# 2.3: Group Velocity

## Tutorial 2.3: Group Velocity versus Phase Velocity

In previous simulations the forward speed of a wave moving in the $x$-direction was determined by $v=\omega/k$ (see simulation three on wave speed). But what value do we use for speed if we add two waves together, each of which has a different value of $v=\omega/k$? In cases where several waves add together to form a single wave shape (called the **envelope**) we can quantify the speed with two numbers, the **group velocity** of the combined wave and the **phase velocity**.

In simulation 2.2 the waves that we added had the same speed so the places where there were destructive or constructive interference (beats for example) were fixed relative to the envelope. In the following simulation the component waves travel at different speeds so there will be internal oscillations in the envelope. The speed of these internal oscillations relative to the envelope is called the phase velocity and is given by $v_{\text{phase}}=\omega_{\text{ave}}/k_{\text{ave}}$.

The group velocity is velocity of the envelope. For two waves group velocity is defined by $v_{\text{group}}=\Delta\omega/\Delta k$ where $\Delta\omega=\omega_1-\omega_2$ and $\Delta k=k_1-k_2$. This expression for group velocity is the slope of a frequency versus wavenumber graph. In the case of adding many waves, each with its own angular velocity and wave vector, $\omega$ and $k$ become continuous variables and we define the group velocity as a partial derivative; $v_{\text{group}}=\partial\omega(k)/\partial k$. Angular velocity as a function of the wave vector ($\omega(k)$, called the dispersion relation) will be examined in a later simulation.

Since this simulation has waves traveling in the $x$-direction only we will talk about **group speed** and **phase speed**.

## Group and Phase Velocity

### Questions:

**Exercise 2.3.1**

The simulation starts with two identical waves. The bottom graph shows the sum of the two top graphs. Change both the wave number ($k_1$) and the angular frequency ($\omega_1$) for the first wave to $8.0$, click 'set values' and 'play'. Describe what you see. Notice that the envelope moves to the right at the same speed as the components.

**Exercise 2.3.2**

Two numbers appear in the bottom of the simulation window. These show the phase speed and the group speed. What are the group and phase speed for the case $k_1=8.0\text{ rad/m}$, $\omega_1=8.0\text{ rad/s}$ and $k_2=8.4\text{ rad/m}$, $\omega_2=8.4\text{ rad/s}$?

**Exercise 2.3.3**

What are the group and phase speeds for the case $k_1=8.0\text{ rad/m}$, $\omega_1=8.4\text{ rad/s}$ with $k_2=8.4\text{ rad/m}$, $\omega_2=8.4\text{ rad/s}$? Describe what you see.

**Exercise 2.3.4**

What are the group and phase speeds for the case $k_1=8.8\text{ rad/m}$, $\omega_1=8.0\text{ rad/s}$ with $k_2=8.4\text{ rad/m}$, $\omega_2=8.4\text{ rad/s}$? Describe what you see.

**Exercise 2.3.5**

For $k_1=8.0\text{ rad/m}$, $k_2=8.4\text{ rad/m}$, $\omega_2=8.4\text{ rad/s}$ try several values of $\omega_1$ between $8.4$ and $9.0\text{ rad/s}$. What can you conclude about the group velocity as $\omega_1$ gets larger?

**Exercise 2.3.6**

For $k_1=8.0\text{ rad/m}$, $k_2=8.4\text{ rad/m}$, $\omega_2=8.4\text{ rad/s}$ try several values of $\omega_1$ between $8.4$ and $7.6\text{ rad/s}$. What can you conclude about the group velocity as $\omega_1$ gets smaller?

You will have noticed by now that the phase velocity can be greater than the group velocity. In the case of electromagnetic waves the phase velocity can be greater than $c = 3 \times 10^9\ \mathrm{m/s}$, the speed of light. This does not break the rules of special relativity because information is transmitted at the group velocity, which is never greater than $c$.

**Exercise 2.3.7**

Based on the previous few questions, what is the general rule for when group velocity is larger than the phase velocity? Under what conditions is phase velocity greater than the group velocity?

**Exercise 2.3.8**

For some material the wave speed is fixed by the medium in which the wave is traveling (see simulation 1.2). In these cases the ratio of $\omega/k$ is always the same number although $\omega$ and $k$ might be different for different waves. For $k_2 = 8.4\ \mathrm{rad/m}$, $\omega_2 = 8.4\ \mathrm{rad/s}$ try several values of $\omega_1$ and $k_1$ such that the ratio $\omega_1/k_1$ is always equal to one (the same as $\omega_2/k_2$). What can you conclude about the group velocity as compared to the phase velocity in cases where all the components travel at the same speed?

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