03. Using the Calculus Another Example

1. Another Example

Another Example

Investigate the scenario described below.

> *A sports car can accelerate from rest to a speed of 40 m/s while traveling a distance of 200 m. Assume the acceleration of the car can be modeled as a decreasing linear function of time, with a maximum acceleration of 10.4 m/s^2.*

Event 1: The car begins from rest	Event 2: The car reaches 40 m/s
$t_1 = 0$ s	$t_2 =$
$r_1 = 0$ m	$r_2 = 200$ m
$v_1 = 0$ m/s	$v_2 = 40$ m/s
$a_1 = 10.4$ m/s^2	$a_2 =$

Between event 1 and 2, the car's acceleration can be modeled by a generic linear function of time, or

$$a(t) = At + B$$

Since the acceleration is decreasing, the maximum value occurs at t = 0 s,

$$a(0) = A(0) + B = 10.4$$
$$B = 10.4$$

Since we don't know the value of the acceleration at t_2, or even the value of t_2, we can't determine A, and all we can currently say about the acceleration function is that it is given by:

$$a(t) = At + 10.4$$

Nonetheless, we can still integrate the acceleration to determine the velocity,

$$v(t) = \int a(t)dt$$
$$v(t) = \int (At + 10.4)dt$$
$$v(t) = \frac{1}{2}At^2 + 10.4t + C$$

Since we know v = 0 m/s when t = 0 s, we can determine the integration constant:

$$v(0) = \frac{1}{2}A(0)^2 + 10.4(0) + C = 0$$
$$C = 0$$

We also know that v = 40 m/s at t_2, so:

$$v(t_2) = \frac{1}{2}At_2{}^2 + 10.4t_2 = 40$$

This equation can't be solved, since it involves two unknowns. However, if we can generate a second equation involving the same two unknowns, we can solve the two equations simultaneously. This second equation must involve the position function of the car:

$$r(t) = \int v(t)dt$$
$$r(t) = \int (\frac{1}{2}At^2 + 10.4t)dt$$
$$r(t) = \frac{1}{6}At^3 + 5.2t^2 + D$$

Since we know r = 0 when t = 0 s, we can determine the integration constant:

$$r(0)=\frac{1}{6}A(0)^3+5.2(0)^2+D=0$$
$$D=0$$

We also know that r = 200 m at t_2, so:

$$r(t_2)=\frac{1}{6}At_2{}^3+5.2t_2{}^2=200$$

These two equations,

$$\frac{1}{2}At_2{}^2+10.4t_2=40$$
$$\frac{1}{6}At_2{}^3+5.2t_2{}^2=200$$

can be solved by substitution (or by using a solver). Solve the first equation for A, and substitute this expression into the second equation. This will result in a quadratic equation for t_2. The solution is t_2 = 7.57 s, the time for the car to reach 40 m/s. Plugging this value back into the original equations allows you to complete the description of the car's motion.
