UNIVERSITY PHYSICS

Lumen

Lumen Learning

Book: University Physics (Lumen)

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This text was compiled on 04/15/2025

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CHAPTER OVERVIEW

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1.1: Kinematics

Kinematics

Concepts and Principles

Kinematics is the formal language physicists use to describe motion. The need for a formal language is evidenced by a simple experiment: drop an object from about shoulder height and ask two people to independently describe the motion of the object. Chances are that the descriptions will not be in perfect agreement, even though both observers described the same motion. Obviously, a more formal way of describing motion is necessary to eliminate this type of descriptive ambiguity. Kinematics is the formal method of describing motion.

Three parameters are carefully defined and used by physicists to describe motion. Specifying these three parameters at all times forms a complete description of the motion of an object.

Position

The position of an object is its location relative to a well-defined coordinate system at a particular instant of time. Without a specified coordinate system, position is a meaningless concept. A coordinate system is comprised of a *zero*, a specified *positive direction*, and a *scale*.

For example, in the hypothetical experiment in which the object was dropped from shoulder height, a coordinate system could have been defined in which the zero position was at ground level, the positive direction was up, and the scale used was meters. Using this coordinate system, the position of the object could have been specified at any particular instant of time. Of course, choosing the zero at the location at which the object was dropped, the positive direction as down, and the scale in feet is also perfectly acceptable. It doesn't matter what you choose as a coordinate system, only that you explicitly choose one. Depending on the coordinate system chosen, the position of an object can be positive, negative, or zero.

We will use the symbol r to designate position, and measure it in meters (m).

Velocity

Although the word velocity is often used loosely in everyday conversation, its meaning in physics is specific and well-defined. To physicists, the velocity is the rate at which the position is changing. The velocity can be specified at any particular instant of time.

For example, if the position is changing quickly the velocity is large and if the position is not changing at all the velocity is zero. A mathematical way to represent this definition is

$$v = \frac{\Delta r}{\Delta t} = \frac{r_{final} - r_{initial}}{t_{final} - t_{initial}}$$

Thus, velocity is measured in meters per second (m/s).

Actually, this is the definition of the *average* velocity of the object over the time interval Dt, but as the time interval becomes smaller and smaller the value of this expression becomes closer and closer to the actual rate at which the position is changing at one particular instant of time.

Since the final position of the object (rfinal) may be either positive, negative, or zero, and either larger, smaller, or the same as the initial position (rinitial), the velocity may be positive, negative, or zero. The sign of the velocity depends on the coordinate system chosen to define the position. A positive velocity simply means that the object is moving in the positive direction, as defined by the coordinate system, while a negative velocity means the object is traveling in the other direction.

Acceleration

Again, although the word acceleration is often used loosely in everyday conversation, its meaning in physics is specific and well-defined. To physicists, the acceleration is the rate at which the velocity is changing. Again, the acceleration can be specified at any particular instant of time.

For example, if the velocity is changing quickly the acceleration is large in magnitude, and if the velocity is not changing the acceleration is zero. If an object has non-zero acceleration, it does *not* mean that the object is speeding up. It simply means that the velocity is changing. Moreover, even if an object has a *positive* acceleration, it does not mean that the object is speeding up! A positive acceleration means that the change in the velocity points in the positive direction. (I can almost guarantee you will experience confusion about this. Take some time to think about the preceding statement right now.)

A mathematical way to represent acceleration is

$$a = \frac{\Delta v}{\Delta t} = \frac{v_{final} - v_{initial}}{t_{final} - t_{initial}}$$

Thus, acceleration is measured in meters per second per second (m/s2).

Again, this is actually the average acceleration of the object over the time interval Dt, but as the time interval becomes smaller and smaller, the value of this expression becomes closer and closer to the actual rate at which the velocity is changing at one particular instant of time.

Since vfinal may be either positive, negative, or zero, and either larger, smaller, or the same as vinitial, the acceleration may be positive, negative, or zero. The algebraic sign of the acceleration depends on the coordinate system chosen to define the position. A negative acceleration means that the change in the velocity points in the negative direction. For example, the velocity could be in the positive direction and the object slowing down *or* the velocity could be in the negative direction and the object speeding up. Both of these scenarios would result in a negative acceleration. Conversely, a positive acceleration means that the change in the velocity points in the positive direction.

Kinematics is the correct use of the parameters position, velocity, and acceleration to describe motion. Learning to use these three terms correctly can be made much easier by learning a few tricks of the trade. These tricks, or analysis tools, are detailed in the following section.

Analysis Tools

Drawing Motion Diagrams

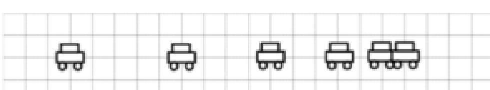
The words used by physicists to describe the motion of objects are defined above. However learning to use these terms correctly is more difficult than simply memorizing definitions. An extremely useful tool for bridging the gap between a normal, conversational description of a situation and a physicists' description is the motion diagram. A motion diagram is the first step in translating a verbal description of a phenomenon into a physicists' description.

Start with the following verbal description of a physical situation:

> The driver of an automobile traveling at 15 m/s, noticing a red-light 30 m ahead, applies the brakes of her car until she stops just short of the intersection.

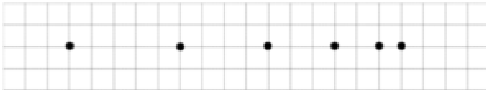
Determining the position from a motion diagram

A motion diagram can be thought of as a multiple-exposure photograph of the physical situation, with the image of the object displayed at equal time intervals. For example, a multiple-exposure photograph of the situation described above would look something like this:



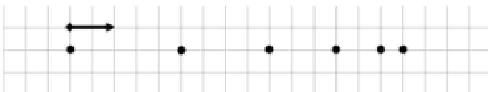
Note that the images of the automobile are getting closer together near the end of its motion because the car is traveling a smaller distance between the equally-timed exposures.

In general, in drawing motion diagrams it is better to represent the object as simply a dot, unless the actual shape of the object conveys some interesting information. Thus, a better motion diagram would be:



Since the purpose of the motion diagram is to help us describe the car's motion, a coordinate system is necessary. Remember, to define a coordinate system you must choose a zero, define a positive direction, and select a scale. We will always use meters as our position scale in this course, so you must only select a zero and a positive direction. Remember, there is no correct answer. Any coordinate system is as correct as any other.

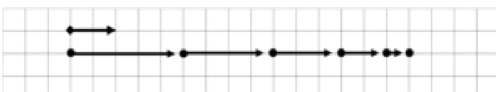
The choice below indicates that the initial position of the car is the origin, and positions to the right of that are positive. (In the text I'll always use a little diamond to indicate the zero with an arrow pointing in the positive direction.)



We can now describe the position of the car. The car starts at position zero and then has positive, increasing positions throughout the remainder of its motion.

Determining the velocity from a motion diagram

Since velocity is the change in position of the car during a corresponding time interval, and we are free to select the time interval as the time interval between exposures on our multiple-exposure photograph, the velocity is simply the change in the position of the car "between dots." Thus, the arrows (vectors) on the motion diagram below represent the velocity of the car.



We can now describe the velocity of the car. Since the velocity vectors always point in the positive direction, the velocity is always positive. The car starts with a large, positive velocity which gradually declines until the velocity of the car is zero at the end of its motion.

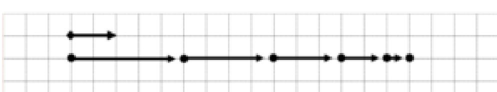
Determining the acceleration from a motion diagram

Since acceleration is the change in velocity of the car during a corresponding time interval, and we are free to select the time interval as the time interval between exposures on our multiple-exposure photograph, we can determine the acceleration by comparing two successive velocities. The change in these velocity vectors will represent the acceleration.

To determine the acceleration,

- select two successive velocity vectors,
- draw them starting from the same point,
- construct the vector (arrow) that connects the tip of the *first* velocity vector to the tip of the *second* velocity vector.
- The vector you have constructed represents the acceleration.

Comparing the first and second velocity vectors leads to the acceleration vector shown below:



Thus, the acceleration points to the left and is therefore negative. You could construct the acceleration vector at every point in time, but hopefully you can see that as long as the velocity vectors continue to point toward the right and decrease in magnitude, the acceleration will remain negative.

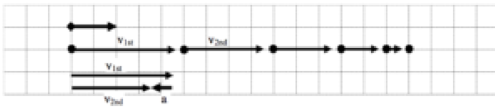
Thus, with the help of a motion diagram, you can extract lots of information about the position, velocity, and acceleration of an object. You are well on your way to a complete kinematic description.

Drawing Motion Graphs

Another useful way to describe the motion of an object is by constructing graphs of the object's position, velocity, and acceleration vs. time. A graphical representation is a very effective means of presenting information concerning an object's motion and, moreover, it is relatively easy to construct motion graphs if you have a correct motion diagram.

Examine the same situation as before:

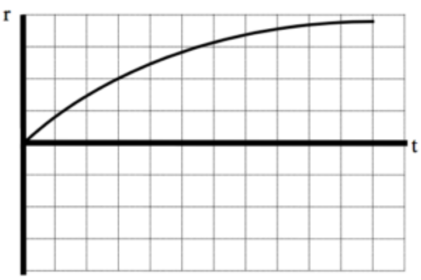
> The driver of an automobile traveling at 15 m/s, noticing a red-light 30 m ahead, applies the brakes of her car until she stops just short of the intersection.



The verbal representation of the situation has already been translated into a motion diagram. A careful reading of the motion diagram allows the construction of the motion graphs.

Drawing the position vs. time graph

We already know, from the motion diagram, that the car starts at position zero, then has positive, increasing positions throughout the remainder of its motion. This information can be transferred onto a position vs. time graph.



Notice that the position is zero when the time is equal to zero, the position is always positive, and the position increases as time increases. Also note that in each subsequent second, the car changes its position by a smaller amount. This leads to the graph of position vs. time gradually decreasing in slope until it achieves a slope of zero. Once the car stops, the position of the car should not change.

Drawing the velocity vs. time graph

From the motion diagram, we know that the velocity of the car is always positive, starts large in magnitude, and decreases until it is zero. This information can be transferred onto a velocity vs. time graph.

How do we know that the slope of the line is constant? The slope of the line represents the rate at which the velocity is changing, and the rate at which the velocity is changing is termed the acceleration. Since in this model of mechanics we will only consider particles undergoing constant acceleration, the slope of a line on a velocity vs. time graph must be constant.

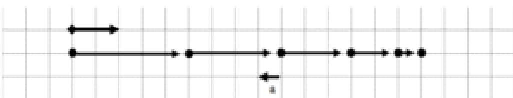
Drawing the acceleration vs. time graph

From the motion diagram, the acceleration of the car can be determined to be negative at every point. Again, in this pass through mechanics we will only be investigating scenarios in which the acceleration is constant. Thus, a correct acceleration vs. time graph is shown below.

Tabulating Motion Information

After constructing the two qualitative representations of the motion (the motion diagram and the motion graphs), we are ready to tackle the quantitative aspects of the motion.

Utilizing the motion diagram,



you can now assign numerical values to several of the kinematic variables. A glance at the situation description should indicate that information is presented about the car at two distinct events. Information is available about the car at the instant the driver applies the brakes (the velocity is given), and the instant the driver stops (the position is given). Other information can also be determined by referencing the motion diagram. To tabulate this information, you should construct a *motion table.*

Event 1: The instant the driver first applies the brakes. t1 = 0 s r1 = 0 m v1 = +15 m/s a12 =	Event 2: The instant the car finally stops. t2 = r2 = +30 m v2 = 0 m/s

In addition to the information explicitly given, the velocity at the first event and the position at the second event, other information can be extracted from the problem statement and the motion diagram.

For example,

- the position of the car at the first event is zero because the origin of the coordinate system is at this point,
- the time at the first instant can be "set to zero" by imagining a hypothetical stopwatch that is clicked on as the car begins to brake,
- and the velocity at the second event is zero because the car is stopped.

Since you are working under the assumption in this model that the acceleration is constant, the acceleration *between* the two instants in time is some unknown, constant value. To remind you that this assumption is in place, the acceleration is not labeled at the first instant, a1, or the second instant, a2, but rather as the acceleration *between* the two instants in time, a12.

You now have a complete tabulation of all the information presented, both explicitly and implicitly, in the situation description. Moreover, you now can easily see that the only kinematic information not known about the situation is the assumed constant acceleration of the auto and the time at which it finally stops. Thus, to complete a kinematic description of the situation these two quantities must be determined. What you may not know is that you have already been presented with the information needed to determine these two unknowns.

Doing the Math

In the concepts and principles portion of this unit, you were presented with two formal, mathematical relationships, the definitions of velocity and acceleration. In the example that you are working on, there are two unknown kinematic quantities. You should remember from algebra that two equations are sufficient to calculate two unknowns. Thus, by applying the two definitions you should be able to determine the acceleration of the car and the time at which it comes to rest.

Although you can simply apply the two definitions directly, normally the two definitions are rewritten, after some algebraic re-arranging, into two different relationships. This rearrangement is simply to make the algebra involved in solving for the unknowns easier. It is by no means necessary to solve the problem. In fact, the two definitions can be written in a large number of different ways, although this does not mean that there are a large number of different formulas you must memorize in order to analyze kinematic situations. There are only two independent kinematic relationships. The two kinematic relationships[1] we will use when the acceleration is constant are:

$$v_2 = v_1 + a_{12}(t_2 - t_1)$$

and

$$r_2 = r_1 + v_1(t_2 - t_1) + \frac{1}{2}a_{12}(t_2 - t_1)^2$$

To finish the analysis of this situation,

Event 1: The instant the driver first applies the brakes. t1 = 0 s r1 = 0 m v1 = +15 m/s a12 =	Event 2: The instant the car finally stops. t2 = r2 = +30 m v2 = 0 m/s

simply write the two kinematic relationships, input the known kinematic variables from the motion table, and solve the two relations for the two unknowns. (This process is not physics, it's algebra.)

$v_2 = v_1 + a_{12}(t_2 - t_1)$ $0 = 15 + a_{12}(t_2 - 0)$ $a_{12} = \frac{-15}{t_2}$ Now substitute this expression into the other equation:	$r_2 = r_1 + v_1(t_2 - t_1) + \frac{1}{2}a_{12}(t_2 - t_1)^2$ $30 = 0 + 15(t_2 - 0) + \frac{1}{2}a_{12}(t_2 - 0)^2$ $30 = 15t_2 + \frac{1}{2}a_{12}t_2^{\,2}$ $30 = 15t_2 + \frac{1}{2}(\frac{-15}{t_2})t_2^{\,2}$ $30 = 15t_2 - 7.5t_2$ $30 = 7.5t_2$ $t_2 = 4.0s$ Substitute this result back into the original equation:
$a_{12} = \frac{-15}{4}$ $a_{12} = -3.8m/s^2$	

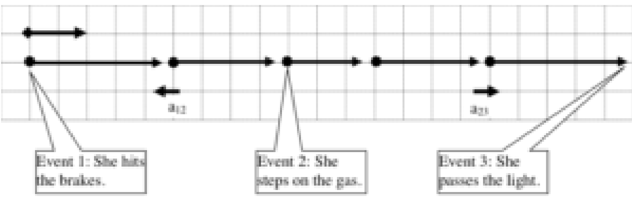
Thus, the car must have accelerated at 3.8 m/s2 in the negative direction, and stopped after 4.0 seconds. The kinematic description of the situation is complete.

Analyzing a More Complex Motion

Let’s re-visit our scenario, although this time the light turns green while the car is slowing down:

> The driver of an automobile traveling at 15 m/s, noticing a red-light 30 m ahead, applies the brakes of her car. When she is 10 m from the light, and traveling at 8.0 m/s, the light turns green. She instantly steps on the gas and is back at her original speed as she passes under the light.

Our first step in analyzing this motion should be to draw a motion diagram.



I’ve noted on the motion diagram the important events that take place during the motion. Notice that between the instant she hits the brakes and the instant she steps on the gas the acceleration is negative, while between the instant she steps on the gas and the instant she passes the light the acceleration is positive. Thus, in tabulating the motion information and applying the kinematic relations we will have to be careful not to confuse kinematic variables between these two intervals. Below is a tabulation of motion information using the coordinate system established in the motion diagram.

Event 1: She hits the brakes.	Event 2: She steps on the gas.	Event 3: She passes the light
t1 = 0 s	t2 =	t3 =
r1 = 0 m	r2 = +20 m	r3 = +30 m
v1 = +15 m/s	v2 = +8.0 m/s	v3 = +15 m/s
a12 =	a23 =	

First, notice that during the time interval between “hitting the brakes” and “stepping on the gas” there are two kinematic variables that are unknown. Recall that by using your two kinematic relations you should be able to determine these values. Second, notice that during the second time interval again two variables are unknown. Once again, the two kinematic relations will allow you to determine these values. Thus, before I actually begin to *do* the algebra I *know* the unknown variables can be determined!

First let’s examine the motion between hitting the brakes and stepping on the gas:

$v_2 = v_1 + a_{12}(t_2 - t_1)$ $8 = 15 + a_{12}(t_2 - 0)$ $a_{12} = \frac{-7}{t_2}$ Now substitute this expression into the other equation:	$r_2 = r_1 + v_1(t_2 - t_1) + \frac{1}{2}a_{12}(t_2 - t_1)^2$ $20 = 0 + 15(t_2 - 0) + \frac{1}{2}a_{12}(t_2 - 0)^2$ $20 = 15t_2 + \frac{1}{2}a_{12}t_2^{\ 2}$ $20 = 15t_2 + \frac{1}{2}(\frac{-7}{t_2})t_2^{\ 2}$ $20 = 15t_2 - 3.5t_2$ $20 = 11.5t_2$ $t_2 = 1.74s$
$a_{12} = \frac{-7}{1.74}$ $a_{12} = -4.03m/s^2$	Substitute this result back into the original equation:

Now, using these results, examine the kinematics between stepping on the gas and passing the light. Note that the initial values of the kinematic variables are denoted by ‘2’ and the final values by ‘3’, since we are examining the interval between event 2 and event 3.

$v_3 = v_2 + a_{23}(t_3 - t_2)$ $15 = 8 + a_{23}(t_3 - 1.74)$ $a_{23} = \frac{7}{(t_3 - 1.74)}$ Now substitute this expression into the other equation:	$r_3 = r_2 + v_2(t_3 - t_2) + \frac{1}{2}a_{23}(t_3 - t_2)^2$ $30 = 20 + 8(t_3 - 1.74) + \frac{1}{2}a_{23}(t_3 - 1.74)$ $10 = 8(t_3 - 1.74) + \frac{1}{2}a_{23}(t_3 - 1.74)^2$ $10 = 8(t_3 - 1.74) + \frac{1}{2}(\frac{7}{(t_3 - 1.74)})(t_3 - 1$ $10 = 8(t_3 - 1.74) + 3.5(t_3 - 1.74)$ $10 = 11.5(t_3 - 1.74)$ $0.87 = t_3 - 1.74$ $t_3 = 2.61s$
$a_{23} = \frac{7}{(2.61 - 1.74)}$ $a_{12} = 8.0m/s^2$	Substitute this result back into the original equation:

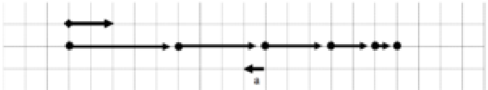
We now have a complete kinematic description of the motion.

Symbolic Analysis

Consider the following situation:

> The driver of an automobile suddenly sees an obstacle blocking her lane. Determine the total distance the auto travels between seeing the obstacle and stopping (d) as a function of the initial velocity of the car (vi) and the magnitude of its acceleration while stopping (as).

As always, the first step in analyzing motion is to draw a motion diagram.



Rather than calculate the stopping distance for particular values of initial velocity and acceleration, the goal of this activity is to determine, in general, how the stopping distance depends on these two parameters. If we can construct this function we can then use the result to calculate the stopping distance for *any* car if we know its initial velocity and stopping acceleration.

Although this sounds like a different task from what we've done in the previous two examples we will approach this task exactly the same way, by tabulating what we know about the situation,

Event 1: The instant the driver hits the brakes.	Event 2: The instant the car stops.
t1 = 0 s r1 = 0 m v1 = vi a12 = -as	t2 = r2 = d v2 = 0 m/s

and then applying our two kinematic relationships:

$$v_2 = v_1 + a_{12}(t_2 - t_1)$$
$$0 = v_i - a_s t_2$$

$$r_2 = r_1 + v_1(t_2 - t_1) + \frac{1}{2}a_{12}(t_2 - t_1)^2$$
$$d = 0 + v_i t_2 - \frac{1}{2}a_s t_2^{\,2}$$

Since our goal is to determine d as a function of vi and as, we must eliminate t2. To do this, solve for t2 in the left equation and substitute this expression into the right equation.

$$t_2 = \frac{v_i}{a_s}$$

$$d = v_i\left(\frac{v_i}{a_s}\right) - \frac{1}{2}a_s\left(\frac{v_i}{a_s}\right)^2$$
$$d = \frac{v_i^{\,2}}{a_s} - \frac{1}{2}\frac{v_i^{\,2}}{a_s}$$
$$d = \frac{v_i^{\,2}}{2a_s}$$

Thus, the stopping distance appears to be proportional to the *square* of the initial velocity and inversely proportional to the stopping acceleration. Does this make sense?

To determine if a symbolic expression is sensible it is often useful to check *limiting cases*. A limiting case is when one of the variables in the expression takes on an extreme value, typically zero or infinity. For example, if the initial velocity of the car was zero the stopping distance would have to also be zero, since the car was never moving! Allowing vi to equal zero in the above expression results in a stopping distance of zero, so our expression "passes" this logical test.

Another limiting case would be setting the acceleration of the car equal to zero. With no acceleration, the car should never stop. In our expression, setting the acceleration equal to zero results in an infinite stopping distance, which again agrees with commonsense. If our expression didn't give the correct results in these limiting cases we would know we made an error somewhere in the derivation (and, of course, we would then go back and find our mistake and fix it because we are good students …).

Hints and Suggestions

Algebraic Signs

Confusion about the meaning of algebraic signs is common among beginning physics students. The best way to clarify this confusion is to remember that algebraic signs are simply a mathematical way to describe direction. Instead of saying up and down, or east and west, physicists construct coordinate systems and translate the words east and west into the symbols ‘+’ and ‘-’, or even ‘-’ and ‘+’ if we choose a different coordinate system. The key to the translation is the coordinate system. A coordinate system is very similar to the English-French dictionary you might take with you on your first trip to France. When you see a ‘-’, use your coordinate system to translate it into a verbal description of direction.

Do not fall into the common habit of translating a ‘-’ into the word “decreasing”. A negative acceleration, for example, does *NOT* imply that the object is slowing down. It implies an acceleration that points in the negative direction. It is *impossible* to determine whether an object is speeding up or slowing down by looking at the sign of the acceleration! Conversely, the word “deceleration”, which does mean that an object is slowing down, does *not* give any information regarding the sign of the acceleration. I can decelerate in the positive direction as easily as I can decelerate in the negative direction.

Addendum

Deriving the kinematic relationships

Let’s construct the two independent kinematic relationships that you will use whenever the acceleration is constant. In a later chapter, we will return to the case in which the acceleration is not constant.

From the definition of acceleration:

$$a = \frac{v_f - v_i}{t_f - t_i}$$

$$v_f - v_i = a(t_f - t_i)$$

$$v_f = v_i + a(t_f - t_i)$$

The above relationship is our first kinematic relationship. The acceleration in this relationship is really the average acceleration. However, since the acceleration is constant in this model the average acceleration is the same as the acceleration at any instant between the initial and final state.

From the definition of velocity:

$$v = \frac{r_f - r_i}{t_f - t_i}$$

$$r_f - r_i = v(t_f - t_i)$$

$$r_f = r_i + v(t_f - t_i)$$

You must remember, however, that the velocity in this formula is really the average velocity of the object over the time interval selected. To keep you from having to remember this fact, we can rewrite the average velocity as the sum of the initial velocity and the final velocity divided by two:

$$r_f = r_i + (\frac{v_i + v_f}{2})(t_f - t_i)$$

$$r_f = r_i + \frac{1}{2}(v_i + v_f)(t_f - t_i)$$

Substituting in the first kinematic relationship for the final velocity yields:

$$r_f = r_i + \frac{1}{2}(v_i + v_i + a(t_f - t_i))(t_f - t_i)$$

$$r_f = r_i + \frac{1}{2}(2v_i(t_f - t_i) + a(t_f - t_i)^2)$$

$$r_f = r_i + v_i(t_f - t_i) + \frac{1}{2}a(t_f - t_i)^2$$

The above relationship is our second kinematic relationship.

Although we could keep deriving new kinematic equations forever, it is *impossible* that any other derived equation could allow us to calculate some quantity that these equations do not allow us to calculate.

Activities

Construct motion diagrams for the motions described below on a separate piece of paper.

a. A subway train in Washington, D.C., starts from rest and accelerates at 2.0 m/s^2 for 12 seconds.

b. The driver of a car traveling at 35m/s suddenly sees a police car. The driver attempts to reach the speed limit of 25 m/s by accelerating at 2.5 m/s^2. The driver has a reaction time of 0.55 s. (The reaction time is the time between first seeing the police car and pressing the brake.)

c. *A pole-vaulter, just before touching the cushion on which she lands after a jump, is falling downward at a speed of 10 m/s. The pole-vaulter sinks about 2.0 m into the cushion before stopping.*

d. *An elevator is moving downward at 4.0 m/s for 3.5 s before someone presses the emergency stop button. The elevator comes to rest after traveling 2.9 m.*

Construct motion diagrams for the motions described below.

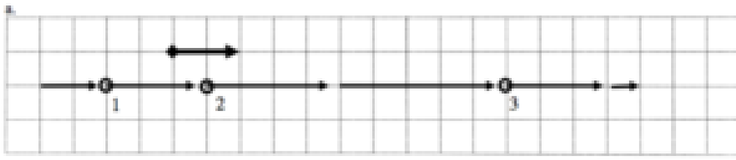
a. An automobile comes to rest after skidding 35 m. The car's acceleration while skidding is known to be 6.0 m/s^2.

b. A car, initially traveling at 20 m/s to the east, accelerates toward the west at 2.0 m/s^2. At the same time that the car starts moving, a truck, 60 m west of the car and moving at 16 m/s toward the east, starts to move faster, accelerating at 1.0 m/s^2. It's a one-lane road and both drivers are too busy texting to notice each other.

c. *A child is hanging from a rope by her hands. She exerts a burst of strength and 2.0 s later is traveling at 1.4 m/s up the rope.*

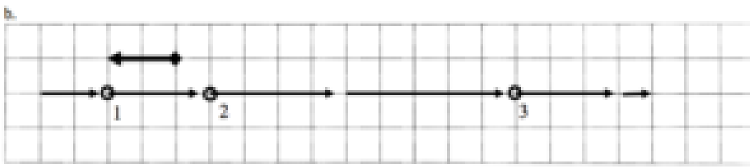
d. *A two-stage rocket initially accelerates upward from rest at 13 m/s^2 for 5.0s before the second stage initiates a 14 s long upward acceleration of 25 m/s^2.*

For each of the motion diagrams below, determine the algebraic sign (+, – or zero) of the position, velocity, and acceleration of the object at the location of the three open circles. Describe an actual motion that could be represented by each motion diagram.

a. 

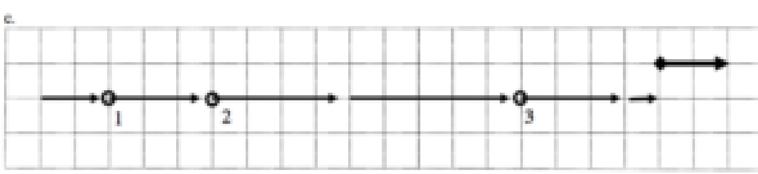
Description:

	1	2	3
r			
v			
a			

b. 

Description:

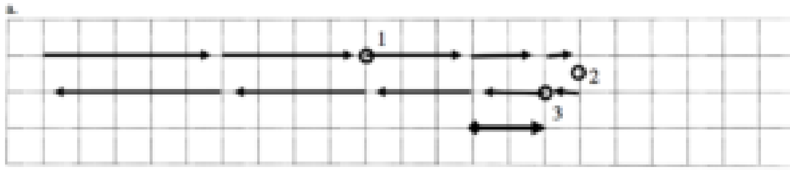
	1	2	3
r			
v			
a			

c. 

Description:

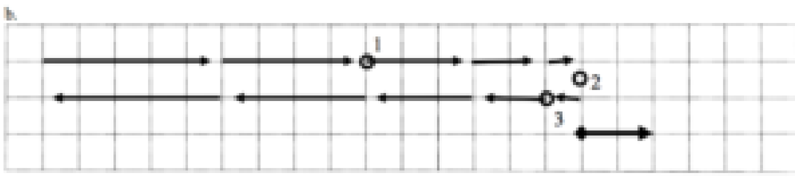
	1	2	3
r			
v			
a			

For each of the motion diagrams below, determine the algebraic sign (+, – or zero) of the position, velocity, and acceleration of the object at the location of the three open circles. Describe an actual motion that could be represented by each motion diagram.


a.

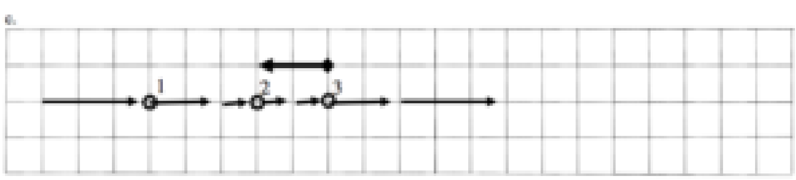
Description:

	1	2	3
r			
v			
a			


b.

Description:

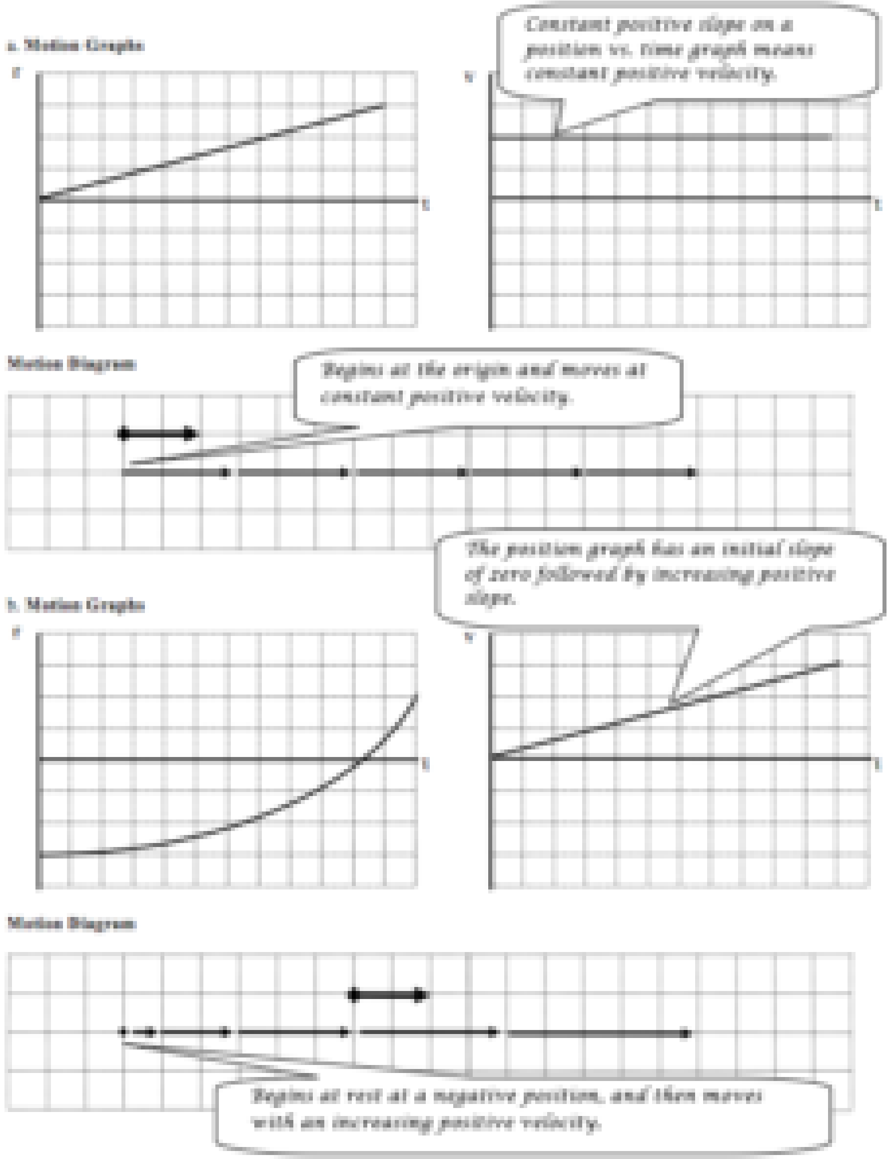
	1	2	3
r			
v			
a			


c.

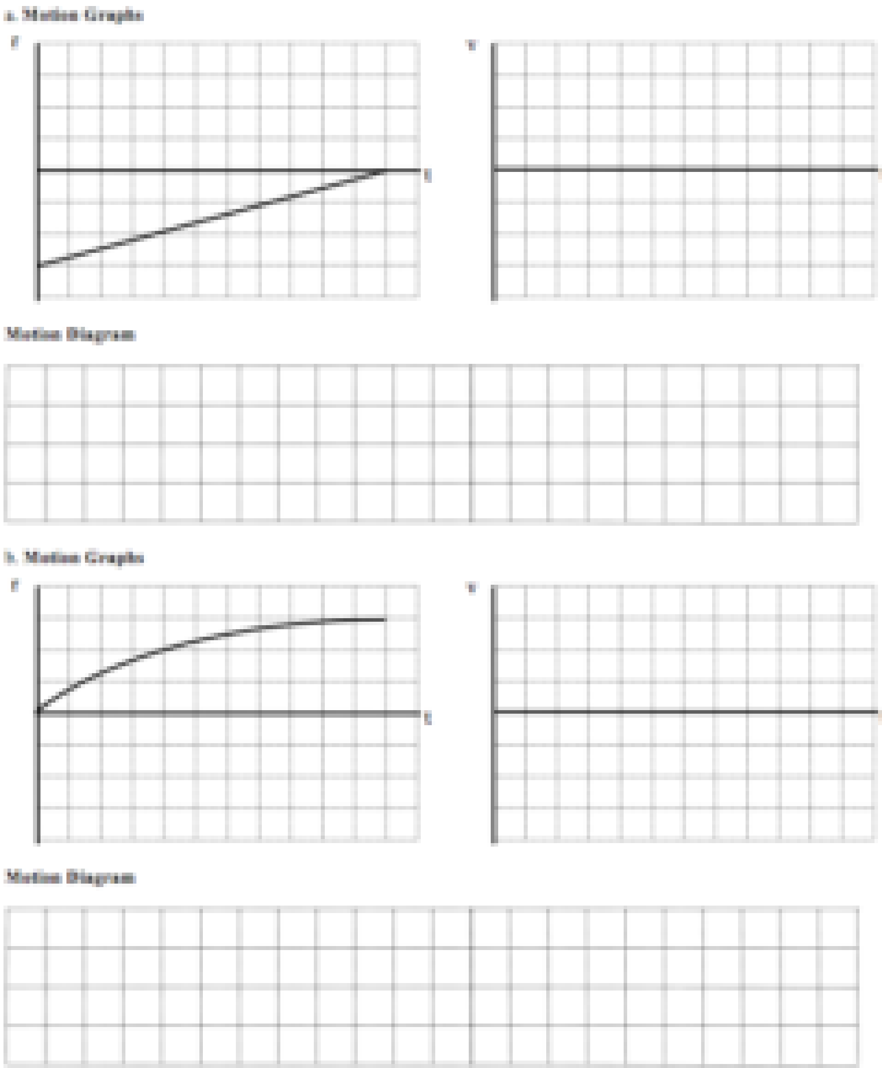
Description:

	1	2	3
r			
v			
a			

For each of the position vs. time graphs below, construct a corresponding motion diagram and velocity vs. time graph.

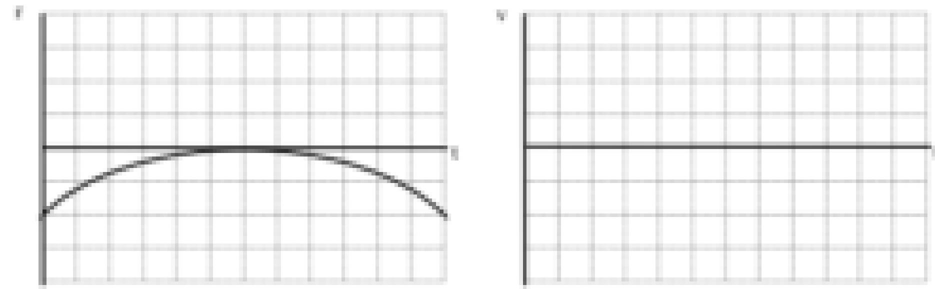


For each of the position vs. time graphs below, construct a corresponding motion diagram and velocity vs. time graph.

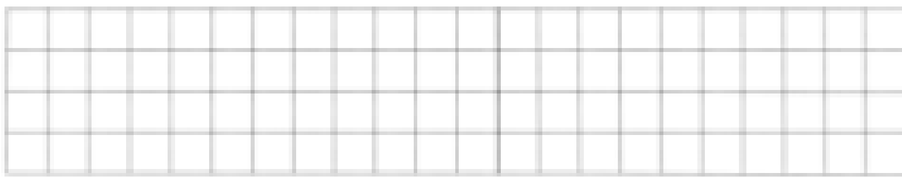


For each of the graphs below, construct a corresponding graph and motion diagram.

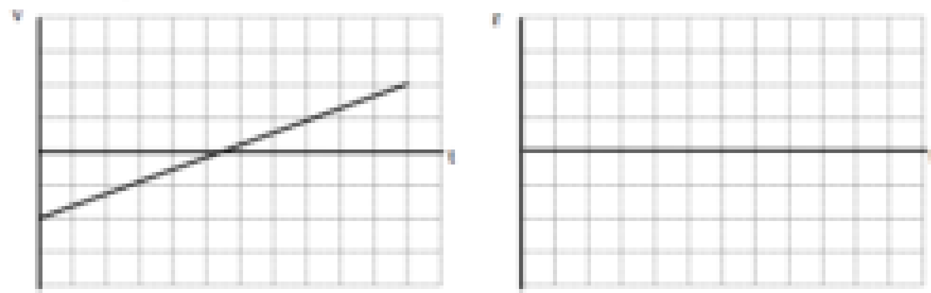
a. Motion Graphs



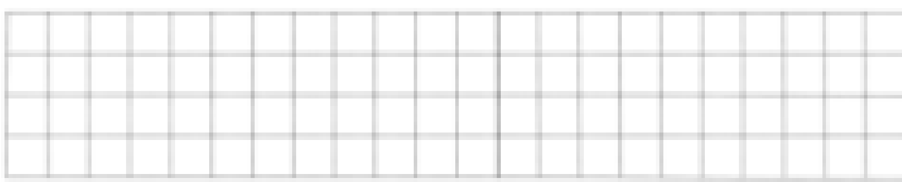
Motion Diagram



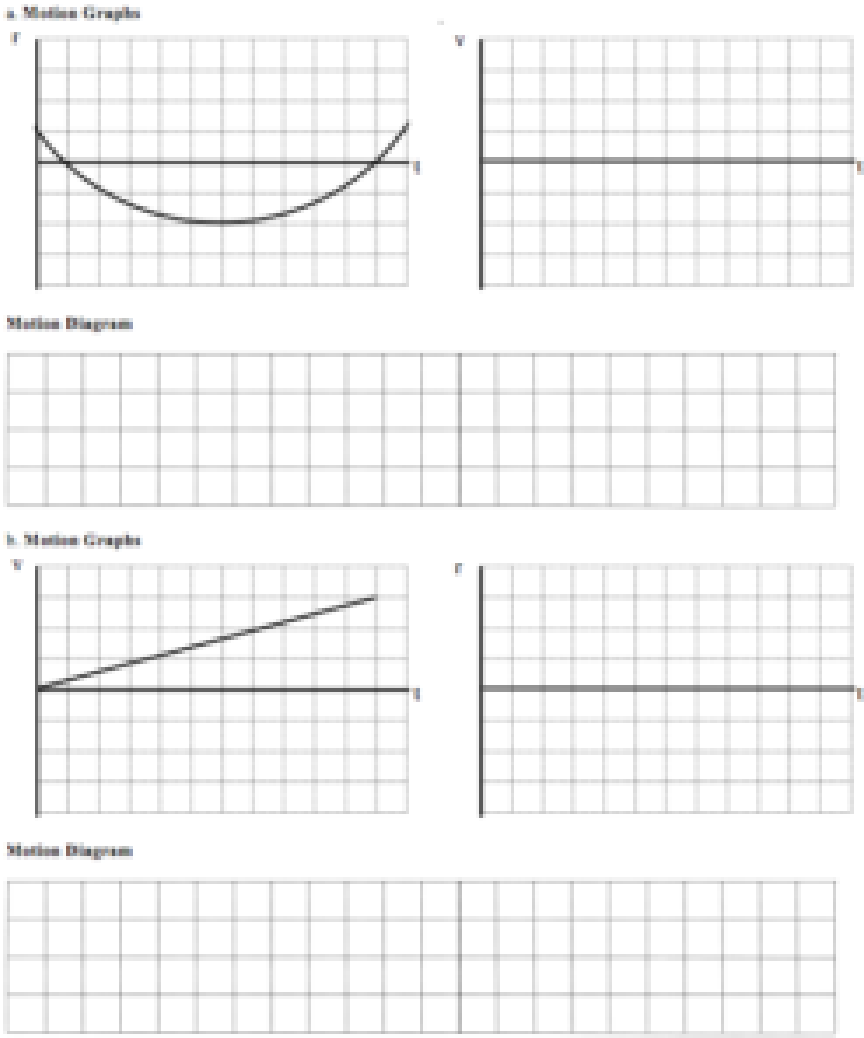
b. Motion Graphs



Motion Diagram

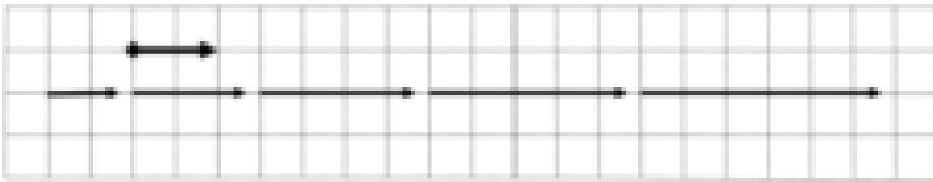


For each of the graphs below, construct a corresponding graph and motion diagram.

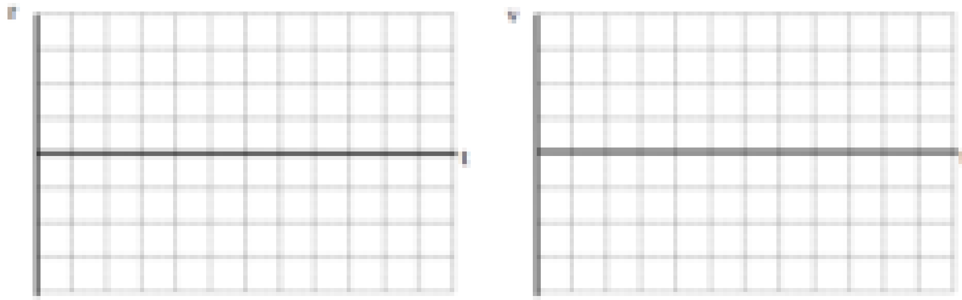


For each of the motion diagrams below, construct the corresponding motion graphs.

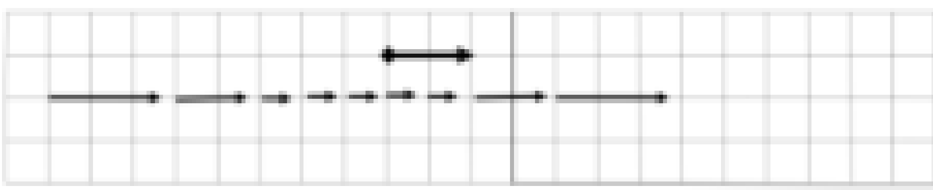
a. Motion Diagram



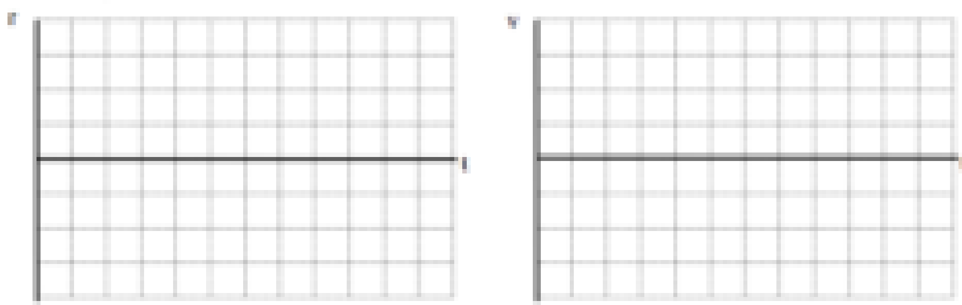
Motion Graphs



b. Motion Diagram

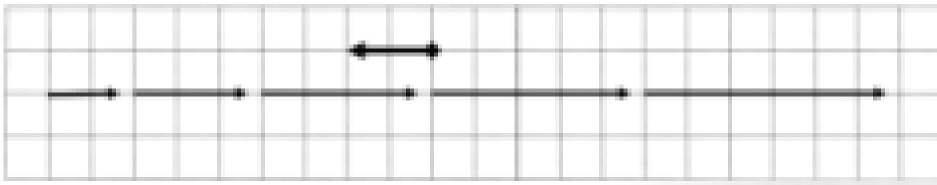


Motion Graphs

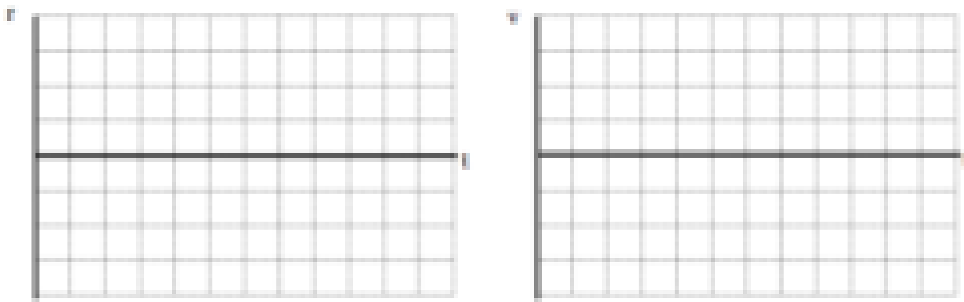


For each of the motion diagrams below, construct the corresponding motion graphs.

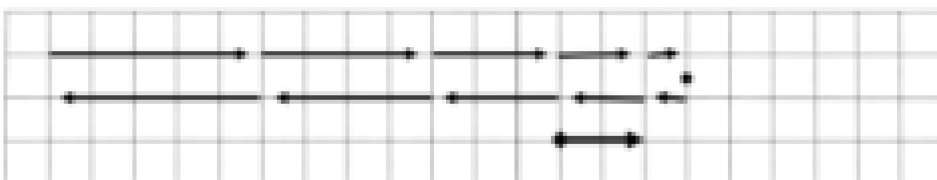
a. Motion Diagram



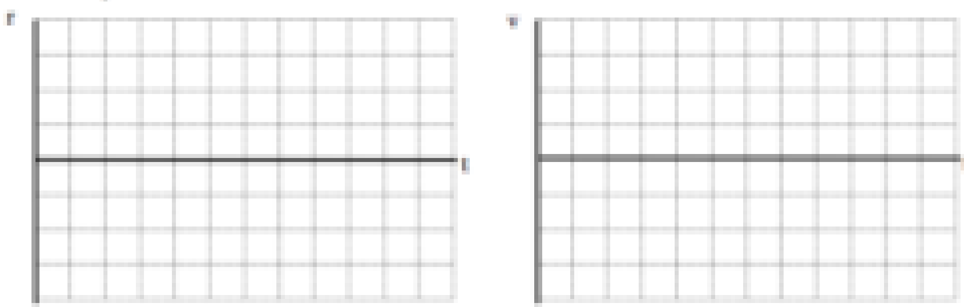
Motion Graphs

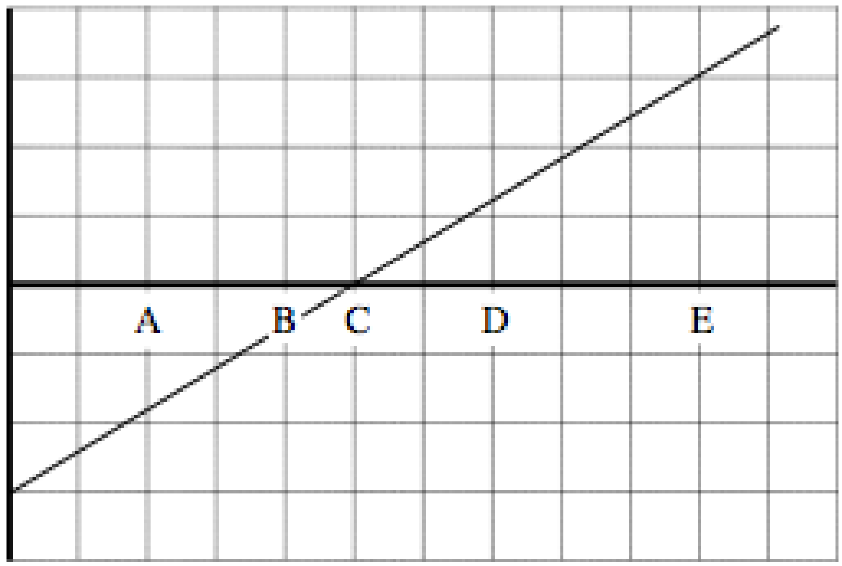


b. Motion Diagram



Motion Graphs





An object's motion is represented by the position vs. time graph above.

a. Rank the object's position at the lettered times.

Largest Positive 1. E 2. D 3. C 4. B 5. A Largest Negative

b. Rank the object's velocity at the lettered times.

Largest Positive 1. ABCDE 2. 3. 4. 5. Largest Negative

Constant slope means constant velocity (and zero acceleration)

c. Rank the object's acceleration at the lettered times.

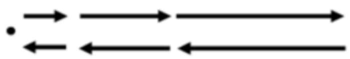
Largest Positive 1. 2. 3. ABCDE 4. 5. Largest Negative

An object's motion is represented by the velocity vs. time graph above.

d. Rank the object's position at the lettered times.

Largest Positive 1. E 2. A 3. D 4. B 5. C Largest Negative

The motion diagram for the object is sketched below. Notice that regardless of where the origin is located, the turn-around point (C) is the smallest position.



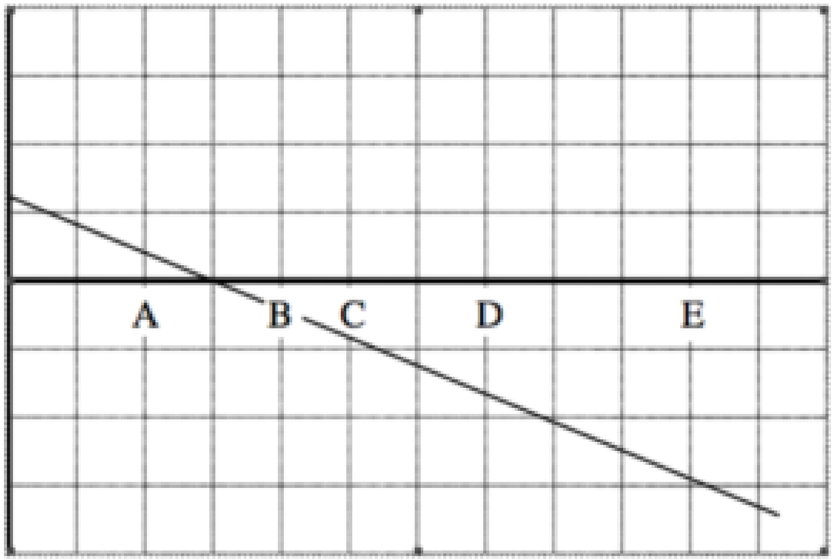
e. Rank the object's velocity at the lettered times.

Largest Positive 1. E 2. D 3. C 4. B 5. A Largest Negative

f. Rank the object's acceleration at the lettered times.

Largest Positive 1. ABCDE 2. 3. 4. 5. Largest Negative

Constant slope means constant acceleration.



An object's motion is represented by the position vs. time graph above.

a. Rank the object's position at the lettered times.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Largest Negative

b. Rank the object's velocity at the lettered times.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Largest Negative

c. Rank the object's acceleration at the lettered times.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Largest Negative

An object's motion is represented by the velocity vs. time graph above.

a. Rank the object's position at the lettered times.

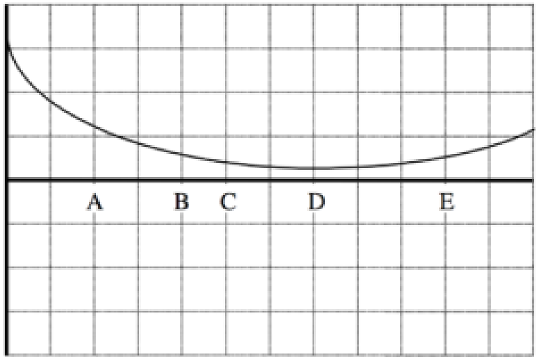
Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Largest Negative

b. Rank the object's velocity at the lettered times.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Largest Negative

c. Rank the object's acceleration at the lettered times.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Largest Negative



An object's motion is represented by the position vs. time graph at right.

a. Rank the object's position at the lettered times.

Largest Positive — Largest Negative

1. _____ 2. _____ 3. _____ 4. _____ 5. _____

b. Rank the object's velocity at the lettered times.

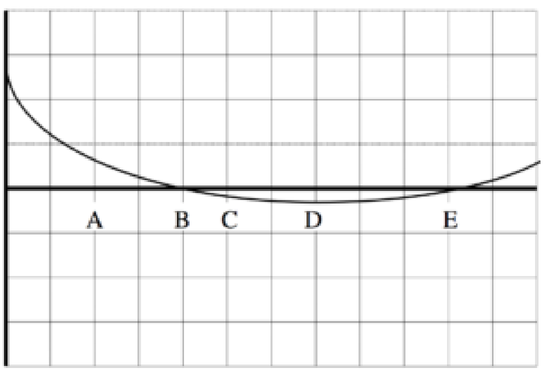
Largest Positive — Largest Negative

1. _____ 2. _____ 3. _____ 4. _____ 5. _____

c. Rank the object's acceleration at the lettered times.

Largest Positive — Largest Negative

1. _____ 2. _____ 3. _____ 4. _____ 5. _____



An object's motion is represented by the position vs. time graph at right.

d. Rank the object's position at the lettered times.

Largest Positive — Largest Negative

1. _____ 2. _____ 3. _____ 4. _____ 5. _____

e. Rank the object's velocity at the lettered times.

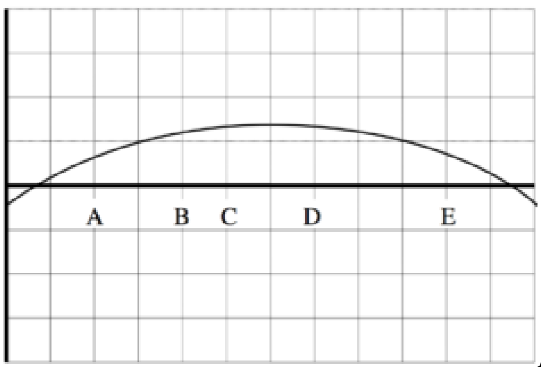
Largest Positive | Largest Negative

1. _____ 2. _____ 3. _____ 4. _____ 5. _____

f. Rank the object's acceleration at the lettered times.

Largest Positive | Largest Negative

1. _____ 2. _____ 3. _____ 4. _____ 5. _____



An object's motion is represented by the position vs. time graph at right.

a. Rank the object's position at the lettered times.

Largest Positive | Largest Negative

1. _____ 2. _____ 3. _____ 4. _____ 5. _____

b. Rank the object's velocity at the lettered times.

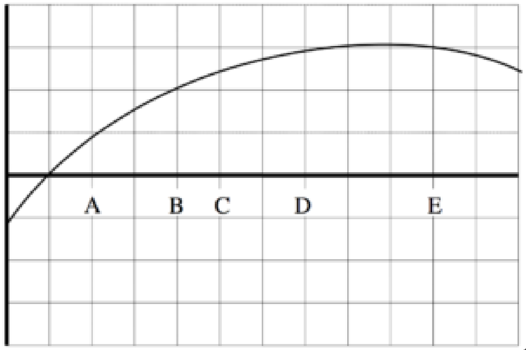
Largest Positive | Largest Negative

1. _____ 2. _____ 3. _____ 4. _____ 5. _____

c. Rank the object's acceleration at the lettered times.

Largest Positive | Largest Negative

1. _____ 2. _____ 3. _____ 4. _____ 5. _____



An object's motion is represented by the position vs. time graph at right.

d. Rank the object's position at the lettered times.

Largest Positive | Largest Negative

1. _____ 2. _____ 3. _____ 4. _____ 5. _____

e. Rank the object's velocity at the lettered times.

Largest Positive | Largest Negative

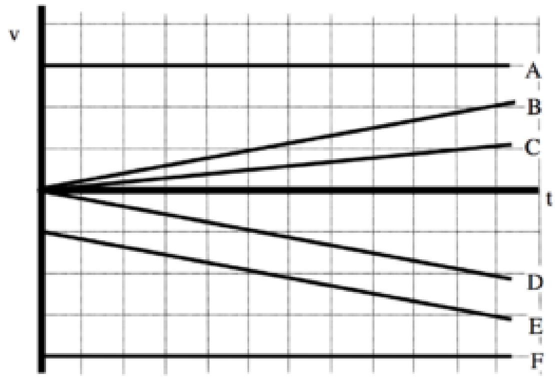
1. _____ 2. _____ 3. _____ 4. _____ 5. _____

f. Rank the object's acceleration at the lettered times.

Largest Positive | Largest Negative

1. _____ 2. _____ 3. _____ 4. _____ 5. _____

Below are velocity vs. time graphs for six different objects.



Rank these graphs on the basis of the distance traveled by each object.

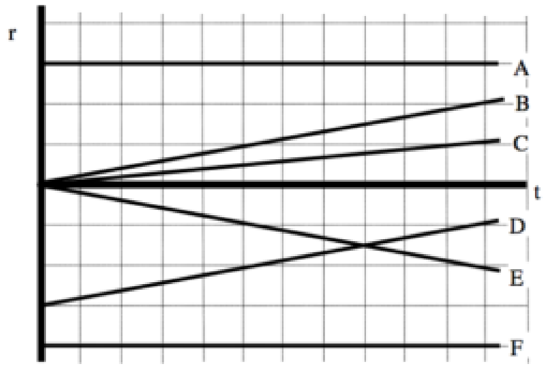
Largest 1. ___F___ 2. ___A___ 3. ___E___ 4. ___B D___ 5. ___C___ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

The faster you travel, the more distance you travel. The direction you are headed is not important. Therefore, F travels the largest distance because it is always moving the fastest, followed by A. E is consistently traveling faster than D, so it covers a larger distance than D. B and D always travel at the same speed (although in opposite directions) so they cover the same distance, therefore they are ranked as equal. C travels the slowest so it covers the least distance.

Below are position vs. time graphs for six different objects.



1. Rank these graphs on the basis of the velocity of the object.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

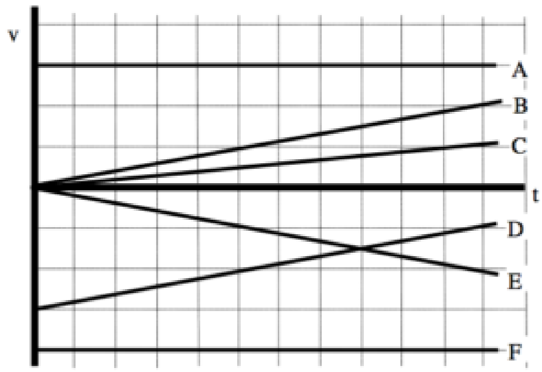
1. Rank these graphs on the basis of the acceleration of the object.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are velocity vs. time graphs for six different objects.



1. Rank these graphs on the basis of the final position of each object.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

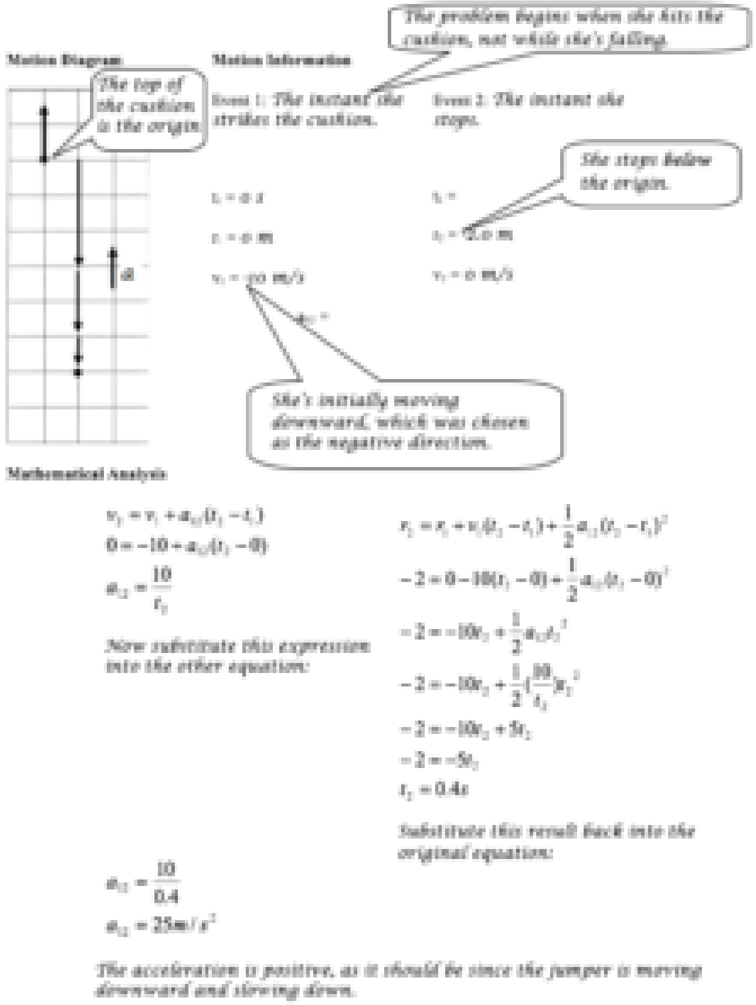
1. Rank these graphs on the basis of the acceleration of the object.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

A pole-vaulter, just before touching the cushion on which she lands after a jump, is falling downward at a speed of 10 m/s. The pole-vaulter sinks about 2.0 m into the cushion before stopping.



A child is hanging from a rope by her hands. She exerts a burst of strength and 2.0 s later is traveling at 1.4 m/s up the rope.

Motion Diagram **Motion Information**

	Event 1: t1 = r1 = v1 = a12 =	Event 2: t2 = r2 = v2 =

Mathematical Analysis

An elevator is moving downward at 4.0 m/s when someone presses the emergency stop button. The elevator comes to rest after traveling 2.9 m.

Motion Diagram **Motion Information**

	Event 1: t1 = r1 = v1 = a12 =	Event 2: t2 = r2 = v2 =

Mathematical Analysis

The driver of a car traveling at 16 m/s suddenly sees a truck that has entered from a side street and blocks the car's path. The car's maximum magnitude acceleration while braking is 6.0 m/s2 and the driver has a reaction time of 0.75 s. (The reaction time is the time between first seeing the truck and pressing the brake.) The driver stops just in time to avoid an accident.

Motion Diagram

Motion Information

Event 1: t1 = r1 = v1 = a12 =	Event 2: t2 = r2 = v2 = a23 =	Event 3: t3 = r3 = v3 =

Mathematical Analysis

The driver of a car traveling at 35 m/s suddenly sees a police car. The driver attempts to reach the speed limit of 25 m/s by accelerating at 2.5 m/s2. The driver has a reaction time of 0.55 s. (The reaction time is the time between first seeing the police car and pressing the brake.)

Motion Diagram

Motion Information

Event 1: t1 = r1 = v1 = a12 =	Event 2: t2 = r2 = v2 = a23 =	Event 3: t3 = r3 = v3 =

Mathematical Analysis

An automobile, initially traveling at 15 m/s, begins to slow down as it approaches a red light. After traveling 15 m, and slowing to 3.0 m/s, the light turns green and the driver steps on the gas and accelerates for 2.4 seconds until she reaches her original speed.

Motion Diagram

Motion Information

Event 1: t1 = r1 = v1 = a12 =	Event 2: t2 = r2 = v2 = a23 =	Event 3: t3 = r3 = v3 =

Mathematical Analysis

A two-stage rocket initially accelerates upward from rest at 13 m/s2 for 5.0s before the second stage ignites. The second stage "burns" for 14 s and results in the rocket achieving a speed of 415 m/s at the end of this stage.

Motion Diagram **Motion Information**

	Event 1: t1 = r1 = v1 = a12 =	Event 2: t2 = r2 = v2 = a23 =	Event 3: t3 = r3 = v3 =		

Mathematical Analysis

Rather than pushing the button for the correct floor a man prefers to hit the emergency stop button when an elevator approaches his floor, and then pry the doors apart. An elevator is moving at a constant speed of 2.8 m/s when it is 16 m from his floor. With uncanny timing, and an elevator that can slow at 3.5 m/s2, he makes the elevator stop precisely at his floor.

Motion Diagram **Motion Information**

	Event 1: t1 = r1 = v1 = a12 =	Event 2: t2 = r2 = v2 = a23 =	Event 3: t3 = r3 = v3 =		

Mathematical Analysis

A subway train in Washington, D.C., starts from rest and accelerates at 2.0 m/s2 for 12 s. The train travels at a constant speed for 65 s. The speed of the train then decreases for 25 s until it reaches the next station.

Motion Diagram

Motion Information

Event 1: t1 = r1 = v1 = a12 =	Event 2: t2 = r2 = v2 = a23 =	Event 3: t3 = r3 = v3 = a34 =	Event 4: t4 = r4 = v4 =

Mathematical Analysis

A rocket ship is launched from rest from a space station. Its destination is 1.0 x 1011 m away. The ship is programmed to accelerate at 7.4 m/s2 for 12 hours. After 12 hours, the ship will travel at constant velocity until it comes within 1.0 x 1010 m of its *destination. Then, it will fire its retrorockets to land safely.*

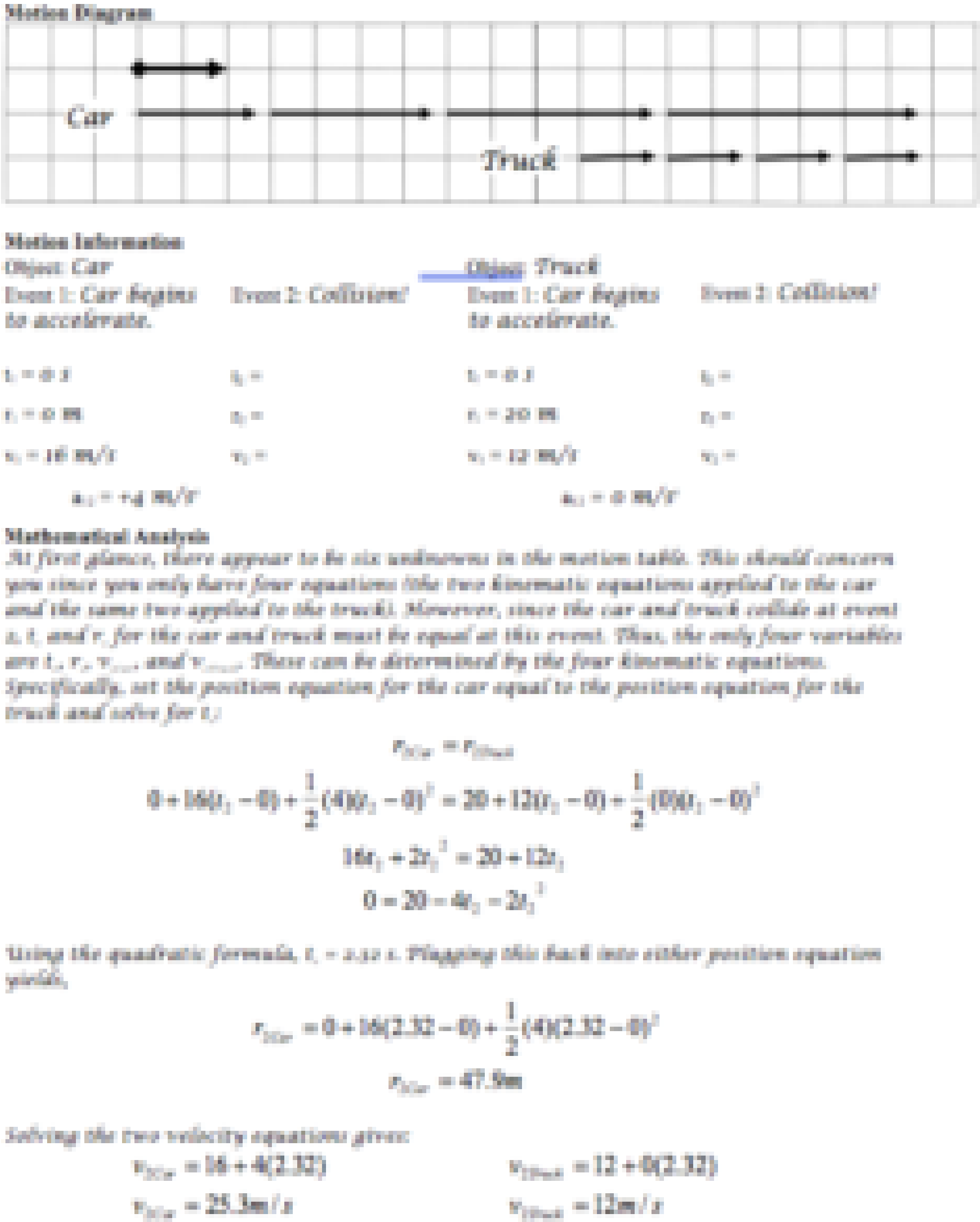
Motion Diagram

Motion Information

Event 1:	Event 2:	Event 3:	Event 4:
t1 =	t2 =	t3 =	t4 =
r1 =	r2 =	r3 =	r4 =
v1 =	v2 =	v3 =	v4 =
a12 =	a23 =	a34 =	

Mathematical Analysis

The driver of a car traveling at 16 m/s sees a truck 20 m ahead traveling at a constant speed of 12 m/s. The car starts without delay to accelerate at 4.0 m/s2 in an attempt to rear-end the truck. The truck driver is too busy talking on his cell phone to notice the car.



Motion Diagram

Motion Information

Object: *Car*		Object: *Truck*	
Event 1: Car begins to accelerate.	*Event 2: Collision!*	*Event 1: Car begins to accelerate.*	*Event 2: Collision!*
$t_1 = 0$ s	$t_2 =$	$t_1 = 0$ s	$t_2 =$
$r_1 = 0$ m	$r_2 =$	$r_1 = 20$ m	$r_2 =$
$v_1 = 16$ m/s	$v_2 =$	$v_1 = 12$ m/s	$v_2 =$
$a_{12} = +4$ m/s²		$a_{12} = 0$ m/s²	

Mathematical Analysis

At first glance, there appear to be six unknowns in the motion table. This should concern you since you only have four equations (the two kinematic equations applied to the car and the same two applied to the truck). However, since the car and truck collide at event 2, t_2 and r_2 for the car and truck must be equal at this event. Thus, the only four variables are t_2, r_2, $v_{2\,car}$ and $v_{2\,truck}$. These can be determined by the four kinematic equations. Specifically, set the position equation for the car equal to the position equation for the truck and solve for t_2:

$$r_{2Car} = r_{2Truck}$$

$$0 + 16(t_2 - 0) + \frac{1}{2}(4)(t_2 - 0)^2 = 20 + 12(t_2 - 0) + \frac{1}{2}(0)(t_2 - 0)^2$$

$$16t_2 + 2t_2^2 = 20 + 12t_2$$

$$0 = 20 - 4t_2 - 2t_2^2$$

Using the quadratic formula, $t_2 = 2.32$ s. Plugging this back into either position equation yields,

$$r_{2Car} = 0 + 16(2.32 - 0) + \frac{1}{2}(4)(2.32 - 0)^2$$

$$r_{2Car} = 47.9\text{m}$$

Solving the two velocity equations gives:

$$v_{2Car} = 16 + 4(2.32) \qquad v_{2Truck} = 12 + 0(2.32)$$

$$v_{2Car} = 25.3\text{m}/s \qquad v_{2Truck} = 12\text{m}/s$$

A car, initially at rest, accelerates toward the west at 2.0 m/s2. At the same time that the car starts, a truck, 350 m west of the car and moving at 16 m/s toward the east, starts to move slower, accelerating at 1.0 m/s2. The car and truck pass safely.

Motion Diagram

Motion Information

Object: Object:

Event 1:	Event 2:	Event 1:	Event 2:
t1 = r1 = v1 = a12 =	t2 = r2 = v2 =	t1 = r1 = v1 = a12 =	t2 = r2 = v2 =

Mathematical Analysis

A car, initially traveling at 20 m/s to the east, accelerates toward the west at 2.0 m/s2. At the same time that the car starts, a truck, 60 m west of the car and moving at 16 m/s toward the east, starts to move faster, accelerating at 1.0 m/s2. It's a one-lane road and both drivers are too busy texting to notice each other.

Motion Diagram

Motion Information

Object: Object:

Event 1:	Event 2:	Event 1:	Event 2:
t1 = r1 = v1 = a12 =	t2 = r2 = v2 =	t1 = r1 = v1 = a12 =	t2 = r2 = v2 =

Mathematical Analysis[xi]

A helium balloon begins to rise from rest with an acceleration of 2.0 m/s2 until it reaches a height of 50 m, and then continues upward at constant speed. However, a passenger in the balloon forgot their cell phone on the ground and threatens to jump from the balloon because of separation anxiety. Therefore a second balloon is launched 5.0 s after the first and accelerates upwards at 3.0 m/s2 to reunite the passenger and his phone.

Motion Diagram **Motion Information**

	Object: Event 1: t1 = r1 = v1 = a12 =	Event 2: t2 = r2 = v2 = a23 =	Event 3: t3 = r3 = v3 =	Object: Event 1: t1 = r1 = v1 = a12 =	Event 2: t2 = r2 = v2 = a23 =	Event 3: t3 = r3 = v3 =

Mathematical Analysis[xii]

A car traveling at 16 m/s sees a truck 20 m ahead traveling at a constant speed of 12 m/s. After thinking it over for 0.80 s, the car starts to accelerate at 4.0 m/s2 in an attempt to rear-end the truck. The truck driver has paid-up insurance so doesn't care. The car driver's motive is unknown.

Motion Diagram

Motion Information

Object: Object:

Event 1: t1 = r1 = v1 = a12 =	Event 2: t2 = r2 = v2 = a23 =	Event 3: t3 = r3 = v3 =	Event 1: t1 = r1 = v1 = a12 =	Event 2: t2 = r2 = v2 = a23 =	Event 3: t3 = r3 = v3 =

Mathematical Analysis[xiii]

A pole-vaulter lands on a cushion after a vault. Determine her acceleration as she sinks into the cushion (acushion) as a function of her velocity when she hits the cushion (vi) and the distance she sinks into the cushion (d).

Motion Diagram **Motion Information**

	Event 1: t1 = r1 = v1 = a12 = **Mathematical Analysis**	Event 2: t2 = r2 = v2 =

Questions

If vi = 0 m/s, what should acushion equal? Does your function agree with this observation?

If d = 0 m, what should acushion equal? Does your function agree with this observation?

What would result in a larger magnitude acceleration, hitting the cushion twice as fast or sinking one-half as far into the cushion?

The driver of an automobile suddenly sees an obstacle blocking her lane. Determine the total distance the car travels between seeing the obstacle and stopping (d) as a function of the initial velocity of the car (vi) and the magnitude of the car's acceleration while stopping (as). Ignore the driver's reaction time.

Motion Diagram

Motion Information

Event 1:	Event 2:
t1 = r1 = v1 = a12 =	t2 = r2 = v2 =

Mathematical Analysis

Questions

If vi = 0 m/s, what should d equal? Does your function agree with this observation?

If as = 0 m/s2, what should d equal? Does your function agree with this observation?

If the car was traveling twice as fast how much further would the car travel before stopping?

The driver of an automobile suddenly sees an obstacle blocking his lane. Determine the total distance the car travels between seeing the obstacle and stopping (d) as a function of his reaction time (tR), the initial velocity of the car (vi), and the magnitude of the car's acceleration while stopping (as).

Motion Diagram

Motion Information

Event 1:	Event 2:	Event 3:
t1 = r1 = v1 = a12 =	t2 = r2 = v2 = a23 =	t3 = r3 = v3 =

Mathematical Analysis

Questions

If vi = 0 m/s, what should d equal? Does your function agree with this observation?

If as = 0 m/s2, what should d equal? Does your function agree with this observation?

Does the car travel a greater distance during the reaction phase or the braking phase? Estimate necessary quantities assuming a panic stop from highway speeds.

Two automobiles are involved in a race over level ground. Both cars begin at rest. The Audi accelerates at A for T seconds and then travels at constant velocity for the rest of the race. The Buick accelerates at A/2 for 2T seconds and then travels at constant velocity for the rest of the race. Determine the distance (d) between the two cars, at the instant t = 2T, as a function of A and T.

Motion Diagram

Motion Information

Object: Object:

Event 1: t1 = r1 = v1 = a12 =	Event 2: t2 = r2 = v2 = a23 =	Event 3: t3 = r3 = v3 =	Event 1: t1 = r1 = v1 = a12 =	Event 2: t2 = r2 = v2 = a23 =	Event 3: t3 = r3 = v3 =

Mathematical Analysis

Questions

If A = 0 m/s^2, what should d equal? Does your function agree with this observation?

Assume the race takes longer than 2T to finish. What is the difference in speed between the two cars as they cross the finish line?

Assume the race takes longer than 2T to finish. What is the distance between the two cars as they cross the finish line?

Two automobiles are involved in a game of chicken. The two cars start from rest at opposite ends of a long, one-lane road. The Audi accelerates at A for T seconds and then travels at constant velocity. The Buick accelerates at A/2 for 2T seconds and then travels at constant velocity. Determine the elapsed time when the car's collide ($t_{collide}$) as a function of the length of the road (d), A and T. The collision takes place after both cars have reached constant velocity.

Motion Diagram

Motion Information

Object: Object:

Event 1: t1 = r1 = v1 = a12 =	Event 2: t2 = r2 = v2 = a23 =	Event 3: t3 = r3 = v3 =	Event 1: t1 = r1 = v1 = a12 =	Event 2: t2 = r2 = v2 = a23 =	Event 3: t3 = r3 = v3 =

Mathematical Analysis

Questions

If A = 0 m/s2, what should tcollide equal? Does your function agree with this observation?

If d = ∞, what should tcollide equal? Does your function agree with this observation?

What is the minimum length of road needed to guarantee the two cars collide at top speed?

[i] r2 = 1.4 m

[ii] t2 = 1.45 m

[iii] t3 = 3.4 s

[iv] t3 = 4.55 s

[v] r3 = 36.6 m

[vi] r3 = 3520 m

[vii] r2 = 14.9 m

[viii] r4 = 2000 m

[ix] t4 = 3.7 x 105 s

[x] t2 = 14.9 s

[xi] t2 = 7.8 s

[xii] t3 = 15.7 s

[xiii] t3 = 2.87 s

Homework 1 – Model 1: 25, 31, 32, 35, 39, 45, 47, 53, 55, and 59.

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1. These two relationships are simply the definitions of velocity and acceleration rearranged. I show all the steps in the addendum to this chapter. ↵

1.2: Dynamics

Dynamics

Concepts and Principles

Newton’s First Law

Dynamics is the study of the cause of motion, or more precisely the cause of *changes* in motion. In the late 1600’s Isaac Newton hypothesized that motion does not require a cause, rather *changes* in motion require causes. An object experiences a change in motion only when it interacts with some aspect of its surroundings. This bold hypothesis, referred to as Newton’s First Law of Motion, is summarized by the idea that an object will maintain its state of motion, whether at rest or traveling at high speed, unless acted upon by some aspect of its surroundings.

Using the kinematic terminology developed in the last unit, this means that an object’s velocity (state of motion) is constant unless it interacts with some outside agent. An external interaction is not necessary for an object to move, it is only necessary if the object’s velocity changes. Thus, what is *caused* is not velocity, but acceleration. This concept is one of the most subtle, and complex, in all of physics.

