3.6.7.1: Adding Two Linear Waves Simulation

The waves we have been discussing so far and the ones that are most often seen in everyday life, such as light and sound, are for the most part ***linear waves***. Linear waves have the property, called ***superposition***, that their amplitudes add linearly if they arrive at the same point at the same time. This gives rise to several interesting phenomena in nature which we will investigate in this and the next few simulations.

The simulation shows the function $f(x,t)$ in red, $g(x,t)$ in the blue and $u(x,t) = f(x,t) + g(x,t)$ in grey. The check boxes on the lower right determine which functions are visible. You can enter your own functions for $f(x,t)$ and $g(x,t)$ using the same notations used for spreadsheets and calculators.

Simulation Questions:

1. Uncheck the $g(x,t)$ function so that just $f(x,t)$ is showing. What is the amplitude (half the height from highest point to lowest point) of $f(x,t)$? Now measure the amplitude of $g(x,t)$ (they should be the same). Now find the amplitude of the sum, $u(x,t)$. How does the amplitude of the sum compare with the amplitude of $f(x,t)$ or $g(x,t)$? This is an example of ***constructive interference***; the two waves add to give a wave with an amplitude which is the sum of the two amplitudes.
2. Start the simulation (button on lower left). How does the wavelength, frequency and speed of $f(x,t)$ or $g(x,t)$ compare with the wavelength, frequency and speed of $u(x,t)$?
3. Change $f(x,t)$ to have a phase of π (the simulation reads pi as π; type or cut and paste $2.0*\sin(\mathrm{x - t + pi})$ for $f(x,t)$ and press Return or Enter). Run the simulation. What happens to the amplitude of the sum of the two waves, $u(x,t)$? This is an example of destructive interference. Write a definition for destructive interference in your own words.
4. Experiment with cases in between total destructive and total constructive interference by changing the phase of $f(x,t)$ to be $\pi/2,\ \pi/3$,and $\pi/4$. Stop the simulation each time and record the amplitude of the sum compared to the amplitude of $f(x,t)$ or $g(x,t)$.
5. Click the reset button (fourth button on lower left) and then change the amplitude of $f(x,t)$ from 2.0 to 3.0 and the amplitude of $g(x,t)$ from 2.0 to 1.0 (Hit Return or Enter to update the values). What is the amplitude of $f(x,t) + g(x,t)$ in this case? How does this amplitude compare to the original case?
6. Go back to the original functions but change one of the minus signs to a plus sign (so now $f(x,t) = 2.0*\sin(x+t)$ and $g(x,t) = 2.0*\sin(x-t)$). The sum $u(x,t)$ is called a ***standing wave*** in this case (an example would be the waves on a guitar string as we will see later). Describe the behavior of $u(x,t)$. How does the period and wavelength of the combined wave compare to the period and wavelength of two components? How is the maximum amplitude of the sum related to the amplitudes of the two components? What can you say about the speed of the sum?
7. For standing waves on a string a ***node*** is a location where there is no motion and an ***anti-node*** is a location where there is maximum motion. For the standing wave in the previous exercise, how many nodes are there? How many anti-nodes?

Advanced Questions:

1. Use trigonometric identities to show that the sum of $f(x,t) = A\sin(kx+\omega t)$ and $g(x,t) = A\sin(kx-\omega t)$ equals $2A\cos(\omega t)\sin(kx)$. We can interpret this as a time dependent amplitude, $2A\cos(\omega t)$, multiplying a sine wave which is fixed in space. What happens to the amplitude as time increases? What fixes the location of the maximums and minimums of the standing wave (Hint: $k = 2\pi/\lambda$)?
2. Notice that the standing wave has zero amplitude on both ends in the simulation. This means that only certain wavelengths will "fit" on a given length. See if you can adjust x min and x max so that you have a wave with more nodes and anti nodes that fits on a longer string with the amplitude still zero on the ends. (Hint: $6.28 = 2\pi$.)
3. Now enter the following functions: $f(x,t) = 2.0*\sin(x-t)$ and $g(x,t) = 2.0*\sin(1.1*x - 1.1*t)$ (you can cut and paste instead of typing). Watch the just the sum $u(x,t) = f(x,t) + g(x,t)$ for a while and describe what happens (it changes slowly). Two waves with slightly different frequencies added together give rise to the phenomena of ***beats***. Now turn $f(x,t)$ and $g(x,t)$ on. Are these waves still traveling at the same speed as $u(x,t)$? Find the beat frequency the following way: Stop the simulation when the two source waves exactly cancel ($f(x,t) + g(x,t)$ is a straight line) and record the time (use the step buttons if you overshoot). Start the simulation and stop it again the next time the waves cancel. Record the new time and subtract to get the elapsed time. The beat frequency is 1/(time elapsed). How does this compare to the frequency of $f(x,t)$ subtracted from the frequency of $g(x,t)$?