30.3: D’Alembert’s Principle

The “better way” is simply to write down Newton’s equations, $\vec{F}=m\vec{a}$ and the rotational equivalent $\vec{K}=I\dot{\vec{\Omega}}$ for each component of the system, now using, of course, *total* force and torque, including constraint reaction forces, etc. This approach Landau calls “d’Alembert’s principle”.

Footnote: We’re not going to pursue this here, but the “principle” stems from the concept of **virtual work**: if a system is in equilibrium, then making tiny displacements of all parameters, subject to the system constraints (but not necessarily an infinitesimal set of displacements that would arise in ordinary dynamical development in time), the total work done by all forces acting on parts of the system is zero. This is just saying that in equilibrium, it is at a local minimum (or stationary point if we allow unstable equilibrium) in the energy “landscape”. D’Alembert generalized this to the dynamical case by adding in effective forces corresponding to the coordinate accelerations, he wrote essentially $\sum\vec{F}-m\vec{a}=0,$ representing $m\vec{a}$ as a “force”, equivalent to Newton’s laws of motion.

Having written down the equations, the reaction forces can be cancelled out to derive equations of motion.