15.1: Electromagnetic Four-Potential

Electromagnetism is a gauge theory. Particles that have a property called electric charge are subject to forces exerted by the gauge fields of electromagnetism. The potential four-momentum $Q = (\mathbf{Q}, U/c)$ of a particle with charge q in the presence of the electromagnetic four-potential $\underline{a}$ is just

$$\underline{Q} = q\underline{a} \tag{15.1.1}$$

In the simplest case the four-potential represents the amplitude for finding the intermediary particle associated with the electromagnetic gauge field. This particle has zero mass and is called the photon. If more than one photon is present, the interpretation of $\underline{a}$ becomes more complicated. This issue will be considered later.

The four-potential has space and time components $\mathbf{A}$ and ϕ/c such that $\underline{\mathrm{a}} = (\mathbf{A}, \phi/\mathrm{c})$ . . The quantity $\mathbf{A}$ is called the vector potential and ϕ is called the scalar potential. The scalar and vector potential are related to the potential energy U and potential momentum $\mathbf{Q}$ of a particle of charge q by

$$U = q\phi \quad \mathbf{Q} = q\mathbf{A} \tag{15.1.2}$$

The Lorenz condition written in terms of $\mathbf{A}$ and ϕ is

$$\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} + \frac{1}{c^2}\frac{\partial \phi}{\partial t} = 0 \tag{15.1.3}$$