19.6: Allowed Wavenumbers from Boundary Conditions

The usual way of representing a wave on a line in physics is to have displacement proportional to e^{ikx}, and k is called the *wavenumber*. For our discretized system, the displacement parameter for the n^{th} atom, at position na, would therefore be proportional to e^{ikna}.

But we know this is an eigenvector of a circulant, so we must have $e^{iNka} = 1$, and the allowed values of k are

$$k_n = \frac{2\pi}{Na} n = \frac{2\pi}{L} n \tag{19.6.1}$$

with n an integer.

The circulant structure of the matrix has determined the eigenvectors, but not the eigenvalues ω_n