

## 10.2: Gravity and Curvature

### ? Curving Space and Time



### 10.2.1: Gravity Curves Space

In this section we will explore some of the consequences of the weak equivalence principle. Let us again consider a rocket ship far away from any source of gravity. Earlier, we imagined the motions of material objects. Now we will consider what happens when light passes through the rocket ship under these same conditions.

First imagine that a small window is opened in the side of the rocket, just enough to let in a beam of light through the side of the ship. We know that the light beam will travel in a straight line through the ship. Upon reaching the other side it will pass out of the ship, as long as we provide a strategically placed window that allows it to do so. The light will then continue in a straight line in its original direction.

We can imagine three different cases when light shines through the ship, and we consider them in turn. In each case, the light enters through one side of the ship and passes out the opposite side. We consider each case from two different perspectives. In the first, we consider the point of view of an occupant of the rocket ship. In the second, we look at the situation from the point of view of an observer outside the rocket who at rest with respect to the light source.

#### 10.2.1.1: Case One: The ship is stationary with respect to the light source.

This is the case we just considered. The light will travel straight through the ship and pass out directly across from where it entered. Figure 10.3 shows how this might look to observers both on the outside of the ship and inside it.

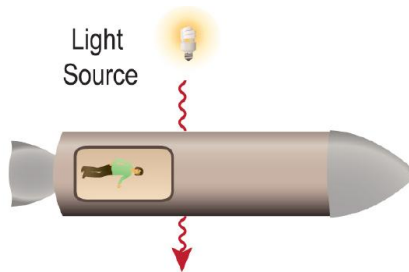


Figure 10.2.1: In the case that the ship and the light source are at rest with respect to one another, a light beam sent perpendicularly through a wall will travel straight across the ship, exiting directly opposite of the point it entered. Credit: NASA/SSU/Aurore Simonnet

### 10.2.1.2: Case Two: The ship moves at a constant speed, perpendicular to the direction of the light as seen by an observer at rest with respect to the light's source.

In this situation, the light will travel in a straight line, but it will exit the ship at a point displaced from its entry point because the ship moves some distance while the light is traveling across the ship. Figure 10.2.2 shows what this situation might look like to our two observers.

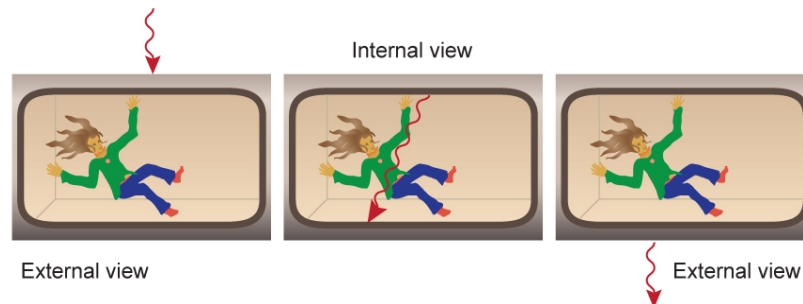


Figure 10.2.2: In the case that the ship and the light source move at a constant velocity with respect to one another, a light beam sent perpendicularly through a wall will travel at an angle as viewed by an occupant of the ship. It will exit at a point displaced in a direction opposite to that of the ship's motion. On the other hand, an outside observer thinks it is the ship that has been displaced, while the light itself has again traveled in a straight line. Credit: NASA/SSU/Aurore Simonnet

### 10.2.1.3: Case Three: The ship moves at a constant acceleration, perpendicular to the direction of the light according to the same observer in Case Two.

In this case the situation is markedly different for the two observers. Our outside observer still sees the light travel in a straight line as the ship accelerates forward. But the observer on the inside of the ship sees a very different path for the light. For that observer the light beam seems to curve toward the back of the ship as shown in Figure 10.2.3 In any given time interval, a second, say, the light travels a constant distance across the ship, but the ship moves an ever-increasing distance forward as its speed increases. As a result, the light makes a parabolic path relative to the ship and its occupants. Since light travels in straight lines in freely falling frames, it appears to an observer in an accelerated frame that "straight" lines are curved. We will return to this idea below.

