

23.3: Two Bricks in Thermal Contact

Where does the idea of temperature fit into the picture? This concept has come up informally, but we need to give it a precise definition. If two objects at different temperatures are placed in contact with each other, we observe that internal energy flows from the warmer object to the cooler object, as illustrated in Figure 23.3.3.

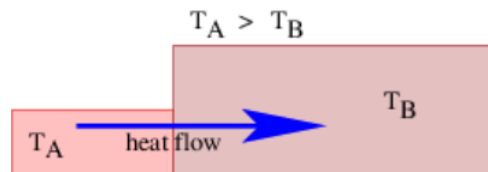


Figure 23.3.3:: Two bricks in thermal contact, one at temperature T_A , the other at temperature T_B . If $T_A > T_B$, internal energy flows from brick A to brick B.

We wish to see if the role of temperature differences in the flow of internal energy can be related to the ideas developed in the previous section. Let us consider two bricks as before, but possibly of different size, and therefore containing different numbers of harmonic oscillators. Suppose brick A has N_A oscillators and energy E_A while brick B has N_B oscillators and energy E_B . The two bricks have entropies

$$S_A = k_B N_A \left[\ln \left(\frac{E_A}{N_A E_0} \right) + 1 \right] \quad (23.3.1)$$

and

$$S_B = k_B N_B \left[\ln \left(\frac{E_B}{N_B E_0} \right) + 1 \right]. \quad (23.3.2)$$

If the two bricks are thermally isolated from each other but are nevertheless considered together as one system, then the total number of states available to this combined system is just the product of the numbers of states available to each brick separately:

$$\Delta \mathcal{N} = \Delta \mathcal{N}_A \Delta \mathcal{N}_B \quad (23.3.3)$$

To make an analogy, the total number of ways of arranging two coins, each of which may either be heads up or tails up, is $4 = 2 \times 2$, or heads-heads, heads-tails, tails-heads, and tails-tails. We compute the states of the combined system just as we compute the total number of ways of arranging the coins, i. e., by taking the product of the numbers of states of the individual systems.

Taking the logarithm of $\Delta \mathcal{N}$ and multiplying by Boltzmann's constant results in an equation for the combined entropy S of the two bricks:

$$S = S_A + S_B \quad (23.3.4)$$

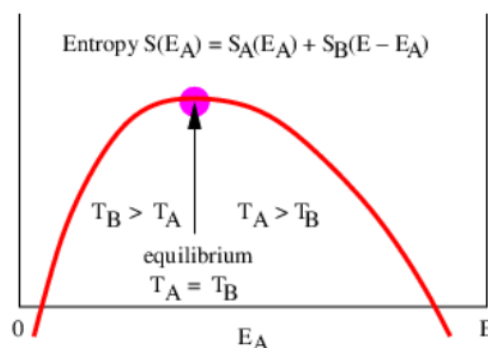


Figure 23.3.4:: Total entropy of two systems for fixed total energy $E = E_A + E_B$ as a function of E_A , the energy of system A.

We can determine how the total entropy of the two bricks depends on the distribution of energy between them by using equations (23.3.1) and (23.3.2). Plotting the sum of the entropies of the two bricks $S_A(E_A) + S_B(E_B)$ versus the energy E_A of brick A under the constraint that the total energy $E = E_A + E_B$ is constant yields a curve that typically looks something like Figure 23.3.4. Notice that the total entropy reaches a maximum for some critical value of E_A . Since the slope of $S(E_A)$ is zero at this

point, we can determine the corresponding value of E_A by setting the derivative to zero of the total entropy with respect to E_A , subject to the condition that the total energy is constant. Under the constraint of constant total energy E , we have $dE_B/dE_A = d(E - E_A)/dE_A = -1$, so

$$\frac{\partial S}{\partial E_A} = \frac{\partial S_A}{\partial E_A} + \frac{\partial S_B}{\partial E_A} = \frac{\partial S_A}{\partial E_A} + \frac{\partial S_B}{\partial E_B} \frac{dE_B}{dE_A} = \frac{\partial S_A}{\partial E_A} - \frac{\partial S_B}{\partial E_B} = 0. \quad (23.3.5)$$

(The partial derivatives indicate that parameters besides the energy are held constant while taking the derivative of entropy.) Thus,

$$\frac{\partial S_A}{\partial E_A} = \frac{\partial S_B}{\partial E_B} \quad (\text{equilibrium condition}) \quad (23.3.6)$$

at the point of maximum entropy.

Once the equilibrium values of E_A and E_B are found, we can calculate the total entropy $S = S_A + S_B$ of two thermally isolated bricks. We now assert that this entropy doesn't change when two bricks in equilibrium are brought into thermal contact. Why is this so?

The derivative of the entropy of a system with respect to energy turns out to be one over the temperature of the system. Thus, the temperatures of the bricks can be found from

$$\frac{1}{T} \equiv \frac{\partial S}{\partial E} \quad (\text{definition of temperature}) \quad (23.3.7)$$

The condition for equilibrium (23.3.6) therefore reduces to $1/T_A = 1/T_B$, or $T_A = T_B$. This is consistent with observations of the behavior of real systems. Thus, at the equilibrium point the temperatures of the two bricks are the same and bringing them together causes no heat flow to occur. The process of bringing two bricks at the same temperature into thermal contact is thus completely reversible, since separating them leaves each with the same amount of energy it started with.

The temperature of a brick is easily calculated using equation (23.3.7):

$$T = \frac{E}{k_B N} \quad (\text{temperature of } N \text{ harmonic oscillators}) \quad (23.3.8)$$

We see that the temperature of a brick is just the average energy per harmonic oscillator in the brick divided by Boltzmann's constant.

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