3.1: Another Minimization Problem…

Here's another minimization problem from the 1600's, even earlier than the brachistochrone. Fermat famously stated in the 1630's that a ray of light going from point A to point B always takes the route of least time -- OK, it's trivially trivially true in a single medium, light rays go in a straight line, but it's a lot less obvious if, say, A is in air and B in glass. Notice that this is closely related to our previous topic, the calculus of variations -- if this is a minimal time path, varying the path by a small amount will not change the time taken to first order. (*Historical note*: actually what amounted to Fermat's principle was first stated by Alhazen, in Baghdad, around 1000 AD.)

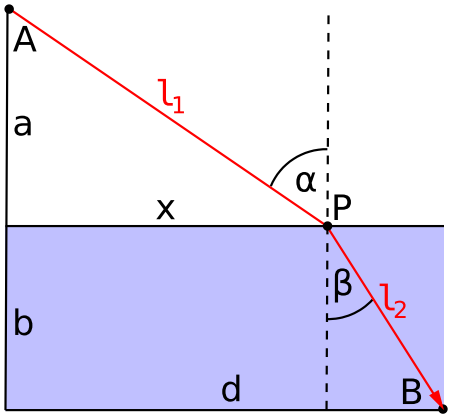

Figure 3.1.1: Fermat's principle in the case of refraction of light at a flat surface between (say) air and water. Given an object-point A in the air, and an observation point B in the water, the refraction point P is that which minimizes the time taken by the light to travel the path APB. If we seek the required value of x, we find that the angles α and β satisfy Snell's law. (Public Domain; Klaus-Dieter Keller via Wikipedia)

This seemed very mysterious when first extensively discussed, in the 1600's. In the last part of that century, and through the 1700's, Newton was the dominant figure, and he believed that light was a stream of particles. But how could the particle figure out the shortest time path from A to B?

In fact, there was one prominent physicist, Huygens', who thought light might be a wave, and, much later, this turned out to be the crucial insight. The main objection was that waves go around corners, at least to some extent, it seemed that light didn't. (Also, they exhibit diffraction effects, which no one thought they'd seen for light, although in fact Newton himself had observed diffraction -- Newton's rings -- but had an ingenious explanation, as always, of why his particle picture could explain what he saw.) Anyway, in 1678, Huygens' suggested the following picture: it's a simple beginning to understanding wave propagation, most notably it omits phases (later added by Fresnel) but it was a beginning.