19.2: The Circulant Matrix- Nature of its Eigenstates

The matrix we've constructed above has a very special property: each row is identical to the preceding row with the elements moved over one place, that is, it has the form

$$\begin{pmatrix} c_0 & c_1 & c_2 & c_3 \\ c_3 & c_0 & c_1 & c_2 \\ c_2 & c_3 & c_0 & c_1 \\ c_1 & c_2 & c_3 & c_0 \end{pmatrix} \tag{19.2.1}$$

Such matrices are called **circulants**, and their properties are well known. In particular, we'll show that the *eigenvectors* have the form $\left(1, \omega_j, \omega_j^2, \omega_j^3, \ldots, \omega_j^{N-1}\right)^T$ where $\omega_j^N = 1$.

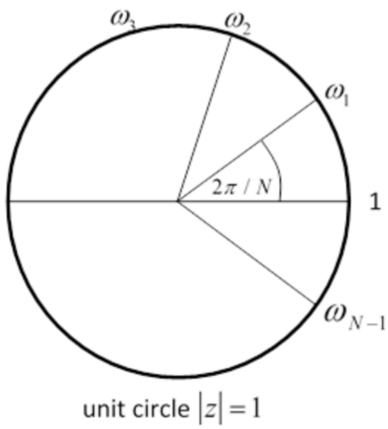

Figure 19.2.1

Recall the roots of the equation $z^N = 1$ are N points equally spaced around the unit circle,

$$e^{2\pi i n/N}, n = 0, 1, 2, \ldots N-1 \tag{19.2.2}$$

The standard mathematical notation is to label these points $1, \omega_1, \omega_2, \omega_3, \ldots, \omega_{N-1}$ as shown in the figure, but notice that $\omega_j = \omega_1^j$