02. Applying the Work-Energy Relation including Rotation - I

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> *A mischievous child releases his mother's bowling ball from the top of the family's 25 m long, 15^0 above horizontal driveway. The ball rolls without slipping down the driveway and at the bottom plows into the mailbox. The 6.4 kg ball has a diameter of 24 cm.*

Let's determine the speed of the ball when it hits the mailbox. To determine this value, we can apply the work-energy relation to the ball between:

Event 1: The instant the ball is released.

Event 2: The instant the ball hits the mailbox.

For these two events, work-energy looks like this:

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where the bottom of the driveway is the zero for gravitiational potential energy and the rotational inertia of the bowling ball is taken to be that of a sphere.

Now we must carefully determine which, if any, of the forces on the bowling ball do work on the bowling ball.

First, we don't have to worry about the force of gravity. The gravitational potential energy function was developed to automatically incorporate the work done by the force of gravity.

Second, the contact force can do no work on the ball because the contact force is always perpendicular to the motion of the ball.

Finally, what about the force of friction? It does appear, at first glance, that the force of friction is applied over the entire motion of the ball down the driveway. However, let's pay closer attention to the actual point at which the force acts.

The frictional force is static in nature, because since the ball rolls without slipping down the driveway the bottom of the ball is always in static contact with the ground (i.e., there is no relative velocity between the bottom of the ball and the ground). If the bottom of the ball has a velocity of zero, then the force that acts on the bottom of the ball (static friction) can act through no distance. During the instant at which the frictional force acts on a particular point on the bottom of the ball, that point is not moving. That point on the ball only moves when it is no longer in contact with the ground, but by that time the frictional force is acting on a *different* point. To summarize (and stop saying the same thing over and over), *the force of static friction can do no work because it acts on a point that does not move.*

Thus, our equation simplifies to:

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The linear and angular velocity of the ball must be related. Since the ball rolls without slipping, the bottom of the ball has no linear velocity. Since the velocity of the bottom of the ball is the sum of velocity due to translation of the CM and the velocity due to rotation about the CM, the velocity due to rotation must be equal in magnitude to the velocity of the CM:

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Substituting this into our equation yields:

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Applying the Work-Energy Relation including Rotation - II

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The crate is descending with speed v_0 when a brake shoe is applied to the disk-shaped pulley of mass M and radius R. The coefficient of friction between the shoe and the pulley is (m_s, m_k). Determine the distance (D) the crate moves before stopping as a function of M, m, F, R, v_0, g, and the appropriate coefficient of friction.

To determine this function, let's apply the work-energy relation to both the crate and the pulley between:

Event 1: The instant the brake shoe is applied.

Event 2: The instant the crate comes to rest.

In addition to defining the two instants of interest, we'll need free-body diagrams for both the crate and the pulley.

Pulley

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To find the work on the pulley, note that $F_{contact}$ and F_{pin} do no work since the distance over which these forces act is zero. However, both F_{rope} and $F_{friction}$ do work.

If the crate falls a distance D, both of these forces act over a distance D. However, note that this displacement is in the smae direction as F_{rope} (f= 180) on the right side of the pulley but in the opposite direction on the other side of the pulley (opposite to $F_{friction}$ and therefore f = 180.) Since these forces are constant, there's no need to actually integrate:

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Note that since the brake shoe does not accelerate, the external force applied to the shoe, F, is the same magnitude as the contact force between the shoe and the pulley.

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Crate

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where the final position of the crate is the zero for gravitational potential energy.

The two equations can be added together to yield:

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Thus, in order for D to have a physical (positive) value, the frictional force on the pulley must be greater in magnitude than the force of gravity on the crate, which agrees with common sense. Note that the numerator is simply the kinetic energy of the pulley plus crate. The larger this sum, the larger the distance needed to stop the crate's fall, which again agrees with common sense.
