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For a rigid body rotating about a fixed axle, the moment of inertia is

$$I=\sum_{i} M_{i} d_{i}^{2} \tag{11.6.1}$$

where d_i is the perpendicular distance of the ith particle from the axle. Equations (11.16)-(11.18) are valid for a rigid body consisting of many particles. Furthermore, the moment of inertia is constant in this case, so it can be taken out of the time derivative:

$$\tau=\frac{d I \omega}{d t}=I \frac{d \omega}{d t}=I \alpha \quad (\text{ fixed axle, constant } I) \tag{11.6.2}$$

The quantity $a=d \omega / d t$ is called the angular acceleration.

The sum in the equation for the moment of inertia can be converted to an integral for a continuous distribution of mass. We shall not pursue this here, but simply quote the results for a number of solid objects of uniform density:

- For rotation of a sphere of mass M and radius R about an axis piercing its center: $I=2 M R^{2} / 5$.
- For rotation of a cylinder of mass M and radius R about its axis of symmetry: $I=M R^{2} / 2$.
- For rotation of a thin rod of mass M and length L about an axis perpendicular to the rod passing through its center: $\mathrm{I}=\mathrm{ML}^{2} / 12$.
- For rotation of an annulus of mass M, inner radius R_a, and outer radius R_b about its axis of symmetry: $I=M\left(R_{a}^{2}+R_{b}^{2}\right) / 2$.