4.7: First Integral- Energy Conservation and the Hamiltonian

Since Lagrange's equations are precisely a calculus of variations result, it follows from our earlier discussion that *if the Lagrangian has no explicit time dependence* then:

$$\sum_i \dot{q}_i \frac{\partial L}{\partial \dot{q}_i} - L = \text{constant} \tag{4.7.1}$$

(This is just the first integral $y' \partial f / \partial y' - f = \text{ constant }$ discussed earlier, now with n variables.)

This constant of motion is called the *energy* of the system, and denoted by E. We say the energy is *conserved*, even in the presence of external potentials—provided those potentials are time-independent.

(We'll just mention that the function on the left-hand side, $\sum_i \dot{q}_i \partial L / \partial \dot{q}_i - L$ is the Hamiltonian. We don't discuss it further at this point because, as we'll find out, it is more naturally treated in other variables.)

We'll now look at a couple of simple examples of the Lagrangian approach.