Newton’s Second Law

Newton also hypothesized that the sum total of all interactions with the external environment, which he termed *forces*, is directly proportional to the acceleration of the object. Moreover, the proportionality constant between the sum of all forces acting on an object and the acceleration of the object measures the “resistance” of an object to changes in its motion. This resistance to changes in motion is termed the *inertia*.

For example, an object with great inertia (quantified by a large proportionality constant) responds to the application of forces with a relatively small acceleration. An object with little inertia (a small proportionality constant) responds to the application of the same forces with a relatively large acceleration. The amount of inertia an object has is measured by the *inertial mass* of the object.

In summary, this relationship, known as Newton’s Second Law of Motion, and can be written mathematically as:

$$\Sigma F = ma$$

where

- F is a force acting on the object from its surroundings, measured in Newtons (N),
- S (sigma) is a shorthand reminder to sum all of the forces acting on the object,
- and m is the mass of the object, measured in kilograms (kg).

The sum of all of the forces acting on an object will be referred to as the *total force* acting on the object.

Newton’s Third Law

Newton’s third great contribution to the study of dynamics was his vision of force, defined to be the interaction between an object and some aspect of its surroundings. Newton theorized that since objects *inter*act with other objects in their surroundings, always in pairs, a certain symmetry exists in nature. The distinction between the actor and the acted-upon is arbitrary. It would be just as easy to switch focus and consider the object in the surroundings as the acted-upon and the original object of interest the actor.

If nature exhibits this symmetry, then the force that one object exerts on another must *always* be equal in magnitude to the force that the second object exerts on the first. To speak of one object as exerting a force on another is to speak of only one-half of the picture. This idea, known as Newton’s Third Law of Motion, is of central importance in the study of forces. In summary, objects *interact* with each other, and equal magnitude forces are exerted on each of the two objects interacting. A simplistic way of picturing this is the idea that you cannot touch something without being touched, and moreover that the harder you touch the harder you will be touched in return.

Investigating the dynamics of a situation involves the identification of all interactions an object experiences with other objects in its surroundings. To help in the identification of these interactions, and to use this information to better describe the ultimate motion of the object, a number of useful analysis tools are detailed below.

Analysis Tools

Drawing Free-Body Diagrams

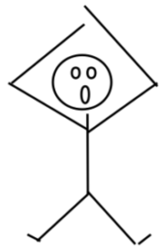
The free-body diagram is by far the most important analysis tool for determining the interactions between an object and its surroundings. There are three distinct steps to creating a free-body diagram. Let’s walk through the steps for the situation described below:

> A child pulls herself up a rope using only her hands.

1. Select the object you would like to study.

In this example, it is probably safe to assume that the object we would like to study is the child. However, depending on what we are investigating it may be the rope or even the ceiling we are interested in. Selecting the correct object to represent by a free-body diagram is a crucial step, especially in more complicated situations. With practice you will develop a knack for selecting the correct object to represent.

2. Draw a picture of the object of interest *free from all other objects*.



Notice that the rope does not appear in the diagram. As the name *free-body* implies, the object is drawn free of all external constraints.

3. Indicate on the diagram all interactions of the object with its environment.

Now comes the most difficult part of constructing a free-body diagram. It is crucial not to miss an interaction. If an interaction is overlooked, than the total of the forces will be incorrect, and the acceleration will be incorrect, and your entire analysis will be incorrect.

Also, only the portion of the interaction that acts *on the girl* should be indicated on a free-body diagram of the girl. For example, she is interacting with the rope. The rope's action *on the girl* will be indicated, not the action of the girl *on the rope*.

To aid in the search for interactions, we will divide the types of interactions that the girl can be part of into two types, non-contact and contact.

- Non-Contact Interactions

  Non-contact interactions include all interactions that can occur between the girl and objects in her surroundings that do not require direct physical contact between the two objects. Non-contact interactions include the interaction of the girl with the gravitational and electromagnetic fields in her vicinity. (How these fields are created and how they can affect the girl will slowly be incorporated into our physics model.)

  At the current level of complexity, however, the only non-contact interaction you need worry about is the interaction of the girl with the gravitational field created by the earth, which we will simply term the force of gravity. The direction of this force is down, toward the center of the earth.

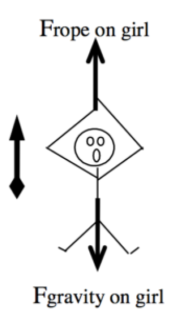
- Contact Interactions

  Contact interactions occur at every point on the girls' body in which she is in direct physical contact with an external object. The most obvious of these is the rope. The girl is in contact with the rope, so the rope and girl exert forces on each other. These forces are equal in magnitude. Remember, however, that it is only the force exerted *on* the girl that is indicated on a free-body diagram of the girl. The location of this force is at the girls' hands, and the direction of this force is up. (The direction of this force cannot be down, because that would imply that the rope is *pushing* the girl, as opposed to *pulling* her. It is impossible for a rope to push someone, unless it is a very stiff rope. Very stiff ropes will be called rods.)

  The only other objects to actually make contact with the girl are air molecules. The air molecules interact with the girl on all sides, each exerting a small force directly inward, perpendicular to the girl's body. Although each of these forces is very small, their sum is not always small. For example, if the girl were falling freely from an airplane the vast numbers of air molecules colliding with the girl from underneath, versus the rather small number colliding from above, and the strength of these collisions, would add to a very large force acting upward on the girl. This force could easily be equal in magnitude to the force of gravity on the girl. The force of air molecules on an object, referred to as air resistance, is often ignored in analyzing scenarios simply because of the difficulty of dealing with the complexity.

  However, in many cases the effects of the air molecules are negligible compared to the other forces acting on the object. This is the case with the girl climbing the rope. The forces exerted by the air molecules are probably very close to being uniformly distributed around the girl's surface. Thus, for every air molecule pushing her to the right, there is probably an air molecule pushing her to the left. These forces will add to a total force very close to zero.

A correct free-body diagram for the girl is shown below:



Since a coordinate system is crucial for translating motion diagrams and free-body diagrams into mathematical relationships, a coordinate system has been added to the free-body diagram. It is always a good idea to use the same coordinate system for both the free-body diagram and the motion diagram.

Calculating the Force of Gravity near the Surface of the Earth

In addition to creating the three laws of motion mentioned earlier, Newton also postulated the Law of Universal Gravitation. This law states that every object of mass in the universe creates a gravitational field, and every object of mass in the universe senses and interacts with every other objects' field. That's an awful lot of forces! To try to identify and estimate the magnitude of all of these forces on an object near the surface of the earth would be a lifelong task.

Luckily, the strength of the gravitational field depends on the mass of the object producing the field, and inversely as the square of the distance from the object. The more massive the object, the stronger the field. The closer the object, the stronger the field. Thus, since the earth is much more massive than any other nearby object, when creating free-body diagrams for objects near the surface of the earth we can safely include just the gravitational field due to the earth, ignoring all the other, relatively small, gravitational fields.

The magnitude of the gravitational field of a massive object (g) depends on the mass of the object (M), the distance from the center of the object (d), and a constant called, appropriately, the gravitational constant (G). The relationship is:

$$g = \frac{GM}{d^2}$$

Near the surface of the earth, the gravitational field has a magnitude of approximately 9.8 N/kg. Although the gravitational field strength varies with the distance from the surface of the earth, we will ignore this slight variation unless explicitly told to include its effects.

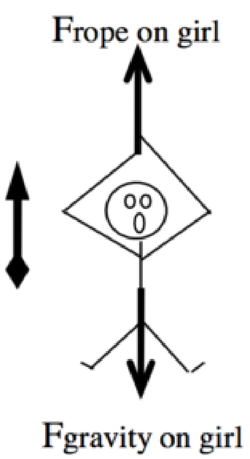
The gravitational force felt by a massive object in the presence of a gravitational field is given by the product of the object's mass and the magnitude of the gravitational field at the location of the object:

$$F_{gravity} = mg$$

Applying Newton's Second Law

Let's return to the scenario under investigation and make some quantitative information more explicit. Then, we can attempt to further investigate the situation using Newton's Second Law.

> A 30 kg child pulls herself up a rope at approximately constant speed using only her hands.



Newton's Second Law states:

$$\Sigma F = ma$$

refers to the sum of all of the forces acting on the girl, the force of the rope (which is positive in our coordinate system) and the force of gravity (which is negative in our coordinate system). Thus,

$$F_{rope} - F_{gravity} = ma$$

Since m = 30 kg, and a = 0 m/s2 (since she climbs at constant speed), the equation becomes:

$$F_{rope} - F_{gravity} = 0$$

By Newton's relationship for the force of gravity:

$$F_{gravity} = mg$$

$$F_{gravity} = (30kg)(9.8\frac{N}{kg})$$

$$F_{gravity} = 294N$$

Therefore:

$$F_{rope} - 294N = 0$$

$$F_{rope} = 294N$$

Thus, Newton's Second Law allows us to determine the force with which the rope pulls on the girl. Of course, by Newton's Third Law, the force with which the girl pulls on the rope is equal in magnitude, so the girl exerts a 294 N force on the rope.

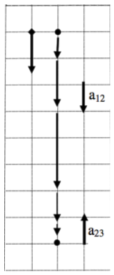
If the girl had not climbed the rope at approximately constant speed her acceleration would have to be determined, either from an explicit mention in the description or through using the kinematic relations developed in the last unit, and then inserted into Newton's Second Law. If her acceleration had been directed upwards (positive) the force of the rope on the girl would have had to be larger. If her acceleration had been directed downwards (negative) the force of the rope on the girl would have had to be smaller.

Analyzing a More Complex Scenario

Before you start analyzing dynamics scenarios on your own, let's work our way through a more complex scenario.

> To practice falling, a 55 kg pole-vaulter falls from rest off of a wall 5.0 m above a foam cushion. The pole-vaulter sinks about 1.8 m into the cushion before stopping.

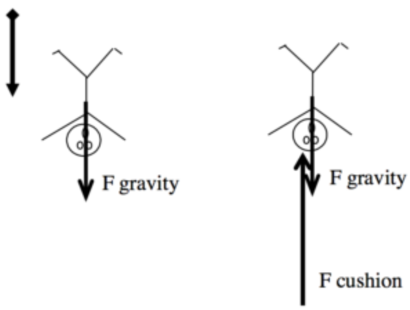
Before we begin analyzing the forces acting on this pole-vaulter, I think we should try to get a handle on the kinematics of the situation. Therefore, our first step in analyzing this situation is to draw a motion diagram and tabulate motion information.

	Event 1: The instant she leaves the wall. t1 = 0 s r1 = 0 m v1 = 0 m/s a12 =	Event 2: The instant she hits the cushion. t2 = r2 = 5.0 m v2 = a23 =	Event 3: The instant she comes to rest. t3 = r3 = 6.8 m v3 = 0 m/s

Notice that between the instant she leaves the wall and the instant she hits the cushion the acceleration is positive (down), while between the instant she hits the cushion and the instant she comes to rest the acceleration is negative (up). Thus, when applying the kinematic relationships and Newton's Second Law we will have to be careful not to confuse variables between these two intervals.

What should jump out at you is the fact that this kinematic scenario cannot be solved! There are *five* unknown kinematic quantities and only *four* kinematic equations. Something else is needed in order to complete the kinematic description. Let's look at the forces acting on the pole-vaulter to see if we can figure out another piece of kinematic information.

Between the first two instants, the only force acting on the pole-vaulter is the force of gravity. Once she hits the cushion, however, there are two forces acting on the pole-vaulter, the force of gravity and the force of the cushion. Correct free-body diagrams for these *two distinct phases of her motion* are given below.



The diagram on the left corresponds to the first time interval and the diagram on the right to the second time interval. For *each* of these free-body diagrams, I will apply Newton's Second Law:

$$\Sigma F = ma_{12} \qquad\qquad \Sigma F = ma_{23}$$

$$+F_{gravity} = (55kg)a_{12} \qquad\qquad +F_{gravity} - F_{cushion} = (55kg)a_{23}$$

Since

$$F_{gravity} = mg$$

$$F_{gravity} = (55kg)(9.8\frac{N}{kg})$$

$$F_{gravity} = 539N$$

$$539N = (55kg)a_{12} \qquad\qquad 539N - F_{cushion} = (55kg)a_{23}$$

$$a_{12} = 9.8m/s^2$$

Thus, from Newton's Second Law, we know that the acceleration *during the fall* is 9.8 m/s2. (We still don't know what the acceleration was during the impact portion of the motion.) Substituting this value back into the motion table yields:

Event 1: The instant she leaves the wall. t1 = 0 s r1 = 0 m v1 = 0 m/s a12 = 9.8 m/s2	Event 2: The instant she hits the cushion. t2 = r2 = +5.0 m v2 = a23 =	Event 3: The instant she comes to rest. t3 = r3 = +6.8 m v3 = 0 m/s

This is now solvable, using strictly kinematics, for the four remaining unknowns. Try to do the math on your own, and compare your result to:

Event 1: The instant she leaves the wall.	Event 2: The instant she hits the cushion.	Event 3: The instant she comes to rest.
t1 = 0 s r1 = 0 m v1 = 0 m/s a12 = 9.8 m/s2	t2 = 1.0 s r2 = +5.0 m v2 = +9.9 m/s a23 = -27 m/s2	t3 = 1.36 s r3 = +6.8 m v3 = 0 m/s

We now have a complete kinematic description of the motion.

Returning to Newton's Second Law for the impact portion of the motion,

$$539N - F_{cushion} = (55kg)a_{23}$$

$$539N - F_{cushion} = (55kg)(-27m/s^2)$$

$$539N - F_{cushion} = -1485N$$

$$F_{cushion} = 2024N$$

The cushion exerts a force of about 2000 N on the pole-vaulter to stop her fall.

Hints and Suggestions

The Magnitude of the Gravitational Field

Quite often, students make a pair of mistakes when dealing with the magnitude of the gravitational field, g.

1. 'g' is never negative.

Since g is the *magnitude* of the gravitational field, it cannot be a negative number. As a magnitude, *it does not have a direction associated with it!* Resist all temptation to replace 'g' with the value "-9.8 N/kg"!

Part of the confusion lies with the fact that the gravitational *field* does have an associated direction. The gravitational field of the earth is directed downward toward the center of the earth. Even so, the gravitational field is *not* negative. Negative only makes sense relative to a coordinate system, and since you are always free to choose any system you want, the gravitational field is just as likely to be oriented in the positive as the negative direction.

2. 'g' is not an acceleration.

Often, students have learned that 'g' is the "acceleration due to gravity." However, as I sit here in a chair writing this book, the force of gravity *is* acting on me and I am most definitely *not* accelerating at 9.8 m/s2! In fact, the force of gravity has acted on me for every second of my life and only very rarely have I accelerated at 9.8 m/s2. 'g' measures the strength of the gravitational field. As such, it is related to the gravitational force, which, like all forces, can give raise to accelerations. However, it is the *total* force acting on an object that determines its acceleration, not simply the force of gravity.

It is true that the units of 'g', N/kg, are also the units of acceleration, since a Newton is defined to be a kg m/s2. It is also true that in a *very specific scenario[1]*, when the only force acting on an object is the force of gravity, the magnitude of the object's acceleration is numerically equal to 'g'. However, there are also very specific scenarios in which the acceleration of an object is numerically equal to 4.576 m/s2, or 62.31452 m/s2. The strength of physics is its ability to analyze diverse scenarios with the same small set of tools, not to develop specialized tools tailored to every different specific scenario. Newton's Second Law will always allow you to determine an object's acceleration, whether the force of gravity acts alone or not.

Newton's Third Law

Many physics students have heard the saying, "For every action there is an equal and opposite re-action." I was forced to memorize this statement in a middle-school science class, and was told it was called Newton's Third Law. I'm sure I had no idea what it really meant. It states that there is a *reaction* to every action, which seems to imply the "action" happens first. This isn't what the law means. There really is no separation or possible distinction between action and reaction. A better way to look at it is that there is an *interaction* between two objects, and the two "sides" of this interaction experience exactly the same force. Of course, the *effect* of this mutually symmetrical force acting on the two objects need not be identical.

As a test of your understanding of Newton's Third Law, try to answer the following question:

> As you drive along the highway, a mosquito splats against your car windshield. During the collision between the mosquito and the car,
>
> a. the force on the mosquito was greater in magnitude than the force on the car.
>
> b. the force on the car was greater in magnitude than the force on the mosquito.
>
> c. the force on the mosquito was equal in magnitude to the force on the car.
>
> d. it is impossible to determine the relative sizes of the forces without more information.

As strange as it may seem, the correct answer is 'c'. The forces exerted on the mosquito and the car are equal in magnitude. In the terminology used in this chapter, the mosquito and car interact (probably an unpleasant interaction for the mosquito), and in an interaction the two agents involved exert equal forces on each other.

However, obviously *something* is different about the interaction from the mosquito’s perspective. What is different is not the force acting on the mosquito but rather its acceleration. Although the forces acting on the mosquito and car are the same, the mosquito’s acceleration is *much greater* than the car’s acceleration because the mosquito’s mass is *much smaller* than the car’s mass. The acceleration of the car is so small that it is not even noticed by the driver, while the acceleration of the mosquito is certainly noticed by the mosquito!

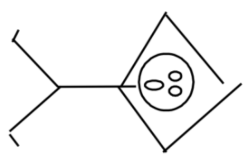
Activities

Construct free-body diagrams for the objects described below.

a. When throwing a ball vertically upward, my hand moves through a distance of about 1.0 m before the ball leaves my hand. The 0.80 kg ball reaches a maximum height of about 20 m above my hand.

while the ball is in my hand	*after the ball leaves my hand*
	

b. To practice falling, a 55 kg pole-vaulter falls off of a wall 6.0 m above a 2.0 m thick foam cushion resting on the ground. However, he misses the cushion.

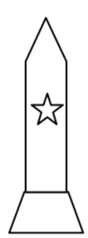
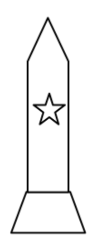
while falling through the air	*while being stopped by the ground*
	

c. A decorative light fixture in an elevator consists of a 2.0 kg light suspended by a cable from the ceiling of the elevator. From this light, a separate cable suspends a second 0.80 kg light.

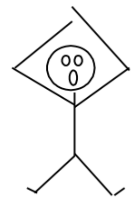
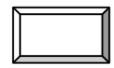
the top light	*the bottom light*
	

Construct free-body diagrams for the objects described below.

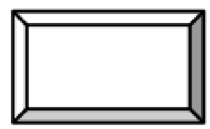
a. A 4000 kg rocket’s engine produces a thrust of 70,000 N for 15 s. The rocket is fired vertically upward.

while the engine is firing	*after the engine turns off*
	

b. Tired of walking up the stairs, an 80 kg engineering student designs an ingenious device for reaching his third floor dorm room. An 84 kg block is attached to a rope that passes over a pulley. The student holds the other end of the rope. When the 84 kg block is released, the student is pulled up to his dorm room.

the student	*the block*
	

c. A 1.0 kg block is stacked on top of a 2.0 kg block on the floor of an elevator moving downward at constant speed.

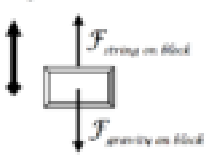
the 1.0 kg block	*the 2.0 kg block*
	

A block hangs from the ceiling of an elevator by a string. For each of the following situations, circle the correct relationship symbol between the magnitude of the force of the string on the block and the magnitude of the force of gravity on the block and explain your reasoning.

a. The elevator is at rest.

F string on block > (=) < ? F gravity on block

Explanation:



Since the block is not accelerating, the two forces acting on it must be equal in magnitude.

b. The elevator is moving upward at an constant speed.

F string on block > (=) < ? F gravity on block

Explanation:
Since the block is still not accelerating, the two forces acting on it must be equal in magnitude.

c. The elevator is moving downward at a decreasing speed.

F string on block (>) = < ? F gravity on block

Explanation:
Since the block is accelerating upward, the force directed upward (the force of the string) must be larger than the force directed downward (the force of gravity).

d. The elevator is moving upward at an increasing speed.

F string on block (>) = < ? F gravity on block

Explanation:
The block is accelerating upward, so the force directed upward must be larger than the force directed downward.

A man stands on a bathroom scale inside of an elevator. For each of the following situations, circle the correct relationship symbol between the magnitude of the force of the scale on the man and the magnitude of the force of gravity on the man and explain your reasoning.

a. The elevator is at rest.

Fscale on man   >   =   <   ?   Fgravity on man

Explanation:

b. The elevator is moving downward at a constant speed.

Fscale on man   >   =   <   ?   Fgravity on man

Explanation:

c. The elevator is moving downward at a increasing speed.

Fscale on man   >   =   <   ?   Fgravity on man

Explanation:

d. The elevator is moving upward at a decreasing speed.

Fscale on man   >   =   <   ?   Fgravity on man

Explanation:

Two blocks are stacked on top of each other on the floor of an elevator. For each of the following situations, circle the correct relationship symbol between the two force magnitudes and explain your reasoning.

a. The elevator is moving downward at a constant speed.

Fbottom block on top block   >   =   <   ?   Ftop block on bottom block

Explanation:

Fbottom block on top block   >   =   <   ?   Fgravity on top block

Explanation:

b. The elevator is moving downward at an increasing speed.

Fbottom block on top block   >   =   <   ?   Ftop block on bottom block

Explanation:

Fbottom block on top block   >   =   <   ?   Fgravity on top block

Explanation:

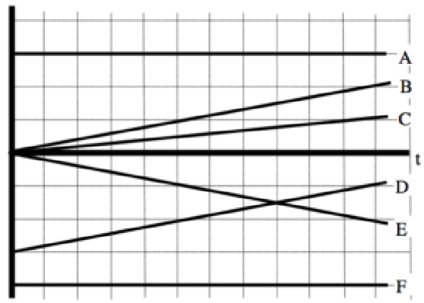
c. The elevator is moving upward at a decreasing speed.

Fbottom block on top block > = < ? Ftop block on bottom block

Explanation:

Fbottom block on top block > = < ? Fgravity on top block

Explanation:



If the graph is of position vs. time, rank these graphs on the basis of the total force acting on the object.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

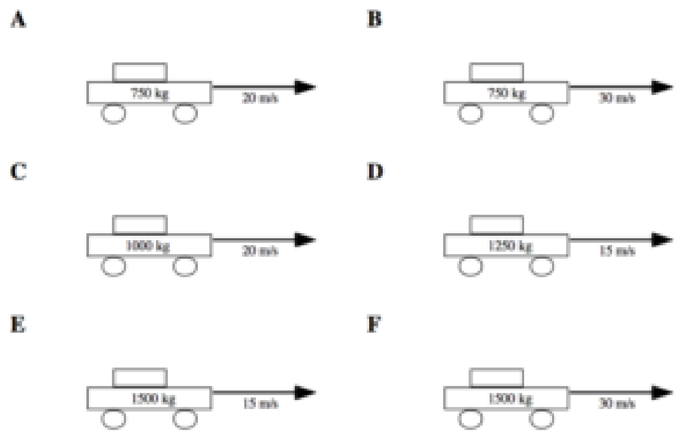
If the graph is of velocity vs. time, rank these graphs on the basis of the total force acting on the object.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are six automobiles traveling at constant velocity. The automobiles have different masses and velocities. Rank these automobiles on the basis of the magnitude of the total force acting on them.

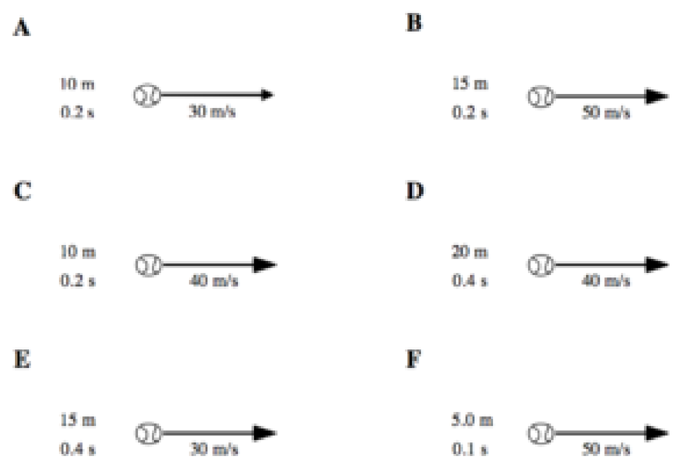


Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are six identical baseballs shortly after being thrown. At the instant shown, the baseball's velocity is indicated, along with the distance the ball has traveled and the elapsed time since leaving the thrower's hand. Rank these baseballs on the basis of the magnitude of the force of the thrower's hand currently acting on them.

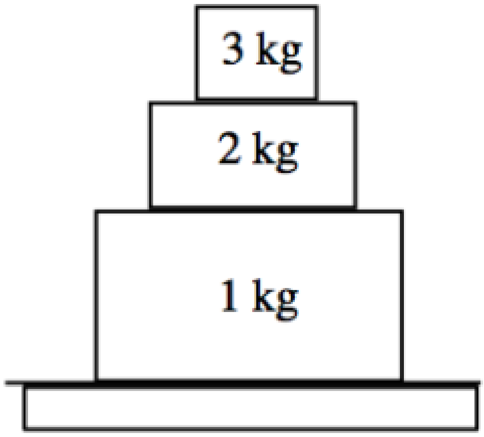


Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are three blocks stacked on top of each other at rest. Rank the magnitude of the forces referred to below from largest to smallest.



A The force of the 3 kg block on the 2 kg block

B The force of the 2 kg block on the 3 kg block

C The force of the 3 kg block on the 1 kg block

D The force of the 1 kg block on the 3 kg block

E The force of the 2 kg block on the 1 kg block

F The force of the 1 kg block on the 2 kg block

G The force of the 1 kg block on the floor

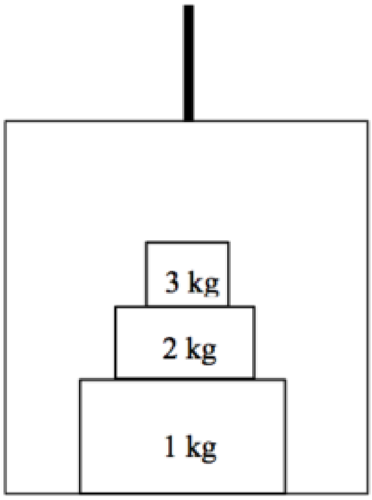
H The force of the floor on the 1 kg block

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ 7. _____ 8. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are three blocks stacked on top of each other inside of an elevator moving upward at increasing speed. Rank the magnitude of the forces referred to below from largest to smallest.



A The force of the 3 kg block on the 2 kg block

B The force of the 2 kg block on the 3 kg block

C The force of the 3 kg block on the 1 kg block

D The force of the 1 kg block on the 3 kg block

E The force of the 2 kg block on the 1 kg block

F The force of the 1 kg block on the 2 kg block

G The force of the 1 kg block on the floor of the elevator

H The force of the floor of the elevator on the 1 kg block

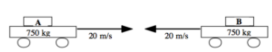
Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ 7. _____ 8. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

For each of the collisions illustrated below, circle the correct relationship symbol between the magnitude of the force of car A on car B and the magnitude of the force of car B on car A and explain your reasoning.

a.

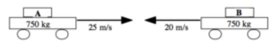


Fcar A on car B > = < ? Fcar B on car A

Explanation:

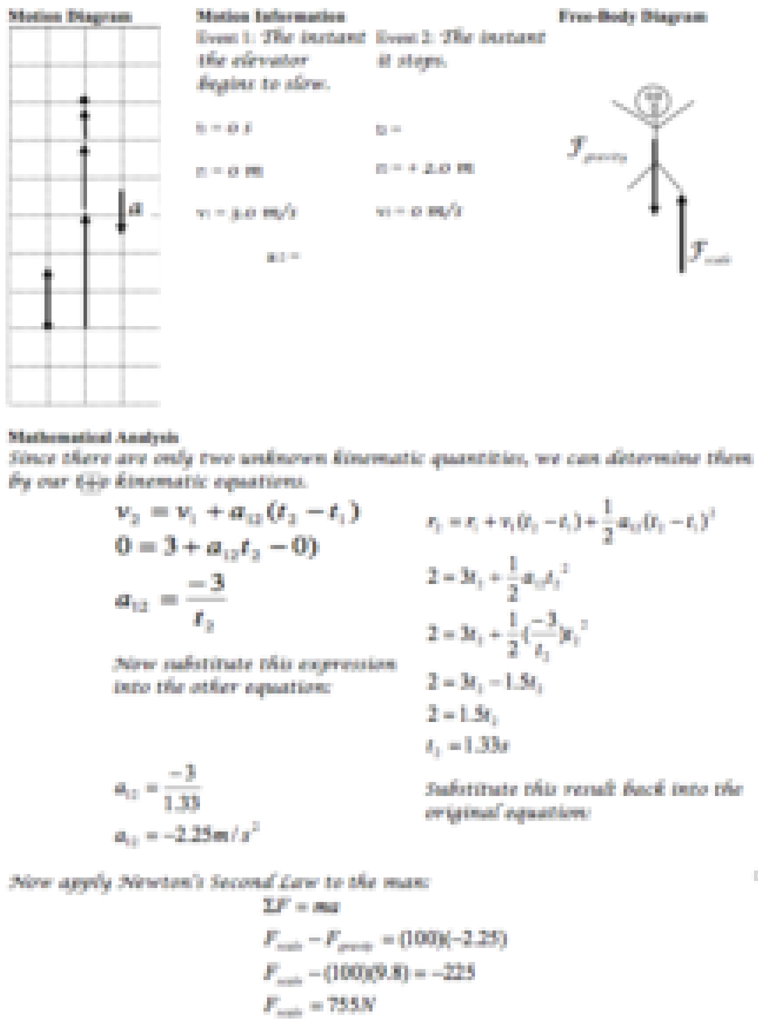
b. 

Fcar A on car B > = < ? Fcar B on car A
Explanation:

c. 

Fcar A on car B > = < ? Fcar B on car A
Explanation:

A 100 kg man concerned about his weight decides to weigh himself in an elevator. He stands on a bathroom scale in an elevator that is moving upward at 3.0 m/s. As the elevator reaches his floor, it slows to a stop over a distance of 2.0 m.



Motion Diagram **Motion Information** **Free-Body Diagram**

Event 1: *The instant the elevator begins to slow.*

Event 2: *The instant it stops.*

$t_1 = 0\ s$ $t_2 =$

$r_1 = 0\ m$ $r_2 = +2.0\ m$

$v_1 = 3.0\ m/s$ $v_2 = 0\ m/s$

$a_{12} =$

Mathematical Analysis

Since there are only two unknown kinematic quantities, we can determine them by our two kinematic equations.

$$v_2 = v_1 + a_{12}(t_2 - t_1)$$
$$0 = 3 + a_{12}(t_2 - 0)$$
$$a_{12} = \frac{-3}{t_2}$$

$$r_2 = r_1 + v_1(t_2 - t_1) + \frac{1}{2}a_{12}(t_2 - t_1)^2$$
$$2 = 3t_2 + \frac{1}{2}a_{12}t_2^2$$
$$2 = 3t_2 + \frac{1}{2}\left(\frac{-3}{t_2}\right)t_2^2$$

Now substitute this expression into the other equation:

$$2 = 3t_2 - 1.5t_2$$
$$2 = 1.5t_2$$
$$t_2 = 1.33s$$

Substitute this result back into the original equation:

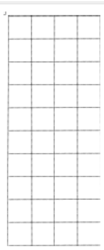
$$a_{12} = \frac{-3}{1.33}$$
$$a_{12} = -2.25m/s^2$$

Now apply Newton's Second Law to the man:

$$\Sigma F = ma$$
$$F_{scale} - F_{gravity} = (100)(-2.25)$$
$$F_{scale} - (100)(9.8) = -225$$
$$F_{scale} = 755N$$

A 40 kg child is hanging from a rope by her hands. She exerts a burst of strength and 2.0 s later is traveling at 1.4 m/s up the rope.

Motion Diagram **Motion Information** **Free-Body Diagram**

	Event 1: t1 = r1 = v1 = a12 =	Event 2: t2 = r2 = v2 =	

Mathematical Analysis[i]

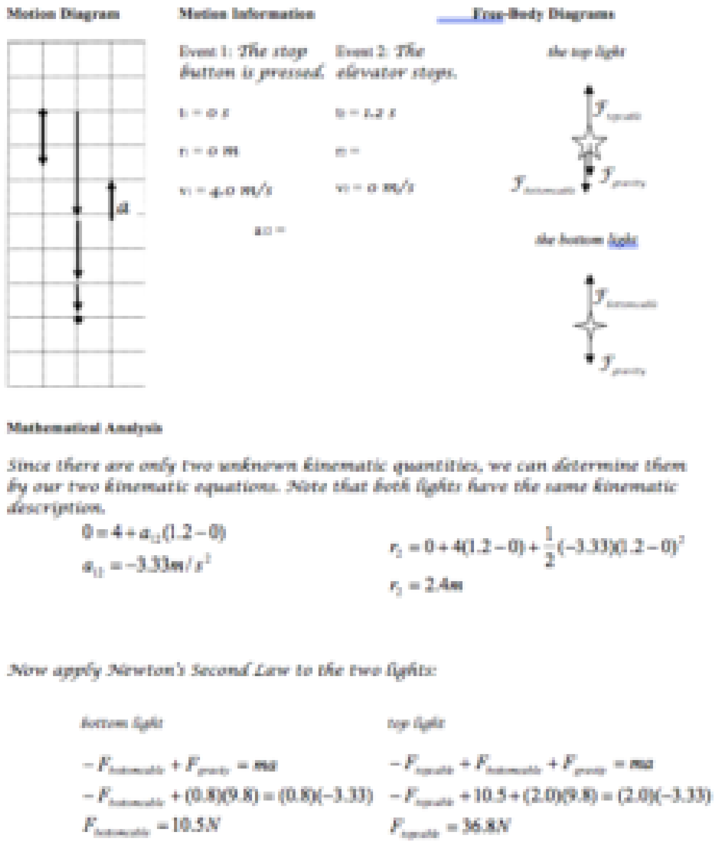
A 55 kg pole-vaulter, just before touching the cushion on which she lands after a jump, is falling downward at a speed of 10 m/s. The pole-vaulter sinks about 2.0 m into the cushion before stopping.

Motion Diagram **Motion Information** **Free-Body Diagram**

	Event 1: t1 = r1 = v1 = a12 =	Event 2: t2 = r2 = v2 =	

Mathematical Analysis[ii]

A decorative light fixture in an elevator consists of a 2.0 kg light suspended by a cable from the ceiling of the elevator. From this light, a separate cable suspends a second 0.80 kg light. The elevator is moving downward at 4.0 m/s when someone presses the emergency stop button. The elevator comes to rest in 1.2 seconds.



Motion Diagram **Motion Information** **Free-Body Diagrams**

Event 1: *The stop button is pressed.* Event 2: *The elevator stops.*

t1 = 0 s t2 = 1.2 s

r1 = 0 m r2 =

v1 = 4.0 m/s v2 = 0 m/s

a12 =

the top light

the bottom light

Mathematical Analysis

Since there are only two unknown kinematic quantities, we can determine them by our two kinematic equations. Note that both lights have the same kinematic description.

$$0 = 4 + a_{12}(1.2 - 0)$$
$$a_{12} = -3.33 m/s^2$$

$$r_2 = 0 + 4(1.2 - 0) + \frac{1}{2}(-3.33)(1.2 - 0)^2$$
$$r_2 = 2.4 m$$

Now apply Newton's Second Law to the two lights:

bottom light

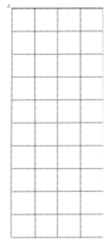
$$-F_{bottom cable} + F_{gravity} = ma$$
$$-F_{bottom cable} + (0.8)(9.8) = (0.8)(-3.33)$$
$$F_{bottom cable} = 10.5 N$$

top light

$$-F_{top cable} + F_{bottom cable} + F_{gravity} = ma$$
$$-F_{top cable} + 10.5 + (2.0)(9.8) = (2.0)(-3.33)$$
$$F_{top cable} = 36.8 N$$

A decorative light fixture in an elevator consists of a 2.0 kg light suspended by a cable from the ceiling of the elevator. From this light, a separate cable suspends a second 0.80 kg light. The elevator is moving downward at 4.0 m/s when someone presses the emergency stop button. During the stop, the upper cable snaps. The elevator engineer says that the cable could withstand a force of 40 N without breaking.

Motion Diagram **Motion Information** **Free-Body Diagrams**

	Event 1: t1 = r1 = v1 = a12 =	Event 2: t2 = r2 = v2 =	*the top light* *the bottom light*

Mathematical Analysis[iii]

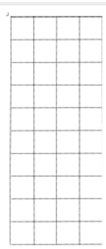
A 70 kg student is 120 m above the ground, moving upward at 3.5 m/s, while hanging from a rope hanging from a 280 kg helium balloon. The lift on the balloon due to the buoyant force is 3000 N.

Motion Diagram **Motion Information** **Free-Body Diagrams**

Mathematical Analysis[iv]	Event 1: t1 = r1 = v1 = a12 =	Event 2: t2 = r2 = v2 =	*student* *balloon*

To practice falling, a 55 kg pole-vaulter falls off of a wall 6.0 m above a foam cushion. The pole-vaulter sinks about 1.4 m into the cushion before stopping.

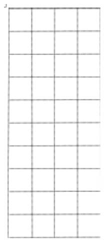
Motion Diagram **Motion Information** **Free-Body Diagrams**

	Event 1: t1 = r1 = v1 = a12 =	Event 2: t2 = r2 = v2 = a23 =	Event 3: t3 = r3 = v3 =	*before hitting cushion* *after hitting cushion*

Mathematical Analysis[v]

To practice falling, a 55 kg pole-vaulter falls off of a wall 6.0 m above a 2.0 m thick foam cushion resting on the ground. However, he misses the cushion. The pole-vaulter sinks about 0.10 m into the ground before stopping.

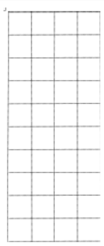
Motion Diagram **Motion Information** **Free-Body Diagrams**

	Event 1: t1 = r1 = v1 = a12 =	Event 2: t2 = r2 = v2 = a23 =	Event 3: t3 = r3 = v3 =	*before hitting cushion* *after hitting ground*

Mathematical Analysis[vi]

When throwing a 0.80 kg ball vertically upward, my hand moves through a distance of about 1.0 m before the ball leaves my hand. The ball leaves my hand at 35 m/s.

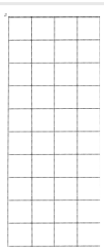
Motion Diagram **Motion Information** **Free-Body Diagrams**

	Event 1: t1 = r1 = v1 = a12 =	Event 2: t2 = r2 = v2 = a23 =	Event 3: Ball reaches its max height t3 = r3 = v3 =	*before ball leaves hand* *after ball leaves hand*

Mathematical Analysis[vii]

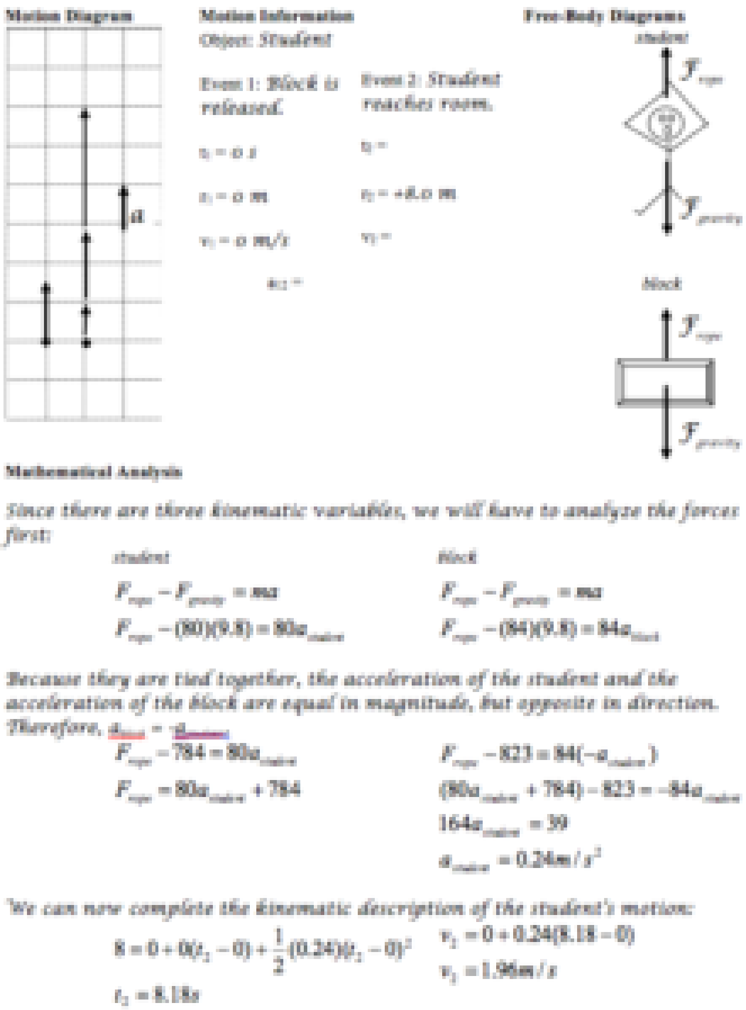
A 4000 kg rocket's engine produces a thrust of 70,000 N for 15 s. The rocket is fired vertically upward.

Motion Diagram **Motion Information** **Free-Body Diagrams**

	Event 1: t1 = r1 = v1 = a12 =	Event 2: t2 = r2 = v2 = a23 =	Event 3: Rocket reaches its max height t3 = r3 = v3 =	*before engine turns off* *after engine turns off*

Mathematical Analysis[viii]

Tired of walking up the stairs, an 80 kg engineering student designs an ingenious device for reaching his third floor dorm room. An 84 kg block is attached to a rope that passes over a pulley. The student holds the other end of the rope. When the 84 kg block is released, the student is pulled up to his dorm room, 8.0 m off the ground.



Motion Diagram

Motion Information

Object: *Student*

Event 1: *Block is released.*

$t_1 = 0$ s

$r_1 = 0$ m

$v_1 = 0$ m/s

$a_{12} =$

Event 2: *Student reaches room.*

$t_2 =$

$r_2 = +8.0$ m

$v_2 =$

Free-Body Diagrams

Mathematical Analysis

Since there are three kinematic variables, we will have to analyze the forces first:

student

$$F_{rope} - F_{gravity} = ma$$
$$F_{rope} - (80)(9.8) = 80a_{student}$$

block

$$F_{rope} - F_{gravity} = ma$$
$$F_{rope} - (84)(9.8) = 84a_{block}$$

Because they are tied together, the acceleration of the student and the acceleration of the block are equal in magnitude, but opposite in direction. Therefore, $a_{block} = -a_{student}$

$$F_{rope} - 784 = 80a_{student}$$
$$F_{rope} = 80a_{student} + 784$$

$$F_{rope} - 823 = 84(-a_{student})$$
$$(80a_{student} + 784) - 823 = -84a_{student}$$
$$164a_{student} = 39$$
$$a_{student} = 0.24 m/s^2$$

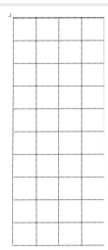
We can now complete the kinematic description of the student's motion:

$$8 = 0 + 0(t_2 - 0) + \frac{1}{2}(0.24)(t_2 - 0)^2$$
$$t_2 = 8.18s$$

$$v_2 = 0 + 0.24(8.18 - 0)$$
$$v_2 = 1.96 m/s$$

Tired of walking up the stairs, an 80 kg engineering student designs an ingenious device for reaching his third floor dorm room. A block is attached to a rope that passes over a pulley. The student holds the other end of the rope. When the block is released, the student is pulled up to his dorm room, 8 m off the ground, in a time of 1.8 s.

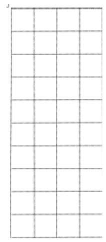
Motion Diagram **Motion Information** **Free-Body Diagrams**

	Object: Event 1: t1 = r1 = v1 = a12 =	Event 2: t2 = r2 = v2 =	*student* *block*

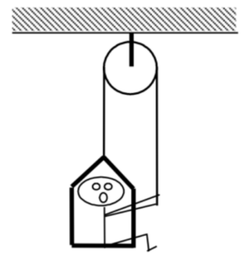
Mathematical Analysis[ix]

Tired of walking down the stairs, an 80 kg engineering student designs an ingenious device for reaching the ground from his third floor dorm room. A block, at rest on the ground, is attached to a rope that passes over a pulley. The student holds the other end of the rope. When the student steps out of the window, she falls the 8 m to the ground in a time of 1.8 s.

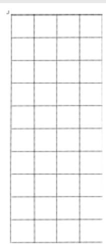
Motion Diagram **Motion Information** **Free-Body Diagrams**

	Object: Event 1: t1 = r1 = v1 = a12 =	Event 2: t2 = r2 = v2 =	*student* *block*

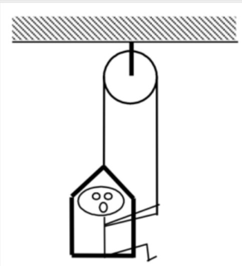
Mathematical Analysis[x]

A 60 kg student lifts herself from rest to a speed of 1.5 m/s in 2.1 s. The chair has a mass of 35 kg.	

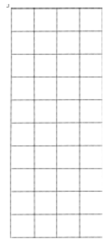
Motion Diagram **Motion Information** **Free-Body Diagrams**

	Object: Event 1: t1 = r1 = v1 = a12 =	Event 2: t2 = r2 = v2 =	*student* *chair*

Mathematical Analysis[xi]

A 60 kg student lowers herself down 40 m at a constant speed of 1.0 m/s. The chair has a mass of 35 kg.	

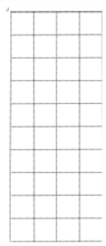
Motion Diagram **Motion Information** **Free-Body Diagrams**

	Object: Event 1: t1 = r1 = v1 = a12 =	Event 2: t2 = r2 = v2 =	*student* *chair*

Mathematical Analysis[xii]

A man of mass m, concerned about his weight, decides to weigh himself in an elevator. He stands on a bathroom scale in an elevator moving upward at speed v. As the elevator reaches his floor, it slows to a stop over a distance, d. Determine the reading on the bathroom scale (Fscale) as a function of m, v, d, and g.

Motion Diagram **Motion Information** **Free-Body Diagram**

	Event 1: t1 = r1 = v1 = a12 =	Event 2: t2 = r2 = v2 =	

Mathematical Analysis

Questions

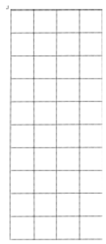
If v = 0 m/s, what should Fscale equal? Does your function agree with this observation?

If d = ∞, what should Fscale equal? Does your function agree with this observation?

For what stopping distance, d, would the bathroom scale read 0 N? Would the scale also read 0 N for this stopping distance if the elevator was initially moving downward?

A falling pole-vaulter of mass m lands on a cushion at speed v. The pole-vaulter sinks a distance d into the cushion before stopping. Determine the force exerted on the pole-vaulter due to the cushion (Fcushion) as a function of m, v, d, and g.

Motion Diagram **Motion Information** **Free-Body Diagram**

	Event 1: t1 = r1 = v1 = a12 =	Event 2: t2 = r2 = v2 =	

Mathematical Analysis

Questions

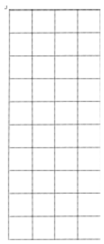
If v = 0 m/s, what should Fcushion equal? Does your function agree with this observation?

If d = 0 m, what should Fcushion equal? Does your function agree with this observation?

What would be worse for the pole-vaulter, hitting the cushion at twice her original speed or sinking half of the original distance into the cushion?

A pole-vaulter of mass m falls off a wall a distance D above a cushion. The pole-vaulter sinks a distance d into the cushion before stopping. Determine the force exerted on the pole-vaulter due to the cushion (Fcushion) as a function of m, D, d, and g.

Motion Diagram **Motion Information** **Free-Body Diagrams**

	Event 1: t1 = r1 = v1 = a12 =	Event 2: t2 = r2 = v2 = a23 =	Event 3: t3 = r3 = v3 =	*before hitting cushion* *after hitting cushion*

Mathematical Analysis

Questions

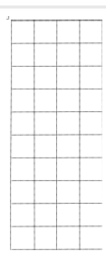
If D = ∞, what should Fcushion equal? Does your function agree with this observation?

If d = 0 m, what should Fcushion equal? Does your function agree with this observation?

What would be worse for the pole-vaulter, starting at twice the initial distance above the cushion or sinking half of the original distance into the cushion?

Tired of walking up the stairs, an engineering student of mass m designs an ingenious device for reaching his third floor dorm room. A block of mass M is attached to a rope that passes over a pulley. The student holds the other end of the rope. When the block is released, the student is pulled up to his dorm room in a time T. Determine the velocity of the student (v) when he reaches his room as a function of m, M, T and g.

Motion Diagram **Motion Information** **Free-Body Diagrams**

	Object: Event 1: t1 = r1 = v1 = a12 =	Event 2: t2 = r2 = v2 =	*student*	*block*

Mathematical Analysis

Questions

If g = 0 m/s2, what should v equal? Does your function agree with this observation?

If m = M, what should v equal? Does your function agree with this observation?

If M = ∞, what should v equal? Does your function agree with this observation?

A rocket of mass m is fired vertically upward from rest. The rocket's engine produces a thrust of constant magnitude Fthrust for tthrust seconds. Determine the maximum height reached by the rocket (H) as a function of Fthrust, tthrust, m, and g.

Motion Information **Free-Body Diagrams**

Event 1: t1 = r1 = v1 = a12 =	Event 2: t2 = r2 = v2 = a23 =	Event 3: t3 = r3 = v3 =	*before engine turns off*	*after engine turns off*

Mathematical Analysis

Questions

If g = 0 m/s2, what should H equal? Does your function agree with this observation?

If Fthrust = mg, what should H equal? Does your function agree with this observation?

A rocket of mass m is fired vertically upward from rest. The rocket's engine produces a thrust of constant magnitude Fthrust for tthrust seconds. Determine the time it takes the rocket to reach its apex (tapex) as a function of Fthrust, tthrust, m, and g.

Motion Information **Free-Body Diagrams**

Event 1: t1 = r1 = v1 = a12 =	Event 2: t2 = r2 = v2 = a23 =	Event 3: t3 = r3 = v3 =	*before engine turns off*	*after engine turns off*

Mathematical Analysis

Questions

If g = 0 m/s2, what should tapex equal? Does your function agree with this observation?

If Fthrust = mg, what should tapex equal? Does your function agree with this observation?

[1] When the only force acting on an object is the force of gravity, the situation is termed *freefall*.

[i] F rope = 420 N

[ii] F cushion = 1910 N

[iii] a ≥ 4.49 m/s2

[iv] t2 = 17.1 s to reach ground

[v] F cushion = 2830 N

[vi] F ground = 43700 N

[vii] r3 = 63.5 m

[viii] r3 = 1550 m

[ix] m block = 240 kg

[x] m block = 26 kg

[xi] F rope = 500 N

[xii] F rope = 466 N

http://www.compadre.org/IVV/vignettes/newtonsFirstLaw.cfm

http://www.compadre.org/IVV/vignettes/newtonsSecondLaw.cfm

http://www.compadre.org/IVV/vignettes/newtonsThirdLaw.cfm

Homework 2 – Model 1: 74, 77, 78, 81, 82, 90, 95, 96, 97, and 102.

1.3: Conservation Laws

Conservation Laws

Concepts and Principles

What is a Conservation Law?

In general, a conservation law is a statement that a certain quantity does not change over time. If you know how much of this quantity you have today, you can be assured that the exact same amount of the quantity will be available tomorrow. A famous (at least to physicists) explanation of the nature of a conservation law was given by Richard Feynman.

> *Imagine your child has a set of 20 wooden blocks. Every day before bedtime you gather up your child's blocks to put them away. As you gather up the blocks, you keep count in your head. Once you reach 20, you know you have found all of the blocks and it is unnecessary for you to search any longer. This is because the number of blocks is conserved. It is the same today as it was yesterday.*
>
> *If one day you only find 18 blocks, you know to keep looking until you find the missing 2 blocks. Also, with experience, you discover the typical hiding places for the blocks. You know to check under the sofa, or behind the curtains.*
>
> *If your child is rambunctious, you may even have to look outside of the room. Perhaps he threw a block or two out of the window. Even though blocks can disappear from inside of the room, and appear out in the yard, if you search everywhere you will always find the 20 blocks.*

Physicists have discovered a number of quantities that behave exactly like the number of wooden blocks. We will examine two of these quantities, energy and momentum, below.

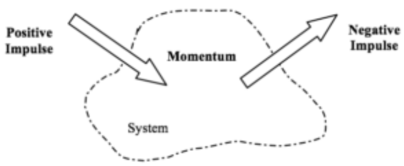
The Impulse-Momentum Relation

While Newton's Second Law directly relates the total force that acts on an object at a specific time to the object's acceleration at that exact same time, conservation laws relate the amount of a certain quantity present at one time to the amount present at a later time.

The first conserved quantity we will investigate is *momentum*. Of course, just because momentum is conserved doesn't mean that the momentum of any particular object or system of objects is always constant. The momentum of a single object, like the number of blocks in the playroom, can change. Just as blocks can be thrown out of the window of the playroom, the momentum of a single object can be changed by applying *impulse* to it. The relationship between impulse and momentum is, conceptually,

$$\textit{initial momentum} + \textit{impulse} = \textit{final momentum}$$

Pictorially, we can visualize this as



In practice, we will identify an object or collection of objects (a *system*) and determine the amount of momentum the system contains at some initial time. This quantity cannot change unless impulse is done to the system. We call processes that bring momentum into the system as positive impulses, and processes that remove momentum from the system as negative impulses.

Mathematically this is written as

$$\textit{initial momentum} + \textit{impulse} = \textit{final momentum}$$

$$P_i + J_{if} = P_f$$

where

- momentum (P) is the product of an object's mass and velocity,
- impulse (J) is the product of a force *external to the system* and the time interval over which it acts,
- and S indicates that you must sum the momentum of all of the objects in the system and all of the impulses acting on the system.

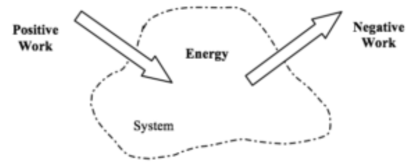
In short, if no impulse is applied to a system, its momentum will remain constant. However, if an impulse is applied to the system, its momentum will change by an amount exactly equal to the impulse applied. This momentum does not appear or disappear without a trace. It is simply transferred to the object *supplying* the impulse. In this sense, impulse is the transfer of momentum into or out of a system, analogous to tossing blocks into or out of a playroom.

The Work-Energy Relation

The second conserved quantity we will investigate is *energy*. Just like momentum, or wooden blocks, the conservation of energy doesn't mean that the energy of any particular object is always constant. The energy of a single object or system of objects can be changed by doing *work* to it. The relationship between work and energy is, conceptually,

$$\textit{initial energy} + \textit{work} = \textit{final energy}$$

Pictorially, we can visualize this as



The similarity between momentum and energy is not complete, however. While there is only one form of momentum (i.e., one hiding place for momentum "blocks") there are several forms of energy. These different forms of energy will be introduced as you progress through more and more complicated models of the physical world. For now, the only "hiding place" I want to discuss is *kinetic energy*. In terms of kinetic energy, the above conceptual relationship between work and energy becomes, expressed mathematically,

$$\textit{initial energy} + \textit{work} = \textit{final energy}$$

$$KE_i + W_{if} = KE_f$$

$$\Sigma \frac{1}{2} m v_i^2 + \Sigma |F| |\Delta r| \cos\phi = \Sigma \frac{1}{2} m v_f^2$$

where

- kinetic energy (KE) is the product of one-half an object's mass and squared velocity,
- work (W) is the product of a force *(even an internal force)* and the displacement over which it acts (with more subtle details discussed below),
- S indicates that you must sum the kinetic energy of all of the objects in the system and all of the work done to the system,
- and we define a new unit, Joule (J), as J = kg $(m/s)^2$ = N m.

Unlike anything we've studied up to this point, the work-energy relation is a *scalar* equation. This will become especially important when we study objects moving in more than one dimension. For now, all this means is that in the expression for work, $|F||\Delta r|\cos\phi$, we should use the *magnitude* of the force and the *magnitude* of the change in position. This product is then multiplied by cos f , where f is defined to be the angle between the applied force and the displacement of the object. If the force and displacement are in the same direction f = 0°, and the work is positive (the object gains energy). If the force and displacement are in the opposite direction f = 180°, and the work is negative (the object loses energy). Note that the actual directions of the force and the displacement are unimportant, only their directions *relative to each other* affect the work.

In general, if no work is done to a system, its kinetic energy will remain constant. However, if work is done to system, its total energy will change by an amount exactly equal to the work done. Work is the transfer of energy from one system to another, again analogous to tossing blocks from the playroom into the yard.

Analysis Tools

Applying the Impulse-Momentum Relation to a Single Object

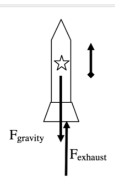
Let's investigate the following scenario:

> A 0.35 kg model rocket is fitted with an engine that produces a thrust of 11.8 N for 1.8 s. The rocket is launched vertically upward.

To apply the impulse-momentum relation, you must clearly specify the initial and final events at which you will tabulate the momentum. For example:

Event 1: The instant the engine is ignited. $P_1 = 0$	Event 2: The instant the rocket reaches maximum height. $P_2 = (0.35)\ v_2$

$J_{12} = +F_{exhaust}(1.8) - F_{gravity}(1.8)$



Note that each external force acting on the rocket is multiplied by the time interval over which it acts. (Also note that the rocket's engine does not produce a force on the rocket! The engine produces a downward force on the hot exhaust gases emitted from the engine and these hot gases exert an equal magnitude force back up on the rocket. That is why the force on the rocket is labeled as Fexhaust rather than Fengine.)

Applying impulse-momentum to the rocket during this time interval yields:

$$P_1 + J_{12} = P_2$$

$$0 + F_{exhaust}(1.8) - F_{gravity}(1.8) = 0.35 v_2$$

$$0 + (11.8)(1.8) - (0.35)(9.8)(1.8) = 0.35 v_2$$

$$v_2 = 43.0\ m/s$$

Thus, the rocket is traveling at 43.0 m/s at the instant the engine shuts off.
Of course, there is no reason why we had to analyze the rocket's motion between the two instants of time we selected above. We could have selected the events:

Event 1: The instant the engine is ignited.	Event 2: The instant the rocket reaches maximum height.
$P_1 =$	$P_2 = 0$

$$J_{12} = +F_{exhaust}(1.8) - F_{gravity}(\Delta t)$$

During this time interval, the force of the exhaust gases only act on the rocket for a *portion* of the entire time interval. Noting that the rocket's velocity when it reaches its maximum height is zero, impulse-momentum would look like this:

$$P_1 + J_{12} = P_2$$
$$0 + F_{exhaust}(1.8) - F_{gravity}(\Delta t) = 0$$
$$0 + (11.8)(1.8) - (0.35)(9.8)(\Delta t) = 0$$
$$\Delta t = 6.19 s$$

Thus, the rocket is in the air for 6.19 s before reaching its maximum height.

Applying the Work-Energy Relation to a Single Object

The work-energy relation also has many uses for investigating physical scenarios. For example, let's look again at our model rocket:

> *A 0.35 kg model rocket is fitted with an engine that produces a thrust of 11.8 N for 1.8 s. The rocket is launched vertically upward.*

Assuming we've already analyzed this scenario using impulse-momentum, what additional information can we extract using work-energy?

Event 1: The instant the engine is ignited.	Event 2: The instant the engine shuts off.
$KE_1 = 0$	$KE_2 = ½ (0.35) (43)^2$

$$W_{12} = F_{exhaust}(\Delta r)\cos 0 + F_{gravity}(\Delta r)\cos 180$$

Therefore,

$$KE_1 + W_{12} = KE_2$$
$$0 + |F_{exhaust}||\Delta r|\cos 0 + |F_{gravity}||\Delta r|\cos 180 = \frac{1}{2}(0.35)$$
$$0 + 11.8(\Delta r)(1) + (0.35)(9.8)(\Delta r)(-1) = 327$$
$$11.8\Delta r - 3.43\Delta r = 327$$
$$\Delta r = 39.1 m$$

What if we apply work-energy between the following two events:Thus, the rocket rises to a height of 39.1 m before the engines shuts off.

Event 1: The instant the engine is ignited.	Event 2: The instant the rocket reaches its maximum height
$KE_1 = 0$	$KE_2 = 0$

$$W_{12} = F_{exhaust}(39)\cos 0 + F_{gravity}(\Delta r)\cos 180$$

During this time interval, the force of the exhaust gases only act on the rocket for a *portion* of the entire displacement, namely 39 m, while the force of gravity acts over the entire displacement.

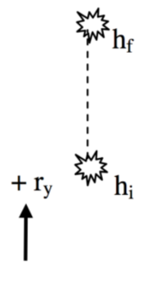
$$KE_1 + W_{12} = KE_2$$
$$0 + (11.8)(39)\cos 0 + (0.35)(9.8)(\Delta r)\cos 180 = 0$$
$$0 + 460 - 3.43\Delta r = 0$$
$$\Delta r = 134 m$$

Thus, the maximum height reached by the rocket is 134 m.

Gravitational Potential Energy

In any situation in which an object changes its height above the surface of the earth, the force of gravity does work on the object. It is possible to calculate this work in general, and to rewrite the work-energy relation in such a way as to incorporate the effects of this work. This is referred to as constructing a *potential energy function* for the work done by gravity.

Let's imagine an object of mass, m, located an initial height, hi, above the zero of a vertical coordinate system, with the upward direction designated positive. It moves to a final height of hf.



To calculate the work done by gravity on this object:

$$W_{gravity} = |F||\Delta r|\cos\phi$$
$$W_{gravity} = (mg)(h_f - h_i)\cos 180$$
$$W_{gravity} = -mgh_f + mgh_i$$

The "mgh" terms are referred to as *gravitational potential energy*. Thus, the work done by gravity can be thought of as changing the gravitational potential energy of the object. Let's plug the above result into the work-energy relation:

$$\frac{1}{2}mv_i^2 + \Sigma|F||\Delta r|\cos\phi = \frac{1}{2}mv_f^2$$
$$\frac{1}{2}mv_i^2 + \Sigma|F||\Delta r|\cos\phi + W_{gravity} = \frac{1}{2}mv_f^2$$
$$\frac{1}{2}mv_i^2 + \Sigma|F||\Delta r|\cos\phi - mgh_f + mgh_i = \frac{1}{2}mv_f^2$$
$$\frac{1}{2}mv_i^2 + mgh_i + \Sigma|F||\Delta r|\cos\phi = \frac{1}{2}mv_f^2 + mgh_f$$

Therefore, this final relation:

$$KE_i + GE_i + W_{if} = KE_f + GE_f$$
$$\frac{1}{2}mv_i^2 + mgh_i + \Sigma|F||\Delta r|\cos\phi = \frac{1}{2}mv_f^2 + mgh_f$$

can (and will) be used in place of the standard work-energy relation provided:

1. You do not include the force of gravity a second time by calculating the work done by gravity. Basically, in this relationship gravity is no longer thought of as a force that does work on objects but rather as a source of potential energy.
2. You calculate the initial and final heights, hi and hf, using a coordinate system in which the upward direction is positive.

Applying Work-Energy with Gravitational Potential Energy

Let's use the work-energy relation, with gravitational potential energy terms, to re-analyze the previous scenario:

> *A 0.35 kg model rocket is fitted with an engine that produces a thrust of 11.8 N for 1.8 s. The rocket is launched vertically upward.*

Let's apply work-energy between the following two events, setting the initial elevation of the rocket equal to zero:

Event 1: The instant the engine is ignited.	Event 2: The instant the rocket reaches its maximum height
$KE_1 = 0$ $GE_1 = 0$	$KE_2 = 0$ $GE_2 = (0.35)(9.8)\ h^2$

$W_{12} = F_{exhaust}(39)\cos 0$

During this time interval, the force of the exhaust gases only act on the rocket for a *portion* of the entire displacement, namely 39 m. Remember, the force of gravity *does not do work* in this way of modeling nature, rather the gravitational energy of the rocket changes as it changes its elevation.

$$KE_i + GE_i + W_{if} = KE_f + GE_f$$
$$0 + 0 + (11.8)(39)\cos 0 = 0 + (0.35)(9.8)h_f$$
$$0 + 0 + 460 = 0 + 3.43h_f$$
$$h_f = 134m$$

results in, of course, the same maximum height reached by the rocket.

Applying the Impulse-Momentum Relation to a Collision

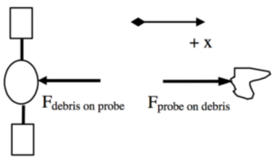
Probably the most useful application of the impulse-momentum relation is in the study of collisions. For example:

> *Far from the earth, a 250 kg space probe, moving at 5 km/s, collides head-on with a 60 kg piece of space debris initially at rest. The debris becomes entangled in the probe's solar collectors.*

Let's choose:

Event 1: The instant before the collision.	Event 2: The instant the debris and probe reach a common velocity.
Object: Space Probe P1 = (250)(5000) $J_{12} = -F_{debrisonprobe}(\Delta t)$	P2 = 250 v2
Object: Debris P1 = 0 $J_{12} = +F_{probeondebris}(\Delta t)$	P2 = 60 v2

The free-body diagrams for the two objects during this time interval are shown below.



Applying the impulse-momentum relation to each object separately yields:

Probe

$$P_1 + J_{12} = P_2$$
$$250(5000) - F_{debrisonprobe}(\Delta t) = 250v_2$$
$$1250000 - F_{debrisonprobe}(\Delta t) = 250v_2$$

Debris

$$P_1 + J_{12} = P_2$$
$$0 + F_{probeondebris}(\Delta t) = 60v_2$$
$$F_{probeondebris}(\Delta t) = 60v_2$$

Notice that the final velocities of the two objects are the same, because they remain joined together following the collision. Also, the Dt's are the same because the time interval over which the force of the debris acts on the probe must be the same as the time interval over which the force of the probe acts on the debris. In fact, these two forces must be equal to each other in magnitude by Newton's Third Law.

Thus, the impulses must cancel if the two equations are added together:

$$1250000 - F_{debrisonprobe}(\Delta t) = 250v_2$$
$$\underline{F_{probeondebris}(\Delta t) = 60v_2}$$
$$1250000 = 310v_2$$
$$v_2 = 4032 m/s$$

The probe slows to a speed of 4032 m/s (and the debris changes direction and accelerates to a speed of 4032 m/s) via the collision. Thus, even though we do not know the magnitude of the force involved, or the duration of the collision, we can calculate the final velocities of the two objects colliding. This is because the forces involved comprise an interaction, and by Newton's Third Law forces that comprise an interaction are always equal in magnitude and opposite in direction.

In fact, in problems involving collisions (or explosions, which to physicists are merely collisions played backward in time!), you should almost always apply the impulse-momentum relation to the interacting objects because the forces involved comprise an interaction. Thus, by adding your equations together, these terms will always add to zero. This will often allow you to determine the final velocities of the colliding objects.

In conclusion, I should point out that the probe loses momentum during the collision and that the debris gains the exact same amount of momentum. (Check the numbers to verify this statement.) The momentum is transferred from the probe to the debris through the action of the impulse the probe and debris exert on each other. The momentum transfer from the probe to the debris is analogous to throwing a wooden block from the playroom into the yard: The playroom now has one less block and the yard has one more!

Applying the Work-Energy Relation to the Same Collision

Let's return to the collision scenario discussed above and attempt to investigate it using work-energy.

> *Far from the earth, a 250 kg space probe, moving at 5 km/s, collides head-on with a 60 kg piece of space debris initially at rest. The debris becomes entangled in the probe's solar collectors.*

Event 1: The instant before the collision.

Event 2: The instant the debris and probe reach a common velocity.

Object: Space Probe

$KE_1 = \frac{1}{2}(250)5000^2$ | $KE_2 = \frac{1}{2}(250)v_2^2$

$GE_1 = 0$ | $GE_2 = 0$

$$W_{12} = F_{on\,P}(\Delta r_P)\cos 180$$

Object: Debris

$KE_1 = 0$ | $KE_2 = \frac{1}{2}(60)v_2^2$

$GE_1 = 0$ | $GE_2 = 0$

$$W_{12} = F_{onD}(\Delta r_D)\cos 0$$

Applying the work-energy relation to each object separately yields:

Probe

$$KE_i + GE_i + W_{if} = KE_f + GE_f$$
$$\frac{1}{2}(250)(5000)^2 + (F_{onP})(\Delta r_{probe})\cos 180 = \frac{1}{2}(250)(v_{2probe})^2$$
$$3.13\text{x}10^9 - F_{onP}(\Delta r_{probe}) = 125 v_{2probe}{}^2$$

Debris

$$KE_i + GE_i + W_{if} = KE_f + GE_f$$
$$0 + (F_{onD})(\Delta r_{debris})\cos 0 = \frac{1}{2}(60)(v_{2debris})^2$$
$$F_{onD}(\Delta r_{debris}) = 30 v_{2debris}{}^2$$

The final velocities of the two objects are the same, because they remain joined together following the collision, and the two forces are the same by Newton's Third Law. **However, these two equations cannot be added together and solved because the two distances over which the forces act, Drprobe and Drdebris, are not necessarily equal.** During the collision, the center of the probe will move a different distance that the center of the debris[1]. Since these two distances are different, the works will *not* cancel as the impulses did, and the equations are *not* solvable!

In fact, since we know v2 = 4032 m/s from our momentum analysis,

Probe

$$3.13\text{x}10^9 - F_{onP}(\Delta r_{probe}) = 125(4032)^2$$
$$3.13\text{x}10^9 - F_{onP}(\Delta r_{probe}) = 2.03\text{x}10^9$$
$$W_{onprobe} = -F_{onP}(\Delta r_{probe}) = -1.1\text{x}10^9\,J$$

Debris

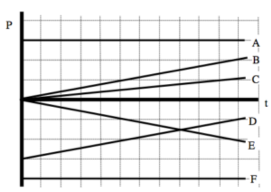
$$F_{onD}(\Delta r_{debris}) = 30(4032)^2$$
$$W_{ondebris} = F_{onD}(\Delta r_{debris}) = 0.49\text{x}10^9\,J$$

Obviously, the two works do not cancel. In fact, the *internal work*, or work done by the objects on each other, totals -0.61 x 109 J. This means that there is 0.61 x 109 J *less* kinetic energy in the system of the probe and the debris after the collision than before the collision. This is sometimes referred to as the energy lost in the collision, although the energy is not lost but rather converted into other forms of energy (i.e., other hiding places for the wooden blocks that have yet to be discussed), such as thermal energy.

In short, the work-energy relation (as it now stands) cannot be used to effectively analyze collisions unless additional information regarding the internal energy is available. Occasionally, an approximation is made in which the total internal work is zero. When this approximation is made, the collision is referred to as an *elastic* collision. Realistic collisions, in which the total internal energy is not zero and kinetic energy is "lost", are referred to as *inelastic* collisions.

Activities

Below are momentum vs. time graphs for six different objects.



a. Rank these graphs on the basis of the change in momentum of the object over the time interval indicated.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

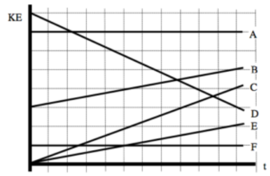
b. Rank these graphs on the basis of the total impulse on the object over the time interval indicated.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are kinetic energy vs. time graphs for six different objects. All of the objects move horizontally.



Rank these graphs on the basis of the change in kinetic energy of the object over the time interval indicated.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

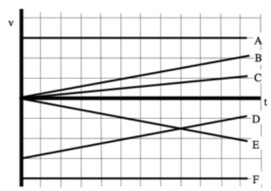
Rank these graphs on the basis of the total work on the object over the time interval indicated.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are velocity vs. time graphs for six equal-mass objects.



Rank these graphs on the basis of the change in momentum of the object over the time interval indicated.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

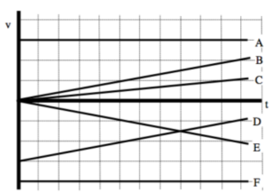
Rank these graphs on the basis of the change in kinetic energy of the object over the time interval indicated.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are velocity vs. time graphs for six equal-mass objects. All of the objects move horizontally.



Rank these graphs on the basis of the total impulse on the object over the time interval indicated.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

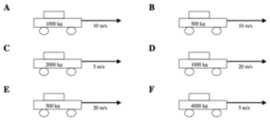
Rank these graphs on the basis of the total work on the object over the time interval indicated.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are six automobiles initially traveling at the indicated velocity. The automobiles have different masses and velocities.



All automobiles will be stopped in the same amount of time. Rank these automobiles on the basis of the magnitude of the force needed to stop them.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

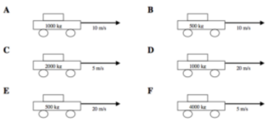
All automobiles will be stopped in the same amount of distance. Rank these automobiles on the basis of the magnitude of the force needed to stop them.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are six automobiles initially traveling at the indicated velocity. The automobiles have different masses and velocities.



Rank these automobiles on the basis of the magnitude of the force needed to stop them.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Rank these automobiles on the basis of the magnitude of the work needed to stop them.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

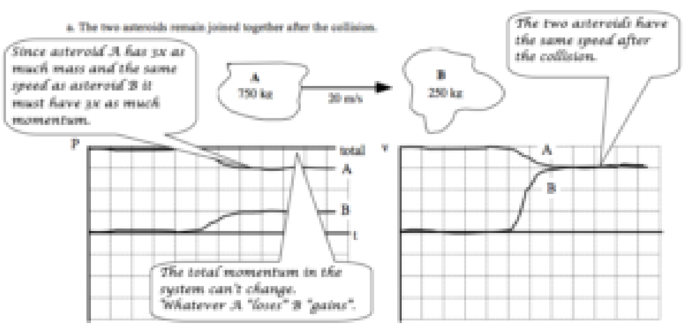
Rank these automobiles on the basis of the magnitude of the impulse needed to stop them.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

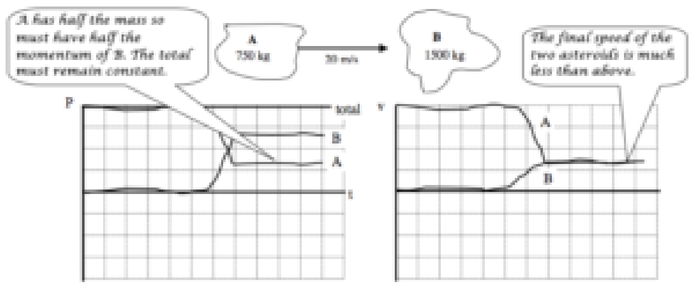
_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

For each of the collisions illustrated below, sketch a graph of the momentum and velocity of asteroid A, the momentum and velocity of asteroid B, and the total momentum in the system of the two asteroids. Begin your graph before the collision takes place and continue it after the collision is over. Use a consistent scale on all graphs.

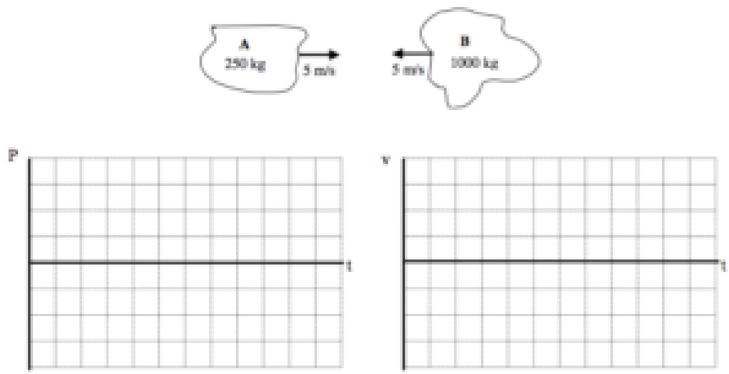


b. The two asteroids remain joined together after the collision.

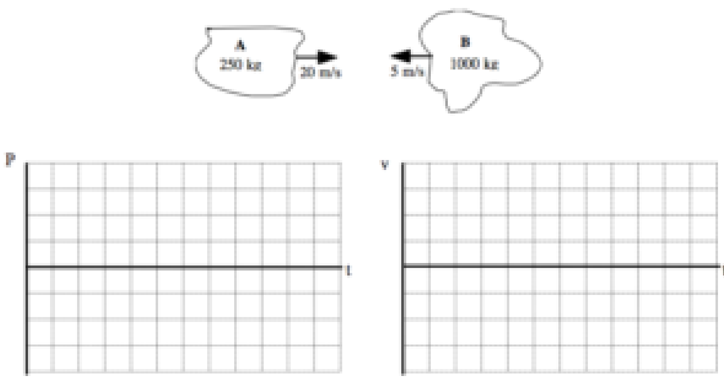


For each of the collisions illustrated below, sketch a graph of the momentum and velocity of asteroid A, the momentum and velocity of asteroid B, and the total momentum in the system of the two asteroids. Begin your graph before the collision takes place and continue it after the collision is over. Use a consistent scale on all graphs.

a. The two asteroids remain joined together after the collision.

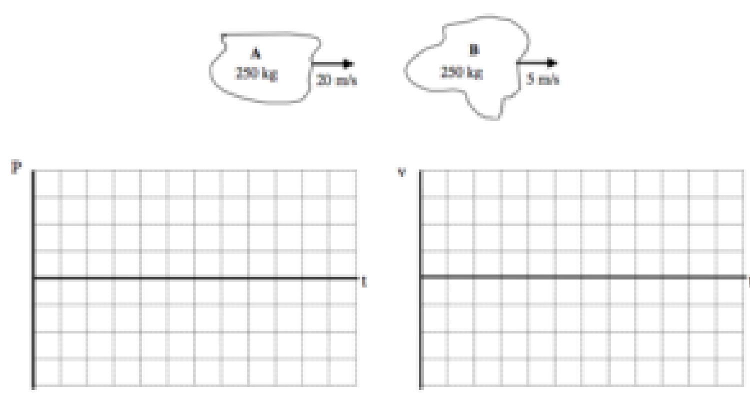


b. The two asteroids remain joined together after the collision.

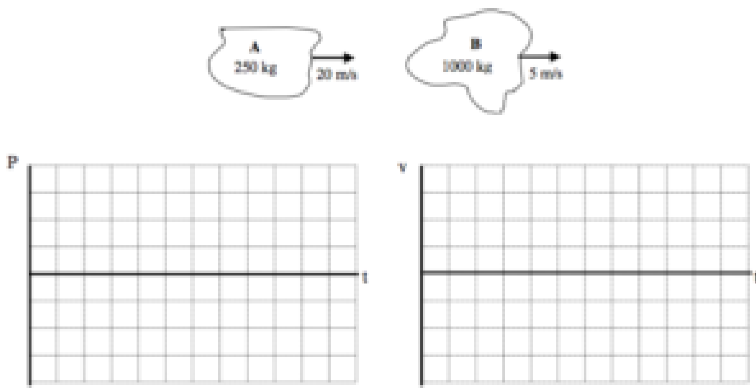


For each of the collisions illustrated below, sketch a graph of the momentum and velocity of asteroid A, the momentum and velocity of asteroid B, and the total momentum in the system of the two asteroids. Begin your graph before the collision takes place and continue it after the collision is over. Use a consistent scale on all graphs.

a. The two asteroids remain joined together after the collision.

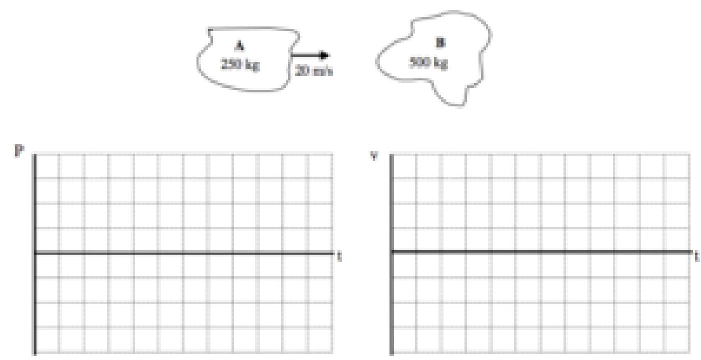


b. The two asteroids remain joined together after the collision.

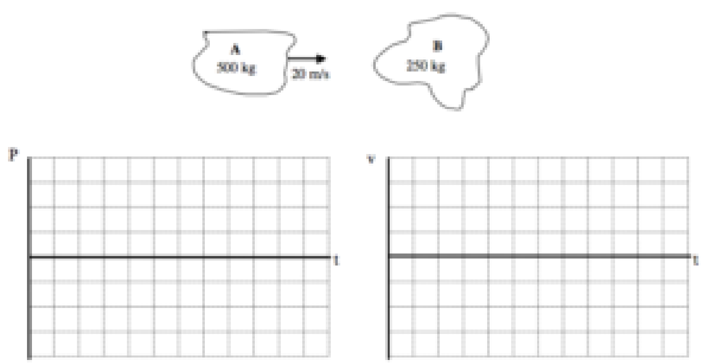


For each of the collisions illustrated below, sketch a graph of the momentum and velocity of asteroid A, the momentum and velocity of asteroid B, and the total momentum in the system of the two asteroids. Begin your graph before the collision takes place and continue it after the collision is over. Use a consistent scale on all graphs.

a. Asteroid A rebounds at 5 m/s after the collision.

