9.5: Applications of Spacetime

Learning Objectives

- You will explore aspects of the feasibility of interstellar travel.
- You will understand that events that are simultaneous in one frame are not necessarily simultaneous in another.
- You will understand that events are real, that the same events happen in both frames, but that observers in different frames disagree about the spacetime coordinates of the events.

What Do You Think: Interstellar Travel

In this section, we will see several applications of the ideas introduced in the previous sections. These examples illustrate how our thinking must be modified when we are dealing with objects traveling close to the speed of light, such that the relativistic gamma factor differs appreciably from 1.

9.5.1: Interstellar Travel

Movies and on television often depict journeys between the stars. As we discussed in previous chapters, the stars are extraordinarily far away. Even the closest star to the Sun is about 4 light-years distant. We can use special relativity to investigate the practicality of such a journey and decide if the depictions of space travel from Hollywood are anything near realistic, and whether such trips are at all feasible.

Investigating Interstellar Space Travel

In this activity, you will get a chance to calculate some realistic situations involving space travel.

Questions:

VACATION TO SIRIUS?

In the distant future, when space travel is advanced, your descendants have 3 weeks of vacation time and want to visit the star Sirius, in honor of their favorite *Harry Potter* character. Sirius is 8.6 light-years away.

You might find the gamma graph helpful when thinking about how time is measured differently for observers moving at high relative velocities.

USE GRAPH

9.5.2: The Twin Paradox

One of the apparent inconsistencies in special relativity arises when thinking about extensions to interstellar trips, such as the ones described above. In the previous activity, we took an imaginary trip to Sirius at half light-speed. Imagine that after arriving at Sirius, we turn our spaceship around and return to Earth. This leg of the trip will be identical to the first because there is nothing special about the direction we travel. Due to time dilation, we will again measure a shorter time on our own wristwatch (15 years) than our friends back on Earth, who will again measure 17.2 years for our return trip. However, if the total time that they measure for the round trip is 34.4 years, and the time we measure is smaller (30 years), then we will be younger than we would have been had we foregone the trip and remained on Earth.

This scenario is often called the Twin Paradox because it can be devised in such a way that the space traveler has a twin sibling who remains on Earth. Upon returning to Earth, the traveling twin is younger than the twin who remained. But, that is not the paradox. The paradox is that, since the effects of special relativity are symmetric, both twins can equally claim that they themselves were at rest the whole time, while the other twin was in motion. Thus, they can both equally claim to be the older twin after the trip has been completed. However, this paradox, like most, is the result of not thinking carefully about the situation. Can you think of a reason why special relativity might not work in this case?

To complete a round trip from Earth to Sirius and back, it is necessary to violate one of the conditions of special relativity, namely, that all observations are made in reference frames that move uniformly. This means that neither frame may accelerate. However, that is not the case for the scenario described. For the traveling twin to reverse direction and return to Earth, the spaceship must slow down, stop, and then accelerate back up to half light-speed. We ignored the acceleration parts in our initial phrasing of the trip, but of course, this is a cheat. We cannot change our state of motion from rest to moving without undergoing some acceleration.

Nor can we complete a round trip to another star without accelerating, by virtue of the simple fact that we must turn around to return to our starting point. Once we accelerate, we have stepped out of the realm of special relativity, at least as far as we have been thinking about it. More than that, we have broken the symmetry that exists between nonaccelerating frames.

You most certainly have some experience with this sort of broken symmetry. Think of a time when you were riding in a car on a curving highway. You could feel yourself being pushed around as the car (hopefully) followed the highway. Similarly, if you have ever been forced to stop rapidly, you will recall feeling the car slowing beneath you as you continue moving forward toward the dashboard, perhaps held back by your seat belt. In both situations, it is quite clear that you do not “remain at rest” in the frame of the car. You are slammed back and forth, you fly forward or backward.

Contrast this scenario with the case of two cars moving down a straight, smooth freeway. One car travels at 60 mph, while the other travels past it at 65 mph. If the road is smooth enough, passengers in either car might have the illusion of standing still, while the other car glides slowly past.

