12.S: Waves (Summary)

Key Terms

antinode	location of maximum amplitude in standing waves
constructive interference	when two waves arrive at the same point exactly in phase; that is, the crests of the two waves are p
destructive interference	when two identical waves arrive at the same point exactly out of phase; that is, precisely
fixed boundary condition	when the medium at a boundary is fixed in place so it cannot mov
free boundary condition	exists when the medium at the boundary is free to move
fundamental frequency	lowest frequency that will produce a standing wave
intensity (I)	power per unit area
interference	overlap of two or more waves at the same point and time
linear wave equation	equation describing waves that result from a linear restoring force of the medium; any function that is a s wave moving in the positive x-direction or the negative x-direction with a consta
longitudinal wave	wave in which the disturbance is parallel to the direction of propaga
mechanical wave	wave that is governed by Newton’s laws and requires a medium
node	point where the string does not move; more generally, nodes are where the wave disturbanc
normal mode	possible standing wave pattern for a standing wave on a string
overtone	frequency that produces standing waves and is higher than the fundamenta
pulse	single disturbance that moves through a medium, transferring energy but
standing wave	wave that can bounce back and forth through a particular region, effectively bec
superposition	phenomenon that occurs when two or more waves arrive at the same
transverse wave	wave in which the disturbance is perpendicular to the direction of prop
wave	disturbance that moves from its source and carries energy
wave function	mathematical model of the position of particles of the medium
wave number	$\frac{2\pi}{\lambda}$
wave speed	magnitude of the wave velocity
wave velocity	velocity at which the disturbance moves; also called the propagation v
wavelength	distance between adjacent identical parts of a wave

Key Equations

Wave speed	$v = \frac{\lambda}{T} = \lambda f$
Linear mass density	$\mu = \frac{mass\ of\ the\ string}{length\ of\ the\ string}$
Speed of a wave or pulse on a string under tension	$\lvert v \rvert = \sqrt{\frac{F_T}{\mu}}$
Speed of a compression wave in a fluid	$v = \sqrt{\frac{B}{\rho}}$
Resultant wave from superposition of two sinusoidal waves that are identical except for a phase shift	\[y_{R} (x,t) = \Bigg[2A \cos \left(\dfrac{\phi}{2}\right) \Bigg] \sin \left(kx - \omega t -
Wave number	$k = \equiv \frac{2\pi}{\lambda}$
Wave speed	$v = \frac{\omega}{k}$
Periodic wave	$y(x, t) = A \sin(kx \mp \omega + \phi)$
Phase of a wave	$kx \mp \omega t + \phi$
Linear wave equation	$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v_w^2} \frac{\partial^2 y(x,t)}{\partial t^2}$
Power in a wave for one wavelength	$P_{ave} = \frac{E_\lambda}{T} = \frac{1}{2}\mu A^2 \omega^2 \frac{\lambda}{T} = \frac{1}{2}\mu A^2 \omega^2 v$
Intensity	$I = \frac{P}{A}$

Intensity for a spherical wave	$I=\frac{P}{2\pi r^2}$
Equation of a standing wave	\[y (x,t) = [2A \sin (kx)] \cos (\omega t)$$
Wavelength for symmetric boundary conditions	$\lambda_n=\frac{2}{n}L, \qquad n=1,2,3,4,5\ldots$
Frequency for symmetric boundary conditions	$f_n=n\frac{v}{2L}=nf_1, \qquad n=1,2,3,4,5\ldots$

Summary

16.1 Traveling Waves

- A wave is a disturbance that moves from the point of origin with a wave velocity v.
- A wave has a wavelength λ, which is the distance between adjacent identical parts of the wave. Wave velocity and wavelength are related to the wave's frequency and period by v = $\frac{\lambda}{T}$ = λf.
- Mechanical waves are disturbances that move through a medium and are governed by Newton's laws.
- Electromagnetic waves are disturbances in the electric and magnetic fields, and do not require a medium.
- Matter waves are a central part of quantum mechanics and are associated with protons, electrons, neutrons, and other fundamental particles found in nature.
- A transverse wave has a disturbance perpendicular to the wave's direction of propagation, whereas a longitudinal wave has a disturbance parallel to its direction of propagation.

16.2 Mathematics of Waves

- A wave is an oscillation (of a physical quantity) that travels through a medium, accompanied by a transfer of energy. Energy transfers from one point to another in the direction of the wave motion. The particles of the medium oscillate up and down, back and forth, or both up and down and back and forth, around an equilibrium position.
- A snapshot of a sinusoidal wave at time t = 0.00 s can be modeled as a function of position. Two examples of such functions are y(x) = A sin (kx + ϕ) and y(x) = A cos (kx + ϕ).
- Given a function of a wave that is a snapshot of the wave, and is only a function of the position x, the motion of the pulse or wave moving at a constant velocity can be modeled with the function, replacing x with x $\mp$ vt. The minus sign is for motion in the positive direction and the plus sign for the negative direction.
- The wave function is given by y(x, t) = A sin (kx − ωt + ϕ) where k = $\frac{2\pi}{\lambda}$ is defined as the wave number, $\omega=\frac{2\pi}{T}$ is the angular frequency, and ϕ is the phase shift.
- The wave moves with a constant velocity v_w, where the particles of the medium oscillate about an equilibrium position. The constant velocity of a wave can be found by v = $\frac{\lambda}{T}$ = $\frac{\omega}{k}$.

16.3 Wave Speed on a Stretched String

- The speed of a wave on a string depends on the linear density of the string and the tension in the string. The linear density is mass per unit length of the string.
- In general, the speed of a wave depends on the square root of the ratio of the elastic property to the inertial property of the medium.
- The speed of a wave through a fluid is equal to the square root of the ratio of the bulk modulus of the fluid to the density of the fluid.
- The speed of sound through air at T = 20 °C is approximately v_s = 343.00 m/s.

16.4 Energy and Power of a Wave

- The energy and power of a wave are proportional to the square of the amplitude of the wave and the square of the angular frequency of the wave.
- The time-averaged power of a sinusoidal wave on a string is found by P_{ave} = $\frac{1}{2}\mu A^2\omega^2 v$, where μ is the linear mass density of the string, A is the amplitude of the wave, ω is the angular frequency of the wave, and v is the speed of the wave.
- Intensity is defined as the power divided by the area. In a spherical wave, the area is A = 4πr^2 and the intensity is I = $\frac{P}{4\pi r^2}$. As the wave moves out from a source, the energy is conserved, but the intensity decreases as the area increases.

16.5 Interference of Waves

- Superposition is the combination of two waves at the same location.
- Constructive interference occurs from the superposition of two identical waves that are in phase.
- Destructive interference occurs from the superposition of two identical waves that are 180° (π radians) out of phase.
- The wave that results from the superposition of two sine waves that differ only by a phase shift is a wave with an amplitude that depends on the value of the phase difference.

16.6 Standing Waves and Resonance

- A standing wave is the superposition of two waves which produces a wave that varies in amplitude but does not propagate.
- Nodes are points of no motion in standing waves.
- An antinode is the location of maximum amplitude of a standing wave.
- Normal modes of a wave on a string are the possible standing wave patterns. The lowest frequency that will produce a standing wave is known as the fundamental frequency. The higher frequencies which produce standing waves are called overtones.

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