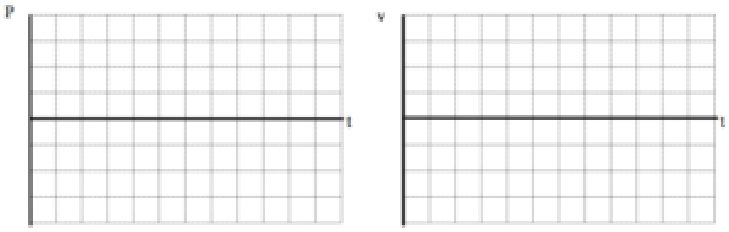


b. Asteroid B moves at 20 m/s after the collision.

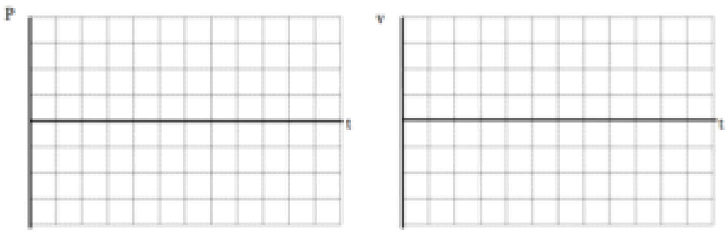


For each of the explosions illustrated below, sketch a graph of the momentum and velocity of fragment A, the momentum and velocity of fragment B, and the total momentum in the system of the two fragments. Begin your graph before the explosion takes place and continue it while the fragments travel away from the sight of the explosion. Use a consistent coordinate system and scale on all graphs.

a. The exploding egg is initially at rest.

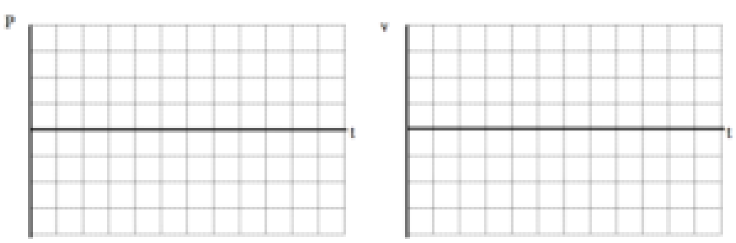


b. The exploding egg is initially at rest.

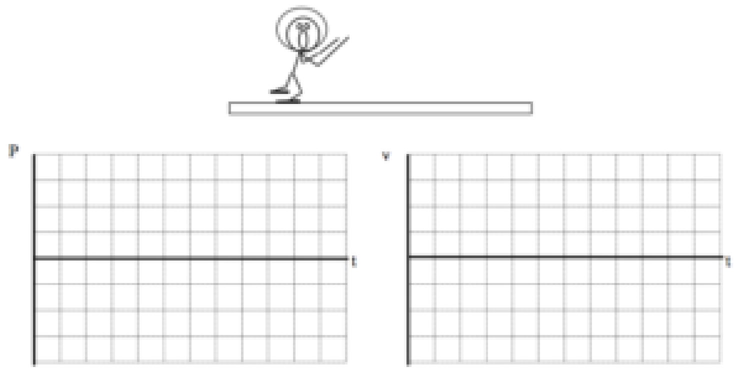


A 200 kg astronaut is initially at rest on the extreme edge of a 1000 kg space platform. She wears special magnetic shoes that allow her to walk along the metal platform. For each of the situations illustrated below, sketch a graph of the momentum and velocity of the astronaut, the momentum and velocity of the platform, and the total momentum in the system of the two objects. Begin your graph before the astronaut begins to walk and continue it while she walks along the platform. Use a consistent coordinate system and scale on all graphs.

a. The astronaut and platform are initially at rest.

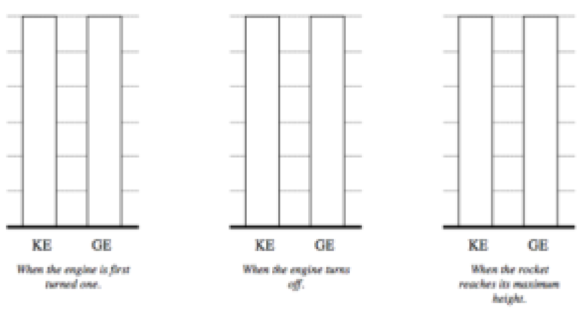


b. The astronaut and platform are initially drifting to the right.

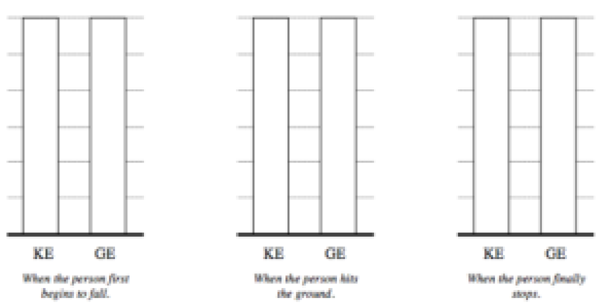


For each of the scenarios described below, indicate the amount of kinetic energy and gravitational potential energy of the object at each of the events listed. Use a consistent scale throughout each motion. Set the lowest point of the motion as the zero-point of gravitational potential energy

a. A 4000 kg rocket's engine produces a thrust of 70,000 N for 15 s. The rocket is fired vertically upward.

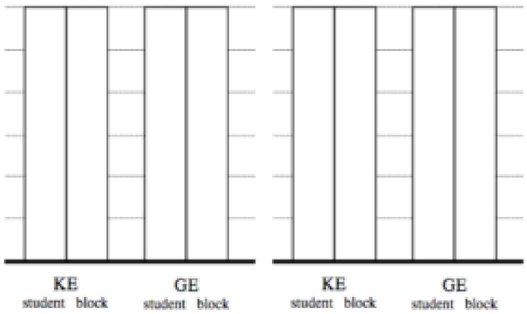


b. To practice falling, a 55 kg pole-vaulter falls off of a wall 6.0 m above a 2.0 m thick foam cushion resting on the ground. However, he misses the cushion. The pole-vaulter sinks about 0.10 m into the ground before stopping.



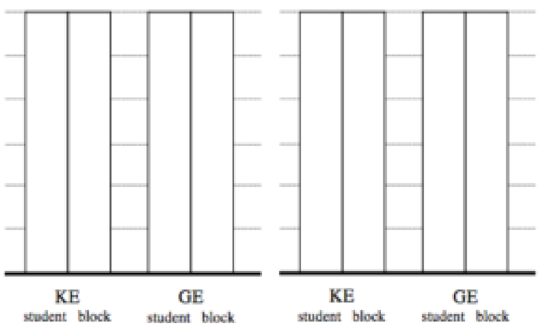
For each of the scenarios described below, indicate the amount of kinetic energy and gravitational potential energy of each object at each of the events listed. Use a consistent scale throughout each motion. Set the lowest point of the motion as the zero-point of gravitational potential energy

a. Tired of walking up the stairs, an 80 kg engineering student designs an ingenious device for reaching his third floor dorm room. An 84 kg block is attached to a rope that passes over a pulley. The student holds the other end of the rope. When the 84 kg block is released, the student is pulled up to his dorm room, 8.0 m off the ground.



When the block is released. When the student reaches his room.

b. Tired of walking down the stairs, a 75 kg engineering student designs an ingenious device for reaching the ground from his third floor dorm room. A 60 kg block at rest on the ground is attached to a rope that passes over a pulley. The student grabs the other end of the rope and steps out of his window.



When the student steps out of the window. When the student reaches the ground.

For the following problems,

A 100 kg man concerned about his weight decides to weigh himself in an elevator. He stands on a bathroom scale in an elevator that is moving upward at 3.0 m/s. As the elevator reaches his floor, it slows to a stop.

a. If the elevator slows to a stop over a distance of 2.0 m, what is the reading on the bathroom scale?

b. If the elevator slows to a stop in 1.5 s, what is the reading on the bathroom scale?

Motion Diagram	**a. Motion Information** Event 1: KE1 = GE1 = W12 =	Event 2: KE2 = GE2 =	**Mathematical Analysis**[i]

Free-Body Diagram	**b. Motion Information** Event 1: P1 = J12 =	Event 2: P2 =	**Mathematical Analysis**

A 70 kg student is 120 m above the ground, moving upward at 3.5 m/s, while hanging from a rope hanging from a 280 kg helium balloon. The lift on the balloon due to the buoyant force is 3000 N.

a. With what speed does the student hit the ground?

b. How long does it take the student to reach the ground?

Motion Diagram	**a. Motion Information** Event 1: KE1 = GE1 = W12 =	Event 2: KE2 = GE2 =	**Mathematical Analysis**[ii]

Free-Body Diagram *student & balloon*	**b. Motion Information** Event 1: P1 = J12 =	Event 2: P2 =	**Mathematical Analysis**

A 4000 kg rocket's engine produces a thrust of 70,000 N for 15 s. The rocket is fired vertically upward.

a. What is the speed of the rocket when its engine turns off?

b. How long does it take the rocket to reach its maximum height?

Motion Diagram	**a. Motion Information** Event 1: P1 = J12 =	Event 2: P2 =	**Mathematical Analysis**[iii]

Free-Body Diagrams *while engine fires* *after engine turns off*	**b. Motion Information** Event 1: P1 = J12 =	Event 2: P2 =	**Mathematical Analysis**

To practice falling, a 55 kg pole-vaulter falls off of a wall 6.0 m above a 2.0 m thick foam cushion resting on the ground. However, he misses the cushion. The pole-vaulter sinks about 0.10 m into the ground before stopping.

a. What is the speed of the pole-vaulter when he hits the ground?

b. What is the force exerted on the pole-vaulter by the ground as he comes to rest?

Motion Diagram	**a. Motion Information** Event 1: KE1 = GE1 = W12 =	Event 2: KE2 = GE2 =	**Mathematical Analysis**[iv]

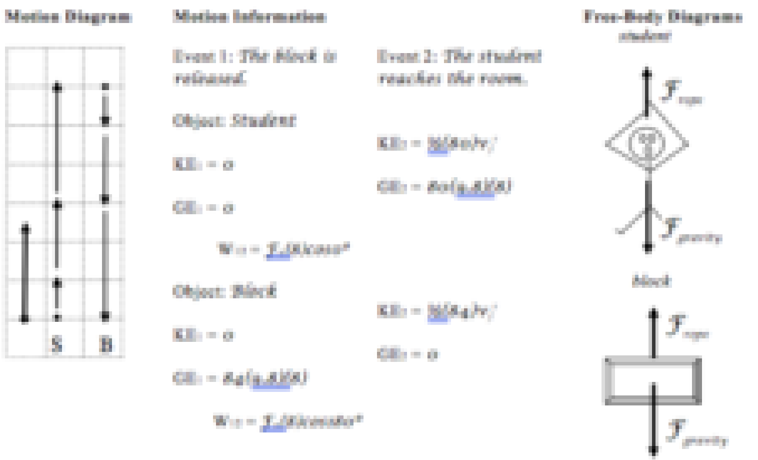
Free-Body Diagrams *while falling* *while dying*	**b. Motion Information** Event 1: KE1 = GE1 = W12 =	Event 2: KE2 = GE2 =	**Mathematical Analysis**

A decorative light fixture in an elevator consists of a 2.0 kg light suspended by a cable from the ceiling of the elevator. From this light, a separate cable suspends a second 0.80 kg light. The elevator is moving downward at 4.0 m/s when someone presses the emergency stop button. During the stop, the upper cable snaps. The elevator engineer says that the cable could withstand a force of 40 N without breaking. Find the maximum time and distance over which the elevator stopped.

Motion Diagram	**Motion Information** Event 1: P1 = J12 = KE1 = GE1 = W12 =	Event 2: P2 = KE2 = GE2 =	**Mathematical Analysis**[v]

Free-Body Diagrams *the two lights*

Tired of walking up the stairs, an 80 kg engineering student designs an ingenious device for reaching his third floor dorm room. An 84 kg block is attached to a rope that passes over a pulley. The student holds the other end of the rope. When the 84 kg block is released, the student is pulled up to his dorm room, 8.0 m off the ground.



Motion Diagram **Motion Information** **Free-Body Diagrams**

Event 1: *The block is released.* Event 2: *The student reaches the room.*

Object: *Student*

KE1 = 0 KE2 = ½(80)v_f^2

GE1 = 0 GE2 = 80(9.8)(8)

W12 = F_{rope}(8)cos0°

Object: *Block*

KE1 = 0 KE2 = ½(84)v_f^2

GE1 = 84(9.8)(8) GE2 = 0

W12 = F_{rope}(8)cos180°

Since the distance the student and block travel is known, applying work-energy should allow us to solve the problem. I'll apply it separately to each object.

student *block*

$$0 + F_{rope}(8) = \frac{1}{2}80v_f^2 + 80(9.8)(8) \qquad 84(9.8)(8) - F_{rope}(8) = \frac{1}{2}84v_f^2$$

$$8F_{rope} = 40v_f^2 + 6272 \qquad 6586 - 8F_{rope} = 42v_f^2$$

F_{rope} is the same in both equations, as is the final speed. Thus the two equations can be added together to yield:

$$6586 = 40v_f^2 + 42v_f^2 + 6272$$

$$314 = 82v_f^2$$

$$v_f = 1.96 m/s$$

Notice that if you applied work-energy to the entire system you would have generated this same equation. Initially, the only form of energy present is the gravitational energy of the block (mgh = 6586 J). At the second event, both objects have kinetic energy plus the student has gravitational potential energy (mgh = 6272 J).

Tired of walking up the stairs, an engineering student designs an ingenious device for reaching his third floor dorm room. A 100 kg block is attached to a rope that passes over a pulley. The student holds the other end of the rope. When the block is released, the student is pulled up to his dorm room, 8.0 m off the ground. He is traveling at 2.2 m/s when he reaches his room.

Motion Diagram	**Motion Information** Event 1: Object: KE1 = GE1 = W12 = Object: KE1 = GE1 = W12 =	Event 2: KE2 = GE2 = KE2 = GE2 =	**Free-Body Diagrams** *student* *block*

Mathematical Analysis[vi]

Tired of walking down the stairs, a 75 kg engineering student designs an ingenious device for reaching the ground from his third floor dorm room. A 60 kg block at rest on the ground is attached to a rope that passes over a pulley. The student grabs the other end of the rope and steps out of his window. He falls for 5.5 s before reaching the ground.

Motion Diagram	**Motion Information** Event 1: Object: P1 = J12 = Object: P1 = J12	Event 2: P2 = P2 =	**Free-Body Diagrams** *student* *block*

Mathematical Analysis[vii]

Tired of walking down the stairs, a 75 kg engineering student designs an ingenious device for reaching the ground from his dorm room. A 60 kg block at rest on the ground is attached to a rope that passes over a pulley. The student grabs the other end of the rope and steps out of his window. He hits the ground at 3.3 m/s.

Motion Diagram	**Motion Information** Event 1: Object: P1 = J12 = KE1 = GE1 = W12 =	Event 2: P2 = KE2 = GE2 =	Object: P1 = J12 = KE1 = GE1 = W12 =	P2 = KE2 = GE2 =

Free-Body Diagrams	**Mathematical Analysis**[viii]
student *block*	

Far from any other masses, a 2000 kg asteroid traveling at 12 m/s collides with a 1200 kg asteroid traveling in the other direction at 16 m/s. After the collision they remain joined together and move with a common velocity.

Motion Diagram	**Free-Body Diagrams** *2000 kg asteroid*	*1200 kg asteroid*

Motion Information Event 1: Object: P1 = J12 = Object: P1 = J12 =	Event 2: P2 = P2 =	**Mathematical Analysis**[ix]

On a remote stretch of intergalactic highway, a 7.5 x 106 kg spaceship traveling at 10 percent the speed of light (0.10c = 3.0 x 107 m/s) doesn't notice the slower spaceship ahead clogging the lane. The fast-moving ship rear-ends the slower ship, an older 5.5 x 106 kg model, and the two ships get entangled and drift forward at 0.07c.

Motion Diagram	**Free-Body Diagrams** *fast ship*	*slow ship*
Motion Information Event 1: Object: P1 = J12 = Object: P1 = J12 =	Event 2: P2 = P2 =	**Mathematical Analysis**[x]

On a remote stretch of intergalactic highway, a 7.5 x 106 kg spaceship traveling at 10 percent the speed of light (0.10c = 3.0 x 107 m/s) doesn't notice the slower spaceship ahead, moving at 0.05c, clogging the lane. The fast-moving ship rear-ends the slower ship, an older 4.5 x 106 kg model, and the slower ship gets propelled forward at 0.13c.

Motion Diagram	**Free-Body Diagrams** *fast ship*	*slow ship*
Motion Information Event 1: Object: P1 = J12 = Object: P1 = J12 =	Event 2: P2 = P2 =	**Mathematical Analysis**[xi]

In the farthest reaches of deep space, an 8000 kg spaceship, including contents, is at rest relative to a space station. The spaceship recoils after it launches a 600 kg scientific probe with a speed of 300 m/s relative to the space station.

Motion Diagram	Free-Body Diagrams *spaceship*	*probe*
Motion Information Event 1: Object: P1 = J12 = Object: P1 = J12 =	Event 2: P2 = P2 =	**Mathematical Analysis**[xii]

In the farthest reaches of deep space, an 8000 kg spaceship, including contents, is drifting at 50 m/s relative to a space station. The spaceship is brought to rest, relative to the space station, by the recoil from launching a 600 kg scientific probe.

Motion Diagram	Free-Body Diagrams *spaceship*	*probe*
Motion Information Event 1: Object: P1 = J12 = Object: P1 = J12 =	Event 2: P2 = P2 =	**Mathematical Analysis**[xiii]

A 140 kg astronaut is standing on the extreme edge of a 1000 kg space platform, at rest relative to the mother ship. She begins to walk toward the other edge of the platform, reaching a speed of 2.0 m/s relative to the mother ship. (She wears special magnetic shoes that allow her to walk along the metal platform.)

Motion Diagram	Free-Body Diagrams *astronaut*	*platform*

Motion Information Event 1: Object: P1 = J12 = Object: P1 = J12 =	Event 2: P2 = P2 =	**Mathematical Analysis**[xiv]

Two astronauts, 140 kg Andy and 170 kg Bob, are standing on opposite edges of a 1000 kg space platform, at rest relative to the mother ship. They each begin to walk toward the opposite ends of the platform, Andy reaching a speed of 2.0 m/s and Bob 1.5 m/s, both relative to the mother ship. (They wear special magnetic shoes that allow them to walk along the metal platform.)

Motion Diagram

Free-Body Diagrams

Andy	*platform*	*Bob*
Motion Information Event 1: Object: P1 = J12 = Object: P1 = J12 = Object: P1 = J12 =	Event 2: P2 = P2 = P2 =	**Mathematical Analysis**[xv]

A 70 kg student is hanging from a 280 kg helium balloon. The balloon is rising at a constant speed of 8.0 m/s relative to the ground. The lift on the balloon due to the buoyant force is constant. The student begins to climb up the rope at a speed of 15 m/s relative to the ground. The balloon's upward speed is decreased as the student climbs.

Motion Diagram	**Motion Information** Event 1: Object: P1 = J12 = Object: P1 = J12 =	Event 2: P2 = P2 =	**Free-Body Diagrams** *student* *balloon* **Mathematical Analysis**

A man of mass m, concerned about his weight, decides to weigh himself in an elevator. He stands on a bathroom scale in an elevator which is moving upward at v. As the elevator reaches his floor, it slows to a stop over a time interval, T. Determine the reading on the bathroom scale (Fscale) as a function of m, v, T, and g.

Motion Diagram	**Motion Information** Event 1: P1 = J12 =	Event 2: P2 =	Free-Body Diagram

Questions

If T = ∞, what should Fscale equal? Does your function agree with this observation?

For what combination of v and T would the bathroom scale read 0 N?

If the elevator were initially going down, would the above combination of v and T also lead to a scale reading of 0 N?

A rocket of mass m is fired vertically upward from rest. The rocket's engine produces a thrust of constant magnitude F for tthrust seconds. Determine the time it takes the rocket to reach its apex (tapex) as a function of F, tthrust, m, and g.

Motion Diagram	**Motion Information** Event 1: P1 = J12 =	Event 2: P2 =	Free-Body Diagram

Questions

If g = 0 m/s2, what should tapex equal? Does your function agree with this observation?

If F = mg, what should tapex equal? Does your function agree with this observation?

For what value of F would tapex = 2tthrust?

To practice falling, a pole-vaulter of mass m falls off of a wall a distance D above a thick foam cushion. The pole-vaulter sinks a distance d into the cushion before stopping. Determine the force exerted on the pole-vaulter due to the cushion (Fcushion) as a function of m, D, d, and g.

Motion Diagram	**Motion Information** Event 1: KE1 = GE1 = W12 =	Event 2: KE2 = GE2 =	**Free-Body Diagrams** *while falling* *while dying*

Questions

If D = ∞, what should Fcushion equal? Does your function agree with this observation?

If d = 0 m, what should Fcushion equal? Does your function agree with this observation?

What would be worse for the pole-vaulter, starting at twice the initial distance above the cushion or sinking half of the original distance into the cushion?

Tired of walking up the stairs, an engineering student of mass m designs an ingenious device for reaching his third floor dorm room. A block of mass M is attached to a rope that passes over a pulley. The student holds the other end of the rope. When the block is released, the student is pulled up to his dorm room in a time T. Determine the velocity of the student (v) when he reaches his room as a function of m, M, T and g.

Motion Diagram **Motion Information** **Free-Body Diagrams**

	Event 1: Object: P1 = J12 = Object: P1 = J12 =	Event 2: P2 = P2 =	*student* *block*

Questions

If g = 0 m/s2, what should v equal? Does your function agree with this observation?

If m = M, what should v equal? Does your function agree with this observation?

If M = ∞, what should v equal? Does your function agree with this observation?

Tired of walking down the stairs, an engineering student of mass m designs an ingenious device for reaching the ground from her dorm room. A block of mass M at rest on the ground is attached to a rope that passes over a pulley. The student grabs the other end of the rope and steps out of her window a distance D above the ground. Determine the velocity of the student (v) when she reaches the ground as a function of m, M, D and g.

Motion Diagram **Motion Information** **Free-Body Diagrams**

	Event 1: Object: KE1 = GE1 = W12 = Object: KE1 = GE1 = W12 =	Event 2: KE2 = GE2 = KE2 = GE2 =	*student* *Block*

Questions

If g = 0 m/s2, what should v equal? Does your function agree with this observation?

If m = M, what should v equal? Does your function agree with this observation?

If M = ∞, what should v equal? Does your function agree with this observation?

In the farthest reaches of deep space, a spaceship of mass M, including contents, is at rest relative to a space station. The spaceship recoils after it launches a scientific probe of mass m at a speed v relative to the space station. Determine the recoil speed of the spaceship (V) as a function of M, m, and v.

Motion Diagram	**Free-Body Diagrams** *spaceship*	*probe*	
Motion Information Event 1: Object: P1 = J12 = Object: P1 = J12 =	Event 2: P2 = P2 =		

Questions

If M = 2m, what should V equal? Does your function agree with this observation?

If M = ∞, what should V equal? Does your function agree with this observation?

[1] If the two objects were *actually* particles, rather than being *approximated* as particles, then the two distances would have to be the same and the two works would cancel when the equation were added together.

[i] a. F scale = 755 N b. F scale = 780 N

[ii] a. v = 17.5 m/s b. t = 17.1 s

[iii] a. v = 116 m/s b. t = 26.8 s

[iv] a. v = 12.5 m/s b. F ground = 43700 N

[v] t2 = 0.89 s r2 = 1.78 m

[vi] m student = 94 kg

[vii] v2 = 6.0 m/s

[viii] Student falls 5.0 m in 3.03 s

[ix] v2 = 1.5 m/s

[x] v1 slowship = 0.029c = 8.73 x 106 m/s

[xi] v2 fastship = 0.052c = 1.56 x 107 m/s

[xii] v2 ship = 24.3 m/s

[xiii] v probe = 667 m/s

[xiv] v2 platform = 0.28 m/s

[xv] v2 platform = 0.025 m/s

[xvi] v2 balloon = 6.3 m/s

Homework 3 – Model 1: 117, 118, 120, 121, 127, 131, 134, 138, 144, and 147.

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1. If the two objects were actually particles, rather than being approximated as particles, then the two distances would have to be the same and the two works would cancel when the equation were added together. ↵

CHAPTER OVERVIEW

2: Model 2

2.1: Kinematics

Kinematics

Concepts and Principles

An empirical fact about nature is that motion in one direction (for example, the horizontal) does not appear to influence aspects of the motion in a perpendicular direction (the vertical). Imagine a coin dropped from shoulder height. The elapsed time for the coin to hit the ground, the rate at which its vertical position is changing, and its vertical acceleration are the same whether you do this experiment in a stationary bus or one traveling down a smooth, level highway at 65 mph. The horizontal motion of the coin does not affect these aspects of its vertical motion.[1]

[1] Actually, at extremely high speeds the horizontal and vertical motions are not independent. At speeds comparable to the speed of light, the interdependence between horizontal and vertical motion (because of time dilation) becomes noticeable.

Thus, to completely describe the motion of an object moving both horizontally and vertically you can first ignore the horizontal motion, and describe only the vertical *component* of the motion, and then ignore the vertical motion, and describe the horizontal component. Putting these kinematic components together gives you a complete description of the motion. This experimental fact about nature will make analyzing multi-dimensional motion no more conceptually difficult than analyzing one-dimensional motion.

Given this independence between motions in perpendicular directions, the same kinematic concepts and relationships utilized in one-dimensional motion will be utilized for multi-dimensional motion.

Position

The position of an object is its location relative to a well-defined coordinate system. In multi-dimensional situations, however, you must designate coordinate systems for all perpendicular directions of interest. The zero and positive direction for one dimension is completely independent of the zero and positive direction for another direction. The location at which all coordinate system zeros intersect is referred to as the origin of the coordinate system.

Velocity

The velocity is the rate at which the position is changing. Thus, we will define the velocity component in the vertical direction, for example, as the rate at which the vertical position is changing. The velocity component in the vertical direction is *completely* independent of the horizontal position or the rate at which the horizontal position changes.

As long as the coordinate directions are perpendicular, the speed, or magnitude of the object's velocity, can be determined by:

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

The direction of the object's velocity can be determined via right-angle trigonometry.

Acceleration

The acceleration is the rate at which the velocity is changing. Thus, we will define the acceleration component in the vertical direction, for example, as the rate at which the velocity component in the vertical direction is changing. The acceleration component in the vertical direction is *completely* independent of the velocity component in the horizontal direction or the rate at which the velocity component in the horizontal direction changes.

As long as the coordinate directions are perpendicular, the magnitude of the object's acceleration can always be determined by:

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

The direction of the object’s acceleration can be determined via right-angle trigonometry

Doing kinematics in multiple dimensions involves a concerted effort on your part to disregard motion in one direction when considering motion in a perpendicular direction. The ability to mentally break down a complicated motion into its component motions requires considerable practice.

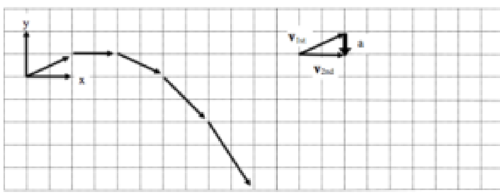
Analysis Tools

Drawing Motion Diagrams

Beginning your analysis by drawing a motion diagram is always the correct first step:

In the shot put, a large mass is thrown at an angle of 22o above horizontal, from a position of 2 m above the ground, a horizontal distance of 25 m.

A motion diagram for this scenario is sketched below.



- Horizontal (x) and vertical (y) coordinate systems are clearly indicated.
- In constructing the motion diagram, only a portion of the entire motion of the shot put is illustrated. For this motion diagram, analysis begins **the instant after the shot put leaves the putter’s hand**, and analysis ends **the instant before the shot put hits the ground**. It is of extreme importance to clearly understand the beginning and the end of the motion that you will describe. The acceleration of the shot put while in the putter’s hand, and the acceleration upon contact with the ground, has been conveniently left out of this analysis. Unless explicit information is either provided or desired about these accelerations, it is best to focus analysis on the simplest portion of the motion, i.e., when it is flying freely through the air.
- The acceleration is determined by the same method as in one-dimensional motion. In this case, the acceleration was determined near the beginning of the motion. Determining the acceleration at any other time will also indicate that its direction is straight downward, since we have focused our analysis on the time interval when the shot put is being acted on by only the force of gravity.

Drawing Motion Graphs

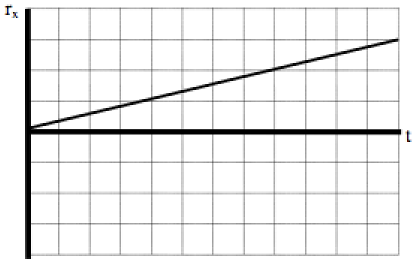
The verbal representation of the situation has already been translated into a pictorial representation, the motion diagram. A careful reading of the motion diagram allows the construction of the motion graphs.

Drawing the position vs. time graph

First, examine the position of the shot put as it moves through the air. Remember, the analysis of the horizontal position must be independent of the analysis of the vertical position.

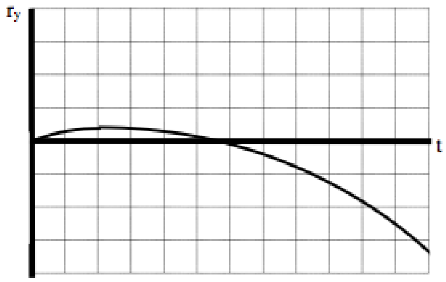
Horizontal Position

From the motion diagram, the shot put starts at position zero, and then has positive, increasing positions throughout the remainder of its motion. The horizontal position increases by even amounts in even time intervals.

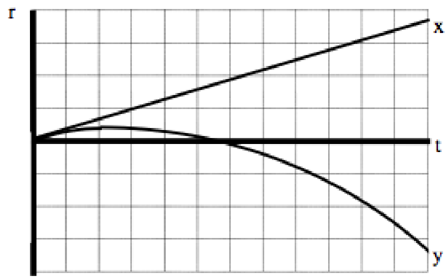


Vertical Position

The shot put starts at position zero, increases its vertical position at a rate that is decreasing, then begins to decrease its vertical position at a rate that is increasing, even as it drops to negative positions.

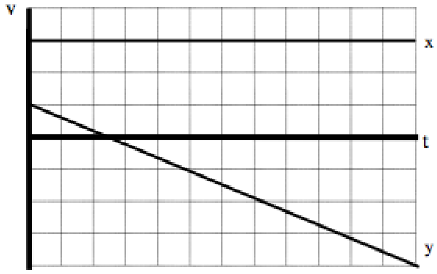


Typically, both the horizontal and vertical positions are displayed on the same axis.



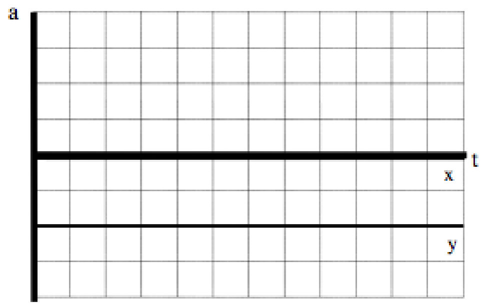
Drawing the velocity vs. time graph

In the horizontal direction, the rate at which the position changes is constant. Hence, the horizontal component of velocity is constant, and positive. In the vertical direction, the velocity component begins positive, decreases to zero, and then increases in the negative direction.



Drawing the acceleration vs. time graph

From the motion diagram, the acceleration of the shot-put can be determined to be directed downward at every point. Thus, the horizontal component of acceleration is zero and the vertical component is negative, and approximately constant due to our model's approximations.



Tabulating Motion Information

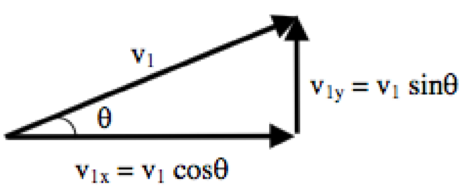
In the shot put, a large mass is thrown at an angle of 22o above horizontal, from a position of 2 m above the ground, a horizontal distance of 25 m.

Now that you have constructed a motion diagram and motion graphs, you should be able to assign numerical values to several of the kinematic variables. A glance at the situation description should indicate that information is presented about the shot put at two distinct events: when the shot put leaves the putter's hand and when the shot put strikes the ground. Other information can also be determined about these events by referencing the motion diagram. To organize this information, you should construct a motion table.

Event 1: The instant after the shot put leaves the hand. t1 = 0 s r1x = 0 m r1y = 0 m v1x = v1 cos 22° v1y = v1 sin 22° a12x = 0 m/s2 a12y = -9.8 m/s2	Event 2: The instant before the shot put hits the ground. t2 = r2x = +25 m r2y = -2 m v2x = v2y =

In addition to the information explicitly given (the initial and final positions), information is available about both the initial velocity and the acceleration.

- **Initial velocity:** Although the magnitude of the initial velocity (v1) is unknown, its orientation in space is known. Thus, via the right-angle trigonometry shown below, the components of this unknown magnitude velocity in the horizontal and vertical directions can be determined. Since we will analyze the x- and y-motion separately, we *must* break the initial velocity into its x- and y-components.



- **Acceleration:** The only force acting on the shot-put *during the time interval of interest* is the force of gravity, which acts directly downward. This is because the analysis of the motion is restricted to the time interval *after* leaving the thrower's hand and *before* striking the ground. Thus, there is no horizontal acceleration of the shot-put and the vertical acceleration has a magnitude of 9.8 m/s2.

Doing the Math

In Model 1, you were presented with two kinematic relationships. These relationships are valid in both the horizontal and vertical directions. Thus, you have a total of four relationships with which to analyze the scenario given. In the example above, there are four unknown kinematic variables. You should remember from algebra that four equations are sufficient to calculate four unknowns. Thus, by applying the kinematic relations in both the horizontal and vertical directions, you should be able to determine the initial velocity of the shot-put, the time in the air, and the final horizontal and vertical velocity components.

First, let's examine the horizontal component of the motion. Note that the positions, velocities, and accelerations in the following equations are all horizontal components.

x-direction

$$v_2 = v_1 + a_{12}(t_2 - t_1)$$
$$v_{2x} = (v_1 \cos 22) + 0(t_2 - 0)$$
$$v_{2x} = 0.927 v_1$$

$$r_2 = r_1 + v_1(t_2 - t_1) + \frac{1}{2}a_{12}(t_2 - t_1)^2$$
$$25 = 0 + (v_1 \cos 22)(t_2 - 0) + \frac{1}{2}(0)(t_2 - 0)^2$$
$$25 = 0.927 v_1 t_2$$
$$v_1 = \frac{27.0}{t_2}$$

Now let's examine the vertical component of the motion. All the positions, velocities, and accelerations in the following equations are now vertical components.

y-direction

$$v_2 = v_1 + a_{12}(t_2 - t_1)$$
$$v_{2y} = (v_1 \sin 22) + (-9.8)(t_2 - 0)$$
$$v_{2y} = 0.375 v_1 - 9.8 t_2$$

$$r_2 = r_1 + v_1(t_2 - t_1) + \frac{1}{2}a_{12}(t_2 - t_1)^2$$
$$-2 = 0 + (v_1 \sin 22)(t_2 - 0) + \frac{1}{2}(-9.8)(t_2 - 0)^2$$
$$-2 = 0.375 v_1 t_2 - 4.9 t_2^2$$

Substituting the value of v_1 from above yields:

$$-2 = 0.375\left(\frac{27.0}{t_2}\right)t_2 - 4.9 t_2^2$$
$$-2 = 10.1 - 4.9 t_2^2$$
$$-12.1 = -4.9 t_2^2$$
$$2.47 = t_2^2$$
$$t_2 = 1.57 s$$

Plugging t2 = 1.57 s into all of the remaining equations gives:

v1 = 17.2 m/s v2x = 15.9 m/s v2y = -8.94 m/s

Hints and Suggestions

Selecting Events

Let's look again at the shot-putter.

> In the shot put, a large mass is thrown at an angle of 22º above horizontal, from a position of 2 m above the ground, a horizontal distance of 25 m.

Imagine a video of the shot put event. Fast-forward over the frames showing the shot putter picking up the shot and stepping into the ring. Begin to watch the imaginary video frame-by-frame as the shot putter begins to push the shot off of her shoulder and forward. Stop the video on the frame when the shot first leaves the putter's hand.

Why is it so important that we begin the analysis at this frame and **explicitly disregard** all the motion that has taken place before this frame? The reason is that in every frame preceding this frame, the shot put was in contact with the putter. Thus, the putter was exerting a force on the shot. Since no information is presented concerning this force, we have no way to determine the acceleration during these frames and hence no way to determine any other kinematic variables. Thus, we disregard all motion preceding the instant the shot leaves the putter's hand because that portion of the motion is simply impossible to analyze with the information provided. Once the shot leaves her hand, the only force acting on the shot is the force of gravity, which greatly simplifies the analysis.

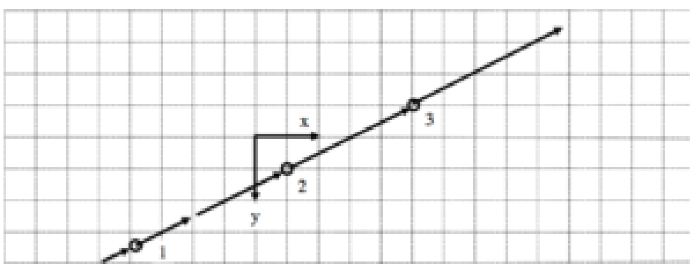
Continue playing the imaginary video forward. Begin playing the tape frame-by-frame as the shot approaches the ground. Stop the video the frame before the shot hits the ground. We will stop our analysis at this frame. Why? Because starting with the next frame, the shot is in contact with the ground. Once in contact with the ground, an additional, unknown magnitude force begins to act on the shot. Once an unknown magnitude force begins to act, the acceleration of the shot becomes unknown and we are stuck. Thus, we conveniently stop our analysis before things get too complicated!

Since our analysis stops the instant before contact, note that the shot is still moving at this instant. (If it wasn't, how could it ever reach the ground?) Thus, resist the temptation to think that the velocity of the shot is zero at the end of analysis. The velocity of the shot is ultimately equal to zero (after it makes a big divot into the ground) but that happens long after it strikes the ground and hence long after our analysis is finished.

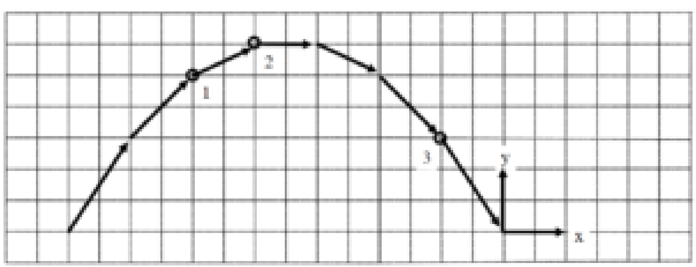
Activities

For each of the motion diagrams below, determine the algebraic sign (+, – or zero) of the x- and y-position, velocity, and acceleration of the object at location of the three open circles.

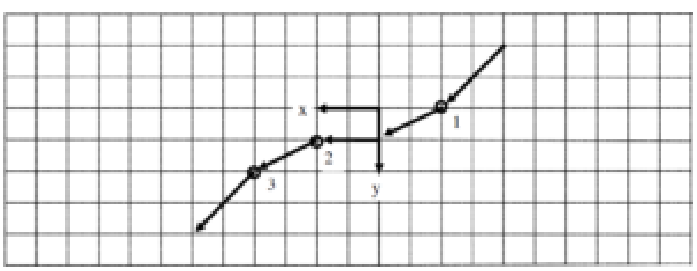
a.



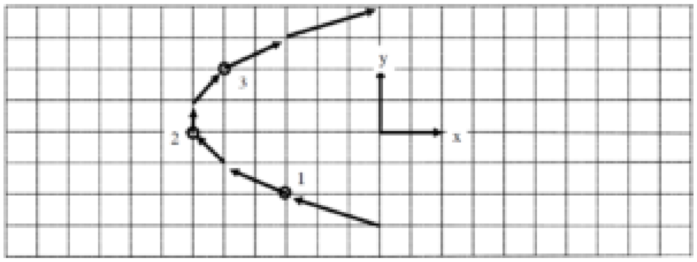
b.

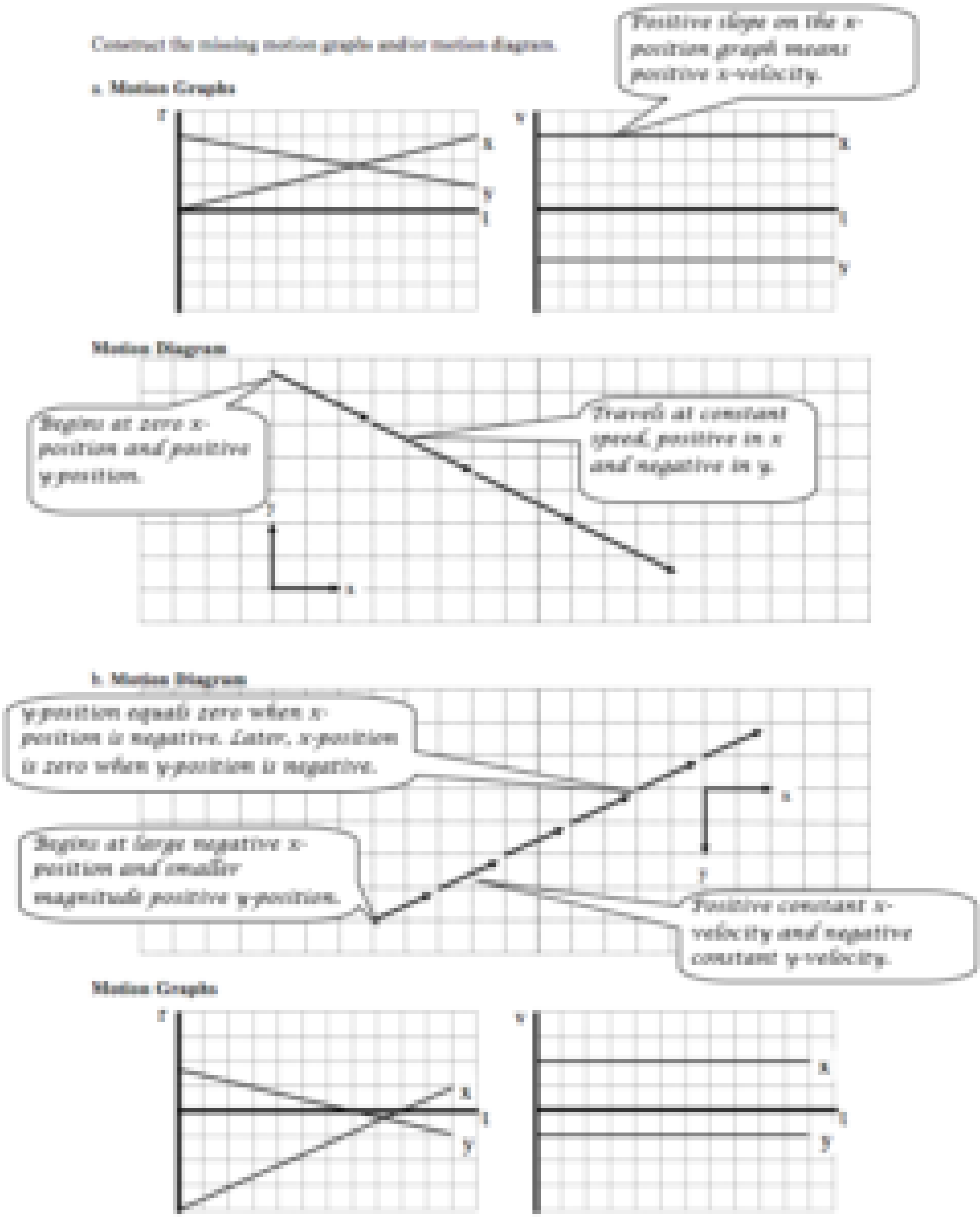


c.



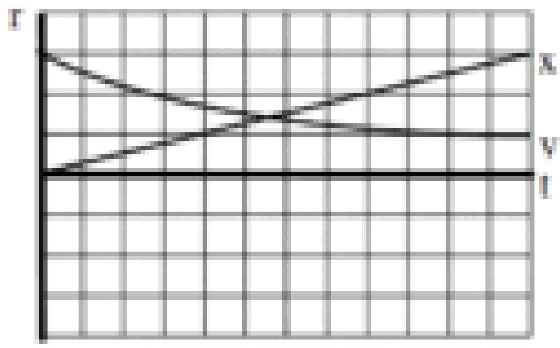
d.

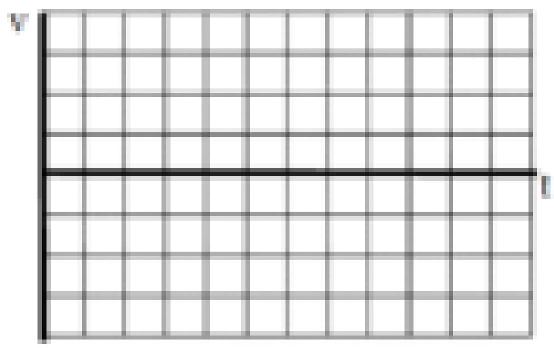

y
x
1
2
3



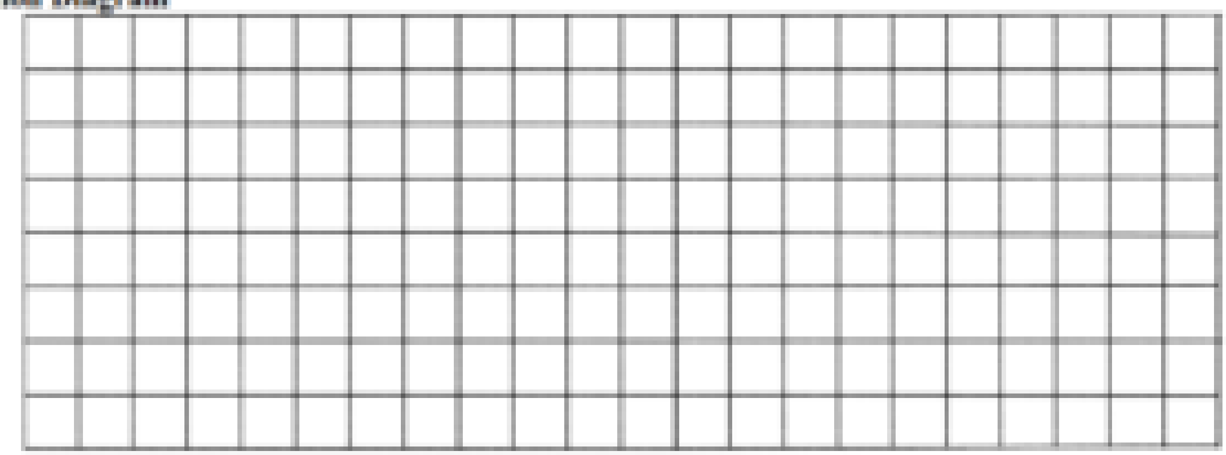
Construct the missing motion graphs and/or motion diagram.

a. **Motion Graphs**

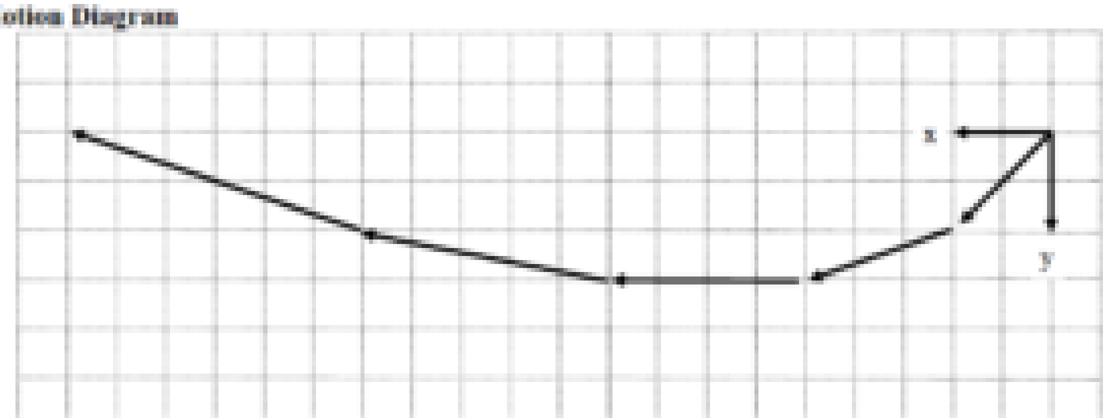




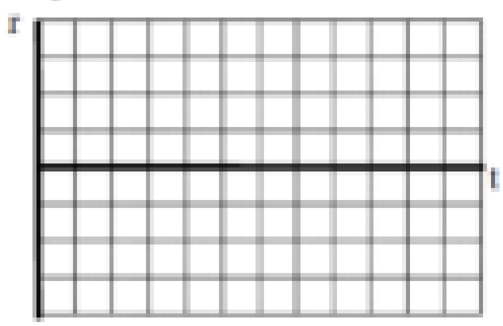
Motion Diagram

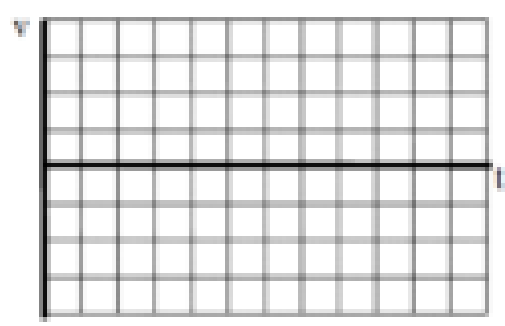


b. **Motion Diagram**

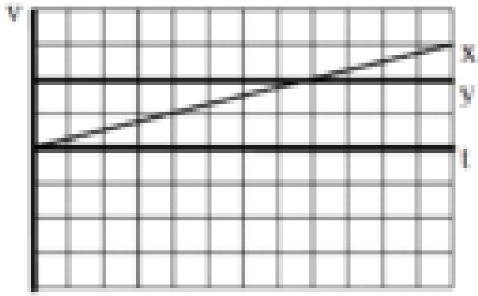


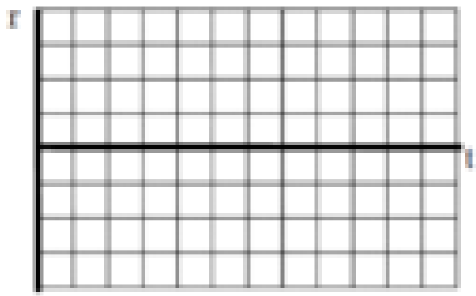
Motion Graphs



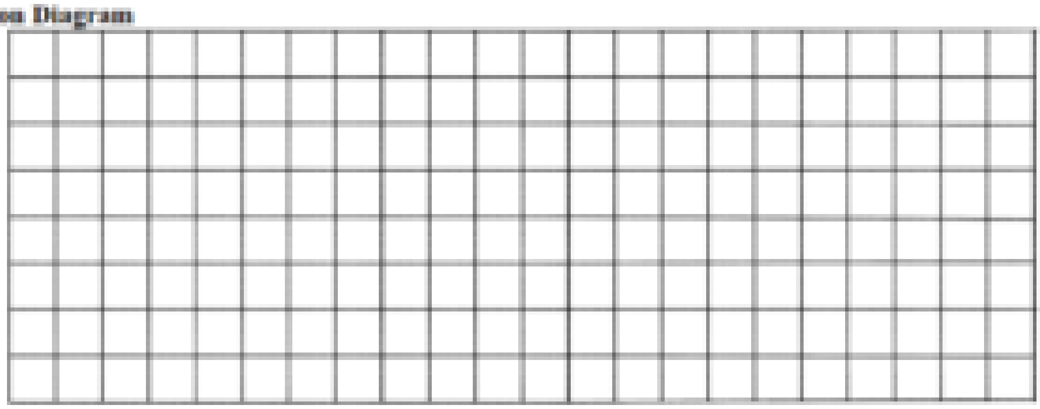


a. Motion Graphs

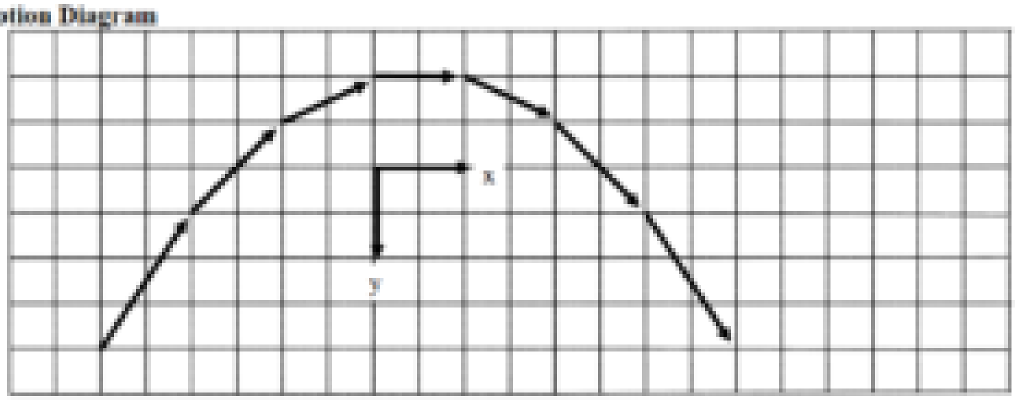




Motion Diagram



b. Motion Diagram



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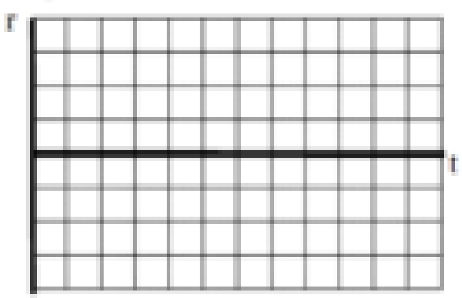

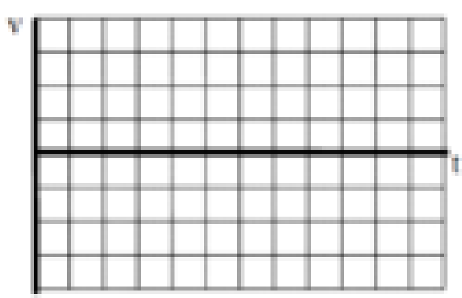

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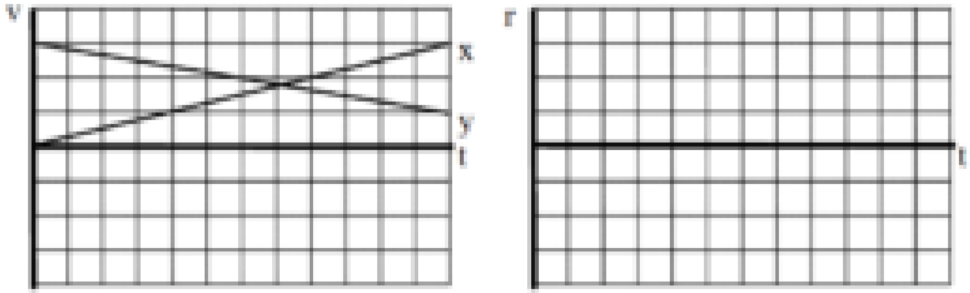
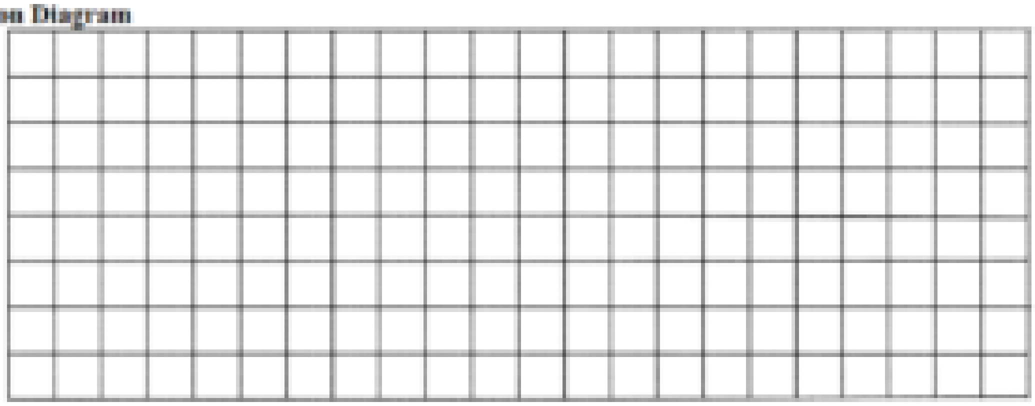


Motion Diagram



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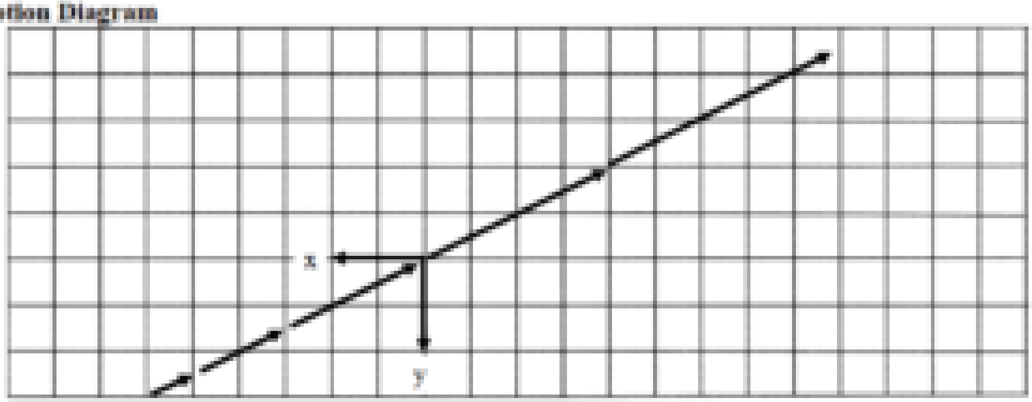
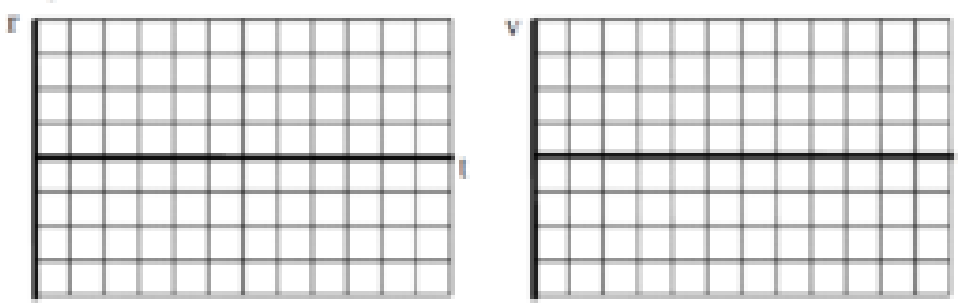
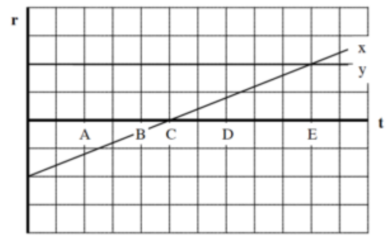


Motion Graphs



An object’s motion is represented by the position vs. time graph below. Both the x- and y-position components are indicated on the graph.



a. Rank the object's distance from the origin at the lettered times.

Largest 1. E 2. A 3. D 4. B 5. C Smallest

_____ The ranking cannot be determined based on the information provided.

Since distance is given by Pythagoras' Theorem, $D = \sqrt{r_x^2 + r_y^2}$, and r_y is constant, the distance from the origin is proportional to the magnitude of the x-position.

b. Rank the object's speed at the lettered times.

Largest 1. ABCDE 2. 3. 4. 5. Smallest

_____ The ranking cannot be determined based on the information provided.

The object moves with constant speed in the positive x-direction.

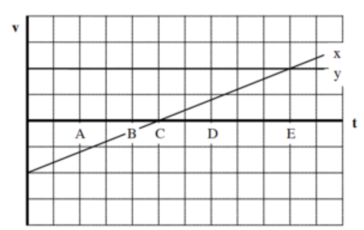
c. Rank the angle between the object's velocity and the x-axis at the lettered times, measuring all angles counterclockwise from +x. (Thus, the +y axis is at 90^0.)

Largest 1. ABCDE 2. 3. 4. 5. Smallest

_____ The ranking cannot be determined based on the information provided.

Since the object moves with constant speed in the positive x-direction, the angle of its velocity vector is 0°

An object's motion is represented by the velocity vs. time graph below. Both the x- and y-velocity components are indicated on the graph.



a. Rank the object's distance from the origin at the lettered times.

Largest 1. 2. 3. 4. 5. Smallest

__X__ The ranking cannot be determined based on the information provided.

Since a velocity graph doesn't specify the location of the coordinate system, you can't determine the distance from the origin of the coordinate system.

b. Rank the object's speed at the lettered times.

Largest 1. E 2. A 3. D 4. B 5. C Smallest

_____ The ranking cannot be determined based on the information provided.

Since speed is given by $v = \sqrt{v_x^2 + v_y^2}$, and v_y is constant, the speed is proportional to the magnitude of the x-position velocity.

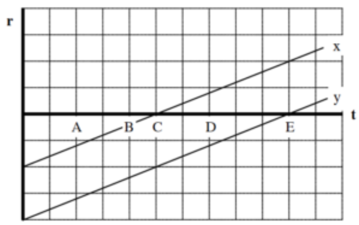
c. Rank the angle between the object's velocity and the x-axis at the lettered times, measuring all angles counterclockwise from +x. (Thus, the +y axis is at 900.)

Largest 1. A 2. B 3. C 4. D 5. E Smallest

_____ The ranking cannot be determined based on the information provided.

Draw a motion diagram! The y-velocity is constant and positive, so all of the vectors are in the first and second quadrant. A and B are at > 90°, C is at 90°, and D and E are at < 90°.

An object's motion is represented by the position vs. time graph below. Both the x- and y-position components are indicated on the graph.



Rank the object's distance from the origin at the lettered times.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Rank the object's speed at the lettered times.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Smallest

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Rank the angle between the object's velocity and the x-axis at the lettered times, measuring all angles counterclockwise from +x. (Thus, the +y axis is at 900.)

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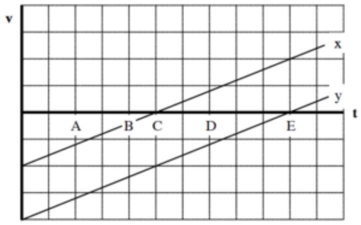

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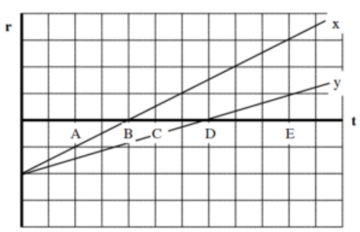

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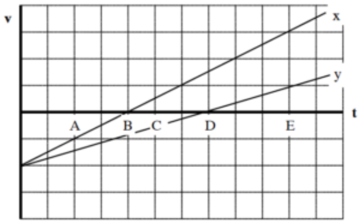

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Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Rank the object's speed at the lettered times.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Smallest

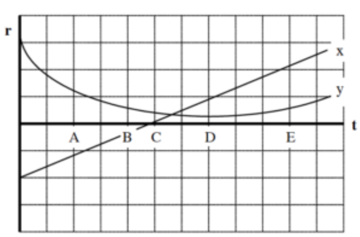
_____ The ranking cannot be determined based on the information provided.

Rank the angle between the object's velocity and the x-axis at the lettered times, measuring all angles counterclockwise from +x. (Thus, the +y axis is at 900.)

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

An object's motion is represented by the position vs. time graph below. Both the x- and y-position components are indicated on the graph.



Rank the object's distance from the origin at the lettered times.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Rank the object's speed at the lettered times.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Smallest

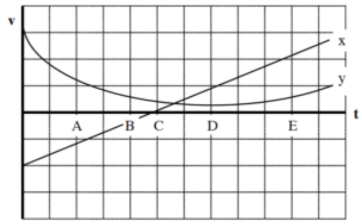
_____ The ranking cannot be determined based on the information provided.

Rank the angle between the object's velocity and the x-axis at the lettered times, measuring all angles counterclockwise from +x. (Thus, the +y axis is at 900.)

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

An object's motion is represented by the velocity vs. time graph below. Both the x- and y-velocity components are indicated on the graph.



Rank the object's distance from the origin at the lettered times.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Rank the object's speed at the lettered times.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Smallest

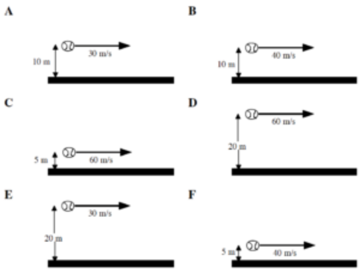
_____ The ranking cannot be determined based on the information provided.

Rank the angle between the object's velocity and the x-axis at the lettered times, measuring all angles counterclockwise from +x. (Thus, the +y axis is at 900.)

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Below are six identical baseballs thrown horizontally at different speeds from different heights above the ground. Assume the effects of air resistance are negligible.



Rank these baseballs on the basis of the elapsed time before they hit the ground.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking.

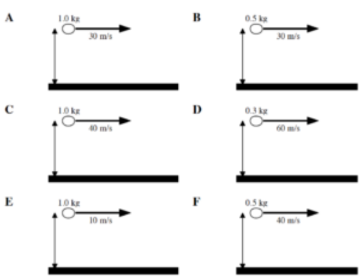
Rank these baseballs on the basis of the magnitude of their vertical velocity when they hit the ground.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking.

Below are six balls of different mass thrown horizontally at different speeds from the same height above the ground. Assume the effects of air resistance are negligible.



Rank these baseballs on the basis of the elapsed time before they hit the ground.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking.

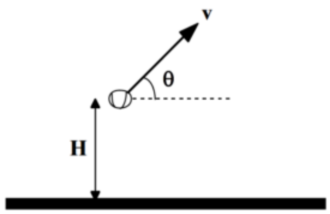
Rank these baseballs on the basis of the horizontal distance traveled before they hit the ground.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking.

Below are six different directions and speeds at which a baseball can be thrown. In all cases the baseball is thrown at the same height, H, above the ground. Assume the effects of air resistance are negligible.



	v	θ
A	30 m/s	30^0
B	45 m/s	0^0
C	30 m/s	60^0
D	15 m/s	60^0
E	20 m/s	45^0
F	15 m/s	90^0

Rank these baseballs on the basis of the maximum height the baseball reaches above the ground.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking.

Rank these baseballs on the basis of the elapsed time before they hit the ground.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking.

At a circus, a human cannonball is shot from a cannon at 15 m/s at an angle of 400 above horizontal. She leaves the cannon 1.0 m off the ground and lands in a net 2.0 m off the ground.

Motion Diagram

Motion Information

Event 1:	Event 2:
t1 = r1x = r1y = v1x = v1y = a12x = a12y =	t2 = r2x = r2y = v2x = v2y =

Mathematical Analysis[i]

x-direction

y-direction

At the buzzer, a basketball player shoots a desperation shot. She is 10 m from the basket and the ball leaves her hands exactly 1.2 m below the rim. She shoots at 350 above the horizontal and the ball goes in!

Motion Diagram

Motion Information

Event 1: t1 = r1x = r1y = v1x = v1y = a12x = a12y =	Event 2: t2 = r2x = r2y = v2x = v2y =

Mathematical Analysis[ii]

x-direction *y-direction*

With 1.0 s left on the clock, a basketball player shoots a desperation shot. The ball leaves her hands exactly 0.9 m below the rim at an angle of 350 above the horizontal and the ball goes in just as the buzzer sounds!

Motion Diagram

Motion Information

Event 1: t1 = r1x = r1y = v1x = v1y = a12x = a12y =	Event 2: t2 = r2x = r2y = v2x = v2y =

Mathematical Analysis[iii]

x-direction *y-direction*

A mountaineer must leap across a 3.0 m wide crevasse. The other side of the crevasse is 4.0 m below the point from which the mountaineer leaps. The mountaineer leaps at 350 above horizontal and successfully makes the jump.

Motion Diagram

Motion Information

Event 1:	Event 2:
t1 = r1x = r1y = v1x = v1y = a12x = a12y =	t2 = r2x = r2y = v2x = v2y =

Mathematical Analysis[iv]

x-direction *y-direction*

The right fielder flawlessly fields the baseball and throws a perfect strike to the catcher who tags out the base runner trying to score. The right fielder is approximately 300 feet (90 m) from home plate and throws the ball at an initial angle of 30° above horizontal. The catcher catches the ball on the fly exactly 1.7 m below the height from which it was thrown.

Motion Diagram

Motion Information

Event 1:	Event 2:
t1 = r1x = r1y = v1x = v1y = a12x = a12y =	t2 = r2x = r2y = v2x = v2y =

Mathematical Analysis[v]

x-direction *y-direction*

The right fielder flawlessly fields the baseball and throws the ball at 94 mph (42 m/s) at an initial angle of 20° above horizontal toward home plate. The fielder is 80 m from the catcher and the ball leaves his hand exactly 1.6 m above the ground.

Motion Diagram

Motion Information

Event 1: t1 = r1x = r1y = v1x = v1y = a12x = a12y =	Event 2: t2 = r2x = r2y = v2x = v2y =

Mathematical Analysis[vi]

x-direction

y-direction

A fire hose, with muzzle velocity of 24 m/s, is used to put out an apartment building fire. The fire is raging inside an apartment 5.0 m above the level of the hose and 10 m, measured horizontally, from the end of the hose. Ignore the effects of air resistance on the water.

Motion Diagram

Motion Information

Event 1: Water leaves hose	*Event 2: Water hits flames*
$t_1 = 0$ s	$t_2 =$
$r_{1x} = 0$ m	$r_{2x} = 10$ m
$r_{1y} = 0$ m	$r_{2y} = 5$ m
$v_{1x} = (24 \text{ m/s}) \cos\theta$	$v_{2x} =$
$v_{1y} = (24 \text{ m/s}) \sin\theta$	$v_{2y} =$

$a_{12x} = 0 \text{ m/s}^2$

$a_{12y} = -9.8 \text{ m/s}^2$

Mathematical Analysis

x-direction

$$r_2 = r_1 + v_1(t_2 - t_1) + \frac{1}{2}a_{12}(t_2 - t_1)^2$$

$$10 = 0 + (24\cos\theta)t_2 + 0$$

$$t_2 = \frac{10}{24\cos\theta}$$

substitute into y-equation:

y-direction

$$r_2 = r_1 + v_1(t_2 - t_1) + \frac{1}{2}a_{12}(t_2 - t_1)^2$$

$$5 = 0 + (24\sin\theta)t_2 + \frac{1}{2}(-9.8)t_2^2$$

$$5 = (24\sin\theta)(\frac{10}{24\cos\theta}) - 4.9(\frac{10}{24\cos\theta})^2$$

$$5 = 10\tan\theta - \frac{0.851}{\cos^2\theta}$$

$$0 = 10\tan\theta - \frac{0.851}{\cos^2\theta} - 5$$

This equation can be solved by using a "solver" program, by knowing a few trig identities, or, most conveniently, by graphing the righthand side of the equation and finding where it crosses zero. The solution is $\theta = 32°$.

Therefore, $t_2 = 0.49$ s, and

$$v_2 = v_1 + a_{12}(t_2 - t_1)$$

$$v_{2x} = 24\cos 32 + 0$$

$$v_{2x} = 20.4 m/s$$

$$v_2 = v_1 + a_{12}(t_2 - t_1)$$

$$v_{2y} = 24\sin 32 - 9.8(0.49)$$

$$v_{2y} = 7.92 m/s$$

A mountaineer must leap across a 3.0 m wide crevasse. The other side of the crevasse is 4.0 m below the point from which the mountaineer leaps. The mountaineer leaps at a speed of 3.5 m/s and barely makes the jump.

Motion Diagram

Motion Information

Event 1:	Event 2:
t1 = r1x = r1y = v1x = v1y = a12x = a12y =	t2 = r2x = r2y = v2x = v2y =

Mathematical Analysis[vii]

x-direction

y-direction

The right fielder flawlessly fields the baseball and must throw a perfect strike to the catcher, 90 m away, to tag out the base runner trying to score. The right fielder knows she can throw a baseball at 80 mph (36 m/s) and calculates the proper angle at which to throw so that the catcher will catch the ball on the fly exactly 1.8 m below the height from which it was thrown. However, her calculation is so time-consuming that the ball arrives too late and the runner scores.

Motion Diagram

Motion Information

Event 1:	Event 2:
t1 = r1x = r1y = v1x = v1y = a12x = a12y =	t2 = r2x = r2y = v2x = v2y =

Mathematical Analysis[viii]

x-direction

y-direction

At the buzzer, a basketball player shoots a desperation shot. She is 14 m from the basket and the ball leaves her hands exactly 1.4 m below the rim. She throws the ball at 18 m/s. Can she make the shot?

Motion Diagram

Motion Information

Event 1:	Event 2:
t1 = r1x = r1y = v1x = v1y = a12x = a12y =	t2 = r2x = r2y = v2x = v2y =

Mathematical Analysis[ix]

x-direction *y-direction*

At a circus, a human cannonball will be shot from a cannon at 15 m/s. She will leave the cannon 1.0 m off the ground and hopefully land in a net 3.0 m off the ground, after flying a horizontal distance of 22 m. Do you want this job?

Motion Diagram

Motion Information

Event 1:	Event 2:
t1 = r1x = r1y = v1x = v1y = a12x = a12y =	t2 = r2x = r2y = v2x = v2y =

Mathematical Analysis[x]

x-direction *y-direction*

At a circus, a human cannonball will be shot from a cannon at 24 m/s. She will leave the cannon 1.0 m off the ground and hopefully land in a net 3.0 m off the ground, after flying a horizontal distance of 22 m. Do you want this job?

Motion Diagram

Motion Information

Event 1:	Event 2:
t1 = r1x = r1y = v1x = v1y = a12x = a12y =	t2 = r2x = r2y = v2x = v2y =

Mathematical Analysis[xi]

x-direction

y-direction

A ball is rolled off a level 0.80 m high table at 15 m/s. The floor beyond the table slopes down at a constant 50 below the horizontal.

Motion Diagram

Motion Information

Event 1:	Event 2:
t1 = r1x = r1y = v1x = v1y = a12x = a12y =	t2 = r2x = r2y = v2x = v2y =

Mathematical Analysis[xii]

x-direction

y-direction

A golf ball leaves the club at 18 m/s at an angle of 650 above the horizontal. The ground ahead slopes upward at 4°.

Motion Diagram

Motion Information

Event 1: t1 = r1x = r1y = v1x = v1y = a12x = a12y =	Event 2: t2 = r2x = r2y = v2x = v2y =

Mathematical Analysis[xiii]

x-direction *y-direction*

A ski-jumper leaves the ramp at an angle of 110 above the horizontal, 3.0 m above the ground. The ground slopes downward at 33° from this point. The jumper lands 140 m down the slope.

Motion Diagram

Motion Information

Event 1: t1 = r1x = r1y = v1x = v1y = a12x = a12y =	Event 2: t2 = r2x = r2y = v2x = v2y =

Mathematical Analysis[xiv]

x-direction *y-direction*

A kayaker 120 m east and 350 m north of his campsite is moving with the current at 2 m/s to the south. He begins to paddle west, giving the kayak an acceleration of 0.2 m/s2 for 15 s.

Motion Diagram

Motion Information

Event 1:	Event 2:
t1 = r1x = r1y = v1x = v1y = a12x = a12y =	t2 = r2x = r2y = v2x = v2y =

Mathematical Analysis[xv]

x-direction *y-direction*

A kayaker 120 m east and 80 m north of her campsite is moving with the current at 2 m/s to the south. She begins to paddle west, giving the kayak a constant acceleration. She lands right at her campsite.

Motion Diagram

Motion Information

Event 1:	Event 2:
t1 = r1x = r1y = v1x = v1y = a12x = a12y =	t2 = r2x = r2y = v2x = v2y =

Mathematical Analysis[xvi]

x-direction *y-direction*

An astronaut on a spacewalk is 30 m from her spaceship and moving at 0.8 m/s away from her ship, at an angle of 17° from a line between her and the ship. She engages her Manned Maneuvering Unit (MMU) for 15 s. The MMU imparts an acceleration of 0.1 *m/s2 to her in the direction she was originally moving.*

Motion Diagram

Motion Information

Event 1:	Event 2:
t1 = r1x = r1y = v1x = v1y = a12x = a12y =	t2 = r2x = r2y = v2x = v2y =

Mathematical Analysis[xvii]

x-direction *y-direction*

An astronaut on a spacewalk is 30 m from her spaceship and moving at 0.8 m/s away from her ship, at an angle of 17° from a line between her and the ship. She engages her Manned Maneuvering Unit (MMU) for 20 s. The MMU imparts an acceleration of 0.1 m/s2 to her in the direction initially toward her ship. (She does not change this direction during the maneuver.)

Motion Diagram

Motion Information

Event 1:	Event 2:
t1 = r1x = r1y = v1x = v1y = a12x = a12y =	t2 = r2x = r2y = v2x = v2y =

Mathematical Analysis[xviii]

x-direction *y-direction*

An astronaut on a spacewalk is 30 m from his spaceship and moving at 0.8 m/s away from his ship, at an angle of 17° from a line between him and the ship. He engages his Manned Maneuvering Unit (MMU), which supplies an acceleration of 0.1 m/s2 in a constant direction. He returns to his ship safely.

Motion Diagram

Motion Information

Event 1:	Event 2:
t1 = r1x = r1y = v1x = v1y = a12x = a12y =	t2 = r2x = r2y = v2x = v2y =

Mathematical Analysis[xix]

x-direction

y-direction

Determine the time-of-flight (T) of a rock thrown horizontally off of a cliff as a function of the initial velocity (vi), the height of the cliff (H), and g. Assume the ground at the base of the cliff is level.

Motion Diagram

Motion Information

Event 1:	Event 2:
t1 = r1x = r1y = v1x = v1y = a12x = a12y =	t2 = r2x = r2y = v2x = v2y =

Mathematical Analysis

x-direction

y-direction

Questions

If H = ∞, what should T equal? Does your function agree with this observation?

If g = 0 m/s2, what should T equal? Does your function agree with this observation?

If vi is doubled, what happens to T?

Determine the horizontal range (R) of a rock thrown horizontally off of a cliff as a function of the initial velocity (vi), the height of the cliff (H), and g. Assume the ground at the base of the cliff is level.

Motion Diagram

Motion Information

Event 1:	Event 2:
t1 = r1x = r1y = v1x = v1y = a12x = a12y =	t2 = r2x = r2y = v2x = v2y =

Mathematical Analysis

x-direction

y-direction

Questions

If H = ∞, what should R equal? Does your function agree with this observation?

If g = ∞, what should R equal? Does your function agree with this observation?

If vi is doubled, what happens to R?

Determine the maximum height (H) of a projectile launched over level ground as a function of the initial velocity (vi), the launch angle (q), and g.

Motion Diagram

Motion Information

Event 1:	Event 2:
t1 = r1x = r1y = v1x = v1y = a12x = a12y =	t2 = r2x = r2y = v2x = v2y =

Mathematical Analysis

x-direction *y-direction*

Questions

If g = ∞, what should H equal? Does your function agree with this observation?

If q = 0°, what should H equal? Does your function agree with this observation?

If vi is doubled, what happens to H?

Determine the range (R) of a projectile launched over level ground as a function of the initial velocity (vi), the launch angle (q), and g.

Motion Diagram

Motion Information

Event 1:	Event 2:
t1 = r1x = r1y = v1x = v1y = a12x = a12y =	t2 = r2x = r2y = v2x = v2y =

Mathematical Analysis

x-direction *y-direction*

Questions

If g = 0 m/s2, what should R equal? Does your function agree with this observation?

If q = 90°, what should R equal? Does your function agree with this observation?

If vi is doubled, what happens to R?

A projectile is launched from the top of an decline of constant angle f. Determine the distance the projectile travels along the decline (D) as a function of the initial velocity (vi), the launch angle above horizontal (q), the decline angle (f), and g.

Motion Diagram

Motion Information

Event 1: t1 = r1x = r1y = v1x = v1y = a12x = a12y =	Event 2: t2 = r2x = r2y = v2x = v2y =

Mathematical Analysis

x-direction

y-direction

Questions

If f = 90°, what should D equal? Does your function agree with this observation?

If q = 90°, what should D equal? Does your function agree with this observation?

[i] t2 = 1.86 s

[ii] v1 = 11.2 m/s

[iii] Drx = 8.3 m

[iv] v1 = 3.28 m/s

[v] v1 = 31.4 m/s

[vi] The ball hits people in the stands behind home plate. (It sails 10.7 m above home plate.)

[vii] q = 44°

[viii] q = 20.1°, 68.8°

[ix] q = 18.6°, 77.2°

[x] No.

