14.6: Conservation of Four-Momentum

We earlier introduced the ideas of energy and momentum conservation. In other words, if we have a number of particles isolated from the rest of the universe, each with momentum $\mathbf{p_i}$ and energy $\mathrm{E_i}$, then particles may be created and destroyed and they may collide with each other. In these interactions the energy and momentum of each particle may change, but the sum total of all the energy and the sum total of all the momentum remains constant with time:

$$E=\sum_i E_i=\text{ const } \quad \mathbf{p}=\sum_i \mathbf{p}_i=\text{ const } . \tag{14.6.1}$$

The expression is simpler in terms of four-momentum:

$$\underline{p}=\sum_i \underline{p}_i=\text{ const} \tag{14.6.2}$$

At this point a statement such as the one above should ring alarm bells. Just what does it mean to say that the total energy and momentum remain constant with time in the context of relativity? *Which time?* The time in *which reference frame*?

Figure $14.6.4$ illustrates the problem. Suppose two particles exchange four-momentum remotely at the time indicated by the fat horizontal bar in the left panel of Figure $14.6.4$. Conservation of four-momentum implies that

$$\underline{p}_A+\underline{p}_B=\underline{p}_A^{\prime}+\underline{p}_B^{\prime} \tag{14.6.3}$$

where the subscripted letters correspond to the particle labels in Figure $14.6.4$. Primed values refer to the momentum after the exchange while no primes indicates values before the exchange.

Now view the exchange from the reference frame in the right panel of Figure $14.6.4$. A problem with four-momentum conservation exists in the region between the thin horizontal lines. In this region particle B has already transferred its four-momentum, but it has yet to be received by particle A. In other words, four-momentum is *not* conserved in this reference frame!

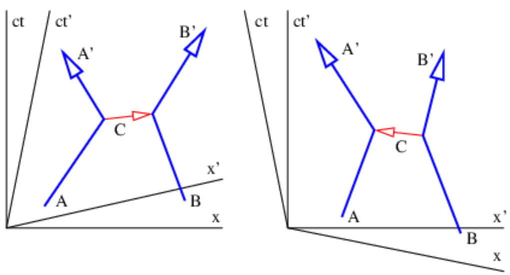

Figure 14.6.5:: Mediation of action at a distance by a third particle. Notice that the world line of the intermediary particle has a slope less than unity, which means that it is nominally moving faster than the speed of light.

This problem is so serious that we must eliminate the concept of force at a distance from the repertoire of physics. The only way to have particles interact remotely and still conserve four-momentum in all reference frames is to assume that all remote interactions are mediated by *another particle*, as indicated in figure 14.5. In other words, momentum and energy are transferred from particle A to particle B in a two step process. First, particle A emits particle C in a manner which conserves the four-momentum. Second, particle C is absorbed by particle B in a similarly conservative interaction. Four-momentum is conserved at all times in all reference frames in this picture.