet a sizeable percentage of their electricity needs from renewable resources.
- The United States obtains only about 10% of its energy from renewable sources, mostly hydroelectric power.
- Economic well-being is dependent upon energy use, and in most countries higher standards of living, as measured by GDP (Gross Domestic Product) per capita, are matched by higher levels of energy consumption per capita.
- Even though, in accordance with the law of conservation of energy, energy can never be created or destroyed, energy that can be used to do work is always partly converted to less useful forms, such as waste heat to the environment, in all of our uses of energy for practical purposes.

Glossary

renewable forms of energy

those sources that cannot be used up, such as water, wind, solar, and biomass

fossil fuels

oil, natural gas, and coal

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6.E: Work, Energy, and Energy Resources (Exercise)

Conceptual Questions

7.1: Work: The Scientific Definition

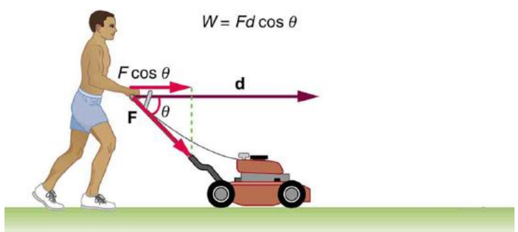
1. Give an example of something we think of as work in everyday circumstances that is not work in the scientific sense. Is energy transferred or changed in form in your example? If so, explain how this is accomplished without doing work.

2. Give an example of a situation in which there is a force and a displacement, but the force does no work. Explain why it does no work.

3. Describe a situation in which a force is exerted for a long time but does no work. Explain.

7.2: Kinetic Energy and the Work-Energy Theorem

4. The person in Figure does work on the lawn mower. Under what conditions would the mower gain energy? Under what conditions would it lose energy?



5. Work done on a system puts energy into it. Work done by a system removes energy from it. Give an example for each statement.

6. When solving for speed in Example, we kept only the positive root. Why?

7.3: Gravitational Potential Energy

7. In Example, we calculated the final speed of a roller coaster that descended 20 m in height and had an initial speed of 5 m/s downhill. Suppose the roller coaster had had an initial speed of 5 m/s uphill instead, and it coasted uphill, stopped, and then rolled back down to a final point 20 m below the start. We would find in that case that its final speed is the same as its initial speed. Explain in terms of conservation of energy.

8. Does the work you do on a book when you lift it onto a shelf depend on the path taken? On the time taken? On the height of the shelf? On the mass of the book?

7.4: Conservative Forces and Potential Energy

9. What is a conservative force?

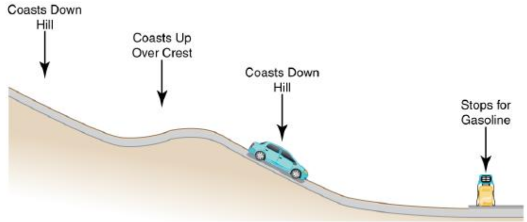
10. The force exerted by a diving board is conservative, provided the internal friction is negligible. Assuming friction is negligible, describe changes in the potential energy of a diving board as a swimmer dives from it, starting just before the swimmer steps on the board until just after his feet leave it.

11. Define mechanical energy. What is the relationship of mechanical energy to nonconservative forces? What happens to mechanical energy if only conservative forces act?

12. What is the relationship of potential energy to conservative force?

7.6: Conservation of Energy

13. Consider the following scenario. A car for which friction is not negligible accelerates from rest down a hill, running out of gasoline after a short distance. The driver lets the car coast farther down the hill, then up and over a small crest. He then coasts down that hill into a gas station, where he brakes to a stop and fills the tank with gasoline. Identify the forms of energy the car has, and how they are changed and transferred in this series of events. (See Figure.)



A car experiencing non-negligible friction coasts down a hill, over a small crest, then downhill again, and comes to a stop at a gas station.

14. Describe the energy transfers and transformations for a javelin, starting from the point at which an athlete picks up the javelin and ending when the javelin is stuck into the ground after being thrown.

15. Do devices with efficiencies of less than one violate the law of conservation of energy? Explain.

16. List four different forms or types of energy. Give one example of a conversion from each of these forms to another form.

17. List the energy conversions that occur when riding a bicycle.

7.7: Power

18. Most electrical appliances are rated in watts. Does this rating depend on how long the appliance is on? (When off, it is a zero-watt device.) Explain in terms of the definition of power.

19. Explain, in terms of the definition of power, why energy consumption is sometimes listed in kilowatt-hours rather than joules. What is the relationship between these two energy units?

20. A spark of static electricity, such as that you might receive from a doorknob on a cold dry day, may carry a few hundred watts of power. Explain why you are not injured by such a spark.

7.8: Work, Energy, and Power in Humans

21. Explain why it is easier to climb a mountain on a zigzag path rather than one straight up the side. Is your increase in gravitational potential energy the same in both cases? Is your energy consumption the same in both?

22. Do you do work on the outside world when you rub your hands together to warm them? What is the efficiency of this activity?

23. Shivering is an involuntary response to lowered body temperature. What is the efficiency of the body when shivering, and is this a desirable value?

24. Discuss the relative effectiveness of dieting and exercise in losing weight, noting that most athletic activities consume food energy at a rate of 400 to 500 W, while a single cup of yogurt can contain 1360 kJ (325 kcal). Specifically, is it likely that exercise alone will be sufficient to lose weight? You may wish to consider that regular exercise may increase the metabolic rate, whereas protracted dieting may reduce it.

7.9: World Energy Use

25. What is the difference between energy conservation and the law of conservation of energy? Give some examples of each.

26. If the efficiency of a coal-fired electrical generating plant is 35%, then what do we mean when we say that energy is a conserved quantity?

Problems & Exercises

7.1: Work: The Scientific Definition

27. How much work does a supermarket checkout attendant do on a can of soup he pushes 0.600 m horizontally with a force of 5.00 N? Express your answer in joules and kilocalories.

Solution

$3.00 J = 7.17 \times 10^{-4} kcal$

28. A 75.0-kg person climbs stairs, gaining 2.50 meters in height. Find the work done to accomplish this task.

https://phys.libretexts.org/@go/page/26531

29. (a) Calculate the work done on a 1500-kg elevator car by its cable to lift it 40.0 m at constant speed, assuming friction averages 100 N.

(b) What is the work done on the lift by the gravitational force in this process?

(c) What is the total work done on the lift?

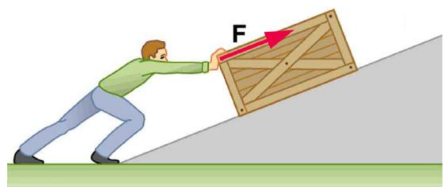
Solution
(a) $5.92 \times 10^5 J$
(b) $-5.88 \times 10^5 J$
(c) The net force is zero.

30. Suppose a car travels 108 km at a speed of 30.0 m/s, and uses 2.0 gal of gasoline. Only 30% of the gasoline goes into useful work by the force that keeps the car moving at constant speed despite friction. (See [link] for the energy content of gasoline.)

(a) What is the magnitude of the force exerted to keep the car moving at constant speed?

(b) If the required force is directly proportional to speed, how many gallons will be used to drive 108 km at a speed of 28.0 m/s?

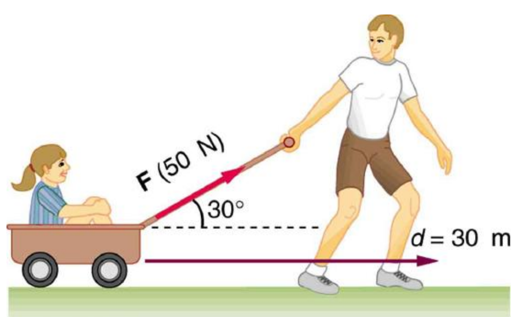
31. Calculate the work done by an 85.0-kg man who pushes a crate 4.00 m up along a ramp that makes an angle of $20.0°$ with the horizontal. (See Figure.) He exerts a force of 500 N on the crate parallel to the ramp and moves at a constant speed. Be certain to include the work he does on the crate and on his body to get up the ramp.



A man pushes a crate up a ramp.

Solution
$3.14 \times 10^3 J$

32. How much work is done by the boy pulling his sister 30.0 m in a wagon as shown in Figure? Assume no friction acts on the wagon.



The boy does work on the system of the wagon and the child when he pulls them as shown.

33. A shopper pushes a grocery cart 20.0 m at constant speed on level ground, against a 35.0 N frictional force. He pushes in a direction $25.0°$ below the horizontal.

(a) What is the work done on the cart by friction?

(b) What is the work done on the cart by the gravitational force?

(c) What is the work done on the cart by the shopper?

(d) Find the force the shopper exerts, using energy considerations.

(e) What is the total work done on the cart?

Solution
(a) $-700J$
(b) 0
(c) 700 J
(d) 38.6 N
(e) 0

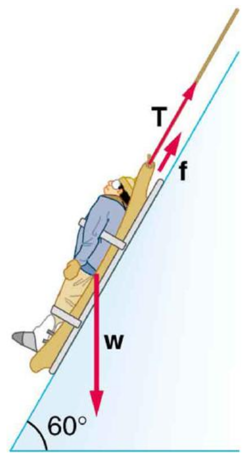
34. Suppose the ski patrol lowers a rescue sled and victim, having a total mass of 90.0 kg, down a 60.0^{o} slope at constant speed, as shown in Figure. The coefficient of friction between the sled and the snow is 0.100.

(a) How much work is done by friction as the sled moves 30.0 m along the hill?

(b) How much work is done by the rope on the sled in this distance?

(c) What is the work done by the gravitational force on the sled?

(d) What is the total work done?



A rescue sled and victim are lowered down a steep slope.

7.2: Kinetic Energy and the Work-Energy Theorem

35. Compare the kinetic energy of a 20,000-kg truck moving at 110 km/h with that of an 80.0-kg astronaut in orbit moving at 27,500 km/h.

Solution
$1/250$

36. (a) How fast must a 3000-kg elephant move to have the same kinetic energy as a 65.0-kg sprinter running at 10.0 m/s?

(b) Discuss how the larger energies needed for the movement of larger animals would relate to metabolic rates.

37. Confirm the value given for the kinetic energy of an aircraft carrier in [link]. You will need to look up the definition of a nautical mile (1 knot = 1 nautical mile/h).

Solution
$1.1 \times 10^{10} J$

38. (a) Calculate the force needed to bring a 950-kg car to rest from a speed of 90.0 km/h in a distance of 120 m (a fairly typical distance for a non-panic stop).

(b) Suppose instead the car hits a concrete abutment at full speed and is brought to a stop in 2.00 m. Calculate the force exerted on the car and compare it with the force found in part (a).

39. A car's bumper is designed to withstand a 4.0-km/h (1.1-m/s) collision with an immovable object without damage to the body of the car. The bumper cushions the shock by absorbing the force over a distance. Calculate the magnitude of the average force on a bumper that collapses 0.200 m while bringing a 900-kg car to rest from an initial speed of 1.1 m/s.

Solution
$2.8 \times 10^3 N$

40. Boxing gloves are padded to lessen the force of a blow.

(a) Calculate the force exerted by a boxing glove on an opponent's face, if the glove and face compress 7.50 cm during a blow in which the 7.00-kg arm and glove are brought to rest from an initial speed of 10.0 m/s.

(b) Calculate the force exerted by an identical blow in the gory old days when no gloves were used and the knuckles and face would compress only 2.00 cm.

(c) Discuss the magnitude of the force with glove on. Does it seem high enough to cause damage even though it is lower than the force with no glove?

41. Using energy considerations, calculate the average force a 60.0-kg sprinter exerts backward on the track to accelerate from 2.00 to 8.00 m/s in a distance of 25.0 m, if he encounters a headwind that exerts an average force of 30.0 N against him.

Solution
102 N

7.3: Gravitational Potential Energy

42. A hydroelectric power facility (see Figure) converts the gravitational potential energy of water behind a dam to electric energy.

(a) What is the gravitational potential energy relative to the generators of a lake of volume $50.0 km^3 (mass = 5.00 \times 10^{13} kg)$, given that the lake has an average height of 40.0 m above the generators?

(b) Compare this with the energy stored in a 9-megaton fusion bomb.

Hydroelectric facility (credit: Denis Belevich, Wikimedia Commons)

Solution
(a) $1.96 \times 10^{16} J$
(b) The ratio of gravitational potential energy in the lake to the energy stored in the bomb is 0.52. That is, the energy stored in the lake is approximately half that in a 9-megaton fusion bomb.

43. (a) How much gravitational potential energy (relative to the ground on which it is built) is stored in the Great Pyramid of Cheops, given that its mass is about $7 \times 10^9 kg$ and its center of mass is 36.5 m above the surrounding ground?

(b) How does this energy compare with the daily food intake of a person?

44. Suppose a 350-g kookaburra (a large kingfisher bird) picks up a 75-g snake and raises it 2.5 m from the ground to a branch.

(a) How much work did the bird do on the snake?

(b) How much work did it do to raise its own center of mass to the branch?

Solution
(a) 1.8 J

(b) 8.6 J

45. In Example, we found that the speed of a roller coaster that had descended 20.0 m was only slightly greater when it had an initial speed of 5.00 m/s than when it started from rest. This implies that $\Delta PE >> KE_i$. Confirm this statement by taking the ratio of ΔPE to KE_i . (Note that mass cancels.)

46. A 100-g toy car is propelled by a compressed spring that starts it moving. The car follows the curved track in Figure. Show that the final speed of the toy car is 0.687 m/s if its initial speed is 2.00 m/s and it coasts up the frictionless slope, gaining 0.180 m in altitude.

A toy car moves up a sloped track. (credit: Leszek Leszczynski, Flickr)

Solution

$v_f = \sqrt{2gh + v_0^2} = \sqrt{2(9.80 m/s^2)(-0.180 m) + (2.00 m/s)^2} = 0.687 m/s$

47. In a downhill ski race, surprisingly, little advantage is gained by getting a running start. (This is because the initial kinetic energy is small compared with the gain in gravitational potential energy on even small hills.) To demonstrate this, find the final speed and the time taken for a skier who skies 70.0 m along a 30^o slope neglecting friction:

(a) Starting from rest.

(b) Starting with an initial speed of 2.50 m/s.

(c) Does the answer surprise you? Discuss why it is still advantageous to get a running start in very competitive events.

7.4: Conservative Forces and Potential Energy

48. A $5.00 \times 10^5 - kg$ subway train is brought to a stop from a speed of 0.500 m/s in 0.400 m by a large spring bumper at the end of its track. What is the force constant k of the spring?

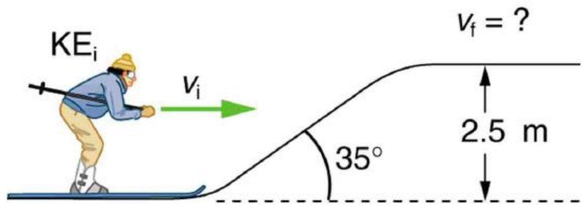
Solution

$7.81 \times 10^5 N/m$

49. A pogo stick has a spring with a force constant of $2.50 \times 10^4 N/m$, which can be compressed 12.0 cm. To what maximum height can a child jump on the stick using only the energy in the spring, if the child and stick have a total mass of 40.0 kg? Explicitly show how you follow the steps in the Problem-Solving Strategies for Energy.

7.5: Nonconservative Forces

50. A 60.0-kg skier with an initial speed of 12.0 m/s coasts up a 2.50-m-high rise as shown in Figure. Find her final speed at the top, given that the coefficient of friction between her skis and the snow is 0.0800. (Hint: Find the distance traveled up the incline assuming a straight-line path as shown in the figure.)



The skier's initial kinetic energy is partially used in coasting to the top of a rise.

Solution
9.46 m/s

51. (a) How high a hill can a car coast up (engine disengaged) if work done by friction is negligible and its initial speed is 110 km/h?

(b) If, in actuality, a 750-kg car with an initial speed of 110 km/h is observed to coast up a hill to a height 22.0 m above its starting point, how much thermal energy was generated by friction?

(c) What is the average force of friction if the hill has a slope 2.5° above the horizontal?

7.6: Conservation of Energy

52. Using values from Table, how many DNA molecules could be broken by the energy carried by a single electron in the beam of an old-fashioned TV tube? (These electrons were not dangerous in themselves, but they did create dangerous x rays. Later model tube TVs had shielding that absorbed x rays before they escaped and exposed viewers.)

Solution
4×10^4 molecules

53. Using energy considerations and assuming negligible air resistance, show that a rock thrown from a bridge 20.0 m above water with an initial speed of 15.0 m/s strikes the water with a speed of 24.8 m/s independent of the direction thrown.

Solution
Equating ΔPE_g and ΔKE, we obtain $v = \sqrt{2gh + v_0^2} = \sqrt{2(9.80 m/s^2)(20.0 m) + (15.0 m/s)^2} = 24.8 m/s$

54. If the energy in fusion bombs were used to supply the energy needs of the world, how many of the 9-megaton variety would be needed for a year's supply of energy (using data from Table)? This is not as far-fetched as it may sound—there are thousands of nuclear bombs, and their energy can be trapped in underground explosions and converted to electricity, as natural geothermal energy is.

55. (a) Use of hydrogen fusion to supply energy is a dream that may be realized in the next century. Fusion would be a relatively clean and almost limitless supply of energy, as can be seen from Table. To illustrate this, calculate how many years the present energy needs of the world could be supplied by one millionth of the oceans' hydrogen fusion energy.

(b) How does this time compare with historically significant events, such as the duration of stable economic systems?

Solution
(a) $25 \times 10^6 years$
(b) This is much, much longer than human time scales.

7.7: Power

56. The Crab Nebula (see Figure) pulsar is the remnant of a supernova that occurred in A.D. 1054. Using data from Table, calculate the approximate factor by which the power output of this astronomical object has declined since its explosion.

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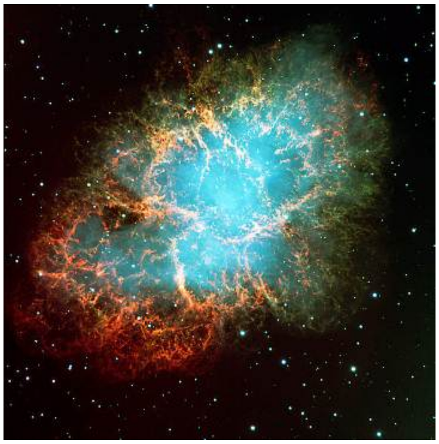


Crab Nebula (credit: ESO, via Wikimedia Commons)

Solution
2×10^{-10}

57. Suppose a star 1000 times brighter than our Sun (that is, emitting 1000 times the power) suddenly goes supernova. Using data from Table:

(a) By what factor does its power output increase?

(b) How many times brighter than our entire Milky Way galaxy is the supernova?

(c) Based on your answers, discuss whether it should be possible to observe supernovas in distant galaxies. Note that there are on the order of 10^{11} observable galaxies, the average brightness of which is somewhat less than our own galaxy.

58. A person in good physical condition can put out 100 W of useful power for several hours at a stretch, perhaps by pedaling a mechanism that drives an electric generator. Neglecting any problems of generator efficiency and practical considerations such as resting time:

(a) How many people would it take to run a 4.00-kW electric clothes dryer?

(b) How many people would it take to replace a large electric power plant that generates 800 MW?

Solution
(a) 40
(b) 8 million

59. What is the cost of operating a 3.00-W electric clock for a year if the cost of electricity is $0.0900 per $kW \cdot h$?

60. A large household air conditioner may consume 15.0 kW of power. What is the cost of operating this air conditioner 3.00 h per day for 30.0 d if the cost of electricity is $0.110 per $kW \cdot h$?

Solution
$149

61. (a) What is the average power consumption in watts of an appliance that uses $5.00 kW \cdot h$ of energy per day?

(b) How many joules of energy does this appliance consume in a year?

62. (a) What is the average useful power output of a person who does $6.00 \times 10^6 J$ of useful work in 8.00 h?

(b) Working at this rate, how long will it take this person to lift 2000 kg of bricks 1.50 m to a platform? (Work done to lift his body can be omitted because it is not considered useful output here.)

Solution
(a) 208 W
(b) 141 s

63. A 500-kg dragster accelerates from rest to a final speed of 110 m/s in 400 m (about a quarter of a mile) and encounters an average frictional force of 1200 N. What is its average power output in watts and horsepower if this takes 7.30 s?

64. (a) How long will it take an 850-kg car with a useful power output of 40.0 hp (1 hp = 746 W) to reach a speed of 15.0 m/s, neglecting friction? (b) How long will this acceleration take if the car also climbs a 3.00-m-high hill in the process?

Solution
(a) 3.20 s
(b) 4.04 s

65. (a) Find the useful power output of an elevator motor that lifts a 2500-kg load a height of 35.0 m in 12.0 s, if it also increases the speed from rest to 4.00 m/s. Note that the total mass of the counterbalanced system is 10,000 kg—so that only 2500 kg is raised in height, but the full 10,000 kg is accelerated.

(b) What does it cost, if electricity is $0.0900 per $kW \cdot h$?

66. (a) What is the available energy content, in joules, of a battery that operates a 2.00-W electric clock for 18 months?

(b) How long can a battery that can supply $8.00 \times 10^4 J$ run a pocket calculator that consumes energy at the rate of $1.00 \times 10^{-3} W$?

Solution
(a) $9.46 \times 10^7 J$
(b) $2.54y$

67. (a) How long would it take a 1.50×10^5-kg airplane with engines that produce 100 MW of power to reach a speed of 250 m/s and an altitude of 12.0 km if air resistance were negligible?

(b) If it actually takes 900 s, what is the power?

(c) Given this power, what is the average force of air resistance if the airplane takes 1200 s? (Hint: You must find the distance the plane travels in 1200 s assuming constant acceleration.)

68. Calculate the power output needed for a 950-kg car to climb a 2.00° slope at a constant 30.0 m/s while encountering wind resistance and friction totaling 600 N. Explicitly show how you follow the steps in the Problem-Solving Strategies for Energy.

Solution
Identify knowns: $m = 950kg$, **slope angle**$\theta = 2.00^{\circ}$, $v = 3.00m/s$, $f = 600N$
Identify unknowns: power P of the car, force F that car applies to road
Solve for unknown:
$P = \frac{W}{t} = \frac{Fd}{t} = F(\frac{d}{t}) = Fv$,
where F is parallel to the incline and must oppose the resistive forces and the force of gravity:
$F = f + w = 600N + mgsin\theta$
Insert this into the expression for power and solve:
$P = (f + mgsin\theta)v$
$= [600N + (950kg)(9.80m/s^2)sin2^{\circ}](30.0m/s)$
$= 2.77 \times 10^4 W$
About 28 kW (or about 37 hp) is reasonable for a car to climb a gentle incline.

69. (a) Calculate the power per square meter reaching Earth's upper atmosphere from the Sun. (Take the power output of the Sun to be $4.00 \times 10^{26} W$.)

(b) Part of this is absorbed and reflected by the atmosphere, so that a maximum of $1.30kW/m^2$ reaches Earth's surface. Calculate the area in km^2 of solar energy collectors needed to replace an electric power plant that generates 750 MW if the collectors convert an average of 2.00% of the maximum power into electricity. (This small conversion efficiency is due to the devices themselves, and the fact that the sun is directly overhead only briefly.) With the same assumptions, what area would be needed to meet the United States' energy needs $(1.05 \times 10^{20} J)$? Australia's energy needs $(5.4 \times 10^{18} J)$? China's energy needs $(6.3 \times 10^{19} J)$? (These energy consumption values are from 2006.)

70. (a) How long can you rapidly climb stairs (116/min) on the 93.0 kcal of energy in a 10.0-g pat of butter?

(b) How many flights is this if each flight has 16 stairs?

Solution
(a) 9.5 min
(b) 69 flights of stairs

71. (a) What is the power output in watts and horsepower of a 70.0-kg sprinter who accelerates from rest to 10.0 m/s in 3.00 s?

(b) Considering the amount of power generated, do you think a well-trained athlete could do this repetitively for long periods of time?

72. Calculate the power output in watts and horsepower of a shot-putter who takes 1.20 s to accelerate the 7.27-kg shot from rest to 14.0 m/s, while raising it 0.800 m. (Do not include the power produced to accelerate his body.)

Shot putter at the Dornoch Highland Gathering in 2007. (credit: John Haslam, Flickr)

Solution
641 W, 0.860 hp

73. (a) What is the efficiency of an out-of-condition professor who does $2.10 \times 10^5 J$ of useful work while metabolizing 500 kcal of food energy?

(b) How many food calories would a well-conditioned athlete metabolize in doing the same work with an efficiency of 20%?

74. Energy that is not utilized for work or heat transfer is converted to the chemical energy of body fat containing about 39 kJ/g. How many grams of fat will you gain if you eat 10,000 kJ (about 2500 kcal) one day and do nothing but sit relaxed for 16.0 h and sleep for the other 8.00 h? Use data from Table for the energy consumption rates of these activities.

Solution
31 g

75. Using data from Table, calculate the daily energy needs of a person who sleeps for 7.00 h, walks for 2.00 h, attends classes for 4.00 h, cycles for 2.00 h, sits relaxed for 3.00 h, and studies for 6.00 h. (Studying consumes energy at the same rate as sitting in class.)

76. What is the efficiency of a subject on a treadmill who puts out work at the rate of 100 W while consuming oxygen at the rate of 2.00 L/min? (Hint: See Table.)

Solution
14.3%

77. Shoveling snow can be extremely taxing because the arms have such a low efficiency in this activity. Suppose a person shoveling a footpath metabolizes food at the rate of 800 W.

(a) What is her useful power output?

(b) How long will it take her to lift 3000 kg of snow 1.20 m? (This could be the amount of heavy snow on 20 m of footpath.)

(c) How much waste heat transfer in kilojoules will she generate in the process?

78. Very large forces are produced in joints when a person jumps from some height to the ground.

(a) Calculate the magnitude of the force produced if an 80.0-kg person jumps from a 0.600–m-high ledge and lands stiffly, compressing joint material 1.50 cm as a result. (Be certain to include the weight of the person.)

(b) In practice the knees bend almost involuntarily to help extend the distance over which you stop. Calculate the magnitude of the force produced if the stopping distance is 0.300 m.

(c) Compare both forces with the weight of the person.

Solution
(a) $3.21 \times 10^4 N$
(b) $2.35 \times 10^3 N$
(c) Ratio of net force to weight of person is 41.0 in part (a); 3.00 in part (b)

79. Jogging on hard surfaces with insufficiently padded shoes produces large forces in the feet and legs.

(a) Calculate the magnitude of the force needed to stop the downward motion of a jogger's leg, if his leg has a mass of 13.0 kg, a speed of 6.00 m/s, and stops in a distance of 1.50 cm. (Be certain to include the weight of the 75.0-kg jogger's body.)

(b) Compare this force with the weight of the jogger.

80. (a) Calculate the energy in kJ used by a 55.0-kg woman who does 50 deep knee bends in which her center of mass is lowered and raised 0.400 m. (She does work in both directions.) You may assume her efficiency is 20%.

(b) What is the average power consumption rate in watts if she does this in 3.00 min?

Solution
(a) 108 kJ
(b) 599 W

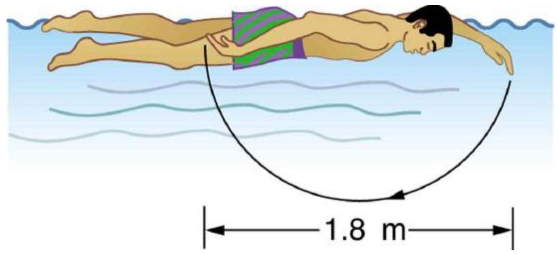
81. Kanellos Kanellopoulos flew 119 km from Crete to Santorini, Greece, on April 23, 1988, in the Daedalus 88, an aircraft powered by a bicycle-type drive mechanism (see Figure). His useful power output for the 234-min trip was about 350 W. Using the efficiency for cycling from [link], calculate the food energy in kilojoules he metabolized during the flight.

The Daedalus 88 in flight. (credit: NASA photo by Beasley)

82. The swimmer shown in Figure exerts an average horizontal backward force of 80.0 N with his arm during each 1.80 m long stroke.

(a) What is his work output in each stroke?

(b) Calculate the power output of his arms if he does 120 strokes per minute.



Solution
(a) 144 J
(b) 288 W

83. Mountain climbers carry bottled oxygen when at very high altitudes.

(a) Assuming that a mountain climber uses oxygen at twice the rate for climbing 116 stairs per minute (because of low air temperature and winds), calculate how many liters of oxygen a climber would need for 10.0 h of climbing. (These are liters at sea level.) Note that only 40% of the inhaled oxygen is utilized; the rest is exhaled.

(b) How much useful work does the climber do if he and his equipment have a mass of 90.0 kg and he gains 1000 m of altitude?

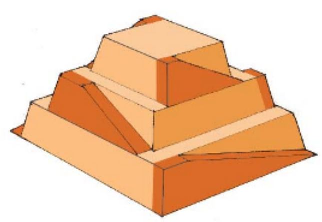
(c) What is his efficiency for the 10.0-h climb?

84. The awe-inspiring Great Pyramid of Cheops was built more than 4500 years ago. Its square base, originally 230 m on a side, covered 13.1 acres, and it was 146 m high, with a mass of about $7 \times 10^{9} kg$. (The pyramid's dimensions are slightly different today due to quarrying and some sagging.) Historians estimate that 20,000 workers spent 20 years to construct it, working 12-hour days, 330 days per year.

(a) Calculate the gravitational potential energy stored in the pyramid, given its center of mass is at one-fourth its height.

(b) Only a fraction of the workers lifted blocks; most were involved in support services such as building ramps (see Figure), bringing food and water, and hauling blocks to the site. Calculate the efficiency of the workers who did the lifting, assuming there were 1000 of them and they consumed food energy at the rate of 300 kcal/h. What does your answer imply about how much of their work went into block-lifting, versus how much work went into friction and lifting and lowering their own bodies?

(c) Calculate the mass of food that had to be supplied each day, assuming that the average worker required 3600 kcal per day and that their diet was 5% protein, 60% carbohydrate, and 35% fat. (These proportions neglect the mass of bulk and nondigestible materials consumed.)



Ancient pyramids were probably constructed using ramps as simple machines. (credit: Franck Monnier, Wikimedia Commons)

Solution
(a) $2.50 \times 10^{12} J$
(b) 2.52%
(c) $1.4 \times 10^{4} kg$ (14 metric tons)

85. (a) How long can you play tennis on the 800 kJ (about 200 kcal) of energy in a candy bar?

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(b) Does this seem like a long time? Discuss why exercise is necessary but may not be sufficient to cause a person to lose weight.

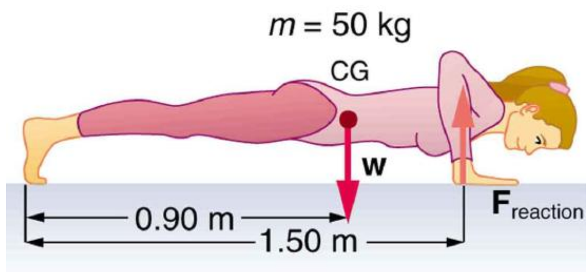
7.9: World Energy Use

86. *Integrated Concepts*

(a) Calculate the force the woman in Figure exerts to do a push-up at constant speed, taking all data to be known to three digits.

(b) How much work does she do if her center of mass rises 0.240 m?

(c) What is her useful power output if she does 25 push-ups in 1 min? (Should work done lowering her body be included? See the discussion of useful work in Work, Energy, and Power in Humans.



Forces involved in doing push-ups. The woman's weight acts as a force exerted downward on her center of gravity (CG).

Solution
(a) 294 N
(b) 118 J
(c) 49.0 W

87. *Integrated Concepts*

A 75.0-kg cross-country skier is climbing a 3.0^{o} slope at a constant speed of 2.00 m/s and encounters air resistance of 25.0 N. Find his power output for work done against the gravitational force and air resistance.

(b) What average force does he exert backward on the snow to accomplish this?

(c) If he continues to exert this force and to experience the same air resistance when he reaches a level area, how long will it take him to reach a velocity of 10.0 m/s?

88. *Integrated Concepts*

The 70.0-kg swimmer in [link] starts a race with an initial velocity of 1.25 m/s and exerts an average force of 80.0 N backward with his arms during each 1.80 m long stroke.

(a) What is his initial acceleration if water resistance is 45.0 N?

(b) What is the subsequent average resistance force from the water during the 5.00 s it takes him to reach his top velocity of 2.50 m/s?

(c) Discuss whether water resistance seems to increase linearly with velocity.

Solution
(a) $0.500 m/s^2$
(b) $62.5 N$
(c) Assuming the acceleration of the swimmer decreases linearly with time over the 5.00 s interval, the frictional force must therefore be increasing linearly with time, since $f = F - ma$. If the acceleration decreases linearly with time, the velocity will contain a term dependent on time squared (t^2). Therefore, the water resistance will not depend linearly on the velocity.

89. *Integrated Concepts*

A toy gun uses a spring with a force constant of 300 N/m to propel a 10.0-g steel ball. If the spring is compressed 7.00 cm and friction is negligible:

(a) How much force is needed to compress the spring?

(b) To what maximum height can the ball be shot?

(c) At what angles above the horizontal may a child aim to hit a target 3.00 m away at the same height as the gun?

(d) What is the gun's maximum range on level ground?

90. *Integrated Concepts*

(a) What force must be supplied by an elevator cable to produce an acceleration of $0.800 m/s^2$ against a 200-N frictional force, if the mass of the loaded elevator is 1500 kg?

(b) How much work is done by the cable in lifting the elevator 20.0 m?

(c) What is the final speed of the elevator if it starts from rest?

(d) How much work went into thermal energy?

Solution
(a) $16.1 \times 10^3 N$
(b) $3.22 \times 10^5 J$
(c) $5.66 m/s$
(d) 4.00 kJ

91. *Unreasonable Results*

A car advertisement claims that its 900-kg car accelerated from rest to 30.0 m/s and drove 100 km, gaining 3.00 km in altitude, on 1.0 gal of gasoline. The average force of friction including air resistance was 700 N. Assume all values are known to three significant figures.

(a) Calculate the car's efficiency.

(b) What is unreasonable about the result?

(c) Which premise is unreasonable, or which premises are inconsistent?

92. *Unreasonable Results*

Body fat is metabolized, supplying 9.30 kcal/g, when dietary intake is less than needed to fuel metabolism. The manufacturers of an exercise bicycle claim that you can lose 0.500 kg of fat per day by vigorously exercising for 2.00 h per day on their machine.

(a) How many kcal are supplied by the metabolization of 0.500 kg of fat?

(b) Calculate the kcal/min that you would have to utilize to metabolize fat at the rate of 0.500 kg in 2.00 h.

(c) What is unreasonable about the results?

(d) Which premise is unreasonable, or which premises are inconsistent?

Solution
(a) $4.65 \times 10^3 kcal$
(b) 38.8 kcal/min
(c) This power output is higher than the highest value on [link], which is about 35 kcal/min (corresponding to 2415 watts) for sprinting.
(d) It would be impossible to maintain this power output for 2 hours (imagine sprinting for 2 hours!).

93. *Construct Your Own Problem*

Consider a person climbing and descending stairs. Construct a problem in which you calculate the long-term rate at which stairs can be climbed considering the mass of the person, his ability to generate power with his legs, and the height of a single stair step. Also consider why the same person can descend stairs at a faster rate for a nearly unlimited

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time in spite of the fact that very similar forces are exerted going down as going up. (This points to a fundamentally different process for descending versus climbing stairs.)

94. *Construct Your Own Problem*

Consider humans generating electricity by pedaling a device similar to a stationary bicycle. Construct a problem in which you determine the number of people it would take to replace a large electrical generation facility. Among the things to consider are the power output that is reasonable using the legs, rest time, and the need for electricity 24 hours per day. Discuss the practical implications of your results.

95. Integrated Concepts

A 105-kg basketball player crouches down 0.400 m while waiting to jump. After exerting a force on the floor through this 0.400 m, his feet leave the floor and his center of gravity rises 0.950 m above its normal standing erect position.

(a) Using energy considerations, calculate his velocity when he leaves the floor.

(b) What average force did he exert on the floor? (Do not neglect the force to support his weight as well as that to accelerate him.)

(c) What was his power output during the acceleration phase?

Solution
(a) 4.32 m/s
(b) $3.47 \times 10^3 N$
(c) 8.93 kW

Contributors and Attributions

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CHAPTER OVERVIEW

7: Linear Momentum and Collisions

We use the term momentum in various ways in everyday language, and most of these ways are consistent with its precise scientific definition. We speak of sports teams or politicians gaining and maintaining the momentum to win. We also recognize that momentum has something to do with collisions. For example, looking at the rugby players in the photograph colliding and falling to the ground, we expect their momenta to have great effects in the resulting collisions. Generally, momentum implies a tendency to continue on course—to move in the same direction—and is associated with great mass and speed.

- 7.1: Prelude to Linear Momentum and Collisions
- 7.2: Linear Momentum and Force
- 7.3: Impulse
- 7.4: Conservation of Momentum
- 7.5: Elastic Collisions in One Dimension
- 7.6: Inelastic Collisions in One Dimension
- 7.7: Collisions of Point Masses in Two Dimensions
- 7.8: Introduction to Rocket Propulsion
- 7.E: Linear Momentum and Collisions (Exercises)

Thumbnail: A pool break-off shot. (CC-SA-BY; No-w-ay).

