

4.2: The Ohm Law

As was mentioned above, the relations spelled out in Sec. 1 are sufficient for forming a closed system of equations for finding electric current and field in a system only if they are complemented with some constitutive relations between the scalars I and V in each lumped circuit element, or alternatively between the macroscopic (atomic-scale-averaged) vectors \mathbf{j} and \mathbf{E} at each point of the material of such an element. The simplest of such relations is the famous Ohm law whose differential (or “local”) form is

$$\mathbf{j} = \sigma \mathbf{E}, \quad \text{Ohm law} \quad (4.8)$$

where σ is a constant called the Ohmic conductivity (or just the “conductivity” for short).⁵ Though the Ohm law (discovered, in its simpler form, by Georg Simon Ohm in 1827) is one of constitutive rather than fundamental relations, and is approximate for any conducting medium, we can argue that if:

- (i) the medium carries no current at $\mathbf{E} = 0$ (mind superconductors!),
- (ii) the medium is isotropic or virtually isotropic (a notable exception: some organic conductors),
- (iii) the mean free path l of the current carriers (the notion to be discussed in detail in SM Ch. 6) in this medium is much smaller than the characteristic scale a of the spatial variations of \mathbf{j} and \mathbf{E} ,

then the law may be viewed as the leading, linear term of the Taylor expansion of the local relation $\mathbf{j}(\mathbf{E})$, and thus is general for relatively low fields.

Table 1 gives approximate experimental values of σ for some representative (and/or practically important) materials. Note that the range of the values is very broad, even without going to such extremes as very pure metallic crystals at very low temperatures, where σ may reach $\sim 10^{12}$ S/m.

Table 4.1. Ohmic dc conductivities for some materials at 20°C.

Material	$\sigma(\text{S/m})$
Teflon (PTFE, $[\text{C}_2\text{F}_4]_n$)	$10^{-22} - 10^{-24}$
Silicon dioxide	$10^{-16} - 10^{-19}$
Various glasses	$10^{-10} - 10^{-14}$
Deionized water	$\sim 10^{-6}$
Seawater	5
Silicon n -doped to 10^{16} cm^{-3}	2.5×10^2
Silicon n -doped to 10^{19} cm^{-3}	1.6×10^4
Silicon p -doped to 10^{19} cm^{-3}	1.1×10^4
Nichrome (alloy 80%Ni + 20%Cr)	0.9×10^6
Aluminum	3.8×10^7
Copper	6.0×10^7
Zinc crystal along a -axis	1.65×10^7
Zinc crystal along c -axis	1.72×10^7

In order to get a better feeling of what do these values mean, let us consider a very simple system (Fig. 4): a plane capacitor of area $A \gg d^2$, filled with a material that has not only a dielectric constant κ , but also some Ohmic conductivity σ , with much more conductive electrodes.

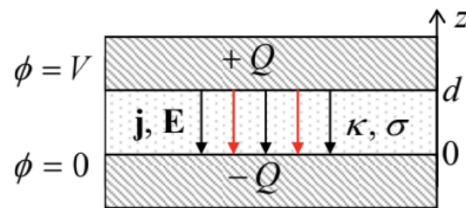


Fig. 4.4. A “leaky” plane capacitor.

Assuming that these properties are compatible with each other,⁶ we may assume that the distribution of the electric potential (not too close to the capacitor’s edges) still obeys Eq. (2.39), so that the electric field is normal to the plates and uniform, with $E = V/d$. Then, according to Eq. (6), the current density is also uniform, $j = \sigma E = \sigma V/d$. From here, the total current between the plates is

$$I = jA = \sigma EA = \sigma \frac{V}{d} A. \quad (4.9)$$

On the other hand, from Eqs. (2.26) and (3.45), the instant value of the total charge of the top electrode is $Q = CV = (\kappa \epsilon_0 A/d) V$. Plugging these relations into Eq. (1), we see that the speed of charge (and voltage) relaxation does not depend on the geometric parameters A and d of the capacitor:

$$\frac{dV}{dt} = -\frac{V}{\tau_r}, \quad \text{with } \tau_r \equiv \frac{\epsilon_0 \kappa}{\sigma} \equiv \frac{\epsilon}{\sigma}, \quad (4.10)$$

so that the relaxation time constant τ_r may be used to characterize the gap-filling material as such.

As we already know (see Table 3.1), for most practical materials the dielectric constant κ is within one order of magnitude from 10, so that the numerator in the second of Eqs. (10) is of the order of 10^{-10} (SI units). As a result, according to Table 1, the charge relaxation time ranges from $\sim 10^{14}$ s (more than a million years!) for the best insulators like Teflon (polytetrafluoroethylene, PTFE),⁷ to $\sim 10^{-18}$ s for the least resistive metals. What is the physics behind such a huge range of σ , and why, for some materials, Table 1 gives them with such a large uncertainty? As in Chapters 2 and 3, in this course I have time only for a brief, admittedly superficial discussion of these issues.⁸

If the charge carriers move as classical particles (e.g., in plasmas or non-degenerate semiconductors), a very reasonable description of the conductivity is given by the famous Drude formula.⁹ In his picture, due to a weak electric field, the charge carriers are accelerated in its direction (on the top of their random motion in all directions, with a vanishing average velocity vector):

$$\frac{d\mathbf{v}}{dt} = \frac{q}{m} \mathbf{E}, \quad (4.11)$$

and as a result, their velocity acquires the average value

$$\mathbf{v} = \frac{d\mathbf{v}}{dt} \tau = \frac{q}{m} \mathbf{E} \tau, \quad (4.12)$$

where the phenomenological parameter $\tau = l/2\nu$ (not to be confused with τ_r !) may be understood as half of the average time between carrier scattering events. From here, the current density:¹⁰

$$\mathbf{j} = q n \mathbf{v} = \frac{q^2 n \tau}{m} \mathbf{E}, \quad \text{i.e. } \sigma = \frac{q^2 n \tau}{m}. \quad (4.13a)$$

(Notice the independence of σ of the charge sign.) Another form of the same result, more popular in the physics of semiconductors, is

$$\sigma = q^2 n \mu, \quad \text{with } \mu = \frac{\tau}{m}, \quad \text{Drude formula: two versions} \quad (4.13b)$$

where the parameter μ , defined by the relation $\mathbf{v} \equiv \mu \mathbf{E}$, is called the charge carrier mobility.

