7.6: Conservation of Energy

Sliding Blocks

As we have already discussed (and you have probably noticed), falling objects speed up as they fall. On the other hand, if you throw an object upward, it slows down, finally stops, and then begins to fall back toward the ground. We already described this motion in terms of the acceleration caused by the force of gravity. There is always a gravitational force downward on objects due to the Earth, even when they are in the air, even when they are traveling upward, even when they are stopped. In fact, it is the gravitational pull that causes them to slow down in the first place. If there were no gravitational force between Earth and, say, a ball, you could throw the ball up and it would just keep going. There would be no forces to slow it down (Newton's first law).

There is also another way to think about why these types of motion occur. In addition to thinking about forces acting on them, we can consider their energy.

7.6.1: Conversion of Kinetic and Potential Energy Near Earth's Surface

In the previous section, we learned that gravitational potential energy increases when an object is placed at an elevated location and that kinetic energy increases as an object moves faster. You might be wondering if the motion of a falling object can be understood in terms of its energy. In fact, in can be. In the example of the pinecone, we imagined that the cone starts out at the top of the tree with a lot of potential energy. It then falls to the ground, and just before it hits the ground we know from experience that it is moving quite fast. How does its potential energy at the top of the tree compare to its kinetic energy just before it impacts the ground? The two are equal.

Energy, it turns out, is one of several quantities in nature that are said to be conserved. This conservation law for energy is one of the most important in science and can be applied in many different situations. (Exceptions only come up in quantum mechanics and are so rare that we need not consider them here, though we will touch upon some of them in later chapters.)

In its simplest form, the law of conservation of energy states that:

Energy can be neither created nor destroyed. It can only be moved from one part of a system to another, or changed from one form to another.

Mathematically, this can be expressed as:

$$E_{final} = E_{initial}$$

where E_{inital} is the total starting energy of a system and E_{final} is the total ending energy of a system. The “forms” of energy referred to are those we have already mentioned: potential, kinetic, and radiation. Conversion of mass to energy can also take place.

Applying this to our pinecone, we can say that its initial energy when it is hanging at the top of the tree, which is entirely gravitational potential energy, *mgh*, is equal to its final energy, entirely kinetic, $\frac{1}{2}mv^2$, just before it hits the ground. The total amount of energy the pinecone has stays the same, but it changes form. As the pinecone falls, the potential energy is converted into kinetic energy.

We can use the law of conservation of energy to trace how energy is transferred and transformed in everyday processes. These include the dropping of pinecones, but also astronomical processes, too.

Choose any everyday task. If you consider it carefully, you will see that the history of the energy involved in it goes back farther than you might at first think. For example, if you walked to class today, where did that energy come from? It came from the chemical bonds in the food you ate for lunch or for breakfast. Where did the food get that energy? From the Sun, directly if it was a plant, and through plants if it was meat. Where did the Sun get its energy? Read Chapter 3 to find out... And so on.

So energy a very useful number. We can calculate how much energy we need for a particular task, and then compare that to the amount of energy we have in a readily available form. We can also trace its origins back in time to see how things in the Universe are (or are not) connected in chains of events, one leading to the next, each acting as a conduit for energy as it moves from the past, through the present, into the future.

As a practical example, we can use the energy method to determine the speed of falling objects. Starting with the law of conservation of energy:

$$E_{final} = E_{initial}$$

And using the case where all of the final energy is kinetic and all of the initial energy is gravitational potential energy, we get:

$$\frac{1}{2}mv^2 = mgh$$

We can solve this equation to find the speed that a pinecone, or any other falling object, is traveling when it impacts the ground:

$$v = \sqrt{2gh}$$

Notice that the mass has again canceled; all objects will have the same final speed regardless of their mass.

In the next activity, we will plug in some numbers from our pinecone example and others to explore the law of conservation of energy.

Potential Energy to Kinetic Energy

Worked Examples:

1. A pinecone of mass 70 g starts out 50 meters above the ground and falls out of a tree. What is its final speed?

- Given: initial height h = 50 m, m = 0.07 kg
- Find: final speed v
- Concept: using conservation of energy and plugging in for *KE* and *PE*, we can use the formula derived above: $v = \sqrt{(2gh)}$
- Solution: $v = [\ (2)\ (9.8\ m/s^2)\ (50\ m)\ ]^{1/2} = (980\ m^2/s^2)^{1/2} = 31\ m/s$

Math Exploration 7.3: Potential Energy to Kinetic Energy

Questions

If you have been following our argument closely, you might have a question: Where does the energy go when the pinecone stops on the ground? After all, both the potential and kinetic energy are gone once the pinecone has stopped. If energy cannot be created or destroyed, it must have gone somewhere. And indeed it did.