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7.1: Prelude to Linear Momentum and Collisions

We use the term momentum in various ways in everyday language, and most of these ways are consistent with its precise scientific definition. We speak of sports teams or politicians gaining and maintaining the momentum to win. We also recognize that momentum has something to do with collisions. For example, looking at the rugby players in the photograph colliding and falling to the ground, we expect their momenta to have great effects in the resulting collisions. Generally, momentum implies a tendency to continue on course—to move in the same direction—and is associated with great mass and speed.

Figure 7.1.1: Each rugby player has great momentum, which will affect the outcome of their collisions with each other and the ground. (credit: ozzzie, Flickr)

Momentum, like energy, is important because it is conserved. Only a few physical quantities are conserved in nature, and studying them yields fundamental insight into how nature works, as we shall see in our study of momentum.

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7.2: Linear Momentum and Force

Learning Objectives

By the end of this section, you will be able to:

- Define linear momentum.
- Explain the relationship between momentum and force.
- State Newton's second law of motion in terms of momentum.
- Calculate momentum given mass and velocity.

The scientific definition of linear momentum is consistent with most people's intuitive understanding of momentum: a large, fast-moving object has greater momentum than a smaller, slower object. Linear momentum is defined as the product of a system's mass multiplied by its velocity.

Linear Momentum

Linear momentum is defined as the product of a system's mass multiplied by its velocity:

$$p = mv \tag{7.2.1}$$

Momentum is directly proportional to the object's mass and also its velocity. Thus the greater an object's mass or the greater its velocity, the greater its momentum. Momentum p is a vector having the same direction as the velocity v. The SI unit for momentum is $kg \cdot m/s$.

Example 7.2.1: Calculating Momentum: A Football Player and a Football

a. Calculate the momentum of a 110-kg football player running at 8.00 m/s.
b. Compare the player's momentum with the momentum of a hard-thrown 0.410-kg football that has a speed of 25.0 m/s.

Strategy

No information is given regarding direction, and so we can calculate only the magnitude of the momentum, p (As usual, a symbol that is in italics is a magnitude, whereas one that is italicized, boldfaced, and has an arrow is a vector.) In both parts of this example, the magnitude of momentum can be calculated directly from the definition of momentum given in Equation 7.2.1, which becomes

$$p = mv$$

when only magnitudes are considered.

Solution for (a)

To determine the momentum of the player, substitute the known values for the player's mass and speed into the equation.

$$\begin{aligned} p_{player} &= (110\, kg)(8.00\, m/s) \\ &= 880\, kg \cdot m/s \end{aligned}$$

Solution for (b)

To determine the momentum of the ball, substitute the known values for the ball's mass and speed into the equation.

$$\begin{aligned} p_{ball} &= (0.410\, kg)(25.0\, m/s) \\ &= 10.3\, kg \cdot m/s \end{aligned}$$

The ratio of the player's momentum to that of the ball is

$$\frac{p_{player}}{p_{ball}} = \frac{880}{10.3} = 85.0$$

Discussion

Although the ball has greater velocity, the player has a much greater mass. Thus the momentum of the player is much greater than the momentum of the football, as you might guess. As a result, the player's motion is only slightly affected if he catches the ball. We shall quantify what happens in such collisions in terms of momentum in later sections.

Momentum and Newton's Second Law

The importance of momentum, unlike the importance of energy, was recognized early in the development of classical physics. Momentum was deemed so important that it was called the "quantity of motion." Newton actually stated his second law of motion in terms of momentum: The net external force equals the change in momentum of a system divided by the time over which it changes.

Newton's Second Law of Motion in Terms of Momentum

The net external force equals the change in momentum of a system divided by the time over which it changes.

$$F_{net} = \frac{\Delta p}{\Delta t} \tag{7.2.2}$$

where F_{net} is the net external force, Δp is the change in momentum, and Δt is the change in time.

Making Connections: Force and Momentum

Force and momentum are intimately related. Force acting over time can change momentum, and Newton's second law of motion, can be stated in its most broadly applicable form in terms of momentum. Momentum continues to be a key concept in the study of atomic and subatomic particles in quantum mechanics.

This statement of Newton's second law of motion includes the more familiar $F_{net} = ma$ as a special case. We can derive this form as follows. First, note that the change in momentum Δp is given by

$$\Delta p = \Delta(mv) \tag{7.2.3}$$

If the mass of the system is constant, then

$$\Delta(mv) = m\Delta v. \tag{7.2.4}$$

So that for constant mass, Newton's second law of motion becomes

$$F_{net} = \frac{\Delta p}{\Delta t} = \frac{m\Delta v}{\Delta t}. \tag{7.2.5}$$

Because $\frac{\Delta v}{\Delta t} = a,$ we get the familiar equation

$$F_{net} = ma \tag{7.2.6}$$

when the mass of the system is *constant*.

Newton's second law of motion stated in terms of momentum is more generally applicable because it can be applied to systems where the mass is changing, such as rockets, as well as to systems of constant mass. We will consider systems with varying mass in some detail**;** however, the relationship between momentum and force remains useful when mass is constant, such as in the following example.

Example 7.2.2: Calculating Force: Venus Williams' Racquet

During the 2007 French Open, Venus Williams hit the fastest recorded serve in a premier women's match, reaching a speed of 58 m/s (209 km/h). What is the average force exerted on the 0.057-kg tennis ball by Venus Williams' racquet, assuming that the ball's speed just after impact is 58 m/s, that the initial horizontal component of the velocity before impact is negligible, and that the ball remained in contact with the racquet for 5.0 ms (milliseconds)?

Strategy

This problem involves only one dimension because the ball starts from having no horizontal velocity component before impact. Newton's second law stated in terms of momentum is then written as

$$F_{net} = \frac{\Delta p}{\Delta t}$$

As noted above, when mass is constant, the change in momentum is given by

$$\Delta p = m\Delta v = m(v_f - v_i).$$

In this example, the velocity just after impact and the change in time are given; thus, once Δp s calculated, $F_{net} = \frac{\Delta p}{\Delta t}$ can be used to find the force.

Solution

To determine the change in momentum, substitute the values for the initial and final velocities into the equation above.

$$\begin{aligned} \Delta p &= m(v_f - v_i) \\ &= (0.057\,kg)(58\,m/s - 0\,m/s) \\ &= 3.306\,kg \cdot m/s = 3.3\,kg \cdot m/s \end{aligned}$$

Now the magnitude of the net external force can determined by using $F_{net} = \frac{\Delta p}{\Delta t}$

$$\begin{aligned} F_{net} &= \frac{\Delta p}{\Delta t} = \frac{3.306\,kg}{5.0 \times 10^{-3}} \\ &= 661\,N, \end{aligned}$$

where we have retained only two significant figures in the final step.

Discussion

This quantity was the average force exerted by Venus Williams' racquet on the tennis ball during its brief impact (note that the ball also experienced the 0.56-N force of gravity, but that force was not due to the racquet). This problem could also be solved by first finding the acceleration and then using $F = ma$ but one additional step would be required compared with the strategy used in this example.

Summary

- Linear momentum (*momentum* for brevity) is defined as the product of a system's mass multiplied by its velocity.
- In symbols, linear momentum p is defined to be

  $$p = mv$$

  where m is the mass of the system and v is its velocity.
- The SI unit for momentum is $kg \cdot m/s$.
- Newton's second law of motion in terms of momentum states that the net external force equals the change in momentum of a system divided by the time over which it changes.
- In symbols, Newton's second law of motion is defined to be

  $$F_{net} = \frac{\Delta p}{\Delta t}$$

  where F_{net} is the net external force, Δp s the change in momentum, and Δt is change in time.

Glossary

linear momentum

the product of mass and velocity

second law of motion

physical law that states that the net external force equals the change in momentum of a system divided by the time over which it changes

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7.3: Impulse

Learning Objectives

By the end of this section, you will be able to:

- Define impulse.
- Describe effects of impulses in everyday life.
- Determine the average effective force using graphical representation.
- Calculate average force and impulse given mass, velocity, and time.

The effect of a force on an object depends on how long it acts, as well as how great the force is. In [link], a very large force acting for a short time had a great effect on the momentum of the tennis ball. A small force could cause the same change in momentum, but it would have to act for a much longer time. For example, if the ball were thrown upward, the gravitational force (which is much smaller than the tennis racquet's force) would eventually reverse the momentum of the ball. Quantitatively, the effect we are talking about is the change in momentum Δp.

By rearranging the equation $\Delta F_{net} = \frac{\Delta p}{\Delta t}$ to be

$$\Delta p = F_{net} \Delta t, \tag{7.3.1}$$

we can see how the change in momentum equals the average net external force multiplied by the time this force acts. The quantity $F_{net} \Delta t$ is given the name impulse. Impulse is the same as the change in momentum.

Impulse: Change in Momentum

Change in momentum equals the average net external force multiplied by the time this force acts.

$$\Delta p = F_{net} \Delta t \tag{7.3.2}$$

The quantity $F_{net} \Delta t$ is given the name impulse.

There are many ways in which an understanding of impulse can save lives, or at least limbs. The dashboard padding in a car, and certainly the airbags, allow the net force on the occupants in the car to act over a much longer time when there is a sudden stop. The momentum change is the same for an occupant, whether an air bag is deployed or not, but the force (to bring the occupant to a stop) will be much less if it acts over a larger time. Cars today have many plastic components. One advantage of plastics is their lighter weight, which results in better gas mileage. Another advantage is that a car will crumple in a collision, especially in the event of a head-on collision. A longer collision time means the force on the car will be less. Deaths during car races decreased dramatically when the rigid frames of racing cars were replaced with parts that could crumple or collapse in the event of an accident.

Bones in a body will fracture if the force on them is too large. If you jump onto the floor from a table, the force on your legs can be immense if you land stiff-legged on a hard surface. Rolling on the ground after jumping from the table, or landing with a parachute, extends the time over which the force (on you from the ground) acts.

Example 7.3.1: Calculating Magnitudes of Impulses: Two Billiard Balls Striking a Rigid Wall

Two identical billiard balls strike a rigid wall with the same speed, and are reflected without any change of speed. The first ball strikes perpendicular to the wall. The second ball strikes the wall at an angle of 30^o from the perpendicular, and bounces off at an angle of 30^o from perpendicular to the wall.

a. Determine the direction of the force on the wall due to each ball.
b. Calculate the ratio of the magnitudes of impulses on the two balls by the wall.

Strategy for (a)

In order to determine the force on the wall, consider the force on the ball due to the wall using Newton's second law and then apply Newton's third law to determine the direction. Assume the $x-$axis to be normal to the wall and to be positive in the initial direction of motion. Choose the $y-$axis to be along the wall in the plane of the second ball's motion. The momentum direction and the velocity direction are the same.

Strategy for (a)

In order to determine the force on the wall, consider the force on the ball due to the wall using Newton's second law and then apply Newton's third law to determine the direction. Assume the $x-$axis to be normal to the wall and to be positive in the initial direction of motion. Choose the $y-$axis to be along the wall in the plane of the second ball's motion. The momentum direction and the velocity direction are the same.

Solution for (a)

The first ball bounces directly into the wall and exerts a force on it in the +x direction. Therefore the wall exerts a force on the ball in the -y direction. The second ball continues with the same momentum component in the $y-$ direction, but reverses its x-component of momentum, as seen by sketching a diagram of the angles involved and keeping in mind the proportionality between velocity and momentum.

These changes mean the change in momentum for both balls is in the $-x$ direction, so the force of the wall on each ball is along the $-x$ direction.

Strategy for (b)

Calculate the change in momentum for each ball, which is equal to the impulse imparted to the ball.

Solution for (b)

Let μ be the speed of each ball before and after collision with the wall, and m the mass of each ball. Choose the $x-$axis and $y-$axis as previously described, and consider the change in momentum of the first ball which strikes perpendicular to the wall.

$$p_{xi} = m\mu;\ p_{yi} = 0 \tag{7.3.3}$$

$$p_{xf} = -m\mu;\ p_{yf} = 0 \tag{7.3.4}$$

Impulse is the change in momentum vector. Therefore the $x-$component of impulse is equal to -2$m\mu$ and the $y-$component of impulse is equal to zero.

Now consider the change in momentum of the second ball.

$$p_{xi} = m/mu\, cos 30^o;\ p_{yi} = -m\mu\, 30^o \tag{7.3.5}$$

$$p_{xf} = -m/mu\, cos 30^o;\ p_{yf} = -m\mu\, 30^o \tag{7.3.6}$$

It should be noted here that while p_x changes sign after the collision, p_y does not. Therefore the -component of impulse is equal to $-2m\mu\, cos\, 30^o$ and the $y-$component of impulse is equal to zero.

The ratio of the magnitudes of the impulse imparted to the balls is

$$\frac{2m\mu}{2m\mu\, 30^o} = \frac{2}{sqrt3} = 1.155. \tag{7.3.7}$$

Discussion

The direction of impulse and force is the same as in the case of (a); it is normal to the wall and along the negative $x-$direction. Making use of Newton's third law, the force on the wall due to each ball is normal to the wall along the positive $x-$direction.

Our definition of impulse includes an assumption that the force is constant over the time interval Δt.*Forces are usually not constant*. Forces vary considerably even during the brief time intervals considered. It is, however, possible to find an average effective force F_{eff} that produces the same result as the corresponding time-varying force. Figure shows a graph of what an actual force looks like as a function of time for a ball bouncing off the floor. The area under the curve has units of momentum and is equal to the impulse or change in momentum between times t_1 and t_2. That area is equal to the area inside the rectangle bounded by F_{eff}, t_1, and t_2 Thus the impulses and their effects are the same for both the actual and effective forces.

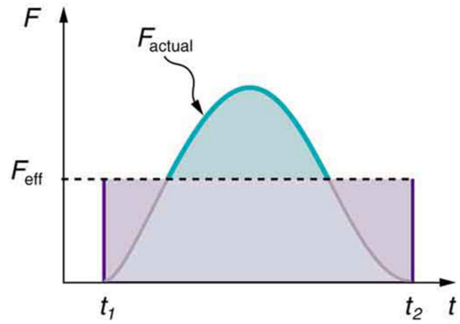

Figure 7.3.1: A graph of force versus time with time along the $x-$axis and force along the $y-$axis for an actual force and an equivalent effective force. The areas under the two curves are equal.

MAKING CONNECTIONS: Take-Home Investigation—Hand Movement and Impulse

Try catching a ball while "giving" with the ball, pulling your hands toward your body. Then, try catching a ball while keeping your hands still. Hit water in a tub with your full palm. After the water has settled, hit the water again by diving your hand with your fingers first into the water. (Your full palm represents a swimmer doing a belly flop and your diving hand represents a swimmer doing a dive.) Explain what happens in each case and why. Which orientations would you advise people to avoid and why?

MAKING CONNECTIONS: Constant Force anD Constant Acceleration

The assumption of a constant force in the definition of impulse is analogous to the assumption of a constant acceleration in kinematics. In both cases, nature is adequately described without the use of calculus.

Summary

- Impulse, or change in momentum, equals the average net external force multiplied by the time this force acts:

$$\Delta p = F_{net}\Delta t. \tag{7.3.8}$$

- Forces are usually not constant over a period of time.

Glossary

change in momentum

the difference between the final and initial momentum; the mass times the change in velocity

impulse

the average net external force times the time it acts; equal to the change in momentum

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7.4: Conservation of Momentum

Learning Objectives

By the end of this section, you will be able to:

- Describe the principle of conservation of momentum.
- Derive an expression for the conservation of momentum.
- Explain conservation of momentum with examples.
- Explain the principle of conservation of momentum as it relates to atomic and subatomic particles.

Momentum is an important quantity because it is conserved. Yet it was not conserved in the examples in Impulse and Linear Momentum and Force, where large changes in momentum were produced by forces acting on the system of interest. Under what circumstances is momentum conserved?

The answer to this question entails considering a sufficiently large system. It is always possible to find a larger system in which total momentum is constant, even if momentum changes for components of the system. If a football player runs into the goalpost in the end zone, there will be a force on him that causes him to bounce backward. However, the Earth also recoils —conserving momentum—because of the force applied to it through the goalpost. Because Earth is many orders of magnitude more massive than the player, its recoil is immeasurably small and can be neglected in any practical sense, but it is real nevertheless.

Consider what happens if the masses of two colliding objects are more similar than the masses of a football player and Earth—for example, one car bumping into another, as shown in Figure $7.4.1$. Both cars are coasting in the same direction when the lead car (labeled m_2 is bumped by the trailing car (labeled m_1. The only unbalanced force on each car is the force of the collision. (Assume that the effects due to friction are negligible.) Car 1 slows down as a result of the collision, losing some momentum, while car 2 speeds up and gains some momentum. We shall now show that the total momentum of the two-car system remains constant.

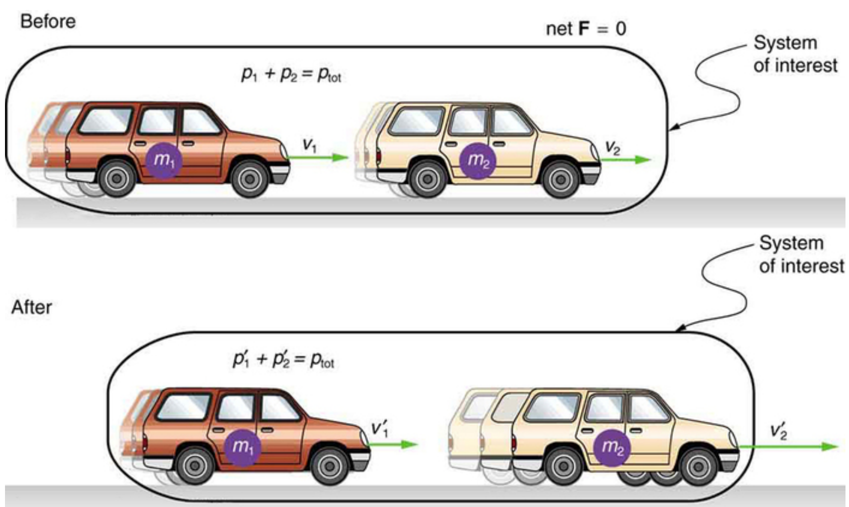

Figure $7.4.1$: A car of mass m_1 moving with a velocity of v_1 bumps into another car of mass m_2 and velocity v_2 that it is following. As a result, the first car slows down to a velocity of v_1' and the second speeds up to a velocity of v_2' The momentum of each car is changed, but the total momentum p_{tot} of the two cars is the same before and after the collision (if you assume friction is negligible).

Using the definition of impulse, the change in momentum of car 1 is given by

$$\Delta p_1 = F_1 \Delta t, \tag{7.4.1}$$

is the force on car 1 due to car 2, and Δt

where F_1 is the time the force acts (the duration of the collision). Intuitively, it seems obvious that the collision time is the same for both cars, but it is only true for objects traveling at ordinary speeds. This assumption must be modified for objects travelling near the speed of light, without affecting the result that momentum is conserved.

Similarly, the change in momentum of car 2 is

$$\Delta p_2 = F_2 \Delta t, \tag{7.4.2}$$

where F_2 is the force on car 2 due to car 1, and we assume the duration of the collision Δt is the same for both cars. We know from Newton's third law that $F_2 = -F_1$, and so

$$\Delta p_2 = -F_1 \Delta t = -\Delta p_1. \tag{7.4.3}$$

Thus, the changes in momentum are equal and opposite, and

$$\Delta p_1 + \Delta p_2 = 0. \tag{7.4.4}$$

Because the changes in momentum add to zero, the total momentum of the two-car system is constant. That is,

$$p_1 + p_2 = constant \tag{7.4.5}$$

$$p_1 + p_2 = p'_1 + p'_2, \tag{7.4.6}$$

where p'_1 and p'_2 are the momenta of cars 1 and 2 after the collision. (We often use primes to denote the final state.)

This result—that momentum is conserved—has validity far beyond the preceding one-dimensional case. It can be similarly shown that total momentum is conserved for any isolated system, with any number of objects in it. In equation form, the conservation of momentum principle for an isolated system is written

$$p_{tot} = constant, \tag{7.4.7}$$

or

$$p_{tot} = p_{tot}, \tag{7.4.8}$$

where p_{tot} is the total momentum (the sum of the momenta of the individual objects in the system) and p_{tot}, is the total momentum some time later. (The total momentum can be shown to be the momentum of the center of mass of the system.) An isolated system is defined to be one for which the net external force is zero $(F_{net} = 0)$.

Conservation of Momentum Principle

$$p_{tot} = constant \tag{7.4.9}$$

$$p_{tot} = p'_{tot} \; (isolated \; system) \tag{7.4.10}$$

Isolated System

An isolated system is defined to be one for which the net external force is zero $(F_{net} = 0)$.

Perhaps an easier way to see that momentum is conserved for an isolated system is to consider Newton's second law in terms of momentum, $F_{net} = \frac{\Delta p_{tot}}{\Delta t}$. For an isolated system, $(F_{net} = 0)$; thus $\Delta p_{tot} = 0$ and Δp is constant.

We have noted that the three length dimensions in nature x, y and z are independent, and it is interesting to note that momentum can be conserved in different ways along each dimension. For example, during projectile motion and where air resistance is negligible, momentum is conserved in the horizontal direction because horizontal forces are zero and momentum is unchanged. But along the vertical direction, the net vertical force is not zero and the momentum of the projectile is not conserved (Figure 7.4.2). However, if the momentum of the projectile-Earth system is considered in the vertical direction, we find that the total momentum is conserved.

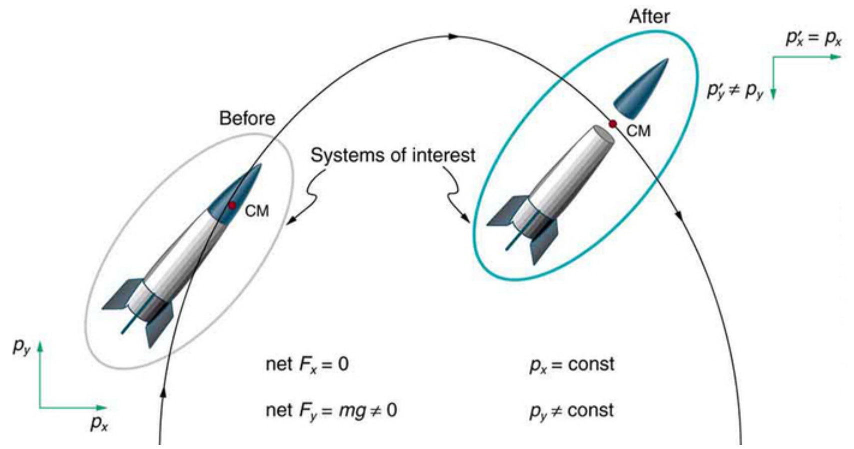

Figure 7.4.2: The horizontal component of a projectile's momentum is conserved if air resistance is negligible, even in this case where a space probe separates. The forces causing the separation are internal to the system, so that the net external horizontal force F_{x-net} is still zero. The vertical component of the momentum is not conserved, because the net vertical force F_{y-net} is not zero. In the vertical direction, the space probe-Earth system needs to be considered and we find that the total momentum is conserved. The center of mass of the space probe takes the same path it would if the separation did not occur.

The conservation of momentum principle can be applied to systems as different as a comet striking Earth and a gas containing huge numbers of atoms and molecules. Conservation of momentum is violated only when the net external force is not zero. But another larger system can always be considered in which momentum is conserved by simply including the source of the external force. For example, in the collision of two cars considered above, the two-car system conserves momentum while each one-car system does not.

MAKING CONNECTIONS: TAKE-HOME Investigation—Drop of Tennis Ball and a Basketball

Hold a tennis ball side by side and in contact with a basketball. Drop the balls together. (Be careful!) What happens? Explain your observations. Now hold the tennis ball above and in contact with the basketball. What happened? Explain your observations. What do you think will happen if the basketball ball is held above and in contact with the tennis ball?

MAKING CONNECTIONS: TAKE-HOME Investigation—Two Tennis Balls in a Ballistic Trajectory

Tie two tennis balls together with a string about a foot long. Hold one ball and let the other hang down and throw it in a ballistic trajectory. Explain your observations. Now mark the center of the string with bright ink or attach a brightly colored sticker to it and throw again. What happened? Explain your observations.

Some aquatic animals such as jellyfish move around based on the principles of conservation of momentum. A jellyfish fills its umbrella section with water and then pushes the water out resulting in motion in the opposite direction to that of the jet of water. Squids propel themselves in a similar manner but, in contrast with jellyfish, are able to control the direction in which they move by aiming their nozzle forward or backward. Typical squids can move at speeds of 8 to 12 km/h.

The ballistocardiograph (BCG) was a diagnostic tool used in the second half of the 20th century to study the strength of the heart. About once a second, your heart beats, forcing blood into the aorta. A force in the opposite direction is exerted on the rest of your body (recall Newton's third law). A ballistocardiograph is a device that can measure this reaction force. This measurement is done by using a sensor (resting on the person) or by using a moving table suspended from the ceiling. This technique can gather information on the strength of the heart beat and the volume of blood passing from the heart. However, the electrocardiogram (ECG or EKG) and the echocardiogram (cardiac ECHO or ECHO; a technique that uses ultrasound to see an image of the heart) are more widely used in the practice of cardiology.

Making Connections: Conservation of Momentum and Collision

Conservation of momentum is quite useful in describing collisions. Momentum is crucial to our understanding of atomic and subatomic particles because much of what we know about these particles comes from collision experiments.

Subatomic Collisions and Momentum

The conservation of momentum principle not only applies to the macroscopic objects, it is also essential to our explorations of atomic and subatomic particles. Giant machines hurl subatomic particles at one another, and researchers evaluate the results by assuming conservation of momentum (among other things).

On the small scale, we find that particles and their properties are invisible to the naked eye but can be measured with our instruments, and models of these subatomic particles can be constructed to describe the results. Momentum is found to be a property of all subatomic particles including massless particles such as photons that compose light. Momentum being a property of particles hints that momentum may have an identity beyond the description of an object's mass multiplied by the object's velocity. Indeed, momentum relates to wave properties and plays a fundamental role in what measurements are taken and how we take these measurements. Furthermore, we find that the conservation of momentum principle is valid when considering systems of particles. We use this principle to analyze the masses and other properties of previously undetected particles, such as the nucleus of an atom and the existence of quarks that make up particles of nuclei. Figure $7.4.3$ below illustrates how a particle scattering backward from another implies that its target is massive and dense. Experiments seeking evidence that quarks make up protons (one type of particle that makes up nuclei) scattered high-energy electrons off of protons (nuclei of hydrogen atoms). Electrons occasionally scattered straight backward in a manner that implied a very small and very dense particle makes up the proton—this observation is considered nearly direct evidence of quarks. The analysis was based partly on the same conservation of momentum principle that works so well on the large scale.

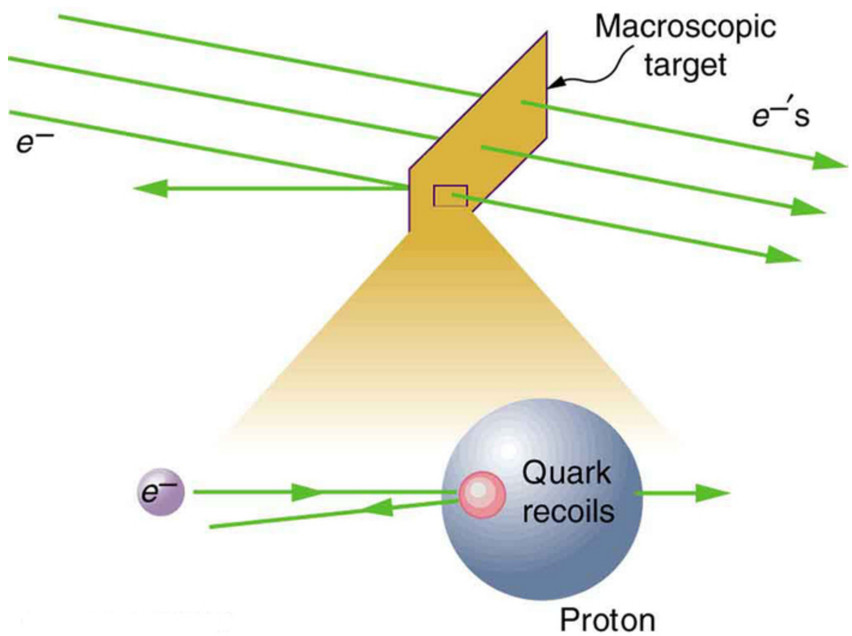

Figure $7.4.3$: A subatomic particle scatters straight backward from a target particle. In experiments seeking evidence for quarks, electrons were observed to occasionally scatter straight backward from a proton.

Summary

- The conservation of momentum principle is written

$$p_{tot} = constant \tag{7.4.11}$$

or

$$p_{tot} = p'_{tot}\ (isolated\ system), \tag{7.4.12}$$

- p_{tot} is the initial total momentum and p'_{tot} is the total momentum some time later. An isolated system is defined to be one for which the net external force is zero $(F_{net} = 0$
- During projectile motion and where air resistance is negligible, momentum is conserved in the horizontal direction because horizontal forces are zero.
- Conservation of momentum applies only when the net external force is zero.
- The conservation of momentum principle is valid when considering systems of particles.

Glossary

conservation of momentum principle
when the net external force is zero, the total momentum of the system is conserved or constant

isolated system
a system in which the net external force is zero

quark
fundamental constituent of matter and an elementary particle

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7.5: Elastic Collisions in One Dimension

Learning Objectives

By the end of this section, you will be able to:

- Describe an elastic collision of two objects in one dimension.
- Define internal kinetic energy.
- Derive an expression for conservation of internal kinetic energy in a one dimensional collision.
- Determine the final velocities in an elastic collision given masses and initial velocities.

Let us consider various types of two-object collisions. These collisions are the easiest to analyze, and they illustrate many of the physical principles involved in collisions. The conservation of momentum principle is very useful here, and it can be used whenever the net external force on a system is zero.

We start with the elastic collision of two objects moving along the same line—a one-dimensional problem. An elastic collision is one that also conserves internal kinetic energy. Internal kinetic energy is the sum of the kinetic energies of the objects in the system. Figure illustrates an elastic collision in which internal kinetic energy and momentum are conserved.

Truly elastic collisions can only be achieved with subatomic particles, such as electrons striking nuclei. Macroscopic collisions can be very nearly, but not quite, elastic—some kinetic energy is always converted into other forms of energy such as heat transfer due to friction and sound. One macroscopic collision that is nearly elastic is that of two steel blocks on ice. Another nearly elastic collision is that between two carts with spring bumpers on an air track. Icy surfaces and air tracks are nearly frictionless, more readily allowing nearly elastic collisions on them.

Elastic Collision

An elastic collision is one that conserves internal kinetic energy.

Internal Kinetic Energy

Internal kinetic energy is the sum of the kinetic energies of the objects in the system.

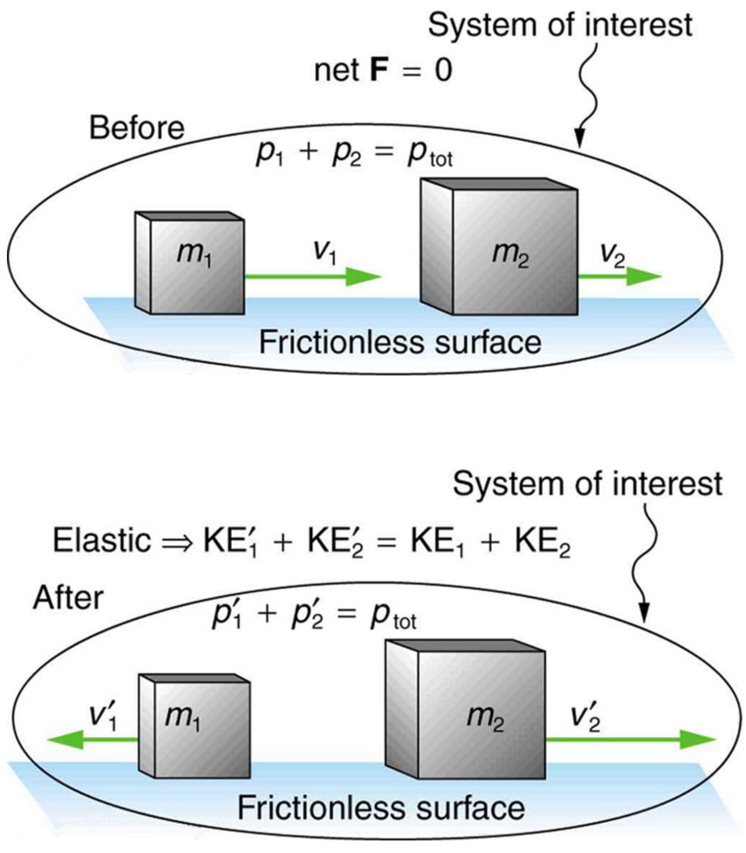

Figure 7.5.1: An elastic one-dimensional two-object collision. Momentum and internal kinetic energy are conserved.

Now, to solve problems involving one-dimensional elastic collisions between two objects we can use the equations for conservation of momentum and conservation of internal kinetic energy. First, the equation for conservation of momentum for two objects in a one-dimensional collision is

$$p_1 + p_2 = p'_1 + p'_2 \,(F_{net} = 0) \tag{7.5.1}$$

or

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m'_2 v'_2 \,(F_{net} = 0), \tag{7.5.2}$$

where the primes (') indicate values after the collision. By definition, an elastic collision conserves internal kinetic energy, and so the sum of kinetic energies before the collision equals the sum after the collision. Thus,

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2 \tag{7.5.3}$$

expresses the equation for conservation of internal kinetic energy in a one-dimensional collision.

Example 7.5.1: Calculating Velocities Following an Elastic Collision

Calculate the velocities of two objects following an elastic collision, given that

$$m_1 = 0.500\, kg,\ m_2 = 3.50\, kg,\ v_1 = 4.00\, m/s,\ and\, v_2 = 0, \tag{7.5.4}$$

Strategy and Concept

First, visualize what the initial conditions mean—a small object strikes a larger object that is initially at rest. This situation is slightly simpler than the situation shown in Figure where both objects are initially moving. We are asked to find two unknowns (the final velocities v'_1 and v'_2). To find two unknowns, we must use two independent equations. Because this collision is elastic, we can use the above two equations. Both can be simplified by the fact that object 2 is initially at rest, and thus $v_2 = 0$. Once we simplify these equations, we combine them algebraically to solve for the unknowns.

Solution

For this problem, note that $v_2 = 0$ and use conservation of momentum. Thus,

$$p_1 = p'_1 + p'_2 \tag{7.5.5}$$

or

$$m_1 v_1 = m_1 v'_1 + m_2 v'_2. \tag{7.5.6}$$

Using conservation of internal kinetic energy and that $v_2 = 0$,

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m v_1 v_1'^2 + \frac{1}{2} m v_2 v_2'^2. \tag{7.5.7}$$

Solving the first equation (momentum equation) for v'_2, we obtain

$$v'_2 = \frac{m_1}{m_2}(v_1 - v'_1). \tag{7.5.8}$$

Substituting this expression into the second equation (internal kinetic energy equation) eliminates the variable v'_2, leaving only v'_1 as an unknown (the algebra is left as an exercise for the reader). There are two solutions to any quadratic equation; in this example, they are as an unknown (the algebra is left as an exercise for the reader). There are two solutions to any quadratic equation; in this example, they are

$$v'_1 = 4.00\, m/s \tag{7.5.9}$$

and

$$v'_1 = -3.00\, m/s. \tag{7.5.10}$$

As noted when quadratic equations were encountered in earlier chapters, both solutions may or may not be meaningful. In this case, the first solution is the same as the initial condition. The first solution thus represents the situation before the collision and is discarded. The second solution ($v'_1 = -3.00\, m/s$) is negative, meaning that the first object bounces backward. When this negative value of v'_1 is used to find the velocity of the second object after the collision, we get

$$v'_2 = \frac{m_1}{m_2}(v_1 - v'_1) = \frac{0.500\, kg}{3.50\, kg}[4.00 - (-3.00)]\, m/s \tag{7.5.11}$$

or

$$v'_2 = 1.00\, m/s. \tag{7.5.12}$$

Discussion

The result of this example is intuitively reasonable. A small object strikes a larger one at rest and bounces backward. The larger one is knocked forward, but with a low speed. (This is like a compact car bouncing backward off a full-size SUV that is initially at rest.) As a check, try calculating the internal kinetic energy before and after the collision. You will see that the internal kinetic energy is unchanged at 4.00 J. Also check the total momentum before and after the collision; you will find it, too, is unchanged.

The equations for conservation of momentum and internal kinetic energy as written above can be used to describe any one-dimensional elastic collision of two objects. These equations can be extended to more objects if needed.$'\, 4.00\, \mathrm{m/s}$

Making Connections: Take-Home Investigation—Ice Cubes and Elastic Collision

Find a few ice cubes which are about the same size and a smooth kitchen tabletop or a table with a glass top. Place the ice cubes on the surface several centimeters away from each other. Flick one ice cube toward a stationary ice cube and observe the path and velocities of the ice cubes after the collision. Try to avoid edge-on collisions and collisions with rotating ice cubes. Have you created approximately elastic collisions? Explain the speeds and directions of the ice cubes using momentum.

