5.5: Magnetic Materials

In order to form a complete system, the macroscopic Maxwell equations (109) have to be complemented with the constitutive relations describing the medium: $\mathbf{D} \leftrightarrow \mathbf{E}$, $\mathbf{j} \leftrightarrow \mathbf{E}$, and $\mathbf{B} \leftrightarrow \mathbf{H}$. The first two of them were discussed, in brief, in the last two chapters; let us proceed to the last one.

A major difference between the dielectric and magnetic constitutive relations $\mathbf{D}(\mathbf{E})$ and $\mathbf{B}(\mathbf{H})$ is that while a dielectric medium always reduces the external field, magnetic media may either reduce or enhance it. To quantify this fact, let us consider the most widespread materials – linear magnetics in that $\mathbf{M}$ (and hence $\mathbf{H}$) is proportional to $\mathbf{B}$. For isotropic materials, this proportionality is characterized by a scalar – either the magnetic permeability μ, defined by the following relation:

$$\mathbf{B} \equiv \mu \mathbf{H}, \qquad \text{Magnetic permeability} \tag{5.110}$$

or the magnetic susceptibility[48] defined as

$$\mathbf{M} = \chi_{\mathrm{m}} \mathbf{H}. \qquad \text{Magnetic susceptibility} \tag{5.111}$$

Plugging these relations into Eq. (108), we see that these two parameters are not independent, but are related as

$$\mu = (1 + \chi_{\mathrm{m}})\, \mu_0. \qquad \chi_m \text{ vs. } \mu \tag{5.112}$$

Note that despite the superficial similarity between Eqs. (110)-(112) and the corresponding relations (3.43)-(3.47) for linear dielectrics:

$$\mathbf{D} = \varepsilon \mathbf{E}, \quad \mathbf{P} = \chi_{\mathrm{e}} \varepsilon_0 \mathbf{E}, \quad \varepsilon = (1 + \chi_{\mathrm{e}})\, \varepsilon_0, \tag{5.113}$$

there is an important conceptual difference between them. Namely, while the vector $\mathbf{E}$ on the right-hand sides of Eqs. (113) is the actual (though macroscopic) electric field, the vector $\mathbf{H}$ on the right-hand side of Eqs. (110)-(111) represents a "would-be" magnetic field, in most aspects similar to $\mathbf{D}$ rather than $\mathbf{E}$ – see, for example, Eqs. (109). This historic difference in the traditional way to write the constitutive relations for the electric and magnetic fields is not without its physical reasons. Most key experiments with electric and magnetic materials are performed by placing their samples into nearly-uniform electric and magnetic fields, and the simplest systems for their implementation are, respectively, the plane capacitor (Fig. 2.3) and the long solenoid (Fig. 6). The field in the former system may be most conveniently controlled by measuring the voltage V between its plates, which is proportional to the electric field $\mathbf{E}$. On the other hand, the field provided by the solenoid may be controlled by the current I in it. According to Eq. (107), the field proportional to this stand-alone current is $\mathbf{H}$, rather than $\mathbf{B}$.[49]

Table 1 lists the magnetic susceptibility values for several materials. It shows that in contrast to linear dielectrics whose susceptibility χ_{e} is always positive, i.e. the dielectric constant $\kappa = \chi_{\mathrm{e}} + 1$ is always larger than 1 (see Table 3.1), linear magnetic materials may be either paramagnets (with $\chi_m > 0$, i. e. $\mu > \mu_0$) or diamagnets (with $\chi_{\mathrm{m}} < 0$, i.e. $\mu < \mu_0$).

Table 5.1. Magnetic susceptibility $(\chi_{\mathrm{m}})_{\mathrm{SI}}$ of a few representative and/or important materials[(a)]

"Mu-metal" (75% Ni + 15% Fe + a few %% of Cu and Mo)	~20,000[(b)]
Permalloy (80% Ni + 20% Fe)	~8,000[(b)]
"Electrical" (or "transformer") steel (Fe + a few %% of Si)	~4,000[(b)]
Nickel	~100
Aluminum	$+2 \times 10^{-5}$
Oxygen (at ambient conditions)	$+0.2 \times 10^{-5}$
Water	-0.9×10^{-5}
Diamond	-2×10^{-5}
Copper	-7×10^{-5}
Bismuth (the strongest non-superconducting diamagnet)	-17×10^{-5}

[(a)]The table does not include bulk superconductors, which may be described, in a so-called coarse-scale approximation, as perfect diamagnets (with $\mathbf{B}=0$, i.e. formally with $\chi_{\mathrm{m}}=-1$ and $\mu=0$), though the actual physics of this phenomenon is different – see Sec. 6.3 below.

