9.5: Exercise Problems

9.1. Generalize the reasoning of Sec. 1 to an arbitrary 1D map $q_{n+1}=f\left(q_n\right)$, with a function $f(q)$ differentiable at all points of interest. In particular, derive the condition of stability of an N-point limit cycle $q^{(1)} \rightarrow q^{(2)} \rightarrow \ldots \rightarrow q^{(N)} \rightarrow q^{(1)} \ldots$

9.2. Use the stability condition, derived in the previous problem, to analyze the possibility of the deterministic chaos in the so-called tent map, with

$$f(q)= \begin{cases}r q, & \text { for } 0 \leq q \leq 1 / 2, \\ r(1-q), & \text { for } 1 / 2 \leq q \leq 1,\end{cases} \quad \text { with } 0 \leq r \leq 2 \tag{9.5.1}$$

9.3. A dynamic system is described by the following system of differential equations:

$$\begin{aligned}
& \dot{q}_1=-q_1+a_1 q_2^3, \\
& \dot{q}_2=a_2 q_2-a_3 q_2^3+a_4 q_2\left(1-q_1^2\right) .
\end{aligned}$$

Can it exhibit chaos at some set of constant parameters a_1-a_4 ?

9.4. A periodic function of time has been added to the right-hand side of the first equation of the system considered in the previous problem. Is deterministic chaos possible now?