## 3.11.1.1: Tube Resonance

In a tube based instrument waves are created at one end of the tube by something that vibrates and travel to the end of the tube and reflects as show in the animations at the bottom of this page. A standing wave is created by the waves traveling in each direction. Since the wave is traveling in air it will move at the speed of sound in air at the ambient temperature. It will also be true for waves in a tube that $v = f\lambda$ where $f$ is in Hertz and $\lambda$ is the wavelength in meters. As was the case for strings, the length of a tube determines the frequency of a standing wave in the tube. There are several complications, however, depending on if one or both ends are closed or open.

Strings always have displacement nodes at each end since the string is fixed there and cannot move. For a tube, however, if an end is open the air can move freely and so there is a ***displacement anti-node*** (the point of maximum air movement) at that end.

The pressure in the tube is different than the displacement, however. Since the air can move at the ends it does not build up any pressure so at these locations (the ends) there will be a ***pressure node*** where the pressure stays constant. At the center where the air cannot move the pressure has large oscillations and there is a pressure anti-node. The following two graphs show the displacement wave and the pressure wave of the fundamental for a tube open at both ends.

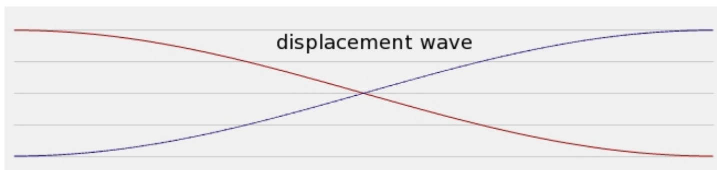


Figure $3.11.1.1.1$

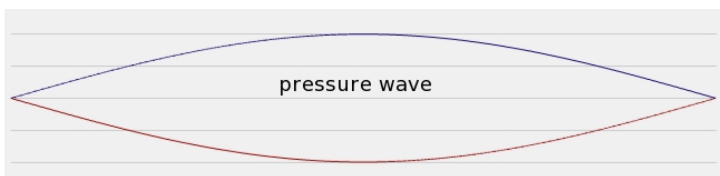


Figure $3.11.1.1.2$

For a tube of length $L$ to have a pressure node on each end it has to be the case that $L = n\lambda/2$ where $n$ is a whole number. In other words, you can have half a wave in the tube ($n = 1$), one wave ($n = 2$), one and a half waves ($n = 3$), etc. But you never have any other fraction of a wave because that would require not having a node at both ends. The pressure in a tube *that is open on both ends* gives harmonics *exactly* the same as a string (look back to the previous chapter to verify this):

| **Frequencies for a tube open on both ends** | | |
|---|---|---|
| **Harmonic number** | **Wavelength** | **Frequency** |
| $n = 1$ | $\lambda_1 = 2L$ | $f_1 = v/\lambda_1$ |
| $n = 2$ | $\lambda_2 = L = \lambda_1/2$ | $f_2 = v/\lambda_2 = 2f_1$ |
| $n = 3$ | $\lambda_3 = \frac{2}{3}L = \lambda_1/3$ | $f_3 = v/\lambda_3 = 3f_1$ |
| $n = 4$ | $\lambda_4 = \frac{1}{2}L = \lambda_1/4$ | $f_4 = v/\lambda_4 = 4f_1$ |
| $n$ | $\lambda_n = \frac{2}{n}L = \lambda_1/n$ | $f_n = v/\lambda_n = nf_1$ |

Table $3.11.1.1.1$

For many musical instruments that are made of a tube, the tube is closed at one end but opened at the other. If the end is closed the air cannot move so the pressure fluctuates and there is a pressure anti-node (see the simulation below). This same location is a displacement node because the air is not moving. The graphs below are of the displacement and the pressure waves for a tube closed on one end.

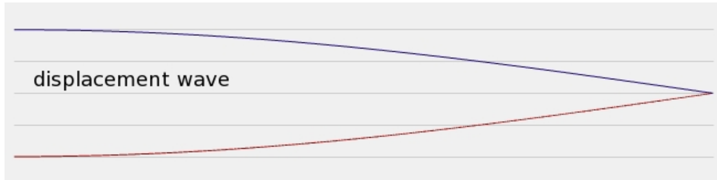


Figure 3.11.1.1.3

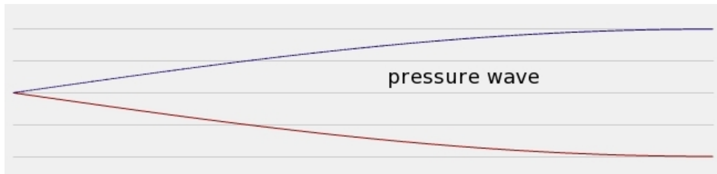


Figure 3.11.1.1.4

For a tube of length $L$ to have a pressure node on one end but a pressure anti-node at the other the lowest possible wavelength is given by $L = n\lambda/4$ where $n$ is a whole number. Notice this is half the lowest wavelength available to a pipe with two open ends. The requirement of having a pressure anti-node at the closed end means the even numbered frequencies will be missing from a tube closed at one end as shown in the following graphs of the first three harmonics available to a pipe closed on one end.

