11.5: Many Particles

The generalization from two particles to many particles is quite easy in principle. If a subscripted *i* indicates the value of a quantity for the *i*th particle, then the center of mass is given by

$$\mathbf{R}_{cm}=\frac{1}{M_{\text{total}}}\sum_i M_i\mathbf{r}_i \tag{11.5.1}$$

where

$$M_{\text{total}}=\sum_i M_i \tag{11.5.2}$$

Furthermore, if we define $\mathbf{r}_{\mathrm{i}}^{\prime}=\mathbf{r}_{\mathrm{i}}-\mathbf{R}_{\mathrm{cm}}$, etc., then the kinetic energy is just

$$K_{\text{total}}=M_{\text{total}}V_{\text{cm}}^2/2+\sum_i M_i v_i^{\prime 2}/2 \tag{11.5.3}$$

and the angular momentum is

$$\mathbf{L}_{\text{total}}=M_{\text{total}}\mathbf{R}_{cm}\times\mathbf{V}_{cm}+\sum_i M_i\mathbf{r}_i^{\prime}\times\mathbf{v}_i^{\prime} \tag{11.5.4}$$

In other words, both the kinetic energy and the angular momentum can be separated into two parts: one part is related to the overall motion of the system and the other is due to motions of system components relative to the center of mass, just as for the case of the dumbbell.