1.6: Longitudinal Waves

In comparing simulations on transverse waves (Tutorial 1.3) with vertical harmonic motion (Tutorial 1.4) we discovered that particles in a transverse wave move up with simple harmonic motion. In the previous exercise (Tutorial 1.5) we saw that harmonic motion can also occur in the horizontal direction. Can we also have a wave moving horizontally where the particles move with harmonic motion in the horizontal direction?

YES! **Longitudinal waves** are waves where the motion of the material in the wave is back and forth in the same direction that the wave moves. Sound waves (in air and in solids) are examples of longitudinal waves. When a tuning fork or stereo speaker vibrates it moves back and forth creating regions of compressed air (where the pressure is slightly higher) and regions in between where the air has a lower pressure (called a rarefaction). These compressions and rarefactions move out away from the tuning fork or speaker at the speed of sound. When they reach your ear they cause your eardrum to vibrate, sending signals through the rest of the ear to the brain.

Longitudinal waves can be described with the same mathematical functions as transverse waves: $y(x,t) = A\sin(kx - \omega t + \varphi)$ where now $y(x,t)$ is the *horizontal* (or longitudinal) displacement from equilibrium at location x and time t instead of the vertical displacement from equilibrium. As was the case for transverse waves the forward velocity of a longitudinal wave is given by $v = \lambda/T = \omega/k$.

The following simulation shows a graph of the longitudinal motion of one molecule, the red circle, in a collection of molecules which has a longitudinal wave passing through it, much like sound passing through air. A vertical line marks the equilibrium location of the red circle. Random thermal motions are not shown.

Longitudinal Waves

Questions:

Exercise 1.6.1

Click on 'Position' and then 'play'. Left clicking on the upper panel gives the time and amplitude of points on the graph in the yellow box. Do any of the circles travel all the way across the simulation to the other side? Explain.

Exercise 1.6.2

Left clicking on the upper panel gives the time and amplitude of points on the graph in the yellow box. Determine the maximum amplitude and the period of oscillation from the graph.

Exercise 1.6.3

Left clicking on the lower panel gives the x and y locations of points on the wave in a yellow box. Pause and step the animation until the red circle at its equilibrium position. Find the wavelength of the wave using the mouse by finding the distance between one place where the circles are clumped together to the next location (or from two successive locations where the circles are furthest apart). What is the wavelength?

Exercise 1.6.4

From the period and wavelength find the speed of this wave (Hint: The same equations work for both longitudinal and transverse waves).

For sound the frequency (inversely proportional to the wavelenght) tells us something about the **pitch** of the sound. There are other aspects of pitch percption which inolve other physical features of the wave but the main component of pitch is the frequency.

Exercise 1.6.5

What is the frequency of the wave in the simulation?

Exercise 1.6.6

Write an equation of the form $y(x,t) = A\sin(kx - \omega t + \varphi)$, filling in the values of A, k and ω for this wave. Assume the phase angle is zero.

Notice that the circles in the simulation move back and forth with a variable speed around an equilibrium position while the wave moves only in one direction with a constant speed. The velocity of the individual particles is given as before by the derivative of the amplitude: $v(x,t) = \partial y(x,t)/\partial t = -A\omega\cos(kx - \omega t + \varphi)$.

Exercise 1.6.7

Click on 'Velocity' and then 'play'. The upper graph now gives the velocity of the red circle as a function of time. What is the maximum velocity (approximately) of the red dot according to the graph? How does this compare with the velocity of the wave which you found in 1.6.4? How does it compare with $v_{\max} = A\omega$?

Exercise 1.6.8

In your own words, explain the difference between wave speed and particle velocity for a longitudinal wave.

Exercise 1.6.9

Where is the red dot relative to the vertical line when the maximum velocity occurs? Where is it when the velocity is approximately zero? What is the relationship between position and velocity.

Exercise 1.6.10

Take a derivative of velocity to find an expression for acceleration of particles in the material (the red dot). Show that the maximum acceleration is given by $a_{\max} = A\omega^2$.

Exercise 1.6.11

Calculate the maximum acceleration of the red dot using $a_{\max} = A\omega^2$. If amplitude is in meters and angular frequency in radians per second, what are the units of this acceleration?

As a sound wave moves through the air the air molecules do not move forward at the speed of sound but rather, oscillate back and forth as harmonic oscillators in the same general location while the sound wave passes (see question one). For sound waves the displacement amplitude (distance from the equilibrium location) tells us something about the pressure of the air at that location. Pressure is measured in pascals $(1\text{ Pa} = 1\text{ N/m}^2)$ and pressure squared is proportional to the **intensity** of the sound wave, measured in W/m^2.

The relationship between sound intensity, I measured in watts per meter squared and **loudness**, or sound intensity level (SIL) measured in decibels, is given by $SIL = 10\log(I/I_0)$. Here log is the logarithm and $I_0 = 10^{-12}\text{ W/m}^2$ is a reference sound intensity at about the threashold of human hearing.

Exercise 1.6.12

For the following sound intensities, what is the equivalent SIL in decibels: Jet enginge, 100 W/m^2; pain threshold, 1 W/m^2; vacuum cleaner, 10^{-4} W/m^2; converstaion, 10^{-6} W/m^2; the rustle of leaves, 10^{-11} W/m^2.