2.1.4: Problems

1. Correct use of a calculator: (a) Calculate $\frac{74658}{53222+97554}$ on a calculator. [Self-check: The most common mistake results in 97555.40.] (answer check available at lightandmatter.com)

(b) Which would be more like the price of a TV, and which would be more like the price of a house, $\$3.5 \times 10^5$ or $\$3.5^5$?

2. Compute the following things. If they don't make sense because of units, say so.
(a) 3 cm + 5 cm
(b) 1.11 m + 22 cm
(c) 120 miles + 2.0 hours
(d) 120 miles / 2.0 hours

3. Your backyard has brick walls on both ends. You measure a distance of 23.4 m from the inside of one wall to the inside of the other. Each wall is 29.4 cm thick. How far is it from the outside of one wall to the outside of the other? Pay attention to significant figures.

4. The speed of light is 3.0×10^8 m/s. Convert this to furlongs per fortnight. A furlong is 220 yards, and a fortnight is 14 days. An inch is 2.54 cm.(answer check available at lightandmatter.com)

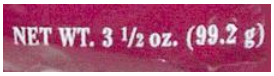
5. Express each of the following quantities in micrograms:
(a) 10 mg, (b) 10^4 g, (c) 10 kg, (d) 100×10^3 g, (e) 1000 ng. (answer check available at lightandmatter.com)

6. (solution in the pdf version of the book) Convert 134 mg to units of kg, writing your answer in scientific notation.

7. In the last century, the average age of the onset of puberty for girls has decreased by several years. Urban folklore has it that this is because of hormones fed to beef cattle, but it is more likely to be because modern girls have more body fat on the average and possibly because of estrogen-mimicking chemicals in the environment from the breakdown of pesticides. A hamburger from a hormone-implanted steer has about 0.2 ng of estrogen (about double the amount of natural beef). A serving of peas contains about 300 ng of estrogen. An adult woman produces about 0.5 mg of estrogen per day (note the different unit!). (a) How many hamburgers would a girl have to eat in one day to consume as much estrogen as an adult woman's daily production? (b) How many servings of peas? (answer check available at lightandmatter.com)

8. (solution in the pdf version of the book) The usual definition of the mean (average) of two numbers a and b is $(a+b)/2$. This is called the arithmetic mean. The geometric mean, however, is defined as $(ab)^{1/2}$ (i.e., the square root of ab). For the sake of definiteness, let's say both numbers have units of mass. (a) Compute the arithmetic mean of two numbers that have units of grams. Then convert the numbers to units of kilograms and recompute their mean. Is the answer consistent? (b) Do the same for the geometric mean. (c) If a and b both have units of grams, what should we call the units of ab? Does your answer make sense when you take the square root? (d) Suppose someone proposes to you a third kind of mean, called the superduper mean, defined as $(ab)^{1/3}$. Is this reasonable?

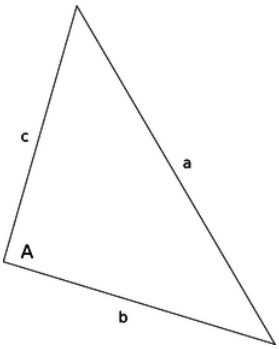
9. In an article on the SARS epidemic, the May 7, 2003 New York Times discusses conflicting estimates of the disease's incubation period (the average time that elapses from infection to the first symptoms). “The study estimated it to be 6.4 days. But other statistical calculations ... showed that the incubation period could be as long as 14.22 days.” What's wrong here?



a / Problem 10.

10. The photo shows the corner of a bag of pretzels. What's wrong here?

11. The distance to the horizon is given by the expression $\sqrt{2rh}$, where r is the radius of the Earth, and h is the observer's height above the Earth's surface. (This can be proved using the Pythagorean theorem.) Show that the units of this expression make sense. (See example 2 on p. 26 for an example of how to do this.) Don't try to prove the result, just check its units.



b / Problem 12

12. (solution in the pdf version of the book) (a) Based on the definitions of the sine, cosine, and tangent, what units must they have? (b) A cute formula from trigonometry lets you find any angle of a triangle if you know the lengths of its sides. Using the notation shown in the figure, and letting $s = (a+b+c)/2$ be half the perimeter, we have

$$\tan A/2 = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}.$$

Show that the units of this equation make sense. In other words, check that the units of the right-hand side are the same as your answer to part a of the question.