Most good conductors (e.g., metals) are essentially degenerate Fermi gases (or liquids), in which the average thermal energy of a particle, $k_B T$ is much lower than the Fermi energy ϵ_F . In this case, a quantum theory is needed for the calculation of σ . Such theory was developed by the quantum physics’ godfather A. Sommerfeld in 1927 (and is sometimes called the Drude-Sommerfeld model). I have no time to discuss it in this course,¹¹ and here will only notice that for a nearly-ideal, isotropic Fermi gas the result is reduced to Eq. (13), with a certain effective value of τ , so it may be used for estimates of σ , with due respect to the quantum

theory of scattering. In a typical metal, n is very high ($\sim 10^{23} \text{ cm}^{-3}$) and is fixed by the atomic structure, so that the sample quality may only affect σ via the scattering time τ .

At room temperature, the scattering of electrons by thermally-excited lattice vibrations (phonons) dominates, so that τ and σ are high but finite, and do not change much from one sample to another. (Hence, the relatively accurate values given for metals in Table 1.) On the other hand, at $T \rightarrow 0$, a perfect crystal should not exhibit scattering at all, and its conductivity should be infinite. In practice, this is never true (for one, due to electron scattering from imperfect boundaries of finite size samples), and the effective conductivity σ is infinite (or practically infinite, at least above the largest measurable values $\sim 10^{20} \text{ S/m}$) only in superconductors.¹²

On the other hand, the conductivity of quasi-insulators (including deionized water) and semiconductors depends mostly on the carrier density n , which is much lower than in metals. From the point of view of quantum mechanics, this happens because the ground-state wavefunctions of charge carriers are localized within an atom (or molecule), and their energies are separated from those of excited states, with space-extended wavefunctions, by a large energy gap – often called the bandgap. For example, in SiO_2 the bandgap approaches 9 eV, equivalent to $\sim 4,000 \text{ K}$. This is why even at room temperatures the density of thermally-excited free charge carriers in good insulators is negligible. In these materials, n is determined by impurities and vacancies, and may depend on a particular chemical synthesis or other fabrication technology, rather than on the fundamental properties of the material. (On the contrary, the carrier mobility μ in these materials is almost technology-independent.)

The practical importance of the fabrication technology may be illustrated by the following example. In the cells of the so-called floating-gate memories, in particular the flash memories, which currently dominate the nonvolatile digital memory technology, data bits are stored as small electric charges ($Q \sim 10^{-16} \text{ C} \sim 10^3 e$) of highly doped silicon islands (so-called floating gates) separated from the rest of the integrated circuit with $\sim 10\text{-nm}$ -thick layers of the silicon dioxide, SiO_2 . Such layers are fabricated by high-temperature oxidation of virtually perfect silicon crystals. The conductivity of the resulting high-quality (though amorphous) material is so low, $\sigma \sim 10^{-19} \text{ S/m}$, that the relaxation time τ_r , defined by Eq. (10), is well above 10 years – the industrial standard for data retention in nonvolatile memories. To appreciate how good this technology is, the cited value should be compared with the typical conductivity $\sigma \sim 10^{-16} \text{ S/m}$ of the usual, bulk SiO_2 ceramics.¹³

Reference

⁵ In SI units, the conductivity is measured in S/m, where one siemens (S) is the reciprocal of the ohm: $1 \text{ S} \equiv (1 \Omega)^{-1} \equiv 1 \text{ A/V}$. The constant reciprocal to conductivity, $1/\sigma$, is called resistivity and is commonly denoted by the letter ρ . I will, however, try to avoid using this notion, because in these notes this letter is already overused.

⁶ As will be discussed in Chapter 6, this is true is only valid if σ is not too high.

⁷ This polymer is broadly used in engineering and physical experiment, due to its many remarkable properties.

⁸ A more detailed discussion may be found in SM Chapter 6.

⁹ It was suggested by Paul Drude in 1900.

¹⁰ Note that \mathbf{j} in Eq. (8) is defined as a macroscopic variable, averaged over inter-particle distances, so that no additional average sign is necessary in the fist of Eqs. (13a).

¹¹ For such a discussion see, e.g., SM Sec. 6.3.

¹² The electrodynamic properties of superconductors are so interesting (and fundamentally important) that I will discuss them in more detail in Chapter 6.

¹³ This course is not an appropriate platform to discuss details of the floating-gate memory technology. However, I think that every educated physicist should know its basics, because such memories are presently the driver of all semiconductor integrated circuit technology development, and hence of the whole information technology progress. Perhaps the best available general book on this topic is still the relatively old review collection by J. Brewer and M. Gill (eds.), *Nonvolatile Memory Technologies with Emphasis on Flash*, IEEE Press, 2008.

This page titled [4.2: The Ohm Law](#) is shared under a [CC BY-NC-SA 4.0](#) license and was authored, remixed, and/or curated by [Konstantin K. Likharev](#) via [source content](#) that was edited to the style and standards of the LibreTexts platform.