[xi] Yes, as long as the cannon is set to 16.4°.

[xii] t2 = 0.56 s

[xiii] t2 = 3.22 s

[xiv] t2 = 3.22 s

[xv] 335 m from home

[xvi] t2 = 40 s a = 0.15 m/s2

[xvii] 52.6 m from ship

[xviii] 25.7 m from ship

[xix] t2 = 33.5 s q = 188° from line initially between ship and man

http://www.compadre.org/IVV/vignettes/projectileMotion.cfm

Homework 4 – Model 2: 14, 18, 22, 23, 28, 31, 34, 42, 45, and 53.

2.2: Dynamics

Dynamics

Concepts and Principles

Just like in kinematics, it's an empirical fact about nature that when a force acts on an object in one direction (for example, the horizontal) this action does not appear to cause changes in the motion in a perpendicular direction (the vertical). Therefore, to investigate the effects of forces on the motion of an object in the vertical direction, you can ignore all forces acting in the horizontal direction. Of course, many forces will simultaneously act in both the horizontal and vertical directions. As in kinematics, the effect of these forces can be examined by concentrating on the *components* of the forces in the various directions. Again, as long as the directions of interest are perpendicular, the force components can be determined through right-angle trigonometry, and the magnitude of the force can always be determined by:

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

Thus, Newton's second law,

$$\Sigma F = ma$$

is independently valid in any member of a set of perpendicular directions. The total force in the horizontal direction, for example, is equal to the mass times the acceleration in that direction. Note that the mass has been verified to be independent of direction, meaning that objects possess the same inertia in all directions.

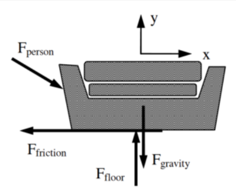
Analysis Tools

Drawing Free-Body Diagrams

The free-body diagram is still the most important analysis tool for determining the forces that act on a particular object. As an example, start with a verbal description of a situation:

> While rearranging furniture, a 600 N force is applied at an angle of 250 below horizontal to a 100 kg sofa at rest.

A free-body diagram for the sofa is sketched below:



The only non-contact interaction is the force of gravity, directed vertically downward.
The couch is in contact with two external objects, the person, pushing the couch across the floor, and the floor, exerting a force directed upward to prevent the couch from sinking into the floor. In addition, experience tells us that the interaction between the couch and the floor also hinders the motion of the couch in the direction of the person's push. This portion of the couch-floor interaction is commonly referred to as *friction*.

This is a complete free-body diagram for the couch.

The Force of Friction

The interaction between objects in direct contact typically consists of two parts. One part of the interaction is directed perpendicular to the surface of contact.[1] The other part of the interaction is the portion commonly called friction. The frictional portion of the interaction depends on many variables.

For most situations, a *model* of friction limiting the number of variables effecting the interaction to two is adequate. These two variables are the magnitude of the perpendicular portion of the interaction, generically called the *contact force*, and a unit-less constant that reflects the relative roughness of the surface-to-surface contact, termed the *coefficient of friction*. This linear model of sliding friction further differentiates between the frictional interaction when the two surfaces are moving with respect to each other, termed *kinetic friction*, and when they are not, termed *static friction*.

Kinetic friction

The kinetic friction model states that the frictional interaction between the surfaces is approximately equal to the product of the contact force, $F_{contact}$ and the coefficient of friction for kinetic situations, μ_k:

$$F_{friction} = \mu_k F_{contact}$$

The direction of this force on a particular object is in opposition to the relative motion of the two surfaces in contact.

Static friction

The static friction model states that the frictional interaction between the surfaces must be less than, or at most equal to, the product of the contact force, $F_{contact}$, and the coefficient of friction for static situations, μ_s:

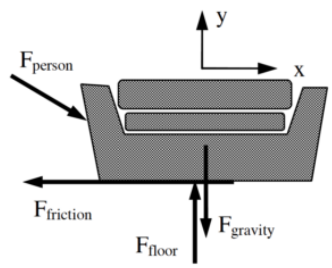
$$F_{friction} \le \mu_s F_{contact}$$

The direction of this force on a particular object is in opposition to the motion that *would* result if the frictional interaction were not present.

Applying Newton's Second Law

Using this model for friction, we can now quantitatively analyze the original situation. Note that the two coefficients of friction will typically be given as an ordered pair, (ms, mk).

> While rearranging furniture, a 600 N force is applied at an angle of 250 below horizontal to a 100 kg sofa at rest. The coefficient of friction between the sofa and the floor is (0.5, 0.4).



Hopefully you realize that two quite different outcomes can result from this push. Either the person pushes too weakly to move the couch or the push is sufficient to make the couch move. Since the frictional forces acting in these two cases are quite different, we can't really numerically analyze the situation until we make an *assumption* as to the outcome of the push. Of course, we will then have to check the validity of our assumption once we have completed our analysis. If our assumption turns out to be incorrect, we will then have to re-analyze the situation using the other possible outcome.

I will assume the couch doesn't move for the analysis below, and then later check the assumption. Assuming the couch doesn't move is equivalent to assuming ax = 0 m/s2 and that the relevant type of friction to use is static friction.

Applying Newton's Second Law independently in the horizontal (x) and vertical (y) directions yields:

x-direction	**y-direction**
$\Sigma F = ma$	$\Sigma F = ma$
$F_{person}\cos 25 - F_{staticfriction} = ma_x$	$-F_{person}\sin 25 - F_{gravity} + F_{floor} = ma_y$
$600\cos 25 - F_{staticfriction} = 100(0)$	$-600\cos\sin 25 - (100)(9.8) + F_{floor} = 100(0)$
$F_{staticfriction} = 544N$	$F_{floor} = 1234N$

Notice that the acceleration of the couch in the vertical direction must be zero regardless of my assumption, unless the couch begins to levitate or crash through the floor.

Assuming the couch doesn't move leads to a calculated value of static friction equal to 544 N. Can static friction create a force of this magnitude to prevent the couch's motion? I can check this calculated value against the allowed values for static friction:

$$F_{staticfriction} \le \mu_s F_{contact}$$
$$F_{staticfriction} \le (0.5)(1234)$$
$$F_{staticfriction} \le 617N$$

Since the calculated value of the static frictional force is below the maximum possible value of the static frictional force, my analysis and assumption are valid, the couch does not budge. The person is not pushing hard enough to overcome the static frictional force that acts to prevent the couch's motion relative to the floor.

Therefore, in this scenario the actual value of the static frictional force is 544 N (remember, it can be any value less than or equal to 617 N) and the acceleration of the couch is equal to zero.

How would the analysis change if the couch *was* initially in motion? Assume you enlisted a friend to help get the couch moving, but as soon as it began to move your friend stopped pushing. Would the couch stop immediately, gradually slow down to a stop, or could you keep the couch in motion across the room?

If the couch was initially moving, two things must change in our analysis. First, the horizontal acceleration of the couch is no longer necessarily zero. Second, the frictional force acting on the couch is kinetic.

Applying Newton's Second Law independently in the horizontal (x) and vertical (y) directions now yields:

x-direction	**y-direction**
$\Sigma F = ma$	$\Sigma F = ma$
$F_{person}\cos 25 - F_{kineticfriction} = 100a_x$	$-F_{person}\sin 25 - F_{gravity} + F_{floor} = ma_y$
$600\cos 25 - F_{kineticfriction} = 100a_x$	$-600\cos\sin 25 - (100)(9.8) + F_{floor} = 100(0)$
$544 - F_{kineticfriction} = 100a_x$	$F_{floor} = 1234N$

To finish the analysis, we need to calculate the kinetic frictional force.

$$F_{kineticfriction} = \mu_k F_{contact}$$
$$F_{kineticfriction} = (0.4)(1234)$$
$$F_{kineticfriction} = 494N$$

Substituting this into the x-equation above yields:

$$544 - 494 = 100a_x$$
$$50 = 100a_x\ 50 = 100a_x$$
$$a_x = 0.50m/s^2$$

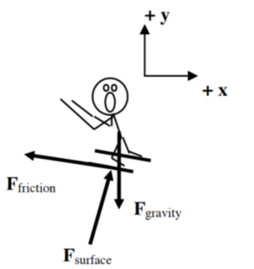
Thus, if the couch is already moving the kinetic frictional force is 494 N and the couch accelerates toward the right at 0.50 m/s2. In summary, if the couch is initially moving it will continue to move and accelerate at 0.50 m/s2 to the right. If it is initially at rest, the person pushing on it will not be able to get it to move.

Choosing a Coordinate System

In analyzing a scenario, you are always free to choose whatever coordinate system you like. If you make up negative, or left positive, this will not make you get the wrong answer. However, certain coordinate systems may make the mathematical analysis simpler than other coordinate systems. For example;

> A 75 kg skier starts from rest at the top of a 200 slope. He's a show-off, so he skies down the hill backward. The frictional coefficient between his skies and the snow is (0.10,0.05).

In attempting to analyze this situation, first draw a free-body diagram.



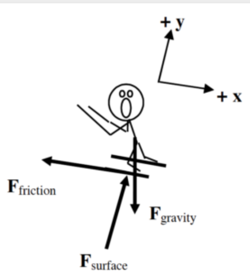
Notice that I have chosen the traditional horizontal and vertical coordinate system. I could analyze the situation using this coordinate system, but there are two difficulties with this choice.

1. Neither the force of the surface nor the force of friction is oriented in the x- or y-direction. (The force of gravity is oriented in the negative y-direction.) Therefore, I will have to use trigonometry to determine the x- and y-components of both of these forces.

2. The skier is accelerating down the inclined slope. Thus, I will also need trigonometry to determine the x- and y-components of the acceleration.

Although these difficulties are by no means insurmountable, why make the task more difficult than it has to be?

Contrast the above choice of coordinate system with a coordinate system in which the x-direction is tilted parallel to the surface on which the skier slides and the y-direction, remaining perpendicular to the x, is perpendicular to the surface.

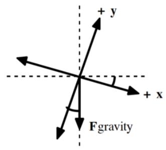


1. Using the tilted coordinate system, the only force not oriented in the x- or y-direction is the force of gravity. Therefore, I will only need to use trigonometry to determine the x- and y-components of one force rather than two.

2. The skier is accelerating down the inclined slope. Since the x-direction is oriented parallel to the slope, the skier has an acceleration in the x-direction and zero acceleration in the y-direction.

This simple rotation of the coordinate system has made the mathematical analysis of this situation much easier. Applying Newton's second law in the x- and y-direction leads to:

x-direction	y-direction
$\Sigma F = m\ a$	$\Sigma F = m\ a$
$F_{gravity} (\sin 20) - F_{friction} = m\ a_x$	$-F_{gravity} (\cos 20) + F_{surface} = m\ (0)$
$(75)(9.8)(\sin 20) - F_{friction} = 100\ a_x$	$-(75)(9.8)(\cos 20) + F_{surface} = m\ (0)$
$251 - F_{friction} = 100\ a_x$	$F_{surface} = 691\ N$

	Notice that if the x-axis is rotated by 200 from horizontal to become parallel to the slope, the y-axis is rotated by 200 from vertical. Since the force of gravity is always oriented vertically downward, it's now 200 from the y-axis. Thus, the force of gravity has a component in the positive x-direction of Fgravity (sin 200) and a component in the negative y-direction of Fgravity (cos 200).

Now that the contact force between the skier and the slope is known, the static friction force can be determined.

$$F_{friction} \leq \mu_s F_{contact}$$

$$F_{friction} \leq (0.10)(691)$$

$$F_{friction} \leq 69\ \text{N}$$

Since the x-component of the force of gravity on the skier (251 N) is larger than the force of static friction (69 N), the skier will accelerate down the hill. Once he begins to move, the frictional force must be calculated using the kinetic friction model.

$$F_{friction} = \mu_k F_{contact}$$

$$F_{friction} = (0.05)(691)$$

$$F_{friction} = 35\ \text{N}$$

Examining the x-component of Newton's second law:

$$251 - F_{friction} = 100\ a_x$$

$$251 - 35 = 100\ a_x$$

$$a_x = 2.2\ \text{m/s}^2$$

The skier accelerates down the slope with an acceleration of 2.2 m/s2.

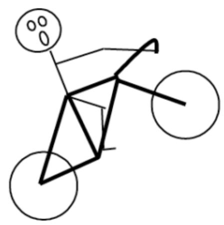
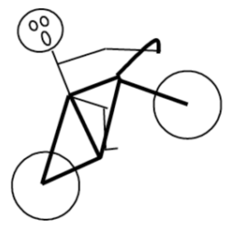
Activities

Construct free-body diagrams for the objects described below.

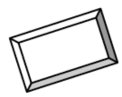
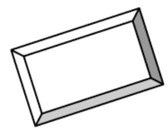
Someone mistakenly put a lovely couch out at the curb on garbage day and you decide to take it back to your apartment. You push horizontally on the 80 kg couch with a force of 320 N. The frictional coefficient is (0.40,0.35).

assuming the couch does not move	*assuming the couch does move*
	

A 100 kg bicycle and rider initially move at 16 m/s up a 150 hill. The rider slams on the brakes and skids to rest. The coefficient of friction is (0.8,0.7).

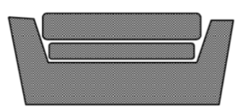
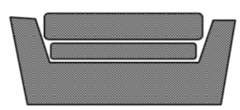
while skidding	when the bike is at rest on the incline
	

A 10 kg box is stacked on top of a 25 kg box. The boxes are at rest on an 8° incline.

the top box	the bottom box
	

Construct free-body diagrams for the objects described below.

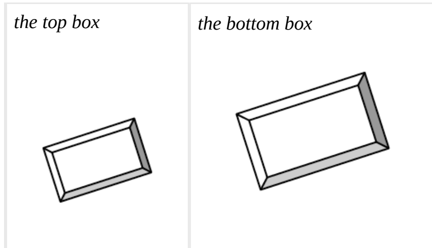
Someone mistakenly put a lovely couch out at the curb on garbage day and you decide to take it back to your apartment. You pull on the 110 kg couch with a force of 410 N directed at 350 above horizontal. The frictional coefficient is (0.40,0.35).

assuming the couch does not move	assuming the couch does move
	

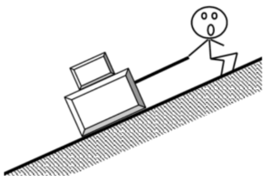
A 60 kg skier starts from rest at the top of a 100 m, 250 slope. He doesn't push with his poles because he's afraid of going too fast. The frictional coefficient is (0.10,0.05).

while skiing downhill	while being pulled back uphill by the towrope
	

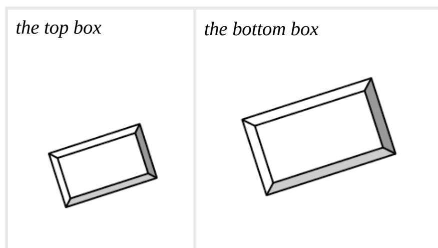
A 10 kg box is stacked on top of a 25 kg box. The boxes are sliding down an 18° incline at increasing speed. The top box is not moving relative to the bottom box.



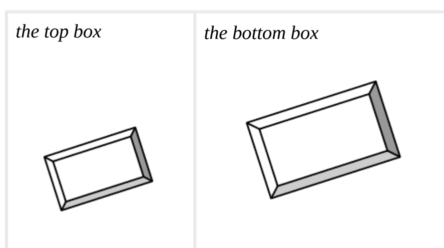
The strange man below is trying to pull the pair of boxes up the incline. Construct the requested free-body diagrams.



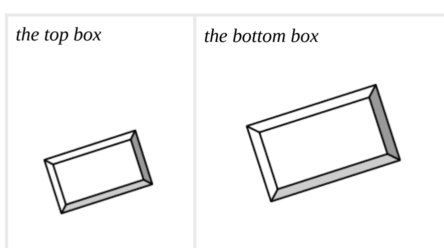
The boxes *almost* move up the incline.



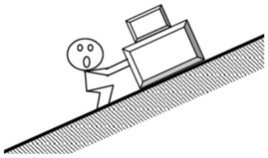
The boxes move up the incline.



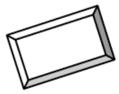
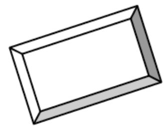
The bottom box moves up the incline but the top box slides off the bottom box.



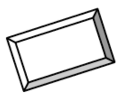
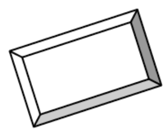
The strange man below is trying to prevent himself from getting crushed by the boxes. Construct the requested free-body diagrams.



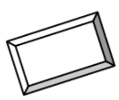
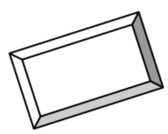
The boxes *almost* move down the incline.

the top box	*the bottom box*
	

The boxes *almost* move up the incline.

the top box	*the bottom box*
	

The bottom box *almost* moves down the incline but the top box slides off the bottom box.

the top box	*the bottom box*
	

A constant magnitude force is applied to a rope attached to a crate. The crate is on a level surface. For each of the following situations, circle the correct relationship symbol between the two force magnitudes and explain your reasoning.

The crate moves at constant speed and the rope is horizontal.

Fgravity	> = < **?**	Fsurface
Frope	> = < **?**	Ffriction

Explanation:

The crate does not move and the rope is horizontal.

Fgravity	> = < ?	Fsurface
Frope	> = < ?	Ffriction

Explanation:

The crate moves at constant speed and the rope is inclined above the horizontal.

Fgravity	> = < ?	Fsurface
Frope	> = < ?	Ffriction

Explanation:

A constant magnitude force is applied to a rope attached to a crate. The crate is on an inclined surface. For each of the following situations, circle the correct relationship symbol between the two force magnitudes and explain your reasoning.

The crate does not move and the rope is parallel to the incline and directed up the incline.

Fgravity	> = < ?	Fsurface
Frope	> = < ?	Ffriction

Explanation:

The crate does not move and the rope is parallel to the incline and directed down the incline.

Fgravity	> = < ?	Fsurface
Frope	> = < ?	Ffriction

Explanation:

The crate moves at constant speed up the incline and the rope is parallel to the incline and directed up the incline.

Fgravity	> = < ?	Fsurface
Frope	> = < ?	Ffriction

Explanation:

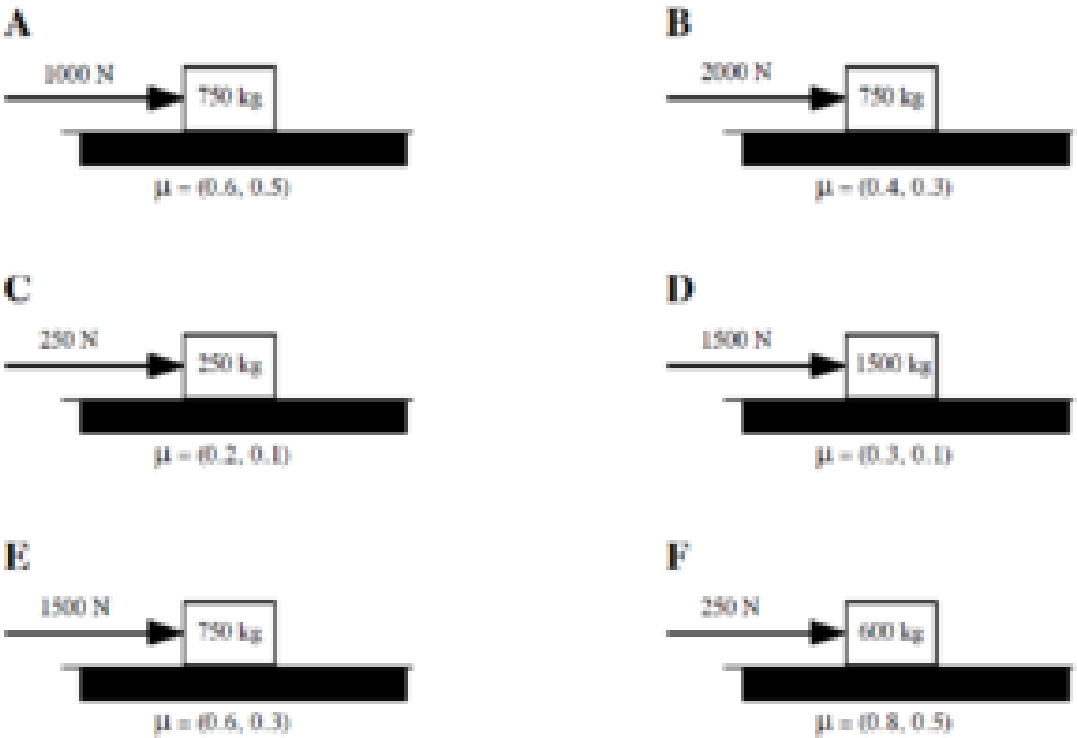
Below are six crates at rest on level surfaces. The crates have different masses and the frictional coefficients between the crates and the surfaces differ. The same external force is applied to each crate, but none of the crates move. Rank the crates on the basis of the magnitude of the frictional force acting on them.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are six crates at rest on level surfaces. The masses, frictional coefficients between the crates and the surfaces, and the external applied force all differ.



If none of the crates move, rank the crates on the basis of the magnitude of the frictional force acting on them.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

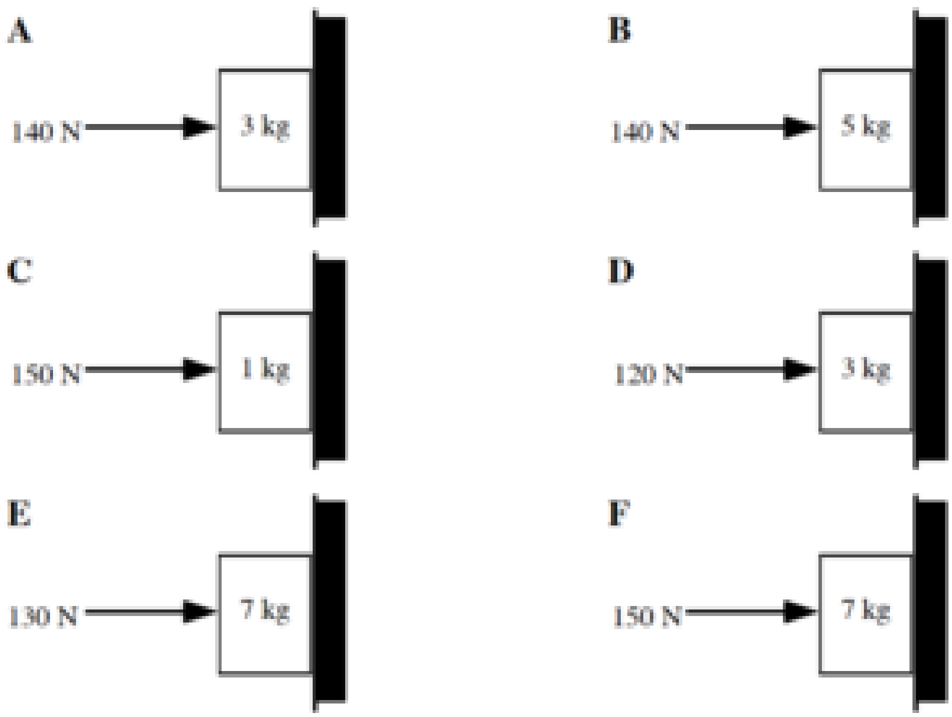
If the crates are moving, rank the crates on the basis of the magnitude of the frictional force acting on them.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are six boxes held at rest against a wall. The coefficients of friction between each box and the wall are identical.



Rank the boxes on the basis of the magnitude of the force of the wall acting on them.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

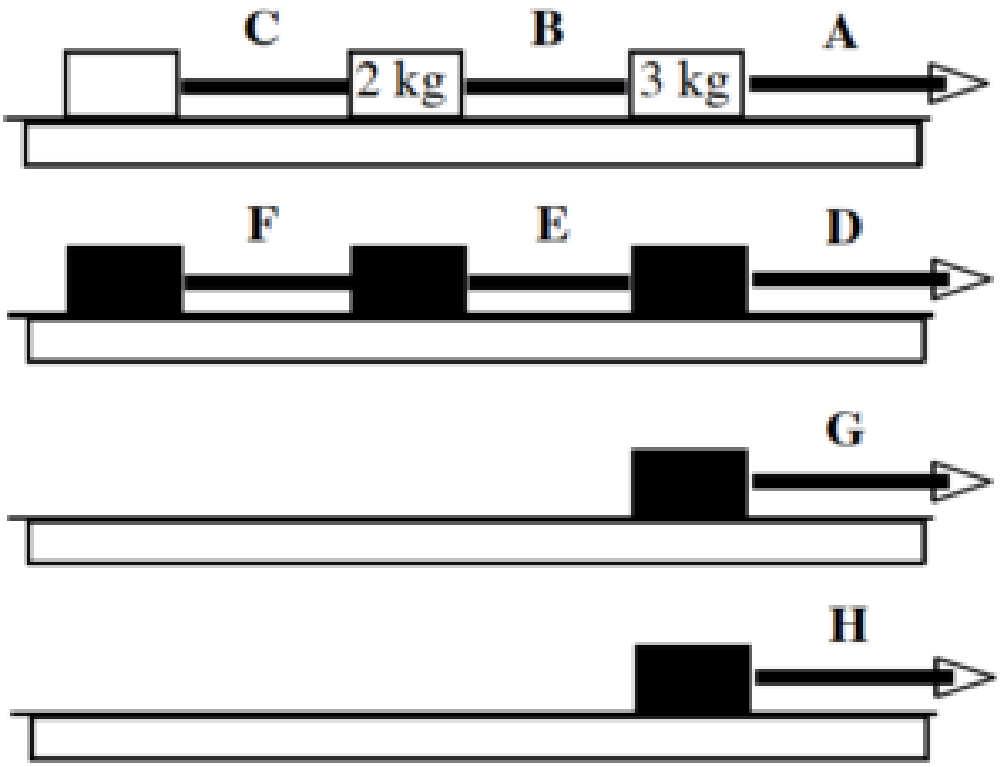
Rank the boxes on the basis of the magnitude of the frictional force acting on them.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are eight crates of differing mass. Each crate is being pulled to the right at the same constant speed.



Rank the magnitude of the force exerted by each rope on the crate immediately to its left if the frictional coefficient between each crate and the surface is the same non-zero value.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

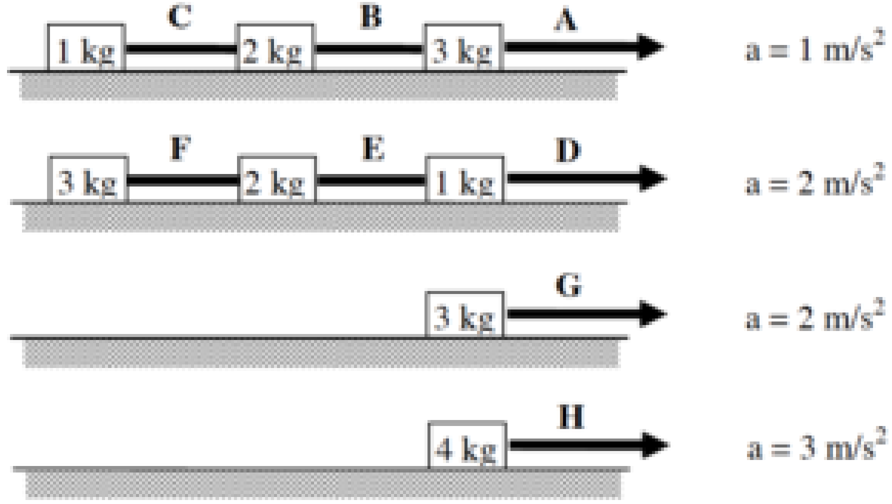
Rank the magnitude of the force exerted by each rope on the crate immediately to its left if the frictional coefficient between each crate and the surface is zero.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are eight crates of differing mass. The frictional coefficients between each crate and the surface on which they slide are so small that the force of friction is negligible on all crates. Each crate is being pulled to the right and accelerating. The acceleration of each crate or chain of crates is given. Rank the magnitude of the force exerted by each rope on the crate immediately to its left.



Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____7. _____ 8. _____Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Someone mistakenly put a lovely couch out at the curb on garbage day and you decide to take it back to your apartment. You push on the 110 kg couch with a force of 410 N directed at 350 below horizontal. The couch doesn't move.

Free-Body Diagram

Mathematical Analysis [i]

x-direction

y-direction

Someone mistakenly put a lovely couch out at the curb on garbage day and you decide to take it back to your apartment. You push on the 80 kg couch with a force of 320 N directed at 150 below horizontal. The frictional coefficient is (0.40,0.35).

Free-Body Diagram

Mathematical Analysis [ii]

x-direction *y-direction*

Someone mistakenly put a lovely couch out at the curb on garbage day and you decide to take it back to your apartment. You pull on the 110 kg couch with a force of 510 N directed at 350 above horizontal. The frictional coefficient is (0.40,0.35).

Free-Body Diagram

Mathematical Analysis [iii]

x-direction *y-direction*

You get into a fight with another person over a garbage-day couch. You push on the 80 kg couch with a force of 660 N directed at 150 below horizontal. She claims ownership by sitting on the couch while you try to push it. You still manage to just barely get the couch moving. The frictional coefficient is (0.40,0.35).

Free-Body Diagram

Mathematical Analysis [iv]

x-direction *y-direction*

You get into a fight with another person over a garbage-day couch. You push on the 80 kg couch with a force of 420 N directed at 150 below horizontal. She pushes on the other side of the couch with a force of 510 N directed at 250 below horizontal. The frictional coefficient is (0.40,0.35).

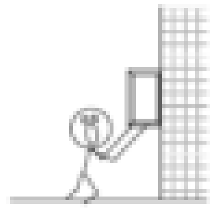
Free-Body Diagram

Mathematical Analysis [v]

x-direction *y-direction*

The person at right exerts the minimum force necessary to support the 100 kg block. He pushes at an angle of 50° above the horizontal. The coefficient of friction is (0.6,0.3).

Since we are looking for the minimum force needed to hold the block in place, the block is almost moving downward. This means that the frictional force is static and directed upward.



Free-Body Diagram

Mathematical Analysis

x-direction

$F_{push}\cos 50 - F_{wall} = 100(0)$

$F_{wall} = 0.643F_{push}$

Since the block is not moving, both accelerations equal zero.

y-direction

$F_{push}\sin 50 - (100)(9.8) + F_{static friction} = 100(0)$

friction

$F_{static friction} \le 0.6F_{wall}$

$F_{static friction} \le 0.6(0.643F_{push})$

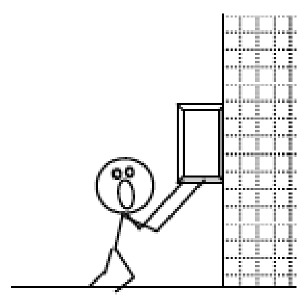
$F_{static friction} \le 0.386F_{push}$

Since we are looking for the minimum force needed to support the block, the block is almost moving. This means that static friction is at its maximum value. Therefore we can substitute $0.386F_{push}$ into the y-equation and solve.

$$0.766F_{push} - 980 + 0.386F_{push} = 0$$
$$1.152F_{push} = 980$$
$$F_{push,min} = 851N$$

If we were looking for the maximum force, everything would be the same except for the direction of the frictional force (it would be downward). Therefore, the maximum force would be

$$0.766F_{push} - 980 - 0.386F_{push} = 0$$
$$0.380F_{push} = 980$$
$$F_{push,max} = 2579N$$

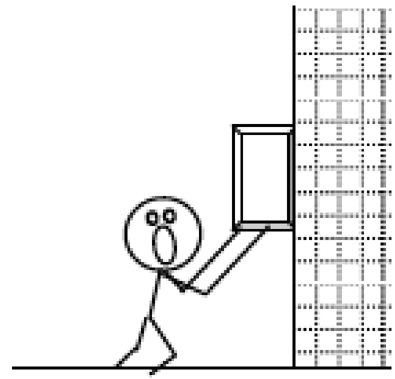
The person at right exerts an 850 N force on the 90 kg block at an angle of 550 above the horizontal. The coefficient of friction is (0.6,0.5).	

Free-Body Diagram

Mathematical Analysis [vi]

x-direction

y-direction

The person at right exerts a 620 N force on the 70 kg block at an angle of 400 above the horizontal. The coefficient of friction is (0.5,0.4).	

Free-Body Diagram

Mathematical Analysis [vii]

x-direction *y-direction*

A boy pulls a 30 kg sled, including the mass of his kid brother, along ice. The boy pulls on the tow rope, oriented at 600 above horizontal, with a force of 110 N until his kid brother begins to cry. Like clockwork, his brother always cries upon reaching a speed of 2.0 m/s. The frictional coefficient is (0.20,0.15).

Motion Information **Free-Body Diagram**

Event 1: t1 = r1x = r1y = v1x = v1y = a12x = a12y =	Event 2: t2 = r2x = r2y = v2x = v2y =	

Mathematical Analysis [viii]

x-direction *y-direction*

Starting from rest, a girl can pull a sled, carrying her kid brother, 20 m in 8 s. The girl pulls on the tow rope, oriented at 300 above horizontal, with a force of 90 N. The frictional coefficient is (0.15,0.10).

Motion Information **Free-Body Diagram**

Event 1: t1 = r1x = r1y = v1x = v1y = a12x = a12y =	Event 2: t2 = r2x = r2y = v2x = v2y =	

Mathematical Analysis [ix]

x-direction

y-direction

A 60 kg skier starts from rest at the top of a 100 m, 250 slope. He doesn't push with his poles because he's afraid of going too fast. The frictional coefficient is (0.10,0.05).

Motion Information

Free-Body Diagram

Event 1: t1 = r1x = r1y = v1x = v1y = a12x = a12y =	Event 2: t2 = r2x = r2y = v2x = v2y =	

Mathematical Analysis [x]

x-direction

y-direction

A 100 kg bicycle and rider initially move at 16 m/s up a 150 hill. The rider slams on the brakes and skids to rest. The coefficient of friction is (0.8,0.7).

Motion Information

Free-Body Diagram

Event 1: t1 = r1x = r1y = v1x = v1y = a12x = a12y =	Event 2: t2 = r2x = r2y = v2x = v2y =	

Mathematical Analysis [xi]

x-direction

y-direction

A 70 kg snowboarder starts from rest at the top of a 270 m, 200 slope. She reaches the bottom of the slope in 14.5 seconds.

Motion Information

Free-Body Diagram

Event 1: t1 = r1x = r1y = v1x = v1y = a12x = a12y =	Event 2: t2 = r2x = r2y = v2x = v2y =	

Mathematical Analysis [xii]

x-direction

y-direction

At a UPS distribution center, a 60 kg crate is at rest on an 8° ramp. A worker applies the minimum horizontal force needed to push the crate up the ramp. The coefficient of friction between the crate and the ramp is (0.3, 0.2).

Free-Body Diagram

Mathematical Analysis [xiii]

x-direction

y-direction

At a UPS distribution center, a 40 kg crate is sliding down an 8° ramp at 3 m/s. A worker applies a horizontal force to the crate and brings the crate to rest in 1.5 s. The coefficient of friction between the crate and the ramp is (0.3, 0.2).

Motion Information

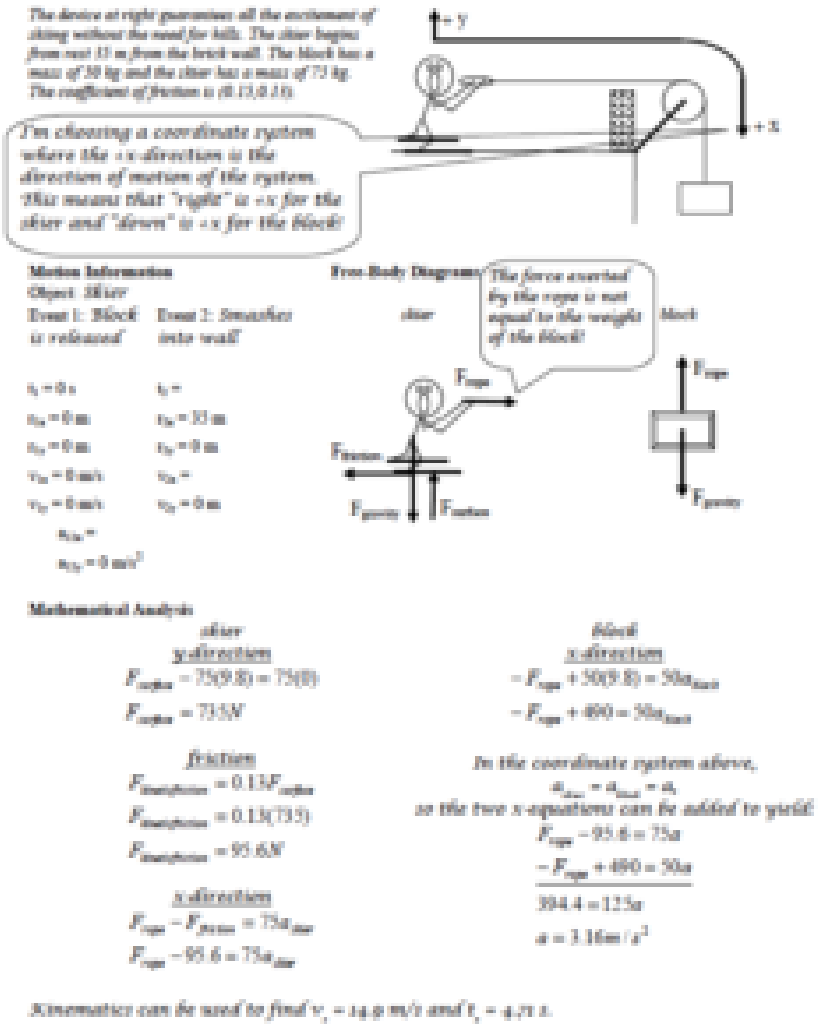
Free-Body Diagram

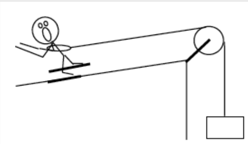
Event 1: t1 = r1x = r1y = v1x = v1y = a12x = a12y =	Event 2: t2 = r2x = r2y = v2x = v2y =	

Mathematical Analysis [xiv]

x-direction

y-direction



The device at right allows novices to ski downhill at reduced speeds. The block has a mass of 15 kg and the skier has a mass of 70 kg. The coefficient of friction is (0.05,0.04). The skier starts from rest at the top of a 30 m, 200 slope.	

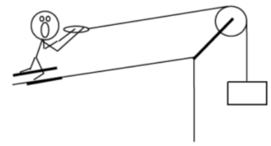
Motion Information **Free-Body Diagrams**

Object:

Event 1: t1 = r1x = r1y = v1x = v1y = a12x = a12y =	Event 2: t2 = r2x = r2y = v2x = v2y =	*skier*	*block*

Mathematical Analysis [xv]

The device at right allows you to ski uphill. The ballast block has a mass of 30 kg and the skier has a mass of 60 kg. The coefficient of friction is (0.07,0.06). The skier starts from rest at the bottom of a 30 m, 200 slope.



Motion Information

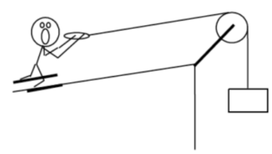
Object:

Free-Body Diagrams

Event 1:	Event 2:		
t1 = r1x = r1y = v1x = v1y = a12x = a12y =	t2 = r2x = r2y = v2x = v2y =	*skier*	*block*

Mathematical Analysis [xvi]

*The device at right **may** allow you to ski uphill (or it **may** allow you to ski downhill backward). The ballast block has a mass of 20 kg and the skier has a mass of 70 kg. The coefficient of friction is (0.1,0.09). The ramp is inclined at 200 above horizontal.*

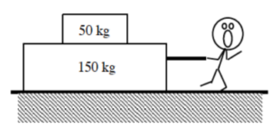


Free-Body Diagrams

skier	*block*

Mathematical Analysis [xvii]

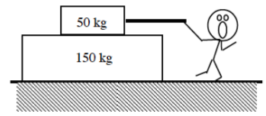
The strange man at right wants to pull the two blocks to the other side of the room in as short a time as possible. However, he doesn't want the top block to slide relative to the bottom block. The coefficient of friction between the bottom block and the floor is (0.25,0.20) and the coefficient of friction between the top block and the bottom block is (0.30,0.25). The blocks start from rest.



Free-Body Diagrams

top block	*bottom block*

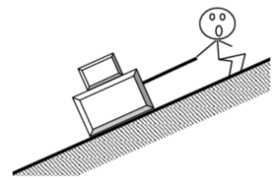
Mathematical Analysis [xviii]

The strange man at right wants to pull the two blocks to the other side of the room in as short a time as possible by pulling on the top block. However, he doesn't want the top block to slide relative to the bottom block. The coefficient of friction between the bottom block and the floor is (0.10,0.05) and the coefficient of friction between the top block and the bottom block is (0.60,0.50). The blocks start from rest.	

Free-Body Diagrams

top block	*bottom block*

Mathematical Analysis [xix]

The strange man at right wants to pull the two blocks to the top of the hill in as short a time as possible. However, he doesn't want the top block to slide relative to the bottom block. The coefficient of friction between the 150 kg bottom block and the floor is (0.25,0.20) and the coefficient of friction between the 50 kg top block and the bottom block is (0.30,0.25). The hill is inclined at 150 above horizontal. The blocks start from rest.	

Free-Body Diagrams

top block	*bottom block*

Mathematical Analysis [xx]

The strange man at right applies the minimum force necessary to not get crushed by the bottom block. (The top block may or may not crush him.) The coefficient of friction between the 150 kg bottom block and the floor is (0.25,0.20) and the coefficient of friction between the 50 kg top block and the bottom block is (0.40,0.35). The hill is inclined at 200 above horizontal. The blocks are initially at rest.	

Free-Body Diagrams

top block	*bottom block*

Mathematical Analysis [xxi]

You should know the story by now. You push on a garbage-day couch at an angle q below horizontal. Determine the minimum force (F_{min}) needed to move the couch as a function of the couch's mass (m), q, the appropriate coefficient of friction, and g.

Free-Body Diagram

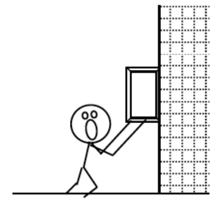
Mathematical Analysis

Questions

If q = 0°, what should F_{min} equal? Does your function agree with this observation?

If q = 90°, what should F_{min} equal? Does your function agree with this observation?

If m = ∞, what should F_{min} equal? Does your function agree with this observation?

The man at right exerts a force on the block at an angle q above horizontal. Determine the minimum force (F_{min}) needed to begin to slide the block up the wall as a function of the block's mass (m), q, the appropriate coefficient of friction, and g.	

Free-Body Diagram

Mathematical Analysis

Questions

If q = 90°, what should F_{min} equal? Does your function agree with this observation?

If q = 0°, what should F_{min} equal? Does your function agree with this observation?

Below what angle q is it impossible to slide the block up the wall?

A crate is held at rest on a ramp inclined at q from horizontal. Determine the minimum force (Fmin), applied parallel to the incline, needed to prevent the crate from sliding down the ramp as a function of the crate's mass (m), q, the appropriate coefficient of friction, and g.

Free-Body Diagram

Mathematical Analysis

Questions

If q = 0°, what should F_{min} equal? Does your function agree with this observation?

If q = 90°, what should F_{min} equal? Does your function agree with this observation?

If m = ∞, what should F_{min} equal? Does your function agree with this observation?

A crate is held at rest on a ramp inclined at q from horizontal. Determine the maximum force (Fmax), applied horizontally, before the crate begins to move as a function of the crate's mass (m), q, the appropriate coefficient of friction, and g.

Free-Body Diagram

Mathematical Analysis

Questions

If m = ∞, what should F_{max} equal? Does your function agree with this observation?

If g = ∞, what should F_{min} equal? Does your function agree with this observation?

If q = 0°, what should F_{max} equal? Does your function agree with this observation?

A skier of mass m starts from rest at the top of a ski run of incline q. Determine the minimum angle (qmin) such that the skier will begin to slide down the slope without pushing off as a function of m, the appropriate coefficient of friction, and g.

Free-Body Diagram

Mathematical Analysis

Questions

If m = 0, what should q_{min} equal? Does your function agree with this observation?

If g = 0 m/s2, what should q_{min} equal? Does your function agree with this observation?

If m was twice as large, what should q_{min} equal? Does your function agree with this observation?

[1] The portion of the interaction directed perpendicular to the surface of contact is sometimes referred to as the *normal* force, where normal has its mathematical definition of perpendicular.

[i] ms ≥ 0.256

[ii] a = 0 m/s2

[iii] a = 0.94 m/s2

[iv] m = 65.2 kg

[v] Fsf = 56 N

[vi] Fsf = 186 N up

[vii] a = 1.39 m/s2 down

[viii] a = 0.84 m/s2

[ix] m = 51.4 kg

[x] t2 = 7.4 s

[xi] r2x = 14 m

[xii] mk = 0.085

[xiii] F = 270 N

[xiv] F = 55.9 N

[xv] a = 0.73 m/s2

[xvi] a = 0.66 m/s2

[xvii] a = 0 m/s2

[xviii] Fmax = 980 N

[xix] Fmax = 359 N

[xx] Fmax = 947 N

[xxi] Fmin = 210 N

Homework 5 – Model 2: 64, 68, 70, 71, 73, 77, 85, 92, 95, and 101.

1. The portion of the interaction directed perpendicular to the surface of contact is sometimes referred to as the normal force, where normal has its mathematical definition of perpendicular. ↵

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2.3: Conservation Laws

Conservation Laws

Concepts and Principles

The Impulse-Momentum Relation

Just like the kinematic relations and Newton's second law, the impulse-momentum relation is independently valid in any member of a set of perpendicular directions. Thus, we will typically apply the impulse-momentum relation in its component forms:

$$mv_{x_i} + \Sigma(F_x(\Delta t)) = mv_{x\,f}$$

$$mv_{y_i} + \Sigma(F_y(\Delta t)) = mv_{y\,f}$$

$$mv_{z_i} + \Sigma(F_z(\Delta t)) = mv_{z\,f}$$

The Work-Energy Relation

From Model 1, our expression for the Work-Energy Relation, with gravitational potential energy terms, is:

$$\frac{1}{2}mv_i^{\,2} + mgh_i + \Sigma(|F||\Delta r|\cos\phi) = \frac{1}{2}mv_f^{\,2} + mgh_f$$

It's very important to remember that the work-energy relation is a *scalar* equation, meaning it cannot be broken into components and "solved" separately in the x-, y-, and z-directions. This is even more important to remember now that we are working in multiple dimensions. This observation results in two important points:

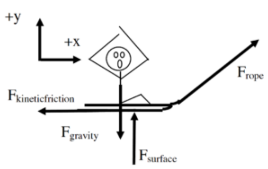
- The work-energy relation involves the actual initial and final velocities, *not* their components. The kinetic energy of an object does not depend on the direction of travel of the object.
- In the expression for work, $|F||\Delta r|\cos\phi$, the product of the *magnitude* of the force and the *magnitude* of the displacement is multiplied by cos ϕ, where ϕ is defined to be the angle between the applied force and the displacement of the object. If the force and displacement are in the same direction ϕ = 0°, and the work is positive (the object gains energy). If the force and displacement are in the opposite direction ϕ = 180°, and the work is negative (the object loses energy). If the force and displacement are perpendicular, no work is done. Note that the actual directions of the force and the displacement are unimportant, only their directions *relative to each other* affect the work.

Analysis Tools

Applying the Impulse-Momentum Relation to a Single Object

Let's investigate the following scenario:

> A boy pulls a 30 kg sled, including the mass of his kid brother, along ice. The boy pulls on the tow rope, oriented at 600 above horizontal, with a force of 110 N until his kid brother begins to cry. Like clockwork, his brother always cries upon reaching a speed of 2.0 m/s. The frictional coefficient is (0.20,0.15).



To apply the impulse-momentum relation, you must clearly specify the initial and final events at which you will tabulate the momentum. For example:

Event 1: The instant before the sled begins to move.	Event 2: The instant the sled reaches 2.0 m/s
P_{1x} = 0 P_{1y} = 0	P_{2x} = 30 (2.0) = 60 P_{2y} = 0

$J_{12x} = 110\cos 60(\Delta t) - F_{kf}(\Delta t)$

$J_{12y} = 110\sin 60(\Delta t) - (30)(9.8)(\Delta t) + F_{surface}(\Delta t)$

Applying impulse-momentum separately in the x- and y-directions yields:

<u>x-direction</u>	<u>y-direction</u>
$P_1 + J_{12} = P_2$	$P_1 + J_{12} = P_2$
$0 + 110\cos 60(\Delta t) - F_{kf}(\Delta t) = 60$	$0 + 110\sin 60(\Delta t) - (30)(9.8)(\Delta t) + F_{surface}(\Delta t) = 0$
$55(\Delta t) - 0.15F_{surface}(\Delta t) = 60$	$95(\Delta t) - 294(\Delta t) + F_{surface}(\Delta t) = 0$
	$F_{surface} = 199N$

Substituting the value for the force of the surface into the x-equation,

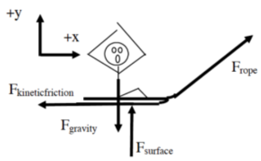
$$55(\Delta t) - 0.15(199)(\Delta t) = 60$$
$$55(\Delta t) - 30(\Delta t) = 60$$
$$25(\Delta t) = 60$$
$$\Delta t = 2.4s$$

The kid brother begins to cry after only 2.4 s.

Applying the Work-Energy Relation to a Single Object

What will the work-energy relation tell us about the same scenario?

> A boy pulls a 30 kg sled, including the mass of his kid brother, along ice. The boy pulls on the tow rope, oriented at 600 above horizontal, with a force of 110 N until his kid brother begins to cry. Like clockwork, his brother always cries upon reaching a speed of 2.0 m/s. The frictional coefficient is (0.20,0.15).



To apply the work-energy relation, you must clearly specify the initial and final events at which you will tabulate the energy. For example:

Event 1: The instant before the sled begins to move.	Event 2: The instant the sled reaches 2.0 m/s
KE1 = 0 GE1 = 0	KE2 = ½ 30 (2.0)2 = 60 GE2 = 0

$$W_{12} = 110(\Delta r)\cos 60 + F_{surface}(\Delta r)\cos 90 + F_{kineticfriction}(\Delta r)\cos 180$$

Applying the work-energy relation yields:

$$KE_1 + GE_1 + W_{12} = KE_2 + GE_2$$
$$0 + 0 + 110(\Delta r)\cos 60 + F_{surface}(\Delta r)\cos 90 + F_{kineticfriction}(\Delta r)\cos 180 = 60 + 0$$
$$55(\Delta r) - 0.15F_{surface}(\Delta r) = 60$$

Notice that the force of the surface does no work, the force of the rope does positive work, and the force of friction does negative work. Each of these terms should make sense if you remember that work is the transfer of energy into (positive) or out of (negative) the system of interest. Also recall that in this form of the work-energy relation we conceptualize gravity as a source of potential energy, not as a force that does work.

Using the result for the force of the surface determined in the first example, $F_{surface}$ = 199 N, gives:

$$55(\Delta r) - 0.15F_{surface}(\Delta r) = 60$$
$$55(\Delta r) - 0.15(199)(\Delta r) = 60$$
$$55(\Delta r) - 30(\Delta r) = 60$$
$$25(\Delta r) = 60$$
$$\Delta r = 2.4m$$

The kid brother begins to cry after traveling 2.4 m.

Applying Work-Energy with Gravitational Potential Energy

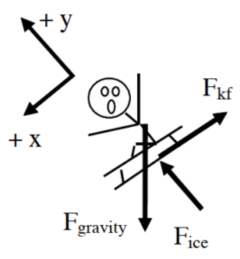
Let's use the work-energy relation, with gravitational potential energy terms, to analyze the following scenario:

> A 30 kg child on his 15 kg sled slides down his parents' 10 m long, 150 above horizontal driveway after an ice storm. The coefficient of friction between the sled and the driveway is (0.10, 0.08).

To calculate the gravitational energy terms let the bottom of the driveway be zero and up positive. The coordinate system used to calculate gravitational energy does not in general have to be the same as the system you use for the rest of the problem. In fact, since the work-energy relation is a scalar equation, the other portions of the equation should not depend on your choice of coordinate system at all!

Event 1: The instant before the sled begins to move.	Event 2: The instant the sled reaches the bottom of the driveway
KE1 = 0 GE1 = 45 (9.8) (10 sin150) = 1140	KE2 = ½ 45 v2 GE2 = 0

$$W_{12} = F_{ice}(10)\cos 90 + F_{kineticfriction}(10)\cos 180$$

	$KE_1 + GE_1 + W_{12} = KE_2 + GE_2$ $0 + 1140 + F_{ice}(10)\cos 90 + F_{kf}(10)\cos 180 = \frac{1}{2}45 v_2^{\ 2} + 0$ $1140 - 10F_{kf} = 22.5 v_2^{\ 2}$

Note:

- The only forces that *could* do work are the force of the ice and the force of friction, since the action of the force of gravity is already incorporated into the gravitational potential energy terms.
- The heights in the gravitational potential energy function were measured from the bottom of the driveway, with the positive direction as upward, as required. Notice that the initial height is not the same as the length of the driveway. Since the driveway is 10 m long, at an angle of 15°, the height of the top of the driveway relative to the bottom is (10 m) sin 15°. The height at the bottom of the driveway is defined to be 0 m.

To finish the analysis we need to determine the kinetic frictional force. Since this depends on the force of the ice, apply Newton's Second Law in the y-direction and find:

$$\begin{aligned}
&\Sigma F = ma \\
&+F_{ice} - F_{gravity}\cos 15 = 45(0) \\
&F_{ice} - (45)(9.8)\cos 15 = 0 \\
&F_{ice} = 426N
\end{aligned}$$

$$\begin{aligned}
&F_{kf} = \mu_s F_{ice} \\
&F_{kf} = (0.08)(426) \\
&F_{kf} = 34N
\end{aligned}$$

Plugging this value into the work-energy relation yields:

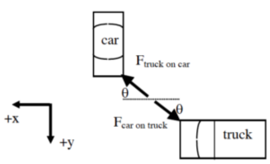
$$\begin{aligned}
&1140 - 10(34) = 22.5 v_2^{\ 2} \\
&1140 - 340 = 22.5 v_2^{\ 2} \\
&800 = 22.5 v_2^{\ 2} \\
&v_2 = 5.96 m/s
\end{aligned}$$

A Two-Dimensional Collision

Let's try a two-dimensional collision.

> At a busy intersection, an impatient driver heading south runs a red-light and collides with a delivery truck originally moving at 15 m/s west. The vehicles become entangled and the skid marks from the wreckage are at 22o south of west. The auto mass is 755 kg and the truck mass is 1250 kg.

Partial free-body diagrams (top view) for both the car and the truck during the time interval during the collision are shown below.



These are only partial free-body diagrams because:

- Forces perpendicular to the earth's surface (the force of gravity and the force of the road) are not shown.These are only partial free-body diagrams because:
- During a collision, the force *between* the colliding objects is normally much greater in magnitude than any other forces acting on the objects. Therefore we will often ignore the other forces acting on colliding objects for the duration of a collision. This approximation is termed the *impulse approximation*. Under the impulse approximation, the frictional forces between the car and truck and the road are ignored.

Also note that the direction of the force acting between the car and truck is unknown. The angle q is not determined from the situation description.

Event 1: The instant before the car and truck collide	Event 2: The instant they reach a common velocity
Object: Car P1x = 0 P1y = 755 vcar	P2x = 755 (v2 cos 22º) = 700 v2 P2y = 755 (v2 sin 22º) = 283 v2

$J_{12x} = F_{truckoncar} \cos\theta(\Delta t)$
$J_{12y} = -F_{truckoncar} \sin\theta(\Delta t)$

Object: Truck P1x = 1250 (15) = 18750 P1y = 0	P2x = 1250 (v2 cos 22º) = 1160 v2 P2y = 1250 (v2 sin 22º) = 468 v2

$J_{12x} = -F_{carontruck} \cos\theta(\Delta t)$
$J_{12y} = F_{carontruck} \sin\theta(\Delta t)$

Applying the impulse-momentum relation to the car and truck yields:

Car

x-direction	**y-direction**
$P_1 + J_{12} = P_2$ $0 + F_{truckoncar} \cos\theta(\Delta t) = 700 v_2$	$P_1 + J_{12} = P_2$ $755 v_{car} - F_{truckoncar} \sin\theta(\Delta t) = 283 v_2$

Truck

x-direction	**y-direction**
$P_1 + J_{12} = P_2$ $18750 - F_{carontruck} \cos\theta(\Delta t) = 1160 v_2$	$P_1 + J_{12} = P_2$ $0 + F_{carontruck} \sin\theta(\Delta t) = 468 v_2$

Since the magnitude of the force on the car due to the truck and the force on the truck due to the car are equal, when the x-equations for the car and truck are added, the impulses cancel!

x-direction

$$0 + 18750 = 700 v_2 + 1160 v_2$$
$$18750 = 1860 v_2$$
$$v_2 = 10.1 m/s$$

This is the speed of the wreckage immediately after the collision. Note that this is exactly the same equation we would have written if we had considered the *system* of the car and truck right from the start. Try it!

Adding the two y-equations yields:

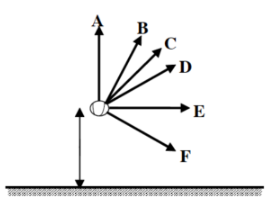
y-direction

$$755 v_{car} + 0 = 283 v_2 + 468 v_2$$
$$755 v_{car} = 751(10.1)$$
$$v_{car} = 10.0 m/s$$

This is the speed of the car immediately before colliding with the truck.

Activities

Below are six different directions in which a baseball can be thrown. In all cases the baseball is thrown at the same initial speed from the same height above the ground. Assume the effects of air resistance are negligible.



Rank these baseballs on the basis of their horizontal speed the instant before they hit the ground.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Rank these baseballs on the basis of their vertical speed the instant before they hit the ground.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

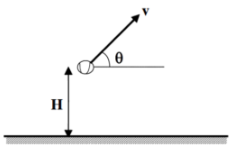
Rank these baseballs on the basis of their speed the instant before they hit the ground.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your rankings:

Below are six different directions and heights from which a baseball can be thrown. In all cases the baseball is thrown at the same speed, v. Assume the effects of air resistance are negligible.



	H	θ
A	10 m	30^0
B	10 m	0^0
C	10 m	60^0
D	20 m	0^0
E	15 m	45^0
F	5 m	90^0

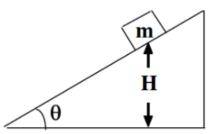
Rank these baseballs on the basis of their speed the instant before they hit the ground.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

A crate is released from rest along an inclined surface. The mass of the crate and the angle of the incline vary. The frictional coefficients between the crates and the surfaces are identical and so small that the effect of friction is negligible. All crates are released from the same vertical height, H, above the bottom of the incline.



	m	θ
A	10 kg	30^0
B	20 kg	15^0
C	10 kg	60^0
D	20 kg	60^0
E	15 kg	45^0
F	5 kg	85^0

Rank these scenarios on the basis of the kinetic energy of the crate the instant it reaches the bottom of the incline.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

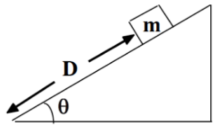
Rank these scenarios on the basis of the speed of the crate the instant it reaches the bottom of the incline.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

A crate is released from rest along an inclined surface. The mass of the crate and the angle of the incline vary. The frictional coefficients between the crates and the surfaces are identical and so small that the effect of friction is negligible. All crates are released from the same distance, D, along the incline.



	m	θ
A	10 kg	30^0
B	20 kg	15^0
C	10 kg	60^0
D	20 kg	60^0
E	15 kg	45^0
F	5 kg	85^0

Rank these scenarios on the basis of the kinetic energy of the crate the instant it reaches the bottom of the incline.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

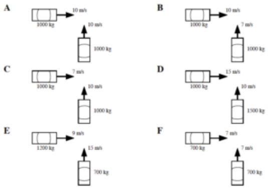
Rank these scenarios on the basis of the speed of the crate the instant it reaches the bottom of the incline.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are bird's-eye views of six automobile crashes an instant before they occur. The automobiles have different masses and velocities. All automobiles will remain joined together after the impact and skid to rest. Rank these automobile crashes on the basis of the angle at which the wreckage skids. Let 0^0 be the angle oriented directly toward the right and measure angles counterclockwise from 0^0.

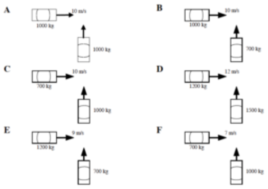


Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are bird's-eye views of six automobile crashes an instant before they occur. The automobiles have different masses and velocities. All automobiles will remain joined together after the impact and skid to rest at the same angle, as measured from a line oriented directly toward the right. Rank these scenarios on the basis of the initial speed of the auto traveling toward the top of the page.



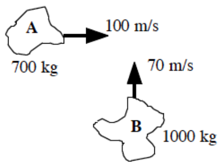
Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

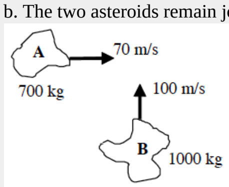
Explain the reason for your ranking:

For each of the collisions illustrated below, sketch a graph of the momentum of asteroid A, the momentum of asteroid B, and the total momentum in the system of the two asteroids. Sketch the horizontal and vertical momentum separately. Begin your graph before the collision takes place and continue it after the collision is over. Use a consistent scale on all graphs.

a. The two asteroids remain joined together after the collision.

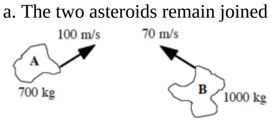


b. The two asteroids remain joined together after the collision.

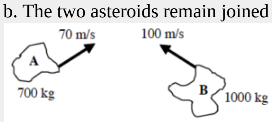


For each of the collisions illustrated below, sketch a graph of the momentum of asteroid A, the momentum of asteroid B, and the total momentum in the system of the two asteroids. Sketch the horizontal and vertical momentum separately. Begin your graph before the collision takes place and continue it after the collision is over. The asteroids' initial velocities are both oriented at the same angle from horizontal. Use a consistent scale on all graphs.

a. The two asteroids remain joined together after the collision.

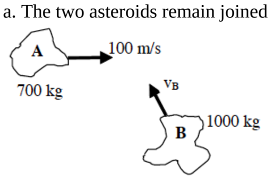


b. The two asteroids remain joined together after the collision.

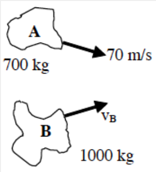


For each of the collisions illustrated below, sketch a graph of the momentum of asteroid A, the momentum of asteroid B, and the total momentum in the system of the two asteroids. Sketch the horizontal and vertical momentum separately. Begin your graph before the collision takes place and continue it after the collision is over.

a. The two asteroids remain joined together after the collision and move directly toward the top of the page.

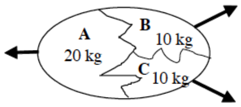


b. The two asteroids remain joined together after the collision and move directly toward the right. The asteroids' initial velocities are both oriented at the same angle from horizontal.

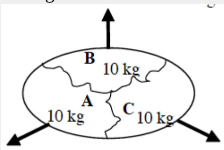


For each of the explosions illustrated below, sketch a graph of the momentum of fragment A, B, and C, and the total momentum in the system of the three asteroids. Sketch the horizontal and vertical momentum separately. Begin your graph before the explosion takes place and continue it as the fragments move apart. The exploding egg is initially at rest.

a. Fragment A moves horizontally and fragments B and C move at the same angle from horizontal after the explosion.



b. Fragment B moves vertically and fragments A and C move at the same angle from vertical after the explosion.



For each of the scenarios described below, indicate the amount of kinetic energy and gravitational potential energy of the object at each of the events listed. Use a consistent scale throughout both motions. Set ground level as the zero-point of gravitational potential energy

a. A baseball is thrown at 30 m/s at an angle of 30 ° above horizontal over level ground.

b. A baseball is thrown at 30 m/s at an angle of 60 ° above horizontal over level ground.

For each of the scenarios described below, indicate the amount of kinetic energy and gravitational potential energy of the object at each of the events listed. Use a consistent scale throughout both motions.

a. A 60 kg skier starts from rest at the top of a 100 m, 250 slope. He doesn't push with his poles because he's afraid of going too fast. Set the bottom of the slope as the zero-point of gravitational potential energy.

b. A 60 kg skier starts from rest at the top of a 100 m, 250 slope. He doesn't push with his poles because he's afraid of going too fast. Set the top of the slope as the zero-point of gravitational potential energy.

For each of the scenarios described below, indicate the amount of kinetic energy and gravitational potential energy of each object at each of the events listed. Use a consistent scale throughout both motions. Set the initial positions of the skier and block as the zero-points of gravitational potential energy

a. In a horizontal skiing device, the skier begins from rest 35 m from the end of the skiing run. The ballast block has a mass of 50 kg and the skier has a mass of 75 kg. The coefficient of friction is extremely small.

b. In an inclined skiing device, the skier begins from rest 35 m from the end of the 200 above horizontal inclined skiing run. The ballast block has a mass of 50 kg and the skier has a mass of 75 kg. The coefficient of friction is extremely small.

A girl pulls a 35 kg sled, including the mass of her kid sister, along ice. The girl pulls on the tow rope, oriented at 400 above horizontal, with a force of 120 N until her sister begins to cry. Like clockwork, her sister always cries upon reaching a speed of 3.0 m/s. The frictional coefficient is (0.10,0.08).

Free-Body Diagram	**Mathematical Analysis**[i] Event 1: P1x = P1y = J12x = J12y = KE1 = GE1 = W12 =	Event 2: P2x = P2y = KE2 = GE2 =

a. How far has the sled moved before the little sister begins to cry?

b. What is the elapsed time before the little sister begins to cry?

A boy pulls a 30 kg sled, including the mass of his kid brother, along ice. The boy pulls on the tow rope, oriented at 60° above horizontal, with a force of 110 N for 3.0 s. At the end of the 3.0 s pull, his kid brother begins to cry. The frictional coefficient is (0.20,0.15).

Free-Body Diagram	**Mathematical Analysis**[ii] Event 1: P1x = P1y = J12x = J12y = KE1 = GE1 = W12 =	Event 2: P2x = P2y = KE2 = GE2 =

a. How fast is the sled moving before the little brother begins to cry?

b. How far has the sled moved before the little brother begins to cry?

Starting from rest, a girl can pull a sled, carrying her kid brother, 20 m in 8 s. The girl pulls on the tow rope, oriented at 300 above horizontal, with a force of 90 N. The frictional coefficient is (0.15,0.10).

Free-Body Diagram	**Mathematical Analysis**[iii] Event 1: P1x = P1y = J12x = J12y = KE1 = GE1 = W12 =	Event 2: P2x = P2y = KE2 = GE2 =

A 100 kg bicycle and rider initially move at 16 m/s up a 150 hill. The rider slams on the brakes and skids to rest. The coefficient of friction is (0.8,0.7).

Free-Body Diagram	Mathematical Analysis[iv] Event 1: P1x = P1y = J12x = J12y = KE1 = GE1 = W12 =	Event 2: P2x = P2y = KE2 = GE2 =

How far does the bike skid?

What is the elapsed time before the bike stops skidding?

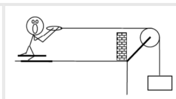
A 60 kg skier starts from rest at the top of a 100 m, 250 slope. He doesn't push with his poles because he's afraid of going too fast. The frictional coefficient is (0.10,0.05).

Free-Body Diagram	Mathematical Analysis[v] Event 1: P1x = P1y = J12x = J12y = KE1 = GE1 = W12 =	Event 2: P2x = P2y = KE2 = GE2 =

A 70 kg snowboarder starts from rest at the top of a 270 m, 200 slope. At the bottom of the hill she's moving at 33 m/s.

Free-Body Diagram	Mathematical Analysis[vi] Event 1: P1x = P1y = J12x = J12y = KE1 = GE1 = W12 =	Event 2: P2x = P2y = KE2 = GE2 =

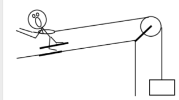
The device at right guarantees all the excitement of skiing without the need for hills. The skier begins from rest 35 m from the brick wall. The block has a mass of 50 kg and the skier has a mass of 75 kg. The coefficient of friction is (0.15,0.13).



Free-Body Diagrams

Mathematical Analysis[vii]

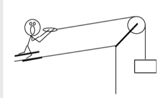
skier	*block*	Event 1: Object: KE1 = GE1 = W12 = Object: KE1 = GE1 = W12 =	Event 2: KE2 = GE2 = KE2 = GE2 =

The device at right allows novices to ski downhill at reduced speeds. The block has a mass of 10 kg and the skier has a mass of 80 kg. The coefficient of friction is (0.08,0.07). The skier starts from rest at the top of a 30 m, 200 slope.	

Free-Body Diagrams

Mathematical Analysis[viii]

skier	*block*	Event 1: Object: KE1 = GE1 = W12 = Object: KE1 = GE1 = W12 =	Event 2: KE2 = GE2 = KE2 = GE2 =

The device at right allows you to ski uphill (until you smash into the pulley). The ballast block has a mass of 60 kg and the skier has a mass of 70 kg. The coefficient of friction is (0.1,0.09). The ramp is inclined at 200 above horizontal and the pulley is 45 m away.	

Free-Body Diagrams

Mathematical Analysis[ix]

skier	*block*	Event 1: Object: KE1 = GE1 = W12 = Object: KE1 = GE1 = W12 =	Event 2: KE2 = GE2 = KE2 = GE2 =

Two identical 800 kg automobiles, one moving east at 10 m/s and the other moving north at 15 m/s, collide. After the collision they remain joined together and move with a common velocity.

Free-Body Diagrams **Mathematical Analysis**[x]

Event 1:	Event 2:
Object: P1x = P1y = J12x = J12y =	P2x = P2y =
Object: P1x = P1y = J12x = J12y =	P2x = P2y =

Two identical 750 kg automobiles, one moving east at 10 m/s and the other moving north, collide. After the collision they remain joined together and move with a common velocity. The wreckage skids at 300 north of east.

Free-Body Diagrams **Mathematical Analysis**[xi]

Event 1:	Event 2:
Object: P1x = P1y = J12x = J12y =	P2x = P2y =
Object: P1x = P1y = J12x = J12y =	P2x = P2y =

In a demolition derby, a 700 kg Audi is traveling at 15 m/s 300 north of east. An 800 kg BMW is traveling at 5.0 m/s south. They collide. After the collision, the Audi is redirected to 100 north of east and the BMW is redirected to 400 east of south.

Free-Body Diagrams **Mathematical Analysis**[xii]

Event 1:	Event 2:
Object: P1x = P1y = J12x = J12y =	P2x = P2y =
Object: P1x = P1y = J12x = J12y =	P2x = P2y =

In a demolition derby, a 600 kg Audi is traveling at 15 m/s 300 west of south. A 700 kg BMW is traveling at 10 m/s 400 north of east. They collide. After the collision, the Audi is redirected to 200 north of west and the BMW is redirected to 500 south of east.

Free-Body Diagrams

Mathematical Analysis[xiii]

Event 1: Object: P1x = P1y = J12x = J12y = Object: P1x = P1y = J12x = J12y =	Event 2: P2x = P2y = P2x = P2y =

A boy pulls an initially stationary sled of mass m (including the mass of the strange neighborhood kid riding the sled) along a level surface. He exerts a force of magnitude F at an angle of q above the horizontal. Determine the velocity (v) of the sled as a function of the distance pulled (d), the appropriate coefficient of friction between the sled and the surface, m, F, q , and g.

Free-Body Diagram	**Mathematical Analysis** Event 1: KE1 = GE1 = W12 =	Event 2: KE2 = GE2 =

Questions

If d = 0 m, what should v equal? Does your function agree with this observation?

If m = 0 kg, what should v equal? Does your function agree with this observation?

If F = 0 N, what should v equal? Does your function agree with this observation?

A skier of mass m starts from rest at the top of a slope of length D inclined at q above horizontal. She does not push with her poles. Determine the speed of the skier at the bottom of the slope (v) as a function of the appropriate coefficient of friction between the skies and the snow, D, m, q , and g.

Free-Body Diagram	**Mathematical Analysis** Event 1: KE1 = GE1 = W12 =	Event 2: KE2 = GE2 =

Questions

If g = 0 m/s2, what should v equal? Does your function agree with this observation?

If q = 0°, what should v equal? Does your function agree with this observation?

If the D is doubled, what will happen to v?

The driver of an automobile of mass m, traveling down an incline of angle q, suddenly sees an obstacle blocking her lane. Ignoring her reaction time, determine the time elapsed (T) before the car skids to a stop as a function of the initial velocity (v), the appropriate coefficient of friction between the tires and the road, m, q, and g.

Free-Body Diagram	**Mathematical Analysis** Event 1: P1x = P1y = J12x = J12y =	Event 2: P2x = P2y =

Questions

If m = 0, what should T equal? Does your function agree with this observation?

If m is doubled, what will happen to T?

If v is doubled, what will happen to T?

Is there a maximum angle above which the car will not stop? If so, determine an expression for this angle.

The driver of an automobile of mass m, traveling down an incline of angle q, suddenly sees an obstacle blocking her lane. Ignoring her reaction time, determine the distance (D) the car skids before stopping as a function of the initial velocity (v), the appropriate coefficient of friction between the tires and the road, m, q, and g.

Free-Body Diagram	**Mathematical Analysis** Event 1: KE1 = GE1 = W12 =	Event 2: KE2 = GE2 =

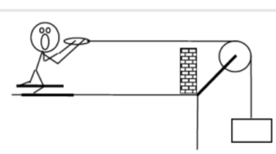
Questions

If m = 0, what should D equal? Does your function agree with this observation?

If m is doubled, what will happen to D?

If v is doubled, what will happen to D?

Is there a maximum angle above which the car will not stop? If so, determine an expression for this angle.

The device at right guarantees all the excitement of skiing without the need for hills. A skier of mass m begins from rest a distance D from the brick wall. Determine the speed of the skier as he crashes into the wall (v) as a function of the appropriate coefficient of friction between the skies and the snow, the mass of the block (M), D, m, and g.	

Free-Body Diagrams

Mathematical Analysis

skier	*block*	Event 1: Object: KE1 = GE1 = W12 = Object: KE1 = GE1 = W12 =	Event 2: KE2 = GE2 = KE2 = GE2 =

Questions

If M = 0, what should v equal? Does your function agree with this observation?

If D is doubled, what will happen to v?

What is the minimum block mass needed for the skier to move?

Two identical automobiles, one moving east at vE and the other moving north at vN, collide. After the collision they remain joined together and move with a common velocity. Determine the angle at which the wreckage skids (q), measured counterclockwise from east, as a function of vE and vN.

Free-Body Diagrams **Mathematical Analysis**

Event 1: Object: P1x = P1y = J12x = J12y = Object: P1x = P1y = J12x = J12y =	Event 2: P2x = P2y = P2x = P2y =

Questions

If vE = ∞, what should q equal? Does your function agree with this observation?

If vE = vN, what should q equal? Does your function agree with this observation?

[i] a. r2 = 2.23 m b. t2 = 1.49 s

[ii] a. v2 = 2.52 m/s b. r2 = 3.78 m

[iii] m = 51.4 kg

[iv] a. r2 = 14 m b. t2 = 1.75 s

[v] v2 = 27 m/s

[vi] m = 0.14

[vii] v2 = 14.9 m/s

[viii] v2 = 8.89 m/s

[ix] v2 = 14.3 m/s

[x] q = 56°

[xi] v2 = 5.8 m/s

[xii] v2audi = 12.8 m/s

[xiii] v2audi = 4.86 m/s

Homework 6 – Model 2: 110, 111, 112, 113, 114, 127, 129, 130, 132, 139.

CHAPTER OVERVIEW

3: Model 3

3.1: Kinematics

Kinematics

Concepts and Principles

If the forces acting on an object are not constant, then the acceleration of the object is not constant. To analyze the kinematics of an object undergoing non-constant acceleration requires the use of calculus. By re-examining our original definitions, valid in the limit of very small time intervals during which the acceleration is *approximately* constant, the relationships between position, velocity, and acceleration can be constructed in terms of the derivative.

Position

Let r(t) be the location of the object at every time, t, in the time interval of interest.

Velocity

Our original definition of velocity,

$$v=\frac{r_{final}-r_{initial}}{t_{final}-t_{initial}}$$

remains valid if the acceleration is constant. The acceleration will always be constant in the limit of infinitesimally small time intervals. By the fundamental definition of calculus, the above expression, in the limit of infinitesimally small time intervals, becomes the derivative of the position function. Thus, v(t), the velocity of the object at every time, t, is defined to be the derivative of the position function, r(t).

$$v(t)=\frac{dr(t)}{dt}$$

Acceleration

Our original definition of acceleration,

$$a=\frac{v_{final}-v_{initial}}{t_{final}-t_{initial}}$$

also remains valid only in the limit of infinitesimally small time intervals. By the fundamental definition of calculus, the above expression, in the limit of infinitesimally small time intervals, becomes the derivative of the velocity function. Thus, a(t), the acceleration of the object at every time, t, is defined to be the derivative of the velocity function, v(t).

$$a(t)=\frac{dv(t)}{dt}$$

Thus, if the position of the object is known as a function of time, the velocity and acceleration functions can be constructed through differentiation of r(t). On the other hand, if the acceleration of the object is known as a function of time, the velocity and position functions can be constructed through anti-differentiation, or integration, of a(t).

An important distinction, however, is that when integrating a(t) to form v(t), an arbitrary constant will be introduced into the expression for v(t). This constant can often be determined from knowledge of the object's velocity at some specific instant in time. Another integration, to form r(t), will introduce an additional arbitrary constant that can often be determined from knowledge of the object's position at some specific instant in time.

In closing, please remember that the kinematic relations that have been used throughout this course were derived assuming a constant acceleration. If the acceleration is not constant, those relations are *incorrect*, and the correct kinematic relationships *must* be determined through direct integration and differentiation.

Analysis Tools

Using the Calculus

Investigate the scenario described below.

> In a 100 m dash, detailed video analysis indicates that a particular sprinter's speed can be modeled as a quadratic function of time at the beginning of the race, reaching 10.6 m/s in 2.70 s, and as a decreasing linear function of time for the remainder of the race. She finished the race in 12.6 seconds.

To analyze this situation, we should first carefully determine and define the sequence of events that take place. At each of these instants, let's tabulate what we know about the motion.

Event 1: The race begins	Event 2: She reaches 10.6 m/s	Event 3: She crosses the finish line
t1 = 0 s	t2 = 2.7 s	t3 = 12.6 s
r1 = 0 m	r2 =	r3 = 100 m
v1 = 0 m/s	v2 = 10.6 m/s	v3 =
a1 =	a2 =	a3 =

Since the acceleration is no longer necessarily constant between instants of interest, it is no longer useful to speak of a_{12} or a_{23}. The acceleration, like the position and the velocity, is a *function*. What the table represents is the *value* of that function at specific instants of time.

Between event 1 and 2, the sprinter's velocity can be modeled by a generic quadratic function of time[1], or

$$v(t) = At^2 + B$$

Our job is to first determine (if possible) the arbitrary constants A and B, and then use this velocity function to find the position and acceleration functions.

Since the sprinter starts from rest, we can evaluate the function at t = 0 s and set the result equal to 0 m/s:

$$v(0) = A(0)^2 + B = 0$$
$$B = 0$$

Since we also know the sprinter reaches a speed of 10.6 m/s in 2.7 s, we can evaluate the function at t = 2.7 s and set the result equal to 10.6 m/s:

$$v(2.7) = A(2.7)^2 + 0 = 10.6$$
$$A = 1.45$$

Now that we know the two constants in the velocity function, we have a complete description of the sprinter's speed during this time interval:

$$v(t) = 1.45t^2$$

Once the velocity function is determined, we can differentiate to determine her acceleration function.

$$a(t) = \frac{dv(t)}{dt}$$
$$a(t) = \frac{d}{dt}1.45t^2$$
$$a(t) = 2.90t$$

Evaluating this function at t = 0 s and t = 2.7 s yields $a_1 = 0\ m/s^2$ and $a_2 = 7.83\ m/s^2$.

We can also integrate to determine her position function.

$$r(t) = \int v(t)dt$$
$$r(t) = \int 1.45t^2 dt$$
$$r(t) = \frac{1.45}{3}t^3 + C$$

Since we know r = 0 m when t = 0 s, we can determine the integration constant:

$$r(0) = \frac{1.45}{3}(0)^3 + C = 0$$
$$C = 0$$

Therefore, the position of the sprinter is given by the function:

$$r(t) = \frac{1.45}{3}t^3$$

Evaluating this function at t = 0 s and t = 2.7 s yields $r_1 = 0$ m and $r_2 = 9.51$ m.

During the second portion of the race, when her speed is decreasing linearly, her acceleration is constant. Therefore, we can use the kinematic relations developed for constant acceleration, and her acceleration is simply a constant value, denoted a_{23}.

$$r_3 = r_2 + v_2(t_3 - t_2) + \frac{1}{2}a_{23}(t_3 - t_2)^2$$
$$100 = 9.51 + 10.6(12.6 - 2.7) + \frac{1}{2}a_{23}(12.6 - 2.7)^2$$
$$a_{23} = -0.295 m/s^2$$

$$v_3 = v_2 + a_{23}(t_3 - t_2)$$
$$v_3 = 10.6 - 0.295(12.6 - 2.7)$$
$$v_3 = 7.68 m/s$$

She crosses the finish line running at 7.68 m/s.

