1.2: Speed of a Wave

Tutorial 1.2: Speed of a Wave

There are three different velocities involved with describing a wave, one of which will be introduced here. The **velocity** of the wave, v, is a constant determined by the properties of the medium in which the wave is moving. The velocity is a vector which gives the forward **speed** of the wave and the direction the wave is traveling. For now we will not worry about direction since the waves being discussed will all be assumed to travel along the x-axis. The speed of a sine wave is given by $v = \lambda/T$ in meters per second where wavelength and period for a sine wave were defined in the previous exercise.

In this simulation the original wave will remain in the window so that as you make changes to $f(x,t)$ you can see how the new wave (in red) compares to the original ($g(x,t)$, in blue).

Wave Speed Simulation

Questions:

Exercise 1.2.1

Determine the speed of the wave in the simulation using $v = \lambda/T$ where wavelength and period are determined from the simulation as you did in the previous exercise using the mouse to find the wavelength and the time to find the period. What is the forward speed of this wave?

Exercise 1.2.2

The speed of this wave is also given mathematically by $v = \omega/k$ since $\omega = 2\pi f = 2\pi/T$ and $k = 2\pi/\lambda$. What is the speed of this wave based on the values of ω and k in the equation? Does this match the speed you got from the simulation?

Exercise 1.2.3

Reload the initial conditions and experiment with values of the wavenumber both smaller and larger than $2.0\ \mathrm{rad/m}$ keeping the angular frequency fixed. How does the wavenumber change the speed of the wave?

Exercise 1.2.4

Reload the initial conditions and experiment with values of the angular frequency both smaller and larger than $0.8\ \mathrm{rad/s}$ keeping the wavenumber fixed. How does the angular frequency change the speed of the wave?

This simulation is *misleading* in one important way. In the simulation you can set any combination of angular frequency and wavenumber you choose and so have any speed you want for the wave. But for mechanical and acoustic waves the speed is determined by the medium in which the wave travels. As we will see, for these waves it is often the case that $v = \omega/k$ so that the angular frequency and wavenumber are inversely proportional with v a constant. Examples:

- For sound waves in a fluid (for example air) the speed is determined by $v = (B/\rho)^{1/2}$ where B is the bulk modulus or compressibility of the fluid in newtons per meter squared and ρ is the density in kilograms per cubic meter.
- For waves in a solid the speed is determined by $v = (Y/\rho)^{1/2}$ where Y is Young's modulus or stiffness in newtons per meter squared and and ρ is the density in kilograms per meter cubed.
- For waves on a string the speed is determined by $v = (T/\mu)^{1/2}$ where T is the tension in the string in newtons and μ is the mass per length in kilograms per meter.
- Although electromagnetic waves do not need a medium to travel (they can travel through a vacuum) their speed in a vacuum, $c = (1/\mu_0\varepsilon_0)^{1/2}$ is governed by two physical constants, the permeability μ_0 and the permittivity, ε_0.
- If an electromagnetic wave enters a medium (such as light going through air, or water or glass) the speed changes because the permittivity and/or permeability may be different. The amount the speed changes is given by the **index of refraction** $n = c/v$ where c is the speed of light and v is the speed in the medium.

As we will see later, it is the case that speed can sometimes depend on the frequency of the wave, a phenomenon know as dispersion.

Exercise 1.2.5

Does sound travel slightly faster on a hot day or a cool day? Does sound travel faster or slower if the humidity is high?

Exercise 1.2.6

Density is relatively easy to measure. What would be a clever way to measure Young's modulus, Y for a solid?

Exercise 1.2.7

Do waves on a string travel faster or slower if the string is tighter? Do waves on a string travel faster or slower if the string is thicker?

Exercise 1.2.8

In general the index of refraction is larger than one. What does this tell you about the speed of light in glass?

Exercise 1.2.9

Reload the initial conditions with the 'reload' button. For a wavenumber of $4.0\ \mathrm{rad/m}$ experiment to find the correct angular frequency which gives the original speed of the wave you found in Exercises $1.2.1$ and $1.2.2$ (you should be able to see from the simulation when the new wave is traveling at the same speed as the original).

Exercise 1.2.10

Calculate the wavenumber which gives the speed of the original wave for angular frequencies of $0.4,\ 0.6,\ 1.0$,and $1.2\ \mathrm{rad/s}$ using the relationship in Exercise $1.2.2$. Check your answers with the simulation if you are in doubt.