8.3: Work and Power

When a force is exerted on an object, energy is transferred to the object. The amount of energy transferred is called the *work* done on the object. However, energy is only transferred if the object moves. The work W done is

$$W = F\Delta x \tag{8.3.1}$$

where the distance moved by the object is Δx and the force exerted on it is F. Notice that work can either be positive or negative. The work is positive if the object being acted upon moves in the same direction as the force, with negative work occurring if the object moves opposite to the force.

Equation (8.3.1) assumes that the force remains constant over the full displacement Δx. If it is not, then it is necessary to break up the displacement into a number of smaller displacements, over each of which the force can be assumed to be constant. The total work is then the sum of the works associated with each small displacement.

If more than one force acts on an object, the works due to the different forces each add or subtract energy, depending on whether they are positive or negative. The total work is the sum of these individual works.

There are two special cases in which the work done on an object is related to other quantities. If F is the total force acting on the object, then $\mathrm{W} = \mathrm{F}\Delta\mathrm{x} = \mathrm{ma}\Delta\mathrm{x}$ by Newton's second law. However, $\mathrm{a} = \mathrm{dv}/\mathrm{dt}$ where v is the velocity of the object, and $\Delta\mathrm{x} = (\Delta\mathrm{x}/\Delta\mathrm{t})\Delta\mathrm{t} \approx \mathrm{v}\Delta\mathrm{t}$, where Δt is the time required by the object to move through distance Δx. The approximation becomes exact when Δx and Δt become very small. Putting all of this together results in

$$W_{\text{total}} = m\frac{dv}{dt}v\Delta t = \frac{d}{dt}\left(\frac{mv^2}{2}\right)\Delta t = \Delta K \quad \text{(total work)} \tag{8.3.2}$$

where K is the kinetic energy of the object. Thus, when F is the only force, $W = W_{\text{total}}$ is the total work on the object, and this equals the change in kinetic energy of the object. This is called the work-energy theorem, and it demonstrates that work really is a transfer of energy to an object.

The other special case occurs when the force is conservative, but is not necessarily the total force acting on the object. In this case

$$W_{\text{cons}} = -\frac{dU}{dx}\Delta x = -\Delta U \quad \text{(conservative force)} \tag{8.3.3}$$

where ΔU is the change in the potential energy of the object associated with the force of interest.

The *power* associated with a force is simply the amount of work done by the force divided by the time interval Δt over which it is done. It is therefore the energy per unit time transferred to the object by the force of interest. From equation (8.3.1) we see that the power is

$$P = \frac{F\Delta x}{\Delta t} = Fv \quad \text{(power)} \tag{8.3.4}$$

where v is the velocity at which the object is moving. The *total power* is just the sum of the powers associated with each force. It equals the time rate of change of kinetic energy of the object:

$$P_{\text{total}} = \frac{W_{\text{total}}}{\Delta t} = \frac{dK}{dt} \quad \text{(total power).} \tag{8.3.5}$$