# 17.4: Principle of Superposition

The equations of motion are *linear* equations, meaning that if you multiply a solution by a constant, that's still a solution, and if you have two different solutions to the equation, the sum of the two is also a solution. This is called the *principle of superposition*.

The general motion of the system is therefore

$$\begin{aligned} \theta_1(t) &= Ae^{i\omega_0 t} + Be^{i\sqrt{\omega_0^2+2k}t} \\ \theta_2(t) &= Ae^{i\omega_0 t} - Be^{i\sqrt{\omega_0^2+2k}t} \end{aligned} \tag{17.4.1}$$

where it is understood that A,B are complex numbers and the physical motion is the real part.

This is a four-parameter solution: the initial positions and velocities can be set arbitrarily, completely determining the motion.

*Exercise*: begin with one pendulum straight down, the other displaced, both momentarily at rest. Find values for A,B and describe the subsequent motion.

*Solution*: At $t=0$, the pendulums are at $A \pm B$. Take $A=B=1$ . This ensures zero initial velocities too, since the physical parameters are the real parts of the complex solution, and at the initial instant the derivatives are pure imaginary.

The solution for the motion of the first pendulum is

$$\theta_1(t) = \cos\omega_0 t + \cos\left(\omega_0^2+2k\right)^{\frac{1}{2}} t = 2\cos\frac{1}{2}\left(\left(\omega_0^2+2k\right)^{\frac{1}{2}}+\omega_0\right)t\cos\frac{1}{2}\left(\left(\omega_0^2+2k\right)^{\frac{1}{2}}-\omega_0\right)t \tag{17.4.2}$$

and for small $k$,

$$\theta_1(t) \cong 2\cos\omega_0 t\cos(k/\omega_0)t \tag{17.4.3}$$

Here the pendulum is oscillating at approximately $\omega_0$, but the second term sets the overall oscillation amplitude: it's slowly varying, going to zero periodically (at which point the *other* pendulum has maximum kinetic energy).

*To think about*: What happens if the two have different masses? Do we still get these beats -- can the larger pendulum transfer all its kinetic energy to the smaller?

*Exercise*: try pendulums of different lengths, hung so the bobs are at the same level, small oscillation amplitude, same spring as above.