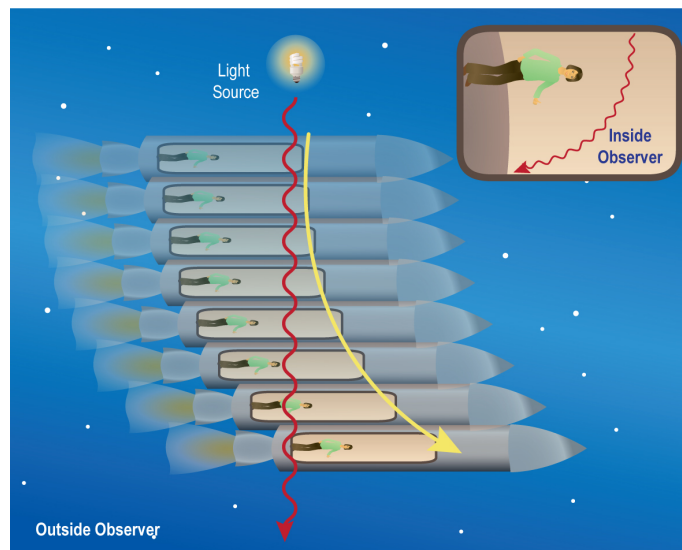


Figure 10.2.3: In this case, the ship moves at an ever increasing speed, while the light moves across it at a constant speed. The result is that the ship's occupant sees the light follow a parabolic path, not a straight one (shown as the red curve in the inset). To an outside observer though, the light still travels in a straight line (red line), while the ship moves ever faster in the forward direction (yellow curve). Credit: NASA/SSU/Aurore Simonnet

### 📌 A Seagull Flies Across a Ship as Seen by Two Different Observers

Relativity does not require us to consider spaceships and light beams. We can use other analogies, too. For example, imagine a ship at sea that moves forward at a constant speed of 5 m/s. A seagull glides past at 1 m/s in a direction perpendicular to the motion of the ship, as shown in Figure A.10.1.

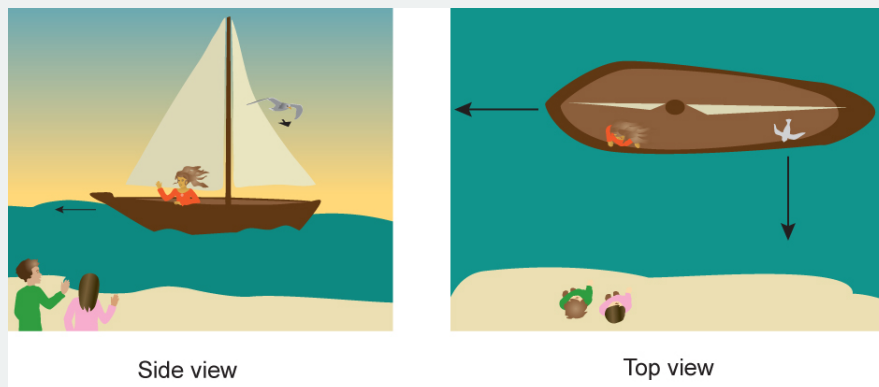


Figure 10.2.4: A seagull flies over a ship moving at a constant velocity. An observer on the ship is watching the seagull. A second observer watches from the shore. The panel on the left shows the view from the side, and the panel on the right shows the view from above. Credit: NASA/SSU/Aurore Simonnet

1.

2.

3.

### Accelerating Ship and a Seagull

Consider the ship in the previous activity, but now imagine that it starts from rest and then accelerates at a constant rate of  $1 \text{ m/s}^2$  (Figure A.10.2).

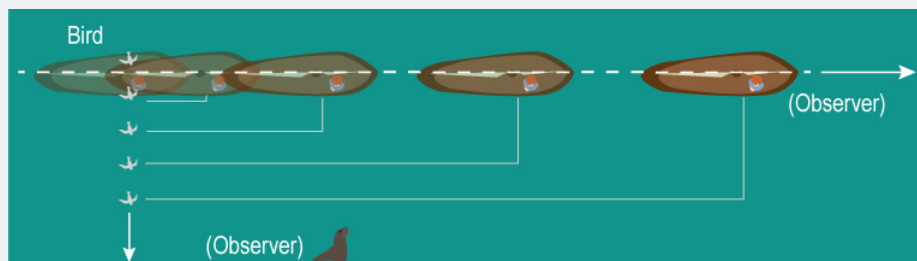


Figure 10.2.5: A seagull flies over a ship that is accelerating. An observer on the ship is watching the seagull. A second observer watches from nearby the shore. Credit: NASA/SSU/Aurore Simonnet

#### Worked Examples:

1. If the same seagull glides past at the same speed,  $1 \text{ m/s}$ , how much time will it take to fly across the boat?

