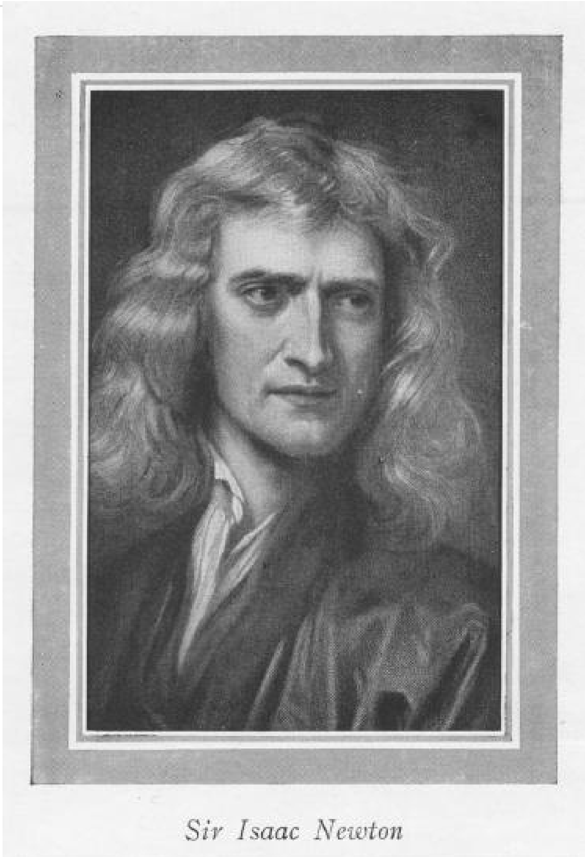
3.5: Isaac Newton (1642-1724) and the Laws of Motion



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Few people have made as many contributions to science as Sir Isaac Newton. He experimented with optics to determine that white light could be separated in the colors of the rainbow. He invented the reflecting telescope and the mathematics of calculus. But it is his studies of motion and gravity for which he most remembered. Newton studied natural philosophy at Trinity College, Cambridge and later became a professor at Cambridge, becoming the Lucasian Chair of Mathematics at Cambridge, one of the most prestigious positions at the university.

Newton's great insight was that the same laws that govern the motion of objects on Earth also govern objects in the Solar System and beyond. No longer would the heavens be regarded as mysterious bodies moved by unseen hands, but as real objects that obey the same laws of physics we do here on Earth. In 1687, Newton published his *Philosophiæ Naturalis Principia Mathematica* (Mathematical Principles of Natural Philosophy) which became the foundation of classical mathematics. Newton's Principia outlined his three laws of motion, which in modern terminology, are as follows:

Newton's First Law of Motion: An object at rest will remain at rest while an object in motion will continue in motion with a constant velocity unless acted on by outside force.

Place a ball on a flat, level table. It will not move unless you give it a nudge. This much was consistent with Aristotle's idea that objects naturally come to rest. But Newton figured that a moving object would continue moving so long as an outside force does not interfere with its motion. If we had an infinite, frictionless surface and gave the ball a push, it would roll forever. In the real world, a rolling ball slows down, not because its nature is to come to rest, but because friction with the table's surface acts as a force to slow it down. Newton's first law is sometimes referred to as the Law of Inertia, where inertia is an object's resistance to changes in its motion.

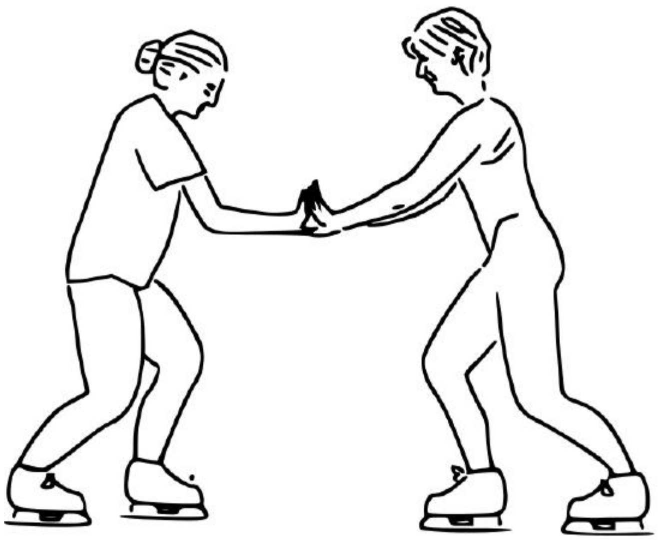
Newton's Second Law of Motion: When a force acts on an object, its acceleration is inversely proportional to its mass.