13. A physics homework question asks, "If you start from rest and accelerate at 1.54 $\mathrm{m/s^2}$ for 3.29 s, how far do you travel by the end of that time?" A student answers as follows:

$$1.54 \times 3.29 = 5.07 \text{ m}$$

His Aunt Wanda is good with numbers, but has never taken physics. She doesn't know the formula for the distance traveled under constant acceleration over a given amount of time, but she tells her nephew his answer cannot be right. How does she know?

14. You are looking into a deep well. It is dark, and you cannot see the bottom. You want to find out how deep it is, so you drop a rock in, and you hear a splash 3.0 seconds later. How deep is the well? (answer check available at lightandmatter.com)

15. You take a trip in your spaceship to another star. Setting off, you increase your speed at a constant acceleration. Once you get half-way there, you start decelerating, at the same rate, so that by the time you get there, you have slowed down to zero speed. You see the tourist attractions, and then head home by the same method.
(a) Find a formula for the time, T, required for the round trip, in terms of d, the distance from our sun to the star, and a, the magnitude of the acceleration. Note that the acceleration is not constant over the whole trip, but the trip can be broken up into constant-acceleration parts.
(b) The nearest star to the Earth (other than our own sun) is Proxima Centauri, at a distance of $d = 4 \times 10^{16}$ m . Suppose you use an acceleration of $a = 10 \text{ m/s}^2$, just enough to compensate for the lack of true gravity and make you feel comfortable. How long does the round trip take, in years?
(c) Using the same numbers for d and a, find your maximum speed. Compare this to the speed of light, which is 3.0×10^8 \ \textup{m}/\textup{s}. (Later in this course, you will learn that there are some new things going on in physics when one gets close to the speed of light, and that it is impossible to exceed the speed of light. For now, though, just use the simpler ideas you've learned so far.) (answer check available at lightandmatter.com)

16. You climb half-way up a tree, and drop a rock. Then you climb to the top, and drop another rock. How many times greater is the velocity of the second rock on impact? Explain. (The answer is not two times greater.)

17. (solution in the pdf version of the book) If the acceleration of gravity on Mars is 1/3 that on Earth, how many times longer does it take for a rock to drop the same distance on Mars? Ignore air resistance.

18. A person is parachute jumping. During the time between when she leaps out of the plane and when she opens her chute, her altitude is given by an equation of the form

$$y = b - c\left(t + ke^{-t/k}\right),$$

where e is the base of natural logarithms, and b, c, and k are constants. Because of air resistance, her velocity does not increase at a steady rate as it would for an object falling in vacuum.
(a) What units would b, c, and k have to have for the equation to make sense?
(b) Find the person's velocity, v, as a function of time. [You will need to use the chain rule, and the fact that $d(e^x)/dx = e^x$.] (answer check available at lightandmatter.com)
(c) Use your answer from part (b) to get an interpretation of the constant c. [Hint: e^{-x} approaches zero for large values of x.]
(d) Find the person's acceleration, a, as a function of time.(answer check available at lightandmatter.com)
(e) Use your answer from part (d) to show that if she waits long enough to open her chute, her acceleration will become very small.

19. (solution in the pdf version of the book) In July 1999, Popular Mechanics carried out tests to find which car sold by a major auto maker could cover a quarter mile (402 meters) in the shortest time, starting from rest. Because the distance is so short, this type of test is designed mainly to favor the car with the greatest acceleration, not the greatest maximum speed (which is irrelevant to the average person). The winner was the Dodge Viper, with a time of 12.08 s. The car's top (and presumably final) speed was 118.51 miles per hour (52.98 \ \textup{m}/\textup{s}). (a) If a car, starting from rest and moving with *constant* acceleration, covers a quarter mile in this time interval, what is its acceleration? (b) What would be the final speed of a car that covered a quarter mile with the constant acceleration you found in part a? (c) Based on the discrepancy between your answer in part b and the actual final speed of the Viper, what do you conclude about how its acceleration changed over time?

