2.8: Multivariable First Integral

Following and generalizing the one-variable derivation, multiplying the above equations one by one by the corresponding $y_i' = dy_i/dx$ we have the n equations

$$\frac{\partial f\left(\vec{y}, \vec{y}'\right)}{\partial y_i} \frac{dy_i}{dx} - \frac{d}{dx}\left(\frac{\partial f\left(\vec{y}, \vec{y}'\right)}{\partial y_i'}\right) y_i' = 0 \tag{2.8.1}$$

Since f doesn't depend explicitly on x, we have

$$\frac{df}{dx} = \sum_{i=1}^{n}\left(\frac{\partial f}{\partial y_i}\frac{dy_i}{dx} + \frac{\partial f}{\partial y_i'}\frac{dy_i'}{dx}\right) \tag{2.8.2}$$

and just as for the one-variable case, these equations give

$$\frac{d}{dx}\left(\sum_{i=1}^{n} y_i' \frac{\partial f}{\partial y_i'} - f\right) = 0 \tag{2.8.3}$$

and the (important!) **first integral** $\sum_{i=1}^{n} y_i' \frac{\partial f}{\partial y_i'} - f = \text{constant}.$