1.2: Activities

Things You Will Need

All you will need for this lab is the PhET simulator, which you can run now in a separate window.

Configuring the Simulator

Begin with the "Energy" panel of the simulator. Configure it as follows:

- **Open the "Particles" and "Injection Temperature" dropdowns (the "Average Speed" and "Speed" dropdowns should already be open, but if they are not, then open them as well).**
- **Select the units for the Pressure gauge to be kilopascals (kPa).**
- **Uncheck the "Collisions" box in the "Particles" dropdown.**

A few useful features of the simulator to know about:

- All the particles injected into the chamber come in with the same kinetic energy (which is proportional to their "temperature"). The injection temperature can be selected, or by default, the injection temperature is the same as the current temperature of the gas in the chamber.
- You can inject 50 particles at a time of a single variety with the button (fully extending the pump does the same, if you are into the animation). Using the button injects one particle at a time. The similar buttons whose arrows point to the left remove particles in the same number. [These removal buttons are of little use, as they choose particles at random to remove, and may select more "hot" particles than "cold" ones, meaning that while we can add particles without changing the gas temperature, we cannot remove them without changing the gas temperature.]
- You can remove all of the particles from the chamber at once with the button.

Deducing the Simulated Gases

We will start by assuming that this simulator is designed to demonstrate the three-dimensional kinetic theory of gases. Two gases are selected (one that is "heavy," and the other that is "light"), and in an effort to be as accurate as possible, the quantitative read-outs of the simulator are based on the correct masses of these gases.

1. Your task is to determine which of the five gases listed below is represented by the light particles, and which one is represented by the heavy particles. You are of course free to look up properties of these atoms and molecules to help you make this determination.

$$H_2\;,\quad He\;,\quad N_2\;,\quad O_2\;,\quad Ne$$

Timeout for Some Math

We now turn toward the question raised in the Background Material about whether this simulator actually does represent a gas in three dimensions. The Maxwell-Boltzmann distribution function gives us a measure of the probability of finding a random particle that has a speed in a given infinitesimal range:

$$P\left(v\leftrightarrow v+dv\right)=\mathcal{D}\left(v\right)dv \tag{1.2.1}$$

This Maxwell-Boltzmann distribution $\mathcal{D}\left(v\right)$ is derived using the assumption that the particles can "randomly" change their individual velocities, *subject to the constraint* that the sum of their kinetic energies remains fixed. In other words, all collisions (with the walls of the container, and with each other) must be elastic. Our simulator is programmed to exactly follow this constraint. Something else that is hidden within the distribution function is the dimension of the space within which the particles are constrained. What this means is that if you select a random particle from a gas, the probability that it will have a speed between v and $v+dv$ is different if that gas is constrained to two dimensions than if it occupies three dimensions. This is evidenced by the two distribution functions:

$$\begin{aligned}\mathcal{D}_{2d}\left(v\right)&=\alpha\;v\,e^{-\frac{1}{2}\alpha v^2}\\ \mathcal{D}_{3d}\left(v\right)&=\sqrt{\frac{2}{\pi}}\;\alpha^{\frac{3}{2}}\;v^2\;e^{-\frac{1}{2}\alpha v^2}\end{aligned} \tag{1.2.2}$$

In both cases, the constant α is defined in terms of the temperature of the gas and the mass of the particles (and the Boltzmann constant k_B):

$$\alpha \equiv \frac{m}{k_B T} \tag{1.2.3}$$

Multiplying each of these by dv gives the probability of selecting a random particle with a speed that falls in the range $v \leftrightarrow v+dv$ for its particular dimension. For what follows below, the following *definite gaussian integrals* will be useful:

$$\begin{aligned} \int_0^\infty x\ e^{-\beta x^2}\ dx &= \tfrac{1}{2}\beta^{-\frac{2}{2}} \\ \int_0^\infty x^2\ e^{-\beta x^2}\ dx &= \tfrac{1}{4}\sqrt{\pi}\ \beta^{-\frac{3}{2}} \\ \int_0^\infty x^3\ e^{-\beta x^2}\ dx &= \tfrac{1}{2}\beta^{-\frac{4}{2}} \\ \int_0^\infty x^4\ e^{-\beta x^2}\ dx &= \tfrac{3}{8}\sqrt{\pi}\ \beta^{-\frac{5}{2}} \end{aligned} \tag{1.2.4}$$

2. The probability of finding a random particle with *any* speed whatsoever (from $v=0$ to $v=\infty$) must obviously equal 1, and this is the sum of the probabilities of all the tiny ranges (which is of course an integral). Show that each of the two distributions given satisfies this important requirement.

The value of having a probability distribution is that it can be used to compute averages. An average of several possible values that can arise randomly is the product of each value and it corresponding probability, summed over all of the possible values:

$$\text{average of } x = \langle x\rangle = x_1 \cdot P(x_1) + x_2 \cdot P(x_2) + \dots \tag{1.2.5}$$

When the values lie on a continuum (like particle speeds), this sum becomes an integral. So multiplying every possible speed v by its corresponding probability $\mathcal{D}(v)\,dv$, and adding all of these products together gives a means for computing the average speed:

$$\langle v\rangle = \int_0^\infty v\,\mathcal{D}(v)\,dv \tag{1.2.6}$$

We can similarly take averages of other quantities, most notably the square of the velocity:

$$\langle v^2\rangle = \int_0^\infty v^2\ \mathcal{D}(v)\,dv \tag{1.2.7}$$

3. Determine the relationship (i.e. the ratio) between the average and rms particle speeds for both a two and three-dimensional gas.

Simulated Evaluation

We are at last ready to examine the simulator. The average speed comes as a direct read-out in the simulator. The rms speed we can determine using the trick indicated in the Background Material – if we start with the "no collisions" configuration given above, the particle speeds remain constant and uniform. Then, without changing anything else about the gas, we can turn on collisions to introduce randomness.

4. Determine whether the simulator replicates a two or three-dimensional gas. This consists of several parts:
   a. Run the simulator "without randomness" to get the particle rms speed.
   b. Introduce randomness and get the average speed. Does the rms value change when you do this? Why or why not?
   c. The average speed fluctuates with time, so you will need to "sample" these numbers at random times, say by closing your eyes, waiting a second, and clicking the pause button, then writing down the number. Then un-pause the simulator and repeat several times. An average of these samples should be a good measure of the "true" average speed.
   d. The data obtained from the sampling method just described also allows you to calculate an uncertainty interval for the measurement. For a refresher on how to compute standard deviation, review this from your very first 9HA lab.
   e. Determine whether the relationship between the average and rms speeds is (to within uncertainty) characteristic of a two or three-dimensional gas.