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Physics

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CHAPTER OVERVIEW

5.1: The Basics of Physics

Topic hierarchy

5.1.1: The Basics of Physics

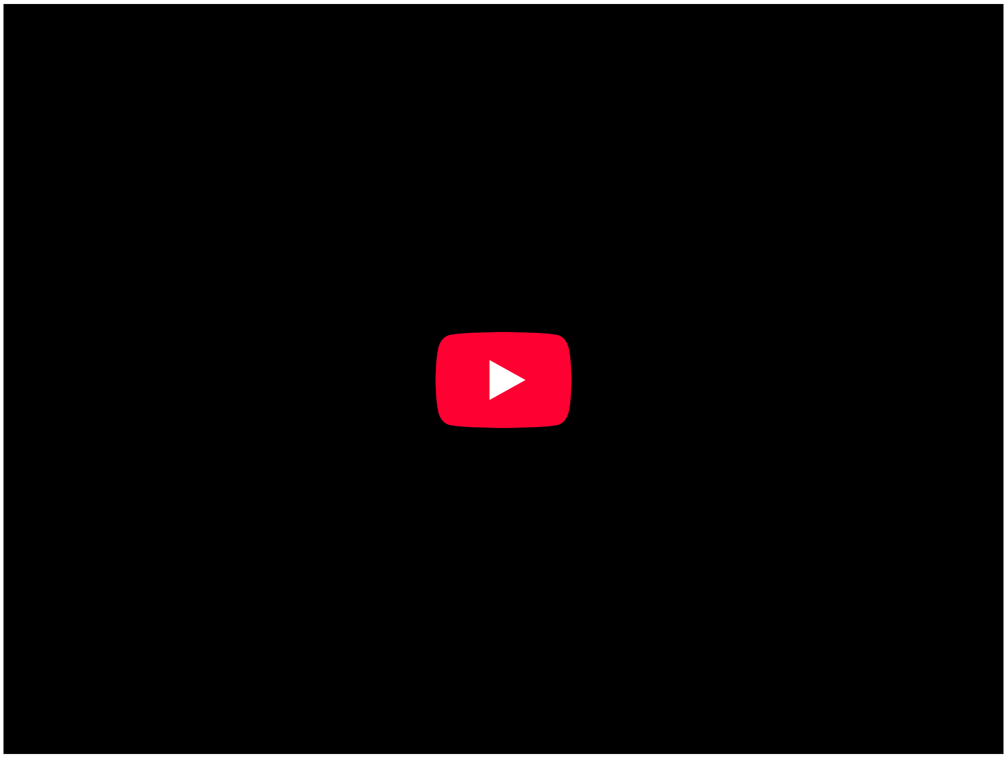
Introduction: Physics and Matter

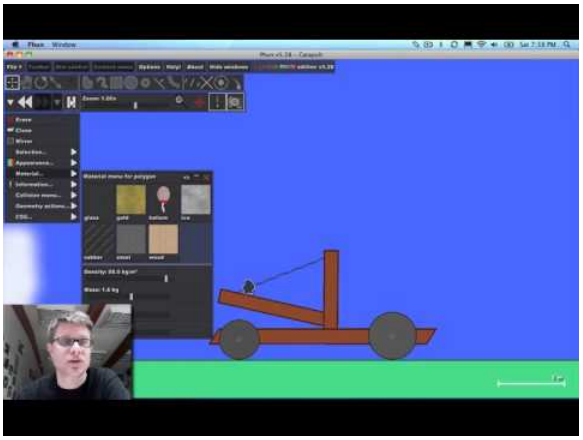
Physics is a study of how the universe behaves.

learning objectives

- Apply physics to describe the function of daily life

Physics is a natural science that involves the study of matter and its motion through space and time, along with related concepts such as energy and force. More broadly, it is the study of nature in an attempt to understand how the universe behaves.



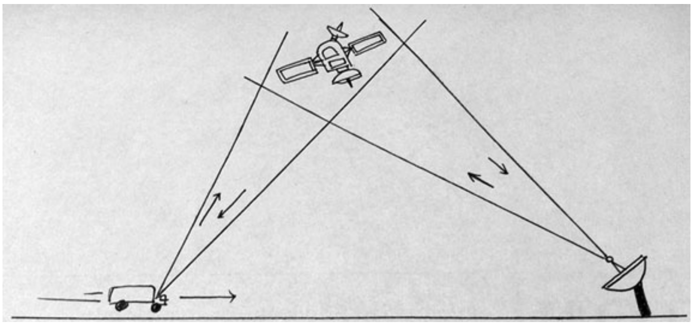


What is Physics?: Mr. Andersen explains the importance of physics as a science. History and virtual examples are used to give the discipline context.

Physics uses the scientific method to help uncover the basic principles governing light and matter, and to discover the implications of those laws. It assumes that there are rules by which the universe functions, and that those laws can be at least partially understood by humans. It is also commonly believed that those laws could be used to predict everything about the universe's future if complete information was available about the present state of all light and matter.

Matter is generally considered to be anything that has mass and volume. Many concepts integral to the study of classical physics involve theories and laws that explain matter and its motion. The law of conservation of mass, for example, states that mass cannot be created or destroyed. Further experiments and calculations in physics, therefore, take this law into account when formulating hypotheses to try to explain natural phenomena.

Physics aims to describe the function of everything around us, from the movement of tiny charged particles to the motion of people, cars, and spaceships. In fact, almost everything around you can be described quite accurately by the laws of physics. Consider a smart phone; physics describes how electricity interacts with the various circuits inside the device. This knowledge helps engineers select the appropriate materials and circuit layout when building the smart phone. Next, consider a GPS system; physics describes the relationship between the speed of an object, the distance over which it travels, and the time it takes to travel that distance. When you use a GPS device in a vehicle, it utilizes these physics equations to determine the travel time from one location to another. The study of physics is capable of making significant contributions through advances in new technologies that arise from theoretical breakthroughs.



Global Positioning System: GPS calculates the speed of an object, the distance over which it travels, and the time it takes to travel that distance using equations based on the laws of physics.

Physics and Other Fields

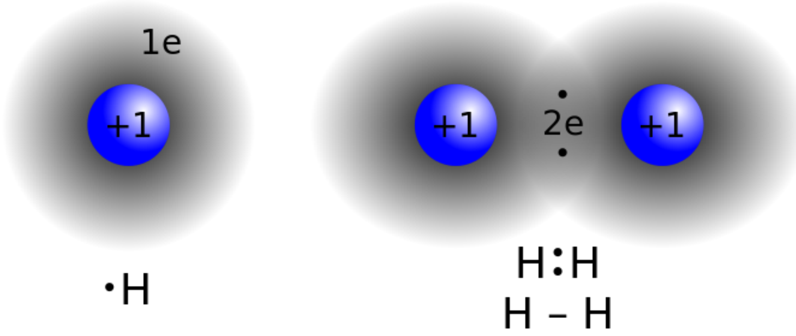
Physics is the foundation of many disciplines and contributes directly to chemistry, astronomy, engineering, and most scientific fields.

learning objectives

- Explain why the study of physics is integral to the study of other sciences

Physics and Other Disciplines

Physics is the foundation of many important disciplines and contributes directly to others. Chemistry deals with the interactions of atoms and molecules, so it is rooted in atomic and molecular physics. Most branches of engineering are applied physics. In architecture, physics is at the heart of structural stability and is involved in acoustics, heating, lighting, and the cooling of buildings. Parts of geology rely heavily on physics, such as the radioactive dating of rocks, earthquake analysis, and heat transfer in the Earth. Some disciplines, such as biophysics and geophysics, are hybrids of physics and other disciplines.



Physics in Chemistry: The study of matter and electricity in physics is fundamental towards the understanding of concepts in chemistry, such as the covalent bond.

Physics has many applications in the biological sciences. On the microscopic level, it helps describe the properties of cell walls and cell membranes. On the macroscopic level, it can explain the heat, work, and power associated with the human body. Physics is involved in medical diagnostics, such as X-rays, magnetic resonance imaging (MRI), and ultrasonic blood flow measurements. Medical therapy sometimes directly involves physics: cancer radiotherapy uses ionizing radiation, for instance. Physics can also explain sensory phenomena, such as how musical instruments make sound, how the eye detects color, and how lasers can transmit information.

The boundary between physics and the other sciences is not always clear. For instance, chemists study atoms and molecules, which are what matter is built from, and there are some scientists who would be equally willing to call themselves physical chemists or chemical physicists. It might seem that the distinction between physics and biology would be clearer, since physics seems to deal with inanimate objects. In fact, almost all physicists would agree that the basic laws of physics that apply to molecules in a test tube work equally well for the combination of molecules that constitutes a bacterium. What differentiates physics from biology is that many of the scientific theories that describe living things ultimately result from the fundamental laws of physics, but cannot be rigorously derived from physical principles.

It is not necessary to formally study all applications of physics. What is most useful is the knowledge of the basic laws of physics and skill in the analytical methods for applying them. The study of physics can also improve your problem-solving skills. Furthermore, physics has retained the most basic aspects of science, so it is used by all of the sciences. The study of physics makes other sciences easier to understand.

Models, Theories, and Laws

The terms *model*, *theory*, and *law* have exact meanings in relation to their usage in the study of physics.

learning objectives

- Define the terms model, theory, and law

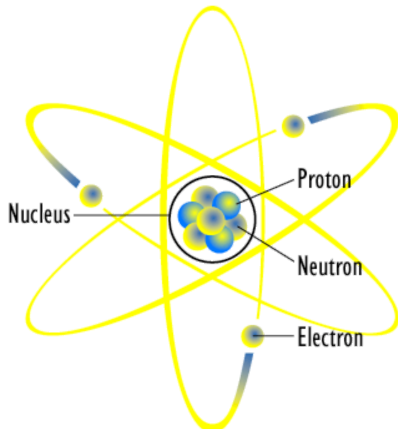
Definition of Terms: Model, Theory, Law

In colloquial usage, the terms *model*, *theory*, and *law* are often used interchangeably or have different interpretations than they do in the sciences. In relation to the study of physics, however, each term has its own specific meaning.

The *laws of nature* are concise descriptions of the universe around us. They are not explanations, but human statements of the underlying rules that all natural processes follow. They are intrinsic to the universe; humans did not create them and we cannot change them. We can only discover and understand them. The cornerstone of discovering natural laws is observation; science must describe the universe as it is, not as we may imagine it to be. Laws can never be known with absolute certainty, because it is impossible to perform experiments to establish and confirm a law in every possible scenario without exception. Physicists operate under the assumption that all scientific laws and theories are valid until a counterexample is observed. If a good-quality, verifiable experiment contradicts a well-established law, then the law must be modified or overthrown completely.

Models

A *model* is a representation of something that is often too difficult (or impossible) to display directly. While a model's design is justified using experimental information, it is only accurate under limited situations. An example is the commonly used "planetary model" of the atom, in which electrons are pictured as orbiting the nucleus, analogous to the way planets orbit the Sun. We cannot observe electron orbits directly, but the mental image helps explain the observations we can make, such as the emission of light from hot gases. Physicists use models for a variety of purposes. For example, models can help physicists analyze a scenario and perform a calculation, or they can be used to represent a situation in the form of a computer simulation.



Planetary Model of an Atom: The planetary model of the atom in which electrons are pictured as orbiting the nucleus, analogous to the way planets orbit the Sun

Theories

A *theory* is an explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers. *Some theories include models to help visualize phenomena, whereas others do not*. Newton's theory of gravity, for example, does not require a model or mental image, because we can observe the objects directly with our own senses. The kinetic theory of gases, on the other hand, makes use of a model in which a gas is viewed as being composed of atoms and molecules. Atoms and molecules are too small to be observed directly with our senses—thus, we picture them mentally to understand what our instruments tell us about the behavior of gases.

Laws

A law uses concise language to describe a generalized pattern in nature that is supported by scientific evidence and repeated experiments. Often, a law can be expressed in the form of a single mathematical equation. Laws and theories are similar in that they are both scientific statements that result from a tested hypothesis and are supported by scientific evidence. However, the designation law is reserved for a concise and very general statement that describes phenomena in nature, such as the law that energy is conserved during any process, or Newton's second law of motion, which relates force, mass, and acceleration by the simple equation $F = ma$. A theory, in contrast, is a less concise statement of observed phenomena. For example, the Theory of Evolution and the Theory of Relativity cannot be expressed concisely enough to be considered a law. The biggest difference between a law and a theory is that a law is much more complex and dynamic, and a theory is more explanatory. A law describes a single observable point of fact, whereas a theory explains an entire group of related phenomena. And, whereas a law is a postulate that forms the foundation of the scientific method, a theory is the end result of that process.

Key Points

- Physics is a natural science that involves the study of matter and its motion through space and time, along with related concepts such as energy and force.
- Matter is generally considered to be anything that has mass and volume.
- Scientific laws and theories express the general truths of nature and the body of knowledge they encompass. These laws of nature are rules that all natural processes appear to follow.
- Many scientific disciplines, such as biophysics, are hybrids of physics and other sciences.
- The study of physics encompasses all forms of matter and its motion in space and time.