When the pinecone hits the ground you will notice several things happen. First, it will make a sound. Sound is a vibration in the air (otherwise known as a sound wave), and it has energy. Some of the pinecone's energy goes into the energy of the sound wave. In addition, some dirt, pine needles and other objects will probably be kicked up and flung away from the point of impact. They all have energy, too. Finally, the pinecone itself might break or be deformed in some way, as may the ground itself if a small crater or indentation is created. The ground might also be slightly heated when the pinecone hits, though the heating will likely be too small to notice. This heating and the deformations of the pinecone and ground also require energy. That energy comes from the kinetic energy the pinecone has when it hits. So none of the energy goes missing after the pinecone comes to rest, it just goes into other forms. That is what is expected from the energy conservation law.

Of course, energy conservation works the other way, too. If we throw the pinecone upward at 31 m/s, it will rise to a height of 50 meters before it finally stops. In general, an object thrown upward with a speed v will rise to a height given by $h = ½ v^2/g$ before it stops and falls back down again.

So how far up can we throw an object? Are there limits, or will it always eventually fall back down? We can use what we have learned to try to predict the answer to this question.

We know that the conservation of energy says that the kinetic energy we give an object thrown straight up must equal the potential energy the object has at the top of its trajectory. As before, we can write this mathematically:

$$\frac{1}{2}mv^2 = mgh$$

According to this expression, as we increase the kinetic energy the object starts with (we throw it upward faster and faster), it will go higher and higher, but it always comes back down after some amount of time. The height h can become arbitrarily large as long as we increase the initial speed of the particle. However, there is a problem with this line of reasoning.

7.6.2: Conversion of Kinetic and Potential Energy Far From Earth's Surface

If you recall, when we introduced the expression for the potential energy we said it was valid as long as the gravitational acceleration, g, was essentially constant. This is true only as long as our distance from Earth's center does not change much. However, as we rise higher and higher above Earth's surface, g does begin to change. We calculated that at the altitude of the

International Space Station, the value of g has dropped from 9.8 m/s^2 to about 8.75 m/s^2, and by the time we get out to the distance of the Moon, g has dropped to only a couple *millimeters per second per second*. Clearly we cannot use the expression valid for a constant g once we have risen a significant distance off Earth's surface.

Not to worry. Physicists know that the expression we have used for potential energy is only an approximation. If the gravitational force changes appreciably along an object's path, due to its changing distance from another massive object, then we must use the correct expression for potential energy. It looks a lot like Newton's law of gravitation, but not quite.

$$PE = -\frac{GMm}{r}$$

Unlike in the expression for the gravitational force, the gravitational potential energy depends on the reciprocal of the distance separating the two objects, not the square of the distance. Also, there are no vector signs here. Unlike force, energy is not a vector. It has no direction. Finally, and this is important, there is a negative sign in front of the potential energy. The negative sign has to do with how we have defined the zero-point for gravitational potential energy. Notice that potential energy is zero when r is infinitely large, otherwise it is *always* negative. This might seem strange at first, but here is a way to think about it: From experience we know that objects tend to move from higher potential energy to lower potential energy. That is why pinecones fall out of trees onto the ground, but they do not fall from the ground up into the trees. With the definition of potential energy as written above, an object at a large elevation will lower its potential energy by moving to a smaller elevation. Notice that as r decreases, the potential energy becomes a larger negative number, or in other words, it becomes *smaller*. (In absolute value it is larger, of course, but negative numbers with large absolute value are smaller than negative numbers with small absolute value.)

So, now we can return to our question, How far up can we throw something so that it will always fall back down? Or to put it another way, how fast must we throw something so that it does not fall back down? In this situation, we have to realize that zero potential energy can no longer be assumed to be at Earth's surface. Now we must define zero potential energy to occur at $r = \infty$. Remember, the zero-point for potential energy is arbitrary, and we are only interested in changes in the potential energy. At Earth's surface the initial energy, call it $E_{initial}$, of the object will be its potential energy there (which is negative) plus whatever kinetic energy we give to it. Assuming the object has mass m and starts out with speed v, we can write its initial energy as below.

$$E_{initial} = \frac{1}{2}mv^2 - \frac{GM_E m}{R_E}$$

The final energy of the particle will be zero because it will be out at $r = \infty$, so its potential energy will be zero, and it will not be moving anymore, so its kinetic energy will also be zero:

$$E_{final} = 0$$

In the following activity, we will work out the details of the answer to our question for Earth, as well as how fast we can throw objects on other planetary bodies before they will no longer fall back down.

How High Can We Throw an Object?

Worked Example:

We want to determine how fast we can launch an object from Earth and have it fall back down. Anything going faster will keep going and anything slower will come back. The speed right on the border is known as the escape velocity.

- The final state of an object in this case will be when it has traveled as far as it can go ($r = \infty$), and has used up all its kinetic energy ($v = 0$) in doing so. In that case, its final energy will be zero: $E_{final} = 0$.
- Since energy must be conserved : $E_{final} = E_{initial}$, its initial energy must also be zero.
- So we can set our expression for $E_{initial}$ above equal to zero:
  $½mv^2 - GM_E m / R_E = 0$
- Rearranging terms and solving for the velocity we see that
  $½ v^2 = GM_E / R_E$
  $v = (2 GM_E / R_E)^{½}$
- This is the escape velocity. For Earth, the value is
  $v = [(2) (6.67384e\text{-}11 \text{ N m}^2 \text{ kg}^{-2}) (5.945e24 \text{ kg}) / (6.368e6 \text{ m})]^{½}$
  v = 11,160 m/s = 11.16 km/s.