Since the seagull is still flying directly from one side of the boat to the other, it must traverse 10 meters to get all the way across. It therefore will take 10 seconds for the bird to cross the boat, just as before.

2. Why can we still ignore the motion of the boat when we are finding the time required for the bird to cross the boat?

We can ignore the motion of the boat because the seagull is still flying directly from one side of the boat to the other. Therefore it must still traverse 10 meters to get all the way across.

3. In Math Exploration 10.1, we show how to calculate the distance the accelerating boat travels.

#### Math Exploration 10.1

#### Questions

- 1.

2.

3.

4.

#### Going Further 10.1: Accelerated Motion

We have seen that the distance traveled by an object moving with a constant velocity is given by the following equation.

$$d = vt$$

We cannot generally use this expression if the velocity is changing. However, we can use a similar relation if we know the average speed a particle moves. For example, if you traveled 30 km/hr for half an hour, and then you sped up to 90 km/hr for an additional half hour, you would have traveled a total of 60 km in one hour:

$$(30 \text{ km/hr})(0.5 \text{ hr}) + (90 \text{ km/hr})(0.5 \text{ hr}) = 15\text{km} + 45\text{km} = 60\text{km}$$

Since you completed the trip in one hour, and you traveled a total distance of 60 km, your average speed must have been 60 km/hr, though you did not spend any of the trip actually traveling 60 km/hr. In general, if we know the average speed of an object, then we can use the following relation to find how far it moves in a time  $t$ :

$$d = \bar{v}t$$

The bar over the  $v$  means that we are using the average speed for a trip. In effect, this is one definition of what we mean by average speed.

We can rewrite the previous numerical example to show explicitly how the distance for that trip is related to the average speed for the trip. We start as we did before, then rearrange terms slightly by writing 0.5 hours as (1 hr/2):

$$(30 \text{ km/hr})(0.5 \text{ hr}) + (90 \text{ km/hr})(0.5 \text{ hr}) = \left( \frac{30 \text{ km/hr} + 90 \text{ km/hr}}{2} \right) (1 \text{ hr}) = 60 \text{ km}$$

The term in the big parentheses is by definition the average speed for the trip in our example. Multiplying by the total time for the trip gives us the distance traveled during the trip.

This example was simple because there were only two relevant speeds during the trip. However, even if the speed constantly changes we can use this notion of average speed to deduce how far an object moves in a given time. If the object accelerates at a constant rate, then we can find a general expression for the distance it travels, as we now show.

First, imagine an object that starts from rest and accelerates at a constant rate. We do not have to start from rest, but it makes the argument a little simpler to follow; adding an initial speed would shift the distance up or down, but it would not change the outline of the argument we present.

For the sake of argument, imagine that the object accelerates at 1 m/s/s, for 10 seconds. How fast will this object be moving at the end of the 10 seconds? Well, after one second it is moving 1 m/s, after 2 seconds it is moving 2 m/s, and so on. After 10 seconds it will be moving 10 m/s. If we plot its speed against time, it will look like the graph below.

