7.2: Interlude - a Bit of History of Quantum Mechanics

It should be clear at this point that the Poisson bracket is very closely related to the commutator in quantum mechanics. In the usual quantum mechanical notation, the momentum operator

[Math Processing Error]

so the commutator (which acts on a wavefunction, remember)

[Math Processing Error]

identical to the Poisson bracket result multiplied by the constant *[Math Processing Error]*.

The first successful mathematical formulation of quantum mechanics, in 1925 (before Schrödinger's equation!) was by Heisenberg. As you know, he was the guy with the Uncertainty Principle: he realized that you couldn't measure momentum and position of anything simultaneously. He represented the states of a quantum system as vectors in some Hilbert space, and the dynamical variables as matrices acting on these vectors. He interpreted the result of a measurement as finding an eigenvalue of a matrix. If two variables couldn't be measured at the same time, the matrices had a nonzero commutator. In particular, for a particle's position and momentum the matrix representations satisfied *[Math Processing Error]*.

Dirac made the connection with Poisson brackets on a long Sunday walk, mulling over Heisenberg's uv−vu (as it was written). He suddenly but dimly remembered what he called "these strange quantities"—the Poisson brackets—which he felt might have properties corresponding to the quantum mathematical formalism Heisenberg was building. But he didn't have access to advanced dynamics books until the college library opened the next morning, so he spent a sleepless night. First thing Monday, he read the relevant bit of Whittaker's *Analytical Dynamics*, and saw he was correct. (From the biography by Helge Kragh.)

Dirac went on to adapt the equation

[Math Processing Error]

to quantum mechanics: for time-independent functions,

[Math Processing Error]

becomes

[Math Processing Error]

for time development of an operator in the Heisenberg picture, where state vectors of closed systems do not vary in time (as opposed to the Schrödinger picture, where the vectors vary and the operators remain constant).