3.7.1.3: The Decibel Scale

The human ear is an amazing instrument that can detect intensities as low as $10^{-12}\ \mathrm{W/m^2}$ and can hear intensities as high as $10^3\ \mathrm{W/m^2}$ (although this is loud enough to cause damage to the ear). To make this huge range easier to write down, a second scale of loudness was created called the ***sound intensity level***, measured in *decibels*. The relationship between sound intensity, I measured in watts per meter squared and sound intensity level (SIL) measured in decibels (dB), is given by $SIL = 10\log(I/I_o)$. Here log is the logarithm and $I_o = 10^{-12}\ \mathrm{W/m^2}$ is a reference sound intensity at about the threshold of human hearing.

We know sound waves are longitudinal fluctuations in the pressure of the air (or other medium) in which the sound is traveling. This is a fluctuation above and below normal atmospheric pressure and is measured in pascals (Pa), the standard unit of pressure. If you tried to report an average value of the fluctuation you would get zero since the pressure varies from above to below normal atmospheric pressure. We could just give the (maximum) amplitude of the fluctuation but what is usually done is to report the ***rms*** value which is the square root of the average of the squares of the amplitude. By first squaring the amplitude and then taking an average and then a square root you have a kind of average value that is not zero. In the chart below the root mean square (rms) variation in pressure from normal atmospheric pressure (in pascals Pa), the sound intensity level (SIL in dB) and the intensity (I in $\mathrm{W/m^2}$) are given for several sounds.

Source	**Pressure** rms (Pa)	**Sound Intensity Level** SIL (dB)	**Intensity** $(\mathrm{W/m^2})$
Jet engine at 10 m		150	10^3
Jet engine	200	140	100
Jack hammer	60	130	10
Car horn	20	120 (pain threshold)	1
Rock band	6	110	0.1
Machine shop	2	100	0.01
Train	0.6	90	10^{-3}
Vacuum cleaner	0.2	80	10^{-4}
TV	0.06	70	10^{-5}
Conversation	0.02	60	10^{-6}
Office	0.006	50	10^{-7}
Library	0.002	40	10^{-8}
Hospital	0.0006	30	10^{-9}
Broadcast studio	0.0002	20	10^{-10}
Rustle of leaves	0.00006	10	10^{-11}
Threshold of hearing	0.00002	0	10^{-12}

Table 3.7.1.3.1

Here are a few examples and rules of thumb for converting intensity ($\mathrm{W/m^2}$) into intensity levels (in dB):

- A 10 fold increase in intensity equals an addition of 10 dB. So going from a car horn to a jackhammer *multiplies* the intensity by 10 (1 $\mathrm{W/m^2}$ to 10 $\mathrm{W/m^2}$) but *adds* 10 dB to the intensity level (120 dB to 130 dB).
- A two fold increase in intensity (twice as loud in $\mathrm{W/m^2}$) equals and addition of $3\ \mathrm{dB}$ to the SIL. Suppose one trombone produces a sound level of 40 dB. How loud are four trombones? Doubling the number of trombones to two adds $3\ \mathrm{dB}$, doubling again to four adds $3\ \mathrm{dB}$ more so the new sound level is 46 dB.

- Suppose the sound intensity is $100\ \mathrm{W/m^2}$. What is the sound level? $\mathrm{SIL} = 10\log(100/10^{-12}) = 10\log(10^{14}) = 10 * 14 = 140\ \mathrm{dB}$.
- Suppose the sound level is $110\ \mathrm{dB}$. What is the sound intensity? $110\ \mathrm{dB} = 10\log(I/10^{-12})$. Divide both sides by 10 to get $11 = \log(I/10^{-12})$. Now take inverse $\log\ 11$ (same as 10^{11}) to get $10^{11} = I/10^{-12}$. Multiply both sides by 10^{-12} to get $0.1\ \mathrm{W/m^2} = I$.
- A Sound Conversion web site that converts between sound level, sound pressure and sound intensity.

Both sound intensity ($\mathrm{W/m^2}$) and sound intensity level (SIL) are numbers that can be measured precisely in the laboratory (objective). The human ear, however, is an imperfect measuring instrument. We hear better at a mid-range of frequencies than we do at very low or very high frequencies. The ***Loudness Level*** (L_L) scale is a *subjective* measurement of loudness. This scale is arrived at by asking real humans to compare the loudness of different notes and an average is taken for many people (subjective). The units of the Loudness Level are the *phon*.

