7.4: Gravity and Orbits

What Do You Think: Orbiting Planets

As you saw in the video that starts the chapter, astronauts in the Space Station are seemingly weightless. We will now explore this phenomenon.

The International Space Station orbits Earth at an altitude of about 380 km. We can calculate the gravitational acceleration at this altitude using Newton's law for the gravitational acceleration: $g = GM_E / r^2$. In this case, the distance r from the center of Earth is the sum of the ISS altitude and Earth's radius.

g = (6.67384 x 10^{-11} N m^2 kg^{-2}) (5.972 x 10^{24} kg) / (6.368 x 10^6 m + 3.80 x 10^5 m $)^2$

g = 8.75 m/s^2

This is almost as large as the acceleration at Earth's surface, and certainly not equal to zero. So why then, do the astronauts appear weightless?

The answer to that question has to do with how orbits work. The astronauts appear weightless because they are in free fall. That means they are falling freely toward Earth. It is a bit like when sky divers jump out of an airplane, except in the case of sky divers the air eventually pushes upward on them as hard as their weight pulls them down, so they are not really falling freely. You should be able to understand why this is if you have ever stuck your hand out of the window of a car moving at high speed. Air exerts a strong drag force on any object moving through it at high speed. This is also similar to the feather example we discussed toward the beginning of this chapter.

In contrast, astronauts do not encounter any drag force, first because they are inside a container, and second, that container is high above most of Earth's atmosphere. Astronauts, and the satellites they inhabit, fall freely toward Earth under the influence of gravity and nothing else. But if astronauts are falling toward Earth, shouldn't they eventually hit it? No, it turns out.

You might be aware that the ISS orbits Earth approximately once every 90 minutes. But it is only a couple hundred miles up, so that means it goes around Earth at an incredibly high speed. In fact, it is moving so fast that, even though it is constantly falling toward Earth, its sideways motion causes it to "miss." On the other hand, it cannot get away from Earth because the gravitational interaction between the space station and Earth bends the path of the ISS, keeping in a circle. It is constantly falling toward Earth, and constantly missing. That is what an orbit is, whether it is the orbit of a satellite (including the Moon) around Earth or the orbit of Earth around the Sun.

Without the gravitational interaction between orbiting bodies, their relative speed would very rapidly separate them. The ISS would go flying off into space, Earth and the other planets would go flying off into space, etc. But the balance between their relative motion and the acceleration caused by their gravitational interaction keeps orbiting objects bound to each other, constantly falling toward each other and constantly missing.

Orbiting Around Planets

Worked Example:

Imagine you could orbit just above Earth's surface. What would your orbital speed have to be, and how long would it take you to go around Earth once? We will assume that Earth has no atmosphere, so we can ignore drag forces. We will also assume that there are no mountains for us to run into; Earth is perfectly smooth for this example.

We know that the gravitational acceleration at Earth's surface is 9.8 m/s^2. For us to be in orbit this would also have to be our centripetal acceleration, just as in our previous examples. Setting these two equal we have:

$$a_c = g \tag{7.4.1}$$

$$\frac{v^2}{R_E} = 9.8m/s^2$$

Solving for the velocity, v:

v = [(9.8 m/s^2) (6.368e6 m)]$^{1/2}$ = 7900 m/s = 7.9 km/s

Using this velocity, we can find the orbital period, the time required to circle Earth once: It is the Earth's circumference divided by this speed.

$T = C / v = 2\pi R_E$ / v = (2π) (6368 ~~km~~) / (7.9 ~~km~~/s) = 5065 s

Converting this to minutes we get

T = (5065 ~~s~~) / (60 ~~s~~ / min) = 84 min

Around the World in 80 Minutes! That is fast. It would be the time required to orbit once around Earth if you could orbit just above the surface. Notice that this time is just a little bit shorter than the time required for the ISS, reflecting the somewhat higher gravity at the surface (and therefore the higher orbital velocity required), as well as the slightly shorter circumference of the proposed orbit. But if you could run 8 km/s (about 5 miles per second), on a perfectly smooth and airless Earth, you could put yourself into orbit just above the ground! You would be moving so fast that, as you fell toward Earth, the curvature of the surface would cause the ground to get out of your way, barely.

