

7.3: Gravity Is a Universal Force

? What Do You Think: Gravity on the International Space Station



Thus far we have looked at two aspects of gravity. The first is the acceleration due to gravity near the surface of Earth. We know that gravity makes everything fall at the same rate. The second aspect we have considered is the force of gravity, again near the surface of Earth; this force is also called weight. We can now explain to some extent how objects move under the influence of gravity, but we do not really know why they move the way they do. More particularly, we have not learned anything about where gravity itself comes from. Isaac Newton had the first insight into that aspect of gravity.

7.3.1: Newton's Universal Law of Gravitation

One legend suggests that Newton's epiphany on gravity came as he was sitting under an apple tree on the Cambridge University campus where he was a professor. As he sat staring at the Moon, pondering what made it go around Earth, suddenly an apple is supposed to have fallen from the tree onto his head, causing him to exclaim "Eureka!" This story is probably untrue, but it does accurately relate the essence of Newton's insight into gravity: The same force that causes apples to fall to the ground, and keeps people firmly on the ground, also causes the Moon to circle Earth (and Earth to circle the Sun). This was the vital connection to make.

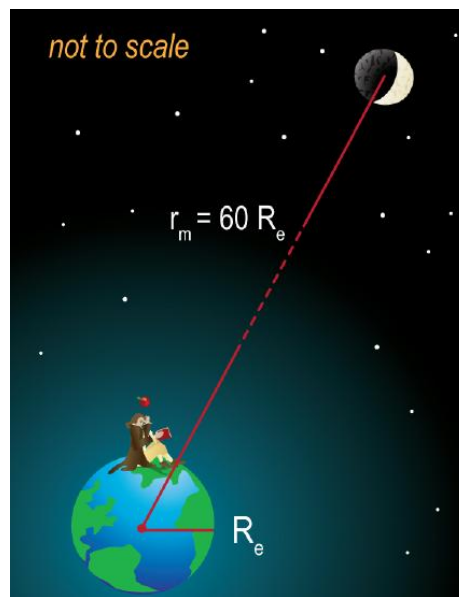


Figure 7.4: Isaac Newton had an insight that the same force that causes an apple to fall to the ground must also keep the Moon in its circular path about Earth. He used this insight and his new laws of motion to deduce the Law of Universal Gravitation. Credit: NASA/SSU/Aurore Simonnet

Newton knew that the Moon circled Earth in 27.5 days. (This is the time required to go 360 degrees around Earth, not the time from new moon to new moon.) Newton also knew that the Moon's orbit around Earth is roughly circular, and that the orbital radius of the Moon is about 60 times larger than Earth's own radius.

To keep an object moving in a circular path, it must be accelerated. Why? Because as it travels along its path, the direction of its velocity is constantly changing, and changing velocity implies an acceleration. If we consider such motion, we realize that a faster object has a larger acceleration than a slower one. Its velocity must change more quickly than the slower object. Furthermore, if an object travels on a very big circle, say like the orbit of the Moon, its acceleration will be smaller than if it travels at the same speed on a smaller circle. The equation that describes the acceleration for circular motion is:

$$a_c = \frac{v^2}{r}$$

where a_c is the acceleration, v is the velocity, and r is the distance between the object and the center of the circle.

With this equation, we see that a larger velocity causes a larger acceleration, while a larger radius for the motion causes a smaller acceleration. You might be surprised that the velocity is squared, but that is how this type of circular motion works.

We know from our earlier discussions that acceleration is a vector and has both size and direction. What is the direction of this acceleration? It is not obvious from this expression, but it turns out to be pointed along the radius of the circle, directly toward its center. For this reason, acceleration of this type is called centripetal (center-seeking) acceleration, hence the subscript "c." It is sensible that the acceleration should point to the center if you recall that the acceleration is just the change in velocity over time. For circular motion like this, where the only thing changing is the direction of the velocity vector, the vector is constantly turning around toward the center of the circle. The velocity vector never turns all the way toward the center of the circle, of course. It continues to point in a direction tangent to the circular path, and the moving object travels farther along the circumference of the circle. See Figure 7.5 for an animated representation of this situation.

