6.4: On the us of Involutions

The existence of the three involutions (see Equations A.1.1 above), provides a great deal of flexilbity. However, the most efficient use of these concepts calls for some care.

For any matrix of $\mathcal{A}_2$,

$$A^{-1}=\frac{\tilde{A}}{|A|} \quad |A|=\frac{1}{2}\operatorname{Tr}(A\tilde{A}) \tag{6.4.1}$$

In the case of Hermitian matrices we have two alternatives:

$$k_0 r_0-\vec{k}\cdot\vec{r}=\frac{1}{2}\operatorname{Tr}(K\tilde{R}) \tag{6.4.2}$$

or

$$k_0 r_0-\vec{k}\cdot\vec{r}=\frac{1}{2}\operatorname{Tr}(K\bar{R}) \tag{6.4.3}$$

It will appear, however from later discussions, that the complex reflection of Equation 6.4.3 is more appropriate to describe the transition from contravariant to covariant entities.

A case in point is the formal representation of the mirroring of a four-vector in a plane with the normal along $\hat{x}_1$. We have

$$\begin{aligned} K' &=\sigma_1\bar{K}\sigma_1=\sigma_1\left(k_0 1-k_1\sigma_1-k_2\sigma_2-k_3\sigma_3\right)\sigma_1 \\ &=\sigma_1^2\left(k_0 1-k_1\sigma_1+k_2\sigma_2+k_3\sigma_3\right) \\ &=k_0 1-k_1\sigma_1+k_2\sigma_2+k_3\sigma_3 \end{aligned} \tag{6.4.4}$$

More generally the mirroring in a plane with normal x is achieve by means of the operation

$$K'=\hat{a}\cdot\vec{\sigma}\bar{K}\hat{a}\cdot\vec{\sigma} \tag{6.4.5}$$

Again, we could have chosen $\tilde{K}$ instead of $\bar{K}$.

However, Eq (22) generalizes to the inversion of the electromagnetic six-vector $\vec{f}=\vec{E}+i\vec{B}$:

$$\left(\vec{E}'+i\vec{B}'\right)\cdot\vec{\sigma}=\overline{(\vec{E}+i\vec{B})\cdot\vec{\sigma}}=(-\vec{E}+i\vec{B})\cdot\vec{\sigma} \tag{6.4.6}$$

This relation takes into account the fact that $\vec{E}$ is a polar and $\vec{B}$ an axial vector.