5.3: Modeling Model Rockets

Name: ______________________________________

Date: ______________________

Partners:____

Equipment

- Laptop computer
- *LabPro* Interface
- Force sensor with rocket engine adaptor
- *ModelRocket* Excel file

Introduction

The motion of a rocket is quite complex. Typically the thrust force is not constant in time, the mass of the rocket decreases as the fuel burns, and substantial drag forces act on the rocket. This is as true of model rockets as full-scale rockets.

Creating a mathematical model that incorporates all of these aspects of the motion can be challenging. You will begin by creating a simplified model (constant thrust, constant mass, no drag) for a model rocket and gradually eliminate these simplifications.

I. Engine Thrust vs. Time

In this activity, you will model the flight of a 230 gram model rocket which initially carries 25 grams of fuel. (The fuel is the solid propellant in an Estes model D12 model rocket engine.) Thus, the total initial mass of the system is 255 g. The rocket will begin at rest on the ground.

Your instructor may test-fire a sample engine attached to a force probe to collect force vs. time data for the engine. If not, data from a previous test-firing is available in the file *ModelRocket*. If an engine is test-fired, import the data into the *ModelRocket* file in place of the saved data.

Create a graph of engine thrust vs. time. The thrust is obviously not constant.

II. Constant Thrust, Constant Mass, No Drag

A. Finding the Average Thrust

Although the thrust is not constant, it is useful to first analyze the motion of the rocket assuming the thrust *is* constant, because when the thrust is constant the motion can be analyzed by the familiar equations of introductory physics. We will assume the thrust is constant at its average value. Therefore, to proceed we must determine the average value of the thrust.

If the thrust *was* constant, the total impulse supplied by the engine would be given by:

where the constant thrust () can be “removed” from the integral.

This results in:

Therefore, determining the average thrust requires the determination of the integral of the thrust with respect to time. Since the mathematical function describing the thrust is not known, we will have to approximate the integral. The simplest way to do this is by replacing the thrust integral with a sum:

where F_{ave} is the average value of the force during each time step, t.

For example, if you are using the previously saved data during the first time step (from 0 s to 0.04 s) the average force is 0.5 N (the average of 1.0 and 0.0 N). Therefore, the value of the sum during this time step is

F_{ave} (t) = (0.5 N)(0.04 s) = 0.02 Ns

Question: Use the spreadsheet to determine the approximate value of the thrust integral. Record your result, with units, below.

Question: Based on the value of the thrust integral, determine the average thrust acting on the rocket. Record your result, with units, below. (Hint: If you are using the previously saved data, you should get a value between 11 and 12 N. If you don’t, do not proceed until you find your mistake!)

B. Finding the Maximum Height “by Hand”

A 255 g rocket is launched vertically upward from rest. The thrust acting on the rocket is equal to the value calculated above. Determine the maximum height reached by the rocket and the time needed to reach this height. Assume the mass of the rocket is constant and the drag acting on the rocket is zero.

Freebody Diagram Motion Information

	Event 1: t_1 = r_1 = v_1 = a_{12} =	Event 2: t_2 = r_2 = v_2 = a_{23} =	Event 3: t_2 = r_2 = v_2 =

Mathematical Analysis

C. Finding the Maximum Height by Spreadsheet

The kinematics equations you used to solve the above problem are only valid when the acceleration is constant. Since varying the thrust, mass and drag on the rocket all lead to non-constant accelerations, they will not be very useful in our ultimate model. Rather, a general technique for solving any motion problem, no matter how complicated, will serve us better. In this approach, we will apply the basic concepts of kinematics in a step-by-step manner to the motion.

In a nutshell, the velocity of an object at one instant can always be determined if you know the velocity of the object at a *previous* instant and the object's acceleration.

Also, the position of an object at one instant can always be determined if you know the position of the object at a *previous* instant and the object's velocity.

These equations are only approximate, but become exact in the limit as the *time-step*, t, approaches zero. As long as we use a very small t, the approximation made will be very small.

Input the initial position, velocity, and mass of the rocket. Assume the drag coefficient is zero. Enter the correct formulas and complete the spreadsheet *assuming the thrust is constant* (i.e., do not use the column containing the actual values of the thrust). Since you are trying to find the maximum height reached by the rocket, you will need to continue your time column beyond 1.76 s.

Question: Based on your spreadsheet, determine the maximum height reached by the rocket and the time it takes to reach this height. Compare these values to the values calculated by hand. If they are not *extremely* close, correct the problem.

III. Variable Thrust, Constant Mass, No Drag

Real rocket engines produce a thrust that is not constant in time. We will now attempt to include this factor into our model of a model rocket.

Question: Based on the actual variation of thrust vs. time, do you think that the model with variable thrust will fly higher, lower or the same as your original, constant thrust, prediction? Explain your rationale in detail.

