

## 11.2: Spacetime Near Black Holes

### Learning Objectives

- You will understand that in general relativity black holes affect space and time.
- You will understand that outside observers never actually see anything cross the event horizon because of time dilation.
- You will understand that outside observers see a clock get redder, dimmer, and tick slower as it approaches a black hole.
- You will be able to describe “spaghettification” and compression that occurs when something falls into a black hole.
- You will understand that observers falling into a black hole do cross the event horizon and that time runs differently for them compared to outside observers.
- You will understand that material that falls into a black hole cannot get back out.
- You will understand that an object can orbit a black hole in a variety of ways without inevitably falling into the black hole.
- You will understand that it is only close to black holes that the GR effects become important; far from the event horizon Newtonian gravity is a good description.
- You will understand that while black holes are primarily described by their mass, including spin and charge provides a complete description of them.

### ? What Do You Think: The Shape of Black Holes



### 11.2.1: The Geometry Near Black Holes

In general relativity, the spacetime around stars or planets is described by a spherically symmetric spacetime interval. The spacetime interval tells us what the separation is between points in spacetime; it is analogous to finding the distance in space using the Pythagorean theorem in flat space, but including a time part, and accounting for the curvature of spacetime.

We can make the spherical symmetry more explicit by separating out the angular and radial parts of the spacetime interval, where we are using the fact that any point in space can be described by its distance from the origin in a particular direction. In Figure 11.1, we compare different types of coordinate systems (Cartesian:  $(x,y,z,t)$  vs. spherical:  $(d,\theta,\phi,t)$  ).

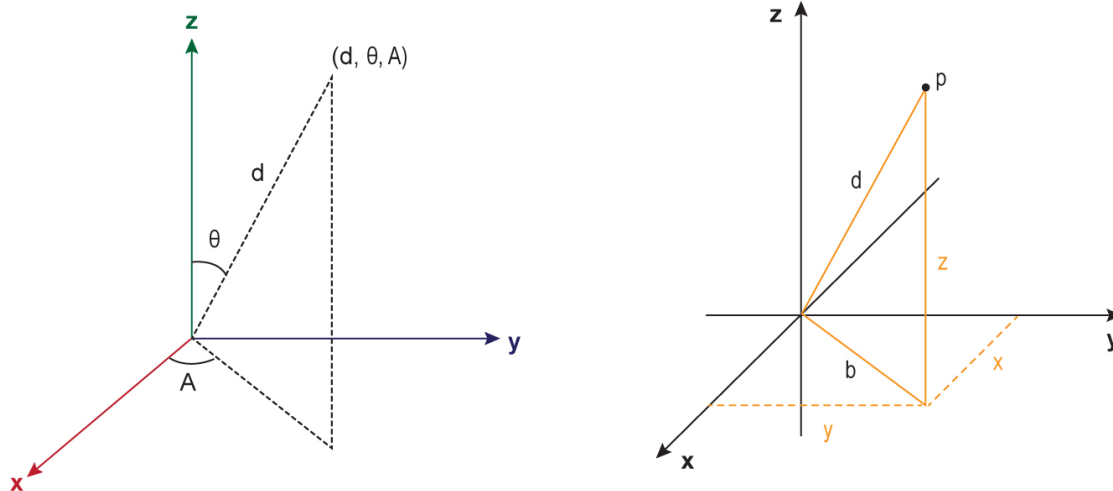


Figure 11.1: Points in space can be described by an xyz position, called Cartesian coordinates (after the French mathematician René Descartes), or by an  $d\theta A$  set of coordinates, as shown. The angles  $\theta$  and  $A$  are like longitude and latitude, respectively. However, since we are not restricting ourselves to the surface of a globe, as on Earth, we must also specify how far from the origin the point is located: that is what the coordinate  $d$  does. Credit: NASA/SSU/Aurore Simonnet

In spherical coordinates, the angle  $\theta$  is like latitude on Earth and the angle  $A$  is like longitude. For a spherically symmetric space (one that is the same in every direction) the orientation of the poles is arbitrary. Only the distance from the origin is important. Under this circumstance we can ignore the angular parts of the coordinates in spacetime because they don't tell us anything meaningful about the geometry of the space.

