3.2: The Corpuscular Aspects of Light

As a first step in carrying out the problem just stated, we start with a precise, even though schematic, formulation of wave kinematics.

We consider first a spherical wave front

$$r_o^2 - \vec{r}^2 = 0 \tag{3.2.1}$$

where

$$r_o = ct \tag{3.2.2}$$

and t is the time elapsed since emission to the instantaneous wave front.

In order to describe propagation in a definite direction, say along the unit vector $\hat{k}$ we introduce an appropriately chosen tangent plane corresponding to a monochromatic plane wave.

$$k_o r_o - \vec{k} \cdot \vec{r} = 0 \tag{3.2.3}$$

with

$$k_o = \omega/c \tag{3.2.4}$$

$$\vec{k} = \frac{2\pi}{\lambda}\hat{k} \tag{3.2.5}$$

and

$$k_o^2 - \vec{k}^2 = 0 \tag{3.2.6}$$

where the symbols have their conventional meanings.

Next we postulate that radiation has a granular character, as it is expressed already in Definition 1 of Newton's Optics! However, in a more quantitative sense we state the standard quantum condition according to which a quantum of light pulse with the wave vector $\left(k_o, \vec{k}\right)$ is associated with a four-momentum.

$$(p_o, \vec{p}) = \hbar\left(k_o, \vec{k}\right) \tag{3.2.7}$$

$$p_o^2 - \vec{p}^2 = 0 \tag{3.2.8}$$

with

$$p_o = \frac{E}{c} \tag{3.2.9}$$

where E is the energy of the light quantum, or photon.

The proper coordination of the two descriptions involving spherical and plane waves, presents problems to which we shall return later. At this point it is sufficient to note that individual photons have directional properties described by a wave vector,and a spherical wave can be considered as an assembly of photons emitted isotropically from a small source.

As the next step in our procedure we argue that the photon as a particle should be associated with an object group, as introduced in Sec. 1.7. Assuming with Einstein that light velocity is unaffected by an inertial transformation, the passive kinematic group that leaves Eq's (3.2.1) - (3.2.3) invariant is the Lorentz group.

There are few if any principles in physics which are as thoroughly justified by their implications as the principle of Lorentz invariance. Our objective is to develop these implications in a systematic fashion.

In the early days of relativity the consequences of Lorentz invariance involved mostly effects of the order of $(v/c)^2$, a quantity that is small for the velocities attainable at that time. The justification is much more dramatic at present when we can refer to the operation of high energy accelerators operating near light velocity.

Yet this is not all. Lorentz invariance has many consequences which are valid even in nonrelativistic physics, but classically they require a number of independent postulates for their justification. In such cases the Lorentz group is used to achieve conceptual economy.

In-view of this far-reaching a posteriori verification of the constancy of light velocity, we need not be unduly concerned with its a priori justification. It is nevertheless a puzzling question, and it has given rise to much speculation: what was Einstein's motivation in advancing this postulate?

Einstein himself gives the following account of a paradox on which he hit at the age of sixteen:

"If I pursue a beam of light with the velocity c (velocity of light in vacuum), I should observe such a beam of light' as a spatially oscillatory electromagnetic field at rest. However, there seems to be no such thing, whether on the basis of experience or according to Maxwell's equations."

The statement could be actually even sharpened: on overtaking a travelling wave, the resulting phenomenon would simply come to rest, rather than turn into a standing wave.

However it may be, if the velocity of propagation were at all affected by the motion of the observer, it could be "transformed away." Should we accept such a radical change from an inertial transformation? At least in hindsight, we know that the answer is indeed no.

Note that the essential point in the above argument is that a light quantum cannot be transformed to rest. This absence of a preferred rest system with respect to the photon does not exclude the existence of a preferred frame defined from other considerations. Thus it has been recently suggested that a preferred frame be defined by the requirement that the $3K$ radiation be isotropic in it [Wei72].

Since Einstein and his contemporaries emphasized the absence of any preferred frame of reference, one might have wondered whether the aforementioned radiation, or some other cosmologically defined frame,might cause difficulties in the theory of relativity.

Our formulation, based on weaker assumptions, shows that such concern is unwarranted.

We observe finally, that we have considered thus far primarily wave kinematics, with no reference to the electrodynamic interpretation of light. This is only a tactical move. We propose to derive classical electrodynamics (CED) within our scheme, rather than suppose its validity.

Problems of angular momentum and polarization are also left for later inclusion.

However, we are ready to widen our context by being more explicit with respect to the properties of the four-momentum.

Eq ($3.2.5$) provides us with a definition of the four-momentum, but only for the case of the photon, that is for a particle with zero rest mass and the velocity c.

This relation is easily generalized to massive particles that can be brought to rest. We make use of the fact that the Lorentz transformation leaves the left-hand side of Eq ($3.2.5$) invariant, whether or not it vanishes. Therefore we set

$$p_o^2 - \vec{p}\cdot\vec{p} = m^2c^2 \tag{3.2.10}$$

and define the mass m of a particle as the invariant "length" of the four-momentum according to the Minkowski metric (with $c = 1$).

We can now formulate the postulate: The four-momentum is conserved. This principle includes the conservation of energy and that of the three momentum components. It is to be applied to the interaction between the photon and a massive particle and also to collision processes in general.

In order to make use of the conservation law, we need explicit expressions for the velocity dependence of the four-momentum components. These shall be obtained from the study of the Lorentz group.