02. Concepts and Principles 2

Charge and Charge Density

Macroscopic objects are normally neutral (or very close to neutral) because they contain equal numbers of protons and electrons. All charged objects are charged because of either an excess of lack of electrons. (It's much easier to add or remove electrons from an object than trying to add or remove the protons tightly bound inside the nuclei of its atoms.) Thus, the electric charge of any object is always an integer multiple of the electric charge on an electron.

Because of its fundamental importance, the magnitude of the charge on an electron is termed the *elementary charge* and denoted by the symbol e. In a purely logical world, the charge on any object would be reported as a multiple of e. However, since the charge on a macroscopic system can be *many* multiples of e, a more user-friendly unit, the coulomb (C), typically used to quantify electric charge. In this system,

$$e = 1.6 \times 10^{-19} C \tag{1}$$

Thus, you can consider the charge on an electron as an incredibly small fraction of a coulomb, or a coulomb of charge as an incredibly large number of electrons.

In many applications, in addition to knowing the total charge on an object you will need to know how the charge is distributed. The distribution of charge on an object can be defined in several different ways. For objects such as wires or other thin cylinders, a *linear charge density*, l, will often be defined. This is the amound of charge per unit length of the object. if the charge is uniformly distributed, this is simply

$$\lambda = \frac{Q}{L} \tag{2}$$

where Q is the total charge on the object[1] and L its total length. However, if the charge density varies over the length of the object, its value at any point must be defined as the ratio of the charge on a differential element at that location to the length of the element:

$$\lambda(x) = \frac{dQ}{dx} \tag{3}$$

For objects such as flat plates or the surfaces of cylinders and spheres, a *surface charge density*, s, can be defined. This is the amount of charge per unit area of the object. If the charge is uniformly distributed, this is

$$\sigma = \frac{Q}{Area} \tag{4}$$

or if the charge density varies over the surface:

$$\sigma(x, y) = \frac{dQ(x, y)}{dx dy} \tag{5}$$

Lastly, for objects that have charge distributed throughout their volume, a *volume charge density*, r, can be defined. This is the amount of charge per unit volume of the object. If the charge is uniformly distributed, this is

$$\rho = \frac{Q}{V} \tag{6}$$

or if the charge density varies inside the object:

$$\rho = \frac{dQ}{dV} \tag{7}$$

To add to the confusion, you must realize that the same object can be described as having two different charge densities. For example, consider a plastic rod with charge distributed throughout its volume. Obviously, the charge per unit volume, r, can be defined for this object. However, you can also define the object as having linear charge density, l, reporting the amount of charge present per meter of length. These two parameters will have different values but refer to exactly the same object.

Perfect Conductors and Perfect Insulators

Determining how electric charges in real materials respond to electric fields in incredibly important but also incredibly complicated. In light of this, we will restrict ourselves to two types of hypothetical materials.

In a perfect conductor, electric charges are free to move without any resistance to their motion. Metals provide a reasonable approximation ot perfect conductors, although, of course, in a real metal a small amount of resistance to motion is present. When I refer to a material as a metal, we will approximate the metal as a perfect conductor.

In a perfect insulator electric charges can not move, regardless of the amount of force applied to them. Many materials act as insulators, but all real materials experience electrical breakdown if the forces acting on charges become so great that the charges begin to move. When I refer to an insulating material, like plastic, for example, we will approximate the material as a perfect insulator.

Since electric fields create forces on electric charges, there can not be static electric fields present inside perfect conductors. If a field was present inside a perfect conductor, the charges inside the conductor would feel an electric force and hence move in response to that force. They would continue to move until they redistributed themselves inside of the conductor in such a way as to cancel the electric field. The system could not reach equilibrium as long as an electric field was present. This re-arranging process would typically occur *very* quickly and we will always assume our analysis takes place after it is completed.

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[1]I will use lowercase q to designate the charge on a point particle and uppercase Q to designate the total charge distributed on macroscopic objects.