Another Example

Investigate the scenario described below.

> A sports car can accelerate from rest to a speed of 40 m/s while traveling a distance of 200 m. Assume the acceleration of the car can be modeled as a decreasing linear function of time, with a maximum acceleration of 10.4 m/s^2.

Event 1: The car begins from rest	Event 2: The car reaches 40 m/s
t_1 = 0 s	t_2 =
r_1 = 0 m	r_2 = 200 m
v_1 = 0 m/s	v_2 = 40 m/s
a_1 = 10.4 m/s2	a_2 =

Between event 1 and 2, the car's acceleration can be modeled by a generic linear function of time, or

$$a(t) = At + B$$

Since the acceleration is decreasing, the maximum value occurs at t = 0 s,

$$a(0) = A(0) + B = 10.4$$
$$B = 10.4$$

Since we don't know the value of the acceleration at t_2, or even the value of t_2, we can't determine A, and all we can currently say about the acceleration function is that it is given by:

$$a(t) = At + 10.4$$

Nonetheless, we can still integrate the acceleration to determine the velocity,

$$v(t) = \int a(t)dt$$
$$v(t) = \int (At + 10.4)dt$$
$$v(t) = \frac{1}{2}At^2 + 10.4t + C$$

Since we know v = 0 m/s when t = 0 s, we can determine the integration constant:

$$v(0) = \frac{1}{2}A(0)^2 + 10.4(0) + C = 0$$
$$C = 0$$

We also know that v = 40 m/s at t_2, so:

$$v(t_2) = \frac{1}{2}At_2{}^2 + 10.4t_2 = 40$$

This equation can't be solved, since it involves two unknowns. However, if we can generate a second equation involving the same two unknowns, we can solve the two equations simultaneously. This second equation must involve the position function of the car:

$$r(t) = \int v(t)dt$$
$$r(t) = \int (\frac{1}{2}At^2 + 10.4t)dt$$
$$r(t) = \frac{1}{6}At^3 + 5.2t^2 + D$$

Since we know r = 0 when t = 0 s, we can determine the integration constant:

$$r(0) = \frac{1}{6}A(0)^3 + 5.2(0)^2 + D = 0$$
$$D = 0$$

We also know that r = 200 m at t_2, so:

$$r(t_2) = \frac{1}{6}At_2{}^3 + 5.2t_2{}^2 = 200$$

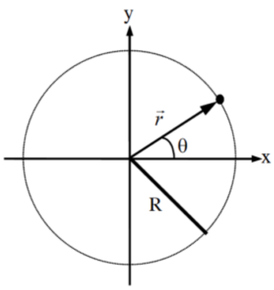
These two equations,

$$\frac{1}{2}At_2{}^2 + 10.4t_2 = 40$$
$$\frac{1}{6}At_2{}^3 + 5.2t_2{}^2 = 200$$

can be solved by substitution (or by using a solver). Solve the first equation for A, and substitute this expression into the second equation. This will result in a quadratic equation for t_2. The solution is t_2 = 7.57 s, the time for the car to reach 40 m/s. Plugging this value back into the original equations allows you to complete the description of the car's motion.

Circular Motion

If the acceleration of an object is not constant, in either magnitude or direction, the development of a kinematic description necessitates the use of calculus. A very common class of motion, in which the acceleration is guaranteed to change in at least direction, is the motion of an object on a circular path. Let's examine general circular motion in more detail before we attempt to describe a specific situation.



The x- and y-position of the object moving along a circular path of radius R can always be described by the functions:

$$r_x(t) = R\cos\theta(t)$$

$$r_y(t) = R\sin\theta(t)$$

assuming the origin of the coordinate system is placed at the center of the circle.

Defining angular position, angular velocity and angular acceleration

Angular Position

The function θ(t) specifies the *angular position* of the object, and is typically measured in radians. It specifies where, *along the circle,* the object is at every instant of time. For example, if θ(t) is a constant, the object doesn't move. If θ(t) is a linear function of time, the object moves with constant velocity around the circle. If θ(t) is a more complex function, the object speeds up or slows down as it moves around the circle.

Angular Velocity

The rate at which θ(t) is changing, $\frac{d\theta(t)}{dt}$, is termed the *angular velocity* of the object, and denoted ω(t). (ω is the lower-case Greek letter "omega".[2]) Since ω(t) is the rate at which the angular position is changing, it has units of rad/s.

Angular Acceleration

The rate at which ω(t) is changing, $\frac{d\omega(t)}{dt}$, is termed the *angular acceleration* of the object, and denoted α(t). (α is the lower-case Greek letter "alpha".) Since α(t) is the rate at which the angular velocity is changing, it has units of rad/s^2.

Deriving relationships for velocity and acceleration

Now that we have the definitions of the angular quantities out of the way, let's determine the velocity and acceleration of an object undergoing circular motion. I'll begin by writing the position function in $\hat{i}\hat{j}\hat{k}$ notation, a common "short-hand" method of writing the x-, y-, and z- components of a vector all together. In this notation, the $\hat{i}$ simply stands for the x-component, the $\hat{j}$ for the y-component, and the $\hat{k}$ for any z-component. Hold onto your hat and try not to get lost in the calculus.

Position

$$\vec{r}(t) = R(\cos\theta(t)\hat{i} + \sin\theta(t)\hat{j})$$

Velocity

$$\vec{v}(t) = \frac{d}{dt}\vec{r}(t)$$
$$\vec{v}(t) = \frac{d}{dt}[R(\cos\theta(t)\hat{i} + \sin\theta(t)\hat{j})]$$
$$\vec{v}(t) = R(\frac{d}{dt}\cos\theta(t)\hat{i} + \frac{d}{dt}\sin\theta(t)\hat{j})$$
$$\vec{v}(t) = R(-\sin\theta(t)\frac{d}{dt}\theta(t)\hat{i} + \cos\theta(t)\frac{d}{dt}\theta(t)\hat{j})$$
$$\vec{v}(t) = R(-\sin\theta(t)\omega(t)\hat{i} + \cos\theta(t)\omega(t)\hat{j})$$
$$\vec{v}(t) = R\omega(t)(-\sin\theta(t)\hat{i} + \cos\theta(t)\hat{j})$$

Acceleration

$$\vec{a}(t) = \frac{d}{dt}\vec{v}(t)$$
$$\vec{a}(t) = \frac{d}{dt}[R\omega(t)(-\sin\theta(t)\hat{i} + \cos\theta(t)\hat{j})]$$

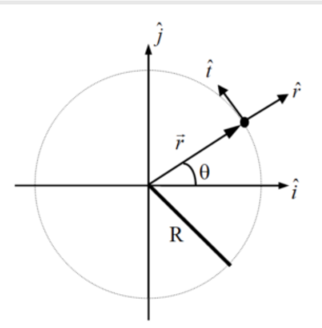
Remembering to use the chain rule for differentiation,

$$\vec{a}(t) = R[\frac{d}{dt}\omega(t)](-\sin\theta(t)\hat{i} + \cos\theta(t)\hat{j}) + R\omega(t)[\frac{d}{dt}(-\sin\theta(t)\hat{i} + \cos\theta(t)\hat{j})]$$
$$\vec{a}(t) = R\alpha(t)(-\sin\theta(t)\hat{i} + \cos\theta(t)\hat{j}) + R\omega(t)(-\cos\theta(t)\frac{d}{dt}\theta(t)\hat{i} + -\sin\theta(t)\frac{d}{dt}\theta(t)\hat{j})$$
$$\vec{a}(t) = R\alpha(t)(-\sin\theta(t)\hat{i} + \cos\theta(t)\hat{j}) + R\omega(t)(-\cos\theta(t)\omega(t)\hat{i} + -\sin\theta(t)\omega(t)\hat{j})$$
$$\vec{a}(t) = R\alpha(t)(-\sin\theta(t)\hat{i} + \cos\theta(t)\hat{j}) - R\omega^2(t)(\cos\theta(t)\hat{i} + \sin\theta(t)\hat{j})$$

These relationships for velocity and acceleration *look* intimidating, but are actually rather simple. (You don't have to believe me just yet …) The problem is that they are written using an awkward choice of coordinate system. In a previous Model, we used inclined coordinates for situations involving objects moving on an inclined surface. For an object moving in a circle, it's almost as if the surface upon which the object moves is continually changing its angle of incline! Perhaps we should use a coordinate system in which the orientation of the system continually changes, always keeping one axis parallel and one axis perpendicular to the motion. This is exactly what we will do. This coordinate system is referred to as the *polar coordinate system*.

Polar Coordinates

In the polar coordinate system, one axis (the radial axis, or $\hat{r}$) is perpendicular to the surface of the circular path pointing radially away from the center, and the other axis (the tangential, or $\hat{t}$) is parallel to the surface of the circular path pointing in the counterclockwise direction.



Notice that $\hat{r}$ is inclined by an angle q from the positive x-axis. Therefore, in terms of $\hat{i}$ and $\hat{j}$;

$$\hat{r} = \cos\theta(t)\hat{i} + \sin\theta(t)\hat{j}$$

$\hat{t}$, on the other hand, is inclined by an angle θ to the left of the positive y-axis. Therefore, in terms of $\hat{i}$ and $\hat{j}$;

$$\hat{t} = -\sin\theta(t)\hat{i} + \cos\theta(t)\hat{j}$$

Now re-examine the relationships for position, velocity and acceleration.

Position

$$\vec{r}(t) = R(\cos\theta(t)\hat{i} + \sin\theta(t)\hat{j})$$

becomes[3]

$$\vec{r}(t) = R\hat{r}$$

In component form this is:

$$r_r = R$$

$$r_t = 0$$

This means that the position of an object undergoing circular motion is *only* in the radial direction, and has a constant magnitude equal to the radius of the circle. Basically, the coordinate system is constructed so that the location of the object *defines* the radial direction.

Velocity

$$\vec{v}(t) = R\omega(t)(-\sin\theta(t)\hat{i} + \cos\theta(t)\hat{j})$$

becomes

$$\vec{v}(t) = R\omega(t)\hat{t}$$

In component form this is:

$$v_r = 0$$

$$v_t = R\omega$$

This means that the velocity of an object undergoing circular motion is *only* in the tangential direction, and has a magnitude equal to the product of the radius and angular velocity. The only way an object can have a radial velocity is if the radius of its path changes, but that can't happen for an object moving along a circular path. If the object moved along an elliptical path, for example, then it would have both tangential and radial velocities.

Acceleration

$$\vec{a}(t) = R\alpha(t)(-\sin\theta(t)\hat{i} + \cos\theta(t)\hat{j}) - R\omega^2(t)(\cos\theta(t)\hat{i} + \sin\theta(t)\hat{j})$$

becomes

$$\vec{a}(t) = R\alpha(t)\hat{t} - R\omega^2(t)\hat{r}$$

In component form this is:

$$a_r = -R\omega^2$$

$$a_t = R\alpha$$

The acceleration of an object undergoing circular motion has two components. If the object is speeding up or slowing down, the angular acceleration does not equal zero and there is an acceleration component in the tangential direction. The magnitude of the tangential acceleration is equal to the product of the radius and angular acceleration.

However, *even if the object is moving at constant speed*, there is an acceleration component in the negative radial direction, i.e., pointing *toward* the center of the circle. By virtue of traveling in a circle, the velocity vector of an object continually changes its orientation. This change in orientation is directed toward the center of the circle. Draw a motion diagram and convince yourself of this fact!

The magnitude of the radial acceleration is equal to the product of the radius and the *square* of the angular *velocity*.

The Kinematics of Circular Motion

Let's try out our new tools by examining the following scenario.

> An automobile enters a U-turn of constant radius of curvature 95 m. The car enters the U-turn traveling at 33 m/s north and exits at 22 m/s south. Assume the speed of the car can be modeled as a quadratic function of time.

To analyze this situation, we should first carefully determine and define the sequence of events that take place. At each of these events we will tabulate the position, velocity and acceleration in polar coordinates as well as the angular position, angular velocity, and angular acceleration.

Event 1: The auto enters the turn.	Event 2: The auto exits the turn.
$t_1 = 0$ s	$t_2 =$
$r_{1r} = 95$ m	$r_{2r} = 95$ m
$r_{1t} = 0$ m	$r_{2t} = 0$ m
$\theta_1 = 0$ rad	$\theta_2 = \pi$ rad
$v_{1r} = 0$ m/s	$v_{2r} = 0$ m/s
$v_{1t} = 33$ m/s	$v_{2t} = 22$ m/s
$\omega_1 =$	$\omega_2 =$
$a_{1r} =$	$a_{2r} =$
$a_{1t} =$	$a_{2t} =$
$\alpha_1 =$	$\alpha_2 =$

Notice that the position of the car is strictly in the radial direction and the velocity of the car is strictly in the tangential direction. It is *impossible*, for an object moving along a circular path, to have a non-zero tangential position or radial velocity.

Also notice that since the car completely reverses its direction of travel, it must have traveled halfway around a circular path. Thus, $\theta_2 = \pi$ rad.

We can quickly determine the angular velocity of the car at the two events by using the relationship:

$$v_t = R\omega$$

With R = 95 m, the angular velocity of the car as it enters the turn is 0.35 rad/s, and as it exits the turn, 0.23 rad/s.

Between event 1 and 2, the car's speed can be modeled by a generic quadratic function of time, or

$$v_t(t) = At^2 + B$$

Since the car enters the turn at 33 m/s, we can evaluate the function at t = 0 s and set the result equal to 33 m/s:

$$v_t(0) = A(0)^2 + B = 33$$
$$B = 33$$

The car exits the turn at 22 m/s, but since we don't know the value of t_2 we can't determine A. We do know, however, that:

$$v_t(t_2) = At_2^{\ 2} + 33 = 22$$

If we can determine another equation involving A and t_2 we can solve these two equations simultaneously.

The only other important piece of information regarding the motion is that $\theta_2 = \pi$. To make use of that information, we must "convert" our velocity equation into an angular velocity (ω) equation and then integrate the resulting equation into an equation for θ. To determine the angular velocity function,

$$v_t = R\omega$$

$$\omega = \frac{v_t}{R}$$

$$\omega(t) = \frac{At^2 + 33}{95}$$

$$\omega(t) = \frac{A}{95}t^2 + 0.35$$

The angular position (θ) is the integral of the angular velocity,

$$\theta(t) = \int \omega(t)dt$$

$$\theta(t) = \int (\frac{A}{95}t^2 + 0.35)dt$$

$$\theta(t) = \frac{A}{285}t^3 + 0.35t + D$$

Since we know $\theta = 0$ when t = 0 s, we can determine the integration constant:

$$\theta(0) = \frac{A}{285}(0)^3 + 0.35(0) + D = 0$$

$$D = 0$$

We also know that $\theta = \pi$ at t_2, so:

$$\theta(t_2) = \frac{A}{285}t_2^{\ 3} + 0.35t_2 = \pi$$

These two equations,

$$\frac{A}{285}t_2^{\ 3} + 0.35t_2 = \pi$$

$$At_2^{\ 2} + 33 = 22$$

can be solved by substitution (or by using a solver). Solve the second equation for A, and substitute this expression into the first equation. You can solve the resulting equation for t_2. The solution is t_2 = 10.0 s, the time for the car to complete the turn. Plugging this value back into the original equation allows you to determine A = -0.11.

Once you know A, you can complete the rest of the motion table. For example, since

$$\omega(t) = \frac{A}{95}t^2 + 0.35$$
$$\omega(t) = -0.00116t^2 + 0.35$$

We can now determine angular acceleration,

$$\alpha(t) = \frac{d\omega(t)}{dt}$$
$$\alpha(t) = \frac{d}{dt}(-0.00116t^2 + 0.35)$$
$$\alpha(t) = -0.00232t$$

radial acceleration,

$$a_r(t) = -R\omega(t)^2$$
$$a_r(t) = -95(-0.00116t^2 + 0.35)^2$$

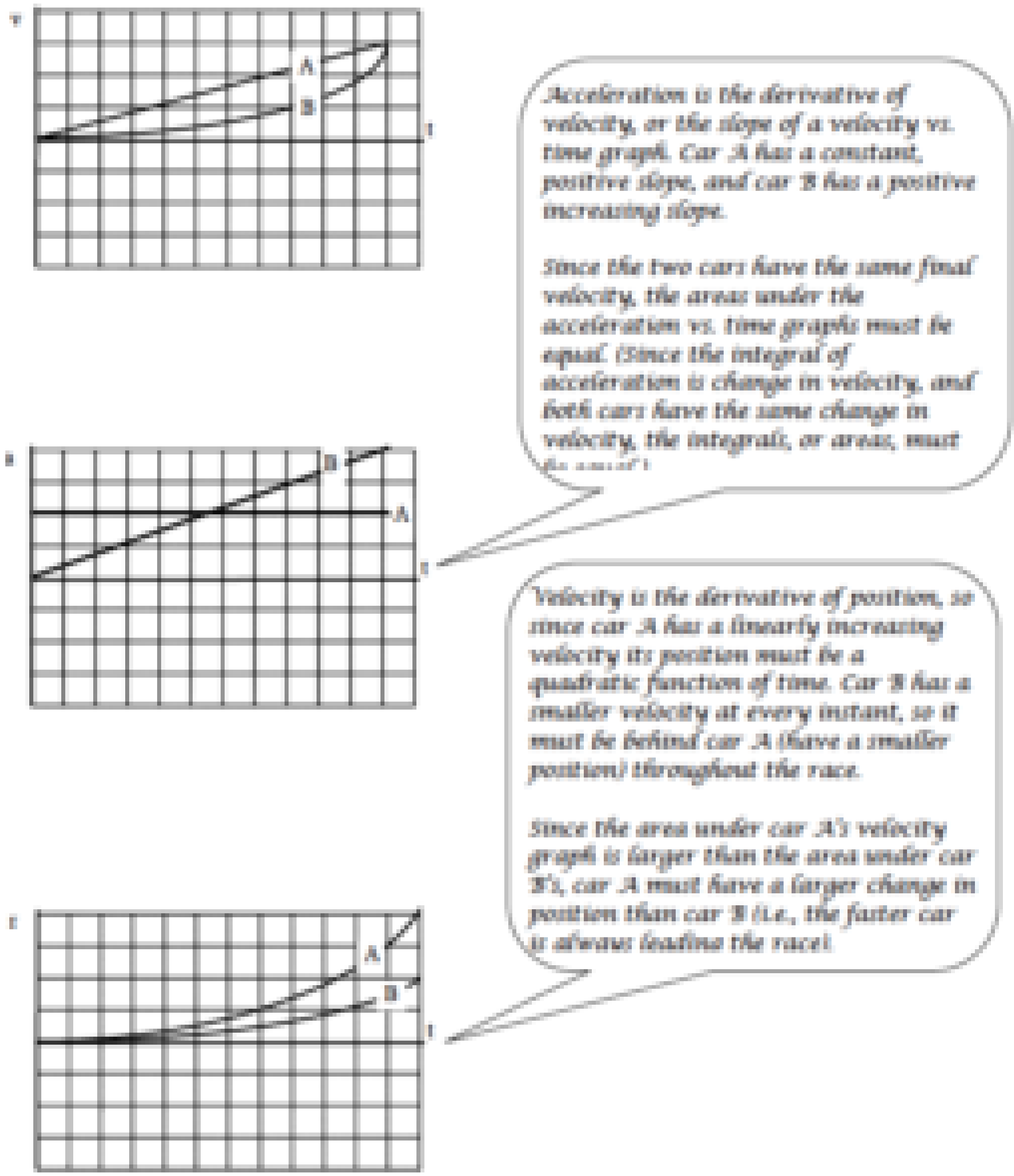
and tangential acceleration.

$$a_t(t) = R\alpha(t)$$
$$a_t(t) = 95(-0.00232t)$$
$$a_t(t) = -0.22t$$

Each of these functions could be evaluated at t_1 = 0 s and t_2 = 10.0 s to complete the table.

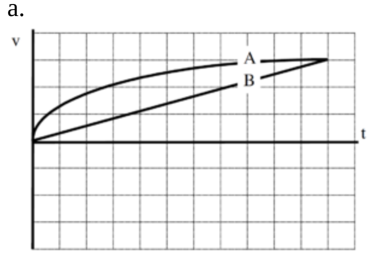
Activities

In a drag race, two cars begin from rest at the starting line and move according to the velocity vs. time graph below. Construct the position and acceleration vs. time graphs for each car.

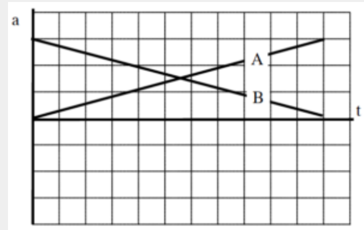


In a drag race, two cars begin from rest at the starting line and move according to the given kinematic graph. Construct the other two kinematic graphs for each car.

a.

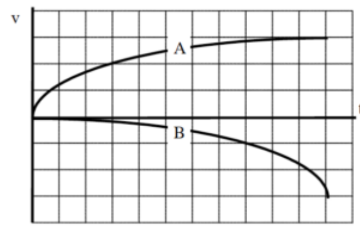


b.

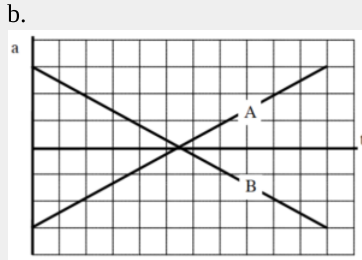


In a strange type of race, two cars begin from rest at the starting line and move according to the given kinematic graph. Construct the other two kinematic graphs for each car.

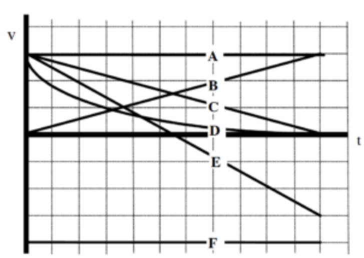
a.



b.



Below are velocity vs. time graphs for six different objects.



Rank these objects on the basis of their change in position during the time interval shown.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

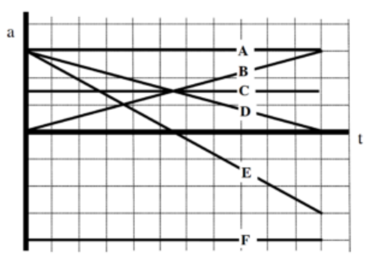
Rank these objects on the basis of their change in acceleration during the time interval shown.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are acceleration vs. time graphs for six different cars. All six cars begin a race at rest at the starting line.



Rank these cars on the basis of their final velocity at the end of the time interval shown.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Rank these cars on the basis of their final position at the end of the time interval shown.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Construct motion diagrams for the motions described below.

A satellite has been programmed to circle a stationary space station at a radius of 10 km and a constant angular speed of 0.02 rad/s.

A rider on a merry-go-round, 3 m from the axis, is traveling at 4 m/s. The merry-go-round slows, and the rider reaches a speed of 0.5 m/s in 11 seconds.

In a device built to acclimate astronauts to large accelerations, astronauts are strapped into a pod that is swung in a 6 m radius circle at high speed. The linear speed of the pod is increased from rest to a speed of 17 m/s in a time interval of 25 seconds.

Sketch position, velocity, and acceleration vs. time graphs for one complete cycle of a rider on a merry-go-round turning at constant angular speed. Set the origin of the coordinate system at the center of the merry-go-round.

Sketch the x- and y-components of this motion separately.

Sketch the r- and t-components of this motion separately.

Sketch position, velocity, and acceleration vs. time graphs for one complete cycle of a rider on a Ferris wheel as it increases its angular speed from rest. Set the origin of the coordinate system at the center of the Ferris wheel.

Sketch the x- and y-components of this motion separately.

Sketch the r- and t-components of this motion separately.

A car drives around a semi-circular U-turn at constant speed. Set the origin of the coordinate system at the center-of-curvature of the U-turn.

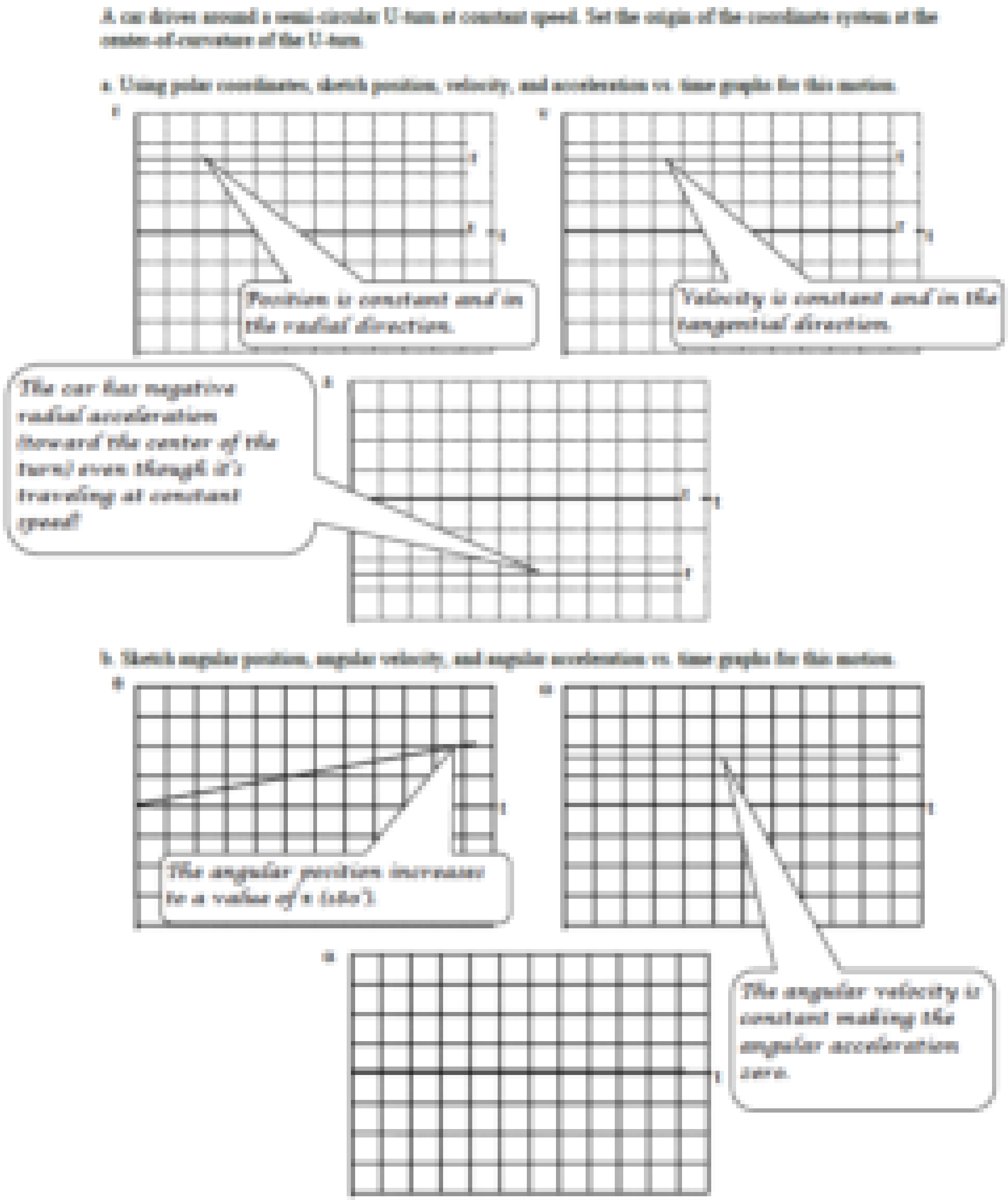
Using polar coordinates, sketch position, velocity, and acceleration vs. time graphs for this motion.

Sketch angular position, angular velocity, and angular acceleration vs. time graphs for this motion.

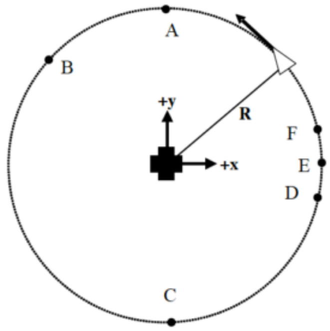
A car drives around a semi-circular U-turn at decreasing speed. Set the origin of the coordinate system at the center-of-curvature of the U-turn.

Using polar coordinates, sketch position, velocity, and acceleration vs. time graphs for this motion.

Sketch angular position, angular velocity, and angular acceleration vs. time graphs for this motion.



An artificial satellite circles a space station at constant speed. The satellite passes through the six labeled points. For all questions below, use the indicated coordinate system.



Rank the x-velocity of the satellite at each of the labeled points.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Rank the y-velocity of the satellite at each of the labeled points.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Rank the radial velocity of the satellite at each of the labeled points.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Rank the tangential velocity of the satellite at each of the labeled points.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

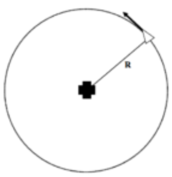
_____ The ranking cannot be determined based on the information provided.

Rank the angular velocity of the satellite at each of the labeled points.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Six artificial satellites of identical mass circle a space station with constant period T. The satellites are located a distance R from the space station.



	R	T
A	5000 m	160 hrs
B	2500 m	40 hrs
C	2500 m	80 hrs
D	10000 m	160 hrs
E	5000 m	120 hrs
F	10000 m	80 hrs

Rank these satellites on the basis of the magnitude of their angular velocity.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Rank these satellites on the basis of the magnitude of their radial velocity.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

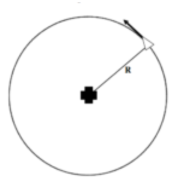
Rank these satellites on the basis of the magnitude of their tangential velocity.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your rankings:

Six artificial satellites of identical mass circle a space station at constant speed v. The satellites are located a distance R from the space station.



	R	v
A	5000 m	160 m/s
B	2500 m	40 m/s
C	2500 m	80 m/s
D	10000 m	160 m/s
E	5000 m	120 m/s
F	10000 m	80 m/s

Rank these satellites on the basis of the magnitude of their angular acceleration.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Rank these satellites on the basis of the magnitude of their radial acceleration.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Rank these satellites on the basis of the magnitude of their tangential acceleration.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your rankings:

In a 100 m dash, detailed video analysis indicated that a particular sprinter's speed can be modeled as a quadratic function of time at the beginning of a race, reaching a speed of 12.1 m/s in 1.7 s, and then as a linear function of time for the remainder of the race. She finished the race in 10.6 seconds.

Motion Information

Event 1:	Event 2:	Event 3:
t1 =	t2 =	t3 =
r1 =	r2 =	r3 =
v1 =	v2 =	v3 =
a1 =	a2 =	a3 =

Mathematical Analysis[i]

In a 400 m race, detailed video analysis indicated that a particular sprinter's speed can be modeled as a cubic function of time at the beginning of a race, reaching a speed of 8.5 m/s in 7.1 s, and as a linear function of time for the remainder of the race. She crossed the finish line traveling at 7.4 m/s.

Motion Information

Event 1:	Event 2:	Event 3:
t1 =	t2 =	t3 =
r1 =	r2 =	r3 =
v1 =	v2 =	v3 =
a1 =	a2 =	a3 =

Mathematical Analysis[ii]

Two cars, an Audi and a BMW, can accelerate from rest to a speed of 25 m/s in a time of 6.2 s. The velocity of the Audi increases as a linear function of time and the velocity of the BMW increases as a quadratic function of time.

Motion Graph **Motion Information**

	Audi Event 1:	Event 2:	BMW Event 1:	Event 2:
	t1 =	t2 =	t1 =	t2 =
	r1 =	r2 =	r1 =	r2 =
	v1 =	v2 =	v1 =	v2 =
	a1 =	a2 =	a1 =	a2 =

Question

Based only on the graph, which car travels a larger distance in 6.2 s? Explain.

Mathematical Analysis[iii]

Two cars, an Audi and a BMW, can accelerate from rest to a speed of 40 m/s after traveling a distance of 400 m. The velocity of the Audi increases as a linear function of time and the velocity of the BMW increases as a quadratic function of time.

Motion Graph **Motion Information**

	Audi Event 1:	Event 2:	BMW Event 1:	Event 2:
	t1 =	t2 =	t1 =	t2 =
	r1 =	r2 =	r1 =	r2 =
	v1 =	v2 =	v1 =	v2 =
	a1 =	a2 =	a1 =	a2 =

Question

Based only on the graph, which car will take longer to reach 40 m/s? Explain.

Mathematical Analysis[iv]

Two cars, an Audi and a BMW, accelerating from rest, can travel a distance of 400 m in 16.2 s. The velocity of the Audi increases as a linear function of time and the velocity of the BMW increases as a quadratic function of time.

Motion Graph

Motion Information

	Audi Event 1:	Event 2:	BMW Event 1:	Event 2:
	t1 =	t2 =	t1 =	t2 =
	r1 =	r2 =	r1 =	r2 =
	v1 =	v2 =	v1 =	v2 =
	a1 =	a2 =	a1 =	a2 =

Question

Based only on the graph, which car is traveling faster at 16.2 s? Explain.

Mathematical Analysis[v]

Two cars, an Audi and a BMW, can accelerate from 15 m/s to 25 m/s in a time of 3.9 s. The velocity of the Audi increases as a linear function of time and the velocity of the BMW increases as a quadratic function of time.

Motion Graph

Motion Information

	Audi Event 1:	Event 2:	BMW Event 1:	Event 2:
	t1 =	t2 =	t1 =	t2 =
	r1 =	r2 =	r1 =	r2 =
	v1 =	v2 =	v1 =	v2 =
	a1 =	a2 =	a1 =	a2 =

Question

Based only on the graph, which car travels a larger distance while accelerating? Explain.

Mathematical Analysis[vi]

Two cars, an Audi and a BMW, can slow from 35 m/s to 5 m/s over a distance of 70 m. The velocity of the Audi decreases as a linear function of time and the velocity of the BMW decreases as a quadratic function of time.

Motion Graph

Motion Information

	Audi Event 1:	Event 2:	BMW Event 1:	Event 2:
	t1 =	t2 =	t1 =	t2 =
	r1 =	r2 =	r1 =	r2 =
	v1 =	v2 =	v1 =	v2 =
	a1 =	a2 =	a1 =	a2 =

Question

Based only on the graph, which car takes a longer time to slow to 5 m/s? Explain.

Mathematical Analysis[vii]

In a hypothetical universe, the acceleration of an object subject to the gravitational force field of an Earth-like planet decreases with the amount of time in seconds, t, spent in the field as

$$a = (9.8\ m/s^2)\ e^{-0.1\ t}$$

A ball is released from rest 100 m above the ground.

Motion Information

Event 1: The ball is released.	Event 2: The ball hits the ground.
t1 =	t2 =
r1 =	r2 =
v1 =	v2 =
a1 =	a2 =

Mathematical Analysis[viii]

In a hypothetical universe, the acceleration of an object subject to the gravitational force field of an Earth-like planet decreases with the amount of time in seconds, t, spent in the field as

$$a = (9.8\ m/s^2)\ (1 - e^{-0.1\ t})$$

A ball is thrown vertically upward at 40 m/s.

Motion Information

Event 1: The ball is released.	Event 2: The ball reaches its apex.	Event 3: The ball returns to your hand.
t1 =	t2 =	t3 =
r1 =	r2 =	r3 =
v1 =	v2 =	v3 =
a1 =	a2 =	a3 =

Mathematical Analysis[ix]

In a hypothetical universe, the acceleration of an object subject to the gravitational force field of an Earth-like planet increases with the amount of time in seconds, t, spent in the field as

a = (9.8 m/s2) (1 – e -0.1 t)

A ball is released from rest 100 m above the ground.

Motion Information

Event 1: The ball is released.	Event 2: The ball hits the ground.
t1 = r1 = v1 = a1 =	t2 = r2 = v2 = a2 =

Mathematical Analysis[x]

In a hypothetical universe, the acceleration of an object subject to the gravitational force field of an Earth-like planet increases with the amount of time in seconds, t, spent in the field as

$a = (9.8\ m/s^2)(1 - e^{-0.1t})$

A ball is thrown vertically upward at 40 m/s.

Motion Information

Event 1: The ball is released.	Event 2: The ball reaches its apex.	Event 3: The ball returns to your hand.
t1 = r1 = v1 = a1 =	t2 = r2 = v2 = a2 =	t3 = r3 = v3 = a3 =

Mathematical Analysis[xi]

A rider on a merry-go-round is traveling at a constant speed of 4.0 m/s, and completes three revolutions in 14 s.

Motion Information

Mathematical Analysis[xii]

Event 1:	Event 2:
t1 = r1r = r1t = q1 = v1r = v1t = w1 = a1r = a1t = a1 =	t2 = r2r = r2t = q2 = v2r = v2t = w2 = a2r = a2t = a2 =

A rider on a merry-go-round, 3.0 m from the axis, is traveling at 4.0 m/s. The merry-go-round slows to rest over three complete revolutions. The rider's speed decreases as a linear function of time.

Motion Information **Mathematical Analysis**[xiii]

Event 1:	Event 2:
t1 = r1r = r1t = q1 = v1r = v1t = w1 = a1r = a1t = a1 =	t2 = r2r = r2t = q2 = v2r = v2t = w2 = a2r = a2t = a2 =

A rider on a 15 m diameter Ferris wheel is initially at rest. The angular speed of the Ferris wheel is increased to 1.5 rad/s over a time interval of 3.5 s. The angular acceleration of the Ferris wheel increases from 0 rad/s2 as a square-root function of time.

Motion Information **Mathematical Analysis**[xiv]

Event 1:	Event 2:
t1 =	t2 =
r1r =	r2r =
r1t =	r2t =
q1 =	q2 =
v1r =	v2r =
v1t =	v2t =
w1 =	w2 =
a1r =	a2r =
a1t =	a2t =
a1 =	a2 =

A rider on a 16 m diameter Ferris wheel is initially at traveling at 10 m/s. The Ferris wheel slows to rest over two complete revolutions. During the slow-down, the magnitude of the angular acceleration of the Ferris wheel decreases linearly from its maximum value to 0 rad/s2.

Motion Information **Mathematical Analysis**[xv]

Event 1:	Event 2:
t1 =	t2 =
r1r =	r2r =
r1t =	r2t =
q1 =	q2 =
v1r =	v2r =
v1t =	v2t =
w1 =	w2 =
a1r =	a2r =
a1t =	a2t =
a1 =	a2 =

An automobile enters a constant 90 m radius of curvature turn traveling at 25 m/s north and exits the curve traveling at 35 m/s east. Assume the speed of the car can be modeled as a square-root function of time.

Motion Information **Mathematical Analysis**[xvi]

Event 1:	Event 2:
t1 =	t2 =
r1r =	r2r =
r1t =	r2t =
q1 =	q2 =
v1r =	v2r =
v1t =	v2t =
w1 =	w2 =
a1r =	a2r =
a1t =	a2t =
a1 =	a2 =

An automobile enters a constant 90 m radius of curvature turn traveling at 25 m/s north and exits the curve traveling east. The car completes the turn in 5.4 s. Assume the speed of the car can be modeled as a quadratic function of time.

Motion Information **Mathematical Analysis**[xvii]

Event 1:	Event 2:
t1 = r1r = r1t = q1 = v1r = v1t = w1 = a1r = a1t = a1 =	t2 = r2r = r2t = q2 = v2r = v2t = w2 = a2r = a2t = a2 =

In a device built to acclimate astronauts to large accelerations, astronauts are strapped into a pod that is swung in a 6.0 m radius circle at high speed. The angular speed of the pod is increased quadratically from rest to an angular speed of 2.8 rad/s in a time interval of 20 seconds. The device is then linearly slowed to rest over a time interval of 40 seconds.

Motion Information

Mathematical Analysis[xviii]

Event 1:	Event 2:	Event 3:
t1 = r1r = r1t = q1 = v1r = v1t = w1 = a1r = a1t = a1 =	t2 = r2r = r2t = q2 = v2r = v2t = w2 = a2r = a2t = a2 =	t3 = r3r = r3t = q3 = v3r = v3t = w3 = a3r = a3t = a3 =

In a device built to acclimate astronauts to large accelerations, astronauts are strapped into a pod that is swung in a 6.0 m radius circle at high speed. The linear speed of the pod is increased quadratically from rest to a speed of 17 m/s after three complete *revolutions of the pod. The device is then linearly slowed to rest over a time interval of 35 seconds.*

Motion Information

Mathematical Analysis[xix]

Event 1:	Event 2:	Event 3:
t1 = r1r = r1t = q1 = v1r = v1t = w1 = a1r = a1t = a1 =	t2 = r2r = r2t = q2 = v2r = v2t = w2 = a2r = a2t = a2 =	t3 = r3r = r3t = q3 = v3r = v3t = w3 = a3r = a3t = a3 =

[i] v3 = 8.83 m/s

[ii] t3 = 55.5 s

[iii] r2 bmw = 51.7 m

[iv] t2 bmw = 30 s

[v] v2 bmw = 74.1 m/s

[vi] r2 bmw = 71.5 m

[vii] t2 bmw = 2.8 s

[viii] t = 4.9 s

[ix] t3 = 11.6 s

[x] t2 = 9.3 s

[xi] t3 = 19.3 s

[xii] rr = 2.97 m

[xiii] t2 = 28.3 s

[xiv] q2 = 2.1 rad

[xv] t2 = 30.2 s

[xvi] t2 = 4.46 s

[xvii] v2 = 28.5 m/s

[xviii] q3 = 74.7 rad

[xix] q3 = 68.4 rad

http://www.compadre.org/IVV/vignettes/circularMotion.cfm

Homework 7 – Model 3: 25, 27, 32, 33, 34, 36, 37, 38, 47, 51.

1. When I say a “quadratic function of time”, I mean a function that only contains a term in which the time variable is squared (along with a time-independent constant). I don’t mean a polynomial of degree two, i.e., $At^2 + Bt + C$. This may or may not agree with the terminology you learned in math class. ↵
2. Call it “omega” don’t call it “double-you”. It will make you sound smarter. ↵
3. There’s always a bit of complaining regarding this formula. It looks like it says “r is equal to r times r” but these three “r’s” have completely different meanings. $\vec{r}$ is the position of the object in space and has components in 2 (or more generally 3) directions. It can be expressed in any coordinate system. R is the magnitude of the position in polar coordinates. $\hat{r}$ is simply notation telling you that the position magnitude is measured radially away from the origin. ↵

3.2: Dynamics

Dynamics

Concepts and Principles

We are now prepared to investigate forces that are free to vary in magnitude and direction. However, you may recall that at no time in the explanation of Newton's laws was the conversation restricted to either constant acceleration or constant force. In fact, Newton's laws are completely valid regardless of the nature of the forces acting on an object, or its acceleration.[1] Thus, the relation

$$\Sigma F = ma$$

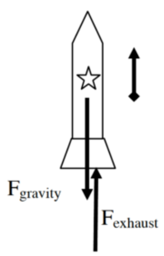
remains valid, and is still the central relationship in the study of mechanics.

Analysis Tools

Time-Dependent Forces

A 2.0 kg toy rocket is fitted with an engine that provides a thrust roughly modeled by the function F(t) = (60 N/s) t – (15 N/s2) t2, for 0 < t < 4.0 s, and zero thereafter. The rocket is launched directly upward.

Of course, the first step to analyzing any situation involving forces is to construct a free-body diagram. Below is a free-body diagram for the rocket during the time interval 0 < t < 4.0 s.



Given that you are not presented with information pertaining to the frictional force acting on the rocket, you must analyze the rocket's trajectory in a hypothetical, friction-free environment.
From Newton's second law:

$$\Sigma F = ma$$
$$F_{exhaust} - F_{gravity} = ma$$
$$60t - 15t^2 - 2(9.8) = 2a$$
$$-7.5t^2 + 30t - 9.8 = a$$

Thus, the acceleration of the rocket is not constant, and the velocity and position of the rocket must be determined by integrating the acceleration.

Before we do this, however, note that the acceleration of the rocket is initially directed downward. (At t = 0s, a = -9.8 m/s2.) This is because the supporting force exerted on the rocket by the launch platform has been ignored. This force would hold the rocket in place until the force of the exhaust gases on the rocket is equal to, and then exceeds, the force of gravity on the rocket. In reality, the acceleration of the rocket is zero until $F_{exhaust} = F_{gravity}$. This occurs when:

$$F_{exhaust} = F_{gravity}$$
$$60t - 15t^2 = 2(9.8)$$
$$-15t^2 + 60t - 19.6 = 0$$
$$t = 0.36s$$

The correct motion information is tabulated below.

Event 1: The engine is ignited	Event 2: Rocket leaves the pad	Event 3: The thrust ends	Event 4: The rocket reaches its apex
$t_1 = 0$ s	$t_2 = 0.36$ s	$t_3 = 4.0$ s	$t_4 =$
$r_1 = 0$ m	$r_2 = 0$ m	$r_3 =$	$r_4 =$
$v_1 = 0$ m/s	$v_2 = 0$ m/s	$v_3 =$	$v_4 = 0$ m/s
$a_1 = 0$ m/s^2	$a_2 = 0$ m/s^2	$a_3 =$	$a_4 = -9.8$ m/s^2

Between event 2 and 3, the rocket's acceleration is given by the function above:

$$a(t) = -7.5t^2 + 30t - 9.8$$

We can integrate the acceleration to determine the velocity,

$$v(t)=\int a(t)dt$$
$$v(t)=\int(-7.5t^2+30t-9.8)dt$$
$$v(t)=-2.5t^3+15t^2-9.8t+C$$

Since we know v = 0 m/s when t = 0.36 s, we can determine the integration constant:

$$v(0.36)=-2.5(0.36)^3+15(0.36)^2-9.8(0.36)+C=0$$
$$C=1.70m/s$$

Therefore,

$$v(t)=-2.5t^3+15t^2-9.8t+1.7$$

and when the thrust ends, at t3 = 4.0 s, v3 = 42.5 m/s.

To find the position of the rocket when the thrust ends, integrate the velocity function:

$$r(t)=\int v(t)dt$$
$$r(t)=\int(-2.5t^3+15t^2-9.8t+1.7)dt$$
$$r(t)=-0.625t^4+5t^3-4.9t^2+1.7t+D$$

Since we know r = 0 m when t = 0.36 s, we can determine the integration constant:

$$r(0.36)=-0.625(0.36)^4+5(0.36)^3-4.9(0.36)^2+1.7(0.36)+D=0$$
$$D=-0.178$$

Therefore,

$$r(t)=-0.625t^4+5t^3-4.9t^2+1.7t-0.178$$

and when the thrust ends, at t3 = 4.0 s, r3 = 88.2 m.

Between event 3 and 4 the acceleration is constant, so we can use our constant-acceleration kinematic equations:

$v_4 = v_3 + a_{34}(t_4 - t_3)$	$r_4 = r_3 + v_3(t_4 - t_3) + {}^1/_2\, a_{34}(t_4 - t_3)^2$
$0 = 42.5 + (-9.8)(t_4 - 4)$	$r_4 = 88.2 + 42.5(8.34 - 4) + {}^1/_2(-9.8)(8.34 - 4)^2$
$t_4 = 8.34$ s.	$r_4 = 180$ m.

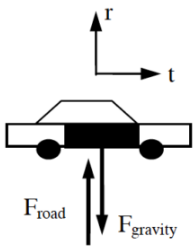
Thus, the rocket would achieve a maximum height of 180 m in a friction-free environment.

Applying Newton's Second Law to Circular Motion

Investigate the situation below, in which an object travels on a curved path.

> During a high speed auto chase in San Francisco, a 1745 kg police cruiser traveling at constant speed loses contact with the road as it goes over the crest of a hill with radius of curvature 80 m.

A free-body diagram for the car at the instant it's at the crest of the hill is sketched below.



In general you should apply Newton's second law independently in the radial (r) and tangential (t) directions, although since the cruiser is moving at constant speed there is no acceleration in the tangential direction and therefore no net force in the tangential direction.It's doubtful that the shape of the hill that the car travels on is circular. However, at every point on the hill we can approximate the path of the car with a circle of appropriate radius. This "instantaneous circular path" has a radius equal to the given radius of curvature. *Any* curved path can be approximated as a series of circular paths. Therefore, since the car is traveling along a circular path, it's best to analyze the situation using polar coordinates. Remember, in polar coordinates the radial direction points away from the center of the circular path.

r–direction

$$\Sigma F = ma$$
$$F_{road} - F_{gravity} = ma_r$$
$$F_{road} - F_{gravity} = m(-R\omega^2)$$

Since the cruiser loses contact with the road, $F_{road} = 0$ N.

$$0 - 1745(9.8) = -1745(80)\omega^2$$
$$\omega = 0.35 rad/s$$

Since $v = R\,\omega$,

$$v = (80m)(0.35 rad/s)$$
$$v = 28 m/s$$

For the car to lose contact with the road, it must be traveling at a speed of 28 m/s or greater when it reaches the crest of the hill. At lower speeds, the car would remain in contact with the road.

Since it is often useful to directly relate the radial acceleration of an object to its velocity, let's construct a direct relationship between these two variables:

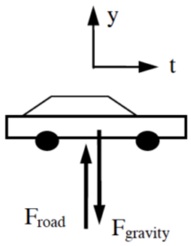
$$a_r = -R\omega^2$$
$$a_r = -R\left(\frac{v}{R}\right)^2$$
$$a_r = -\frac{v^2}{R}$$

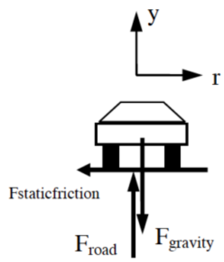
This relationship, although mathematically equivalent to $a_r = -R\omega^2$, is in a more "useful" form.

Another Circular Motion Scenario

A 950 kg car, traveling at a constant 30 m/s, safely makes a lefthand-turn with radius of curvature 75 m.

First, let's draw a pair of free-body diagrams for the car, a side-view (on the left) and a rear-view (on the right)





The free-body diagram on the left is a side-view of the car. Notice that the upward direction is the y-direction, and the forward direction, tangent to the turn, is the t-direction.

The free-body diagram on the right is a rear-view of the car. This is what you would see if you stood directly behind the car. Notice that the upward direction is still the y-direction, and the horizontal direction, *perpendicular* to the direction of travel and hence directed radially outward, is the r-direction. The frictional force indicted is *perpendicular* to the tread on the tire. This force causes the car to accelerate *toward* the center of the turn. Remember, if the car is going to travel along a circular path, it must have an acceleration directed toward the center of the circle. *Something* has to be supplying the force that creates this acceleration. This something is the *static* friction between the tire and the road that acts to prevent the car from sliding out of the turn. Since the car has no velocity in the radial direction, the frictional force that points in this direction must be static!

Now that we have all that straightened out (maybe), let's apply Newton's Second Law.

y–direction

$$\Sigma F = ma$$
$$F_{road} - F_{gravity} = ma_y$$
$$F_{road} - 950(9.8) = 950(0)$$
$$F_{road} = 9310N$$

r–direction

$$\Sigma F = ma$$
$$-F_{staticfriction} = ma_r$$
$$-F_{staticfriction} = m(-\frac{v^2}{R})$$
$$F_{staticfriction} = 950(\frac{30^2}{75})$$
$$F_{staticfriction} = 11400N$$

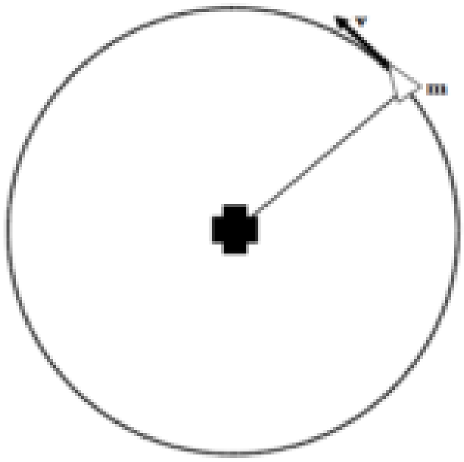
Thus, to safely make this turn requires at least 11400 N of static friction. Using this value, I should be able to compute the minimum coefficient of friction necessary for the car to safely round this turn at this speed.

$$F_{staticfriction} \leq \mu_s F_{road}$$
$$11400 \leq \mu_s (9310)$$
$$\mu_s \geq 1.22$$

Although this is a large value for the coefficient of static friction, it is an attainable value for a sports car with performance tires.

Activities

Six artificial satellites of mass, m, circle a space station at constant speed, v.



	m	v
A	200 kg	160 m/s
B	100 kg	160 m/s
C	400 kg	80 m/s
D	800 kg	40 m/s
E	200 kg	120 m/s
F	300 kg	80 m/s

If the distance between the space station and the satellites is the same for all satellites, rank the magnitude of the net force acting on each satellite.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

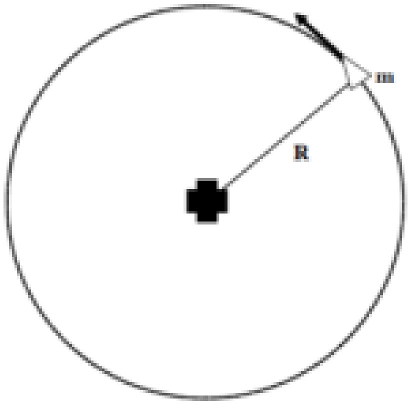
If the period is the same for all satellites, rank the magnitude of the net force acting on each satellite.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Six artificial satellites of mass, m, circle a space station at distance, R.



	m	R
A	200 kg	5000 m
B	100 kg	10000 m
C	400 kg	2500 m
D	800 kg	5000 m
E	100 kg	2500 m
F	300 kg	7500 m

If the tangential speed is the same for all satellites, rank the magnitude of the net force acting on each satellite.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

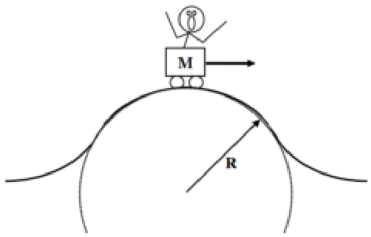
If the period is the same for all satellites, rank the magnitude of the net force acting on each satellite.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Six rollercoaster carts pass over semi-circular “bumps”. The mass of each cart (including passenger), M, and the force of the track on the cart at the apex of each bump, F, are given below.



	M	F
A	200 kg	400 N
B	100 kg	400 N
C	400 kg	200 N
D	800 kg	800 N
E	100 kg	800 N
F	300 kg	300 N

If the radius of each bump is the same, rank the speed of the cart as it passes over the apex of the bump.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

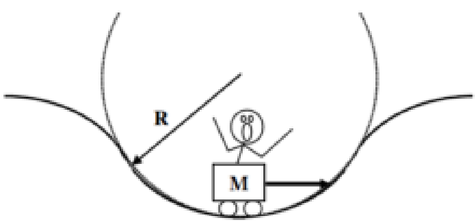
If the speed of each cart is the same at the apex, rank the radius of each bump.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

A rollercoaster track has six semi-circular “dips” with different radii of curvature, R. The rollercoaster cart rides through each dip at a different speed, v.



	R	v
A	30 m	16 m/s
B	60 m	16 m/s
C	15 m	8 m/s
D	30 m	4 m/s
E	15 m	12 m/s
F	45 m	4m/s

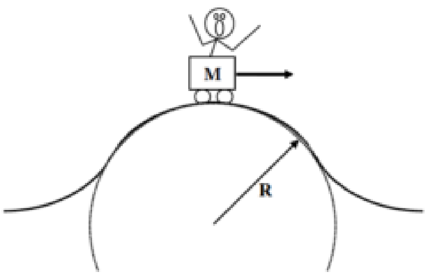
Rank the magnitude of the force of the rollercoaster track on the cart at the bottom of each dip.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

A rollercoaster track has a semi-circular “bump” of radius of curvature R. A rollercoaster cart (including passenger) of total mass M rides over the bump.



	M	R
A	200 kg	30 m
B	100 kg	60 m
C	400 kg	15 m
D	800 kg	30 m
E	100 kg	15 m
F	300 kg	45 m

Rank the minimum speed necessary for the cart to momentarily lose contact with the track at the top of the bump.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

A 100 kg rocket is fired vertically upward. Its engine supplies an upward force of magnitude $F = (5000 - 5.0t^2)$ N (where t is in seconds) until F = 0 N, then the engine shuts off.

Motion Information

Event 1:	Event 2:	Event 3:
t1 =	t2 =	t3 =
r1 =	r2 =	r3 =
v1 =	v2 =	v3 =
a1 =	a2 =	a3 =

Free-Body Diagrams

before engine turns off *after engine turns off*

Mathematical Analysis[i]

A 75 kg rocket is launched directly upward. The force on the rocket due to its engine increases from 0 N to 5000 N over 8.0 s as a linear function of time. The thrust then drops to zero almost instantaneously.

Motion Information

Event 1:	Event 2:	Event 3:	Event 4:
t1 =	t2 =	t3 =	t4 =
r1 =	r2 =	r3 =	r4 =
v1 =	v2 =	v3 =	v4 =
a1 =	a2 =	a3 =	a4 =

Free-Body Diagrams

before engine turns off *after engine turns off*

Mathematical Analysis[ii]

A 75 kg rocket is launched directly upward. The force on the rocket due to its engine decreases from 5000 N to 0 N as a quadratic function of time. The rocket reaches a speed of 200 m/s during the time interval that its engine fires.

Motion Information **Free-Body Diagrams**

Event 1:	Event 2:	Event 3:	*before engine turns off*	*after engine turns off*
t1 =	t2 =	t3 =		
r1 =	r2 =	r3 =		
v1 =	v2 =	v3 =		
a1 =	a2 =	a3 =		

Mathematical Analysis[iii]

Two 75 kg rockets, the A-57X and the B-44ZC, are launched directly upward. The force on each rocket due to its engine decreases from 5000 N to 0 N over 8.0 s. The force on the A-57X decreases as a linear function of time while the force on the B-44ZC decreases as a quadratic function of time.

Motion Information **Force Graph**

A-57X Event 1:	Event 2:	B-44ZC Event 1:	Event 2:	**DELETE**
t1 =	t2 =	t1 =	t2 =	
r1 =	r2 =	r1 =	r2 =	
v1 =	v2 =	v1 =	v2 =	
a1 =	a2 =	a1 =	a2 =	

Question

Based only on the graph, which rocket is traveling faster after 8.0 s? Explain.

Mathematical Analysis[iv]

Two 75 kg rockets, the A-57X and the B-44ZC, are launched directly upward. Both rockets reach a speed of 200 m/s after traveling for 7.1 s. The force on the A-57X due to its engine decreases to 0 N in 7.1 s as a linear function of time while the force on the B-44ZC due to its engine decreases to 0 N in 7.1 s as a quadratic function of time.

Motion Information **Force Graph**

A-57X Event 1:	Event 2:	B-44ZC Event 1:	Event 2:	**DELETE**
t1 =	t2 =	t1 =	t2 =	
r1 =	r2 =	r1 =	r2 =	

v1 =	v2 =	v1 =	v2 =
a1 =	a2 =	a1 =	a2 =

Question

Based only on the graph, which rocket's engine exerts the larger maximum force? Explain.

Mathematical Analysis[v]

The horizontal thrust acting on a 270 kg rocket sled increases as a cubic function of time from 0 N to 5400 N in 4.3 s. The thrust then drops to zero almost instantaneously. The effective frictional coefficient acting between the rocket sled and the ground is (0.4, 0.3). The rocket sled starts from rest on a level surface.

Motion Information

Free-Body Diagram

Event 1:	Event 2:	Event 3:	Event 4:	*before engine turns off*
t1 =	t2 =	t3 =	t4 =	
r1 =	r2 =	r3 =	r4 =	
v1 =	v2 =	v3 =	v4 =	
a1 =	a2 =	a3 =	a4 =	

Mathematical Analysis[vi]

The horizontal thrust acting on a 270 kg rocket sled decreases from 5000 N to 0 N as a quadratic function of time. The rocket sled is traveling at 195 m/s when the engine shuts down. The effective frictional coefficient acting between the rocket sled and the ground is (0.4, 0.3). The rocket sled starts from rest on a level surface.

Motion Information

Free-Body Diagram

Event 1:	Event 2:	Event 3:	*before engine turns off*
t1 =	t2 =	t3 =	
r1 =	r2 =	r3 =	
v1 =	v2 =	v3 =	
a1 =	a2 =	a3 =	

Mathematical Analysis[vii]

A rollercoaster cart travels through a semicircular "bump" of radius 20 m followed by a "dip" of radius 25 m. At the apex of the bump, the 50 kg passenger momentarily loses contact with her seat, and at the bottom of the dip the bathroom scale she's sitting on reads 1020 N.

How fast is the cart moving at the apex of the bump?

How fast is the cart moving at the bottom of the dip?

A 60 kg woman rides in a Ferris wheel of radius 16 m. In order to better understand physics, she takes along a bathroom scale and sits on it. When at the top, the scale reads 540 N. The Ferris wheel maintains the same speed throughout its motion. She checks the scale again at the bottom of the motion.

What is the angular speed of the Ferris wheel?

What does the bathroom scale read at the bottom of the motion?

A 1.5 kg pendulum bob swings from the end of a 2 m long string. The maximum angle from vertical reached by the pendulum is 15°, and the bob reaches a speed of 1.16 m/s as it passes through its lowest point.

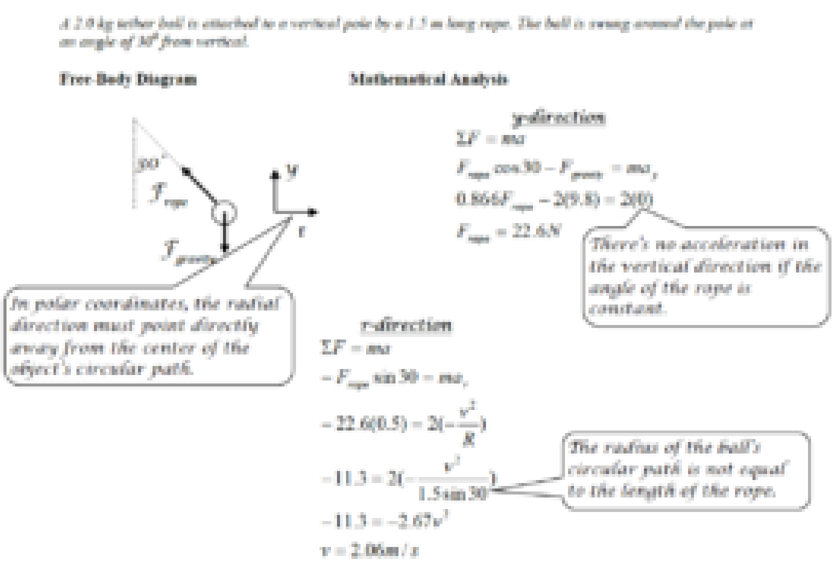
What is the force exerted on the bob by the string at the maximum angle?

What is the force exerted on the bob by the string at the lowest point?

The 65 kg strange man at right is skateboarding in a half-pipe. The half-pipe is a half-circle with a radius of 5 m. At the instant shown he is 2 m from the bottom of the half-pipe, measured vertically, and is moving at 7.5 m/s. He reaches a speed of 9.5 m/s at the bottom.

What is the force exerted by the skateboard on the man at the instant shown?

What is the force exerted by the skateboard on the man at the bottom of the half-pipe?



A 2.0 kg tether ball swings around a vertical pole attached to two 1.5 m long ropes, each at an angle of 300 from vertical. One rope is attached to the top of the ball and the top of the pole, the other rope is attached to the bottom of the ball and the bottom of the pole. The ball is traveling at 3.5 m/s.

In the hammer throw, a 7.3 kg steel ball at the end of a 1.22 m wire is swung in an approximately circular path around a thrower's head. Assume the ball is traveling at a constant speed of 7 m/s.

A 1000 kg car rounds a 50 m radius curve at constant speed without slipping. The coefficient of friction between the car's tires and the road is (0.6,0.5).

An 400 m radius, banked, highway curve is designed to allow cars to drive through the curve at a speed of 20 m/s and not slip, even when the road is extremely icy.

A 100 m radius, 8^0 *banked, highway curve has just been built. You decide to find maximum velocity with which you can drive your 1200 kg pick-up truck through this curve without sliding up or down the incline. The coefficient of friction between your enormous tires and the road is (0.7, 0.6) .*

You enter an 80 m radius, 6^0 *banked, highway curve at 30 m/s. The coefficient of friction between your tires and the road is (0.9, 0.8) .*

In an amusement park ride called the Rotor, a circular, 4.0 m radius room is spun around at high speed and then the floor is removed. The people riding the Rotor feel that they are being pressed against the wall with such a large force that they do not slide down the wall to the floor. (Obviously they do not understand physics.) The coefficient of friction between the riders and the wall is (0.6,0.5).

A rollercoaster track has a semicircular "bump" of radius R. A passenger of mass m sits on a bathroom scale in a rollercoaster cart. Determine the reading of the bathroom scale (F_{scale}) *as the cart goes over the top of the bump as a function of the speed of the cart (v), m, R, and g.*

Questions

If $R = \infty$, *what should* F_{scale} *equal? Does your function agree with this observation?*

If $v = 0$ *m/s, what should* F_{scale} *equal? Does your function agree with this observation?*

At what speed will the rider lose contact with the scale?

A woman of mass m rides in a Ferris wheel of radius R. In order to better understand physics, she takes along a bathroom scale and sits on it. Determine the ***difference*** *in scale readings between the bottom and top of the Ferris wheel* (ΔF_{scale}) *as a function of the constant angular speed of the Ferris wheel* (ω)*, m, R, and g.*

Questions

If $\omega = 0$ *rad/s, what should* ΔF_{scale} *equal? Does your function agree with this observation?*

If ω *was twice as large, what would happen to* ΔF_{scale}*?*

A banked highway curve with radius of curvature R has just been built. With bald tires on an icy morning, determine the maximum speed (v_{max}) with which you can make the turn without skidding as a function of the banking angle from horizontal (θ), the car's mass, R, and g.

Questions

If the mass of the car was twice as large, what would happen to v_{max}?

If θ = 0°, what should v_{max} equal? Does your function agree with this observation?

If θ = 90°, what should v_{max} equal? Does your function agree with this observation?

A tether ball of mass m is attached to a vertical pole by a rope of length L. Determine the angle the rope makes with the pole (θ) as an implicit function of the speed of the ball (v), m, L, and g.

Questions

If the mass of the ball was twice as large, what would happen to θ ?

If v = 0 m/s, what should θ equal? Does your function agree with this observation?

If v = ∞, what should θ equal? Does your function agree with this observation?

[i] t3 = 108 s

[ii] t4 = 27.8 s

[iii] t3 = 26.2 s

[iv] v2B = 278 m/s

[v] r2B = 950 m

[vi] t4 = 8.96 s

[vii] t3 = 87.1 s

[viii] a. v = 14.0 m/s b. v = 16.3 m/s

[ix] a. w = 0.224 rad/s b. F = 636 N

[x] a. F = 14.2 N b. F = 15.7 N

[xi] a. F = 1114 N b. F = 1810 N

[xii] F top rope = 44 N

[xiii] q = 76.8°

[xiv] vmax = 17.1 m/s

[xv] q = 5.8°

[xvi] vmax = 30 m/s

[xvii] too fast!

[xviii] wmin = 2 rad/s

Homework 8 – Model 3: 62, 63, 64, 65, 66, 75, 76, 80, 88, 89.

1. Actually, Newton's Second Law is known to be inadequate to explain phenomenon occurring at extremely high speeds or at extremely small distances. The speeds must be comparable to the speed of light or the distances must be comparable to the size of the atom for these effects to be noticeable. ↵

3.3: Conservation Laws

Concepts and Principles

The Impulse-Momentum Relation

We've already used the impulse-momentum relation to analyze situations involving constant forces. The relation is typically applied in its component form:

$$mv_{x\,i} + \Sigma(F_x(\Delta t)) = mv_{x\,f}$$

$$mv_{y\,i} + \Sigma(F_y(\Delta t)) = mv_{y\,f}$$

$$mv_{z\,i} + \Sigma(F_z(\Delta t)) = mv_{z\,f}$$

Hopefully it's not too much of a stretch to argue that for forces that vary in magnitude or direction the simple summation over a time interval (Dt) must be replaced by an integral over an infinitesimal time (dt):

$$mv_{x\,i} + \int_{t_i}^{t_f} F_x dt = mv_{x\,f}$$

$$mv_{y\,i} + \int_{t_i}^{t_f} F_y dt = mv_{y\,f}$$

$$mv_{z\,i} + \int_{t_i}^{t_f} F_z dt = mv_{z\,f}$$

Regardless of whether the forces acting on an object are constant or not, the impulse they exert on the object is precisely equal to the change in the object's momentum.

The Work-Energy Relation

Our previous encounter with the work-energy relation resulted in:

$$\frac{1}{2}mv_i^2 + mgh_i + \Sigma(|F||\Delta r|\cos\phi) = \frac{1}{2}mv_f^2 + mgh_f$$

where the forces acting on the object of interest were constant in both magnitude and direction. Again, for forces that vary, I will generalize this result to:

$$\frac{1}{2}mv_i^2 + mgh_i + \int_{r_i}^{r_f}(F\cos\phi)dr = \frac{1}{2}mv_f^2 + mgh_f$$

where the angle f is the angle between the instantaneous force acting on the object and the instantaneous displacement of the object (dr). Thus, this angle can change as the object moves along its path. Technically, this integral is termed a *line integral* and its evaluation can be rather complicated.

Also recall from our previous discussion of work-energy that this is *not* a vector equation, meaning it is not applied independently in each of the coordinate directions.

Analysis Tools

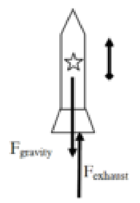
Applying the Impulse-Momentum Relation

Let's re-examine the same situation we examined at the beginning of the previous chapter, a rocket launched directly upward with a time-dependent thrust.

> A 2.0 kg toy rocket is fitted with an engine that provides a thrust roughly modeled by the function F(t) = (60 N/s) t – (15 N/s2) t2, for 0 < t < 4.0 s, and zero thereafter. The rocket is launched directly upward.

For analysis, we'll apply the impulse-momentum relation between:

> Event 1: The instant the rocket leaves the launch-pad.
>
> Event 2: The instant the thrust drops to zero.



$$mv_{y_i} + \int_{t_i}^{t_f} F_y dt = mv_{y_f}$$

$$2(0) + \int_{t_i}^{t_f} (F_{thrust} - F_{gravity})dt = 2v_{y_f}$$

$$\int_{0.36}^{4} (60t - 15t^2 - 2(9.8))dt = 2v_{y_f}$$

Remember from last chapter that the rocket does not leave the launch pad until 0.36 s after the engine is ignited.

$$\int_{0.36}^{4} (60t - 15t^2 - 19.6)dt = 2v_{y_f}$$

$$\left[30t^2 - 5t^3 - 19.6t\right]_{0.36}^{4} = 2v_{y_f}$$

$$(30(4)^2 - 5(4)^3 - 19.6(4)) - (30(0.36)^2 - 5(0.36)^3 - 19.6(0.36)) = 2v_{y_f}$$

$$v_{y_f} = 42.5 m/s$$

When the engine shuts off, the rocket is traveling at 42.5 m/s upward.

We could also apply the impulse-momentum relation between:

Event 1: The instant the thrust drops to zero.

Event 2: The instant the rocket reaches its maximum height.

During this interval, the only force acting on the rocket is the force of gravity, and the impulse-momentum relation is:

$$mv_{y_i} + \int_{t_i}^{t_f} F_y dt = mv_{y_f}$$

$$2(42.5) + \int_{4}^{t_f} (-F_{gravity})dt = 2(0)$$

$$85 + \int_{4}^{t_f} (-19.6)dt = 0$$

$$85 - 19.6(t_f) + 19.6(4) = 0$$

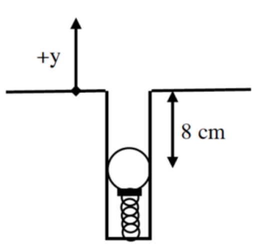
$$t_f = 8.34s$$

Thus, the rocket reaches its highest altitude 8.34 s after launch.

It's important to note that when a force is a function of time, it's relatively easy to integrate the function and determine the impulse. However, it should be clear that it would *not* be easy to determine the work done by a force of this type. Since work is expressed as an integral of a force with respect to a displacement (dr), the force function has to be expressed in terms of position, r. In general, it's not an easy (or sometimes possible) task to "convert" a function of time into a function of position, so work-energy is not a particularly useful way to analyze systems when the forces acting are time dependent. However, if the forces depend upon the *position* of the object, work-energy is a powerful analysis tool.

Applying the Work-Energy Relation

A 0.15 kg ball is launched vertically upward by means of a spring-loaded plunger, pulled back 8.0 cm and released. It requires a force of about 10 N to push the plunger back 8.0 cm.



The Force Exerted by a SpringThe force that the spring exerts on the ball depends on the amount by which the spring is compressed. The more the spring is compressed, the larger the force it exerts on the ball. A common *model* is that the force exerted by a spring is directly proportional to the amount of deformation of the spring, in this case compression. Deformation (s) is defined to be the difference between the current length of the spring (L) and the equilibrium length (L0):

$$s = L - L_0$$

If we define the positive coordinate direction to point in the same direction as positive deformation (stretch), then

$$F_{spring} = -ks$$

with the proportionality constant, k, referred to as the *spring constant*.

For the plunger, since it takes 10 N to compress the spring by 8.0 cm,

$$F_{spring} = -ks$$
$$10 = -k(-0.08)$$
$$k = 125\frac{N}{m}$$

For analysis, we'll apply the work-energy relation between:

Event 1: The instant the plunger is released.

Event 2: The instant the ball reaches its maximum height.

For these two events, work-energy looks like this:

$$\frac{1}{2}mv_i^2 + mgh_i + \int_{r_i}^{r_f}(F\cos\phi)dr = \frac{1}{2}mv_f^2 + mgh_f$$
$$0 + 0.15(9.8)(-0.08) + \int_{y_i}^{y_f}(F_{spring}\cos\phi)dy = 0 + 0.15(9.8)h_f$$

To do the integral, we must express Fspring in terms of the variable of integration, y. For the coordinate system chosen, s and y are identical. Also note that the force of the spring and the direction of motion of the ball point in the same direction. Thus, f = 0.

$$-0.118 + \int_{-0.08}^{0}(-125s)(\cos 0)ds = 1.47h_f$$
$$-0.118 + \left[-62.5s^2\right]_{-0.08}^{0} = 1.47h_f$$
$$-0.118 + \left[0 + 62.5(-0.08)^2\right] = 1.47h_f$$
$$-0.118 + 0.40 = 1.47h_f$$
$$h_f = 0.19m$$

The ball reaches a maximum height of 19 cm above the top of the plunger.

Elastic Potential Energy

When an object interacts with a spring, or other elastic material, a common model is that the material reacts linearly, i.e., with a force directly proportional to the deformation of the material. It is possible to calculate the work done by the linear material in general, and to rewrite the work-energy relation in such a way as to incorporate the effects of this work from the start. This is referred to as constructing a *potential energy function* for the work done by the elastic material. We did exactly the same thing earlier for the force of gravity.

Imagine a spring of spring constant k, initially deformed by a distance si. It changes its deformation, ultimately resulting in deformation sf. To calculate the work done by the spring on the object causing the deformation:

$$Work = \int_{r_i}^{r_f} (F\cos\phi)dr$$

$$Work = \int_{r_i}^{r_f} (-ks)(\cos\phi)dr$$

Choosing a coordinate system in which s and r are interchangeable (the origin is located at the point where the spring is at its natural length and the direction of elongation is positive) and allowing the force and the displacement to be in the same direction (f = 0) results in,

$$Work = \int_{s_i}^{s_f} (-ks)(\cos 0)ds$$

$$Work = -\frac{1}{2}ks_f^{\ 2} + \frac{1}{2}ks_i^{\ 2}$$

The terms, 1/2ks2, are referred to as *elastic potential energy*.

Inserting this result into the work-energy relation results in

$$\frac{1}{2}mv_i^{\ 2} + \frac{1}{2}ks_i^{\ 2} + mgh_i + \int_{r_i}^{r_f}(F\cos\phi)dr = \frac{1}{2}mv_f^{\ 2} + \frac{1}{2}ks_f^{\ 2} + mgh_f$$

with the understanding that the forces remaining in the equation, which may do work on the system, do not include the force of gravity or the force of the spring. The work done by the force of gravity and the force of the spring are already included in the relation via the inclusion of the potential energy terms.

Applying the Work-Energy Relation with Elastic Potential Energy

> A 0.15 kg ball is launched vertically upward by means of a spring-loaded plunger, pulled back 8.0 cm and released. It requires a force of about 10 N to push the plunger back 8.0 cm.

The spring constant of the plunger is known to be 125 N/m from above. Applying the work-energy relation with potential energy terms between the instant the plunger is released and the instant the ball reaches its maximum height results in,

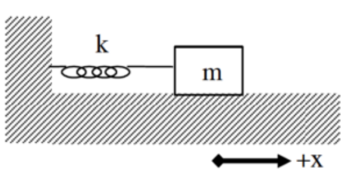
$$\frac{1}{2}mv_i^{\ 2} + \frac{1}{2}ks_i^{\ 2} + mgh_i + \int_{r_i}^{r_f}(F\cos\phi)dr = \frac{1}{2}mv_f^{\ 2} + \frac{1}{2}ks_f^{\ 2} + mgh_f$$

$$0 + \frac{1}{2}(125)(-0.08)^2 + 0.15(9.8)(-0.08) + 0 = 0 + 0 + 0.15(9.8)h_f$$

$$h_f = 0.19m$$

Of course, this results in the same answer as before.

Applying Newton's Second Law to a Spring-Mass System

	Since we didn't analyze systems involving springs in the previous dynamics section, we should make up for that omission now. The system at left consists of a spring with spring constant k attached to a block of mass m resting on a frictionless surface. The origin of the coordinate system is located at the position in which the spring is unstretched.

Now imagine the block is pulled to the right and let go. Hopefully you can convince yourself that the block will oscillate back and forth. Let's apply Newton's Second Law at the instant the mass is at an arbitrary position, x. The only force acting on the mass in the x-direction is the force of the spring.

$$\Sigma F = ma$$
$$F_{spring} = ma$$
$$-ks = ma$$

Because of our choice of coordinate system, the stretch of the spring (s) is exactly equal to the location of the block (x). Therefore,

$$-kx = ma$$

Note that when the block is at a positive position, the force of the spring is in the negative direction and when the block is at a negative position, the force of the spring is in the positive direction. Thus, the force of the spring always acts to return the block to equilibrium.

Rearranging gives

$$-\frac{k}{m}x = a$$
$$-\frac{k}{m}x = \frac{d^2x}{dt^2}$$

and defining a constant, w2, as

$$\omega^2 = \frac{k}{m}$$

(Granted, it seems pretty silly to define k/m as the square of a constant, but just play along. You may also find it frustrating to learn that this “omega” is *not* an angular velocity. The block doesn’t even have an angular velocity!)

yields,

$$-\omega^2 x = \frac{d^2x}{dt^2}$$

Therefore, the position function for the block must have a second time derivative equal to the product of (- w2) and itself. The only functions whose second time derivative is equal to the product of a negative constant and itself are the sine and the cosine functions. Therefore, a solution to this differential equation[1]

$$-\omega^2 x = \frac{d^2x}{dt^2}$$

can be written:

$$x(t) = A\cos(\omega t + \phi)$$

or equivalently with the sine function, where A and f are arbitrary constants.[2]

- **A** is the *amplitude* of the oscillation. The amplitude is the maximum displacement of the object from equilibrium.
- φ is the *phase angle*. The phase angle is used to adjust the function forward or backward in time. For example, if the particle is at the origin at t = 0 s, f must equal +p/2 or -p/2 to ensure that the cosine function evaluates to zero at t = 0 s. If the particle is at its maximum position at t = 0 s, then the phase angle must be zero or p to ensure that the cosine function evaluates to +1 or -1 at t = 0 s.
- **ω** is the *angular frequency* of the oscillation.[3]

Note that the cosine function repeats itself when its argument increases by 2p. Thus, when

$$\Delta(\omega t + \phi) = 2\pi$$

the function repeats. Since **ω** and φ are constant,

$$\Delta(\omega t + \phi) = \omega\Delta t$$

Therefore, the time interval when

$$\omega \Delta t = 2\pi$$

is the time interval for one complete cycle of the oscillatory motion. The time for one complete cycle of the motion is termed the *period*, T. Thus,

$$T = \frac{2\pi}{\omega}$$

Therefore, the physical significance of the angular frequency is that it is inversely proportional to the period.

Substituting in the definition of w:

$$\omega = \sqrt{\frac{k}{m}}$$

yields

$$T = 2\pi\sqrt{\frac{m}{k}}$$

In summary, a mass attached to a spring will oscillate about its equilibrium position with a position function given by:

$$x(t) = A\cos(\omega t + \phi)$$

This function repeats with a period of

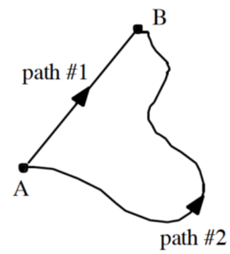
$$T = 2\pi\sqrt{\frac{m}{k}}$$

Potential Energy Functions

The potential energy functions for the work done by gravity and by springs make analyzing many situations much easier. In light of this, why don’t we construct potential energy functions for the work done by every different type of force that could possibly act on an object? Of course, one reason is that there are too many different types of forces. Having a potential energy function for the work done by every one of them would lead to so many potential energy functions that it would be hard to keep them all straight.

