7.8: Applications of Magnetic Forces and Fields

Learning Objectives

By the end of this section, you will be able to:

- Explain how a mass spectrometer works to separate charges
- Explain how a cyclotron works

Being able to manipulate and sort charged particles allows deeper experimentation to understand what matter is made of. We first look at a mass spectrometer to see how we can separate ions by their charge-to-mass ratio. Then we discuss cyclotrons as a method to accelerate charges to very high energies.

Mass Spectrometer

The mass spectrometer is a device that separates ions according to their charge-to-mass ratios. One particular version, the Bainbridge mass spectrometer, is illustrated in Figure $7.8.1$. Ions produced at a source are first sent through a velocity selector, where the magnetic force is equally balanced with the electric force. These ions all emerge with the same speed $v = E/B$ since any ion with a different velocity is deflected preferentially by either the electric or magnetic force, and ultimately blocked from the next stage. They then enter a uniform magnetic field B_0 where they travel in a circular path whose radius **R** is given by Equation 11.4.2, $r = \frac{mv}{qB}$. The radius is measured by a particle detector located as shown in the figure.

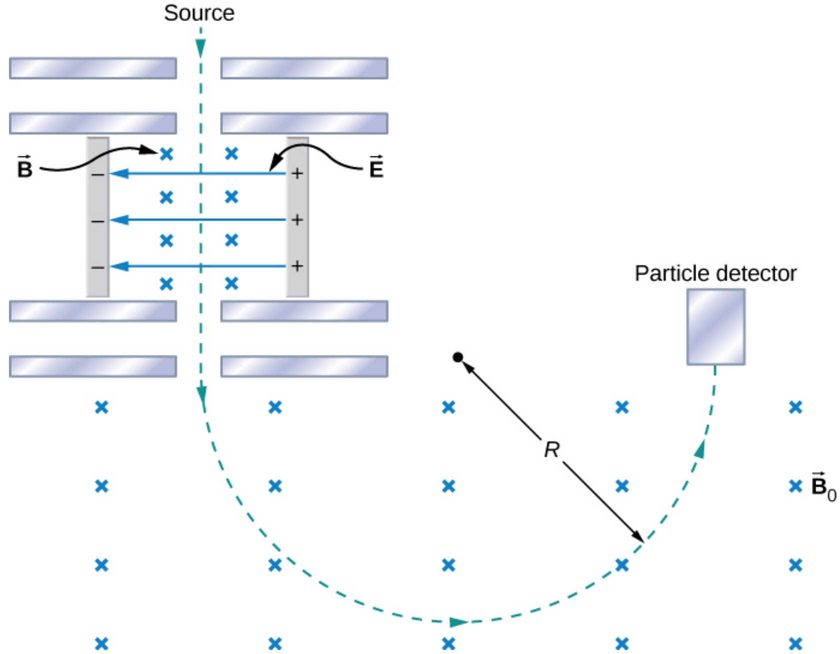

Figure $7.8.1$: A schematic of the Bainbridge mass spectrometer, showing charged particles leaving a source, followed by a velocity selector where the electric and magnetic forces are balanced, followed by a region of uniform magnetic field where the particle is ultimately detected.

The relationship between the charge-to-mass ratio **q/m** and the radius **R** is determined by combining Equation 11.4.2 and Equation 11.7.2:

$$\frac{q}{m} = \frac{E}{BB_0R}.$$

Since most ions are singly charged $(q = 1.6 \times 10^{-10} C)$, measured values of **R** can be used with this equation to determine the mass of ions. With modern instruments, masses can be determined to one part in 10^8.

An interesting use of a spectrometer is as part of a system for detecting very small leaks in a research apparatus. In low-temperature physics laboratories, a device known as a dilution refrigerator uses a mixture of He-3, He-4, and other cryogens to reach temperatures well below 1 K. The performance of the refrigerator is severely hampered if even a minute leak between its various components occurs. Consequently, before it is cooled down to the desired temperature, the refrigerator is subjected to a leak test. A small quantity of gaseous helium is injected into one of its compartments, while an adjacent, but supposedly isolated, compartment is connected to a high-vacuum pump to which a mass spectrometer is attached. A heated filament ionizes any helium atoms evacuated by the pump. The detection of these ions by the spectrometer then indicates a leak between the two compartments of the dilution refrigerator.

In conjunction with gas chromatography, mass spectrometers are used widely to identify unknown substances. While the gas chromatography portion breaks down the substance, the mass spectrometer separates the resulting ionized molecules. This technique is used with fire debris to ascertain the cause, in law enforcement to identify illegal drugs, in security to identify explosives, and in many medicinal applications.

Cyclotron

The cyclotron was developed by E.O. Lawrence to accelerate charged particles (usually protons, deuterons, or alpha-particles) to large kinetic energies. These particles are then used for nuclear-collision experiments to produce radioactive isotopes. A cyclotron is illustrated in Figure $7.8.2$. The particles move between two flat, semi-cylindrical metallic containers D1 and D2, called **dees**. The dees are enclosed in a larger metal container, and the apparatus is placed between the poles of an electromagnet that provides a uniform magnetic field. Air is removed from the large container so that the particles neither lose energy nor are deflected because of collisions with air molecules. The dees are connected to a high-frequency voltage source that provides an alternating electric field in the small region between them. Because the dees are made of metal, their interiors are shielded from the electric field.