[(b)] The exact values of $\chi_{\mathrm{m}} >> 1$ for soft ferromagnetic materials (see, e.g., the upper three rows of the table) depend not only on their composition but also on their thermal processing ("annealing"). Moreover, due to unintentional vibrations, the extremely high values of χ_{m} of such materials may decay with time, though they may be restored to the original values by new annealing. The reason for such behavior is discussed in the text below.

The reason for this difference is that in dielectrics, two different polarization mechanisms (schematically illustrated by Fig. 3.7) lead to the same sign of the average polarization – see the discussion in Sec. 3.3. One of these mechanisms, illustrated by Fig. 3.7b, i.e. the ordering of spontaneous dipoles by the applied field, is also possible for magnetization – for the atoms and molecules with spontaneous internal magnetic dipoles of magnitude $m_0 \sim \mu_{\mathrm{B}}$, due to their net spins. Again, in the absence of an external magnetic field the spins, and hence the dipole moments $\mathbf{m}_0$ may be disordered, but according to Eq. (100), the external magnetic field tends to align the dipoles along its direction. As a result, the average direction of the spontaneous elementary moments $\mathbf{m}_0$, and hence the direction of the arising magnetization $\mathbf{M}$, is the same as that of the microscopic field $\mathbf{B}$ at the points of the dipole location (i.e., for a diluted media, of $\mathbf{H} \approx \mathbf{B}/\mu_0$), resulting in a positive susceptibility χ_{m}, i.e. in the paramagnetism, such as that of oxygen and aluminum – see Table 1.

However, in contrast to the electric polarization of atoms/molecules with no spontaneous electric dipoles, which gives the same sign of $\chi_{\mathrm{e}} \equiv \kappa - 1$ (see Fig. 3.7a and its discussion), the magnetic materials with no spontaneous atomic magnetic dipole moments have $\chi_{\mathrm{m}} < 0$ – the effect called the orbital (or "Larmor"[50]) diamagnetism. As the simplest model of this effect, let us consider the orbital motion of an atomic electron about an atomic nucleus as that of a classical particle of mass m_0, with an electric charge q, about an immobile attracting center. As the classical mechanics tells us, the central attractive force does not change particle's angular momentum $\mathbf{L} \equiv m_0 \mathbf{r} \times \mathbf{v}$, but the applied magnetic field $\mathbf{B}$ (that may be taken uniform on the atomic scale) does, due to the torque (101) it exerts on the magnetic moment (95):

$$\frac{d\mathbf{L}}{dt} = \boldsymbol{\tau} = \mathbf{m} \times \mathbf{B} = \frac{q}{2m_0}\mathbf{L} \times \mathbf{B}. \tag{5.114}$$

The diagram in Fig. 13 shows that in the limit of relatively weak field, when the magnitude of the angular momentum $\mathbf{L}$ may be considered constant, this equation describes the rotation (called the torque-induced precession[51]) of the vector $\mathbf{L}$ about the direction of the vector $\mathbf{B}$, with the angular
frequency $\mathbf{\Omega} = -q\mathbf{B}/2m_0$, independent of the angle θ. According to Eqs. (91), (114), the resulting additional (field-induced) magnetic moment $\Delta\mathbf{m} \propto q\mathbf{\Omega} \propto -q^2\mathbf{B}/m_0$ has, irrespectively of the sign of q, a direction opposite to the field. Hence, according to Eq. (111) with $\mathbf{H} \approx \mathbf{B}/\mu_0, \chi_{\mathrm{m}} \propto \chi \equiv \Delta\mathbf{m}/\mathbf{H}$ is indeed negative. (Let me leave its quantitative estimate within this model for the reader's exercise.) The quantum-mechanical treatment confirms this qualitative picture of the Larmor diamagnetism, while giving quantitative corrections to the classical result for χ_{m}.[52]

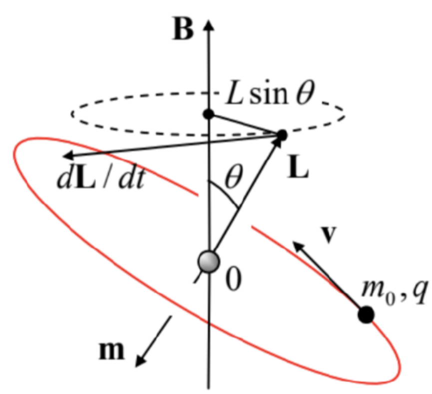

Fig. 5.13. The torque-induced precession of a classical charged particle in a magnetic field.

