15.S: Oscillations (Summary)

Key Terms

amplitude (A)	maximum displacement from the equilibrium position of an object oscillating around the equilibrium position
critically damped	condition in which the damping of an oscillator causes it to return as quickly as possible to its equilibrium position without oscillating back and forth about this position
elastic potential energy	potential energy stored as a result of deformation of an elastic object, such as the stretching of a spring
equilibrium position	position where the spring is neither stretched nor compressed
force constant (k)	characteristic of a spring which is defined as the ratio of the force applied to the spring to the displacement caused by the force
frequency (f)	number of events per unit of time
natural angular frequency	angular frequency of a system oscillating in SHM
oscillation	single fluctuation of a quantity, or repeated and regular fluctuations of a quantity, between two extreme values around an equilibrium or average value
overdamped	condition in which damping of an oscillator causes it to return to equilibrium without oscillating; oscillator moves more slowly toward equilibrium than in the critically damped system
period (T)	time taken to complete one oscillation
periodic motion	motion that repeats itself at regular time intervals
phase shift	angle, in radians, that is used in a cosine or sine function to shift the function left or right, used to match up the function with the initial conditions of data
physical pendulum	any extended object that swings like a pendulum
resonance	large amplitude oscillations in a system produced by a small amplitude driving force, which has a frequency equal to the natural frequency
restoring force	force acting in opposition to the force caused by a deformation
simple harmonic motion (SHM)	oscillatory motion in a system where the restoring force is proportional to the displacement, which acts in the direction opposite to the displacement
simple harmonic oscillator	a device that oscillates in SHM where the restoring force is proportional to the displacement and acts in the direction opposite to the displacement
simple pendulum	point mass, called a pendulum bob, attached to a near massless string
stable equilibrium point	point where the net force on a system is zero, but a small displacement of the mass will cause a restoring force that points toward the equilibrium point
torsional pendulum	any suspended object that oscillates by twisting its suspension

underdamped	condition in which damping of an oscillator causes the amplitude of oscillations of a damped harmonic oscillator to decrease over time, eventually approaching zero

Key Equations

Relationship between frequency and period	$$f=\frac{1}{T} \tag{15.S.1}$$
Position in SHM with ϕ = 0.00	$$x(t)=A\cos(\omega t) \tag{15.S.2}$$
General position in SHM	$$x(t)=A\cos(\omega t+\phi) \tag{15.S.3}$$
General velocity in SHM	$$v(t)=-A\omega\sin(\omega t+\phi) \tag{15.S.4}$$
General acceleration in SHM	$$a(t)=-A\omega^2\cos(\omega t+\phi) \tag{15.S.5}$$
Maximum displacement (amplitude) of SHM	$$x_{max}=A \tag{15.S.6}$$
Maximum velocity of SHM	$$\lvert v_{max}\rvert=A\omega \tag{15.S.7}$$
Maximum acceleration of SHM	$$\lvert a_{max}\rvert=A\omega^2 \tag{15.S.8}$$
Angular frequency of a mass-spring system in SHM	$$\omega=\sqrt{\frac{k}{m}} \tag{15.S.9}$$
Period of a mass-spring system in SHM	$$T=2\pi\sqrt{\frac{m}{k}} \tag{15.S.10}$$
Frequency of a mass-spring system in SHM	$$f=\frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{15.S.11}$$
Energy in a mass-spring system in SHM	$$E_{Total}=\frac{1}{2}kx^2+\frac{1}{2}mv^2=\frac{1}{2}kA^2 \tag{15.S.12}$$
The velocity of the mass in a spring-mass system in SHM	$$v=\pm\sqrt{\frac{k}{m}(A^2-x^2)} \tag{15.S.13}$$
The x-component of the radius of a rotating disk	$$x(t)=A\cos(\omega t+\phi) \tag{15.S.14}$$
The x-component of the velocity of the edge of a rotating disk	$$v(t)=-v_{max}\sin(\omega t+\phi) \tag{15.S.15}$$
The x-component of the acceleration of the edge of a rotating disk	$$a(t)=-a_{max}\cos(\omega t+\phi) \tag{15.S.16}$$
Force equation for a simple pendulum	$$\frac{d^2\theta}{dt^2}=-\frac{g}{L}\theta \tag{15.S.17}$$