- The application of physics is fundamental towards significant contributions in new technologies that arise from theoretical breakthroughs.
- Concepts in physics cannot be proven, they can only be supported or disproven through observation and experimentation.
- A model is an evidence-based representation of something that is either too difficult or impossible to display directly.
- A theory is an explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers.
- A law uses concise language, often expressed as a mathematical equation, to describe a generalized pattern in nature that is supported by scientific evidence and repeated experiments.

Key Terms

- **matter**: The basic structural component of the universe. Matter usually has mass and volume.
- **scientific method**: A method of discovering knowledge about the natural world based in making falsifiable predictions (hypotheses), testing them empirically, and developing peer-reviewed theories that best explain the known data.
- **application**: the act of putting something into operation
- **Model**: A representation of something difficult or impossible to display directly
- **Law**: A concise description, usually in the form of a mathematical equation, used to describe a pattern in nature
- **theory**: An explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers

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5.1.2: Units

Length

Length is a physical measurement of distance that is fundamentally measured in the SI unit of a meter.

learning objectives

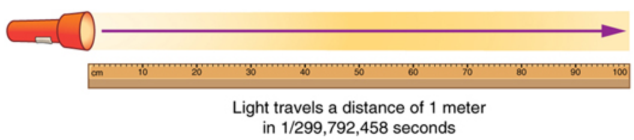
- Distinguish SI and customary units of length

Length can be defined as a measurement of the physical quantity of distance. Many qualitative observations fundamental to physics are commonly described using the measurement of length. The distance between objects, the rate at which objects are traveling, and how much force an object exerts are all dependent on length as a variable. In order to describe length in a standardized and quantitative manner, an accepted unit of measurement must be utilized.

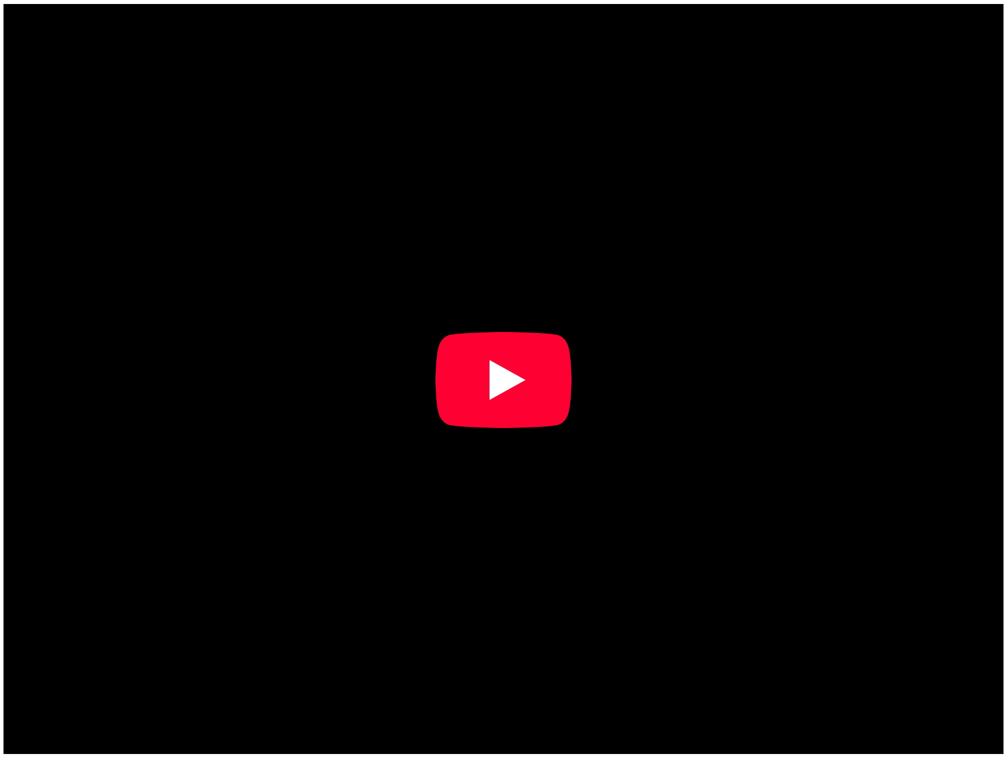
Many different units of length are used around the world. In the United States, the U.S. customary units operationally describe length in terms of the basic unit of an inch. Varying lengths are thus described in relation to the inch, such as a foot equaling 12 inches, a yard equaling three feet, and a mile equaling 1,760 yards.

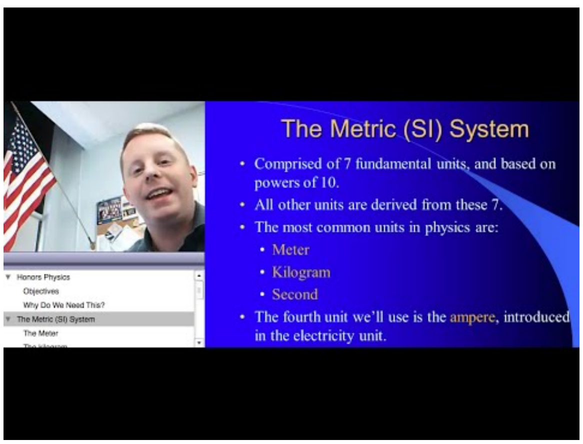
Though regional use of different measurement units is not generally problematic, it can raise issues of compatibility and understanding when working abroad or collaboratively with international partners. As such, a standard unit of measurement that is internationally accepted is needed. The basic unit of length as identified by the International System of Units (SI) is the meter. The meter is expressed more specifically in terms of speed of light.

One meter is defined as the distance that light travels in a vacuum in $\frac{1}{299,792,458}$ of a second. All lengths are measured in terms related to the meter, where its multiples are devised around the convenience of the number 10. For example, a centimeter is equal to $\frac{1}{100}$ of a meter (or 10^{-2} meters), and a kilometer is equal to 1,000 meters (or 10^{3} meters).



Meter Defined by Speed of Light: The meter is defined to be the distance that light travels in $\frac{1}{299,792,458}$ of a second in a vacuum. Distance traveled is speed multiplied by time.





Metric System – Length: A brief introduction to the metric system and unit conversions.

Mass

Mass is the quantity of matter that an object contains, as measured by its resistance to acceleration.

learning objectives

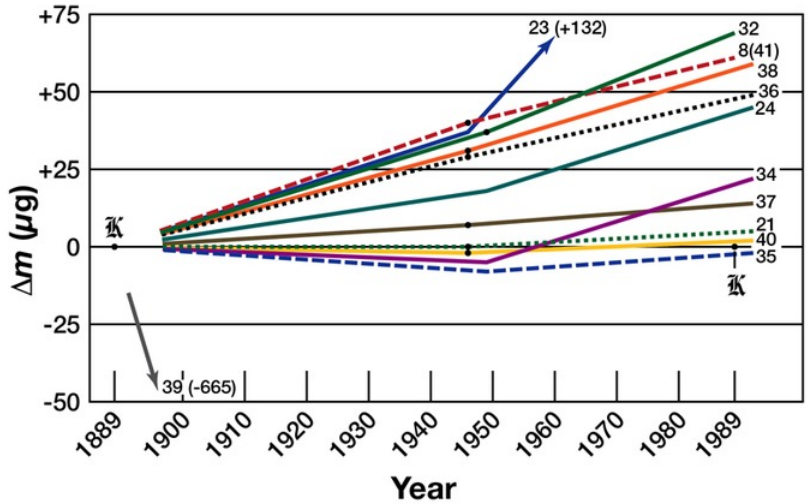
- Explain the difference between mass and weight

Mass

Mass, specifically inertial mass, is a quantitative measure of an object's resistance to acceleration. It is an intrinsic property of an object and does not change because of the environment. The SI unit of mass is the kilogram (kg).

The kilogram is defined as being equal to the mass of the International Prototype Kilogram (IPK), which is almost exactly equal to the mass of one liter of water. It is also the only SI unit that is directly defined by an artifact, rather than a fundamental physical property that can be reproduced in different laboratories. Four of the seven base units in the SI system are defined relative to the kilogram, so the stability of this measurement is crucial for accurate and consistent measurements.

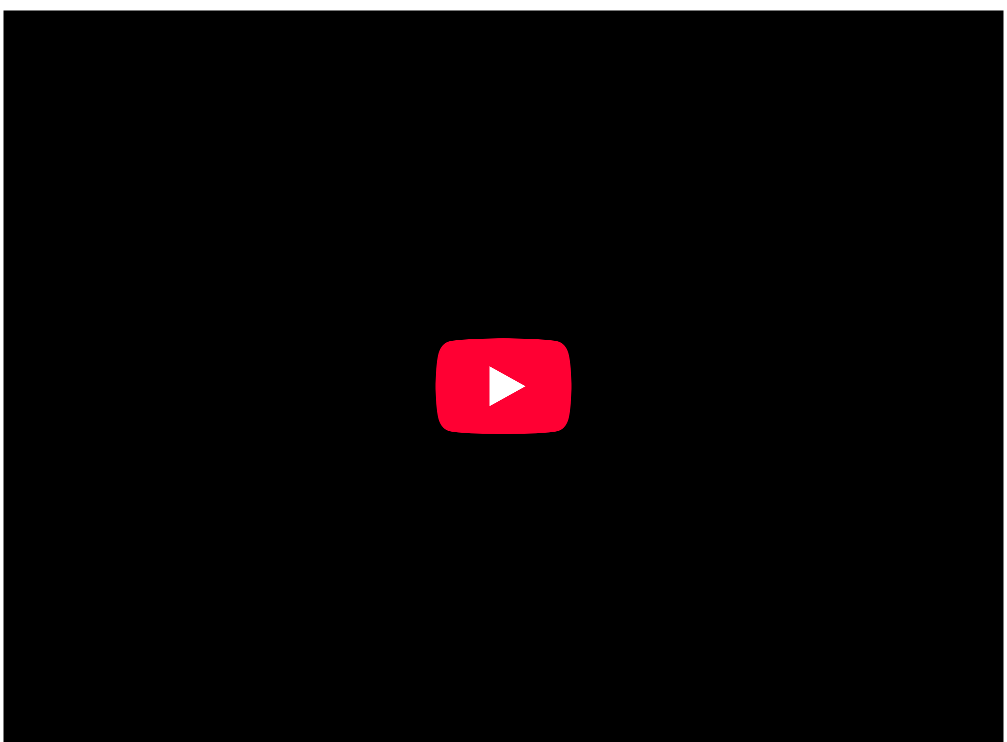
In 2005, the International Committee for Weights and Measures (CIPM) recommended that the kilogram be redefined in terms of a fundamental constant of nature, due to evidence that the International Prototype Kilogram will vary in mass over time. At its 2011 meeting, the General Conference on Weights and Measures (CGPM) agreed that the kilogram should be redefined in terms of the Planck constant. The conference deferred a final decision until its next meeting in 2014.

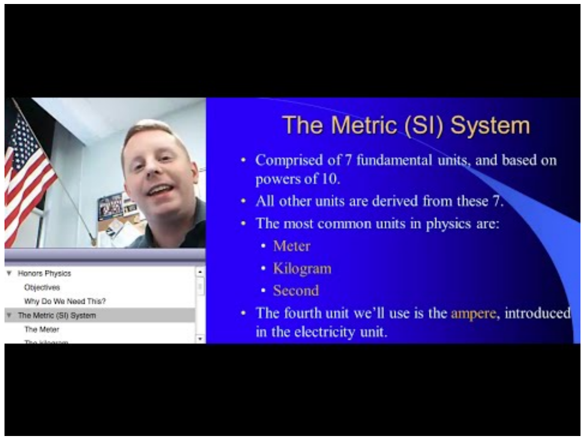


Prototype Mass Drifts: A graph of the relative change in mass of selected kilogram prototypes.

Mass and Weight

In everyday usage, the mass of an object in kilograms is often referred to as its weight. This value, though given in kilograms, is actually the non-SI unit of measure known as the kilogram- force. In scientific terms, 'weight' refers to the gravitational force acting on a given body. This measurement changes depending on the gravitational pull of the opposing body. For example, a person's weight on the Earth is different than a person's weight on the moon because of the differences in the gravitational pull of each body. In contrast, the mass of an object is an intrinsic property and remains the same regardless of gravitational fields. Accordingly, astronauts in microgravity must exert 10 times more force to accelerate a 10-kg object at the same rate as a 1-kg object, even though the differences in weight are imperceptible.





Metric System – Mass: A brief introduction to the metric system and unit conversions.

Time

Time is the fundamental physical quantity of duration and is measured by the SI Unit known as the second.