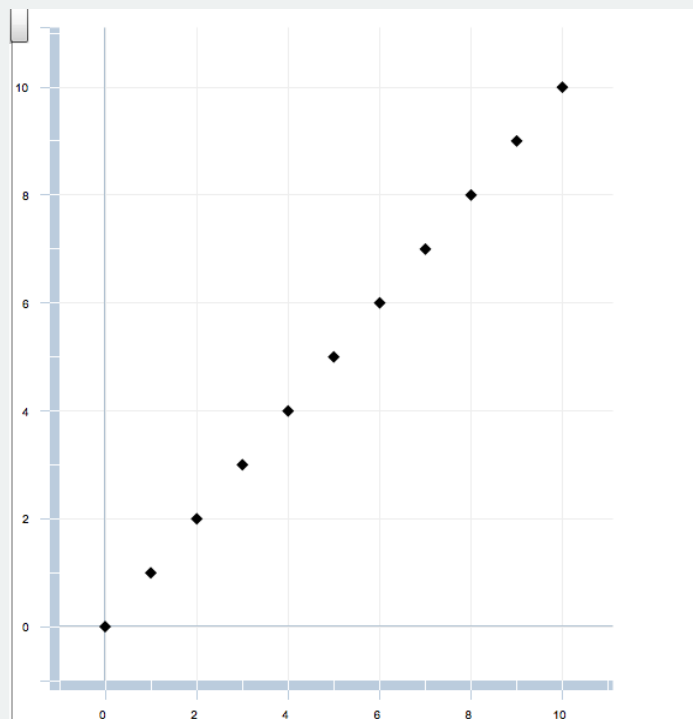


Figure 10.2.6: Graph shows velocity (vertical) vs. time (horizontal) for an object with constant acceleration.

The average speed for the trip is right in the middle: 5 m/s. Notice how the object spends as much time above its average speed as below it. This is because the speed of the particle is symmetric about its average. Under these circumstances the average can be computed simply from the mean of the initial and final speeds. Check that this formula works for the example above.

$$\bar{v} = \frac{v_0 + v_f}{2}$$

But the final speed ( $v_f$ ) is just the initial speed ( $v_0$ ), plus whatever change in speed is due to the object's acceleration,  $v_f = v_0 + at$ . So we can rewrite the last expression as follows.

$$\bar{v} = \frac{v_0 + (v_0 + at)}{2}$$

or

$$\bar{v} = \frac{v_0 + v_0 + at}{2}$$

We can simplify this expression slightly.



$$\bar{v} = v_0 + \frac{1}{2}at$$

Now we can substitute this into our earlier expression for the distance in terms of the average speed.

$$d = \left( v_0 + \frac{1}{2}at \right) t$$

This leads to the following expression for the distance traveled by an object that accelerates at a constant rate.

$$d = v_0 t + \frac{1}{2}at^2$$

We can use this expression to find how far an object moves when it starts from some initial speed and accelerates at a constant rate. An example would be an object falling under the influence of gravity near Earth's surface. Recall that the gravitational acceleration is essentially constant if we remain near the surface.

In the preceding examples we saw how the path of light was dramatically different for observers in an accelerated frame than for those in a frame moving at constant velocity. According to the equivalence principle, there should not be a difference between what is seen in an accelerated frame and what is seen in the presence of a gravitational acceleration. So does that mean that if we view light moving where there is a gravitational acceleration we should see it bend? According to the equivalence principle, the answer is “yes.” This is the sort of thought experiment that Einstein used to come to the realization that space must be curved. To oversimplify for the moment, light, by definition, follows a “straight path,” but in the presence of gravity, that “straight path” must follow the curvature created by the gravitational acceleration. But we do not observe light curving due to Earth's gravitational acceleration. Is there a contradiction here between the equivalence principle and reality? The next activity addresses this apparent contradiction.

### Light Deflection

Consider the ship at sea one more time. Now replace the gliding seagull with a speeding photon, as in Figure 10.2.7.

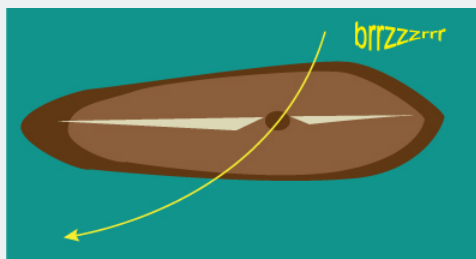


Figure 10.2.7: A photon passes over a ship that is accelerating, and is seen to be deflected by 1 mm. Drawing is not to scale.  
Credit: NASA/SSU/Aurore Simonnet

1.

2.

3.

The idea of curved space seems strange, but if you take a trip from New York to Tokyo, how do you go? The straightest path between the two cities must follow the curvature of Earth's surface. It is not a "straight line" in the usual sense. Even if we do not think about it very much, we are already somewhat familiar with curved spaces because we happen to live on one: Earth's surface! And just as in general relativity, the curvature is only apparent when we travel long distances. On small scales the surface of Earth seems flat. We might create an "equivalence principle" for Earth's surface that says the world is flat on small scales. Only on larger scales does its curvature become apparent. We will explore these ideas in more detail below, but first we will look at how gravity affects time.

