8.7: How Can p, q Really Be Independent Variables?

It may seem a little odd at first that varying p,q as independent variables leads to the same equations as the Lagrangian minimization, where we only varied q, and that variation "locked in" the variation of *[Math Processing Error]*. And, isn't p *defined* in terms of *[Math Processing Error]* which is some function of *[Math Processing Error]*? So wouldn't varying q automatically determine the variation of p?

The answer is, no, p is *not* defined as *[Math Processing Error]* from the start in Hamilton's formulation. In this Hamiltonian approach, p,q really are taken as independent variables, then varying them to find the minimum path gives the equations of motion, *including the relation between p and [Math Processing Error]*

This comes about as follows: Along the minimum action path, we just established that

[Math Processing Error]

We also have that *[Math Processing Error]* so (Legendre transformation!)

[Math Processing Error]

from which, along the physical path, *[Math Processing Error]*. So this identity, previously written as the *definition* of p, now arises as a consequence of the action minimization in phase space.
