4.6: Non-uniqueness of the Lagrangian

The Lagrangian is not uniquely defined: *two Lagrangians differing by the total derivative with respect to time of some function will give the same identical equations on minimizing the action,*

$$S' = \int_{t_1}^{t_2} L'(q, \dot{q}, t) dt = \int_{t_1}^{t_2} L(q, \dot{q}, t) dt + \int_{t_1}^{t_2} \frac{df(q,t)}{dt} dt = S + f\left(q\left(t_2\right), t_2\right) - f\left(q\left(t_1\right), t_1\right) \tag{4.6.1}$$

and since $q\left(t_1\right), t_1, q\left(t_2\right), t_2$ are all fixed, the integral over df/dt is trivially independent of path variations, and varying the path to minimize S' gives the same result as minimizing S. This turns out to be important later—it gives us a useful new tool to change the variables in the Lagrangian.