University of California, Davis

UCD: Physics 9HA Lab

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CHAPTER OVERVIEW

1: Uncertainty and Confirmation of Hypotheses

1.1: Background Material

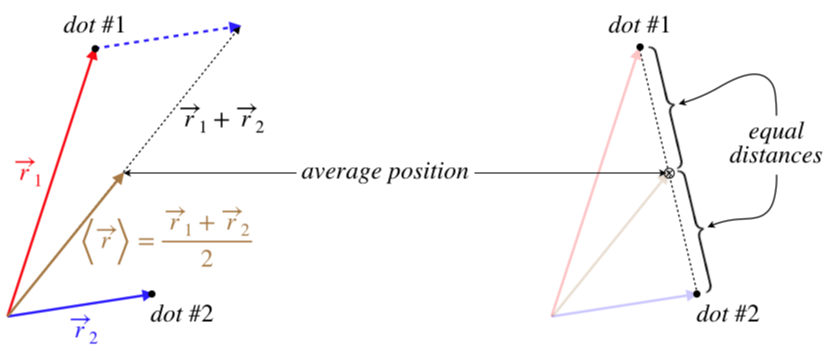
Average Position

In this lab we will be doing several trials that all produce dots on a piece of paper that measure the position of a marble as it strikes the floor. These dots can all be thought of as existing at the heads of position vectors, $\overrightarrow{r}_1$, $\overrightarrow{r}_2$, $\overrightarrow{r}_3$, and so on. As is typically the case for experiments, we will be interested in an average quantity over many trials – in this case the average position at which the marble lands. Finding an average vector is no different from finding an average number, namely:

$$\text{average position} = \left\langle \overrightarrow{r} \right\rangle = \frac{\overrightarrow{r}_1 + \overrightarrow{r}_2 + \dots + \overrightarrow{r}_n}{n} \tag{1.1.1}$$

We will not want to actually choose an origin and draw all these vectors, so it is helpful to come up with some way to find an average position directly from the positions of the dots. It isn't hard to show that the average position for two trials is just the point that is located halfway between the positions of the two trials.

Figure 1.1.1 – Average of Two Position Vectors



Finding the average position of more than two dots does not have quite as simple of a method, but we will use a trick to keep things from getting too complicated. Notice that if we have four dots, we can write the average position vector this way:

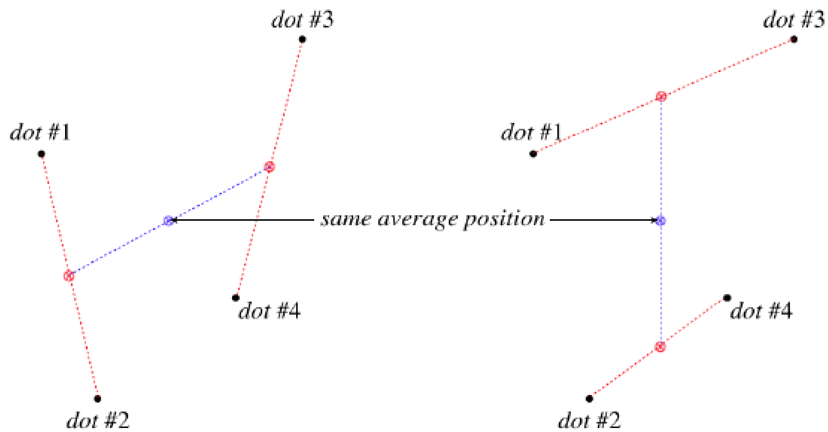
$$\left\langle \overrightarrow{r} \right\rangle = \frac{\overrightarrow{r}_1 + \overrightarrow{r}_2 + \overrightarrow{r}_3 + \overrightarrow{r}_4}{4} = \frac{\frac{\overrightarrow{r}_1 + \overrightarrow{r}_2}{2} + \frac{\overrightarrow{r}_3 + \overrightarrow{r}_4}{2}}{2} \tag{1.1.2}$$

This shows that we can get the average position of four dots by first finding the average positions of two pairs of dots, and then finding the average of those averages. This allows us to just use a ruler to locate the halfway points between pairs of dots to find the average position of all the dots. Notice that this procedure requires that we have some power-of-2 number of dots (2, 4, 8, 16, etc.). We could do it for a different number, but then we lose the "halfway between points" simplicity, and since we have control over the number of trials, we will stick with this method.

It should also be noted that it doesn't matter how we pair-off the points – in the end we end up with the same average position:

$$\left\langle \overrightarrow{r} \right\rangle = \frac{\frac{\overrightarrow{r}_1 + \overrightarrow{r}_2}{2} + \frac{\overrightarrow{r}_3 + \overrightarrow{r}_4}{2}}{2} = \frac{\frac{\overrightarrow{r}_1 + \overrightarrow{r}_3}{2} + \frac{\overrightarrow{r}_2 + \overrightarrow{r}_4}{2}}{2} \tag{1.1.3}$$

Figure 1.1.2 – Average Position of Four Dots Found Two Ways



Statistical Uncertainty

When we perform an experiment, we are interested in more than just the average value we obtain from many trials, we want to know to what extent this average can be trusted. That is, we want to know how *uncertain* we are that what have measured can be applied to any conclusions we might wish to draw. In the experiment we will perform, we will be "aiming" the marbles at a particular point on the paper, and the scatter of the dots is a result of uncertainty in our aim.

Whenever experimental runs have results that are scattered either because of human involvement or because the apparatus is not good at repeating a run very precisely, we determine the uncertainty *statistically*. This consists of computing what is called the *standard deviation*, which goes as follows:

- compute the average of all the data points

$$\langle x\rangle=\frac{x_1+x_2+\cdots+x_n}{n} \tag{1.1.4}$$

- compute how far each data point deviates from the average

$$\Delta x_1=x_1-\langle x\rangle\,,\;\Delta x_2=x_2-\langle x\rangle\,,\;\ldots \tag{1.1.5}$$

- square all the deviations from the average

$$\Delta x_1^2,\;\Delta x_2^2,\;\ldots \tag{1.1.6}$$

- average the square deviations

$$\left\langle\Delta x^2\right\rangle=\frac{\Delta x_1^2+\Delta x_2^2+\cdots+\Delta x_n^2}{n} \tag{1.1.7}$$

- compute the square root of the average

$$\sigma_x=\sqrt{\frac{\Delta x_1^2+\Delta x_2^2+\cdots+\Delta x_n^2}{n}} \tag{1.1.8}$$

This description of the computation of standard deviation makes it easy to remember, as we are just computing averages (first of the data points, then of the squares of the deviation of the data point values from the mean), but technically in these situations where we compute a mean from the actual data, there is a actually a slightly more accurate formula for standard deviation. It involves dividing the sum of the square deviations by $n-1$, rather than by n. We won't go into the technical details of why this is so, but it is important to note that the difference between these can become significant when n is quite small, as it often will be in our experiments. We will therefore henceforth use the so-called "unbiased" version of the standard deviation:

$$\sigma_x=\sqrt{\frac{\Delta x_1^2+\Delta x_2^2+\cdots+\Delta x_n^2}{n-1}} \tag{1.1.9}$$

The way this method of measuring uncertainty works for our present experiment should be clear: First use the method described above to determine the place on the paper that is the average landing point. Second, measure the distance from each dot to the average landing point. This is the "deviation from the mean" (Δx, measured in centimeters) of each data point. Then do the math from there.

Estimated Uncertainty

Another place where we introduce uncertainty in our results is in measurements. For example, if we are measuring a distance with a ruler, we would not expect our measurements to be accurate down to the micron ($\frac{1}{1000}$ millimeter), and we would estimate the uncertainty of these measurements to be more like in the range of perhaps a few millimeters. So in the experiment described above, our measurements of distances between dots and between the average landing points and dots introduces uncertainty into our results, because our measuring device does not measure these distances exactly. However, in this case we find that the tiny "few millimeter" estimated uncertainty of these distance measurements is insignificant compared to the statistical uncertainty associated with human aim (which is in the "few centimeter" range). We can therefore ignore the estimated uncertainty associated with ruler measurements for this experiment, as it contributes a negligible amount. Though both types of uncertainty typically occur in any experiment, it is usually true that only one type is the dominant version, allowing us to ignore the other. The simplest way to get a sense of this is to do a few repetitions of (what should be identical) runs, to get a sense of how much the results "scatter." While this experiment has this scatter greatly exceed the measurement uncertainty, more often than not, the reverse will be true. This is because we will use apparatuses that do a decent job of repeating runs.

So how do we make a decent estimate of a measurement uncertainty? Without going into the details that you may have encountered in a statistics class (like the nuances of the central limit theorem and the assumption of a normal distribution for our measurements), we will say that the range of uncertainty is such that we will expect that *roughly two-thirds of the data points will land within one standard deviation of the average*. While this works out automatically when we do the uncertainty statistically, we will use this as the standard for making our uncertainty estimates for measurements as well. That is, estimate the uncertainty of a measurement such that you would expect the true value of the measurement to lie within the uncertainty range of your recorded measurement roughly two-thirds of the time.

A Good Example to Keep in Mind

If you find the notions of statistical and estimated uncertainties confusing, here is a good model to keep in mind for them. Suppose you perform an experiment that involves measuring both a distance between two well-defined points, and a time interval between two events. So for example, a bouncing marble has pretty well-defined landing points, and the times between the landings is a time interval. The separation of the two points involves using a ruler or tape measure, and you can look at the markings on the measuring device to get an idea of the uncertainty in that measurement. This is an estimated uncertainty. The time interval is different – the device that launched the marble may not be consistent, but more importantly, if a human being is pressing a stopwatch when they see the marble land, then the uncertainty in this measurement is more amenable to statistical calculation – measure the time interval for several "identical" cases many times and compute the standard deviation.