### 10.2.2: Gravity Curves Time

To understand the effect of gravity on the time part of spacetime, we can again consider the motion of light in a rocket ship, as in our previous example. For this example, however, we imagine what happens if we shine the light beam parallel to the ship's motion. For instance, imagine that we shine light forward from a source at the back of the ship. If an observer at the back measures the frequency of the light to be  $f_0$ , what frequency will a different observer, placed at the front of the ship, measure? Will it be different from, or the same as, the frequency measured by the observer at the back? There are three possible outcomes: either the frequency at the front is larger, smaller, or unchanged. We can reason our way to the answer.

In the case of no acceleration, the answer is straightforward: we know that the velocity of the source and observer are the same at all times. Since they are both attached to the same rocket ship their relative velocity is zero. The critical aspect of this case is that we know the speed of both sources remains unchanged from the moment the light is emitted to the moment it is detected. Therefore, the frequency of the light is the same at both the source and the detector. It does not matter if the rocket is "moving" or "stationary" according to an outside observer: the observers on the ship are not moving relative to one another.

Now imagine what happens if we do the same experiment on a ship that accelerates in the forward direction (Figure 10.2.8). At the instant the light leaves the source, the ship has a certain velocity, call it  $v_0$ . However, in the time required for the light to travel from the source to the detector the ship speeds up. Its final velocity,  $v$ , is larger than the velocity it had when the light was emitted. Note the difference between this case and the previous one. If the rocket does not accelerate, then there is a single reference frame in which both source and detector are at rest at all times. If the rocket accelerates, then there is no single reference frame in which both are always at rest. At the instant the light is emitted there is a reference frame in which both are at rest. Then, when the light is detected there is a different reference frame in which they are at rest. The two reference frames are different because the rocket has increased its speed in the short time required by the light to travel from source to detector. It is as if the source and detector have different velocities, and this is because of the time delay caused by the finite speed of light.

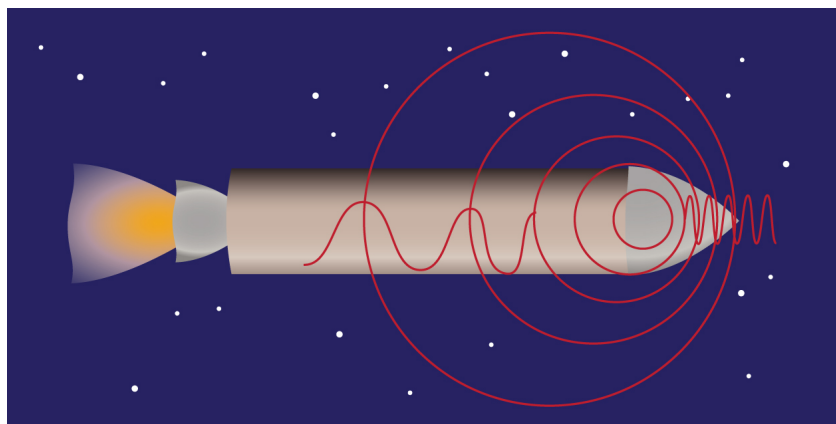


Figure 10.2.8: On a ship that is accelerating, the velocity of the ship at the time the light is sent is less than the velocity of the ship at the time the light is received. This causes a shift in the frequency of the light. Credit: NASA/SSU/Aurore Simonnet

From the Doppler shift formula we know that if a source of light and a detector of light have different velocities, then there will be a shift in the frequency of the light as measured at the source and detector. In this case, since the detector is accelerating away from the point where the light was emitted, the frequency will be lower (there will be a redshift) than it was at the source—which is also accelerating away from the point of emission, of course. This is the critical difference between the accelerating rocket and the constant velocity rocket. The following example shows the size of the shift when the acceleration is small.

In terms of frequency, the Doppler relation is described by the following equation.

$$f = f_0 \left( 1 - \frac{v}{c} \right)$$

The frequency  $f$  is that measured by an observer moving at a speed  $v$  relative to the source of the light, and  $f_0$  is the frequency of the light measured in the frame of the source. For positive velocities (when we are moving away from the source, inducing a redshift) the frequency decreases. For negative velocities (when we move toward the source, inducing a blueshift) the frequency increases, just as we expect.