PHET EXPLORATIONS: COLLISIONS LAB

Investigate collisions on an air hockey table. Set up your own experiments: vary the number of discs, masses and initial conditions. Is momentum conserved? Is kinetic energy conserved? Vary the elasticity and see what happens.

Figure 7.5.2: Collision Lab

Summary

- An elastic collision is one that conserves internal kinetic energy.
- Conservation of kinetic energy and momentum together allow the final velocities to be calculated in terms of initial velocities and masses in one dimensional two-body collisions.

Glossary

elastic collision

a collision that also conserves internal kinetic energy

internal kinetic energy

the sum of the kinetic energies of the objects in a system

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7.6: Inelastic Collisions in One Dimension

Learning Objectives

By the end of this section, you will be able to:

- Define inelastic collision.
- Explain perfectly inelastic collision.
- Apply an understanding of collisions to sports.
- Determine recoil velocity and loss in kinetic energy given mass and initial velocity.

We have seen that in an elastic collision, internal kinetic energy is conserved. An inelastic collision is one in which the internal kinetic energy changes (it is not conserved). This lack of conservation means that the forces between colliding objects may remove or add internal kinetic energy. Work done by internal forces may change the forms of energy within a system. For inelastic collisions, such as when colliding objects stick together, this internal work may transform some internal kinetic energy into heat transfer. Or it may convert stored energy into internal kinetic energy, such as when exploding bolts separate a satellite from its launch vehicle.

Definition: Inelastic Collisions

An inelastic collision is one in which the internal kinetic energy changes (it is not conserved).

Figure $7.6.1$ shows an example of an inelastic collision. Two objects that have equal masses head toward one another at equal speeds and then stick together. Their total internal kinetic energy is initially

$$\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2. \tag{7.6.1}$$

The two objects come to rest after sticking together, conserving momentum. But the internal kinetic energy is zero after the collision. A collision in which the objects stick together is sometimes called a perfectly inelastic collision because it reduces internal kinetic energy more than does any other type of inelastic collision. In fact, such a collision reduces internal kinetic energy to the minimum it can have while still conserving momentum.

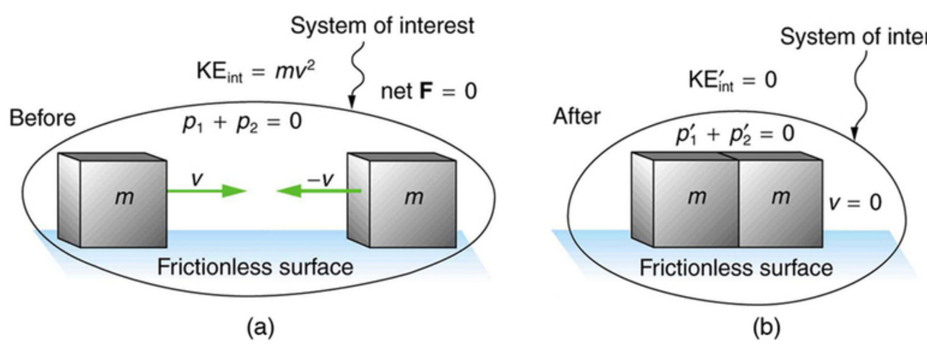

Figure $7.6.1$: An inelastic one-dimensional two-object collision. Momentum is conserved, but internal kinetic energy is not conserved. (a) Two objects of equal mass initially head directly toward one another at the same speed. (b) The objects stick together (a perfectly inelastic collision), and so their final velocity is zero. The internal kinetic energy of the system changes in any inelastic collision and is reduced to zero in this example.

Definition: Perfectly Inelastic Collisions

A collision in which the objects stick together is sometimes called "perfectly inelastic."

Example $7.6.1$: Calculating Velocity and Change in Kinetic Energy - Inelastic Collision of a Puck and a Goalie

a. Find the recoil velocity of a 70.0-kg ice hockey goalie, originally at rest, who catches a 0.150-kg hockey puck slapped at him at a velocity of 35.0 m/s.
b. How much kinetic energy is lost during the collision? Assume friction between the ice and the puck-goalie system is negligible (Figure $7.6.2$)

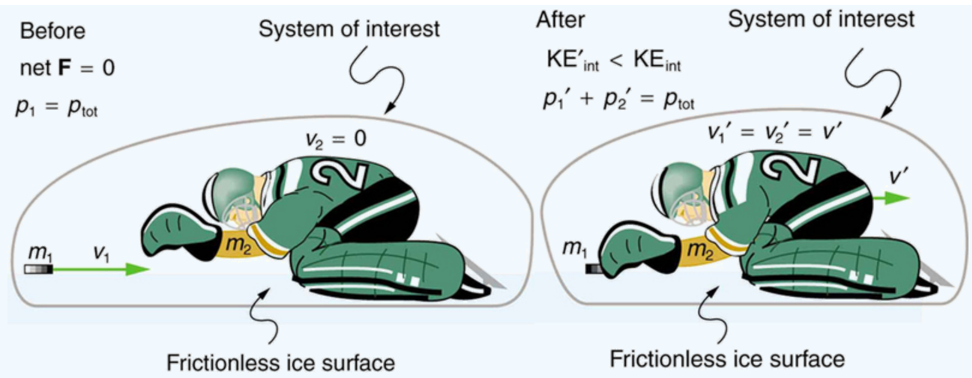

Figure 7.6.2: An ice hockey goalie catches a hockey puck and recoils backward. The initial kinetic energy of the puck is almost entirely converted to thermal energy and sound in this inelastic collision.

Strategy

Momentum is conserved because the net external force on the puck-goalie system is zero. We can thus use conservation of momentum to find the final velocity of the puck and goalie system. Note that the initial velocity of the goalie is zero and that the final velocity of the puck and goalie are the same. Once the final velocity is found, the kinetic energies can be calculated before and after the collision and compared as requested.

Solution for (a)

Momentum is conserved because the net external force on the puck-goalie system is zero.

Conservation of momentum is

$$p_1 + p_2 = p'_1 + p'_2$$

or

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2.$$

Because the goalie is initially at rest, we know $v_2 = 0$. Because the goalie catches the puck, the final velocities are equal, or $v'_1 = v'_2 = v'$. Thus, the conservation of momentum equation simplifies to

$$m_1 v_1 = (m_1 + m_2) v'.$$

Solving for v' yields

$$v' = \frac{m_1}{m_1 + m_2} v_1.$$

Entering known values in this equation, we get

$$\begin{aligned} v' &= \left(\frac{0.150\, kg}{70.0\, kg + 0.150\, kg} \right) (35.0\, m/s) \\ &= 7.48 \times 10^{-2} m/s)^2 \\ &= 0.196\, J. \end{aligned}$$

The change in internal kinetic energy is thus

$$\begin{aligned} KE'_{int} - KE_{int} &= 0.196\, J - 91.9\, J \\ &= -91.7\, J \end{aligned}$$

where the minus sign indicates that the energy was lost.

Discussion for (b)

Nearly all of the initial internal kinetic energy is lost in this perfectly inelastic collision. KE_{int} is mostly converted to thermal energy and sound.

During some collisions, the objects do not stick together and less of the internal kinetic energy is removed—such as happens in most automobile accidents. Alternatively, stored energy may be converted into internal kinetic energy during a collision. Figure $7.6.3$ shows a one-dimensional example in which two carts on an air track collide, releasing potential energy from a compressed spring. Example $7.6.2$ deals with data from such a collision.

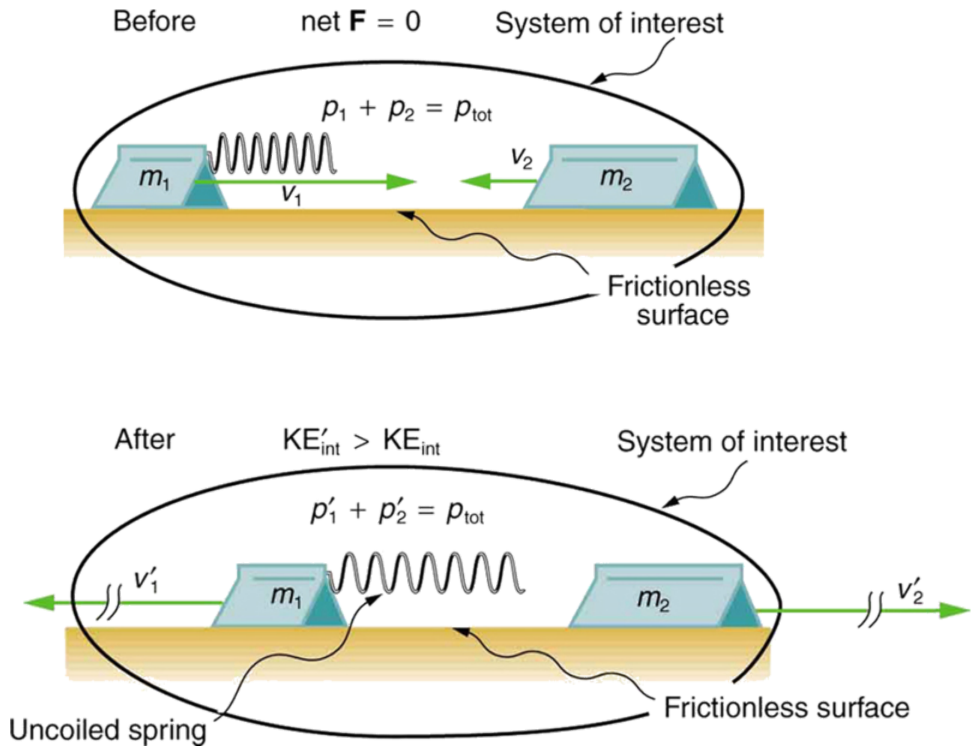

Figure $7.6.3$: An air track is nearly frictionless, so that momentum is conserved. Motion is one-dimensional. In this collision, examined in Example $7.6.2$, the potential energy of a compressed spring is released during the collision and is converted to internal kinetic energy.

Collisions are particularly important in sports and the sporting and leisure industry utilizes elastic and inelastic collisions. Let us look briefly at tennis. Recall that in a collision, it is momentum and not force that is important. So, a heavier tennis racquet will have the advantage over a lighter one. This conclusion also holds true for other sports—a lightweight bat (such as a softball bat) cannot hit a hardball very far.

The location of the impact of the tennis ball on the racquet is also important, as is the part of the stroke during which the impact occurs. A smooth motion results in the maximizing of the velocity of the ball after impact and reduces sports injuries such as tennis elbow. A tennis player tries to hit the ball on the "sweet spot" on the racquet, where the vibration and impact are minimized and the ball is able to be given more velocity. Sports science and technologies also use physics concepts such as momentum and rotational motion and vibrations.

Take-Home Experiment—Bouncing of Tennis Ball

1. Find a racquet (a tennis, badminton, or other racquet will do). Place the racquet on the floor and stand on the handle. Drop a tennis ball on the strings from a measured height. Measure how high the ball bounces. Now ask a friend to hold the racquet firmly by the handle and drop a tennis ball from the same measured height above the racquet. Measure how high the ball bounces and observe what happens to your friend's hand during the collision. Explain your observations and measurements.
2. The coefficient of restitution (c) is a measure of the elasticity of a collision between a ball and an object, and is defined as the ratio of the speeds after and before the collision. A perfectly elastic collision has a c of 1. For a ball bouncing off the floor (or a racquet on the floor), c can be shown to be $c = (h/H)^{1/2}$ where h is the height to which the ball bounces and H is the height from which the ball is dropped. Determine c for the cases in Part 1 and for the case of a tennis ball bouncing off a concrete or wooden floor ($c = 0.85$ for new tennis balls used on a tennis court).

Example 7.6.2: Calculating Final Velocity and Energy Release - Two Carts Collide

In the collision pictured in Figure $7.6.3$, two carts collide inelastically. Cart 1 (denoted m_1 carries a spring which is initially compressed. During the collision, the spring releases its potential energy and converts it to internal kinetic energy. The mass of cart 1 and the spring is 0.350 kg, and the cart and the spring together have an initial velocity of $-0.500\,m/s$. After the collision, cart 1 is observed to recoil with a velocity of $-4.00\,m/s$.

a. What is the final velocity of cart 2?
b. How much energy was released by the spring (assuming all of it was converted into internal kinetic energy)?

Strategy

We can use conservation of momentum to find the final velocity of cart 2, because $F_{net} = 0$ (the track is frictionless and the force of the spring is internal). Once this velocity is determined, we can compare the internal kinetic energy before and after the collision to see how much energy was released by the spring.

Solution for (a)

As before, the equation for conservation of momentum in a two-object system is

$$m_1v_1 + m_2v_2 = m_1v'_1 + m_2v'_2.$$

The only unknown in this equation is v'_2. Solving for v'_2 and substituting known values into the previous equation fields

$$\begin{aligned} v'_2 &= \frac{m_1v_1 + m_2v_2 - m_1v'_1}{m_2} \\ &= \frac{0.350\,kg)(2.00\,m/s) + (0.500\,kg)(-0.500\,m/s)}{0.500\,kg} - \frac{(0.350\,kg)(-4.00\,m/s)}{0.500\,kg} \\ &= 3.70\,m/s. \end{aligned}$$

Solution for (b)

The internal kinetic energy before the collision is

$$\begin{aligned} KE_{int} &= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \\ &= \frac{1}{2}(0.350\,kg)(2.00\,m/s)^2 + \frac{1}{2}(0.500\,kg)(-0.500\,m/s)^2 \\ &= 0.763\,J. \end{aligned}$$

After the collision, the internal kinetic energy is

$$\begin{aligned} KE'_{int} &= \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2 \\ &= \frac{1}{2}(0.350\,kg)(-4.00\,m/s)^2 + \frac{1}{2}(0.500\,kg)(0.370\,m/s)^2 \\ &= 6.22\,J. \end{aligned}$$

The change in internal kinetic energy is thus

$$\begin{aligned} KE' - KE &= 6.22\,J - 0.763\,J \\ &= 5.46\,J. \end{aligned}$$

Discussion

The final velocity of cart 2 is large and positive, meaning that it is moving to the right after the collision. The internal kinetic energy in this collision increases by 5.46 J. That energy was released by the spring.

Summary

- An inelastic collision is one in which the internal kinetic energy changes (it is not conserved).
- A collision in which the objects stick together is sometimes called perfectly inelastic because it reduces internal kinetic energy more than does any other type of inelastic collision.
- Sports science and technologies also use physics concepts such as momentum and rotational motion and vibrations.

Glossary

inelastic collision

a collision in which internal kinetic energy is not conserved

perfectly inelastic collision

a collision in which the colliding objects stick together

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7.7: Collisions of Point Masses in Two Dimensions

Learning Objectives

By the end of this section, you will be able to:

- Discuss two dimensional collisions as an extension of one dimensional analysis.
- Define point masses.
- Derive an expression for conservation of momentum along *x*-axis and *y*-axis.
- Describe elastic collisions of two objects with equal mass.
- Determine the magnitude and direction of the final velocity given initial velocity, and scattering angle.

In the previous two sections, we considered only one-dimensional collisions; during such collisions, the incoming and outgoing velocities are all along the same line. But what about collisions, such as those between billiard balls, in which objects scatter to the side? These are two-dimensional collisions, and we shall see that their study is an extension of the one-dimensional analysis already presented. The approach taken (similar to the approach in discussing two-dimensional kinematics and dynamics) is to choose a convenient coordinate system and resolve the motion into components along perpendicular axes. Resolving the motion yields a pair of one-dimensional problems to be solved simultaneously.

One complication arising in two-dimensional collisions is that the objects might rotate before or after their collision. For example, if two ice skaters hook arms as they pass by one another, they will spin in circles. We will not consider such rotation until later, and so for now we arrange things so that no rotation is possible. To avoid rotation, we consider only the scattering of point masses—that is, structureless particles that cannot rotate or spin.

We start by assuming that $F_{net}=0,$ so that momentum p is conserved. The simplest collision is one in which one of the particles is initially at rest. (See Figure.) The best choice for a coordinate system is one with an axis parallel to the velocity of the incoming particle, as shown in Figure. Because momentum is conserved, the components of momentum along the $x-$ and $y-$ axes (p_x and p_y) will also be conserved, but with the chosen coordinate system, p_y is initially zero and p_x is the momentum of the incoming particle. Both facts simplify the analysis. (Even with the simplifying assumptions of point masses, one particle initially at rest, and a convenient coordinate system, we still gain new insights into nature from the analysis of two-dimensional collisions.)

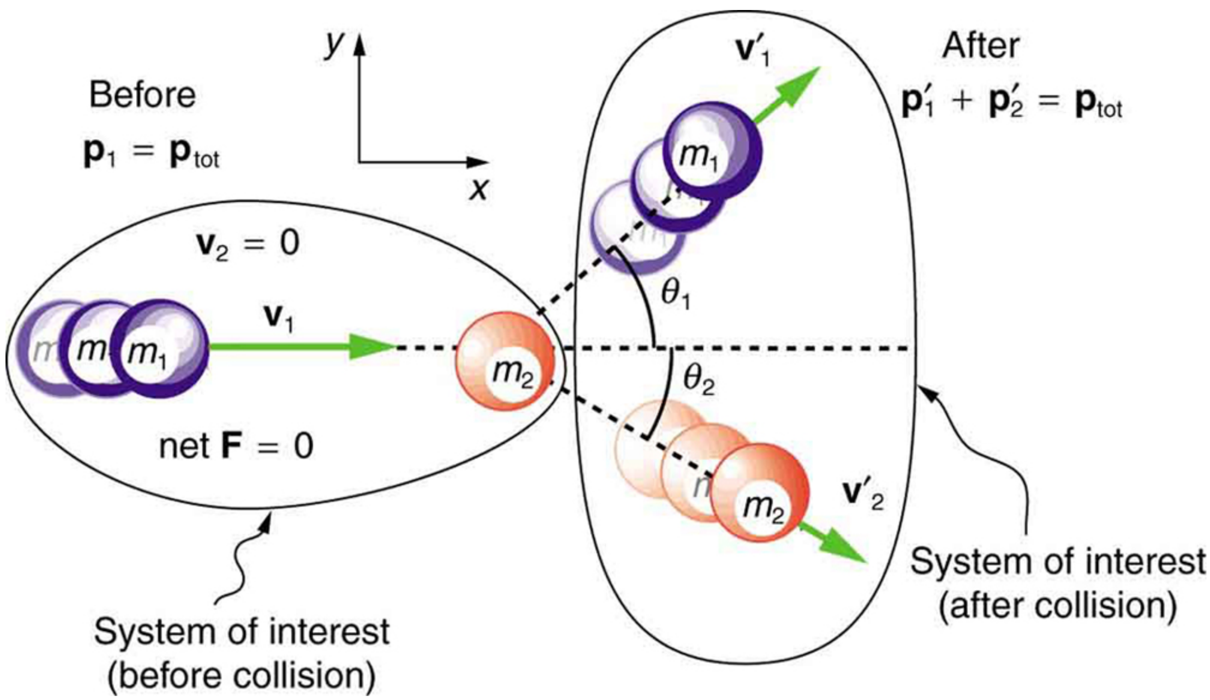

Figure 7.7.1: A two-dimensional collision with the coordinate system chosen so that m_2 is initially at rest and v_1 is parallel to the x-axis. This coordinate system is sometimes called the laboratory coordinate system, because many scattering experiments have a target that is stationary in the laboratory, while particles are scattered from it to determine the particles that make-up the target and how they are bound together. The particles may not be observed directly, but their initial and final velocities are.

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Along the x-axis, the equation for conservation of momentum is

$$p_{1x}+p_{2x}=p'_{1x}+p'_{2x}. \tag{7.7.1}$$

Where the subscripts denote the particles and axes and the primes denote the situation after the collision. In terms of masses and velocities, this equation is

$$m_1v_1+m_2v_{2x}=m_1v'_{1x}+m_2v'_{2x}. \tag{7.7.2}$$

But because particle 2 is initially at rest, this equation becomes

$$m_1v_1=m_1v'_{1x}+m_2v'_{2x}. \tag{7.7.3}$$

The components of the velocities along the x-axis have the form $v\,cos\,\theta$. Because particle 1 initially moves along the x-axis, we find $v_{1x}=v_1$.

Conservation of momentum along the x-axis gives the following equation

$$m_1v_1=m_1v'_1\,cos\,\theta_1+m_2v'_2\,cos\,\theta_2, \tag{7.7.4}$$

where θ_1 and θ_2 are as shown in Figure.

Conservation of Momentum Along the x-axis

$$m_1v_1=m_1v'_1\,cos\,\theta_1+m_2v'_2\,cos\,\theta_2 \tag{7.7.5}$$

Along the y-axis, the equation for conservation of momentum is

$$p_{1y}+p_{2y}=p'_{1y}+p'_{2y}, \tag{7.7.6}$$

or

$$m_1v_1+m_2v_{2y}=m_1v'_{1y}+m_2v'_{2y}. \tag{7.7.7}$$

But v_{1y} is zero, because particle 1 initially moves along the x-axis. Because particle 2 is initially at rest, v_{2y} is also zero. The equation for conservation of momentum along the y-axis becomes

$$0=m_1v'_{1y}+m_2v'_{2y}. \tag{7.7.8}$$

The components of the velocities along the y-axis have the form $v\,sin\,\theta$.

Thus, conservation of momentum along the y-axis gives the following equation:

$$0=m_1v'_{1y}\,sin\,\theta_1+m_2v'_{2y}\,sin\,\theta_2. \tag{7.7.9}$$

Conservation of Momentum Along y-axis

$$0=m_1v'_{1y}\,sin\,\theta_1+m_2v'_{2y}\,sin\,\theta_2. \tag{7.7.10}$$

The equations of conservation of momentum along the x-axis and y-axis are very useful in analyzing two-dimensional collisions of particles, where one is originally stationary (a common laboratory situation). But two equations can only be used to find two unknowns, and so other data may be necessary when collision experiments are used to explore nature at the subatomic level.

Example 7.7.1: Determining the Final Velocity of an Unseen Object from the Scattering of Another Object

Suppose the following experiment is performed. A 0.250-kg object (m_1) is slid on a frictionless surface into a dark room, where it strikes an initially stationary object with mass of 0.400 kg (m_2). The 0.250-kg object emerges from the room at an angle of 45^o with its incoming direction.The speed of the 0.250-kg object is originally 2.00 m/s and is 1.50 m/s after the collision. Calculate the magnitude and direction of the velocity $(v'_2$ and $\theta_2)$ of the 0.400-kg object after the collision.

Strategy

Momentum is conserved because the surface is frictionless. The coordinate system shown in Figure is one in which m_2 is originally at rest and the initial velocity is parallel to the x-axis, so that conservation of momentum along the x- and y-axes is applicable. Everything is known in these equations except v'_2 and θ_2, which are precisely the quantities we wish to find. We can find two unknowns because we have two independent equations: the equations describing the conservation of momentum in the x- and y-directions.

Solution

Solving $m_1 v_1 = m_1 v'_1 \, cos\,\theta_1 + m_2 v'_2 \, cos\,\theta_2$ for $v'_2 \, cos\,\theta_2$ and $0 = m_1 v'_{1y} \, sin\,\theta_1 + m_2 v'_{2y} \, sin\,\theta_2$ for $v'_2 \, sin\,\theta_2$ and taking the ratio yields an equation (in which θ_2 is the only unknown quantity. Applying the identity $\left(tan\,\theta = \frac{sin\,\theta}{cos\,\theta}\right)$, we obtain

$$tan\,\theta_2 = \frac{v'_1 \, sin\,\theta_1}{v'_1 \, cos\,\theta_1 - v_1}. \tag{7.7.11}$$

Entering known values into the previous equation gives

$$tan\,\theta_2 = \frac{(1.50\,m/s)(0.7071)}{(1.50\,m/s)(0.7071) - 2.00\,m/s} = -1.129. \tag{7.7.12}$$

Thus,

$$\theta_2 = tan^{-1} 9 - 1.129) = 311.5^o \approx 312^o. \tag{7.7.13}$$

Angles are defined as positive in the counter clockwise direction, so this angle indicates that m_2 is scattered to the right in Figure, as expected (this angle is in the fourth quadrant). Either equation for the x- or y-axis can now be used to solve for v_2, but the latter equation is easiest because it has fewer terms.

$$v'_2 = -\left(\frac{0.250\,kg}{0.400\,kg}\right)(1.50\,m/s)\left(\frac{0.7071}{-0.7485}\right). \tag{7.7.14}$$

Thus,

$$v'_2 = 0.886\,m/s. \tag{7.7.15}$$

Discussion

It is instructive to calculate the internal kinetic energy of this two-object system before and after the collision. (This calculation is left as an end-of-chapter problem.) If you do this calculation, you will find that the internal kinetic energy is less after the collision, and so the collision is inelastic. This type of result makes a physicist want to explore the system further.

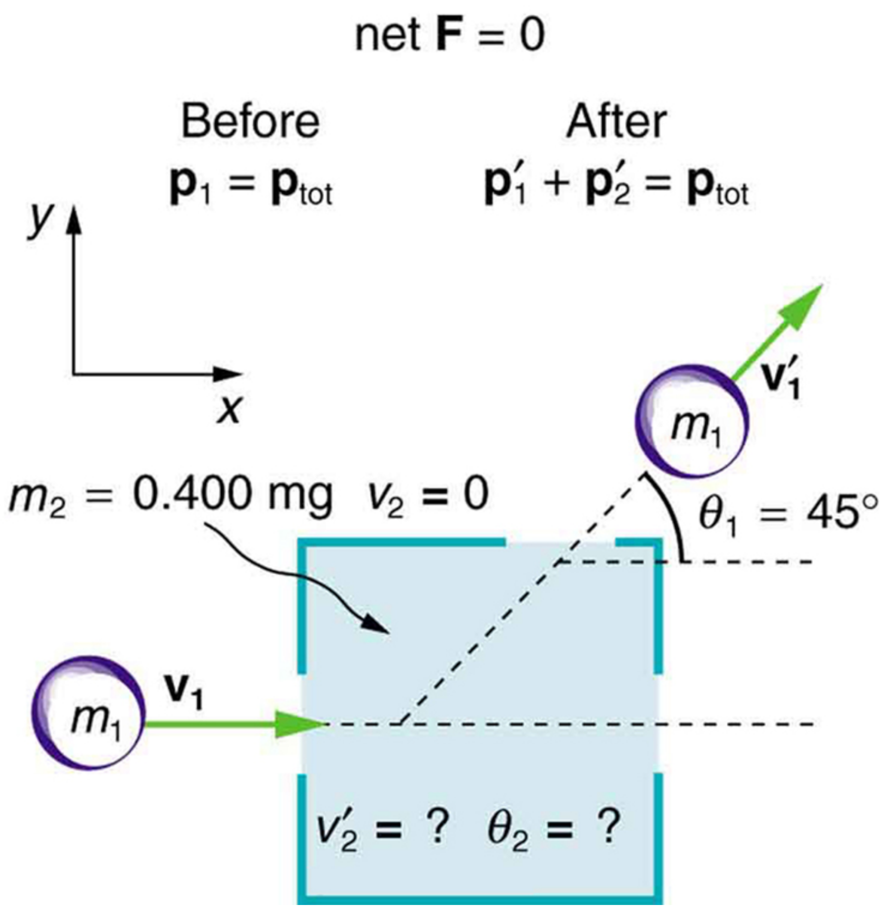

Figure 7.7.2: A collision taking place in a dark room is explored in Example. The incoming object m_1 is scattered by an initially stationary object. Only the stationary object's mass m_2 is known. By measuring the angle and speed at which m_1 emerges from the room, it is possible to calculate the magnitude and direction of the initially stationary object's velocity after the collision.

Elastic Collisions of Two Objects with Equal Mass

Some interesting situations arise when the two colliding objects have equal mass and the collision is elastic. This situation is nearly the case with colliding billiard balls, and precisely the case with some subatomic particle collisions. We can thus get a mental image of a collision of subatomic particles by thinking about billiards (or pool). (Refer to Figure for masses and angles.) First, an elastic collision conserves internal kinetic energy. Again, let us assume object 2 m_2 is initially at rest. Then, the internal kinetic energy before and after the collision of two objects that have equal masses is

$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_1'^2 + \frac{1}{2}mv_2'^2. \tag{7.7.16}$$

Because the masses are equal, $m_1 = m_2 = m$. Algebraic manipulation (left to the reader) of conservation of momentum in the x- and y-directions can show that

$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_1'^2 + \frac{1}{2}mv_2'^2 + mv_1'v_2'\,cos(\theta_1 - \theta_2). \tag{7.7.17}$$

(Remember that θ_2 is negative here.) The two preceding equations can both be true only if

$$mv_1'v_2'\,cos(\theta_1 - \theta_2) = 0. \tag{7.7.18}$$

There are three ways that this term can be zero. They are

$v_1' = 0$: head-on collision; incoming ball stops;

$v_2' = 0$: no collision; incoming ball continues unaffected

All three of these ways are familiar occurrences in billiards and pool, although most of us try to avoid the second. If you play enough pool, you will notice that the angle between the balls is very close to 90º after the collision, although it will vary from this value if a great deal of spin is placed on the ball. (Large spin carries in extra energy and a quantity called *angular momentum*, which must also be conserved.) The assumption that the scattering of billiard balls is elastic is reasonable based on the correctness of the three results it produces. This assumption also implies that, to a good approximation, momentum is conserved for the two-ball system in billiards and pool. The problems below explore these and other characteristics of two-dimensional collisions.

CONNECTIONS TO NUCLEAR AND PARTICLE PHYSICS

Two-dimensional collision experiments have revealed much of what we know about subatomic particles, as we shall see in Medical Applications of Nuclear Physics and Particle Physics. Ernest Rutherford, for example, discovered the nature of the atomic nucleus from such experiments.

Summary

- The approach to two-dimensional collisions is to choose a convenient coordinate system and break the motion into components along perpendicular axes. Choose a coordinate system with the x-axis parallel to the velocity of the incoming particle.
- Two-dimensional collisions of point masses where mass 2 is initially at rest conserve momentum along the initial direction of mass 1 (the x-axis), stated by $m_1v_1 = m_1v'_1 cos\theta_1 + m_2v'_2 cos\theta_2$ and along the direction perpendicular to the initial direction (the y-axis) stated by $0 = m_1v'_1y + m_2v'2_y$.
- The internal kinetic before and after the collision of two objects that have equal masses is

$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv'^2_1 + \frac{1}{2}mv'^2_2 + mv'_1v'_2 cos(\theta_1 - \theta_2) \quad .$$

- Point masses are structureless particles that cannot spin.

Glossary

point masses

structureless particles with no rotation or spin

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7.8: Introduction to Rocket Propulsion

Learning Objectives

By the end of this section, you will be able to:

- State Newton's third law of motion.
- Explain the principle involved in propulsion of rockets and jet engines.
- Derive an expression for the acceleration of the rocket.
- Discuss the factors that affect the rocket's acceleration.
- Describe the function of a space shuttle.

Rockets range in size from fireworks so small that ordinary people use them to immense Saturn Vs that once propelled massive payloads toward the Moon. The propulsion of all rockets, jet engines, deflating balloons, and even squids and octopuses is explained by the same physical principle—Newton's third law of motion. Matter is forcefully ejected from a system, producing an equal and opposite reaction on what remains. Another common example is the recoil of a gun. The gun exerts a force on a bullet to accelerate it and consequently experiences an equal and opposite force, causing the gun's recoil or kick.

Making Connections: Take-Home Experiment —Propulsion of a Balloon

- Hold a balloon and fill it with air. Then, let the balloon go. In which direction does the air come out of the balloon and in which direction does the balloon get propelled? If you fill the balloon with water and then let the balloon go, does the balloon's direction change? Explain your answer.

Figure $7.8.1$ shows a rocket accelerating straight up. In part (a), the rocket has a mass m and a velocity v relative to Earth, and hence a momentum mv In part (b), a time Δt has elapsed in which the rocket has ejected a mass Δm of hot gas at a velocity v_e relative to the rocket. The remainder of the mass $(m-\Delta m)$ now has a greater velocity $(v+\Delta v)$. The momentum of the entire system (rocket plus expelled gas) has actually decreased because the force of gravity has acted for a time Δt, producing a negative impulse $\Delta p=-mg\Delta t$. (Remember that impulse is the net external force on a system multiplied by the time it acts, and it equals the change in momentum of the system.) So, the center of mass of the system is in free fall but, by rapidly expelling mass, part of the system can accelerate upward. It is a commonly held misconception that the rocket exhaust pushes on the ground. If we consider thrust; that is, the force exerted on the rocket by the exhaust gases, then a rocket's thrust is greater in outer space than in the atmosphere or on the launch pad. In fact, gases are easier to expel into a vacuum.By calculating the change in momentum for the entire system over Δt, and equating this change to the impulse, the following expression can be shown to be a good approximation for the acceleration of the rocket.

$$a=\frac{v_e}{m}\frac{\Delta m}{\Delta t}-g, \tag{7.8.1}$$

where a is the acceleration of the rocket, v_e is the escape velocity, m is the mass of the rocket, $Deltam$ is the mass of the ejected gas, and Δt is the time in which the gas is ejected.

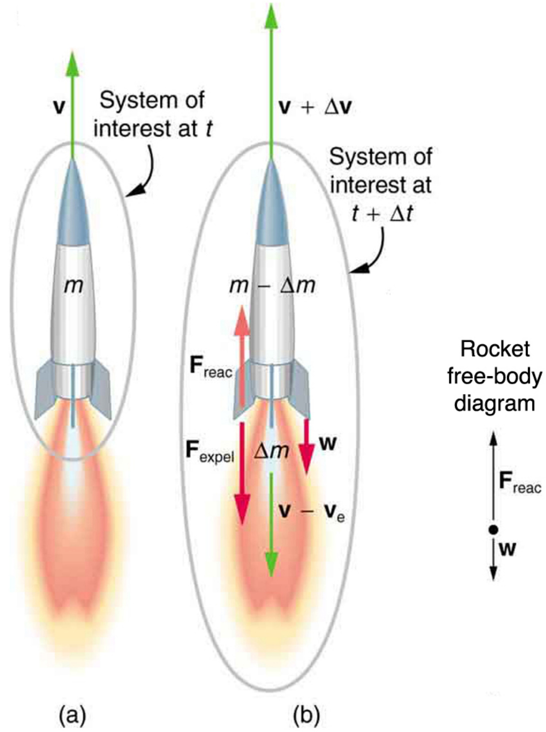

Figure 7.8.1: (a) This rocket has a mass m and an upward velocity v. The net external force on the system is $-mg$, if air resistance is neglected. (b) A time Δt later the system has two main parts, the ejected gas and the remainder of the rocket. The reaction force on the rocket is what overcomes the gravitational force and accelerates it upward.

A rocket's acceleration depends on three major factors, consistent with the equation for acceleration of a rocket **.** First, the greater the exhaust velocity of the gases relative to the rocket, v_e, the greater the acceleration is. The practical limit for v_e is about $2.5 \times 10^3\ m/s$ for conventional (non-nuclear) hot-gas propulsion systems. The second factor is the rate at which mass is ejected from the rocket. This is the factor $(\Delta m/\Delta t)v_e$, with units of newtons, is called "thrust." The faster the rocket burns its fuel, the greater its thrust, and the greater its acceleration. The third factor is the mass m of the rocket. The smaller the mass is (all other factors being the same), the greater the acceleration. The rocket mass m decreases dramatically during flight because most of the rocket is fuel to begin with, so that acceleration increases continuously, reaching a maximum just before the fuel is exhausted.

Factors Affecting a Rocket's Acceleration

- The greater the exhaust velocity v_e of the gases relative to the rocket, the greater the acceleration.
- The faster the rocket burns its fuel, the greater its acceleration.
- The smaller the rocket's mass (all other factors being the same), the greater the acceleration.

Example 7.8.1: Calculating Acceleration: Initial Acceleration of a Moon Launch

A Saturn V's mass at liftoff was $2.80 \times 10^6\ kg$, its fuel-burn rate was $1.40 \times 10^4 \times kg/s$, and the exhaust velocity was $2.40 \times 10^3 m/s$. Calculate its initial acceleration.

Strategy

This problem is a straightforward application of the expression for acceleration because is the unknown and all of the terms on the right side of the equation are given.

Solution

Substituting the given values into the equation for acceleration yields

$$\begin{aligned} a &= \frac{v_e}{m}\frac{\Delta m}{\delta t} - g \\ &= \frac{2.40 \times 10^3\, m/s}{2.80 \times 10^6\, kg}(1.40 \times 10^4\, kg/s) - 9.8\, m/s^2 \\ &= 2.20\, m/s^2. \end{aligned}$$

Discussion

This value is fairly small, even for an initial acceleration. The acceleration does increase steadily as the rocket burns fuel, because m decreases while v_e and $\frac{\Delta m}{\Delta t}$ remain constant. Knowing this acceleration and the mass of the rocket, you can show that the thrust of the engines was $3.36 \times 10^7\, N$.

