1.3: Postulates' Discussion

The wave mechanics' postulates listed in the previous section (hopefully, familiar to the reader from their undergraduate studies) may look very simple. However, the physics of these axioms is very deep, leading to some counter-intuitive conclusions, and their in-depth discussion requires solutions of several key problems of wave mechanics. This is why in this section I will give only an initial, admittedly superficial discussion of the postulates, and will be repeatedly returning to the conceptual foundations of quantum mechanics throughout the course, especially in Chapter 10 .

First of all, the fundamental uncertainty of observables, which is in the core of the first postulate, is very foreign to the basic ideas of classical mechanics, and historically has made quantum mechanics so hard to swallow for many star physicists, notably including Albert Einstein - despite his 1905 work, which essentially launched the whole field! However, this fact has been confirmed by numerous experiments, and (more importantly) there has not been a single confirmed experiment that would contradict this postulate, so that quantum mechanics was long ago promoted from a theoretical hypothesis to the rank of a reliable scientific theory.

One more remark in this context is that Eq. (25) itself is deterministic, i.e. conceptually enables an exact calculation of the wavefunction's distribution in space at any instant t, provided that its initial distribution, and the particle's Hamiltonian, are known exactly. Note that in the classical statistical mechanics, the probability density distribution $w(\mathbf{r}, t)$ may be also calculated from deterministic differential equations, for example, the Liouville equation. [31] The quantum-mechanical description differs from that situation in two important aspects. First, in the perfect conditions outlined above (the best possible initial state preparation and measurements), the Liouville equation is reduced to the 2^{nd} Newton law of classical mechanics, i.e. the statistical uncertainty of its results disappears. In quantum mechanics this is not true: the quantum uncertainly, such as described by Eq. (35), persists even in this limit. Second, the wavefunction $\Psi(\mathbf{r}, t)$ gives more information than just $w(\mathbf{r}, t)$, because besides the modulus of Ψ, involved in Eq. (22), this complex function also has the phase $\varphi \equiv \arg \Psi$, which may affect some observables, describing, in particular, interference of the de Broglie waves.

Next, it is very important to understand that the relation between the quantum mechanics and experiment, given by the second postulate, necessarily involves another key notion: that of the corresponding statistical ensemble, in this case, a set of many experiments carried out at apparently (macroscopically) similar conditions, including the initial conditions - which nevertheless may lead to different measurement results (outcomes). Indeed, the probability of a certain $\left(n^{\text{th}}\right)$ outcome of an experiment may be only defined for a certain statistical ensemble, as the limit

$$W_n \equiv \lim_{M \rightarrow \infty} \frac{M_n}{M}, \quad \text{with } M \equiv \sum_{n=1}^{N} M_n, \tag{1.3.1}$$

where M is the total number of experiments, M_n is the number of outcomes of the n^{th} type, and N is the number of different outcomes.

Note that a particular choice of statistical ensemble may affect probabilities W_n very significantly. For example, if we pull out playing cards at random from a standard pack of 52 different cards of 4 suits, the probability W_n of getting a certain card (e.g., the queen of spades) is $1/52$. However, if the cards of a certain suit (say, hearts) had been taken out from the pack in advance, the probability of getting the queen of spades is higher, $1/39$. It is important that we would also get the last number for the probability even if we had used the full 52-card pack, but by some reason discarded results of all experiments giving us any rank of hearts. Hence, the ensemble definition (or its redefinition in the middle of the game) may change outcome probabilities.

In wave mechanics, with its fundamental relation (22) between w and Ψ, this means that not only the outcome probabilities, but the wavefunction itself also may depend on the statistical ensemble we are using, i.e. not only on the preparation of the system and the experimental setup, but also on the subset of outcomes taken into account. The sometimes accounted attribution of the wavefunction to a single experiment, both before and after the measurement, may lead to very unphysical interpretations of the results, including some wavefunction's evolution which is not described by the Schrödinger equation (the so-called wave packet reduction), subluminal action on distance, etc. Later in the course, we will see that minding the fundamentally statistical nature of quantum mechanics, and in particular the

Note, however, again that the standard quantum mechanics, as discussed in Chapters 1-6 of this course, is limited to statistical ensembles with the least possible uncertainty of the considered systems, i.e. with the best possible knowledge of their state. [32] This condition requires, first, the least uncertain initial preparation of the system, and second, its total isolation from the rest of the world, or at least from its disordered part (the "environment"), in the course of its evolution in time. Only such ensembles may be described by certain wavefunctions. A detailed discussion of more general ensembles, which are necessary if these conditions are not satisfied, will be given in Chapters 7,8 , and 10 .

Finally, regarding Eq. (23): a better feeling of this definition may be obtained by its comparison with the general definition of the expectation value (i.e. the statistical average) in the probability theory. Namely, let each of N possible outcomes in a set of M experiments give a certain value A_n of observable A; then

$$\langle A\rangle \equiv \lim_{M\to\infty} \frac{1}{M}\sum_{n=1}^{N} A_n M_n = \sum_{n=1}^{N} A_n W_n. \tag{1.3.2}$$

Taking into account Eq. (22), which relates W and Ψ, the structures of Eq. (23) and the final form of Eq. (37) are similar. Their exact relation will be further discussed in Sec. 4.1.

[31] See, e.g., SM Sec. 6.1. dependence of wavefunctions on the statistical ensembles' definition (or redefinition), readily resolves some, though not all, paradoxes of quantum measurements.