

## 27.6: Stability of Top Spinning about Vertical Axis

(from Landau) For  $\theta = \dot{\theta} = 0$ ,  $L_3 = L_Z$ ,  $E' = 0$ . Near  $\theta = 0$ ,

$$\begin{aligned}
 V_{\text{effective}}(\theta) &= \frac{(L_Z - L_3 \cos \theta)^2}{2I_1' \sin^2 \theta} - Mgl(1 - \cos \theta) \\
 &\cong \frac{L_3^2 \left(\frac{1}{2}\theta^2\right)^2}{2I_1'\theta^2} - \frac{1}{2}Mgl\theta^2 \\
 &= \left(L_3^2/8I_1' - \frac{1}{2}Mgl\right)\theta^2
 \end{aligned} \tag{27.6.1}$$

The vertical position is stable against small oscillations provided  $L_3^2 > 4I_1'Mgl$ , or, or  $\Omega_3^2 > 4I_1'Mgl/I_3^2$

### Exercise 27.6.1

Suppose you set the top vertical, but spinning at less than  $\Omega_{3 \text{ crit}}$ , the value at which it is just stable. It will fall away, but bounce back, and so on. Show the maximum angle it reaches is given by  $\cos(\theta/2) = \Omega_3/\Omega_{3 \text{ crit}}$ .

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