To achieve the high speeds needed to hop continents, obtain orbit, or escape Earth's gravity altogether, the mass of the rocket other than fuel must be as small as possible. It can be shown that, in the absence of air resistance and neglecting gravity, the final velocity of a one-stage rocket initially at rest is

$$v = v_e\, ln\frac{m_0}{m_r}, \tag{7.8.2}$$

where $ln(m_0/m_r)$ is the natural logarithm of the ratio of the initial mass of the rocket (m_0) to what is left (m_r) after all of the fuel is exhausted. (Note that v is actually the change in velocity, so the equation can be used for any segment of the flight. If we start from rest, the change in velocity equals the final velocity.) For example, let us calculate the mass ratio needed to escape Earth's gravity starting from rest, given that the escape velocity from Earth is about $11.2 \times 10^3\, m/s$, and assuming an exhaust velocity $v_e = 2.5 \times 10^3\, m/s$.

$$ln\frac{m_0}{m_r} = \frac{v}{v_e} = \frac{11.2 \times 10^3\, m/s}{2.5 \times 10^3\, m/s} = 4.48 \tag{7.8.3}$$

Solving for m_0/m_r gives

$$\frac{m_0}{m_r} = e^{4.48} = 88. \tag{7.8.4}$$

Thus, the mass of the rocket is

$$m_r = \frac{m_0}{88}. \tag{7.8.5}$$

This result means that only $1/88$ of the mass is left when the fuel is burnt, and $87/88$ of the initial mass was fuel. Expressed as percentages, 98.9% of the rocket is fuel, while payload, engines, fuel tanks, and other components make up only 1.10%. Taking air resistance and gravitational force into account, the mass m_r remaining can only be about $m_0/180$. It is difficult to build a rocket in which the fuel has a mass 180 times everything else. The solution is multistage rockets. Each stage only needs to achieve part of the final velocity and is discarded after it burns its fuel. The result is that each successive stage can have smaller engines and more payload relative to its fuel. Once out of the atmosphere, the ratio of payload to fuel becomes more favorable, too.The space shuttle was an attempt at an economical vehicle with some reusable parts, such as the solid fuel boosters and the craft itself. (See Figure) The shuttle's need to be operated by humans, however, made it at least as costly for launching satellites as expendable, unmanned rockets. Ideally, the shuttle would only have been used when human activities were required for the success of a mission, such as the repair of the Hubble space telescope. Rockets with satellites can also be launched from airplanes. Using airplanes has the double advantage that the initial velocity is significantly above zero and a rocket can avoid most of the atmosphere's resistance.

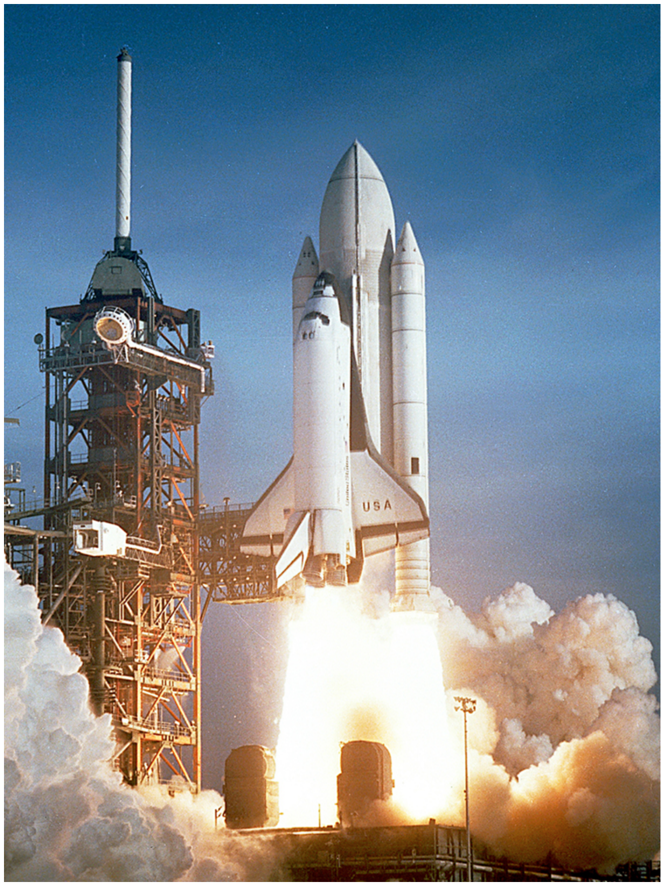

Figure 7.8.2:The space shuttle had a number of reusable parts. Solid fuel boosters on either side were recovered and refueled after each flight, and the entire orbiter returned to Earth for use in subsequent flights. The large liquid fuel tank was expended. The space shuttle was a complex assemblage of technologies, employing both solid and liquid fuel and pioneering ceramic tiles as reentry heat shields. As a result, it permitted multiple launches as opposed to single-use rockets. (credit: NASA)

Phet Explorations: Lunar Lander

Can you avoid the boulder field and land safely, just before your fuel runs out, as Neil Armstrong did in 1969? Our version of this classic video game accurately simulates the real motion of the lunar lander with the correct mass, thrust, fuel consumption rate, and lunar gravity. The real lunar lander is very hard to control.

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PhET Interactive Simulation

Figure 7.8.3: Lunar Lander

Summary

This page connects force to thrust and weight of a rocket and foxes on Newton's second law: $a = \frac{F}{mass}$.

- Newton’s third law of motion states that to every action, there is an equal and opposite reaction.
- Acceleration of a rocket is $a = \frac{v_e}{m}\frac{\Delta m}{\Delta t} - g$.
- A rocket’s acceleration depends on three main factors. They are

1. The greater the exhaust velocity of the gases, the greater the acceleration.
2. The faster the rocket burns its fuel, the greater its acceleration.
3. The smaller the rocket's mass, the greater the acceleration.

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7.E: Linear Momentum and Collisions (Exercises)

Conceptual Questions

8.1: Linear Momentum and Force

1. An object that has a small mass and an object that has a large mass have the same momentum. Which object has the largest kinetic energy?

2. An object that has a small mass and an object that has a large mass have the same kinetic energy. Which mass has the largest momentum?

3. *Professional Application*

4. Football coaches advise players to block, hit, and tackle with their feet on the ground rather than by leaping through the air. Using the concepts of momentum, work, and energy, explain how a football player can be more effective with his feet on the ground.

5. How can a small force impart the same momentum to an object as a large force?

8.2: Impulse

6. *Professional Application*

Explain in terms of impulse how padding reduces forces in a collision. State this in terms of a real example, such as the advantages of a carpeted vs. tile floor for a day care center.

7. While jumping on a trampoline, sometimes you land on your back and other times on your feet. In which case can you reach a greater height and why?

8. *Professional Application*

Tennis racquets have “sweet spots.” If the ball hits a sweet spot then the player's arm is not jarred as much as it would be otherwise. Explain why this is the case.

8.3: Conservation of Momentum

9. *Professional Application*

If you dive into water, you reach greater depths than if you do a belly flop. Explain this difference in depth using the concept of conservation of energy. Explain this difference in depth using what you have learned in this chapter.

10. Under what circumstances is momentum conserved?

11. Can momentum be conserved for a system if there are external forces acting on the system? If so, under what conditions? If not, why not?

12. Momentum for a system can be conserved in one direction while not being conserved in another. What is the angle between the directions? Give an example.

13. *Professional Application*

Explain in terms of momentum and Newton’s laws how a car’s air resistance is due in part to the fact that it pushes air in its direction of motion.

14. Can objects in a system have momentum while the momentum of the system is zero? Explain your answer.

15. Must the total energy of a system be conserved whenever its momentum is conserved? Explain why or why not.

8.4: Elastic Collisions in One Dimension

16. What is an elastic collision?

8.5: Inelastic Collisions in One Dimension

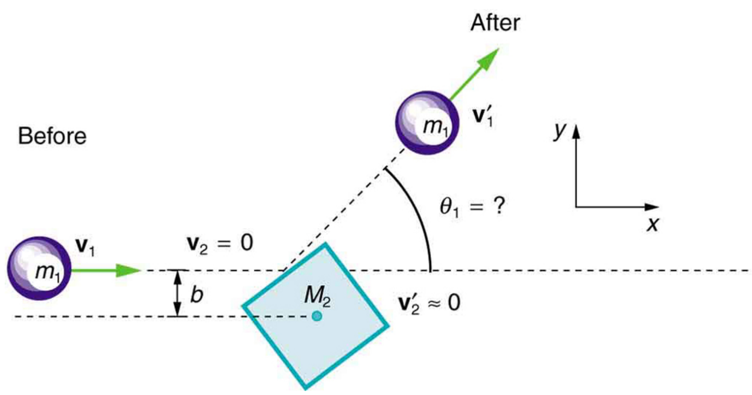
17. What is an inelastic collision? What is a perfectly inelastic collision?

18. Mixed-pair ice skaters performing in a show are standing motionless at arms length just before starting a routine. They reach out, clasp hands, and pull themselves together by only using their arms. Assuming there is no friction between the blades of their skates and the ice, what is their velocity after their bodies meet?

19. A small pickup truck that has a camper shell slowly coasts toward a red light with negligible friction. Two dogs in the back of the truck are moving and making various inelastic collisions with each other and the walls. What is the effect of the dogs on the motion of the center of mass of the system (truck plus entire load)? What is their effect on the motion of the truck?

8.6: Collisions of Point Masses in Two Dimensions

19. Figure shows a cube at rest and a small object heading toward it. (a) Describe the directions (angle θ_1) at which the small object can emerge after colliding elastically with the cube. How does θ_1 depend on b, the so-called impact parameter? Ignore any effects that might be due to rotation after the collision, and assume that the cube is much more massive than the small object. (b) Answer the same questions if the small object instead collides with a massive sphere.



A small object approaches a collision with a much more massive cube, after which its velocity has the direction θ_1. The angles at which the small object can be scattered are determined by the shape of the object it strikes and the impact parameter b.

8.7: Introduction to Rocket Propulsion

20. *Professional Application*

Suppose a fireworks shell explodes, breaking into three large pieces for which air resistance is negligible. How is the motion of the center of mass affected by the explosion? How would it be affected if the pieces experienced significantly more air resistance than the intact shell?

21. *Professional Application*

During a visit to the International Space Station, an astronaut was positioned motionless in the center of the station, out of reach of any solid object on which he could exert a force. Suggest a method by which he could move himself away from this position, and explain the physics involved.

22. *Professional Application*

It is possible for the velocity of a rocket to be greater than the exhaust velocity of the gases it ejects. When that is the case, the gas velocity and gas momentum are in the same direction as that of the rocket. How is the rocket still able to obtain thrust by ejecting the gases?

Problems & Exercises

8.1: Linear Momentum and Force

23. (a) Calculate the momentum of a 2000-kg elephant charging a hunter at a speed of 7.50 m/s size 12{7 "." "50"``"m/s"} {}.

(b) Compare the elephant’s momentum with the momentum of a 0.0400-kg tranquilizer dart fired at a speed of 600 m/s size 12{"600"``"m/s"} {}.

(c) What is the momentum of the 90.0-kg hunter running at 7.40 m/s size 12{7 "." "40"``"m/s"} {} after missing the elephant?

Solution

(a) $1.50 \times 10^{4} kg \cdot m/s$

(b) 625 to 1

(c) $6.66 \times 10^{2} kg \cdot m/s$

24. (a) What is the mass of a large ship that has a momentum of $1.60 \times 10^{9} kg \cdot m/s$, when the ship is moving at a speed of $48.0 km/h$?

(b) Compare the ship's momentum to the momentum of a 1100-kg artillery shell fired at a speed of $1200 m/s$.

25. (a) At what speed would a $2.00 \times 10^{4} - kg$ airplane have to fly to have a momentum of $1.60 \times 10^{9} kg \cdot m/s$ (the same as the ship's momentum in the problem above)?

(b) What is the plane's momentum when it is taking off at a speed of $60.0 m/s$?

(c) If the ship is an aircraft carrier that launches these airplanes with a catapult, discuss the implications of your answer to (b) as it relates to recoil effects of the catapult on the ship.

Solution

(a) $8.00 \times 10^{4} m/s$

(b) $1.20 \times 10^{6} kg \cdot m/s$

(c) Because the momentum of the airplane is 3 orders of magnitude smaller than of the ship, the ship will not recoil very much. The recoil would be $-0.0100 m/s$, which is probably not noticeable.

26. (a) What is the momentum of a garbage truck that is $1.20 \times 10^{4} kg$ and is moving at $10.0 m/s$

(b) At what speed would an 8.00-kg trash can have the same momentum as the truck?

27. A runaway train car that has a mass of 15,000 kg travels at a speed of $5.4 m/s$ down a track. Compute the time required for a force of 1500 N to bring the car to rest.

Solution

54 s

28. The mass of Earth is $5.972 \times 10^{24} kg$ and its orbital radius is an average of $1.496 \times 10^{11} m$. Calculate its linear momentum.

8.2: Impulse

29. A bullet is accelerated down the barrel of a gun by hot gases produced in the combustion of gun powder. What is the average force exerted on a 0.0300-kg bullet to accelerate it to a speed of 600 m/s in a time of 2.00 ms (milliseconds)?

Solution

$9.00 \times 10^{3} N$

30. *Professional Application*

A car moving at 10 m/s crashes into a tree and stops in 0.26 s. Calculate the force the seat belt exerts on a passenger in the car to bring him to a halt. The mass of the passenger is 70 kg.

31. A person slaps her leg with her hand, bringing her hand to rest in 2.50 milliseconds from an initial speed of 4.00 m/s.

(a) What is the average force exerted on the leg, taking the effective mass of the hand and forearm to be 1.50 kg?

(b) Would the force be any different if the woman clapped her hands together at the same speed and brought them to rest in the same time? Explain why or why not.

Solution

a) $2.40 \times 10^{3} N$ toward the leg

b) The force on each hand would have the same magnitude as that found in part (a) (but in opposite directions by Newton's third law) because the change in momentum and the time interval are the same.

32. *Professional Application*

A professional boxer hits his opponent with a 1000-N horizontal blow that lasts for 0.150 s.

(a) Calculate the impulse imparted by this blow.

(b) What is the opponent's final velocity, if his mass is 105 kg and he is motionless in midair when struck near his center of mass?

(c) Calculate the recoil velocity of the opponent's 10.0-kg head if hit in this manner, assuming the head does not initially transfer significant momentum to the boxer's body.

(d) Discuss the implications of your answers for parts (b) and (c).

33. *Professional Application*

Suppose a child drives a bumper car head on into the side rail, which exerts a force of 4000 N on the car for 0.200 s.

(a) What impulse is imparted by this force?

(b) Find the final velocity of the bumper car if its initial velocity was 2.80 m/s and the car plus driver have a mass of 200 kg. You may neglect friction between the car and floor.

Solution

a) $800 kg \cdot m/s$ away from the wall
b) $1.20 m/s$ away from the wall

34. *Professional Application*

One hazard of space travel is debris left by previous missions. There are several thousand objects orbiting Earth that are large enough to be detected by radar, but there are far greater numbers of very small objects, such as flakes of paint. Calculate the force exerted by a 0.100-mg chip of paint that strikes a spacecraft window at a relative speed of $4.00 \times 10^3 m/s$, given the collision lasts $6.00 \times 10^{-8} s$.

35. *Professional Application*

A 75.0-kg person is riding in a car moving at 20.0 m/s when the car runs into a bridge abutment.

(a) Calculate the average force on the person if he is stopped by a padded dashboard that compresses an average of 1.00 cm.

(b) Calculate the average force on the person if he is stopped by an air bag that compresses an average of 15.0 cm.

Solution

(a) $1.50 \times 10^6 N$ away from the dashboard
(b) $1.00 \times 10^5 N$ away from the dashboard

36. *Professional Application*

Military rifles have a mechanism for reducing the recoil forces of the gun on the person firing it. An internal part recoils over a relatively large distance and is stopped by damping mechanisms in the gun. The larger distance reduces the average force needed to stop the internal part.

(a) Calculate the recoil velocity of a 1.00-kg plunger that directly interacts with a 0.0200-kg bullet fired at 600 m/s from the gun.

(b) If this part is stopped over a distance of 20.0 cm, what average force is exerted upon it by the gun?

(c) Compare this to the force exerted on the gun if the bullet is accelerated to its velocity in 10.0 ms (milliseconds).

37. A cruise ship with a mass of $1.00 \times 10^7 kg$ strikes a pier at a speed of 0.750 m/s. It comes to rest 6.00 m later, damaging the ship, the pier, and the tugboat captain's finances. Calculate the average force exerted on the pier using the concept of impulse. (Hint: First calculate the time it took to bring the ship to rest.)

Solution

$4.69 \times 10^5 N$ in the boat's original direction of motion

38. Calculate the final speed of a 110-kg rugby player who is initially running at 8.00 m/s but collides head-on with a padded goalpost and experiences a backward force of $1.76 \times 10^4 N$ for $5.50 \times 10^{-2} s$.

39. Water from a fire hose is directed horizontally against a wall at a rate of 50.0 kg/s and a speed of 42.0 m/s. Calculate the magnitude of the force exerted on the wall, assuming the water's horizontal momentum is reduced to zero.

Solution

$2.10 \times 10^3 N$ away from the wall

40. A 0.450-kg hammer is moving horizontally at 7.00 m/s when it strikes a nail and comes to rest after driving the nail 1.00 cm into a board.

(a) Calculate the duration of the impact.

(b) What was the average force exerted on the nail?

41. Starting with the definitions of momentum and kinetic energy, derive an equation for the kinetic energy of a particle expressed as a function of its momentum.

Solution

$$p = mv \Rightarrow p^2 = m^2v^2 \Rightarrow \frac{p^2}{m} = mv^2 \Rightarrow \frac{p^2}{2m} = \frac{1}{2}mv^2 = KE$$

$$KE = \frac{p^2}{2m}$$

42. A ball with an initial velocity of 10 m/s moves at an angle 60° above the $+x$-direction. The ball hits a vertical wall and bounces off so that it is moving 60° above the $-x$-direction with the same speed. What is the impulse delivered by the wall?

43. When serving a tennis ball, a player hits the ball when its velocity is zero (at the highest point of a vertical toss). The racquet exerts a force of 540 N on the ball for 5.00 ms, giving it a final velocity of 45.0 m/s. Using these data, find the mass of the ball.

Solution

60.0 g

44. A punter drops a ball from rest vertically 1 meter down onto his foot. The ball leaves the foot with a speed of 18 m/s at an angle 55° size 12{"55"°} {} above the horizontal. What is the impulse delivered by the foot (magnitude and direction)?

8.3: Conservation of Momentum

45. *Professional Application*

Train cars are coupled together by being bumped into one another. Suppose two loaded train cars are moving toward one another, the first having a mass of 150,000 kg and a velocity of 0.300 m/s, and the second having a mass of 110,000 kg and a velocity of $-0.120 m/s$. (The minus indicates direction of motion.) What is their final velocity?

Solution

0.122 m/s

46. Suppose a clay model of a koala bear has a mass of 0.200 kg and slides on ice at a speed of 0.750 m/s. It runs into another clay model, which is initially motionless and has a mass of 0.350 kg. Both being soft clay, they naturally stick together. What is their final velocity?

47. *Professional Application*

Consider the following question: *A car moving at 10 m/s crashes into a tree and stops in 0.26 s. Calculate the force the seatbelt exerts on a passenger in the car to bring him to a halt.* The mass of the passenger is 70 kg. Would the answer to this question be different if the car with the 70-kg passenger had collided with a car that has a mass equal to and is traveling in the opposite direction and at the same speed? Explain your answer.

Solution

In a collision with an identical car, momentum is conserved. Afterwards $v_f = 0$ for both cars. The change in momentum will be the same as in the crash with the tree. However, the force on the body is not determined since the time is not known. A padded stop will reduce injurious force on body.

48. What is the velocity of a 900-kg car initially moving at 30.0 m/s, just after it hits a 150-kg deer initially running at 12.0 m/s in the same direction? Assume the deer remains on the car.

49. A 1.80-kg falcon catches a 0.650-kg dove from behind in midair. What is their velocity after impact if the falcon's velocity is initially 28.0 m/s and the dove's velocity is 7.00 m/s in the same direction?

Solution
22.4 m/s in the same direction as the original motion

8.4: Elastic Collisions in One Dimension

50. Two identical objects (such as billiard balls) have a one-dimensional collision in which one is initially motionless. After the collision, the moving object is stationary and the other moves with the same speed as the other originally had. Show that both momentum and kinetic energy are conserved.

51. *Professional Application*

Two manned satellites approach one another at a relative speed of 0.250 m/s, intending to dock. The first has a mass of $4.00 \times 10^3 kg$, and the second a mass of $7.50 \times 10^3 kg$. If the two satellites collide elastically rather than dock, what is their final relative velocity?

Solution
0.250 m/s

52. A 70.0-kg ice hockey goalie, originally at rest, catches a 0.150-kg hockey puck slapped at him at a velocity of 35.0 m/s. Suppose the goalie and the ice puck have an elastic collision and the puck is reflected back in the direction from which it came. What would their final velocities be in this case?

8.5: Inelastic Collisions in One Dimension

53. A 0.240-kg billiard ball that is moving at 3.00 m/s strikes the bumper of a pool table and bounces straight back at 2.40 m/s (80% of its original speed). The collision lasts 0.0150 s.

(a) Calculate the average force exerted on the ball by the bumper.

(b) How much kinetic energy in joules is lost during the collision?

(c) What percent of the original energy is left?

Solution
(a) 86.4 N perpendicularly away from the bumper
(b) 0.389 J
(c) 64.0%

54. During an ice show, a 60.0-kg skater leaps into the air and is caught by an initially stationary 75.0-kg skater.

(a) What is their final velocity assuming negligible friction and that the 60.0-kg skater's original horizontal velocity is 4.00 m/s?

(b) How much kinetic energy is lost?

55. *Professional Application*

Using mass and speed data from [link] and assuming that the football player catches the ball with his feet off the ground with both of them moving horizontally, calculate:

(a) the final velocity if the ball and player are going in the same direction and

(b) the loss of kinetic energy in this case.

(c) Repeat parts (a) and (b) for the situation in which the ball and the player are going in opposite directions. Might the loss of kinetic energy be related to how much it hurts to catch the pass?

Solution
(a) 8.06 m/s

(b) -56.0 J
(c)(i) 7.88 m/s; (ii) -223 J

56. A battleship that is $6.00 \times 10^{7} kg$ and is originally at rest fires a 1100-kg artillery shell horizontally with a velocity of 575 m/s.

(a) If the shell is fired straight aft (toward the rear of the ship), there will be negligible friction opposing the ship's recoil. Calculate its recoil velocity.

(b) Calculate the increase in internal kinetic energy (that is, for the ship and the shell). This energy is less than the energy released by the gun powder—significant heat transfer occurs.

57. *Professional Application*

Two manned satellites approaching one another, at a relative speed of 0.250 m/s, intending to dock. The first has a mass of $4.00 \times 10^{3} kg$, and the second a mass of $7.50 \times 10^{3} kg$.

(a) Calculate the final velocity (after docking) by using the frame of reference in which the first satellite was originally at rest.

(b) What is the loss of kinetic energy in this inelastic collision?

(c) Repeat both parts by using the frame of reference in which the second satellite was originally at rest. Explain why the change in velocity is different in the two frames, whereas the change in kinetic energy is the same in both.

Solution
(a) 0.163 m/s in the direction of motion of the more massive satellite
(b) 81.6 J
(c) $8.70 \times 10^{-2} m/s$ in the direction of motion of the less massive satellite, 81.5 J. Because there are no external forces, the velocity of the center of mass of the two-satellite system is unchanged by the collision. The two velocities calculated above are the velocity of the center of mass in each of the two different individual reference frames. The loss in KE is the same in both reference frames because the KE lost to internal forces (heat, friction, etc.) is the same regardless of the coordinate system chosen.

58. *Professional Application*

A 30,000-kg freight car is coasting at 0.850 m/s with negligible friction under a hopper that dumps 110,000 kg of scrap metal into it.

(a) What is the final velocity of the loaded freight car?

(b) How much kinetic energy is lost?

59. *Professional Application*

Space probes may be separated from their launchers by exploding bolts. (They bolt away from one another.) Suppose a 4800-kg satellite uses this method to separate from the 1500-kg remains of its launcher, and that 5000 J of kinetic energy is supplied to the two parts. What are their subsequent velocities using the frame of reference in which they were at rest before separation?

Solution
0.704 m/s
–2.25 m/s

60. A 0.0250-kg bullet is accelerated from rest to a speed of 550 m/s in a 3.00-kg rifle. The pain of the rifle's kick is much worse if you hold the gun loosely a few centimeters from your shoulder rather than holding it tightly against your shoulder.

(a) Calculate the recoil velocity of the rifle if it is held loosely away from the shoulder.

(b) How much kinetic energy does the rifle gain?

(c) What is the recoil velocity if the rifle is held tightly against the shoulder, making the effective mass 28.0 kg?

(d) How much kinetic energy is transferred to the rifle-shoulder combination? The pain is related to the amount of kinetic energy, which is significantly less in this latter situation.

(e) Calculate the momentum of a 110-kg football player running at 8.00 m/s. Compare the player's momentum with the momentum of a hard-thrown 0.410-kg football that has a speed of 25.0 m/s. Discuss its relationship to this problem.

Solution
(a) 4.58 m/s away from the bullet
(b) 31.5 J
(c) –0.491 m/s
(d) 3.38 J

61. *Professional Application*

One of the waste products of a nuclear reactor is plutonium-239 (^{239}Pu). This nucleus is radioactive and decays by splitting into a helium-4 nucleus and a uranium-235 nucleus ($^{4}He + ^{235}U$), the latter of which is also radioactive and will itself decay some time later. The energy emitted in the plutonium decay is (8.40×10^{–13}J\) and is entirely converted to kinetic energy of the helium and uranium nuclei. The mass of the helium nucleus is $6.68 \times 10^{-27} kg$, while that of the uranium is $3.92 \times 10^{-25} kg$ (note that the ratio of the masses is 4 to 235).

(a) Calculate the velocities of the two nuclei, assuming the plutonium nucleus is originally at rest.

(b) How much kinetic energy does each nucleus carry away? Note that the data given here are accurate to three digits only.

62. *Professional Application*

The Moon's craters are remnants of meteorite collisions. Suppose a fairly large asteroid that has a mass of \(\displaystyle 5.00×10^{12}kg\(\displaystyle about a kilometer across) strikes the Moon at a speed of 15.0 km/s.

(a) At what speed does the Moon recoil after the perfectly inelastic collision (the mass of the Moon is $7.36 \times 10^{22} kg$) ?

(b) How much kinetic energy is lost in the collision? Such an event may have been observed by medieval English monks who reported observing a red glow and subsequent haze about the Moon.

(c) In October 2009, NASA crashed a rocket into the Moon, and analyzed the plume produced by the impact. (Significant amounts of water were detected.) Answer part (a) and (b) for this real-life experiment. The mass of the rocket was 2000 kg and its speed upon impact was 9000 km/h. How does the plume produced alter these results?

Solution
(a) $1.02 \times 10^{-6} m/s$
(b) $5.63 \times 10^{20} J$ (almost all KE lost)
(c) Recoil speed is $6.79 \times 10^{-17} m/s$, energy lost is $6.25 \times 10^{9} J$. The plume will not affect the momentum result because the plume is still part of the Moon system. The plume may affect the kinetic energy result because a significant part of the initial kinetic energy may be transferred to the kinetic energy of the plume particles.

63. *Professional Application*

Two football players collide head-on in midair while trying to catch a thrown football. The first player is 95.0 kg and has an initial velocity of 6.00 m/s, while the second player is 115 kg and has an initial velocity of –3.50 m/s. What is their velocity just after impact if they cling together?

64. What is the speed of a garbage truck that is $1.20 \times 10^{4} kg$ and is initially moving at 25.0 m/s just after it hits and adheres to a trash can that is 80.0 kg and is initially at rest?

Solution
24.8 m/s

65. During a circus act, an elderly performer thrills the crowd by catching a cannon ball shot at him. The cannon ball has a mass of 10.0 kg and the horizontal component of its velocity is 8.00 m/s when the 65.0-kg performer catches it. If the performer is on nearly frictionless roller skates, what is his recoil velocity?

66. (a) During an ice skating performance, an initially motionless 80.0-kg clown throws a fake barbell away. The clown's ice skates allow her to recoil frictionlessly. If the clown recoils with a velocity of 0.500 m/s and the barbell is thrown with a velocity of 10.0 m/s, what is the mass of the barbell?

(b) How much kinetic energy is gained by this maneuver?

(c) Where does the kinetic energy come from?

Solution
(a) 4.00 kg
(b) 210 J
(c) The clown does work to throw the barbell, so the kinetic energy comes from the muscles of the clown. The muscles convert the chemical potential energy of ATP into kinetic energy.

8.6: Collisions of Point Masses in Two Dimensions

67. Two identical pucks collide on an air hockey table. One puck was originally at rest.

(a) If the incoming puck has a speed of 6.00 m/s and scatters to an angle of 30.0^{o},what is the velocity (magnitude and direction) of the second puck? (You may use the result that $\theta_1 - \theta_2 = 90^{o}$ for elastic collisions of objects that have identical masses.)

(b) Confirm that the collision is elastic.

Solution
(a) 3.00 m/s, 60^{o} below x-axis

(b) Find speed of first puck after collision: $0 = mv_1' sin30^{o} - mv_2' sin60^{o} \Rightarrow v_1' = v'_2 \frac{sin60^{o}}{sin30^{o} =} 5.196 m/s$

Verify that ratio of initial to final KE equals one: $KE = \frac{1}{2} mv_1^2 = 18 mJ$

$KE = \frac{1}{2} mv_1'^2 + \frac{1}{2} mv_2'^2 = 18 mJ \quad \frac{KE}{KE'} = 1.00$

68. Confirm that the results of the example Example do conserve momentum in both the x- and y-directions.

69. A 3000-kg cannon is mounted so that it can recoil only in the horizontal direction.

(a) Calculate its recoil velocity when it fires a 15.0-kg shell at 480 m/s at an angle of 20.0^{o} above the horizontal.

(b) What is the kinetic energy of the cannon? This energy is dissipated as heat transfer in shock absorbers that stop its recoil.

(c) What happens to the vertical component of momentum that is imparted to the cannon when it is fired?

Solution
(a) $-2.26 m/s$
(b) $7.63 \times 10^3 J$
(c) The ground will exert a normal force to oppose recoil of the cannon in the vertical direction. The momentum in the vertical direction is transferred to the earth. The energy is transferred into the ground, making a dent where the cannon is. After long barrages, cannon have erratic aim because the ground is full of divots.

70. *Professional Application*

A 5.50-kg bowling ball moving at 9.00 m/s collides with a 0.850-kg bowling pin, which is scattered at an angle of 85.0^{o} to the initial direction of the bowling ball and with a speed of 15.0 m/s.

(a) Calculate the final velocity (magnitude and direction) of the bowling ball.

(b) Is the collision elastic?

(c) Linear kinetic energy is greater after the collision. Discuss how spin on the ball might be converted to linear kinetic energy in the collision.

71. *Professional Application*

Ernest Rutherford (the first New Zealander to be awarded the Nobel Prize in Chemistry) demonstrated that nuclei were very small and dense by scattering helium-4 nuclei (^4He) from gold-197 nuclei (^{197}Au). The energy of the incoming helium nucleus was $8.00 \times 10^{-13} J$, and the masses of the helium and gold nuclei were $6.68 \times 10^{-27} kg$ and $3.29 \times 10^{-25} kg$, respectively (note that their mass ratio is 4 to 197).

(a) If a helium nucleus scatters to an angle of 120^o during an elastic collision with a gold nucleus, calculate the helium nucleus's final speed and the final velocity (magnitude and direction) of the gold nucleus.

(b) What is the final kinetic energy of the helium nucleus?

Solution
(a) $5.36 \times 10^5 m/s$ at -29.5^o
(b) $7.52 \times 10^{-13} J$

72. *Professional Application*

Two cars collide at an icy intersection and stick together afterward. The first car has a mass of 1200 kg and is approaching at $8.00 m/s$ due south. The second car has a mass of 850 kg and is approaching at $17.0 m/s$ due west.

(a) Calculate the final velocity (magnitude and direction) of the cars.

(b) How much kinetic energy is lost in the collision? (This energy goes into deformation of the cars.) Note that because both cars have an initial velocity, you cannot use the equations for conservation of momentum along the x-axis and y-axis; instead, you must look for other simplifying aspects.

73. Starting with equations $m_1 v_1 = m_1 v_1' cos\theta_1 + m_2 v_2' cos\theta_2$ and $0 = m_1 v_1' sin\theta_1 + m_2 v_2' sin\theta_2$ for conservation of momentum in the $x-$ and $y-$directions and assuming that one object is originally stationary, prove that for an elastic collision of two objects of equal masses,

$$\frac{1}{2} m v_1^2 = \frac{1}{2} m v_1'^2 + \frac{1}{2} m v_2'^2 + m v_1' v_2' cos(\theta_1 - \theta_2)$$

as discussed in the text.

Solution
We are given that $m_1 = m_2 \equiv m$. The given equations then become:

$$v_1 = v_1 cos\theta_1 + v_2 cos\theta_2$$

and

$$0 = v_1' sin\theta_1 + v_2' sin\theta_2.$$

Square each equation to get

$$v_1^2 = v_1'^2 cos^2\theta_1 + v_2'^2 cos^2\theta_2 + 2 v_1' v_2' cos\theta_1 cos\theta_2$$
$$0 = v_1'^2 sin^2\theta_1 + v_2'^2 sin^2\theta_2 + 2 v_1' v_2' sin\theta_1 sin\theta_2.$$

Add these two equations and simplify:

$$\begin{aligned} v_1^2 &= v_1'^2 + v_2'^2 + 2 v_1' v_2' (cos\theta_1 cos\theta_2 + sin\theta_1 sin\theta_2) \\ &= v_1'^2 + v_2'^2 + 2 v_1' v_2' [\frac{1}{2} cos(\theta_1 - \theta_2) + \frac{1}{2} cos(\theta_1 + \theta_2) + \frac{1}{2} cos(\theta_1 - \theta_2) - \frac{1}{2} cos(\theta_1 + \theta_2)] \\ &= v_1'^2 + v_2'^2 + 2 v_1' v_2' cos(\theta_1 - \theta_2). \end{aligned}$$

Multiply the entire equation by $\frac{1}{2} m$ to recover the kinetic energy:

$$\frac{1}{2} m v_1^2 = \frac{1}{2} m v_1'^2 + \frac{1}{2} m v_2'^2 + m v_1' v_2' cos(\theta_1 - \theta_2)$$

74. *Integrated Concepts*

A 90.0-kg ice hockey player hits a 0.150-kg puck, giving the puck a velocity of 45.0 m/s. If both are initially at rest and if the ice is frictionless, how far does the player recoil in the time it takes the puck to reach the goal 15.0 m away?

8.7: Introduction to Rocket Propulsion

75. *Professional Application*

Antiballistic missiles (ABMs) are designed to have very large accelerations so that they may intercept fast-moving incoming missiles in the short time available. What is the takeoff acceleration of a 10,000-kg ABM that expels 196 kg of gas per second at an exhaust velocity of $2.50 \times 10^3 m/s$?

Solution
$39.2 m/s^2$

76. *Professional Application*

What is the acceleration of a 5000-kg rocket taking off from the Moon, where the acceleration due to gravity is only $1.6 m/s^2$, if the rocket expels 8.00 kg of gas per second at an exhaust velocity of $2.20 \times 10^3 m/s$?

77. *Professional Application*

Calculate the increase in velocity of a 4000-kg space probe that expels 3500 kg of its mass at an exhaust velocity of $2.00 \times 10^3 m/s$. You may assume the gravitational force is negligible at the probe's location.

Solution
$4.16 \times 10^3 m/s$

78. *Professional Application*

Ion-propulsion rockets have been proposed for use in space. They employ atomic ionization techniques and nuclear energy sources to produce extremely high exhaust velocities, perhaps as great as $8.00 \times 10^6 m/s$. These techniques allow a much more favorable payload-to-fuel ratio. To illustrate this fact:

(a) Calculate the increase in velocity of a 20,000-kg space probe that expels only 40.0-kg of its mass at the given exhaust velocity.

(b) These engines are usually designed to produce a very small thrust for a very long time—the type of engine that might be useful on a trip to the outer planets, for example. Calculate the acceleration of such an engine if it expels $4.50 \times 10^{-6} kg/s$ at the given velocity, assuming the acceleration due to gravity is negligible.

79. Derive the equation for the vertical acceleration of a rocket.

Solution
The force needed to give a small mass Δman acceleration $a_{\Delta m}$ is $F = \Delta m a_{\Delta m}$. To accelerate this mass in the small time interval Δt at a speed $v_e = a_{\Delta m} \Delta t$, so $F = v_e \frac{\Delta m}{\Delta t}$. By Newton's third law, this force is equal in magnitude to the thrust force acting on the rocket, so $F_{thrust} = v_e \frac{\Delta m}{\Delta t}$, where all quantities are positive. Applying Newton's second law to the rocket gives $F_{thrust} - mg = ma \Rightarrow a = \frac{v_e}{m} \frac{\Delta m}{\Delta t} - g$, where m is the mass of the rocket and unburnt fuel.

80. *Professional Application*

(a) Calculate the maximum rate at which a rocket can expel gases if its acceleration cannot exceed seven times that of gravity. The mass of the rocket just as it runs out of fuel is 75,000-kg, and its exhaust velocity is $2.40 \times 10^3 m/s$. Assume that the acceleration of gravity is the same as on Earth's surface $(9.80 m/s^2)$.

