22.1: Introduction to Resonant Nonlinear Oscillations

Landau' next sections (Chapter 6, sections 28,29) address nonlinear one-dimensional systems. In particular, he focusses on driven damped oscillators with nonlinear, but small, added potential terms. Using ingenious semiquantitative techniques, he predicts some unexpected results: for example, a *discontinuity* in the oscillation amplitude on slowly varying the driving frequency at constant driving force (and constant damping). He also finds resonances when the driving frequency is a fraction, for example a third, of the oscillator's natural frequency.

Fortunately, this system is easy to analyze numerically, and we have an applet to do just that. The parameters are set by sliders, and one can immediately find the large discontinuity in amplitude (factor of two or so) as the frequency is slightly changed. At the end of this lecture, we show simple plots of amplitude response to a constant driving force as the frequency is varied. These were found using the applet, the reader can easily check them, and venture into parts of the parameter space. The applet provides a measure of Landau's (semiquantitative) accuracy, of course surprisingly good (of order 20% error or less) given the nature of the problem.

It should be added that this is one area where, thanks to computers, major advances have been made since Landau wrote the book, in particular the discovery for some systems of period doubling and chaos as the driving force is increased. We've added a lecture (22a) on a particular system, the driven damped pendulum, a natural extension of Landau's oscillator. This illustrates some of the novel features. We will follow part of chapter 12 of Taylor's excellent text, *Classical Mechanics*. Taylor provides many computer-generated graphs of the pendulum's response as parameters are varied. We provide applets that can generate these graphs. The reader can easily use these applets to explore other parameter inputs.

In this lecture, to gain a bit of intuition about these nonlinear potentials, we'll begin (following Landau) with no driving and no damping: just a particle oscillating in a potential that's simple harmonic plus small and (positive) terms. The basic questions are, how do these terms change the frequency of oscillation, and how does that frequency depend on the amplitude of oscillation? The answers will guide us in understanding how a particle in such a potential will respond to a harmonic driving term, plus damping.

Next, we briefly review the driven damped *linear* oscillator (covered in detail in lecture 18, this is really just a reminder of the notation). Then we add small cubic and quartic terms. We present Landau's argument that--above a certain driving force--gradually increasing the driving frequency leads at a critical value to a *discontinuous drop* in the amplitude of the response, then use an applet to confirm and quantify his result.