24.8: Principal Axes Form of Moment of Inertia Tensor

We already know that the transformed matrix is diagonal, so its form has to be

$$\sum_n m_n \begin{pmatrix} x_{n2}^2 + x_{n3}^2 & 0 & 0 \\ 0 & x_{n3}^2 + x_{n1}^2 & 0 \\ 0 & 0 & x_{n1}^2 + x_{n2}^2 \end{pmatrix} = \begin{pmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{pmatrix} \tag{24.8.1}$$

The moments of inertia, the diagonal elements, are of course all positive. Note that no one of them can exceed the sum of the other two, although it can be equal in the (idealized) case of a two-dimensional object. For that case, taking it to lie in the (x, y) plane,

$$I_z = \sum_n \left(x_n^2 + y_n^2 \right) = I_x + I_y \tag{24.8.2}$$