Another, more subtle reason is that it is impossible to construct potential energy functions for certain forces. A potential energy function must, by definition, be a *function*. Mathematically speaking, a potential energy *function* of position must assign a single, specific value of potential energy to every position. Functions must be single-valued. Notice that the gravitational and elastic potential energies are single valued. If you specify a height off of the ground, or the deformation of the spring, the potential energy function tells you exactly how much energy the system possesses at that position, regardless of the path the object took to reach that position.

In light of this observation, let’s try to create a potential energy function to replace the work done by friction.



Imagine sliding an object along a rough surface from point A to point B. If you slide the object along path #1, the force of friction will do a certain amount of work on the object. (This work will be negative because the direction of the force of friction is always in opposition to the change in position of the object.)
If you slide the object along path #2, you should see that the magnitude of the work done by friction will be greater. (Although the frictional force will be the same in magnitude, the distance over which the frictional force acts will be larger. Thus, a larger amount of negative work will be done by friction.)

Now let’s try to create a potential energy function for the work done by friction. Since we are always free to choose a coordinate system, we can choose a system in which the potential energy at A is zero. What is the value for the potential energy at B? Since the work done by friction depends on the path taken from A to B, so must the potential energy. However, this leaves us with a potential energy at B that can be either one of two values, depending on the path taken! Since a function must be single-valued, the work done by friction cannot be represented by a function. You cannot create a potential energy function for the work done by friction!

There are also other forces whose work cannot be represented by a potential energy function. In general, forces whose work can be represented by a potential energy function are termed *conservative* forces, while those for which potential energy functions cannot be constructed are termed *non-conservative*.

Activities

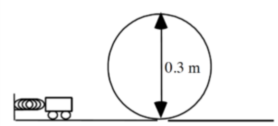
For each of the scenarios described below, indicate the amount of kinetic energy, gravitational potential energy, and elastic energy in the system at each of the events listed. Use a consistent scale throughout both motions.

A 75 kg bungee jumper steps off a platform high above a raging river and plummets downward. The elastic bungee cord has an effective spring constant of 50 N/m and is initially slack, although it begins to stretch the moment the jumper steps off of the platform. Set the lowest point of the jumper as the zero-point of gravitational potential energy

A 75 kg bungee jumper steps off a platform high above a raging river and plummets downward. The elastic bungee cord has an effective spring constant of 50 N/m and is initially slack, although it begins to stretch the moment the jumper steps off of the platform. Set the platform as the zero-point of gravitational potential energy

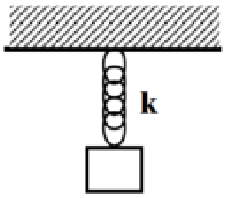
For each of the scenarios described below, indicate the amount of kinetic energy, gravitational potential energy, and elastic energy in the system at each of the events listed. Use a consistent scale throughout each motion. Set the initial position of the object as the zero-point of gravitational potential energy

A 0.27 kg toy car is held at rest against a 84 N/m compressed spring. When released, the car travels around a 0.30 m high loop. The car’s speed at the top of the loop is 2.2 m/s. Assume friction is so small that it can be ignored.



A 0.10 kg pinball is launched into a pinball machine by means of a plunger, pulled back 8.0 cm and released. The surface of the pinball machine is inclined at 130 from horizontal. Assume friction between the ball and the surface of the pinball machine during the launch of the ball is so small it can be ignored.

Six crates of different mass are hanging at rest from six springs. Each spring has a stiffness (k), a natural length (L0), and a current length (L). The natural length of the spring is its length before the crate is hung from it. The current length of the spring is its length when the crate is hung from it.



	k	L_0	L
A	20 N/m	0.4 m	0.8 m
B	20 N/m	0.3 m	0.6 m
C	10 N/m	0.4 m	0.8 m
D	10 N/m	0.3 m	1.2 m
E	30 N/m	0.4 m	0.6 m
F	30 N/m	0.2 m	0.4 m

Rank the magnitude of the force of each spring acting on each crate.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

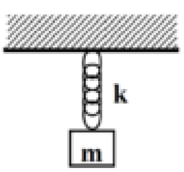
Rank the mass of each crate.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Six crates of different mass (m) are attached to springs of different stiffness (k). The masses are held in place such that none of the springs are initially stretched. All springs are initially the same length. The masses are released and the springs stretch.



	m	k
A	5 kg	20 N/m
B	20 kg	5 N/m
C	10 kg	10 N/m
D	15 kg	20 N/m
E	5 kg	5 N/m
F	15 kg	10 N/m

Rank the maximum elongation of the spring in each system.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Rank the maximum speed of the crate in each system.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

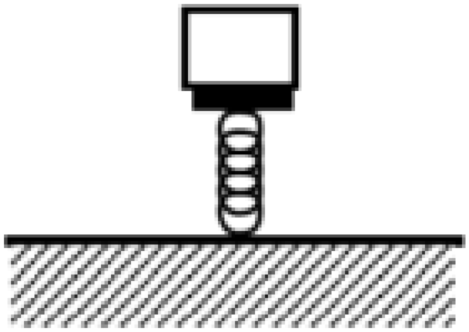
Rank the maximum acceleration magnitude of the crate in each system.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your rankings:

Six identical mass crates are at rest on six springs. Each spring has a natural length (L0) and a current length (L). The natural length of the spring is its length before the crate is placed on top of it. The current length of the spring is its length when the crate is on top of it.



	L_0	L
A	0.8 m	0.4 m
B	0.6 m	0.3 m
C	0.8 m	0.2 m
D	1.2 m	0.3 m
E	0.6 m	0.4 m
F	0.4 m	0.2 m

Rank the magnitude of the force of the spring acting on each crate.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

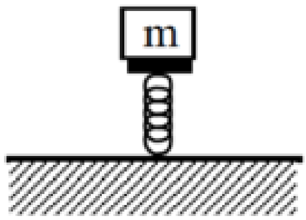
Rank the stiffness of each spring.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Six crates of different mass (m) are at rest on six springs. Each spring has a natural length (L0) and a current length (L). The natural length of the spring is its length before the crate is placed on top of it. The current length of the spring is its length when the crate is on top of it.



	m	L_0	L
A	10 kg	0.8 m	0.4 m
B	20 kg	0.6 m	0.3 m
C	10 kg	0.8 m	0.2 m
D	5 kg	1.2 m	0.3 m
E	5 kg	0.6 m	0.4 m
F	20 kg	0.4 m	0.2 m

Rank the stiffness of each spring.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

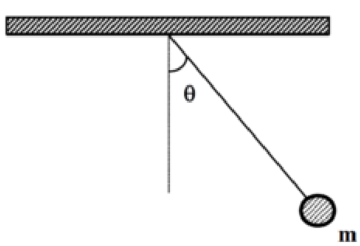
Rank the elastic potential energy stored in each spring.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

A pendulum of mass m is released from rest from an angle q from vertical. All pendulums are the same length.



	m	θ
A	2 kg	60°
B	1 kg	60°
C	4 kg	30°
D	8 kg	15°
E	2 kg	30°
F	3 kg	45°

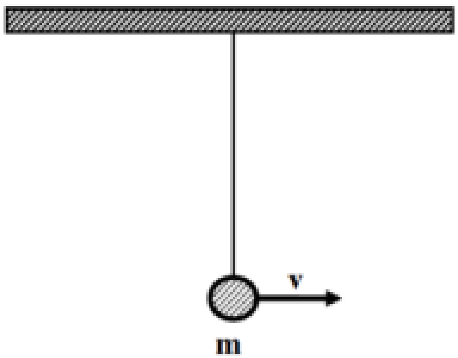
Rank the maximum speed of each pendulum.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

A pendulum of mass m is moving at velocity v as it passes through the vertical. All pendulums are the same length.



	m	v
A	2 kg	4 m/s
B	1 kg	4 m/s
C	4 kg	2 m/s
D	8 kg	1 m/s
E	2 kg	2 m/s
F	3 kg	3 m/s

Rank the maximum angle from vertical reached by the pendulum.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

A 100 kg rocket is fired vertically upward. Its engine supplies an upward force of magnitude F = (5000 – 5.0t2) N (where t is in seconds) until F = 0 N, then the engine shuts off.

How fast is the rocket moving when the engine shuts off?

Free-Body Diagram	**Mathematical Analysis**[i] Event 1: Event 2:

How long does it take the rocket to reach its maximum height?

Free-Body Diagram	**Mathematical Analysis** Event 1: Event 2:

A 75 kg rocket is launched directly upward. The force on the rocket due to its engine increases from 0 N to 5000 N over 8.0 s as a linear function of time. The thrust then drops to zero almost instantaneously.

How fast is the rocket moving when the engine shuts off?

Free-Body Diagram	**Mathematical Analysis**[ii] Event 1: Event 2:

How long does it take the rocket to reach its maximum height?

Free-Body Diagram	**Mathematical Analysis** Event 1: Event 2:

A 75 kg rocket is launched directly upward. The rocket is traveling at a speed of 200 m/s when its engine turns off. The force on the rocket due to its engine decreases to 0 N in 22 s as a quadratic function of time.

What is the maximum thrust acting on the rocket?

Free-Body Diagram	**Mathematical Analysis**[iii] Event 1: Event 2:

The horizontal thrust acting on a 270 kg rocket sled increases as a cubic function of time from 0 N to 5400 N in 4.3 s. The thrust then drops to zero almost instantaneously. The effective frictional coefficient acting between the rocket sled and the ground is (0.4, 0.3). The rocket sled starts from rest on a level surface.

How fast is the rocket sled moving when the engine shuts off?

Free-Body Diagram	**Mathematical Analysis**[iv] Event 1: Event 2:

The horizontal thrust acting on a 270 kg rocket sled decreases from 8000 N to 0 N as a quadratic function of time. The rocket sled is traveling at 195 m/s when the engine shuts down. The effective frictional coefficient acting between the rocket sled and the ground is (0.4, 0.3). The rocket sled starts from rest on a level surface.

What is the total elapsed time between the engine turning on and the sled ultimately coming to rest?

Free-Body Diagram	**Mathematical Analysis**[v] Event 1: Event 2:

A 75 kg bungee jumper is about to step off of a platform high above a raging river and plummet downward. The elastic bungee cord has an effective spring constant of 50 N/m and is initially slack, although it begins to stretch the moment the jumper steps off of the platform.

How far does the bungee jumper fall?

Free-Body Diagram	**Mathematical Analysis**[vi] Event 1: Event 2:

What is the maximum speed of the bungee jumper?

Free-Body Diagram	**Mathematical Analysis** Event 1: Event 2:

A 75 kg bungee jumper is about to step off of a platform 65 m above a raging river and plummet downward. He hopes to get just the top of his head wet. The elastic bungee cord acts as a linear spring and is initially slack, although it begins to stretch the moment the jumper steps off of the platform.

What is the spring constant of the bungee cord?

Free-Body Diagram	**Mathematical Analysis**[vii] Event 1: Event 2:

What is the maximum speed of the bungee jumper?

Free-Body Diagram	**Mathematical Analysis** Event 1: Event 2:

A 45 kg bungee jumper is about to step off of a platform 65 m above a raging river and plummet downward. She hopes to get just the top of her head wet. The elastic bungee cord acts as a linear spring and is initially slack. The cord does not begin to stretch until the jumper has fallen 10 m.

What is the spring constant of the bungee cord?

Free-Body Diagram	**Mathematical Analysis**[viii] Event 1: Event 2:

What is the maximum speed of the bungee jumper?

Free-Body Diagram	**Mathematical Analysis** Event 1: Event 2:

A 55 kg bungee jumper is about to step off of a platform 45 m above a raging river and plummet downward. She hopes to get just the top of her head wet. The elastic bungee cord acts as a linear spring with spring constant 30 N/m and is initially slack, but does not immediately begin to stretch when she steps off the platform.

How far can she safely fall before the bungee cord must begin to stretch?

Free-Body Diagram	**Mathematical Analysis**[ix] Event 1: Event 2:

How fast is she moving when she hits the river if she falls 15 m before the bungee cord begins to stretch?

Free-Body Diagram	**Mathematical Analysis** Event 1: Event 2:

A 0.10 kg pinball is launched into a pinball machine by means of a plunger, pulled back 8.0 cm and released. The surface of the pinball machine is inclined at 130 from horizontal. It requires a force of 12 N to pull the plunger back 5.0 cm. Assume friction between the ball and the surface of the pinball machine during the launch of the ball is so small it can be ignored.

What is the speed of the ball as it leaves the plunger?

Free-Body Diagram	**Mathematical Analysis**[x] Event 1: Event 2:

How far does the ball travel after leaving the plunger, assuming friction is small enough to be ignored?

Free-Body Diagram	**Mathematical Analysis** Event 1: Event 2:

At a UPS distribution center, a 40 kg crate is sliding down an 8° ramp at 3 m/s. At the bottom of the ramp, 8 m away, is a 150 N/m spring designed to bring the crate to rest. The coefficient of friction between the crate and the ramp is (0.2, 0.1).

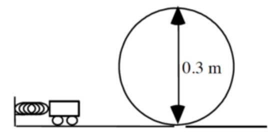
What is the speed of the crate when it hits the spring?

Free-Body Diagram	**Mathematical Analysis**[xi] Event 1: Event 2:

How far does the spring compress before bringing the crate to rest?

Free-Body Diagram	**Mathematical Analysis** Event 1: Event 2:

A 0.27 kg toy car is held at rest against a 84 N/m compressed spring. When released, the car travels around a 0.30 m high loop. The car's speed at the top of the loop is 2.2 m/s. Assume friction is so small that it can be ignored.



What is the initial compression of the launcher?

Free-Body Diagram	**Mathematical Analysis**[xii] Event 1: Event 2:

With what speed does the cart enter the loop?

Free-Body Diagram	**Mathematical Analysis** Event 1: Event 2:

A rocket of mass m is fired vertically upward. The interaction between its engine and the surroundings produces an upward force of magnitude $F = Fmax - Ct2$ N (where t is in seconds) until $F = 0$ N, then the engine shuts off. Determine the speed of the rocket at the instant the engine shuts off, voff, as a function of Fmax , C, m, and g.

Free-Body Diagram	**Mathematical Analysis** Event 1: Event 2:

Questions

If C = 0 N/s2, what should voff equal? Does your function agree with this observation?

If Fmax < mg, what should voff equal? Does your function agree with this observation?

A bungee jumper of mass m is about to step off of a platform a distance D above a rocky ravine and plummet downward. He hopes to not get the top of his head bloody. The elastic bungee cord acts as a linear spring and is initially slack. The cord does not begin to stretch until the jumper has fallen a distance d. Determine the minimum spring constant (k) needed for a safe jump as a function of m, g, D, and d.

Free-Body Diagram	**Mathematical Analysis** Event 1: Event 2:

Questions

If g = 0 m, what should k equal? Does your function agree with this observation?

If d = D, what should k equal? Does your function agree with this observation?

If m is twice as large, what effect will this have on k?

A pinball of mass m is launched into a pinball machine by means of a spring-driven plunger pulled back a distance s and released. The surface of the pinball machine is inclined at q from horizontal. Determine the speed of the ball as it leaves the plunger (v0) as a function of m, s, q , g, and the effective spring constant of the plunger system. Assume friction between the ball and the surface of the pinball machine during the launch of the ball is so small it can be ignored.

Free-Body Diagram	**Mathematical Analysis** Event 1: Event 2:

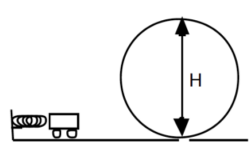
Questions

If s = 0 m, what should v0 equal? Does your function agree with this observation?

If k = 0 N/m, what should v0 equal? Does your function agree with this observation?

If the plunger is pulled back twice as far, what effect will this have on v0?

A toy car of mass m is held at rest against a compressed spring. When released, the car travels around a loop. Determine the speed of the cart (v) as a function of the angular position of the cart on the loop (let straight down be q =0°), the spring constant (k), the initial compression of the spring (s), H, m, and g. Assume friction is so small that it can be ignored.



Free-Body Diagram	**Mathematical Analysis** Event 1: Event 2:

Questions

If s = 0 m, what should v equal? Does your function agree with this observation?

If k = 0 N/m, what should v equal? Does your function agree with this observation?

If q = 0°, what should v equal? Does your function agree with this observation?

[1] A differential equation is an equation involving a function and its derivative(s).

[2] To prove to yourself that this is indeed the solution to the equation, you should substitute the function, x(t), into the left side of the equation and the second derivative of x(t) into the right side. This will verify that the two sides of the equation are equal. In addition to mathematically verifying this solution, you should verify the solution physically by sketching a graph of the motion that you *know* would result if the block were displaced to the right and comparing that sketch to a sketch of the function.

[3] Again, note that w is *not* the angular velocity. The block is *not* rotating; it does not have an angular velocity.

[i] a. v2 = 746 m/s b. t3 = 107.7 s

[ii] a. v2 = 194 m/s b. t3 = 27.8 s

[iii] Fmax = 2125 N

[iv] v = 13.7 m/s

[v] t = 11.6 s

[vi] a. Dr = 29.4 m b. v2 = 12 m/s

[vii] a. k = 22.6 N/m b. v2 = 17.9 m/s

[viii] a. k = 19 N/m b. v2 = 20.6 m/s

[ix] a. d = 4.8 m b. v = 19.8 m/s

[x] a. v2 = 3.87 m/s b. Dr = 3.4 m

[xi] a. v2 = 3.91 m/s b. s = 2.13 m

[xii] a. s = 0.19 m b. v2 = 3.27 m/s

Homework 9 – Model 3: 105, 106, 107, 109, 110, 117, 118, 121.

CHAPTER OVERVIEW

4: Model 4

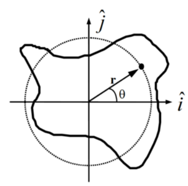
4.1: Kinematics

Kinematics

Concepts and Principles

Rotation about a Fixed Axis (Spinning)

Imagine a rigid body constrained to rotate about a fixed axis. Non-physicists would say that the object is *spinning*.



Place the origin of a coordinate system at the location of the rotation axis. Examine an arbitrary point on the object, denoted by the position vector

$$\vec{r}(t) = r(\cos\theta(t)\hat{i} + \sin\theta(t)\hat{j})$$

or

$$\vec{r}(t) = r\hat{r}$$

Notice that as the object spins, this point undergoes circular motion (denoted by the dashed line). Although the actual object may be of irregular shape, as it spins *every point on the object undergoes circular motion.* Moreover, since each and every point on the object has to complete an entire cycle around the rotation axis in the same amount of time, every point must undergo circular motion with the same angular speed (w(t)) and the same angular acceleration (a(t)).

Since every point on a rigid body must have the same angular speed and the same angular acceleration, we will speak of the angular speed and angular acceleration *of* the object, rather than the angular speed and angular acceleration of some point *on* the object.

Given that every point on a spinning object undergoes circular motion, the results from our study of circular motion will be very important in analyzing spinning objects. Recall that with q(t) defined as the angular position of an arbitrary point on the rigid-body,

$$\omega(t) = \frac{d\theta(t)}{dt}$$

and

$$\alpha(t) = \frac{d\omega(t)}{dt}$$

Moreover, the velocity and acceleration of any point on a spinning, rigid body can be related to the angular quantities:

$$v_t(t) = r\omega(t)$$

and

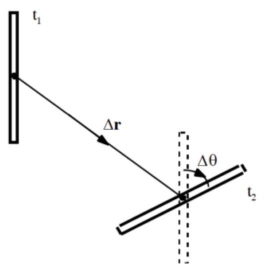
$$a_t(t) = r\alpha(t)$$

$$a_r(t) = -r\omega^2(t)$$

Rotation and Translation

To describe the motion of an object undergoing pure rotation (spinning), we have to describe the circular motion each point on the object undergoes. What must we do if the object is simultaneously rotating and translating (moving in a plane perpendicular to the rotation axis), like a wheel rolling down an incline?

The answer lies in the independence of these two types of motion. In much the same manner as we attacked kinematics in two dimensions by independently analyzing the horizontal and vertical motions, we will attack rigid-body kinematics by independently analyzing the rotational and translational motions. In short, *we will model any motion of a rigid-body as a superposition of a translation of the object's center-of-mass (CM) (which we will analyze by particle kinematics) and a rotation about an axis passing through the CM (which we will analyze by the kinematics of spinning, detailed above).*



For example, examine the motion of the thin rod between t1 and t2. Although the rod may have been spinning crazily through space between these two times, we can model its motion as a superposition of a simple translation of its CM without rotation (denoted by the vector D**r**), which leaves the rod in the orientation denoted by the dashed lines, and a simple rotation about an axis through its CM without translation (denoted by Dq), which leaves the rod in its proper, final orientation. If we imagine the time difference (t2 – t1) shrinking toward zero, hopefully it becomes plausible that we can model any motion through this method.

In summary, to describe the motion of an arbitrary rigid body we will break the motion down into a pure translation of the CM and a pure rotation about the CM. We will use particle kinematics to describe the translational portion of the motion and the kinematics of circular motion to describe the rotational portion. The velocity (or acceleration) of any point on the object is then determined by the sum of the velocity (or acceleration) due to the translation and the velocity (or acceleration) due to the rotation.

Analysis Tools

Pure Rotation

After the off button is pressed, a ceiling fan takes 22 s to come to rest. During this time, it completes 18 complete revolutions.

To analyze this situation, we should first carefully determine and define the sequence of events that take place. At each of these instants, let's tabulate what we know about the motion. Since we are dealing with pure rotation, the relevant kinematic variables are the angular position, velocity, and acceleration of the fan. Also, let's take the direction that the fan is initially rotating to be the positive direction.

Event 1: The 'off' button is pressed	Event 2: The ceiling fan stops
t1 = 0 s	t2 = 22 s
q1 = 0 rad	q2 = 18 (2p) = 113 rad
w1 =	w2 = 0 rad/s
a1 =	a2 =

Since no specific information concerning the angular acceleration of the fan is given, let's assume the angular acceleration is constant. (Thus, a1 = a2 = a12.) Since the relationships between angular position, velocity, and acceleration are the same as the relationships between linear position, velocity and acceleration, the kinematic equations for constant linear acceleration must have direct analogies for constant angular acceleration. Thus,

Now substitute this expression into the other equation:

$$\omega_2 = \omega_1 + \alpha_{12}(t_2 - t_1)$$
$$0 = \omega_1 + \alpha_{12}(22)$$
$$\omega_1 = -22\alpha_{12}$$

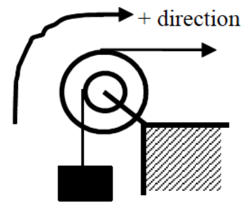
$$\omega_1 = -22(-0.47)$$
$$\omega_1 = 10.3 rad/s$$

$$\theta_2 = \theta_1 + \omega_1(t_2 - t_1) + \frac{1}{2}\alpha_{12}(t_2 - t_1)^2$$
$$113 = 0 + \omega_1(22) + \frac{1}{2}\alpha_{12}(22)^2$$
$$113 = 22(-22\alpha_{12}) + 242\alpha_{12}$$
$$113 = -242\alpha_{12}$$
$$\alpha_{12} = -0.47 rad/s^2$$

Substitute this result back into the original equation:

Notice that the sign of the angular acceleration is negative. This indicates that the angular acceleration is in the opposite direction of the angular velocity, as it should be since the fan is slowing down.

Connecting Pure Rotation to Pure Translation

The device at right is used to lift a heavy load. The free rope is attached to a truck which accelerates from rest at a rate of 1.5 m/s2. The inner radius of the pulley is 20 cm and the outer radius is 40 cm. The load must be raised 15 m.	

The coordinate system chosen indicates that the block moving upward, the pulley rotating clockwise, and the truck moving to the right are all positive.

There are three different objects that we should be able to describe kinematically; the truck, the pulley, and the block. Let's tabulate everything we know about each object:

Truck Event 1: Truck begins to move	Event 2: Block raised 15 m	***Pulley*** Event 1: Truck begins to move	Event 2: Block raised 15 m	***Block*** Event 1: Truck begins to move	Event 2: Block raised 15 m
t1 = 0 s	t2 =	t1 = 0 s	t2 =	t1 = 0 s	t2 =
r1 = 0 m	r2 =	q1 = 0 rad	q2 =	r1 = 0 m	r2 = 15 m
v1 = 0 m/s	v2 =	w1 = 0 rad/s	w2 =	v1 = 0 m/s	v2 =
a1 = 1.5 m/s2	a2 = 1.5 m/s2	a1 =	a2 =	a1 =	a2 =

The truck and block exhibit translational motion, so we only have to tabulate linear variables. The pulley spins, so the relevant variables are the angular variables. Notice that just because the block moves 15 m *doesn't* mean the truck moves 15 m. Also, the acceleration of the block is *not* equal to the acceleration of the truck. However, these variables are related to each other since both objects are attached to the same pulley.

Also notice that we can't currently solve this problem. Each of the objects has three unknown quantities. However, since the kinematics of the three objects are related, we will be able to solve the problem once we've worked out the exact relationship between each object's kinematics.

Let's start with the acceleration. Assuming the rope from the *truck* does not slip on the pulley, the point on the pulley in contact with the rope must be accelerating at the same rate as the truck. Notice that this acceleration is tangent to the pulley, and this rope is located 0.4 m from the center of the pulley. Therefore, from

$$a_t = r\alpha$$

$$1.5 = 0.4\alpha$$

$$\alpha = 3.75 rad/s^2$$

The pulley must have an angular acceleration of 3.75 rad/s2 since it is attached to the truck.

In addition, assuming the rope from the *block* does not slip on the pulley, the point on the pulley in contact with this rope must be accelerating at:

$$a_t = r\alpha$$

$$a_t = 0.2(3.75)$$

$$a_t = 0.75 m/s^2$$

Since the point on the pulley attached to the block is accelerating at 0.75 m/s2, the block itself must be accelerating at 0.75 m/s2. In a nutshell, what we've done is used the acceleration of the truck to find the angular acceleration of the pulley, and then used the angular acceleration of the pulley to find the acceleration of the block. This chain of reasoning is very common.

Now that we know the block's acceleration, we can solve the problem. Applying the two kinematic equations to the block yields:

$$r_2 = r_1 + v_1(t_2 - t_1) + \frac{1}{2}a_{12}(t_2 - t_1)^2$$

$$15 = 0 + 0 + \frac{1}{2}(0.75)t_2^2$$

$$t_2 = 6.32 s$$

$$v_2 = v_1 + a_{12}(t_2 - t_1)$$

$$v_2 = 0 + 0.75(6.32)$$

$$v_2 = 4.74 m/s$$

Now that we know the final speed of the block, we can find the final angular speed of the pulley and the final speed of the truck:

Again assuming the rope from the *block* does not slip on the pulley, the point on the pulley in contact with the rope must be moving at the same rate as the block. Notice that this velocity is tangent to the pulley, and this rope is located 0.2 m from the center of the pulley. Therefore, from

$$v_t = r\omega$$
$$4.74 = 0.2\omega$$
$$\omega = 23.7 rad/s$$

The pulley must have an angular velocity of 23.7 rad/s since it is attached to the block.

In addition, assuming the rope from the *truck* does not slip on the pulley, the point on the pulley in contact with this rope must be moving at:

$$v_t = r\omega$$
$$v_t = 0.4(23.7)$$
$$v_t = 9.48 m/s$$

The truck is moving at 9.48 m/s when the block reaches 15 m.

We can relate the displacement of the block to the angular displacement of the pulley and the displacement of the truck using

$$s = r\theta$$

where s is (technically) the arc length over which a rope is wrapped around the pulley and q is the angular displacement of the pulley. Of course, the amount of rope wrapped around a pulley is exactly equal to the displacement of the object attached to the rope. Therefore, relating the block to the pulley yields

$$s = r\theta$$
$$15 = 0.2\theta$$
$$\theta = 75 rad$$

The pulley must have turned through 75 rad since it is attached to the block.

Relating the pulley to the truck results in:

$$s = r\theta$$
$$s = 0.4(75)$$
$$s = 30m$$

The truck moved 30 m while the block moved 15 m.

Notice that in all cases, the values of the kinematic variables for the truck are exactly twice the values for the block. This is not a coincidence. Since the truck is attached to the pulley at twice the distance from the axle that the block is attached, all of its kinematic variables will have twice the value. Once you understand why this is the case, you can use this insight to simplify your analysis.

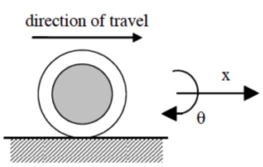
Rotating and Translating

Accelerating from rest, a Cadillac Sedan de Ville can reach a speed of 25 m/s in a time of 6.2 s. During this acceleration, the Cadillac's tires do not slip in their contact with the road. The diameter of the Cadillac's tires is 0.80 m.

Let's examine the motion of one of the Cadillac's tires. Obviously, the tire both translates and rotates. We will imagine the motion to be a superposition of a pure translation of the CM of the tire and a pure rotation about the CM of the tire.

To analyze the situation, we should define the sequence of events that take place and tabulate what we know about the motion at each event. Since we are dealing with translation as well as rotation, we will need to keep track of both the linear kinematic variables and the angular kinematic variables.

Let's take the positive x direction to be the direction that the Cadillac translates and the positive q direction to be the direction in which the tires rotate.



Event 1: The Cadillac begins to accelerate	Event 2: The Cadillac reaches 25 m/s
t1 = 0 s r1 = 0 m q1 = 0 rad v1 = 0 m/s w1 = 0 rad/s a1 = a1 =	t2 = 6.2 s r2 = q2 = v2 = 25 m/s w2 = a2 = a2 =

First, let’s examine the translational portion of the tire’s motion.

$$v_2 = v_1 + a_{12}(t_2 - t_1)$$
$$25 = 0 + a_{12}(6.2)$$
$$a_{12} = 4.03 m/s^2$$

$$r_2 = r_1 + v_1(t_2 - t_1) + \frac{1}{2}a_{12}(t_2 - t_1)^2$$
$$r_2 = 0 + 0 + \frac{1}{2}(4.03)(6.2)^2$$
$$r_2 = 77.5 m$$

What about the rotational kinematics of the tire? The motion of the tire is the superposition of the pure translational motion of the CM and the pure rotational motion about the CM[1]. Thus, the velocity of any point on the tire is given by

$$\vec{v}_{\text{any point on tire}} = \vec{v}_{\text{due to translation of CM}} + \vec{v}_{\text{due to rotation about CM}}$$
$$\vec{v}_{\text{any point on tire}} = \vec{v}_{\text{CM}} + r\omega\hat{t}$$

where r is the distance between the point of interest and the rotation axis, i.e., the distance from the CM.

The key insight into studying the rotation of the tire is to realize that *at any instant the velocity of the point on the tire in contact with the road is zero because the tire never slips in its contact with the road*. Thus, if the CM of the tire is moving forward at

$$\vec{v}_{\text{CM}} = (25 m/s)\hat{i}$$

the point on the bottom of the tire must be moving backward *relative to the CM* at the exact same speed in order for its velocity *relative to the ground* to be zero! Thus, only a very particular value for w will allow the tire to roll without slipping.

From above,

$$\vec{v}_{\text{any point on tire}} = \vec{v}_{\text{CM}} + r\omega\hat{t}$$
$$0 = (25 m/s)\hat{i} + r\omega\hat{t}$$

At the bottom of the tire, , since motion tangent to the tire at the bottom of the tire is motion to the left. Thus,

$$0 = (25 m/s)\hat{i} - r\omega\hat{i}$$
$$r\omega\hat{i} = (25 m/s)\hat{i}$$
$$(0.4 m)\omega = (25 m/s)$$
$$\omega = 62.5 rad/s$$

Therefore, at the end of the acceleration, the tire has an angular speed of 62.5 rad/s.

I included all of the vector notation to ensure I did the calculation correctly, but hopefully we can understand the result without getting bogged down in notation. Basically, the bottom of the tire can’t be moving relative to the ground if the wheel rolls without slipping. Since the CM of the tire is moving forward at 25 m/s, the bottom of the tire must be moving backward at 25 m/s relative to the CM. From the CM frame of reference, the tire is just spinning, and I can apply v = r w to calculate the angular speed of the tire.

$$v_t = r\omega$$
$$25 = 0.4\omega$$
$$\omega = 62.5 rad/s$$

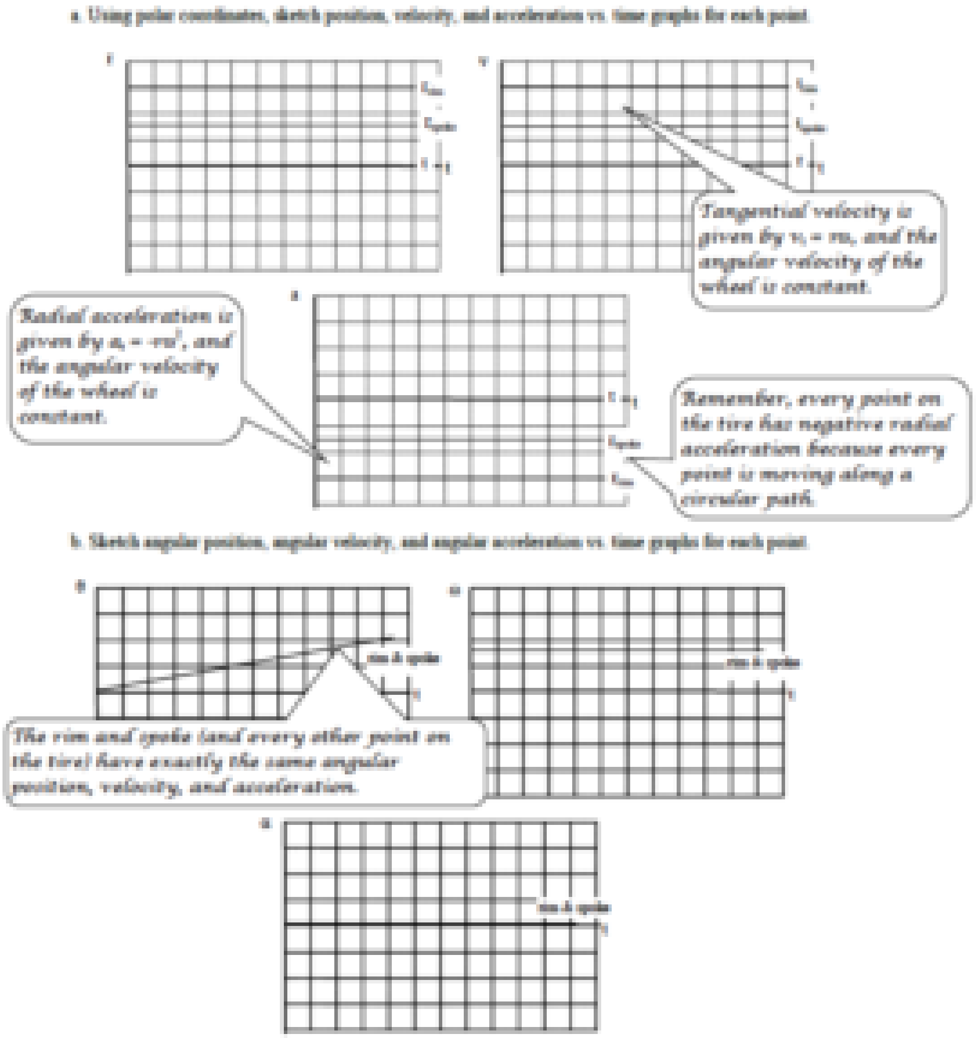
We can find the angular acceleration and angular position of the tire by using the linear variables determined above and the method described in the previous example, or by using the equations for constant angular acceleration.

$$\omega_2 = \omega_1 + \alpha_{12}(t_2 - t_1)$$
$$62.5 = 0 + \alpha_{12}(6.2)$$
$$\alpha_{12} = 10.1 rad/s^2$$

$$\theta_2 = \theta_1 + \omega_1(t_2 - t_1) + \frac{1}{2}\alpha_{12}(t_2 - t_1)^2$$
$$\theta_2 = 0 + 0 + \frac{1}{2}(10.1)(6.2)^2$$
$$\theta_2 = 194 rad$$

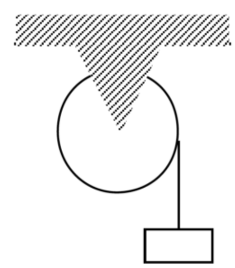
Thus, the wheel rotates through 194 rad, or 30.8 revolutions, while accelerating.

Activities



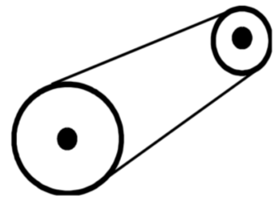
A girl pedals her bicycle, from rest, at a constantly increasing speed along a level path. Examine the motion of a point on the rim of her tire and a point midway along a spoke for one complete cycle, starting from rest. Set the origin of the coordinate system at the center of the tire.

a. Using polar coordinates, sketch position, velocity, and acceleration vs. time graphs for each point.
b. Sketch angular position, angular velocity, and angular acceleration vs. time graphs for each point.

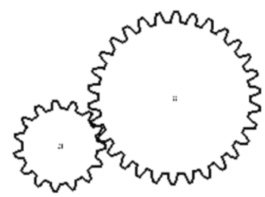
A block is attached to a rope that is wound around a pulley. Sketch velocity vs. time and acceleration vs. time graphs for a point on the rim of the pulley and for the block. Set the origin of the coordinate system at the center of the pulley.	

a. Sketch the polar components of the point on the rim’s motion and the Cartesian components of the block’s motion when the block is lowered at constant speed.

b. Sketch the polar components of the point on the rim’s motion and the Cartesian components of the block’s motion when the block is released from rest.

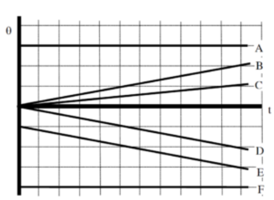
The two wheels at right are coupled together by an elastic conveyor belt that does not slip on either wheel. Examine the motion of a point on the rim of the large wheel and a point on the rim of the small wheel when the conveyor belt’s speed increases from rest. Set the origin of a coordinate system at the center of each pulley.	

a. Using polar components, sketch velocity and acceleration vs. time graphs for each point.
b. Sketch angular velocity and angular acceleration vs. time graphs for each point.

The two gears at right have equal teeth spacing and are well-meshed. Examine the motion of a point on the rim of the large gear and a point on the rim of the small gear when the small gear’s speed increases from rest in the counterclockwise direction. Set the origin of a coordinate system at the center of each gear and let counterclockwise be the positive direction.	

a. Using polar components, sketch velocity and acceleration vs. time graphs for each point.
b. Sketch angular velocity and angular acceleration vs. time graphs for each point.

Below are angular position vs. time graphs for six different objects.



Rank these graphs on the basis of the angular velocity of the object.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

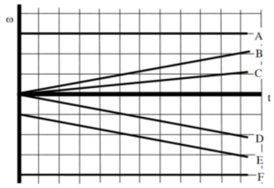
Rank these graphs on the basis of the angular acceleration of the object.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are angular velocity vs. time graphs for six different objects.



Rank these graphs on the basis of the angular acceleration of the object.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

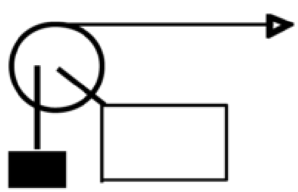
Rank these graphs on the basis of the angular displacement of the object over the time interval shown.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

The pulley below has the outer radius and inner radius indicated. In all cases, the horizontal rope is pulled to the right at the same, constant speed.



	R_{outer}	R_{inner}
A	0.4 m	0.2 m
B	0.4 m	0.3 m
C	0.8 m	0.4 m
D	0.6 m	0.5 m
E	0.2 m	0.1 m
F	0.6 m	0.2 m

Rank these scenarios on the basis of the angular speed of the pulley.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

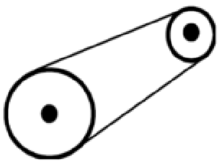
Rank these scenarios on the basis of the speed of the block.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

The two wheels, with radii indicated below, are linked together by an elastic conveyor belt that does not slip on either wheel. In all cases, the small wheel is turning at the same, constant angular speed.



	R_{large}	R_{small}
A	0.4 m	0.2 m
B	0.4 m	0.3 m
C	0.8 m	0.4 m
D	0.6 m	0.5 m
E	0.2 m	0.1 m
F	0.6 m	0.2 m

Rank the scenarios on the basis of the speed of the belt.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

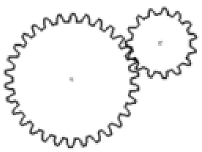
Rank the scenarios on the basis of the angular speed of the large wheel.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

The two gears, with radii indicated below, have equal teeth spacing and are well-meshed. In all cases, the large gear is turning at the same, constant angular speed.



	R_{large}	R_{small}
A	0.4 m	0.2 m
B	0.4 m	0.3 m
C	0.8 m	0.4 m
D	0.6 m	0.5 m
E	0.2 m	0.1 m
F	0.6 m	0.2 m

Rank the scenarios on the basis of the speed of the teeth on the small gear.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

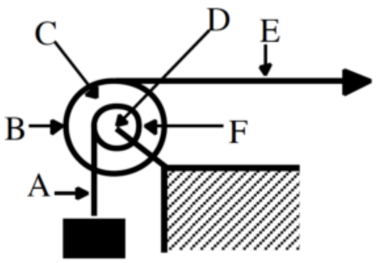
Rank the scenarios on the basis of the angular speed of the small gear.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

A pulley system is illustrated below. The horizontal rope is pulled to the right at constant speed. Each letter designates a point on either the pulley or on one of the two ropes. Neither rope slips in its contact with the pulley.



Rank the designated points on the basis of their speed.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

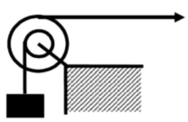
Rank the designated points on the basis of the magnitude of their acceleration.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

At the instant shown, the pulley below has the outer radius, inner radius, angular velocity, and angular acceleration indicated. Positive angular quantities are counterclockwise.



	R_{outer}	R_{inner}	ω	α
A	0.4 m	0.2 m	10 rad/s	0 rad/s^2
B	0.4 m	0.3 m	10 rad/s	1 rad/s^2
C	0.8 m	0.4 m	10 rad/s	0 rad/s^2
D	0.6 m	0.5 m	5 rad/s	-1 rad/s^2
E	0.2 m	0.1 m	20 rad/s	-4 rad/s^2
F	0.6 m	0.2 m	15 rad/s	2 rad/s^2

Rank these scenarios on the basis of the speed of the block.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

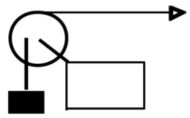
Rank these scenarios on the basis of the magnitude of the acceleration of the block.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

At the instant shown, the pulley below has the outer radius, inner radius, angular velocity, and angular acceleration indicated. Positive angular quantities are counterclockwise.



	R_{outer}	R_{inner}	ω	α
A	0.4 m	0.2 m	10 rad/s	0 rad/s^2
B	0.4 m	0.3 m	10 rad/s	0 rad/s^2
C	0.8 m	0.4 m	10 rad/s	0 rad/s^2
D	0.6 m	0.5 m	5 rad/s	0 rad/s^2
E	0.2 m	0.1 m	20 rad/s	0 rad/s^2
F	0.6 m	0.2 m	15 rad/s	0 rad/s^2

Rank these scenarios on the basis of the magnitude of the acceleration of a point on the inner rim of the pulley.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Rank these scenarios on the basis of the magnitude of the acceleration of the block.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

After being turned on, a record player (an antique device used by hipsters to listen to music) reaches its rated angular speed of 45 rpm (4.71 rad/s) in 1.5 s.

Motion Information

Event 1:	Event 2:
t1 =	t2 =
q1 =	q2 =
w1 =	w2 =
a1 =	a2 =

Mathematical Analysis[i]

A clothes dryer spins clothes at an angular speed of 6.8 rad/s. Starting from rest, the drier reaches its operating speed with an average angular acceleration of 7.0 rad/s2.

Motion Information

Event 1:	Event 2:
t1 =	t2 =
q1 =	q2 =
w1 =	w2 =
a1 =	a2 =

Mathematical Analysis[ii]

In a secret Las Vegas research laboratory, two roulette wheels (Aladdin's and Bally's) are undergoing extensive testing. Both wheels are spun at 25 rad/s. After 6.4 s, both wheels have rotated through 100 rad. However, the angular speed of Aladdin's wheel decreases as a linear function of time while the angular speed of Bally's wheel decreases as an exponential function of time, w = Ae-Bt.

Motion Graph

Motion Information

	Aladdin's Wheel Event 1:	Event 2:	Bally's Wheel Event 1:	Event 2:
	t1 =	t2 =	t1 =	t2 =
	q1 =	q2 =	q1 =	q2 =
	w1 =	w2 =	w1 =	w2 =
	a1 =	a2 =	a1 =	a2 =

Question

Based only on the graph, which roulette wheel is spinning faster after 6.4s? Explain.

Mathematical Analysis[iii]

A diver rotating at approximately constant angular velocity completes four revolutions before hitting the water. She jumped vertically upward with an initial velocity of 5 m/s from a diving board 4 m above the water.

Motion Information

Event 1:	Event 2:
t1 =	t2 =
r1 = q1 =	r2 = q2 =
v1 = w1 =	v2 = w2 =
a1 = a1 =	a2 = a2 =

Mathematical Analysis[iv]

A baton twirler throws a spinning baton directly upward. As it goes up and returns to the twirler's hand, it turns through four complete revolutions. The constant angular speed of the baton while in the air is 10 rad/s.

Motion Information

Event 1:	Event 2:
t1 =	t2 =
r1 = q1 =	r2 = q2 =
v1 = w1 =	v2 = w2 =
a1 = a1 =	a2 = a2 =

Mathematical Analysis[v]

A quarterback throws a pass that is a perfect spiral, spinning at 50 rad/s. The ball is thrown at 19 m/s at an angle of 350 above the horizontal. The ball leaves the quarterback's hand 2.0 m above the Astroturf and is caught just before it hits the turf. (The opposing coach thinks the ball was trapped. We are still waiting for the result of the challenge.)

Motion Information

Event 1:	Event 2:
t1 =	t2 =
r1x = r1y = q1 =	r2x = r2y = q2 =
v1x = v1y = w1 =	v2x = v2y = w2 =
a1x = a1y = a1 =	a2x = a2y = a2 =

Mathematical Analysis[vi]

A car, with 0.75 m diameter tires, slows from 35 m/s to 15 m/s over a distance of 70 m. The car's tires do not slip in their contact with the road.

Motion Information

Event 1:	Event 2:
t1 =	t2 =
r1 = q1 =	r2 = q2 =
v1 = w1 =	v2 = w2 =
a1 = a1 =	a2 = a2 =

Mathematical Analysis[vii]

A yo-yo of inner diameter 0.70 cm and outer diameter 8.0 cm is released from rest. The string is 0.80 m long and the yo-yo is moving downward at 0.5 m/s when it reaches the bottom of its motion. Assume the string does not slip on the inner diameter of the yo-yo during this portion of its motion.

Motion Information

Event 1:	Event 2:
t1 =	t2 =
r1 =	r2 =
q1 =	q2 =
v1 =	v2 =
w1 =	w2 =
a1 =	a2 =
a1 =	a2 =

Mathematical Analysis[viii]

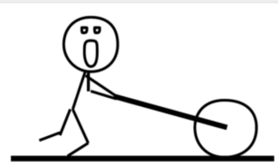
A bowling ball of diameter 21.6 cm is rolled down the alley at 4.7 m/s. The ball slows with an acceleration of 0.2 m/s2 until it strikes the pins. The pins are located 18.3 m from the release point of the ball.

Motion Information

Event 1:	Event 2:
t1 =	t2 =
r1 =	r2 =
q1 =	q2 =
v1 =	v2 =
w1 =	w2 =
a1 =	a2 =
a1 =	a2 =

Mathematical Analysis[ix]

The strange man at right is pretending to be a steamroller by rolling a heavy cylindrical object around his backyard. The cylinder has a diameter of 0.70 m. Starting from rest, the man can make the cylinder rotate through three complete revolutions in 4.5 s. The cylinder does not slip in its contact with the ground.

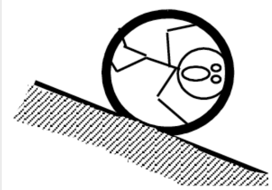


Motion Information

Event 1:	Event 2:
t1 =	t2 =
r1 =	r2 =
q1 =	q2 =
v1 =	v2 =
w1 =	w2 =
a1 =	a2 =
a1 =	a2 =

Mathematical Analysis[x]

The man at right is trapped inside a section of large pipe. If that's not bad enough, the pipe begins to roll from rest down a 35 m long, 180 incline! The pipe has a diameter of 1.2 m. The pipe (and very dizzy man) reach the bottom of the incline after 6.32 s.

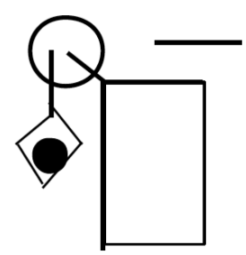


Motion Information

Event 1:	Event 2:
t1 =	t2 =
r1 =	r2 =
q1 =	q2 =
v1 =	v2 =
w1 =	w2 =
a1 =	a2 =
a1 =	a2 =

Mathematical Analysis[xi]

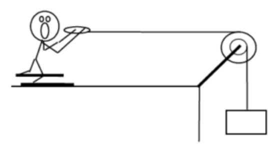
The unlucky man is falling at 20 m/s, 75 m above the crocodile-infested waters below! In an attempt to save him, the brake shoe is pressed against the spinning pulley. The action of the brake shoe gives the pulley an angular acceleration of 7.5 rad/s2. The man is saved! (Barely.)



Motion Information

Lucky Man Event 1:	Event 2:	Pulley Event 1:	Event 2:
t1 =	t2 =	t1 =	t2 =
r1 =	r2 =	q1 =	q2 =
v1 =	v2 =	w1 =	w2 =
a1 =	a2 =	a1 =	a2 =

Mathematical Analysis[xii]

The device at right guarantees all the excitement of skiing without the need for hills. The man begins from rest and reaches a speed of 17 m/s after the block falls 10 m. The inner and outer pulley diameters are 0.40 m and 0.90 m, respectively.	

Motion Information

Skier Event 1:	Event 2:	Pulley Event 1:	Event 2:	Block Event 1:	Event 2:
t1 =	t2 =	t1 =	t2 =	t1 =	t2 =
r1 =	r2 =	q1 =	q2 =	r1 =	r2 =
v1 =	v2 =	w1 =	w2 =	v1 =	v2 =
a1 =	a2 =	a1 =	a2 =	a1 =	a2 =

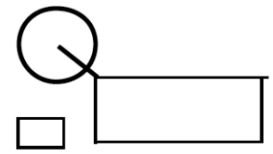
Mathematical Analysis[xiii]

Tired of walking up the stairs, an engineering student designs an ingenious device for reaching her third floor dorm room. An block is attached to a rope that passes over the outer diameter of a 0.7 m outer diameter, disk-shaped compound pulley. The student holds a second rope, wrapped around the inner 0.35 m diameter of the pulley. When the block is released, the student is pulled up to her dorm room, 8 m off the ground, in 11.2 s.

Motion Information

Student Event 1:	Event 2:	Pulley Event 1:	Event 2:	Block Event 1:	Event 2:
t1 =	t2 =	t1 =	t2 =	t1 =	t2 =
r1 =	r2 =	q1 =	q2 =	r1 =	r2 =
v1 =	v2 =	w1 =	w2 =	v1 =	v2 =
a1 =	a2 =	a1 =	a2 =	a1 =	a2 =

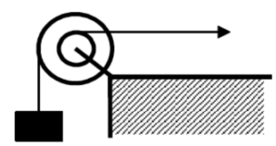
Mathematical Analysis[xiv]

The device at right is used to lift a heavy load. The free rope is attached to a truck that, from rest, accelerates to the right with a constant acceleration. The block is lifted 25 m in 45 s. The inner and outer pulley diameters are 0.40 m and 0.90 m, respectively.	

Motion Information

Truck Event 1:	Event 2:	Pulley Event 1:	Event 2:	Block Event 1:	Event 2:
t1 =	t2 =	t1 =	t2 =	t1 =	t2 =
r1 =	r2 =	q1 =	q2 =	r1 =	r2 =
v1 =	v2 =	w1 =	w2 =	v1 =	v2 =
a1 =	a2 =	a1 =	a2 =	a1 =	a2 =

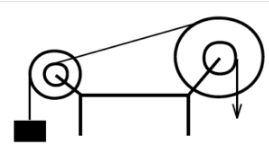
Mathematical Analysis[xv]

The device at right is used to lift a load quickly. The free rope is attached to a truck that, from rest, accelerates to the right with a constant acceleration. The block is lifted 25 m in 15 s. The inner and outer pulley diameters are 0.40 m and 0.90 m, respectively.	

Motion Information

Truck Event 1:	Event 2:	Pulley Event 1:	Event 2:	Block Event 1:	Event 2:
t1 =	t2 =	t1 =	t2 =	t1 =	t2 =
r1 =	r2 =	q1 =	q2 =	r1 =	r2 =
v1 =	v2 =	w1 =	w2 =	v1 =	v2 =
a1 =	a2 =	a1 =	a2 =	a1 =	a2 =

Mathematical Analysis[xvi]

The device at right is used to quickly lift the block shown. The free rope is pulled downward at a constant speed of 2 m/s for 15 s. The inner and outer diameters of the big pulley are 0.40 m and 0.90 m, and the inner and outer diameters of the small pulley are 0.2 m and 0.6 m.	

Motion Information

Big Pulley Event 1:	Event 2:	Little Pulley Event 1:	Event 2:	Block Event 1:	Event 2:
t1 =	t2 =	t1 =	t2 =	t1 =	t2 =
q1 =	q2 =	q1 =	q2 =	r1 =	r2 =
w1 =	w2 =	w1 =	w2 =	v1 =	v2 =
a1 =	a2 =	a1 =	a2 =	a1 =	a2 =

Mathematical Analysis[xvii]

[i] q2 = 3.53 rad

[ii] t = 0.97 s

[iii] wB2 = 8.98 rad/s

[iv] t = 1.55 s

[v] v1 = 12.3 m/s

[vi] t = 2.39 s

[vii] t2 = 2.8 s

[viii] a = 45.7 rad/s2

[ix] w2 = 35.6 rad/s

[x] a = 0.65 m/s2

[xi] a = 2.92 rad/s2

[xii] t2 = 7.5 s

[xiii] t2 = 2.65 s

[xiv] ablock = 0.256 m/s2

[xv] atruck = 0.056 m/s2

[xvi] atruck = 0.099 m/s2

[xvii] Drblock = 203 m

Homework 10 – Model 4: 15, 17, 20, 23, 25, 27, 36, 39, 41, 42.

1. As stated in the text, to describe the general motion of the tire we will break the motion down into a pure translation of the CM and a pure rotation about the CM. Another way to envision the motion is that the tire is rotating about the point on the tire in contact with the ground. The rotation axis of the tire passes through this contact point. (Imagine the point on the tire in contact with the ground as being glued to the ground and the tire rotating about it.) However, an instant later this point is no longer in contact with the ground, and the rotation axis passes through the next point on the tire in contact with the ground. There are advantages and disadvantages to both conceptualizations of the tire's motion, however, we will restrict ourselves to the view that the tire undergoes pure rotation about the CM. ↩

4.2: Dynamics

Dynamics

Concepts and Principles

To study the dynamics of an arbitrary rigid body we will break the motion down into a pure translation of the CM and a pure rotation about the CM. We will use particle dynamics, i.e., Newton's second law applied to the CM of the object, to study the translational portion of the motion. The study of the rotational portion of the motion requires a pair of new concepts. We will "invent" these concepts through the use of an analogy with linear dynamics.

In linear dynamics, Newton's second law states that the linear acceleration of an object is proportional to the total force acting on the object and inversely proportional to the mass, or inertia, of the object. It would seem *plausible* that the angular acceleration of an object would depend on analogous concepts in the same manner.

We will replace the concept of force, often thought of as the push or pull applied to an object, with a quantity measuring the *twist* applied to an object. We will call this new quantity *torque*, symbolized t.

We will replace the concept of mass, the measure of the resistance of the object to changes in its linear velocity, with a quantity measuring the resistance of the object to changes in its angular velocity. We will call this new quantity *rotational inertia*, symbolized I.

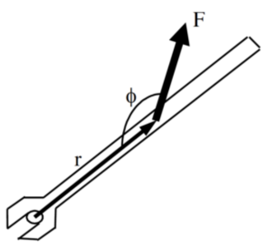
In summary,

$$\Sigma\tau = I\alpha$$

Before we go any further, however, let's define these new concepts more clearly.

Torque

In simple English, torque measures the twist applied to an object. The question remains, however, how do we *quantify* twist?



Let's examine a common device used to generate twist, a wrench. The magnitude, location, and orientation of the force applied to the wrench by the person's hand are indicated. Each of these three parameters effects the amount of twist the person delivers to the wrench (and therefore to the bolt).

If you've turned many bolts in your life, two things about this person's bolt-turning technique should grab you. First, why is this person applying the force at such a silly angle? She would generate much more twist if she applied the same magnitude force perpendicular to the wrench, rather than at an angle far from 900. Second, why is she not applying the force at the far edge of the wrench? She would generate far more twist if she applied the same magnitude force at the far edge of the wrench.

If the preceding paragraph makes sense to you, you understand how to quantify torque. To maximize torque, you should:

1. Apply the force far from the axis of rotation (the bolt).
2. Apply the force perpendicular to the position vector between the axis of rotation and the force.
3. Apply a large magnitude force.

Mathematically, this is summarized by:

$$\tau = rF\sin\phi$$

Note that this function has a maximum when r is large, F is large, and f = 900. Torque will also be assigned a direction, either clockwise or counterclockwise, depending upon the direction of the twist applied to the object.

Rotational Inertia

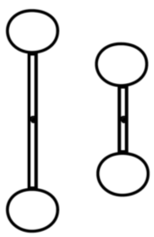
We have constructed a rotational analogy to Newton's second law,

$$\Sigma\tau = I\alpha$$

Our next task is to better define what we mean by I, the rotational inertia.

The rotational inertia is a measure of the resistance of the object to changes in its angular velocity. Imagine applying the same torque to two objects, initially at rest. After applying the torques for some set amount of time, measure the angular velocity of the objects. The object with the smaller angular velocity has the larger rotational inertia, because it has the larger resistance to angular acceleration.

Since more massive objects are harder to get moving linearly, it seems plausible that rotational inertia should depend on the mass of the object. It also seems plausible that rotational inertia should depend on the shape of the object.



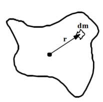
For example, it would be easier to get the smaller dumbbell spinning than the larger dumbbell, even though they have the same total mass.

Let's try to get more quantitative. Examining

$$\Sigma\tau = I\alpha$$

we can see that the units of I must be the units of torque (N.m) divided by the units of angular acceleration (s-2). Remembering that a Newton is equivalent to kg.m.s-2 leads to the units of rotational inertia being kg.m2. Thus, rotational inertia must be the product of a mass and a distance squared.

It seems plausible (there's that word again!) that the distance that is squared in the relationship for rotational inertia is the distance from the rotation axis. The farther a piece of mass is from the rotation axis, the more difficult it is to give the object an angular acceleration.



Enough with the "plausibilities", let's finally just define the rotational inertia to be:

$$I = \int r^2 dm$$

Imagine the object of interest divided into a large number of infinitesimally small chunks of mass, each with mass dm. Each chunk of mass is a distance r from the rotation axis. If you take the product of the mass of each chunk and the distance of the chunk from the rotation axis, squared, and sum this quantity over all the chunks of mass, you have the rotational inertia of the object of interest.

In summary, we now have quantitative relationships for measuring torque and rotational inertia,

$$\tau = rF\sin\phi$$

$$I = \int r^2 dm$$

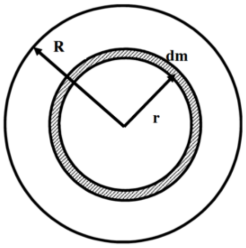
and the hypothesis that these two quantities are related in a manner analogous to Newton's second law,

$$\Sigma\tau = I\alpha$$

Analysis Tools

Calculating the Rotational Inertia

To fully utilize Newton's second law in rotational form, we must be able to set up and evaluate the integral that determines the rotational inertia. (To be honest, this is a lie. For the vast majority of common shapes, and many quite uncommon shapes, these integrals have already been evaluated. A table of selected results is at the end of this section.) To test our understanding of the relationship for rotational inertia, let's calculate the rotational inertia for a thin disk about an axis passing through its center of mass and perpendicular to its circular face.

	**1.** Choose the chunks of mass, dm, to be ring-shaped. This is because you must multiply each dm by the distance of the chunk from the rotation axis squared. To facilitate doing this, it's crucial that every point in the chunk be the same distance from the axis, i.e., have the same r. If the ring-shaped chunk is thin enough, for example dr (infinitesimally) thick, then this is true.

2. Realize that the mass of the little chunk is directly proportional to its volume, assuming the disk has a constant density. If it does have a constant density, the ratio of the chunk's mass to its volume must be the same as the ratio of the total mass of the disk to its volume.

$$\frac{dm}{dV} = \frac{M}{V}$$

$$dm = \frac{M}{V} dV$$

$$dm = \frac{M}{\pi R^2 T} dV$$

where R is the radius of the disk and T is its thickness. The volume of the ring-shaped chunk, dV, is equal to the product of the circumference of the ring (2pr), the thickness of the disk (T), and the thickness of the ring (dr). Thus,

$$dm = \frac{M}{\pi R^2 T} 2\pi r T(dr)$$

3. Plug the expression for dm into

$$I = \int r^2 dm$$

$$I = \int r^2 \frac{M}{\pi R^2 T} 2\pi r T(dr)$$

$$I = \frac{2M}{R^2} \int r^3 (dr)$$

To include all the chunks of mass, the integral must go from r = 0 m up to r = R.

$$I = \frac{2M}{R^2} \int_0^R r^3 (dr)$$

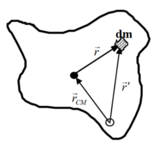
$$I = \frac{2M}{R^2} \frac{R^4}{4}$$

$$I = \frac{1}{2} MR^2$$

Thus, the rotational inertia of a thin disk about an axis through its CM is the product of one-half the total mass of the disk and the square of its radius. Notice that the thickness of the disk does not effect its rotational inertia. A consequence of this fact is that a cylinder has the same rotational inertia as a disk, when rotated about an axis through its CM and perpendicular to its circular face.

The Parallel-Axis Theorem

Now let's imagine we need to calculate the rotational inertia of a thin disk about an axis perpendicular to its circular face and along the edge of the disk. It would be convenient if we could determine the rotational inertia about an axis along the edge using the rotational inertia about an axis through the CM (which we've already calculated). In fact, there is a *very* convenient method to determine the rotational inertia about any axis *parallel* to an axis through the CM if we know the rotational inertia about an axis through the CM.



Imagine you want to determine the rotational inertia of an arbitrarily shaped object about an arbitrary axis. The solid circle denotes an axis through the CM, the hollow circle the axis of interest. The two axes are parallel.
Notice that

$$\vec{r}' = \vec{r}_{CM} + \vec{r}$$

with $\vec{r}_{CM} = x_{CM}\hat{i} + y_{CM}\hat{j}$

and $\vec{r} = x\hat{i} + y\hat{j}$

Thus,

$$\vec{r}' = (x_{CM} + x)\hat{i} + (y_{CM} + y)\hat{j}$$
$$r'^2 = (x_{CM} + x)^2 + (y_{CM} + y)^2$$
$$r'^2 = x_{CM}{}^2 + x^2 + 2x_{CM}x + y_{CM}{}^2 + y^2 + 2y_{CM}y$$

The rotational inertia about the axis of interest is given by:

$$I = \int r'^2 dm$$
$$I = \int (x_{CM}{}^2 + x^2 + 2x_{CM}x + y_{CM}{}^2 + y^2 + 2y_{CM}y)dm$$
$$I = \int ((x_{CM}{}^2 + y_{CM}{}^2) + (x^2 + y^2) + 2x_{CM}x + 2y_{CM}y)dm$$
$$I = \int (r_{CM}{}^2 + r^2 + 2x_{CM}x + 2y_{CM}y)dm$$
$$I = \int r_{CM}{}^2 dm + \int r^2 dm + \int 2x_{CM}xdm + \int 2y_{CM}ydm$$

Note that xCM , yCM , and rCM are constants that depend only on the distance between the two axes. Thus, xCM , yCM , and rCM can by brought outside of the integral.

$$I = r_{CM}{}^2 \int dm + \int r^2 dm + 2x_{CM} \int xdm + 2y_{CM} \int ydm$$

Now comes the key observation in the derivation. Examine the term ò x dm. Remember that x is the horizontal distance from the CM. If this distance is integrated over all the chunks of mass, dm, throughout the entire object, this integral must equal zero because the CM is defined to be in exactly the spot where a mass-weighted average over distance is equal to zero. ò x dm and ò y dm are equal to zero by the definition of CM! (Pretty cool, huh?)

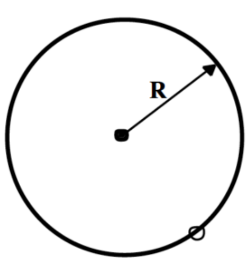
Thus,

$$I = r_{CM}{}^2 \int dm + \int r^2 dm$$

Noting that ò dm is the total mass of the object, M, and ò r2 dm is the rotational inertia about the CM, ICM, then

$$I = Mr_{CM}{}^2 + I_{CM}$$

This result states that the rotational inertia about an axis parallel to an axis through the CM, I, is equal to the rotational inertia about an axis through the CM, ICM, plus the product of the total mass of the object and the distance between the axes, rCM, squared.



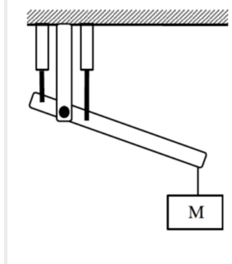
To answer the original question, let's determine the rotational inertia of a thin disk about an axis perpendicular to its circular face and along the edge of the disk using the parallel-axis theorem.
We know that the rotational inertia for a thin disk about an axis passing through its center of mass and perpendicular to its circular face is 1/2 MR2 and that the distance between the CM axis and the axis of interest is R. Thus,

$$I = Mr_{CM}{}^2 + I_{CM}$$
$$I = MR^2 + \frac{1}{2}MR^2$$
$$I = \frac{3}{2}MR^2$$

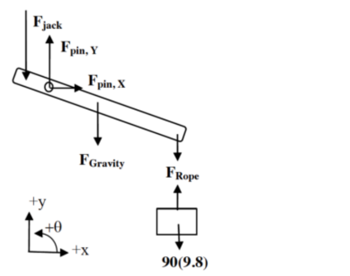
Applying Newton's Second Law in Translational and Rotational Form – I

Investigate the scenario described below.

The robotic arm at right consists of a pair of hydraulic jacks, each attached 10 cm from the pin “elbow”. The elbow is 15 cm from the extreme edge of the 40 kg, 0.90 m long “forearm”. The forearm and attached 90 kg load are held stationary at an angle of 20° below horizontal. Since the jacks primarily exert forces through extension, the front, or “biceps”, jack is exerting negligible force.



To study the dynamics of this situation, we will first need a free-body diagram of the forearm (and the attached load).



The forces due to the jack, the rope, and gravity should not need much explanation. However, the forces due to the pin elbow may need some explanation.

First, the free-body diagram drawn at left is of the forearm, not including the pin that serves as the elbow. Thus, the pin is external to the object of interest, and thus its interactions with the object are forces that must be indicated on the diagram. If you were to include the pin as part of the forearm, then the upper part of the arm would be external to the object of interest (pin plus forearm) and its interactions with the “pin plus forearm” would have to be indicated as forces on the diagram. Both of these approaches are completely valid.

Second, the direction of the force of the pin on the forearm may not be obvious. Does the pin push straight down on the forearm, pull up on the elbow, or push at some unspecified angle? The only thing that’s clear is that this force is directed somewhere in the xy-plane. If the force is in the xy-plane, then it must have components along both the x- and y-axis (although one of these components may turn out to have a magnitude of zero). Thus, to handle the generality of the situation you should include both an x- and y-component for the force of the pin on the forearm. For convenience, I’ll draw these forces as pointing in the positive x and y directions. (If my guess is wrong, the mathematics will tell me.)

Now that we have a free-body diagram, we can apply Newton’s second law, both the linear form (in the x- and y-directions) and the rotational form (in the q-direction).

x-direction	y-direction
$\Sigma F = ma$	$\Sigma F = ma$
$F_{pin,x} = 40a_x$	$-F_{jack} + F_{pin,y} - F_{gravity} - F_{rope} = 40a_y$

Since the forearm is held stationary, both ax and ay are equal to zero. Also, Frope is equal to the force of gravity on the load, 882 N.

$F_{pin,x} = 0$	$-F_{jack} + F_{pin,y} - 40(9.8) - 882 = 40(0)$
	$-F_{jack} + F_{pin,y} = 1274$

Obviously, we need another equation in order to determine the two unknown forces. The obvious choice is Newton’s second law in rotational form.

Before we begin, we should determine the rotational inertia for a thin rod (the closest thing to a forearm in our table) rotated about an axis not at its CM. A thin rod rotated about its CM has rotational inertia 1/12 ML2. We are interested in its rotational inertia about an axis not at its CM, so we must use the parallel-axis theorem with rCM = 0.30 m. (The CM is at the center of the forearm, 0.45 m from either end. Since the elbow is 0.15 m from one end, the distance between the elbow and the CM is 0.30 m.) Thus,

$$I = Mr_{CM}^2 + I_{CM}$$

$$I = (40)(0.30)^2 + \frac{1}{12}(40)(0.90)^2$$

$$I = 6.3kgm^2$$

We must also determine the torque due to each force. (Remember, the angle, f, in the relation for torque is the angle between r (oriented along the forearm) and F.)

Pin, Y	Jack	Gravity	Rope
$\tau = rF\sin\phi$	$\tau = rF\sin\phi$	$\tau = rF\sin\phi$	$\tau = rF\sin\phi$
$\tau = (0)F_{pin,y}\sin(0)$	$\tau = (0.10)F_{jack}\sin(70)$	$\tau = (0.30)(392)\sin(110)$	$\tau = (0.75)(882)\sin(110)$
$\tau = 0$	$\tau = 0.094F_{jack}$	$\tau = 111Nm$	$\tau = 622Nm$

Finally, let’s apply Newton’s second law in rotational form, paying careful attention to the algebraic sign of each torque. Note that our coordinate system indicates that all torques acting counterclockwise are positive. Therefore, all torques acting clockwise are negative.

$$\Sigma\tau = I\alpha$$

$$+\tau_{jack} - \tau_{gravity} - \tau_{rope} = 6.3\alpha$$

$$0.094F_{jack} - 111 - 622 = 6.3\alpha$$

Since the forearm is held stationary, a is equal to zero.

$$0.094F_{jack} - 111 - 622 = 0$$
$$0.094F_{jack} = 733$$
$$F_{jack} = 7796N$$

The jack must exert a force of 7796 N to hold the forearm stationary.

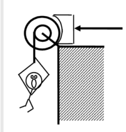
Plugging this value into our y-equation yields:

$$-F_{jack} + F_{pin,y} = 1274$$
$$-7796 + F_{pin,y} = 1274$$
$$F_{pin,y} = 9070N$$

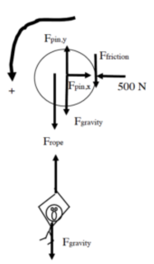
The pin must exert a force of 9070 N upwards, since the mathematics determined that the algebraic sign of FPin,y was positive.

Applying Newton’s Second Law in Translational and Rotational Form – II

A 75 kg man is attached to a rope wrapped around a 35 kg disk-shaped pulley, with inner and outer diameters 0.60 m and 0.90 m, respectively. The man is initially at rest. The brake shoe is pressed against the pulley with a force of 500 N. The coefficient of friction between the brake shoe and the pulley is (0.9, 0.8).



First, we have to determine whether the force applied to the brake shoe is sufficient to hold the man stationary. If it’s not, we have to find the man’s acceleration. To accomplish this, we need free-body diagrams for both the man and the pulley.



We’ll use counterclockwise as the positive q-direction and down as the positive y-direction.
The pin, or axle, at the center of the pulley exerts forces in the x- and y-direction, although the exact directions may be unknown. We’ll just label these forces as Fpin,x and Fpin,y.
The frictional force that acts on the right edge of the pulley acts in a way to prevent the man from falling (or, if he does fall, to slow down the man’s fall).

To begin my analysis, I’ll assume that the man is stationary, solve for the value of the frictional force required to keep him stationary, and then determine whether this frictional force is possible given the applied force of 500 N and the coefficient of static friction.

Let’s apply Newton’s second law to the man in the y-direction. (Remember, we are *assuming* he is not falling.)

$$\Sigma F = ma$$
$$F_{gravity} - F_{rope} = ma$$
$$75(9.8) - F_{rope} = 75(0)$$
$$F_{rope} = 735N$$

Now look at Newton’s second law applied to the pulley in the q-direction.

$$\Sigma\tau = I\alpha$$
$$+\tau_{rope} - \tau_{friction} = I(0)$$

Notice that all of the other forces acting on the pulley do *not* exert torques on the pulley. They are either located at r = 0 m or have angular orientations of f = 180°.

$$(0.30)(735)\sin 90 - (0.45)F_{friction}\sin 90 = 0$$
$$F_{friction} = 490N$$

Therefore, to hold the man stationary requires 490 N of friction. However,

$$F_{friction} \le \mu_s F_{contact}$$
$$F_{friction} \le (0.9)(500)$$
$$F_{friction} \le 450N$$

Therefore, the man must accelerate downward.

Now that we know the man must fall, let's write the same two equations as before, although this time the accelerations (both angular for the pulley and linear for the man) are not zero.

For the pulley:

$$+\tau_{rope} - \tau_{friction} = (\frac{1}{2}MR^2)\alpha$$
$$(0.30)F_{rope}\sin 90 - (0.45)F_{friction}\sin 90 = (\frac{1}{2}35(0.45)^2)\alpha$$
$$0.30F_{rope} - 0.45F_{friction} = 3.54\alpha$$

This assumes that the rotational inertia of a "compound" pulley, one with more than one location where a rope can be wrapped, is the same as a regular pulley, and that the outermost radius of the pulley determines the inertia.

Since the man is falling, the frictional force is now kinetic, and

$$F_{friction} = \mu_k F_{contact}$$
$$F_{friction} = (0.8)(500)$$
$$F_{friction} = 400N$$

Thus our "pulley" equation becomes:

$$0.30F_{rope} - 0.45(400) = 3.54\alpha$$
$$0.30F_{rope} - 180 = 3.54\alpha$$

Examining Newton's second law for the man,

$$F_{gravity} - F_{rope} = ma$$
$$75(9.8) - F_{rope} = 75(a)$$
$$735 - F_{rope} = 75a$$

We now have two equations with three variables. However, the angular acceleration of the pulley and the linear acceleration of the man are directly related. Since the rope that the man is attached to is wrapped 0.3 m from the center of the pulley, the man accelerates at the same rate as the tangential acceleration of a point on the pulley 0.3 m from the center. Thus,

$$a_{man} = r\alpha_{pulley}$$
$$a = 0.3\alpha$$

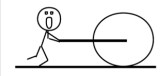
Using this relationship allows me to rewrite the two Newton's second law equations as:

$$0.30F_{rope} - 180 = 3.54(\frac{a}{0.3})$$
$$735 - F_{rope} = 75a$$

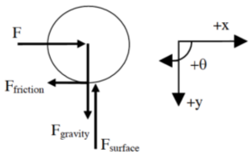
This pair of equations has the solution

$$a = 1.18m/s^2$$
$$F = 647N$$

The man falls with this acceleration and the rope exerts this force (upward on the man and downward on the pulley).

The child at right is pretending to be a steamroller by pushing a 24 kg, 0.60 m radius cylindrical object around his backyard. The boy pushes horizontally with a force of 70 N. The coefficient of friction between the cylinder and the ground is (0.5, 0.4).	

Let's draw a free-body diagram and write Newton's second law in the x-, y-, and q-directions.



x-direction

$$\Sigma F = ma$$
$$F - F_{friction} = ma$$
$$70 - F_{friction} = 24a$$

y-direction

$$\Sigma F = ma$$
$$F_{surface} - F_{gravity} = ma$$
$$F_{surface} - 24(9.8) = 24(0)$$
$$F_{surface} = 235N$$

θ-direction

$$\Sigma \tau = I\alpha$$
$$+\tau_{friction} = I\alpha$$
$$(0.6)F_{friction} \sin 90 = (\frac{1}{2}MR^2)\alpha$$
$$0.6F_{friction} = (\frac{1}{2}(24)(0.6)^2)\alpha$$
$$F_{friction} = 7.2\alpha$$

Note that the description does not specify whether the cylinder *rolls* or *skids* when the child pushes it. We will have to make an assumption, continue with the calculation, and then check our assumption for validity. Let's assume that the bottom of the cylinder *does not slip* in its contact with the ground, which means the cylinder rolls without slipping around the backyard.

If the cylinder bottom does not slip in its contact with the ground, the horizontal acceleration (and velocity) of the cylinder bottom must equal zero. Since the acceleration of the cylinder bottom is the sum of the acceleration due to translation of the CM and the acceleration due to rotation about the CM, for the bottom to have zero horizontal acceleration means that the acceleration due to rotation,

$$a_t = r\alpha$$

must be equal in magnitude, and opposite in direction, to the translational acceleration of the CM. Thus,

$$a = (0.6)\alpha$$

Combining this with our y- and q-equations yields:

$$70 - F_{friction} = 24a$$

$$F_{friction} = 7.2(\frac{a}{0.6})$$

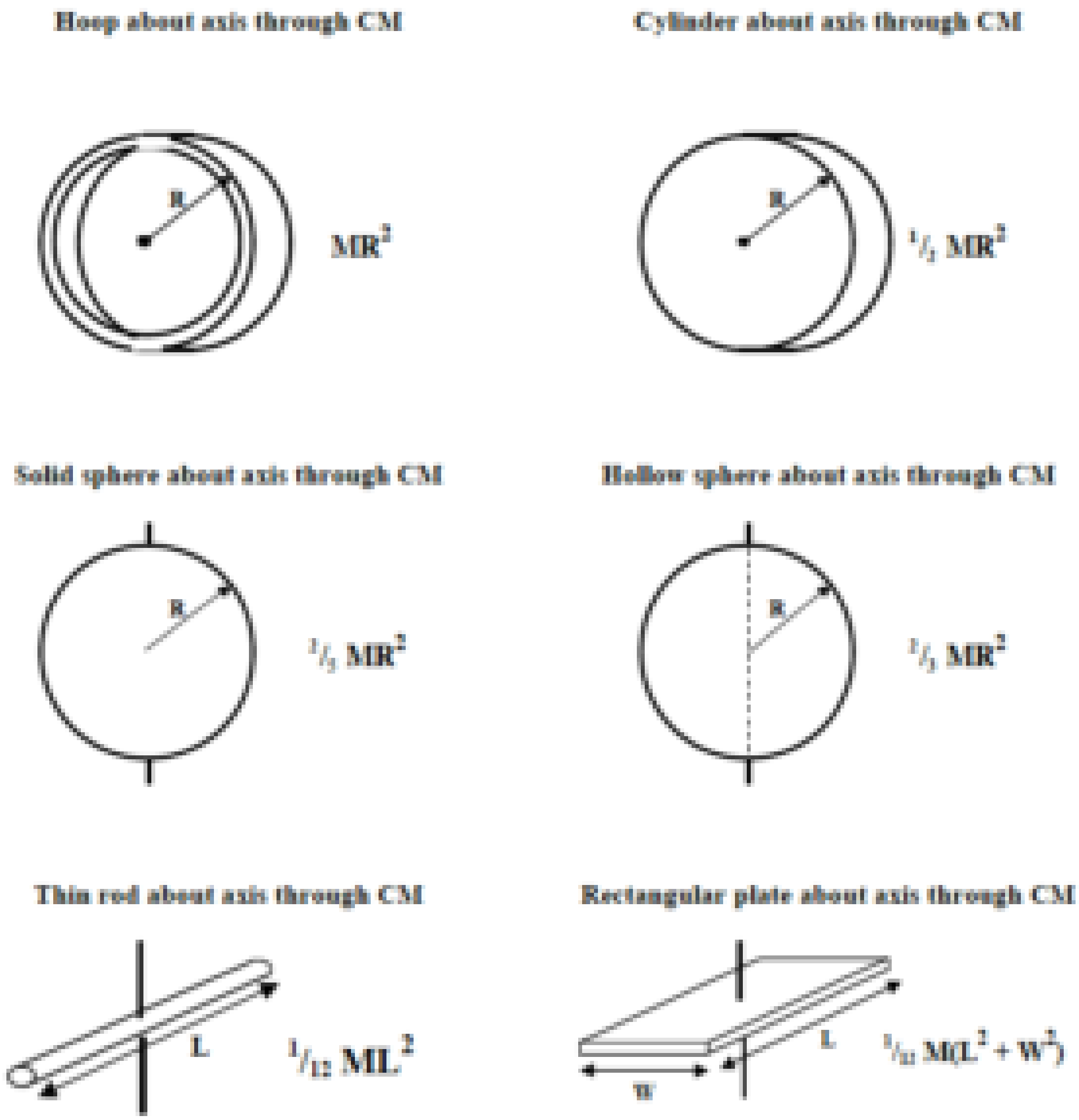
This pair of equations has the solution

$$a = 1.94m/s^2$$
$$F_{friction} = 23.3N$$

Now we must check the validity of our assumption. If the cylinder rolls without slipping, the frictional force is static. Thus,

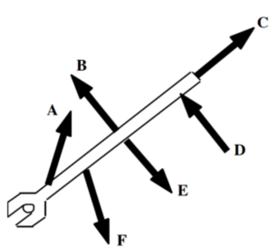
$$F_{friction} \le \mu_s F_{contact}$$
$$F_{friction} \le (0.5)(235)$$
$$F_{friction} \le 118N$$

Since our calculated value for Ffriction is less than 118 N, the cylinder does remain in static contact with the ground during its motion, our assumption is validated, and our numerical results are correct.



