24.4: Problems

1. Following the procedure for a three-dimensional gas, do the following for a two-dimensional gas in a box of area $A = a^2$, where a is the side length of the box.
   1. Find $\mathcal{N}$ for N particles. Eliminate a in favor of the box area A.
   2. Compute the entropy for this gas.
   3. Compute the temperature T, as a function of N and the internal energy E. Invert to obtain the internal energy as a function of N and T.
   4. Solve the entropy equation for energy and compute the "two-dimensional pressure", $\tau = \mathrm{T} = -\partial E/\partial A$. What units does T have?
   5. Find the two-dimensional analog to the ideal gas equation.

   These calculations are relevant to atoms that can move freely around on a surface, but cannot escape it for energetic reasons.
2. Suppose your house has interior volume V. There are a few small air leaks, so that the inside air pressure p always equals the outside air pressure, which is assumed not to change.
   1. Compute the internal energy of the air in your house.
   2. Your roommate, trying to impress you with his knowledge of physics, says that he is going to turn up the thermostat to increase the internal energy of the air in the house. Will this work? Explain.
3. It has been proposed to extract useful work from the ocean by exploiting the temperature difference between deep ocean water at ≈ 0° C and tropical surface water at ≈ 30° C to run a heat engine. What thermodynamic efficiency would this process have?
4. Suppose your house is heated by a Carnot engine working as a refrigerator between an outdoor temperature of 273 K and an indoor temperature of 303 K. (This means you are cooling the outdoors to heat the indoors! Such devices are called heat pumps.) If your house loses heat at a rate of 5 kW, how much electrical energy must be used to power the (perfectly efficient) electrical motor running the Carnot engine? Compare the monthly cost of running this Carnot engine to the cost of direct electric heating, i. e., via a big resistor.
5. Suppose an airplane engine is a heat engine that works between temperatures T_{air} and $T_{air} + \Delta T$, where T_{air} is the air temperature, and where ΔT is fixed. Other things being equal, is this engine more thermodynamically efficient in the summer or winter? Explain.
6. Suppose the spring constant k of a spring varies with temperature, so that $k = CT$, where C is a constant and T is the (Kelvin) temperature. Describe how this spring could be used to construct a heat engine.
7. Suppose a monatomic ideal gas at initial temperature T is allowed to expand very rapidly so that its new volume is twice its original volume. It is then compressed isentropically (i. e., at constant entropy, which means it is done slowly) back to the original volume. What is its new temperature?
8. An inventor claims to have a refrigerator that extracts 100 W of heat from its interior, which is kept at 150 K, rejecting the heat at room temperature (300 K). He claims that the refrigerator only consumes 10 W of externally supplied power. If this device works, does it violate the second law of thermodynamics?