The diagram below (modified from an MIT OpenCourseWare graph) relates sound intensity level (SIL, measured in dB with laboratory instruments), intensity (measured in $\mathrm{W/m^2}$ with laboratory instruments) and Loudness Level (measured in phons). A SIL of $110\ \mathrm{dB}$ is considered painful while a SIL of 0 is at the threshold of hearing. If our ears were the same as laboratory instruments the lines would go straight across. The L_L scale and the SIL scale do give approximately the same number only for frequencies around $1000\ \mathrm{Hz}$. In other words our subjective perception of loudness (in phons) and the laboratory measurement (in dB) agree but only for sounds with a frequency of $1000\ \mathrm{Hz}$.

Notice there is a dip in all the curves between $1000\ \mathrm{Hz}$ and $5000\ \mathrm{Hz}$ indicating we are more sensitive to these frequencies and this is true for all loudness readings. For example suppose we perceive a sound at $4000\ \mathrm{Hz}$ to be $45\ \mathrm{phons}$ (labeled by a blue X in the diagram). The chart shows that at this loudness and frequency the dB reading in the laboratory is actually around $36\ \mathrm{dB}$ (dotted line to the SIL axis). So we perceive a sound of $36\ \mathrm{dB}$ (measured in the lab) as being much louder ($45\ \mathrm{phons}$) if it occurs at $4000\ \mathrm{Hz}$. Most other animals have a similar curve allowing them to hear better in a certain frequency range although the dip is usually much more narrow and does not go as low as for humans. Many animals can hear frequencies above and below what humans can hear, but we can hear much softer sounds in this range of frequencies than almost any other animal, owls being an exception. Your cat or dog can hear higher frequencies than you can but you can hear softer sounds in the $1000\ \mathrm{Hz}$ to $5000\ \mathrm{Hz}$ range.

This improved ability to hear softer sounds in the $1000\ \mathrm{Hz}$ to $5000\ \mathrm{Hz}$ range is not surprising once you realize these are important frequencies for human speech; our hearing mechanism is built to hear human voices better than sound with much higher or much lower frequencies. This greater sensitivity around $3500\ \mathrm{Hz}$ is due to the tube resonance of the auditory canal (see chapter 12 for tube resonance and chapter 10 for a picture of the auditory canal).

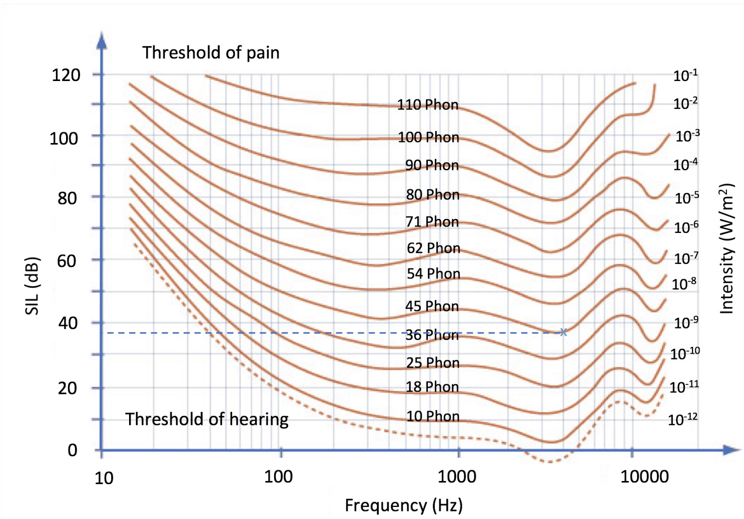

Figure $3.7.1.3.1$

It is also the case that intensity has an effect on perceived frequency; the same laboratory frequency will appear to be a slightly different frequency if the intensity is different. High frequencies are perceived to be a slightly higher pitch than normal if they are very loud. Low frequencies are perceived to be slightly lower than expected if they are very loud. Medium loudness doesn't change the perceived pitch very much.

The above curves are very much like the ***frequency response curves*** of microphones and speakers. No microphone has the same sensitivity to all frequencies and no speaker reproduces all frequencies equally well, as we will see in Chapter 18 on electronics. Likewise our hearing does not have the same sensitivity at all frequencies.

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