18.2: Damped Driven Oscillator

The linear damped driven oscillator:

$$\ddot{x}+2\lambda\dot{x}+\omega_0^2x=(f/m)e^{i\Omega t} \tag{18.2.1}$$

(Following Landau's notation here—note it means the actual frictional drag force is $2\lambda m\dot{x}$)

Looking near resonance for steady state solutions at the driving frequency, with amplitude b, phase lag δ, that is, $x(t)=be^{i(\Omega t+\delta)}$, we find

$$be^{i\delta}\left(-\Omega^2+2i\lambda\Omega+\omega_0^2\right)=(f/m) \tag{18.2.2}$$

For a near-resonant driving frequency $\Omega=\omega_0+\varepsilon$, and assuming the damping to be sufficiently small that we can drop the $\varepsilon\lambda$ term along with ε^2, the leading order terms give

$$be^{i\delta}=-f/2m(\varepsilon-i\lambda)\omega_0 \tag{18.2.3}$$

so the response, the dependence of amplitude of oscillation on frequency, is to this accuracy

$$b=\frac{f}{2m\omega_0\sqrt{(\Omega-\omega_0)^2+\lambda^2}}=\frac{f}{2m\omega_0\sqrt{\varepsilon^2+\lambda^2}} \tag{18.2.4}$$

(We might also note that the resonant frequency is itself lowered by the damping, but this is another second-order effect we ignore here.)

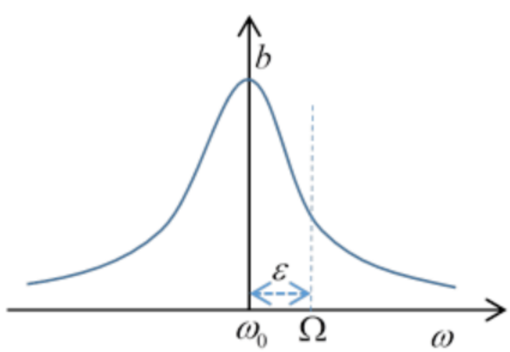

Figure 18.2.1

The rate of absorption of energy equals the frictional loss. The friction force $2\lambda m\dot{x}$ on the mass moving at $\dot{x}$ is doing work at a rate:

$$2\lambda m\dot{x}^2=\lambda mb^2\Omega^2 \tag{18.2.5}$$

The half width of the resonance curve as a function of driving frequency Ω is given by the damping. The total area under the curve is independent of damping.

For future use, we'll write the above equation for the amplitude as

$$b^2\left(\varepsilon^2+\lambda^2\right)=\frac{f^2}{4m^2\omega_0^2} \tag{18.2.6}$$

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