8.1: Background Material

Text References

- moment of inertia of common geometries
- unwinding spools

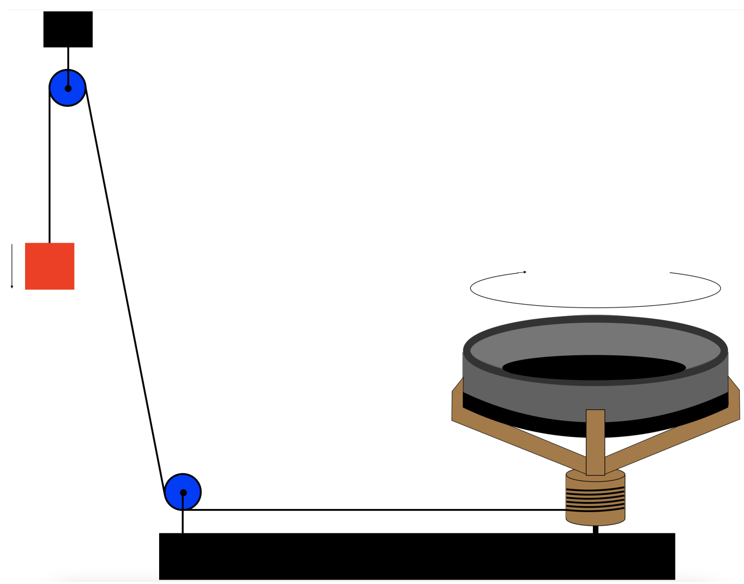
Measuring Moment of Inertia Dynamically

The second half of this lab consists of using dynamics to determine the moment of inertia of a thick circular ring. The first thing that should come to mind when thinking of "dynamics" and "inertia" is Newton's second law. In this case, it is the version of the second law that applies to rotations:

$$\overrightarrow{\alpha} = \frac{\tau_{net}}{I} \tag{8.1.1}$$

Clearly if we can measure the torque and angular acceleration, we immediately have the value of the moment of inertia. But these are not the easiest values to measure directly, so we have an experimental setup that simplifies the task somewhat:

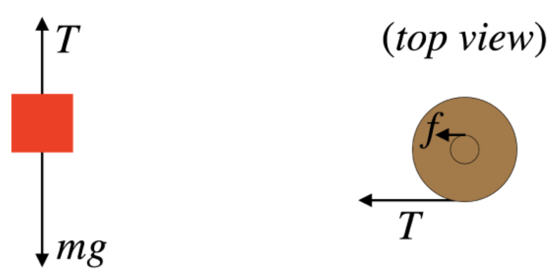
Figure 8.1.1 – Experimental Apparatus



As the mass accelerates downward, it accelerates the ring in the cradle in a rotational fashion. Their motions are linked by the fact that the string does not slip as it unwinds from the hub, and we can pretty easily measure the acceleration of the descending block. As for the torque on the rotating system, we can get this from the tension that pulls on the hub, and the radius of the hub. With the tension also affecting the motion of the mass, we end up with a convenient result.

Let's solve the physics problem. Start with free-body diagrams. For the hub, we will take a top view.

Figure 8.1.2 – Free-Body Diagrams



As the mass descends, the hub rotates clockwise according to the diagram. Summing the forces on the block and the torques (about the axle) on the hub, we get two second-law equations (r is the radius of the hub, and I is the moment of inertia of the rotating system):

$$F_{net} = mg - T = ma \qquad \tau_{net} = Tr = I\alpha \tag{8.1.2}$$

The "string unwinds without slipping" constraint gives a relation between the linear acceleration of the mass and the rotational acceleration of the ring & cradle:

$$a = r\alpha \tag{8.1.3}$$

Putting these three equations together such that we eliminate α and T (neither of which is easy to directly measure in an experiment), we get the following expression for the moment of inertia:

$$I = mr^2\left(\frac{g}{a} - 1\right) \tag{8.1.4}$$

So all we need to do to compute the moment of inertia is measure the acceleration of the falling mass, as well as the amount of mass hanging on the string and the radius of the hub. If we set the mass in motion from rest, it's not difficult to determine its constant acceleration from the distance it travels and the time that elapses.

There is one last thing we need to take into account here. We are interested in the moment of inertia of the ring, not the entire rotating system, which is what is calculated above. So we need to come up with a way to remove the contribution of the cradle to the system's moment of inertia.

Reducing Errors

There are some assumptions involved in the solution to the problem above that we have to take into consideration when we convert this from a physics problem to an real-world experiment. Most notably, friction in the axle and air resistance for both moving objects are going to cause problems for us.

If we were using energy conservation to solve this (which we certainly could do), then the more that the system rotates, the more work is done by the friction force on the axle, and the farther off our calculation based on mechanical energy conservation would be. With our analysis above, we don't have to worry about this cumulative effect, as it is only the instantaneous torque by the friction that introduces error into our results. While there is some friction present, it is very small compared to the tension force. As little as 5 grams will overcome the static friction to get the system rotating (the tension force that starts the system rotating in the experiment is the 200 gram weight), and the static friction is always greater than the kinetic friction. Also, the radius of the axle is smaller than the radius of the hub, so the torque due to friction is fairly negligible.

Air resistance is another matter. Normally we would like to run the process as long as possible (i.e. have the weight drop as much as possible), so that the percentage uncertainty of the distance the mass falls and the time it takes is as small as possible. But the air resistance grows as the speed increases, increasing the error introduced. So we need a fairly short drop time (from rest), keeping the speed of the system small, and we nevertheless need as accurate a measurement of drop distance and drop time as we can muster. Fortunately, we have the advantage of video recording at our disposal.