This means for a spherically symmetric spacetime (one in which the spatial part forms a spherically symmetric 3-space) we do not need to worry about any terms in the spacetime interval with  $\theta$  and  $A$ . They don't change when developing the spacetime interval from the 3-space interval. We can focus solely on the terms with the radial distance ( $d$ ) and time ( $t$ ), since those are the only things that affect the geometry. With these considerations, the spacetime interval ( $s^2$ ) around a spherically symmetric object is that stated below.

$$s^2 = \left(1 - \frac{2GM}{dc^2}\right)^{-1} (\Delta d)^2 - \left(1 - \frac{2GM}{dc^2}\right) (c\Delta t)^2 + r^2 \Delta\Omega^2$$

Here  $G$  is the universal gravitational constant,  $c$  is the speed of light,  $M$  is the mass of the object,  $d$  is the distance from the origin, and  $t$  is the time. The angular part is given by the last term, with which we do not have to concern ourselves; it is the same as for flat space and does not contribute to spacetime curvature.

The spacetime interval may look complicated, but the curvature depends only on the distance from the origin,  $d$ , and time,  $t$ . Those are the only terms where we find a difference from the flat space interval, the one used for special relativity. The curvature has no dependence on the direction considered. This is clear from the lack of modifications to the angular term in the expression for the interval. That is what is meant by spherically symmetric.

The interval describes the Schwarzschild geometry, discovered by the German astronomer Karl Schwarzschild (1873–1916) in 1915. It applies to any spherically symmetric gravitating system, including non-spinning black holes—we will have a look at spinning black holes later.



Figure 11.2: Karl Schwarzschild was a German astrophysicist. He was the first person to solve Einstein's equations for a spherically symmetric system. Credit: Bildarchiv Preussischer, Kulturbesitz, Berlin

An interesting historical aspect of Schwarzschild's discovery is that it was made during his service in the German army during World War I. Schwarzschild had volunteered for the military at the outbreak of the war in 1914. He was stationed on the Russian front as an artillery officer when he wrote his two papers on general relativity in the following year—the very year Einstein published the theory. The papers presented the solution we have presented, that for a spherical, nonrotating body.

Because Schwarzschild was occupied at the front, he was unable to present his papers at scientific meetings in Berlin. Instead, he sent copies of them to Einstein, who presented them in his absence. The papers were well received, and Einstein wrote to Schwarzschild that he had been surprised to see that the solution could be so simple.

While we now use Schwarzschild's solutions to describe the spacetime around black holes, Schwarzschild himself stated that he thought their description of objects whose escape speed exceeded that of light was mere mathematics, with no basis in reality—an opinion which Einstein also held. Schwarzschild never had the chance to explore the idea further. While on the Russian front, he contracted pemphigus, an autoimmune disease in which the immune system attacks the skin. He died of the condition in the spring of 1916, shortly after his relativity papers had been presented by Einstein.

