

7.1: The Poisson Bracket

A function $[Math Processing Error]$ of the phase space coordinates of the system and time has total time derivative

$[Math Processing Error]$

This is often written as

$[Math Processing Error]$

where

$[Math Processing Error]$

is called the *Poisson bracket*.

Caution

Equation $[Math Processing Error]$ is Landau's definition for the Poisson bracket. It *differs in sign* from Goldstein, Wikipedia and others.

If, for a phase space function $[Math Processing Error]$ (that is, no explicit time dependence) $[Math Processing Error]$ is a constant of the motion, also called an *integral of the motion*.

In fact, the Poisson bracket can be defined for *any* two functions defined in phase space:

$[Math Processing Error]$

It's straightforward to check the following properties of the Poisson bracket:

$[Math Processing Error]$

The Poisson brackets of the basic variables are easily found to be:

$[Math Processing Error]$

Now, using

$[Math Processing Error]$

and the basic variable P.B.'s, we find

$[Math Processing Error]$

and, in fact, the bracket of p with any reasonably smooth function of q is:

$[Math Processing Error]$

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