16.7: The Lorenz Condition

We are now in a position to see what the Lorenz condition means. For an isolated stationary charge, the scalar potential is given by equation (16.1.1) and the vector potential **A** is zero. The Lorenz condition reduces to

$$\frac{1}{c^2}\frac{\partial\phi}{\partial t}=\frac{1}{4\pi\epsilon_0 rc^2}\frac{dq}{dt}=0 \tag{16.7.1}$$

From this we see that the Lorenz condition applied to the four-potential for a point charge is equivalent to the statement that **the charge on a point particle is conserved** , i. e., it doesn't change with time. This is extended to any stationary distribution of charge by the superposition principle.

We thus see that the Lorenz condition is closely related to charge conservation for the four-potential of any charge distribution in the reference frame in which the charge is stationary. If we can further show that the Lorenz condition is an equation that is equally valid in all reference frames, then we will have demonstrated that it is true for the four-potential produced by moving charged particles as well.

If the Lorenz condition is valid in one reference frame, it is valid in all frames for the special case of a plane electromagnetic wave. This follows from substituting the four-potential for a plane wave into the Lorenz condition, as was done in equation 16.6.2 in the previous section. In this case the Lorenz condition reduces to $k\cdot a=0$. Since the dot product of two four-vectors is a relativistic scalar, the Lorenz condition is equally valid in all frames.