This gives the classic equation of $a = F/m$ or $F = ma$, where F is the force acting on the object, a is the acceleration or the rate of the change in motion of the object, and m is the object's mass. The unit of force is therefore the $kg\cdot m/s^2$ or the newton (N), in honor of Isaac Newton.

Newton's Third Law of Motion: When object A exerts a force on object B, object B exerts an equal and opposite force on object A. For every force, there is always an equal and opposite reaction force.

It is through the Third Law that rockets can function. A rocket does not launch itself by pushing against the ground. It launches by burning a fuel, which produces, hot expanding gases. The force of the gas escaping the nozzle produces a reaction force in the opposite direction that pushes the rocket upwards.

Newton's laws state that the ball will remain motionless until attacked on by a force from the kicker. Also, as the soccer player exerts a force on the ball, the ball will exert an equal and opposite force on the soccer player.



Two skaters pushing against each other will exert equal and opposite forces on each other.

Newton's Third Law explains how rockets move without having a surface to "push off" of.

https://www.nasa.gov/audience/foredu...-rocketry.html

We use several terms to describe an object’s motion. For example, **speed** is defined as the rate at which an object moves. In metric units, speed is often given in units of meters per second (m/s) or kilometers per second (km/s). On the other hand, **velocity** is defined as both the magnitude of the speed with a specific direction. So, if we describe an object as moving 10 m/s, we are saying its speed is 10 m/s with information about which direction it is moving. If we say the object moving 10 m/s, due north, now we are describing its velocity with both a magnitude (10 m/s) and a direction (north). Velocity is a vector, which is a quantity that has bother magnitude and direction.

Acceleration is the rate change of velocity. Like velocity, acceleration is a vector and can describe any change in an objects rate of motion, whether in magnitude, direction, or both. Physicists also use acceleration to describe the slowing down of an object (negative acceleration). In metric units, acceleration is often given in meters per second squared (m/s^2).

Earth’s gravity accelerates all objects toward the center of the planet. This acceleration of gravity, g, is the same for all objects, not counting friction or air resistance. Near the surface of the Earth, the value of $g = 9.8\ m/s^2$. Galileo’s experiments with the inclined plane demonstrated that g is the same for all objects, regardless of their mass and Newton later expanded on this principle with his law of universal gravity.

Another important term is **momentum**, or a measure of an object’s motion. Mathematically, momentum is equal to an object’s mass times its velocity. A net force, then, will act on an object to change its momentum, resulting in an acceleration or change in velocity.

A rotating object or an orbiting object has the property known as **angular momentum**. Angular momentum describes the motion of all spinning or revolving objects. For an orbiting object like a planet, its angular momentum is equal to its mass times its velocity times the radius of its orbit. Changing angular momentum requires a **torque**, which is equal to the force times its distance from the axis of rotation.

Before a collision, the system has a certain amount of momentum, based on the masses and speeds of the two objects.

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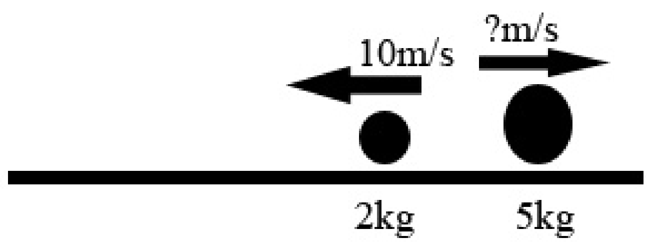

After the collision, the total momentum of the system remains the same.

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As a skater pulls her arms closer to her body, her moment of inertia decreases and her rotational speed increases.

