18.6: Problems

1. If possible, observe the moon through a thin cloud layer and estimate the angular size of the disk of scattered light around the moon. From this, estimate the size of the particles doing the scattering.
2. Which particle can be used to investigate smaller scales, a proton or an electron, at
    1. the same velocity, and
    2. at the same kinetic energy? (Work non-relativistically in both cases.)
    3. Now consider ultra-relativistic protons and electrons with the same total energy. Is there a significant difference between their ability to investigate very small scales?
3. Electron microscope:
    1. What kinetic energy (in electron volts) must electrons in an electron microscope have to match the resolution of an optical microscope? (The resolutions match when the wavelengths of the electrons and the light are the same.)
    2. If the electrons have kinetic energy 50 KeV, how much better resolution does the electron microscope have than the best optical microscope?

    Hint: Use the non-relativistic kinetic energy and check whether this assumption is valid in retrospect.
4. Integrated circuits are made by a system in which the circuit pattern is engraved on a silicon wafer using a photochemical process working with an optical imaging device that projects the circuit image on the wafer.
    1. Assuming visible light is used, estimate the size of the smallest feature that could be produced on the silicon by this system.
    2. Do the same for 1 KeV X-rays.

    Hint: Recall that the smallest feature resolvable by a wave is approximately the wavelength of that wave.
5. The rest energy of two colliding particles is just c^2 multiplied by the mass of the single particle created by the colliding particles sticking together.
    1. Compute the rest energy (in GeV) of a particle resulting from a 100 GeV energy proton colliding with a stationary proton.
    2. Compute the rest energy of the particle resulting from two 50 GeV protons colliding head-on.

    Hint: These calculations are relativistic, since the rest energy of the proton is about 0.9 GeV.
6. Relativistic charged particle in magnetic field: Assume that a relativistic particle of mass m and charge e is moving in a circle under the influence of the magnetic field **B** = (0, 0,-B). The position of the particle as a function of time is given by $\mathbf{x} = [\mathrm{R}\cos(\omega \mathrm{t}), \mathrm{R}\sin(\omega \mathrm{t}), 0]$.
    1. Compute the (vector) velocity of the particle and show that its speed is $v = \omega R$.
    2. Compute the (relativistic) momentum (again in vector form) of the particle using the above results.
    3. Compute the magnetic force **F** on the particle.
    4. Using the relativistic version of Newton's second law, $\mathbf{F} = \mathbf{dp}/\mathbf{dt}$, determine how the rotational frequency ω depends on the speed of the particle, the magnetic field B, and the particle's charge and mass. Examine particularly the limits where $v \ll c$ and $v \approx c$.
    5. Eliminate ω between the above result and the speed formula to get an equation for the radius R of the circle. Show that this takes the particularly simple form $\mathrm{R} = \mathrm{p}/(\mathrm{eB})$ when written in terms of the magnitude of the momentum $p = mv\gamma$.
7. A 30 GeV electron is scattered by a virtual photon through an angle of 60° without changing its energy.
    1. Compute its momentum vector before and after the scattering.
    2. Compute the momentum transfer to the electron by the photon in the scattering event.
    3. Compute the wavelength of the virtual photon.
    4. What is the virtual photon's energy?
    5. What is the virtual photon's mass?
8. Find α, β, γ such that $\hbar^{\alpha}\mathrm{C}^{\beta}\mathrm{G}^{\gamma}$ has the units of length. (G is the universal gravitational constant.) Compute the numerical value of this length, which is called the Planck length. Compare this value to the resolution available today in the highest energy accelerators.