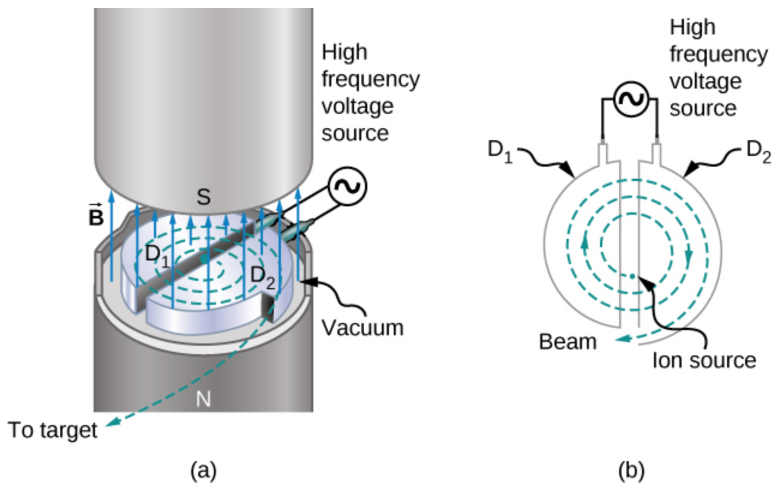

Figure $7.8.2$: The inside of a cyclotron. A uniform magnetic field is applied as circulating protons travel through the dees, gaining energy as they traverse through the gap between the dees.

Suppose a positively charged particle is injected into the gap between the dees when D2 is at a positive potential relative to D1. The particle is then accelerated across the gap and enters D1 after gaining kinetic energy **qV**, where **V** is the average potential difference the particle experiences between the dees. When the particle is inside D1, only the uniform magnetic field $\vec{B}$ of the electromagnet acts on it, so the particle moves in a circle of radius

$$r = \frac{mv}{qB} \tag{7.8.1}$$

with a period of

$$T = \frac{2\pi m}{qB}. \tag{7.8.2}$$

The period of the alternating voltage course is set at **T**, so while the particle is inside D1, moving along its semicircular orbit in a time **T**/2, the polarity of the dees is reversed. When the particle reenters the gap, D1 is positive with respect to D2, and the particle is again accelerated across the gap, thereby gaining a kinetic energy **qV**. The particle then enters D2, circulates in a slightly larger circle, and emerges from D2 after spending a time **T**/2 in this dee. This process repeats until the orbit of the particle reaches the boundary of the dees. At that point, the particle (actually, a beam of particles) is extracted from the cyclotron and used for some experimental purpose.

The operation of the cyclotron depends on the fact that, in a uniform magnetic field, a particle's orbital period is independent of its radius and its kinetic energy. Consequently, the period of the alternating voltage source need only be set at the one value given by Equation 7.8.2. With that setting, the electric field accelerates particles every time they are between the dees.

If the maximum orbital radius in the cyclotron is **R**, then from Equation 7.8.1, the maximum speed of a circulating particle of mass **m** and charge **q** is

$$v_{max} = \frac{qBR}{m}.$$

Thus, its kinetic energy when ejected from the cyclotron is

$$\frac{1}{2}mv_{max}^2 = \frac{q^2B^2R^2}{2m}.$$

The maximum kinetic energy attainable with this type of cyclotron is approximately 30 MeV. Above this energy, relativistic effects become important, which causes the orbital period to increase with the radius. Up to energies of several hundred MeV, the relativistic effects can be compensated for by making the magnetic field gradually increase with the radius of the orbit. However, for higher energies, much more elaborate methods must be used to accelerate particles.

Particles are accelerated to very high energies with either linear accelerators or synchrotrons. The linear accelerator accelerates particles continuously with the electric field of an electromagnetic wave that travels down a long evacuated tube. The Stanford Linear Accelerator (SLAC) is about 3.3 km long and accelerates electrons and positrons (positively charged electrons) to energies of 50 GeV. The synchrotron is constructed so that its bending magnetic field increases with particle speed in such a way that the particles stay in an orbit of fixed radius. The world's highest-energy synchrotron is located at CERN, which is on the Swiss-French border near Geneva. CERN has been of recent interest with the verified discovery of the Higgs Boson (see Particle Physics and Cosmology). This synchrotron can accelerate beams of approximately 10^{13} protons to energies of about 10^3 GeV.

Example 7.8.1: Accelerating Alpha-Particles in a Cyclotron

A cyclotron used to accelerate alpha-particles $(m = 6.64 \times 10^{-27} kg,\ q = 3.2 \times 10^{-19} C)$ has a radius of 0.50 m and a magnetic field of 1.8 T. (a) What is the period of revolution of the alpha-particles? (b) What is their maximum kinetic energy?

Strategy

1. The period of revolution is approximately the distance traveled in a circle divided by the speed. Identifying that the magnetic force applied is the centripetal force, we can derive the period formula.
2. The kinetic energy can be found from the maximum speed of the beam, corresponding to the maximum radius within the cyclotron.

Solution

1. By identifying the mass, charge, and magnetic field in the problem, we can calculate the period:

$$T = \frac{2\pi m}{qB} = \frac{2\pi(6.64 \times 10^{-27} kg)}{(3.2 \times 10^{-19} C)(1.8T)} = 7.3 \times 10^{-8} s.$$

2. By identifying the charge, magnetic field, radius of path, and the mass, we can calculate the maximum kinetic energy:

$$\frac{1}{2}mv_{max}^2 = \frac{q^2B^2R^2}{2m} = \frac{(3.2 \times 10^{-19} C)^2(1.8T)^2(0.50m)^2}{2(6.65 \times 10^{-27} kg)} = 6.2 \times 10^{-12} J = 39\ MeV.$$

Exercise 7.8.1

A cyclotron is to be designed to accelerate protons to kinetic energies of 20 MeV using a magnetic field of 2.0 T. What is the required radius of the cyclotron?

Solution

0.32 m