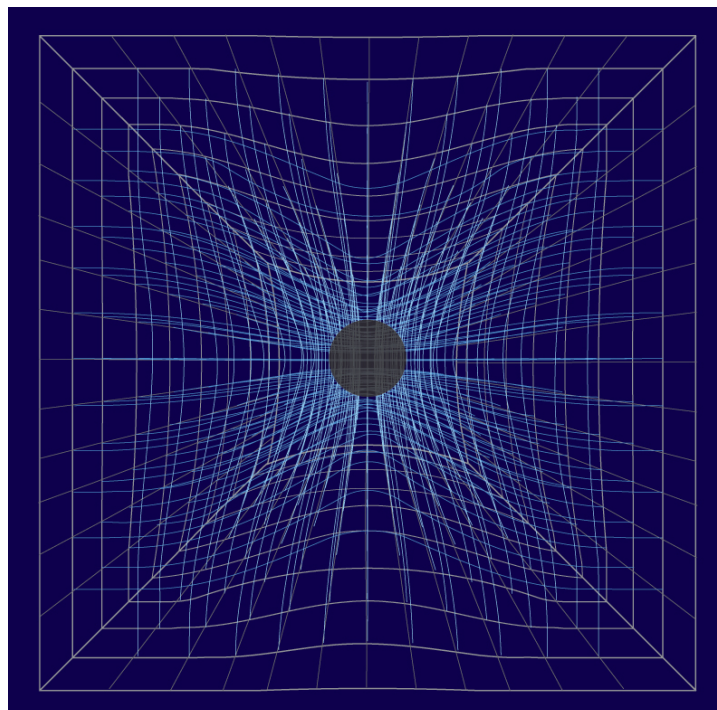


Figure 11.3: We can visualize the spatial part of the stretching of spacetime as shown in this diagram. The stretching occurs in all three spatial dimensions and is spherically symmetric. The time part of the distortion cannot be shown in this diagram, of course. Credit: NASA/SSU/Aurore Simonnet

The Schwarzschild spacetime interval applies to any spherically symmetric gravitating object. We can use it to study the orbits of planets around the Sun, and even the bending of light as it passes through the Sun's gravity. However, there is something a little peculiar about the Schwarzschild geometry. Notice the curvature terms in the large parentheses. They contain the ratio:

$$R_S = \frac{2GM}{c^2}$$

You might have noticed that this expression is the same one we derived in Section 11.1 for the size of a black hole. In the Newtonian derivation, it is the point where the escape speed becomes equal to the speed of light. This quantity has been given the name **Schwarzschild radius**,  $R_S$ , and it is quite important, as we will see. We can rewrite the Schwarzschild interval in terms of the Schwarzschild radius, and its importance will become more apparent.

$$s^2 = \left(1 - \frac{R_S}{d}\right)^{-1} (\Delta d)^2 - \left(1 - \frac{R_S}{d}\right) (c\Delta t)^2 + r^2 \Delta\Omega^2$$

Look at what happens to the curvature terms as the radial coordinate ( $d$ ) approaches  $R_S$ : The time curvature term goes to zero, but the spatial (radial) curvature does not. If  $R_S = d$ , then  $R_S/d = 1$  and  $1 - R_S/d = 1 - 1 = 0$ . In fact, since the radial curvature term is the reciprocal of the time curvature term, it must grow without bound as  $r$  approaches  $R_S$ , because  $1/0$  is infinite. What does this mean? Clearly something interesting is happening as we approach  $R_S$ .

### 11.2.2: Time Dilation

In the next activity, we will examine what happens as a clock moves closer to a black hole. The effects become particularly pronounced as it approaches the Schwarzschild radius.

#### ? Time Dilation

##### A. Predictions

First, predict what you think will happen:

1.

2.

3.

4.

### B. Observations

Now test your predictions by using the slider at the bottom to bring the clock closer and farther away from the Schwarzschild radii of a black hole.

[Play Activity](#)

1.

2.

3.

4.

In the last activity you saw what happens to an object falling into a black hole as seen from an observer far from the black hole. Several different effects were evident. First, the object was gravitationally redshifted as it fell deeper into the gravitational potential well, where the gravity is stronger. This made it shift in color to appear redder. The clock also became dimmer and time seemed to slow down. All of these effects are caused by the time dilation of an object's clocks as seen by an observer outside the region of strong gravity.

The slowing of clocks is very dramatic. So much so that as the black hole event horizon is reached, the clocks are seen to cease running altogether; according to an outside distant observer, time ceases to flow at the event horizon of a black hole. So, according to an outside observer, nothing ever quite crosses the event horizon. Instead, an object falling in slows and seems to freeze just as it reaches the event horizon. It also dims to the point of invisibility at the same time. So does anything ever actually fall into a black hole? Keep that question in mind as we undertake the journey from the perspective of the object that is falling.

