# 03. Applying the Impulse-Momentum Relations (Linear and Angular) - I



## Applying the Impulse-Momentum relations (Linear and Angular) - I

*A bowling ball of mass M and radius R leaves a bowler's hand with CM velocity $v_0$ and no rotation. The ball skids down the alley before it begins to roll without slipping. The coefficient of friction is ($m_s$, $m_k$). Determine the elapsed time (T) before the ball begins to roll without slipping as a function of M, R, $v_0$, g, and the appropriate coefficient of friction.*

To determine this function, let's apply the impulse-momentum relations to the ball between:

Event 1: The instant the ball leaves the bowler's hand.

Event 2: The instant the ball begins to roll without slipping.

pic

Since the ball both translates and rotates, we must write both the linear and rotational forms of the impulse-momentum relation. Remember, the rotation of the bowling ball is modeled to be about an axis throuh the CM. (Since all the forces are constant, we'll write the relations without the use of the integral.)

**x-direction (linear momentum) q-direction(angular momentum)**

pic and pic

The final CM velocity and final angular velocity are related because at this instant the ball begins to roll without slipping. When the ball rolls without slipping, the bottom of the ball is in static contact with the ground, i.e., it has no linear velocity. Since the velocity of the bottom of the ball is the sum of the velocity due to translation of the CM and the velocity due to rotation about the CM, the velocity due to rotation must be equal in magnitude to the velocity of the CM:

pic

Substituting this into our angular equation yields:

pic

Plugging this into the linear equation:

pic

Thus, the faster you throw the ball or the smaller the kinetic coefficient of friction, the longer it will take for the ball to begin to roll.

## Applying the Impulse-Momentum Relations (Linear and Angular) - II

*A 30 kg child running at 4.0 m/s leaps onto the outer edge of an initially stationary merry-go-round. The merry-go-round is a flat disk of radius 2.4 m and mass 70 kg. Ignore the frictional torque in the bearings of the merry-go-round.*

Let's imagine we're interested in determining the final angular velocity of the merry-go-round after the child has safely come to rest on its surface. To determine this angular velocity, we can apply the impulse-momentum relations to both the child and the merry-go-round between:

Event 1: The instant before the child lands on the merry-go-round

Event 2: The instant after the child comes to rest on the merry-go-round.

pic

The child and the merry-go-round interact via some unknown magnitude force. (Contrary to the free-body diagram, this force does not necessarily act on the child's disembodied head.) There are also other forces due to gravity and supporting structures that act on both the child and the merry-go-round, however, these forces are in the vertical direction and supply no torque.

***Child Merry-Go-Round***

**x-direction q-direction**

pic and pic

The final velocity of the child and final angular velocity of the merry-go-round are related because at this instant the child is at rest (hanging on to) the merry-go-round. Therefore,

pic

Substituting this into our angular equation yields:

pic

By Newton's Third Law, the force of the child on the merry-go-round and the force of the merry-go-round on the child must be equal in magnitude. Therefore our two equations can summed and the impulses cancel. This gives:

pic

The child is slowed from 4 m/s to 1.85 m/s by jumping onto the merry-go-round. The merry-go-round, however, is accelerated from rest to:

pic