1.2: Activities

Things You Will Need

[*For the labs we do this quarter that involve activities at home, you are expected to be resourceful. If you don't have precisely the items in these lists, you need to think outside the box to get materials that will do the same job. You may have to test several possibilities to see what works best. In short, you need to think like an engineer. Channel your inner MacGyver!*]

- several $8\frac{1}{2} \times 11$ sheets of paper
- a ruler
- a compass for drawing circles (not for telling which way is north) – a pencil and a string will do in a pinch
- a marble (steel or glass), or a small, round, smooth stone
- (useful, but not necessary) carbon paper
- pens of multiple colors

Data Collection

1. On the line that bisects the paper along its long axis, make a small dot with a pen, slightly (~1 inch, ~2.5 centimeters) off-center.
2. Place the paper onto a hard floor. If you have carbon paper, place it *beneath* the paper with the carbon side up.
3. Raise the marble to a height *over your head*, and do your best to drop it onto the dot you have made on the paper. *You should not be making adjustments between drops, to account for missing on previous drops – each drop should be a sincere attempt to hit the dot on* ***that*** *drop.* If you are not using carbon paper, you will need to find the indentation created by the marble, and mark it with a pen. Do your best to avoid any extraneous marks, or the paper will get overly cluttered.
4. Repeat the marble drop until you have 8 "data points" on the paper, and label each dot from 1 through 8.
5. With a fresh piece of paper, repeat this entire process, this time dropping the marble from waist height.

Data Analysis

For each of your two runs, do the following:

1. Use the "average of averages" method outlined in Background Material to determine the average landing position of the marble. This is where the multiple colors of ink will come in handy, as it can otherwise be difficult to keep straight all the dots on the paper.
2. Compute the standard deviation of the position from the average landing position, using the ruler to measure how far each data point is from the average. Creating a table like the one shown below will be helpful. Also, rounding the measurements of Δx to the nearest half-centimeter and using two significant figures for Δx^2 should be plenty of precision.

3. Use the compass and the ruler to draw a circle (probably best to use pencil) centered at the average landing position, with a radius equal to the uncertainty (standard deviation). If you don't have a compass, a string tied to a pencil and pinned at the other end should be adequate – precision is not really a major issue here.

Hypothesis

In any other experiment, developing an hypothesis comes before the data collection and analysis. In this exercise we have placed it later, because the experiment doesn't test any actual physical principles, but has rather just been contrived to illustrate the concept of uncertainty. Whenever performing an experiment, the hypothesis outlines the expected results. In this experiment, since we are aiming at a specific point on the paper, the hypothesis is as simple as, "The person is dropping the marble on the target point on the paper." While no individual drop actually hits this target point, the hypothesis states what would happen if all of the uncertainty could be removed (i.e. if the person had perfect aim).

So how do we use our data to *confirm* the hypothesis? The short answer is that we check to see if the expected result lands within the uncertainty range of the experimental result (i.e. the average). In terms of what we have done here, the hypothesis is confirmed if the target point lies within the circle we have drawn around the average landing point.

But not so fast. The person may have actually been aiming at any number of points besides the target point – anywhere inside the circle, in fact. So it turns out that this hypothesis is too broad to be useful. A better hypothesis would be one that involves a *comparison*. To see this, add the following to the two papers with all the dots:

1. Label the original target point with the letter "A."
2. Add a second target point, labeled with the letter "B," at the position on the same line lengthwise through the paper as target A, but on the opposite side of the widthwise line (again, about 1 inch / 2.5 centimeters off-center).

Now we can form a more specific hypothesis: "Given that the person is aiming at one of the two targets, the target they are aiming at is A." Not all hypotheses can be boiled-down to two possibilities, but this is a nicely illustrative example. There are several possibilities here:

- ***only target A lies within the circle, confirming the hypothesis*** – It is important to understand that even with this confirmation, it doesn't prove beyond all doubt that the hypothesis is correct. It only confirms it to within our agreed-upon level of certainty.
- ***only target B lies within the circle, refuting the hypothesis*** – This would indicate that, to within uncertainty, the person is actually aiming at the other target.
- ***neither target lies within the circle*** – This result should lead us to reevaluate our starting point, where it was given that one of the two points was the target. Either that assumption is incorrect, or some mistakes were made in the experimental procedure or data analysis, or there was some unseen source of systematic error (e.g. a steady wind that blowing the marble horizontally during its journey).
- ***both targets lie within the circle*** – There are two possibilities for this result. First, it can be "fixed" by adding more trials, which will have the effect of moving the average landing point closer to the actual target (assuming no systematic errors), and farther from the alternative target, such that the latter then lands outside the uncertainty range. The second possibility is that the two targets are simply too close to each other for the degree of uncertainty we have. In this case, we say that the experiment "doesn't provide sufficient resolution" to confirm or refute the hypothesis.

Lab Report

This lab is different from most of the labs we will have this quarter, in that it is more of an exercise in procedure and an illustration of principles of uncertainty than it is a test of physics. As such, the lab report is a little more "cookbook" than future labs will be. That is, your report will focus on some specific questions, rather than depend upon you – with little prompting – to extract and report on the relevant information.

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CHAPTER OVERVIEW

2: Graphical Methods of Data Analysis

2.1: Background Material

Graphical Methods

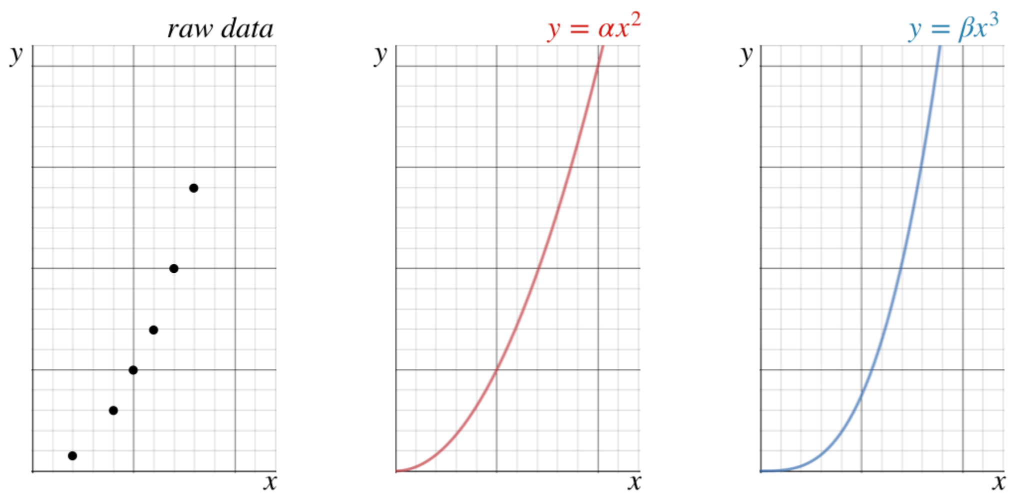
Very often our experiments are intended explore the mathematical relation between two quantities. Such experiments test an hypothesis that posits a certain functional relationship. For example, suppose we wanted to test the theory that objects dropped from rest fall a distance that is proportional to the square of the time elapsed after being released:

$$\text{distance fallen} = \Delta y \propto \Delta t^2 \tag{2.1.1}$$

How does one test such a proposition? Naturally lots of trials are required, where the time of descent and distance fallen are both measured. Multiple trials are needed for two elements of such an experiment. First, as we have already seen, we need to do several runs for fixed values of Δy and Δt in order to determine the uncertainties of our measurements. But we also need to do trials at a variety of values of Δy and Δt, so that we can test the functional dependence. We can never perform enough experiments to prove the relation beyond a doubt. As we saw in the previous lab, hypotheses that are framed as one conclusion against the entire universe of alternatives all suffer from this shortcoming, and it is generally better to do a direct comparison of two specific possibilities (in the previous lab, this consisted of framing the hypothesis in terms of aiming at target A or target B). So rather than use our data to confirm a single mathematical relation, we would use it to determine the relative merits of two competing mathematical relations.

There is a very powerful method for evaluating an hypothesis of this kind using the data acquired from all these trials. This method consists of plotting the data on a graph, and then checking to see which of the perspective formulas produces a graph that most closely fits the data (this is called a *best-fit curve*). There is software that does this for us, but rather than throwing this task into a black box, we get a better understanding if we do this "by hand," at least for awhile. When we plot the data points, we will undoubtedly notice that they follow some sort of curve, but it is not always obvious to the naked eye what function best fits those points. Consider the data points shown in the graph below, and two curves that we think may potentially express the correct functional dependence.

Figure 2.1.1 – Data Points and Two Prospective Curves



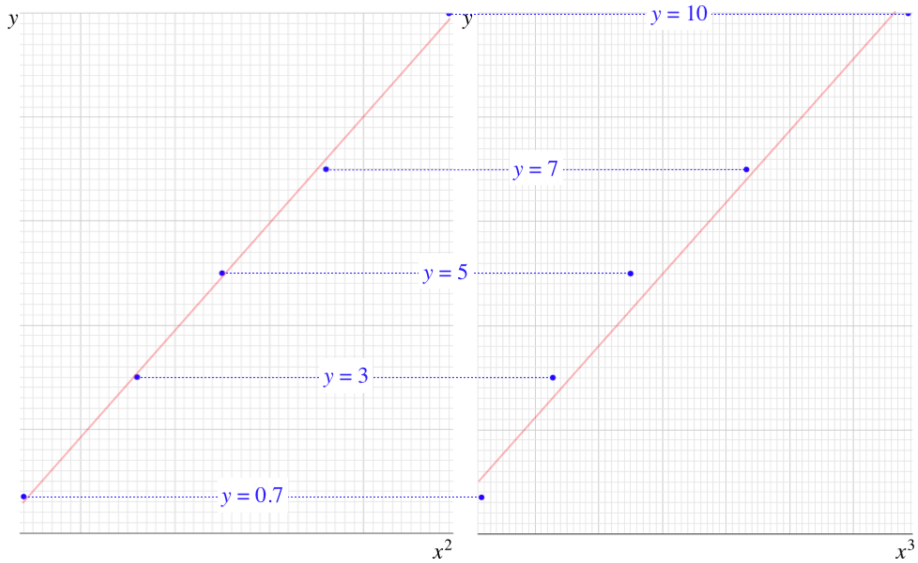
It certainly isn't clear from looking at the data alone which of the two prospective functions best fits it. If we superimpose the data points with the curves, we get a sense that perhaps the quadratic fits better than the cubic, but given the uncertainties in the measurements, it is by no means certain. What is more, creating these prospective curves is tricky business, since we are only testing power-dependence, which means we don't know what the values of α and β are. We have to keep tweaking these parameters until the curves are the best they can be – a very tedious task.

The main problem is that we humans are not particularly good at judging curvature of an array of points, and this can get even worse for functions more complicated than the two shown above. We are, however, pretty good at evaluating straight lines. It is therefore useful to change the graphs above to straight lines by plotting the y value versus the *prospective powers of* x. That is, make two plots of the data like the one on the left of the figure above, but plot y vs. x^2 for one graph, and y vs. x^3 for the other. Then take a straight-edge and see how well these data points can be aligned along it for the two cases. Whichever set of points more closely approximates a line is the one that reflects the functional dependence better.

