

6.1: Useful formulas

$$A = a_0 1 + \vec{a} \cdot \vec{\sigma} \tilde{A} = a_0 1 - \vec{a} \cdot \vec{\sigma} A^\dagger = a_0^* 1 + \vec{a}^* \cdot \vec{\sigma} \tilde{A} = \tilde{A}^\dagger = a_0^* 1 - \vec{a}^* \cdot \vec{\sigma}$$

$$\frac{1}{2} \text{Tr}(A) = a_0, \quad |A| = a_0^2 - \vec{a}^2 1 \frac{1}{2} \text{Tr}(A \tilde{A}) \quad (6.1.1)$$

$$\frac{1}{2} \text{Tr}(A \tilde{B}) = a_0 b_0 - \vec{a} \cdot \vec{b} \quad (6.1.2)$$

$$A^{-1} = \frac{\tilde{A}}{|A|} \quad \text{for} \quad |A| = 1 : A^{-1} = \tilde{A} \quad (6.1.3)$$

$$(\vec{a} \cdot \vec{\sigma})(\vec{b} \cdot \vec{\sigma}) = \vec{a} \cdot \vec{b} 1 + i(\vec{a} \times \vec{b}) \cdot \vec{\sigma} \quad (6.1.4)$$

$$\text{For } \vec{a} \parallel \vec{b} \quad \frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3} \quad \vec{a} \times \vec{b} = 0 \quad (6.1.5)$$

$$(\vec{a} \cdot \vec{\sigma})(\vec{b} \cdot \vec{\sigma}) - (\vec{b} \cdot \vec{\sigma})(\vec{a} \cdot \vec{\sigma}) = [(\vec{a} \cdot \vec{\sigma}), (\vec{b} \cdot \vec{\sigma})] = 0 \quad (6.1.6)$$

$$\text{For } A = a_0 1 + \vec{a} \cdot \vec{\sigma}, \quad B = b_0 1 + \vec{b} \cdot \vec{\sigma} \quad (6.1.7)$$

$$[A, B] = 0 \quad \text{iff} \quad \vec{a} \parallel \vec{b} \quad (6.1.8)$$

$$\text{For } \vec{a} \perp \vec{b}, \quad \vec{a} \cdot \vec{b} \quad (6.1.9)$$

$$\{\vec{a} \cdot \vec{\sigma}, \vec{b} \cdot \vec{\sigma}\} \equiv (\vec{a} \cdot \vec{\sigma})(\vec{b} \cdot \vec{\sigma}) + (\vec{b} \cdot \vec{\sigma})(\vec{a} \cdot \vec{\sigma}) = 0$$

$$A(\vec{b} \cdot \vec{\sigma}) = (\vec{b} \cdot \vec{\sigma})\tilde{A} \quad (6.1.10)$$

$$U = U\left(\hat{n}, \frac{\phi}{2}\right) = \cos \frac{\phi}{2} 1 - i \sin \frac{\phi}{2} \hat{n} \cdot \vec{\sigma} = \exp\left(-i \frac{\phi}{2} \hat{n} \cdot \vec{\sigma}\right) \quad (6.1.11)$$

$$H = H\left(\hat{h}, \frac{\mu}{2}\right) = \cosh \frac{\mu}{2} 1 + \sinh \frac{\mu}{2} \hat{h} \cdot \vec{\sigma} = \exp\left(\frac{\mu}{2} \hat{h} \cdot \vec{\sigma}\right) \quad (6.1.12)$$

U unitary unimodular, H Hermitian and positive.

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