20. The speed required for a low-earth orbit is 7.9×10^3 m/s (see ch. 10). When a rocket is launched into orbit, it goes up a little at first to get above almost all of the atmosphere, but then tips over horizontally to build up to orbital speed. Suppose the horizontal acceleration is limited to $3g$ to keep from damaging the cargo (or hurting the crew, for a crewed flight). (a) What is the minimum distance the rocket must travel downrange before it reaches orbital speed? How much does it matter whether you take into account the initial eastward velocity due to the rotation of the earth? (b) Rather than a rocket ship, it might be advantageous to use a railgun design, in which the craft would be accelerated to orbital speeds along a railroad track. This has the advantage that it isn't necessary to lift a large mass of fuel, since the energy source is external. Based on your answer to part a, comment on the feasibility of this design for crewed launches from the earth's surface.

21. Consider the following passage from Alice in Wonderland, in which Alice has been falling for a long time down a rabbit hole:

Down, down, down. Would the fall *never* come to an end? “I wonder how many miles I've fallen by this time?” she said aloud. “I must be getting somewhere near the center of the earth. Let me see: that would be four thousand miles down, I think” (for, you see, Alice had learned several things of this sort in her lessons in the schoolroom, and though this was not a *very* good opportunity for showing off her knowledge, as there was no one to listen to her, still it was good practice to say it over)...

Alice doesn't know much physics, but let's try to calculate the amount of time it would take to fall four thousand miles, starting from rest with an acceleration of 10 $\mathrm{m/s^2}$. This is really only a lower limit; if there really was a hole that deep, the fall would actually take a longer time than the one you calculate, both because there is air friction and because gravity gets weaker as you get deeper (at the center of the earth, g is zero, because the earth is pulling you equally in every direction at once). (answer check available at lightandmatter.com)

22. How many cubic inches are there in a cubic foot? The answer is not 12.(answer check available at lightandmatter.com)

23. Assume a dog's brain is twice as great in diameter as a cat's, but each animal's brain cells are the same size and their brains are the same shape. In addition to being a far better companion and much nicer to come home to, how many times more brain cells does a dog have than a cat? The answer is not 2.

24. The population density of Los Angeles is about 4000 people/$\mathrm{km^2}$. That of San Francisco is about 6000 people/$\mathrm{km^2}$. How many times farther away is the average person's nearest neighbor in LA than in San Francisco? The answer is not 1.5. (answer check available at lightandmatter.com)

25. A hunting dog's nose has about 10 square inches of active surface. How is this possible, since the dog's nose is only about 1 in $\times$ 1 in $\times$ 1 in = 1 $\mathrm{in^3}$? After all, 10 is greater than 1, so how can it fit?

26. Estimate the number of blades of grass on a football field.

27. In a computer memory chip, each bit of information (a 0 or a 1) is stored in a single tiny circuit etched onto the surface of a silicon chip. The circuits cover the surface of the chip like lots in a housing development. A typical chip stores 64 Mb (megabytes) of data, where a byte is 8 bits. Estimate (a) the area of each circuit, and (b) its linear size.

28. Suppose someone built a gigantic apartment building, measuring 10 km $\times$ 10 km at the base. Estimate how tall the building would have to be to have space in it for the entire world's population to live.

29. A hamburger chain advertises that it has sold 10 billion Bongo Burgers. Estimate the total mass of feed required to raise the cows used to make the burgers.

30. Estimate the volume of a human body, in cm^3.

31. (solution in the pdf version of the book) How many cm^2 is 1 mm^2?

32. (solution in the pdf version of the book) Compare the light-gathering powers of a 3-cm-diameter telescope and a 30-cm telescope.

33. (solution in the pdf version of the book) One step on the Richter scale corresponds to a factor of 100 in terms of the energy absorbed by something on the surface of the Earth, e.g., a house. For instance, a 9.3-magnitude quake would release 100 times more energy than an 8.3. The energy spreads out from the epicenter as a wave, and for the sake of this problem we'll assume we're dealing with seismic waves that spread out in three dimensions, so that we can visualize them as hemispheres spreading out under the surface of the earth. If a certain 7.6-magnitude earthquake and a certain 5.6-magnitude earthquake produce the same amount of vibration where I live, compare the distances from my house to the two epicenters.