Using the data in the figure above with every grid line equal to one unit, we can create a table, and plot the points on a grid for each of the two prospective functional dependences. In this particular case, the y-vs-x^2 plot displays the decidedly more linear form, while the y-vs-x^3 plot is quite obviously convex in shape. In other words, it is easier to fit a straight line to the y-vs-x^2 plot than it is to fit it to the y-vs-x^3. We therefore conclude that between these two choices, the dependence of y on x is quadratic rather than cubic.

Figure 2.1.2 – Creating Prospective Linear Plots

y	x	x^2	x^3
0.7	2	4	8
3	4	16	64
5	5	25	125
7	6	36	216
10	7	49	343



[As it has been expressed here, this is not a particularly quantitative way of determining which of the prospective functions is correct, but there is certainly a way to make it so. This basically consists of computing the aggregate of how far the data points deviate from the best-fit line. This gives a measure of how well the curve fits – called the R-value of that curve that varies from +1 to –1. The closer the absolute value of the R-value comes to 1, the better the curve fits the data. We won't go so far as to calculate R-values in this class, but you will likely run across it in another some other experimental science or statistics class.]

Some Final Comments

We get an added bonus by creating these linear plots. Note that the *slope* of the linear plot is the unknown constant of proportionality for the power law. We can find this constant by drawing our best-fit line, then selecting two points on the line and computing $\dfrac{\Delta y}{\Delta x}$. There are two important things to keep in mind while doing this. First, you are *using the line* to determine the slope, not two actual data points. So the two points used in the slope calculation must lie on the best-fit line. Second, in order to get as accurate a measurement as possible for the slope of the line, it is best to choose two points on the line that are as far apart on the graph as possible – if you choose two points that are close together, then a small absolute error in the reading of the x and y coordinates turns into a large percentage error in the ratio. It's also possible to derive information from the y intercept of the best-fit line. This will obviously give an additive constant, which, if it is not supposed to be part of the physics, can reveal a systematic error (e.g. every measurement of one of the variables was off by the same amount).

Note that if we have only two data points, then literally any curve can be made to fit, which means we get no information about functional for from only two data points. While two points are sufficient to determine the slope of a line, they are insufficient to determine the curvature (second derivative) of a non-line. If there are three points, then the curvature of a parabola that fits these points can be determined, but a cubic function can also be perfectly fit to those three points. The result is that as the power of the prospective curve grows, more data points are required to distinguish prospective functions. Of course it is always better to have more data points, but at a minimum, to distinguish between power laws with powers n and $n+1$, at least $n+2$ data points are needed.

It's also useful to note that the curvature of a graph is more apparent when the points are spread out. That is, a parabola can look like a straight line when points that are close together are plotted. Note that in the two graphs above, the horizontal axes are scaled so that the two most distant data points are at opposite ends of the graph.

2.2: Activities

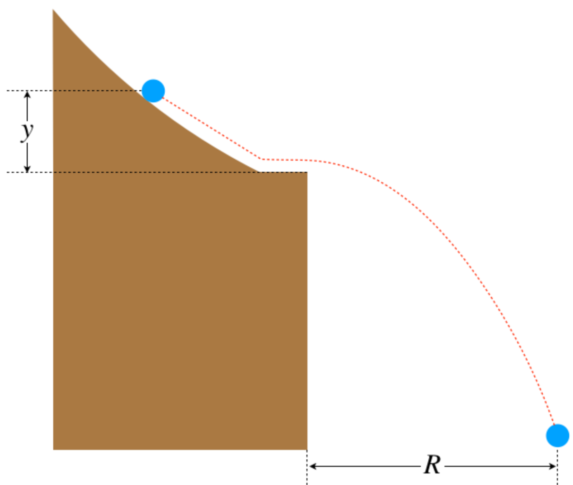
Things You Will Need

- several $8\frac{1}{2} \times 11$ sheets of paper
- a ruler
- a marble (steel or glass), or a coin
- (useful, but not necessary) carbon paper
- something to use as an inclined plane (clipboard, three-ring binder, sheet of cardboard, etc.)

The Problem

A bitter controversy has recently broken out between the residents of a new housing development in the foothills and the Town Council over the safety of a number of large rocks perched on the slope of a hill that ends with a cliff, down to a lake, on the opposite side of which is the housing development. At the heart of the dispute is the relationship between the height above the edge of the cliff at which the rocks are balanced (y) and the distance, horizontally from the base of the cliff, to which the rocks will range (R) should they roll down the hill and fly off the cliff. Neither party is convinced by fancy physics calculations, so they both hired engineering firms to conduct modeled experiments to come to a conclusion.

Figure 2.1.1 – Engineers' Model



One group of engineers concluded that y and R are related by:

$$y \propto R \tag{2.2.1}$$

A second engineering firm concluded that y and R are related by:

$$y \propto R^2 \tag{2.2.2}$$

Further calculations reveal that if the trajectories obey the linear relation, then some of the rocks starting higher on the hill will fly over the lake and land within the housing development, but if the quadratic relation holds, then even the highest rocks on the hill will land in the lake, leaving the housing development undamaged.

Your Task: Set up the same experiment performed by the two engineering firms, and use the graphical technique discussed in the Background Material to draw a conclusion about which of the two relationships given above is the correct one.

Data Collection

The general procedure to this experiment should be clear: Release the marble from different heights on the ramp, and record their landing points. Make plots of y against the two proposed functions of R, and determine from a best-fit line which of these functions is the relationship between y and R that is more likely to be correct. There are several considerations to keep in mind for the data-taking process itself:

- Unlike the previous lab, a small stone is unlikely to work here, as it may not roll very predictably down the plane. If a small sphere is not available, this can be done with a coin on edge. The only mildly-challenging part is releasing it from rest such that it rolls and doesn't fall over.
- Recording the landing points of the marble/coin will work exactly like the previous lab, but the range of heights from which it is released may lead to a spread of points that goes beyond a single sheet of paper, so several sheets placed end-to-end may be needed. You will want to do "warm-up runs" from the range of heights you plan to use, to make sure that your paper covers the full range of landing points.
- As our goal is to measure distances from the edge of the "cliff," we will need to have a line on the paper that indicates the edge of the cliff. Think about how you would most accurately determine that line (is "eyeballing it" a good idea?).
- There is variance in each roll, even from the same height, so perform 4 rolls from each height and use the "averaging" method from the previous lab to determine the landing points. There is no need to compute uncertainties for this lab.
- Keeping control over the experimental conditions is important, so pay close attention to the following:
  - Make sure the ramp is stable, so that it doesn't shift or change its angle between runs. The ramp should also end *before* the edge of the tabletop. That is, this model assumes that the rock is moving horizontally when it leaves the cliff.
  - The ramp doesn't need to be flat (it is only the starting height of the marble above the tabletop that matters), and the data is likely to be "cleaner" if the marble/coin makes a smoother transition from ramp to table. It won't be a disaster if there is a small bounce at this transition point between the ramp and the table, so don't go too far out of your way to make this transition smooth – just don't use an extreme angle for the ramp and it should be fine.
  - Make sure that the paper remains held in place on the floor for all runs, either by taping it there, or placing weights on the corners.
  - The measurement of the height above the tabletop will be hard to do with any precision, and it is especially tedious to measure individually every time you do four runs at the same height, so it is a good idea to measure one time and mark the starting point on the ramp, then release from that point 4 times.
- Try to keep the side-to-side variance as small as possible, and keep in mind that it is the distance from the edge of the cliff that matters, not the distance from a single point (i.e. the exit point from the tabletop will not be exactly the same with every run).
- To make sure that you have sufficient data to discern a curve in the plot, make measurement for at least 5 different starting heights.

Data Analysis and Additional Discussion

Once you have collected the sets of dots on your papers, do the following:

1. Create a table like that shown in Figure 2.1.2, and use it to create plots for the two competing theories.
2. Interpret your results – explain which theory is correct according to information you can draw from your plots.
3. Describe what assumptions were made that might invalidate the result if they turned out to be wrong.
4. Briefly discuss the likely sources of error in your experiment, including the following considerations:
   - how the procedure can be improved to give cleaner results
   - the likelihood that the errors could have led to an inaccurate conclusion
5. (optional) Feel free to confirm your conclusion with the "fancy physics calculations" that your clients don't trust.

Lab Report

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CHAPTER OVERVIEW

3: Testing a Theory of Air Resistance

3.1: Background Material

Text References

- free-body diagrams
- Newton's 2nd law
- air resistance

Using Uncertainties with Graphs

In our previous lab, we were able to use graphical methods to choose between two proposed functional relations, because one plot of points clearly landed closer to a common line than the other. In this lab, we will be asked to test just a single "theory" (functional relation). In such a case, an experiment can never conclusively prove a theory, because there is always small errors in the measurements, which means the proposed function will never run exactly through all the data points. But if the function misses the data points badly, we certainly want to be able to discard the theory. So the question we need to answer is, "How do we know when the data points come 'close enough' to the theoretical function that we conclude that that the theory is worth keeping?"

When we draw a best-fit line through a set of data points in a graph, the line generally misses most or all of the points. This line presumably represents the theoretical linear relationship, and it does not match the data points, but *it doesn't have to* – it only needs to come close to every point, and as we saw in the first lab, "close enough" is defined as being within the uncertainty. So if the best-fit line misses a data point by an amount of 0.12, and the uncertainty in the measurement of that data point is 0.23, then as far as that data point is concerned, the best-fit line is appropriate. Of course, there are many data points, each with its own uncertainty, and the line must pass within the allowed range for every data point in order to be acceptable.

It is common to display this graphically by plotting not only the measured data points, but also the points that represent one standard deviation above and below the data value – i.e. the range of "acceptable" values. So if a data point has an x-value of 1.4, with an uncertainty of $\sigma_x = \pm 0.3$, then not only will the graph include the data point, but also two additional points with the same y-value that have x values of 1.1 and 1.7. Usually a line segment is drawn (called an *error bar*), connecting these two outer points, to indicate the acceptable range for the graph of the function to pass through.

Figure 3.1.1 – Error Bars

Naturally the values measured for the vertical axis also have uncertainties, which then results in vertical error bars as well. When one of these error bars is much smaller than the other, it is often left off the graph, as the fluctuation range of the data point is still represented well under the assumption that the measurement on the axis with the small uncertainty is essentially exact.

