1.1: Background Material

Text References

- kinetic theory of gases
- particle speed in a gas

Understanding the Simulator

To fully grasp what is going on with this lab, a deeper understanding of this simulator is needed. Its intent is to mimic the behavior of a three-dimensional gas, but the medium in which this is done (a flat screen) is two dimensions. This is certainly a visual shortcoming, but is that all, or is something more fundamental involved here? The answer is that it depends upon how you wish to use the program.

The simulator includes both numbers that are displayed and actual particle motions that obey physical laws (elastic collisions). Things get tricky because the numbers displayed can come from either plugging numbers directly into physics equations, or extracted from the results of the simulated motions. In the case of this particular simulator, the gases are modeled accurately, which is to say that the particles of each gas are imbued with their correct masses. These masses, along with a selected user setting (temperature), are used in a physics calculation that assumes three dimensions, and provides another number that is displayed (particle speed). The fact that the simulator is confined to two dimensions never comes into play here, because none of the numbers depends upon the physical nature of two-dimensional motion.

But the simulator allows us to do more, and this additional exploration depends intimately upon the statistical physics of the particle motions, and this includes the fact that they have two degrees of freedom, rather than the previously-assumed three. So although we are inclined to treat this as a three-dimensional simulator based on the purely calculated quantities (and the fact that the masses nicely match those of real gases), we find that this assumption breaks down when we let the simulator behave physically.

One might ask why one would bother to build a laboratory around such a flawed simulator. As you will see, exploring these very flaws turns out to be extremely instructive!

Average Speed vs. RMS Speed

There are many ways to compute a "characteristic" measurement of a quantities from a sample that contains many randomly-distributed such measurements. In this lab, we will be looking at the speeds of particles in a gas. We can define a "typical" speed of such a particle several ways. One way is to take a simple average of the speeds of all the particles. The simulator we are using keeps track of the speeds of all the particles at every moment in time, so computing this average is easy to do, and indeed it displays this average in real time.

Another way to define the speed of a typical particle is in terms of the *root-mean-square*, or *rms* value. This consists of averaging not the speeds of the particles, but rather the *squares* of those speeds. Of course, the average of squares is hardly characteristic of a speed, as this average doesn't even have the proper units. So to complete the calculation, we take the square root of this average. Summarizing the difference between these two measurements of a typical particle's speed:

$$\begin{aligned} v_{ave} &= \frac{1}{N}\left(v_1+v_2+\cdots+v_N\right) \\ v_{rms} &= \sqrt{\frac{1}{N}\left(v_1^2+v_2^2+\cdots+v_N^2\right)} \end{aligned} \tag{1.1.1}$$

There are a couple of important things to point out about these values when it comes to this lab:

1. Notice that if every particle has the same speed ($v_1=v_2=\cdots=v$), then these quantities come out exactly equal. At the beginning of a simulation, it is possible to ensure that all the particles start at the same speed, which means that even though the display shows the average speed per particle, we know that this is also the starting rms speed.
2. Suppose instead of measuring the speeds of the particles, we measured their *kinetic energies*. An average of these measurements would look like this (note: all the particles are the same type, and therefore have equal masses):

$$
\begin{aligned}
KE_{ave} \quad & = \frac{1}{N}(KE_1 + KE_2 + \cdots + KE_N) \\
& = \frac{1}{N}\left(\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \cdots + \frac{1}{2}mv_N^2\right) \\
& = \frac{1}{2}m\left[\frac{1}{N}\left(v_1^2 + v_2^2 + \cdots + v_N^2\right)\right] \\
& = \frac{1}{2}mv_{rms}^2
\end{aligned} \tag{1.1.2}
$$

The simulator is designed so that all of the collisions between particles are elastic, which means that no matter what happens, the total kinetic energy of all the confined particles remains fixed, which in turn assures that the rms speed of the collection remains fixed, no matter what collisions occur. It is for this reason that when we define a typical speed for a particle in a gas (at given temperature), the definition of "typical" that we use is the rms speed.

These two properties will be useful to know, because we can see what happens when we start with a gas with non-random motion (all the particles at the same speed, not affecting each other), and then "randomize" their motions by allowing them to collide with each other, separating their average and rms speeds.