### 11.2.3: Falling Into a Black Hole

#### ? Falling Into a Black Hole

In the previous activity, you discovered the effects of time dilation on an object falling into a black hole as seen from the point of view of an observer outside of the region of strong gravitational acceleration of the black hole.

In this activity, we will consider the viewpoint of an object falling in toward the black hole, in this case a circular ring of balls.

1.

2.

Now play the activity to observe what happens as a circular ring of balls falls into a black hole.

[Play Activity](#)

3.

If you are an intrepid space traveler who decides to take a trip into a black hole, the last activity should have convinced you that you do indeed fall all the way to the center in a finite time. You do not freeze above the event horizon, with clocks completely stopped. That is the viewpoint of someone far outside the black hole, but remember, they experience space and time quite differently from you. That is one of the fundamental results of relativity.

On the way to your doom, you experience some unpleasant side effects of the intense gravity near the black hole. One of these stems from the increasingly strong tidal effects you encounter. Tides, like ocean tides on Earth, are caused by the difference in the strength of gravity across an object: the side of Earth facing the Moon feels stronger gravity than the opposite side, and this difference, when added up over the entire surface of Earth, causes a bulge in the height of sea water that is nearest to (and farthest from) the Moon. The same thing happens to an object falling into a black hole, but on a much more dramatic scale.

Imagine yourself falling feet-first into the black hole. At first you will not register any difference in acceleration across your body. However, if a ring of free-falling balls surrounds you, you will see them begin to separate, as shown in Figure 11.4. This is because the balls nearest to the black hole experience a somewhat larger gravitational effect than the balls farthest from the black hole. As a result, they fall faster than the balls farther away. This differential gravitational effect (gravitational tide) is tiny if you are quite far from the black hole, but it becomes larger as you fall closer to the black hole. By the way, this is not any different for non-collapsed objects like Earth, the Moon, Sun and so on. That is why even the tiny little Moon can exert tidal effects on Earth.

There will eventually come a point where you begin to feel that your feet are being attracted much more strongly than your head toward the center of the black hole, tugging on you to separate your head from your toes. Farther in, the tides would grow large enough that you would begin to feel uncomfortable. As you fell deeper and deeper, the effect would grow to the point that it would begin to rip you apart, stretching you as if you were undergoing some sort of medieval torture. In fact, you would eventually be stretched to the point of breaking. Your feet would be falling so much faster than your head that your body could not overcome the tension; it would break. This would happen over and over again, on smaller and smaller scales. The remaining bits of you would be stretched and rendered into tinier bits, and those bits would in turn be ripped apart as the gravity on their side nearest to the black hole increasingly exceeded the gravity on their far side.

Eventually, even the molecules of which you are made would be ripped asunder, as would the atoms in those molecules. This process would continue to arbitrarily small scales as you approached the center of the black hole, where all matter would be destroyed by the intense gravitational tides.



