11.1: What Are Black Holes?

Learning Objectives

- Explain the following laws within the Ideal Gas Law

Black Holes in Galaxies

The modern notion of a black hole has its origins in general relativity. In fact, the term "black hole" was itself coined by the American physicist John Archibald Wheeler (1911–2006), one of that theory's greatest proponents. Wheeler and his students are largely responsible for the great advances in general relativity that took place after World War II. However, in its simplest form, the idea of black holes predates these advances by almost two centuries.

Toward the end of the 18th century, two scientists had independently considered the possibility of an object whose gravity was so strong that light would be unable to escape it—the essence of what we mean by "black hole." One of these scientists was John Michell (1725–1793), in England. Michell was a colleague of the much more widely known Henry Cavendish (1731–1810), famous for the "Cavendish experiment" to measure the gravitational constant , G. In fact, Cavendish used an experimental apparatus devised by Michell for his classic experiment. In 1784 Michell sent a letter to Cavendish in which he speculated about an object whose escape velocity might exceed the speed of light:

> *"...if the semi-diameter of a sphere of the same density with the Sun were to exceed that of the Sun in the proportion 500 to 1, a body falling from an infinite height towards it, would have acquired at its surface a greater velocity than that of light, and consequently, supposing light to be attracted by the same force in proportion to its vis inertiae, with other bodies, all light emitted from such a body would be made to return towards it, by its own proper gravity."*

This is the earliest known discussion of such an object. At around the same time, the French scientist Pierre-Simon LaPlace was entertaining similar ideas. He published them in a book, which he called *Exposition du Systeme du Monde*, in 1796.

Both Michell and LaPlace were well versed in the Newtonian theory of gravity, and they knew about the concept of escape velocity. The escape velocity from an object of mass M and radius R is given below.

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

If we set the escape velocity to the speed of light (c) and rearrange the expression, we find that the size (R) of such an object - one with $v_{esc} = c$ - given that it has a mass M, is the following.

$$R = \frac{2GM}{c^2}$$

This is the maximum size R an object of mass M must have if its escape velocity is to equal the speed of light. Of course, if the object is smaller, then light still cannot escape. Also, the size of these objects scales linearly with mass, so if we double the mass of an object we double the minimum size needed for it to trap light. Any object whose size is less than or equal to that given by the previous expression will be a so-called "dark star," as Michell referred to them. No light will be able to leave its surface, so it will be completely dark. The notion of a "dark star" (as opposed to black hole) is purely Newtonian. We have not made any reference to general relativity at all yet. However, in section 11.2, we will see that the characteristic size of a black hole, as determined by general relativity, is described by this same expression.

Size of Black Holes

Worked Example:

1. What is the size of a black hole with the same mass as the Sun, 2×10^{30} kg?

- Given: $M = 2 \times 10^{30}\ \mathrm{kg}$
- Find: R, the radius in meters
- Concept(s): R = 2GM/c^2 where the gravitational constant, \(G = 6.67\times 10^-11}\, {\rm Nm^2/kg^2,}\) and the speed of light, $c = 3 \times 10^8\ \mathrm{m/s}$.
- Solution: $R = (2)(6.67 \times 10^{-11}\ \mathrm{Nm^2/kg^2})(2 \times 10^{30}\ \mathrm{kg})/(3 \times 10^8\ \mathrm{m/s})^2 = 2.95 \times 10^3\ \mathrm{m}$

For an object with the Sun's mass to be a black hole, or in the Newtonian view a "dark star," such that light is unable to escape it, it would have to have to be about 6 km across - or less.

Questions:

1.

2.

What if the Sun Were Replaced With a Black Hole?

Our discussion thus far has only considered Newtonian gravitation, and so we should expect that these objects behave gravitationally just as other objects do in Newtonian gravity.

1.

2.

3.

At the time of Michell and LaPlace, most scientists thought of light as being composed of waves, not particles—though Michell was thinking in terms of particles. No one could understand how gravity would be able to affect the motion of waves, and there was no good understanding of the equivalence principle. As a result, nobody really took the ideas of “dark stars” seriously. It turns out the same was true for black holes after general relativity was published, including with Einstein himself. Well, at least at first.

We have deduced the size of black holes (“dark stars” in the Newtonian view) by setting an object’s escape velocity to be the speed of light. The Newtonian intuitions gained this way can be helpful to a certain point; however, the Newtonian form of gravity is not the proper one to use when thinking about black holes. It leaves out much of what we know to be true about gravity. To get a full picture of what black holes are like, we should use the ideas of general relativity. That is what we do in the next section.