Questions

Jupiter's moon Io has an approximately circular orbit at a radius of 4.2 x 10^8 m from Jupiter's center. Jupiter's mass is 1.9 x 10^{27} kg.

Orbit Simulator

In this activity you will experiment with different combinations of velocities and vary the distance between two objects in order to see what it takes to get into a stable orbit. The orange circle within the simulator represents the Sun. The blue circle represents Earth. When the interactive is started, the distance (given as 1 AU) is scaled to be that between Earth and the Sun (but you can change this parameter). Time has been sped up so that it is about one month per second for the initial orbit.

Play Activity

You might be asking a question after the last few examples: How do we know the mass of Earth? After all, we cannot put Earth on a scale, measure its weight, and then convert that to a mass using the second law. In fact, the way we are able to determine the mass of Earth, the Moon, the Sun, and many other objects is by using Newton's law of gravitation. If we know the gravitational acceleration of an orbiting body, we can use it to determine the mass of the object causing that acceleration.

DETERMINING MASSES OF ASTRONOMICAL OBJECTS

Worked Example:

In an earlier activity, we found the centripetal acceleration, *a*, of the Moon to be 2.685 x 10^{-3} m/s^2. Using this value, we can find the mass of Earth.

In that earlier activity, we used a combination of Newton's law of gravitation and Newton's second law to derive an expression for the Moon's acceleration:

$$a = G\frac{M_E}{r_M^2}$$

where r_M is the distance between Earth and the moon, M_E is the mass of Earth, and a is the acceleration of the moon. Solving our expression for the mass of Earth, we have

$$M_E = \frac{ar_M^2}{G}$$

Now we can plug in the appropriate numbers to find Earth's mass. We are using the distance from Earth to the Moon, r_M, because we are using the value of the gravitational acceleration felt at the Moon's orbital distance.

M_E = (2.685e-3 m/s^2) (3.844e8 m)^2 / (6.67384e-11 N m^2 kg^{-2}) = 5.945e24 kg

This is within 1% of the value we used previously. We could find Earth's mass in several other similar ways. We could use the measured value of the surface gravity and use that to find Earth's mass. Or we could use one of the many artificial satellites that have been launched into orbit above Earth. Any of these methods would allow us to determine Earth's mass.

Question

This activity demonstrates how the masses for all objects in the Universe are determined. Newton's universal law of gravitation, as its name implies, is assumed to be universally applicable. It is therefore used to determine the mass of stars, planets, galaxies and galaxy clusters.

Newton's universal law of gravitation makes a very strong prediction for the motions of the planets in the Solar System -- namely, that their centripetal accelerations should diminish as the square of their distance from the Sun. So Saturn, which is nearly 10 times farther from the Sun than is the Earth, should have a centripetal acceleration only about 1% as large as Earth's. On the other hand, Mercury, at just under half the distance from the Sun as Earth, should have about four times larger centripetal acceleration than Earth does. Remember that centripetal acceleration depends on both the orbital speed of the planet and its orbital radius. If we set the centripetal acceleration of a planet equal to its gravitational acceleration, we will have a relationship between its orbital speed and its distance from the Sun.

$$\frac{v^2}{r} = \frac{GM_S}{r^2}$$

We can cancel one factor of r (the distance from planet to Sun) from each side of this equation and then take the square root, which gives us an expression for the velocity of a planet as a function of its distance from the Sun.

$$v^2 = \frac{GM_S}{r}$$

$$v = \sqrt{\frac{GM_S}{r}}$$

G and M_S are the same for all the planets in the solar system, so only r is changing from one planet to the next. This means that planets orbiting the Sun (or any central massive object like the Sun) should have orbital velocities that decrease with the *square root* of their distance from that object. This relationship is plotted for our Solar System in Figure 7.7 below. A plot of velocity vs. distance is known as a rotation curve.

