Resources: Formulae, Constants, and Conversions

Formulae	Relationship	Initial Chapter
$\lambda = c/f$	wavelength, frequency, and speed	Chapter 2
$E = hf$	energy and frequency	
$E_{photon} = \Delta E_{electron}$	energy of photon emitted/absorbed by atom	
$E_n = -13.6/n^2$	energy levels of atom	
$T = 2.9e\text{-}3/\lambda$	temperature and peak wavelength	
$F = \sigma T^4$	brightness and temperature	
$A = \pi r^2$	area and radius	Chapter 3
$\theta = 1.22\ \lambda /D$	resolution	
$v = d/t$	speed, distance, and time	Chapter 4
$z = \Delta\lambda/\lambda$	redshift	
$v = cz$	velocity and redshift	
$t = E/L$	stellar lifetime	Chapter 5
$E = mc^2$	energy and mass	
$t \sim m^{-2}$	stellar lifetime and mass	
$L \sim m^3$	stellar mass and luminosity	
$d = 1/a$	parallax angle and distance	Chapter 6
$d = S/\theta$	small angle formula	
$F = L/4\pi d^2$	inverse square law	
$F = ma$	Newton's second law	Chapter 7
$F_g = mg$	Weight and mass	
$a_c = v^2/r$	Centripetal acceleration	
$F_g = Gm_1m_2/r^2$	Newton's law of gravity	
$PE = mgh$	Potential energy (non-relativistic)	
$KE = \frac{1}{2}\ mv^2$	Kinetic energy (non-relativistic)	
$E_{final} = E_{initial}$	Conservation of energy	

Formulae	Relationship	Initial Chapter
$v_{escape} = (2GM/R)^{½}$	Escape velocity	
$v \propto r$	Rotation of rigid disk	Chapter 8
$v \propto 1/r$	Rotation of water around drain	
$v \propto$ constant	Rotation of cars on roundabout	
$v \propto 1/r^{½}$	Keplerian rotation (planets)	
$v^2 = GM/r$	Relationship of enclosed mass to velocity and distance	
$M = \rho V$	Mass, density, and volume	
$\gamma = 1/\sqrt{(1-v^2/c^2)}$	gamma factor	Chapter 9
$\Delta t' = \gamma \Delta t$	time dilation	
$L' = L/\gamma$	length contraction	
$d^2 = \Delta x^2 + \Delta y^2$	Pythagorean Theorem	
$s^2 = \Delta x^2 - c(\Delta t)^2$	spacetime interval	
$E = mc^2$	mass and rest energy	
$E = \gamma E_0$	total energy and rest energy	
$$g = \frac{GM}{R^2}$$	Surface gravity	Chapter 10
$$d = v_0 t + \frac{1}{2} a t^2$$	Distance and acceleration	
$$v = at$$	Velocity and acceleration	
$$t = \frac{t_0}{\left(1 - \frac{gH}{c^2}\right)}$$	Time dilation (weak field approximation)	
$$f = f_0 \left(1 - \frac{gH}{c^2}\right)$$	Gravitational redshift (weak field approximation, frequency, photon traveling upward)	

Formulae	Relationship	Initial Chapter
$$\lambda = \frac{\lambda_0}{\left(1 - \frac{gH}{c^2}\right)}$$	Gravitational redshift (weak field approximation, wavelength)	
$$f = f_0\sqrt{1 - \frac{2GM}{Rc^2}}$$	Gravitational redshift (full expression, frequency)	
$$\lambda = \frac{\lambda_0}{\sqrt{1 - \frac{2GM}{Rc^2}}}$$	Gravitational redshift (full expression, wavelength)	
$$d^2 = (\Delta x)^2 + (\Delta y)^2$$	Pythagorean theorem	
$$d^2 = (R\Delta\theta)^2 + \cos^2\theta(R\Delta\alpha)^2$$	Distance on a sphere	
$$d^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$$	Pythagorean Theorem in 3-D	
$$s^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - (c\Delta t)^2$$	Spacetime interval in flat space	
$$s^2 = \left(1 - \frac{2GM}{rc^2}\right)^{-1}\left[(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2\right] - \left(1 - \frac{2GM}{rc^2}\right)(c\Delta t)^2$$	Spacetime interval in spherically curved space	
$$\theta = \frac{2GM}{bc^2}$$	Angle of deflection of light	
$$P_{gw} = \frac{2}{5}\left(\frac{GM}{Rc^2}\right)^5\left(\frac{m}{M}\right)^2\left(\frac{c^5}{G}\right)$$	Power emitted by gravitational waves	
$$\mathbf{G} = 8\pi G\,\mathbf{T}/c^4$$	Einstein equation	