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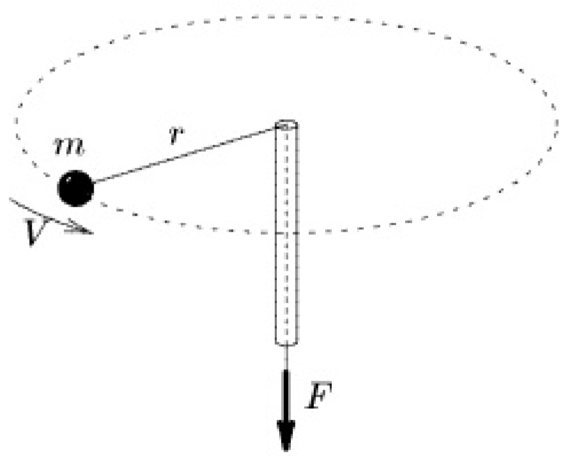
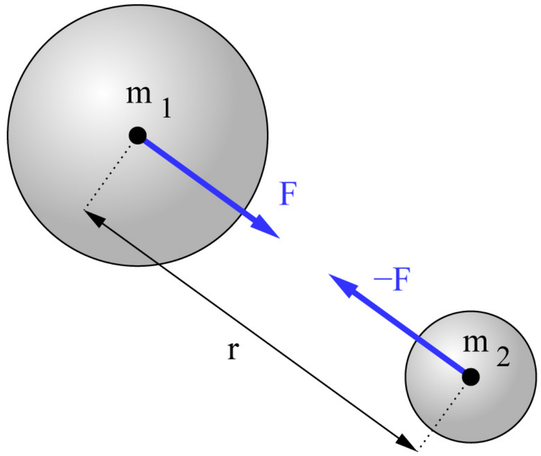


A rotating object has angular momentum and centripetal force that holds it in its path.

LP~commonswiki; MomAng2.jpg

Another important principle is the difference between **mass** and **weight**. Mass is defined as the amount of material “stuff” an object contains, whereas weight is the force of gravity acting on the object. Mass is generally a constant. An object with a mass of 1 kg will be 1 kg whether its on the Earth, on the Moon, or in space. However, an object with a weight of 1 N on the Earth will only weigh 1/6 N on the Moon. In space, it would weightless. Note however, that there is gravity in space. Object in orbit around the Earth is still subject to the Earth’s gravity, however, it is said to be in **free fall**. An object in free fall is weightless.

Objects and people in freefall, such as on board the International Space Station, are weightless.



Newton's Law of Universal Gravitation states that the attractive force between two objects is proportional to the product of their masses and inversely proportional to the square of the distance between them.

"File:Newtons-law-of-universal-gravitation-two-masses.svg" by MikeRun is licensed under CC BY-SA 4.0;

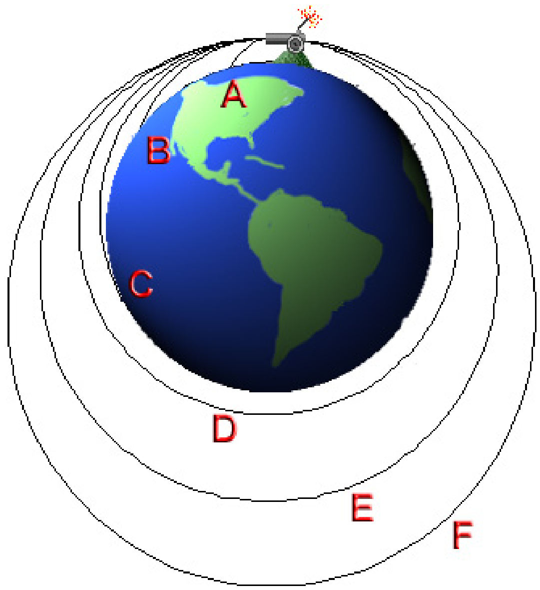
On Earth, the acceleration of gravity becomes readily apparent. Throw a ball and it will travel in a curved path. It will continue moving with the horizontal velocity you gave it, but it will first move vertically if you gave it an upward velocity. The vertical component of its velocity will slow down as the acceleration of gravity acts on it. At the pinnacle of its arc, its vertical velocity reaches zero and then it begins accelerating down until it reaches the ground. This curved path of throw objects and falling objects inspired Newton to formulate his Law of Universal Gravity. The equation for his law of gravity is:

$$F = GM_1M_2/r^2$$

Where F is the force of gravity between two objects. M_1 and M_2 are the respective masses of the two objects. The value r is the distance between their centers and G is the Universal Gravitational constant where $G = 6.67408 \times 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$.

Apply the law of gravity, a planet stays in orbit based on two factors. The force of gravity between the Sun and planet pull on each other. Since the Sun is many times more massive than the planet, the planet is pulled towards the Sun’s center. Meanwhile, the inertia from the planet’s motion keeps it moving forward. The combination of these two factors creates a centripetal acceleration, or an acceleration towards the center. The planet therefore moves around the Sun. We can think of an orbit as a form of free fall, where the object is continuously “falling” toward the Sun, but its forward motion causes it to also “miss” it. A planetary orbit is therefore just like the curved path of the thrown ball where the radius of the curve of its motion is greater than the radius of the Sun.