Play Animation

Animated Figure 7.5: Object moving in a circular path. This object is moving in a circular path with a constant speed, but a constantly changing direction. Since the direction of its motion is changing, the velocity of the object must also be changing. Thus, there must be an acceleration that points in the direction of its change in motion. This acceleration points toward the center of the circular path, as the animation shows. Credit: NASA/SSU/Kevin John

If you know the formula for centripetal acceleration, as Newton did, it is fairly easy to compute the value of the acceleration in a given situation. For the Moon and Earth, you just plug in the Moon's orbital speed and its distance from Earth. We can use the

Moon's orbital size and period to find the Moon's speed, and then its acceleration, as we show in the next activity. Then you will have a chance to do a similar calculation for Earth's orbit around the Sun.

Worked Example: Calculating Orbital Accelerations

Find the centripetal acceleration of the Moon. The radius of the Moon's orbit has long been known from observations. In modern units it is 384,400 km, or 3.844×10^8 meters.

Concept: in order to use the formula $a_c = v^2/r$, we are given r , but we will still need to find the orbital velocity of the Moon, v . We can get this from the circumference around its orbital path and the time period that it takes to go once around.

Step 1. In order to derive the Moon's centripetal acceleration, we first need to find the circumference of its orbit. Given the radius of its orbit, its circumference, C , is:

$$C = 2\pi r = (2\pi)(3.844 \times 10^8 \text{ m}) = 2.415 \times 10^9 \text{ m.} \quad (7.3.1)$$

Step 2. The period, T , for the Moon's orbit around Earth is 27.5 days. Convert this to seconds:

$$T = (27.5 \text{ day}) (24 \text{ hr/day}) (60 \text{ min/hr}) (60 \text{ sec/min}) = 2.376 \times 10^6 \text{ sec}$$

Step 3. Now find the Moon's orbital speed by dividing the distance (circumference) by time (period):

$$v = \frac{C}{T} = \frac{2.415 \times 10^9 \text{ m}}{2.376 \times 10^6 \text{ sec}} = 1,016 \text{ m/s} \quad (7.3.2)$$

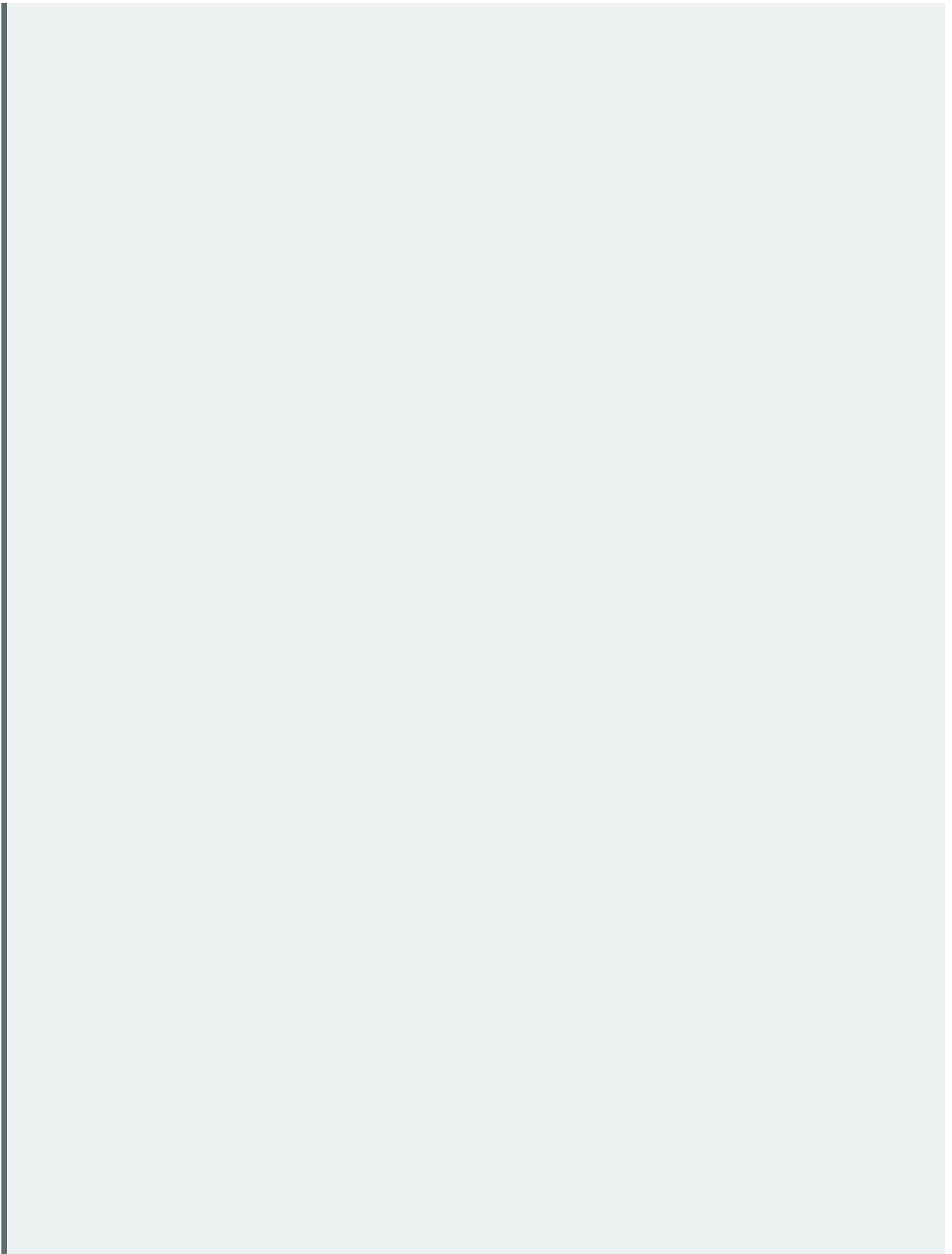
So the Moon moves about 1 km/s.