The same is true for our imagined space traveler and Earthbound twin. The twin on Earth can claim to have always been in an unaccelerated frame; we will ignore gravity for the moment, and Earth's motion. The traveling twin can absolutely not claim to have been always at rest. This is because, at some point, the spaceship had to turn around; that twin had to change from one inertial frame to another. So, in this case, there is an absolute way to tell which of them was not in uniform motion the entire time, and the symmetry is broken.

The traveling twin will always turn out to have aged less than the stationary twin. We need general relativity to examine how clocks tick in accelerating frames. If we looked in detail we would see that the difference between the clocks can be made extremely large, arbitrarily so. In that case, the traveling twin might age only a few months, whereas on Earth, millions of years could have passed for the twin who stayed behind.

If you think carefully about the expression for the spacetime interval $s^2 = \Delta t^2 - \Delta x^2$ you can probably convince yourself that the time is always greatest for the stationary observer. What is Δx for an observer who is always at rest (an observer who is not moving!) with respect to some reference frame? How does this compare to Δx for an observer who is not at rest relative to that frame? What does this imply about the time part (Δt) of the interval for each observer?

9.5.3: Simultaneous, Says Who?

We will explore one more example of how relativity requires us to abandon our usual ideas of space and time. This also takes the form of a paradox, which is resolved only by considering carefully the situation we will illustrate.

Imagine that you are watching a train approach a barn in a rail yard. Like many barns, this one has a door at each end. Further, imagine that the train tracks lead directly through the barn, passing in one door and then out the opposite one. The illustration in Figure 9.14 shows the action as the train approaches the barn. We will assume for this problem that the barn is 100 meters long.

Figure 9.14: In the figure, a train is approaching a barn at a high rate of speed. The tracks pass through the barn, so in a few moments the train will enter the barn and then pass out the opposite side. Credit: NASA/SSU/Aurore Simonnet

Imagine that this is a special high-speed train that can move at speeds close to the speed of light. In this case, we can imagine that the train moves relative to an observer on the ground (and the barn) at 0.7c. Under these circumstances, we know that the train will appear to be shrunk along the direction of its motion by an amount given by the relativistic gamma factor, with $v = 0.7c$. The gamma factor in this case is 1.4 (check this number if you like). The length of the train as seen in the rest frame of the barn will be 1.4 times shorter than in its own frame.

Now further assume that the left door of the barn, the one nearest the oncoming train, is initially open and that the other door is closed. As the train passes into the barn, a rail-yard worker manages to close the left door just as the back of the train passes into the barn. We will call the moment when the back of the train is fully within the barn Event A.

At the *same time*, another worker opens the right-side door to let the train pass out just as the front of the train reaches the right side of the barn. We will call the act of the front of the train reaching the rightmost door Event B. The barn is apparently just long enough to contain the moving train, because for just an instant, the train is inside the barn with both doors closed, and the ends of the train coincide with the left and right of the barn. This situation is shown in Figure 9.15.

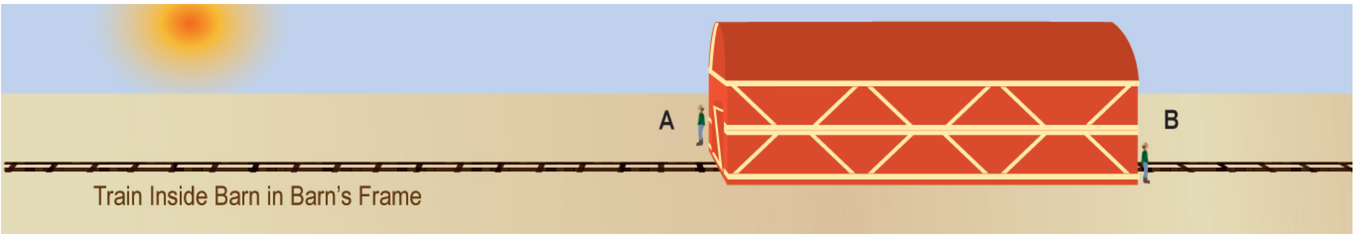

Figure 9.15: In the reference frame of the barn, the train appears to be instantaneously completely within the barn. This view shows the moment right after the rail-yard worker at the left has closed the barn door (Event A), and just before the worker on the right opens the door to let the train pass out (Event B). Credit: NASA/SSU/Aurore Simonnet

