21.1: Introduction to the Ponderomotive Force

Imagine first a particle of mass m moving along a line in a smoothly varying potential $V(x)$, so $m\ddot{x} = -\nabla V(x)$. Now add in a rapidly oscillating force, not necessarily small, acting on the particle:

$$f = f_1 \cos \omega t + f_2 \sin \omega t \tag{21.1.1}$$

where f_1, f_2 are in general functions of position. This force is oscillating much more rapidly than any oscillation of the particle in the original potential, and we'll assume that the position of the particle as a function of time can be written as a sum of a "slow motion" $X(t)$ and a rapid oscillation $\xi(t)$.

$$x(t) = X(t) + \xi(t) \tag{21.1.2}$$

We'll also assume that the amplitude of the oscillations, determined by the strength of the force and the frequency, is small compared with distances over which the original fixed potential and the coefficients f_1, f_2 vary substantially.

You might be thinking at this point, well, isn't $X(t)$ just the path the particle would describe in $V(x)$ alone, and the force f just jiggles it about that path? Surprisingly, the answer is no. For example, a rigid pendulum confined to rotation in a vertical plane, but with its point of support driven in fairly small amplitude rapid up-and-down oscillations from the outside, can be stable pointing *upwards*. For motion on the slow timescale associated with the original potential, the rapidly oscillating imposed force is equivalent to an effective potential.

This turns out to have important practical consequences. For a charged particle in a rapidly oscillating electric field, the effective potential from the oscillation is proportional to $e^2\overline{E^2}$, generating a force driving the particle towards regions of weaker field. It is termed the *ponderomotive force*.

For plasma physicists, the ponderomotive force has one very important property—*it drives the positive and negative particles in the same direction*, and so gives a different tool from the usual electric and magnetic fields for containing a plasma.

In the analysis below, following Landau, we have a fixed potential and a fast oscillating field superimposed. However, we could just have a non-uniform fast oscillating field, with an equation of motion $\ddot{x} = g(x) \cos \omega t$, and still write the particle path as a sum of slow moving and jiggling components, $x(t) = X(t) + \xi(t)$. Fast oscillating electric fields (crossed laser beams) are used to trap ultracold ions and atoms, using the ponderomotive force. It has been suggested that atoms trapped in this way could be part of a quantum computer (Turker, arXiv: 1308.0573v1).