5.15: Two-Dimensional Motion

Name: ____

Date: ______________________

Partners:____

Equipment

• *Movie* software file • *CartonIncline* movie • *TwoBallFall* movie • *Projectile* Excel file	• Projectile launcher • Carbon paper • Wooden board

Introduction

Except for a few limited cases, a motion detector is not useful to analyze motion in two dimensions. A better technique is *video analysis*, in which a video is made of an object's motion and the position of the object is determined on a frame-by-frame basis. Once the movie is scaled, this position data can be used to determine the velocity and acceleration of the object during its motion. The *LoggerPro* software you have been using has integrated video analysis capabilities.

I. Motion on an Incline

The motion of a cart on an incline is a simple example of a two-dimensional motion; the cart moves in both the horizontal and vertical directions. Although this is an example in which a motion detector *can* be used to collect kinematic data (by rotating the coordinate system this motion can be modeled as a one-dimensional motion along the incline), we will use it as an introduction to video analysis techniques.

Open the file *Movie* and select **Insert/Movie**. Find the movie *CartonIncline* and open it. Play the movie. You will analyze the motion of the fan cart rolling down the incline. Return the movie to the first frame.

To extract data from a video:

- Click the icon in the lower right portion of the movie. The video analysis tools will appear.
- Enlarge the movie until it fills as much of the screen as possible. This will allow you to accurately select the pixel representing the cart's position.
- Select and maneuver the cross-hair until it is directly over the white dot representing the center of the cart. Click on the white dot. The movie will automatically advance to the next frame. Click on the position of the cart in every frame of the movie. The software will automatically create a graph of x- and y-position vs. time.
- Currently, the position of the cart is measured in pixels. To convert from pixels to meters, you need to scale the movie using a known length included in the movie. To do this, select to open the scaling tool. Click and drag along the length of the meter stick near the bottom of the frame. When you release the mouse, the dialogue box requests the length of the object used for scaling. Type and enter the appropriate length. The position data will immediately change to meters.
- Select a coordinate system. (The default system is the lower left corner of the video frame.) To move the coordinate system, select and then click where you would like the coordinate axis to appear. For this activity, choose the initial position of the cart.

Arrange a graph of x- and y-position vs. time and a separate graph (**Insert/Graph**) of x- and y-velocity vs. time on the screen. Print these graphs (on the same page) and attach the graphs to the end of this activity.

Question: Describe the relationship between the x- and y-components of motion. Which direction involves the larger change in position? The larger change in velocity? The larger acceleration?

Determine the acceleration of the cart by calculating the slope of the velocity vs. time graph.

	Acceleration (m/s^2)

x-direction	±
y-direction	±

Question: What is the magnitude of the total acceleration of the cart? At what angle is this acceleration vector oriented? (Remember to determine uncertainties for these values.)

Adjust your position graph to create a graph of y-position vs. x-position.

Question: Why is this graph a straight line while a graph of either x- or y-position vs. time is a curve?

Question: Find the slope of the best-fit line to this data, and record it below (with units and uncertainty).

Question: What is the physical meaning of this slope? To what number calculated above can it be compared? Explain.

Question: If you created a graph of y-velocity vs. x-velocity, what shape do you think the data would show? What else can you predict about the best-fit function on this graph?

Create a graph of y-velocity vs. x-velocity.

Question: Find the slope of the best-fit line to this data, and record it below (with units and uncertainty).

Question: What is the physical meaning of this slope?

Return to a graph of x- and y-position vs. time and a separate graph of x- and y-velocity vs. time.

The two-dimensional motion of the cart down the incline can be reduced to one-dimensional motion if we rotate our coordinate system so that the x-axis is along the length of the incline. To do this, select the coordinate system icon and rotate the coordinate system by "grabbing" and rotating the yellow ball on the positive x-axis. Make the direction down the incline the positive x-direction.

Question: Describe how the change in coordinate system affects the position and velocity vs. time graphs.

Question: Determine the acceleration of the cart by calculating the slope of the x-velocity vs. time graph. Record your value below.

Question: Does the cart have the same magnitude acceleration in the rotated coordinate system that it had in the original coordinate system, within measured uncertainties? Should it? Explain.

II. Projectile Motion

Clear all data and delete the movie *CartonIncline*.

Insert the movie *TwoBallFall*. The movie shows a device that simultaneously launches a small ball forward and drops a second identical ball. Play the movie, and then return the movie to the first frame.

Analyze the motion of the launched ball, and then select to add a new point series to analyze the motion of the dropped ball. Scale the movie using the meter stick hanging on the wall and set the origin of the coordinate system at the initial location of the dropped ball.

Arrange a graph of x- and y-position of both balls vs. time (all on the same graph). Print this graph and attach it to the end of this activity.

Question: Clearly describe the results of this experiment. Does the difference in x-motion of the balls have any measurable effect on their y-motion? Explain.

