18.1: More General Energy Exchange

We'll derive a formula for the energy fed into an oscillator by an *arbitrary* time-dependent external force.

The equation of motion can be written

$$\frac{d}{dt}(\dot{x}+i\omega x)-i\omega(\dot{x}+i\omega x)=\frac{1}{m}F(t) \tag{18.1.1}$$

and defining $\xi=\dot{x}+i\omega x$, this is

$$(d\xi/dt-i\omega\xi=F(t)/m \tag{18.1.2}$$

This first-order equation integrates to

$$\xi(t)=e^{i\omega t}\left(\int_0^t \frac{1}{m}F\left(t'\right)e^{-i\omega t'}dt'+\xi_0\right) \tag{18.1.3}$$

The energy of the oscillator is

$$E=\frac{1}{2}m\left(\dot{x}^2+\omega^2x^2\right)=\frac{1}{2}m|\xi|^2 \tag{18.1.4}$$

So if we drive the oscillator over all time, with beginning energy zero,

$$E=\frac{1}{2m}\left|\int_{-\infty}^{\infty}F(t)e^{-i\omega t}dt\right|^2 \tag{18.1.5}$$

This is equivalent to the quantum mechanical time-dependent perturbation theory result: ξ,ξ^* are equivalent to the annihilation and creation operators.

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