5.1: Waves in Spacetime

We now look at the characteristics of waves in spacetime. Recall that a sine wave moving to the right in one space dimension can be represented by

$$A(x,t) = A_0 \sin(kx - \omega t) \tag{5.1.1}$$

where A_0 is the (constant) amplitude of the wave, k is the wavenumber, and ω is the angular frequency, and that the quantity $\phi = kx - \omega t$ is called the phase of the wave. For a plane wave in three space dimensions, the wave is represented in a similar way,

$$A(\mathbf{x},t) = A_0 \sin(\mathbf{k} \cdot \mathbf{x} - \omega t) \tag{5.1.2}$$

where k is now the position vector and k is the wave vector. The magnitude of the wave vector, $|\mathbf{k}| = \mathrm{k}$ is just the wavenumber of the wave and the direction of this vector indicates the direction the wave is moving. The phase of the wave in this case is $\phi = \mathbf{k} \cdot \mathbf{x} - \omega t$.

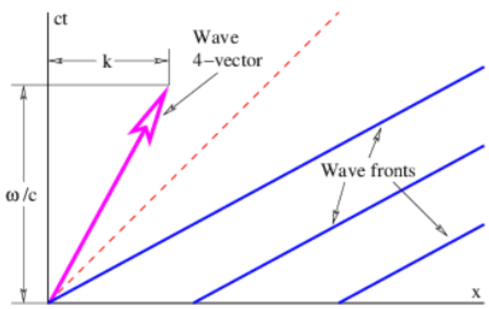

Figure 5.1.1:: Sketch of wave fronts for a wave in spacetime. The large arrow is the associated wave four-vector, which has slope ω/ck. The slope of the wave fronts is the inverse, ck/ω . . The phase speed of the wave is greater than c in this example. (Can you tell why?)

In the case of a one-dimensional wave moving to the right $\phi = kx - \omega t$. A wave front has constant phase ϕ, so solving this equation for t and multiplying by c, the speed of light in a vacuum, gives us an equation for the world line of a wave front:

$$ct = \frac{ckx}{\omega} - \frac{c\phi}{\omega} = \frac{cx}{u_p} - \frac{c\phi}{\omega} \quad (\text{ wave front }) \tag{5.1.3}$$

The slope of the world line in a spacetime diagram is the coefficient of x, or $c/\mathrm{u_p}$, where $\mathrm{u_p} = \omega/\mathrm{k}$ is the phase speed. The world lines of the wave fronts of a wave are illustrated in Figure 5.1.1:.