9.2: It's Not About Time

But that’s *not* what you’re interested in doing! Of course you want the ball to get in the hole, but you’re not obsessed with how long it takes to get there. Yet without that time requirement, there are obviously many possible paths. If you hit it really fast, so its kinetic energy is far greater than the gravitational potential energy variations in the bumpy green, it will go close to a straight line (we’re assuming that when it gets over the hole, it will drop in). As you slow down, the winning path will deviate from a straight line because of the uneven terrain. So the physical path to the hole will vary continuously with initial kinetic energy.

Maupertuis’ principle is about what is the path $y(x)$ *to the hole, say from* (x_1, y_1, t_1) to $(x_2, y_2, \text{ any } t)$ *for a given initial energy* E .

So now we’re fixing the beginning and end points in *space*, but allowing possible variation in the final *time*. Also, we’re *fixing the energy*: $H(x, y, p_x, p_y) = E$ This means that in varying the path to minimize the action, we must restrict ourselves to the class of paths having energy E. In the bumpy putting green, you’re giving the ball a fixed initial speed v_0, and trying different initial directions to get it in the hole.

From the expression for the differential of action in terms of varying the *endpoint* (as well as the rest of the path—remember, that gives the integral term that disappears along the dynamical path), we have all $\delta q_i = 0$ (the endpoint is fixed at the hole), leaving

$$\delta S = \sum_i p_i \delta q_i - H \delta t = -E \delta t \tag{9.2.1}$$

(Since we’re restricting to paths with energy E, H can be replaced by E.)