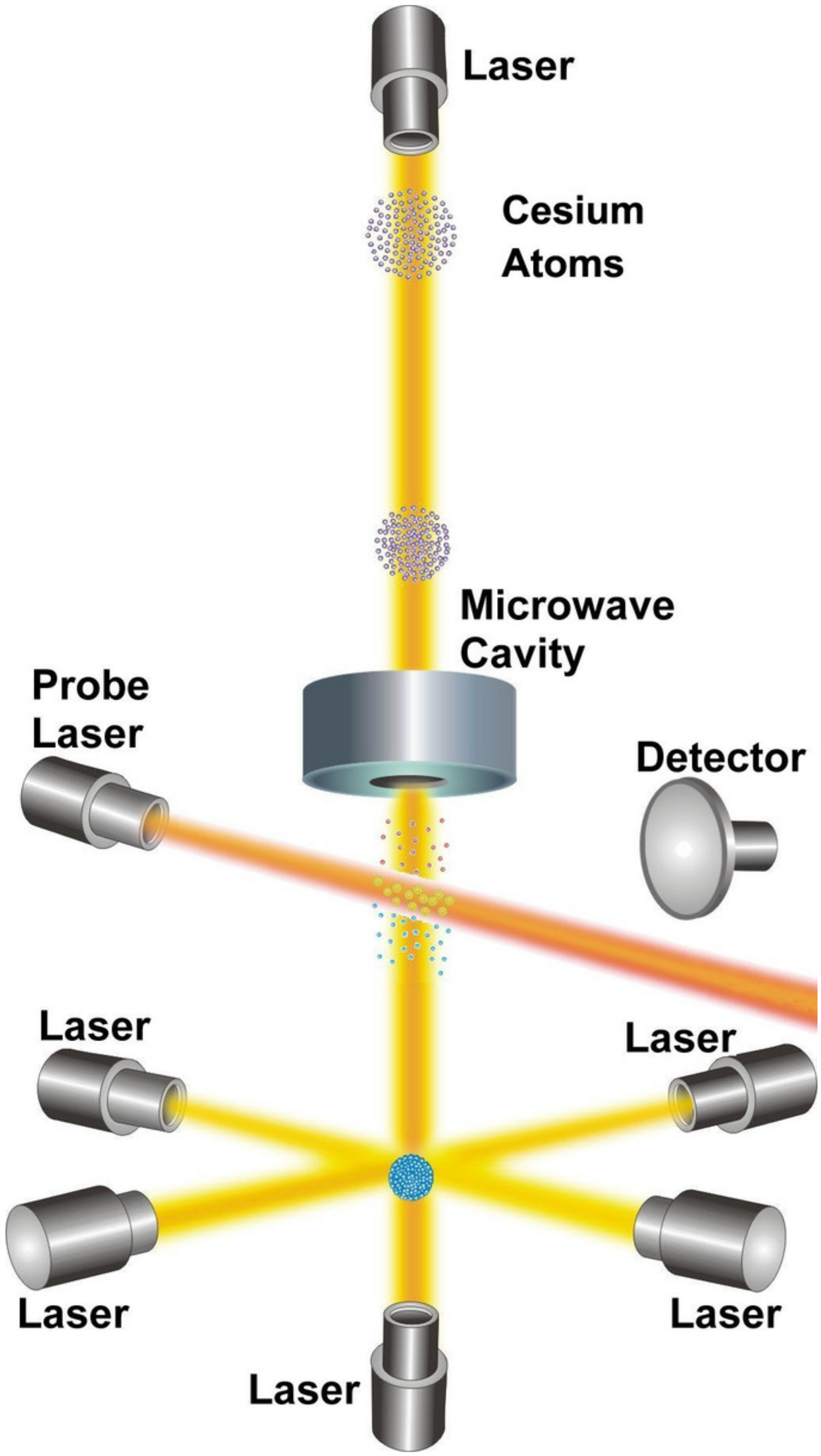
learning objectives

- Relate time with other physical quantities

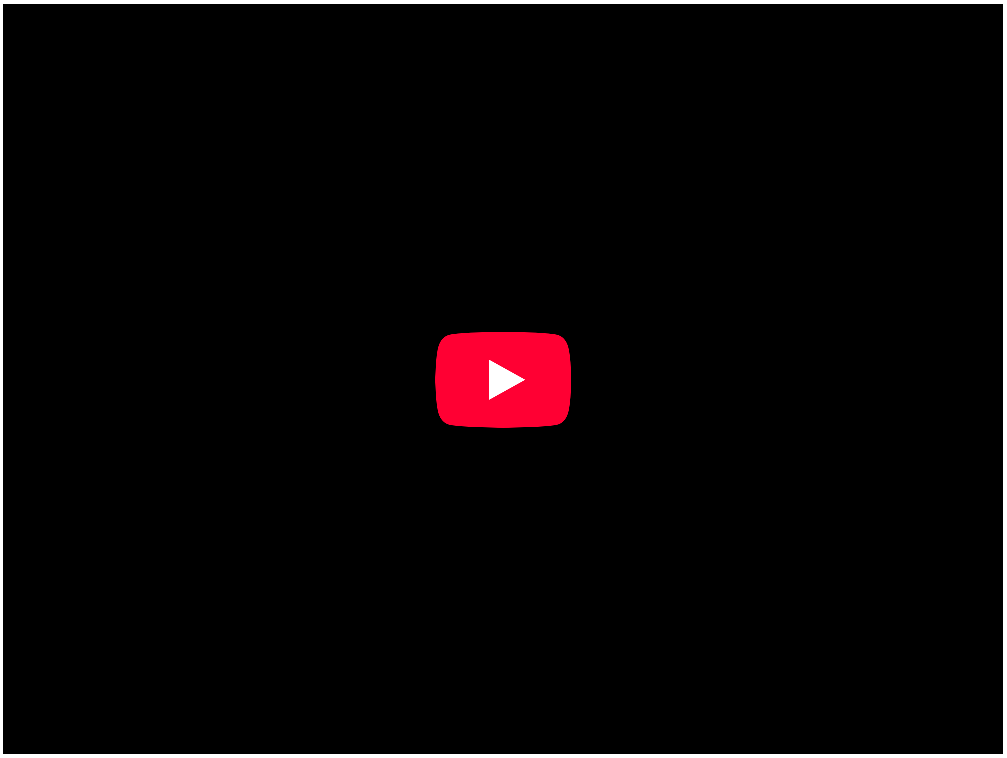
Time is one of the seven fundamental physical quantities in the International System (SI) of Units. Time is used to define other quantities, such as velocity or acceleration, and as such, it is important that it be standardized and quantified precisely. An operational definition of time is highly useful in the conduct of both advanced experiments and everyday affairs of life.

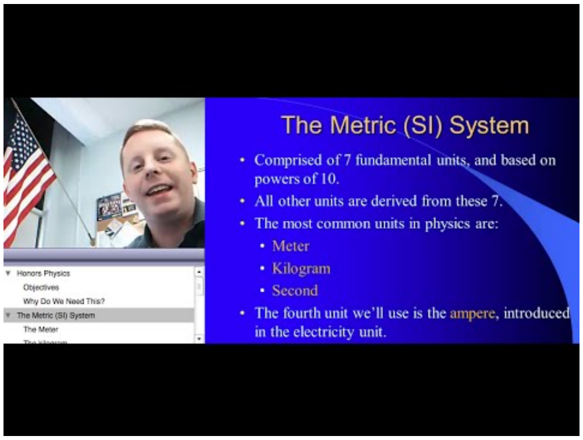
Historically, temporal measurement was a prime motivation in navigation and astronomy. Periodic events and motion have long served as standards for units of time. For example, the movement of the sun across the sky, the phases of the moon, the swing of a pendulum, and the beat of a heart have all been used as a standard for time keeping. These events and standards, however, are highly dynamic in nature and cannot reliably be utilized for accurate quantitative measures. Between 1000 and 1960 the second was defined as $\frac{1}{86,400}$ of a mean solar day. This definition changed between 1960 and 1967 and was defined in terms of the period of the Earth's orbit around the Sun in 1900. Today, the SI Unit of the second is defined in terms of radiation emitted by cesium atoms.

The second is now operationally defined as "the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom." It follows that the hyperfine splitting in the ground state of the cesium 133 atom is exactly 9,192,631,770 hertz. In other words, cesium atoms can be made to vibrate in a very steady way, and these vibrations can be readily observed and counted. The second is the time required for 9,192,631,770 of these vibrations to occur.



NIST-F1 Cesium Clock: NIST-F1 is referred to as a fountain clock because it uses a fountain-like movement of atoms to obtain its improved reckoning of time.





Metric System – Time: A brief introduction to the metric system and unit conversions.

Prefixes and Other Systems of Units

SI prefixes precede a basic unit of measure to indicate a multiple or fraction of the unit.

learning objectives

- Apply prefixes to units and distinguish between SI and customary units

Prefixes

A metric prefix, or SI prefix, is a unit prefix that precedes a basic unit of measure to indicate a multiple or fraction of the unit. Each prefix has a unique symbol that is prepended to the unit symbol. The prefix kilo-, for example, may be added to gram to indicate multiplication by one thousand; one kilogram is equal to one thousand grams (1 kg = 1000 g). The prefix centi-, likewise, may be added to meter to indicate division by one hundred; one centimeter is equal to one hundredth of a meter (1 cm = 0.01 m). Prefixes in varying multiples of 10 are a feature of all forms of the metric system, with many dating back to the system’s introduction in the 1790s. Today, the prefixes are standardized for use in the International System of Units (SI) by the International Bureau of Weights and Measures.There are twenty prefixes officially specified by SI.

Metric prefixes						
Prefix	**Symbol**	**1000^m**	**10^n**	**Decimal**	**Short scale**	**Long scale**
yotta	Y	1000^8	10^{24}	1000000000000000000000000	septillion	quadrillion
zetta	Z	1000^7	10^{21}	1000000000000000000000	sextillion	trilliard
exa	E	1000^6	10^{18}	1000000000000000000	quintillion	trillion
peta	P	1000^5	10^{15}	1000000000000000	quadrillion	billiard
tera	T	1000^4	10^{12}	1000000000000	trillion	billion
giga	G	1000^3	10^9	1000000000	billion	milliard
mega	M	1000^2	10^6	1000000	million	
kilo	k	1000^1	10^3	1000	thousand	
hecto	h	$1000^{2/3}$	10^2	100	hundred	
deca	da	$1000^{1/3}$	10^1	10	ten	
		1000^0	10^0	1	one	
deci	d	$1000^{-1/3}$	10^{-1}	0.1	tenth	
centi	c	$1000^{-2/3}$	10^{-2}	0.01	hundredth	
milli	m	1000^{-1}	10^{-3}	0.001	thousandth	
micro	µ	1000^{-2}	10^{-6}	0.000001	millionth	
nano	n	1000^{-3}	10^{-9}	0.000000001	billionth	milliardth
pico	p	1000^{-4}	10^{-12}	0.000000000001	trillionth	billionth
femto	f	1000^{-5}	10^{-15}	0.000000000000001	quadrillionth	billiardth
atto	a	1000^{-6}	10^{-18}	0.000000000000000001	quintillionth	trillionth
zepto	z	1000^{-7}	10^{-21}	0.000000000000000000001	sextillionth	trilliardth
yocto	y	1000^{-8}	10^{-24}	0.000000000000000000000001	septillionth	quadrillionth

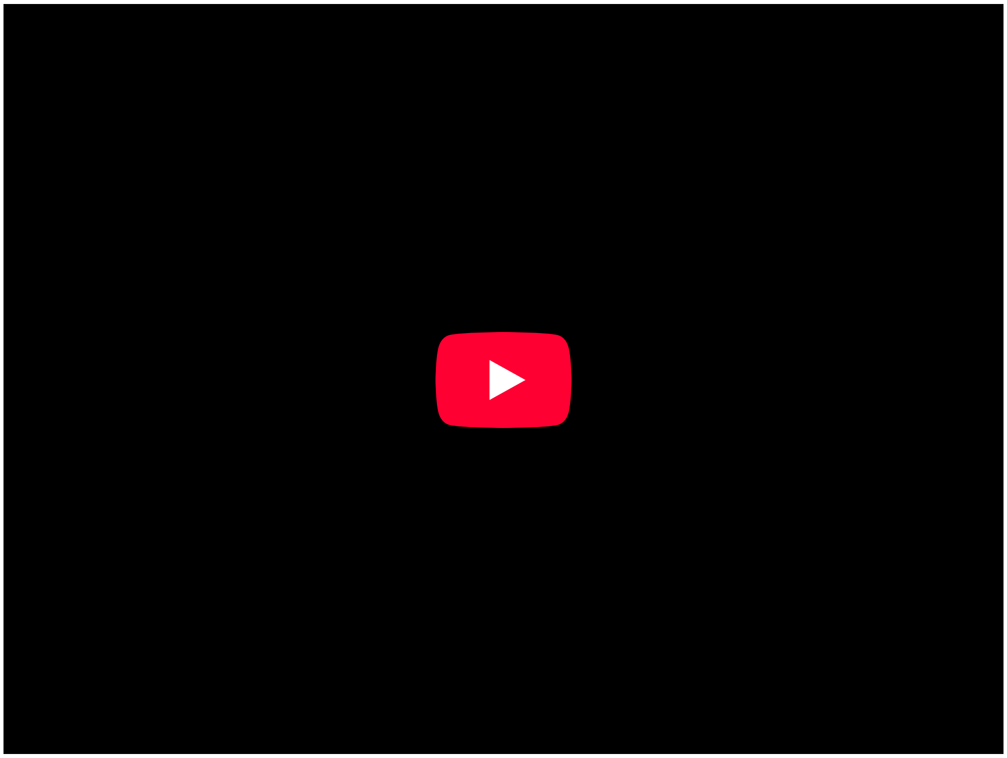
SI Unit Prefixes: The twenty prefixes officially specified by the International System of Units

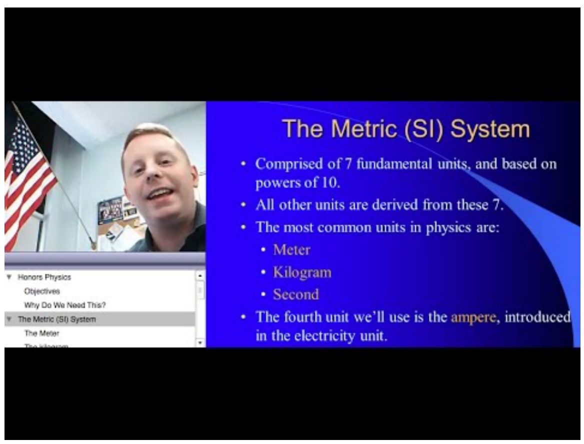
It is important to note that the kilogram is the only SI unit with a prefix as part of its name and symbol. Because multiple prefixes may not be used, in the case of the kilogram the prefix names are used with the unit name “gram” and the prefix symbols are used with the unit symbol “g. ” With this exception, any SI prefix may be used with any SI unit, including the degree Celsius and its symbol °C.

