1.1: Sine Wave

Tutorial 1.1: Sine Waves

Imagine a perfect, smooth wave out on the ocean far enough from shore so that it has not started to break (complications involved in describing real waves will be discussed later in this tutorial). If we take a snapshot picture of this wave at a single instant in time and measure the distance in meters from one peak to the next we have measured the **wavelength**, λ of the wave.

If instead we watch a floating cork at a single location in space and measure the time in seconds between arriving peaks we have measured the **period**, T of the wave. We could also measure the number of times the cork bobs up, down and back up per second which would be the **frequency** in hertz or cycles per second. The period and frequency are inverses of each other: $f = 1/T$.

The height of the wave at any location and time, measured from the middle, or equilibrium position is the **displacement**. The maximum displacement is called the **amplitude**.

As a first approximation, water waves, electromagnetic waves and many other kinds of waves can be modeled by the mathematical functions sine or cosine or some combination of them. For a wave traveling along the x axis the mathematical description of the displacement of a wave at location x and time t can be written as $y(x,t) = A\sin(kx - \omega t + \varphi)$ where A is the **amplitude** (maximum height measured from the middle of the wave). Here we have used the **wavenumber** $k = 2\pi/\lambda$ (measured in radians per meter), the **angular frequency**, $\omega = 2\pi f$ (measured in radians per sec), and φ, the **phase** angle (in radians) which are often easier to use mathematically.

In this and all of the following exercises the angles will be in radians (Hint: Set your calculator to radian mode to avoid problems later on!)

With the following physlet you can explore different values of amplitude, wavenumber, wavelength, angular frequency, frequency, period and phase. Make any changes you wish in the $y(x,t) =$ window, hit enter or return and then the play button, ▶, to see the wave in action. The initial values are amplitude, $A = 1.2\text{ m}$; wavenumber, $k = 2.0\text{ rad/m}$; angular frequency, $\omega = 0.8\text{ rad/s}$; and phase angle, $\varphi = 10.0\text{ radians}$. Clicking in on the graph shows the coordinates of the cursor in a yellow box in the lower left corner.

Sine Wave Simulation

Questions:

Exercise 1.1.1

Stop the simulation, double the maximum amplitude (from 1.2 m to 2.4 m) enter and play. What effect does maximum amplitude, A, have on the wave?

Exercise 1.1.2

Stop the simulation, double the wave number, k, (from 2.0 rad/m to 4.0 rad/m) enter and play. What effect does the wave number have on the wave?

Exercise 1.1.3

Stop the simulation, double the angular frequency, ω, (from 0.8 rad/s to 1.6 rad/s) enter and play. What effect does the angular frequency have on the wave?

Exercise 1.1.4

Stop the simulation, double the phase, φ, (from 10.0 to 20.0 radians) enter and play. Try several different values for the phase. What effect does the phase have on the wave?

Exercise 1.1.5

Go back to the original wave by clicking the reload button, ⮌. Pause the wave and measure the wavelength, λ, on the graph (find the x location of two successive peaks or troughs using the cursor; the wavelength is the x distance between peaks or troughs). Calculate the wavenumber, k, from this wavelength. How does your value for wavenumber compare with the wavenumber in the equation?

Exercise 1.1.6

Now start the original wave in motion with the play button. Use the time numbers in the lower panel to find the period, T, of the wave (the time from when one peak passes a point until the next peak passes the same point). To get an accurate number you can use the step buttons. From the period you measure, calculate the angular frequency, ω. How does your value for angular frequency compare with the angular frequency in the equation?

Exercise 1.1.7

Go back to the original wave using the reload button. Change the minus sign in the equation between kx and ωt to a plus sign and click 'play'. What does changing this sign do to the wave?

Exercise 1.1.8

Now change the plus sign in front of the phase to a minus sign, enter and play. Try several values of phase (you may want to use the pause button to be sure you can tell what is happening). What does changing this sign do to the wave?

Exercise 1.1.9

In general the power transmitted by a wave, measured in watts, is proportional to the amplitude squared. What happens to the power if the amplitude is doubled? What happens to the power if the amplitude is cut in half?
