

## 6.3: Typical Examples

### Example 1

$$K' = HKH, \quad H = \exp\left(\frac{\mu}{2} \hat{h} \cdot \vec{\sigma}\right) \quad (6.3.1)$$

$$\vec{k} = \vec{k}_{\parallel} + \vec{k}_{\perp} \quad \vec{k}_{\parallel} = (\vec{k} \cdot \hat{h}) \hat{h}$$

By using (6a) and (7b):

$$\vec{k}_{\parallel} \cdot \vec{\sigma} H = H \vec{k}_{\parallel} \cdot \vec{\sigma}, \quad \vec{k}_{\perp} \cdot \vec{\sigma} H = H^{-1} \vec{k}_{\perp} \cdot \vec{\sigma} \quad (6.3.2)$$

$$\vec{k}'_{\parallel} = \vec{k}_{\parallel} = k \hat{h}$$

$$\begin{aligned} (k'_0 + \vec{k}'_{\parallel} \cdot \vec{\sigma}) &= H^2 (k_0 + \vec{k}_{\parallel} \cdot \vec{\sigma}) \\ &= (\cosh \mu + \sinh \mu \hat{h} \cdot \vec{\sigma}) (k_0 + \vec{k}_{\parallel} \cdot \vec{\sigma}) \end{aligned} \quad (6.3.3)$$

$$\begin{aligned} k'_0 &= k_0 \cosh \mu + k \sinh \mu \\ k' &= k_0 \sinh \mu + k \cosh \mu \end{aligned} \quad (6.3.4)$$

### Example 2

$$K' = UKU^{-1}, \quad U = \exp\left(-i \frac{\phi}{2} \hat{u} \cdot \vec{\sigma}\right) \quad (6.3.5)$$

$$\vec{k} = \vec{k}_{\parallel} + \vec{k}_{\perp} \quad \vec{k}_{\parallel} = (\vec{k} \cdot \hat{u}) \hat{u}$$

$$\vec{k}_{\parallel} \cdot \vec{\sigma} U^{-1} = U^{-1} \vec{k}_{\parallel} \cdot \vec{\sigma}, \quad \vec{k}_{\perp} \cdot \vec{\sigma} U^{-1} = U \vec{k}_{\perp} \cdot \vec{\sigma} \quad (6.3.6)$$

$$\vec{k}'_{\parallel} = \vec{k}_{\parallel} \quad (6.3.7)$$

$$\begin{aligned} \vec{k}'_{\perp} \cdot \vec{\sigma} &= \left( \cos \frac{\phi}{2} 1 - i \sin \frac{\phi}{2} \hat{u} \cdot \vec{\sigma} \right)^2 \vec{k}_{\perp} \cdot \vec{\sigma} \\ &= (\cos \phi 1 - i \sin \phi \hat{u} \cdot \vec{\sigma}) \vec{k}_{\perp} \cdot \vec{\sigma} \\ \vec{k}'_{\perp} &= \cos \phi \vec{k}_{\perp} + \sin \phi \hat{u} \times \vec{k}_{\perp} \end{aligned} \quad (6.3.8)$$

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