25.4: Cylinder Rolling Inside another Cylinder

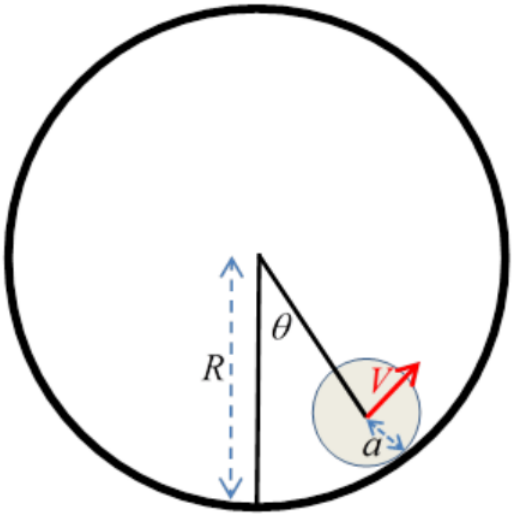

Figure 25.4.1

Now consider a solid cylinder radius a rolling inside a hollow cylinder radius R, angular distance from the lowest point θ, the solid cylinder axis moving at $V=(R-a)\dot{\theta}$ and therefore having angular velocity (compute about the point of contact) $\Omega=V/a$.

The kinetic energy is

$$\frac{1}{2}MV^2+\frac{1}{2}I(V/a)^2=\frac{1}{2}\left(M+\frac{I}{a^2}\right)(R-a)^2\dot{\theta}^2 \tag{25.4.1}$$

The potential energy is $-Mg(R-a)\cos\theta$.

The Lagrangian $L=T-V$, the equation of motion is

$$\left(M+\frac{I}{a^2}\right)(R-a)^2\ddot{\theta}=-Mg(R-a)\sin\theta\cong-Mg(R-a)\theta \tag{25.4.2}$$

so small oscillations are at frequency $\omega=\sqrt{\dfrac{g}{\left(1+\dfrac{I}{Ma^2}\right)(R-a)}}$.