3.4.1.2: Transverse Wave Simulation

The following simulation shows a (frozen position) graph of the motion of one location, the red circle, on a string which has a transverse wave on it. Notice that, while the wave moves forward along the string, the red circle does not (in fact none of the circles move forward). The speed of the wave is how fast the crest of the wave moves forward, not the up and down speed of the circles.

A *second* velocity associated with a wave is how fast the material of the wave moves up and down at a single location (the vertical speed of the circles in the simulation). This velocity, the ***transverse velocity***, is not a constant but is a function of location and time (different places on the wave move upward or downward at different speeds at different times).

The vertical location of points on the string (represented by the circles) as a function of horizontal location along the x-direction and time is described mathematically by $y(x,t) = A\sin(2\pi x/\lambda - 2\pi ft + \phi)$. If the wavelength, frequency, amplitude and phase are known, the height of the wave at any location and any time can be found by substituting time and location into the equation.

Simulation Questions:

1. Play the animation. From the top (frozen location) graph, what are the amplitude and period of the motion of the red dot?
2. Clicking on the lower panel gives the mouse location (in the yellow box) which in this case are the *x* and *y* location of points on the wave. Use these numbers to determine the wavelength of the wave (this is easiest to do with the animation paused or finished).
3. From the period and wavelength that you just measured, calculate the forward speed of the wave.
4. You can check your answer in the following way. Pause the wave just after it starts, click on the top of the wave to find its x-position (first number in the yellow box) and record the time. Let the wave move forward for a while and pause it again and record the x-position and time. Subtract the two x-locations to find the distance traveled and divide by the time interval between the two recorded distances. Does this match what you calculated? Why or why not?
5. Check the 'V' box to see graphs of both position and transverse speed of the red dot. What is the maximum transverse speed of the red dot? How does this compare with the forward speed of the wave you just calculated, are they the same or different?
6. Carefully state the relationship between position and velocity of the red dot. When the position is zero (equilibrium position of the red dot) what is the velocity? When the position is a maximum, what is the velocity?

Advanced Questions:

The forward speed of the wave is constant but the vertical speed of the material of the wave is not. Since velocity is the rate of change of position, the transverse velocity in the y direction is given by the derivative of the displacement with respect to time: $v(x,t) = \partial y(x,t)/\partial t = -A\omega\cos(kx - \omega t + \phi)$ where $k = 2\pi/\lambda$ is called the wave number and $\omega = 2\pi f$ as before. We use a partial derivative here because $y(x,t)$ is a function of two variables.

Notice that the maximum speed of a section of the wave at location x and time t will be given by $v_{max} = A\omega$.

1. Why is it more convenient to use a sine function for the description of the motion of the red dot in this case rather than the cosine function used for the mass in the previous chapter?
2. What is the wave number for the wave on this string?
3. Click the 'V' box to show speed and then 'play'. The upper graph now gives the speed of the red dot in the y-direction as a function of time. What is the maximum speed (approximately) of the red dot according to the graph? How does this compare with the forward speed of the wave which you found in the last question, are they the same or different?
4. Where is the red dot (relative to the rest position before the wave passes) when the maximum transverse speed occurs? Where is it when the transverse speed is approximately zero?
5. Using the amplitude and v_{max} from the graph and $v_{max} = A\omega$, what is the angular frequency? How does this compare with the value calculated from the period?
6. How does the speed amplitude (maximum speed) from the graph compare to the speed amplitude given by $v_{max} = A\omega$?
7. Since points on the wave change their transverse velocity over time there must also be a vertical or *transverse acceleration*. Since acceleration is the time rate of change of velocity we have $a(x,t) = \partial v(x,t)/\partial t = -A\omega^2\sin(kx - \omega t + \phi)$ where the *maximum acceleration* is $a_{max} = A\omega^2$. Calculate the maximum acceleration of the red dot. What are the units of this acceleration if amplitude is in meters and angular frequency in radians per second squared?
8. Based on the equation for acceleration, where will the red dot be when the acceleration is a maximum? Where will it be when the acceleration is approximately zero? What is the phase difference between the acceleration and the speed?

9. Carefully state the relationship between position, speed and acceleration. When the position is zero (equilibrium position of the red dot) what are the speed and acceleration? When the position is a maximum, what are the speed and acceleration?