Math Exploration 13.1

The merging of two neutron stars produces a spectacular explosion called a **gamma-ray burst**. The annihilation of electrons and positrons causes the emission of an enormous amount of high energy radiation at 511 keV (a wavelength of 0.00242 nm). One of the most distant gamma-ray bursts has occurred at a redshift of $z = 4.5$.

a. At what wavelength would we observe the emission line now?

- **Given**: $z = 4.5$ and $\lambda_{emitted} = 0.00242\,\mathrm{nm}$
- **Find**: $\lambda_{observed}$
- **Concept**: The observed and emitted wavelengths are related by:

$$z = \frac{\lambda_{observed} - \lambda_{emitted}}{\lambda_{emitted}}$$

or equivalently

$$\frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} = 1 + z$$

- **Solution**:

$$\begin{aligned} \lambda_{\text{observed}} &= 1 + z(\lambda_{\text{emitted}}) \\ &= (1 + 4.5)(0.00242\,nm) \\ &= (5.5)(0.00242\,nm) \\ &= 0.0133\,nm \end{aligned}$$

b. By what factor has the Universe stretched since the neutron stars merged and emitted the light that we are just seeing now?

- **Given**: $z = 4.5$
- **Find**: The ratio $S(t_{\text{observed}})/S(t_{\text{emitted}})$
- **Concept**:

$$1 + z = \frac{S(t_{\text{observed})}}{S(t_{\text{emitted})}}$$

- **Solution**:

$$\frac{S(t_{\text{observed}})}{S(t_{\text{emitted}})} = 1 + 4.5 = 5.5$$

- The Universe has expanded by a factor of 5.5 since the light we are seeing now was emitted from Galaxy A.