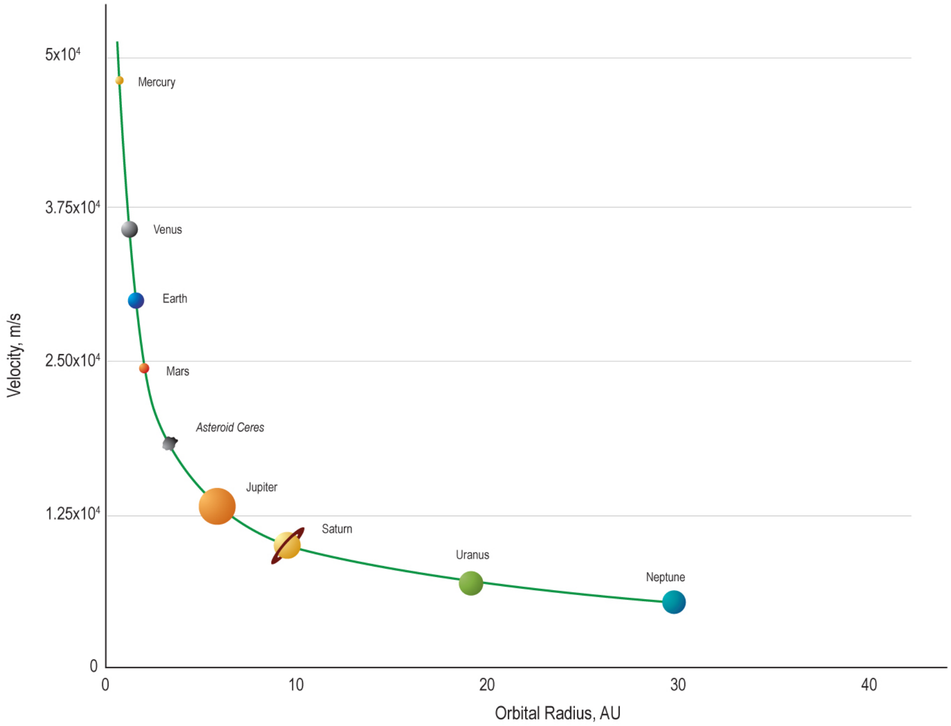

Figure 7.7: The velocities of the planets are plotted as a function of their distance. The rotation curve illustrates that the orbital velocities decrease with the *square root* of their distance from the Sun. Note: the sizes of the planets are not to scale. Credit:NASA/SSU/Aurore Simonnet

All planets in our Solar System follow this relation, but stars in galaxies do not. They follow a different relation that is specific to the details of those systems. However, it is still derived from the laws of gravity.

We can substitute an expression with time for velocity in our previous expressions in order to examine how the orbital time period of planets varies with distance:

$$v=\frac{C}{T}=\frac{2\pi r}{T}=\sqrt{\frac{GM}{r}}$$

where C is the circumference of the orbit, T is the period of the orbit, and M is the mass of the object that is being orbited.

Squaring both sides:

$$\frac{4\pi^2 r^2}{T^2}=\frac{GM}{r}$$

and gathering terms on each side:

$$4\pi^2 r^3 = GMT^2$$

or more simply:

$$r^3=\kappa T^2$$

where the constant $\kappa = GM/4\pi^2$. This means that as the orbital distance (r) of a planet *increases*, the orbital period (T) *increases*. Notice that this expression only depends on the distance to a planet, not its mass. Again, this relationship is true for planets in our Solar System, and the constant κ depends on the mass of the Sun. In other systems, κ will depend on the mass of the object being orbited in that system.

This relationship between the period of an orbit, *T*, and its orbital radius, *r*, in which the square of the orbital period is proportional to the cube of the orbital radius, was discovered by the German astronomer Johannes Kepler (1571 - 1630). For this discovery, Kepler used observational data of planetary motions that had been collected by the Danish astronomer Tycho Brahe (1546 - 1601). Kepler's derivation of this relation (now called Kepler’s third law of planetary motion) was entirely based on these observations. It did not use any theoretical understanding of gravity or motion; at the time there was none.

Kepler also discovered that planetary orbits are ellipses, not perfect circles (his first law of planetary motion), and that planets sweep out equal areas in their orbits in equal times (his second law of planetary motion). All of these laws are predictions of Newton's gravitational theory.

Kepler had no knowledge or understanding of the law of gravity because he died more than a decade before Newton was born. His laws were based solely on mathematical fits to observational data. Just as with Galileo's measurement of Earth's surface gravity, Newton's laws provide a theoretical basis for and a unification of Kepler's observational results. Of course, the ability of Newton's laws to explain earlier measurements strongly encouraged Newton and his contemporaries to recognize the validity of his work.

