5.2: Air Resistance

Name: ____

Date: ________________________

Partners:____

Equipment

- *LabPro* Interface
- Motion detector
- *Motion* software file
- Large coffee filters
- *Drag* Excel file

Introduction

The motion of objects through air is studied in every introductory physics course. Ignoring the effects of air resistance, or *drag*, allows one to derive simple equations to predict the time of flight, range, and various other parameters of the motion. These equations are, however, only approximations to the true motions of real objects. Air drag is seldom so small that it can be ignored in the real world.

Unfortunately, the mathematics of including drag in the study of motion are quite complicated. However, by using an approximation scheme and the calculational power of *Excel*, you will develop a spreadsheet that can calculate motion parameters that are quite close to those measured in the real world.

I. Quantifying Drag

To quantitatively study the effect of drag on the motion of an object, we need to quantify what we mean by drag. Drag is a force that acts to oppose the motion of an object through a fluid. (To a physicist, air and water are both fluids.) This force depends on numerous parameters of the system.

Question: What parameters do you think drag depends on? (Imagine an object moving through air. What variables affect the size of the drag force?) Explain why each parameter affects the drag force.

Often, all of the parameters effecting drag *except the speed of the object* are lumped together into a single number, the *drag coefficient*. The dependence on speed will be determined in the following activity.

Place the motion detector on the floor and open the file *Motion.* Add a velocity graph to the display.

Hold a large coffee filter directly above the motion detector, as close to the ceiling as possible, press **Collect**, and release the filter. If necessary, repeat until you have clean data.

Question: Describe the motion of the coffee filter. Is the acceleration of the filter constant?

Question: Draw a free-body diagram for the coffee filter as it falls. Label the drag force F_{drag}. Apply Newton's Second Law in the vertical direction.

If the drag force was constant, both forces acting on the filter would be constant resulting in a constant acceleration. However, if the drag force depended on the speed of the filter, the drag force would grow in magnitude as the filter fell until the drag force equaled the weight of the filter. At this point, the two forces would be equal and the acceleration of the filter would drop to zero. The speed at which this occurs is termed the *terminal velocity* of the filter.

At terminal velocity, v_T,

- If the drag force was proportional to speed, this would lead to:

and the terminal velocity would be proportional to the mass of the filter.

- If the drag force was proportional to the square of the speed, this would lead to:

and the square of the terminal velocity would be proportional to the mass of the filter.

Thus, we can determine the dependence of drag force on speed by determining the dependence of terminal velocity on mass!

Measure the terminal velocity of the coffee filter by highlighting the time interval over which the velocity is constant. Record your result in the table below. Label, print, and attach the graph illustrating this measurement.

You can increase the mass of the coffee filter (without altering its shape) by stacking coffee filters inside each other. Thus, the number of coffee filters stacked together is proportional to the mass of the falling object. Do this to complete the following table.

Number of Filters	Terminal Velocity (m/s)
1	±
2	±
3	±
4	±
5	±
6	±
7	±

To determine the relationship between drag force and speed, create the following two graphs:

- Terminal velocity, with error bars, vs. number of filters. If a direct proportion exists between these two variables, drag is proportional to speed.
- Terminal velocity squared, with error bars, vs. number of filters. If a direct proportion exists between these two variables, drag is proportional to speed squared. (By calculus, the uncertainty in terminal velocity squared is *twice* the uncertainty in terminal velocity.)

Print and attach these two graphs, with linear-fit functions displayed.

Question: What is the mathematical dependence of drag on the speed of the object (i.e., which of the two graphs provides a better fit to the data)? Clearly explain how your data supports this relationship.

Question: Clearly explain the limitations in determining the dependence of drag force on speed by this method. What could be done to improve this experiment?

II. Drag in One Dimension

A. Creating and Testing the Spreadsheet

To begin, imagine a baseball thrown vertically upward at 35 m/s from the top of a cliff 45 m above a river. If you ignore drag, you should be able to calculate the time it takes for the ball to reach the river and its speed on impact.

Question: Determine the time it takes to reach the river and its speed on impact.

Our ultimate task is to create a spreadsheet that can solve a problem like this one when drag is included. As a first step, we will create a spreadsheet that can solve a kinematic problem *without* drag, and compare its results to the “known” correct answer. Once the spreadsheet passes this test, we will try to add drag to the spreadsheet and use it to solve problems where the answer can not be calculated by hand.

The equations you used to solve the above problem are only valid when the acceleration is constant. Since drag leads to a non-constant acceleration, they are not very useful if the ultimate goal is to incorporate drag. Rather, a general technique for solving any motion problem, no matter how complicated, will serve us better. In this approach, we will apply the basic concepts of kinematics in a step-by-step manner to the motion.

In a nutshell, the velocity of an object at one instant can always be determined if you know the velocity of the object at a *previous* instant and the object’s acceleration, via:

In words, the equation says that the velocity of the object at the “new” time (t + Δt), is equal to the velocity at the previous time (t) plus the product of the acceleration at the previous time and the difference in time.

Also, the position of an object at one instant can always be determined if you know the position of the object at a *previous* instant and the object’s velocity, via:

Again, this equation says that the position of the object at the “new” time (t + Δt), is equal to the position at the previous time (t) plus the product of the velocity at the previous time and the difference in time.

