4.12: Center of Mass

If an inertial frame of reference K' is moving at constant velocity $\vec{V}$ relative to inertial frame K, the velocities of individual particles in the frames are related by $\vec{v}_i = \vec{v}_i' + \vec{V}$, so the total momenta are related by

$$\vec{P} = \sum_i m_i \vec{v}_i = \sum_i m_i \vec{v}_i' + \vec{V} \sum_i m_i = \vec{P}' + M\vec{V}, \quad M = \sum_i m_i \tag{4.12.1}$$

If we choose $\vec{V} = \vec{P}/M$, then $\vec{P}' = \sum_i m_i \vec{v}_i' = 0$, the system is “at rest” in the frame K'. Of course, the individual particles might be moving, what is at rest in $\overline{K'}$ is the *center of mass* defined by

$M\vec{R}_{\text{cm}} = \sum_i m_i \vec{r}_i$

(Check this by differentiating both sides with respect to time.)

The energy of a mechanical system in its rest frame is often called its *internal energy*, we’ll denote it by E_{int} (This includes kinetic and potential energies.) The total energy of a moving system is then

$$E = \frac{1}{2} M\vec{V}^2 + E_{\text{int}} \tag{4.12.2}$$

(*Exercise*: verify this.)