3.6: Path Difference and Interference

Tutorial 3.6: Interference due to Path Difference

In simulation 2.1 we saw constructive and destructive interference which was the result of adding waves that had different phases. How can two identical waves end up out of phase with each other? If two waves travel a different distance they can end up arriving out of phase. This is in fact what causes the interference patterns seen in simulation 2.2 where there were two point sources. Waves arriving at points along a center line between the sources travel the same distance and so arrive in phase. For points not on the central line the waves may arrive in phase or out of phase, depending on the distance they travel.

In this simulation the top two waves are identical but start at different locations. The bottom graph shows the sum of the two waves. Depending on the path difference, D, the two waves may end up exactly in phase (leading to constructive interference), exactly out of phase (destructive interference) or something in between. The wavelength can also be changed by changing the wavenumber, k. The units of $k = 2\pi/\lambda$ and distance, D, are arbitrary.

Path Difference

Questions:

Exercise 3.6.1

Start the simulation with the step thickness, D, equal to zero. Are the waves in phase? Slowly increase the Step Thickness until the the waves are exactly out of phase. What step thickness causes this?

Path differences can occur in a number of different ways. In the Ripple Tank simulation of the double slit experiment (tutorial 2.2) the distance to a point on the screen is different for each source (except for the center of the screen) so the light experiences a path difference. If the path difference is a whole number of wavelengths ($1,\ 2,\ 3\ldots$) there is constructive interference and a bright spot for that color appears on the screen. If the path difference is $1/2,\ 3/2,\ 5/2$etc. of a wavelength then a dark spot appears due to destructive interference.

Exercise 3.6.2

Based on your findings in question $3.6.1$, what is the wavelength (in arbitrary units) of the waves in the simulation? (Recall that $k = 2\pi/\lambda$.)

Exercise 3.6.3

Reset the simulation and slowly increase the step hickness to find the first three thicknesses that cause destructive interference. Verify that destructive interference occurs at step thicknesses given by $1/2\lambda,\ 3/2\lambda$,and $5/2\lambda$.

Path differences can also occur due to reflection from a surface that has multiple layers. In this case the waves that hit the deeper surface travel *twice* the distance between the surfaces before recombining (*twice* the thickness in the simulations above for a wave that comes in from the right and reflects back to the right).

Exercise 3.6.4

Imagine this simulation now represents **monochromatic** (single wavelength) light reflecting off a surface with two levels. Only the reflected waves (going to the right) are show (the incoming waves coming from the right are not shown). Now the path difference is twice the depth of the step. How would this change the results? In which case would there be no reflection from the surface? In which case would there be constructive interference?

The formula for **constructive interference due to a path difference** is given by $\delta = (m + 1/2)\lambda/n$ where n is the index of refraction of the medium in which the wave is traveling, λ is the wavelength, δ is the path difference and $m = 0,\ 1,\ 2,\ 3\ldots$. For a *reflected wave* in the simulation $\delta = 2X\ \text{Step Thickness}$ is the actual path difference; a wave reflected off the upper surface must travel an extra distance equal to twice the Step Thickness to catch up with a wave reflected off the lower surface.

Exercise 3.6.5

Reset the simulation and enter 1.57 for the Step Thickness (this is the case of half a wavelength path difference so the waves cancel). Increase the wave vector, $k=2\pi/\lambda$, until you find the next wavelength that experiences destructive interference (don't change the Step Thickness). What is the wave vector k and wave length of this wave?

Exercise 3.6.6

For light, changing the wavelength changes the color. Can the same Step Thickness cause destructive interference for all wavelengths? Explain.

Exercise 3.6.7

A music CD has information stored on it in the form of tiny divots blasted into the surface with a laser. Suppose you see constructive interference for red light (wavelength of $650\ \mathrm{nm}$). What is the minimum ($m=0$) depth of the divots? (Hint: The path difference is twice the divot depth.)

Exercise 3.6.8

Explain why you only see one particular color when looking at a small region of a CD at a fixed angle. What happens to the other colors?

Exercise 3.6.9

If you look at the colors being reflected from a CD you will notice that the color changes depending on the angle. How does the path difference change as you look at the divots at different angles? (Hint: Imagine the waves in the simulation coming in at different angles instead of horizontally. Now the path difference is the hypotenuse of a triangle, one side of which is the Step height.)

Exercise 3.6.10

Suppose you wanted to make a "stealth" jet plane which was non-reflective to a particular wavelength of radar. Describe one way you might try to do this by modifying the surface of the plane.

Exercise 3.6.11

Some insect wings and the feathers of some birds (for example peacocks) exhibit a feature known as **iridescence**. From a fixed angle only one color of reflected light can be seen. Explain this phenomena given the fact that insect wings and feathers consist of overlapping layers causing the surface to be multi-layered.

Exercise 3.6.12

Soap bubbles show different colors at different places on the bubble. So do oil slicks on water. In both cases light reflects off the upper and lower surfaces of the layer of soap or oil. Explain the different colors in terms of path difference (Hint: draw a picture where the wall of the soap bubble is nearly the same thickness as one wavelength and explain why the path difference is twice the thickness of the soap).

There is one other detail needed to explain the soap bubble and oil slick color phenomena completely. The light reflecting from the top surface is going from a "soft" medium (air) to a "stiff" medium (soap) but the light reflecting from the bottom layer of the soap is going from a "stiff" medium (soap) to a "soft" medium (air inside the bubble).

Exercise 3.6.13

From what you learned about reflection and transmission of waves from "stiff" and "soft" boundaries in simulation 3.2, what happens to a wave reflected from the top layer of a soap bubble which does not occur at the bottom layer?

Exercise 3.6.14

If the path difference for a particular thickness of soap film was just right for destructive interference but there was a 180° phase change for the top reflected wave but not the bottom reflected wave, what would happen to that color?
