6.1: Acceleration

Imagine that you are in a powerful luxury car stopped at a stoplight. As you sit there, gravity pushes you into the comfortable leather seat. The light turns green and you “floor it”. The car accelerates and an additional force pushes you into the seat back. You round a curve, and yet another force pushes you toward the outside of the curve. (But the well designed seat and seat belt keep you from feeling discomfort!)

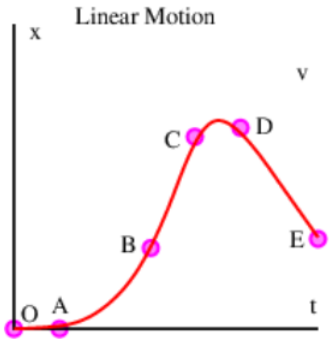

Figure 6.1.1:: Example of linear motion.

Let us examine the idea of acceleration more closely. Considering first acceleration in one dimension, Figure 6.1.1: shows the position of an object as a function of time, $\mathrm{x}(\mathrm{t})$. The velocity is simply the time rate of change of the position:

$$v(t) = \frac{dx(t)}{dt} \tag{6.1.1}$$

The acceleration is the time rate of change of velocity:

$$a(t) = \frac{dv(t)}{dt} = \frac{d^2x(t)}{dt^2} \tag{6.1.2}$$

In Figure 6.1.1:, only the segment OA has zero velocity. Velocity is increasing in AB, and the acceleration is positive there. Velocity is constant in BC, which means that the acceleration is zero. Velocity is decreasing in CD, and the acceleration is negative. Finally, in DE, the velocity is negative and the acceleration is zero.

In two or three dimensions, position **x**, velocity **v**, and acceleration **a** are all vectors, so that the velocity is

$$\mathbf{v}(t) = \frac{d\mathbf{x}(t)}{dt} \tag{6.1.3}$$

while the acceleration is

$$\mathbf{a}(t) = \frac{d\mathbf{v}(t)}{dt} \tag{6.1.4}$$

Thus, over some short time interval Δt, the changes in **x** and **v** can be written

$$\Delta\mathbf{x} = \mathbf{v}\Delta t \quad \Delta\mathbf{v} = \mathbf{a}\Delta t \tag{6.1.5}$$

These are vector equations, so the subtractions implied by the “delta” operations must be done vectorially. An example where the vector nature of these quantities is important is motion in a circle at constant speed, which is discussed in the next section.