Activities

The wrench below has six equal magnitude forces acting on it.



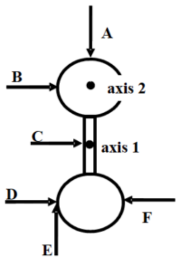
Rank these forces on the basis of the magnitude of the torque they apply to the wrench, measured about an axis centered on the bolt.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Two identical spheres are attached together by a thin rod. The rod lies on a line connecting the centers-of-mass of the two spheres. The length of the rod is equal to the diameter of each sphere. The object has six equal magnitude forces acting on it at the locations shown.



Rank these forces on the basis of the torque they apply to the object, measured about axis 1. Let the counterclockwise direction be positive.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

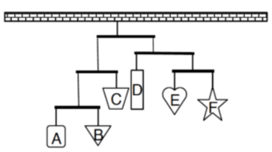
Rank these forces on the basis of the torque they apply to the object, measured about axis 2. Let the counterclockwise direction be positive.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

An artist constructs the mobile shown below. The highest two crossbars are 3 units long, and are hung from a point ⅓ along their length. The lower three crossbars are 2 units long, and are hung from their midpoint. In this configuration, the mobile is perfectly balanced.



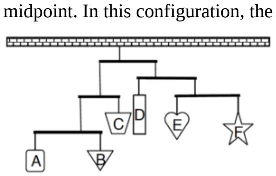
Rank the masses of the six hanging shapes.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

An artist constructs the mobile shown below. The crossbars are either 3 units long, and hung from a point ⅓ along their length, or are 2 units long, and hung from their midpoint. In this configuration, the mobile is perfectly balanced.



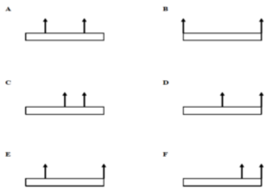
Rank the masses of the six hanging shapes.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

The six platforms below are initially at rest in deep space. The indicated forces act on the platforms at their endpoints, quarter-points, or midpoints. All forces have the same magnitude.



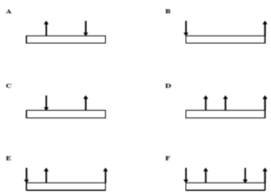
Rank these platforms on the basis of the torque that acts on them, measured about their CM. Let the counterclockwise direction be positive.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

The six platforms below are initially at rest in deep space. The indicated forces act on the platforms at their endpoints, quarter-points, or midpoints. All forces have the same magnitude.



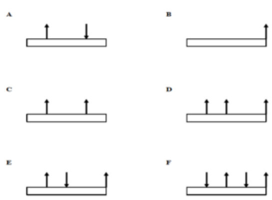
Rank these platforms on the basis of the torque that acts on them, measured about their CM. Let the counterclockwise direction be positive.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

The six platforms below are initially at rest in deep space. The indicated forces act on the platforms at their endpoints, quarter-points, or midpoints. All forces have the same magnitude.



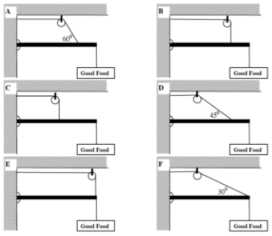
Rank these platforms on the basis of the force that must be applied to them, at their left-edge, to keep them from rotating. Let the positive direction be upward.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

A roadside sign is to be hung from the end of a thin pole, and the pole supported by a single cable. Your design firm brainstorms the following six scenarios. In scenarios A, B, and D, the rope is attached to the pole ¾ of the distance between the hinge and the sign. In C, the rope is attached to the mid-point of the pole.



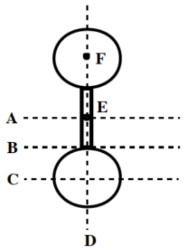
Rank the design scenarios on the basis of the tension in the supporting cable.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Two identical, uniform, solid spheres are attached together by a solid, uniform thin rod. The rod lies on a line connecting the centers-of-mass of the two spheres. The axes A, B, C, and D are in the plane of the page (which contains the centers-of-mass of the spheres and the rod), while axes E and F are perpendicular to the page.



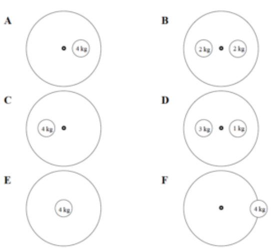
Rank the rotational inertia of this object about the axes indicated.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are top views of six identical turntables, upon which are fastened different masses. The distance from the center of the turntable to the center of the mass is either zero, one-half the radius of the turntable, or the radius of the turntable.



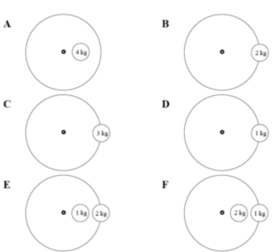
Rank the rotational inertia of these turntable-mass systems.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are top views of six identical turntables, upon which are fastened different masses. The distance from the center of the turntable to the center of the mass is either zero, one-half the radius of the turntable, or the radius of the turntable.



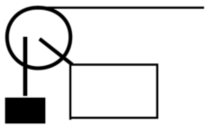
Rank the rotational inertia of these turntable-mass systems.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

A block of mass M is attached to the inner radius (Rinner) of the pulley shown below. A second rope is attached to outer radius (Router) of the pulley. Assume friction in the pulley mount is very small.



	M	R_{inner}	R_{outer}
A	10 kg	10 cm	20 cm
B	5 kg	20 cm	40 cm
C	10 kg	5 cm	20 cm
D	10 kg	5 cm	10 cm
E	20 kg	5 cm	10 cm
F	20 kg	20 cm	30 cm

Rank these scenarios on the basis of the magnitude of the force that must be applied to the free rope to hold the block stationary.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

A 10 kg block is attached to the inner radius (Rinner) of the pulley shown below. A second rope, attached to outer radius (Router) of the pulley, is used to raise the block at constant speed v. Assume friction in the pulley mount is very small.

	v	R_{inner}	R_{outer}
A	10 m/s	10 cm	20 cm
B	20 m/s	20 cm	40 cm
C	5 m/s	5 cm	20 cm
D	10 m/s	5 cm	10 cm
E	5 m/s	5 cm	10 cm
F	10 m/s	20 cm	30 cm

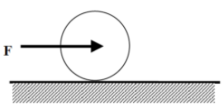
Rank these scenarios on the basis of the magnitude of the force that must be applied to the free rope to raise the block at the speed indicated.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

A disk of mass m is pushed along a level surface by a force acting at its center-of-mass. The disk begins at rest and rolls without slipping along the surface. The coefficient of friction between the disk and the surface, m, is listed below. All disks are pushed by the same magnitude force and have the same radius.



	m	μ
A	1 kg	(0.2, 0.1)
B	2 kg	(0.2, 0.1)
C	3 kg	(0.6, 0.4)
D	2 kg	(0.5, 0.3)
E	1 kg	(0.6, 0.5)
F	2 kg	(0.4, 0.3)

Rank the disks on the magnitude of the frictional force acting on them.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

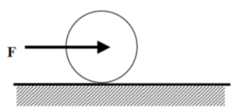
Rank the disks on the magnitude of their acceleration.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

A sphere of radius R is pushed along a level surface by a force acting at its center-of-mass. The sphere begins at rest and rolls without slipping along the surface. The coefficient of friction between the sphere and the surface, m, is listed below. All spheres are pushed by the same magnitude force and have the same mass.



	R	μ
A	10 cm	(0.2, 0.1)
B	20 cm	(0.2, 0.1)
C	30 cm	(0.6, 0.4)
D	20 cm	(0.5, 0.3)
E	10 cm	(0.6, 0.5)
F	20 cm	(0.4, 0.3)

Rank the spheres on the magnitude of the frictional force acting on them.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

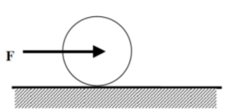
Rank the spheres on the magnitude of their acceleration.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

A hollow sphere is pushed along a level surface by a force, F, acting at its center-of-mass. The hollow sphere begins at rest and rolls without slipping along the surface. The coefficient of friction between the hollow sphere and the surface, m, is listed below. All hollow spheres have the same mass and radius.



	μ	F
A	(0.6, 0.4)	30 N
B	(0.6, 0.5)	30 N
C	(0.7, 0.5)	30 N
D	(0.3, 0.2)	60 N
E	(0.4, 0.2)	45 N
F	(0.9, 0.6)	60 N

Rank the hollow spheres on the magnitude of the frictional force acting on them.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

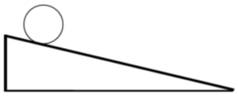
Rank the hollow spheres on the magnitude of their acceleration.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

A disk of mass m is released from rest from the top of an incline. The disk rolls without slipping down the incline. The coefficient of friction between the disk and the incline, m, is listed below. All disks are released from the same point on the incline and have the same radius.



	m	μ
A	1 kg	(0.2, 0.1)
B	2 kg	(0.2, 0.1)
C	3 kg	(0.6, 0.4)
D	2 kg	(0.5, 0.3)
E	1 kg	(0.6, 0.5)
F	2 kg	(0.4, 0.3)

Rank the disks on their speed at the bottom of the incline.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

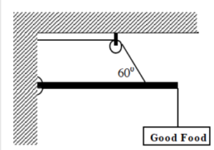
Explain the reason for your ranking:

Rank the disks on the magnitude of the frictional force acting on them.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

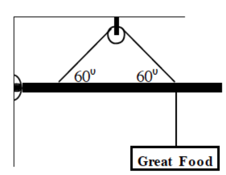
_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

The roadside sign at right has a mass of 22 kg. It hangs from the end of a 1.6 m long, 14 kg support pole hinged to the wall. A support cable is attached to the pole 1.1 m from the wall.	

Free-Body Diagram

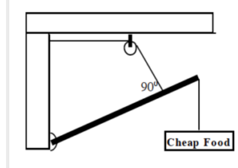
Mathematical Analysis [i]

The roadside sign at right has a mass of 22 kg. It hangs from a 1.6 m long, 14 kg support pole hinged to the wall. A support cable is attached to the pole at both 0.4 m and 1.2 m from the wall. The sign is hung directly below the outer support cable.	

Free-Body Diagram

Mathematical Analysis [ii]

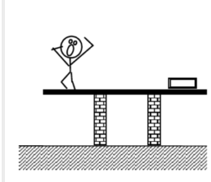
The roadside sign at right has a mass of 22 kg. It hangs from the end of a 1.6 m long, 14 kg support pole hinged to the wall at an angle of 250 above horizontal. A support cable is attached to the pole at a point 1.1 m from the wall along the pole.



Free-Body Diagram

Mathematical Analysis [iii]

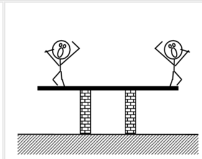
The 70 kg strange man has built a simple scaffold by placing a 6 m long, 35 kg board on top of two piles of bricks. The brick supports are 2 m apart and centered on the CM of the board. The man's 10 kg toolbox is 1 m from the right edge of the board. The man is 1.1 m from the left edge of the board when he stops and realizes he probably can't walk all the way to the left edge of the board.



Free-Body Diagram

Mathematical Analysis [iv]

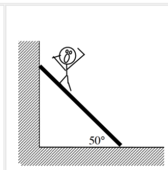
The two 70 kg strange twins have built a simple scaffold by placing a 6 m long, 35 kg board on top of two piles of bricks. The brick supports are 2 m apart and centered on the CM of the board. One twin is at the extreme right edge of the board while the other is 1.2 m from the left edge.



Free-Body Diagram

Mathematical Analysis [v]

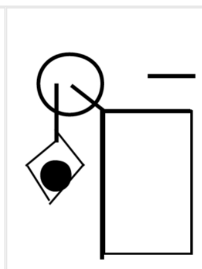
The 70 kg strange man has climbed three-quarters of the way up the 20 kg, 10 m long ladder when he stops and realizes that 50° may not be a very safe angle for a ladder. The coefficient of friction between the base of the ladder and the ground is (0.6, 0.5). The friction between the ladder and the wall is negligible.



Free-Body Diagram

Mathematical Analysis [vi]

The 75 kg man is falling at 20 m/s, 75 m above the crocodile-infested waters below! In an attempt to save him, the brake shoe is pressed against the spinning pulley. The coefficient of friction between the brake shoe and the pulley is (0.9, 0.8), and the 35 kg disk-shaped pulley has inner and outer diameters of 0.60 m and 0.90 m, respectively.



Motion Information

Free-Body Diagrams

<table><tr><td>Object:
Event 1:

t1 =

r1 =
q1 =

v1 =
w1 =

a1 =
a1 =</td><td>
Event 2:

t2 =

r2 =
q2 =

v2 =
w2 =

a2 =
a2 =</td><td></td></tr></table>

Mathematical Analysis [vii]

Tired of walking up the stairs, an 80 kg engineering student designs an ingenious device for reaching his third floor dorm room. An 84 kg block is attached to a rope that passes over a 0.7 m diameter, 15 kg, disk-shaped pulley. The student holds the other end of the rope. When the 84 kg block is released, the student is pulled up to his dorm room, 8 m off the ground.

Motion Information **Free-Body Diagrams**

<table><tr><td>Object:
Event 1:

t1 =

r1 =
q1 =

v1 =
w1 =

a1 =
a1 =</td><td>
Event 2:

t2 =

r2 =
q2 =

v2 =
w2 =

a2 =
a2 =</td><td></td></tr></table>

Mathematical Analysis [viii]

Tired of walking up the stairs, an 80 kg engineering student designs an ingenious device for reaching her third floor dorm room. An 42 kg block is attached to a rope that passes over the outer diameter of a 0.7 m outer diameter, 15 kg, disk-shaped compound pulley. The student holds a second rope, wrapped around the inner 0.35 m diameter of the pulley. When the 42 kg block is released, the student is pulled up to her dorm room, 8 m off the ground.

Motion Information **Free-Body Diagrams**

<table><tr><td>Object:
Event 1:

t1 =

r1 =
q1 =

v1 =
w1 =

a1 =
a1 =</td><td>
Event 2:

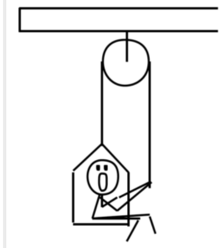
t2 =

r2 =
q2 =

v2 =
w2 =

a2 =
a2 =</td><td></td></tr></table>

Mathematical Analysis [ix]

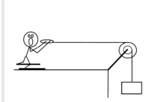
A 60 kg student lifts herself from rest to a speed of 1.5 m/s in 2.1 s. The boson's chair has a mass of 35 kg. The 20 kg, disk-shaped pulley has a diameter of 1.1 m.	

Motion Information

Free-Body Diagrams

Object: Event 1:	Event 2:	
t1 =	t2 =	
r1 = q1 =	r2 = q2 =	
v1 = w1 =	v2 = w2 =	
a1 = a1 =	a2 = a2 =	

Mathematical Analysis [x]

The device at right guarantees all the excitement of skiing without the need for hills. The 80 kg man begins from rest and reaches a speed of 34 m/s in 7.2 s. The 10 kg, disk-shaped pulley has inner and outer diameters of 0.40 m and 0.90 m, respectively. Assume friction is so small that it can be ignored.	

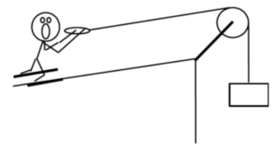
Motion Information

Free-Body Diagrams

Object: Event 1:	Event 2:	
t1 =	t2 =	
r1 = q1 =	r2 = q2 =	
v1 = w1 =	v2 = w2 =	
a1 = a1 =	a2 = a2 =	

Mathematical Analysis [xi]

The device at right allows you to ski uphill. The ballast block has a mass of 30 kg and the skier has a mass of 70 kg. The 10 kg, disk-shaped pulley has diameter 0.90 m. Assume friction is so small that it can be ignored. The ramp is 45 m long and inclined at 200 above horizontal.



Motion Information **Free-Body Diagrams**

Object: Event 1: t1 = r1 = q1 = v1 = w1 = a1 = a1 =	Event 2: t2 = r2 = q2 = v2 = w2 = a2 = a2 =	

Mathematical Analysis [xii]

A yo-yo of mass 0.60 kg, inner diameter 0.70 cm, and outer diameter 8.0 cm is released from rest. The string is 0.80 m long. Assume the rotational inertia of the yo-yo is similar to a simple disk and that the unwinding of the string does not effect the rotational inertia.

Motion Information **Free-Body Diagram**

Event 1: t1 = r1 = q1 = v1 = w1 = a1 = a1 =	Event 2: t2 = r2 = q2 = v2 = w2 = a2 = a2 =	

Mathematical Analysis [xiii]

A bowling ball of mass 7.1 kg and diameter 21.6 cm is thrown down the alley at a speed of 8.7 m/s. The bowling ball is initially skidding, with no angular velocity, down the alley. The coefficient of friction between the ball and the alley is (0.30, 0.25).

Motion Information **Free-Body Diagram**

Event 1:	Event 2:	
t1 =	t2 =	
r1 =	r2 =	
q1 =	q2 =	
v1 =	v2 =	
w1 =	w2 =	
a1 =	a2 =	
a1 =	a2 =	

Mathematical Analysis [xiv]

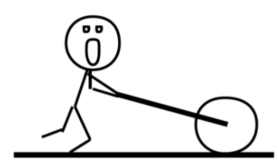
A bowling ball of mass 7.1 kg and diameter 21.6 cm is thrown down the alley at a speed of 9.5 m/s. The bowling ball is initially skidding, with no angular velocity, down the alley. The ball travels 9.6 m before it begins to roll without slipping.

Motion Information **Free-Body Diagram**

Event 1:	Event 2:	
t1 =	t2 =	
r1 =	r2 =	
q1 =	q2 =	
v1 =	v2 =	
w1 =	w2 =	
a1 =	a2 =	
a1 =	a2 =	

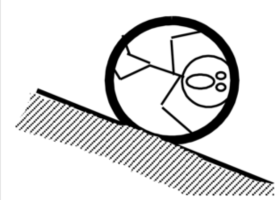
Mathematical Analysis [xv]

The strange man at right is pretending to be a steamroller by pushing a heavy cylindrical object around his backyard. The cylinder has a mass of 50 kg and a diameter of 0.70 m. The man pushes with a force of 400 N at an angle of 180 below horizontal. The coefficient of friction between the cylinder and the ground is (0.6, 0.4).



Free-Body Diagram **Mathematical Analysis** [xvi]

The 65 kg man at right is trapped inside a section of large pipe. If that's not bad enough, the pipe begins to roll, from rest, down a 35 m long, 180 incline! The pipe has a mass of 180 kg and a diameter of 1.2 m. (Assume the man's presence inside the pipe has a negligible effect on the pipe's rotational inertia.) The coefficient of friction between the pipe and the ground is (0.5, 0.4).

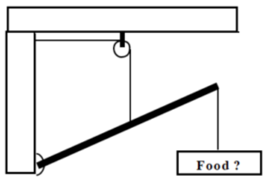


Motion Information **Free-Body Diagram**

Event 1:	Event 2:	
t1 =	t2 =	
r1 =	r2 =	
q1 =	q2 =	
v1 =	v2 =	
w1 =	w2 =	
a1 =	a2 =	
a1 =	a2 =	

Mathematical Analysis [xvii]

The roadside sign at right has mass M. It hangs from the end of a support pole of length L and mass m, hinged to the wall at an angle of q above horizontal. A support cable is attached vertically to the pole at its midpoint. Determine the force exerted by the cable on the pole (Fcable) as a function of M, m, g, L, and q.



Free-Body Diagram

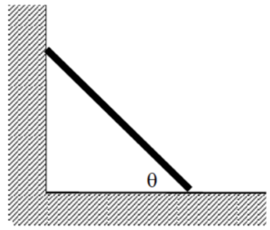
Mathematical Analysis

Questions

If M = 0 kg, what should Fcable equal? Does your function agree with this observation?

If the length of the pole was doubled, what would happen to Fcable?

Determine the minimum angle (qmin) with which the ladder can be leaned against the wall and not slip as a function of the ladder's mass (m), length (L), g, and the appropriate coefficient of friction. Assume the friction between the ladder and the wall is negligible.



Free-Body Diagram

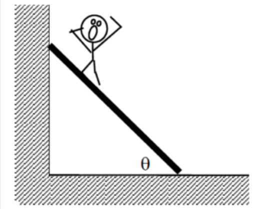
Mathematical Analysis

Questions

If m = 0, what should qmin equal? Does your function agree with this observation?

If the mass of the ladder was doubled, what would happen to qmin?

Determine the minimum coefficient of static friction (mmin) needed for the ladder not to slip as a function of the ladder's mass (m) and length (L), the man's mass (M) and position along the ladder (d), g, and the angle the ladder makes with the floor (q). Assume the friction between the ladder and the wall is negligible.



Free-Body Diagram

Mathematical Analysis

Questions

If q = 90°, what should mmin equal? Does your function agree with this observation?

Tired of walking up the stairs, an engineering student of mass m designs an ingenious device for reaching her third floor dorm room. An block of mass M is attached to a rope that passes over the outer diameter, D, of a disk-shaped compound pulley of mass Mp. The student holds a second rope, wrapped around the inner diameter, d, of the pulley. When the block is released, the student is pulled up to her dorm room. Determine the acceleration of the student (as) as a function of m, M, Mp, d, D, and g.

Free-Body Diagrams

Mathematical Analysis

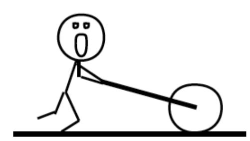
Questions

If Mp = ∞, what should as equal? Does your function agree with this observation?

If d = D and m = M, what should as equal? Does your function agree with this observation?

If md > MD, what should as equal? Does your function agree with this observation?

The strange man at right is pretending to be a steamroller by pushing a heavy cylindrical object around his backyard. The cylinder has a mass, M, and a diameter, D. The man pushes at an angle of q below horizontal. Determine the maximum force (Fmax) with which the man can push and the "steamroller" roll without slipping as a function of M, D, g, q, and the appropriate coefficient of friction between the cylinder and the ground.



Free-Body Diagram

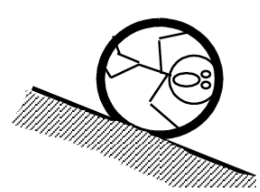
Mathematical Analysis

Questions

If m = 0, what should Fmax equal? Does your function agree with this observation?

Above what angle can the man push as hard as he can and still have the steamroller roll without slipping?

A man of mass m is trapped inside a pipe of mass M and diameter D initially at rest on an incline of angle q. Determine the minimum angle (qmin) above which the pipe will slide down the incline as a function of m, M, D, g, and the appropriate coefficient of friction between the pipe and the ground. Assume the man's presence inside the pipe has a negligible effect on the pipe's rotational inertia.



Free-Body Diagram

Mathematical Analysis

Questions

If m = 0, what should qmin equal? Does your function agree with this observation?

[i] F rope = 477 N

[ii] F rope = 266 N

[iii] F rope = 375 N

[iv] Frightsupport = 9.8 N Fleftsupport = 1117 N

[v] Frightsupport = 1269 N Fleftsupport = 446 N

[vi] Fsf = 514 N

[vii] F minimum = 867 N

[viii] a student = 0.23 m/s2

[ix] a student = 0.14 m/s2

[x] F right rope = 503 N

[xi] m block = 117 kg

[xii] a skier = 0.57 m/s2

[xiii] a = 0.15 m/s2

[xiv] t2 = 1.01 s

[xv] t2 = 1.18 s

[xvi] a = 5.07 m/s2

[xvii] a = 1.75 m/s2

Homework 11 – Model 4: 62, 65, 67, 72, 77, 79, 83, 94, 95, 98.

4.3: Conservation Laws

Conservation Laws

Concepts and Principles

The Angular Impulse-Angular Momentum Relation

We've already used the impulse-momentum relation to analyze situations involving translations through three-dimensional space. The relation is typically applied in its component form:

$$mv_{xi} + \int_{t_i}^{t_f} F_x dt = mv_{xf}$$

$$mv_{y_i} + \int_{t_i}^{t_f} F_y dt = mv_{yf}$$

$$mv_{zi} + \int_{t_i}^{t_f} F_z dt = mv_{zf}$$

Arguing by analogy, if the change in momentum in the x-, y-, and z-directions is equal to the impulse applied to the object in each direction doesn't it seem plausible that a similar relation would hold in the "q-direction"? If so, this relation should be constructed between the torques acting on an object, the time interval over which the torques act, and the change in angular velocity of an object.[1] The relation should be:

$$I\omega_i + \int_{t_i}^{t_f} \tau dt = I\omega_f$$

The product of rotational inertia and angular velocity is termed the *angular momentum* of the object, typically denoted L, and the product of torque and the time interval over which it acts is termed the *angular impulse* applied to the object.

Thus, if no angular impulse is applied to an object, its angular momentum will remain constant. This special case is referred to as *angular momentum conservation.* However, if an angular impulse is applied to the object, the angular momentum will change by an amount exactly equal to the angular impulse applied. *Angular momentum is changed through angular impulse.*

Incorporating Rotation in the Work-Energy Relation

Our previous encounter with the work-energy relation resulted in:

$$\frac{1}{2}mv_i^2 + \frac{1}{2}ks_i^2 + mgh_i + \int_{r_i}^{r_f}(F\cos\phi)dr = \frac{1}{2}mv_f^2 + \frac{1}{2}ks_f^2 + mgh_f$$

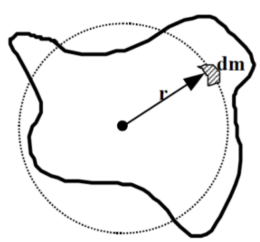
Recall from our previous discussion of work-energy that this is *not* a vector equation, meaning it is not applied independently in each of the coordinate directions. Generalizing this equation to include rotation will involving adding terms to this equation, not creating a separate "angular" energy equation.

Recall that we model the motion of an arbitrary rigid body as a superposition of a pure translation of the CM and a pure rotation about the CM. Let's investigate the effect of this model on our calculation of the kinetic energy of the object.

The translation portion of the motion is easy. We envision the object as a point particle, localized at the CM of the real object, traveling with the velocity of the CM of the object. Thus, this portion of the motion contributes a kinetic energy,

$$KE_{translation} = \frac{1}{2}mv_{CM}^2$$

What about the kinetic energy due to the pure rotation of the object about the axis through the CM?



Every small chunk of the object, dm, moves in a circle around the CM. Each of these pieces of mass has a velocity magnitude given by

$$v = r\omega$$

Thus, the kinetic energy of each piece is

$$KE_{dm} = \frac{1}{2}(dm)v^2$$

$$KE_{dm} = \frac{1}{2}(dm)(r\omega)^2$$

Therefore, the total kinetic energy due to rotation of all the little pieces is:

$$KE_{rotation} = \int \frac{1}{2}(dm)(r\omega)^2$$

$$KE_{rotation} = \frac{1}{2}\omega^2 \int r^2 (dm)$$

$$KE_{rotation} = \frac{1}{2}I\omega^2$$

Combining the kinetic energy due to rotation with the kinetic energy due to translation leads to a total kinetic energy of:

$$KE = \frac{1}{2}mv_{CM}{}^2 + \frac{1}{2}I\omega^2$$

and a work-energy relation of:

$$\frac{1}{2}mv_i{}^2 + \frac{1}{2}I\omega_i{}^2 + \frac{1}{2}ks_i{}^2 + mgh_i + \int_{r_i}^{r_f}(F\cos\phi)dr = \frac{1}{2}mv_f{}^2 + \frac{1}{2}I\omega_f{}^2 + \frac{1}{2}ks_f{}^2 + mgh_f$$

Unless a scenario involves springs or other elastic material, I'll typically write this relationship as:

$$\frac{1}{2}mv_i{}^2 + \frac{1}{2}I\omega_i{}^2 + mgh_i + \int_{r_i}^{r_f}(F\cos\phi)dr = \frac{1}{2}mv_f{}^2 + \frac{1}{2}I\omega_f{}^2 + mgh_f$$

Analysis Tools

Applying the Work-Energy Relation including Rotation – I

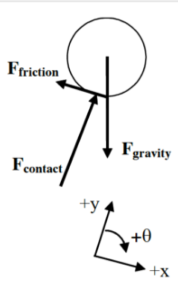
> A mischievous child releases his mother's bowling ball from the top of the family's 25 m long, 150 above horizontal driveway. The ball rolls without slipping down the driveway and at the bottom plows into the mailbox. The 6.4 kg ball has a diameter of 24 cm.

Let's determine the speed of the ball when it hits the mailbox. To determine this value, we can apply the work-energy relation to the ball between:

Event 1: The instant the ball is released.

Event 2: The instant the ball hits the mailbox.

For these two events, work-energy looks like this:



$$\frac{1}{2}mv_i{}^2 + \frac{1}{2}I\omega_i{}^2 + mgh_i + \int_{r_i}^{r_f}(F\cos\phi)dr = \frac{1}{2}mv_f{}^2 + \frac{1}{2}I\omega_f{}^2 + mgh_f$$

$$0 + 0 + 6.4(9.8)(25\sin 15) + \int_{r_i}^{r_f}(F\cos\phi)dr = \frac{1}{2}(6.4)v_f{}^2 + \frac{1}{2}(\frac{2}{5}(6.4)(.12)^2)\omega_f{}^2 + 0$$

$$406 + \int_{r_i}^{r_f}(F\cos\phi)dr = 3.2v_f{}^2 + 0.0184\omega_f{}^2$$

where the bottom of the driveway is the zero for gravitational potential energy and the rotational inertia of the bowling ball is taken to be that of a sphere.

Now we must carefully determine which, if any, of the forces on the bowling ball do work on the bowling ball.

First, we don't have to worry about the force of gravity. The gravitational potential energy function was developed to automatically incorporate the work done by the force of gravity.

Second, the contact force can do no work on the ball because the contact force is always perpendicular to the motion of the ball.

Finally, what about the force of friction? It does appear, at first glance, that the force of friction is applied over the entire motion of the ball down the driveway. However, let's pay closer attention to the actual point at which the force acts.

The frictional force is static in nature, because since the ball rolls without slipping down the driveway the bottom of the ball is always in static contact with the ground (i.e., there is no relative velocity between the bottom of the ball and the ground). If the bottom of the ball has a velocity of zero, then the force that acts on the bottom of the ball (static friction) can act through no distance. During the instant at which the frictional force acts on a particular point on the bottom of the ball, that point is not moving. That point on the ball only moves when it is no longer in contact with the ground, but by that time the frictional force is acting on a *different* point. To summarize (and stop saying the same thing over and over), *the force of static friction can do no work because it acts on a point that does not move*.

Thus, our equation simplifies to:

$$406 = 3.2{v_f}^2 + 0.0184{\omega_f}^2$$

The linear and angular velocity of the ball must be related. Since the ball rolls without slipping, the bottom of the ball has no linear velocity. Since the velocity of the bottom of the ball is the sum of the velocity due to translation of the CM and the velocity due to rotation about the CM, the velocity due to rotation must be equal in magnitude to the velocity of the CM:

$$v = r\omega$$

$$v_{CM} = 0.12\omega$$

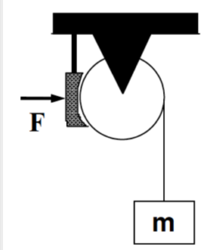
Substituting this into our equation yields:

$$406 = 3.2{v_f}^2 + 0.0184(\frac{v}{0.12})^2$$

$$406 = 3.2{v_f}^2 + 1.28{v_f}^2$$

$$v_f = 9.52 m/s$$

Applying the Work-Energy Relation including Rotation – II

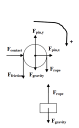
The crate is descending with speed v0 when a brake shoe is applied to the disk-shaped pulley of mass M and radius R. The coefficient of friction between the shoe and the pulley is (ms, mk). Determine the distance (D) the crate moves before stopping as a function of M, m, F, R, v0, g, and the appropriate coefficient of friction.	

To determine this function, let’s apply the work-energy relation to both the crate and the pulley between:

Event 1: The instant the brake shoe is applied.

Event 2: The instant the crate comes to rest.

In addition to defining the two instants of interest, we’ll need free-body diagrams for both the crate and the pulley.



Pulley

$$\frac{1}{2}m{v_i}^2 + \frac{1}{2}I{\omega_i}^2 + mgh_i + \int_{r_i}^{r_f}(F\cos\phi)dr = \frac{1}{2}m{v_f}^2 + \frac{1}{2}I{\omega_f}^2 + mgh_f$$

$$0 + \frac{1}{2}(\frac{1}{2}MR^2)(\frac{v_0}{R})^2 + 0 + \int_{r_i}^{r_f}(F\cos\phi)dr = 0 + 0 + 0$$

To find the work on the pulley, note that Fcontact and Fpin do no work since the distance over which these forces act is zero. However, both Frope and Ffriction do work.

If the crate falls a distance D, both of these forces act over a distance D. However, note that this displacement is in the same direction as Frope (f = 0) on the right side of the pulley but in the opposite direction on the other side of the pulley (opposite to Ffriction and therefore f = 180.) Since these forces are constant, there’s no need to actually integrate:

$$\frac{1}{4}M{v_0}^2 + (F_{rope}\cos 0)D + (F_{friction}\cos 180)D = 0$$

$$\frac{1}{4}M{v_0}^2 + DF_{rope} - DF_{friction} = 0$$

Note that since the brake shoe does not accelerate, the external force applied to the shoe, F, is the same magnitude as the contact force between the shoe and the pulley.

$$\frac{1}{4}M{v_0}^2 + DF_{rope} - D(\mu_k F) = 0$$

Crate

$$\frac{1}{2}mv_i^2 + \frac{1}{2}I\omega_i^2 + mgh_i + \int_{r_i}^{r_f}(F\cos\phi)dr = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 + mgh_f$$
$$\frac{1}{2}mv_0^2 + 0 + mgD + (F_{rope}\cos 180)D = 0 + 0 + 0$$
$$\frac{1}{2}mv_0^2 + mgD - DF_{rope} = 0$$

where the final position of the crate is the zero for gravitational potential energy.

The two equations can be added together to yield:

$$\frac{1}{4}Mv_0^2 - \mu_k FD + \frac{1}{2}mv_0^2 + mgD = 0$$
$$\frac{1}{4}Mv_0^2 + \frac{1}{2}mv_0^2 = \mu_k FD - mgD$$
$$D = \frac{\frac{1}{4}Mv_0^2 + \frac{1}{2}mv_0^2}{\mu_k F - mg}$$

Thus, in order for D to have a physical (positive) value, the frictional force on the pulley must be greater in magnitude than the force of gravity on the crate, which agrees with common sense. Note that the numerator is simply the total kinetic energy of the pulley plus crate. The larger this sum, the larger the distance needed to stop the crate's fall, which again agrees with common sense.

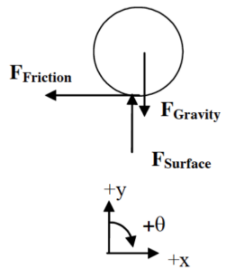
Applying the Impulse-Momentum Relations (Linear and Angular) – I

> A bowling ball of mass M and radius R leaves a bowler's hand with CM velocity v0 and no rotation. The ball skids down the alley before it begins to roll without slipping. The coefficient of friction is (ms, mk). Determine the elapsed time (T) before the ball begins to roll without slipping as a function of M, R, v0, g, and the appropriate coefficient of friction.

To determine this function, let's apply the impulse-momentum relations to the ball between:

Event 1: The instant the ball leaves the bowler's hand.

Event 2: The instant the ball begins to roll without slipping.

	Since the ball both translates and rotates, we must write both the linear and rotational forms of the impulse-momentum relation. Remember, the rotation of the bowling ball is modeled to be about an axis through the CM. (Since all the forces are constant, we'll write the relations without the use of the integral.)

x-direction (linear momentum)

$$mv_{x,i} + \int_{t_i}^{t_f} F_x dt = mv_{x,f}$$
$$Mv_0 - F_{friction}(T) = Mv_f$$
$$Mv_0 - \mu_k MgT = Mv_f$$
$$v_0 - \mu_k gT = v_f$$

θ-direction (angular momentum)

$$I\omega_i + \int_{t_i}^{t_f} \tau dt = I\omega_f$$
$$0 + (RF_{friction}(\sin 90))T = (\frac{2}{5}MR^2)\omega_f$$
$$R(\mu_k Mg)T = \frac{2}{5}MR^2\omega_f$$
$$\mu_k gT = \frac{2}{5}R\omega_f$$

The final CM velocity and final angular velocity are related because at this instant the ball begins to roll without slipping. When the ball rolls without slipping, the bottom of the ball is in static contact with the ground, i.e., it has no linear velocity. Since the velocity of the bottom of the ball is the sum of the velocity due to translation of the CM and the velocity due to rotation about the CM, the velocity due to rotation must be equal in magnitude to the velocity of the CM:

$$v = r\omega$$
$$v_f = R\omega_f$$

Substituting this into our angular equation yields:

$$\mu_k gT = \frac{2}{5}R(\frac{v_f}{R})$$

$$\mu_k gT = \frac{2}{5}v_f$$

$$v_f = \frac{5}{2}\mu_k gT$$

Plugging this into the linear equation:

$$v_0 - \mu_k gT = \frac{5}{2}\mu_k gT$$

$$v_0 = \frac{7}{2}\mu_k gT$$

$$T = \frac{2v_0}{7\mu_k g}$$

Thus, the faster you throw the ball, or the smaller the kinetic coefficient of friction, the longer it will take for the ball to begin to roll.

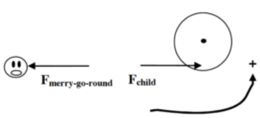
Applying the Impulse-Momentum Relations (Linear and Angular) – II

> A 30 kg child running at 4.0 m/s leaps onto the outer edge of an initially stationary merry-go-round. The merry-go-round is a flat disk of radius 2.4 m and mass 70 kg. Ignore the frictional torque in the bearings of the merry-go-round.

Let’s imagine we’re interested in determining the final angular velocity of the merry-go-round after the child has safely come to rest on its surface. To determine this angular velocity, we can apply the impulse-momentum relations to both the child and the merry-go-round between:

Event 1: The instant before the child lands on the merry-go-round

Event 2: The instant after the child comes to rest on the merry-go-round.



The child and the merry-go-round interact via some unknown magnitude force. (Contrary to the free-body diagram, this force does not necessarily act on the child’s disembodied head.) There are also other forces due to gravity and supporting structures that act on both the child and the merry-go-round, however, these forces are in the vertical direction and supply no torque.

Child	*Merry-Go-Round*
x-direction	θ-direction
$mv_{xi} + \int_{t_i}^{t_f} F_x dt = mv_{xf}$	$I\omega_i + \int_{t_i}^{t_f} \tau dt = I\omega_f$
$30(4) + \int_{t_i}^{t_f} (-F_{merrygoround}) dt = 30v_f$	$0 + \int_{t_i}^{t_f} (2.4F_{child} \sin 90) dt = (\frac{1}{2}70(2.4)^2)\omega_f$
$120 - \int_{t_i}^{t_f} F_{merrygoround} dt = 30v_f$	$\int_{t_i}^{t_f} F_{child} dt = 84\omega_f$

The final velocity of the child and final angular velocity of the merry-go-round are related because at this instant the child is at rest (hanging on to) the merry-go-round. Therefore,

$$v = r\omega$$

$$v_f = 2.4\omega_f$$

Substituting this into our angular equation yields:

$$\int_{t_i}^{t_f} F_{child} dt = 84(\frac{v_f}{2.4})$$

$$\int_{t_i}^{t_f} F_{child} dt = 35v_f$$

By Newton's Third Law, the force of the child on the merry-go-round and the force of the merry-go-round on the child must be equal in magnitude. Therefore our two equations can be summed and the impulses cancel. This gives:

$$120 = 30v_f + 35v_f$$

$$v_f = 1.85 m/s$$

The child is slowed from 4 m/s to 1.85 m/s by jumping onto the merry-go-round. The merry-go-round, however, is accelerated from rest to:

$$1.85 = 2.4\omega_f$$

$$\omega_f = 0.77 rad/s$$

Activities

For each of the scenarios described below, indicate the amount of linear kinetic energy, rotational kinetic energy, and gravitational potential energy in the system at each of the events listed. Use a consistent scale for each motion. Set the lowest point of each motion as the zero-point of gravitational potential energy.

A mischievous child releases his mother's bowling ball from the top of the family's 15 m long, 80 above horizontal driveway. The ball rolls without slipping down the driveway and at the bottom plows into the mailbox.

A yo-yo of inner diameter 0.70 cm and outer diameter 8.0 cm is released from rest. The string is 0.80 m long. Assume the string does not slip on the inner diameter of the yo-yo as the yo-yo falls..

For each of the scenarios described below, indicate the amount of linear kinetic energy, rotational kinetic energy, and gravitational potential energy of each object at each of the events listed. Use a consistent scale for each motion. Set the initial positions of the objects as the zero-points of gravitational potential energy.

Tired of walking up the stairs, an 80 kg engineering student (S) designs an ingenious device for reaching his third floor dorm room. An 84 kg block (B) is attached to a rope that passes over a 0.70 m diameter, 15 kg, disk-shaped pulley (P). The student holds the other end of the rope. When the block is released, the student is pulled up to his dorm room.

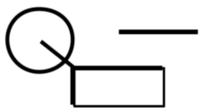
A 75 kg man (M) is falling at 20 m/s, 75 m above crocodile infested waters! He holds a rope attached to a 15 kg simple pulley (P). In an attempt to save him, a brake shoe is pressed against the spinning pulley. The man is saved, but barely.

For each of the scenarios described below, indicate the amount of linear kinetic energy, rotational kinetic energy, and gravitational potential energy of each object at each of the events listed. Use a consistent scale throughout both motions. Set the initial positions of the objects as the zero-points of gravitational potential energy

In a horizontal skiing device, the skier begins from rest 35 m from the end of the skiing run. The skier (S) has a mass of 75 kg, the block (B) has a mass of 50 kg, and the simple pulley (P) has a mass of 15 kg. The coefficient of friction is extremely small.

In an inclined skiing device, the skier begins from rest 35 m from the end of the 200 above horizontal inclined skiing run. The skier (S) has a mass of 75 kg, the block (B) has a mass of 50 kg, and the simple pulley (P) has a mass of 15 kg. The coefficient of friction is extremely small.

A disk-shaped pulley of mass M and radius R is rotating at angular velocity w. The friction in its bearings is so small that it can be ignored. A brake shoe is pressed against the pulley in order to stop it. In all cases, the brake shoe is pressed against the pulley with the same force and the coefficient of friction between the brake shoe and the pulley is the same.



	M	R	ω
A	10 kg	0.8 m	12 rad/s
B	10 kg	0.4 m	24 rad/s
C	20 kg	0.4 m	6 rad/s
D	40 kg	0.2 m	3 rad/s
E	20 kg	0.2 m	3 rad/s
F	30 kg	0.6 m	8 rad/s

Rank the scenarios below on the amount of time it takes to stop the pulley.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

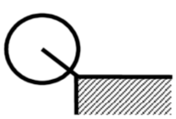
Rank the scenarios below on the angle through which the pulley rotates before stopping.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

A disk-shaped pulley of mass M and radius R is rotating at angular velocity w. The friction in its bearings is constant and ultimately causes the pulley to stop rotating.



	M	R	ω
A	10 kg	0.8 m	12 rad/s
B	10 kg	0.4 m	24 rad/s
C	20 kg	0.4 m	6 rad/s
D	40 kg	0.2 m	3 rad/s
E	20 kg	0.2 m	3 rad/s
F	30 kg	0.6 m	8 rad/s

Rank the scenarios below on the amount of time it takes to stop the pulley.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

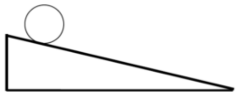
Rank the scenarios below on the angle through which the pulley rotates before stopping.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

A sphere of mass m and radius R is released from rest at the top of an incline and rolls without slipping down the incline. All spheres are released from rest from the same location on the incline.



	m	R
A	1 kg	30 cm
B	2 kg	60 cm
C	3 kg	10 cm
D	2 kg	15 cm
E	1 kg	15 cm
F	2 kg	30 cm

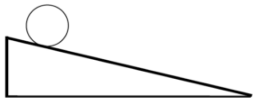
Rank the speed of the sphere at the bottom of the incline.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Six different equal-mass objects are released from rest from the same location at the top of an incline and roll without slipping down the incline.



	Object	R
A	solid sphere	30 cm
B	hollow sphere	60 cm
C	solid disk	10 cm
D	hoop	10 cm
E	solid cylinder	20 cm

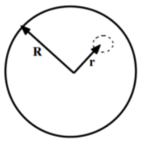
Rank the speed of the objects at the bottom of the incline.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

A merry-go-round of radius R is rotating at constant angular speed. The friction in its bearings is so small that it can be ignored. A sandbag of mass m is dropped onto the merry-go-round, at a position designated by r. The sandbag does not slip or roll upon contact with the merry-go-round.



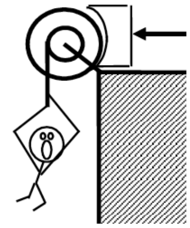
	m	r
A	10 kg	0.50 R
B	10 kg	0.25 R
C	20 kg	0.25 R
D	40 kg	0.25 R
E	10 kg	1.00 R
F	15 kg	0.75 R

Rank the scenarios on the basis of the angular speed of the merry-go-round after the sandbag “sticks” to the merry-go-round.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

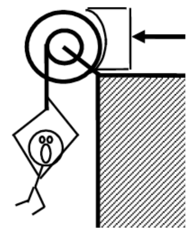
Explain the reason for your ranking:

The 75 kg man is falling at 20 m/s, 75 m above the crocodile infested waters below! In an attempt to save him, the brake shoe is pressed against the spinning pulley. The coefficient of friction between the brake shoe and the pulley is (0.9, 0.8), and the 35 kg disk-shaped pulley has inner and outer diameters of 0.60 m and 0.90 m, respectively. The man is saved, but barely.	

Free-Body Diagrams

Mathematical Analysis[i]

	Event 1: Event 2:

The 75 kg man is falling at 15 m/s! In an attempt to stop his fall, the brake shoe is pressed against the spinning pulley with a force of 1000 N. The coefficient of friction between the brake shoe and the pulley is (0.9, 0.8), and the 35 kg disk-shaped pulley has inner and outer diameters of 0.60 m and 0.90 m, respectively.	

Free-Body Diagrams

Mathematical Analysis[ii]

	Event 1: Event 2:

How long does it take to stop the man's fall?

How far does the man fall?

Tired of walking up the stairs, an 80 kg engineering student designs an ingenious device for reaching his third floor dorm room. An 84 kg block is attached to a rope that passes over a 0.70 m diameter, 15 kg, disk-shaped pulley. The student holds the other end of the rope. When the 84 kg block is released, the student is pulled up to his dorm room in 5.3 s.

Free-Body Diagrams

Mathematical Analysis[iii]

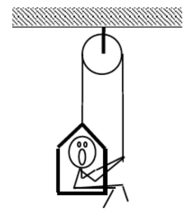
	Event 1: Event 2:

Tired of walking up the stairs, an 80 kg engineering student designs an ingenious device for reaching her third floor dorm room. An 42 kg block is attached to a rope that passes over the outer diameter of a 0.70 m outer diameter, 15 kg, disk-shaped compound pulley. The student holds a second rope, wrapped around the inner 0.35 m diameter of the pulley. When the 42 kg block is released, the student is pulled up to her dorm room, 8.0 m off the ground.

Free-Body Diagrams **Mathematical Analysis**[iv]

Event 1:
Event 2:

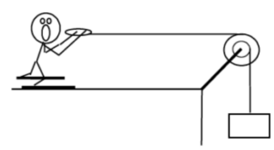
A 60 kg student lifts herself from rest to a speed of 1.5 m/s in 2.1 s. The boson's chair has a mass of 35 kg. The 20 kg, disk-shaped pulley has a diameter of 1.1 m.



Free-Body Diagrams **Mathematical Analysis**[v]

Event 1:
Event 2:

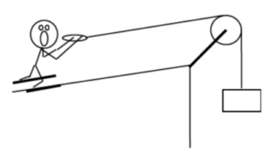
The device at right guarantees all the excitement of skiing without the need for hills. The 80 kg man begins from rest and reaches a speed of 34 m/s in 7.2 s. The 10 kg, disk-shaped pulley has inner and outer diameters of 0.40 m and 0.90 m, respectively. Assume friction is so small that it can be ignored.



Free-Body Diagrams **Mathematical Analysis**[vi]

Event 1:
Event 2:

The device at right allows you to ski uphill. The 85 kg skier begins from rest and reaches a speed of 14 m/s after traveling 25 m up the incline. The 10 kg, disk-shaped pulley has diameter 0.90 m. Assume friction is so small that it can be ignored. The ramp is inclined at 20° above horizontal.



Free-Body Diagrams **Mathematical Analysis**[vii]

Event 1:
Event 2:

A yo-yo of mass 0.60 kg, inner diameter 0.70 cm, and outer diameter 8.0 cm is released from rest. The string is 0.80 m long. Assume the rotational inertia of the yo-yo is similar to a simple disk and that the unwinding of the string does not affect the rotational inertia.

Free-Body Diagram

Mathematical Analysis[viii]

Event 1:
Event 2:

A bowling ball of mass 7.1 kg and diameter 21.6 cm is thrown down the alley at a speed of 9.5 m/s. The bowling ball is initially skidding, with no angular velocity, down the alley. The ball skids for 1.8 s before it begins to roll without slipping.

Free-Body Diagram

Mathematical Analysis[ix]

Event 1:
Event 2:

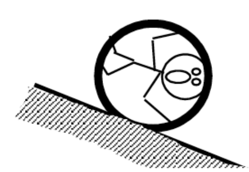
A mischievous child releases his mother's bowling ball from the top of the family's 15 m long, 80 above horizontal driveway. The ball rolls without slipping down the driveway and at the bottom plows into the mailbox. The 6.4 kg ball has a diameter of 21.6 cm.

Free-Body Diagram

Mathematical Analysis[x]

Event 1:
Event 2:

The 65 kg strange man at right is trapped inside a section of large pipe. If that's not bad enough, the pipe begins to roll from rest down a 35 m long, 180 incline! The pipe rolls down the hill without slipping. The pipe has a mass of 180 kg and a diameter of 1.2 m. (Assume the man's presence inside the pipe has a negligible effect on the pipe's rotational inertia.)



Free-Body Diagram

Mathematical Analysis[xi]

	Event 1: Event 2:

A 30 kg child running at 3.0 m/s leaps onto the outer edge of an initially stationary merry-go-round. The merry-go-round is a flat disk of radius 2.4 m and mass 50 kg. Ignore the frictional torque in the bearings of the merry-go-round.

Free-Body Diagrams **Mathematical Analysis**[xii]

child (top view)	*merry-go-round (top view)*	Event 1: Event 2:

A 40 kg child running at 4.0 m/s leaps onto the outer edge of a merry-go-round initially rotating in the opposite direction. The merry-go-round is brought to rest. The merry-go-round is a flat disk of radius 2.4 m and mass 50 kg. Ignore the frictional torque in the bearings of the merry-go-round.

Free-Body Diagrams **Mathematical Analysis**[xiii]

child (top view)	*merry-go-round (top view)*	Event 1: Event 2:

A 30 kg bag of sand is dropped onto a merry-go-round 1.6 m from its center. The merry-go-round was rotating at 3.2 rad/s before the bag was dropped. The merry-go-round is a flat disk of radius 2.4 m and mass 50 kg. Ignore the frictional torque in the bearings of the merry-go-round.

Free-Body Diagrams **Mathematical Analysis**[xiv]

sandbag (top view)	*merry-go-round (top view)*	Event 1: Event 2:

A 30 kg bag of sand is dropped onto a merry-go-round. The merry-go-round was rotating at 2.2 rad/s before the bag was dropped, and 1.3 rad/s after the bag comes to rest. The merry-go-round is a flat disk of radius 2.4 m and mass 60 kg. Ignore the frictional torque in the bearings of the merry-go-round.

Free-Body Diagrams **Mathematical Analysis**[xv]

sandbag (top view)	*merry-go-round (top view)*	Event 1: Event 2:

A child leaps onto the outer edge of an initially stationary merry-go-round. The merry-go-round is a flat disk of radius R and mass M. Determine the angular speed of the merry-go-round (w) after the child jumps on as a function of the child's mass (m), initial speed (v), M, and R. Ignore the frictional torque in the bearings of the merry-go-round.

Free-Body Diagrams **Mathematical Analysis**

child (top view)	*merry-go-round (top view)*	Event 1: Event 2:

Questions

If v = 0 m/s, what should w equal? Does your function agree with this observation?

If M =∞, what should w equal? Does your function agree with this observation?

If m =∞, what should w equal? Does your function agree with this observation?

Tired of walking up the stairs, an engineering student of mass m designs an ingenious device for reaching her third floor dorm room. An block of mass M is attached to a rope that passes over a disk-shaped pulley of mass Mp and radius R. The student holds the other end of the rope. When the block is released, the student is pulled up to her dorm room. Determine the velocity of the student (vS) as she reaches her room as a function of m, M, Mp, R, g, and her distance off the ground (D).

Free-Body Diagrams

Mathematical Analysis

Event 1:
Event 2:

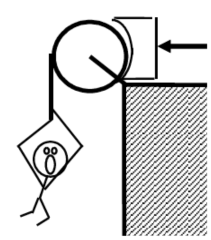
Questions

If D = 0 m, what should vS equal? Does your function agree with this observation?

If m = M, what should vS equal? Does your function agree with this observation?

If Mp= ∞, what should vS equal? Does your function agree with this observation?

The man of mass m is falling at speed v! In an attempt to save him, the brake shoe is pressed against the spinning pulley with force F. The pulley has mass M and radius R. Determine the time needed to stop the man's fall (T) as a function of m, M, v, R, g, F, and the appropriate coefficient of friction between the brake shoe and the pulley.



Free-Body Diagrams

Mathematical Analysis

Event 1:
Event 2:

Questions

If m = 0, what should T equal? Does your function agree with this observation?

If m = ∞, what should T equal? Does your function agree with this observation?

If F = ∞, what should T equal? Does your function agree with this observation?

A yo-yo of mass m, inner diameter d, and outer diameter D is released from rest. Assume the rotational inertia of the yo-yo is similar to a simple disk of diameter D and that the unwinding of the string does not affect the rotational inertia. Determine the angular velocity of the yo-yo (w) as a function of the elapsed time since release (t), g, d, and D.

Free-Body Diagram

Mathematical Analysis

Event 1:
Event 2:

Questions

If D = ∞, what should w equal? Does your function agree with this observation?

If g = 0 m/s2, what should w equal? Does your function agree with this observation?

A bowling ball of mass m and radius R is thrown down the alley at a speed v. The bowling ball is initially skidding, with no angular velocity, down the alley. Determine the time the ball skids (T) before beginning to roll without slipping as a function of m, R, g, v, and the appropriate coefficient of friction between the ball and the lane.

Free-Body Diagram

Mathematical Analysis

Event 1:
Event 2:

Questions

If m = 0, what should T equal? Does your function agree with this observation?

If v = ∞, what should T equal? Does your function agree with this observation?

[i] Fonbrake = 867 N

[ii] a. t = 3.69 s b. r = 27.7 m

[iii] v2 = 1.21 m/s

[iv] v2student = 1.5 m/s

[v] Fright rope = 503 N

[vi] mblock = 116 kg

[vii] mblock = 108 kg

[viii] v2 = 0.49 m/s

[ix] v2 = 6.8 m/s

[x] v2 = 5.4 m/s

[xi] v2 = 11 m/s

[xii] v2 = 1.64 m/s

[xiii] w1 = -2.67 rad/s

[xiv] w2 = 2.1 rad/s

[xv] R = 2.0 m

Homework 12 – Model 4: 119, 130, 131, 132

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1. Please remember that this model, and hence the relationships derived under it, are restricted to rigid bodies and motions in which the rotation axis is perpendicular to the plane in which the center-of-mass moves. Since objects must be rigid, their rotational inertia must remain constant. In addition, motions must either be about a stationary rotation axis or, if not, the rotation axis is taken to be through the CM. ↵

CHAPTER OVERVIEW

5: Labs

5.1: Forces and Conservation Laws in Two Dimensions

Name: ____

Date: ________________________

Partners:____

Equipment

- *Movie* software file
- *PuckCollision* movie
- *StandingJump* movie
- *LabPro* interface
- Force Plate
- *BigForce* software file
- *FrontalCrash* movie

I. Two Dimensional Collision

Open *Movie* and select **Insert/Movie**. Find the movie *PuckCollision* and open it. Play the movie. The movie shows two pucks colliding on an air table. Return the movie to the first frame.

Scale the movie and extract position vs. time data for both pucks.

A. Momentum

To examine the collision in terms of momentum, rather than velocity, *LoggerPro* must be "taught" how to measure momentum.

To determine the x-momentum of puck1:

- Select **Data/New Calculated Column** and enter the appropriate name and units.
- In the equation box, enter the product of the mass of puck1 and its x-velocity. Use the **Variables** pull-down menu to select the appropriate variable for your equation. In this example, the equation should read: 0.050*"X Velocity". (You can set both puck masses to 50 g since the error introduced by this simplification is much less than the error inherent in selecting pixel locations for the two pucks.)

Use this technique to create columns for:

- the x-momentum of each puck,
- the y-momentum of each puck,
- the total x-momentum in the system of the two pucks,
- and the total y-momentum in the system of the two pucks,

Create a graph of the x-momentum of each puck and the total x-momentum vs. time. Print, label, and attach this graph.

Extract data from this graph to complete the first row of the following table.

	before collision	**after collision**
Total x-momentum	±	±
Total y-momentum	±	±

Create a graph of the y-momentum of each puck and the total y-momentum vs. time. Print, label, and attach this graph. Complete the above table.

Question: Which, if either, of the momentum defined above appear to be conserved in the collision? Which, if either, of the momentum *should* be conserved in a collision of this type? Explain.

B. Energy

Using **Data/New Calculated Column** create columns for:

- the kinetic energy of each puck,
- and the total kinetic energy in the system of the two pucks,

Create a graph of kinetic energy of each puck and the total kinetic energy vs. time. Print, label, and attach this graph.

Extract data to complete the following table.

	before collision	**after collision**
Total kinetic energy	±	±

Question: Does the kinetic energy defined above appear to be conserved in the collision? *Should* the kinetic energy be conserved in a collision of this type? Explain.

Question: Carefully watch the two pucks before and after the collision. Can you see where some of the initial kinetic energy in the system is "hiding" after the collision?

II. Vertical Leap

Open the movie *StandingJump*. Play the movie. The movie shows a man (5'8" tall and 160 lbs) performing a standing, vertical jump.

It is quite complicated to accurately analyze this motion. Since his arms and legs move relative to his torso, his center-of-mass is not at a constant location on his body. However, we can approximate the complicated motion of this man by a single point particle, located at his hip, and determine rough answers to a series of interesting questions.

Scale the movie and extract position vs. time data.

As a test of the accuracy of your "clicking" (and a test of the validity of our approximation of a person by a point), fit the appropriate function to the appropriate portion of the y-position vs. time graph to determine the acceleration of the man while he's in the air. Print this graph with the region selected and best-fit function displayed and attach it to the end of this activity.

Question: Based on your best-fit function, what is the acceleration of the man, with uncertainty and units, while he is in the air?

If your result above is reasonable, continue your analysis to answer the following questions. For each question, fit the appropriate function to the appropriate portion of the y-position vs. time graph to answer each question. Print each graph with the region selected and best-fit function displayed and attach it to the end of this activity. Show your work in answering each question. Remember, ***every*** value generated from experimental data has an associated uncertainty!

Question: What is the vertical speed of the man when he leaves the ground?

Question: What is his acceleration during lift-off? What is the force exerted by his legs on the ground during lift-off?

Question: What is his acceleration during landing? What is the force exerted by his legs on the ground during landing?

Use **Data/New Calculated Column** to calculate the man's total energy vs. time. Create a graph of total energy vs. time.

Question: How much energy does the man "create" during lift-off? Where does this energy come from?

Question: How much energy does the man "lose" during landing? Where does this energy go?

Question: Comment on the effectiveness of your analysis. What could you have done to improve your analysis? Do you feel your results are accurate?

III. Live Vertical Leap

Attach the force plate to the *LabPro* interface and open the file *BigForce*.

Rather than watch a video of a person jumping, you are going to perform a vertical jump on top of the force plate. Select your lab group's strongest jumper and collect force vs. time data for a complete lift-off and landing. Print and attach your graph. Use your data to answer the following questions:

Question: What are the maximum force and the average force exerted by your legs during lift-off?

Question: What the maximum force and the average force exerted by your legs during landing?

Question: How do the force exerted by your legs compare to the values calculated for the "professional" jumper?

IV. Car Crash

Open the movie *FrontalCrash.* Play the movie. The movie shows a driver's-side frontal crash test.

This is even more complicated to analyze than the vertical jump! The car is marked by several cross-hairs that can be tracked to gather kinematic data on different parts of the car. For your initial collection of data, extract position vs. time data for the crosshair located slightly in front of where the driver's head would be.

To scale the movie you will have to do a little detective work. Once you figure out the make and model of the car involved in the test, I'd turn Google loose. Once you've scaled the movie, create a graph of x- and y-position vs. time.

As a test of the accuracy of your "clicking" (and your ability to read small print), fit the appropriate function to the appropriate portion of the graph to determine the speed of the car before the crash. Print this graph with the region selected and best-fit function displayed and attach it to the end of this activity.

Question: Based on your best-fit function, what is the speed of the car before the crash, with uncertainty and units? (This crash test was done at 40 mph. If your answer is not within a reasonable uncertainty to 40 mph, you have made a mistake in scaling or analyzing the movie.)

If your result above is reasonable, continue your analysis to answer the following questions. For each question, fit the appropriate function to the appropriate portion of the graph to answer each question. Print each graph with the region selected and best-fit function displayed and attach it to the end of this activity. Show your work in answering each question.

Question: What are the x-, y-, and total acceleration of the crosshair above the driver's head during the collision? How many "g's" of acceleration does this correspond to?

Question: What are the x-, y-, and total acceleration of the crosshair located near the center of the hood of the car? Compare these accelerations to the accelerations of the point near the driver's head. How does the "crumple zone" of the car effect the relative accelerations of these two points?

Use **Data/New Calculated Column** to calculate the car's kinetic energy vs. time. Create a graph of kinetic energy vs. time.

Question: How much energy does the car "lose" during the collision? Where does this energy go?

Question: Comment on the effectiveness of your analysis. What could you have done to improve your analysis? Do you feel your results are accurate?

5.2: Air Resistance

Name: ____

Date: _______________________

Partners:____

Equipment

- *LabPro* Interface
- Motion detector
- *Motion* software file
- Large coffee filters
- *Drag* Excel file

Introduction

The motion of objects through air is studied in every introductory physics course. Ignoring the effects of air resistance, or *drag*, allows one to derive simple equations to predict the time of flight, range, and various other parameters of the motion. These equations are, however, only approximations to the true motions of real objects. Air drag is seldom so small that it can be ignored in the real world.

Unfortunately, the mathematics of including drag in the study of motion are quite complicated. However, by using an approximation scheme and the calculational power of *Excel*, you will develop a spreadsheet that can calculate motion parameters that are quite close to those measured in the real world.

I. Quantifying Drag

To quantitatively study the effect of drag on the motion of an object, we need to quantify what we mean by drag. Drag is a force that acts to oppose the motion of an object through a fluid. (To a physicist, air and water are both fluids.) This force depends on numerous parameters of the system.

Question: What parameters do you think drag depends on? (Imagine an object moving through air. What variables affect the size of the drag force?) Explain why each parameter affects the drag force.

Often, all of the parameters effecting drag *except the speed of the object* are lumped together into a single number, the *drag coefficient*. The dependence on speed will be determined in the following activity.

Place the motion detector on the floor and open the file *Motion.* Add a velocity graph to the display.

Hold a large coffee filter directly above the motion detector, as close to the ceiling as possible, press **Collect**, and release the filter. If necessary, repeat until you have clean data.

Question: Describe the motion of the coffee filter. Is the acceleration of the filter constant?

Question: Draw a free-body diagram for the coffee filter as it falls. Label the drag force F_{drag}. Apply Newton's Second Law in the vertical direction.

If the drag force was constant, both forces acting on the filter would be constant resulting in a constant acceleration. However, if the drag force depended on the speed of the filter, the drag force would grow in magnitude as the filter fell until the drag force equaled the weight of the filter. At this point, the two forces would be equal and the acceleration of the filter would drop to zero. The speed at which this occurs is termed the *terminal velocity* of the filter.

At terminal velocity, v_T,

- If the drag force was proportional to speed, this would lead to:

and the terminal velocity would be proportional to the mass of the filter.

- If the drag force was proportional to the square of the speed, this would lead to:

and the square of the terminal velocity would be proportional to the mass of the filter.

Thus, we can determine the dependence of drag force on speed by determining the dependence of terminal velocity on mass!

Measure the terminal velocity of the coffee filter by highlighting the time interval over which the velocity is constant. Record your result in the table below. Label, print, and attach the graph illustrating this measurement.

You can increase the mass of the coffee filter (without altering its shape) by stacking coffee filters inside each other. Thus, the number of coffee filters stacked together is proportional to the mass of the falling object. Do this to complete the following table.

Number of Filters	Terminal Velocity (m/s)
1	±
2	±
3	±
4	±
5	±
6	±
7	±

To determine the relationship between drag force and speed, create the following two graphs:

- Terminal velocity, with error bars, vs. number of filters. If a direct proportion exists between these two variables, drag is proportional to speed.
- Terminal velocity squared, with error bars, vs. number of filters. If a direct proportion exists between these two variables, drag is proportional to speed squared. (By calculus, the uncertainty in terminal velocity squared is *twice* the uncertainty in terminal velocity.)

Print and attach these two graphs, with linear-fit functions displayed.

Question: What is the mathematical dependence of drag on the speed of the object (i.e., which of the two graphs provides a better fit to the data)? Clearly explain how your data supports this relationship.

Question: Clearly explain the limitations in determining the dependence of drag force on speed by this method. What could be done to improve this experiment?

II. Drag in One Dimension

A. Creating and Testing the Spreadsheet

To begin, imagine a baseball thrown vertically upward at 35 m/s from the top of a cliff 45 m above a river. If you ignore drag, you should be able to calculate the time it takes for the ball to reach the river and its speed on impact.

Question: Determine the time it takes to reach the river and its speed on impact.

Our ultimate task is to create a spreadsheet that can solve a problem like this one when drag is included. As a first step, we will create a spreadsheet that can solve a kinematic problem *without* drag, and compare its results to the "known" correct answer. Once the spreadsheet passes this test, we will try to add drag to the spreadsheet and use it to solve problems where the answer can not be calculated by hand.

The equations you used to solve the above problem are only valid when the acceleration is constant. Since drag leads to a non-constant acceleration, they are not very useful if the ultimate goal is to incorporate drag. Rather, a general technique for solving any motion problem, no matter how complicated, will serve us better. In this approach, we will apply the basic concepts of kinematics in a step-by-step manner to the motion.

In a nutshell, the velocity of an object at one instant can always be determined if you know the velocity of the object at a *previous* instant and the object's acceleration, via:

In words, the equation says that the velocity of the object at the “new” time (t + Δt), is equal to the velocity at the previous time (t) plus the product of the acceleration at the previous time and the difference in time.

Also, the position of an object at one instant can always be determined if you know the position of the object at a *previous* instant and the object’s velocity, via:

Again, this equation says that the position of the object at the “new” time (t + Δt), is equal to the position at the previous time (t) plus the product of the velocity at the previous time and the difference in time.

These equations are only approximate, but become exact in the limit as the *time-step*, Δt, approaches zero. As long as we use a very small Δt, the approximation made will be very small.

Open the Excel file *Drag*.

Input the initial time and the initial position, velocity and acceleration of the baseball. The mass of a baseball is 0.145 kg and for this problem the drag coefficient is assumed to be zero. Use a time step of 0.05 s.

For each of the four calculated spreadsheet columns, write below the appropriate *Excel* formula needed to calculate the second row of each column (the first row is determined by the initial inputs):

time:

position:

velocity:

acceleration:

Enter the correct formulas and complete the spreadsheet. Create a graph of position, velocity, and acceleration vs. time, from release until striking the river. Print and attach your graph.

Question: Based on your spreadsheet, determine the time it takes to reach the river and its speed on impact. Compare these values to the values calculated by hand. If they are not *extremely* close, correct the problem.

B. Adding Drag

Earlier, you determined that the drag force is proportional to the square of the object’s speed. Thus

where b is the drag coefficient. This force is directed opposite to the object’s velocity. This directional information must be correctly included in the equation for drag.

For example, consider the equation

Question: If the velocity is in the positive direction, what direction does this equation produce for the drag force? Is this the correct direction for the drag?

Question: If the velocity is in the negative direction, what direction does this equation produce for the drag force? Is this the correct direction for the drag?

Question: Can the formula above correctly represent the drag force? Explain.

Now consider the equation

Question: If the velocity is in the positive direction, what direction does this equation produce for the drag force? Is this the correct direction for the drag?

Question: If the velocity is in the negative direction, what direction does this equation produce for the drag force? Is this the correct direction for the drag?

Question: Can the formula above correctly represent the drag force? Explain.

Neither of the equations given correctly model drag. The correct equation is:

Question: Explain why this equation gives the correct direction for the drag force.

To incorporate drag into your spreadsheet, you need to calculate the acceleration of the baseball at each and every time. (The velocity and position columns use general results that should not need to be changed.)

Question: Draw a free-body diagram for the baseball during its motion. Using the equation for drag given above, apply Newton's Second Law in the vertical direction and solve for the acceleration.

Enter the correct formula for acceleration and complete the spreadsheet. The drag coefficient for a baseball has been measured to be 1.2×10^{-3} Ns^2/m^2.

Create a graph of position, velocity, and acceleration vs. time, from release until striking the river. Print and attach your graph.

Question: Immediately after being thrown, the acceleration of the baseball is greater in magnitude than the acceleration due to gravity. How is this possible? Explain.

Question: Does the baseball reach terminal velocity? How do you know? Explain.

Complete the following table.

	Without drag	**With drag**
Maximum height		
Time of flight		
Impact speed		

Question: Based on the results above, comment on how drag changes the motion of the baseball.

III. Drag in Two Dimensions

After being hit, imagine a baseball is traveling at 50 m/s at an angle of 53^0 above horizontal.

To handle two-dimensional drag, you will need to calculate the x- and y-components of each of the kinematic variables. Thus, you will need to create columns to calculate a_x, a_y, v_x, v_y, r_x, and r_y. You will also need to set the initial values for all of these functions.

The correct equation for drag force, written as a vector equation, is:

This is equivalent to the two separate x- and y-component equations:

where the absolute value signs now represent the magnitude of a two-dimensional vector, which can be determined by Pythagoras' Theorem,

Question: Draw a free-body diagram for the baseball during its two-dimensional motion. Apply Newton's Second Law in the x- and y-direction and solve for the x- and y-accelerations.

Enter the correct formulas for x- and y-acceleration and complete the spreadsheet.

Construct a graph of the trajectory (y-position vs. x-position) of the baseball, showing the entire path of the ball until it hits the ground. Print and attach your graph.

Question: How far from home plate does the ball land? Is it an out or a home run? The fence is 110 m (360 ft) from home plate.

Question: Compare the above result to the same situation ignoring drag (set b = 0 and re-run the spreadsheet). How far from home plate does the ball now land? Is it an out or a home run? Is drag an important effect in the flight of this baseball?

Question: The drag coefficient is proportional to air density. Air density in Denver is about 88% of air density at sea level. How much farther will the ball described above travel in Denver than at sea level?

IV. Using Your Model

The launch angle that produces maximum range over level ground is commonly thought to be 45°. However, this is only true if drag is ignored. For a thrown (or hit) baseball, maximum range does not occur at 45°. The range of the ball has a rather complicated dependence on launch angle and launch velocity.

To explore this dependence, use your spreadsheet to determine the range of a ball launched at 30 m/s with launch angle 25, 30, 32, 34, 36, 38, 40, 42, 45, and 50°. Repeat for a ball launched at 40, 50 m/s and 60 m/s. Tabulate your results below.

Launch Speed (m/s)

	30	40	50	60	
Launch Angle (º)	*25*				
	30				
	32				
	34				
	36				
	38				
	40				
	42				
	45				
	50				

Using this data, create a graph of range vs. launch angle displaying all four sets of data. Print and attach your table and graph.

Question: Clearly explain why the maximum range occurs for a launch angle less than 45°.

Question: How does the launch angle for maximum range depend on launch velocity? Describe why this dependence is plausible.

5.3: Modeling Model Rockets

Name: ____

Date: _________________________

Partners:____

Equipment

- Laptop computer
- *LabPro* Interface
- Force sensor with rocket engine adaptor
- *ModelRocket* Excel file

Introduction

The motion of a rocket is quite complex. Typically the thrust force is not constant in time, the mass of the rocket decreases as the fuel burns, and substantial drag forces act on the rocket. This is as true of model rockets as full-scale rockets.

Creating a mathematical model that incorporates all of these aspects of the motion can be challenging. You will begin by creating a simplified model (constant thrust, constant mass, no drag) for a model rocket and gradually eliminate these simplifications.

I. Engine Thrust vs. Time

In this activity, you will model the flight of a 230 gram model rocket which initially carries 25 grams of fuel. (The fuel is the solid propellant in an Estes model D12 model rocket engine.) Thus, the total initial mass of the system is 255 g. The rocket will begin at rest on the ground.

Your instructor may test-fire a sample engine attached to a force probe to collect force vs. time data for the engine. If not, data from a previous test-firing is available in the file *ModelRocket*. If an engine is test-fired, import the data into the *ModelRocket* file in place of the saved data.

Create a graph of engine thrust vs. time. The thrust is obviously not constant.

II. Constant Thrust, Constant Mass, No Drag

A. Finding the Average Thrust

Although the thrust is not constant, it is useful to first analyze the motion of the rocket assuming the thrust *is* constant, because when the thrust is constant the motion can be analyzed by the familiar equations of introductory physics. We will assume the thrust is constant at its average value. Therefore, to proceed we must determine the average value of the thrust.

If the thrust *was* constant, the total impulse supplied by the engine would be given by:

where the constant thrust () can be “removed” from the integral.

This results in:

Therefore, determining the average thrust requires the determination of the integral of the thrust with respect to time. Since the mathematical function describing the thrust is not known, we will have to approximate the integral. The simplest way to do this is by replacing the thrust integral with a sum:

where F_{ave} is the average value of the force during each time step, t.

For example, if you are using the previously saved data during the first time step (from 0 s to 0.04 s) the average force is 0.5 N (the average of 1.0 and 0.0 N). Therefore, the value of the sum during this time step is

F_{ave} (t) = (0.5 N)(0.04 s) = 0.02 Ns

Question: Use the spreadsheet to determine the approximate value of the thrust integral. Record your result, with units, below.

Question: Based on the value of the thrust integral, determine the average thrust acting on the rocket. Record your result, with units, below. (Hint: If you are using the previously saved data, you should get a value between 11 and 12 N. If you don’t, do not proceed until you find your mistake!)

B. Finding the Maximum Height “by Hand”

A 255 g rocket is launched vertically upward from rest. The thrust acting on the rocket is equal to the value calculated above. Determine the maximum height reached by the rocket and the time needed to reach this height. Assume the mass of the rocket is constant and the drag acting on the rocket is zero.

Freebody Diagram Motion Information

	Event 1: t_1 = r_1 = v_1 = a_{12} =	Event 2: t_2 = r_2 = v_2 = a_{23} =	Event 3: t_2 = r_2 = v_2 =

Mathematical Analysis

C. Finding the Maximum Height by Spreadsheet

The kinematics equations you used to solve the above problem are only valid when the acceleration is constant. Since varying the thrust, mass and drag on the rocket all lead to non-constant accelerations, they will not be very useful in our ultimate model. Rather, a general technique for solving any motion problem, no matter how complicated, will serve us better. In this approach, we will apply the basic concepts of kinematics in a step-by-step manner to the motion.

In a nutshell, the velocity of an object at one instant can always be determined if you know the velocity of the object at a *previous* instant and the object's acceleration.

Also, the position of an object at one instant can always be determined if you know the position of the object at a *previous* instant and the object's velocity.

These equations are only approximate, but become exact in the limit as the *time-step*, t, approaches zero. As long as we use a very small t, the approximation made will be very small.

Input the initial position, velocity, and mass of the rocket. Assume the drag coefficient is zero. Enter the correct formulas and complete the spreadsheet *assuming the thrust is constant* (i.e., do not use the column containing the actual values of the thrust). Since you are trying to find the maximum height reached by the rocket, you will need to continue your time column beyond 1.76 s.

Question: Based on your spreadsheet, determine the maximum height reached by the rocket and the time it takes to reach this height. Compare these values to the values calculated by hand. If they are not *extremely* close, correct the problem.

III. Variable Thrust, Constant Mass, No Drag

Real rocket engines produce a thrust that is not constant in time. We will now attempt to include this factor into our model of a model rocket.

Question: Based on the actual variation of thrust vs. time, do you think that the model with variable thrust will fly higher, lower or the same as your original, constant thrust, prediction? Explain your rationale in detail.

Now it's time to determine the maximum height reached by the rocket, incorporating the fact that the thrust is not constant. You should be able to accomplish this by simply changing the formula in the acceleration column of your spreadsheet.

Once you correct the acceleration column, you should notice something odd immediately after launch. The rocket initially moves downward because $F_{thrust} < mg$. Obviously, the launch platform will prevent the rocket from moving downward. To correct this, replace the negative values for acceleration with zeros. This should correct the problem.

Question: Based on your spreadsheet, determine the maximum height reached by the rocket and the time it takes to reach this height. Compare these values to the values calculated assuming the thrust is constant. Carefully explain why the maximum height is different when the variation in thrust is taken into account.

IV. Variable Thrust, Variable Mass, No Drag

The mass of the rocket is not constant. At liftoff, it contains 25 grams of propellant that is not present once the thrust drops to zero. Further complicating the analysis is that since the thrust is not constant, the rate at which the rocket's mass decreases is not constant either.

Question: In light of the fact that the mass is not constant, do you think that the rocket will fly higher, lower or the same as your original, constant mass, prediction? Explain your rationale in detail.

Insert a column into your spreadsheet for the mass of the rocket. You should insert this column before the column for acceleration, because you need the correct mass to properly determine the acceleration. (Since you originally determined the acceleration by dividing the net force by a constant 0.255 kg, you must change your acceleration column to take into account the changing mass. To determine acceleration don't divide the net force by a constant, divide it by the mass column.)

The first entry in the mass column should be the initial mass of the rocket plus propellant, or 0.255 kg. The entry in this column once the thrust stops, and thereafter, should be .230 kg. What goes in between? A good guess is that the rate of mass loss is greatest when the thrust is greatest, because the rate of fuel consumption is greatest at this time. If we assume that the rate of mass loss is proportional to thrust, then we can set up the following ratios:

total mass lost total impulse	=	mass lost in each time step impulse in each time step

Question: Based on this relationship, you should be able to write an expression for the rocket mass at a given time in terms of the mass at a previous time and the necessary constants. Carefully write this expression below.

Incorporate this relationship into your spreadsheet. If incorporated correctly, your spreadsheet should now indicate that the mass decreases down to a value of .230 kg once the thrust is complete. If your spreadsheet doesn't accurately calculate the decreasing mass, do not proceed.

Finish altering your spreadsheet to incorporate the variable mass of the rocket.

Question: Based on your spreadsheet, determine the maximum height reached by the rocket and the time it takes to reach this height. Compare these values to the values calculated assuming the mass is constant. Carefully explain why the maximum height is different when the variation in mass is taken into account.

