

17.4: Energy of Electric and Magnetic Fields

In this section we calculate the energy stored by a capacitor and an inductor. It is most profitable to think of the energy in these cases as being stored in the electric and magnetic fields produced respectively in the capacitor and the inductor. From these calculations we compute the energy per unit volume in electric and magnetic fields. These results turn out to be valid for any electric and magnetic fields — not just those inside parallel plate capacitors and inductors!

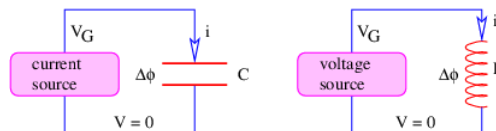


Figure 17.4.7:: Capacitor (left) and inductor (right) being charged respectively by constant sources of current and voltage.

Let us first consider a capacitor starting in a discharged state at time $t = 0$. A constant current i is caused to flow through the capacitor by some device such as a battery or a generator, as shown in the left panel of Figure 17.4.7. As the capacitor charges up, the potential difference across it increases with time:

$$\Delta\phi = \frac{q}{C} = \frac{it}{C} \quad (17.4.1)$$

The EMF supplied by the generator has to increase to match this value.

The generator does work on the positive charges moving around the circuit in the direction indicated by the arrow. We assume that $\Delta\phi$ equals the EMF or work per unit charge done by the generator V_G , so the work done in time dt by the generator is $dW = V_G dq = V_G i dt$. Using the equation for the potential difference across a capacitor, we see that the power input is

$$P = \frac{dW}{dt} = \Delta\phi i = \frac{i^2 t}{C}. \quad (17.4.2)$$

Integrating this in time yields the total energy U_E supplied to the capacitor by the generator:

$$U_E = \frac{i^2 t^2}{2C} = \frac{q^2}{2C} \quad (\text{capacitor}) \quad (17.4.3)$$

Assuming that we have a parallel plate capacitor, let's insert the formula for the capacitance of such a device, $C = \epsilon_0 S/d$. Let us further recall that the electric field in a parallel plate capacitor is $E = \sigma/\epsilon_0 = q/(\epsilon_0 S)$, so that $q = \epsilon_0 ES$ and

$$U_E = \frac{(E\epsilon_0 S)^2}{2(\epsilon_0 S/d)} = \frac{\epsilon_0 E^2 Sd}{2} \quad (17.4.4)$$

The combination Sd is just the volume between the capacitor plates. The **energy density** in the capacitor is therefore

$$u_E = \frac{U_E}{Sd} = \frac{\epsilon_0 E^2}{2} \quad (\text{electric energy density}) \quad (17.4.5)$$

This formula for the energy density in the electric field is specific to a parallel plate capacitor. However, it turns out to be valid for any electric field.

A similar analysis of a current increasing from zero in an inductor yields the energy density in a magnetic field. Imagine that the generator in the right panel of Figure 17.4.7 produces a constant EMF, V_G , starting at time $t = 0$ when the current is zero. The work done by the generator in time dt is $dW = V_G dq = V_G i dt$ so that the power is

$$P = \frac{dW}{dt} = V_G i = Li \frac{di}{dt} = \frac{d}{dt} \left(\frac{Li^2}{2} \right) \quad (17.4.6)$$

We have assumed that the EMF supplied by the generator, V_G , balances the voltage drop across the inductor: $V_G = \Delta\phi = L (di/dt)$.

If we integrate the above equation in time, we get the energy added to the inductor as a result of increasing the current through it. Substituting the formula for the inductance of a parallel plate inductor, $L = \mu_0 dl/w$, we arrive at the equation for the energy stored by the inductor:

$$U_B = \frac{Li^2}{2} = \frac{\mu_0 i^2 l d}{2w} \quad (\text{parallel plate inductor}). \quad (17.4.7)$$

Finally, using the relationship between the current and the magnetic field in a parallel plate inductor, $B = \mu_0 i / w$, we can eliminate the current i and write

$$U_B = \frac{B^2 l w d}{2\mu_0} \quad (17.4.8)$$

The volume between the inductor plates is just dlw , so again we can write an energy density, this time for the magnetic field:

$$u_B = \frac{U_B}{l w d} = \frac{B^2}{2\mu_0} \quad (\text{magnetic energy density}). \quad (17.4.9)$$

Though we only proved this equation for the magnetic field inside a parallel plate inductor, it turns out to be true for any magnetic field.

The total energy density is just the sum of the electric and magnetic energy densities:

$$u_T = u_E + u_B = \frac{\epsilon_0 E^2}{2} + \frac{B^2}{2\mu_0} \quad (17.4.10)$$

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