So given that a graph of the theoretical function must lie within the error bars of all the data points in order to not be rejected, one can think of the set of error bars produced by the data as a sort of "channel" through which the graph must pass. Assuming we have used our method of creating a straight line from our function, we are allowed to move and rotate the line all we want in an effort to fit it within this channel. If no amount of movement or rotation will do the trick, then the experiment refutes the theory. This also explains why we seek to reduce the errors as much as possible. If the error bars are huge, then the channel they create will fit a very large number of graphs, and we are not able to narrow-down the number of viable theories.

Figure 3.1.2 – Using Error Bar "Channel" to Confirm or Refute Theory

3.2: Activities

Things You Will Need

- a stopwatch that measures hundredths of seconds (like found on a smartphone)
- a meter stick or ruler (or really, just a length of string roughly $20 cm$ long)
- a light object that exhibits the effects of air resistance as it falls (though not *too* much – no feathers!):
  - should require no less than about $0.6 s$ and no more than about $1.0 s$ to fall a distance of about $1 m$ from rest
  - should be easy to release quickly/cleanly, and should not drift too much side-to-side
  - examples: coffee filter (flat side down), cotton ball, under-inflated balloon (or inflated zip-lock bag), crumpled tissue paper (this should be a last resort, as its air-resistance properties can change during the course of the experiment if the "degree of crumple" changes (such as if it unravels between runs).

The Problem

It is well-known that an object dropped from rest near the Earth's surface with negligible air resistance will fall a distance as a function of time given by the equation:

$$y = \frac{1}{2} g t^2 \tag{3.2.1}$$

A physics paper written by a classmate seeks to determine the equation of motion that applies when air resistance *isn't* negligible. The author of this paper argues that the opposing force of air resistance reduces the downward net force on the falling object. Then, according to Newton's 2nd law, a smaller net force results in a smaller acceleration. Since the constant g in the equation above assumes acceleration due to a net force that is only due to gravity, we can simply replace this constant with a smaller value (which we will call k) to account for the smaller net force. You classmate therefore proposes that the equation of motion for an object in free fall from rest under the influence of both gravity and air resistance looks like:

$$y = \frac{1}{2} k t^2 \tag{3.2.2}$$

where k depends upon details specific to the falling object, such as its density and its cross-sectional area.

Your task is to perform an experiment that either confirms or refutes this thesis, and if it is confirmed, compute the k-value of the item used in your experiment.

Data Collection

The procedure here is straightforward: Drop an object from several known heights, measure the time elapsed for each journey, and use the best-fit-line method with error bars to draw conclusions. Here are some suggestions that should help you achieve reasonable results:

- Perform at least 5 drops from each height, so that the uncertainty (standard deviation) of the time elapsed for each height can be computed.
- If you "spaz" during a time measurement and know you have made a mistake, there's no reason that you need to keep that data point. But don't keep trying over and over to get close to what you think is about the right value! There *is* supposed to be some uncertainty, after all. [*It generally works better if you don't look at the timer at all, and focus on the falling object – then you know if you made a good measurement without tainting your opinion by seeing the actual number.*]
- Perform drops from 6 different heights, with the lowest being about 1 meter (there is no need for precision on this, since we are looking for a functional dependence as the heights change). The other heights should be equally-spaced by about $20 cm$, so that the highest drop height is about 2 meters. The easiest way to mark these heights is on a wall with tape or a light, erasable pencil mark.
- There is uncertainty in both the drop height and the time of drop, but the latter much more significant (you can measure the height accurately to within about $1 cm$, which is $\leq 1\%$ of the total height, and time uncertainties will be a much larger percentage). For this reason, you do not need to include error bars for the height variable on the graph – treat height measurements as "exact."
- This lab is designed to be performed in the very basic manner described above (and works very well that way), but if you have the resources and are so inclined, you are free to greatly reduce the uncertainties by having someone assist you by taking video

of the dropping process (with a stopwatch running in the video).

Data Analysis and Additional Discussion

1. Create a table of your data.
2. Use the data in your table to compute the standard deviations for each data point.
3. Plot the points and error bars on a graph.
4. If you find it is possible to sketch an acceptable best-fit line, do so. If you cannot, sketch a line that shows an acceptable best-fit line is not possible. Draw a conclusion about the author's theory.
5. Assume for a moment that the theory is correct. In this case, what physical properties of the system do the slope and intercept of the line represent?
6. Discuss any issues that you feel need to be addressed regarding the physics behind this experiment. If you find that the theory is confirmed, this is your opportunity to explain why it had to be so, and if you find that it is refuted, you can explain the theory's fatal flaw.

Lab Report

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CHAPTER OVERVIEW

4: Static Friction

4.1: Background Material

Text References

- static friction
- pulleys and inclined planes

Percentage Uncertainty

In our previous labs, we discussed the importance of measuring and accounting for uncertainty in experimental results. In those cases, we calculated uncertainty from a range of measurements for a single quantity (landing position of a marble and the time elapsed to fall a known distance). Usually the uncertainty in a quantity has little meaning out of context. For example, if we are measuring the speed of an object, and compute the uncertainty in that speed to be $\pm 1.0\frac{cm}{s}$, then our knowledge of this object's speed is quite impressive if we are talking about a bullet, and not so impressive if we are talking about a tortoise. It is therefore useful to define *percentage uncertainty*, which is the ratio of the *absolute uncertainty* (the standard deviation we talked about previously) and the quantity in question:

$$\text{percentage uncertainty in measured quantity } x = e_x = \frac{\sigma_x}{x} \tag{4.1.1}$$

Uncertainty Propagation

In this lab, we will not be measuring the physical quantity in question directly. Instead, we will measure multiple quantities, and put them together mathematically to compute what we are looking for. This poses us with a new problem – there will be uncertainties in all of our measurements, so how do we use these to determine the uncertainty of their combination? We will virtually never be adding or subtracting quantities, so we really only have to worry about multiplication/division and raising to powers.

Without going into the mathematical details behind it, we will simply state that whenever two uncertain quantities are multiplied or divided, the percentage uncertainty in the product or ratio is given by the *quadrature* of their individual percentage uncertainties:

$$\left.\begin{array}{c} z = x \cdot y \\ or \\ z = \dfrac{x}{y} \end{array}\right\} \quad \Rightarrow \quad e_z = \sqrt{e_x^2 + e_y^2} \tag{4.1.2}$$

If the quantity we are calculating involves a power, then the rule is a little different. For example, if we have $z = x^2$, it is *not* correct to simply use the quadrature formula above with x replacing the y (this would result in an uncertainty for z that is $\sqrt{2}$ times the uncertainty of x). Instead, the rule is to multiply the percentage uncertainty of the measured quantity by the power:

$$z = x^n \quad \Rightarrow \quad e_z = n \cdot e_x \tag{4.1.3}$$

Weakest Link Rule

Given that this is a physics lab, we don't want to be spending all of our time doing uncertainty calculations (not withstanding the focus of these first two labs), so we will employ a shortcut that will reduce our workload somewhat. For just about every case where we will need to propagate uncertainty associated with multiple measurements, one of the measurements will have a significantly larger percentage uncertainty than the others. Say for example that we make measurements of two quantities that are multiplied, where one of the percentage uncertainties is 1% and the other is 4%. Putting these together gives:

$$\left.\begin{array}{c} z = x \cdot y \\ e_x = 1\% \\ e_y = 4\% \end{array}\right\} \quad \Rightarrow \quad e_z = \sqrt{(1\%)^2 + (4\%)^2} = \sqrt{17}\% = 4.1\% \tag{4.1.4}$$

As you can see, the resulting percentage uncertainty differs very little from the larger of the two percentage uncertainties. We will therefore use the shortcut we call the *weakest link rule*, which consists of simply finding the component that has the largest percentage uncertainty, and using that as the total uncertainty. Note that we still need to include the power rule shown above, however. For example, if the quantity we are computing looks like $z = xy^2$ and x has a 4% uncertainty, while the uncertainty of y is 3%, the square of y in the computation of z makes its 6% contribution the weakest link.

Comparing Two Uncertain Results

We know how to determine whether an experimental result agrees with an "exact" (theoretical) number – we just check to see if the experimental result lands within the absolute uncertainty of the exact value. But something we will do in several labs is perform two different experiments to find the same value (this is most common when we don't actually have a theoretical number to check against). We will want to know if these two experiments confirm each other's results, but how do we do this, when both provide inexact answers? The answer to this (again, without going into details) is to compare the two results (which are of course both averages of the data), and determine whether they lie within a certain range, which is defined by the quadrature of the *absolute* uncertainties generated for each of the results:

$$range = \sqrt{\sigma_1^2 + \sigma_2^2} \tag{4.1.5}$$

Let's look at a quick example. One experiment yields a (unitless) result of $7.40 \pm 3\%$, while the result of the other experiment is $7.63 \pm 2\%$ (perhaps these percentages were found for each experiment using the weakest link method). Do these two experiments agree to within uncertainty? Well, if we add 3% to the first result, we get 7.622, so the second result does not land within the uncertainty of the first. Conversely, the first result does not lie within the uncertainty of the second result. But the real question is whether their difference of 0.23 lands within the range:

$$\left.\begin{array}{l} 0.03 \cdot 7.40 = 0.222 \\ 0.02 \cdot 7.63 = 0.1526 \end{array}\right\} \quad range = \sqrt{0.222^2 + 0.1526^2} = 0.269 > 0.23 \tag{4.1.6}$$

So these experimental results are consistent with each other to within uncertainty. Notice that if one of the results is "exact," (whether it is a theoretical answer or an experiment with very small errors) its uncertainty is zero, and the range is just the uncertainty of the other experiment.

4.2: Activities

Things You Will Need

Nothing! All the data has been meticulously collected for you.

Finding μ_S Two Ways

We will be seeking to find the coefficient of static friction between a block of wood and a swatch of carpet. We will do this with two separate experiments, a check for consistency between the two results. The first experiment will consist of gradually adding weight to a string that (with the help of pulleys) pulls the block up an inclined plane whose surface is the carpet swatch. We will add weight gradually, until the block just starts to slide up the plane, recording both the last weight that didn't move the block and the weight that does move it, and declare the weight halfway between these as the "trigger weight" (m) that hits the maximum static friction force. We will do this at several heights (y) for the top of the plane, to diversify our data, since we don't expect the coefficient of friction to depend upon the angle of the plane.