The difference in velocity of the rocket between the time the light is emitted at the source and when it is detected is due only to the acceleration of the rocket over the time the light is in transit. If we call the light travel time  $T$ , and the acceleration  $a$ , then the velocity of the rocket at the time the light is detected can be written as follows.

$$v = aT$$

We are assuming that the acceleration is low, and we further assume that the length of the rocket is not very big. Under these assumptions the rocket does not move very far while the light travels. Then the time required for the light to get from the back of the ship to the front is the rocket's length divided by the speed of light. If the length/height of the rocket is  $H$  we have:

$$T = \frac{H}{c}$$

Combining this with the previous two relations we get:

$$f = f_0 \left( 1 - \frac{aH}{c^2} \right)$$

This is the shift in frequency we expect for light if it is on an accelerating rocket ship. (For more information, see [Going Further 10.2: Another Look at Gravitational Redshift](#).) But remember, according to the equivalence principle we should see the same shift wherever there is a gravitational acceleration. That is, if we shine a beam of light directly upward against the acceleration of gravity, just as in the last example where the light traveled against the acceleration, there should be a shift in its wavelength as it travels upward.

As an example, on Earth's surface where the acceleration is  $1g$  ( $9.8\text{m/s}^2$ ), there should be a small shift in the frequency of a beam of light that is directed upward as shown in Figure 10.2.9



Figure 10.2.9: Because of the acceleration due to gravity, there is a small shift in the frequency if we shine a light upward from Earth's surface (blue dot). The effect is exaggerated in the illustration. Credit: NASA/SSU/Aurore Simonnet

### The Effects of a Gravitational Acceleration on Light

Imagine that you are shooting a laser beam up away from Earth.

1.

2.

Now imagine that you are on an orbiting spaceship and were shooting the laser beam down toward the Earth.

3.

4.

The next activity illustrates why we generally do not notice these shifts.

### Gravitational Redshift and Blueshift

#### Worked Example:

1. Imagine that we have a tower 100 meters high, and that we shine a green laser pointer from the bottom to the top of the tower. If the wavelength of the laser light is 420 nm at the bottom of the tower, what will its wavelength be upon reaching the top?

- Given:  $\lambda = 420\text{e-}9\text{ m}$ ,  $a = 9.8\text{ m/s}^2$ ,  $H = 100\text{ m}$
- Find: The shifted frequency,  $f$
- Concepts:  $f_0 = c/\lambda$ ,  $f = f_0(1 - aH/c^2)$
- Solution: First we need to find the frequency associated with the emitted wavelength:  $f_0 = c/\lambda = (3\text{E}8\text{ m/s}) / (420\text{E-}9\text{ m}) = 7\text{E}14\text{ Hz}$
- Now we can find the new frequency:

$$\begin{aligned} f &= (7\text{E}14\text{ Hz}) [(1 - (9.8\text{m/s}^2) (100\text{ m})/(3\text{E}8\text{ m/s})^2)] \\ &= (7\text{E}14\text{ Hz}) (1 - 1.1\text{E-}14) \\ &= (7\text{E}14\text{ Hz}) \end{aligned}$$

The shift in frequency is far too small to be noticeable by an observer in the weak gravity of Earth. It is only about a part in 100 trillion! However, with a stronger gravitational acceleration the effect does become noticeable. We will consider those cases later on.

#### Questions

In the worked example we considered the shift of frequency for a green laser pointer that is fired from the ground to the top of a 100 meter tall tower. Now imagine that we have a red laser pointer with wavelength 650 nm at the top of the tower. We fire straight down toward the ground.

1.

2.

3.

4.

5.

#### Going Further 10.2: Another Look at Gravitational Redshift

In the examples so far we have assumed that the equivalence principle holds, and so we have computed the shift in wavelength of light in a gravitational acceleration based upon what we expect on an accelerating rocket ship. We will now find what the shift will be for a photon on Earth if we fire it from the bottom of a tower to the top. We will again use the equivalence principle to accomplish this.

One way to state the equivalence principle is that an accelerated frame is indistinguishable from a frame where there is a gravitational acceleration, at least for small scales. Another way to state it is as follows: On small scales, freely falling frames are equivalent to the inertial frames of special relativity. So if we are in a frame that is freely falling in a region with a gravitational acceleration, then we expect all our measurements to be the same as we would have in an inertial frame. We will now follow this reasoning through.

