4.4: Energy Dissipation

Let me conclude this brief chapter with an ultra-short discussion of energy dissipation in conductors. In contrast to the electrostatics situations in insulators (vacuum or dielectrics), at dc conduction, the electrostatic energy U is “dissipated” (i.e. transferred to heat) at a certain rate $\mathscr{P} \equiv -dU/dt$, with the dimensionality of power.[17] This so-called energy dissipation may be evaluated by calculating the power of the electric field’s work on a single moving charge:

$$\mathscr{P}_1 = \mathbf{F} \cdot \mathbf{v} = q\mathbf{E} \cdot \mathbf{v}. \tag{4.37}$$

After the summation over all charges, Eq. (37) gives us the average dissipation power. If the charge density n is uniform, multiplying by it both parts of this relation, and taking into account that $qn\mathbf{v} = \mathbf{j}$, for the energy dissipation in a unit volume we get the differential form of the Joule law[18]

$$\text{General Joule law} \qquad \mathscr{I} \equiv \frac{\mathscr{P}}{V} = \frac{\mathscr{P}_1 N}{V} = \mathscr{P}_1 n = q\mathbf{E} \cdot \mathbf{v}n = \mathbf{E} \cdot \mathbf{j}. \tag{4.38}$$

In the case of the Ohmic conductivity (8), this expression may be also rewritten in two other forms:

$$\text{Joule law for Ohmic conductivity} \qquad \mathscr{I} = \sigma E^2 = \frac{j^2}{\sigma}. \tag{4.39}$$

With our electrostatics background, it is also straightforward (and hence left for the reader’s exercise) to prove that the dc current distribution in a uniform Ohmic conductor, at a fixed voltage distribution along its borders, corresponds to the minimum of the total dissipation in the sample,

$$\mathscr{P} \equiv \int_V \mathscr{I} d^3r = \sigma \int_V E^2 d^3r. \tag{4.40}$$

Reference

[17] Since this electric field and hence the electrostatic energy are time-independent, this means that the energy is replenished at the same rate from the current source(s).

[18] Named after James Prescott Joule, who quantified this effect in 1841.