Figure 4.2.1 – Block Pulled Up the Plane

The second experiment will involve the same block on the same inclined plane of carpet, but instead of changing the angle and measuring the weight required to move it up the plane, we will remove the string, pulleys and hanging mass, and just raise the plane very slowly until the block starts to slide down, measuring the height (y) at which it begins to slide. We'll repeat this process 4 times, to get enough data for an average and an approximate statistical uncertainty.

Figure 4.2.2 – Block Slides Down the Plane

We can use free-body diagrams and Newton's laws to solve for μ_S in terms of y, L, and m for these two cases, and compare the results to see if they agree.

The Data

All the raw data for these two experiments is given in the table below. An extra column for the angle θ that the plane makes with the horizontal has been included to aid in the calculation of μ_S. [*Note: In upcoming labs, you will be expected to create your own tables from scratch!*]

Figure 4.2.3 – Data Table

It should be noted that normally the uncertainties in the measured quantities L, M, m, and y would be noted, and then "propagated" (perhaps using the "weakest link rule" described in the Background Material) to determine the uncertainty in the calculated quantity μ_S. In this case, however, the rug involved could display different coefficients of static friction for different runs, depending upon whether it is fluffed-up or matted down, or if the grain was slanting one way or another. These random elements that vary across runs can only be accounted-for statistically, so it is easier to do the statistics after the final values are computed for each run.

Data Analysis and Additional Discussion

1. Fill in the empty spaces in the data table given above. There is quite a bit of "physics work" that needs to be done to get from the raw data to the eventual result (free-body diagrams, application of the maximum static friction condition, etc.), and all of that needs to be included for the analysis to be considered complete. The text references given in the Background Material should be very helpful in this regard.
2. Determine (and explain) whether the results of the two experiments are in agreement, to within uncertainty. See the Background Material for the proper method for determining this.
3. It is mentioned above that there are additional uncertainties brought into the experiment beyond the inexactitude in the measurements taken. For example, in the second experiment, the only measurements needed are the length of the ramp L and the height to which it is raised y. The length of the ramp has a very small uncertainty, so we can treat it as essentially exact. The only uncertainty in measurement of consequence is therefore that of the variable y. These measurements were less precise (raising the ramp slowly and noting the height at which the block begins sliding is not the same as measuring the length of the motionless plank), so they are rounded to the nearest half-centimeter. Compute the percentage uncertainty of the height measurement and the percentage uncertainty of the coefficient of static friction for that experiment, and show that the latter is significantly larger, confirming that random error likely creeps into the condition of the rug between runs.
4. While it is difficult to do without actually being present to take the data, speculate about what might be sources of error for the experiment. We have already mentioned the changing condition of the rug, so focus your consideration on some of the other elements of the experiment. Make suggestions for methods that might reduce these errors.

Lab Report

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CHAPTER OVERVIEW

5: Energy Forms

5.1: Background Material

Text References

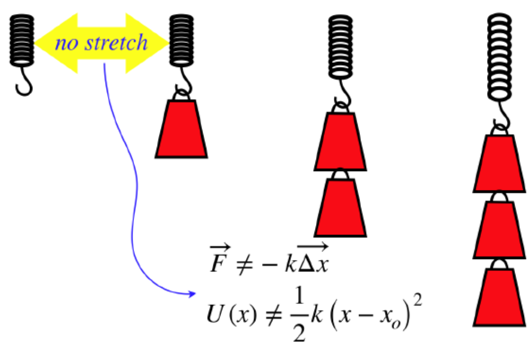
- work done by a general force
- mechanical energy conservation

Non-Ideal Springs

We found the potential energy function for a spring by computing the work done by that spring. This assumes that the spring is "ideal," which means that it perfectly obeys Hooke's law. It turns out that many real springs don't closely approximate this behavior. The springs used in this lab are no exception.

The spring used in this lab pulls its coils tightly together. That is, the pull of the spring is not zero when the spring is at its minimum length – the pull is balanced by the repulsive normal force between the coils that are in contact. If we hang a small weight from these springs, the coils don't separate. With no change in coil separation, the spring force doesn't change – the contact forces between them just get a bit smaller. Eventually, when we add sufficient weight, the coils do separate, causing the spring force to increase (and of course the contact force between coils vanishes, as they are no longer in contact). But the force exerted on the spring does not exhibit Hooke's law, which means that the potential energy stored in the spring can not be computed in the usual way.

Figure 5.1.1 Coil Behavior for Our Non-Ideal Spring



This doesn't mean we can't do anything with these springs, because we can still measure the displacement for various forces, which means that we can still compute the work done on the spring by stretching it. Of course, we need to know how the force changes as a *function* of the displacement to do the work integral – we can't simply multiply the force by the displacement.

Figure 5.1.2 Graph of Applied Force vs. Spring Stretch

The graph above shows how the spring behaves when certain forces are applied to it (in our experiment, we will do this by hanging weights). Imagine applying force F_A first, and noting the amount that the spring stretches x_A. When we reduce the force to F_B, naturally the spring stretch decreases (to x_B). As long as the coils don't touch each other, this follows a linear relationship, as we would expect for any spring. But when we reduce the force to the point where the coils are in contact (F_C), then every applied force from there down to zero produces the same stretch – zero.

When we store potential energy in a spring, we do this by doing work on the spring, and for a perfect spring, this work happened to equal the potential energy function $\frac{1}{2}kx^2$ that we are so familiar with. But this is not such a spring, so to determine the potential energy stored in the spring, we must do a new calculation of the work done on it, using the function $F(x)$ that we will determine experimentally.

Gravitational Potential Energy of Extended Objects

When an object changes heights, its gravitational potential energy changes, which is proportional to its change in height. When the object is not a point mass (i.e. it has extension in space), different parts of the object can be at different heights. How do we measure the change in potential energy, when there are so many points to choose from? What happens if the object rotates as it rises? The answer (which we will proved later in the course) is that this potential energy is computed using the change in height of the object's *center of mass*. Keeping this in mind for the spring, which actually changes length during its journey will be useful.

5.2: Activities

Things You Will Need

Nothing! All the data has been meticulously collected for you.

A Jumping Spring

The physical system we will be examining is that of a spring that is threaded over a vertical post. The spring is closed at the top, so that the top is prevented from going lower than the top of the post. The bottom of the spring is then pulled downward, stretching it, and is released, after which the spring leaps upward. Our goal is to test energy conservation by measuring the energy stored in the spring before it is released, and compare it to the gravitational potential energy the spring gains at its peak height (at both of these stages, the kinetic energy is zero).

When confronted with the details of this experiment, a theorist works on the problem, and declares that because the coils collide when the spring returns to it original shape, the mechanical energy lost from this "inelastic collision" (a topic we will cover in this class soon) is substantial. A simple model reveals that in fact *one quarter* of the mechanical energy is lost during this process. We now seek to test this hypothesis with an experiment.

The Data

The first collection of data collected relates to the spring. The photos below show the position of the bottom of the spring with weights in 20 gram increments hanging from it (the top of the spring is at the same location in every case).

Figure 5.2.1 – Spring Data

The second collection of data comes from the launch of the spring. The photo below shows the stretch of the spring before launching, and the gif shows the result of the launch. The mass of the spring is measured to be 9.8 grams.

Figure 5.2.2 – Launch Data

Data Analysis and Additional Discussion

1. Compute the elastic potential energy stored in the spring before launch:
   a. Make a table and plot the data for the applied force vs. spring stretch (you do not need to include error bars).
   b. Use the plot to create a best-fit line for the data, and write the equation for this line.
   c. From the equation for the line, derive a formula for the work done in stretching this spring by an amount x.
   d. Apply the work formula to determine the elastic potential energy stored in the spring just before the launch.
2. Compute the gravitational potential energy change by the spring at the peak of its flight:
   a. Measure the vertical distance traveled by the bottom of the spring. [*Naturally it is better to do several launches and take an average, but we will treat the launch shown as being representative of a typical launch.*]
   b. Adjust the vertical displacement to account for the height to which the *center of mass* rises. [*Hint: When the spring is stretched, the center of mass moves down half as far as the amount of stretch.*]
   c. Use the vertical displacement of the center of mass of the spring to compute its change in gravitational potential energy.
3. Compute the mechanical energy loss.
   a. Express the energy loss as a percentage.
   b. Compare the percentage loss measured by the experiment with the predicted loss.
4. Check uncertainties to see if the experiment "comes close enough" to confirm the prediction:
   a. Which part of the launch data do you think is susceptible to the greatest absolute error? Which provides the greatest percent error? [*For this, you will first be making estimates of absolute uncertainties based on what you can see – not by doing statistics. And to get the percent uncertainties, you need to keep in mind what is being measured in each case.*] Explain your reasoning.
   b. Calculating the correct percent uncertainty for the spring potential energy before the launch is a tricky matter, so we will just use the percent uncertainty of the stretch of the spring prior to the launch. Is the hypothesis confirmed within this percent uncertainty?

Lab Report

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CHAPTER OVERVIEW

6: Momentum and Impulse

6.1: Background Material

Text References

- impulse and momentum
- elastic and inelastic collisions in 1-dimension

Force and Motion Sensors

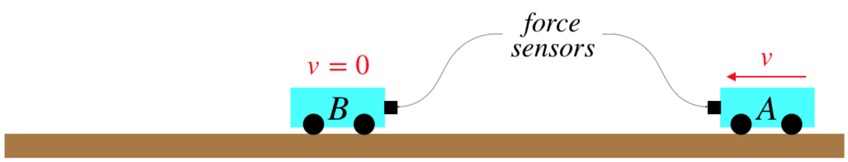
The equipment used in this lab includes two sensing devices that take data in real time. One of these is a force sensor, which measures force exerted in Newtons, and the other a motion sensor, which measures velocity in meters per second.

Both of these sensors electronically send their readings to a laptop at regular intervals. Force measurements are received by the computer 100 times per second, and velocity measurements are received 25 times per second. Upon receiving this information, the computer plots the values versus time on a graph, the details of which we will use for our analysis. Forces are recorded as positive values when they push on the sensor, and negative values when they pull on it. Motion detected by the motion sensors is recorded as a positive value when the object is moving away from the sensor, and a negative value when the object if approaching the sensor.

