21.5: Pendulum with Top Point Oscillating Rapidly in a Horizontal Direction

Take the coordinates of m to be

$$x=a\cos\Omega t+\ell\sin\phi,\ y=\ell\cos\phi \tag{21.5.1}$$

The Lagrangian, omitting the term depending only on time, and performing an integration by parts and dropping the total derivative term, (following the details of the analysis above for the vertically driven pendulum) is

$$L=\frac{1}{2}m\ell^2\dot{\phi}^2+ma\ell\Omega^2\cos\Omega t\sin\phi+mg\ell\cos\phi \tag{21.5.2}$$

It follows that $f=m\ell a\Omega^2\cos\Omega t\cos\phi$ (the only difference in f from the *vertically* driven point of support is the final $\cos\phi$ instead of $\sin\phi$)) and

$$V_{\text{eff}}=mg\ell\left[-\cos\phi+\overline{f^2}/2m\omega^2\right]=mg\ell\left[-\cos\phi+\left(a^2\Omega^2/4g\ell\right)\cos^2\phi\right] \tag{21.5.3}$$

If $a^2\Omega^2<2g\ell,\quad\phi=0$ is stable. If $a^2\Omega^2>2g\ell$ the stable position is $\cos\phi=2g\ell/a^2\Omega^2$

That is, at high frequency, the rest position is *at an angle* to the vertical!

In this case, the ponderomotive force towards the direction of least angular quiver (in this case the *horizontal* direction) is balanced by the gravitational force.