1.10: Problems

1. Measure your pulse rate. Compute the ordinary frequency of your heart beat in cycles per second. Compute the angular frequency in radians per second. Compute the period.
2. An important wavelength for radio waves in radio astronomy is 21 cm. (This comes from neutral hydrogen.) Compute the wavenumber of this wave. Compute the ordinary and angular frequencies. (The speed of light is 3×10^8 m s^{-1}.)
3. Sketch the resultant wave obtained from superimposing the waves $A = \sin(2x)$ and $B = \sin(3x)$. By using the trigonometric identity given in equation (1.17), obtain a formula for *A*+*B* in terms of $\sin(5x2)$ and $\cos(x/2)$. Does the wave obtained from sketching this formula agree with your earlier sketch?
4. Two sine waves with wavelengths λ_1 and λ_2 are superimposed, making wave packets of length *L*. If we wish to make *L* larger, should we make λ_1 and λ_2 closer together or farther apart? Explain your reasoning.
5. By examining Figure $1.10.9$ versus Figure $1.10.10$ and then Figure $1.10.11$ versus Figure $1.10.12$, determine whether equation (1.18) works at least in an approximate sense for isolated wave packets.
6. The frequencies of the chromatic scale in music are given by

$$f_i = f_0 2^{i/12}, \quad i = 0, 1, 2, \ldots, 11 \tag{1.10.1}$$

   where f_0 is a constant equal to the frequency of the lowest note in the scale.

   1. Compute f_1 through f_{11} if f_0 = 440 Hz (the "A" note)
   2. Using the above results, what is the beat frequency between the "A" (i = 0) and "B" (i = 2) notes? (The frequencies are given here in cycles per second rather than radians per second.)
   3. Which pair of the above frequencies f_0 - f_{11} yields the smallest beat frequency? Explain your reasoning.

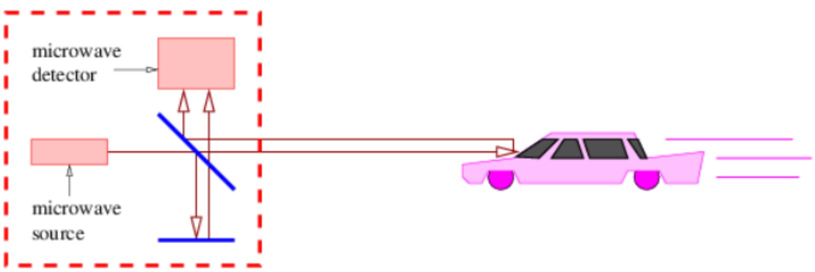


Figure $1.10.19$: Sketch of a police radar.

7. Large ships in general cannot move faster than the phase speed of surface waves with a wavelength equal to twice the ship's length. This is because most of the propulsive force goes into making big waves under these conditions rather than accelerating the ship.
   1. How fast can a 300 m long ship move in very deep water?
   2. As the ship moves into shallow water, does its maximum speed increase or decrease? Explain.
8. Given the formula for refractive index of light quoted in this section, for what range of *k* does the phase speed of light in a transparent material take on real values which exceed the speed of light in a vacuum?
9. A police radar works by splitting a beam of microwaves, part of which is reflected back to the radar from your car where it is made to interfere with the other part which travels a fixed path, as shown in Figure $1.10.19$.
   1. If the wavelength of the microwaves is λ, how far do you have to travel in your car for the interference between the two beams to go from constructive to destructive to constructive?
   2. If you are traveling toward the radar at speed v = 30 m s^{-1}, use the above result to determine the number of times per second constructive interference peaks will occur. Assume that λ = 3 cm.

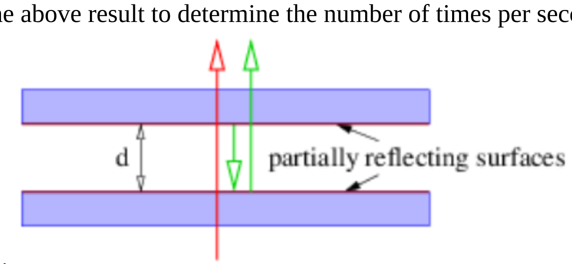


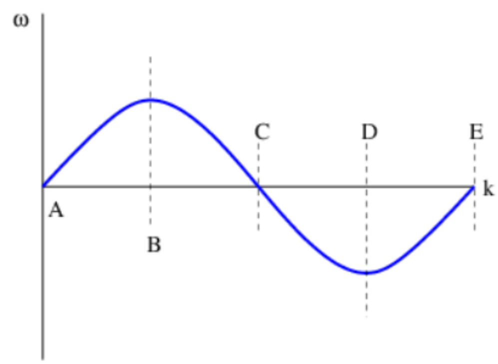
Figure $1.10.20$: Sketch of a Fabry-Perot interferometer.

10. Suppose you know the wavelength of light passing through a Michelson interferometer with high accuracy. Describe how you could use the interferometer to measure the length of a small piece of material.
11. A Fabry-Perot interferometer (see Figure 1.10.20) consists of two parallel half-silvered mirrors placed a distance *d* from each other as shown. The beam passing straight through interferes with the beam which reflects once off of both of the mirrored surfaces as shown. For wavelength λ, what values of *d* result in constructive interference?
12. A Fabry-Perot interferometer has spacing *d* = 2 cm between the glass plates, causing the direct and doubly reflected beams to interfere (see Figure 1.10.20). As air is pumped out of the gap between the plates, the beams go through 23 cycles of constructive-destructive-constructive interference. If the wavelength of the light in the interfering beams is 5×10^{-7} m, determine the index of refraction of the air initially in the interferometer.
13. Measurements on a certain kind of wave reveal that the angular frequency of the wave varies with wavenumber as shown in the following table:

$\omega\left(\mathrm{s}^{-1}\right)$	$k\left(\mathrm{m}^{-1}\right)$
5	1
20	2
45	3
80	4
125	5

(1.10.2)

    1. Compute the phase speed of the wave for $k = 3\ \mathrm{m}^{-1}$ and for $k = 4\ \mathrm{m}^{-1}$.
    2. Estimate the group velocity for $k = 3.5\ \mathrm{m}^{-1}$ using a finite difference approximation to the derivative.
14. Suppose some type of wave has the (admittedly weird) dispersion relation shown in Figure 1.10.21.
    1. For what values of *k* is the phase speed of the wave positive?



    2. For what values of *k* is the group velocity positive?

Figure 1.10.21:: Sketch of a weird dispersion relation.

15. Group velocities of various waves.
    1. Compute the group velocity for shallow water waves. Compare it with the phase speed of shallow water waves. (Hint: You first need to derive a formula for $\omega(k)$ from $c(k)$.)
    2. Repeat the above problem for deep water waves.
    3. Repeat for sound waves. What does this case have in common with shallow water waves?