The experiment involves collisions of nearly-frictionless rolling carts, each of which carries a force sensor to record the force exerted on it by the other cart. As we are interested in the forces as a function of time, we don't want the collision to be between two hard surfaces, or the period of time over which the forces act will be too brief to measure, even at the impressive sampling rate of our force sensors. We have therefore engineered the two types of collisions we will study so that the force of the collision occurs at the leisurely pace of more than one tenth of a second!

The motion sensors are located on opposite ends of the track on which the carts are confined to roll, and measure the velocities of the carts closest to them. For simplicity, all of the cases we will study will involve a stationary target cart, but the conclusions of the experiment will be independent of this choice.

Figure 6.1.1 – Anatomy of the Collisions



We therefore will end up with four graphs plotted at the same time – a force and velocity for each cart. The information we extract from these plots will allow us to draw conclusions about impulse, momentum conservation, and kinetic energy conservation.

6.2: Activities

Things You Will Need

Nothing! All the data has been meticulously collected for you.

Elastic Collisions

As was stated in the Background Material, the forces exerted between the carts have been engineering so that they act over a long enough period of time that we can measure them as they change. For the first two collisions we will examine, this is achieved by placing strong repulsing magnets on the fronts of the two carts. As the carts get closer together, this repulsive force grows, and the carts "gradually" (the whole collision is over in under 0.2 seconds) push on each other with increasing magnitude, until they finally push each other apart. The repulsive magnetic force gets so strong when the magnets get close that they never actually touch each other (at least not at the speed that we provide to the incoming cart). The gif below shows an example of one of these collisions, with the laptop recording the two force-vs-time graphs at the top of the screen and the two velocity-vs-time graphs at the bottom, as the collision unfolds.

Figure 6.2.1 – Elastic Collisions

The graphs of the data for two elastic collisions is given below. The graphing software provides tools for more easily pulling numerical values from the graphs. First, the graph can be greatly enlarged from what you see on the laptop screen above, so that finer details can be discerned. Second, a pinpoint utility provides the ordered pair coordinates of any point on the graph. And finally, by selecting a section of the graph one can measure the area under the curve. All of these have been used in the graphs of our data for your convenience.

- **Equal Masses:** $\boldsymbol{m_A = m_B = 1.307 kg}$

Answer all the questions below, and include explanations or calculations based on the data to backup your answers in each case. Without access to the equipment, it is not possible to get a good sense of the uncertainties we are dealing with, so with this handicap of "performing" the experiment online, we'll arbitrarily use 10% as the percentage uncertainty within which our results must lie to declare victory.

1. Consider the impulses delivered to the carts in the collision.
   a. Is Newton's third law, expressed in terms of impulse, confirmed for this collision?
   b. Is the impulse-momentum theorem confirmed?
2. Examine the data in terms of momentum conservation.
   a. Determine whether momentum is conserved using the before and after pinpoint values in the graphs.
   b. Does the momentum of the system appear to remain constant *during* the collision? Should it be? Explain.
3. Examine the data in terms of kinetic energy conservation.
   a. Determine whether kinetic energy is conserved using the before and after pinpoint values in the graphs.
   b. Does the kinetic energy of the system appear to remain constant *during* the collision? Should it be? Explain.

- **Unequal Masses:** $\boldsymbol{m_A = 1.307 kg, \;\; m_B = 0.838 kg}$

4. 5. 6. Answer questions 1-3 again for this case of a collision of unequal masses.

Inelastic Collision

For our inelastic collision, we will be using a "perfectly-inelastic collision," where the two carts stick together and move off together at one speed. A bottle brush and tube are used, to once again ensure that the force exerted between the carts changes slowly enough to observe its details. The gif below shows an example of one of these collisions.

Figure 6.2.2 – Inelastic Collisions

The only collision of this kind for which we have data features a heavier incoming cart than target cart. The data for this collision is given below.

Unequal Masses: $m_A = 1.335 kg, \;\; m_B = 0.847 kg$

Answer all the questions below, and include explanations or calculations based on the data to backup your answers in each case.

7. Consider the impulses delivered to the carts in the collision.
    a. Is Newton's third law, expressed in terms of impulse, confirmed for this collision?
    b. Is the impulse-momentum theorem confirmed?
    c. In this run there is a small but undeniable dip in the force-vs.-time curve for both carts, just after the main "bump." Interpret what this apparent anomaly is telling us is happening physically.
8. Examine the data in terms of momentum conservation.
    a. Determine whether momentum is conserved using the before and after pinpoint values in the graphs.
    b. Does the momentum of the system appear to remain constant *during* the collision? Should it be? Explain.
9. Examine the data in terms of kinetic energy conservation.
    a. Find the kinetic energy lost using the before and after pinpoint values in the graphs.
    b. Confirm that this matches what is supposed to be lost for such a collision.

Lab Report

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CHAPTER OVERVIEW

7: Navigating the Conservation Laws

7.1: Background Material

Text References

- ballistic pendulum
- comparing two uncertain results

Estimating Uncertainty

In lab #4, you were tasked with finding the coefficient of static friction two different ways, and comparing the results, to see if they agree within uncertainty. This lab consists of doing the same for kinetic friction. One of the procedures is a simple as can be, while the second is quite convoluted. In both cases, you will need to estimate the uncertainty, determine the "weakest link" percentage uncertainties, make the necessary calculations, and compare the results.

The usual approach to estimating an uncertainty (i.e. when one cannot determine it statistically with repeated trials) is to "take half the smallest grade of measurement." For example, this means that if you have a balance scale that can measure mass down to the nearest gram (i.e. trying one gram more or one gram less clearly unbalances the scale), then the uncertainty for this measurement is taken to be ± 0.5 grams.

Very often this is mistakenly taken to be half the smallest grade available on the measuring device, but this is not quite the same thing. As an example, consider once again a balance scale that measures mass. It may have a built-in mechanism for measuring down to 0.1 grams, but if the object being weighed doesn't become unbalanced with this small of a change, then this level of precision doesn't really express our uncertainty.

This means that we can't *really* make a good estimate of uncertainty for a measurement without having the measuring device in front of us so that we can tinker with it, to test its true sensitivity. This lab will provide photos and videos for measurements, and you will be expected to read off measurements. There are some measurements in this lab where the measurement is fairly clear, and others where you will have to make a reasonable guess of both the measurement and the wiggle room you need to give it in the form of absolute uncertainty.

7.2: Activities

Things You Will Need

Nothing! All the data has been meticulously collected for you.

Finding μ_K Two Ways

As stated in the Background Material, the goal of this lab is to measure the coefficient of kinetic friction between a cardboard box and the tabletop across which it slides. We will cause the box to slide in two different manners, and compare the results for confirmation.

The first method is as straightforward as it gets. We will pull the box horizontally across the horizontal tabletop, and measure the magnitude of the pull. From measurements of the box's mass and the reading on the scale, we will do the calculation to determine μ_K. We will also include an analysis of uncertainty in this measurement.

Figure 7.2.1 – Block Pulled Horizontally by Spring Scale

The second method involves a more complicated mechanism. We will swing a pendulum from a horizontal position, and allow it to strike the box when it is vertical. The impulse imparted to the box will cause it to slide some distance before coming to rest. The end of the pendulum consists of a bag of sand, so it deforms during the collision. With measurements of the mass of the box, the mass of the bag of sand, and the distance the box is displaced, we once again will compute (\mu_K\). Again a measure of the uncertainty in this experiment will be included.

Figure 7.2.2 – Block Slides after Being Struck by Pendulum

The Data

The data for these experiments are given below in the form of pictures and gifs, and you are expected to extract the necessary information from them.

Experiment #1

- **the physical process** – In case it isn't clear from the video, it should be noted that the spring scale is pulled horizontally at a constant speed, and the markings on it indicate that it measures grams-force (1 gram-force is the force that the Earth's gravity exerts on 1 gram of mass).

mass of box – Can be read directly from the scale.

Experiment #2

- **the physical process –** There may be important clues here regarding the kind of collision that occurs. Also note that the box starts with its front-facing side flush with the tape on the table. The picture of the box displacement later comes directly from this run.
- **mass of sandbag –** Can be read directly from the scale.

- **maximum height of sandbag** – The sandbag is released from a height that is level with the top of the string.

- **minimum height of sandbag** – This measurement is taken by the same meter stick in the same position as the previous measurement.

- **displacement of box** – The box rotated slightly after the collision, but the displacement of the center of mass is approximately measured.

Data Analysis and Additional Discussion

In each of the experiments, the first step is to "solve the physics problem." You are encouraged to do this using *variables only*, and only after you have a solution should you plug in the numbers to have from the data.

1. Experiment #1 – dragging the box with a scale
   a. Solve the physics problem (free-body diagram, Newton's laws, etc.) and solve for the coefficient of kinetic friction using the data from experiment #1. Along the way, comment on the important features of the experimental procedure that make this calculation correct.
   b. Estimate the absolute uncertainties for the two measured quantities in this experiment, and convert them into percent uncertainties.
   c. Determine the "weakest link" percent uncertainty and record the result of the experiment in the form:

$$\mu_K = 0.\times\times\times\ \pm\times\times.\times\ \% \tag{7.2.1}$$

2. Experiment #2 – hitting the box with a pendulum
   a. Solve the physics problem (work, energy conservation, momentum conservation, etc.) and determine the coefficient of kinetic friction using the data from experiment #2. Along the way, comment on the important features of the experimental procedure that make this calculation correct.
   b. Estimate the absolute uncertainties for the four measured quantities in this experiment (note: the two height measurements for the sandbag combine into a single useful measurement), and convert them into percent uncertainties.
   c. Determine the "weakest link" percent uncertainty and record the result of the experiment in the same form as above.
3. Compare the results, and determine whether they are in agreement to within uncertainties.