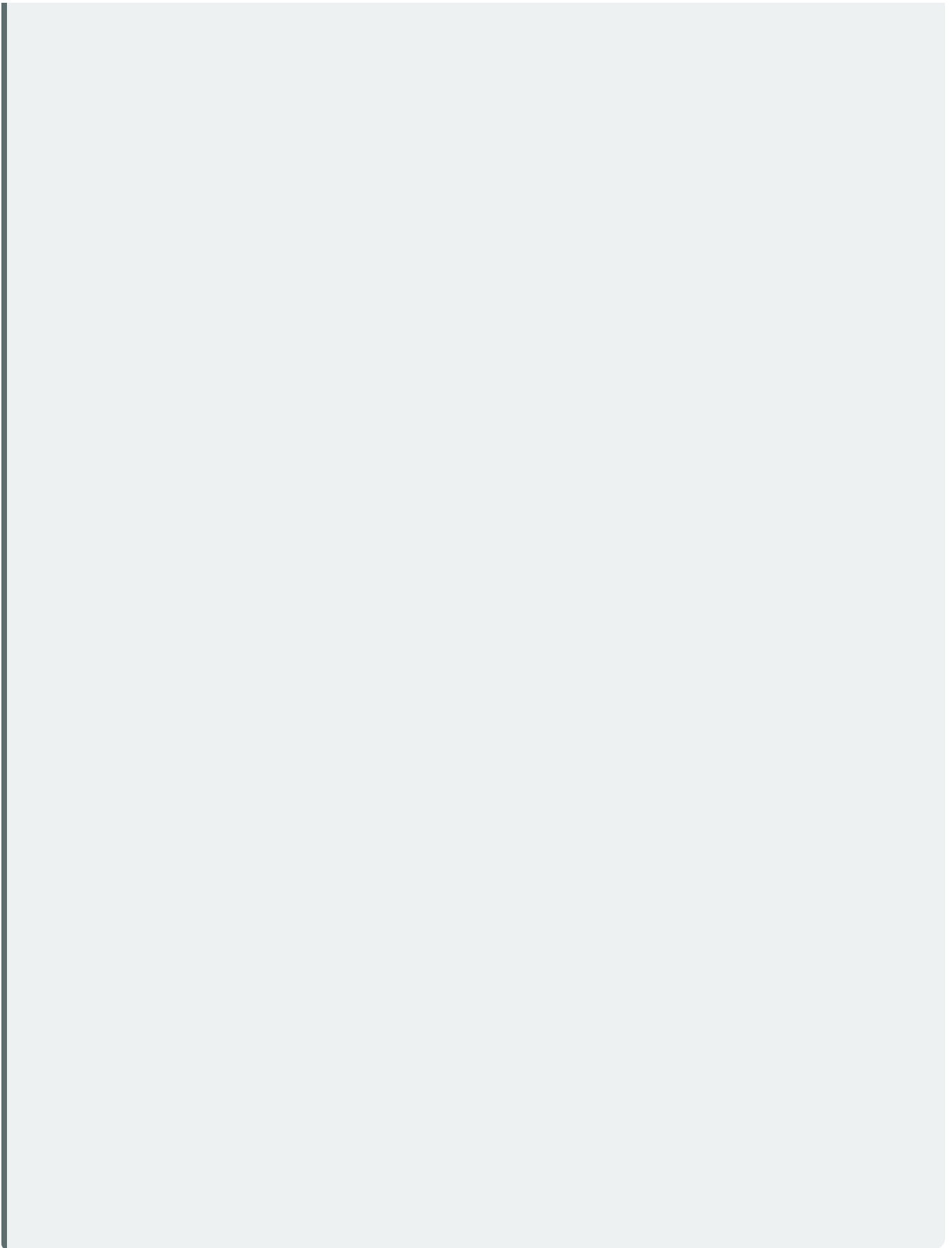
Step 4. Now we can use the centripetal acceleration formula to get the Moon's orbital acceleration:

$$a_c = \frac{v^2}{r} = \frac{(1,016 \text{ m/s})^2}{3.844 \times 10^8 \text{ m}} = 2.685 \times 10^{-3} \text{ m/s}^2 \quad (7.3.3)$$

Questions

Now it is your turn to calculate the orbital acceleration for Earth's motion around the Sun. Follow the steps below to do the calculation:





Newton (and others) deduced (guessed, really) that the reason the acceleration of the Moon toward Earth is so small is because the Moon is very far away. He correctly surmised that gravitational attraction must depend on distance .

How Does the Force of Gravity Depend on Distance?

In this activity, you will make measurements to determine how gravitational force depends on distance.

Play Activity

- The masses are set to 100 kg for this activity.
- Move the slider bar to select different distances between 200 and 600 units. Enter the distance in the x-column and the force into y-column of the graphing tool. For example, for a distance of 200 units, the force is 1250.
- Now use the graphing tool to plot your data and compare your data to the plots provided below in Figure A.7.3.

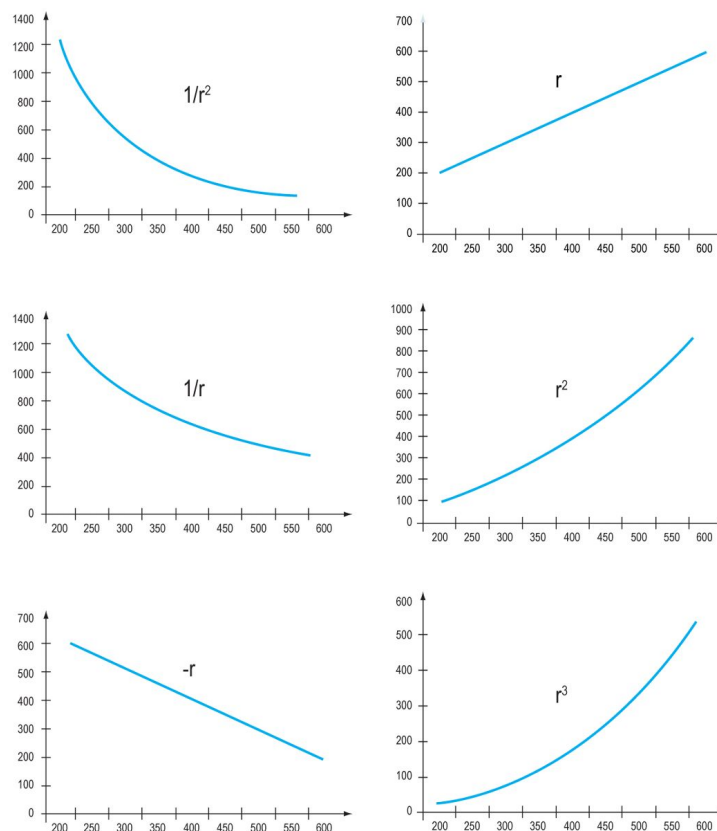


Figure A.7.3: Each of these graphs plots distance on the x-axis, and force on the y-axis. Compare these graphs to the plot you have made in the graphing tool to choose the best match below. Credit: NASA/SSU/Aurore Simonnet