To calculate the escape velocity from other astronomical bodies, you would use:

$$v_{escape} = \sqrt{\frac{2GM}{R}}$$

where M is the mass of the object you were launching from and R is its radius.

Questions

If we accelerate an object to its escape velocity, it will never fall back down. It continues to move outward forever, more and more slowly over time. It never comes completely to rest until it has traveled an infinite distance, which of course, it cannot do. In practical terms, this is the speed you must give an object in order for it to escape the gravitational influence of whatever object it is gravitationally bound to.

In the previous activity we found that for Earth, escape velocity is about 11 km/s (about 7 miles per second). Similar calculations lead to values that range from 1.2 km/s for Pluto to close to 60 km/s for Jupiter.

In order to escape the gravity of Earth, an object ejected from its surface must have at least as much kinetic energy as it has potential energy when sitting in the surface. Of course, it could have more kinetic energy. What would happen then? Well, even after moving outward an infinite distance, it would still have some kinetic energy left over. That means the object would still be moving. Under these circumstances we would say that its total energy, potential plus kinetic, is positive:

$$KE + PE > 0$$

On the other hand, if the kinetic energy is less than the potential energy, then the total energy is negative:

\nonumber \]KE+PE<0 \nonumber \]

The boundary case, where the total energy is zero, is the one we used to define escape velocity.

In general, though, objects (or systems of objects) that have negative total energy are considered gravitationally bound. Objects with zero or positive total energy are gravitationally unbound. We don't have to know anything about the details of the motions within a system to make these determinations.

We will explore this idea further in the following activity.

Energy of Binary Systems

Worked Example:

Consider the system consisting of Earth and the Sun: Earth orbits the Sun at a speed of about 30 km/s. Its distance from the Sun is 1.49598 X 10^{11} m. Earth's mass is 5.945 X 10^{24} kg, and the Sun's mass is 1.9884 X 10^{30} kg.

Using these numbers, we can find the total energy of the Earth-orbital system. We ignore the Moon's motion and Earth's spin for this exercise.

- The total energy is a combination of the kinetic and potential energy:
  $E_{tot} = KE + PE = ½M_E v_E^2 - GM_E M_s / r_E$
- First calculate the kinetic energy piece:

KE = (½) (5.945e24 kg) [(30 ~~km~~/s) (1000 m / ~~km~~)]^2 = 2.675e33 J

- Now calculate the potential energy piece:

  PE = -(6.67384e-11 N m^2 kg^{-2}) (5.945e24 kg) (1.9884e30 kg)/ (1.49598e11 m) = -5.273e33 J
- Adding these together we have

  E_{tot} = 2.675e33 J - 5.273e33 J = -2.598e33 J

As we expect, the total energy of Earth as it orbits the Sun, potential plus kinetic, is negative. This means Earth is in a bound orbit around the Sun. If this were not the case, Earth would not be bound to the Sun and would go flying off into space. Also, notice that the magnitude of the potential energy is roughly twice as large as the magnitude of the kinetic energy. This is typical for gravitationally bound dynamical systems.

Questions

A. Calculate the total energy of the Earth-Moon binary system using a similar procedure to that above:

B. Now consider a comet approaching the Sun

We have assumed so far that orbits are circular for simplicity’s sake. However, orbits are not usually circular. If they are bound, then orbits are generally ellipses, of which a circle is a special case. Two kinds of unbound orbits exist: parabolic and hyperbolic. But we will concentrate on the bound orbits for the moment. The following activity asks you to answer some questions about the energy of an object orbiting the Sun.

ORBITAL ENERGETICS

Figure A.7.5 shows a possible bound orbit. We are looking directly down upon the orbit from above, so this is not a perspective drawing of a circular orbit. Notice that the Sun is not in the center of the orbit, but off to one side at a point called a focus of the ellipse. The figure shows an orbiting body, perhaps a comet or an asteroid, at several points along its orbit.

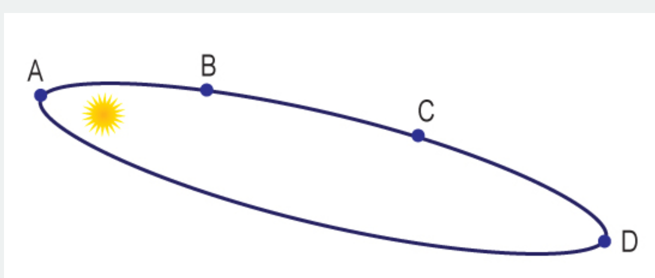

Figure A.7.5: Elliptical orbit of an object around the Sun. The orbiting object is shown at four different points in its orbit around the Sun. Credit: NASA/SSU/Aurore Simonnet

In Figure A.7.5, four different positions of the object are labeled A, B, C, and D.

Questions