(b) Why might it be necessary to limit the acceleration of a rocket?

Solution
Given the following data for a fire extinguisher-toy wagon rocket experiment, calculate the average exhaust velocity of the gases expelled from the extinguisher. Starting from rest, the final velocity is 10.0 m/s. The total mass is initially 75.0 kg and is 70.0 kg after the extinguisher is fired.

81. How much of a single-stage rocket that is 100,000 kg can be anything but fuel if the rocket is to have a final speed of $8.00 km/s$, given that it expels gases at an exhaust velocity of $2.20 \times 10^3 m/s$?

Solution
$2.63 \times 10^3 kg$

82. *Professional Application*

(a) A 5.00-kg squid initially at rest ejects 0.250-kg of fluid with a velocity of 10.0 m/s. What is the recoil velocity of the squid if the ejection is done in 0.100 s and there is a 5.00-N frictional force opposing the squid’s movement.

(b) How much energy is lost to work done against friction?

Solution
(a) 0.421 m/s away from the ejected fluid.
(b) $0.237 J$

83. *Unreasonable Results*

Squids have been reported to jump from the ocean and travel $30.0 m$ (measured horizontally) before re-entering the water.

(a) Calculate the initial speed of the squid if it leaves the water at an angle of 20.0°, assuming negligible lift from the air and negligible air resistance.

(b) The squid propels itself by squirting water. What fraction of its mass would it have to eject in order to achieve the speed found in the previous part? The water is ejected at $12.0 m/s$; gravitational force and friction are neglected.

(c) What is unreasonable about the results?

(d) Which premise is unreasonable, or which premises are inconsistent?

84. *Construct Your Own Problem*

Consider an astronaut in deep space cut free from her space ship and needing to get back to it. The astronaut has a few packages that she can throw away to move herself toward the ship. Construct a problem in which you calculate the time it takes her to get back by throwing all the packages at one time compared to throwing them one at a time. Among the things to be considered are the masses involved, the force she can exert on the packages through some distance, and the distance to the ship.

85. *Construct Your Own Problem*

Consider an artillery projectile striking armor plating. Construct a problem in which you find the force exerted by the projectile on the plate. Among the things to be considered are the mass and speed of the projectile and the distance over which its speed is reduced. Your instructor may also wish for you to consider the relative merits of depleted uranium versus lead projectiles based on the greater density of uranium.

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CHAPTER OVERVIEW

8: Heat and Heat Transfer Methods

Energy can exist in many forms and heat is one of the most intriguing. Heat is often hidden, as it only exists when in transit, and is transferred by a number of distinctly different methods. Heat transfer touches every aspect of our lives and helps us understand how the universe functions. It explains the chill we feel on a clear breezy night, or why Earth's core has yet to cool. This chapter defines and explores heat transfer, its effects, and the methods by which heat is transferred. These topics are fundamental, as well as practical, and will often be referred to in the chapters ahead.

- 8.1: Prelude to Heat and Heat Transfer Methods
- 8.2: Heat
- 8.3: Temperature Change and Heat Capacity
- 8.4: Phase Change and Latent Heat
- 8.5: Heat Transfer Methods
- 8.6: Conduction
- 8.7: Convection
- 8.8: Radiation
- 8.E: Heat and Heat Transfer Methods (Exercise)

Thumbnail: Different flame types of a Bunsen burner depend on oxygen supply. On the left a rich fuel with no premixed oxygen produces a yellow sooty diffusion flame; on the right a lean fully oxygen premixed flame produces no soot. (GNU Free Documentation License, Version 1.2; Jan Fijałkowski).

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8.1: Prelude to Heat and Heat Transfer Methods

Energy can exist in many forms and heat is one of the most intriguing. Heat is often hidden, as it only exists when in transit, and is transferred by a number of distinctly different methods. Heat transfer touches every aspect of our lives and helps us understand how the universe functions. It explains the chill we feel on a clear breezy night, or why Earth's core has yet to cool. This chapter defines and explores heat transfer, its effects, and the methods by which heat is transferred. These topics are fundamental, as well as practical, and will often be referred to in the chapters ahead.

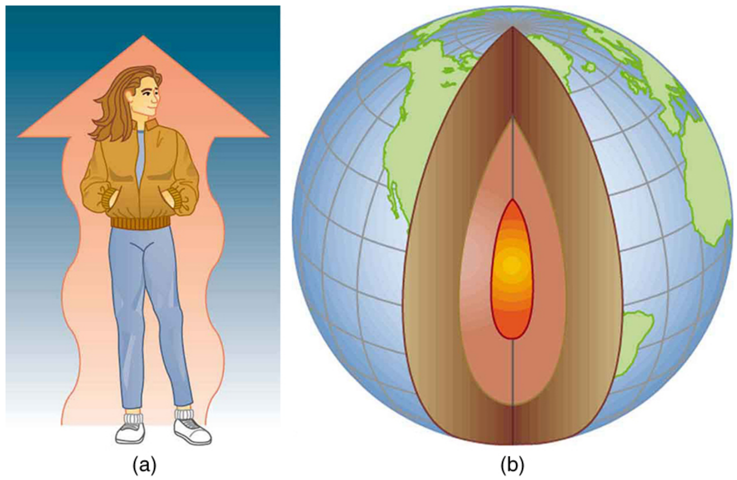

Figure 8.1.1. (a) The chilling effect of a clear breezy night is produced by the wind and by radiative heat transfer to cold outer space. (b) There was once great controversy about the Earth's age, but it is now generally accepted to be about 4.5 billion years old. Much of the debate is centered on the Earth's molten interior. According to our understanding of heat transfer, if the Earth is really that old, its center should have cooled off long ago. The discovery of radioactivity in rocks revealed the source of energy that keeps the Earth's interior molten, despite heat transfer to the surface, and from there to cold outer space.

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8.2: Heat

Learning Objectives

By the end of this section, you will be able to:

- Define heat as transfer of energy.

In Work, Energy, and Energy Resources, we defined work as force times distance and learned that work done on an object changes its kinetic energy. We also saw in Temperature, Kinetic Theory, and the Gas Laws that temperature is proportional to the (average) kinetic energy of atoms and molecules. We say that a thermal system has a certain internal energy: its internal energy is higher if the temperature is higher. If two objects at different temperatures are brought in contact with each other, energy is transferred from the hotter to the colder object until equilibrium is reached and the bodies reach thermal equilibrium (i.e., they are at the same temperature). No work is done by either object, because no force acts through a distance. The transfer of energy is caused by the temperature difference, and ceases once the temperatures are equal. These observations lead to the following definition of heat: Heat is the spontaneous transfer of energy due to a temperature difference.

As noted in Temperature, Kinetic Theory, and the Gas Laws, heat is often confused with temperature. For example, we may say the heat was unbearable, when we actually mean that the temperature was high. Heat is a form of energy, whereas temperature is not. The misconception arises because we are sensitive to the flow of heat, rather than the temperature.

Owing to the fact that heat is a form of energy, it has the SI unit of *joule* (J). The *calorie* (cal) is a common unit of energy, defined as the energy needed to change the temperature of 1.00 g of water by $1.00^{o}C$ —specifically, between $14.5^{o}C$ and $15.5^{o}C$, since there is a slight temperature dependence. Perhaps the most common unit of heat is the **kilocalorie** (kcal), which is the energy needed to change the temperature of 1.00 kg of water by $1.00^{o}C$. Since mass is most often specified in kilograms, kilocalorie is commonly used. Food calories (given the notation Cal, and sometimes called "big calorie") are actually kilocalories $(1\; kilocalorie = 1000\; calories)$, a fact not easily determined from package labeling.

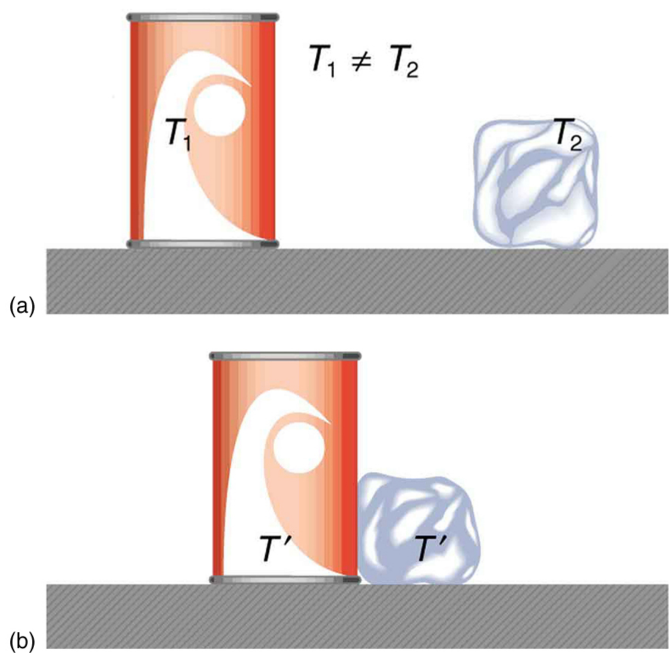

Figure 8.2.1: In panel (a) the soft drink and the ice have different temperatures, T_1 and T_2, and are not in thermal equilibrium. In panel (b), when the soft drink and ice are allowed to interact, energy is transferred until they reach the same temperature T', achieving equilibrium. Heat transfer occurs due to the difference in temperatures. In fact, since the soft drink and ice are both in contact with the surrounding air and bench, the equilibrium temperature will be the same for both.

Mechanical Equivalent of Heat

It is also possible to change the temperature of a substance by doing work. Work can transfer energy into or out of a system. This realization helped establish the fact that heat is a form of energy. James Prescott Joule (1818–1889) performed many experiments to establish the **mechanical equivalent of heat**—*the work needed to produce the same effects as heat transfer*. In terms of the units used for these two terms, the best modern value for this equivalence is

$$1.000\ kcal = 4186\ J. \tag{8.2.1}$$

We consider this equation as the conversion between two different units of energy.

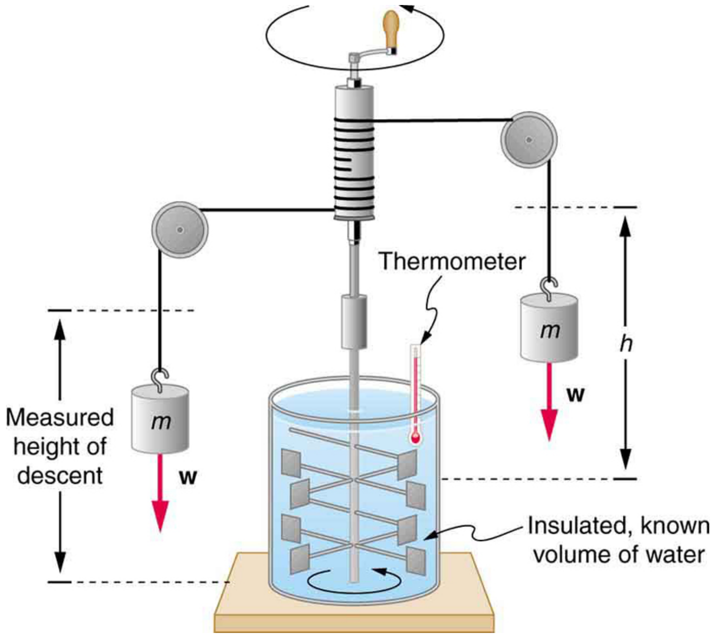

Figure 8.2.2: Schematic depiction of Joule's experiment that established the equivalence of heat and work.

Figure 8.2.2 shows one of Joule's most famous experimental setups for demonstrating the mechanical equivalent of heat. It demonstrated that work and heat can produce the same effects, and helped establish the principle of conservation of energy. Gravitational potential energy (PE) (work done by the gravitational force) is converted into kinetic energy (KE), and then randomized by viscosity and turbulence into increased average kinetic energy of atoms and molecules in the system, producing a temperature increase. His contributions to the field of thermodynamics were so significant that the SI unit of energy was named after him.

Heat added or removed from a system changes its internal energy and thus its temperature. Such a temperature increase is observed while cooking. However, adding heat does not necessarily increase the temperature. An example is melting of ice; that is, when a substance changes from one phase to another. Work done on the system or by the system can also change the internal energy of the system. Joule demonstrated that the temperature of a system can be increased by stirring. If an ice cube is rubbed against a rough surface, work is done by the frictional force. A system has a well-defined internal energy, but we cannot say that it has a certain "heat content" or "work content". We use the phrase "heat transfer" to emphasize its nature.

Exercise 8.2.1

Two samples (A and B) of the same substance are kept in a lab. Someone adds 10 kilojoules (kJ) of heat to one sample, while 10 kJ of work is done on the other sample. How can you tell to which sample the heat was added?

Answer

Heat and work both change the internal energy of the substance. However, the properties of the sample only depend on the internal energy so that it is impossible to tell whether heat was added to sample A or B.

Summary

- Heat and work are the two distinct methods of energy transfer.
- Heat is energy transferred solely due to a temperature difference.
- Any energy unit can be used for heat transfer, and the most common are kilocalorie (kcal) and joule (J).
- Kilocalorie is defined to be the energy needed to change the temperature of 1.00 kg of water between 14.5^oC and 15.5^oC.

- The mechanical equivalent of this heat transfer is $1.00\ kcal = 4186\ J$.

Glossary

heat
: the spontaneous transfer of energy due to a temperature difference

kilocalorie
: 1kilocalorie=1000caloriessize 12{1`"kilocalorie=1000"`"calories"} {}

mechanical equivalent of heat
: the work needed to produce the same effects as heat transfer

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8.3: Temperature Change and Heat Capacity

Learning Objectives

By the end of this section, you will be able to:

- Observe heat transfer and change in temperature and mass.
- Calculate final temperature after heat transfer between two objects.

One of the major effects of heat transfer is temperature change: heating increases the temperature while cooling decreases it. We assume that there is no phase change and that no work is done on or by the system. Experiments show that the transferred heat depends on three factors—the change in temperature, the mass of the system, and the substance and phase of the substance.

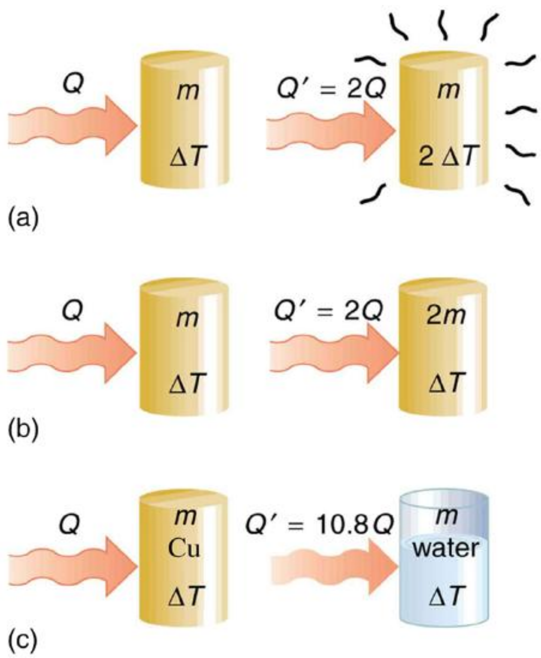

Figure 8.3.1: The heat Q transferred to cause a temperature change depends on the magnitude of the temperature change, the mass of the system, and the substance and phase involved. (a) The amount of heat transferred is directly proportional to the temperature change. To double the temperature change of a mass m, you need to add twice the heat. (b) The amount of heat transferred is also directly proportional to the mass. To cause an equivalent temperature change in a doubled mass, you need to add twice the heat. (c) The amount of heat transferred depends on the substance and its phase. If it takes an amount Q of heat to cause a temperature change ΔT in a given mass of copper, it will take 10.8 times that amount of heat to cause the equivalent temperature change in the same mass of water assuming no phase change in either substance.

The dependence on temperature change and mass are easily understood. Owing to the fact that the (average) kinetic energy of an atom or molecule is proportional to the absolute temperature, the internal energy of a system is proportional to the absolute temperature and the number of atoms or molecules. Owing to the fact that the transferred heat is equal to the change in the internal energy, the heat is proportional to the mass of the substance and the temperature change. The transferred heat also depends on the substance so that, for example, the heat necessary to raise the temperature is less for alcohol than for water. For the same substance, the transferred heat also depends on the phase (gas, liquid, or solid).

Heat Transfer and Temperature Change

The quantitative relationship between heat transfer and temperature change contains all three factors:

$$Q = mc\Delta T, \tag{8.3.1}$$

where Q is the symbol for heat transfer, m is the mass of the substance, and ΔT is the change in temperature. The symbol c stands for specific heat and depends on the material and phase. The specific heat is the amount of heat necessary to change the temperature of 1.00 kg of mass by 1.00^oC. The specific heat c is a property of the substance; its SI unit is $J/(kg \cdot K)$ or

$J/(kg \cdot^o C)$. Recall that the temperature change (ΔT) is the same in units of kelvin and degrees Celsius. If heat transfer is measured in kilocalories, then *the unit of specific heat* is $kcal/(kg \cdot^o C)$.

Values of specific heat must generally be looked up in tables, because there is no simple way to calculate them. In general, the specific heat also depends on the temperature. Table $8.3.1$ lists representative values of specific heat for various substances. Except for gases, the temperature and volume dependence of the specific heat of most substances is weak. We see from this table that the specific heat of water is five times that of glass and ten times that of iron, which means that it takes five times as much heat to raise the temperature of water the same amount as for glass and ten times as much heat to raise the temperature of water as for iron. In fact, water has one of the largest specific heats of any material, which is important for sustaining life on Earth.

Example $8.3.1$: Calculating the Required Heat: Heating Water in an Aluminum Pan

A 0.500 kg aluminum pan on a stove is used to heat 0.250 liters of water from 20.0^oC to 80.0^oC.

(a) How much heat is required? What percentage of the heat is used to raise the temperature of (b) the pan and (c) the water?

Strategy

The pan and the water are always at the same temperature. When you put the pan on the stove, the temperature of the water and the pan is

increased by the same amount. We use the equation for the heat transfer for the given temperature change and mass of water and aluminum. The specific heat values for water and aluminum are given in Table $8.3.1$.

Solution

Because water is in thermal contact with the aluminum, the pan and the water are at the same temperature.

1. Calculate the temperature difference:

$$\Delta T = T_f - T_i = 60.0^oC. \tag{8.3.2}$$

2. Calculate the mass of water. Because the density of water is 1000 kg/m^3, one liter of water has a mass of 1 kg, and the mass of 0.250 liters of water is $m_w = 0.250\,kg$.
3. Calculate the heat transferred to the water. Use the specific heat of water in Table $8.3.1$

$$Q_w = m_w c_w \Delta T = (0.250\,kg)(4186\,J/kg^oC)(60.0^oC) = 62.8\,kJ. \tag{8.3.3}$$

4. Calculate the heat transferred to the aluminum. Use the specific heat for aluminum in Table \(\PageIndex{1}\:

$$Q_{Al} = m_{Al} c_{Al} \Delta T = (0.500\,kg)(900\,J/kg^oC)(60.0^oC) = 27.0 kJ. \tag{8.3.4}$$

5. Compare the percentage of heat going into the pan versus that going into the water. First, find the total transferred heat:

$$Q_{Total} = Q_W + Q_{Al} = 62.8\,kJ + 27.0\,kJ = 89.8\,kJ. \tag{8.3.5}$$

6. Thus, the amount of heat going into heating the pan is

$$\frac{62.8\,kJ}{89.8\,kJ} \times 100\% = 69.9\% \tag{8.3.6}$$

Discussion

In this example, the heat transferred to the container is a significant fraction of the total transferred heat. Although the mass of the pan is twice that of the water, the specific heat of water is over four times greater than that of aluminum. Therefore, it takes a bit more than twice the heat to achieve the given temperature change for the water as compared to the aluminum pan.

Figure 8.3.2: The smoking brakes on this truck are a visible evidence of the mechanical equivalent of heat.

Example 8.3.2: Calculating the Temperature Increase from the Work Done on a Substance: Truck Brakes Overheat on Downhill Runs

Truck brakes used to control speed on a downhill run do work, converting gravitational potential energy into increased internal energy (higher temperature) of the brake material. This conversion prevents the gravitational potential energy from being converted into kinetic energy of the truck. The problem is that the mass of the truck is large compared with that of the brake material absorbing the energy, and the temperature increase may occur too fast for sufficient heat to transfer from the brakes to the environment.

Calculate the temperature increase of 100 kg of brake material with an average specific heat of $800.0\, J/kg \cdot^o C$ if the material retains 10% of the energy from a 10,000-kg truck descending 75.0 m (in vertical displacement) at a constant speed.

Strategy

If the brakes are not applied, gravitational potential energy is converted into kinetic energy. When brakes are applied, gravitational potential energy is converted into internal energy of the brake material. We first calculate the gravitational potential energy (Mgh) that the entire truck loses in its descent and then find the temperature increase produced in the brake material alone.

Solution

1. Calculate the change in gravitational potential energy as the truck goes downhill

$$Mgh = (10,000\, kg)(9.80\, m/s^2)(75.0\, m) = 7.35 \times 10^6\, J. \tag{8.3.7}$$

2. Calculate the temperature from the heat transferred using $Q = Mgh$ and

$$\Delta T = \frac{Q}{mc}, \tag{8.3.8}$$

where m is the mass of the brake material. Insert the values $m = 100\, kg$ and $c = 800\, J/kg \cdot^o C$ to find

$$\Delta T = \frac{(7.35v \times 10^6\, J)}{(100\, kg)(800\, J/kg^oC)} = 92^oC. \tag{8.3.9}$$

Discussion

This temperature is close to the boiling point of water. If the truck had been traveling for some time, then just before the descent, the brake temperature would likely be higher than the ambient temperature. The temperature increase in the descent would likely raise the temperature of the brake material above the boiling point of water, so this technique is not practical. However, the same idea underlies the recent hybrid technology of cars, where mechanical energy (gravitational potential energy) is converted by the brakes into electrical energy (battery).

Table 8.3.1: Specific Heats1 of Various Substances

Substances	Specific heat (c)	
Solids	$J/kg\cdot^{o}C$	$kcal/kg\cdot^{o}C$
Aluminum	900	0.215
Asbestos	800	0.19
Concrete, granite (average)	840	0.20
Copper	387	0.0924
Glass	840	0.20
Gold	129	0.0308
Human body (average at 37 °C)	3500	0.83
Ice (average, -50°C to 0°C)	2090	0.50
Iron, steel	452	0.108
Lead	128	0.0305
Silver	235	0.0562
Wood	1700	0.4
Liquids		
Benzene	1740	0.415
Ethanol	2450	0.586
Glycerin	2410	0.576
Mercury	139	0.0333
Water (15.0 °C)	4186	1.000
Gases [3]		
Air (dry)	721 (1015)	0.172 (0.242)
Ammonia	1670 (2190)	0.399 (0.523)
Carbon dioxide	638 (833)	0.152 (0.199)
Nitrogen	739 (1040)	0.177 (0.248)
Oxygen	651 (913)	0.156 (0.218)
Steam (100°C)	1520 (2020)	0.363 (0.482)

Note that Example $8.3.2$ is an illustration of the mechanical equivalent of heat. Alternatively, the temperature increase could be produced by a blow torch instead of mechanically.

Example $8.3.3$: Calculating the Final Temperature When Heat Is Transferred Between Two Bodies: Pouring Cold Water in a Hot Pan

Suppose you pour 0.250 kg of $20.0^{o}C$ water (about a cup) into a 0.500-kg aluminum pan off the stove with a temperature of $150^{o}C$. Assume that the pan is placed on an insulated pad and that a negligible amount of water boils off. What is the temperature when the water and pan reach thermal equilibrium a short time later?

Strategy

The pan is placed on an insulated pad so that little heat transfer occurs with the surroundings. Originally the pan and water are not in thermal equilibrium: the pan is at a higher temperature than the water. Heat transfer then restores thermal equilibrium once the water and pan are in contact. Because heat transfer between the pan and water takes place rapidly, the mass of evaporated water is negligible and the magnitude of the heat lost by the pan is equal to the heat gained by the water. The exchange of heat stops once a thermal equilibrium between the pan and the water is achieved. The heat exchange can be written as $|Q_{hot}| = Q_{cold}$.

Solution

1. Use the equation for heat transfer $Q = mc\Delta T$ to express the heat lost by the aluminum pan in terms of the mass of the pan, the specific heat of aluminum, the initial temperature of the pan, and the final temperature:
$$Q_{hot} = m_{Al}c_{Al}(T_f - 150^oC). \tag{8.3.10}$$
2. Express the heat gained by the water in terms of the mass of the water, the specific heat of water, the initial temperature of the water and the final temperature:
$$Q_{cold} = m_w c_w (T_f - 20.0^oC). \tag{8.3.11}$$
3. Note that $Q_{hot} < 0$ and $Q_{cold} > 0$ and that they must sum to zero because the heat lost by the hot pan must be the same as the heat gained by the cold water:
$$Q_{cold} + Q_{hot} = 0, \tag{8.3.12}$$
$$Q_{cold} = -Q_{hot}, \tag{8.3.13}$$
$$m_w c_w (T_f - 20.0^oC) = -m_{Al}c_{Al}(T_f - 150.0^oC). \tag{8.3.14}$$
4. Bring all terms involving T_f on the left hand side and all other terms on the right hand side. Solve for T_f,
$$T_f = \frac{m_{Al}c_{Al}(150^oC) + m_W c_W (20.0^oC)}{m_{Al}c_{Al} + m_W c_W},$$
and insert the numerical values:
$$T_f = \frac{(0.500kg)(900J/kg^oC)(150^oC) + (0.250kg)(4186J/kg^oC)(20.0^oC)}{(0.500kg)(900J/kg^oC) + (0.250kg)(4186J/kg^oC)} = \frac{88430J}{1496.5J/^oC} = 59.1^oC.$$

Discussion

This is a typical *calorimetry* problem—two bodies at different temperatures are brought in contact with each other and exchange heat until a common temperature is reached. Why is the final temperature so much closer to **20.0ºC** than **150ºC**? The reason is that water has a greater specific heat than most common substances and thus undergoes a small temperature change for a given heat transfer. A large body of water, such as a lake, requires a large amount of heat to increase its temperature appreciably. This explains why the temperature of a lake stays relatively constant during a day even when the temperature change of the air is large. However, the water temperature does change over longer times (e.g., summer to winter).

TAKE-HOME EXPERIMENT: TEMPERATURE CHANGE OF LAND AND WATER

What heats faster, land or water?

To study differences in heat capacity:

- Place equal masses of dry sand (or soil) and water at the same temperature into two small jars. (The average density of soil or sand is about 1.6 times that of water, so you can achieve approximately equal masses by using 50 more water by volume.)
- Heat both (using an oven or a heat lamp) for the same amount of time.
- Record the final temperature of the two masses.
- Now bring both jars to the same temperature by heating for a longer period of time.
- Remove the jars from the heat source and measure their temperature every 5 minutes for about 30 minutes.

Which sample cools off the fastest? This activity replicates the phenomena responsible for land breezes and sea breezes.

Exercise $8.3.1$

If 25 kJ is necessary to raise the temperature of a block from **25ºC** to **30ºC**, how much heat is necessary to heat the block from **45ºC** to **50ºC**?

Answer

The heat transfer depends only on the temperature difference. Since the temperature differences are the same in both cases, the same 25 kJ is necessary in the second case.

Summary

- The transfer of heat Q that leads to a change ΔT in the temperature of a body with mass m is $Q = mc\Delta T$, where c is the specific heat of the material. This relationship can also be considered as the definition of specific heat.

Footnotes

1 The values for solids and liquids are at constant volume and at **25ºC**, except as noted.
2 These values are identical in units of **cal/g⋅ºC**.
3 cv at constant volume and at **20.0ºC**, except as noted, and at 1.00 atm average pressure. Values in parentheses are c_p at a constant pressure of 1.00 atm.

Glossary

specific heat
the amount of heat necessary to change the temperature of 1.00 kg of a substance by 1.00 ºC

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8.4: Phase Change and Latent Heat

Learning Objectives

By the end of this section, you will be able to:

- Examine heat transfer.
- Calculate final temperature from heat transfer.

So far we have discussed temperature change due to heat transfer. No temperature change occurs from heat transfer if ice melts and becomes liquid water (i.e., during a phase change). For example, consider water dripping from icicles melting on a roof warmed by the Sun. Conversely, water freezes in an ice tray cooled by lower-temperature surroundings.

Figure $8.4.1$. Heat from the air transfers to the ice causing it to melt. (credit: Mike Brand)

Energy is required to melt a solid because the cohesive bonds between the molecules in the solid must be broken apart such that, in the liquid, the molecules can move around at comparable kinetic energies; thus, there is no rise in temperature. Similarly, energy is needed to vaporize a liquid, because molecules in a liquid interact with each other via attractive forces. There is no temperature change until a phase change is complete. The temperature of a cup of soda initially at $0^o C$ stays at $0^o C$ until all the ice has melted. Conversely, energy is released during freezing and condensation, usually in the form of thermal energy. Work is done by cohesive forces when molecules are brought together. The corresponding energy must be given off (dissipated) to allow them to stay together (see Figure $8.4.2$)

The energy involved in a phase change depends on two major factors: the number and strength of bonds or force pairs. The number of bonds is proportional to the number of molecules and thus to the mass of the sample. The strength of forces depends on the type of molecules. The heat Q required to change the phase of a sample of mass m is given by

$$Q = mL_f (melting/freezing), \tag{8.4.1}$$

$$Q = mL_v (vaporization/condensation), \tag{8.4.2}$$

where the latent heat of fusion, L_f, and latent heat of vaporization, L_v, are material constants that are determined experimentally. See (Table $8.4.1$).

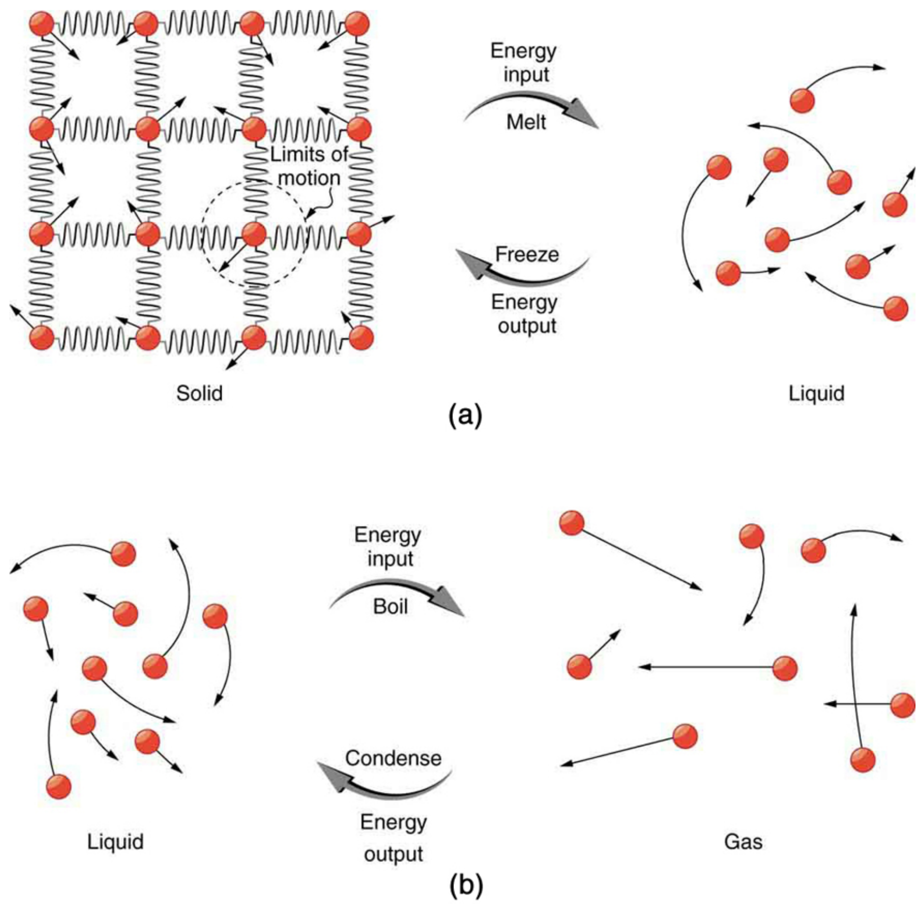

Figure 8.4.2. (a) Energy is required to partially overcome the attractive forces between molecules in a solid to form a liquid. That same energy must be removed for freezing to take place. (b) Molecules are separated by large distances when going from liquid to vapor, requiring significant energy to overcome molecular attraction. The same energy must be removed for condensation to take place. There is no temperature change until a phase change is complete.

Latent heat is measured in units of J/kg. Both L_f and L_v depend on the substance, particularly on the strength of its molecular forces as noted earlier. L_f and L_v are collectively called **latent heat coefficients**. They are *latent*, or hidden, because in phase changes, energy enters or leaves a system without causing a temperature change in the system; so, in effect, the energy is hidden. Table 8.4.1 lists representative values of L_f and L_v, together with melting and boiling points.

The table shows that significant amounts of energy are involved in phase changes. Let us look, for example, at how much energy is needed to melt a kilogram of ice at 0^oC to produce a kilogram of water at 0^oC. Using the equation for a change in temperature and the value for water from Table 8.4.1, we find that

$$Q = mL_f = (1.0\,kg)(334\,kJ/kg) = 334\,kJ \tag{8.4.3}$$

is the energy to melt a kilogram of ice. This is a lot of energy as it represents the same amount of energy needed to raise the temperature of 1 kg of liquid water from 0^oC to 79.8^oC. Even more energy is required to vaporize water; it would take 2256 kJ to change 1 kg of liquid water at the normal boiling point (100^oC at atmospheric pressure) to steam (water vapor). This example shows that the energy for a phase change is enormous compared to energy associated with temperature changes without a phase change.

Table 8.4.1: Heats of Fusion and Vaporization 1

		L_f			L_v	
Substance	**Melting point (°C)**	**kJ/kg**	**kcal/kg**	**Boiling point (°C)**	**kJ/kg**	**kcal/kg**
Helium	−269.7	5.23	1.25	−268.9	20.9	4.99
Hydrogen	−259.3	58.6	14.0	−252.9	452	108

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	L_f			L_v		
Nitrogen	−210.0	25.5	6.09	−195.8	201	48.0
Oxygen	−218.8	13.8	3.30	−183.0	213	50.9
Ethanol	−114	104	24.9	78.3	854	204
Ammonia	−75		108	−33.4	1370	327
Mercury	−38.9	11.8	2.82	357	272	65.0
Water	0.00	334	79.8	100.0	2256[2]	539[3]
Sulfur	119	38.1	9.10	444.6	326	77.9
Lead	327	24.5	5.85	1750	871	208
Antimony	631	165	39.4	1440	561	134
Aluminum	660	380	90	2450	11400	2720
Silver	961	88.3	21.1	2193	2336	558
Gold	1063	64.5	15.4	2660	1578	377
Copper	1083	134	32.0	2595	5069	1211
Uranium	1133	84	20	3900	1900	454
Tungsten	3410	184	44	5900	4810	1150

Phase changes can have a tremendous stabilizing effect even on temperatures that are not near the melting and boiling points, because evaporation and condensation (conversion of a gas into a liquid state) occur even at temperatures below the boiling point.Take, for example, the fact that air temperatures in humid climates rarely go above 35.0^oC,

which is because most heat transfer goes into evaporating water into the air. Similarly, temperatures in humid weather rarely fall below the dew point because enormous heat is released when water vapor condenses.

We examine the effects of phase change more precisely by considering adding heat into a sample of ice at -20^oC (Figure $8.4.3$). The temperature of the ice rises linearly, absorbing heat at a constant rate of $0.50\, cal/g\cdot ^o C$ until it reaches 0^oC. Once at this temperature, the ice begins to melt until all the ice has melted, absorbing 79.8 cal/g of heat. The temperature remains constant at 0^oC during this phase change. Once all the ice has melted, the temperature of the liquid water rises, absorbing heat at a new constant rate of $1.00\, cal/g\cdot ^o C$. At 100^oC, the water begins to boil and the temperature again remains constant while the water absorbs 539 cal/g of heat during this phase change. When all the liquid has become steam vapor, the temperature rises again, absorbing heat at a rate of $0.482\, cal/g\cdot ^o C$.

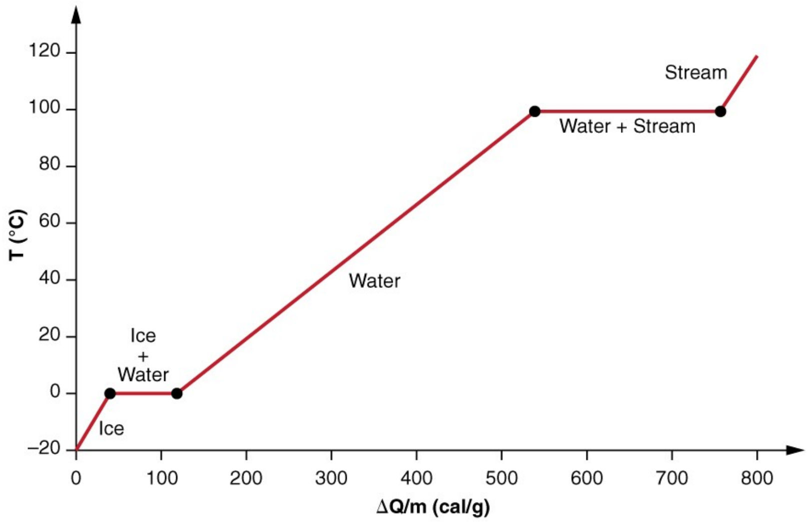


Figure 8.4.3. A graph of temperature versus energy added. The system is constructed so that no vapor evaporates while ice warms to become liquid water, and so that, when vaporization occurs, the vapor remains in of the system. The long stretches of constant temperature values at 0^oC and 100^oC reflect the large latent heat of melting and vaporization, respectively.

