27.3: Free Motion of a Symmetrical Top

As a warm up in using Euler's angles, we'll redo the free symmetric top covered in the last lecture. With *no external torques acting* the top will have constant angular momentum $\vec{L}$.

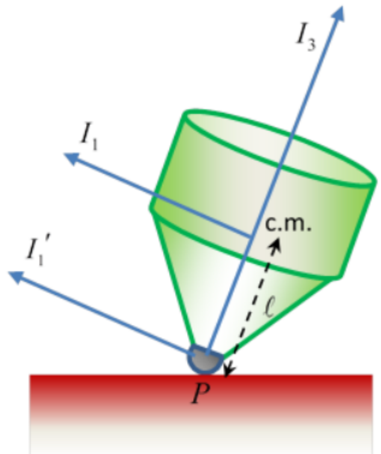

Figure 27.3.1:

We'll take $\vec{L}$ in the fixed Z direction. The axis of the top is along x_3.

Taking the x_1 axis along the line of nodes ON (Figure 27.3.1) at the instant considered, the constant angular

$$\begin{aligned}\vec{L} &= (I_1\Omega_1, I_1\Omega_2, I_3\Omega_3) \\ &= \left(I_1\dot{\theta}, \quad I_1\dot{\phi}\sin\theta, \quad I_3(\dot{\phi}\cos\theta+\dot{\psi})\right)\end{aligned}$$

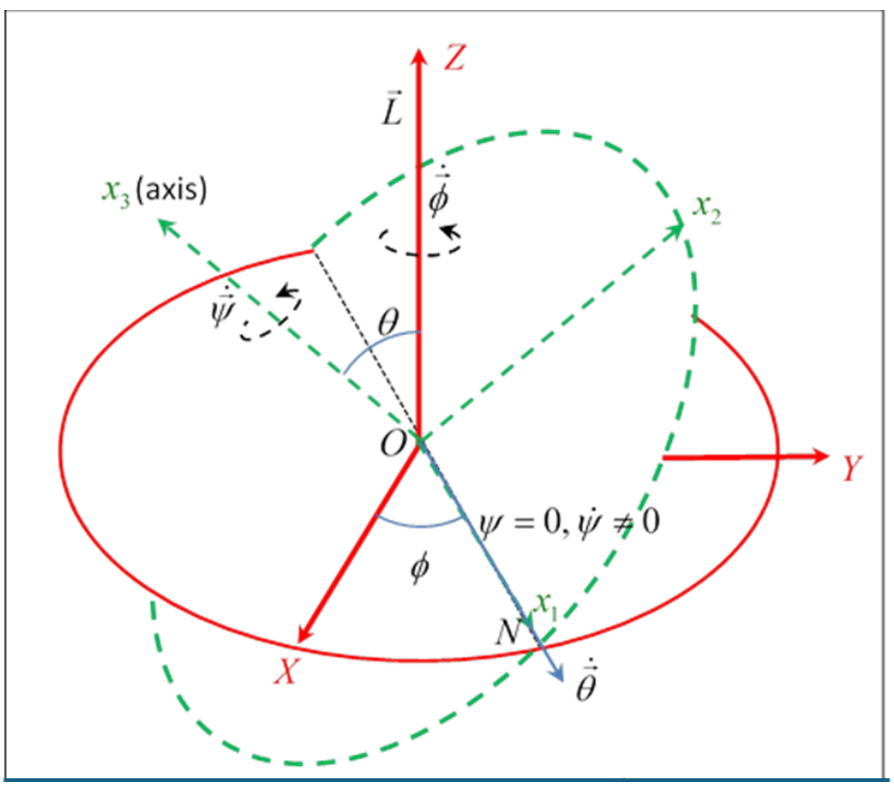

Figure 27.3.1: Free motion of symmetric top: Constant $vecL$ along fixed Z

Remember, this new x_1 axis (Figure $27.3.1$) is perpendicular to the Z axis we've taken $\vec{L}$ along, so $L_1 = I_1\dot{\theta} = 0$, and θ is constant, meaning that the principal axis x_3 describes a cone around the constant angular momentum vector $\vec{L}$. The rate of precession follows from the constancy of $L_2 = I_1\dot{\phi}\sin\theta$. Writing the absolute magnitude of the angular momentum as L, $L_2 = L\sin\theta$ (remember L is in the Z direction, and x_1 is momentarily along ON) so the rate of precession $\dot{\phi} = L/I_1$. Finally, the component of $\vec{L}$ along the x_3 axis of symmetry of the top is $L\cos\theta = I_3\Omega_3$, so the top's spin along its own axis is $\Omega_3 = (L/I_3)\cos\theta$.

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