Lab Report

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CHAPTER OVERVIEW

8: Rotational Dynamics

8.1: Background Material

Text References

- moment of inertia of common geometries
- unwinding spools

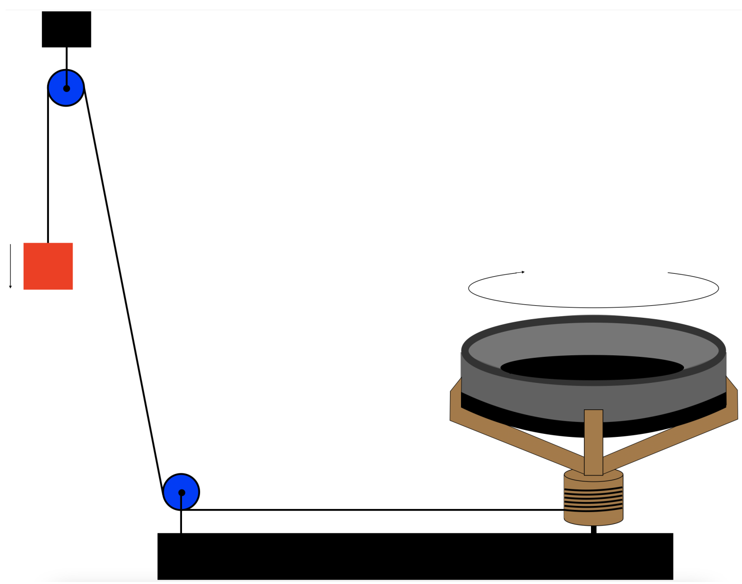
Measuring Moment of Inertia Dynamically

The second half of this lab consists of using dynamics to determine the moment of inertia of a thick circular ring. The first thing that should come to mind when thinking of "dynamics" and "inertia" is Newton's second law. In this case, it is the version of the second law that applies to rotations:

$$\overrightarrow{\alpha}=\frac{\tau_{net}}{I} \tag{8.1.1}$$

Clearly if we can measure the torque and angular acceleration, we immediately have the value of the moment of inertia. But these are not the easiest values to measure directly, so we have an experimental setup that simplifies the task somewhat:

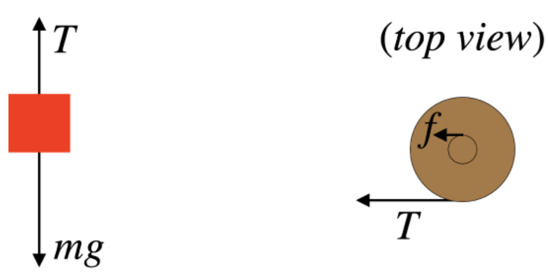
Figure 8.1.1 – Experimental Apparatus



As the mass accelerates downward, it accelerates the ring in the cradle in a rotational fashion. Their motions are linked by the fact that the string does not slip as it unwinds from the hub, and we can pretty easily measure the acceleration of the descending block. As for the torque on the rotating system, we can get this from the tension that pulls on the hub, and the radius of the hub. With the tension also affecting the motion of the mass, we end up with a convenient result.

Let's solve the physics problem. Start with free-body diagrams. For the hub, we will take a top view.

Figure 8.1.2 – Free-Body Diagrams



As the mass descends, the hub rotates clockwise according to the diagram. Summing the forces on the block and the torques (about the axle) on the hub, we get two second-law equations (r is the radius of the hub, and I is the moment of inertia of the rotating system):

$$F_{net} = mg - T = ma \qquad \tau_{net} = Tr = I\alpha \tag{8.1.2}$$

The "string unwinds without slipping" constraint gives a relation between the linear acceleration of the mass and the rotational acceleration of the ring & cradle:

$$a = r\alpha \tag{8.1.3}$$

Putting these three equations together such that we eliminate α and T (neither of which is easy to directly measure in an experiment), we get the following expression for the moment of inertia:

$$I = mr^2\left(\frac{g}{a} - 1\right) \tag{8.1.4}$$

So all we need to do to compute the moment of inertia is measure the acceleration of the falling mass, as well as the amount of mass hanging on the string and the radius of the hub. If we set the mass in motion from rest, it's not difficult to determine its constant acceleration from the distance it travels and the time that elapses.

There is one last thing we need to take into account here. We are interested in the moment of inertia of the ring, not the entire rotating system, which is what is calculated above. So we need to come up with a way to remove the contribution of the cradle to the system's moment of inertia.

Reducing Errors

There are some assumptions involved in the solution to the problem above that we have to take into consideration when we convert this from a physics problem to an real-world experiment. Most notably, friction in the axle and air resistance for both moving objects are going to cause problems for us.

If we were using energy conservation to solve this (which we certainly could do), then the more that the system rotates, the more work is done by the friction force on the axle, and the farther off our calculation based on mechanical energy conservation would be. With our analysis above, we don't have to worry about this cumulative effect, as it is only the instantaneous torque by the friction that introduces error into our results. While there is some friction present, it is very small compared to the tension force. As little as 5 grams will overcome the static friction to get the system rotating (the tension force that starts the system rotating in the experiment is the 200 gram weight), and the static friction is always greater than the kinetic friction. Also, the radius of the axle is smaller than the radius of the hub, so the torque due to friction is fairly negligible.

Air resistance is another matter. Normally we would like to run the process as long as possible (i.e. have the weight drop as much as possible), so that the percentage uncertainty of the distance the mass falls and the time it takes is as small as possible. But the air resistance grows as the speed increases, increasing the error introduced. So we need a fairly short drop time (from rest), keeping the speed of the system small, and we nevertheless need as accurate a measurement of drop distance and drop time as we can muster. Fortunately, we have the advantage of video recording at our disposal.

8.2: Activities

Things You Will Need

Nothing! All the data has been meticulously collected for you.

Finding Moment of Inertia Two Ways

Once again we undertake to measure a physical quantities with two methods. In this case, it is moment of inertia of a thick circular ring, and we are doing it by direct measurement of its mass and dimensions, and also by analyzing it dynamically. As usual, you are expected to estimate uncertainties for each experiment, determine the "weakest link" percentage uncertainties for each experiment, and draw a conclusion at the end about whether the two experiments agree with each other to within the overall uncertainty.

The Data

All of the measured values are given below, either explicitly, or in the form of pictures/videos.

Direct Measurement

- **mass of the ring** – Read this quantity (in grams) directly from the scale.

- **inner and outer diameters of the ring** – Note that the meter stick lies across the center of the ring such that one side of the measurement is aligned with the 30 cm hash. This allows us to not only measure the distance across the inside edges of the ring, but also the thickness of the ring, giving both the inner and outer diameters.

Experiment, part 1

- **mass hanging from string** – 205 grams
- **radius of the hub around which the string is wound** – 2.38 cm
- **the physical process** – The mass is released from rest, and the ring is in the cradle. The large number markings on the meter stick occur every 10 cm.

Experiment, part 2

- **mass hanging from string** – 55 grams
- **the physical process** – The mass is released from rest, and the cradle spins by itself, without the ring present.

Data Analysis and Additional Discussion

1. Direct Measurement – computing the moment of inertia from the mass and dimensions
   a. Use the data provided above to compute the moment of inertia of the ring.
   b. Compute an estimate for the percentage uncertainty of the moment of inertia.
      - The digital scale measures mass to the nearest gram.
      - While it isn't clear from the pictures provided, there is some difficulty in getting the locations of the edges of the ring, due to the effects of parallax on the thick meter stick. Moving the camera only slightly can give a reading that differs by as much as a couple millimeters. Correctly centering the meter stick is also difficult to do. These are considerations you can only truly judge by being present to do the measurements, so in absentia, you will have to be satisfied with **estimating the absolute uncertainty of the diameter to be 2 millimeters**.
      - With percentage uncertainties for both diameters, use the one that is the "weaker link."
      - Don't forget to include the effect of raising uncertain measurements to a power (see here for a reminder).
2. Experiment – testing the moment of inertia dynamically
   a. In the Background Material, we derived an expression for the moment of inertia in terms of the hanging mass, the radius of the hub, and the acceleration of the hanging mass. We have direct measurements of the first two of these quantities. Determine the acceleration from the data given for each of the two parts of the experiment.
   b. Compute the moment of inertia of the ring. Explain why the two parts of the experiment are needed.
   c. Compute an estimate for the "weakest link" percentage uncertainty of the moment of inertia. The hanging mass is certain to within about 2 grams (it would be lower, if not for the accumulation of hanging string as the weight descends). The radius of the hub is certain to within about 0.5 millimeters. Repeats of the experiment (which are not shown here) indicates that the time measurement appears to be accurate in both cases to within about 0.04 seconds (thanks to video recording). This uncertainty in the time measurement more-or-less takes into account the uncertainty in the starting and ending positions of the descending mass, so the distance the mass falls can be treated as exact.

3. Compare the result of the direct measurement to the experiment , and determine whether they are in agreement to within uncertainties.

Lab Report

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CHAPTER OVERVIEW

9: Static Equilibrium

9.1: Background Material

Text References

- balancing on the center of gravity

A Classic Physics Problem

You are given N identical uniform bricks of length L, and are asked to arrange them on top of one another such that the front end of the top brick extends as far over the edge of a table as possible.

Figure 9.1.1 – Stacking Bricks to Extend off a Table's Edge

The typical questions asked for this physical system are:

1. What is the farthest the front end of the top of these N bricks can extend beyond the edge of the table? That is, what is the maximum value of x in terms of L for N bricks?
2. Is there a limit to this extension? That is, what is the maximum value of x in terms of L as $N \rightarrow \infty$)?

The trick to analyzing this problem is to start with the top brick and work your way down. Suppose $N = 1$ – there is only one brick. The farthest this brick can extend beyond the edge of the table is of course $\frac{1}{2}L$. This is because the normal force from the table needs to align with the gravity force on the brick (which acts through its center), for there to be no net torque and the brick not to rotate off the table.

Now suppose there are two bricks. Well, the top brick still can't be allowed to rotate off the bottom brick, and the maximum extension it can have from the edge of that brick is again $\frac{1}{2}L$. We also need to arrange the bricks so that the combination of both bricks doesn't rotate off the table edge, so how far out can the second brick extend from the table edge? We can treat the two bricks as a single system, and as we know from the first brick, the we maximize the extension for which the system will not rotate if the system's center of mass is right at the edge. So we need to calculate where the *two-brick center of mass* is, and place *that* at the edge of the table. It isn't hard to determine that the center of mass of the two bricks is $\frac{3}{4}L$ from the open end of the top brick.

We can continue this process as we add more bricks, basically treating each new brick as if it is the table's edge for the previous bricks, and computing the new center of gravity of the full collection of bricks to determine the correct placement, and with it the maximum extension. Note that this means that the distance from the center of mass of all the bricks to the front end of the top brick the maximum x value we are looking for.

