1.4: Simple Harmonic Motion

Tutorial 1.4: Simple Harmonic Motion I

The following is a simulation of a mass on a spring. The graph shows the y location of the mass. The force acting on the mass in this case is called a **Hooke's Law** force: $F=-\kappa y$ where κ is called the **spring constant**, in $\mathrm{N/m}$ indicating the stiffness of the spring and y is the location of the mass from some equilibrium position. (Careful! κ is **not** the same as the wavenumber, $k=2\pi/\lambda$.)

Simple Harmonic Motion

Questions:

Exercise 1.4.1

Drag the mass to some initial location using the mouse. Click 'play' button (lower left) to see the motion and the graph of the mass location. How is the graph you see here similar to the graph you saw for the motion of the red circle in the transverse wave case?

Exercise 1.4.2

Clicking on the graph shows the coordinates of the mouse in a yellow box at the lower left. Determine the period, frequency and angular frequency of this motion from the values on the graph (Hint: Frequency in Hz is the inverse of period; angular frequency is $\omega=2\pi f$).

Exercise 1.4.3

Check the 'show velocity' box to see graphs of both position and velocity. Where is the mass when the velocity is a maximum? Where is the mass when the velocity becomes zero?

Exercise 1.4.4

Try different spring constant values, κ, between $0.5\ \mathrm{N/m}$ to $5.0\ \mathrm{N/m}$, reseting and releasing the mass at the same point each time. What is the relationship between spring constant and frequency?

Exercise 1.4.5

For the same spring constant try releasing the ball at several different amplitudes. What is the relationship between amplitude and frequency?

For a given spring the frequency is determined by the mass hanging on it and the stiffness of the spring; $f=(\kappa/m)^{1/2}/2\pi$.

Exercise 1.4.6

Measure the frequency for the case of a spring constant, κ equal to 2.0. What must be the mass on the end of this spring? Verify that you get the same mass by measuring the fequency for several different spring constants (this is equivalent to hanging the same mass on several different springs).

Exercise 1.4.7

What happens to the period of oscillation of a spring-mass system if the mass is doubled?

In the previous simulation (1.3: Transverse Waves) the red circle moved up and down as the result of a transverse wave traveling horizontally along the string of particles. The equation for the motion of the entire string is $y(x,t)=A\sin(kx-\omega t+\varphi)$. If we were to assume that the red particle was located at $x=0$ the equation describing the motion of just the red circle would be

$y(t) = A\sin(-\omega t+\varphi)$ where A is the maximum amplitude, ω is the angular velocity and φ is the phase. This is also the formula for **simple harmonic motion** which describes the location of a mass on a spring as a function of time. In other words the motion of each point on a transverse wave is exactly the same as if each of those points was undergoing simple harmonic motion but with a slightly different phase from its neighbor.

Exercise 1.4.8

Using $y(t) = A\sin(-\omega t+\varphi)$ and a calculator, what is the location of the mass when $t=0$ and $\varphi=0$? What is the location of the mass when $t=0$ and $\varphi=\pi$ (don't forget we are in radian mode)? Explain what the phase angle, φ tells you about the intial position ($t=0$) of the mass on the spring.

Exercise 1.4.9

If $y(t) = A\sin(-\omega t+\varphi)$ is the location of the mass on the spring and the time derivative of location is velocity, then the velocity of the mass is given by $v(t) = A\omega\cos(-\omega t+\varphi)$. Check the 'show velocity' box and run a simulation with an initial amplitude of 6.0 m and a spring constant of 2. From the graph find the angular frequency and calculate the maximum amplitude $v_{\max} = A\omega$. How does this number compare with the maximum value on the velocity graph; are they the same?

Exercise 1.4.10

Write down an expression for the acceleration of a mass on a spring, based on the position given by $y(t) = A\sin(-\omega t+\varphi)$ and the fact that acceleration is the second derivative of position with respect to time.