15.1: Preliminary- Polar Equations for Conic Section Curves

As we shall find, Newton's equations for particle motion in an inverse-square central force give orbits that are conic section curves. Properties of these curves are fully discussed in the accompanying "Math for Orbits" lecture, here for convenience we give the relevant polar equations for the various possibilities.

For an ellipse, with eccentricity e and semilatus rectum (perpendicular distance from focus to curve) ℓ:

$$\frac{\ell}{r}=1+e\cos\theta \tag{15.1.1}$$

Recall the eccentricity e is defined by the distance from the center of the ellipse to the focus being ae, where a is the semi-major axis, and $\ell=a\left(1-e^2\right)=b^2/a$.

For a parabola,

$$\ell=r(1+\cos\theta) \tag{15.1.2}$$

For a hyperbolic orbit with an attractive inverse square force, the polar equation with origin at the center of attraction is

$$\frac{\ell}{r}=1-e\cos\theta \tag{15.1.3}$$

where $\theta_{\text{asymptote}}<\theta<2\pi-\theta_{\text{asymptote}}$ (Of course, the physical path of the planet (say) is only one branch of the hyperbola.)

The (r,θ) origin is at the center of attraction (the Sun), geometrically this is one focus of the hyperbola, and for this attractive case it's the focus "inside" the curve.

For a hyperbolic orbit with a *repulsive* inverse square force (such as Rutherford scattering), the origin is the focus "outside" the curve, and to the right (in the usual representation):

$$\frac{\ell}{r}=-e\cos\theta-1 \tag{15.1.4}$$

with angular range $-\theta_{\text{asymptote}}<\theta<\theta_{\text{asymptote}}$.