6.3: Typical Examples

Example 1

$$\begin{gathered}K' = HKH, \quad H = \exp\left(\frac{\mu}{2}\hat{h}\cdot\vec{\sigma}\right) \\ \vec{k} = \vec{k}_{\|} + \vec{k}_{\perp} \quad \vec{k}_{\|} = (\vec{k}\cdot\hat{h})\hat{h}\end{gathered} \tag{6.3.1}$$

By using (6a) and (7b):

$$\begin{aligned}\vec{k}_{\|}\cdot\vec{\sigma}H = H\vec{k}_{\|}\cdot\vec{\sigma}, \quad \vec{k}_{\perp}\cdot\vec{\sigma}H = H^{-1}\vec{k}_{\perp}\cdot\vec{\sigma} \\ \vec{k}'_{\|} = \vec{k}_{\|} = k\hat{h}\end{aligned} \tag{6.3.2}$$

$$\begin{aligned}\left(k'_0 + \vec{k}'_{\|}\cdot\vec{\sigma}\right) &= H^2\left(k_0 + \vec{k}_{\|}\cdot\vec{\sigma}\right) \\ &= (\cosh\mu + \sinh\mu\hat{h}\cdot\vec{\sigma})\left(k_0 + \vec{k}_{\|}\cdot\vec{\sigma}\right)\end{aligned} \tag{6.3.3}$$

$$\begin{aligned}k'_0 &= k_0\cosh\mu + k\sinh\mu \\ k' &= k_0\sinh\mu + k\cosh\mu\end{aligned} \tag{6.3.4}$$

Example 2

$$\begin{gathered}K' = UKU^{-1}, U = \exp\left(-i\frac{\phi}{2}\hat{u}\cdot\vec{\sigma}\right) \\ \vec{k} = \vec{k}_{\|} + \vec{k}_{\perp} \quad \vec{k}_{\|} = (\vec{k}\cdot\hat{u})\hat{u}\end{gathered} \tag{6.3.5}$$

$$\vec{k}_{\|}\cdot\vec{\sigma}U^{-1} = U^{-1}\vec{k}_{\|}\cdot\vec{\sigma}, \quad \vec{k}_{\perp}\cdot\vec{\sigma}U^{-1} = U\vec{k}_{\perp}\cdot\vec{\sigma} \tag{6.3.6}$$

$$\vec{k}'_{\|} = \vec{k}_{\|} \tag{6.3.7}$$

$$\begin{aligned}\vec{k}'_{\perp}\cdot\vec{\sigma} &= \left(\cos\frac{\phi}{2}1 - i\sin\frac{\phi}{2}\hat{u}\cdot\vec{\sigma}\right)^2\vec{k}_{\perp}\cdot\vec{\sigma} \\ &= (\cos\phi 1 - i\sin\phi\hat{u}\cdot\vec{\sigma})\vec{k}_{\perp}\cdot\vec{\sigma} \\ \vec{k}'_{\perp} &= \cos\phi\vec{k}_{\perp} + \sin\phi\hat{u}\times\vec{k}_{\perp}\end{aligned} \tag{6.3.8}$$