Imagine we have a colleague who directs a laser beam upward from the bottom of a tower to the top, where we are located. Further imagine that our colleague measures some wavelength for the beam, call it  $\lambda_0$  — it might be 420 nm, like in the worked example from the activity. What wavelength will we observe at the top of the tower? To answer this question, we make a rash decision and step off the tower platform at the instant the beam is fired upward.

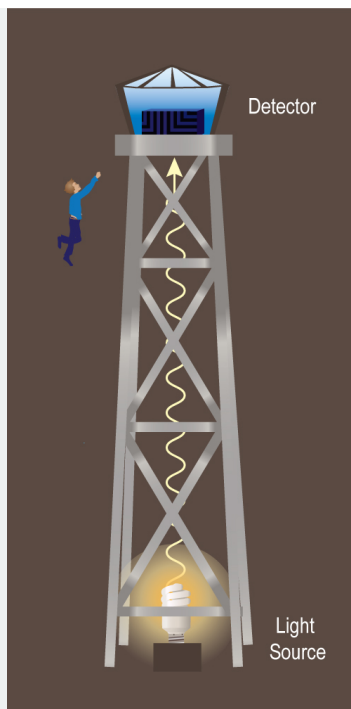


Figure 10.2.10: Light is emitted at the base of a tower and travels upward as the observer steps off the top of the tower and freely falls. Credit: NASA/SSU/Aurore Simonnet

According to the equivalence principle, once we leave the support of the top of the tower and begin to fall, we are in an inertial frame because we are only under the influence of gravity. The light is also freely falling, influenced only by gravity. We neglect the effects of the air, as usual.

For a moment imagine another observer, next to us but traveling upward at a speed  $v$ . From the point of view of such an observer, the wavelength of the light would be increased by an amount given by the observer's speed multiplied by the time required for one wavelength to go by the observer. We will call this time  $T$ ; it is the period of the wave. To understand why this happens, you must realize that the observer would move slightly, in the same direction as the light wave, as the wave moves rapidly upward.

So for the tiny shift in wavelength, we may write:

$$\Delta\lambda = vT$$

We can rewrite the time period,  $T$ , in terms of the frequency,  $f$ , of the light using the relation  $T = 1/f$ , and then substitute for  $T$ .

$$\Delta\lambda = \frac{v}{f}$$

We can also rewrite the shift in wavelength using the relation  $\Delta\lambda = \lambda - \lambda_0$ , where  $\lambda_0$  is the unshifted wavelength of the light (the wavelength in the frame of the source) and  $\lambda$  is the wavelength we measure. Our expressions is now the following.

$$\lambda - \lambda_0 = \frac{v}{f}$$

And now, since we know that the product of wavelength and frequency is the speed of the wave, or ( $\lambda = c/f$ ), we can combine.

$$\frac{c}{f} - \frac{c}{f_0} = \frac{v}{f}$$

Rearranging this leads to the expression below.

$$\frac{f}{f} - \frac{f}{f_0} = \frac{v}{c}$$

This equation simplifies as follows.

$$1 - \frac{f}{f_o} = \frac{v}{c}$$

Now we can solve this for the frequency,  $f$ , measured by the observer.

$$\frac{f}{f_o} = 1 - \frac{v}{c}$$

Or...

$$f = f_o \left( 1 - \frac{v}{c} \right)$$

So an observer moving upward at the instant we step off the tower would measure a shift of this size for the light's wavelength. Of course we are not moving upward; we have just stepped off the tower and are beginning to fall to its base. However, if a friend of ours remains on the tower, as we begin to fall, it looks to us as though our friend is moving upward. Since we are in free fall, and therefore in the inertial frame of the also freely falling light, it must also look to the light as if our friend is moving upward relative to its own frame of reference. And just as in the case of the rocket, the velocity we attain downward is very small in the short time needed for the light to reach the top of the tower: this means that the frame of the top of the tower is only moving slowly relative to the free-falling frame of the light.

We can calculate our speed when both the acceleration due to gravity ( $g$ ) and the height of the tower ( $H$ ) are "small," and thus determine the speed of our friend's reference frame. We realize that the time we have been falling is just the height of the tower divided by the speed of light:  $t = H/c$ . Our speed is therefore  $v = gt = gH/c$ . This gives us the following expression for this new frequency.

$$f = f_o \left( 1 - \frac{gH}{c^2} \right)$$

We have found exactly the expression we got before for the accelerating rocket. In this example we explicitly used the symbol  $g$  for Earth's gravity. Of course it is valid not only on Earth, but on any other objects for which our assumption of low gravity and small  $H$  are valid.