These equations are only approximate, but become exact in the limit as the *time-step*, Δt, approaches zero. As long as we use a very small Δt, the approximation made will be very small.

Open the Excel file *Drag*.

Input the initial time and the initial position, velocity and acceleration of the baseball. The mass of a baseball is 0.145 kg and for this problem the drag coefficient is assumed to be zero. Use a time step of 0.05 s.

For each of the four calculated spreadsheet columns, write below the appropriate *Excel* formula needed to calculate the second row of each column (the first row is determined by the initial inputs):

time:

position:

velocity:

acceleration:

Enter the correct formulas and complete the spreadsheet. Create a graph of position, velocity, and acceleration vs. time, from release until striking the river. Print and attach your graph.

Question: Based on your spreadsheet, determine the time it takes to reach the river and its speed on impact. Compare these values to the values calculated by hand. If they are not *extremely* close, correct the problem.

B. Adding Drag

Earlier, you determined that the drag force is proportional to the square of the object’s speed. Thus

where b is the drag coefficient. This force is directed opposite to the object’s velocity. This directional information must be correctly included in the equation for drag.

For example, consider the equation

Question: If the velocity is in the positive direction, what direction does this equation produce for the drag force? Is this the correct direction for the drag?

Question: If the velocity is in the negative direction, what direction does this equation produce for the drag force? Is this the correct direction for the drag?

Question: Can the formula above correctly represent the drag force? Explain.

Now consider the equation

Question: If the velocity is in the positive direction, what direction does this equation produce for the drag force? Is this the correct direction for the drag?

Question: If the velocity is in the negative direction, what direction does this equation produce for the drag force? Is this the correct direction for the drag?

Question: Can the formula above correctly represent the drag force? Explain.

Neither of the equations given correctly model drag. The correct equation is:

Question: Explain why this equation gives the correct direction for the drag force.

To incorporate drag into your spreadsheet, you need to calculate the acceleration of the baseball at each and every time. (The velocity and position columns use general results that should not need to be changed.)

Question: Draw a free-body diagram for the baseball during its motion. Using the equation for drag given above, apply Newton's Second Law in the vertical direction and solve for the acceleration.

Enter the correct formula for acceleration and complete the spreadsheet. The drag coefficient for a baseball has been measured to be 1.2 x 10^{-3} Ns^2/m^2.

Create a graph of position, velocity, and acceleration vs. time, from release until striking the river. Print and attach your graph.

Question: Immediately after being thrown, the acceleration of the baseball is greater in magnitude than the acceleration due to gravity. How is this possible? Explain.

Question: Does the baseball reach terminal velocity? How do you know? Explain.

Complete the following table.

	Without drag	**With drag**
Maximum height		
Time of flight		
Impact speed		

Question: Based on the results above, comment on how drag changes the motion of the baseball.

III. Drag in Two Dimensions

After being hit, imagine a baseball is traveling at 50 m/s at an angle of 53^0 above horizontal.

To handle two-dimensional drag, you will need to calculate the x- and y-components of each of the kinematic variables. Thus, you will need to create columns to calculate a_x, a_y, v_x, v_y, r_x, and r_y. You will also need to set the initial values for all of these functions.

The correct equation for drag force, written as a vector equation, is:

This is equivalent to the two separate x- and y-component equations:

where the absolute value signs now represent the magnitude of a two-dimensional vector, which can be determined by Pythagoras' Theorem,

Question: Draw a free-body diagram for the baseball during its two-dimensional motion. Apply Newton's Second Law in the x- and y-direction and solve for the x- and y-accelerations.

Enter the correct formulas for x- and y-acceleration and complete the spreadsheet.

Construct a graph of the trajectory (y-position vs. x-position) of the baseball, showing the entire path of the ball until it hits the ground. Print and attach your graph.

Question: How far from home plate does the ball land? Is it an out or a home run? The fence is 110 m (360 ft) from home plate.

Question: Compare the above result to the same situation ignoring drag (set b = 0 and re-run the spreadsheet). How far from home plate does the ball now land? Is it an out or a home run? Is drag an important effect in the flight of this baseball?

Question: The drag coefficient is proportional to air density. Air density in Denver is about 88% of air density at sea level. How much farther will the ball described above travel in Denver than at sea level?

IV. Using Your Model

The launch angle that produces maximum range over level ground is commonly thought to be 45°. However, this is only true if drag is ignored. For a thrown (or hit) baseball, maximum range does not occur at 45°. The range of the ball has a rather complicated dependence on launch angle and launch velocity.

To explore this dependence, use your spreadsheet to determine the range of a ball launched at 30 m/s with launch angle 25, 30, 32, 34, 36, 38, 40, 42, 45, and 50°. Repeat for a ball launched at 40, 50 m/s and 60 m/s. Tabulate your results below.

Launch Speed (m/s)

	30	*40*	*50*	*60*	
Launch Angle (º)	*25*				
	30				
	32				
	34				
	36				
	38				
	40				
	42				
	45				
	50				

Using this data, create a graph of range vs. launch angle displaying all four sets of data. Print and attach your table and graph.

Question: Clearly explain why the maximum range occurs for a launch angle less than 45°.

Question: How does the launch angle for maximum range depend on launch velocity? Describe why this dependence is plausible.