11.5: Jacobian for Time Evolution

As we’ve established, time development is equivalent to a canonical coordinate transformation,

$$\left(p_t, q_t\right) \rightarrow \left(p_{t+\tau}, q_{t+\tau}\right) \equiv (P, Q) \tag{11.5.1}$$

Since we already know that the number of points inside a closed volume is constant in time, Liouville’s theorem is proved if we can show that the volume enclosed by the closed surface is constant, that is, with V'

denoting the volume V evolves to become, we must prove

$$\int_{V'} dQ_1 \ldots dQ_s dP_1 \ldots dP_s = \int_V dq_1 \ldots dq_s dp_1 \ldots dp_s ? \tag{11.5.2}$$

If you’re familiar with Jacobians, you know that (by definition)

$$\int dQ_1 \ldots dQ_s dP_1 \ldots dP_s = \int D dq_1 \ldots dq_s dp_1 \ldots dp_s \tag{11.5.3}$$

where the Jacobian

$$D = \frac{\partial\left(Q_1, \ldots, Q_s, P_1, \ldots, P_s\right)}{\partial\left(q_1, \ldots, q_s, p_1, \ldots, p_s\right)} \tag{11.5.4}$$

Liouville’s theorem is therefore proved if we can establish that D=1. If you’re not familiar with Jacobians, or need reminding, read the next section!