27.6: Stability of Top Spinning about Vertical Axis

(from Landau) For $\theta=\dot{\theta}=0, \quad L_3=L_Z, E'=0$. Near $\theta=0$,

$$\begin{aligned}
V_{\text{effective}}(\theta) &= \frac{\left(L_Z-L_3\cos\theta\right)^2}{2I_1'\sin^2\theta}-Mg\ell(1-\cos\theta) \\
&\cong \frac{L_3^2\left(\frac{1}{2}\theta^2\right)^2}{2I_1'\theta^2}-\frac{1}{2}Mg\ell\theta^2 \\
&= \left(L_3^2/8I_1'-\frac{1}{2}Mg\ell\right)\theta^2
\end{aligned} \tag{27.6.1}$$

The vertical position is stable against small oscillations provided $L_3^2>4I_1'Mg\ell, \text{ or }$, or $\Omega_3^2>4I_1'Mg\ell/I_3^2$

Exercise 27.6.1

Suppose you set the top vertical, but spinning at less than $\Omega_{3\text{ crit}}$, the value at which it is just stable. It will fall away, but bounce back, and so on. Show the maximum angle it reaches is given by $\cos(\theta/2)=\Omega_3/\Omega_{3\text{crit}}$.