5.1: Some Examples

Similar triangles are just scaled up (or down) versions of each other, meaning they have the same angles. Scaling means the same thing in a mechanical system: if a planet can go around the sun in a given elliptical orbit, another planet can go in a scaled up version of that ellipse (the sun remaining at the focus). But it will take longer: so we can't just scale the spatial dimensions, to get the same equation of motion we must scale time as well, and not in general by the same factor.

In fact, we can establish the relative scaling of space and time in this instance with very simple dimensional analysis. We know the planet's radial acceleration goes as the inverse square of the distance, so

$$(\text{radial acceleration}) \times (\text{distance})^{\,2} = \text{constant} \tag{5.1.1}$$

the dimensionality of this expression is

$$LT^{-2}L^2 = L^3T^{-2}, \tag{5.1.2}$$

so

$$T^2 \propto L^3. \tag{5.1.3}$$

the square of the time of one orbit is proportional to the cube of the size of the orbit. A little more explicitly, the acceleration

$$\propto GM_{\text{Sun}}/r^2, \tag{5.1.4}$$

so for the same GM_{Sun}, if we double the orbit size, the equation will be the same but with orbital time up by $2\sqrt{2}$.

Galileo established that real mechanical systems, such as a person, are *not* scale invariant. A giant ten times the linear dimensions of a human would break his hip on the first step. The point is that the weight would be up by a factor of 1,000, the bone strength, going as cross sectional area, only by 100.

Mechanical similarity is important is constructing small models of large systems. A particularly important application is to fluid flow, for example in assessing fluid drag forces on a moving ship, plane or car. There are two different types of fluid drag: viscous frictional drag, and inertial drag, the latter caused by the body having to deflect the medium as it moves through. The patterns of flow depend on the relative importance of these two drag forces, this dimensionless ratio, inertial/viscous, is called the Reynolds number. To give meaningful results, airflow speeds around models must be adjusted to give the model the same Reynolds number as the real system.