23.5: Specific Heat

How can we compute the specific heat of a collection of harmonic oscillators? Starting from the temperature of a brick, as given by equation (23.21), we solve for the brick's internal energy:

$$E = Nk_BT \quad \text{(internal energy of } N \text{ oscillators)}. \tag{23.5.1}$$

Recall that the specific heat is the heat required per unit mass to increase the temperature of the brick by one degree. For a solid body, essentially all the heat added to the body goes into increasing its internal energy. Thus, if the mass of the brick is $M = Nm$ where m is the mass per oscillator, then the predicted specific heat of the brick is

$$C \equiv \frac{1}{M}\frac{dQ}{dT} \approx \frac{1}{M}\frac{dE}{dT} = \frac{k_B}{m} \quad \text{(specific heat of harmonic oscillators)}. \tag{23.5.2}$$

This formula is in reasonable agreement with measurements when the temperature is high enough so that all the harmonic oscillators are in excited states, i. e., with $r > 1$. (We equate $dQ = dE$ using the first law of thermodynamics, since no work is being done by the brick.)