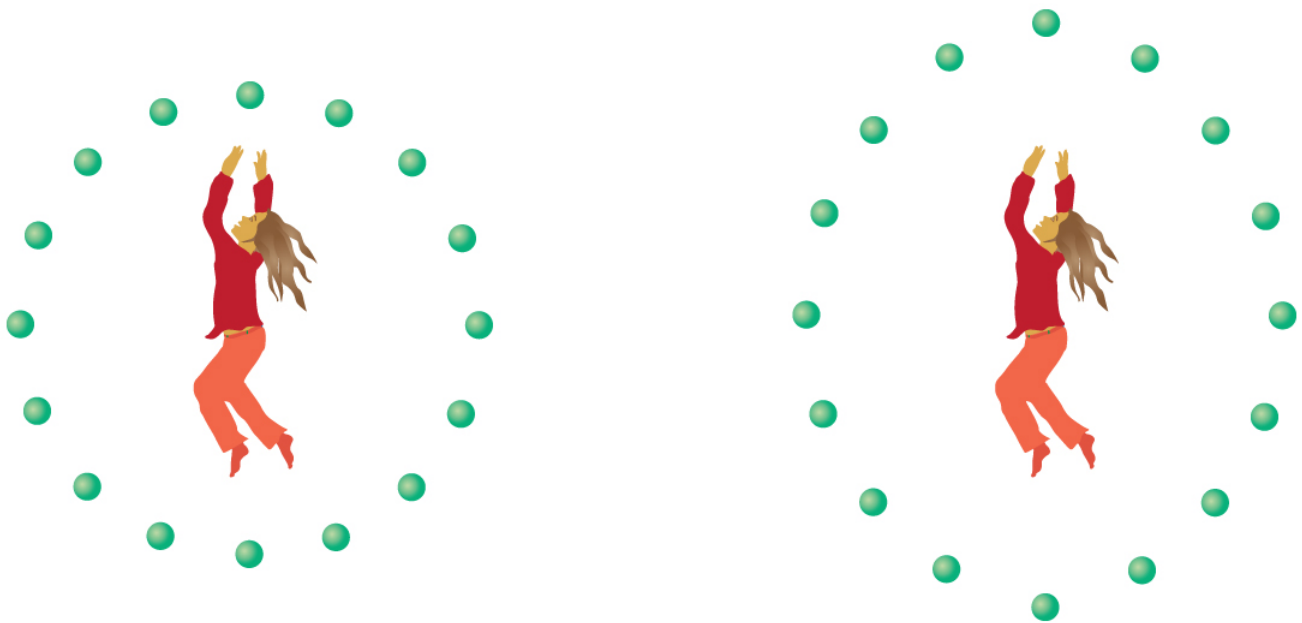


Figure 11.4: If you fell freely into a black hole while surrounded by a ring of freely falling balls, you would eventually begin to see the balls deviate from their initial circular arrangement. This is because of the growing tidal forces of the black hole. Eventually the tides would be strong enough to begin to rearrange you and your atoms in a manner similar to the rearrangement of the balls. Credit: NASA/SSU/Aurore Simonnet

Tidal stretching is not the only unpleasantness in store for anyone venturing into a black hole. Not only does the gravitational acceleration increase the closer you get to the center, but it changes direction as well. Gravity always points directly toward the center of the black hole, but that is different in different places. The gravitational acceleration on your left side is not exactly in the same direction as the acceleration on your right side. When you are far from the center of attraction, the two directions are nearly the same—consider the direction to Earth’s center from your left and right shoulders as an example. However, as you approach the center the difference in direction begins to make itself felt. The strong gravity begins to pull your shoulders inward toward your center. You are squeezed from the sides. This effect, too, just as the tides, grows stronger as you get closer to the center. Eventually you are crushed from the sides by the converging attraction of gravity.



Figure 11.5: If you were to fall feet-first into a black hole, the stronger gravitational force at your feet and weaker gravitational force at your head would stretch you out like a thin strand of spaghetti. Credit: NASA/SSU/Aurore Simonnet

This process of being crushed from the sides while being stretched from head to toe as you approach the center of a black hole has been given the fanciful name **spaghettification**, since you would tend to be drawn out into a long, thin noodle-like shape -

spaghetti - if you fell into a black hole. It is not a pleasant notion to contemplate.

### 11.2.4: Orbiting a Black Hole

A person need not suffer the dramatic (and fatal) effects encountered upon falling into a black hole to see the effects of general relativity. Even in the relatively much weaker gravity near the Sun, the orbit of Mercury is slightly different from what Newtonian gravity predicts it should be. Objects orbiting a black hole also have modified orbits, and much more profoundly so. We will explore some of these effects in the next activity.