Now imagine these events as seen by the passengers on the train. Since the train appears to be 100 m long in the barn's frame, we can deduce that it must be 140 m long in its own frame; recall that it must be a factor of gamma longer. According to an observer in that reference frame, the train is not moving at all. Instead, the barn is rushing toward him at 0.7c. The barn will thus be shorter by a gamma factor of 1.4 than in its own frame. The barn's rest length is 100 m, so its length must be 100 m / 1.4 = 71 meters in the frame of the train. If this calculation is true, how can the train possibly be inside the barn with both doors closed? After all, in the train's frame, the train is 140 meters long, but the barn is only about 71 m long. Figure 9.16 summarizes the problem of a barn that is too short to contain the train from the viewpoint of the train.

Figure 9.16: In the reference frame of the train, the barn is much too short to contain the train. In fact, the train is about twice as long as the barn is deep. How can relativity reconcile these two points of view? Credit: NASA/SSU/Aurore Simonnet

Clearly, this is a paradox. In one frame, the train fits completely within the barn, while in the other frame, it cannot. But, both observers must agree on events. If an event happens in one frame, it must happen in all frames. What is the resolution to this puzzle? To answer that question, we can use the *Spacetime Diagram Tool*.

SIMULTANEITY AND THE TRAIN IN THE BARN

Play Activity

1. To solve the mystery of the train in the barn, reset the *Spacetime Diagram Tool* and create two events, one for Event A, when the back of the train coincides with the left side of the barn, and another for Event B, when the front of the train coincides with the right side of the barn.

- In the rest frame of the barn, these events occur at the same time, since the train fits entirely within the barn. Call this time t = 0. It is convenient to locate Event A at the origin.
- If you make each tick-mark of measure on the graph be 10 meters rather than one meter , then Event B will be located at the x = 10 point along the x-axis. (The axes only extend out to about 30, so it works better to rescale in this way.)

2. Create axes for the frame of the train, which is moving at 0.7 c.

From the activity, we see that the solution to the paradox is to realize that two events that appear to be simultaneous to one observer do not necessarily appear to be simultaneous to an observer in another frame. In our example, from the point of view of passengers on the train, the front of the train reaches the right side of the barn, and the rail-yard worker flings open the door (Event B), before the back of the train reaches the left side of the barn and the other worker slams shut that door (Event A). The resolution of the paradox comes in realizing that, in the train's rest frame, only after the right door has opened and the front of the train has moved out of the barn does the back of the train enter the barn, allowing the worker to close the left-side door of the barn (Figure 9.17).

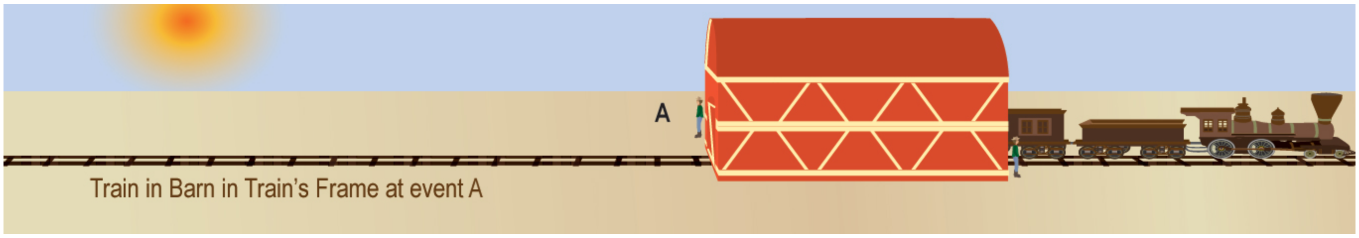

Figure 9.17: In the reference frame of the train, the back of the train has not reached the left of the barn when the front of the train reaches the right of the barn. Thus, event B occurs before event A in the reference frame of the train. Credit: NASA/SSU/Aurore Simonnet

The idea of absolute time, time that is shared by everyone everywhere regardless of their state of motion, is probably the most difficult bias that we must shed if we are to understand special relativity. We have a lifetime of experience built upon the apparent absolute nature of time. This experience is based on an illusion. It only works because we are not accustomed to dealing with objects that move close to the speed of light. The passage of time, the simultaneity of two events, the lengths of objects—all of these things are malleable, and our perceptions of them depend vitally on our state of motion.