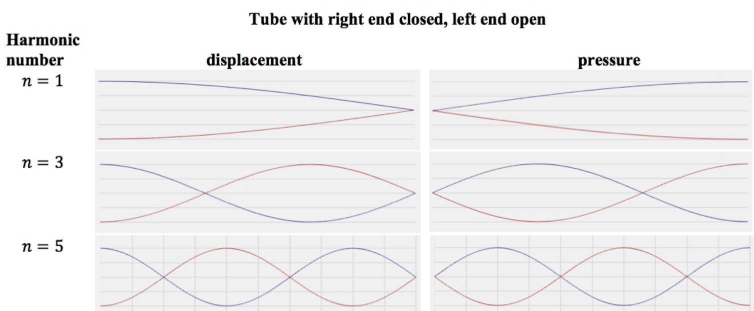


Figure 3.11.1.1.5

So for a tube open on *both* ends the available frequencies are the fundamental, $f_1, 2 \times f_1, 3 \times f_1,$ etc. But for a tube that is *closed* on one end only *odd* multiples of the fundamental $f_1$ are available: $3 \times f_1, 5 \times f_1,$ etc. The following table gives the frequencies available to a tube that is closed at one end.

| Frequencies for a tube open on one end | | |
|---|---|---|
| **Harmonic number** | **Wavelength** | **Frequency** |
| $n = 1$ | $\lambda_1 = 4L$ | $f_1 = v/\lambda_1$ |
| $n = 2$ | doesn't exist | missing |
| $n = 3$ | $\lambda_3 = \frac{4}{3} = \lambda_1/3$ | $f_3 = v/\lambda_3 = 3f_1$ |
| $n = 4$ | doesn't exist | missing |
| $n$ (odd) | $\lambda_n = \frac{4}{n}L = \lambda_1/n$ | $f_n = v/\lambda_n = nf_1$ . $n$ odd |

Table 3.11.1.1.2

If you compare the above table with the one for a tube open on both ends you will notice two things. For a given length, $L$ the tube with one end closed has a lower fundamental frequency (longer wavelength) and has only odd frequencies. This affects the available frequencies of the instrument and so affects the timbre. Most tube instruments are closed on one end because the musician playing the instrument has their mouth over one end. Pan flutes are interesting in that the bottom end is closed and the performer blows across the open end.

As mentioned earlier, the auditory canal of the ear acts as a tube with one end closed. In the chart for the harmonic $n=1$ we have $L=n\lambda/4$ which determines the fundamental wavelength for the auditory canal. The fundamental frequency for this tube turns out to be around $3500\ \mathrm{Hz}$ and this is where human hearing is the most sensitive as we saw in the Sound Intensity Level chart in Chapter 8.

Unless the pipe is very narrow compared to its length, a slight correction to the above formulas needs to be made. We know a standing wave inside a tube is formed from waves being reflected from the ends (just like standing waves on a string). It turns out that for an open end, the wave doesn't reflect exactly at the end of the tube. In order to "feel" the pressure difference at the end and be reflected it goes a little past the end before being reflected. The effect is as if the pipe is just a little bit longer and this extra length depends on the radius of the tube. For a pipe of radius $r$, the extra amount turns out to be (after some sophisticated calculations) $0.61r$. The formulas for the tube open on one end can be corrected by replacing $L$ in the table above with $L+0.61r$. For a single open end the fundamental will be given by $\lambda_1=4(L+0.61r)$. In the case of two open ends the extra length is added twice so the fundamental is $\lambda_1=2(L+1.22r)$.

Unlike a stringed instrument where the speed of the wave on the string can be changed by changing the density and/or tension, the speed of sound in air at a given temperature is fixed. The fundamental frequency of a string can be changed by changing density, tension or length but the only way to change the fundamental frequency of a tube instrument is to change the length of the tube. There are several ways to do that, as we will see below.

## Video/audio examples:

- Actually, the previous paragraph is not quite right; there is a way to change the frequency of a tube without changing its length. If the density of the gas inside changes, the speed of sound will change and this changes the frequencies in the tube. Tube instruments will be slightly out of tune on a cold day compared to a hot day due to the change in the density of air with temperature. The frequency will also change if the tube is filled with a gas other than air as shown in this YouTube of trombone and trumpet filled with different gasses.
- Animations of open ended and closed ended tubes.
- Visualization of sound in a tube using a Kundts Tube (filled with Styrofoam balls).
- Visualization of sound in a tube using a Rubens' Tube.