Newton proposed a model using a cannon. Firing a cannon causes the cannon ball to travel in a curved path. The more powerful a cannon, the further the ball traveled before it hit the ground. Newton imagined that if he had a cannon powerful enough, the curve of the ball’s path would become greater than the radius of the Earth. This would put the ball in orbit, an object in free fall that never reaches the ground as its momentum carried along its orbital path.



Newton's cannon: A ball fired from a cannon travels in a curved path. If you could give a cannonball a high enough velocity, its curved path would carrying around the world, putting it into orbit.

FrankH at English Wikipedia/Public domain; OrbitingCannonBalls.jpg

Newton's law of gravity describes orbits a little differently than Kepler's laws do. Recall that Kepler's First Law stated that planets orbit in ellipses with the Sun at one focus. Kepler derived his law empirically. The elliptical path fit the data he inherited from Tycho, but he did not have any idea what forces governed this motion. Because Newton's law describes the force between two objects at the product of their masses, both objects orbit around their common **center of mass**.

Put two objects of equal mass on a balance. The two will perfectly balance each other at a point equidistant from each other. This point is called the center of the mass of the system. If you add more mass to one side of the balance, the center of mass will shift, moving closer to end with more mass. The Sun and Earth are balanced by their mutual attraction. Because the Sun is much more massive than the Earth, the center of mass of the Earth-Sun system is very closer the center of the Sun. So, while the Earth makes a wide orbit around the center of mass, the Sun only makes a tiny "wobble" around this same point. This wobble also accounts for the slight change in distance between the Earth and the Sun between aphelion and perihelion while Kepler's law assumes a stationary Sun.

While Kepler discovered his third law of planetary motion empirically, we can derive it from Newton's laws mathematically. First, we start with the second law and put in the value of centripetal acceleration:

$$F = ma = mv^2/r$$

Where F is the net force on the planet, m is its mass, v is its average speed, and r is its average distance from the Sun. Next, we set it equal to the force of gravity:

$$GmM/r^2 = mv^2/r$$

Where G is the universal gravitational constant and M is the mass of the Sun. The m's cancel out, as well as one r, leaving:

$$GM/r = v^2$$

Next, using the definition of period, P, being the time for one complete orbit, we find that the average speed, v, is equal to the circumference divided by the period (we're assuming a circular orbit since we're following Newton's law of gravity in which the planet and Sun revolve around their common center of mass).

$$v = 2\pi r/P$$

Substituting this value for v in the equation above yields:

$$GM/r = 4\pi^2 r^2/P^2$$

Solving for P and replacing r with a, the semimajor axis, as we are now considering an elliptical orbit, gives us:

$$P^2 = 4\pi^2 a^3/GM$$

Since 4, π, G, and M are all constants, we have a relationship in which the square of the period is proportional to the cube of the semimajor axis, just as predicted by Kepler’s third law.

Newton’s law of gravitation and Kepler’s laws do not just predict elliptical orbits. An elliptical orbit is simply a bound orbit, where the planet orbits the star indefinitely. There can also be unbound orbits that follow parabolic or hyperbolic paths as they make a close approach to a strong source of gravity. Such unbound orbits can be used to accelerate a satellite to higher or lower velocities by dipping into a planet’s gravity to “borrow” a little bit of energy.

We can also use Newton’s law of gravity to demonstrate the principle discovered by Galileo that all objects experience the same acceleration, regardless of their math. For example, a falling object will experience an acceleration as defined by Netwon’s second law:

$$a = F/m_1$$

Where $m_1$is the object’s mass. The force of gravity is given again as:

$$F = GM_e m_1/r^2$$

Where M_e is the mass of the Earth Putting this value for force into the second law equation gives us:

$$a = F/m_1 = GM_e m_1/r^2 m_1$$

Note that the value for the mass of the object cancels out, leaving a value for acceleration as:

$$a = GM_e/r^2$$

Thus, the acceleration due to gravity is independent of a body’s mass.

Of course, the Solar System does not contain just two bodies. The planets pull on each other and their respective moons also pull on the planets. These actions may cause perturbations as they tug on each other. This can cause deviations from paths predicted by Kepler’s third law. In fact, it was the perturbations on Uranus’ orbit from Neptune’s gravity that led to its discovery.