Determine the acceleration of each ball by calculating the slope of the y-velocity vs. time graphs.

Vertical Motion	**Acceleration (m/s^2)**
launched ball	±
dropped ball	±

Question: Does the data above conform to your expectations? Explain.

Question: Determine the launching speed of the device, with units and uncertainty. Explain how you determined this value.

III. Projectile Range

The horizontal distance a projectile travels is referred to as its *range*. The range of a projectile depends on several parameters, including launch height (relative to landing height), launch speed and launch angle.

Using a projectile launcher, you will collect range data as you vary launch angle. Then, using your data, you will create and validate a mathematical model of projectile motion. Once this model is validated, you can use it to analyze a large number of different scenarios.

A. Collecting Data

Securely mount the projectile launcher to the edge of your lab bench. Make sure the launch direction is free of nearby obstacles.

The photogate apparatus attached to the front of the launcher measures the elapsed time between passing through the two gates. Since the distance between these gates is 10 cm, the elapsed time can be used to determine the launch speed of the launcher.

Securely attach carbon paper to the wooden board. By using this board as a landing area for the projectile, you can accurately measure its range.

Measure the launch height relative to the floor. Record it below.

launch height, y_{launch} = ____________ m

For the launch angles measured below, launch the projectile from the smallest spring setting and record the range of the projectile. You do not need to determine an uncertainty for these values.

Launch Angle (°)	Range (m)
0	
5	
10	
15	
20	
25	
30	
35	
40	
45	
50	
55	
60	
65	
70	
75	

80	
85	
90	

Question: To determine the launch speed, set the launcher to 0° and fire the gun several times, resetting the timer between launches. Based on these measurements, what is the elapsed time (including uncertainty) for the ball to travel 10 cm? How did you determine the uncertainty in this time?

Question: Based on your answer above, what is the launch speed (including uncertainty) for the gun?

B. Mathematical Model

Open the Excel file *Projectile*. Input the launch height, mean and uncertainty in launch speed, and the experimental values for the range of the projectile. You will create a spreadsheet that calculates the range for both the minimum launch speed (mean – uncertainty) and the maximum launch speed (mean + uncertainty).

For each of the four calculated spreadsheet columns, write below the appropriate Excel formula needed to calculate the first row of each column:

initial x-velocity:

initial y-velocity:

time of flight: (Hint: Consider the y-position kinematic equation.)

range: (Hint: Consider the x-position kinematic equation.)

Create a graph showing the minimum and maximum calculated ranges as well as the experimental range. Print your spreadsheet and your graph and attach them to the end of this activity. As always, you will be graded on the quality of your data and on how well you present your data and graph.

Question: Does your mathematical model adequately fit your experimental data? Explain.

Question: At what angle did the launcher produce the largest range? Why is this angle not 45°? Explain.

IV. Using Your Model

Now that you have evidence that your mathematical model is correct (assuming it fits your experimental data), you can use your model to explore a wide variety of projectile motion scenarios.

A. The dependence of the angle of maximum range on launch height

On level ground, the angle of maximum range is 45°. In the experiment you just completed, the angle of maximum range was less than 45°. This should lead you to question how the angle of maximum range depends on launch height. For example, does the angle continue to decrease as you get higher and higher or does it reach some limiting value that it does not drop below? You can use your spreadsheet to address this question.

To do this, hold the launch speed constant at 10 m/s and determine the range vs. angle for launch heights 0, 2, 4, 8, and 16 m. To see the dependence more clearly, change your spreadsheet to determine the range at every angle from 0° to 45°. Plot all five sets of data on the same graph, and see if you can see a pattern developing between the angle of maximum range and launch height. Print this graph and attach it to the end of this activity, and use your results to complete the table at right.	**Launch Height (m)**	**Angle of Max Range (º)**
	0	
	2	
	4	
	8	

	16	

Question: How does the angle of maximum range depend on launch height? Describe why this dependence is plausible.

B. The dependence of the angle of maximum range on launch speed

In the experiment you just completed, the angle of maximum range was less than 45°. Would this value change if the ball was launched at a greater speed? If the ball was launched at a large enough speed, would the angle of maximum range approach 45° or would it remain constant? You can use your spreadsheet to address this question.

To do this, hold the launch height constant at 4 m and determine the range vs. angle for launch speeds of 2, 4, 8, and 12 m/s. Again, examine every angle from 0° to 45°. Plot all four sets of data on the same graph, and see if you can see a pattern developing between the angle of maximum range and launch speed. Print this graph and attach it to the end of this activity, and use your results to complete the table at right.	**Launch Speed (m/s)**	**Angle of Max Range (°)**
	2	
	4	
	8	
	12	

Question: How does the angle of maximum range depend on launch speed? Describe why this dependence is plausible.