29.3: Coriolis Effect - Particle Moving near Earth's Surface

The Earth's surface is a rotating frame of reference, but the angular velocity is sufficiently small that we can often drop second order terms. Recall the equation of motion in a rotating frame is

$$m d\vec{v}/dt = -\partial U/\partial \vec{r} + 2m\vec{v}\times\vec{\Omega} + m(\vec{\Omega}\times\vec{r})\times\vec{\Omega} \tag{29.3.1}$$

which becomes, close to the Earth and dropping the second-order term,

$$d\vec{v}/dt = \vec{g} + 2\vec{v}\times\vec{\Omega} \tag{29.3.2}$$

The leading order solution, ignoring the small rotation term, is the familiar

$$\vec{v}_1 = \vec{v}_0 + \vec{g}t \tag{29.3.3}$$

The second term is much smaller than the first, so it's OK to replace the $\vec{v}$ there by $\vec{v}_1$, and to find the leading correction to the path put

$$\vec{v} = \vec{v}_1 + \vec{v}_2 \tag{29.3.4}$$

in the equation $d\vec{v}/dt = \vec{g} + 2\vec{v}\times\vec{\Omega}$, giving

$$d\vec{v}_2/dt = 2\vec{v}_1\times\vec{\Omega} \cong 2t\vec{g}\times\vec{\Omega} + 2\vec{v}_0\times\vec{\Omega} \tag{29.3.5}$$

The full equation can now be integrated to give

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{g}t^2 + \frac{1}{3}t^3\vec{g}\times\vec{\Omega} + t^2\vec{v}_0\times\vec{\Omega} \tag{29.3.6}$$

Let's try some numbers: in 1803, an experiment was conducted in Schlebusch, Germany that attracted the interest of the scientific community. Twenty-nine iron pebbles were dropped into a 90-meter deep mineshaft.

In 1831 the experiment was repeated in a 158.5 m deep mine in Freiburg, Saxony. From 106 drops an average deflection of 28.3 mm was estimated, close to the theoretical value of 27.5 mm. (This agrees exactly with our formula, from Landau's book.)

Exercise 29.3.1

Where would you expect the particle to fall, compared with a straight down plumb line? To make visualizing a little easier, imagine the mine to be on the equator. Then the ground is moving east faster than the bottom of the mine— so the pebble will fall to the east.

Exercise 29.3.2: Naval Gunnery - HMS Dreadnought, 1906

The BL 12 inch Mk X gun on the HMS Dreadnought. Shells at 800 m/sec, range about 23 km. For vertical velocity of say 400 m/sec, time in air of order 80 secs. The two terms are about equal magnitude, around 100 meters.

I pick this ship because there is a rumor that in a 1915 naval battle near the Falkland islands, off Argentina, between the British and German navies, the British kept missing because they corrected their aim for Coriolis deflections appropriate to the northern hemisphere. I'm not sure if it's true.