

2.8: Multivariable First Integral

Following and generalizing the one-variable derivation, multiplying the above equations one by one by the corresponding $y'_i = dy_i/dx$ we have the n equations

$$\frac{\partial f(\vec{y}, \vec{y}')}{\partial y_i} \frac{dy_i}{dx} - \frac{d}{dx} \left(\frac{\partial f(\vec{y}, \vec{y}')}{\partial y'_i} \right) y'_i = 0 \quad (2.8.1)$$

Since f doesn't depend explicitly on x , we have

$$\frac{df}{dx} = \sum_{i=1}^n \left(\frac{\partial f}{\partial y_i} \frac{dy_i}{dx} + \frac{\partial f}{\partial y'_i} \frac{dy'_i}{dx} \right) \quad (2.8.2)$$

and just as for the one-variable case, these equations give

$$\frac{d}{dx} \left(\sum_{i=1}^n y'_i \frac{\partial f}{\partial y'_i} - f \right) = 0 \quad (2.8.3)$$

and the (important!) **first integral** $\sum_{i=1}^n y'_i \frac{\partial f}{\partial y'_i} - f = \text{constant}$.

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