4.1: Introduction- Galileo and Newton

In the discussion of calculus of variations, we anticipated some basic dynamics, using the potential energy mgh for an element of the catenary, and conservation of energy $\frac{1}{2}mv^2 + mgh = E$ for motion along the brachistochrone. Of course, we haven't actually covered those things yet, but you're already very familiar with them from your undergraduate courses, and my aim was to give easily understood physical realizations of minimization problems, and to show how to find the minimal shapes using the calculus of variations.

At this point, we'll begin a full study of dynamics, starting with the laws of motion. The text, Landau, begins (page 2!) by stating that the laws come from the principle of least action, Hamilton's principle. This is certainly one possible approach, but confronted with it for the first time, one might well wonder where *it* came from. I prefer a gentler introduction, more or less following the historical order: Galileo, then Newton, then Lagrange and his colleagues, then Hamilton. The two approaches are of course equivalent. Naturally, you've seen most of this earlier stuff before, so here is a very brief summary.

To begin, then, with Galileo. His two major contributions to dynamics were:

1. The realization, and experimental verification, that falling bodies have constant acceleration (provided air resistance can be ignored) and *all falling bodies accelerate at the same rate*.

2. *Galilean relativity*. As he put it himself, if you are in a closed room below decks in a ship moving with steady velocity, no experiment on dropping or throwing something will look any different because of the ship's motion: you can't detect the motion. As we would put it now, the laws of physics are the same in all inertial frames.

Newton's major contributions were his *laws of motion*, and his *law of universal gravitational attraction*.

His laws of motion:

1. *The law of inertia*: a body moving at constant velocity will continue at that velocity unless acted on by a force. (Actually, Galileo essentially stated this law, but just for a ball rolling on a horizontal plane, with zero frictional drag.)

2. $\vec{F} = m\vec{a}$

3. *Action = reaction*.

In terms of Newton's laws, Galilean relativity is clear: if the ship is moving at steady velocity $\vec{v}$ relative to the shore, than an object moving at $\vec{u}$ relative to the ship is moving at $\vec{u} + \vec{v}$ relative to the shore. If there is no force acting on the object, it is moving at steady velocity in both frames: both are inertial frames, defined as frames in which Newton's first law holds. And, since $\vec{v}$ is constant, the acceleration is the same in both frames, so if a force is introduced the second law is the same in the two frames.

(Needless to say, all this is classical, meaning nonrelativistic, mechanics.)

Newton's Laws Explain Everything (in Principle...)

...in a classical mechanical system. Any dynamical system can be analyzed as a (possibly infinite) collection of parts, or particles, having mutual interactions, so in principle Newton's laws can provide a description of the motion developing from an initial configuration of positions and velocities.

The problem is, though, that the equations may be intractable—we can't do the mathematics. It is evident that in fact the Cartesian coordinate positions and velocities might not be the best choice of parameters to specify the system's configuration. For example, a simple pendulum is obviously more naturally described by the angle the string makes with the vertical, as opposed to the Cartesian coordinates of the bob. After Newton, a series of French mathematicians reformulated his laws in terms of more useful coordinates—culminating in Lagrange's equations.

The Irish mathematician Hamiltonian then established that these improved dynamical equations could be derived using the calculus of variations to minimize an integral of a function, the *Lagrangian*, along a path in the system's configuration space.

This integral is called the *action*, so the rule is that the system follows the path of least action from the initial to the final configuration.

Note

Nitpicking footnote: strictly, we need the path to be a stationary point in the space of possible paths, usually it's the least-action path.

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