8.2: Velocities, Mass, and Gravity - The Solar System

What Do You Think: Rotation of the Solar System

The systems we have talked about so far in this chapter (wheels and water swirling around a drain) may not seem like they have anything to do with astronomy. But as you will see, these everyday examples will help us understand the motions of planets in the Solar System, the motions of stars and gas in galaxies, and the motions of galaxies in galaxy clusters. Here, we will look at our first astronomical example of rotation, but we will start locally, astronomically speaking— we will look at how the Solar System rotates.

8.2.1: Rotation of the Solar System

We will start our investigation of the Solar System's rotation by listing the average velocities of the planets as they orbit the Sun, and the distances of the planets from the Sun, and see what rotation curve results. Later in this section, we will examine why the Solar System has this kind of rotation curve, using the laws of gravity. Below is a table of average distances of the planets from the Sun, and their average orbital speeds:

Table 8.1: Distances and Velocities for Solar System Objects

PLANET	AVERAGE DISTANCE FROM SUN (AU)	AVERAGE ORBITAL SPEED (KM/S)
Mercury	0.39	47.9
Venus	0.72	35.0
Earth	1.00	29.8
Mars	1.52	24.0
Jupiter	5.20	13.1
Saturn	9.58	9.69
Uranus	19.2	6.81
Neptune	30.1	5.43

We can use the numbers above to look at the rotation curve of the Solar System (Figure 8.6). Does this look like any of the rotation curves we discussed in section 8.1? We will discuss why the rotation curve of the Solar System looks the way it does as we move further into this chapter.

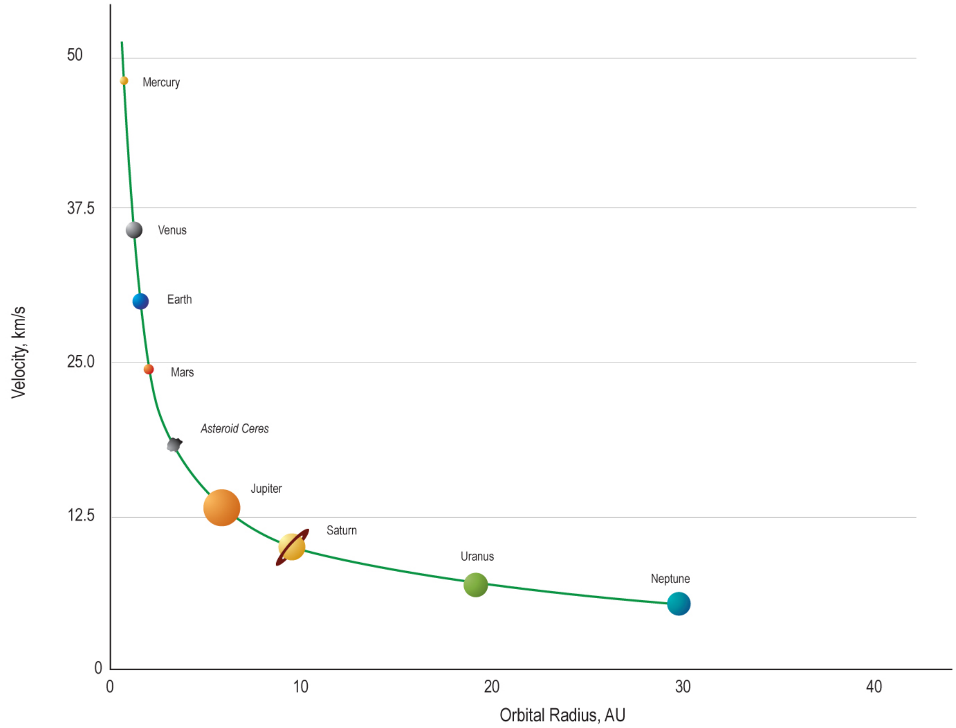

Figure 8.6: The rotation curve of the Solar System shows that the inner planets rotate around the Sun with faster velocities than the outer planets. Credit: NASA/SSU/Aurore Simonnet

Velocities in the Solar System

The dwarf planet Pluto has an average distance of 39.3 AU from the Sun, and an average orbital speed of 4.67 km/s around the Sun. Dwarf planet Eris has an average distance of 68.0 AU from the Sun, and an average orbital speed of 3.44 km/s.

Use Figure 8.6 above, or add these data to your own graph (using the graphing tool), and answer the questions below.