We should note that much of our analysis has been simplified by assuming circular orbits, with the central object not moving. In fact, orbits are elliptical -- or hyperbolic or parabolic, in some cases. The results we have derived still work for the general case of ellipses and other shapes. However a greater level of mathematical sophistication is needed to handle more general orbital shapes. If you wish to see the general treatment of orbits you should take an advanced course in physics.

Finally, it is not quite correct to assume that Earth orbits the Sun, or that the Moon orbits Earth. Instead, we should say that both Earth and the Sun orbit a point between them, as do Earth and the Moon. Recall that the force between Earth and the Sun exerts the same pull on both; Earth pulls on the Sun and the Sun pulls back on Earth just as hard. However, because the Sun is much more massive than Earth, this point for the Earth-Sun interaction is almost at the center of the Sun. Likewise, for the Earth-Moon system it is well within the volume of Earth. But in gravitational interactions, both bodies do move, the more massive one simply moves a lot less. One technique for finding planets around other stars relies on this fact. It uses the tiny back and forth motion of distant stars, evident from a small back and forth shift of their stellar spectral lines, to detect the planets orbiting them.

Orbital Period, Distance, and Velocity

In this activity, you will use the orbit simulator again, but this time to explore how orbital distance affects orbital period and speed.

Play Activity

Keplerian Motion

In this activity, we will use $r^3 = \kappa T^2$ with $\kappa = GM_E/4\pi^2 = 1.01 \times 10^{13}\ m^3/s^2$ to solve problems involving the orbits of satellites around Earth. Using this notation, distance to the satellite from Earth's center (r) should be in meters and the period should be in seconds.

Worked Example:

1. Communications satellites are often put into geosynchronous orbits. This means that from Earth's surface, they appear to hover over the same spot on the ground. A geosynchronous satellite over a location on Earth's equator would therefore have an orbital period of 24 hours. Calculate its height above Earth's surface in meters.

Answer:

- Given: T = 24 ~~hours~~ x (3600 seconds/~~hour~~) = 8.64e4 s
- Find: the distance above Earth's surface
- Concept: $r^3 = \kappa\ T^2$
- Solution: r^3 = (1.01e13 m^3/s^2)(8.64e4 s)^2 = 7.54e22 m^3. Taking the cube root gives us r = 4.22e7 m.
- Note that the distance (r) includes Earth's radius (R_E), so in order to figure out the height above the Earth's surface we must subtract R_E : Height = r - R_E = 4.22e7 m – 6.4 e6 m = 3.6 e7 m

A schematic of the distances is shown in Figure A.7.4.

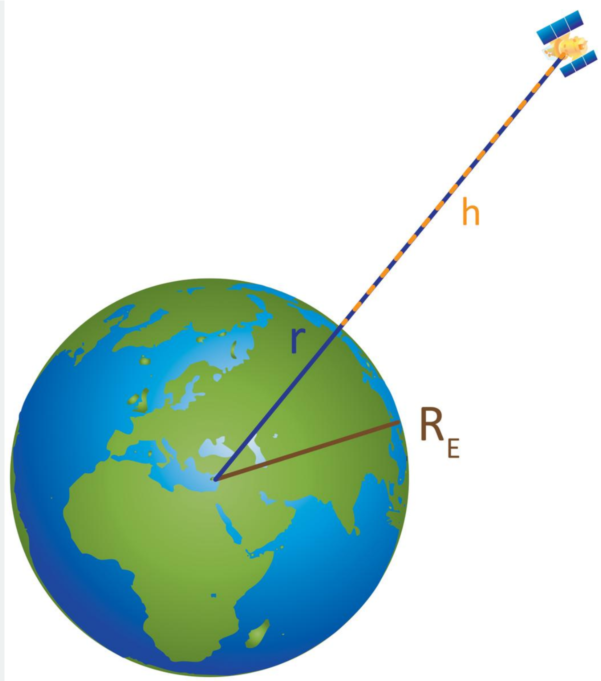

Figure A.7.4: The distance (r) between the center of the Earth and a satellite is a combination of the height of the satellite above Earth's surface (h) and the radius of the Earth (R_E). Credit: NASA/SSU/Aurore Simonnet

Question