A simple estimate (also left for the reader's exercise) shows that in atoms with non-zero net spins, the magnetic dipole orientation mechanism prevails over the orbital diamagnetism, so that the materials incorporating such atoms usually exhibit net paramagnetism – see Table 1. Due to possible strong quantum interaction between the spin dipole moments, the magnetism of such materials is rather complex, with numerous interesting phenomena and elaborate theories. Unfortunately, all this physics is well outside the framework of this course, and I have to refer the interested reader to special literature,[53] but still will mention some key notions.

Most importantly, a sufficiently strong dipole-dipole interaction may lead to their spontaneous ordering, even in the absence of the applied field. This ordering may correspond to either parallel alignment of the magnetic dipoles (ferromagnetism) or anti-parallel alignment of the adjacent dipoles (antiferromagnetism). Evidently, the external effects of ferromagnetism are stronger, because this phase corresponds to a substantial spontaneous magnetization $\mathbf{M}$ even in the absence of an external magnetic field. (The corresponding magnitude of $\mathbf{B}=\mu_0\mathbf{M}$ is called the remanence field, B_{R}). The direction of the vector $\mathbf{B}_R$ may be switched by the application of an external magnetic field, with a magnitude above certain value H_{C} called coercivity, leading to the well-known hysteretic loops on the $[H,B]$ plane (see Fig. 14 for a typical example), similar to those in ferroelectrics, already discussed in Sec. 3.3.

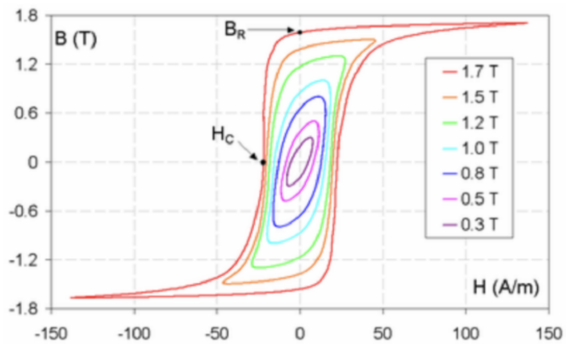

Fig. 5.14. Experimental magnetization curves of specially processed (cold-rolled) electrical steel – a solid solution of ~10% C and ~6% Si in Fe. (Reproduced from www.thefullwiki.org/Hysteresis under the Creative Commons BY-SA 3.0 license.)

Similarly to the ferroelectrics, the ferromagnets may also be hard or soft – in the magnetic rather than mechanical sense. In hard ferromagnets (also called permanent magnets), the dipole interaction is so strong that B stays close to BR in all applied fields below H_{C}, so that the hysteretic loops are virtually rectangular. Hence, in lower fields, the magnetization $\mathbf{M}$ of a permanent magnet may be considered constant, with the magnitude B_{R}/μ_0. Such hard ferromagnetic materials (notably, rare-earth compounds such as $\mathrm{SmCo_5}$, $\mathrm{Sm_2Co_{17}}$, and $\mathrm{Nd_2Fe_{14}\,B}$), with high remanence fields $(\sim 1\ \mathrm{T})$ and high coercivity $(\sim 10^6\,\mathrm{A/m})$, have numerous practical applications. Let me give just two, perhaps most important examples.

First, the permanent magnets are the core components of most electric motors. By the way, this venerable (~150-years-old) technology is currently experiencing a quiet revolution, driven mostly by electric car development. In the most advanced type of motors, called permanent-magnet synchronous machines (PMSM), the remanence magnetic field B_{R} of a permanent-magnet rotating part of the machine (called the rotor) interacts with ac currents passed through wire windings in the external, static part of the motor (called the stator). The resulting torque may drive the rotor to extremely high speeds, exceeding 10,000 rotations per minute, enabling the motor to deliver several kilowatts of mechanical power from each kilogram of its mass.

As the second important example, despite the decades of the exponential (Moore's-law) progress of semiconductor electronics, most computer data storage systems (e.g., in data centers) are still based on hard disk drives whose active medium is a submicron-thin layer of a hard ferromagnet, with the data bits stored in the form of the direction of the remanent magnetization of small film spots. This technology has reached fantastic sophistication, with the recorded data density of the order of 10^{12} bits per square inch.[54] (Only recently it has started to be seriously challenged by the so-called solid-state drives based on the floating-gate semiconductor memories already mentioned in Chapter 3.)[55]

In contrast, in soft ferromagnets, with their lower magnetic dipole interactions, the magnetization is constant only inside each of spontaneously formed magnetic domains, while the volume and shape of the domains are affected by the applied magnetic field. As a result, the hysteresis loop's shape of soft ferromagnets is dependent on the cycled field's amplitude and cycling history – see Fig. 14. At high fields, their $\mathbf{B}$ (and hence $\mathbf{M}$) is driven into saturation, with $B\approx B_{\mathrm{R}}$, but at low fields, they behave essentially as linear magnetics with very high values of χ_{m} and hence μ – see the top rows of Table 1. (The magnetic domain interaction, and hence the low-field susceptibility of such soft ferromagnets are highly dependent on the material's fabrication technology and its post-fabrication thermal and mechanical treatments.) Due to these high values of μ, the soft ferromagnets, especially iron and its alloys (e.g., various special steels), are extensively used in electrical engineering – for example in the cores of transformers – see the next section.

