8.5: How this Classical Action Relates to Phase in Quantum Mechanics

The link between classical and quantum mechanics is particularly evident in the expression for the action integral given above. In the so-called semi-classical regime of quantum mechanics, the de Broglie waves oscillate with wavelengths much smaller than typical sizes in the system. This means that locally it's an adequate approximation to treat the Schrödinger wave function as a plane wave,

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where the amplitude function *[Math Processing Error]* only varies over distances much greater than the wavelength, and times far longer than the oscillation period. This expression is valid in almost all the classically accessible regions, invalid in the neighborhood of turning points, but the size of those neighborhoods goes to zero in the classical limit.

As we've discussed earlier, in the Dirac-Feynman formulation of quantum mechanics, to find the probability amplitude of a particle propagating from one point to another, we add contributions from all possible paths between the two points, each path contributing a term with phase equal to *[Math Processing Error]* times the action integral along the path.

From the semi-classical Schrödinger wave function above, it's clear that the change in phase from a small change in the endpoint is *[Math Processing Error]* coinciding exactly with the incremental contribution to the action in

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So again we see, here very directly, how the action along a classical path is a multiple of the quantum mechanical phase change along the path.