19.5: Eigenvectors of the Linear Chain

Let's get back to our chain, with eigenfunction equation of motion the N dimensional equivalent of

$$-m\Omega^2\begin{pmatrix}A_1\\A_2\\A_3\\A_4\end{pmatrix}=\begin{pmatrix}-2\kappa & \kappa & 0 & \kappa\\ \kappa & -2\kappa & \kappa & 0\\ 0 & \kappa & -2\kappa & \kappa\\ \kappa & 0 & \kappa & -2\kappa\end{pmatrix}\begin{pmatrix}A_1\\A_2\\A_3\\A_4\end{pmatrix} \tag{19.5.1}$$

. We see the matrix is a circulant, so we know the eigenvectors are of the form

$$\left(1,\omega^j,\left(\omega^j\right)^2,\left(\omega^j\right)^3,\ldots,\left(\omega^j\right)^{N-1}\right)^T \tag{19.5.2}$$

, which we'll now write

$$\left(1,e^{2\pi ij/N},e^{2\pi i2j/N},e^{2\pi i3j/N},\ldots,e^{2\pi ij(N-1)/N}\right) \tag{19.5.3}$$

What does this mean for our chain system? Remember that the n^{th} element of the eigenvector represents the displacement of the n^{th} atom of the chain from its equilibrium position, that would be proportional to $e^{2\pi ijn/N}$

The steady phase progression on going around the chain $n=0,1,2,3,\ldots$ makes clear that this is essentially a (longitudinal) wave. (The actual n^{th} particle displacement is the real part of the n^{th} element, but there could be an overall complex factor fixing the phase.)