Of course, Newton was not able to make measurements in the laboratory in which he could determine this dependence. Instead, he used mathematical reasoning to infer the relationship, which could then be experimentally verified. Newton's reasoning is traced in [Going Further 7.3: Determining the Distance Dependence of Gravity](#).

Going Further 7.3: Determination of the Distance Dependence of Gravity

Newton reasoned that the easiest dependence to consider is one where the strength of the gravitational acceleration (a_g) decreases with distance (r) raised to some power. In equation form, we use “ n ” to denote “some power”, where n stands for an integer that we still need to determine here. The symbol \propto means “proportional to.”

We expect that the value of n will be negative because only negative values will cause the gravitational acceleration to get smaller with r . By comparing the gravitational acceleration at Earth's surface to the acceleration of the Moon toward Earth, it is possible to determine the value of n .

The r here is understood to be the distance between the centers of the objects involved. So, for example, if we are standing on the surface of Earth, our distance is taken from Earth's center, not its surface. Similarly, the distance of the Moon from Earth is taken to be the distance from Earth's center (not its surface) to the center of the Moon. This is not obvious, and it was one of

Newton's great insights into gravity; he was able to demonstrate its validity mathematically. In the case of the Moon-Earth distance, the distinction between the center and surface is not very important because their separation is much larger than their sizes. However, for we who stand upon Earth's surface, the difference is vital: we are not zero meters from Earth, we are about 6400 km away from its center, at least as far as gravity is concerned.

You should have recognized in the previous interactive activity that the value of n turns out to be -2. However, Newton had to derive this, and we will now show how.

We can set up a ratio between the acceleration at Earth's surface and at the Moon's orbit.

$$\frac{a_E}{a_M} = \left(\frac{r_E}{r_M} \right)^n \quad (7.3.4)$$

Then we just have to plug the appropriate numbers into this expression.

$$\frac{9.8 \text{ m/s}^2}{2.685 \times 10^{-3} \text{ m/s}^2} = \left(\frac{6.638 \times 10^6 \text{ m}}{3.844 \times 10^8 \text{ m}} \right)^n \quad (7.3.5)$$

Note how units on both sides cancel, leaving no units in any resulting expression. We are left with the following...

$$3,650 = (1.657 \times 10^{-2})^n \quad (7.3.6)$$

Now, to solve for n it will help if we take the log of both sides. Then we have an expression we can easily solve algebraically. Here are the steps:

$$\log(3,650) = \log(1.657 \times 10^{-2})^n \quad (7.3.7)$$

$$\log(3,650) = n \log(1.657 \times 10^{-2}) \quad (7.3.8)$$

And so...

$$n = \frac{\log(3,650)}{\log(1.657 \times 10^{-2})} \quad (7.3.9)$$

$$n = \frac{3.552}{-1.781} \quad (7.3.10)$$

$$n = -2 \quad (7.3.11)$$

This is essentially the reasoning that Isaac Newton used to deduce the $1/r^2$ dependence of gravity. Such a dependence had been suspected by others, but Newton was the first to be able to demonstrate it in this manner.

Gravity does not only depend on the distance between two objects, it also depends upon their masses. Again using an interactive activity, you will explore this dependence.

How Does the Force of Gravity Depend on Mass?

In this activity, you will make measurements to determine how gravitational force depends on mass.

Play Activity

- Set the distance between the masses to 400 units for this activity.
- We will call the mass of the blue object m_1 and the mass of the red object m_2 .

Worked Examples:

1. Move the slider bar such that both masses are 50 units. What is the value of the force on each object in this case?

We see that the force on blue = 78.13 units and the force on red = 78.13 units. Note that the force is the same on both objects.

