27.1: Definition of Euler Angles

The rotational motion of a rigid body is completely defined by tracking the set of principal axes (x_1, x_2, x_3), with origin at the center of mass, as they turn relative to a set of fixed axes (X,Y,Z). The principal axes can be completely defined relative to the fixed set by three angles: the two angles (θ, ϕ) fix the direction of x_3, but that leaves the pair x_1, x_2 free to turn in the plane perpendicular to x_3, the angle ψ fixes their orientation.

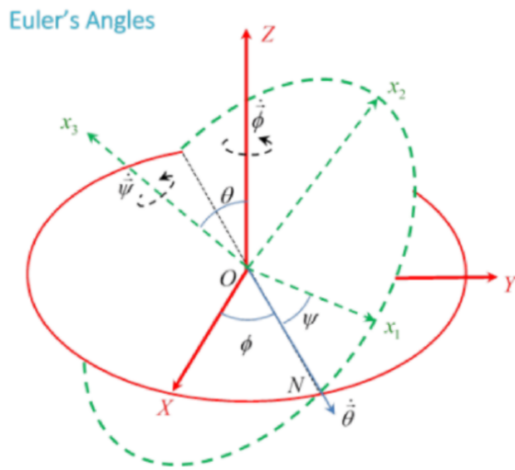

Figure 27.1.1: θ, ϕ follow standard physics practice for labeling the direction of body axis x_3 relative to lab axes X, Y, Z, ψ is the body rotation angle from ON to the x_1 axis in the x_1, x_2 plane, about its x_3 axis.

To see these angles, start with the fixed axes, draw a circle centered at the origin in the horizontal X,Y plane. Now draw a circle of the same size, also centered at the same origin, but in the principal axes x_1, x_2 plane. Landau calls the line of intersection of these circles (or discs) the *line of nodes*. It goes through the common origin, and is a diameter of both circles.

The angle between these two planes, which is also the angle between Z, x_3 (since they're the perpendiculars to the planes) is labeled θ.

The angle between this line of nodes and the X axis is ϕ. It should be clear that θ, ϕ together fix the direction of x_3, then the other axes are fixed by giving ψ, the angle between x_1 and the line of nodes ON. The direction of measurement of ϕ, ψ around Z, x_3 are given by the right-hand or corkscrew rule.