04. Applying Newton's Second Law in Translational and Rotational Form - II

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A 75 kg man is attached to a rope wrapped around a 35 kg disk-shaped pulley, with inner and outer diameters 0.60 m and 0.90 m, respectively. The man is initially at rest. The brake shoe is pressed against the pulley with a force of 500 N. The coefficient of friction between the brake shoe and the pulley is (0.9, 0.8).

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First, we have to determine whether the force applied to the brake shoe is sufficient to hold the man stationary. If it's not, we have to find the man's acceleration. To a accomplish this, we need free-body diagrams for both the man and the pulley.

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We'll use counterclockwise as the positive q-direction and down as the positive y-direction.

The pin, or axle, at the center of the pulley exerts forces in x- and y-direction, although the exact directions may be unknown. We'll just label these forces as $F_{pin\ x}$ and $F_{pin\ y}$.

The frictional force that acts on the right edge of the pulley acts in a way to prevent the man from falling (or, if he does fall, to slow down the man's fall).

To begin my analysis, I'll assuem that the man is stationary, solve for the value of the frictional force required to keep him stationary, and then determine whether this frictional force is possible given the applied force of 500 N and the coefficient of static friction.

Let's apply Newton's second law to the man in the y-direction. (Remember, we are *assuming* he is not falling.)

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Now look at Newton's second law applied to the pulley in the q-direction.

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Notice that all of the other forces acting on the pulley do *not* exert torques on the pulley. They are either located at r = 0 m or have angular orientations of f = 180°.

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Therefore, to hold the man stationary requires 490 N of friction. However,

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Therefore, the man must accelerate downward.

Now that we know the man must fall, let's write the same two equations as before, although this time the accelerations (both angular for the pulley and linear for the man) are not zero.

For the pulley:

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This assumes that the rotational inertia of a "compound" pulley, one with more than one location where a rope can be wrapped, is the same as a regular pulley, and that the outermost radius of the pulley determines the inertia.

Since the man is falling, the frictional force is now kinetic, and

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Thus our "pulley" equation becomes:

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Examining Newton's second law for the man, pic

We now have two equations with three variables. However, the angular acceleration of the pulley and the linear acceleration of the man are directly related. Since the rope that the man is attached to is wrapped 0.3 m from the center of the pulley, the man accelerates at the same rate as the tangential acceleration of a point on the pulley 0.3 m from the center. Thus,

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Using this relationship allows me to rewrite the two Newton's second law equation as:

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This pair of equations has the solution

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The man falls with this acceleration and the rope exerts this force (upward on the man and downward on the pulley).