34. In Europe, a piece of paper of the standard size, called A4, is a little narrower and taller than its American counterpart. The ratio of the height to the width is the square root of 2, and this has some useful properties. For instance, if you cut an A4 sheet from left to right, you get two smaller sheets that have the same proportions. You can even buy sheets of this smaller size, and they're called A5. There is a whole series of sizes related in this way, all with the same proportions. (a) Compare an A5 sheet to an A4 in terms of area and linear size. (b) The series of paper sizes starts from an A0 sheet, which has an area of one square meter. Suppose we had a series of boxes defined in a similar way: the B0 box has a volume of one cubic meter, two B1 boxes fit exactly inside an B0 box, and so on. What would be the dimensions of a B0 box? (answer check available at lightandmatter.com)

c / Albert Einstein, and his moustache, problem 35.

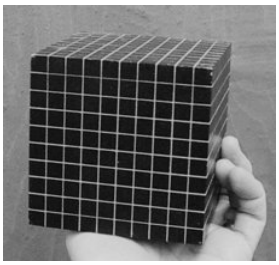
35. Estimate the mass of one of the hairs in Albert Einstein's moustache, in units of kg.

36. According to folklore, every time you take a breath, you are inhaling some of the atoms exhaled in Caesar's last words. Is this true? If so, how many?

37. The Earth's surface is about 70% water. Mars's diameter is about half the Earth's, but it has no surface water. Compare the land areas of the two planets.(answer check available at lightandmatter.com)

38. (solution in the pdf version of the book) The traditional Martini glass is shaped like a cone with the point at the bottom. Suppose you make a Martini by pouring vermouth into the glass to a depth of 3 cm, and then adding gin to bring the depth to 6 cm. What are the proportions of gin and vermouth?

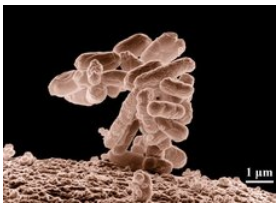
39. The central portion of a CD is taken up by the hole and some surrounding clear plastic, and this area is unavailable for storing data. The radius of the central circle is about 35% of the outer radius of the data-storing area. What percentage of the CD's area is therefore lost? (answer check available at lightandmatter.com)



d / Problem 40.

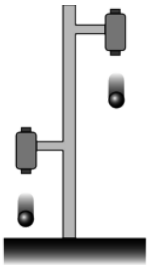
40. The one-liter cube in the photo has been marked off into smaller cubes, with linear dimensions one tenth those of the big one. What is the volume of each of the small cubes?(solution in the pdf version of the book)

41. Estimate the number of man-hours required for building the Great Wall of China. (solution in the pdf version of the book)



e / Problem 42.

42. (a) Using the microscope photo in the figure, estimate the mass of a one cell of the *E. coli* bacterium, which is one of the most common ones in the human intestine. Note the scale at the lower right corner, which is 1 μm. Each of the tubular objects in the column is one cell. (b) The feces in the human intestine are mostly bacteria (some dead, some alive), of which *E. coli* is a large and typical component. Estimate the number of bacteria in your intestines, and compare with the number of human cells in your body, which is believed to be roughly on the order of 10^{13}. (c) Interpreting your result from part b, what does this tell you about the size of a typical human cell compared to the size of a typical bacterial cell?



f / Problem 43.

43. The figure shows a practical, simple experiment for determining g to high precision. Two steel balls are suspended from electromagnets, and are released simultaneously when the electric current is shut off. They fall through unequal heights Δx_1 and Δx_2. A computer records the sounds through a microphone as first one ball and then the other strikes the floor. From this recording, we can accurately determine the quantity T defined as $T = \Delta t_2 - \Delta t_1$, i.e., the time lag between the first and second impacts. Note that since the balls do not make any sound when they are released, we have no way of measuring the individual times Δt_2 and Δt_1.

(a) Find an equation for g in terms of the measured quantities T, Δx_1 and Δx_2.(answer check available at lightandmatter.com)
(b) Check the units of your equation.
(c) Check that your equation gives the correct result in the case where Δx_1 is very close to zero. However, is this case realistic?
(d) What happens when $\Delta x_1 = \Delta x_2$? Discuss this both mathematically and physically.

44. (solution in the pdf version of the book) Estimate the number of jellybeans in figure o on p. 44.

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