7.5: Example- Angular Momentum Components

A moving particle has angular momentum about the origin $\vec{L} = \vec{r} \times \vec{p},$ so

$$L_1 = r_2 p_3 - r_3 p_2, \quad L_2 = r_3 p_1 - r_1 p_3 \tag{7.5.1}$$

Using the Poisson brackets found above,

$$[r_i, r_j] = [p_i, p_j] = 0, \quad [p_i, r_j] = \delta_{ij} \tag{7.5.2}$$

we have

$$\begin{aligned} &= [r_2 p_3, r_3 p_1] + [r_3 p_2, r_1 p_3] \\ &= r_2 p_1 - p_2 r_1 \\ &= -L_3 \end{aligned} \tag{7.5.3}$$

(*Note*: we remind the reader that we are following Landau's convention, in which the Poisson brackets have the *opposite sign* to the more common use, for example in Goldstein and Wikipedia.)

We conclude that if two components of angular momentum are conserved, so is the third.