13.4: Flux from Multiple Masses

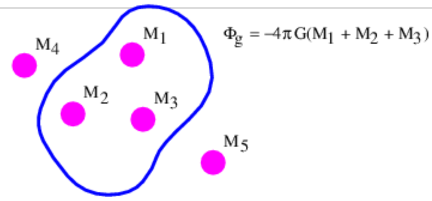

Figure 13.4.6:: Gauss's law applied to more than one mass. The masses M_1, M_2, and M_3 contribute to the outward gravitational flux through the surface shown. The masses M_4 and M_5 don't contribute.

Gauss's law extends trivially to more than one mass. As Figure 13.4.6 shows, the outward flux through a closed surface is just

$$\Phi_g = -4\pi G \sum_{inside} M_i \quad (\text{ Gauss's law }) \tag{13.4.1}$$

In other words, all masses inside the closed surface contribute to the flux, while no masses outside the surface contribute. This is the most general statement of Gauss's law as it applies to gravity.

An important application of Gauss's law is to show that the gravitational field outside of a spherically symmetric extended mass M is exactly the same as if all the mass were concentrated at a point at the center of the sphere. The proof goes as follows: Imagine a sphere concentric with the center of the extended mass, but with larger radius. The gravitational flux from the mass is just $\Phi_g = -4\pi GM$ as before. However, because of the assumed spherical symmetry, we know that the gravitational field points normally inward at every point on the spherical surface and is equal in magnitude everywhere on the sphere. Thus we can infer that $\Phi_g = -4\pi R^2 g$, where R is the radius of the sphere and g is the magnitude of the gravitational field at radius R. From these two equations we immediately infer that the field magnitude is

$$g = \frac{GM}{R^2} \tag{13.4.2}$$

Expressing this in vector form for arbitrary radius r, and remembering that the gravitational field points inward, we find that

$$\mathbf{g} = -\frac{GM\mathbf{r}}{r^3} \tag{13.4.3}$$

which is precisely the equation for g resulting from a point mass M. Recall that r points from the mass to the test point.