LibreTexts

5.1.3: Significant Figures and Order of Magnitude

Scientific Notation

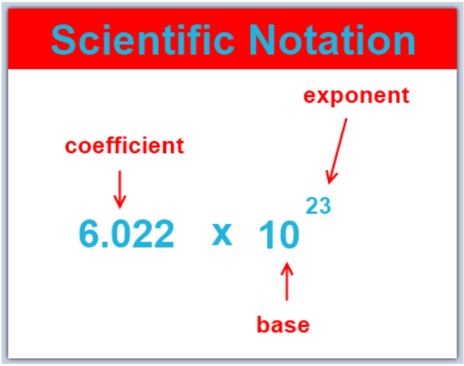
Scientific notation is a way of writing numbers that are too big or too small in a convenient and standard form.

learning objectives

- Convert properly between standard and scientific notation and identify appropriate situations to use it

Scientific Notation: A Matter of Convenience

Scientific notation is a way of writing numbers that are too big or too small in a convenient and standard form. Scientific notation has a number of useful properties and is commonly used in calculators and by scientists, mathematicians and engineers. In scientific notation all numbers are written in the form of $a \cdot 10^b$ (a multiplied by ten raised to the power of b), where the exponent b) is an integer, and the coefficient (a is any real number.



Scientific Notation: There are three parts to writing a number in scientific notation: the coefficient, the base, and the exponent.

Most of the interesting phenomena in our universe are not on the human scale. It would take about 1,000,000,000,000,000,000,000 bacteria to equal the mass of a human body. Thomas Young's discovery that light was a wave preceded the use of scientific notation, and he was obliged to write that the time required for one vibration of the wave was "$\frac{1}{500}$ of a millionth of a millionth of a second"; an inconvenient way of expressing the point. Scientific notation is a less awkward and wordy way to write very large and very small numbers such as these.

A Simple System

Scientific notation means writing a number in terms of a product of something from 1 to 10 and something else that is a power of ten.

For instance, $32 = 3.2 \cdot 10^1$

$320 = 3.2 \cdot 10^2$

$3200 = 3.2 \cdot 10^3$, and so forth…

Each number is ten times bigger than the previous one. Since 10^1 is ten times smaller than 10^2, it makes sense to use the notation 10^0 to stand for one, the number that is in turn ten times smaller than 10^1. Continuing on, we can write 10^{-1} to stand for 0.1, the number ten times smaller than 10^0. Negative exponents are used for small numbers:

$3.2 = 3.2 \cdot 10^0$

$0.32 = 3.2 \cdot 10^{-1}$

$0.032 = 3.2 \cdot 10^{-2}$

Scientific notation displayed calculators can take other shortened forms that mean the same thing. For example, $3.2 \cdot 10^6$(written notation) is the same as $3.2\mathrm{E}+6$ (notation on some calculators) and 3.2^6 (notation on some other calculators).

Round-off Error

A round-off error is the difference between the calculated approximation of a number and its exact mathematical value.

learning objectives

- Explain the impact round-off errors may have on calculations, and how to reduce this impact

Round-off Error

A round-off error, also called a rounding error, is the difference between the calculated approximation of a number and its exact mathematical value. Numerical analysis specifically tries to estimate this error when using approximation equations, algorithms, or both, especially when using finitely many digits to represent real numbers. When a sequence of calculations subject to rounding errors is made, errors may accumulate, sometimes dominating the calculation.

Calculations rarely lead to whole numbers. As such, values are expressed in the form of a decimal with infinite digits. The more digits that are used, the more accurate the calculations will be upon completion. Using a slew of digits in multiple calculations, however, is often unfeasible if calculating by hand and can lead to much more human error when keeping track of so many digits. To make calculations much easier, the results are often 'rounded off' to the nearest few decimal places.

For example, the equation for finding the area of a circle is $\mathrm{A}=\pi \mathrm{r}^{2}$. The number π(pi) has infinitely many digits, but can be truncated to a rounded representation of as 3.14159265359. However, for the convenience of performing calculations by hand, this number is typically rounded even further, to the nearest two decimal places, giving just 3.14. Though this technically decreases the accuracy of the calculations, the value derived is typically 'close enough' for most estimation purposes.

However, when doing a series of calculations, numbers are rounded off at each subsequent step. This leads to an accumulation of errors, and if profound enough, can misrepresent calculated values and lead to miscalculations and mistakes.

The following is an example of round-off error:

$$\sqrt{4.58^{2}+3.28^{2}}=\sqrt{21.0+10.8}=5.64$$

Rounding these numbers off to one decimal place or to the nearest whole number would change the answer to 5.7 and 6, respectively. The more rounding off that is done, the more errors are introduced.