Other Systems of Units

The SI Unit system, or the metric system, is used by the majority of countries in the world, and is the standard system agreed upon by scientists and mathematicians. Colloquially, however, other systems of units are used in many countries. The United States, for example, teaches and uses the *United States customary units*. This system of units was developed from the English, or Imperial, unit standards of the United Kingdom.The United States customary units define measurements using different standards than those used in SI Units. The system for measuring length using the United States customary system is based on the inch, foot, yard, and mile. Likewise, units of area are measured in terms of square feet, and units of capacity and volume are measured in terms of cubic inches, cubic feet, or cubic yards. Units of mass are commonly defined in terms of ounces and pounds, rather than the SI unit of kilograms.Other commonly used units from the United States customary system include the fluid volume units of the teaspoon, tablespoon, fluid ounce, US cup, pint, quart, and gallon, as well as the degrees Fahrenheit used to measure temperature.

Some units that are widely used are not a part of the International System of Units and are considered Non-SI Units. These units, though not officially part of SI Units, are generally accepted for use in conjunction with SI units. These can include the minute, hour, and day used in temporal measurements, the liter for volumetric measurements, and the degree, minute, and second used to measure angles.





Metric System – Prefixes: A brief introduction to the metric system and unit conversions.

Converting Units

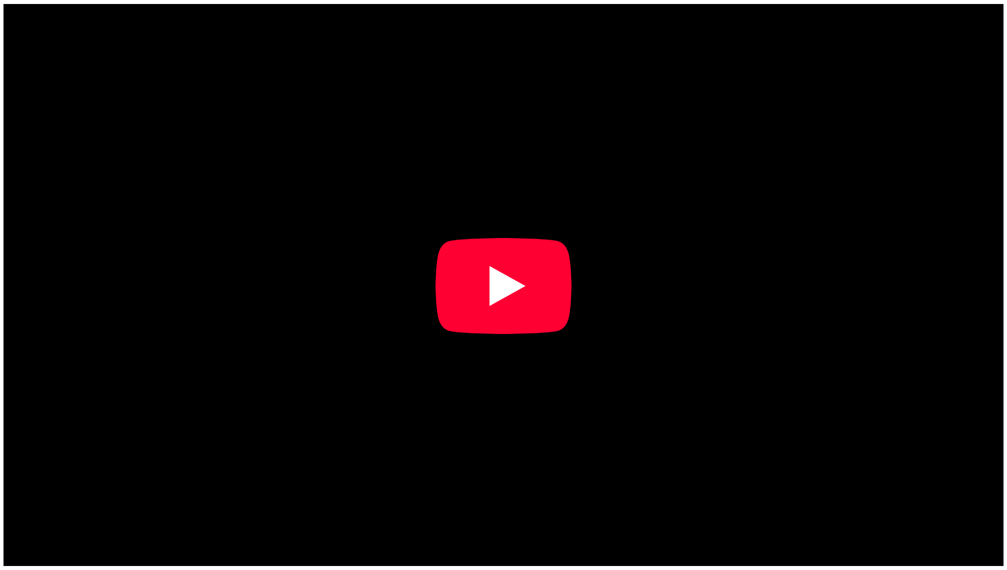
Converting between units can be done through the use of conversion factors or specific conversion formulas.

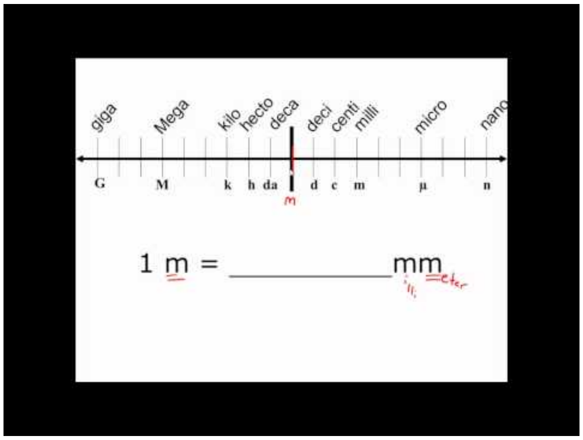
learning objectives

- Apply factor-label method for converting units

Translating Systems of Measurement

It is often necessary to convert from one type of unit to another. Conversion of units is the conversion of different units of measurement for the same quantity, typically using conversion factors. For example, if you are reading a European cookbook, some quantities may be expressed in units of liters; if you're cooking in the US in a standard kitchen with standard tools, you will need to convert those measurements to cups. Or, perhaps you are reading walking directions from one location to another and you are interested in how many miles you will be walking. In this case, you will need to convert units of feet to miles. This is a bit like translating a substitution code, using a formula that helps you understand what one measure means in terms of another system.





Unit Conversion in the Metric System: EASY Unit Conversion in the Metric System – This simple extra help video tutorial explains the metric system and how to make simple metric conversions.

Conversion Methods

There are several ways to approach doing conversions. One commonly used method is known as the Factor-label method for converting units, or the “railroad method. ”

The factor-label method is the sequential application of conversion factors expressed as fractions and arranged so that any dimensional unit appearing in both the numerator and denominator of any of the fractions can be cancelled out until only the desired set of dimensional units is obtained. For example, 10 miles per hour can be converted to meters per second by using a sequence of conversion factors.

Each conversion factor is equivalent to the value of one. For example, starting with 1 mile = 1609 meters and dividing both sides of the equation by 1 mile yields $\frac{1 \text{ mile}}{1 \text{ mile}} = \frac{1609 \text{ meters}}{1 \text{ mile}}$, which when simplified yields $1 = \frac{1609 \text{ meters}}{1 \text{ mile}}$. Physically crossing out the units that cancel each other out will also help visualize what’s left over.

$$1\ \cancel{\text{year}} \times \frac{365\ \cancel{\text{days}}}{1\ \cancel{\text{year}}} \times \frac{24\ \cancel{\text{hours}}}{1\ \cancel{\text{day}}} \times \frac{60\ \cancel{\text{min}}}{1\ \cancel{\text{hour}}} \times \frac{60\ \text{s}}{1\ \cancel{\text{min}}} = \\ = 3.15 \times 10^7\ \text{s}$$

Converting 1 year into seconds using the Factor-Label Method: Physically crossing out units that cancel out helps visualize the “leftover” unit(s).

So, when the units mile and hour are cancelled out and the arithmetic is done, 10 miles per hour converts to 4.47 meters per second.

A limitation of the factor-label method is that it can only convert between units that have a constant ratio that can be multiplied, or a multiplication factor. This method cannot be used between units that have a displacement, or difference factor. An example is the conversion between degrees Celsius and kelvins, or between Celsius and Fahrenheit. For these, it is best to use the specific conversion formulas.

For example, if you are planning a trip abroad in Spain and the weather forecast predicts the weather to be mostly cloudy and 16°C, you may want to convert the temperature into °F, a unit that you are more comfortable interpreting. In order to do this, you would need to know the conversion formula from Celsius to Fahrenheit. This formula is: $[^\circ\mathrm{F}] = [^\circ\mathrm{C}] \times \frac{9}{5} + 32$.

$$[^\circ\mathrm{F}] = [^\circ\mathrm{C}] \times \frac{9}{5} + 32 \tag{5.1.2.1}$$
$$[^\circ\mathrm{F}] = 28.8 + 32 \tag{5.1.2.2}$$
$$[^\circ\mathrm{F}] = 60.8 + 32 \tag{5.1.2.3}$$

So you would then know that 16°C is equivalent to 60.8°F and be able to pack the right type of clothing to be comfortable.

Key Points

- The SI unit for length is the meter.
- One meter is defined as the distance that light travels in a vacuum in $\frac{1}{299{,}792{,}458}$ of a second.
- Derivatives of measurement units related to the meter are devised around the convenience of the number 10.
- The kilogram is the only SI unit directly defined by the artifact itself.
- Mass is a property that does not depend on gravitational fields, unlike weight.
- One kilogram is defined as the mass of the International Prototype Kilogram (IPK), a platinum-iridium alloy cylinder.
- One kilogram is almost exactly equal to the mass of one liter of water.
- Time is a physical quantity of duration.
- The SI Unit for time is the second.
- The second is operationally defined in terms of radiation emitted by cesium atoms.
- The twenty standardized prefixes for use in the International System of Units are derived from multiples of 10.
- The kilogram is the only SI unit with a prefix as part of its name and symbol; as such, SI unit prefixes are prepended to the unit gram.
- The United States customary units define measurements based on the English, or Imperial, unit standards.
- Conversion of units is the conversion between different units of measurement for the same quantity, typically through multiplicative conversion factors.
- The factor-label method is the sequential application of conversion factors expressed as fractions in which units appearing in both the numerator and denominator can be cancelled out, leaving only the desired set of units.
- For conversions that have a difference factor, specific conversion formulas should be used.

Key Terms

- **Length**: How far apart objects are physically.
- **acceleration**: the rate at which the velocity of a body changes with time
- **inertia**: the tendency of an object to resist any change in its motion
- **Radiation**: the emission of energy as electromagnetic waves or as moving or oscillating subatomic particles.
- **prefix**: That which is prefixed; especially one or more letters or syllables added to the beginning of a word to modify its meaning; as, pre- in prefix, con- in conjure.
- **conversion**: a change between different units of measurement for the same quantity.

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5.1.3: Significant Figures and Order of Magnitude

Scientific Notation

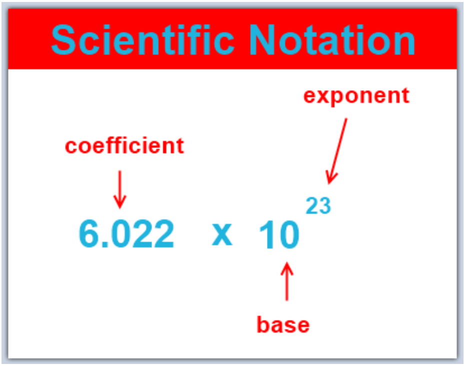
Scientific notation is a way of writing numbers that are too big or too small in a convenient and standard form.

learning objectives

- Convert properly between standard and scientific notation and identify appropriate situations to use it

Scientific Notation: A Matter of Convenience

Scientific notation is a way of writing numbers that are too big or too small in a convenient and standard form. Scientific notation has a number of useful properties and is commonly used in calculators and by scientists, mathematicians and engineers. In scientific notation all numbers are written in the form of $a \cdot 10^{b}$ (a multiplied by ten raised to the power of b), where the exponent b) is an integer, and the coefficient (a is any real number.



Scientific Notation: There are three parts to writing a number in scientific notation: the coefficient, the base, and the exponent.

Most of the interesting phenomena in our universe are not on the human scale. It would take about 1,000,000,000,000,000,000,000 bacteria to equal the mass of a human body. Thomas Young's discovery that light was a wave preceded the use of scientific notation, and he was obliged to write that the time required for one vibration of the wave was "$\frac{1}{500}$ of a millionth of a millionth of a second"; an inconvenient way of expressing the point. Scientific notation is a less awkward and wordy way to write very large and very small numbers such as these.

A Simple System

Scientific notation means writing a number in terms of a product of something from 1 to 10 and something else that is a power of ten.

