23.1: Introduction

We've previously discussed the driven damped simple harmonic oscillator, and in the last lecture we extended that work (following Landau) to an anharmonic oscillator, adding a quartic term to the potential. Here we make a different extension of the simple oscillator: we go to a driven damped *pendulum*. (A weight at one end of a light rigid rod, the other end of the rod being at a fixed position but the rod free to rotate about that fixed point in a vertical plane). That is, we replace the potential term $-\omega_0^2 x$ in the linear oscillator with $-\omega_0^2 \sin x$, or rather $-\omega_0^2 \sin \phi$, to make clear we have an *angular* system. Going to a driven damped pendulum leads to many surprises!

For a sufficiently weak driving force, the behavior of the driven damped pendulum is of course close to that of the driven damped linear oscillator, but on gradually increasing the driving force, at a certain strength the period of the response *doubles*, then, with further increase, it doubles again and again, at geometrically decreasing intervals, going to a chaotic (nonperiodic) response at a definite driving strength. But that is not the end of the story—the chaotic response regime has a lot of structure: many points within a generally chaotic region are in fact nonchaotic, well-defined cyclical patterns. And, as we'll see later, the response pattern itself can be fractal in nature, see for example the strange attractor discussed at the end of this lecture. You can use the accompanying applet to generate this attractor and its cousins easily.

Obviously, this is a very rich subject, we provide only a brief overview. We closely follow the treatment in Taylor's *Classical Mechanics*, but with the addition of our applets for illustrative purposes, and also to encourage further exploration: the applets accurately describe the motion and exhibit the strange attractors in the chaotic regime. With the applets, it is easy to check how these strange attractors change (or collapse) on varying the driving force or the damping. We end with some discussion of the fractal dimension of the attractors, and how it relates to the dynamics, in particular to the rate of divergence of initially close trajectories, here following Baker and Gollub, *Chaotic Dynamics*.