The Surprising Result

The general solution to this problem for N bricks requires a bit of fancy footwork, so it will be presented here. We'll start by numbering the bricks from the top-down, with the N^{th} brick resting on the table. The distance that the n^{th} brick extends beyond the surface below it we will call:

$$\Delta x_n = \alpha_n L \; , \tag{9.1.1}$$

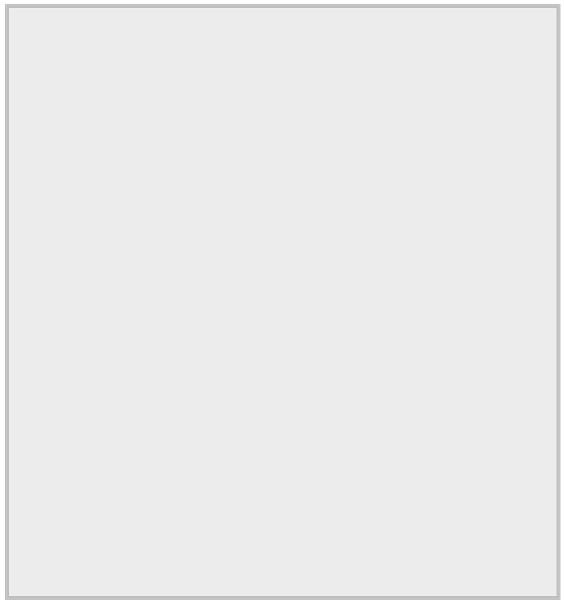
where the value α_n is the fractional overhang, and as we said above, $\alpha_1 = \frac{1}{2}$ and $\alpha_2 = \frac{1}{4}$.

The maximum overhang we are looking for is therefore:

$$x = \Delta x_1 + \Delta x_2 + \cdots + \Delta x_N = (\alpha_1 + \alpha_2 + \cdots + \alpha_N)\, L \tag{9.1.2}$$

Perhaps a diagram is called for here:

Figure 9.1.2 – N Bricks



For maximum extension, the center of gravity of the top $N-1$ bricks is directly above the front edge of the bottom brick, as shown above. Let's choose the front end of the top brick as the origin along the x-direction. We'll call the distance separating the origin and the center of gravity of the top $N-1$ bricks $x_{cm}(N-1)$. We can use this to determine the distance from the origin of the center of gravity of all N bricks, in the usual way. Calling the mass of a single brick m, then the mass of the $N-1$ bricks is $(N-1)m$, and the center of mass of the bottom brick is $\frac{1}{2}L$ farther from the origin than the center of gravity of the $N-1$ bricks. [*For a reminder of how to find the center of mass of two extended objects with their own centers of mass, go here.*]

$$x_{cm}(N) = \frac{[m]\left[x_{cm}(N-1) + \frac{1}{2}L\right] + [(N-1)\,m]\left[x_{cm}(N-1)\right]}{Nm} = x_{cm}(N-1) + \frac{1}{2N}L \tag{9.1.3}$$

So this shows that we can obtain the distance from the origin of the center of gravity of N bricks if we know the distance of the center of gravity of the first $N-1$ bricks. This means we can express the distance as a sum by adding more and more terms (bricks). Clearly the formula works for $N=1$ (where $x_{cm}(0)=0$). Then we use this answer to solve for two bricks, and that answer to solve for three bricks, and so on. For N bricks, we have the answer to question #1:

$$x(N) = \frac{1}{2}\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{N}\right) L \tag{9.1.4}$$

[*Note: Using our earlier notation,* $\alpha_n = \frac{1}{2n}$.]

The answer to question #2 is surprising. When the series in parentheses is taken to the limit as $N \to \infty$, it is called the *harmonic series*, and it diverges. This means that there is no limit to how far bricks can be extended, if we have enough of them! What makes this counterintuitive is how slowly this sum diverges. One can get the stack of bricks extended to more than one brick length ($x > L$) with just 4 bricks, but to get it to two brick lengths requires 31 bricks, and three brick lengths requires 227 bricks!

9.2: Activities

Things You Will Need

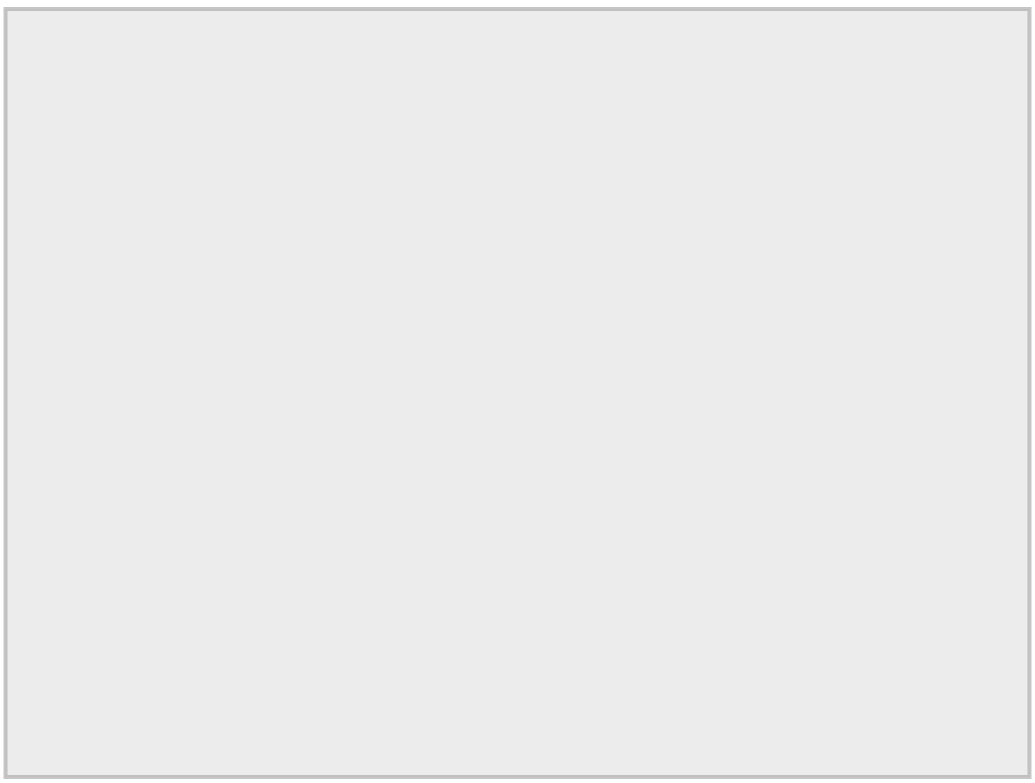
- an old deck of cards (or any uniform rectangular prisms)
- a ruler
- two forks
- a drinking glass
- anything else that you think would be cool

Have Some Fun!

This lab is all about static equilibrium, and more specifically, balancing extended objects at their centers of gravity. Below you will see two demonstrations of this idea. You should try these for yourself. The card structure will require some calculations and meticulous measuring to get a good result, while the fork balance is all about trial-and error. If you have an idea to jazz-up these examples, or an idea for another balancing act, by all means, go for it. In the end, your Gradescope submission will be photographs of two things that you managed – make them as spectacular as you can!

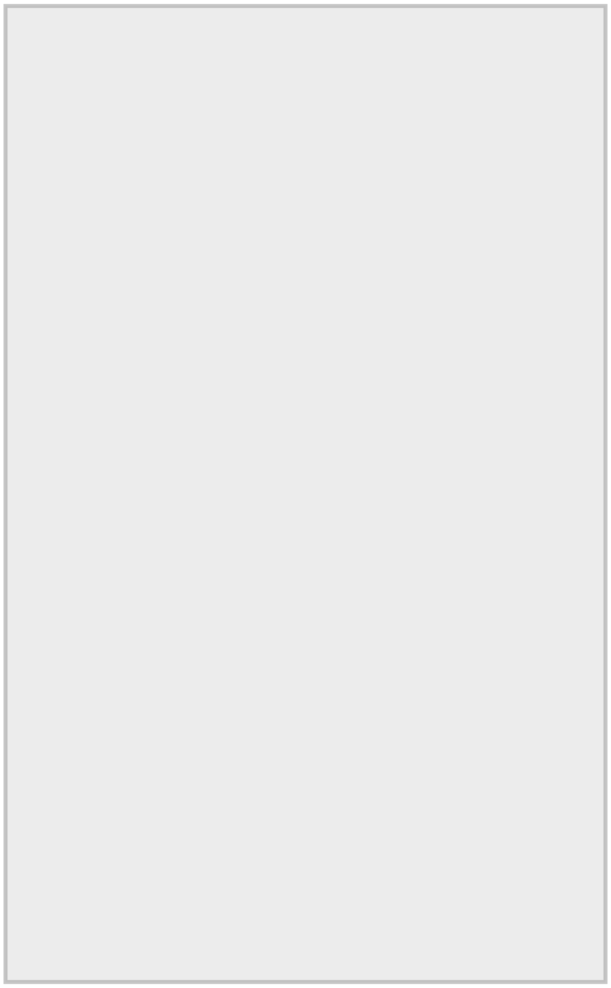
Card Cantilever

The Background Material gives a method for organizing bricks (in this case, playing cards, or some other similar objects) such that they stick out well past the edge of a table. Precision measurements and marking the proper positions of cards on top of each other with a fine point pen are useful in reaching success in this task. You will want to use an old deck, so that you don't have to ruin a new one with marks, but also an older deck of cards will have more friction between cards, preventing them from sliding off each other due to the unavoidable bend that occurs in the card extension. In the photo below, 6 such cards were marked, and then many others are added beneath with what can only be described as "extremely approximate" spacing. In theory, 31 cards is enough to get an extension of 2 full card lengths, while this one used a whole deck, and still fell somewhat short of two card lengths. Still, the effect is quite striking!



Flying Forks

In this example, we create an object that balances on the edge of a drinking glass because its center of gravity aligns with that edge (draw a line tangent to the rim of the glass at the point of contact, and that line divides the forks along their combined center of gravity). The small white rectangle that you see coming through the top tines of the forks is just an index card that has been folded-over many times (you could use a folded playing card, since you have already ruined a deck for the previous example). Very often this "trick" is done with a quarter instead of this stiff paper. In this particular case, the handles of the forks are very heavy, so the center of gravity is farther down the handles than with other forks, which means that a coin does not provide a sufficiently long moment arm for the forks to balance. The point is, you may need to make creative adjustments compared to the picture (don't bend the forks too much!) in order to get this to balance.



Lab Report

Your "lab report" consists of pictures of two balancing acts – they can be your versions of those given above, or something entirely different. Let's see how extreme you can make these (but no trick photography, please)! Please note that Canvas allows for submissions of jpegs, which is probably going to be an easier format to use for pictures than pdf.

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