5.4: Characteristics of Relativistic Waves

Light in a vacuum is an example of a wave for which no special reference frame exists. For light, $\mu=0$, and we have (taking the positive root) $\omega=\mathrm{ck}$. This simply states what we know already, namely that the phase speed of light in a vacuum is c.

If $\mu \neq 0$, waves of this type are dispersive. The phase speed is

$$u_p=\frac{\omega}{k}=\left(c^2+\mu^2/k^2\right)^{1/2} \tag{5.4.1}$$

This phase speed always exceeds c, which at first seems like an unphysical conclusion. However, the group velocity of the wave is

$$u_g=\frac{d\omega}{dk}=\frac{c^2 k}{\left(k^2 c^2+\mu^2\right)^{1/2}}=\frac{kc^2}{\omega}=\frac{c^2}{u_p} \tag{5.4.2}$$

which is always less than c. Since wave packets and hence signals propagate at the group velocity, waves of this type are physically reasonable even though the phase speed exceeds the speed of light.

Another interesting property of such waves is that the wave four-vector is parallel to the world line of a wave packet in spacetime. This is easily shown by the following argument. As Figure $5.4.1$: shows, the spacelike component of a wave four-vector is k, while the timelike component is ω/c. The slope of the four-vector on a spacetime diagram is therefore ω/kc. However, the slope of the world line of a wave packet moving with group velocity $\mathrm{u_g}$ is $\mathrm{c}/\mathrm{u_g}=\omega/(\mathrm{kc})$, which is the same as the slope of the $\underline{k}$ four-vector.

Note that when $\mathrm{k}=0$ we have $\omega=\mu$. In this case the group velocity of the wave is zero. For this reason we call μ the rest frequency of the wave.