5.4.2: Work Done by a Constant Force

Force in the Direction of Displacement

The work done by a constant force is proportional to the force applied times the displacement of the object.

learning objectives

- Contrast displacement and distance in constant force situations

Work Done by a Constant Force

When a force acts on an object over a distance, it is said to have done work on the object. Physically, the work done on an object is the change in kinetic energy that that object experiences. We will rigorously prove both of these claims.

The term work was introduced in 1826 by the French mathematician Gaspard-Gustave Coriolis as “weight lifted through a height,” which is based on the use of early steam engines to lift buckets of water out of flooded ore mines. The SI unit of work is the newton-meter or joule (J).

Units

One way to validate if an expression is correct is to perform dimensional analysis. We have claimed that work is the change in kinetic energy of an object and that it is also equal to the force times the distance. The units of these two should agree. Kinetic energy – and all forms of energy – have units of joules (J). Likewise, force has units of newtons (N) and distance has units of meters (m). If the two statements are equivalent they should be equivalent to one another.

$$\mathrm{N \cdot m = kg\frac{m}{s^2} \cdot m = kg\frac{m^2}{s^2} = J} \tag{5.4.2.1}$$

Displacement versus Distance

Often times we will be asked to calculate the work done by a force on an object. As we have shown, this is proportional to the force and the distance which the object is displaced, not moved. We will investigate two examples of a box being moved to illustrate this.

Example Problems

Here are a few example problems:

(1.a) Consider a constant force of two newtons (F = 2 N) acting on a box of mass three kilograms (M = 3 kg). Calculate the work done on the box if the box is displaced 5 meters.

(1.b) Since the box is displaced 5 meters and the force is 2 N, we multiply the two quantities together. The object’s mass will dictate how fast it is accelerating under the force, and thus the time it takes to move the object from point a to point b. Regardless of how long it takes, the object will have the same displacement and thus the same work done on it.

(2.a) Consider the same box (M = 3 kg) being pushed by a constant force of four newtons (F = 4 N). It begins at rest and is pushed for five meters (d = 5m). Assuming a frictionless surface, calculate the velocity of the box at 5 meters.

(2.b) We now understand that the work is proportional to the change in kinetic energy, from this we can calculate the final velocity. What do we know so far? We know that the block begins at rest, so the initial kinetic energy must be zero. From this we algebraically isolate and solve for the final velocity.

$$\mathrm{Fd = \Delta KE = KE_f - 0 = \frac{1}{2}mv_f^2} \tag{5.4.2.2}$$

$$\mathrm{v_f} = \sqrt{2\frac{\mathrm{Fd}}{\mathrm{m}}} = \sqrt{2\frac{\mathrm{4N \cdot 5m}}{\mathrm{2kg}}} = \sqrt{10}\mathrm{m/s} \tag{5.4.2.3}$$

We see that the final velocity of the block is approximately 3.15 m/s.

Force at an Angle to Displacement

A force does not have to, and rarely does, act on an object parallel to the direction of motion.

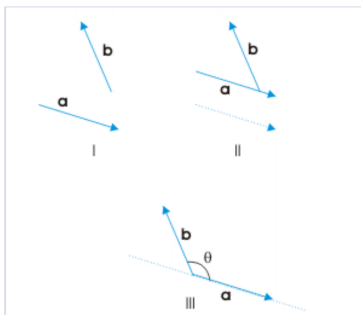
learning objectives

- Infer how to adjust one-dimensional motion for our three-dimensional world

The Fundamentals

Up until now, we have assumed that any force acting on an object has been parallel to the direction of motion. We have considered our motion to be one dimensional, only acting along the x or y axis. To best examine and understand how nature operators in our three-dimensional world, we will first discuss work in two dimensions in order to build our intuition.

A force does not have to, and rarely does, act on an object parallel to the direction of motion. In the past, we derived that $\mathrm{W=Fd}$; such that the work done on an object is the force acting on the object multiplied by the displacement. But this is not the whole story. This expression contains an assumed cosine term, which we do not consider for forces parallel to the direction of motion. "Why would we do such a thing? " you may ask. We do this because the two are equivalent. If the angle of the force along the direction of motion is zero, such that the force is parallel to the direction of motion, then the cosine term equals one and does not change the expression. As we increase the force's angle with respect to the direction of motion, less and less work is done along the direction that we are considering; and more and more work is being done in another, perpendicular, direction of motion. This process continues until we are perpendicular to our original direction of motion, such that the angle is 90, and the cosine term would equal zero; resulting in zero work being done along our original direction. Instead, we are doing work in another direction!



Angle: Recall that both the force and direction of motion are vectors. When the angle is 90 degrees, the cosine term goes to zero. When along the same direction, they equal one.

Let's show this explicitly and then look at this phenomena in terms of a box moving along the x and y directions.

We have discussed that work is the integral of the force and the dot product respect to x. But in fact, dot product of force and a very small distance is equal to the two terms times cosine of the angle between the two. $\mathrm{F \times dx = Fd\cos(\theta)}$. Explicitly,

$$\int_{\mathrm{t_2}}^{\mathrm{t_1}} \mathrm{F\cdot dx} = \int_{\mathrm{t_2}}^{\mathrm{t_1}} \mathrm{Fd}\cos\theta \mathrm{dx} = \mathrm{Fd}\cos\theta \tag{5.4.2.4}$$

A Box Being Pushed

Consider a coordinate system such that we have x as the abscissa and y as the ordinate. More so, consider a box being pushed along the x direction. What happens in the following three scenarios?

- The box is being pushed parallel to the x direction?
- The box is being pushed at an angle of 45 degrees to the x direction?
- The box is being pushed at an angle of 60 degrees to the x direction?
- The box is being pushed at an angle of 90 degrees to the x direction?

In the first scenario, we know that all of the force is acting on the box along the x-direction, which means that work will only be done along the x-direction. More so, a vertical perspective the box is not moving – it is unchanged in the y direction. Since the force is acting parallel to the direction of motion, the angle is equal to zero and our total work is simply the force times the displacement in the x-direction.

In the second scenario, the box is being pushed at an angle of 45 degrees to the x-direction; and thus also a 45 degree angle to the y-direction. When evaluated, the cosine of 45 degrees is equal to $\frac{1}{\sqrt{2}}$, or approximately 0.71. This means is that 71% of the force is contributing to the work along the x-direction. The other 29% is acting along the y-direction.

In the third scenario, we know that the force is acting at a 60 degree angle to the x-direction; and thus also a 30 degree angle to the y-direction. When evaluated, cosine of 60 degrees is equal to 1/2. This means that the force is equally acting in the x and y-direction! The work done is linear with respect to both x and y.

In the last scenario, the box is being pushed at an angle perpendicular to the x direction. In other words, we are pushing the box in the y-direction! Thus, the box's position will be unchanged and experience no displacement along the x-axis. The work done in the x direction will be zero.

Key Points

- Understanding work is quintessential to understanding systems in terms of their energy, which is necessary for higher level physics.
- Work is equivalent to the change in kinetic energy of a system.
- Distance is not the same as displacement. If a box is moved 3 meters forward and then 4 meters to the left, the total displacement is 5 meters, not 7 meters.
- Work done on an object along a given direction of motion is equal to the force times the displacement times the cosine of the angle.
- No work is done along a direction of motion if the force is perpendicular.
- When considering force parallel to the direction of motion, we omit the cosine term because it equals 1 which does not change the expression.

Key Terms

- **work**: A measure of energy expended in moving an object; most commonly, force times displacement. No work is done if the object does not move.
- **dot product**: A scalar product.

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