26.1: Angular Momentum and Angular Velocity

In contrast to angular velocity, the angular momentum of a body depends on the point with respect to which it is defined. For now, we take it (following Landau, of course) as relative to the center of mass, but we denote it by $\vec{L}$, following modern usage. This "intrinsic" angular momentum is like the Earth's angular momentum from its diurnal rotation, as distinct from its orbital angular momentum in going around the Sun.

That is

$$\vec{L}=\sum_{n} \vec{r}_{n} \times m_{n} \vec{v}_{n}=\sum_{n} \vec{r}_{n} \times m_{n}\left(\vec{\Omega} \times \vec{r}_{n}\right)=\sum_{n} m_{n}\left[r_{n}^{2} \vec{\Omega}-\vec{r}_{n}\left(\vec{r}_{n} \cdot \vec{\Omega}\right)\right]=\mathbf{I} \vec{\Omega} \tag{26.1.1}$$

where I is the inertia tensor: this just means $L_{i}=I_{ik} \Omega_{k}$

Explicitly, taking the principal axes as the (x_{1}, x_{2}, x_{3}) axes,

$$L_{1}=I_{1} \Omega_{1}, \quad L_{2}=I_{2} \Omega_{2}, \quad L_{3}=I_{3} \Omega_{3} \tag{26.1.2}$$

For anything with spherical inertial symmetry (such as a cube or a tetrahedron!) $\vec{L}=I \vec{\Omega}$

Landau defines a *rotator* as a collection of massive particles all on a line. (I guess that includes diatomic molecules, and, for example, CO_{2}, neglecting electrons and nuclear size). We know there are only two physical rotational degrees of freedom for these molecular rotators (thanks to quantum mechanics) and obviously the two principal axes are perpendicular to the line of masses, and degenerate. Again, then, $\vec{L}=I \vec{\Omega}$.