14.7: Virtual Particles

Another problem is evident from Figure 14.7.5. As drawn, the velocity of the intermediary particle exceeds the speed of light. This is reflected in the fact that different reference frames yield contradictory results as to whether the intermediary particle moves from A to B or B to A. These difficulties turn out to be much less severe than those arising from non-locality. Let us address them in sequence.

For sake of definiteness, let us view the emission of particle C by particle A in a reference frame in which the velocity of particle A is just reversed in the emission process. In this case the four-momentum before the emission is $\mathrm{p_A = (p, E/c),\ where\ E = \left(p^2c^2 + m^2c^4\right)^{1/2}}$. After the emission we have $\mathrm{p'_A = (-p, E/c)}$. Conservation of four-momentum in the emission process requires that

$$\underline{p}_A = \underline{p}'_A + \underline{q} \tag{14.7.1}$$

where $\underline{\mathrm{q}}$ is the four-momentum of particle C. From the above assumptions it is clear that

$$\underline{q} = \underline{p}_A - \underline{p}'_A = (p + p, E/c - E/c) = (2p, 0). \tag{14.7.2}$$

Suppose that the real, measured mass of particle C is $\mathrm{m_C}$. This conflicts with the apparent or virtual mass of this particle in its flight from A to B, which is

$$m' = (-\underline{q} \cdot \underline{q})^{1/2}/c \equiv iq/c \tag{14.7.3}$$

where q ≡|$\underline{\mathrm{q}}$| is the momentum transfer. Note that the apparent mass is imaginary because the four-momentum is spacelike.

Classically, this discrepancy in the apparent and actual masses of the particle C would simply indicate that the process wasn't possible. However, recall that the uncertainty principle allows there to be an uncertainty in the mass if it doesn't persist for too long in terms of the proper time interval along the particle's world line. The statement of this law is $\Delta\mu\Delta\mathrm{T} \approx 1$. Expressed in terms of mass, this becomes

$$\Delta m \Delta\tau \approx \hbar/c^2 \tag{14.7.4}$$

Let us convert the proper time to an interval since the world line of particle C is horizontal in the reference frame in which we are viewing it. Ignoring the factor of i, $\Delta\mathrm{T} = \Delta\mathrm{I}/\mathrm{C}$. We finally compute the absolute value of the mass discrepancy as follows: $|m_C - iq/c| = [(m_C - iq/c)(m_C + iq/c)]^{1/2} = \left(mc^2 + q^2/c^2\right)^{1/2}$. Solving for I yields the approximate maximum invariant interval that particle C can move from its source point while keeping its erroneous mass hidden by the uncertainty principle:

$$\Delta I \approx \frac{\hbar}{\left(m_C^2c^2 + q^2\right)^{1/2}} \tag{14.7.5}$$

A particle forced into having an apparent mass different from its actual mass is called a virtual particle. The interaction shown in Figure 14.7.5 can only take place if particles A and B come closer to each other than the distance ΔI. This argument thus produces an estimate for the "range" of an interaction with momentum transfer 2p and intermediary particle mass $\mathrm{m_C}$.

Two distinct possibilities exist. If the intermediary particle is massless (a photon, for instance), then the range of the interaction is inversely related to the momentum transfer: $\Delta\mathrm{I} \approx \hbar/\mathrm{q}.$. Thus, small momentum transfers can occur at large distances. An interaction of this type is called "long range". On the other hand, if the intermediary particle has mass, the range is simply $\mathrm{\Delta I \approx \hbar/m_C c\ when\ q \ll m_C C}$. The range is thus constant and inversely proportional to the mass of the intermediary particle for low momentum transfers. For large momentum transfer, i. e., when $\mathrm{q \gg m_C C}$, the range decreases from this value with increasing momentum transfer, as in the case of a massless intermediary particle.

Virtual Particles and Gauge Theory

According to quantum mechanics, particles are represented by waves. The absolute square of the wave amplitude represents the probability of finding the particle. In gauge theory the potential four-momentum performs this role for the virtual particles intermediary interactions. Thus a larger potential four-momentum at some point means a higher probability of finding the related virtual particles at that point.