28.3: Using Energy and Angular Momentum Conservation

We can also gain some insight into the motion of the free spinning top just from conservation of energy and angular momentum.

The equations are:

$$\begin{gathered} \frac{L_1^2}{I_1}+\frac{L_2^2}{I_1}+\frac{L_3^2}{I_3}=2E \\ L_1^2+L_2^2+L_3^2=L^2 \end{gathered} \tag{28.3.1}$$

Visualize these equations as surfaces in (L_1, L_2, L_3) space.

The second is a sphere, radius L, centered at the origin.

The first is an ellipsoid, also centered at the origin, with semimajor axes

$$\left(\sqrt{2EI_1}, \sqrt{2EI_1}, \sqrt{2EI_3}\right) \tag{28.3.2}$$

Do these two surfaces intersect?

The answer is *yes*, they always do.

To see that, assume first that $I_1 > I_3, \text{ then } 2EI_1 \geq L, \text{ and further } L \geq 2EI_3$. The sphere intersects the ellipsoid in two circles. These degenerate to one circle, the equator, when $L_3 = 0$, and two points (the poles) when $L_1 = L_2 = 0$.

We conclude that the path of the angular momentum vector is a circle around the axis of symmetry in the body coordinate system, and since we know that relative to the fixed space axes the angular momentum is in fact constant, this means that actually the *body* is precessing about its axis of symmetry as seen by an observer.