8.2.2: Gravity and the Mass Distribution of the Solar System

By looking at the rotation curve of the Solar System and comparing it to the examples we discussed in Section 8.1, you will notice that the motion of the planets in orbit around the Sun resembles the motion of water swirling around a drain. More specifically, the planets' motion resembles the model we discussed in which velocities decrease with increasing radius. Why do the planets orbit the Sun in this way? Or put more generally, why does the Solar System rotate this way? The reason for this has to do with gravity.

The force of gravity depends on mass, so it will help our understanding to think about how mass is distributed in the Solar System. We know that the most massive objects in the Solar System are the Sun and the planets. But really, the Sun is so massive— far more massive than all the planets combined— that we can ignore the masses of the planets as we model the mass distribution of the Solar System. We can also, of course, ignore the masses of asteroids, comets, and dwarf planets.

Despite having nearly all the mass in the solar system, the sun is relatively tiny in extent; the diameter of the Sun is much, much smaller than the distances between the planets and the Sun. Given these circumstances, we may model the Solar System's mass distribution very simply. To high precision, we can assume that all the mass in the Solar System is concentrated in a point at the position of the center of the Sun. Because we are modeling all the mass as being at a single point, this is called a point mass model.

We can directly use Newton's laws of gravitation and motion to determine the orbital velocity of an object under these assumptions.

$$\frac{GM}{r^2} = \frac{v^2}{r}$$

On the left side of the equation, we have the expression for acceleration due to gravity (from Newton's law of gravitation). On the right, we have the expression for centripetal acceleration. Here, *v* refers to the speed of an orbiting object, *r* refers to the radius of the object's orbit (the distance between the planet and the Sun), and M refers to the total mass enclosed by the orbit. We are assuming circular orbits for simplicity's sake, but this assumption is not necessary. It just makes the math easier to manage.

In our point mass model, *M* is the mass of the Sun. The masses of any planets inside the orbit do not contribute enough to be important, and it is only the mass of the Sun that will be enclosed by the planet's orbit. In other examples that we look at later, we will see that the entire mass of a system is not necessarily enclosed by an object's orbit. Here, however, we can simply plug the mass of the Sun into the equation above.

Continuing with our Solar System example, we cancel one factor of r on the bottom of both sides of the equation. We are then left with the following equation.

$$v^2 = \frac{GM}{r}$$

or

$$v \propto \frac{1}{r^{1/2}}$$

In this case, velocity is proportional to the square root of the distance (r) from the point mass. Velocity will decrease with increasing distance, though in a slightly different way than the example we discussed in Section 8.1.2.

You should complete the numerical activities below to get a better sense for what the proportionality above means. Note that astronomers refer to this relationship between velocity and distance (orbital radius) as Keplerian rotation because it describes the motion of the planets. It is Johannes Kepler who is famous for having derived this expression empirically from observations of the planets in the 17th century.

Keplerian Motion

1. The Earth is 1 AU from the Sun, and Jupiter is 5.2 AU from the Sun. Using the proportionality expression for Keplerian rotation, calculate how much faster we would expect Earth's orbital velocity to be than Jupiter's. How does that compare to the observed ratio between the two planets' velocities?

- Given: r_{Earth} = 1 AU, $r_{Jupiter}$ = 5.2 AU
- Find: v_{Earth} / $v_{Jupiter}$
- Concept: In Math Exploration 8.1, we find that:

$$\frac{v_{Earth}}{v_{Jupiter}} = \frac{r^{1/2}_{Jupiter}}{r^{1/2}_{Earth}}$$

- Solve: Putting in the numbers, we find:

$$\frac{r^{1/2}_{Earth}}{r^{1/2}_{Jupiter}} = \sqrt{\frac{(5.20AU)}{(1AU)}} = \sqrt{5.20} = 2.28$$

The actual ratio between the two planets' observed velocities from Table 8.1 is:

$$\frac{v_{Earth}}{v_{Jupiter}} = \frac{29.8 km/s}{13.1 km/s} = 2.27$$

This is in good agreement with the predicted ratio we calculated based on the Keplerian rotation model.

Math Exploration 8.1: Keplerian Motion

Inverse Square-root Proportionality

Now we can look at the inverse square-root relationship in another mathematical way— by graphing. First, we will assume we have an inverse square-root relationship between v and r like this:

$$v = \sqrt{\frac{5}{r}}$$

Now, use the graphing tool to plot the above six data points (where radius corresponds to the x-coordinate, and velocity corresponds to the y-coordinate). In other words, use the data in the table above to plot a rotation curve.

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