For instance, $32 = 3.2 \cdot 10^{1}$

$320 = 3.2 \cdot 10^{2}$

$3200 = 3.2 \cdot 10^{3}$, and so forth…

Each number is ten times bigger than the previous one. Since 10^{1} is ten times smaller than 10^{2}, it makes sense to use the notation 10^{0} to stand for one, the number that is in turn ten times smaller than 10^{1}. Continuing on, we can write 10^{-1} to stand for 0.1, the number ten times smaller than 10^{0}. Negative exponents are used for small numbers:

$3.2 = 3.2 \cdot 10^{0}$

$0.32 = 3.2 \cdot 10^{-1}$

$0.032 = 3.2 \cdot 10^{-2}$

Scientific notation displayed calculators can take other shortened forms that mean the same thing. For example, $3.2 \cdot 10^{6}$ (written notation) is the same as $3.2\mathrm{E}+6$ (notation on some calculators) and 3.2^{6} (notation on some other calculators).

Round-off Error

A round-off error is the difference between the calculated approximation of a number and its exact mathematical value.

learning objectives

- Explain the impact round-off errors may have on calculations, and how to reduce this impact

Round-off Error

A round-off error, also called a rounding error, is the difference between the calculated approximation of a number and its exact mathematical value. Numerical analysis specifically tries to estimate this error when using approximation equations, algorithms, or both, especially when using finitely many digits to represent real numbers. When a sequence of calculations subject to rounding errors is made, errors may accumulate, sometimes dominating the calculation.

Calculations rarely lead to whole numbers. As such, values are expressed in the form of a decimal with infinite digits. The more digits that are used, the more accurate the calculations will be upon completion. Using a slew of digits in multiple calculations, however, is often unfeasible if calculating by hand and can lead to much more human error when keeping track of so many digits. To make calculations much easier, the results are often 'rounded off' to the nearest few decimal places.

For example, the equation for finding the area of a circle is $\mathrm{A}=\pi \mathrm{r}^{2}$. The number π(pi) has infinitely many digits, but can be truncated to a rounded representation of as 3.14159265359. However, for the convenience of performing calculations by hand, this number is typically rounded even further, to the nearest two decimal places, giving just 3.14. Though this technically decreases the accuracy of the calculations, the value derived is typically 'close enough' for most estimation purposes.

However, when doing a series of calculations, numbers are rounded off at each subsequent step. This leads to an accumulation of errors, and if profound enough, can misrepresent calculated values and lead to miscalculations and mistakes.

The following is an example of round-off error:

$$\sqrt{4.58^{2}+3.28^{2}}=\sqrt{21.0+10.8}=5.64$$

Rounding these numbers off to one decimal place or to the nearest whole number would change the answer to 5.7 and 6, respectively. The more rounding off that is done, the more errors are introduced.

Order of Magnitude Calculations

An order of magnitude is the class of scale of any amount in which each class contains values of a fixed ratio to the class preceding it.

learning objectives

- Choose when it is appropriate to perform an order-of-magnitude calculation

Orders of Magnitude

An order of magnitude is the class of scale of any amount in which each class contains values of a fixed ratio to the class preceding it. In its most common usage, the amount scaled is 10, and the scale is the exponent applied to this amount (therefore, to be an order of magnitude greater is to be 10 times, or 10 to the power of 1, greater). Such differences in order of magnitude can be measured on the logarithmic scale in "decades," or factors of ten. It is common among scientists and technologists to say that a parameter whose value is not accurately known or is known only within a range is "on the order of" some value. The order of magnitude of a physical quantity is its magnitude in powers of ten when the physical quantity is expressed in powers of ten with one digit to the left of the decimal.

Orders of magnitude are generally used to make very approximate comparisons and reflect very large differences. If two numbers differ by one order of magnitude, one is about ten times larger than the other. If they differ by two orders of magnitude, they differ by a factor of about 100. Two numbers of the same order of magnitude have roughly the same scale — the larger value is less than ten times the smaller value.

It is important in the field of science that estimates be at least in the right ballpark. In many situations, it is often sufficient for an estimate to be within an order of magnitude of the value in question. Although making order-of-magnitude estimates seems simple and natural to experienced scientists, it may be completely unfamiliar to the less experienced.

Example 5.1.3.1:

Some of the mental steps of estimating in orders of magnitude are illustrated in answering the following example question: Roughly what percentage of the price of a tomato comes from the cost of transporting it in a truck?

Guessing the Number of Jelly Beans: Can you guess how many jelly beans are in the jar? If you try to guess directly, you will almost certainly underestimate. The right way to do it is to estimate the linear dimensions and then estimate the volume indirectly.

Incorrect solution: Let's say the trucker needs to make a profit on the trip. Taking into account her benifits, the cost of gas, and maintenance and payments on the truck, let's say the total cost is more like 2000. You might guess about 5000 tomatoes would fit in the back of the truck, so the extra cost per tomato is 40 cents. That means the cost of transporting one tomato is comparable to the cost of the tomato itself.

The problem here is that the human brain is not very good at estimating area or volume — it turns out the estimate of 5000 tomatoes fitting in the truck is way off. (This is why people have a hard time in volume-estimation contests, such as the one shown below.) When estimating area or volume, you are much better off estimating linear dimensions and computing the volume from there.

So, here's a better solution: As before, let's say the cost of the trip is \$2000. The dimensions of the bin are probably 4m by 2m by 1m, for a volume of $8\ \mathrm{m}^3$. Since our goal is just an order-of-magnitude estimate, let's round that volume off to the nearest power of ten: $10\ \mathrm{m}^3$. The shape of a tomato doesn't follow linear dimensions, but since this is just an estimate, let's pretend that a tomato is an 0.1m by 0.1m by 0.1m cube, with a volume of $1 \cdot 10^{-3}\ \mathrm{m}^3$. We can find the total number of tomatoes by dividing the volume of the bin by the volume of one tomato: $\frac{10^3\ \mathrm{m}^3}{10^{-3}\ \mathrm{m}^3} = 10^6$ tomatoes. The transportation cost per tomato is $\frac{\$2000}{10^6\ \text{tomatoes}} = \$0.002$ per tomato. That means that transportation really doesn't contribute very much to the cost of a tomato. Approximating the shape of a tomato as a cube is an example of another general strategy for making order-of-magnitude estimates.

Key Points

- Scientific notation means writing a number in terms of a product of something from 1 to 10 and something else that is a power of 10.
- In scientific notation all numbers are written in the form of $\mathrm{a} \cdot 10^{\mathrm{b}}$ (a times ten raised to the power of b).
- Each consecutive exponent number is ten times bigger than the previous one; negative exponents are used for small numbers.
- When a sequence of calculations subject to rounding error is made, these errors can accumulate and lead to the misrepresentation of calculated values.
- Increasing the number of digits allowed in a representation reduces the magnitude of possible round-off errors, but may not always be feasible, especially when doing manual calculations.
- The degree to which numbers are rounded off is relative to the purpose of calculations and the actual value.
- Orders of magnitude are generally used to make very approximate comparisons and reflect very large differences.
- In the field of science, it is often sufficient for an estimate to be within an order of magnitude of the value in question.

- When estimating area or volume, you are much better off estimating linear dimensions and computing volume from those linear dimensions.

Key Terms

- **exponent**: The power to which a number, symbol or expression is to be raised. For example, the 3 in x^3.
- **Scientific notation**: A method of writing, or of displaying real numbers as a decimal number between 1 and 10 followed by an integer power of 10
- **approximation**: An imprecise solution or result that is adequate for a defined purpose.
- **Order of Magnitude**: The class of scale or magnitude of any amount, where each class contains values of a fixed ratio (most often 10) to the class preceding it. For example, something that is 2 orders of magnitude larger is 100 times larger; something that is 3 orders of magnitude larger is 1000 times larger; and something that is 6 orders of magnitude larger is one million times larger, because $10^2 = 100$, $10^3 = 1000$, and $10^6 =$ one million

5.1.4: Solving Physics Problems

Dimensional Analysis

Any physical quantity can be expressed as a product of a combination of the basic physical dimensions.

> **learning objectives**
>
> - Calculate the conversion from one kind of dimension to another

Dimensions

The dimension of a physical quantity indicates how it relates to one of the seven basic quantities. These fundamental quantities are:

- [M] Mass
- [L] Length
- [T] Time
- [A] Current
- [K] Temperature
- [mol] Amount of a Substance
- [cd] Luminous Intensity

As you can see, the symbol is enclosed in a pair of square brackets. This is often used to represent the dimension of individual basic quantity. An example of the use of basic dimensions is speed, which has a dimension of 1 in length and -1 in time; $\frac{[L]}{[T]} = [LT^{-1}]$. Any physical quantity can be expressed as a product of a combination of the basic physical dimensions.

Dimensional Analysis

Dimensional analysis is the practice of checking relations between physical quantities by identifying their dimensions. The dimension of any physical quantity is the combination of the basic physical dimensions that compose it. Dimensional analysis is based on the fact that physical law must be independent of the units used to measure the physical variables. It can be used to check the plausibility of derived equations, computations and hypotheses.

Derived Dimensions

The dimensions of derived quantities may include few or all dimensions in individual basic quantities. In order to understand the technique to write dimensions of a derived quantity, we consider the case of force. Force is defined as:

$$F = m \cdot a \tag{5.1.4.1}$$

$$F = [M][a] \tag{5.1.4.2}$$

The dimension of acceleration, represented as [a], is itself a derived quantity being the ratio of velocity and time. In turn, velocity is also a derived quantity, being ratio of length and time.

$$F = [M][a] = [M][vT^{-1}] \tag{5.1.4.3}$$

$$F = [M][LT^{-1}T^{-1}] = [MLT^{-2}] \tag{5.1.4.4}$$

Dimensional Conversion

In practice, one might need to convert from one kind of dimension to another. For common conversions, you might already know how to convert off the top of your head. But for less common ones, it is helpful to know how to find the conversion factor:

$$Q = n_1 u_1 = n_2 u_2 \tag{5.1.4.5}$$

where n represents the amount per u dimensions. You can then use ratios to figure out the conversion:

$$n_2 = \frac{u_2}{u_1} \cdot n_1 \tag{5.1.4.6}$$

Trigonometry

Trigonometry is central to the use of free body diagrams, which help visually represent difficult physics problems.

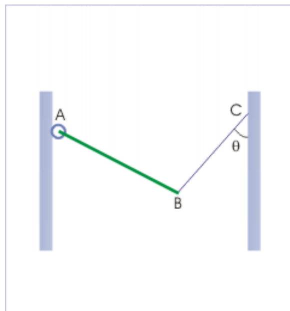
learning objectives

- Explain why trigonometry is useful in determining horizontal and vertical components of forces

Trigonometry and Solving Physics Problems

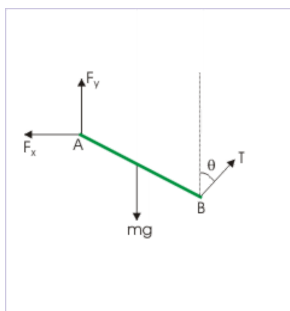
In physics, most problems are solved much more easily when a free body diagram is used. Free body diagrams use geometry and vectors to visually represent the problem. Trigonometry is also used in determining the horizontal and vertical components of forces and objects. Free body diagrams are very helpful in visually identifying which components are unknown and where the moments are applied. They can help analyze a problem, whether it is static or dynamic.

When people draw free body diagrams, often not everything is perfectly parallel and perpendicular. Sometimes people need to analyze the horizontal and vertical components of forces and object orientation. When the force or object is not acting parallel to the *x* or *y* axis, people can employ basic trigonometry to use the simplest components of the action to analyze it. Basically, everything should be considered in terms of *x* and *y*, which sometimes takes some manipulation.



Free Body Diagram: The rod is hinged from a wall and is held with the help of a string.

A rod 'AB' is hinged at 'A' from a wall and is held still with the help of a string, as shown in. This exercise involves drawing the free body diagram. To make the problem easier, the force *F* will be expressed in terms of its horizontal and vertical components. Removing all other elements from the image helps produce the finished free body diagram.



Free Body Diagram: The free body diagram as a finished product

Given the finished free body diagram, people can use their knowledge of trigonometry and the laws of sine and cosine to mathematically and numerical represent the horizontal and vertical components:

General Problem-Solving Tricks

Free body diagrams use geometry and vectors to visually represent the problem.

learning objectives

- Construct a free-body diagram for a physical scenario

In physics, most problems are solved much more easily when a free body diagram is used. This uses geometry and vectors to visually represent to problem, and trigonometry is also used in determining horizontal and vertical components of forces and objects.

Purpose: Free body diagrams are very helpful in visually identifying which components are unknown, where the moments are applied, and help analyze a problem, whether static or dynamic.

How to Make A Free Body Diagram

To draw a free body diagram, do not worry about drawing it to scale, this will just be what you use to help yourself identify the problems. First you want to model the body, in one of three ways:

- As a particle. This model may be used when any turning effects are zero or have zero interest even though the body itself may be extended. The body may be represented by a small symbolic blob and the diagram reduces to a set of concurrent arrows. A force on a particle is a *bound* vector.
- *rigid extended*. Stresses and strains are of no interest but turning effects are. A force arrow should lie along the line of force, but where along the line is irrelevant. A force on an extended rigid body is a*sliding* vector.
- *non-rigid extended*. The *point of application* of a force becomes crucial and has to be indicated on the diagram. A force on a non-rigid body is a *bound* vector. Some engineers use the tail of the arrow to indicate the point of application. Others use the tip.