Angular frequency for a simple pendulum	$\omega=\sqrt{\frac{g}{L}}$	(15.S.18)
Period of a simple pendulum	$T=2\pi\sqrt{\frac{L}{g}}$	(15.S.19)
Angular frequency of a physical pendulum	$\omega=\sqrt{\frac{mgL}{I}}$	(15.S.20)
Period of a physical pendulum	$T=2\pi\sqrt{\frac{I}{mgL}}$	(15.S.21)
Period of a torsional pendulum	$T=2\pi\sqrt{\frac{I}{\kappa}}$	(15.S.22)
Newton's second law for harmonic motion	$m\frac{d^2x}{dt^2}+b\frac{dx}{dt}+kx=0$	(15.S.23)
Solution for underdamped harmonic motion	$x(t)=A_0e^{-\frac{b}{2m}t}\cos(\omega t+\phi)$	(15.S.24)
Natural angular frequency of a mass-spring system	$\omega_0=\sqrt{\frac{k}{m}}$	(15.S.25)
Angular frequency of underdamped harmonic motion	$\omega=\sqrt{\omega_0^2-\left(\frac{b}{2m}\right)^2}$	(15.S.26)
Newton's second law for forced, damped oscillation	$-kx-b\frac{dx}{dt}+F_0\sin(\omega t)=m\frac{d^2x}{dt^2}$	(15.S.27)
Solution to Newton's second law for forced, damped oscillations	$x(t)=A\cos(\omega t+\phi)$	(15.S.28)
Amplitude of system undergoing forced, damped oscillations	$A=\frac{F_0}{\sqrt{m(\omega^2-\omega_0^2)^2+b^2\omega^2}}$	(15.S.29)

Summary

15.1 Simple Harmonic Motion

- Periodic motion is a repeating oscillation. The time for one oscillation is the period T and the number of oscillations per unit time is the frequency f. These quantities are related by $f=\frac{1}{T}$.
- Simple harmonic motion (SHM) is oscillatory motion for a system where the restoring force is proportional to the displacement and acts in the direction opposite to the displacement.
- Maximum displacement is the amplitude A. The angular frequency ω, period T, and frequency f of a simple harmonic oscillator are given by $\omega=\sqrt{\frac{k}{m}}$, T = $2\pi\sqrt{\frac{m}{k}}$, and f = $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$, where m is the mass of the system and k is the force constant.
- Displacement as a function of time in SHM is given by x(t) = Acos$\left(\frac{2\pi}{T}t+\phi\right)$ = Acos$(\omega t+\phi)$.

- The velocity is given by v(t) = -Aωsin($\omega t+\phi$) = -v_{max}sin($\omega t+\phi$), where v_{max} = Aω = A$\sqrt{\frac{k}{m}}$.
- The acceleration is given by a(t) = -Aω^2cos($\omega t+\phi$) = -a_{max}cos($\omega t+\phi$), where a_{max} = Aω^2 = A$\frac{k}{m}$.

15.2 Energy in Simple Harmonic Motion

- The simplest type of oscillations are related to systems that can be described by Hooke's law, F = −kx, where F is the restoring force, x is the displacement from equilibrium or deformation, and k is the force constant of the system.
- Elastic potential energy U stored in the deformation of a system that can be described by Hooke's law is given by U = $\frac{1}{2}$kx^2.
- Energy in the simple harmonic oscillator is shared between elastic potential energy and kinetic energy, with the total being constant:

$$E_{Total}=\frac{1}{2}kx^2+\frac{1}{2}mv^2=\frac{1}{2}kA^2=constant. \tag{15.S.30}$$

- The magnitude of the velocity as a function of position for the simple harmonic oscillator can be found by using

$$v=\sqrt{\frac{k}{m}(A^2-x^2)}. \tag{15.S.31}$$

15.3 Comparing Simple Harmonic Motion and Circular Motion

- A projection of uniform circular motion undergoes simple harmonic oscillation.
- Consider a circle with a radius A, moving at a constant angular speed ω. A point on the edge of the circle moves at a constant tangential speed of v_{max} = Aω. The projection of the radius onto the x-axis is x(t) = Acos(ωt + ϕ), where (ϕ) is the phase shift. The x-component of the tangential velocity is v(t) = −Aωsin(ωt + ϕ).

15.4 Pendulums

- A mass m suspended by a wire of length L and negligible mass is a simple pendulum and undergoes SHM for amplitudes less than about 15°. The period of a simple pendulum is T = 2$\pi\sqrt{\frac{L}{g}}$, where L is the length of the string and g is the acceleration due to gravity.
- The period of a physical pendulum T = 2$\pi\sqrt{\frac{I}{mgL}}$ can be found if the moment of inertia is known. The length between the point of rotation and the center of mass is L.
- The period of a torsional pendulum T = 2$\pi\sqrt{\frac{I}{\kappa}}$ can be found if the moment of inertia and torsion constant are known.

15.5 Damped Oscillations

- Damped harmonic oscillators have non-conservative forces that dissipate their energy.
- Critical damping returns the system to equilibrium as fast as possible without overshooting.
- An underdamped system will oscillate through the equilibrium position.
- An overdamped system moves more slowly toward equilibrium than one that is critically damped.

15.6 Forced Oscillations

- A system's natural frequency is the frequency at which the system oscillates if not affected by driving or damping forces.
- A periodic force driving a harmonic oscillator at its natural frequency produces resonance. The system is said to resonate.
- The less damping a system has, the higher the amplitude of the forced oscillations near resonance. The more damping a system has, the broader response it has to varying driving frequencies.

Contributors and Attributions

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