18.2: The Ring Around the Moon

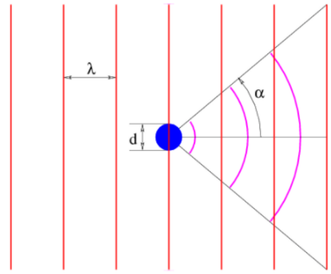

Figure 18.2.2:: Scattering of an incident plane wave by a water droplet. The opening half-angle of the scattered wave is $a \approx \lambda/(2\,d)$.

Sometimes at night one sees a diffuse disk of light around the moon if it happens to be shining through a thin layer of cloud. This disk consists of light diffracted by the water or ice particles in the cloud. The diameter of the disk contains information about the size of the cloud particles doing the diffraction. In particular, if the particles have diameter *d* and the light has wavelength λ, then the diffraction half-angle shown in Figure 18.2.2 is approximately

$$\alpha \approx \lambda/(2d) \tag{18.2.1}$$

This equation comes from the problem of passage of light through a hole or slit of diameter or width *d*. This problem was treated in the chapter on waves, and the above formula was concluded to hold in that case. One can think of the diffraction of light by a particle to be the linear superposition of a plane wave minus the diffraction of light by a hole in a mask, as illustrated in Figure 18.2.2. The angular spread of the diffracted light is the same in both cases.

The interesting point about equation (18.2.1) is that the opening angle of the diffraction cone is inversely proportional to the diameter of the diffracting particles. Thus, for a given wavelength, smaller particles cause diffraction through a wider angle.

Note that when the wavelength exceeds the diameter of the particle by a significant amount, equation (18.2.1) fails, since scattering through an angle greater than π doesn't make physical sense. In this case the diffracted photons tend to be isotropic, i. e., they are scattered with equal probability into any direction.

If one wishes to measure the size of an object by observing the diffraction of a wave around the object, the lesson is clear; the wavelength of the wave must be less than or equal to the dimensions of the object — otherwise the scattering of the wave by the object is largely isotropic and equation (18.2.1) yields no information. Since wavelength is inversely related to momentum by the de Broglie relationship, this condition implies that the momentum must satisfy

$$p = h/\lambda > h/d \tag{18.2.2}$$

in order that the size of an object of diameter *d* be resolved.