Water can evaporate at temperatures below the boiling point. More energy is required than at the boiling point, because the kinetic energy of water molecules at temperatures below 100^oC is less than that at 100^oC, hence less energy is available from random thermal motions. Take, for example, the fact that, at body temperature, perspiration from the skin requires a heat input of 2428 kJ/kg, which is about 10 percent higher than the latent heat of vaporization at 100^oC. This heat comes from the skin, and thus provides an effective cooling mechanism in hot weather. High humidity inhibits evaporation, so that body temperature might rise, leaving unevaporated sweat on your brow.

Example 8.4.1: Calculate Final Temperature from Phase Change: Cooling Soda with Ice Cubes

Three ice cubes are used to chill a soda at 20^oC with mass $m_{soda} = 0.25\ kg$. The ice is at 0^oC and each ice cube has a mass of 6.0 g. Assume that the soda is kept in a foam container so that heat loss can be ignored. Assume the soda has the same heat capacity as water. Find the final temperature when all ice has melted.

Strategy

The ice cubes are at the melting temperature of 0^oC. Heat is transferred from the soda to the ice for melting. Melting of ice occurs in two steps: first the phase change occurs and solid (ice) transforms into liquid water at the melting temperature, then the temperature of this water rises. Melting yields water at 0^oC, so more heat is transferred from the soda to this water until the water plus soda system reaches thermal equilibrium,

$$Q_{ice} = -Q_{soda}. \tag{8.4.4}$$

The heat transferred to the ice is $Q_{ice} = m_{ice}L_f + m_{ice}c_w(T_f - 0^oC)$. . The heat given off by the soda is $Q_{soda} = m_{soda}c_w(T_f - 20^oC)$. Since no heat is lost, $Q_{ice} = -Q_{soda}$, so that

$$m_{ice}Lf + m_{ice}c_w(T_f - 0^oC) = -m_{soda}c_w(T_f - 20^oC). \tag{8.4.5}$$

Bring all terms involving T_f on the left-hand-side and all other terms on the right-hand-side. Solve for the unknown quantity T_f:

$$T_f = \frac{m_{soda}c_w(20^oC) - m_{ice}L_f}{(m_{soda} + m_{ice})c_w}. \tag{8.4.6}$$

Solution

1. Identify the known quantities. The mass of ice is $m_{ice} = 3 \times 6.0\ g = 0.018\ kg$ and the mass of soda is $m_{soda} = 0.25\ kg$.
2. Calculate the terms in the numerator:

$$m_{soda}c_w(20^oC) = (0.25\ kg)(4186\ J/kg \cdot^o C)(20^oC) = 20,930\ J \tag{8.4.7}$$

and

$$m_{ice}L_f = (0.018\,kg)(334,000\,J/kg) = 6012\,J. \tag{8.4.8}$$

3. Calculate the denominator:

$$(m_{soda} + m_{ice})c_w = (0.25\,kg + 0.018\,kg)(4186\,J/kg \cdot {}^o C) = 1122\,J/{}^oC \tag{8.4.9}$$

4. Calculate the final temperature:

$$T_f = \frac{20,930\,J - 6012\,J}{1122\,J/{}^oC} = 13^oC. \tag{8.4.10}$$

Discussion

This example illustrates the enormous energies involved during a phase change. The mass of ice is about 7 percent the mass of water but leads to a noticeable change in the temperature of soda. Although we assumed that the ice was at the freezing temperature, this is incorrect: the typical temperature is -6^oC. However, this correction gives a final temperature that is essentially identical to the result we found. Can you explain why?

We have seen that vaporization requires heat transfer to a liquid from the surroundings, so that energy is released by the surroundings. Condensation is the reverse process, increasing the temperature of the surroundings. This increase may seem surprising, since we associate condensation with cold objects—the glass in the figure, for example. However, energy must be removed from the condensing molecules to make a vapor condense. The energy is exactly the same as that required to make the phase change in the other direction, from liquid to vapor, and so it can be calculated from $Q = mL_v$.

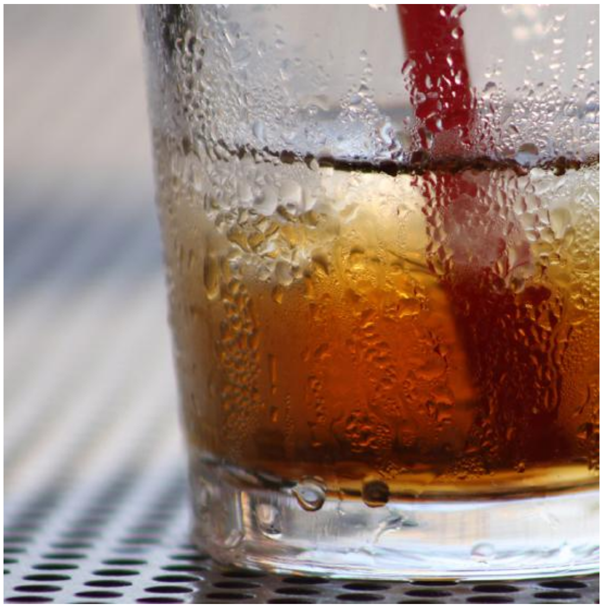


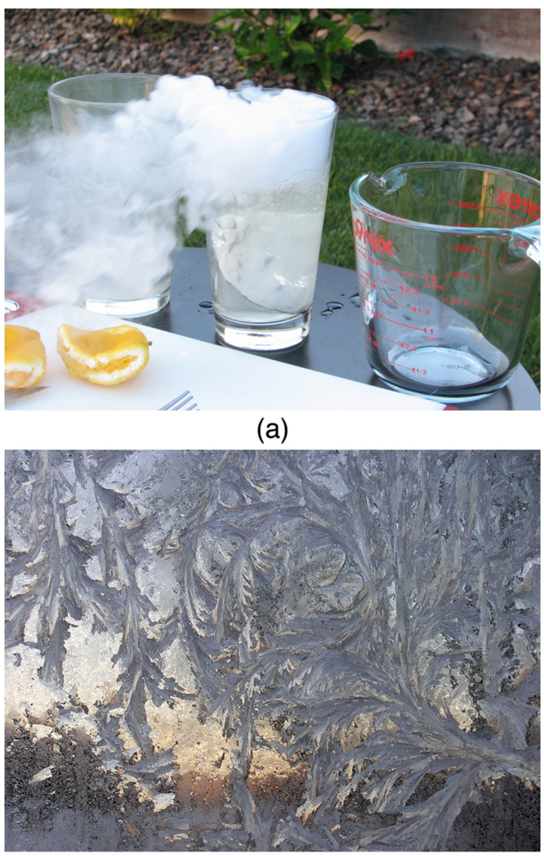
Figure 8.4.4. Condensation forms on this glass of iced tea because the temperature of the nearby air is reduced to below the dew point. The rate at which water molecules join together exceeds the rate at which they separate, and so water condenses. Energy is released when the water condenses, speeding the melting of the ice in the glass. (credit: Jenny Downing)

Real-World Application

Energy is also released when a liquid freezes. This phenomenon is used by fruit growers in Florida to protect oranges when the temperature is close to the freezing point (0^oC). Growers spray water on the plants in orchards so that the water freezes and heat is released to the growing oranges on the trees. This prevents the temperature inside the orange from dropping below freezing, which would damage the fruit.

Figure 8.4.5. The ice on these trees released large amounts of energy when it froze, helping to prevent the temperature of the trees from dropping below $0^{o}C$. Water is intentionally sprayed on orchards to help prevent hard frosts. (credit: Hermann Hammer)

Sublimation is the transition from solid to vapor phase. You may have noticed that snow can disappear into thin air without a trace of liquid water, or the disappearance of ice cubes in a freezer. The reverse is also true: Frost can form on very cold windows without going through the liquid stage. A popular effect is the making of "smoke" from dry ice, which is solid carbon dioxide. Sublimation occurs because the equilibrium vapor pressure of solids is not zero. Certain air fresheners use the sublimation of a solid to inject a perfume into the room. Moth balls are a slightly toxic example of a phenol (an organic compound) that sublimates, while some solids, such as osmium tetroxide, are so toxic that they must be kept in sealed containers to prevent human exposure to their sublimation-produced vapors.



(a)

(b)

Figure 8.4.6. Direct transitions between solid and vapor are common, sometimes useful, and even beautiful. (a) Dry ice sublimates directly to carbon dioxide gas. The visible vapor is made of water droplets. (credit: Windell Oskay) (b) Frost forms patterns on a very cold window, an example of a solid formed directly from a vapor. (credit: Liz West)

All phase transitions involve heat. In the case of direct solid-vapor transitions, the energy required is given by the equation $Q = mL_s$, where L_s is the heat of sublimation, which is the energy required to change 1.00 kg of a substance from the solid phase to the vapor phase. L_s is analogous to L_f and L_v, and its value depends on the substance. Sublimation requires energy input, so that dry ice is an effective coolant, whereas the reverse process (i.e., frosting) releases energy. The amount of energy required for sublimation is of the same order of magnitude as that for other phase transitions.

The material presented in this section and the preceding section allows us to calculate any number of effects related to temperature and phase change. In each case, it is necessary to identify which temperature and phase changes are taking place and then to apply the appropriate equation. Keep in mind that heat transfer and work can cause both temperature and phase changes.

Problem-Solving Strategies for the Effects of Heat Transfer

1. *Examine the situation to determine that there is a change in the temperature or phase. Is there heat transfer into or out of the system?* When the presence or absence of a phase change is not obvious, you may wish to first solve the problem as if there were no phase changes, and examine the temperature change obtained. If it is sufficient to take you past a boiling or melting point, you should then go back and do the problem in steps—temperature change, phase change, subsequent temperature change, and so on.
2. *Identify and list all objects that change temperature and phase.*
3. *Identify exactly what needs to be determined in the problem (identify the unknowns).* A written list is useful.
4. *Make a list of what is given or what can be inferred from the problem as stated (identify the knowns).*
5. *Solve the appropriate equation for the quantity to be determined (the unknown).* If there is a temperature change, the transferred heat depends on the specific heat (see Temperature Change and Heat Capacity) whereas, for a phase change, the transferred heat depends on the latent heat. See Table $8.4.1$.
6. *Substitute the knowns along with their units into the appropriate equation and obtain numerical solutions complete with units.* You will need to do this in steps if there is more than one stage to the process (such as a temperature change followed by a phase change).
7. *Check the answer to see if it is reasonable: Does it make sense?* As an example, be certain that the temperature change does not also cause a phase change that you have not taken into account.

Exercise $8.4.1$

Why does snow remain on mountain slopes even when daytime temperatures are higher than the freezing temperature?

Answer

Snow is formed from ice crystals and thus is the solid phase of water. Because enormous heat is necessary for phase changes, it takes a certain amount of time for this heat to be accumulated from the air, even if the air is above 0^oC. The warmer the air is, the faster this heat exchange occurs and the faster the snow melts.

Summary

- Most substances can exist either in solid, liquid, and gas forms, which are referred to as “phases.”
- Phase changes occur at fixed temperatures for a given substance at a given pressure, and these temperatures are called boiling and freezing (or melting) points.
- During phase changes, heat absorbed or released is given by $Q = mL$, where L is the latent heat coefficient.

Footnotes

1 Values quoted at the normal melting and boiling temperatures at standard atmospheric pressure (1 atm).
2 At 37.0ºC (body temperature), the heat of vaporization L_v for water is 2430 kJ/kg or 580 kcal/kg
3 At 37.0ºC (body temperature), the heat of vaporization L_v for water is 2430 kJ/kg or 580 kcal/kg

Glossary

heat of sublimation
the energy required to change a substance from the solid phase to the vapor phase

latent heat coefficient

a physical constant equal to the amount of heat transferred for every 1 kg of a substance during the change in phase of the substance

sublimation

the transition from the solid phase to the vapor phase

Contributors and Attributions

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8.5: Heat Transfer Methods

Learning Objectives

By the end of this section, you will be able to:

- Discuss the different methods of heat transfer.

Equally as interesting as the effects of heat transfer on a system are the methods by which this occurs. Whenever there is a temperature difference, heat transfer occurs. Heat transfer may occur rapidly, such as through a cooking pan, or slowly, such as through the walls of a picnic ice chest. We can control rates of heat transfer by choosing materials (such as thick wool clothing for the winter), controlling air movement (such as the use of weather stripping around doors), or by choice of color (such as a white roof to reflect summer sunlight). So many processes involve heat transfer, so that it is hard to imagine a situation where no heat transfer occurs. Yet every process involving heat transfer takes place by only three methods:

1. **Conduction** is heat transfer through stationary matter by physical contact. (The matter is stationary on a macroscopic scale—we know there is thermal motion of the atoms and molecules at any temperature above absolute zero.) Heat transferred between the electric burner of a stove and the bottom of a pan is transferred by conduction.
2. **Convection** is the heat transfer by the macroscopic movement of a fluid. This type of transfer takes place in a forced-air furnace and in weather systems, for example.
3. Heat transfer by **radiation** occurs when microwaves, infrared radiation, visible light, or another form of electromagnetic radiation is emitted or absorbed. An obvious example is the warming of the Earth by the Sun. A less obvious example is thermal radiation from the human body.

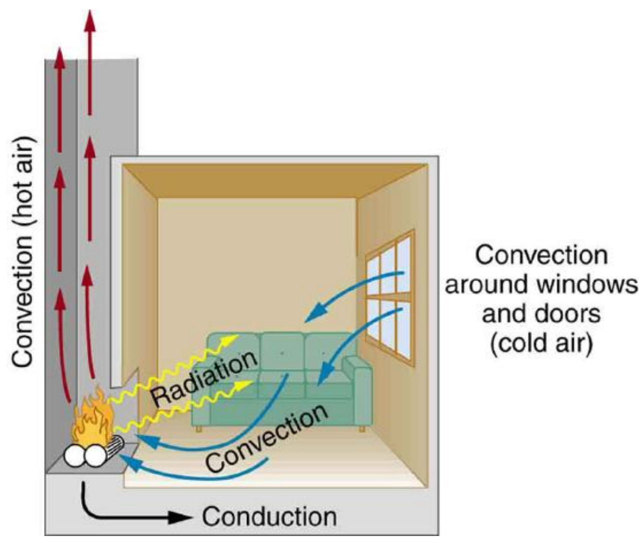

Figure 8.5.1: In a fireplace, heat transfer occurs by all three methods: conduction, convection, and radiation. Radiation is responsible for most of the heat transferred into the room. Heat transfer also occurs through conduction into the room, but at a much slower rate. Heat transfer by convection also occurs through cold air entering the room around windows and hot air leaving the room by rising up the chimney.

We examine these methods in some detail in the three following modules. Each method has unique and interesting characteristics, but all three do have one thing in common: they transfer heat solely because of a temperature difference Figure $8.5.1$.

Exercise $8.5.1$

Name an example from daily life (different from the text) for each mechanism of heat transfer.

Answer

- Conduction: Heat transfers into your hands as you hold a hot cup of coffee.
- Convection: Heat transfers as the barista “steams” cold milk to make hot cocoa.
- Radiation: Reheating a cold cup of coffee in a microwave oven.

Summary

- Heat is transferred by three different methods: conduction, convection, and radiation.

Glossary

conduction

heat transfer through stationary matter by physical contact

convection

heat transfer by the macroscopic movement of fluid

radiation

heat transfer which occurs when microwaves, infrared radiation, visible light, or other electromagnetic radiation is emitted or absorbed

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8.6: Conduction

Learning Objectives

By the end of this section, you will be able to:

- Calculate thermal conductivity.
- Observe conduction of heat in collisions.
- Study thermal conductivities of common substances.

Your feet feel cold as you walk barefoot across the living room carpet in your cold house and then step onto the kitchen tile floor. This result is intriguing, since the carpet and tile floor are both at the same temperature. The different sensation you feel is explained by the different rates of heat transfer: the heat loss during the same time interval is greater for skin in contact with the tiles than with the carpet, so the temperature drop is greater on the tiles.

Figure 8.6.1: Insulation is used to limit the conduction of heat from the inside to the outside (in winters) and from the outside to the inside (in summers). (credit: Giles Douglas)

Some materials conduct thermal energy faster than others. In general, good conductors of electricity (metals like copper, aluminum, gold, and silver) are also good heat conductors, whereas insulators of electricity (wood, plastic, and rubber) are poor heat conductors. Figure 8.6.2 shows molecules in two bodies at different temperatures. The (average) kinetic energy of a molecule in the hot body is higher than in the colder body. If two molecules collide, an energy transfer from the molecule with greater kinetic energy to the molecule with less kinetic energy occurs. The cumulative effect from all collisions results in a net flux of heat from the hot body to the colder body. The heat flux thus depends on the temperature difference $\Delta T = T_{hot} - T_{cold}$ Therefore, you will get a more severe burn from boiling water than from hot tap water. Conversely, if the temperatures are the same, the net heat transfer rate falls to zero, and equilibrium is achieved. Owing to the fact that the number of collisions increases with increasing area, heat conduction depends on the cross-sectional area. If you touch a cold wall with your palm, your hand cools faster than if you just touch it with your fingertip.

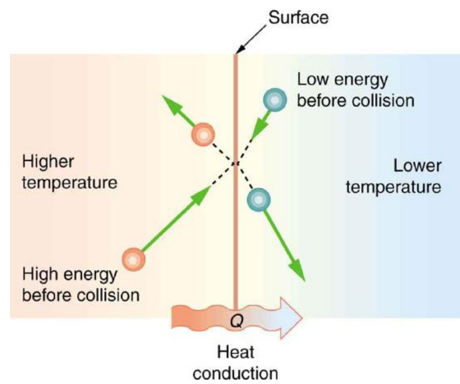

Figure 8.6.2: The molecules in two bodies at different temperatures have different average kinetic energies. Collisions occurring at the contact surface tend to transfer energy from high-temperature regions to low-temperature regions. In this illustration, a molecule in the lower temperature region (right side) has low energy before collision, but its energy increases after colliding with the contact surface. In contrast, a molecule in the higher temperature region (left side) has high energy before collision, but its energy decreases after colliding with the contact surface.

A third factor in the mechanism of conduction is the thickness of the material through which heat transfers. The figure below shows a slab of material with different temperatures on either side. Suppose that T_2 is greater than T_1 so that heat is transferred from left to right. Heat transfer from the left side to the right side is accomplished by a series of molecular collisions. The thicker the material, the more time it takes to transfer the same amount of heat. This model explains why thick clothing is warmer than thin clothing in winters, and why Arctic mammals protect themselves with thick blubber.

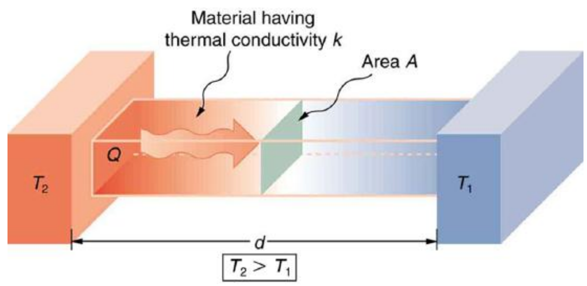

Figure 8.6.3: Heat conduction occurs through any material, represented here by a rectangular bar, whether window glass or walrus blubber. The temperature of the material is T_2 on the left and T_1 on the right, where T_2 is greater than T_1 The rate of heat transfer by conduction is directly proportional to the surface area A, the temperature difference $T_2 - T_1$, and the substance's conductivity k. The rate of heat transfer is inversely proportional to the thickness d.

Lastly, the heat transfer rate depends on the material properties described by the coefficient of **thermal conductivity**. All four factors are included in a simple equation that was deduced from and is confirmed by experiments. The rate of conductive heat transfer through a slab of material, such as the one in Figure $8.6.3$, is given by

$$\frac{Q}{t} = \frac{kA(T_2 - T_1)}{d}, \tag{8.6.1}$$

where Q/t is the rate of heat transfer in watts or kilocalories per second, k s the thermal conductivity of the material, A and d are its surface area and thickness and $(T_2 - T_1)$ is the temperature difference across the slab. Table $8.6.1$gives representative values of thermal conductivity.

Example $8.6.1$: Calculating Heat Transfer Through Conduction: Conduction Rate Through an Ice Box

A Styrofoam ice box has a total area of $0.950\,m^2$ and walls with an average thickness of 2.50 cm. The box contains ice, water, and canned beverages at 0^oC. The inside of the box is kept cold by melting ice. How much ice melts in one day if the ice box is kept in the trunk of a car at 35.0^oC?

Strategy

This question involves both heat for a phase change (melting of ice) and the transfer of heat by conduction. To find the amount of ice melted, we must find the net heat transferred. This value can be obtained by calculating the rate of heat transfer by conduction and multiplying by time.

Solution

1. Identify the knowns.

$$A = 0.950\, m^2;\ d = 2.50\, cm = 0.0250\, m;\ T_1 = 35.0^oC,\ t = 1\ day = 24\ hours = 86,400\ sec. \tag{8.6.2}$$

2. Identify the unknowns. We need to solve for the mass of the ice m. We will also need to solve for the net heat transferred to melt the ice, Q.
3. Determine which equations to use. The rate of heat transfer by conduction is given by

$$\frac{Q}{t} = \frac{kA(T_2 - T_1)}{d}. \tag{8.6.3}$$

4. The heat is used to melt the ice: $Q = mL_f$.
5. Insert the known values:

$$\frac{Q}{t} = \frac{(0.010\, J/s \cdot m \cdot^o C)(0.950\, m^2)(35.0^oC - 0^oC)}{0.0250\, m} = 13.3\, J/s. \tag{8.6.4}$$

6. Multiply the rate of heat transfer by the time ($1\ day = 86,400 s$):

$$Q = (Q/t)t = (13.3\, J/s)(86,400\ s) = 1.15 \times 10^6\ J. \tag{8.6.5}$$

7. Set this equal to the heat transferred to melt the ice: $Q = mL_f$. Solve for the mass m:

$$m = \frac{Q}{L_f} = \frac{1.15 \times 10^6\ J}{334 \times 10^3\ J/kg} = 3.44\ kg. \tag{8.6.6}$$

Discussion

The result of 3.44 kg, or about 7.6 lbs, seems about right, based on experience. You might expect to use about a 4 kg (7–10 lb) bag of ice per day. A little extra ice is required if you add any warm food or beverages.

Inspecting the conductivities in Table 8.6.1 shows that Styrofoam is a very poor conductor and thus a good insulator. Other good insulators include fiberglass, wool, and goose-down feathers. Like Styrofoam, these all incorporate many small pockets of air, taking advantage of air's poor thermal conductivity.

Table 8.6.1: Thermal Conductivities of Common Substances

Substance	Thermal conductivity
Air	0.023
Aluminum	220
Asbestos	0.16
Concrete brick	0.84
Copper	390
Cork	0.042
Down feathers	0.025
Fatty tissue (without blood)	0.2
Glass (average)	0.84
Glass wool	0.042
Gold	318
Ice	2.2
Plasterboard	0.16
Silver	420

Substance	Thermal conductivity
Snow (dry)	0.10
Steel (stainless)	14
Steel iron	80
Styrofoam	0.010
Water	0.6
Wood	0.08–0.16
Wool	0.04

Thermal Conductivities of Common Substances$_1$

A combination of material and thickness is often manipulated to develop good insulators—the smaller the conductivity k and the larger the thickness d, the better. The ratio of d/k will thus be large for a good insulator. The ratio d/k is called the R **factor.** The rate of conductive heat transfer is inversely proportional to R. The larger the value of R, the better the insulation. R factors are most commonly quoted for household insulation, refrigerators, and the like—unfortunately, it is still in non-metric units of $ft^2 \cdot^o F \cdot h/Btu$, although the unit usually goes unstated (1 British thermal unit [Btu] is the amount of energy needed to change the temperature of 1.0 lb of water by 1.0 °F). A couple of representative values are an R factor of 11 for 3.5-in-thick fiberglass batts (pieces) of insulation and an R factor of 19 for 6.5-in-thick fiberglass batts. Walls are usually insulated with 3.5-in batts, while ceilings are usually insulated with 6.5-in batts. In cold climates, thicker batts may be used in ceilings and walls.

Figure 8.6.4:The fiberglass batt is used for insulation of walls and ceilings to prevent heat transfer between the inside of the building and the outside environment. (CC BY-SA 3.0; Radomil).

Note that in Table 8.6.1, the best thermal conductors—silver, copper, gold, and aluminum—are also the best electrical conductors, again related to the density of free electrons in them. Cooking utensils are typically made from good conductors.

Example 8.6.1: Calculating the Temperature Difference Maintained by a Heat Transfer: Conduction Through an Aluminum Pan

Water is boiling in an aluminum pan placed on an electrical element on a stovetop. The sauce pan has a bottom that is 0.800 cm thick and 14.0 cm in diameter. The boiling water is evaporating at the rate of 1.00 g/s. What is the temperature difference across (through) the bottom of the pan?

Strategy

Conduction through the aluminum is the primary method of heat transfer here, and so we use the equation for the rate of heat transfer and solve for the temperature difference.

$$T_2 - T_1 = \frac{Q}{t}\left(\frac{d}{kA}\right). \tag{8.6.7}$$

Solution

1. Identify the knowns and convert them to the SI units.

   The thickness of the pan, $d = 0.800\,cm = 8.0 \times 10^{-3} m$, the area of the pan, $A = \pi(0.14/2)^2\,m^2 = 1.54 \times 10^{-2}\,m^2$, and the thermal conductivity, $k = 220\,J/s \cdot m \cdot^o C$.
2. Calculate the necessary heat of vaporization of 1 g of water:

$$Q = mL_v = (1.00 \times 10^{-3}\ kg)(2256 \times 10^3\ J/kg) = 2256\ J. \tag{8.6.8}$$

3. Calculate the rate of heat transfer given that 1 g of water melts in one second:

$$Q/t = 2256\ J/s\ or 2.26 kW. \tag{8.6.9}$$

4. Insert the knowns into the equation and solve for the temperature difference:

$$T_2 - T_1 = \frac{Q}{t}\left(\frac{d}{kA}\right) = (2256\ J/s)\frac{8.00 \times 10^{-3} m}{(220\ J/s \cdot m \cdot {}^o C)(1.54 \times 10^{-2}\ m^2)} = 5.33^o C. \tag{8.6.10}$$

Discussion

The value for the heat transfer $Q/t = 2.26\ kW\ or 2256\ J/s$ is typical for an electric stove. This value gives a remarkably small temperature difference between the stove and the pan. Consider that the stove burner is red hot while the inside of the pan is nearly $100^o C$ because of its contact with boiling water. This contact effectively cools the bottom of the pan in spite of its proximity to the very hot stove burner. Aluminum is such a good conductor that it only takes this small temperature difference to produce a heat transfer of 2.26 kW into the pan.

Conduction is caused by the random motion of atoms and molecules. As such, it is an ineffective mechanism for heat transport over macroscopic distances and short time distances. Take, for example, the temperature on the Earth, which would be unbearably cold during the night and extremely hot during the day if heat transport in the atmosphere was to be only through conduction. In another example, car engines would overheat unless there was a more efficient way to remove excess heat from the pistons.

Exercise 8.6.1: Check your understanding

How does the rate of heat transfer by conduction change when all spatial dimensions are doubled?

Answer

Because area is the product of two spatial dimensions, it increases by a factor of four when each dimension is doubled $(A_{final} = (2d)^2 = 4d^2 = 4A_{initial})$. The distance, however, simply doubles. Because the temperature difference and the coefficient of thermal conductivity are independent of the spatial dimensions, the rate of heat transfer by conduction increases by a factor of four divided by two, or two:

$$\left(\frac{Q}{t}\right)_{final} = \frac{kA_{final}(T_2 - T_1)}{d_{final}} = \frac{k(4A_{initial})(T_2 - T_1)}{2d_{initial}} = 2\frac{kA_{initial}(T_2 - T_1)}{d_{initial}} = 2\left(\frac{Q}{t}\right)_{initial} \tag{8.6.11}$$

Summary

Heat conduction is the transfer of heat between two objects in direct contact with each other. The rate of heat transfer Q/t (energy per unit time) is proportional to the temperature difference $T_2 - T_1$ and the contact area A and inversely proportional to the distance between the objects:

$$\frac{Q}{t} = \frac{kA(T_2 - T_1)}{d}. \tag{8.6.12}$$

Footnotes

1. At temperatures near 0ºC.

Glossary

***R* factor**

the ratio of thickness to the conductivity of a material

rate of conductive heat transfer

rate of heat transfer from one material to another

thermal conductivity
: the property of a material’s ability to conduct heat

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8.7: Convection

Learning Objectives

By the end of this section, you will be able to:

- Discuss the method of heat transfer by convection.

Convection is driven by large-scale flow of matter. In the case of Earth, the atmospheric circulation is caused by the flow of hot air from the tropics to the poles, and the flow of cold air from the poles toward the tropics. (Note that Earth's rotation causes the observed easterly flow of air in the northern hemisphere). Car engines are kept cool by the flow of water in the cooling system, with the water pump maintaining a flow of cool water to the pistons. The circulatory system is used the body: when the body overheats, the blood vessels in the skin expand (dilate), which increases the blood flow to the skin where it can be cooled by sweating. These vessels become smaller when it is cold outside and larger when it is hot (so more fluid flows, and more energy is transferred).

The body also loses a significant fraction of its heat through the breathing process.

While convection is usually more complicated than conduction, we can describe convection and do some straightforward, realistic calculations of its effects. Natural convection is driven by buoyant forces: hot air rises because density decreases as temperature increases. The house in Figure $8.7.1$ is kept warm in this manner, as is the pot of water on the stove in Figure $8.7.2$. Ocean currents and large-scale atmospheric circulation transfer energy from one part of the globe to another. Both are examples of natural convection.

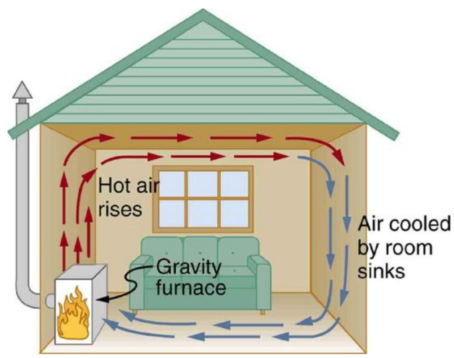

Figure $8.7.1$: Air heated by the so-called gravity furnace expands and rises, forming a convective loop that transfers energy to other parts of the room. As the air is cooled at the ceiling and outside walls, it contracts, eventually becoming denser than room air and sinking to the floor. A properly designed heating system using natural convection, like this one, can be quite efficient in uniformly heating a home.

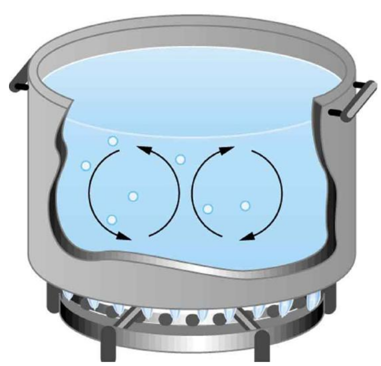

Figure $8.7.2$: Convection plays an important role in heat transfer inside this pot of water. Once conducted to the inside, heat transfer to other parts of the pot is mostly by convection. The hotter water expands, decreases in density, and rises to transfer heat to other regions of the water, while colder water sinks to the bottom. This process keeps repeating.

Take-Home Experiment: Convection Rolls in a Heated Pan

Take two small pots of water and use an eye dropper to place a drop of food coloring near the bottom of each. Leave one on a bench top and heat the other over a stovetop. Watch how the color spreads and how long it takes the color to reach the top. Watch how convective loops form.

Example 8.7.1: Calculating Heat Transfer by Convection: Convection of Air Through the Walls of a House

Most houses are not airtight: air goes in and out around doors and windows, through cracks and crevices, following wiring to switches and outlets, and so on. The air in a typical house is completely replaced in less than an hour. Suppose that a moderately-sized house has inside dimensions $12.0m \times 18.0m \times 3.00m$ high, and that all air is replaced in 30.0 min. Calculate the heat transfer per unit time in watts needed to warm the incoming cold air by 10.0^oC thus replacing the heat transferred by convection alone.

Strategy

Heat is used to raise the temperature of air so that $Q = mc\Delta T$. The rate of heat transfer is then Q/t, where t is the time for air turnover. We are given that ΔT is 10.0^oC, but we must still find values for the mass of air and its specific heat before we can calculate Q. The specific heat of air is a weighted average of the specific heats of nitrogen and oxygen, which gives $c = c_p \approx 1000\ J/kg \cdot^o C$ from the table (note that the specific heat at constant pressure must be used for this process).

Solution

1. Determine the mass of air from its density and the given volume of the house. The density is given from the density ρ and the volume

$$m = \rho V = (1.29\ kg/m^3)912.0\ m \times 18.0\ m \times 3.00\ m) = 836\ kg \tag{8.7.1}$$

2. Calculate the heat transferred from the change in air temperature $Q = mc\Delta T$ so that

$$Q = (836\ kg)(1000\ J/kg \cdot^o C)(10.0^oC) = 8.36 \times 10^6\ J. \tag{8.7.2}$$

3. Calculate the heat transfer from the heat Q and the turnover time t. Since air is turned over in $t = 0.500\ h = 1800\ s$, the heat transferred per unit time is

$$\frac{Q}{t} = \frac{8.36 \times 10^6\ J}{1800\ s} = 4.64\ kW. \tag{8.7.3}$$

Discussion

This rate of heat transfer is equal to the power consumed by about forty-six 100-W light bulbs. Newly constructed homes are designed for a turnover time of 2 hours or more, rather than 30 minutes for the house of this example. Weather stripping, caulking, and improved window seals are commonly employed. More extreme measures are sometimes taken in very cold (or hot) climates to achieve a tight standard of more than 6 hours for one air turnover. Still longer turnover times are unhealthy, because a minimum amount of fresh air is necessary to supply oxygen for breathing and to dilute household pollutants. The term used for the process by which outside air leaks into the house from cracks around windows, doors, and the foundation is called “air infiltration.”

A cold wind is much more chilling than still cold air, because convection combines with conduction in the body to increase the rate at which energy is transferred away from the body. The table below gives approximate wind-chill factors, which are the temperatures of still air that produce the same rate of cooling as air of a given temperature and speed. Wind-chill factors are a dramatic reminder of convection’s ability to transfer heat faster than conduction. For example, a 15.0 m/s wind at 0^oC has the chilling equivalent of still air at about -18^oC.

Moving air temperature Wind speed (m/s)

^oC	2	5	10	15	20
5	3	-1	-8	-10	-12
2	0	-7	-12	-16	-18

^oC	2	5	10	15	20
0	-2	-9	-15	-18	-20
-5	-7	-15	-22	-26	-29
-10	-12	-21	-29	-34	-36
-20	-23	-34	-44	-50	-52
-40	-44	-59	-73	-82	-84

Although air can transfer heat rapidly by convection, it is a poor conductor and thus a good insulator. The amount of available space for airflow determines whether air acts as an insulator or conductor. The space between the inside and outside walls of a house, for example, is about 9 cm (3.5 in) —large enough for convection to work effectively. The addition of wall insulation prevents airflow, so heat loss (or gain) is decreased. Similarly, the gap between the two panes of a double-paned window is about 1 cm, which prevents convection and takes advantage of air's low conductivity to prevent greater loss. Fur, fiber, and fiberglass also take advantage of the low conductivity of air by trapping it in spaces too small to support convection, as shown in the figure. Fur and feathers are lightweight and thus ideal for the protection of animals.

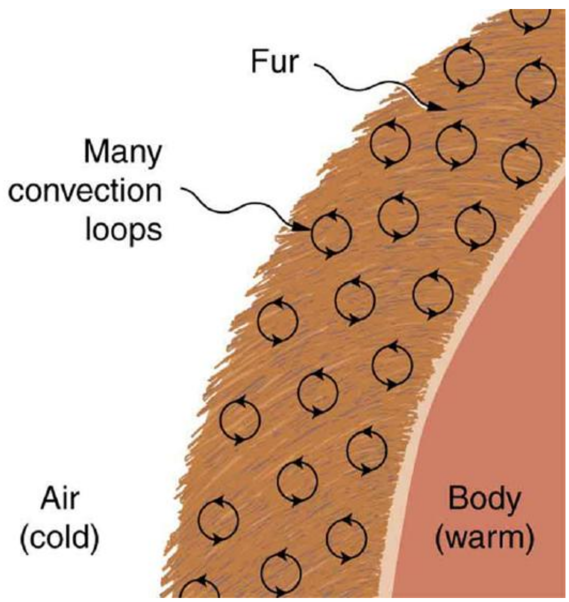

Figure $8.7.3$: Fur is filled with air, breaking it up into many small pockets. Convection is very slow here, because the loops are so small. The low conductivity of air makes fur a very good lightweight insulator.

