10.2: General and Canonical Transformations

In the Hamiltonian approach, we're in *phase space* with a coordinate system having positions and momenta on an equal footing. It is therefore possible to think of more general transformations than the point transformation (which was restricted to the position coordinates).

We can have transformations that mix up position and momentum variables:

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where *[Math Processing Error]* means the whole set of the original variables.

In those original variables, the equations of motion had the nice *canonical* Hamilton form,

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Things won't usually be that simple in the new variables, but it does turn out that many of the "natural" transformations that arise in dynamics, such as that corresponding to going forward in time, *do* preserve the form of Hamilton's canonical equations, that is to say

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A transformation that retains the canonical form of Hamilton's equations is said to be ***canonical***.

(*Jargon note*: these transformations are occasionally referred to as *contact* transformations.)