The redshift relationship we have just found for frequency is known as *gravitational redshift*. It has an important implication: it does not only relate to light; it is true for any periodic phenomenon. For instance, imagine we make a clock that keeps time by measuring the oscillation of a specific frequency of light, ticking once for each wave that goes by. An observer at the top of a tower will see the hands on a clock sitting on the ground moving more slowly than an identical local clock that uses a local light source. This is because light on the ground oscillates more slowly from the observer's vantage point, or equivalently, *time runs more slowly where gravity is stronger*. From this result we can deduce that an observer in Earth orbit will notice that clocks on the ground and on the tower move more slowly than his local clock. The effect is called gravitational time dilation. It is one of the important predictions/tests of general relativity. Written in terms of time, the relationship is the following.

$$t = \frac{t_0}{\left( 1 - \frac{gH}{c^2} \right)}$$

Our derivation is only appropriate for very weak gravitational accelerations, like that of Earth. And we assumed that the tower was not very big. In that case the shift in frequency/wavelength of the light is not very big. A more general derivation would give a result that could also be used in strong fields, like those around black holes and neutron stars. Such a derivation is beyond the mathematical techniques appropriate for these modules. It requires the full mathematical apparatus of general relativity. However, derivation given here gives us a flavor for the physical reality underlying some of the more bizarre-seeming effects of general relativity. We will use the results of the full calculation in the rest of the modules.

As an aside, it is good to realize that popular devices like GPS navigation tools make use of effects from general relativity. GPS relies on signals from satellites in Earth orbit. Without taking account of general (and special) relativistic effects, the precision attainable by GPS is quite noticeably degraded. In the next activity, we will explore the time dilation effects that must be taken into account for a GPS.



## Time Dilation and GPS

### Worked Examples:

1. GPS satellites are in orbit at about 36,000 km above the Earth's surface. Due to general relativistic effects, clocks at a distance above the Earth run faster than those closer to the Earth. The fractional change in the time is given by  $(t - t_0)/t$ . For a clock on a GPS satellite, what is the fractional change in time?

- Given:  $g = 9.8 \text{ m/s}^2$ ,  $H = 3.6\text{E}4 \text{ km} = 3.6\text{E}7\text{m}$
- Find: The fractional change in time,  $(t - t_0)/t$
- Concepts:  $t = t_0/(1 - gH/c^2)$
- Solution: First let's rewrite the expression for time dilation in terms of the fractional time:  
 $t = t_0/(1 - gH/c^2) \rightarrow t_0/t = (1 - gH/c^2) \rightarrow 1 - t_0/t = gH/c^2 \rightarrow (t - t_0)/t = gH/c^2$   
 Plugging in the numbers for the GPS satellite:  
 $(t - t_0)/t = (9.8\text{m/s}^2) (3.6\text{E}7 \text{ m}) / (3\text{E}8 \text{ m/s})^2 = 3.9\text{E}-9$

2. By how many seconds will the clock on a GPS satellite lead an identical clock on the ground after one day in orbit?

The fractional change affects each interval of time, so we multiply the fractional change by the duration, in this case, a day. To get the answer in seconds, we also need to convert by the number of seconds in a day. The final answer is:

$$(3.9\text{E}-9)(1 \text{ day})(24 \text{ hours/day})(3600 \text{ seconds/hour}) = 3.4\text{E}-4 \text{ seconds}$$

To see a comparison of special and general relativistic effects, see Math Exploration 10.2.

3. Due to special relativistic effects, the fractional change in the time  $(t - t_0)/t_0$  for a GPS satellite is about 1 part in ten billion ( $1 \times 10^{-10}$ ). How much smaller is this than the general relativistic effect?

- To compare, take a ratio:
- GR time dilation/SR time dilation =  $3.9\text{e}-9/1\text{E}-10 = 39$
- The effect due to special relativity is nearly 40 times smaller, or the effect of general relativity is about 40 bigger, for a GPS at this distance in Earth's gravitational acceleration.

### Questions

1.

2.