Order of Magnitude Calculations

An order of magnitude is the class of scale of any amount in which each class contains values of a fixed ratio to the class preceding it.

learning objectives

- Choose when it is appropriate to perform an order-of-magnitude calculation

Orders of Magnitude

An order of magnitude is the class of scale of any amount in which each class contains values of a fixed ratio to the class preceding it. In its most common usage, the amount scaled is 10, and the scale is the exponent applied to this amount (therefore, to be an order of magnitude greater is to be 10 times, or 10 to the power of 1, greater). Such differences in order of magnitude can be measured on the logarithmic scale in "decades," or factors of ten. It is common among scientists and technologists to say that a parameter whose value is not accurately known or is known only within a range is "on the order of" some value. The order of magnitude of a physical quantity is its magnitude in powers of ten when the physical quantity is expressed in powers of ten with one digit to the left of the decimal.

Orders of magnitude are generally used to make very approximate comparisons and reflect very large differences. If two numbers differ by one order of magnitude, one is about ten times larger than the other. If they differ by two orders of magnitude, they differ by a factor of about 100. Two numbers of the same order of magnitude have roughly the same scale — the larger value is less than ten times the smaller value.

It is important in the field of science that estimates be at least in the right ballpark. In many situations, it is often sufficient for an estimate to be within an order of magnitude of the value in question. Although making order-of-magnitude estimates seems simple and natural to experienced scientists, it may be completely unfamiliar to the less experienced.

Example 5.1.3.1:

Some of the mental steps of estimating in orders of magnitude are illustrated in answering the following example question: Roughly what percentage of the price of a tomato comes from the cost of transporting it in a truck?

Guessing the Number of Jelly Beans: Can you guess how many jelly beans are in the jar? If you try to guess directly, you will almost certainly underestimate. The right way to do it is to estimate the linear dimensions and then estimate the volume indirectly.

Incorrect solution: Let's say the trucker needs to make a profit on the trip. Taking into account her benifits, the cost of gas, and maintenance and payments on the truck, let's say the total cost is more like 2000. You might guess about 5000 tomatoes would fit in the back of the truck, so the extra cost per tomato is 40 cents. That means the cost of transporting one tomato is comparable to the cost of the tomato itself.

The problem here is that the human brain is not very good at estimating area or volume — it turns out the estimate of 5000 tomatoes fitting in the truck is way off. (This is why people have a hard time in volume-estimation contests, such as the one shown below.) When estimating area or volume, you are much better off estimating linear dimensions and computing the volume from there.

So, here's a better solution: As before, let's say the cost of the trip is $2000. The dimensions of the bin are probably 4m by 2m by 1m, for a volume of $8\ \mathrm{m}^3$. Since our goal is just an order-of-magnitude estimate, let's round that volume off to the nearest power of ten: $10\ \mathrm{m}^3$. The shape of a tomato doesn't follow linear dimensions, but since this is just an estimate, let's pretend that a tomato is an 0.1m by 0.1m by 0.1m cube, with a volume of $1 \cdot 10^{-3}\ \mathrm{m}^3$. We can find the total number of tomatoes by dividing the volume of the bin by the volume of one tomato: $\frac{10^3\ \mathrm{m}^3}{10^{-3}\ \mathrm{m}^3} = 10^6$ tomatoes. The transportation cost per tomato is $\frac{\$2000}{10^6\ \text{tomatoes}} = \$0.002$ per tomato. That means that transportation really doesn't contribute very much to the cost of a tomato. Approximating the shape of a tomato as a cube is an example of another general strategy for making order-of-magnitude estimates.

Key Points

- Scientific notation means writing a number in terms of a product of something from 1 to 10 and something else that is a power of 10.
- In scientific notation all numbers are written in the form of $a \cdot 10^b$ (a times ten raised to the power of b).
- Each consecutive exponent number is ten times bigger than the previous one; negative exponents are used for small numbers.
- When a sequence of calculations subject to rounding error is made, these errors can accumulate and lead to the misrepresentation of calculated values.
- Increasing the number of digits allowed in a representation reduces the magnitude of possible round-off errors, but may not always be feasible, especially when doing manual calculations.
- The degree to which numbers are rounded off is relative to the purpose of calculations and the actual value.
- Orders of magnitude are generally used to make very approximate comparisons and reflect very large differences.
- In the field of science, it is often sufficient for an estimate to be within an order of magnitude of the value in question.

- When estimating area or volume, you are much better off estimating linear dimensions and computing volume from those linear dimensions.

Key Terms

- **exponent**: The power to which a number, symbol or expression is to be raised. For example, the 3 in x^3.
- **Scientific notation**: A method of writing, or of displaying real numbers as a decimal number between 1 and 10 followed by an integer power of 10
- **approximation**: An imprecise solution or result that is adequate for a defined purpose.
- **Order of Magnitude**: The class of scale or magnitude of any amount, where each class contains values of a fixed ratio (most often 10) to the class preceding it. For example, something that is 2 orders of magnitude larger is 100 times larger; something that is 3 orders of magnitude larger is 1000 times larger; and something that is 6 orders of magnitude larger is one million times larger, because $10^2 = 100$, $10^3 = 1000$, and $10^6 =$ one million