7.7: Problems

1. An electron with wavelength $\lambda=1.2\times10^{-10}\ \mathrm{m}$ undergoes Bragg diffraction from a single crystal with atomic plane spacing of $\mathrm{d}=2\times10^{-10}\ \mathrm{m}$
    1. Calculate the Bragg angles (all of them!) for which constructive interference occurs.
    2. Calculate the speed of the electron.
2. Suppose that electrons impinge on two slits in a plate, resulting in a two slit diffraction pattern on a screen on the other side of the plate. The probability for an electron to pass through either one of the slits and reach point A on the screen is P, assuming that the other slit is blocked.
    1. If there are two slits open and A is a point of constructive interference, what is the probability of an electron reaching A? Hint: Remember that amplitudes, not probabilities add.
    2. If there are two slits open and A is a point of destructive interference, what is the probability of an electron reaching A?
    3. If there are two slits open, what is the probability for an electron to reach point A according to the conventional rule that probabilities add? (This is the result one would expect if, for instance, the particles were machine gun bullets and the slits were, say, 5 cm apart.)
    4. If the slit separation is very much greater than the electron wavelength, how does this affect the spacing of regions of constructive and destructive interference? Explain how the results of parts (a) and (b) become approximately consistent with those of part (c) in this case.
3. Compute the (angular) rest frequency of an electron and a neutron. (Look up their masses.)
4. How does the dispersion relation for relativistic waves simplify if the rest frequency (and hence the particle mass) is zero? What is the group velocity in this case?
5. X-rays are photons with frequencies about 2000 times the frequencies of ordinary light photons. From this information and what you know about light, infer the approximate velocity of electrons which have Bragg diffraction properties similar to X-rays. Are the electrons relativistic or non-relativistic?
6. Electrons with velocity $v=0.6c$ are diffracted with a 0.2 radian half-angle of diffraction when they hit an object. What is the approximate size of the object? Hint: Diffraction of a wave by an object of a certain size is quite similar to diffraction by a hole in a screen of the same size.
7. Work out an approximate formula for the kinetic energy of a particle as a function of mass m and velocity $\mathrm{u_g}$ which is valid when $u_g^2\ll c^2$. Hint: Use the approximation $(1+\epsilon)^{\mathrm{x}}\approx1+\mathrm{x}\epsilon$, which is valid for $|\epsilon|\ll1.$ As $\mathrm{u_g}/\mathrm{C}$ becomes larger, how does this approximate formula deviate from the exact formula?
8. Work out an approximate formula for the momentum of a particle as a function of m and $\mathrm{u_g}$ in the case where $\mathrm{u_g^2}\ll\mathrm{c^2}$. You may wish to use the approximation mentioned in the previous problem.
9. If a photon is localized to within a distance Δx, what is the uncertainty in the photon energy?
10. If an electron is localized to within a distance Δx, what is the uncertainty in the electron kinetic energy? Hint: As long as $\Delta\Pi\ll\Pi,\Delta\Pi^2\approx2\sqcap\Delta\Pi$. To see why, compute $\mathrm{d}\Pi^2/\mathrm{d}\Pi$.
11. A grocer dumps some pinto beans onto a scale, estimates their mass as 2 kg, and then dumps them off after 5 s. What is the quantum mechanical uncertainty in this measurement? Assume this occurs in Quantum World where the speed of light is 10 m $\mathrm{s^{-1}}$ (speed of a fast buggy) and Planck's constant $\hbar=1\ \mathrm{kg\ m^2\ s^{-1}}$.
12. Mary's physics text (mass 0.3 kg) has to be kept on a leash (length 0.5 m) to prevent it from wandering away from her in Quantum World ($\hbar=1\ \mathrm{kg\ m^2\ s^{-1}}$).
    1. If the leash suddenly breaks, what is the maximum speed at which the book is likely to move away from its initial location?
    2. In order to reduce this speed, should Mary make the new leash shorter or longer than the old one? Explain.
13. A proton (mass $M=1.7\times10^{-27}\ \mathrm{kg}$) is confined to an atomic nucleus of diameter $\mathrm{D}=2\times10^{-15}\ \mathrm{m}$.
    1. What is the uncertainty in the proton's momentum?
    2. Roughly what kinetic energy might you expect the proton to have? Planck's constant is $\hbar=1.06\times10^{-34}\ \mathrm{kg\ m^2\ s^{-1}}$. You may use the non-relativistic equation for the energy.