2. Move the slider bar such that m_1 is twice as much as it was before, i.e. 100 units. The other mass should remain at 50 units. What is the value of the force on each object in this case? By what factor has it changed?

We see that the force on blue = 156.3 units and the force on red = 156.3 units. This means the force is twice as much as it was before. The force on blue is equal to the force on red in this case too.

7.3.1.1: Questions

Now increase m_2 by a factor of 3, i.e. to 150 units while m_1 remains at 100 units.

Again, Newton deduced that the masses of the objects and the distance between them were responsible for the gravitational attraction. He was able to write an expression for the gravitational force between two objects as:

$$F_g = G \frac{m_1 m_2}{r^2}$$

This equation has been given the grandiose title Newton's Universal Law of Gravitation. It describes the gravitational force (F_g) between two objects, one with mass m_1 and the other with mass m_2 . The objects are separated (center to center) by a distance r from each other. The geometry is shown in Figure 7.6. The constant of proportionality G is called the gravitational constant. Its value in SI units is $G = 6.67384 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$. The constant of proportionality was not determined by Newton, but by Henry Cavendish (1731 - 1810) in a famous experiment carried out just before the turn of the 19th century, more than 50 years after Newton's death. At first, Newton's law was a guess, and it required verification by experiments or observations. Now, Newton's gravitational law forms the basis for our classical understanding of gravity and is accurate enough to predict the motions of the planets around the Sun and to send spacecraft, peopled or not, to other bodies in the solar system.



Figure 7.6: Two objects with masses m_1 and m_2 , respectively, are separated by a distance r . Each will feel a force from gravity that is proportional to the product of their masses and inversely proportional to their separation squared. Object 1 feels a force directly toward object 2, and object 2 feels a force directly toward object 1. Credit: NASA/SSU/Aurore Simonnet

The masses, m_1 and m_2 , could be any two objects in the universe: Earth and the Moon, Earth and the Sun, Earth and you, you and a friend, two tiny particles of interstellar dust, or two galaxies. This is what makes the law universal: it works for everything in the Universe that has mass, no matter how large or small the mass might be.

Sometimes astronomers will be more descriptive in the subscripts they use; instead of calling the objects m_1 and m_2 , they might call them M_E and M_S for Earth and Sun for example, or M and m if the objects are different masses. The subscripts are simply a matter of notational preferences.

In the next set of activities, you will practice using Newton's law of gravitation and explore it in more detail.

Working With Newton's Law of Gravitation

Use Newton's law of gravitation to answer the following questions:

Exploring the Force and Acceleration Due to Gravity

We can use the Newtonian Gravity tool to take a closer look at how the masses and distances between two objects affect the gravitational forces between them, as well as their acceleration toward each other. In this activity, we have arbitrarily set the G constant equal to 1 in order to avoid dealing with very small numbers. This does not matter at all for understanding gravity. It is equivalent to using a different unit to measure force, rather than using newtons, and units are only a matter of convenience and convention anyway.

[Play Activity](#)

A. Force and Acceleration

First, look at the force and acceleration between two objects of the same mass (50 units), and at a given distance (200 units). Use the slider bars to adjust the masses of the objects and the distance between them.

B. The Effect of Distance

Now change the distance between the objects to 500 units, and see how this changes the gravitational force between them, and their acceleration.

C. The Effect of Mass

Now, we will see what happens if we adjust the problem so that the objects have unequal masses. Keep the distance between the objects set to 500 units, and keep the mass of the blue object at 50 units. Adjust the mass of the red object to 150 units.

In the previous activity, you should have noticed a few important points from working with Newton's laws. First, the strength of the gravitational force between two objects is always the same for both of them, even if their masses are different. So, the strength of the force on object 1 from object 2 is the same as the strength of the force on object 2 from object 1. This is because a force is an action that happens between two objects, not a property of a single object. Next, you should have noticed that the directions of the forces go in opposite directions. This ensures that the force is always attractive, which Newtonian gravity certainly is.

