16.3: The Differential Cross Section

In a real scattering experiment, information about the scatterer can be figured out from the different rates of scattering to different angles. Detectors are placed at various angles (θ, ϕ). Of course, a physical detector collects scattered particles over some nonzero solid angle. The usual notation for infinitesimal solid angle is $d\Omega = \sin\theta d\theta d\phi$. The full solid angle (all possible scatterings) is $\int d\Omega = 4\pi$ the area of a sphere of unit radius. (*Note*: Landau uses do for solid angle increment, but $d\Omega$ has become standard.)

The differential cross section, written $d\sigma/d\Omega$ is the fraction of the total number of scattered particles that come out in the solid angle $d\Omega$, so the rate of particle scattering to this detector is $nd\sigma/d\Omega, \text{ with } n$ the beam intensity as defined above.

Now, we'll assume the potential is spherically symmetric. Imagine a line parallel to the incoming particles going through the center of the atom. For a given ingoing particle, its **impact parameter** is defined as the distance its ingoing line of flight is from this central line. Landau calls this ρ, we'll follow modern usage and call it b.

A particle coming in with impact parameter between b and $b+db$ will be scattered through an angle between χ and $\chi + d\chi$ where we're going to calculate, $\chi(b)$ by solving the equation of motion of a single particle in a repulsive inverse-square force.

Note: we've switched for this occasion from θ to χ for the angle scattered through because we want to save θ for the (r, θ) coordinates describing the complete trajectory, or orbit, of the scattered particle.

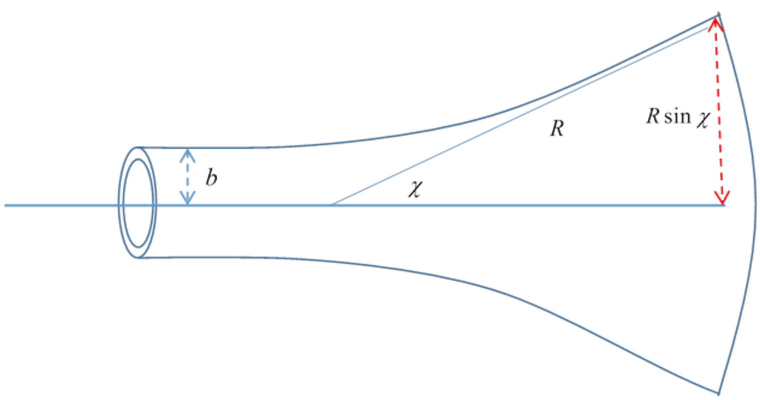

Figure 16.3.1

So, an ingoing cross section $d\sigma = 2\pi b db$ scatters particles into an outgoing spherical area (centered on the scatterer) $2\pi R \sin\chi R d\chi$, that is, a solid angle $d\Omega = 2\pi \sin\chi d\chi$

Therefore the scattering *differential cross section*

$$\frac{d\sigma}{d\Omega} = \frac{b(\chi)}{\sin\chi} \left| \frac{db}{d\chi} \right| \tag{16.3.1}$$

(Note that $d\chi/db$ is clearly negative—increasing b means increasing distance from the scatterer, so a smaller χ)