03. Applying Newton's Second Law in Translational and Rotational Form - I

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Investigate the scenario described below.

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The robotic arm at right consists of a pair of hydraulic jacks, each attached 10 cm from the pin "elbow". The elbow is 15 cm from the extreme edge of the 40 kg, 0.90 m long "forearm". The forearm and attached 90 kg load are held stationary at an angle of 20°below horizontal. Since the jacks primarily exert forces through extension, the front, or "biceps", jack is exerting negligible force.

To study the dynamics of this situation, we will first need a free-body diagram of the forearm (and the attached load).

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The forces due to the jack, the rope, and gravity should not need much explanation. However, the forces due to the pin elbow may need some explanation.

First, the free-body diagram drawn at left is of the forearm, not including the pin that serves as the elbow. Thus, the pin is external to the object of interest, and thus its interactions with the object are forces that must be indicated on the diagram. If you were to include the pin as part of the forearm, then the upper part of the arm would be external to the object of interest (pin plus forearm) and its interactions with the "pin plus forearm" would have to be indicated as forces on the diagram. Both of these approaches are completely valid.

Second, the direction of the force of the pin on the forearm may not be obvious. Does the pin push straight down on the forearm, pull up on the elbow, or push at some unspecified angle? The only thing that's clear is that this force is directed somewhere in the xy-plane. If the force is in the xy-plane, then it must have components along both the x- and y-axis (although one of these components may turn out to have a magnitude of zero). Thus, to handle the generality of the situation you should include both an x- and y-component for the force of the pin on the forearm. For convenience, I'll draw these forces as pointing in the positive x and y directions. (If my guess is wrong, the mathematics will tell me.)

Now that we have a free-body diagram, we can apply Newton's second law, both the linear form (in the x- and y-directions) and the rotational form (in the q-direction).

x-direction y-direction

pic and pic

Since the forearm is held stationary, both a_x and a_y are equal to zero. Also, F_{rope} is equal to the force of gravity on the load, 882 N.

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Obviously, we need another equation in order to determine the two unknown forces. The obvious choice is Newton's second law in rotational form.

Before we begin, we should determine the rotational inertia for a thin rod (the closest thing to a forearm in our table) rotated about an axis not at its CM. A thin rod rotated about its CM has rotational inertia $^1/_{12}$ ML^2. We are interested in its rotational inertia about an axis not at its CM, so we must use the parallel-axis theorem with r_{CM} = 0.30 m. (The CM is at the center of the forearm, 0.45 m from either end. Since the elbow is 0.15 m from one end, the distance between the elbow and the CM is 0.30 m.) Thus,

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We must also determine the torque due to each force. (Remember, the angle, f, in the relation for torque is the angle between r (oriented along the forearm) and F.)

Pin, Y Jack Gravity Rope

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Finally, let's apply Newton's second law in rotational form, paying careful attention to the algebraic sign of each torque. Note that our coordinate system indicates that all torques acting counterclockwise are positive. Therefore, all torques acting clockwise are negative.

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Since the forearm is held stationary, a is equal to zero.

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The jack must exert a force of 7796 N to hold the forearm stationary.

Plugging this value into our y-equation yields:

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The pin must exert a force of 9070 N upwards, since the mathematics determined that the algebraic sign of $F_{Pin\ y}$ was positive.