Formulae	Relationship	Initial Chapter
$$R_S = \frac{2GM}{c^2}$$	Schwarzschild radius	Chapter 11
$$s^2 = \left(1 - \frac{R_S}{d}\right)^{-1} (\Delta d)^2 - \left(1 - \frac{R_S}{d}\right) (c\Delta t)^2$$	Spacetime interval in a spherically symmetric space (Schwarzschild interval)	
$$T_{bh} = \frac{1.23 \times 10^{23}}{M}$$	Temperature of a black hole	
$$\Delta E \Delta t \geq \frac{h}{4\pi}$$	Uncertainty principle	
$$L = \frac{3.56 \times 10^{32}}{M^2}$$	Luminosity of a blackhole	
$$t \approx 2.5 \times 10^{-16} M^3$$	Evaporation time	
$$\frac{\Delta m}{\Delta t} = -\frac{2L}{c^2}$$	Accretion rate	
$$L_{edd} = 6.3 M_{BH}$$	Eddington luminosity	
$$\alpha = \frac{4GM}{bc^2}$$	Deflection angle (full)	Chapter 12
$$\theta_E = \sqrt{\left(\frac{4GM(b)}{c^2}\right)\left(\frac{D_{LS}}{D_{LO}D_{SO}}\right)}$$	Einstein radius	
$$\theta^2 - x\theta - \theta_E^2 = 0$$	Lens equation	

Formulae	Relationship	Initial Chapter
$$m = \frac{1}{\left[1 - \left(\frac{\theta_E}{\theta}\right)^4\right]}$$	Magnification for a point-mass lens	
$$v = H_0 d$$	Hubble law	Chapter 13
$$v = cz$$	Cosmological redshift	
$$d_{\text{physical}}(t) = d_{\text{comoving}}(t) S(t)$$	Comoving coordinates	
$$1 + z = \frac{S(t_{\text{observed}})}{S(t_{\text{emitted}})}$$	Ratio of scale factors	
$$t = \frac{1}{H_0}$$	Hubble time (age)	
$$H^2 - \frac{8\pi G\rho}{3} = -\frac{kc^2}{S^2}$$	Friedman equation	
$d = cz/H_0$	distance and redshift	Chapter 14
$T_e / T_o = 1 + z = S_o / S_e$	Temperature, redshift, and scale factor	Chapter 15
E ~ kT	energy and temperature	Chapter 16
T ~ mc^2/k	temperature of Universe and mass of particle in reaction	
Constants	**Name**	
c = 3 x 10^8 m/s = 3 x 10^5 km/s	speed of light	
h = 6.63 x 10^{-34} J s = 4.136e-15 eV s	Planck's constant	
G = 6.67 x 10^{-11} N m^2/kg^2	Universal gravitational constant	
k_B = 1.38 × 10^{-23} J/K	Boltzmann's constant	
t_{planck} ~ 10^{-43} s	Planck time	
t_{planck} ~ 4 × 10^{-35} m	Planck length	

Formulae	Relationship	Initial Chapter
$m_e = 9.1 \times 10^{-31}$ kg	mass of an electron	
Conversions	Units	
1 km = 1000 m	km and meters	
1 km = 0.6 mi	km and miles	
1 AU = 1.5 x 10^{11} m = 1.5 x 10^{8} km	AU and meters and km	
1 ly = 9.5 x 10^{15} m = 9.5 x 10^{12} km	light-years and meters and km	
1 ly = 6.3 x 10^{4} AU	light-years and AU	
1 eV = 1.6 x 10^{-19} J	eV and joules	
1 degree = 60 arcmin	degrees and arcminutes	
1 arcmin = 60 arcsec	arcminutes and arcseconds	
1 angstrom (Å) = 1 x 10^{-10} meters	angstroms and meters	
1 pc = 3.26 ly	parsecs and light-years	
1 N = 0.2248 pounds	newtons and pounds	
1 kpc = 3.086 x 10^{19} m	kiloparsecs and meters	
1 solar mass = 2 × 10^{30} kg	solar masses and kg	
1 Mpc = 3.09 x 10^{22} m	megaparsecs and meters	
1 radian = 2.06 x 10^{5} arcsecond	radians and arcsec	
1 Mpc = 3.09 × 10^{19} km	megaparsecs and km	
Units (abbreviation)	Type of quantity	
meters (m)	length (SI)	
kilograms (kg)	mass (SI)	
second (s)	time (SI)	
meters per second (m/s)	speed (SI)	
kelvin (K)	temperature (SI)	
miles (mi)	length	
astronomical unit (AU)	length	
year (yr)	time	
light-year (ly)	length	
light-minutes	length	
light-seconds	length	
g/cm^3	density	

Formulae	Relationship	Initial Chapter
solar masses	mass	
hertz (Hz) = cycles/s = 1/s = s^{-1}	frequency (SI)	
joules (J)	energy (SI)	
electron volts (eV)	energy	
watts (W) = J/s	power	
radians	angle	
degrees	angle	
arcmin	angle	
arcsec	angle	
angstrom (Å)	length	
parsec (pc)	length	
m/s^2	acceleration (SI)	
newton (N) = kg m/s^2	force (SI)	
joules (J) = N m	energy (SI)	
μK micro Kelvin = 10^{-6} K	temperature	

Prefix	Meaning (in USA)	Exponent	Symbol
Tera	trillion	10^{12}	T
Giga	billion	10^{9}	G
Mega	million	10^{6}	M
kilo	thousand	10^{3}	k
centi	one-hundredth	10^{-2}	c
milli	one-thousandth	10^{-3}	m
micro	one-millionth	10^{-6}	μ
nano	one-billionth	10^{-9}	n
pico	one-trillionth	10^{-12}	p