Newton's gravitational law depends *only* on the masses of the two objects and their separation. It does not depend on their physical size (their diameters or radii), nor does it depend on whether they spin or not, whether they have magnetic fields, or whether they have atmospheres. Furthermore, the gravitational force between two objects can be felt infinitely far away. It never cuts off, it just becomes smaller and smaller. That means that, in principle, every object in the universe feels a gravitational attraction for every other object, no matter how far. In practical terms, of course, the gravitational interaction between widely separated objects becomes so small as to be negligible.

Even though the strength of the gravitational force felt between two objects is always the same for both of them, you should have noticed that their accelerations are different if their masses are different. We will explore this phenomenon in more detail next.

7.3.2: Applying Newton's Laws

In Math Exploration 7.2, we see with astronomical objects the same phenomenon we noticed with objects on Earth: even though the force is the same, the acceleration depends on the mass. Objects with a larger mass will have smaller accelerations and objects with a smaller mass will have larger accelerations.

Math Exploration 7.2: Gravitational Acceleration between Two Astronomical Objects

Comparing Newton's gravitational law to his second law, we can find an expression for the gravitational acceleration of any body. For example, if we take one of the masses in the equation to be that of Earth, 5.98×10^{24} kg, we can find Earth's surface gravity. We can also determine the surface gravity of other objects in the Solar System, as in the following activity.

Surface Gravity of Objects in the Solar System

In this activity, we will use Newton's law of gravitation to derive the value of g , which we have previously measured empirically as 9.8 m/s^2 on the surface of Earth. Then you will have the opportunity to determine the local " g " constant on other planets.

Worked Example:

Consider an object of mass m at the surface of Earth. We can use Newton's gravitational law to find the force Earth's gravity exerts upon the object. In the equation below, M_E and R_E are the mass and radius of Earth, respectively:

$$F_g = G \frac{M_E m}{R_E^2}$$

Since any force can always be expressed in terms of Newton's Second Law ($F = ma$), we can also write the force in terms of the gravitational acceleration: $F_g = mg$. Setting the two expressions for force equal to each other we get:

$$mg = G \frac{M_E m}{R_E^2}$$

Canceling the common factor of m we get:

$$g = \frac{GM_E}{R_E^2}$$

This is an expression for the gravitational acceleration at Earth's surface (g). Notice that it is the same for any object, regardless of mass since, just as before, the mass of the object itself has dropped out of the expression. Numerically this is:

$$g = (6.67384 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}) (5.972 \times 10^{24} \text{ kg}) / (6.368 \times 10^6 \text{ m})^2 = 9.828 \text{ m/s}^2$$

This is the value of the acceleration of gravity (g) at Earth's surface, as measured originally by Galileo. Newton has given us a theoretical understanding of the origin of the acceleration.

Questions

Now use the table to answer the following questions:

The next activity shows that gravity is really a fairly weak force, and it is only because Earth and other planets have large masses (relative to human scales) that we notice a gravitational force between them. Newton's law of gravitation says that all masses in the Universe attract all other masses in the Universe, no matter how small or far apart. It is just that in everyday life, we do not notice the gravitational forces between small objects because they are smaller than the gravitational forces between large objects. But they are there.

Weight and Gravitational Attraction for Small Objects

Worked Examples: Weight and force for two people

1. We have already seen that the gravitational force of Earth on an object is called that object's weight and that we can calculate the weight using $F_g = mg$ for an object of mass m .

a. A person might have a mass of about 50 kg, though that varies a lot from person to person. What is the weight of such a person?

$$F_g = mg = (50 \text{ kg})(9.8 \text{ m/s}^2) = 490 \text{ N} \quad (7.3.12)$$

2. But what would be the gravitational force of two such people on each other? We can use Newton's law to answer that question. We will assume that they are a meter apart, which seems like a reasonable distance for two people.

$$F = \frac{(6.67384e-11 \text{ N m}^2 \text{ kg}^{-2})(50 \text{ kg})(50 \text{ kg})}{(1 \text{ m})^2} = 1.67e-7 \text{ N} \quad (7.3.13)$$

This should make it pretty clear why we are not aware of the gravitational forces we exert upon each other. After all, the electromagnetic force exerted by a strong magnet can be enough to pick up a car - several thousand pounds. Even the magnets we might have on our refrigerators can exert several newtons of force. In comparison, gravity is an anemic force. It generally becomes noticeable only when the masses involved are very large.

Questions

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