Now it's time to determine the maximum height reached by the rocket, incorporating the fact that the thrust is not constant. You should be able to accomplish this by simply changing the formula in the acceleration column of your spreadsheet.

Once you correct the acceleration column, you should notice something odd immediately after launch. The rocket initially moves downward because $F_{thrust} < mg$. Obviously, the launch platform will prevent the rocket from moving downward. To correct this, replace the negative values for acceleration with zeros. This should correct the problem.

Question: Based on your spreadsheet, determine the maximum height reached by the rocket and the time it takes to reach this height. Compare these values to the values calculated assuming the thrust is constant. Carefully explain why the maximum height is different when the variation in thrust is taken into account.

IV. Variable Thrust, Variable Mass, No Drag

The mass of the rocket is not constant. At liftoff, it contains 25 grams of propellant that is not present once the thrust drops to zero. Further complicating the analysis is that since the thrust is not constant, the rate at which the rocket's mass decreases is not constant either.

Question: In light of the fact that the mass is not constant, do you think that the rocket will fly higher, lower or the same as your original, constant mass, prediction? Explain your rationale in detail.

Insert a column into your spreadsheet for the mass of the rocket. You should insert this column before the column for acceleration, because you need the correct mass to properly determine the acceleration. (Since you originally determined the acceleration by dividing the net force by a constant 0.255 kg, you must change your acceleration column to take into account the changing mass. To determine acceleration don't divide the net force by a constant, divide it by the mass column.)

The first entry in the mass column should be the initial mass of the rocket plus propellant, or 0.255 kg. The entry in this column once the thrust stops, and thereafter, should be .230 kg. What goes in between? A good guess is that the rate of mass loss is greatest when the thrust is greatest, because the rate of fuel consumption is greatest at this time. If we assume that the rate of mass loss is proportional to thrust, then we can set up the following ratios:

total mass lost total impulse	=	mass lost in each time step impulse in each time step

Question: Based on this relationship, you should be able to write an expression for the rocket mass at a given time in terms of the mass at a previous time and the necessary constants. Carefully write this expression below.

Incorporate this relationship into your spreadsheet. If incorporated correctly, your spreadsheet should now indicate that the mass decreases down to a value of .230 kg once the thrust is complete. If your spreadsheet doesn't accurately calculate the decreasing mass, do not proceed.

Finish altering your spreadsheet to incorporate the variable mass of the rocket.

Question: Based on your spreadsheet, determine the maximum height reached by the rocket and the time it takes to reach this height. Compare these values to the values calculated assuming the mass is constant. Carefully explain why the maximum height is different when the variation in mass is taken into account.

V. Variable Thrust, Variable Mass, Drag

Our final task is to incorporate air drag. All the corrections we have been making for variable thrust and variable mass are actually quite minor compared to the effect of drag.

Question: In light of the fact that the rocket experiences air drag, do you think that the rocket will fly higher, lower or the same as your original, frictionless, prediction? Explain your rationale in detail.

The standard model for air drag is that the drag force is proportional to the square of the speed of the object. A formula that always gives the correct direction for the drag force is:

Question: Explain why this equation gives the correct direction for the drag force.

Question: Draw a free-body diagram for the rocket. Use the equation for drag given above. Apply Newton's Second Law in the vertical direction and solve for the acceleration.

Enter the correct formula for acceleration and complete the spreadsheet. The drag coefficient for the particular model rocket under investigation has been measured to be $3.0 \times 10^{-4}\ Ns^2/m^2$. Remember to replace any negative accelerations at the start of the launch with zeroes because the rocket stays at rest on the launch pad until the rocket's thrust is greater than the force of gravity.

Finish altering your spreadsheet to incorporate the variable mass of the rocket.

Question: Based on your spreadsheet, determine the maximum height reached by the rocket and the time it takes to reach this height. Compare these values to the values calculated assuming there is no air drag. Carefully explain why the maximum height is different when drag is taken into account.

VI. Summary and Conclusions

Summarize your results in the table below.

thrust	**mass**	**drag**	r_{peak} **(m)**	t_{peak} **(s)**
Constant	*Constant*	*No*		

Variable	*Constant*	*No*		
Variable	*Variable*	*No*		
Variable	*Variable*	*Yes*		

Question: Did the inclusion of variable thrust substantially change the results? If the thrust of the rocket changed much more rapidly, how should we change our time step to maintain accuracy in our models? Explain.

Question: Did the inclusion of variable mass substantially change the results? If the initial mass of the rocket was mostly fuel, would your answer still be the same? Explain.

Question: Did the inclusion of drag substantially change the result? If our rocket had reached altitudes of several kilometers, how would our model of the drag force have to be changed? Explain.

Question: If our rocket reached altitudes of several hundred kilometers, what else in our model would have to be changed? Explain.