16.1: Coulomb's Law and the Electric Field

A stationary point electric charge q is known to produce a scalar potential

$$\phi = \frac{q}{4\pi\epsilon_0 r} \tag{16.1.1}$$

a distance r from the charge. The constant $\epsilon_0 = 8.85 \times 10^{-12} \mathrm{C}^2\ \mathrm{N}^{-1}\ \mathrm{m}^{-2}$ is called the permittivity of free space. The vector potential produced by a stationary charge is zero.

The potential energy between two stationary charges is equal to the scalar potential produced by one charge multiplied by the value of the other charge:

$$U = \frac{q_1 q_2}{4\pi\epsilon_0 r} \tag{16.1.2}$$

Notice that it doesn't make any difference whether one multiplies the scalar potential from charge 1 by charge 2 or vice versa – the result is the same.

Since $r = \left(x^2 + y^2 + z^2\right)^{1/2}$, the electric field produced by a charge is

$$\mathbf{E} = -\left(\frac{\partial\phi}{\partial x}, \frac{\partial\phi}{\partial y}, \frac{\partial\phi}{\partial z}\right) = \frac{q\mathbf{r}}{4\pi\epsilon_0 r^3} \tag{16.1.3}$$

where **r** = (x,y,z) is the vector from the charge to the point where the electric field is being measured. The magnetic field is zero since the vector potential is zero.

The force between two stationary charges separated by a distance r is the value of one charge multiplied by the electric field produced by the other charge. Thus the magnitude of the force is

$$F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2} \quad (\text{ Coulomb's law }), \tag{16.1.4}$$

with the force being repulsive if the charges are of the same sign, and attractive if the signs are opposite. This is called Coulomb's law.

Equation (16.1.4) is the electric equivalent of Newton's universal law of gravitation. Replacing mass by charge and G by $-1/\left(4\pi\epsilon_0\right)$ in the equation for the gravitational force between two point masses gives us equation (16.1.4). The most important aspect of this result is that both the gravitational and electrostatic forces decrease as the square of the distance between the particles.