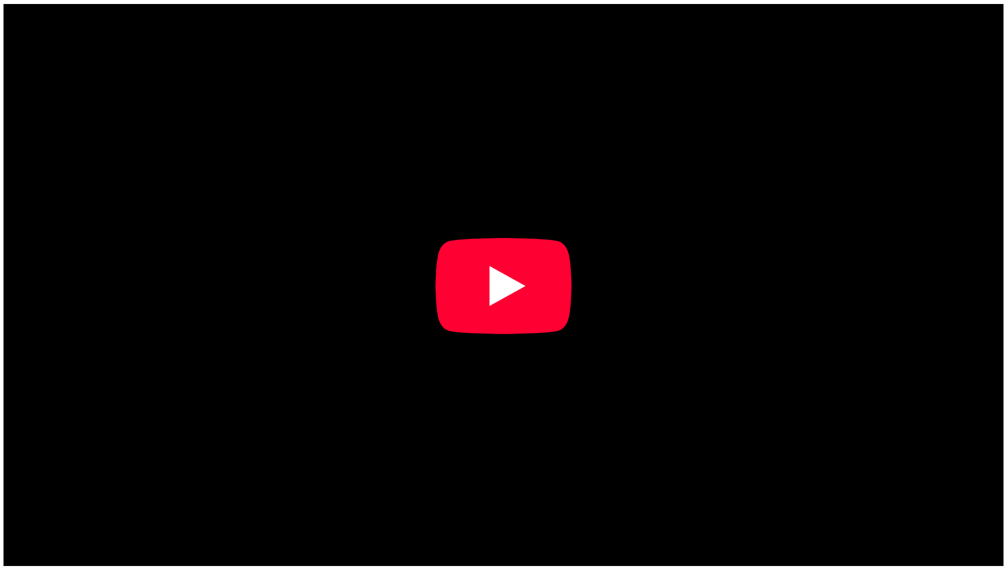
Do's and Don'ts

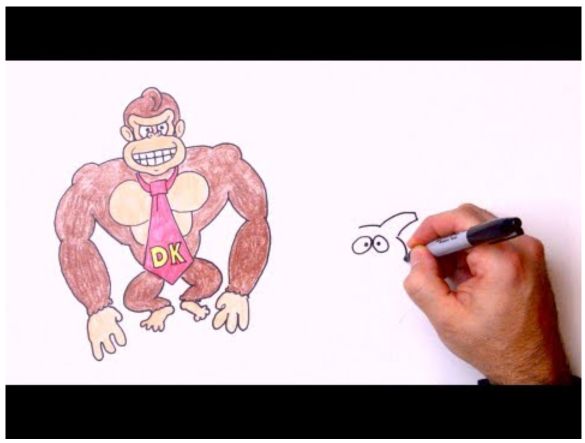
What to include: Since a free body diagram represents the body itself and the external forces on it. So you will want to include the following things in the diagram:

- The body: This is usually sketched in a schematic way depending on the body – particle/extended, rigid/non-rigid – and on what questions are to be answered. Thus if rotation of the body and torque is in consideration, an indication of size and shape of the body is needed.
- The external forces: These are indicated by labelled arrows. In a fully solved problem, a force arrow is capable of indicating the direction, the magnitude the point of application. These forces can be friction, gravity, normal force, drag, tension, etc…

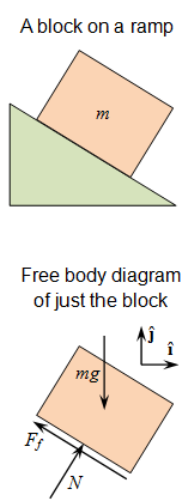
Do not include:

- Do not show bodies other than the body of interest.
- Do not show forces exerted by the body.
- Internal forces acting on various parts of the body by other parts of the body.
- Any velocity or acceleration is left out.





How To Solve Any Physics Problem: Learn five simple steps in five minutes! In this episode we cover the most effective problem-solving method I’ve encountered and call upon some fuzzy friends to help us remember the steps.



Free Body Diagram: Use this figure to work through the example problem.

Key Points

- Dimensional analysis is the practice of checking relations amount physical quantities by identifying their dimensions.
- It is common to be faced with a problem that uses different dimensions to express the same basic quantity. The following equation can be used to find the conversion factor between the two derived dimensions: $n_2 = \frac{u_2}{u_1} \times n_1$.
- Dimensional analysis can also be used as a simple check to computations, theories and hypotheses.
- It is important to identify the problem and the unknowns and draw them in a free body diagram.
- The laws of cosine and sine can be used to determine the vertical and horizontal components of the different elements of the diagram.
- Free body diagrams use geometry and vectors to visually represent physics problems.
- A free body diagram lets you visually isolate the problem you are trying to solve, and simplify it into simple geometry and trigonometry.
- When drawing these diagrams, it is helpful to only draw the body it self, and the forces acting on it.
- Drawing other objects and internal forces can condense the diagram and cause it to be less helpful.

Key Terms

- **dimension**: A measure of spatial extent in a particular direction, such as height, width or breadth, or depth.
- **trigonometry**: The branch of mathematics that deals with the relationships between the sides and the angles of triangles and the calculations based on them, particularly the trigonometric functions.
- **static**: Fixed in place; having no motion.
- **dynamic**: Changing; active; in motion.

- How To Solve Any Physics Problem. **Located at**: **http://www.youtube.com/watch?v=YocWuzi4JhY**. **License**: ***Public Domain: No Known Copyright***. **License Terms**: Standard YouTube license

CHAPTER OVERVIEW

5.2: Kinematics

Topic hierarchy

5.2.1: Basics of Kinematics

Defining Kinematics

Kinematics is the study of the motion of points, objects, and groups of objects without considering the causes of its motion.

learning objectives

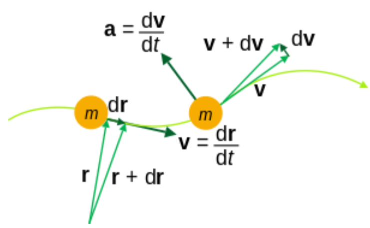
- Define kinematics

Kinematics is the branch of classical mechanics that describes the motion of points, objects and systems of groups of objects, without reference to the causes of motion (i.e., forces). The study of kinematics is often referred to as the "geometry of motion."

Objects are in motion all around us. Everything from a tennis match to a space-probe flyby of the planet Neptune involves motion. When you are resting, your heart moves blood through your veins. Even in inanimate objects there is continuous motion in the vibrations of atoms and molecules. Interesting questions about motion can arise: how long will it take for a space probe to travel to Mars? Where will a football land if thrown at a certain angle? An understanding of motion, however, is also key to understanding other concepts in physics. An understanding of acceleration, for example, is crucial to the study of force.

To describe motion, kinematics studies the trajectories of points, lines and other geometric objects, as well as their differential properties (such as velocity and acceleration). Kinematics is used in astrophysics to describe the motion of celestial bodies and systems; and in mechanical engineering, robotics and biomechanics to describe the motion of systems composed of joined parts (such as an engine, a robotic arm, or the skeleton of the human body).

A formal study of physics begins with kinematics. The word "kinematics" comes from a Greek word "kinesis" meaning motion, and is related to other English words such as "cinema" (movies) and "kinesiology" (the study of human motion). Kinematic analysis is the process of measuring the kinematic quantities used to describe motion. The study of kinematics can be abstracted into purely mathematical expressions, which can be used to calculate various aspects of motion such as velocity, acceleration, displacement, time, and trajectory.



Kinematics of a particle trajectory: Kinematic equations can be used to calculate the trajectory of particles or objects. The physical quantities relevant to the motion of a particle include: mass m, position r, velocity v, acceleration a.

Reference Frames and Displacement

In order to describe an object's motion, you need to specify its position relative to a convenient reference frame.

learning objectives

- Evaluate displacement within a frame of reference.

In order to describe the motion of an object, you must first describe its position — where it is at any particular time. More precisely, you need to specify its position relative to a convenient reference frame. Earth is often used as a reference frame, and we often describe the position of objects related to its position to or from Earth. Mathematically, the position of an object is generally represented by the variable x.

Frames of Reference

There are two choices you have to make in order to define a position variable x. You have to decide where to put $x = 0$ and which direction will be positive. This is referred to as choosing a coordinate system, or choosing a frame of reference. As long as you are consistent, any frame is equally valid. But you don't want to change coordinate systems in the middle of a calculation. Imagine sitting in a train in a station when suddenly you notice that the station is moving backward. Most people would say that they just failed to notice that the train was moving — it only *seemed* like the station was moving. But this shows that there is a *third* arbitrary choice that goes into choosing a coordinate system: valid frames of reference can differ from each other by moving relative to one another. It might seem strange to use a coordinate system moving relative to the earth — but, for instance, the frame of reference moving along with a train might be far more convenient for describing things happening inside the train. Frames of reference are particularly important when describing an object's displacement.

FRAMES OF REFERENCE by Professor Hume and Professor Donald Ivey of the University of Toronto

In this classic film, Professors Hume and Ivey cleverly illustrate reference frames and distinguish between fixed and moving frames of reference.

Frames of Reference (1960) Educational Film: Frames of Reference is a 1960 educational film by Physical Sciences Study Committee. The film was made to be shown in high school physics courses. In the film University of Toronto physics professors Patterson Hume and Donald Ivey explain the distinction between inertial and nonintertial frames of reference, while demonstrating these concepts through humorous camera tricks. For example, the film opens with Dr. Hume, who appears to be upside down, accusing Dr. Ivey of being upside down. Only when the pair flip a coin does it become obvious that Dr. Ivey — and the camera — are indeed inverted. The film's humor serves both to hold students' interest and to demonstrate the concepts being discussed. This PSSC film utilizes a fascinating set consisting of a rotating table and furniture occupying surprisingly unpredictable spots within the viewing area. The fine cinematography by Abraham Morochnik, and funny narration by University of Toronto professors Donald Ivey and Patterson Hume is a wonderful example of the fun a creative team of filmmakers can have with a subject that other, less imaginative types might find pedestrian. Producer: Richard Leacock Production Company: Educational Development Corp. Sponsor: Eric Prestamon

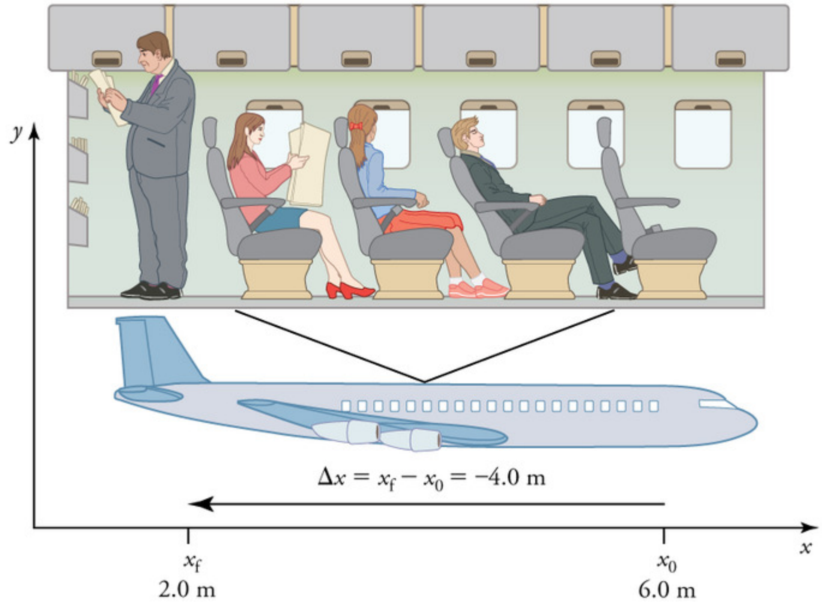
Displacement

Displacement is the change in position of an object relative to its reference frame. For example, if a car moves from a house to a grocery store, its displacement is the relative distance of the grocery store to the reference frame, or the house. The word "displacement" implies that an object has moved or has been displaced. Displacement is the change in position of an object and can be represented mathematically as follows:

$$\Delta \mathrm{x} = \mathrm{x_f} - \mathrm{x_0} \tag{5.2.1.1}$$

where Δx is displacement, x_f is the final position, and x_0 is the initial position.

shows the importance of using a frame of reference when describing the displacement of a passenger on an airplane.



Displacement in Terms of Frame of Reference: A passenger moves from his seat to the back of the plane. His location relative to the airplane is given by x. The -4.0m displacement of the passenger relative to the plane is represented by an arrow toward the rear of the plane. Notice that the arrow representing his displacement is twice as long as the arrow representing the displacement of the professor (he moves twice as far).