#### Orbits Around a Black Hole

In this activity, you will use a simulation created by John Walker and presented by the HubbleSite team (NASA/STScI/eduweb) to explore the different possible types of orbits that are possible around a black hole.

Your challenge is to adjust the position and velocity of an object to create four different orbits. Describe what combinations of position and velocity are needed to achieve each type of orbit. For example, a close distance and a small velocity might cause an object to fall into a black hole.

NOTE: This activity does not currently work properly. We are aware and correcting the issue.

[Play Activity](#)

1.

2.

3.

4.

In the last activity, you studied how objects are able to orbit a black hole. There were some orbits that did not allow the object to continue orbiting forever; eventually it would cross the event horizon of the black hole. Once inside the horizon, no object can escape. On the other hand, you should have found that some orbits are stable. An object in those orbits can persist outside the black hole indefinitely, and it will not fall in. So black holes do not suck objects in. As you have seen with other gravitating bodies, depending on the combination of distance and velocity, an object can have an array of different orbits. Newtonian gravity and general relativity agree on this.

However, Newtonian gravity and general relativity make very different predictions for orbits when gravity becomes strong enough. In Newtonian gravity, all orbits are stable. Near a black hole, general relativity predicts that some of these orbits are not stable. While an object need not fall into a black hole, general relativity predicts that it will if it ventures too close, even if it does not initially cross the event horizon. But if it remains far enough away, an object can pass by the black hole. It can have an elliptical orbit, or it can have a more complicated orbital pattern.

Such possibilities are different from that predicted by Newtonian gravity, which cannot explain the complicated “spirograph” orbits, and which also allows any object to escape no matter how close it approaches the center of attraction. This is not at odds with our discussion of “dark stars” at the beginning of the chapter. In that discussion, we asked how small an object should be for its escape velocity to be the speed of light, we did not consider an object falling in from infinity. Such an object will escape from a classically conceived “dark star” because it will be able to travel faster than light. However, we know that relativity does not allow for that case: no object can travel faster than the speed of light, and so no object can emerge from the event horizon of a black hole.

Earlier we considered the possibility of replacing the Sun with a black hole and asked what the effect would be on Earth’s orbit. Are there any additional effects we must take into account from general relativity that will cause us to change our conclusion? In the case of weak gravity, as is the case far from the Sun in the Solar System, general relativity and Newtonian gravity must give the same results. After all, Newtonian physics works perfectly well in the outer part of the Solar System. If general relativity is correct then it must duplicate the results of Newtonian gravity in the areas where Newtonian physics works. As a result, our conclusion that Earth’s orbit would not change if the Sun were replaced with a black hole remains valid. In fact, for the limiting case in which gravity becomes weak, general relativity simply reduces to Newtonian gravity. Both theories give exactly the same results in that limit.

### 11.2.5: Black Hole Properties

Thus far, we have only considered the effect of mass on the spacetime near a black hole. That was the first solution found for the Einstein equations, as we have already discussed. But might there be other properties of a black hole that can affect the spacetime close to it? It turns out that there are only two.

The first is charge. You could imagine that if a black hole attracted a lot of electrons, for example, then it would eventually build up a large amount of negative charge, and this would create a strong electric field similar to the field created by rubbing a balloon on your head to create “static electricity.” However, the very fact that such a black hole would create a strong electric field means that charged black holes are not likely to exist. In our example, the electric field from all the negatively charged electrons would attract positively charged protons, and they would then fall into the black hole, neutralizing its charge. The same thing would happen if a lot of positively charged particles fell into the black hole: they would attract negative charges that would cancel the total charge. For this reason, we do not expect to observe any black holes in which electric charge plays an important part.

