1.5: Beats

Suppose two sound waves of different frequency but equal amplitude impinge on your ear at the same time. The displacement perceived by your ear is the superposition of these two waves, with time dependence

$$h(t) = \sin(\omega_1 t) + \sin(\omega_2 t) = 2\sin(\omega_0 t)\cos(\Delta\omega t) \tag{1.5.1}$$

where we have used the above math trick, and where $\omega_0 = (\omega_1 + \omega_2)/2$ and $\Delta\omega = (\omega_2 - \omega_1)/2$. What you actually hear is a tone with angular frequency ω_0 which fades in and out with period

$$T_{\rm beat} = \pi/|\Delta\omega| = 2\pi/|\omega_2 - \omega_1| = 1/|f_2 - f_1| \tag{1.5.2}$$

The *beat frequency* is simply

$$f_{\rm beat} = 1/T_{\rm beat} = |f_2 - f_1| \tag{1.5.3}$$

Note how beats are the time analog of wave packets — the mathematics are the same except that frequency replaces wavenumber and time replaces space.