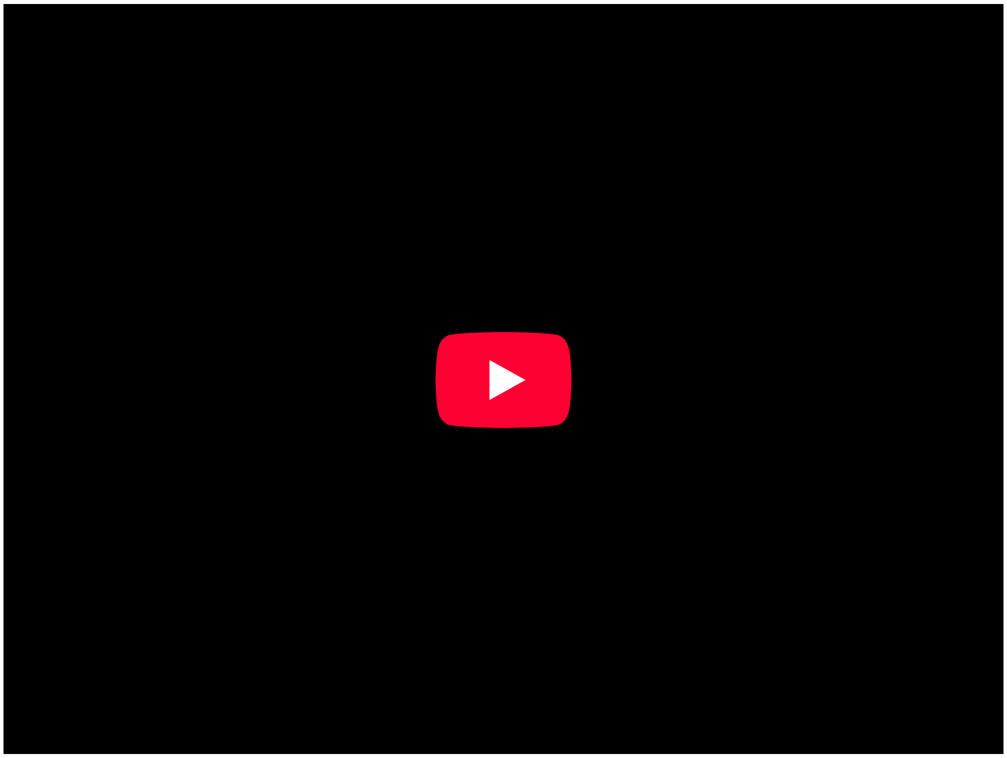
Introduction to Scalars and Vectors

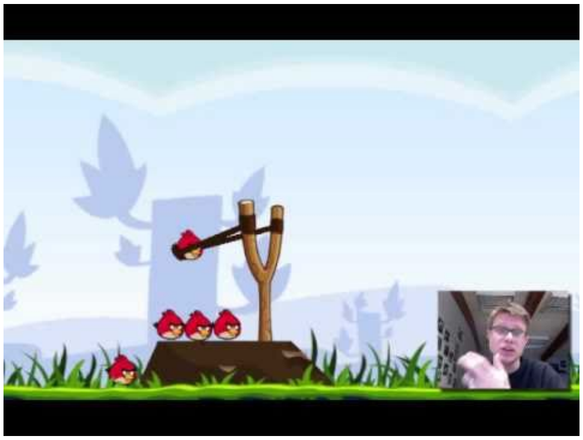
A vector is any quantity that has both magnitude and direction, whereas a scalar has only magnitude.

learning objectives

- Distinguish the difference between scalars and vectors

What is the difference between distance and displacement? Whereas displacement is defined by both direction and magnitude, distance is defined by magnitude alone. Displacement is an example of a vector quantity. Distance is an example of a scalar quantity. A vector is any quantity with both magnitude and direction. Other examples of vectors include a velocity of 90 km/h east and a force of 500 newtons straight down.





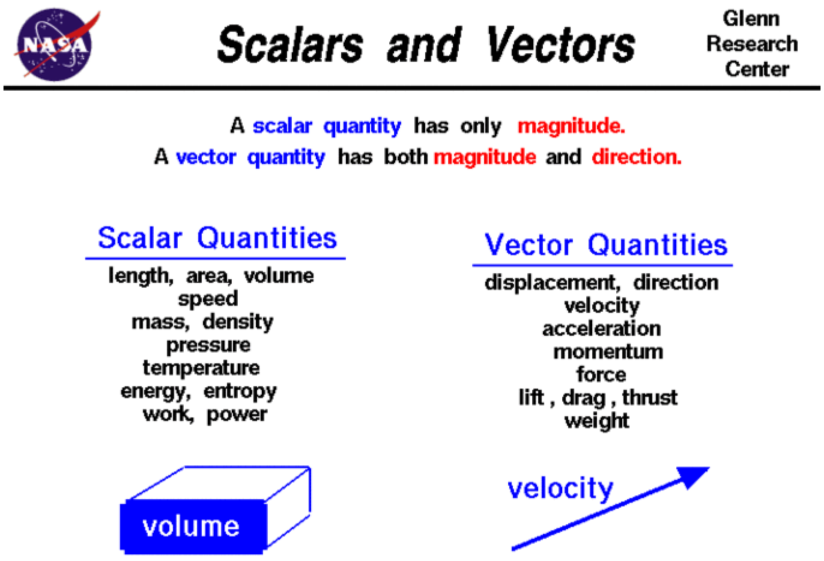
Scalars and Vectors: Mr. Andersen explains the differences between scalar and vectors quantities. He also uses a demonstration to show the importance of vectors and vector addition.

In mathematics, physics, and engineering, a vector is a geometric object that has a magnitude (or length) and direction and can be added to other vectors according to vector algebra. The direction of a vector in one-dimensional motion is given simply by a plus (+) or minus (−) sign. A vector is frequently represented by a line segment with a definite direction, or graphically as an arrow, connecting an initial point A with a terminal point B, as shown in.

$$\overrightarrow{AB}.$$

Vector representation: A vector is frequently represented by a line segment with a definite direction, or graphically as an arrow, connecting an initial point A with a terminal point B.

Some physical quantities, like distance, either have no direction or no specified direction. In physics, a scalar is a simple physical quantity that is not changed by coordinate system rotations or translations. It is any quantity that can be expressed by a single number and has a magnitude, but no direction. For example, a 20ºC temperature, the 250 kilocalories (250 Calories) of energy in a candy bar, a 90 km/h speed limit, a person's 1.8 m height, and a distance of 2.0 m are all scalars, or quantities with no specified direction. Note, however, that a scalar can be negative, such as a −20ºC temperature. In this case, the minus sign indicates a point on a scale rather than a direction. Scalars are never represented by arrows. (A comparison of scalars vs. vectors is shown in.)



Scalars vs. Vectors: A brief list of quantities that are either scalars or vectors.

Key Points

- To describe motion, kinematics studies the trajectories of points, lines and other geometric objects.
- The study of kinematics can be abstracted into purely mathematical expressions.
- Kinematic equations can be used to calculate various aspects of motion such as velocity, acceleration, displacement, and time.
- Choosing a frame of reference requires deciding where the object's initial position is and which direction will be considered positive.
- Valid frames of reference can differ from each other by moving relative to one another.
- Frames of reference are particularly important when describing an object's displacement.
- Displacement is the change in position of an object relative to its reference frame.
- A vector is any quantity that has magnitude and direction.
- A scalar is any quantity that has magnitude but no direction.
- Displacement and velocity are vectors, whereas distance and speed are scalars.

Key Terms

- **kinematics**: The branch of mechanics concerned with objects in motion, but not with the forces involved.
- **displacement**: A vector quantity that denotes distance with a directional component.
- **frame of reference**: A coordinate system or set of axes within which to measure the position, orientation, and other properties of objects in it.
- **scalar**: A quantity that has magnitude but not direction; compare vector.
- **vector**: A directed quantity, one with both magnitude and direction; the between two points.

5.2.2: Speed and Velocity

Average Velocity: A Graphical Interpretation

Average velocity is defined as the change in position (or displacement) over the time of travel.

learning objectives

- Contrast speed and velocity in physics

In everyday usage, the terms "speed" and "velocity" are used interchangeably. In physics, however, they are distinct quantities. Speed is a scalar quantity and has only magnitude. Velocity, on the other hand, is a vector quantity and so has both magnitude and direction. This distinction becomes more apparent when we calculate average speed and velocity.

Average speed is calculated as the distance traveled over the total time of travel. In contrast, average velocity is defined as the change in *position* (or displacement) over the total time of travel.

AVERAGE VELOCITY:

Average velocity is *displacement (change in position) divided by the time of travel,*

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_0}{t_f - t_0},$$

where $\bar{v}$ is the *average* (indicated by the bar over the v) velocity, Δx is the change in position (or displacement), and x_f and x_0 are the final and beginning positions at times t_f and t_0, respectively. If the starting time t_0 is taken to be zero, then the average velocity is simply

$$\bar{v} = \frac{\Delta x}{t}.$$

Average Velocity: The kinematic formula for calculating average velocity is the change in position over the time of travel.

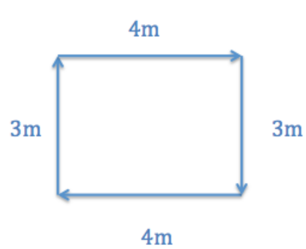
The SI unit for velocity is meters per second, or m/s, but many other units (such as km/h, mph, and cm/s) are commonly used. Suppose, for example, an airplane passenger took five seconds to move -4 m (the negative sign indicates that displacement is toward the back of the plane). His average velocity would be:

$$\mathrm{v} = \frac{\Delta \mathrm{x}}{\mathrm{t}} = \frac{-4\ \mathrm{m}}{5\ \mathrm{s}} = -0.8\frac{\mathrm{m}}{\mathrm{s}} \tag{5.2.2.1}$$

The minus sign indicates that the average velocity is also toward the rear of the plane.

The average velocity of an object does not tell us anything about what happens to it between the starting point and ending point, however. For example, we cannot tell from average velocity whether the airplane passenger stops momentarily or backs up before he gets to the back of the plane. To get more details, we must consider smaller segments of the trip over smaller time intervals.

To illustrate the difference between average speed and average velocity, consider the following additional example. Imagine you are walking in a small rectangle. You walk three meters north, four meters east, three meters south, and another four meters west. The entire walk takes you 30 seconds. If you are calculating average speed, you would calculate the entire distance (3 + 4 + 3 + 4 = 14 meters) over the total time, 30 seconds. From this, you would get an average speed of 14/30 = 0.47 m/s. When calculating average velocity, however, you are looking at the displacement over time. Because you walked in a full rectangle and ended up exactly where you started, your displacement is 0 meters. Therefore, your average velocity, or displacement over time, would be 0 m/s.



Average Speed vs. Average Velocity: If you started walking from one corner and went all the way around the rectangle in 30 seconds, your average speed would be 0.47 m/s, but your average velocity would be 0 m/s.

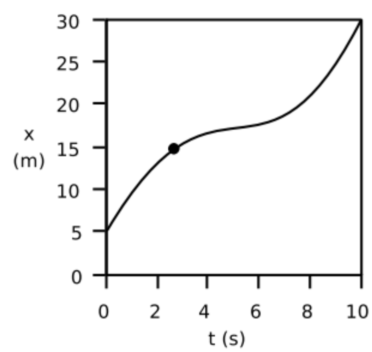
Instananeous Velocity: A Graphical Interpretation

Instantaneous velocity is the velocity of an object at a single point in time and space as calculated by the slope of the tangent line.

learning objectives

- Differentiate instantaneous velocity from other ways of determining velocity

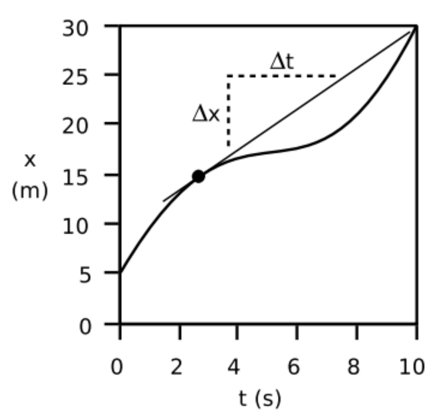
Typically, motion is not with constant velocity nor speed. While driving in a car, for example, we continuously speed up and slow down. A graphical representation of our motion in terms of distance vs. time, therefore, would be more variable or "curvy" rather than a straight line, indicating motion with a constant velocity as shown below. (We limit our discussion to one dimensional motion. It should be straightforward to generalize to three dimensional cases.)



