1.8: Two-Dimensional Waves

Tutorial 1.8: Two-Dimensional Waves

So far these simulations have only shown one dimensional waves. Even though there is motion perpendicular to the direction the wave travels for a transverse wave, the function describing the wave is a function of only one spacial variable, x. Waves can exist in two or three dimensions, however. One example is a **plane wave** where the wave front or crest of the wave makes a line (in two dimensions) or a plane (in three dimensions). **Circular waves** (in two dimensions) and **spherical waves** (in three dimensions) also exist. The present simulation shows plane and circular waves in two dimensions.

The simulation below starts by showing you a plane wave in two dimensions traveling in the $x-y$ plane, in the x direction, viewed from above. In these simulations the amplitude (in the z direction, towards you) is represented in greyscale. When the wave has a positive amplitude the color is white, when the amplitude is zero the color is light gray and when the amplitude is negative the color is black. The wavelength λ is in centimeters and the period, T is in seconds.

Two-Dimensional Waves

Questions:

Exercise 1.8.1

Click on the 'play' button with the plane wave selected. Experiment with wavelength and period. Is the simulation accurate in representing a fixed speed for a real wave? How do you know? Consult 1.2 on wave speed if you are unsure of your answer.

Exercise 1.8.2

Can this representation be used to describe longitudinal waves as well as transverse waves? Why or why not?

Exercise 1.8.3

The answer to the previous question is 'yes'. What would the white, grey and black colors represent if this was a representation of a longitudinal wave?

Keep in mind the wave shown is really a function of just two variables, x and t. For a wave traveling in the x direction the amplitude is the same for any value of y so the equation describing it is exactly the same as the one dimensional wave we have seen before: $z(x, y, t)=z(x, t)=A \sin (k x-\omega t+\varphi)$. This kind of wave is called a **plane wave**.

The equation for a plane wave traveling in an *arbitrary* direction in the $x-y$ plane is given by $z(x, y, t)=A \sin \left(k_1 x+k_2 y-\omega t+\varphi\right)$ where $z(x, y, t)$ is the height of the wave at location (x, y) at time t. Now the wavenumber k has become a *wave vector* with components $k_1=k \cos \theta$ and $k_2=k \sin \theta$ where θ is angle between the direction the wave is traveling and the x axis. This equation can also be writen using a vector dot product as $z(x, y, t)=A \sin (\mathbf{k} \cdot \mathbf{r}-\omega t+\varphi)$ where $\mathbf{r}$ and $\mathbf{k}$ are vectors with components in the x, y and (for waves moving in three dimensions) z directions. The vector $\mathbf{r}$ gives the direction the wave is traveling and the wave vector $\mathbf{k}$ has components of the wavelength in each direction.

Exercise 1.8.4

For what angle, θ, does the equation of a plane wave in an arbitrary direction become the same as the equation for a wave in the x direction?

Exercise 1.8.5

Now select the circular wave button. Suppose this circular wave simulation represents waves in a pan of water. Given a pan of water and other equipment of your choice in the laboratory, how could you create these waves? Explain the proceedure in detail.

Exercise 1.8.6

What is happening to the curvature of the waves as they move away from the source? Is it becoming more or less like a plane wave? Explain why it is reasonable to think of light from the sun as plane waves when the light arrives at the earth, even though they are actually spherical waves moving outwards in all directions from the sun.

Exercise 1.8.7

The circular wave simulation is unphysical in one sense because the amplitude of the circular wave does not change as the diameter gets bigger. Why is this unrealistic? Would you expect the amplitude of a circular wave to be the same once it has spread out over a very large distance from the source? Explain.

As noted previously the **Intensity** of a wave is defined to be the power per meter squared and is measured in $\mathrm{W/m^2}$. Power is energy transmitted per time and the energy of a wave is proportional to the amplitude squared so intensity will also be porportional to amplitude squared.

Exercise 1.8.8

In the simulation, does the intensity of the circular wave change as you get further from the source? How is this different in the case of real waves (for example sound) as you get further from the source?

Many waves spread out in three dimensions from what is essentially a point source, for example a small lamp sends out light in nearly all directions. The surface of a sphere, centered on the point source would catch all of the waves from the source. This imaginary sphere has a surface area of $4\pi r^2$ where r is the distance from the source. So at a distance of r the energy that left the source is spread over area $4\pi r^2$. This means the energy per area is $E/4\pi r^2$ and so decreases as $1/r^2$. We can thus conclude that the intensity of a spherical wave is inversely proportional to distance squared.

Exercise 1.8.9

Suppose the intensity of a spherical wave is $80\ \mathrm{W/m^2}$ at a distance of one meter. What is the intensity at $2\ \mathrm{m}$? $4\ \mathrm{m}$? $1/4\ \mathrm{m}$?

Exercise 1.8.10

If the amplitude squared is proportional to intensity, by what factor does the amplidue of the wave in the previous question change in going from $1\ \mathrm{m}$ to $2\ \mathrm{m}$? How much change is there in going from $1\ \mathrm{m}$ to $1/4\ \mathrm{m}$?