The only other property of a black hole that can affect the nearby spacetime is spin. Like a normal star, a black hole can rotate. In fact, we expect black holes to be rotating. We will discuss why this is the case in Section 11.3. But how does rotation affect the spacetime curvature? The answer, of course, is contained in the spacetime interval for a rotating black hole, first derived by Roy Kerr in 1963. This Kerr spacetime interval differs somewhat from the Schwarzschild interval we have discussed. First of all, it is

not spherically symmetric. Rotation picks out a preferred direction, namely that of the spin axis. Consider how the spinning Earth looks different from different directions (see Animated Figure 11.6): from above the poles you see one particular view, and that is completely different from the view you have above the equator or point in between.

However, if you change your position *around* the axis, the view does not change. We call this type of symmetry, where our view of space depends on our latitude but not on our longitude, azimuthal symmetry. Sometimes it is called cylindrical symmetry or longitudinal symmetry, but these usages are less common. Cylinders and disks both have azimuthal symmetry. So do spinning objects like Earth and spinning (Kerr) black holes, and the spacetime around them reflects this symmetry.

#### Play Animation

Figure 11.2.1: Animated Figure 11.6: Earth looks different depending on the point from which you view it. If you are directly above one of the poles, you see only one hemisphere, and it appears to spin around its center. If you view it from above the equator you will see the entire surface of Earth pass by, but you can only see one half of it at a time. If you are placed intermediate between the equator and one of the poles you will see a hybrid view: Some of the surface will always be visible, some never visible, and some will be visible part of the time, when it is facing your direction. So a spinning Earth is not spherically symmetric because it can look different to you depending on your viewing location. Credit: Animation - NASA/SSU/Kevin John

If we view a black hole that is spinning, several differences from a nonspinning black hole become immediately apparent. These differences are all described by the Kerr spacetime interval. However, with the addition of even something as simple as rotation, the interval becomes more complicated than we would like to deal with in these modules (books on introductory general relativity discuss the Kerr metric for those interested). Instead we will investigate some of the qualitative changes that occur when we add spin to a black hole.

One difference has to do with frame-dragging. For Earth, the effect of its rotation on orbiting satellites was described in [Going Further 10.5: Gravity Probe B](#). A similar effect happens near a spinning black hole, except that it is much stronger than for Earth. The effect of the dragging of frames near a spinning black hole becomes so strong that at some distance it becomes impossible for any object to remain at rest: it must rotate with the black hole. The region where this is true is called the ergosphere of the black hole. In Figure 11.7, you can see the ergosphere depicted; it extends out along the equator of the black hole and is smaller along the spin axis. The shape of the ergosphere is that of an oblate spheroid, the shape you would get if you compressed an inflated spherical ball from its sides. All spinning objects tend to have this shape. Earth and the Sun both do, for example. For slowly spinning things, the departure from pure sphericity is quite small. For spinning black holes the departure can be very large.