V. Variable Thrust, Variable Mass, Drag

Our final task is to incorporate air drag. All the corrections we have been making for variable thrust and variable mass are actually quite minor compared to the effect of drag.

Question: In light of the fact that the rocket experiences air drag, do you think that the rocket will fly higher, lower or the same as your original, frictionless, prediction? Explain your rationale in detail.

The standard model for air drag is that the drag force is proportional to the square of the speed of the object. A formula that always gives the correct direction for the drag force is:

Question: Explain why this equation gives the correct direction for the drag force.

Question: Draw a free-body diagram for the rocket. Use the equation for drag given above. Apply Newton's Second Law in the vertical direction and solve for the acceleration.

Enter the correct formula for acceleration and complete the spreadsheet. The drag coefficient for the particular model rocket under investigation has been measured to be 3.0×10^{-4} Ns^2/m^2. Remember to replace any negative accelerations at the start of the launch with zeroes because the rocket stays at rest on the launch pad until the rocket's thrust is greater than the force of gravity.

Finish altering your spreadsheet to incorporate the variable mass of the rocket.

Question: Based on your spreadsheet, determine the maximum height reached by the rocket and the time it takes to reach this height. Compare these values to the values calculated assuming there is no air drag. Carefully explain why the maximum height is different when drag is taken into account.

VI. Summary and Conclusions

Summarize your results in the table below.

thrust	**mass**	**drag**	**r_{peak} (m)**	**t_{peak} (s)**
Constant	*Constant*	*No*		

Variable	*Constant*	*No*		
Variable	*Variable*	*No*		
Variable	*Variable*	*Yes*		

Question: Did the inclusion of variable thrust substantially change the results? If the thrust of the rocket changed much more rapidly, how should we change our time step to maintain accuracy in our models? Explain.

Question: Did the inclusion of variable mass substantially change the results? If the initial mass of the rocket was mostly fuel, would your answer still be the same? Explain.

Question: Did the inclusion of drag substantially change the result? If our rocket had reached altitudes of several kilometers, how would our model of the drag force have to be changed? Explain.

Question: If our rocket reached altitudes of several hundred kilometers, what else in our model would have to be changed? Explain.

5.4: Force and Circular Motion

Name: ____

Date: ______________________

Partners:____

Equipment

- *LabPro* Interface
- Centripetal Force Apparatus (CFA)
- Force probe
- *CentripetalForce* software file
- *Graphing* software file
- Two each of 5, 10, and 20 g masses

Introduction

According to Newton's first law, a body in motion will remain in motion with constant velocity if the net force acting on it is zero. Constant velocity means that both the speed and direction do not change. An object moving in a circular path with constant speed does not have a constant velocity because the direction of the velocity is constantly changing. This implies that an object moving at a constant speed in a circular path is accelerating.

According to Newton's second law, a non-zero net force is needed to cause acceleration. In the case of an object moving in a circular path the acceleration is directed toward the center of the circle. Therefore the net force is also directed toward the center. This net force is often called the *centripetal force*.

Since the acceleration of an object undergoing uniform circular motion is v^2/R, the net force needed to hold a mass in a circular path is $F = m (v^2/R)$. In this lab you will investigate how changes in m, v, and R affect the net force F needed to keep the mass in a circular path.

I. Changing Mass

1. Carefully observe as your instructor or lab technician demonstrates the proper use of the Centripetal Force Apparatus. **Under no circumstances should you supply the motor with more than 12 volts or 0.40A or you will burn out the motor!**

2. Open the file *CentripetalForce***.**

3. Calibrate the force sensor by selecting **Experiment/Calibrate**. On the pop-up menu, select the force sensor and select **Calibrate Now**. You will now perform a two-point calibration of the force sensors:

- For the first calibration point, do not apply any force to the sensor, enter 0 N, and hit **Keep.**
- For the second calibration point, hang a 500 g mass from the sensor. Enter the weight of this mass (4.90 N) and hit **Keep.**

Note: *It is your responsibility to continually check the calibration of the force probe by first removing all mass and seeing if the probe reads 0N and then by hanging 500 grams on the probe and checking to see if it reads 4.9N. If you do not see these values then you either have to re-zero or re-calibrate the probe.*

4. Place a 5 g mass on each side of the apparatus at the 70 mm marks. The movable mass must be located by having one partner hold the center of the mass at 70 mm while a second partner moves the force probe up until the string is taut. The force probe must then be secured in this position.

5. The values of m and R can be directly measured from the Centripetal Force Apparatus (CFA). The tangential velocity is found by using a photo-gate timer and a calculation done in LoggerPro. The photo-gate will measure the time (T) for the mass to complete one complete revolution. The tangential velocity is thus:

$v = 2\pi R/T$

This formula can be found in Logger Pro under **Data/Column Options,** and then select Velocity. By default a radius value of 0.07 m is used. "Pulse Time" is the time for one revolution of the mass.

Note: *This formula only calculates the correct tangential velocity when the radius of the circle is 0.07 m. When other radii are used the user must edit this formula by entering the current radius in place of 0.07.*

6. Set the voltage to 8.0 volts and observe the apparatus to be sure that the string leading to the force probe is vertical and directly aligned with the pulley below it.

7. Click **Collect** and monitor the Velocity and Force for about 10 seconds, then turn off the power supply.

8. Highlight the area of the graph that shows the most constant values of Velocity and Force. Record the mean Velocity and Force in the table below, along with appropriate uncertainties.

9. Repeat the steps above to complete the table. Be certain that the velocity is the same as in the first trial. If not, adjust the voltage on the power supply.

Effect of Changing Mass on Force (R = 0.070 m)

Mass (kg)	***Velocity (m/s)***	***Force (N)***
0.005	±	±
0.010	±	±
0.015	±	±
0.020	±	±
0.025	±	±
0.030	±	±
0.035	±	±

II. Changing Velocity

1. Place a 20 g mass centered at the 70 mm mark on each side of the apparatus.

2. Check your force probe calibration with 0 N and 4.9 N. Re-zero or re-calibrate if necessary.

3. Set the voltage to 4.0 volts and observe the apparatus to be sure that the string leading to the force probe is vertical and directly aligned with the pulley below it.

4. Click **Collect** and monitor the Velocity and Force for about 10 seconds, then turn off the power supply.

5. Highlight the area of the graph that shows the most constant values of Velocity and Force. Record the mean Velocity and Force in the table below, along with appropriate uncertainties.

6. Repeat the steps above to complete the table.

Effect of Changing Velocity on Force (m = .020 kg and R = 0.070 m)

Voltage (V)	***Velocity (m/s)***	***Force (N)***
4.0	±	±
5.0	±	±
6.0	±	±
7.0	±	±
8.0	±	±

9.0	±	±
10.0	±	±
11.0	±	±
12.0	±	±

III. Changing Radius

1. Place a 20 gram mass centered at the 50 mm mark on each side of the apparatus.

2. Click on **Data/Column Options** and then select **Velocity**. Since our radius is now 0.05 m, edit the expression for mass velocity to **2*pi*(0.05)/"Pulse Time".**

3. Check your force probe calibration with 0N and 4.9N. Re-zero or re-calibrate if necessary.

4. Set the voltage to 12.0 volts and observe the apparatus to be sure that the string leading to the force probe is vertical and directly aligned with the pulley below it.

5. Click **Collect** and monitor the Velocity and Force for about 10 seconds, then turn off the power supply.

6. Highlight the area of the graph that shows the most constant values of Velocity and Force. Record the mean Velocity and Force in the table below, along with appropriate uncertainties.

7. Repeat the steps above to complete the table. For each new radius you need to:

a. Edit the Velocity equation by entering the current radius

b. Adjust the voltage in order to maintain a consistent velocity for each trial

Effect of Changing Radius on Force (m = .020 kg)

Radius (m)	***Velocity (m/s)***	***Force (N)***
0.050	±	±
0.060	±	±
0.070	±	±
0.080	±	±
0.090	±	±
0.100	±	±

IV. Analyzing the Data

A. Force vs. Mass

Open the file *Graphing*. This file is set-up to allow you to enter your data and uncertainties and create a best-fit graph.

- Enter your data into the appropriate column. (Force on the y-axis, mass on the x-axis, and the uncertainties in the appropriate columns.)
- Double-click on each column header to change the label and units for each column.
- Select an appropriate best-fit function (in this case **Analyze/Linear Fit**) and display it on your graph. To determine the uncertainties in the linear fit parameters, right-click in the Linear Fit box and select **Linear Fit Options**. This displays the standard deviation of the slope and y-intercept.
- Finish preparing your graph, then print and attach it to the end of this activity.

Question: Compare the generic form of a linear function, **Y =AX + B**, with the theoretical equation **F = m (v^2/R)**. What should the values of A and B equal if the theory is valid? Hint: If a variable is graphed on the x- or y-axis, it can't be part of A or B.

Question: Based on your observation above, calculate the known value of the constants that comprise A in your best-fit function. Does your experimental value of A, with units and uncertainties, agree with this known value?

B. Force vs. Velocity

Create a graph of Force vs. Velocity as described above. Fit your data with a power function and attach it to the end of this activity.

Question: Compare the generic form of a power function, **Y =AX^B**, with the theoretical equation **F = m (v^2/R)**. What should the values of A and B equal if the theory is valid?

Question: Is the power of your best-fit function equal to 2 within your uncertainty? If not, speculate on *specific* sources of possible error and how you would correct these errors if you repeated the experiment.

Question: Calculate the known value of the constants that comprise A in your best-fit function. Does your experimental value of A, with units and uncertainties, agree with this known value?

C. Force vs. Radius

Create a graph of Force vs. Radius as described above. Fit your data with a power function and attach it to the end of this activity.

Question: Compare the generic form of a power function, **Y =AX^B**, with the theoretical equation **F = m (v^2/R)**. What should the values of A and B equal if the theory is valid?

Question: Is the power of your best-fit function equal to -1 within your uncertainty? If not, speculate on *specific* sources of possible error and how you would correct these errors if you repeated the experiment.

Question: Calculate the known value of the constants that comprise A in your best-fit function. Does your experimental value of A, with units and uncertainties, agree with this known value?

5.5: Rotational Inertia

Name: ____

Date: ______________________

Partners:____

Equipment

- *LabPro* Interface
- *Rotation* software file
- Rotary encoder
- Rotational inertia accessories
- Pulley
- Hanging masses

Introduction

Rotational inertia is a measure of the resistance of an object to changes in its angular velocity. Imagine applying a known torque to an object. While applying the torque, measure the angular acceleration. The smaller the resulting angular acceleration, the larger the object's rotational inertia. Thus, by measuring the applied torque and the resulting angular acceleration, the rotational inertia of an object can be determined.

In this activity, you will hang a known mass from the rotary encoder by means of a string wrapped around the encoder and over a pulley. The encoder will be oriented face-up to enable you to mount different objects on the encoder, and hence determine the rotational inertia of the system. When you release the mass, the string will supply a torque to the encoder, giving rise to an angular acceleration. By measuring this angular acceleration, you will be able to determine the rotational inertia of the encoder and its load.

I. Determining the Rotational Inertia of the Initial System

Open the file *Rotation.*

Orient the rotary encoder and secondary pulley as shown in the illustration above. Remove the 3-step pulley and flip it over so that the largest radius pulley is on the top. Do not yet attach the long rod with adjustable masses.

Spin the encoder to determine the positive direction. Using the small holes in the pulley to securely attach the string, wrap the string around the middle radius pulley to ensure a positive angular acceleration. Adjust the secondary pulley so that the string is horizontal between the pulleys and smoothly passes over the secondary pulley.

Attach a 5 g mass from the end of the string.

Adjust the graph to display angular acceleration vs. time, press **Collect,** and release the mass.

Question: Is the angular acceleration of the encoder approximately constant as the mass falls? Should it be constant? Explain.

Use **Analyze/Statistics** to determine the mean angular acceleration as the mass falls, and record it below. Vary the hanging mass to complete the table.

Hanging Mass (kg)	**Angular Acceleration (rad/s^2)**
0.005	±
0.010	±
0.015	±
0.020	±

The above data can be used to calculate the rotational inertia of the encoder system. To accomplish this requires applying Newton's second law to both the encoder and the falling mass and combining the results. You will be guided through this process step-by-step.

Question: Draw a free-body diagram for the hanging mass and apply Newton's second law. Let the direction of motion of the mass be positive.

Question: Draw a top-view free-body diagram for the encoder and apply the rotational form of Newton's second law. Let the direction of motion of the encoder be positive.

Question: Since the linear acceleration of the mass is unknown, rewrite the hanging mass equation replacing linear with angular acceleration.

Question: Since the force exerted by the string is unknown, combine your two equations to eliminate this term.

Question: Solve the resulting equation for the rotational inertia of the encoder.

You should notice that every variable in the above expression was measured and recorded in the above table, except the radius of the encoder. Measure the radius of the encoder and record it below:

radius of encoder, R = ____________ m

Using the results above, calculate the rotational inertia of the encoder for each trial and record it below.

Hanging Mass (kg)	**Angular Acceleration (rad/s^2)**	**Rotational Inertia (kg m^2)**
0.005	±	±
0.010	±	±
0.015	±	±
0.020	±	±

Question: Is the rotational inertia of the encoder the same in each trial? Should it be? Explain.

Question: What is the mean value, with uncertainty, for the rotational inertia of the encoder.

II. The Dependence of Rotational Inertia on Position

In this activity you will discover the effect that a constant mass, placed at a variable location on an object, has on the rotational inertia of the object. You will begin by placing the mass far from the axis of rotation of the object and then move the mass toward the axis of rotation.

Question: Do you believe the rotational inertia will increase, decrease or stay the same as you move the mass toward the axis of rotation? Explain.

Attach the long metal rod to the top of your encoder. Center the rod between the plastic pins on the 3-step pulley for stability. Attach the two adjustable masses to the rod, each located 16 cm from the center of the rod.

Attach 100 g to the end of the string, press **Collect**, and release the hanging mass. Record the result below. Before completing the following table, consider the question asked below.

Hanging Mass (kg)	**Mass Position (m)**	**Angular Acceleration (rad/s^2)**	**Rotational Inertia (kg m^2)**
0.100	*0.16*	±	±
0.100	*0.12*	±	±
0.100	*0.08*	±	±

0.100	*0.04*	±	±

Question: You may find that the angular acceleration becomes “less constant” as the masses move closer to the axis of rotation. Explain why this is the case and explain what portion of the angular acceleration data should be analyzed to compensate for this effect.

Create a graph of rotational inertia vs. mass position. Based on your data, select an appropriate best-fit function and display it on your graph. (Remember, your best-fit function should both match the data *and* correspond to the known physical relationship between rotational inertia and position.) Print and attach your graph to the end of this activity.

Question: Record the numerical constants in your best-fit function below. Each numerical value must have both uncertainties and units.

Question: Consider the intercept term in your best-fit function. What physical parameter of the system does this term represent? Clearly explain.

Question: Consider the other term(s) in your best-fit function. What physical parameter of the system does each term represent? If you can easily determine the actual value of this parameter, do so and compare it to your best-fit value.

III. Measuring the Rotational Inertia of the Metal Disk

In this last activity you will measure the rotational inertia of the metal disk.

Remove the metal rod from the encoder, flip the 3-step pulley over, and attach the metal disk to the 3-step pulley.

Attach 20 g to the end of the string, press **Collect**, and release the hanging mass. Record the result below.

Complete the table.

Hanging Mass (kg)	**Angular Acceleration (rad/s^2)**	**Rotational Inertia (kg m^2)**
0.020	±	±
0.030	±	±
0.040	±	±
0.050	±	±

Question: What is the mean value, with uncertainty, for the rotational inertia of the metal disk? (Hint: It is not simply the average value of the rotational inertia column above.)

Question: Measure the radius of the metal disk. Using this result, and your result from above, determine the mass of the metal disk, with units and uncertainties.

Question: Determine the actual mass of the metal disk and compare it to the value you determined above. Comment on the agreement between these values.

5.6: Physical Pendulum

Name: ____

Date: ______________________

Partners:____

Equipment

- *LabPro* Interface
- *Rotation* software file
- Rotary encoder
- Rotational inertia accessories
- Meterstick

I. Basic Parameters

Open the file *Rotation.*

Orient the rotary encoder as shown above. Remove the 3-step pulley and flip it over so that the largest radius pulley is furthest from the encoder. Attach the long metal rod to the encoder and center the rod between the plastic pins for stability. Attach an adjustable mass to the rod 24 cm from the pivot point.

Since the initial orientation of the encoder is automatically set as 0°, press **Collect** and, *after the encoder begins to collect data*, rotate the meterstick to about 10° from its natural resting position and let it go.

Rescale your graph until two complete cycles of the motion are clearly visible. Display both angular position and angular velocity vs. time. Print your graph and attach it to the end of this activity.

Question: Describe your graph. What is the relationship between the angular position and angular velocity? How do the maxima, minima and zeroes of the two functions relate to each other?

Display both angular velocity and angular acceleration vs. time.

Question: Describe your graph. What is the relationship between the angular velocity and angular acceleration? How do the maxima, minima and zeroes of the two functions relate to each other?

The period of the motion is the time for one complete cycle of the motion. To very accurately determine the period, rescale your graph until ten complete cycles of the motion are clearly visible. Accurately measure the time needed for ten cycles and divide by ten to determine the period.

Question: What is the period of the motion?

II. Dependence of Period on Mass Position

The pendulum you have been examining is referred to as a *physical pendulum*, as opposed to a *simple pendulum*, which is simply a massive bob on the end of a thin string or rope. In this series of exercises, you will investigate the period of this pendulum as a function of the position of the adjustable mass relative to the pivot point.

Question: Your original period was for the mass located 24 cm from the pivot point. If the mass is slid upward until it is only 20 cm from the pivot point, do you think the period will change? If so, how? Explain.

Move the mass until it is 20 cm from the pivot point and accurately measure the period of the resulting motion. Record your result in the table below.

Question: Does the result agree with your prediction? If not, can you explain why the period changed in the manner that it did.

The dependence of period on clamp position is not a simple one. To analyze this dependence in more detail, take measurements to complete the following table. Always start the motion from the same initial angular orientation.

Mass Position (m)	Period (s)

0.24	±
0.20	±
0.16	±
0.12	±
0.08	±
0.04	±

Create a graph of period vs. mass position.

On your graph, include a line representing the theoretical value for the period of a *simple pendulum* with length given by the location of the adjustable mass. This would be the period if the adjustable mass where simply hung from a string and oscillated back and forth.

Question: Based on your graph, does your pendulum behave like a simple pendulum? If not, is there a region on the graph where the dependence is more similar to a simple pendulum? If so, explain why the pendulum is "simpler" for these values of position than for other values of position.

III. Theoretical Explanation

The period of the pendulum depends upon the position of the adjustable mass in a non-linear manner. Your task is to derive this dependence and see if your data matches this theoretical prediction.

To ease the analysis, we will make a pair of simplifying assumptions. First, although the rod is not pivoted about its actual endpoint we will approximate the system as if the pivot point is at the endpoint of the rod. Second, we will approximate the adjustable mass as a point mass.

Question: Draw a free-body diagram for the pendulum displaced by a small angle, θ, from equilibrium. With L as the length of the metal rod, m as the mass of the rod, d as the location of the adjustable mass measured from the pivot point, and M as the adjustable mass, apply the rotational form of Newton's second law, with the direction of displacement positive.

Question: Use the small-angle approximation:

and the definition of angular acceleration:

to simplify your equation.

Question: Collecting constants, you should now have a differential equation with a known solution. Based on this solution, write an expression for the period of the meterstick.

Question: Determine numerical values for all of your constants and simplify your theoretical expression below.

Add your theoretical model for period to your graph of period vs. mass position. Clearly distinguish between the physical pendulum model, the simple pendulum model, and the actual data. Print your graph and attach it to the end of this activity.

Question: Does you theoretical model accurately represent your experimental data? What could you have done to improve the agreement between theory and experiment?

5.7: Lab Manual

University Physics I

Download the Word Document Laboratory Manual file here

Instructions for Presenting Laboratory Results

All numerical values either directly measured or based on measurements must be reported with an appropriate uncertainty and unit. For example:

Carefully note that:

- The uncertainty has no more than 2 significant digits
- The decimal place of the uncertainty is consistent with the decimal place of the value
- The uncertainty and value are raised to the same power-of-ten, preferably a multiple of 3

In order to determine the uncertainties associated with best-fit parameters, most graphs will need to be constructed using LoggerPro. Use the file "Graphing" in the PHY 262 M-drive folder to construct graphs.

Carefully note that:

- Axes are labeled by variable name and unit
- Data points are not connected and may display appropriate error bars
- The best fit parameters are clearly displayed with uncertainties

Laboratory reports that do not comply with these instructions may not be accepted.

5.8: One-Dimensional Motion

Name: ____

Date: ______________________

Partners:____

Equipment

• *LabPro* Interface • Motion detector • *Motion* software file	• Dynamics track • Dynamics cart • Lab jack

Introduction

A motion detector is a device that can track the position of an object as it moves. In many ways, a motion detector is similar to a bat. The motion detector, like a bat, emits brief "chirps" of very high frequency sound and then "listens" for the echo of this sound. By measuring the amount of time between emitting the chirps and hearing their reflection, the detector, like a bat, can determine the distance to the object that reflected the chirps. The bat then tries to eat the object, while the motion detector merely presents the data as a graph of the object's position vs. time.

In constructing a graph of position vs. time, the motion detector always serves as the origin of the coordinate system, and the direction the detector points is the positive direction. Using this position vs. time data, the *LoggerPro* software can calculate the velocity and acceleration of the moving object.

Note that the motion detector detects the closest object directly in front of it, whether this is the object of interest or not. Additionally, the detector will not correctly measure anything closer than about 15 cm, so objects closer than this distance result in erroneous measurements. Because of these two factors, motion detectors are very prone to extraneous readings. ***It is your responsibility to repeat any measurement until you are sure the detector has properly recorded the motion under investigation. Do not be tentative about repeating observations several times until no extraneous signals are present in your data.***

I. Constant Velocity

A. Position vs. Time Graphs

Place the cart about 15 cm in front of the motion detector. Give the cart a quick push away from the detector. The cart should roll with approximately constant velocity across the track. If necessary, clean the track and/or check the wheels of the cart to ensure a smooth motion. If the wheels of the cart do not spin freely, get a new cart!

Open the file *Motion.*

Collect position vs. time graphs for the following three motions:

- Rolling away from the detector
- Rolling away from the detector at a greater speed
- Rolling toward the detector

To collect data, press the **Collect** button and wait for the detector to begin "clicking", and then give the cart a quick push. If the data is clean, select **Experiment/Store Latest Run**. If the data is noisy, diagnose the problem and repeat the observation until the problem is solved.

Once you have recorded the three motions, prepare your graph for printing. In general, you will be graded on both the *quality* of your data and the *presentation* of your data. To properly present data in graphical form, you must always:

- add a descriptive title to your graph (right-click on the graph and select **Graph Options/Graph Options** tab),
- add you and your partners' names to the footer (**File/Printing Options**),
- make sure each axis is properly labeled with appropriate units,
- annotate your graph if it contains more than one data set (**Insert/Text Annotation**),

- and adjust the x- and y-axis ranges to best present your data (right-click on the graph and select **Graph Options/Axes Options** tab).

Once you've prepared your graph, print your graph and attach it to the end of this activity.

Question: Clearly explain how the direction of travel of an object can be determined from a position vs. time graph.

Question: Clearly explain how the speed of an object can be determined from a position vs. time graph.

B. Velocity vs. Time Graphs

To see a graph of the cart's velocity vs. time, select **Position** on the vertical axis and change the axis to **Velocity**. Adjust the scale to see all three motions (right-click on the graph and select **Graph Options/Axes Options** tab). Delete the text annotations.

Question: Can the direction of travel of the cart be determined from a velocity vs. time graph? Explain.

Question: Can the position of the cart be determined from a velocity vs. time graph? Explain.

Question: Can the *change in position* of the cart during a time interval be determined from a velocity vs. time graph? Explain.

C. Relating the Position and Velocity Graphs

Display both the position and velocity graphs for the rolling away motion. (Select **Velocity** on the vertical axis and select **More…** . Then check both the Position and Velocity boxes for the appropriate run.)

Determine the value of the slope of the position vs. time graph and the value of the mean velocity.

- To determine the slope, click and drag to highlight a large, straight portion of the position vs. time graph, and then select **Analyze/Linear Fit**.
- To determine the uncertainty in the slope, right-click in the Linear Fit box and select **Linear Fit Options**.
- To determine the mean velocity, highlight the *exact same portion* of the velocity vs. time graph, and then select **Analyze/Statistics**.
- The standard deviation can be used as the uncertainty in the mean velocity.

Complete the table below. Remember, all data have units and associated uncertainties. In most cases, we will use the standard deviation as a measure of the uncertainty in the mean value of a set of data.

Motion	**Slope of x vs. t (m/s)**	**Mean Velocity (m/s)**
Rolling away	±	±
Rolling away at a greater speed	±	±
Rolling toward	±	±

Question: Are the two quantities measured above consistent, within measured uncertainties, for each motion analyzed?

E. Relating the Position and Velocity Graphs, Again

For each of the three motions, determine the area under a portion of the velocity graph and the corresponding change in the position of the object. Again, it is easier to do this one data set at a time.

- To determine area under the velocity graph, click and drag to highlight a large, horizontal portion of the velocity vs. time graph, and then select **Analyze/Integral**.
- To determine the change in position over exactly the same interval, select **Analyze/Examine**. A vertical cursor will appear that allows you to determine the position at the two ends of the region highlighted above.

The software does not automatically calculate an uncertainty for these measurements. However, this does not mean that the data is exact. Estimate and record an uncertainty for each measurement below.

Motion	Change in Position (m)	Area under v vs. t (m)
Rolling away	±	±
Rolling away at a greater speed	±	±
Rolling toward	±	±

Question: Describe and defend your method for estimating the uncertainties in your data.

Question: Are the two quantities measured above consistent, within estimated uncertainties, for each motion analyzed?

II. Varying Velocity

Erase your old data (**Data/Clear All Data**), and display only a position graph.

By elevating one end of the track (use a lab jack at its lowest setting) the cart can be made to move with a varying velocity.

Collect position vs. time graphs for the following three motions:

- Speeding up moving away from the detector
- Speeding up moving toward the detector
- Slowing down moving toward the detector

Of course, you will have to move either the lab jack or the motion detector to collect these three distinct motions.

As always, if the data is noisy diagnose the problem and repeat the observation until the data is clean. Once the data is clean select **Experiment/Store Latest Run**.

Once you have recorded the three motions, properly prepare your graph for printing, and then print and attach your graph.

Question: Clearly explain how to distinguish between an object that is speeding up, an object that is slowing down, and an object that is moving at constant speed based *only* on a position vs. time graph.

Display the velocity graph.

Question: Clearly explain how to distinguish between an object that is speeding up, an object that is slowing down, or an object that is moving at constant speed based *only* on a velocity vs. time graph.

Display the acceleration graph.

Question: Is it possible to distinguish between an object that is speeding up and an object that is slowing down based *only* on an acceleration vs. time graph? Explain.

Question: Does the algebraic sign of the acceleration have any relationship to whether the object is speeding up or slowing down? If not, what information does the sign of the acceleration convey?

Question: Is it possible for an object to speed up with a negative acceleration? If so, describe how this could be accomplished with the given lab equipment.

Question: Is it possible for an object to slow down with a positive acceleration? If so, describe how this could be accomplished with the given lab equipment.

For each of the three motions, determine the concavity of the position vs. time graph, the slope of the velocity vs. time graph, and the value of the mean acceleration.

Motion	Slope of v vs. t (m/s^2)	Mean Acceleration (m/s^2)
Speeding up away	±	±

Speeding up toward	±	±
Slowing down toward	±	±

Question: Are the two quantities measured above consistent, within measured uncertainties, for each motion analyzed?

III. Round-Trip Journey

Display a graph of position, velocity, and acceleration vs. time.

With the motion detector at the bottom of the incline, give the cart a quick push up the incline so that it travels almost to the top of the track before rolling back down. Of course, catch the cart before it collides with the motion detector. Once you have cleanly recorded this motion, prepare and print your graph and attach it to the end of this activity.

Label your print-out with the following three vertical lines:

- Draw a vertical line (through all three graphs) at the time the cart first leaves your hand on the way up the incline. Label the line "*Release*".
- Draw a vertical line at the time the cart comes to rest (momentarily) at the top of the track. Label the line "*Apex*".
- Draw a vertical line at the time the cart first strikes your hand on the way down the incline. Label the line "*Catch*".

Question: Clearly explain your reasoning for drawing each line where you did.

for line 1:

for line 2:

for line 3:

Question: Clearly explain why the acceleration is negative for both the motion up the incline and the motion back down the incline.

Question: Clearly explain why the accelerations immediately before *Release* and immediately after *Catch* are large positive values.

Question: Clearly explain why the acceleration is not zero at *Apex*, even though the velocity is zero at this point.

Imagine the motion repeated with the motion detector at the top of the incline.

Question: Clearly explain how this change in coordinate system will affect each of the three graphs.

5.9: Force and Motion I

Name: ____

Date: ______________________

Partners:____

Equipment

• *LabPro* Interface • Motion detector • Force sensor • *Force* software file	• Dynamics track • Dynamics cart • Pulley • Hanging masses

Introduction

The force sensor is a device that measures the amount of force applied to the hook attached to the sensor. The sensor can only measure the force component applied parallel to its length, and forces directed away from the sensor are typically regarded as positive.

The force sensor measures force by means of a strain gauge. As a force is applied to the sensor, a small metal bar inside the sensor deforms. Strain gauges attached to the bar change resistance as the bar deforms. This change in resistance results in a change in voltage. In order to convert this voltage information into force information, the sensor must be calibrated.

The sensor is calibrated by means of a *two-point calibration*. First, no force is applied to the sensor and the resulting voltage is recorded. Then a known force is applied and the resulting voltage recorded. Using these two points as reference, and assuming a linear relationship between force and voltage, any voltage reading can be converted into a force reading. Since the force sensor is accurate enough to detect the weight of the hook, it should always be calibrated in the same orientation in which it will be used.

In this activity, you will measure both the motion of a cart using the motion detector and the force applied to the cart using the force probe. The relationship between the force acting on an object and its subsequent motion is of central importance in many branches of physics.

I. Calibrating the Force Sensor

Open the file *Force.*

Set the sensor to the ±10 N setting and securely attach it to the top of the cart. Place the cart at rest on the track.

To calibrate the force sensor, select **Experiment/Calibrate** and select the Force sensor. Select **Calibrate Now**. You will perform a two-point calibration of the sensor:

- For the first calibration point, do not apply any force to the sensor, enter 0 N, and hit **Keep.**
- For the second calibration point, attach a 200 g mass to the sensor by means of a string passing over the pulley. Enter the force of gravity acting on this mass (1.96 N) and hit **Keep.**

The sensor is now calibrated.

To check the calibration of the sensor, complete the following table. In each case, attach the appropriate hanging mass, hold the cart at rest, and collect force data for several seconds. Find the mean and standard deviation of the force sensor data.

Hanging Mass (g)	Weight (N)	Mean Sensor Reading (N)
0		±
50		±
100		±
150		±

200		±
500		±

You are now going to create a graph, with error bars, of Weight vs. Mean Sensor Reading. To do this in *LoggerPro*:

- Select **Insert/Table** to view your data table. Find the empty columns labeled y-axis, delta y, x-axis, delta x. (The other columns are filled with the last set of data you collected. You can delete or just ignore this data.)
- Enter your data into the appropriate column. (Weight on the y-axis, sensor reading on the x-axis, and the uncertainty in sensor reading as delta x.)
- Double-click on each column header to change the label and units for each column.
- Create a graph of Weight vs. Mean Sensor Reading by changing the variables on the x- and y-axis of one of your pre-existing graphs.
- Right-click on the graph, select **Graph Options/Graph Options**, and deselect the option **Connect Points**.
- Based on your data, select an appropriate best-fit function (**Analyze/Linear Fit** or **Curve Fit**) and display it on your graph. Remember, a best-fit function should be the *simplest possible function* that accurately matches your data.

Finish preparing your graph, then print and attach it to the end of this activity.

Question: Record each constant in your best-fit function below. Each numerical value must have both uncertainties and units. (To determine the uncertainties in linear fit parameters, right-click in the Linear Fit box and select **Linear Fit Options**. This displays the standard deviation of the slope and y-intercept.)

Question: If the force sensor accurately measures the force applied to it, what should the slope and y-intercept of your best-fit function equal? Explain.

Question: Does your force sensor accurately measure the force applied to it? Explain.

II. Force and Initial Motion

Display graphs of force and acceleration vs. time.

Attach 200 g to the sensor via a string passing over the pulley. Hold the cart in place about 20 cm in front of the motion detector, press **Collect**, and after about 1 second of data collection release the cart. Make sure that the track is clean, the wheels of the cart turn freely, and the force sensor cord does not drag behind the cart.

Once you have recorded a clean run of the experiment, properly prepare and print your graph and attach it to the end of this activity. Draw a vertical line (through both graphs) at the time you released the cart. Label the line "*Release*".

Question: Your force vs. time graph should show a clear decrease in force when you released the cart. Is this decrease in force *real* (meaning the force applied to the cart actually does decrease when the cart is released) or is it a *glitch* (due to an imperfect measuring device)? If it is real, explain why the force suddenly decreases. If it is a glitch, hypothesize what is wrong with the measuring device.

Question: If the hanging mass was less massive, would the discontinuity in force be larger or smaller in magnitude? Explain.

Test your prediction by using a smaller magnitude hanging mass. ***Do not proceed until you understand the nature of the discontinuity in the force vs. time graph.***

III. Force and Acceleration

Using the hanging masses listed below, release the cart from rest and measure the mean force acting on the cart (after release) and the mean acceleration of the cart. Make sure that your data is clean and accurate before analyzing it, and only analyze the appropriate portion of the data. ***You must analyze the same time interval on both graphs.***

Print out and attach one of your trials with the region analyzed highlighted and the relevant statistics displayed. ***Show this graph to your instructor to verify that you are analyzing the data correctly!***

If at any time you believe your force sensor is producing erroneous readings, simply check the calibration by hanging a known weight from the sensor. Occasionally the sensor can "drift". This can be corrected by re-zeroing the sensor using the **Zero** button on the toolbar. If the readings are substantially off, you may need to re-calibrate your sensor.

Hanging Mass (g)	Mean Force (N)	Mean Acceleration ()
200	±	±
150	±	±
100	±	±
50	±	±

Create a graph, with error bars for both the y- and x-data, of Mean Force vs. Mean Acceleration. (**Insert/Table** to view your data table and simply "type-over" your previous inputted data. Remember to change the labels and units for each column.)

Based on your data, select an appropriate best-fit function (**Analyze/Linear Fit** or **Curve Fit**) and display it on your graph. Remember, a best-fit function should be the *simplest possible function* that accurately matches your data.

Finish preparing your graph, then print and attach it to the end of this activity.

Question: Record each constant in your best-fit function below. Each numerical value must have both uncertainties and units. (To determine the uncertainties in linear fit parameters, right-click in the Linear Fit box and select **Linear Fit Options**. This displays the standard deviation of the slope and y-intercept.)

Applying Newton's Second Law to the cart in the direction of motion results in:

Rearranging this equation results in:

Question: Your graph involved the force of the string on the y-axis and the acceleration of the cart on the x-axis. Based on this observation, and a simple comparison between your best-fit function and Newton's Second Law, what is the combined mass of your cart and sensor (with units and uncertainties) and what is the mean frictional force acting on your cart (with units and uncertainties)? How did you determine these values?

Question: Using a scale, determine the combined mass of your cart and sensor and record it below. Compare this value to the value determined above. Comment on the accuracy of your experimental data.

IV. Mass and Acceleration

In the previous activity, the force applied to the cart was varied and the mass of the cart was held constant. In general, it's a good idea to vary as few parameters as possible when conducting an experiment. In this experiment, you will hold the force constant while you vary the mass.

This is slightly more difficult since, as you saw earlier in this activity, the force applied to the cart is not equal to the weight of the hanging mass. Therefore, *even if the hanging mass is held constant, the force it exerts on the cart will not be constant*.

For example, if we consider the cart as the system of interest (as we did in the previous activity), the force sensor measures the force applied to the cart, but this force is not equal to the weight of the hanging mass.

Ffriction

$F_{friction}$

$F_{string} \neq 1.96$ N

$F_{string} \neq 1.96$ N

However, if we consider the cart and hanging mass together as the system of interest, the force applied to the system *is* equal to the weight of the hanging mass, which is easy to keep constant. Notice, for this system, that the force sensor doesn't measure anything useful! The two forces acting on the system are the weight of the hanging mass and the friction on the cart. This is the system we will experiment with below.

Ffriction

$F_{friction}$

$F_{gravity} = 1.96$ N

$F_{gravity}$ = 1.96 N

Using a 200 g hanging mass, collect data to complete the table below. Begin with a system of cart, sensor and 200 g hanging mass. Then, continually add 500 g to the system to complete the experiment. Make sure that your data is clean and accurate before analyzing it, and only analyze the appropriate portion of the data.

System	System Mass (kg)	Mean Acceleration ()
cart, sensor, 200 g		±
+500 g		±
+500 g		±
+500 g		±
+500 g		±

Create a graph of System Mass vs. Mean Acceleration. Include the uncertainty in the mean acceleration.

Based on your data, select an appropriate best-fit function (**Analyze/Linear Fit** or **Curve Fit**) and display it on your graph. Remember, a best-fit function should be the *simplest possible function* that accurately matches your data. Additionally, since you know this motion is governed by Newton's Second Law, your best-fit function should agree with Newton's law in functional form.

Question: Starting from Newton's Second Law applied in the direction of motion of the system, rearrange Newton's Law until mass is isolated on the left-side of the equation. (Your best-fit function should have this form.)

Print your graph with best-fit function displayed and attach it to the end of this activity.

Question: Record each constant in your best-fit function below, with units and uncertainties.

Question: Based on your best-fit function, what is the net force (with units and uncertainty) that acts on the system?

Question: Based on your best-fit function, and the observation that you used a 200 g hanging mass to accelerate the system, what is the mean frictional force (with units and uncertainty) that acts on the system? Is this value the same as in the previous experiment? Should it be? Carefully explain.

5.10: Force and Motion II

Name: ______________________________________

Date: _____________________

Partners:____

Equipment

• *LabPro* Interface • Motion detector • Two force sensors • *TwoForces* software file • Dynamics track • Two dynamics carts	• Pulley • Hanging masses • Short loop of string • Rubber band • Force probe rubber bumpers

I. Dueling Force Sensors

Open the file *TwoForces*.

Set each sensor to the ±10 N setting and securely attach it to the top of a cart. Place the carts at rest on the track.

Calibrate both sensors by selecting **Experiment/Calibrate** and then selecting either sensor. On the pop-up menu, check both sensors and select **Calibrate Now**. You will now perform a two-point calibration of both force sensors:

- For the first calibration point, do not apply any force to the sensors, enter 0 N, and hit **Keep.**
- For the second calibration point, hang a 200 g mass from each sensor. Enter the weight of this mass (1.96 N) and hit **Keep.**

When complete, place both sensors back on the track and zero both force sensors.

Connect the force sensors by a short loop of string. Press **Collect** and pull first on one cart and then the other. Sometimes hold the other cart in place and sometimes let it roll freely. Don't go crazy; keep the forces under 10 N.

Question: Describe the relationship between the two measured forces.

Replace the string with a rubber band. Press **Collect** and pull on both carts, then one and then the other. Sometimes hold the other cart in place and sometimes let it roll freely.

Question: Describe the relationship between the two measured forces.

Replace the rubber band with a rubber bumper. Press **Collect** and push on both carts, then one and then the other. Sometimes hold the other cart in place and sometimes let it roll freely.

Question: Describe the relationship between the two measured forces.

Question: Why are the forces displayed not equal in magnitude and *opposite* in direction? What algebraic sign does the sensor give to "pulls"? To "pushes"?

II. A Train

Create a "train" of two carts. The rear cart, closest to the motion detector, should be connected to the front cart by a short loop of string from the hook of the force sensor to a fixed point on the front cart.

The front cart should be attached to a 200 g mass by a string that passes, from the force sensor hook, over a pulley. Thus, you will measure both the force between the two carts and the force applied to the front cart.

Hold the rear cart in place about 20 cm in front of the motion detector, press **Collect**, and after about 1 second of data collection release the cart. Make sure that the force sensor cords do not drag behind the carts nor trigger the motion detector. Repeat until you have clean data.

Question: As you saw in a previous activity, your force vs. time graphs show a clear decrease in force when you released the cart. However, one of the forces decreases by a substantially larger amount than the other. Clearly explain which force decreases by the larger amount and why this larger decrease occurs. (It's probably useful to use Newton's Second Law in your explanation.)

Question: Imagine adding a 500 g mass to one of the two carts. To which cart should the mass be added in order to minimize the large decrease in force? Carefully explain.

Collect the following data from the relevant portion of each graph. **(Note: If your force probes do not both read close to 1.96 N before letting the carts go, something is wrong! Find and correct the problem.)**

Print out and attach one of your trials with the region analyzed highlighted and the relevant statistics displayed.

Trial	Rear Force (N)	Front Force (N)	Acceleration (m/s^2)
no 500 g mass	±	±	±
500 g on rear	±	±	±
500 g on front	±	±	±

Question: To which cart should the mass be added in order to minimize the large decrease in force? Does this agree with your prediction above? If not, provide a correct explanation for this phenomenon.

Question: Does the acceleration depend on which cart carries the 500 g mass? Should it? Explain.

Question: Does the front force depend on which cart carries the 500 g mass? Should it? Explain.

Question: Does the rear force depend on which cart carries the 500 g mass? Should it? Explain.

Measure the mass of each cart and sensor. Record it below.

mass of rear cart and sensor, m_{rear} = ____________ kg

mass of front cart and sensor, m_{front} = ____________ kg

Question: Draw a free-body diagram and apply Newton's Second Law to the rear cart, ignoring friction.

Question: For each of the three trials, substitute your experimental data into your equation above. Is your data consistent with Newton's law (i.e., is the left-hand side of the equation equal to the right-hand side of the equation, within experimental uncertainties)?

Trial	Left-Hand Side; F (N)	Right-Hand Side; ma (N)	Do LHS and RHS overlap?
no 500 g mass	±	±	
500 g on rear	±	±	
500 g on front	±	±	

Question: Is friction small enough to be reasonably ignored? Based on your results, would including friction make the agreement between your data and Newton's law better or worse? Explain.

Question: Draw a free-body diagram and apply Newton's Second Law to the front cart, ignoring friction.

Question: For each of the three trials, substitute your experimental data into your equation above. Is your data consistent with Newton's law (i.e., is the left-hand side of the equation equal to the right-hand side of the equation, within experimental uncertainties)? (Note: Be careful with the uncertainties on the left-hand side of the equation.)

Trial	Left-Hand Side; F (N)	Right-Hand Side; ma (N)	Do LHS and RHS overlap?
no 500 g mass	±	±	

500 g on rear	±	±	
500 g on front	±	±	

Question: Is friction small enough to be reasonably ignored? Based on your results, would including friction make the agreement between your data and Newton's law better or worse? Explain.

Question: Draw a free-body diagram and apply Newton's Second Law to the system of the two carts, ignoring friction.

Question: For each of the three trials, substitute your experimental data into your equation above. Is your data consistent with Newton's law (i.e., is the left-hand side of the equation equal to the right-hand side of the equation, within experimental uncertainties)?

Trial	Left-Hand Side; F (N)	Right-Hand Side; ma (N)	Do LHS and RHS overlap?
no 500 g mass	±	±	
500 g on rear	±	±	
500 g on front	±	±	

Question: Is friction small enough to be reasonably ignored? Based on your results, would including friction make the agreement between your data and Newton's law better or worse? Explain.

5.11: Friction

Name: ____

Date: ______________________

Partners:____

Equipment

• *LabPro* Interface • Motion detector • Force sensor • *Force* software file	• Wooden tracks • Wooden block • Hanging masses

Introduction

The sliding friction between two surfaces is characterized by a single number, the *coefficient of friction*. The coefficient of friction depends on the materials used—for example, ice on metal has a very low coefficient of friction while rubber on pavement has a very high coefficient of friction.

In the standard model of sliding friction, the frictional force is given by the product of the coefficient and the contact force between the two surfaces. In this model, the relative speed between the surfaces and the contact area between the surfaces have no effect on the frictional force. The coefficient of friction is strictly an empirical measurement (i.e., it has to be measured experimentally) and cannot be found through calculations.

I. Static vs. Kinetic Friction

Open the file *Force*. Replace the acceleration graph with a velocity graph.

Set the force sensor to the ±50 N setting and calibrate it using a 1.0 kg mass.

You are going to use the force sensor to pull a wooden block along the track as shown below.

Zero the force sensor with the string slack. Press **Collect** and then very slowly increase the force you apply to the block. It is very important for you to very slowly increase the force, especially in the moments prior to movement of the block. Once the block begins to slide, maintain a constant force on the block in order to keep the block sliding with constant velocity. Repeat the data collection until you feel you have clean data illustrating both the “release” of the block and the motion at constant speed.

Question: Your force vs. time graph should show a clear drop in force when the block first moves. Clearly explain why the force decreases when the block first moves.

Use **Analyze/Statistics** to determine the mean force when the block is moving at constant speed.

Use **Analyze/Examine** to determine the peak force at the moment the block begins to move.

Record your results below and print your graph with the relevant statistics and data displayed. Make sure your velocity graph clearly indicates constant velocity. Attach your graph to the end of this activity.

To get a reasonable idea of the uncertainty involved in these measurements, repeat each trial three times and complete the following tables. (You do not need to record the uncertainty in each individual measurement.) Each of the three trials must be a clean run! If you cannot clearly see the decrease in force, repeat the measurement.

Measure the mass of your wooden block. Record it below.

mass of wooden block, m_{block} = ____________ kg

System	$F_{surface}$ (N)	Force at Impending Motion (N)			
		Trial 1	Trial 2	Trial 3	Average

block + 4.0 kg					±
block + 3.0 kg					±
block + 2.0 kg					±
block + 1.0 kg					±

System	**$F_{surface}$ (N)**	**Force at Constant Speed (N)**			
		Trial 1	**Trial 2**	**Trial 3**	**Average**
block + 4.0 kg					±
block + 3.0 kg					±
block + 2.0 kg					±
block + 1.0 kg					±

Question: At the instant of impending motion, how does the force measured by the force sensor compare to the maximum possible static frictional force? Explain.

Question: When the block moves at constant speed, how does the force measured by the force sensor compare to the kinetic frictional force? Explain.

Create a graph, with error bars, of Maximum Static Friction vs. Surface Force. Based on your data, select an appropriate best-fit function (**Analyze/Linear Fit** or **Curve Fit**) and display it on your graph. Remember, a best-fit function should be the *simplest possible function* that accurately matches your data.

Finish preparing your graph, then print and attach it to the end of this activity.

Question: Record each constant in your best-fit function below. Each numerical value must have both uncertainties and units.

Question: What is the physical meaning of each of the numerical constants in your best-fit function? Explain.

Create a graph of Kinetic Friction vs. Surface Force. Include appropriate error bars and create a best-fit function to your data. Print your graph and attach it to the end of this activity.

Question: Record each constant in your best-fit function below. Each numerical value must have both uncertainties and units.

Question: What is the physical meaning of each of the numerical constants in your best-fit function? Explain.

II. Kinetic Friction, Again

Since frictional coefficients can only be measured experimentally, it's not possible to compare a measured value to a "theoretical" result. Typically, an independent measurement of the coefficient by a different method can be used to corroborate the initial result.

For this activity, the force needed to pull the block up an incline at constant speed will be compared to the force needed to lower the block down the same incline at constant speed. These measurements can be used to determine the kinetic frictional force acting on the block, and hence the coefficient of kinetic friction between the block and the incline.

Question: Draw a free-body diagram for a block of mass m being pulled up an incline θ at constant speed by a force $F_{pulledup}$. Apply Newton's Second Law in the direction parallel to the incline. (Label the frictional force F_{kf} and do not include in your equation.)

Question: Draw a free-body diagram for a block of mass m being lowered down an incline θ at constant speed by a force $F_{lowereddown}$. Apply Newton's Second Law in the direction parallel to the incline. (Label the frictional force F_{kf} and do not include in your equation.)

Question: Combine your two equations above into a single equation for F_{kf} involving only $F_{pulledup}$ and $F_{lowereddown}$. Using this equation, the kinetic friction force can be determined by measuring $F_{pulledup}$ and $F_{lowereddown}$.

Place 1.0 kg on the block and set the incline to 20°.

Carefully pull the block up the incline at constant speed. Repeat this measurement three times and then carefully lower the block down the same incline at constant speed. Complete the tables below. Note that you are *not* trying to determine the maximum static frictional force in this activity.

Incline (°)	**$F_{pulledup}$ (N)**			
	Trial 1	**Trial 2**	**Trial 3**	**Average**
20				±
30				±
40				±
50				±
60				±
70				±

Incline (°)	**$F_{lowereddown}$ (N)**			
	Trial 1	**Trial 2**	**Trial 3**	**Average**
20				±
30				±
40				±
50				±
60				±
70				±

Question: Based on your free-body diagrams above, apply Newton's Second Law in the direction perpendicular to the incline and determine the surface force between the block and the incline.

Using the results derived above, complete the following table.

Incline (°)	**Kinetic Friction Force (N)**	**Surface Force (N)**
20	±	
30	±	
40	±	
50	±	

60	±	
70	±	

Create a graph of Kinetic Friction vs. Surface Force. Include appropriate error bars and create a best-fit function to your data. Print your graph and attach it to the end of this activity.

Question: Record each constant in your best-fit function below. Each numerical value must have both uncertainties and units.

Question: What is the physical meaning of each of the numerical constants in your best-fit function? Compare these values to those determined earlier.

III. Static Friction, Again

Just as for kinetic friction, you need a measurement of the coefficient of static friction by a different method to corroborate your initial result.

To do this, you will place the block at rest on the horizontal wooden track. As you slowly increase the incline of the track, you will reach a point at which the block begins to slide down the incline. Carefully measuring this *angle of slippage* will allow you to determine the coefficient of static friction.

Question: Draw a free-body diagram for a block of mass m that is *almost* slipping down an incline θ. Apply Newton's Second Law in the direction parallel to the incline. (Since the block is almost slipping, the static friction force is at its maximum value. Use this fact to express the static frictional force in terms of the surface force.)

Question: Apply Newton's Second Law in the direction perpendicular to the incline to determine an expression for the surface force.

Question: Substitute your expression for the surface force into your original expression and solve for the coefficient of static friction in terms of the angle at which the block is almost slipping down the incline. Using this equation, the coefficient of friction can be determined by carefully measuring this angle.

Place the block on the horizontal wooden track. Slowly and carefully increase the angle of the track until the block just begins to slide. Record the angle at which the block begins to slide and complete the table below.

Angle of Slippage					
Trial 1	**Trial 2**	**Trial 3**	**Trial 4**	**Trial 5**	**Average**
					±

Question: Using the equation you derived above, use your data to determine the coefficient of static friction, with uncertainty. Compare this value to the value determined earlier.

5.12: Circular Motion

Name: ____

Date: ______________________

Partners:____

Equipment

• *Circle* movie • *Cyclops* movie • *PirateShip* movie • *LabPro* Interface • Motion detector	• Force sensor • *Movie* software file • *Force* software file • Dynamics track • Pulley • Hanging masses

Introduction

It is quite common for objects to move along circular or semi-circular paths. Although these motions can be analyzed using a traditional xy-coordinate system, other coordinate systems, such as polar coordinates, and other sets of kinematic variables (angular variables rather than linear variables) are quite useful. In this activity, the relationships between x- and y-position and velocity to radial and tangential position and velocity, as well as angular position and velocity, will be explored.

I. Circular Motion

Open *Movie* and select **Insert/Movie**. Find the movie *Circle* and open it. Play the movie.

The movie shows a piece of tape with five equally-spaced dots attached to a rotating platform. The central dot is at the center of the platform. Return the movie to the first frame.

Extract position vs. time data for the outermost dot. This dot is 12 cm from the center of the platform. Scale the movie, move your coordinate system to the center of the platform, and rotate your coordinate system so that the initial angular position of the dot is zero.

A. X-Position, Y-Position and Radial Position

Create a graph of x- and y-position vs. time.

Using **Data/New Calculated Column**, create a new column for the radial position (the position in polar coordinates) of the dot. Add the radial position to your graph of x- and y-position vs. time and print and attach your graph to the end of the activity.

Question: Describe your graph. Do the x, y, and radial positions look as you expect?

B. X-Velocity, Y-Velocity, and Tangential Velocity

Create a graph of x- and y-velocity vs. time. Using **Data/New Calculated Column**, create a new column for the tangential velocity (the velocity in polar coordinates) of the dot. Add the tangential velocity to your graph and print and attach your graph to the end of the activity.

Question: Describe your graph. Do the x, y, and tangential velocities look as you expect?

C. X- Position, X-Velocity, and X-Acceleration

Create a graph of x-position, x-velocity, and x-acceleration vs. time. (Use **Data/New Calculated Column**, to create x-acceleration.) Print and attach your graph to the end of the activity.

Question: Describe your graph. Do the x-position, x-velocity, and x-acceleration look as you expect?

D. Angular Position, Angular Velocity, and Angular Acceleration

Create a graph of angular position, angular velocity, and angular acceleration vs. time. (You must first create these columns.) Print and attach your graph to the end of the activity.

Question: Describe your graph. Do the angular position, angular velocity, and angular acceleration look as you expect?

E. Circular Motion with Different Radius

Extract additional position vs. time data, this time for the dot 6 cm from the center.

Question: Carefully explain how the x- and y-position data for the dot 6 cm from the center compares to the same data for the dot 12 cm from the center.

Question: Carefully explain how the x- and y-velocity data for the dot 6 cm from the center compares to the same data for the dot 12 cm from the center.

Question: Carefully explain how the angular kinematic data for the dot 6 cm from the center compares to the same data for the dot 12 cm from the center. **II. Force and Circular Motion**

Open the file *Force.*

Set the force sensor to the ±10 N setting, and calibrate the force sensor with a 200 g mass.

Create a pendulum by attaching a string to the force sensor, passing the string over a pulley, and attaching a 200 g mass to the end of the string. Adjust the endstop of the track to hold the force sensor at rest. You should be able to oscillate the mass back and forth without the force sensor moving. Orient a motion detector to measure the position of the oscillating mass.

Collect data for one complete cycle of the motion. Create graphs of position, velocity, and force vs. time. Print these graphs on the same page. With a vertical line, designate the time(s) at which the mass passes through the equilibrium position.

Question: Clearly explain why the force exerted by the string on the mass is greater than the weight of the mass when the mass passes through equilibrium.

Question: Clearly explain why the force exerted by the string on the mass is less than the weight of the mass when the mass momentarily stops at each end of its swing.

Question: Draw a free-body diagram for the mass as it passes through equilibrium, apply Newton's Second Law, and calculate the theoretical value for the force exerted by the string. Compare this value to the value measured by the force probe. Comment on the agreement between these two values.

Question: Draw a free-body diagram for the mass as it momentarily stops at the end of one swing, apply Newton's Second Law, and calculate the theoretical value for the force exerted by the string. Compare this value to the value measured by the force probe. Comment on the agreement between these two values.

III. The Cyclops

Open the movie *Cyclops.* Play the movie. The movie shows the Cyclops ride at Hershey Park amusement park. The diameter of the wheel, ***not including the cars***, is 17 m.

Although you probably can't recognize me, I'm riding the Cyclops while sitting on a bathroom scale. My mass when the movie was made was 82 kg.

Scale the movie and extract position vs. time data for my motion. Click on where the bathroom scale would be (if you could see it). Notice that I'm upside down at the top of the ride.

To answer the following two questions, fit the appropriate function to the appropriate portion of the data. Print each graph you use with the region selected and best-fit function displayed and attach it to the end of this activity. Show your work in answering each question.

Question: Based on your data, what does the bathroom scale read, with uncertainty and units, at the bottom of the ride's motion?

Question: Based on your data, what does the bathroom scale read, with uncertainty and units, at the top of the ride's motion?

Question: Comment on the effectiveness of your analysis. What could you have done to improve your analysis? Do you feel your results are accurate?

IV. Pirate Ship

Open the movie *PirateShip.* Play the movie. The movie shows the Pirate Ship at Hershey Park amusement park. The people riding the Pirate Ship move along a circular path of radius 20 feet.

Although you probably can't recognize me, I'm riding in the bow of the Pirate Ship while sitting on a bathroom scale. My mass when the movie was made was 82 kg.

Scale the movie and extract position vs. time data for my motion. Click on where the bathroom scale would be (if you could see it).

To answer the following question, fit the appropriate function to the appropriate portion of the data. Print each graph you use with the region selected and best-fit function displayed and attach it to the end of this activity. Show your work in answering the question.

Question: Based on your data, what does the scale read, with uncertainty and units, as I pass through the lowest point on the final swing?

Question: Does my maximum speed occur at the lowest point of my motion? Carefully explain why or why not.

Question: Comment on the effectiveness of your analysis. What could you have done to improve your analysis? Do you feel your results are accurate?

5.13: Rotational Motion

Name: ______________________________________

Date: ______________________

Partners:____

Equipment

- *LabPro* Interface
- Rotary encoder
- *Rotation* software file
- Hanging masses

Introduction

The rotary encoder is a rotational version of a motion detector. As the encoder rotates, its angular position is measured and displayed as a graph of angular position vs. time. In constructing the angular position vs. time graph, the orientation of the encoder when the *LabPro* first begins collecting data always serves as the origin of the coordinate system.

I. Constant Angular Velocity

A. Angular Position vs. Time Graphs

Open the file *Rotation.*

Collect angular position vs. time graphs for the following three motions:

- Rotating clockwise
- Rotating clockwise at a greater angular speed
- Rotating counterclockwise

To collect data, press the **Collect** button, wait for the encoder to begin collecting data, and then give the encoder a quick spin. If the data is clean, select **Experiment/Store Latest Run**. If the data is noisy, diagnose the problem and repeat the observation until the problem is solved.

Once you've prepared your graph for presentation, print your graph and attach it to the end of this activity.

Question: Clearly explain how the direction of rotation of the encoder can be determined from an angular position vs. time graph.

Question: Clearly explain how the angular velocity of the encoder can be determined from an angular position vs. time graph.

B. Angular Velocity vs. Time Graphs

Display a graph of angular velocity vs. time.

Question: Clearly explain how the direction of rotation of the encoder can be determined from an angular velocity vs. time graph.

C. Angular Acceleration vs. Time Graphs

Display a graph of angular acceleration vs. time.

Question: In all three trials, the angular acceleration of the encoder should be approximately constant and close to zero after you release it. Clearly explain both why the angular acceleration is constant and why its magnitude is close to zero.

II. Angular Acceleration

Display angular position, velocity, and acceleration graphs.

Hang 10 g from the end of a string wrapped around the encoder. Wrap the string so that the encoder will rotate in the positive direction, and place a small piece of tape on the end of the string so that the string won't slip off the encoder. Press **Collect**, and then release the mass.

If the data is clean, select **Experiment/Store Latest Run**. If the data is noisy, diagnose the problem and repeat the observation until the problem is solved. Replace the 10 g with 20 g and repeat the data collection.

Once you've prepared your graph for presentation, print your graph and attach it to the end of this activity.

Question: Clearly explain how to determine the angular acceleration of the encoder based *only* on an angular position vs. time graph. Perform this analysis and record the results in the table below.

Question: Clearly explain how to determine the angular acceleration of the encoder based *only* on an angular velocity vs. time graph. Perform this analysis and record the results in the table below.

Question: Clearly explain how to determine the angular acceleration of the encoder based *only* on an angular acceleration vs. time graph. Perform this analysis and record the results in the table below.

Trial	**Angular Acceleration (rad/s^2) determined from:**		
	vs. t	**vs. t**	**vs. t**
10 g	±	±	±
20 g	±	±	±

Question: Are the three quantities measured above consistent, within measured uncertainties, for each motion analyzed?

III. Unbalanced Forces and Torques

Display only an angular acceleration vs. time graph.

In the previous experiment, you measured the angular acceleration of the encoder when an unbalanced torque acted on the encoder. In this activity, you will apply the same unbalanced torque to a slightly different system.

Question: Imagine hanging identical 100 g masses from opposite ends of a string passing over the encoder, and then placing an additional 10 g on one side of the string. If the system is released from rest, will the angular acceleration be greater than, less than, or approximately the same as it was when the two sides of the pulley were unbalanced by 10 g above? Explain.

Perform the experiment described above, and measure the resulting angular acceleration. Record your result and complete the table below.

Hanging Masses	**Angular Acceleration (rad/s^2)**
0 g and 10 g	±
100 g and 110 g	±
200 g and 210 g	±

Question: Although the difference in mass on the two sides of the pulley is equal in all three trials, clearly describe why the resulting angular acceleration is not the same in all three trials.

III. More with Unbalanced Forces and Torques

A. Relating the Outer and Inner Pulley Radii

In this activity, you will flip the encoder around to use the 3-step pulley.

Hang 10 g from the end of a string wrapped around the outer radius pulley. Hang 20 g from the end of a second string wrapped the opposite direction around the inner radius pulley. Secure both strings to the pulley with small pieces of tape.

Press **Collect**, and then release the masses.

Question: Clearly explain how it is possible for a falling 10 g mass to lift a 20 g mass.

Record your result and complete the table below.

Inner Pulley Mass (g)	Angular Acceleration (rad/s^2)
20 g	±
30 g	±
40 g	±
50 g	±

Create a graph of angular acceleration, with error bars, vs. inner pulley mass. Select an appropriate best-fit function and display it on your graph.

Finish preparing your graph, then print and attach it to the end of this activity.

Question: Based on your best-fit function, determine the ratio of the outer radius to the inner radius (R_{outer}/R_{inner}) of the pulley. Describe how you determined this ratio, as well as how you determined the uncertainty in this ratio.

B. Relating the Outer and Middle Pulley Radii

Determine the ratio of the outer radius to the middle radius (R_{outer}/R_{middle}) of the pulley by collecting and analyzing the following data. In all cases, attach 10 g to the outer radius.

Middle Pulley Mass (g)	Angular Acceleration (rad/s^2)
10 g	±
20 g	±
30 g	±
40 g	±

Create a graph of angular acceleration, with error bars, vs. middle pulley mass. Select an appropriate best-fit function and display it on your graph.

Finish preparing your graph, then print and attach it to the end of this activity.

Question: Based on your best-fit function, determine the ratio of the outer radius to the middle radius (R_{outer}/R_{middle}) of the pulley. Describe how you determined this ratio, as well as how you determined the uncertainty in this ratio.

5.14: Conservation Laws

Name: ____

Date: _________________________

Partners:____

Equipment

• *LabPro* Interface • Two motion detectors • Force sensor • *Force* software file	• Dynamics track • Dynamics cart • Pulley • Hanging masses

Introduction

Newton's Second Law directly relates the net force acting on an object to its acceleration. However, the force acting on an object can also be related to the other kinematic parameters of the system (the time, position, velocity, and changes in these variables). The Work-Energy and Impulse-Momentum relations directly relate force to these other parameters. More importantly, in closed systems, these relations generalize to the Laws of Energy and Momentum Conservation. These fundamental laws of physics provide an alternative to Newton's Laws for the study of motion.

I. An Alternative to Newton's Laws

Open the file *Force.*

Set the sensor to the ±10 N setting and securely attach it to the top of the cart.

Measure the mass of your cart and sensor. Record it below.

mass of cart and sensor, m = _____________ kg

Select **Experiment/Calibrate** and select the Force sensor. Select **Calibrate Now**. You will now perform a two-point calibration of the force sensor:

- For the first calibration point, do not apply any force to the sensor, enter 0 N, and hit **Keep.**
- For the second calibration point, attach a 200 g mass to the sensor by means of a string passing over the pulley. Enter the weight of this mass (1.96 N) and hit **Keep.**

You are going to repeat the same series of experiments you conducted while exploring Newton's Laws. However, rather than focusing on the relationship between force and acceleration, you will focus of the interrelationships between force and the other kinematic variables.

Attach 200 g to the sensor via a string passing over the pulley. Hold the cart in place about 20 cm in front of the motion detector, press **Collect**, and then release the cart. Make sure that the track is clean, the wheels of the cart turn freely, and the force sensor cord does not drag behind the cart. The resulting graphs of force and acceleration vs. time should look familiar.

To see a graph of momentum vs. time, you have to "teach" the program how to calculate momentum. To do this,

- Select **Data/New Calculated Column** and enter the appropriate name and units for a momentum column.
- In the equation box, enter the appropriate equation to calculate momentum. (You do not need to enter an equal sign in your equation.) Use the **Variables** pull-down menu to select the appropriate variable in your equation. Press **Done**.
- Replace the graph of acceleration vs. time with a graph of momentum vs. time.

Select the same large region of "good" data on both graphs and complete the first row of the table below. Directly measure the initial and final momentum from your momentum vs. time graph and determine the impulse acting on the cart during this time interval by determining the area under the force vs. time graph. Although the software does not automatically calculate an uncertainty for these measurements, this does not mean that the data is exact. Estimate and record an uncertainty for each measurement.

To see a graph of kinetic energy, you have to "teach" the program how to calculate kinetic energy. To do this,

- Select **Data/New Calculated Column** and enter the appropriate name and units for a kinetic energy column.
- In the equation box, enter the appropriate equation to calculate kinetic energy.
- Since work as the product of the force applied to the cart and the displacement of the cart, change the force vs. time graph to a force vs. position graph and replace the lower graph with a kinetic energy vs. position graph.

Select a large region of “good” data on both graphs and complete the first row of the second table below.

Hanging Mass (g)		**P_{final} (kg m/s)**	**$P_{initial}$ (kg m/s)**	**P (kg m/s)**	**Impulse (N s)**
200				±	±
150				±	±
100				±	±
50				±	±

Hanging Mass (g)		**KE_{final} (kg m^2/s^2)**	**$KE_{initial}$ (kg m^2/s^2)**	**KE (kg m^2/s^2)**	**Work (N m)**
200				±	±
150				±	±
100				±	±
50				±	±

Complete both of the above tables.

Question: Describe and defend your method for estimating the uncertainties in your data.

Question: Is there a relationship, within measured uncertainties, between impulse and change in momentum?

Question: Is friction small enough to be reasonably ignored? Would including friction make the agreement between your data and the impulse-momentum relation better or worse? Explain.

Question: Is there a relationship, within measured uncertainties, between work and change in kinetic energy?

Question: Is friction small enough to be reasonably ignored? Would including friction make the agreement between your data and the work-energy relation better or worse? Explain.

II. Collisions

Remove the force sensor from the interface and connect a second motion detector. Place the motion detectors at opposite ends of the track. Configure *LoggerPro* to display a single graph of both velocities.

Since “away” is considered the positive direction for a motion detector, the two motion detectors have conflicting coordinate systems. To fix this problem, select **Experiment/Set Up Sensors/Show All Interfaces**. Right-click one of your motion detectors and select **Reverse Direction**. Now both motion detectors will refer to the same direction as positive.

A. Velcro-Velcro Collisions

Arrange your two carts such that when they collide they will stick together.

Roll cart 1 in the positive direction, colliding with a stationary cart 2. Roll cart 1 fast enough so that after the collision the two carts roll to the end of the track, but slow enough so that the carts don’t “jump” slightly off the track on contact.

1. Tracking Momentum during a Collision

Measure the mass of each cart and record it below.

mass of cart 1, $m_{cart\ 1}$ = ____________ kg

mass of cart 2, $m_{cart\ 2}$ = ____________ kg

To visualize how momentum behaves during the collision,

- Create a data table with columns for Momentum1, Momentum2, and Total Momentum using **Data/New Calculated Column** and entering the appropriate name and units. In the equation box, enter the appropriate equation to calculate each momentum.
- Create a single full-screen graph of Momentum1, Momentum2, and Total Momentum vs. time.

Question: Is the momentum of cart 1 conserved (i.e. constant) during the collision? If not, does it gain or lose momentum? Explain.

Question: Is the momentum of cart 2 conserved during the collision? If not, does it gain or lose momentum? Explain.

Question: Is the total momentum in the system of the two carts conserved during the collision? Should it be? Explain.

2. Tracking Kinetic Energy during a Collision

To examine the same collision in terms of kinetic energy, rather than momentum, *LoggerPro* can also be "taught" how to measure kinetic energy.

Using the technique described above, create calculated columns for Kinetic Energy1, Kinetic Energy2, and Total Kinetic Energy. Create a single full-screen graph of Kinetic Energy1, Kinetic Energy2, and Total Kinetic Energy vs. time.

Question: Is the kinetic of cart 1 conserved (i.e. constant) during the collision? If not, does it gain or lose kinetic energy? Explain.

Question: Is the kinetic energy of cart 2 conserved during the collision? If not, does it gain or lose kinetic energy? Explain.

Question: Is the total kinetic energy in the system of the two carts conserved during the collision? Should it be? Explain.

Question: If the total kinetic energy in the system of the two carts decreased during the collision, where did this energy go?

Add a second graph to your display. Make the top graph Momentum1, Momentum2, and Total Momentum vs. time and the bottom graph Kinetic Energy1, Kinetic Energy2, and Total Kinetic Energy vs. time. Adjust the scales on your graphs so that they are easy to read. Print this page and attach it to the end of the activity. Note that your graph may look better in landscape mode.

3. Other Types of Collisions

Imagine different types of collisions. For example, what if cart 1 was twice as massive (i.e. carrying a 500 g black bar)? What if cart 2 was moving and cart 1 stationary? What if both carts were moving? Would momentum and kinetic energy behave the same way they did in the simple collision just studied?

For the following three collisions, add a 500 g black bar to cart 1 (and alter your *LoggerPro* equation for Momentum 1 and Kinetic Energy 1):

- Cart 1 colliding with a stationary cart 2.
- Cart 2 colliding with a stationary cart 1.
- Cart 1 and cart 2 in a head-on collision.

For each collision, prepare and print the momentum and kinetic energy graphs described above and attach them to the end of the activity. Below each graph, clearly describe the collision and your findings regarding momentum and kinetic energy conservation for that collision.

Question: What conclusion(s) can you draw about the amount of momentum present before and after each collision? Do all 4 collisions (including the one analyzed in the previous section) support the same conclusion?

Question: What conclusion(s) can you draw about the amount of kinetic energy present before and after each collision? Do all 4 collisions support the same conclusion?

B. Magnet-Magnet Collisions

Arrange your two carts such that they repel each other when they get close together.

Roll cart 1 in the positive direction, colliding with a stationary cart 2. Roll cart 1 fast enough so that after the collision cart 2 rolls to the end of the track, but slow enough so that the carts don't actually touch during the interaction.

Prepare, print, and attach this graph to the end of the activity.

Question: Is the total momentum in the system of the two carts approximately constant during the collision? Should it be? Explain.

Question: Is the total kinetic energy in the system of the two carts approximately constant during the collision? Should it be? Explain.

Question: If the total kinetic energy in the system of the two carts decreased during the collision, did it decrease by a smaller amount than in the velcro-velcro collision?

Repeat the remaining two collisions you analyzed earlier, except using the magnet-magnet interaction.

- Cart 2 colliding with a stationary cart 1.
- Cart 1 and cart 2 in a head-on collision.

For each collision, prepare and print the momentum and kinetic energy graphs described above and attach them to the end of the activity. Below each graph, clearly describe the collision and your findings regarding momentum and kinetic energy conservation for that collision.

Question: What conclusion(s) can you draw about the amount of momentum present before and after each collision? Do all 3 collisions support the same conclusion? Compare these conclusions to the conclusions drawn from the velcro-velcro data.

Question: What conclusion(s) can you draw about the amount of kinetic energy present before and after each collision? Do all 3 collisions support the same conclusion? Compare these conclusions to the conclusions drawn from the velcro-velcro data.

5.15: Two-Dimensional Motion

Name: ____

Date: _________________________

Partners:____

Equipment

• *Movie* software file • *CartonIncline* movie • *TwoBallFall* movie • *Projectile* Excel file	• Projectile launcher • Carbon paper • Wooden board

Introduction

Except for a few limited cases, a motion detector is not useful to analyze motion in two dimensions. A better technique is *video analysis*, in which a video is made of an object's motion and the position of the object is determined on a frame-by-frame basis. Once the movie is scaled, this position data can be used to determine the velocity and acceleration of the object during its motion. The *LoggerPro* software you have been using has integrated video analysis capabilities.

I. Motion on an Incline

The motion of a cart on an incline is a simple example of a two-dimensional motion; the cart moves in both the horizontal and vertical directions. Although this is an example in which a motion detector *can* be used to collect kinematic data (by rotating the coordinate system this motion can be modeled as a one-dimensional motion along the incline), we will use it as an introduction to video analysis techniques.

Open the file *Movie* and select **Insert/Movie**. Find the movie *CartonIncline* and open it. Play the movie. You will analyze the motion of the fan cart rolling down the incline. Return the movie to the first frame.

To extract data from a video:

- Click the icon in the lower right portion of the movie. The video analysis tools will appear.
- Enlarge the movie until it fills as much of the screen as possible. This will allow you to accurately select the pixel representing the cart's position.
- Select and maneuver the cross-hair until it is directly over the white dot representing the center of the cart. Click on the white dot. The movie will automatically advance to the next frame. Click on the position of the cart in every frame of the movie. The software will automatically create a graph of x- and y-position vs. time.
- Currently, the position of the cart is measured in pixels. To convert from pixels to meters, you need to scale the movie using a known length included in the movie. To do this, select to open the scaling tool. Click and drag along the length of the meter stick near the bottom of the frame. When you release the mouse, the dialogue box requests the length of the object used for scaling. Type and enter the appropriate length. The position data will immediately change to meters.
- Select a coordinate system. (The default system is the lower left corner of the video frame.) To move the coordinate system, select and then click where you would like the coordinate axis to appear. For this activity, choose the initial position of the cart.

Arrange a graph of x- and y-position vs. time and a separate graph (**Insert/Graph**) of x- and y-velocity vs. time on the screen. Print these graphs (on the same page) and attach the graphs to the end of this activity.

Question: Describe the relationship between the x- and y-components of motion. Which direction involves the larger change in position? The larger change in velocity? The larger acceleration?

Determine the acceleration of the cart by calculating the slope of the velocity vs. time graph.

	Acceleration (m/s^2)

x-direction	±
y-direction	±

Question: What is the magnitude of the total acceleration of the cart? At what angle is this acceleration vector oriented? (Remember to determine uncertainties for these values.)

Adjust your position graph to create a graph of y-position vs. x-position.

Question: Why is this graph a straight line while a graph of either x- or y-position vs. time is a curve?

Question: Find the slope of the best-fit line to this data, and record it below (with units and uncertainty).

Question: What is the physical meaning of this slope? To what number calculated above can it be compared? Explain.

Question: If you created a graph of y-velocity vs. x-velocity, what shape do you think the data would show? What else can you predict about the best-fit function on this graph?

Create a graph of y-velocity vs. x-velocity.

Question: Find the slope of the best-fit line to this data, and record it below (with units and uncertainty).

Question: What is the physical meaning of this slope?

Return to a graph of x- and y-position vs. time and a separate graph of x- and y-velocity vs. time.

The two-dimensional motion of the cart down the incline can be reduced to one-dimensional motion if we rotate our coordinate system so that the x-axis is along the length of the incline. To do this, select the coordinate system icon and rotate the coordinate system by "grabbing" and rotating the yellow ball on the positive x-axis. Make the direction down the incline the positive x-direction.

Question: Describe how the change in coordinate system affects the position and velocity vs. time graphs.

Question: Determine the acceleration of the cart by calculating the slope of the x-velocity vs. time graph. Record your value below.

Question: Does the cart have the same magnitude acceleration in the rotated coordinate system that it had in the original coordinate system, within measured uncertainties? Should it? Explain.

II. Projectile Motion

Clear all data and delete the movie *CartonIncline*.

Insert the movie *TwoBallFall*. The movie shows a device that simultaneously launches a small ball forward and drops a second identical ball. Play the movie, and then return the movie to the first frame.

Analyze the motion of the launched ball, and then select to add a new point series to analyze the motion of the dropped ball. Scale the movie using the meter stick hanging on the wall and set the origin of the coordinate system at the initial location of the dropped ball.

Arrange a graph of x- and y-position of both balls vs. time (all on the same graph). Print this graph and attach it to the end of this activity.

Question: Clearly describe the results of this experiment. Does the difference in x-motion of the balls have any measurable effect on their y-motion? Explain.

Determine the acceleration of each ball by calculating the slope of the y-velocity vs. time graphs.

Vertical Motion	Acceleration (m/s^2)
launched ball	±
dropped ball	±

Question: Does the data above conform to your expectations? Explain.

Question: Determine the launching speed of the device, with units and uncertainty. Explain how you determined this value.

III. Projectile Range

The horizontal distance a projectile travels is referred to as its *range*. The range of a projectile depends on several parameters, including launch height (relative to landing height), launch speed and launch angle.

Using a projectile launcher, you will collect range data as you vary launch angle. Then, using your data, you will create and validate a mathematical model of projectile motion. Once this model is validated, you can use it to analyze a large number of different scenarios.

A. Collecting Data

Securely mount the projectile launcher to the edge of your lab bench. Make sure the launch direction is free of nearby obstacles.

The photogate apparatus attached to the front of the launcher measures the elapsed time between passing through the two gates. Since the distance between these gates is 10 cm, the elapsed time can be used to determine the launch speed of the launcher.

Securely attach carbon paper to the wooden board. By using this board as a landing area for the projectile, you can accurately measure its range.

Measure the launch height relative to the floor. Record it below.

launch height, y_{launch} = ____________ m

For the launch angles measured below, launch the projectile from the smallest spring setting and record the range of the projectile. You do not need to determine an uncertainty for these values.

Launch Angle (°)	Range (m)
0	
5	
10	
15	
20	
25	
30	
35	
40	
45	
50	
55	
60	
65	
70	
75	

80	
85	
90	

Question: To determine the launch speed, set the launcher to 0° and fire the gun several times, resetting the timer between launches. Based on these measurements, what is the elapsed time (including uncertainty) for the ball to travel 10 cm? How did you determine the uncertainty in this time?

Question: Based on your answer above, what is the launch speed (including uncertainty) for the gun?

B. Mathematical Model

Open the Excel file *Projectile*. Input the launch height, mean and uncertainty in launch speed, and the experimental values for the range of the projectile. You will create a spreadsheet that calculates the range for both the minimum launch speed (mean – uncertainty) and the maximum launch speed (mean + uncertainty).

For each of the four calculated spreadsheet columns, write below the appropriate Excel formula needed to calculate the first row of each column:

initial x-velocity:

initial y-velocity:

time of flight: (Hint: Consider the y-position kinematic equation.)

range: (Hint: Consider the x-position kinematic equation.)

Create a graph showing the minimum and maximum calculated ranges as well as the experimental range. Print your spreadsheet and your graph and attach them to the end of this activity. As always, you will be graded on the quality of your data and on how well you present your data and graph.

Question: Does your mathematical model adequately fit your experimental data? Explain.

Question: At what angle did the launcher produce the largest range? Why is this angle not 45°? Explain.

IV. Using Your Model

Now that you have evidence that your mathematical model is correct (assuming it fits your experimental data), you can use your model to explore a wide variety of projectile motion scenarios.

A. The dependence of the angle of maximum range on launch height

On level ground, the angle of maximum range is 45°. In the experiment you just completed, the angle of maximum range was less than 45°. This should lead you to question how the angle of maximum range depends on launch height. For example, does the angle continue to decrease as you get higher and higher or does it reach some limiting value that it does not drop below? You can use your spreadsheet to address this question.

To do this, hold the launch speed constant at 10 m/s and determine the range vs. angle for launch heights 0, 2, 4, 8, and 16 m. To see the dependence more clearly, change your spreadsheet to determine the range at every angle from 0° to 45°. Plot all five sets of data on the same graph, and see if you can see a pattern developing between the angle of maximum range and launch height. Print this graph and attach it to the end of this activity, and use your results to complete the table at right.	**Launch Height (m)**	**Angle of Max Range (º)**
	0	
	2	
	4	
	8	

	16	

Question: How does the angle of maximum range depend on launch height? Describe why this dependence is plausible.

B. The dependence of the angle of maximum range on launch speed

In the experiment you just completed, the angle of maximum range was less than 45°. Would this value change if the ball was launched at a greater speed? If the ball was launched at a large enough speed, would the angle of maximum range approach 45° or would it remain constant? You can use your spreadsheet to address this question.

To do this, hold the launch height constant at 4 m and determine the range vs. angle for launch speeds of 2, 4, 8, and 12 m/s. Again, examine every angle from 0° to 45°. Plot all four sets of data on the same graph, and see if you can see a pattern developing between the angle of maximum range and launch speed. Print this graph and attach it to the end of this activity, and use your results to complete the table at right.	**Launch Speed (m/s)**	**Angle of Max Range (º)**
	2	
	4	
	8	
	12	

Question: How does the angle of maximum range depend on launch speed? Describe why this dependence is plausible.

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Detailed Licensing

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