26.2: Precession of a Symmetrical Top

A more interesting case is the free rotation (zero external torque) of a symmetrical top, meaning $I_1 = I_2 \neq I_3$.

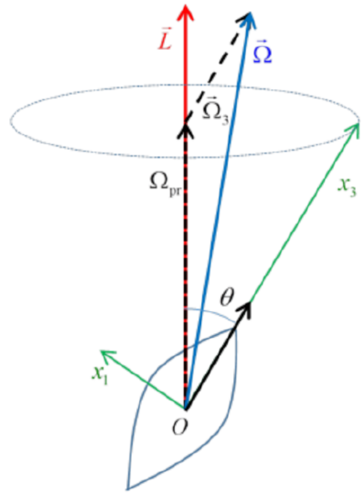

Figure 26.2.1

We can take any pair of orthogonal axes, perpendicular to the body's symmetry axis, as the x_1, x_2 axes. We'll choose x_2 following Landau, as perpendicular to the plane containing $\vec{L}$ and the momentary position of the x_3 axis, so in the diagram here x_2 is perpendicularly out from the paper/screen, towards the viewer.

This means the angular momentum component $L_2 = 0$ and therefore $\Omega_2 = 0$. Hence $\vec{\Omega}$ is in the same plane as $\vec{L}, x_3,$ and so the velocity $\vec{v} = \vec{\Omega} \times \vec{r}$ of every point on the axis of the top is perpendicular to this plane (into the paper/screen). The axis of the top Ox_3 must be rotating uniformly about the direction of $\vec{L}$.

The spin rate of the top around its own axis is

$$\Omega_3 = L_3/I_3 = (L/I_3)\cos\theta \tag{26.2.1}$$

The angular velocity vector $\vec{\Omega}$ can be written as a sum of two components, one along the body's axis Ox_3 and one parallel to the angular momentum $\vec{L}$ (these components are shown dashed in the figure)

$$\vec{\Omega} = \vec{\Omega}_{\text{precession}} + \vec{\Omega}_3 \tag{26.2.2}$$

The component along the body's axis Ox_3 does not contribute to the precession, which all comes from the component along the (fixed in space) angular momentum vector.

The speed of precession follows from

$$\Omega_{\text{precession}} \sin\theta = \Omega_1 \tag{26.2.3}$$

and

$$\Omega_1 = L_1/I_1 = (L/I_1)\sin\theta \tag{26.2.4}$$

so

$$\Omega_{\text{precession}} = L/I_1 \tag{26.2.5}$$

Note also the ratio of precession rate to spin around axis is

$$\Omega_{\text{precession}}/\Omega_3 = (I_3/I_1)\sec\theta \tag{26.2.6}$$

This means the precession rate and the spin are very comparable, except when θ is near $\pi/2$, when the precession becomes much faster. Remember this is the body's precession with *no external torque*, and is clearly completely different—much faster precession—than the familiar case of a fast spinning top under gravity.