17.1: Particle in a Well

We begin with the one-dimensional case of a particle oscillating about a local minimum of the potential energy $V(x)$. We'll assume that near the minimum, call it x_0 the potential is well described by the leading second-order term, $V(x) = \frac{1}{2}V''(x_0)(x-x_0)^2$ so we're taking the zero of potential at x_0, assuming that the second derivative $V''(x_0) \neq 0$, and (for now) neglecting higher order terms.

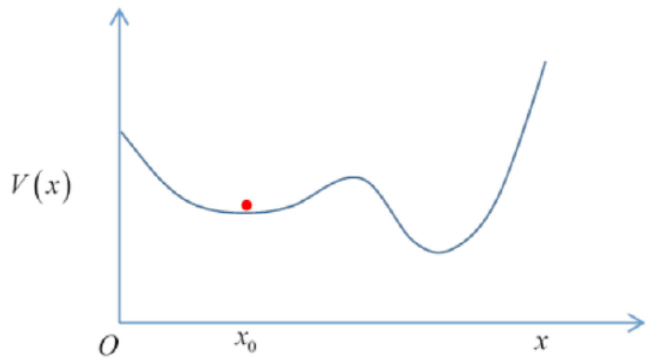

Figure 17.1.1

To simplify the equations, we'll also move the x origin to x_0, so

$$m\ddot{x} = -V''(0)x = -kx \tag{17.1.1}$$

replacing the second derivative with the standard "spring constant" expression.

This equation has solution

$$x = A\cos(\omega t + \delta), \text{ or } x = \mathrm{Re}\left(Be^{i\omega t}\right), \quad B = Ae^{i\delta}, \quad \omega = \sqrt{k/m} \tag{17.1.2}$$

(This can, of course, also be derived from the Lagrangian, easily shown to be $L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}m\omega^2x^2$.