22.3: Resonance in a Damped Driven Linear Oscillator- A Brief Review

This is just to remind you of what we covered in lecture 18, before we add anharmonic terms in the next section.

The *linear* damped driven oscillator has equation of motion:

$$\ddot{x}+2\lambda\dot{x}+\omega_0^2x=(f/m)e^{i\gamma t} \tag{22.3.1}$$

(Following Landau's notation here note it means the actual frictional drag force is $2\lambda m\dot{x}$)

Looking near resonance for steady state solutions at the driving frequency, with amplitude b, phase lag δ, that is, $x(t)=be^{i(\gamma t+\delta)}$, we find

$$be^{i\delta}\left(-\gamma^2+2i\lambda\gamma+\omega_0^2\right)=(f/m) \tag{22.3.2}$$

For a near-resonant driving frequency

$$\gamma=\omega_0+\varepsilon \tag{22.3.3}$$

and assuming the damping to be sufficiently small that we can drop the term along with , the leading order terms give

$$be^{i\delta}=-f/2m(\varepsilon-i\lambda)\omega_0 \tag{22.3.4}$$

so the response, the dependence of amplitude b on driving frequency $\Omega=\omega_0+\varepsilon$ is to this accuracy

$$b=\frac{f}{2m\omega_0\sqrt{\left(\gamma-\omega_0\right)^2+\lambda^2}}=\frac{f}{2m\omega_0\sqrt{\varepsilon^2+\lambda^2}} \tag{22.3.5}$$

(Note also that the resonant frequency is itself lowered by the damping, another second-order effect we'll ignore.)

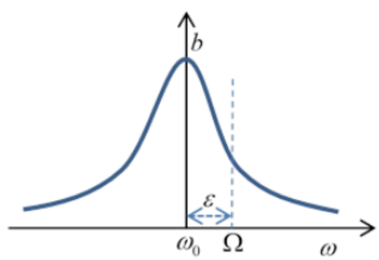

Figure 22.3.1

The rate of absorption of energy equals the frictional loss. The friction force $2\lambda m\dot{x}$ on the mass moving at $\dot{x}$ is doing work at an average rate:

$$2\lambda m\overline{\dot{x}^2}=\lambda mb^2\gamma^2 \tag{22.3.6}$$

The half width of the resonance curve as a function of γ is given by the damping. The total area under the curve is independent of damping.

For future use, we'll write the above equation for the amplitude b in terms of deviation ε from the resonant frequency ω_0.

$$b^2\left(\varepsilon^2+\lambda^2\right)=\frac{f^2}{4m^2\omega_0^2},\quad \varepsilon=\gamma-\omega_0 \tag{22.3.7}$$