Some interesting phenomena happen *when convection is accompanied by a phase change*. It allows us to cool off by sweating, even if the temperature of the surrounding air exceeds body temperature. Heat from the skin is required for sweat to evaporate from the skin, but without air flow, the air becomes saturated and evaporation stops. Air flow caused by convection replaces the saturated air by dry air and evaporation continues.

Example $8.7.2$: Calculate the Flow of Mass during Convection: Sweat-Heat Transfer away from the Body

The average person produces heat at the rate of about 120 W when at rest. At what rate must water evaporate from the body to get rid of all this energy? (This evaporation might occur when a person is sitting in the shade and surrounding temperatures are the same as skin temperature, eliminating heat transfer by other methods.)

Strategy

Energy is needed for a phase change $(Q = mL_v)$. Thus, the energy loss per unit time is

$$\frac{Q}{t} = \frac{mL_v}{t} = 120\,W = 120\,J/s. \tag{8.7.4}$$

We divide both sides of the equation by L_v to find that the mass evaporated per unit time is

$$\frac{m}{t} = \frac{120\ J/s}{L_v}. \tag{8.7.5}$$

Solution

(1) Insert the value of the latent heat from [link], $L_v = 2430\ kJ/kg = 2430\ J/g$. This yields

$$\frac{m}{t} = \frac{120\ J/s}{2430\ J/g} = 0.0494\ g/s = 2.96\ g/min. \tag{8.7.6}$$

Discussion

Evaporating about 3 g/min seems reasonable. This would be about 180 g (about 7 oz) per hour. If the air is very dry, the sweat may evaporate without even being noticed. A significant amount of evaporation also takes place in the lungs and breathing passages.

Another important example of the combination of phase change and convection occurs when water evaporates from the oceans. Heat is removed from the ocean when water evaporates. If the water vapor condenses in liquid droplets as clouds form, heat is released in the atmosphere. Thus, there is an overall transfer of heat from the ocean to the atmosphere. This process is the driving power behind thunderheads, those great cumulus clouds that rise as much as 20.0 km into the stratosphere. Water vapor carried in by convection condenses, releasing tremendous amounts of energy. This energy causes the air to expand and rise, where it is colder. More condensation occurs in these colder regions, which in turn drives the cloud even higher. Such a mechanism is called positive feedback, since the process reinforces and accelerates itself. These systems sometimes produce violent storms, with lightning and hail, and constitute the mechanism driving hurricanes.

Figure 8.7.4: Cumulus clouds are caused by water vapor that rises because of convection. The rise of clouds is driven by a positive feedback mechanism. (credit: Mike Love)

Figure 8.7.5: Convection accompanied by a phase change releases the energy needed to drive this thunderhead into the stratosphere. (credit: Gerardo García Moretti)

Figure 8.7.6: The phase change that occurs when this iceberg melts involves tremendous heat transfer. (credit: Dominic Alves)

The movement of icebergs is another example of convection accompanied by a phase change. Suppose an iceberg drifts from Greenland into warmer Atlantic waters. Heat is removed from the warm ocean water when the ice melts and heat is released to the land mass when the iceberg forms on Greenland.

Exercise 8.7.1

Explain why using a fan in the summer feels refreshing!

Answer

Using a fan increases the flow of air: warm air near your body is replaced by cooler air from elsewhere. Convection increases the rate of heat transfer so that moving air "feels" cooler than still air.

Summary

- Convection is heat transfer by the macroscopic movement of mass. Convection can be natural or forced and generally transfers thermal energy faster than conduction. The table gives wind-chill factors, indicating that moving air has the same chilling effect of much colder stationary air. *Convection that occurs along with a phase change* can transfer energy from cold regions to warm ones.

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8.8: Radiation

Learning Objectives

By the end of this section, you will be able to:

- Discuss heat transfer by radiation.
- Explain the power of different materials.

You can feel the heat transfer from a fire and from the Sun. Similarly, you can sometimes tell that the oven is hot without touching its door or looking inside—it may just warm you as you walk by. The space between the Earth and the Sun is largely empty, without any possibility of heat transfer by convection or conduction. In these examples, heat is transferred by radiation. That is, the hot body emits electromagnetic waves that are absorbed by our skin: no medium is required for electromagnetic waves to propagate. Different names are used for electromagnetic waves of different wavelengths: radio waves, microwaves, infrared radiation, visible light, ultraviolet radiation, X-rays, and gamma rays.

Figure 8.8.1: Most of the heat transfer from this fire to the observers is through infrared radiation. The visible light, although dramatic, transfers relatively little thermal energy. Convection transfers energy away from the observers as hot air rises, while conduction is negligibly slow here. Skin is very sensitive to infrared radiation, so that you can sense the presence of a fire without looking at it directly. (credit: Daniel X. O'Neil)

The energy of electromagnetic radiation depends on the wavelength (color) and varies over a wide range: a smaller wavelength (or higher frequency) corresponds to a higher energy. Because more heat is radiated at higher temperatures, a temperature change is accompanied by a color change. Take, for example, an electrical element on a stove, which glows from red to orange, while the higher-temperature steel in a blast furnace glows from yellow to white. The radiation you feel is mostly infrared, which corresponds to a lower temperature than that of the electrical element and the steel. The radiated energy depends on its intensity, which is represented in the figure below by the height of the distribution.

Electromagnetic Waves explains more about the electromagnetic spectrum and Introduction to Quantum Physics discusses how the decrease in wavelength corresponds to an increase in energy.

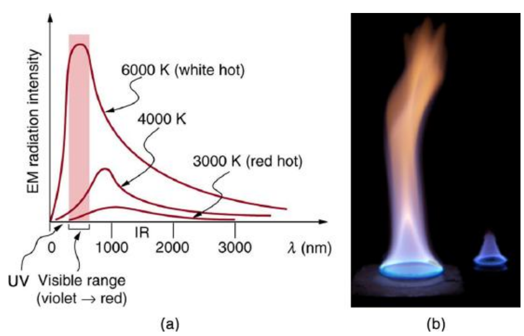

Figure 8.8.2: (a) A graph of the spectra of electromagnetic waves emitted from an ideal radiator at three different temperatures. The intensity or rate of radiation emission increases dramatically with temperature, and the spectrum shifts toward the visible and ultraviolet parts of the spectrum. The shaded portion denotes the visible part of the spectrum. It is apparent that the shift toward the ultraviolet with temperature makes the visible appearance shift from red to white to blue as temperature increases. (b) Note the variations in color corresponding to variations in flame temperature. (credit: Tuohirulla)

All objects absorb and emit electromagnetic radiation. The rate of heat transfer by radiation is largely determined by the color of the object. Black is the most effective, and white is the least effective. People living in hot climates generally avoid wearing black clothing, for instance. Similarly, black asphalt in a parking lot will be hotter than adjacent gray sidewalk on a summer day, because black absorbs better than gray. The reverse is also true—black radiates better than gray. Thus, on a clear summer night, the asphalt will be colder than the gray sidewalk, because black radiates the energy more rapidly than gray. An *ideal radiator* is the same color as an *ideal absorber*, and captures all the radiation that falls on it. In contrast, white is a poor absorber and is also a poor radiator. A white object reflects all radiation, like a mirror. (A perfect, polished white surface is mirror-like in appearance, and a crushed mirror looks white.)

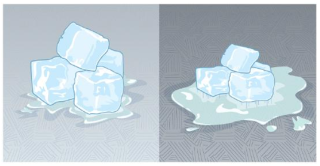

Figure 8.8.3: This illustration shows that the darker pavement is hotter than the lighter pavement (much more of the ice on the right has melted), although both have been in the sunlight for the same time. The thermal conductivities of the pavements are the same.

Gray objects have a uniform ability to absorb all parts of the electromagnetic spectrum. Colored objects behave in similar but more complex ways, which gives them a particular color in the visible range and may make them special in other ranges of the nonvisible spectrum. Take, for example, the strong absorption of infrared radiation by the skin, which allows us to be very sensitive to it.

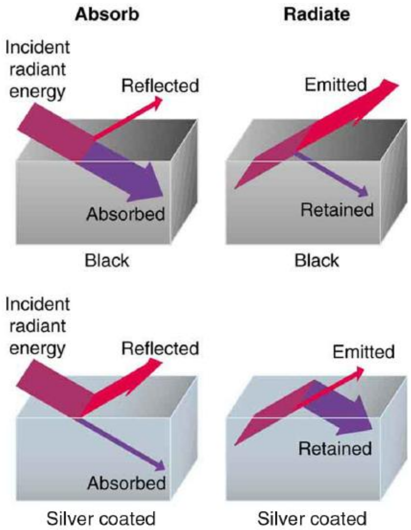

Figure 8.8.4: A black object is a good absorber and a good radiator, while a white (or silver) object is a poor absorber and a poor radiator. It is as if radiation from the inside is reflected back into the silver object, whereas radiation from the inside of the black object is “absorbed” when it hits the surface and finds itself on the outside and is strongly emitted.

The rate of heat transfer by emitted radiation is determined by the **Stefan-Boltzmann law of radiation:**

$$\frac{Q}{t} = \sigma e A T^4, \tag{8.8.1}$$

where $\sigma = 5.67 \times 10^{-8}\ J/s \cdot m^2 \cdot k^4$ is the Stefan-Boltzmann constant, A is the surface area of the object, and T is its absolute temperature in kelvin. The symbol e stands for the emissivity of the object, which is a measure of how well it radiates. An ideal jet-black (or black body) radiator has $e = 1$, whereas a perfect reflector has $e = 0$. Real objects fall between these two values. Take, for example, tungsten light bulb filaments which have an e of about 0.5, and carbon black (a material used in printer toner), which has the (greatest known) emissivity of about 0.99.

The radiation rate is directly proportional to the *fourth power* of the absolute temperature—a remarkably strong temperature dependence. Furthermore, the radiated heat is proportional to the surface area of the object. If you knock apart the coals of a fire, there is a noticeable increase in radiation due to an increase in radiating surface area.

Figure 8.8.5: A thermograph of part of a building shows temperature variations, indicating where heat transfer to the outside is most severe. Windows are a major region of heat transfer to the outside of homes. (credit: U.S. Army)

Skin is a remarkably good absorber and emitter of infrared radiation, having an emissivity of 0.97 in the infrared spectrum. Thus, we are all nearly (jet) black in the infrared, in spite of the obvious variations in skin color. This high infrared emissivity is why we can so easily feel radiation on our skin. It is also the basis for the use of night scopes used by law enforcement and the military to detect human beings. Even small temperature variations can be detected because of the T^4 dependence. Images, called *thermographs*, can be used medically to detect regions of abnormally high temperature in the body, perhaps indicative of disease. Similar techniques can be used to detect heat leaks in homes Figure 8.8.5, optimize performance of blast furnaces, improve comfort levels in work environments, and even remotely map the Earth's temperature profile.

All objects emit and absorb radiation. The *net* rate of heat transfer by radiation (absorption minus emission) is related to both the temperature of the object and the temperature of its surroundings. Assuming that an object with a temperature T_1 is surrounded by an environment with uniform temperature T_2 the net rate of heat transfer by radiation is

$$\frac{Q_{net}}{t} = \sigma e A(T_2^4 - T_1^4), \tag{8.8.2}$$

where e is the emissivity of the object alone. In other words, it does not matter whether the surroundings are white, gray, or black; the balance of radiation into and out of the object depends on how well it emits and absorbs radiation. When $T_2 > T_1$, the quantity Q_{net}/t is positive; that is, the net heat transfer is from hot to cold.

Take-Home Experiment: Temperature in the Sun

Place a thermometer out in the sunshine and shield it from direct sunlight using an aluminum foil. What is the reading? Now remove the shield, and note what the thermometer reads. Take a handkerchief soaked in nail polish remover, wrap it around the thermometer and place it in the sunshine. What does the thermometer read?

Example 8.8.1: Calculate the Net Heat Transfer of a Person: Heat Transfer by Radiation

What is the rate of heat transfer by radiation, with an unclothed person standing in a dark room whose ambient temperature is 22.0^oC. The person has a normal skin temperature of 33.0^oC and a surface area of $1.50\,m^2$. The emissivity of skin is 0.97 in the infrared, where the radiation takes place.

Strategy

We can solve this by using the equation for the rate of radiative heat transfer.

Solution

Insert the temperatures values $T_2 = 295\,K$ and $T_1 = 306\,K$, so that

$$\frac{Q}{t} = \sigma e A(T_2^4 - T_1^4) \tag{8.8.3}$$

$$= (5.67 \times 10^{-8}\,J/s \cdot m^2 \cdot k^4)(0.97)(1.50\,m^2)[(295\,K)^4 - (306\,K)^4] \tag{8.8.4}$$

$$= -99\,J/s = -99\,W. \tag{8.8.5}$$

Discussion

This value is a significant rate of heat transfer to the environment (note the minus sign), considering that a person at rest may produce energy at the rate of 125 W and that conduction and convection will also be transferring energy to the environment. Indeed, we would probably expect this person to feel cold. Clothing significantly reduces heat transfer to the environment by many methods, because clothing slows down both conduction and convection, and has a lower emissivity (especially if it is white) than skin.

The Earth receives almost all its energy from radiation of the Sun and reflects some of it back into outer space. Because the Sun is hotter than the Earth, the net energy flux is from the Sun to the Earth. However, the rate of energy transfer is less than the equation for the radiative heat transfer would predict because the Sun does not fill the sky. The average emissivity (e) of the Earth is about 0.65, but the calculation of this value is complicated by the fact that the highly reflective cloud coverage varies greatly from day to day. There is a negative feedback (one in which a change produces an effect that opposes that change) between clouds and heat transfer; greater temperatures evaporate more water to form more clouds, which reflect more radiation back into space, reducing the temperature. The often mentioned **greenhouse effect** is directly related to the variation of the Earth's emissivity with radiation type (see the figure given below). The greenhouse effect is a natural phenomenon responsible for providing temperatures suitable for life on Earth. The Earth's relatively constant temperature is a result of the energy balance between the incoming solar radiation and the energy radiated from the Earth. Most of the infrared radiation emitted from the Earth is absorbed by carbon dioxide (CO_2) and water (H_2O) in the atmosphere and then re-radiated back to the Earth or into outer space. Re-radiation back to the Earth maintains its surface temperature about 40^oC higher than it would be if there was no atmosphere, similar to the way glass increases temperatures in a greenhouse.

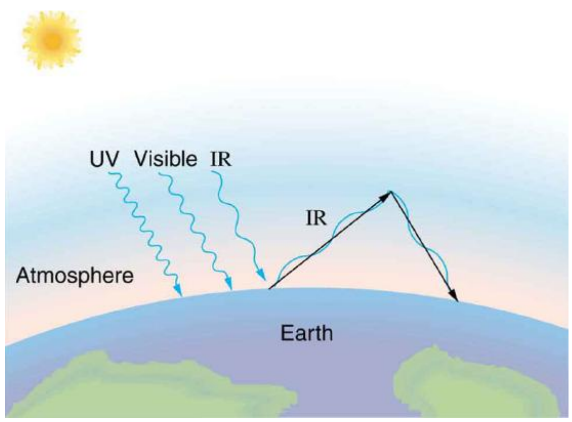

Figure 8.8.6: The greenhouse effect is a name given to the trapping of energy in the Earth's atmosphere by a process similar to that used in greenhouses. The atmosphere, like window glass, is transparent to incoming visible radiation and most of the Sun's infrared. These wavelengths are absorbed by the Earth and re-emitted as infrared. Since Earth's temperature is much lower than that of the Sun, the infrared radiated by the Earth has a much longer wavelength. The atmosphere, like glass, traps these longer infrared rays, keeping the Earth warmer than it would otherwise be. The amount of trapping depends on concentrations of trace gases like carbon dioxide, and a change in the concentration of these gases is believed to affect the Earth's surface temperature.

The greenhouse effect is also central to the discussion of global warming due to emission of carbon dioxide and methane (and other so-called greenhouse gases) into the Earth's atmosphere from industrial production and farming. Changes in global climate could lead to more intense storms, precipitation changes (affecting agriculture), reduction in rain forest biodiversity, and rising sea levels.

Heating and cooling are often significant contributors to energy use in individual homes. Current research efforts into developing environmentally friendly homes quite often focus on reducing conventional heating and cooling through better building materials, strategically positioning windows to optimize radiation gain from the Sun, and opening spaces to allow convection. It is possible to build a zero-energy house that allows for comfortable living in most parts of the United States with hot and humid summers and cold winters.

https://phys.libretexts.org/@go/page/26551

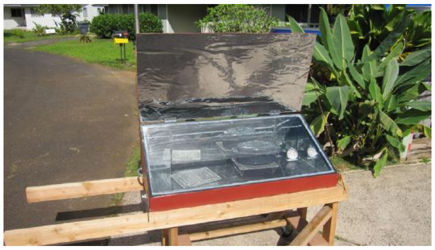

Figure 8.8.7: This simple but effective solar cooker uses the greenhouse effect and reflective material to trap and retain solar energy. Made of inexpensive, durable materials, it saves money and labor, and is of particular economic value in energy-poor developing countries. (credit: E.B. Kauai)

Conversely, dark space is very cold, about $3K(-454^oF)$, so that the Earth radiates energy into the dark sky. Owing to the fact that clouds have lower emissivity than either oceans or land masses, they reflect some of the radiation back to the surface, greatly reducing heat transfer into dark space, just as they greatly reduce heat transfer into the atmosphere during the day. The rate of heat transfer from soil and grasses can be so rapid that frost may occur on clear summer evenings, even in warm latitudes.

Exercise 8.8.1

What is the change in the rate of the radiated heat by a body at the temperature $T_1 = 20^oC$ compared to when the body is at the temperature $T_2 = 40^oC$?

Answer

The radiated heat is proportional to the fourth power of the absolute temperature. Because $T_1 = 293\,K$ and $T_2 = 313\,K$, the rate of heat transfer increases by about 30 percent of the original rate.

Career Connection: Energy Conservation Consultation

The cost of energy is generally believed to remain very high for the foreseeable future. Thus, passive control of heat loss in both commercial and domestic housing will become increasingly important. Energy consultants measure and analyze the flow of energy into and out of houses and ensure that a healthy exchange of air is maintained inside the house. The job prospects for an energy consultant are strong.

Problem Solving Strategies for the Methods of Heat Transfer

1. Examine the situation to determine what type of heat transfer is involved.
2. Identify the type(s) of heat transfer—conduction, convection, or radiation.
3. Identify exactly what needs to be determined in the problem (identify the unknowns). A written list is very useful.
4. Make a list of what is given or can be inferred from the problem as stated (identify the knowns).
5. Solve the appropriate equation for the quantity to be determined (the unknown).
6. For conduction, equation $\frac{Q}{t} = \frac{kA(T_2 - T_1)}{d}$ is appropriate. [link] lists thermal conductivities. For convection, determine the amount of matter moved and use equation $Q = mc\Delta T$, to calculate the heat transfer involved in the temperature change of the fluid. If a phase change accompanies convection, equation $Q = mL_f$ or $Q = mL_v$ is appropriate to find the heat transfer involved in the phase change. [link] lists information relevant to phase change. For radiation, equation $\frac{Q_{net}}{t} = \sigma eA(T_2^4 - T_1^4)$ gives the net heat transfer rate.
7. Insert the knowns along with their units into the appropriate equation and obtain numerical solutions complete with units.
8. Check the answer to see if it is reasonable. Does it make sense?

Summary

- Radiation is the rate of heat transfer through the emission or absorption of electromagnetic waves.
- The rate of heat transfer depends on the surface area and the fourth power of the absolute temperature:

$$\frac{Q}{t} = \sigma eAT^4, \tag{8.8.6}$$

where $\sigma = 5.67 \times 10^{-8}\ J/s \cdot m^2 \cdot K^4$ is the Stefan-Boltzmann constant and e is the emissivity of the body. For a black body, $e = 1$ whereas a shiny white or perfect reflector has $e = 0$, with real objects having values of e between 1 and 0. The net rate of heat transfer by radiation is

$$\frac{Q_{net}}{t} = \sigma e A (T_2^4 - T_1^4) \tag{8.8.7}$$

where T_1 is the temperature of an object surrounded by an environment with uniform temperature T_2 and e is the emissivity of the *object*.

Glossary

emissivity

measure of how well an object radiates

greenhouse effect

warming of the Earth that is due to gases such as carbon dioxide and methane that absorb infrared radiation from the Earth's surface and reradiate it in all directions, thus sending a fraction of it back toward the surface of the Earth

net rate of heat transfer by radiation

is $\frac{Q_{net}}{t} = \sigma e A (T_2^4 - T_1^4)$

radiation

energy transferred by electromagnetic waves directly as a result of a temperature difference

Stefan-Boltzmann law of radiation

$\frac{Q}{t} = \sigma e A T^4$, where σ is the Stefan-Boltzmann constant, A is the surface area of the object, T is the absolute temperature, and e is the emissivity

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8.E: Heat and Heat Transfer Methods (Exercise)

Conceptual Questions

14.1: Heat

1. How is heat transfer related to temperature?

2. Describe a situation in which heat transfer occurs. What are the resulting forms of energy?

3. When heat transfers into a system, is the energy stored as heat? Explain briefly.

14.2: Temperature Change and Heat Capacity

4. What three factors affect the heat transfer that is necessary to change an object's temperature?

5. The brakes in a car increase in temperature by ΔT when bringing the car to rest from a speed v. How much greater would ΔT be if the car initially had twice the speed? You may assume the car to stop sufficiently fast so that no heat transfers out of the brakes.

14.3: Phase Change and Latent Heat

6. Heat transfer can cause temperature and phase changes. What else can cause these changes?

7. How does the latent heat of fusion of water help slow the decrease of air temperatures, perhaps preventing temperatures from falling significantly below 0^oC, in the vicinity of large bodies of water?

8. What is the temperature of ice right after it is formed by freezing water?

9. If you place 0^oC ice into 0^oC water in an insulated container, what will happen? Will some ice melt, will more water freeze, or will neither take place?

10. What effect does condensation on a glass of ice water have on the rate at which the ice melts? Will the condensation speed up the melting process or slow it down?

11. In very humid climates where there are numerous bodies of water, such as in Florida, it is unusual for temperatures to rise above about **35ºC(95ºF).** In deserts, however, temperatures can rise far above this. Explain how the evaporation of water helps limit high temperatures in humid climates.

12. In winters, it is often warmer in San Francisco than in nearby Sacramento, 150 km inland. In summers, it is nearly always hotter in Sacramento. Explain how the bodies of water surrounding San Francisco moderate its extreme temperatures.

13. Putting a lid on a boiling pot greatly reduces the heat transfer necessary to keep it boiling. Explain why.

14. Freeze-dried foods have been dehydrated in a vacuum. During the process, the food freezes and must be heated to facilitate dehydration. Explain both how the vacuum speeds up dehydration and why the food freezes as a result.

15. When still air cools by radiating at night, it is unusual for temperatures to fall below the dew point. Explain why.

16. In a physics classroom demonstration, an instructor inflates a balloon by mouth and then cools it in liquid nitrogen. When cold, the shrunken balloon has a small amount of light blue liquid in it, as well as some snow-like crystals. As it warms up, the liquid boils, and part of the crystals sublimate, with some crystals lingering for awhile and then producing a liquid. Identify the blue liquid and the two solids in the cold balloon. Justify your identifications using data from Table.

14.4: Heat Transfer Methods

17. What are the main methods of heat transfer from the hot core of Earth to its surface? From Earth's surface to outer space?

14.5: Conduction

18. Some electric stoves have a flat ceramic surface with heating elements hidden beneath. A pot placed over a heating element will be heated, while it is safe to touch the surface only a few centimeters away. Why is ceramic, with a conductivity less than that of a metal but greater than that of a good insulator, an ideal choice for the stove top?

19. Loose-fitting white clothing covering most of the body is ideal for desert dwellers, both in the hot Sun and during cold evenings. Explain how such clothing is advantageous during both day and night.


A jellabiya is worn by many men in Egypt. (credit: Zerida)

14.6: Convection

20. One way to make a fireplace more energy efficient is to have an external air supply for the combustion of its fuel. Another is to have room air circulate around the outside of the fire box and back into the room. Detail the methods of heat transfer involved in each.

21. On cold, clear nights horses will sleep under the cover of large trees. How does this help them keep warm?

14.7 Radiation

22. When watching a daytime circus in a large, dark-colored tent, you sense significant heat transfer from the tent. Explain why this occurs.

23. Satellites designed to observe the radiation from cold (3 K) dark space have sensors that are shaded from the Sun, Earth, and Moon and that are cooled to very low temperatures. Why must the sensors be at low temperature?

24. Why are cloudy nights generally warmer than clear ones?

25. Why are thermometers that are used in weather stations shielded from the sunshine? What does a thermometer measure if it is shielded from the sunshine and also if it is not?

26. On average, would Earth be warmer or cooler without the atmosphere? Explain your answer.

Problems & Exercises

14.2: Temperature Change and Heat Capacity

27. On a hot day, the temperature of an 80,000-L swimming pool increases by $1.50^{\circ}C$. What is the net heat transfer during this heating? Ignore any complications, such as loss of water by evaporation.

Solution
$5.02 \times 10^8 J$

28. Show that $1 cal/g \cdot {}^{\circ}C = 1 kcal/kg \cdot {}^{\circ}C$.

29. To sterilize a 50.0-g glass baby bottle, we must raise its temperature from $22.0^{\circ}C$ to $95.0^{\circ}C$. How much heat transfer is required?

Solution
$3.07 \times 10^3 J$

30. The same heat transfer into identical masses of different substances produces different temperature changes. Calculate the final temperature when 1.00 kcal of heat transfers into 1.00 kg of the following, originally at $20.0^{\circ}C$:

(a) water;

(b) concrete;

(c) steel; and

(d) mercury.

31. Rubbing your hands together warms them by converting work into thermal energy. If a woman rubs her hands back and forth for a total of 20 rubs, at a distance of 7.50 cm per rub, and with an average frictional force of 40.0 N, what is the temperature increase? The mass of tissues warmed is only 0.100 kg, mostly in the palms and fingers.

Solution
$0.171^{o}C$

32. A 0.250-kg block of a pure material is heated from $20.0^{o}C$ to $65.0^{o}C$ by the addition of 4.35 kJ of energy. Calculate its specific heat and identify the substance of which it is most likely composed.

33. Suppose identical amounts of heat transfer into different masses of copper and water, causing identical changes in temperature. What is the ratio of the mass of copper to water?

Solution
10.8

34. (a) The number of kilocalories in food is determined by calorimetry techniques in which the food is burned and the amount of heat transfer is measured. How many kilocalories per gram are there in a 5.00-g peanut if the energy from burning it is transferred to 0.500 kg of water held in a 0.100-kg aluminum cup, causing a $54.9^{o}C$ temperature increase?

(b) Compare your answer to labeling information found on a package of peanuts and comment on whether the values are consistent.

35. Following vigorous exercise, the body temperature of an 80.0-kg person is $40.0^{o}C$. At what rate in watts must the person transfer thermal energy to reduce the the body temperature to $37.0^{o}C$ in 30.0 min, assuming the body continues to produce energy at the rate of 150 W? (**1 watt = 1 joule/second or 1 W = 1 J/s**).

Solution
617 W

36. Even when shut down after a period of normal use, a large commercial nuclear reactor transfers thermal energy at the rate of 150 MW by the radioactive decay of fission products. This heat transfer causes a rapid increase in temperature if the cooling system fails (**1 watt = 1 joule/second or 1 W = 1 J/s and 1 MW = 1 megawatt**).

(a) Calculate the rate of temperature increase in degrees Celsius per second ($^{o}C/s$) if the mass of the reactor core is $1.60 \times 10^{5} kg$ and it has an average specific heat of $0.3349 kJ/kg^{o} \cdot C$.

(b) How long would it take to obtain a temperature increase of $2000^{o}C$, which could cause some metals holding the radioactive materials to melt? (The initial rate of temperature increase would be greater than that calculated here because the heat transfer is concentrated in a smaller mass. Later, however, the temperature increase would slow down because the $5 \times 10^{5} - kg$ steel containment vessel would also begin to heat up.)

14.3: Phase Change and Latent Heat

37. How much heat transfer (in kilocalories) is required to thaw a 0.450-kg package of frozen vegetables originally at $0^{o}C$ if their heat of fusion is the same as that of water?

Solution
35.9 kcal

38. A bag containing $0^{o}C$ ice is much more effective in absorbing energy than one containing the same amount of 0ºC water.

a. How much heat transfer is necessary to raise the temperature of 0.800 kg of water from $0^{o}C$ to $30.0^{o}C$?

b. How much heat transfer is required to first melt 0.800 kg of $0^{o}C$ ice and then raise its temperature?

c. Explain how your answer supports the contention that the ice is more effective.

39. (a) How much heat transfer is required to raise the temperature of a 0.750-kg aluminum pot containing 2.50 kg of water from $30.0^{o}C$ to the boiling point and then boil away 0.750 kg of water?

(b) How long does this take if the rate of heat transfer is 500 W **1 watt = 1 joule/second** (**1 W = 1 J/s**)?

Solution
(a) 591 kcal
(b) $4.94 \times 10^{3} s$

40. The formation of condensation on a glass of ice water causes the ice to melt faster than it would otherwise. If 8.00 g of condensation forms on a glass containing both water and 200 g of ice, how many grams of the ice will melt as a result? Assume no other heat transfer occurs.

41. On a trip, you notice that a 3.50-kg bag of ice lasts an average of one day in your cooler. What is the average power in watts entering the ice if it starts at $0^{o}C$ and completely melts to $0^{o}C$ water in exactly one day **1 watt = 1 joule/second** (**1 W = 1 J/s**)?

Solution
13.5 W

42. On a certain dry sunny day, a swimming pool's temperature would rise by $1.50^{o}C$ if not for evaporation. What fraction of the water must evaporate to carry away precisely enough energy to keep the temperature constant?

43. (a) How much heat transfer is necessary to raise the temperature of a 0.200-kg piece of ice from $-20.0^{o}C$ to $130^{o}C$, including the energy needed for phase changes?

(b) How much time is required for each stage, assuming a constant 20.0 kJ/s rate of heat transfer?

(c) Make a graph of temperature versus time for this process.

Solution
(a) 148 kcal
(b) 0.418 s, 3.34 s, 4.19 s, 22.6 s, 0.456 s

44. In 1986, a gargantuan iceberg broke away from the Ross Ice Shelf in Antarctica. It was approximately a rectangle 160 km long, 40.0 km wide, and 250 m thick.

(a) What is the mass of this iceberg, given that the density of ice is $917 kg/m^{3}$?

(b) How much heat transfer (in joules) is needed to melt it?

(c) How many years would it take sunlight alone to melt ice this thick, if the ice absorbs an average of $100 W/m^{2}$, 12.00 h per day?

45. How many grams of coffee must evaporate from 350 g of coffee in a 100-g glass cup to cool the coffee from $95.0^{o}C$ to $45.0^{o}C$? You may assume the coffee has the same thermal properties as water and that the average heat of vaporization is 2340 kJ/kg (560 cal/g). (You may neglect the change in mass of the coffee as it cools, which will give you an answer that is slightly larger than correct.)

Solution
33.0 g

46. (a) It is difficult to extinguish a fire on a crude oil tanker, because each liter of crude oil releases $2.80 \times 10^{7} J$ of energy when burned. To illustrate this difficulty, calculate the number of liters of water that must be expended to absorb the energy released by burning 1.00 L of crude oil, if the water has its temperature raised from $20.0^{o}C$ to $100^{o}C$, it boils, and the resulting steam is raised to $300^{o}C$.

(b) Discuss additional complications caused by the fact that crude oil has a smaller density than water.

Solution
(a) 9.67 L
(b) Crude oil is less dense than water, so it floats on top of the water, thereby exposing it to the oxygen in the air, which it uses to burn. Also, if the water is under the oil, it is less efficient in absorbing the heat generated by the oil.

47. The energy released from condensation in thunderstorms can be very large. Calculate the energy released into the atmosphere for a small storm of radius 1 km, assuming that 1.0 cm of rain is precipitated uniformly over this area.

48. To help prevent frost damage, 4.00 kg of $0^{o}C$ water is sprayed onto a fruit tree.

(a) How much heat transfer occurs as the water freezes?

(b) How much would the temperature of the 200-kg tree decrease if this amount of heat transferred from the tree? Take the specific heat to be $3.35 kJ/kg \cdot ^{\circ}C$, and assume that no phase change occurs.

Solution
a) 319 kcal
b) $2.00^{\circ}C$

49. A 0.250-kg aluminum bowl holding 0.800 kg of soup at $25.0^{\circ}C$ is placed in a freezer. What is the final temperature if 377 kJ of energy is transferred from the bowl and soup, assuming the soup's thermal properties are the same as that of water? Explicitly show how you follow the steps in Problem-Solving Strategies for the Effects of Heat Transfer.

50. A 0.0500-kg ice cube at $-30.0^{\circ}C$ is placed in 0.400 kg of $35.0^{\circ}C$ water in a very well-insulated container. What is the final temperature?

Solution
$20.6^{\circ}C$

51. If you pour 0.0100 kg of $20.0^{\circ}C$ water onto a 1.20-kg block of ice (which is initially at $-15.0^{\circ}C$), what is the final temperature? You may assume that the water cools so rapidly that effects of the surroundings are negligible.

52. Indigenous people sometimes cook in watertight baskets by placing hot rocks into water to bring it to a boil. What mass of $500^{\circ}C$ rock must be placed in 4.00 kg of $15.0^{\circ}C$ water to bring its temperature to $100^{\circ}C$, if 0.0250 kg of water escapes as vapor from the initial sizzle? You may neglect the effects of the surroundings and take the average specific heat of the rocks to be that of granite.

Solution
4.38 kg

53. What would be the final temperature of the pan and water in Calculating the Final Temperature When Heat Is Transferred Between Two Bodies: Pouring Cold Water in a Hot Pan if 0.260 kg of water was placed in the pan and 0.0100 kg of the water evaporated immediately, leaving the remainder to come to a common temperature with the pan?

54. In some countries, liquid nitrogen is used on dairy trucks instead of mechanical refrigerators. A 3.00-hour delivery trip requires 200 L of liquid nitrogen, which has a density of $808 kg/m^3$.

(a) Calculate the heat transfer necessary to evaporate this amount of liquid nitrogen and raise its temperature to $3.00^{\circ}C$. (Use c_p and assume it is constant over the temperature range.) This value is the amount of cooling the liquid nitrogen supplies.

(b) What is this heat transfer rate in kilowatt-hours?

(c) Compare the amount of cooling obtained from melting an identical mass of 0ºC ice with that from evaporating the liquid nitrogen.

Solution
(a) $1.57 \times 10^4 kcal$
(b) $18.3 kW \cdot h$
(c) $1.29 \times 10^4 kcal$

55. Some gun fanciers make their own bullets, which involves melting and casting the lead slugs. How much heat transfer is needed to raise the temperature and melt 0.500 kg of lead, starting from **25.0ºC**?

14.5: Conduction

56. (a) Calculate the rate of heat conduction through house walls that are 13.0 cm thick and that have an average thermal conductivity twice that of glass wool. Assume there are no windows or doors. The surface area of the walls is $120 m^2$ and their inside surface is at $18.0^{\circ}C$, while their outside surface is at $5.00^{\circ}C$.

(b) How many 1-kW room heaters would be needed to balance the heat transfer due to conduction?

Solution
(a) 1.01×10^3W
(b) One

57. The rate of heat conduction out of a window on a winter day is rapid enough to chill the air next to it. To see just how rapidly the windows transfer heat by conduction, calculate the rate of conduction in watts through a $3.00-m^2$ window that is $0.635cm$ thick (1/4 in) if the temperatures of the inner and outer surfaces are 5.00^oC and -10.0^oC, respectively. This rapid rate will not be maintained—the inner surface will cool, and even result in frost formation.

58. Calculate the rate of heat conduction out of the human body, assuming that the core internal temperature is 37.0^oC, the skin temperature is 34.0^oC, the thickness of the tissues between averages $1.00cm$, and the surface area is $1.40m^2$.

Solution
84.0 W

59. Suppose you stand with one foot on ceramic flooring and one foot on a wool carpet, making contact over an area of $80.0cm^2$ with each foot. Both the ceramic and the carpet are 2.00 cm thick and are 10.0^oC on their bottom sides. At what rate must heat transfer occur from each foot to keep the top of the ceramic and carpet at 33.0^oC?

60. A man consumes 3000 kcal of food in one day, converting most of it to maintain body temperature. If he loses half this energy by evaporating water (through breathing and sweating), how many kilograms of water evaporate?

Solution
2.59 kg

61. (a) A firewalker runs across a bed of hot coals without sustaining burns. Calculate the heat transferred by conduction into the sole of one foot of a firewalker given that the bottom of the foot is a 3.00-mm-thick callus with a conductivity at the low end of the range for wood and its density is $300kg/m^3$. The area of contact is $25.0cm^2$, the temperature of the coals is 700^oC, and the time in contact is 1.00 s.

(b) What temperature increase is produced in the $25.0cm^3$ of tissue affected?

(c) What effect do you think this will have on the tissue, keeping in mind that a callus is made of dead cells?