Due to the relative weakness of the magnetic dipole interaction in some materials, their ferromagnetic ordering may be destroyed by thermal fluctuations, if the temperature is increased above some value called the so-called Curie temperature T_{C}. The transition between the ferromagnetic and paramagnetic phase at $T=T_{\mathrm{C}}$ is a classical example of a continuous phase transition, with the average polarization $\mathbf{M}$ playing the role of the so-called order parameter that (in the absence of external fields) becomes different from zero only at $T<T_{\mathrm{C}}$, increasing gradually at the further temperature reduction.[56]

Reference

48 According to Eqs. (110) and (112), i.e. in the SI units, χ_{m} is dimensionless, while μ has the same dimensionality as μ_0. In the Gaussian units, μ is dimensionless: $(\mu)_{\text{Gaussian}}=(\mu)_{\mathrm{SI}}/\mu_0$, and χ_m is also introduced differently, as $\mu=1+4\pi\chi_m$. Hence, just as for the electric susceptibilities, these dimensionless coefficients are different in the two systems: $(\chi_{\mathrm{m}})_{\mathrm{SI}}=4\pi(\chi_{\mathrm{m}})_{\text{Gaussian}}$. Note also that χ_{m} is formally called the volumic magnetic susceptibility, to distinguish it from the atomic (or "molecular") susceptibility χ defined by a similar relation, $\langle\mathbf{m}\rangle\equiv\chi\mathbf{H}$, where $\mathbf{m}$ is the induced magnetic moment of a single dipole – e.g., an atom. (χ is an analog of the electric atomic polarizability α – see Eq. (3.48) and its discussion.) In a dilute medium, i.e. in the absence of a substantial dipole-dipole interaction, $\chi_{\mathrm{m}}=n\chi$, where n is the dipole density.

49 This fact also explains the misleading term "magnetic field" for $\mathbf{H}$

50 Named after Sir Joseph Larmor who was the first (in 1897) to describe this effect mathematically.

51 For a detailed discussion of the effect see, e.g., CM Sec. 4.5.

52 See, e.g., QM Sec. 6.4. Quantum mechanics also explains why in most common (s-) ground states, the average contribution (95) of the orbital angular momentum $\mathbf{L}$ to the net $\mathbf{m}$ vanishes.

53 See, e.g., D. J. Jiles, Introduction to Magnetism and Magnetic Materials, 2nd ed., CRC Press, 1998, or R. C. O'Handley, Modern Magnetic Materials, Wiley, 1999.

54 "A magnetic head slider [the read/write head – KKL] flying over a disk surface with a flying height of 25 nm with a relative speed of 20 meters/second [all realistic parameters – KKL] is equivalent to an aircraft flying at a physical spacing of 0.2 µm at 900 kilometers/hour." B. Bhushan, as quoted in the (generally good) book by G. Hadjipanayis, Magnetic Storage Systems Beyond 2000, Springer, 2001.

55 High-frequency properties of hard ferromagnets are also very non-trivial. For example, according to Eq. (101), an external magnetic field $\mathbf{B}_{\text{ext}}$ exerts torque $\tau=\mathbf{M}\times\mathbf{B}_{\text{ext}}$ on the spontaneous magnetic moment $\mathbf{M}$ of a unit volume of a ferromagnet. In some nearly-isotropic, mechanically fixed ferromagnetic samples, this torque causes the precession around the direction of $\mathbf{B}_{\text{ext}}$ (very similar to that illustrated in Fig. 13) of not the sample as such, but of the magnetization $\mathbf{M}$ inside it, with a certain frequency ω_{r}. If the frequency ω of an additional ac field becomes very close to ω_{r}, its absorption sharply increases – the so-called ferromagnetic resonance. Moreover, if ω is somewhat higher than ω_{r}, the effective magnetic permeability $\mu(\omega)$ of the material for the ac field may become negative, enabling a series of interesting effects and practical applications. Very unfortunately, I do not have time for their discussion and have to refer the interested reader to literature, for example to the monograph by A. Gurevich and G. Melkov, Magnetization Oscillations and Waves, CRC Press, 1996.

56 In this series, a quantitative discussion of such transitions is given in SM Chapter 4.