8.1: Prelude to Quantum Statistical Mechanics

There are two kinds of particles from the point of view of statistics, bosons and fermions. The corresponding statistical distributions are called the **Bose-Einstein distribution** and the **Fermi-Dirac distribution**. Bosons have the property that one can have any number of particles in a given quantum state, while fermions obey the Pauli exclusion principle which allows a maximum of only one particle per quantum state. Any species of particles can be put into one of these two categories. The natural question is, of course, how do we know which category a given species belongs to; is there an a priori way to know this? The answer to this question is yes, and it constitutes one of the deep theorems in quantum field theory, the so-called spin-statistics theorem. The essence of the theorem, although this is not the precise statement, is given as follows.

Theorem 8.1.1: Spin-Statistics Theorem

Identical particles with integer values for spin (or intrinsic angular momentum) are bosons, they obey Bose-Einstein statistics.

$$\text{Identical particles with spin (or intrinsic angular momentum)} = \left(n + \frac{1}{2}\right) \tag{8.1.1}$$

where n is an integer, obey the Pauli exclusion principle and hence they are fermions and obey Fermi-Dirac statistics.

Thus, among familiar examples of particles, photons (which have spin = 1), phonons (quantized version of elastic vibrations in a solid), atoms of He^4, etc. are bosons, while the electron (spin = $\frac{1}{2}$), proton (also spin $-\frac{1}{2}$), atoms of He^3, etc. are fermions. In all cases, particles of the same species are identical.