6.1: Useful formulas

$A = a_0 1 + \vec{a}\cdot\vec{\sigma}\tilde{A} = a_0 1 - \vec{a}\cdot\vec{\sigma}A^{\dagger} = a_0^* 1 + \vec{a}^*\cdot\vec{\sigma}\bar{A} = \tilde{A}^{\dagger} = a_0^* 1 - \vec{a}^*\cdot\vec{\sigma}$

$$\frac{1}{2}\mathrm{Tr}(A) = a_0, \quad |A| = a_0^2 - \vec{a}^2 1 \frac{1}{2}\mathrm{Tr}(A\tilde{A}) \tag{6.1.1}$$

$$\frac{1}{2}\mathrm{Tr}(A\tilde{B}) = a_0 b_0 - \vec{a}\cdot\vec{b} \tag{6.1.2}$$

$$A^{-1} = \frac{\tilde{A}}{|A|} \quad \text{for} \quad |A| = 1 : A^{-1} = \tilde{A} \tag{6.1.3}$$

$$(\vec{a}\cdot\vec{\sigma})(\vec{b}\cdot\vec{\sigma}) = \vec{a}\cdot\vec{b}1 + i(\vec{a}\times\vec{b})\cdot\vec{\sigma} \tag{6.1.4}$$

$$\text{For } \vec{a}\|\vec{b} \quad \frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3} \quad \vec{a}\times\vec{b} = 0 \tag{6.1.5}$$

$$(\vec{a}\cdot\vec{\sigma})(\vec{b}\cdot\vec{\sigma}) - (\vec{b}\cdot\vec{\sigma})(\vec{a}\cdot\vec{\sigma}) = [(\vec{a}\cdot\vec{\sigma}), (\vec{b}\cdot\vec{\sigma})] = 0 \tag{6.1.6}$$

$$\text{For} \quad A = a_0 1 + \vec{a}\cdot\vec{\sigma}, \quad B = b_0 1 + \vec{b}\cdot\vec{\sigma} \tag{6.1.7}$$

$$[A, B] = 0 \quad \text{iff} \quad \vec{a}\|\vec{b} \tag{6.1.8}$$

$$\begin{aligned} \text{For} \quad & \vec{a}\perp\vec{b}, \quad \vec{a}\cdot\vec{b} \\ & \{\vec{a}\cdot\vec{\sigma}, \vec{b}\cdot\vec{\sigma}\} \equiv (\vec{a}\cdot\vec{\sigma})(\vec{b}\cdot\vec{\sigma}) + (\vec{b}\cdot\vec{\sigma})(\vec{a}\cdot\vec{\sigma}) = 0 \end{aligned} \tag{6.1.9}$$

$$A(\vec{b}\cdot\vec{\sigma}) = (\vec{b}\cdot\vec{\sigma})\tilde{A} \tag{6.1.10}$$

$$U = U\left(\hat{u}, \frac{\phi}{2}\right) = \cos\frac{\phi}{2}1 - i\sin\frac{\phi}{2}\hat{n}\cdot\vec{\sigma} = \exp\left(-i\frac{\phi}{2}\hat{n}\cdot\vec{\sigma}\right) \tag{6.1.11}$$

$$H = H\left(\hat{h}, \frac{\mu}{2}\right) = \cosh\frac{\mu}{2}1 + \sinh\frac{\mu}{2}\hat{h}\cdot\vec{\sigma} = \exp\left(\frac{\mu}{2}\hat{h}\cdot\vec{\sigma}\right) \tag{6.1.12}$$

U unitary unimodular, H Hermitian and positive.