62. (a) What is the rate of heat conduction through the 3.00-cm-thick fur of a large animal having a $1.40-m^2$ surface area? Assume that the animal's skin temperature is 32.0^oC, that the air temperature is -5.00^oC, and that fur has the same thermal conductivity as air. (b) What food intake will the animal need in one day to replace this heat transfer?

Solution
(a) 39.7 W
(b) 820 kcal

63. A walrus transfers energy by conduction through its blubber at the rate of 150 W when immersed in -1.00^oC water. The walrus's internal core temperature is 37.0^oC, and it has a surface area of $2.00m^2$. What is the average thickness of its blubber, which has the conductivity of fatty tissues without blood?

Walrus on ice. (credit: Captain Budd Christman, NOAA Corps)

64. Compare the rate of heat conduction through a 13.0-cm-thick wall that has an area of $10.0m^2$ and a thermal conductivity twice that of glass wool with the rate of heat conduction through a window that is 0.750 cm thick and that has an area of $2.00m^2$, assuming the same temperature difference across each.

Solution
35 to 1, window to wall

65. Suppose a person is covered head to foot by wool clothing with average thickness of 2.00 cm and is transferring energy by conduction through the clothing at the rate of 50.0 W. What is the temperature difference across the clothing, given the surface area is $1.40m^2$?

66. Some stove tops are smooth ceramic for easy cleaning. If the ceramic is 0.600 cm thick and heat conduction occurs through the same area and at the same rate as computed in Example, what is the temperature difference across it? Ceramic has the same thermal conductivity as glass and brick.

Solution
$1.05 \times 10^3 K$

67. One easy way to reduce heating (and cooling) costs is to add extra insulation in the attic of a house. Suppose the house already had 15 cm of fiberglass insulation in the attic and in all the exterior surfaces. If you added an extra 8.0 cm of fiberglass to the attic, then by what percentage would the heating cost of the house drop? Take the single story house to be of dimensions 10 m by 15 m by 3.0 m. Ignore air infiltration and heat loss through windows and doors.

68. (a) Calculate the rate of heat conduction through a double-paned window that has a $1.50 - m^2$ area and is made of two panes of 0.800-cm-thick glass separated by a 1.00-cm air gap. The inside surface temperature is $15.0°C$, while that on the outside is $-10.0°C$. (Hint: There are identical temperature drops across the two glass panes. First find these and then the temperature drop across the air gap. This problem ignores the increased heat transfer in the air gap due to convection.)

(b) Calculate the rate of heat conduction through a 1.60-cm-thick window of the same area and with the same temperatures. Compare your answer with that for part (a).

Solution
(a) 83 W
(b) 24 times that of a double pane window.

69. Many decisions are made on the basis of the payback period: the time it will take through savings to equal the capital cost of an investment. Acceptable payback times depend upon the business or philosophy one has. (For some industries, a payback period is as small as two years.) Suppose you wish to install the extra insulation in Exercise. If energy cost $1.00 per million joules and the insulation was $4.00 per square meter, then calculate the simple payback time. Take the average ΔT for the 120 day heating season to be $15.0°C$.

70. For the human body, what is the rate of heat transfer by conduction through the body's tissue with the following conditions: the tissue thickness is 3.00 cm, the change in temperature is $2.00°C$, and the skin area is $1.50m^2$. How does this compare with the average heat transfer rate to the body resulting from an energy intake of about 2400 kcal per day? (No exercise is included.)

Solution
20.0 W, 17.2% of 2400 kcal per day

14.6: Convection

71. At what wind speed does $-10°C$ air cause the same chill factor as still air at $-29°C$?

Solution
10 m/s

72. At what temperature does still air cause the same chill factor as $-5°C$ air moving at 15 m/s?

73. The "steam" above a freshly made cup of instant coffee is really water vapor droplets condensing after evaporating from the hot coffee. What is the final temperature of 250 g of hot coffee initially at $90.0°C$ if 2.00 g evaporates from it? The coffee is in a Styrofoam cup, so other methods of heat transfer can be neglected.

Solution
$85.7°C$

74. (a) How many kilograms of water must evaporate from a 60.0-kg woman to lower her body temperature by $0.750°C$?

(b) Is this a reasonable amount of water to evaporate in the form of perspiration, assuming the relative humidity of the surrounding air is low?

75. On a hot dry day, evaporation from a lake has just enough heat transfer to balance the $1.00kW/m^2$ of incoming heat from the Sun. What mass of water evaporates in 1.00 h from each square meter? Explicitly show how you follow the steps in the Problem-Solving Strategies for the Effects of Heat Transfer.

Solution
1.48 kg

76. One winter day, the climate control system of a large university classroom building malfunctions. As a result, $500m^3$ of excess cold air is brought in each minute. At what rate in kilowatts must heat transfer occur to warm this air by $10.0°C$ (that is, to bring the air to room temperature)?

77. The Kilauea volcano in Hawaii is the world's most active, disgorging about $5 \times 10^5 m^3$ of $1200°C$ lava per day. What is the rate of heat transfer out of Earth by convection if this lava has a density of $2700 kg/m^3$ and eventually cools to $30°C$? Assume that the specific heat of lava is the same as that of granite.

Lava flow on Kilauea volcano in Hawaii. (credit: J. P. Eaton, U.S. Geological Survey)

Solution
$2 \times 10^4 MW$

78. During heavy exercise, the body pumps 2.00 L of blood per minute to the surface, where it is cooled by $2.00°C$. What is the rate of heat transfer from this forced convection alone, assuming blood has the same specific heat as water and its density is $1050 kg/m^3$?

79. A person inhales and exhales 2.00 L of $37.0°C$ air, evaporating $4.00 \times 10^{-2} g$ of water from the lungs and breathing passages with each breath.

(a) How much heat transfer occurs due to evaporation in each breath?

(b) What is the rate of heat transfer in watts if the person is breathing at a moderate rate of 18.0 breaths per minute?

(c) If the inhaled air had a temperature of $20.0°C$, what is the rate of heat transfer for warming the air?

(d) Discuss the total rate of heat transfer as it relates to typical metabolic rates. Will this breathing be a major form of heat transfer for this person?

Solution
(a) 97.2 J
(b) 29.2 W
(c) 9.49 W
(d) The total rate of heat loss would be $29.2W + 9.49W = 38.7W$. While sleeping, our body consumes 83 W of power, while sitting it consumes 120 to 210 W. Therefore, the total rate of heat loss from breathing will not be a major form of heat loss for this person.

80. A glass coffee pot has a circular bottom with a 9.00-cm diameter in contact with a heating element that keeps the coffee warm with a continuous heat transfer rate of 50.0 W

(a) What is the temperature of the bottom of the pot, if it is 3.00 mm thick and the inside temperature is $60.0°C$?

(b) If the temperature of the coffee remains constant and all of the heat transfer is removed by evaporation, how many grams per minute evaporate? Take the heat of vaporization to be 2340 kJ/kg.

14.7 Radiation

81. At what net rate does heat radiate from a $275 - m^2$ black roof on a night when the roof's temperature is $30.0°C$ and the surrounding temperature is $15.0°C$? The emissivity of the roof is 0.900.

Solution
$-21.7kW$
Note that the negative answer implies heat loss to the surroundings.

82. (a) Cherry-red embers in a fireplace are at $850^{\circ}C$ and have an exposed area of $0.200m^2$ and an emissivity of 0.980. The surrounding room has a temperature of $18.0^{\circ}C$. If 50% of the radiant energy enters the room, what is the net rate of radiant heat transfer in kilowatts?

(b) Does your answer support the contention that most of the heat transfer into a room by a fireplace comes from infrared radiation?

83. Radiation makes it impossible to stand close to a hot lava flow. Calculate the rate of heat transfer by radiation from $1.00m^2$ of $1200^{\circ}C$ fresh lava into $30.0^{\circ}C$ surroundings, assuming lava's emissivity is 1.00.

Solution
$-266kW$

84. (a) Calculate the rate of heat transfer by radiation from a car radiator at $110^{\circ}C$ into a $50.0^{\circ}C$ environment, if the radiator has an emissivity of 0.750 and a $1.20-m^2$ surface area.

(b) Is this a significant fraction of the heat transfer by an automobile engine? To answer this, assume a horsepower of $200hp(1.5kW)$ and the efficiency of automobile engines as 25%.

85. Find the net rate of heat transfer by radiation from a skier standing in the shade, given the following. She is completely clothed in white (head to foot, including a ski mask), the clothes have an emissivity of 0.200 and a surface temperature of $10.0^{\circ}C$, the surroundings are at $-15.0^{\circ}C$, and her surface area is $1.60m^2$.

Solution
$-36.0W$

86. Suppose you walk into a sauna that has an ambient temperature of $50.0^{\circ}C$.

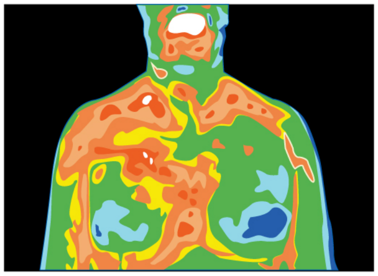
(a) Calculate the rate of heat transfer to you by radiation given your skin temperature is $37.0^{\circ}C$, the emissivity of skin is 0.98, and the surface area of your body is $1.50m^2$.

(b) If all other forms of heat transfer are balanced (the net heat transfer is zero), at what rate will your body temperature increase if your mass is 75.0 kg?

87. Thermography is a technique for measuring radiant heat and detecting variations in surface temperatures that may be medically, environmentally, or militarily meaningful.

(a) What is the percent increase in the rate of heat transfer by radiation from a given area at a temperature of $34.0^{\circ}C$ compared with that at $33.0^{\circ}C$, such as on a person's skin?

(b) What is the percent increase in the rate of heat transfer by radiation from a given area at a temperature of $34.0^{\circ}C$ compared with that at $20.0^{\circ}C$, such as for warm and cool automobile hoods?



Artist's rendition of a thermograph of a patient's upper body, showing the distribution of heat represented by different colors.

Solution
(a) 1.31%
(b) 20.5%

88. The Sun radiates like a perfect black body with an emissivity of exactly 1.

(a) Calculate the surface temperature of the Sun, given that it is a sphere with a $7.00 \times 10^8 - m$ radius that radiates $3.80 \times 10^{26} W$ into 3-K space.

(b) How much power does the Sun radiate per square meter of its surface?

(c) How much power in watts per square meter is that value at the distance of Earth, $1.50 \times 10^{11} m$ away? (This number is called the solar constant.)

89. A large body of lava from a volcano has stopped flowing and is slowly cooling. The interior of the lava is at 1200^oC, its surface is at 450^oC, and the surroundings are at 27.0^oC

(a) Calculate the rate at which energy is transferred by radiation from $1.00 m^2$ of surface lava into the surroundings, assuming the emissivity is 1.00.

(b) Suppose heat conduction to the surface occurs at the same rate. What is the thickness of the lava between the 450^oC surface and the 1200^oC interior, assuming that the lava's conductivity is the same as that of brick?

Solution
(a) $-15.0 kW$
(b) 4.2 cm

90. Calculate the temperature the entire sky would have to be in order to transfer energy by radiation at $1000 W/m^2$—about the rate at which the Sun radiates when it is directly overhead on a clear day. This value is the effective temperature of the sky, a kind of average that takes account of the fact that the Sun occupies only a small part of the sky but is much hotter than the rest. Assume that the body receiving the energy has a temperature of 27.0^oC.

91. (a) A shirtless rider under a circus tent feels the heat radiating from the sunlit portion of the tent. Calculate the temperature of the tent canvas based on the following information: The shirtless rider's skin temperature is 34.0^oC and has an emissivity of 0.970. The exposed area of skin is $0.400 m^2$. He receives radiation at the rate of 20.0 W—half what you would calculate if the entire region behind him was hot. The rest of the surroundings are at 34.0^oC.

(b) Discuss how this situation would change if the sunlit side of the tent was nearly pure white and if the rider was covered by a white tunic.

Solution
(a) 48.5^oC
(b) A pure white object reflects more of the radiant energy that hits it, so a white tent would prevent more of the sunlight from heating up the inside of the tent, and the white tunic would prevent that heat which entered the tent from heating the rider. Therefore, with a white tent, the temperature would be lower than 48.5^oC, and the rate of radiant heat transferred to the rider would be less than 20.0 W.

92. *Integrated Concepts*

One 30.0^oC day the relative humidity is 75.0, and that evening the temperature drops to 20.0^oC, well below the dew point.

(a) How many grams of water condense from each cubic meter of air?

(b) How much heat transfer occurs by this condensation?

(c) What temperature increase could this cause in dry air?

93. *Integrated Concepts*

Large meteors sometimes strike the Earth, converting most of their kinetic energy into thermal energy.

(a) What is the kinetic energy of a 10^9kg meteor moving at 25.0 km/s?

(b) If this meteor lands in a deep ocean and 80 of its kinetic energy goes into heating water, how many kilograms of water could it raise by 5.0^oC?

(c) Discuss how the energy of the meteor is more likely to be deposited in the ocean and the likely effects of that energy.

Solution

(a) $3 \times 10^{17} J$

(b) $1 \times 10^{13} kg$

(c) When a large meteor hits the ocean, it causes great tidal waves, dissipating large amount of its energy in the form of kinetic energy of the water.

94. *Integrated Concepts*

Frozen waste from airplane toilets has sometimes been accidentally ejected at high altitude. Ordinarily it breaks up and disperses over a large area, but sometimes it holds together and strikes the ground. Calculate the mass of $0^{o}C$ ice that can be melted by the conversion of kinetic and gravitational potential energy when a 20.0 piece of frozen waste is released at 12.0 km altitude while moving at 250 m/s and strikes the ground at 100 m/s (since less than 20.0 kg melts, a significant mess results).

95. *Integrated Concepts*

(a) A large electrical power facility produces 1600 MW of "waste heat," which is dissipated to the environment in cooling towers by warming air flowing through the towers by $5.00^{o}C$. What is the necessary flow rate of air in m^3/s?

(b) Is your result consistent with the large cooling towers used by many large electrical power plants?

Solution

(a) $3.44 \times 10^5 m^3/s$

(b) This is equivalent to 12 million cubic feet of air per second. That is tremendous. This is too large to be dissipated by heating the air by only $5^{o}C$. Many of these cooling towers use the circulation of cooler air over warmer water to increase the rate of evaporation. This would allow much smaller amounts of air necessary to remove such a large amount of heat because evaporation removes larger quantities of heat than was considered in part (a).

96. *Integrated Concepts*

(a) Suppose you start a workout on a Stairmaster, producing power at the same rate as climbing 116 stairs per minute. Assuming your mass is 76.0 kg and your efficiency is 20.0, how long will it take for your body temperature to rise $1.00^{o}C$ if all other forms of heat transfer in and out of your body are balanced? (b) Is this consistent with your experience in getting warm while exercising?

97. *Integrated Concepts*

A 76.0-kg person suffering from hypothermia comes indoors and shivers vigorously. How long does it take the heat transfer to increase the person's body temperature by $2.00^{o}C$ if all other forms of heat transfer are balanced?

Solution

20.9 min

98. *Integrated Concepts*

In certain large geographic regions, the underlying rock is hot. Wells can be drilled and water circulated through the rock for heat transfer for the generation of electricity.

(a) Calculate the heat transfer that can be extracted by cooling $1.00 km^3$ of granite by $100^{o}C$.

(b) How long will this take if heat is transferred at a rate of 300 MW, assuming no heat transfers back into the 1.00 km of rock by its surroundings?

99. *Integrated Concepts*

Heat transfers from your lungs and breathing passages by evaporating water.

(a) Calculate the maximum number of grams of water that can be evaporated when you inhale 1.50 L of $37^{o}C$ air with an original relative humidity of 40.0%. (Assume that body temperature is also $37^{o}C$.)

(b) How many joules of energy are required to evaporate this amount?

(c) What is the rate of heat transfer in watts from this method, if you breathe at a normal resting rate of 10.0 breaths per minute?

Solution
(a) $3.96 \times 10^{-2} g$
(b) $96.2 J$
(c) $16.0 W$

100. *Integrated Concepts*

(a) What is the temperature increase of water falling 55.0 m over Niagara Falls?

(b) What fraction must evaporate to keep the temperature constant?

101. Integrated Concepts

Hot air rises because it has expanded. It then displaces a greater volume of cold air, which increases the buoyant force on it. (a) Calculate the ratio of the buoyant force to the weight of 50.0ºC50.0ºC air surrounded by 20.0ºC20.0ºC air. (b) What energy is needed to cause 1.00m31.00 m3 of air to go from 20.0ºC20.0ºC to 50.0ºC50.0ºC? (c) What gravitational potential energy is gained by this volume of air if it rises 1.00 m? Will this cause a significant cooling of the air?

Solution
(a) 1.102
(b) $2.79 \times 10^{4} J$
(c) 12.6 J. This will not cause a significant cooling of the air because it is much less than the energy found in part (b), which is the energy required to warm the air from **20.0ºC** to **50.0ºC**

102. *Unreasonable Results*

(a) What is the temperature increase of an 80.0 kg person who consumes 2500 kcal of food in one day with 95.0% of the energy transferred as heat to the body?

(b) What is unreasonable about this result?

(c) Which premise or assumption is responsible?

Solution
(a) **36ºC**
(b) Any temperature increase greater than about $3^{o}C$ would be unreasonably large. In this case the final temperature of the person would rise to **73ºC**(**163ºF**).
|(c) The assumption of 95 heat retention is unreasonable.

103. *Unreasonable Results*

A slightly deranged Arctic inventor surrounded by ice thinks it would be much less mechanically complex to cool a car engine by melting ice on it than by having a water-cooled system with a radiator, water pump, antifreeze, and so on.

(a) If 80.0 of the energy in 1.00 gal of gasoline is converted into “waste heat” in a car engine, how many kilograms of $0^{o}C$ ice could it melt?

(b) Is this a reasonable amount of ice to carry around to cool the engine for 1.00 gal of gasoline consumption?

(c) What premises or assumptions are unreasonable?

104. *Unreasonable Results*

(a) Calculate the rate of heat transfer by conduction through a window with an area of $1.00 m^{2}$ that is 0.750 cm thick, if its inner surface is at $22.0^{o}C$ and its outer surface is at $35.0^{o}C$.

(b) What is unreasonable about this result?

(c) Which premise or assumption is responsible?

Solution
(a) 1.46 kW
(b) Very high power loss through a window. An electric heater of this power can keep an entire room warm.

(c) The surface temperatures of the window do not differ by as great an amount as assumed. The inner surface will be warmer, and the outer surface will be cooler.

105. *Unreasonable Results*

A meteorite 1.20 cm in diameter is so hot immediately after penetrating the atmosphere that it radiates 20.0 kW of power.

(a) What is its temperature, if the surroundings are at $20.0^{o}C$ and it has an emissivity of 0.800?

(b) What is unreasonable about this result?

(c) Which premise or assumption is responsible?

106. *Construct Your Own Problem*

Consider a new model of commercial airplane having its brakes tested as a part of the initial flight permission procedure. The airplane is brought to takeoff speed and then stopped with the brakes alone. Construct a problem in which you calculate the temperature increase of the brakes during this process. You may assume most of the kinetic energy of the airplane is converted to thermal energy in the brakes and surrounding materials, and that little escapes. Note that the brakes are expected to become so hot in this procedure that they ignite and, in order to pass the test, the airplane must be able to withstand the fire for some time without a general conflagration.

107. *Construct Your Own Problem*

Consider a person outdoors on a cold night. Construct a problem in which you calculate the rate of heat transfer from the person by all three heat transfer methods. Make the initial circumstances such that at rest the person will have a net heat transfer and then decide how much physical activity of a chosen type is necessary to balance the rate of heat transfer. Among the things to consider are the size of the person, type of clothing, initial metabolic rate, sky conditions, amount of water evaporated, and volume of air breathed. Of course, there are many other factors to consider and your instructor may wish to guide you in the assumptions made as well as the detail of analysis and method of presenting your results.

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Index

A

acceleration
2.5: Acceleration

Acceleration due to Gravity
2.8: Falling Objects

air resistance
3.5: Projectile Motion

analytical method
3.4: Vector Addition and Subtraction- Analytical Methods

angular velocity
5.2: Rotation Angle and Angular Velocity

application
1.1: The Basics of Physics
4.10: Further Applications of Newton’s Laws of Motion

approximation
1.2: Scientific Notation and Order of Magnitude

arc length
5.2: Rotation Angle and Angular Velocity

Average Acceleration
2.5: Acceleration

average speed
2.4: Time, Velocity, and Speed

Average Velocity
2.4: Time, Velocity, and Speed

B

banked curve
5.4: Centripetal Force

basal metabolic rate
6.9: Work, Energy, and Power in Humans

base quantities
1.3: Units and Standards

base quantity
1.3: Units and Standards

base unit
1.3: Units and Standards

Base units
1.3: Units and Standards

C

carrier particle
4.11: Extended Topic- The Four Basic Forces—An Introduction

Center of mass
5.6: Newton’s Universal Law of Gravitation

Centrifugal force
5.5: Fictitious Forces and Non-inertial Frames - The Coriolis Force

centripetal acceleration
5.3: Centripetal Acceleration

Centripetal force
5.4: Centripetal Force

change in momentum
7.3: Impulse

chemical energy
6.7: Conservation of Energy

classical relativity
3.6: Addition of Velocities

Collisions
7.7: Collisions of Point Masses in Two Dimensions

Commutative
3.3: Vector Addition and Subtraction- Graphical Methods

component
3.3: Vector Addition and Subtraction- Graphical Methods

conduction
8.5: Heat Transfer Methods
8.6: Conduction

conservation of mechanical energy
6.5: Conservative Forces and Potential Energy

conservation of momentum principle
7.4: Conservation of Momentum

conservative force
6.5: Conservative Forces and Potential Energy

constant acceleration
2.6: Motion Equations for Constant Acceleration in One Dimension

convection
8.5: Heat Transfer Methods
8.7: Convection

conversion factor
1.4: Unit Conversion

Coordinate Systems
2.3: Vectors, Scalars, and Coordinate Systems

Coriolis Force
5.5: Fictitious Forces and Non-inertial Frames - The Coriolis Force

D

deceleration
2.5: Acceleration

dependent variable
2.9: Graphical Analysis of One-Dimensional Motion

derived quantity
1.3: Units and Standards

derived unit
1.3: Units and Standards

derived units
1.3: Units and Standards

Dimensional Analysis
1.5: Dimensional Analysis

dimensionally consistent
1.5: Dimensional Analysis

dimensionless
1.5: Dimensional Analysis

direction
3.3: Vector Addition and Subtraction- Graphical Methods

displacement
2.2: Displacement
2.3: Vectors, Scalars, and Coordinate Systems

distance
2.2: Displacement
2.3: Vectors, Scalars, and Coordinate Systems

distance traveled
2.2: Displacement

drag forces
4.8: Drag Forces

Dynamics
4.2: Development of Force Concept

E

efficiency
6.7: Conservation of Energy

elapsed time
2.4: Time, Velocity, and Speed

Elastic collision
7.5: Elastic Collisions in One Dimension

elastic collisions
7.7: Collisions of Point Masses in Two Dimensions

electrical energy
6.7: Conservation of Energy

emissivity
8.8: Radiation

energy
6.2: Work- The Scientific Definition

English units
1.3: Units and Standards

exponent
1.2: Scientific Notation and Order of Magnitude

external force
4.2: Development of Force Concept

F

falling
2.8: Falling Objects

fictitious force
5.5: Fictitious Forces and Non-inertial Frames - The Coriolis Force

force
4.2: Development of Force Concept

force field
4.11: Extended Topic- The Four Basic Forces—An Introduction

forces
4.4: Newton’s Second Law of Motion- Concept of a System

fossil fuels
6.10: World Energy Use

Friction
4.7: Friction
6.6: Nonconservative Forces

G

Graphical analysis
2.9: Graphical Analysis of One-Dimensional Motion

gravitational constant, G
5.6: Newton’s Universal Law of Gravitation

Gravitational potential energy
6.4: Gravitational Potential Energy

greenhouse effect
8.8: Radiation

H

head
3.3: Vector Addition and Subtraction- Graphical Methods

heat
8.2: Heat

Heat capacity
8.3: Temperature Change and Heat Capacity

heat transfer
8.3: Temperature Change and Heat Capacity

horsepower
6.8: Power

I

ideal angle
5.4: Centripetal Force
ideal banking
5.4: Centripetal Force
ideal speed
5.4: Centripetal Force
Impulse
7.3: Impulse
independent variable
2.9: Graphical Analysis of One-Dimensional Motion
Inelastic collision
7.6: Inelastic Collisions in One Dimension
Inertia
4.3: Newton’s First Law of Motion- Inertia
inertial frame of reference
4.6: Normal, Tension, and Other Examples of Forces
instantaneous acceleration
2.5: Acceleration
Instantaneous velocity
2.4: Time, Velocity, and Speed
internal kinetic energy
7.5: Elastic Collisions in One Dimension
isolated system
7.4: Conservation of Momentum

J

joule
6.2: Work- The Scientific Definition

K

Kepler's first law
5.7: Satellites and Kepler’s Laws- An Argument for Simplicity
Kepler's Second Law
5.7: Satellites and Kepler’s Laws- An Argument for Simplicity
Kepler's third law
5.7: Satellites and Kepler’s Laws- An Argument for Simplicity
kilocalorie
8.2: Heat
kilogram
1.3: Units and Standards
Kinematics
2.2: Displacement
2.6: Motion Equations for Constant Acceleration in One Dimension
3.5: Projectile Motion
Kinematics problems
2.7: Problem-Solving Basics for One-Dimensional Kinematics
kinetic energy
6.3: Kinetic Energy and the Work-Energy Theorem
Kinetic Friction
4.7: Friction

L

latent heat
8.4: Phase Change and Latent Heat
law
1.1: The Basics of Physics
law of conservation of energy
6.7: Conservation of Energy
Law of inertia
4.3: Newton’s First Law of Motion- Inertia
Linear momentum
7.2: Linear Momentum and Force

M

Magnitude
3.3: Vector Addition and Subtraction- Graphical Methods
mass
4.3: Newton’s First Law of Motion- Inertia
matter
1.1: The Basics of Physics
mechanical energy
6.5: Conservative Forces and Potential Energy
mechanical equivalent of heat
8.2: Heat
metabolic rate
6.9: Work, Energy, and Power in Humans
Meter
1.3: Units and Standards
method of adding percents
1.6: Significant Figures
metric system
1.3: Units and Standards
microgravity
5.6: Newton’s Universal Law of Gravitation
model
1.1: The Basics of Physics
2.4: Time, Velocity, and Speed
momentum
7.7: Collisions of Point Masses in Two Dimensions
motion
3.5: Projectile Motion

N

net work
6.3: Kinetic Energy and the Work-Energy Theorem
Newton’s third law of motion
4.5: Newton’s Third Law of Motion- Symmetry in Forces
Newton’s Universal Law of Gravitation
5.6: Newton’s Universal Law of Gravitation
newton's laws
4.10: Further Applications of Newton’s Laws of Motion
nonconservative force
6.6: Nonconservative Forces
normal force
4.6: Normal, Tension, and Other Examples of Forces
nuclear energy
6.7: Conservation of Energy

O

order of magnitude
1.2: Scientific Notation and Order of Magnitude

P

perfectly inelastic collision
7.6: Inelastic Collisions in One Dimension
phase change
8.4: Phase Change and Latent Heat
8.7: Convection
physical quantity
1.3: Units and Standards
pit
5.2: Rotation Angle and Angular Velocity
planetary motion
5.7: Satellites and Kepler’s Laws- An Argument for Simplicity
position
2.2: Displacement
potential energy
6.5: Conservative Forces and Potential Energy
potential energy of a spring
6.5: Conservative Forces and Potential Energy
power
6.8: Power
problem solving
4.9: Problem-Solving Strategies
projectile
3.5: Projectile Motion
Projectile motion
3.5: Projectile Motion

Q

Quarks
7.4: Conservation of Momentum

R

R factor
8.6: Conduction
Radians
5.2: Rotation Angle and Angular Velocity
radiant energy
6.7: Conservation of Energy
radiation
8.5: Heat Transfer Methods
8.8: Radiation
radius of curvature
5.2: Rotation Angle and Angular Velocity
range
3.5: Projectile Motion
rate of conductive heat transfer
8.6: Conduction
relative velocity
3.6: Addition of Velocities
relativity
3.6: Addition of Velocities
renewable forms of energy
6.10: World Energy Use
resultant
3.3: Vector Addition and Subtraction- Graphical Methods
resultant vector
3.3: Vector Addition and Subtraction- Graphical Methods
rocket propulsion
7.8: Introduction to Rocket Propulsion
rocket science
7.8: Introduction to Rocket Propulsion
rockets
7.8: Introduction to Rocket Propulsion

rotation angle
5.2: Rotation Angle and Angular Velocity

S

satellites
5.7: Satellites and Kepler’s Laws- An Argument for Simplicity
Scalar
2.3: Vectors, Scalars, and Coordinate Systems
3.3: Vector Addition and Subtraction- Graphical Methods
scientific method
1.1: The Basics of Physics
scientific notation
1.2: Scientific Notation and Order of Magnitude
second
1.3: Units and Standards
Second Law of Motion
7.2: Linear Momentum and Force
SI Units
1.3: Units and Standards
significant figures
1.6: Significant Figures
slope
2.9: Graphical Analysis of One-Dimensional Motion
system
4.4: Newton’s Second Law of Motion- Concept of a System

T

tail
3.3: Vector Addition and Subtraction- Graphical Methods
temperature change
8.3: Temperature Change and Heat Capacity
tension
4.6: Normal, Tension, and Other Examples of Forces
Terminal velocity
4.8: Drag Forces
theory
1.1: The Basics of Physics
thermal energy
6.7: Conservation of Energy
thrust
4.5: Newton’s Third Law of Motion- Symmetry in Forces
time
2.4: Time, Velocity, and Speed
trajectory
3.5: Projectile Motion

U

ultracentrifuge
5.3: Centripetal Acceleration
units
1.3: Units and Standards
useful work
6.9: Work, Energy, and Power in Humans

V

vector
2.3: Vectors, Scalars, and Coordinate Systems
3.2: Kinematics in Two Dimensions - An Introduction
3.3: Vector Addition and Subtraction- Graphical Methods
Vector Addition
3.4: Vector Addition and Subtraction- Analytical Methods
3.6: Addition of Velocities
Vector subtraction
3.4: Vector Addition and Subtraction- Analytical Methods
velocity
3.6: Addition of Velocities

W

watt
6.8: Power
Work
6.2: Work- The Scientific Definition

Glossary

Sample Word 1 | Sample Definition 1

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By Page

- Algebra Based Physics I - *Undeclared*
  - Front Matter - *Undeclared*
    - TitlePage - *Undeclared*
    - InfoPage - *Undeclared*
    - Table of Contents - *Undeclared*
    - Licensing - *Undeclared*
  - 1: Nature of Physics - *Undeclared*
    - 1.1: The Basics of Physics - *Undeclared*
    - 1.2: Scientific Notation and Order of Magnitude - *Undeclared*
    - 1.3: Units and Standards - *CC BY 4.0*
    - 1.4: Unit Conversion - *CC BY 4.0*
    - 1.5: Dimensional Analysis - *CC BY 4.0*
    - 1.6: Significant Figures - *CC BY 4.0*
    - 1.7: Summary - *CC BY 4.0*
    - 1.8: Exercises - *CC BY 4.0*
    - 1.9: Answers - *CC BY 4.0*
  - 2: One-Dimensional Kinematics - *Undeclared*
    - 2.1: Prelude to One-Dimensional Kinematics - *CC BY 4.0*
    - 2.2: Displacement - *CC BY 4.0*
    - 2.3: Vectors, Scalars, and Coordinate Systems - *CC BY 4.0*
    - 2.4: Time, Velocity, and Speed - *CC BY 4.0*
    - 2.5: Acceleration - *CC BY 4.0*
    - 2.6: Motion Equations for Constant Acceleration in One Dimension - *CC BY 4.0*
    - 2.7: Problem-Solving Basics for One-Dimensional Kinematics - *CC BY 4.0*
    - 2.8: Falling Objects - *CC BY 4.0*
    - 2.9: Graphical Analysis of One-Dimensional Motion - *CC BY 4.0*
    - 2.E: Kinematics (Exercises) - *CC BY 4.0*
  - 3: Two-Dimensional Kinematics - *CC BY 4.0*
    - 3.1: Prelude to Two-Dimensional Kinematics - *CC BY 4.0*
    - 3.2: Kinematics in Two Dimensions - An Introduction - *CC BY 4.0*
    - 3.3: Vector Addition and Subtraction- Graphical Methods - *CC BY 4.0*
    - 3.4: Vector Addition and Subtraction- Analytical Methods - *CC BY 4.0*
    - 3.5: Projectile Motion - *CC BY 4.0*
    - 3.6: Addition of Velocities - *CC BY 4.0*
    - 3.E: Two-Dimensional Kinematics (Exercises) - *CC BY 4.0*
  - 4: Dynamics- Force and Newton's Laws of Motion - *CC BY 4.0*
    - 4.1: Prelude to Dynamics- Newton's Laws of Motion - *CC BY 4.0*
    - 4.2: Development of Force Concept - *CC BY 4.0*
    - 4.3: Newton's First Law of Motion- Inertia - *CC BY 4.0*
    - 4.4: Newton's Second Law of Motion- Concept of a System - *CC BY 4.0*
    - 4.5: Newton's Third Law of Motion- Symmetry in Forces - *CC BY 4.0*
    - 4.6: Normal, Tension, and Other Examples of Forces - *CC BY 4.0*
    - 4.7: Friction - *CC BY 4.0*
    - 4.8: Drag Forces - *CC BY 4.0*
    - 4.9: Problem-Solving Strategies - *CC BY 4.0*
    - 4.10: Further Applications of Newton's Laws of Motion - *CC BY 4.0*
    - 4.11: Extended Topic- The Four Basic Forces—An Introduction - *CC BY 4.0*
    - 4.E: Dynamics- Force and Newton's Laws of Motion (Exercises) - *CC BY 4.0*
  - 5: Uniform Circular Motion and Gravitation - *CC BY 4.0*
    - 5.1: Prelude to Uniform Circular Motion and Gravitation - *CC BY 4.0*
    - 5.2: Rotation Angle and Angular Velocity - *CC BY 4.0*
    - 5.3: Centripetal Acceleration - *CC BY 4.0*
    - 5.4: Centripetal Force - *CC BY 4.0*
    - 5.5: Fictitious Forces and Non-inertial Frames - The Coriolis Force - *CC BY 4.0*

- 5.6: Newton’s Universal Law of Gravitation - *CC BY 4.0*
- 5.7: Satellites and Kepler’s Laws- An Argument for Simplicity - *CC BY 4.0*
- 5.E: Uniform Circular Motion and Gravitation (Excercise) - *CC BY 4.0*

- 6: Work, Energy, and Energy Resources - *CC BY 4.0*
  - 6.1: Prelude to Work, Energy, and Energy Resources - *CC BY 4.0*
  - 6.2: Work- The Scientific Definition - *CC BY 4.0*
  - 6.3: Kinetic Energy and the Work-Energy Theorem - *CC BY 4.0*
  - 6.4: Gravitational Potential Energy - *CC BY 4.0*
  - 6.5: Conservative Forces and Potential Energy - *CC BY 4.0*
  - 6.6: Nonconservative Forces - *CC BY 4.0*
  - 6.7: Conservation of Energy - *CC BY 4.0*
  - 6.8: Power - *CC BY 4.0*
  - 6.9: Work, Energy, and Power in Humans - *CC BY 4.0*
  - 6.10: World Energy Use - *CC BY 4.0*
  - 6.E: Work, Energy, and Energy Resources (Exercise) - *CC BY 4.0*
- 7: Linear Momentum and Collisions - *CC BY 4.0*
  - 7.1: Prelude to Linear Momentum and Collisions - *CC BY 4.0*
  - 7.2: Linear Momentum and Force - *CC BY 4.0*
  - 7.3: Impulse - *CC BY 4.0*
  - 7.4: Conservation of Momentum - *CC BY 4.0*
  - 7.5: Elastic Collisions in One Dimension - *CC BY 4.0*
  - 7.6: Inelastic Collisions in One Dimension - *CC BY 4.0*
  - 7.7: Collisions of Point Masses in Two Dimensions - *CC BY 4.0*
  - 7.8: Introduction to Rocket Propulsion - *CC BY 4.0*
  - 7.E: Linear Momentum and Collisions (Exercises) - *CC BY 4.0*
- 8: Heat and Heat Transfer Methods - *CC BY 4.0*
  - 8.1: Prelude to Heat and Heat Transfer Methods - *CC BY 4.0*
  - 8.2: Heat - *CC BY 4.0*
  - 8.3: Temperature Change and Heat Capacity - *CC BY 4.0*
  - 8.4: Phase Change and Latent Heat - *CC BY 4.0*
  - 8.5: Heat Transfer Methods - *CC BY 4.0*
  - 8.6: Conduction - *CC BY 4.0*
  - 8.7: Convection - *CC BY 4.0*
  - 8.8: Radiation - *CC BY 4.0*
  - 8.E: Heat and Heat Transfer Methods (Exercise) - *CC BY 4.0*
- Back Matter - *Undeclared*
  - Index - *Undeclared*
  - Glossary - *Undeclared*
  - Detailed Licensing - *Undeclared*