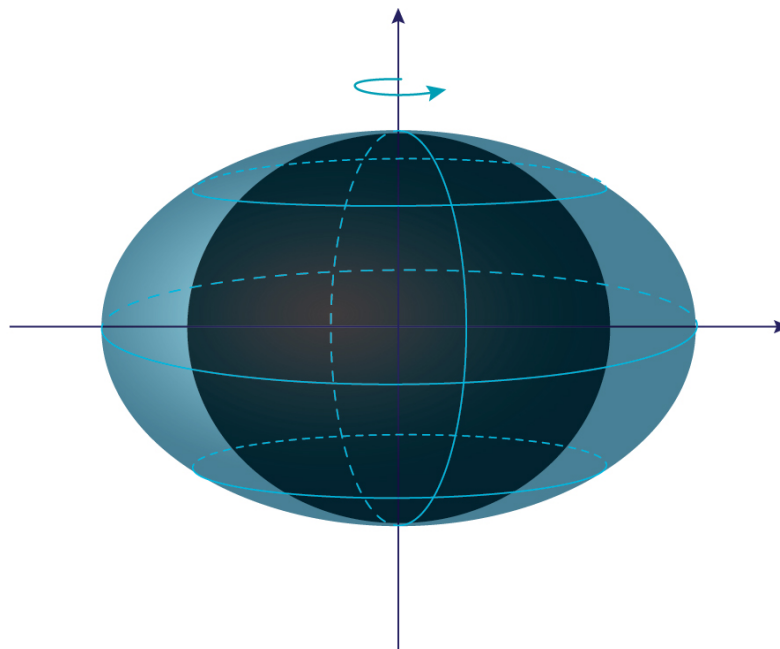


Figure 11.7: A spinning black hole has a different appearance from one that does not spin. It contains an ergosphere (depicted in blue), where spacetime is dragged around by the motion of the hole. Inside that region is a horizon, similar to the one for a nonspinning black hole, but with a slightly oblate shape. Credit: NASA/SSU/Aurore Simonnet

The black hole spin has a strong effect on the surrounding spacetime, and that in turn has a strong effect on the orbits of particles around the black hole. If we confine ourselves to orbits in the equatorial plane of the black hole, then orbits in which the particle moves with the spin of the hole are markedly different than orbits for which the particle moves opposite the spin.

For orbits that are not confined to the equatorial plane, the particle motions become complicated. Such orbits do not have to conserve angular momentum, for example. So a particle that approaches the black hole from some angle can be dragged by the spin of the black hole into an orbit with a completely different orientation. In fact, the effects of the spinning spacetime can cause an orbiting particle to circle the spin axis many times, while it makes only a single circuit from above the equator to below and then back again. To an outside observer such orbits look like a helix, as shown in Figure 11.8. The details of these orbits are not easily computed by simple computer exercises like the ones we are using in these modules.

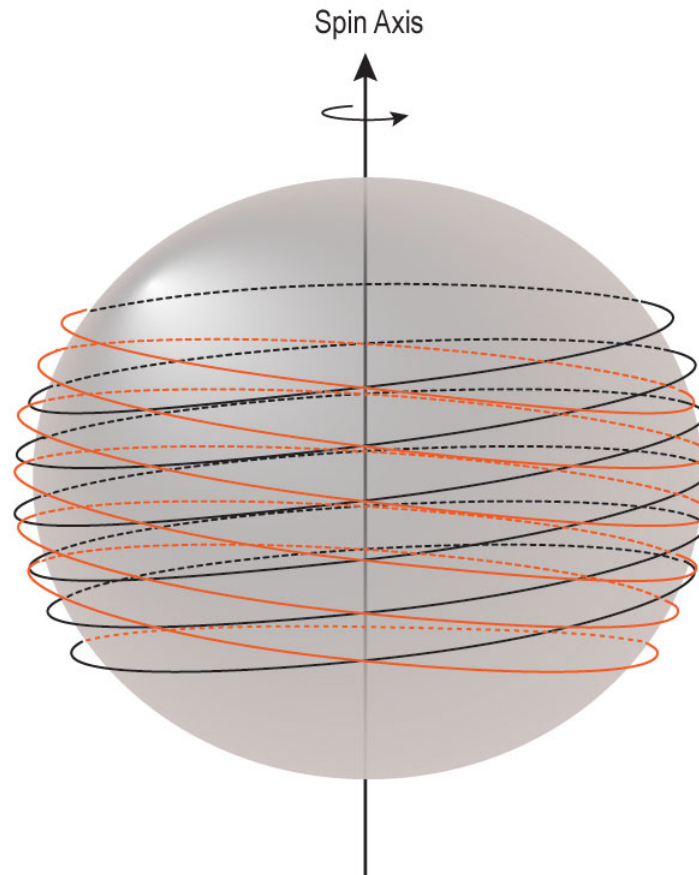


Figure 11.8: For orbits around a spinning black hole that are not confined to the equatorial plane, the orbits can appear quite complicated. The rotating spacetime near the hole can drag the particle around the spin axis much faster than its own motion would carry it. As a result, the orbit can appear to be a helix that circles the black hole as the particle moves up and down parallel to the spin axis. Credit: NASA/SSU/Aurore Simonnet

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