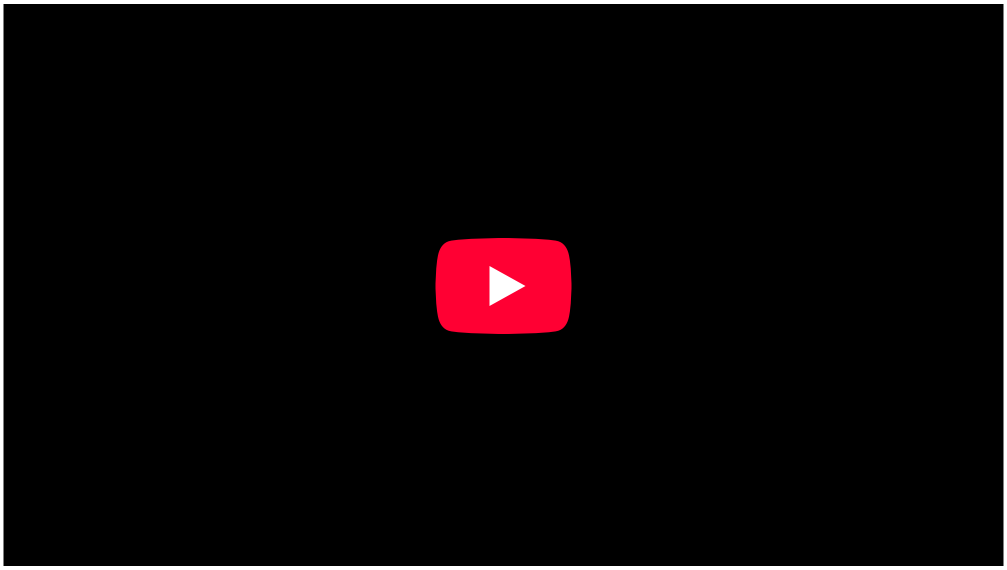
Motion with Changing Velocity: Motion is often observed with changing velocity. This would result in a curvy line when graphed with distance over time.

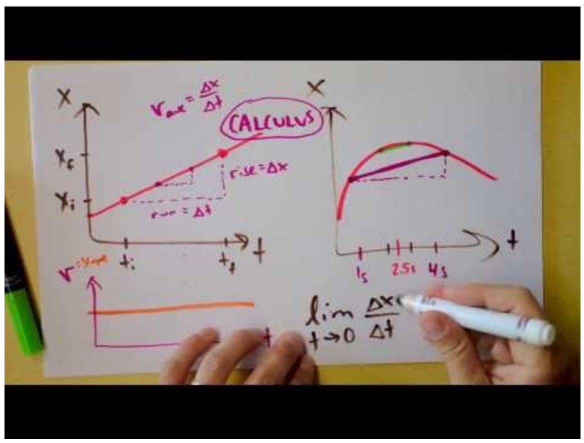
To calculate the speed of an object from a graph representing constant velocity, all that is needed is to find the slope of the line; this would indicate the change in distance over the change in time. However, changing velocity it is not as straightforward.

Since our velocity is constantly changing, we can estimate velocity in different ways. One way is to look at our instantaneous velocity, represented by one point on our curvy line of motion graphed with distance vs. time. In order to determine our velocity at any given moment, we must determine the slope at that point. To do this, we find a line that represents our velocity in that moment, shown graphically in. That line would be the line tangent to the curve at that point. If we extend this line, we can easily calculate the displacement of distance over time and determine our velocity at that given point. The velocity of an object at any given moment is the slope of the tangent line through the relevant point on its x vs. t graph.



Determining instantaneous velocity: The velocity at any given moment is defined as the slope of the tangent line through the relevant point on the graph





Instantaneous Velocity, Acceleration, Jerk, Slopes, Graphs vs. Time: This is how kinematics begins.

In calculus, finding the slope of curve $f(x)$ at $x = x_0$ x=x$_0$ is equivalent to finding the first derivative:

$$\frac{df(x)}{dx}|_{x=x_0}. \tag{5.2.2.2}$$

One interpretation of this definition is that the velocity shows how many meters the object would travel in one second if it continues moving at the same speed for at least one second.

Key Points

- Average velocity can be calculated by determining the total displacement divided by the total time of travel.
- The average velocity of an object does not tell us anything about what happens to it between the starting point and ending point.
- Average velocity is different from average speed in that it considers the direction of travel and the overall change in position.
- When velocity is constantly changing, we can estimate our velocity by looking at instantaneous velocity.
- Instantaneous velocity is calculated by determining the slope of the line tangent to the curve at the point of interest.
- Instantaneous velocity is similar to determining how many meters the object would travel in one second at a specific moment.

Key Terms

- **velocity**: A vector quantity that denotes the rate of change of position with respect to time, or a speed with a directional component.
- **instantaneous**: (As in velocity)—occurring, arising, or functioning without any delay; happening within an imperceptibly brief period of time.

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5.2.3: Acceleration

Graphical Interpretation

The graphical representation of acceleration over time can be derived through the graph of an object's position over time.

learning objectives

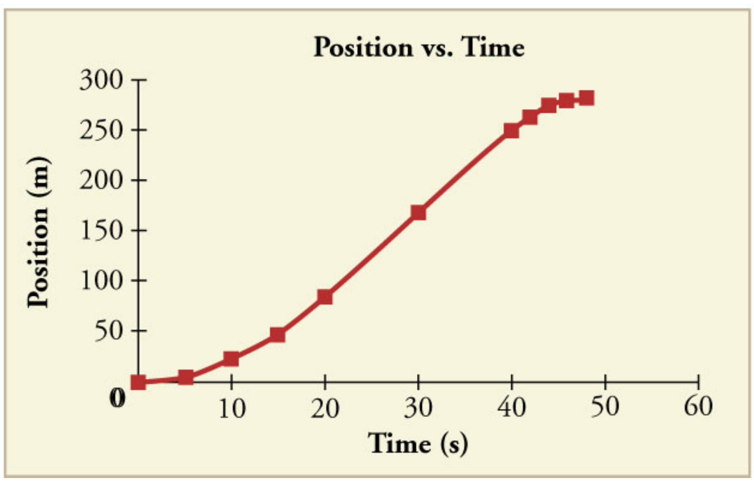
- Distinguish the difference between how to plot a velocity graph and how to plot an acceleration graph

In physics, acceleration is the rate at which the velocity of a body changes with time. It is a vector quantity with both magnitude and direction. Acceleration is accompanied by a force, as described by Newton's Second Law; the force, as a vector, is the product of the mass of the object being accelerated and the acceleration (vector), or $\mathrm{F} = \mathrm{ma}$. The SI unit of acceleration is the meter per second squared: $\frac{\mathrm{m}}{\mathrm{s}^2}$

Acceleration is a vector that points in the same direction as the change in velocity, though it may not always be in the direction of motion. For example, when an object slows down, or decelerating, its acceleration is in the opposite direction of its motion.

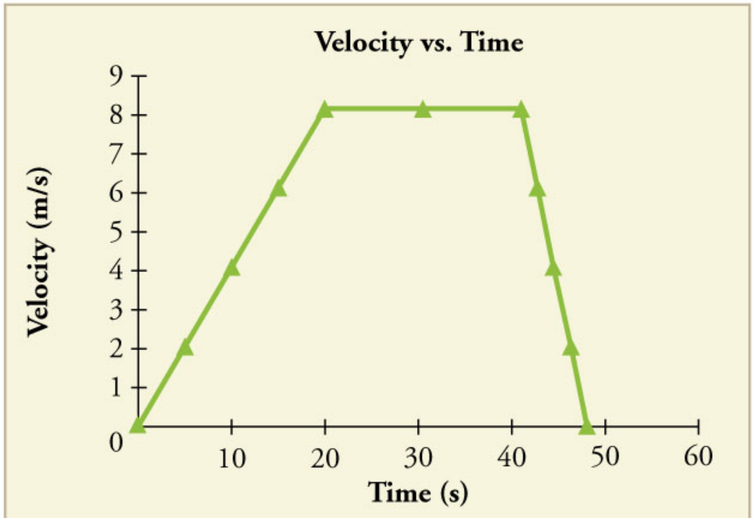
The motion of an object can be depicted graphically by plotting the position of an object over time. This distance-time graph can be used to create another graph that shows changes in velocity over time. Because acceleration is velocity in $\frac{\mathrm{m}}{\mathrm{s}}$ divided by time in s, we can further derive a graph of acceleration from a graph of an object's speed or position.

is a graph of an object's position over time. This graph is similar to the motion of a car. In the beginning, the object's position changes slowly as it gains speed. In the middle, the speed is constant and the position changes at a constant rate. As it slows down toward the end, the position changes more slowly. From this graph, we can derive a velocity vs time graph.



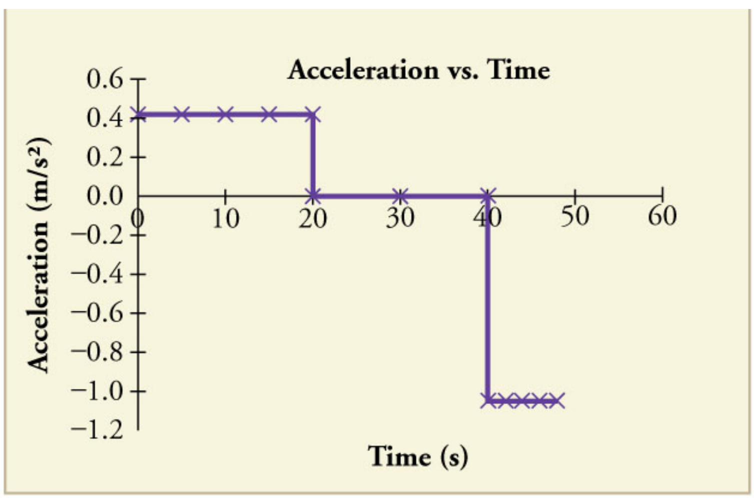
Position vs Time Graph: Notice that the object's position changes slowly at the beginning of the journey, then more and more quickly as it picks up speed. Its position then changes more slowly as it slows down at the end of the journey. In the middle of the journey, while the velocity remains constant, the position changes at a constant rate.

This shows the velocity of the object over time. The object's velocity increases in the beginning as it accelerates at the beginning, then remains constant in the middle before it slows down toward the end. Notice that this graph is a representation of the slope of the previous position vs time graph. From this graph, we can further derive an acceleration vs time graph.

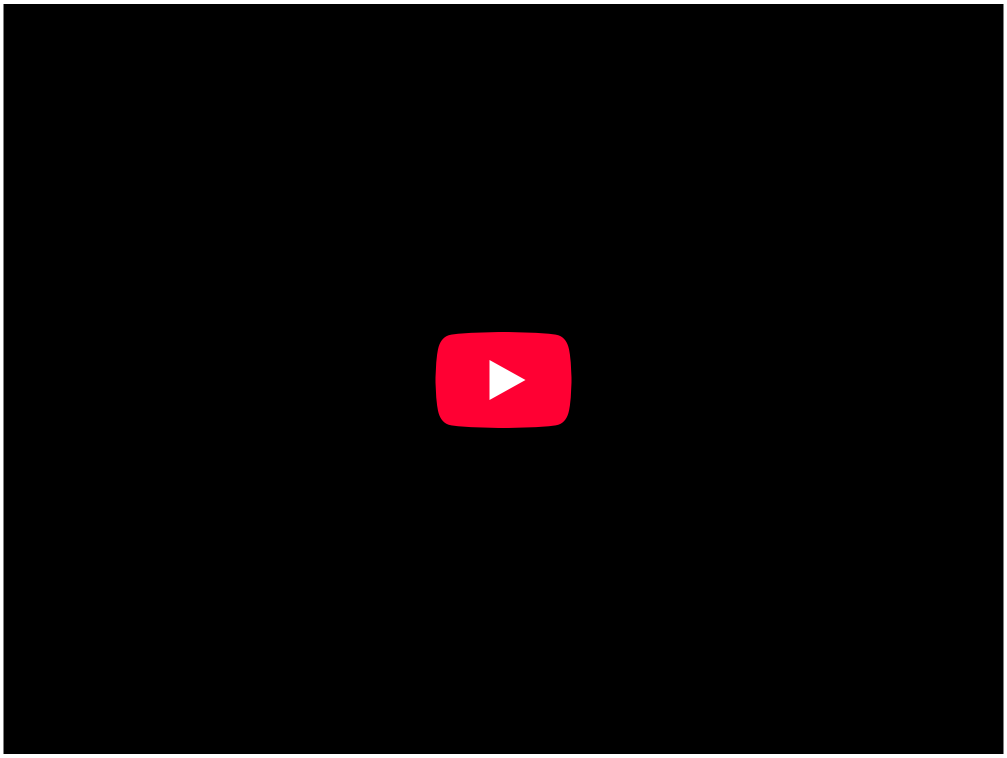


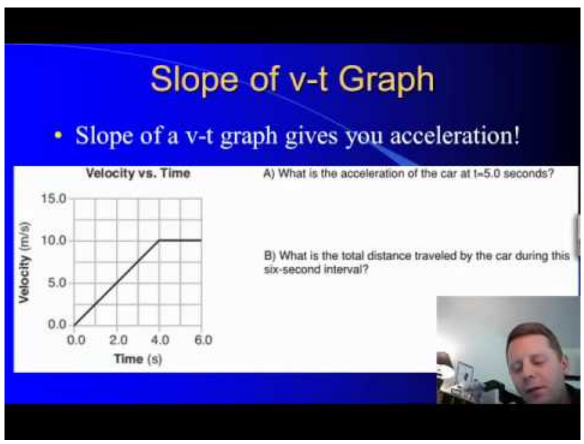
Velocity vs Time: The object's velocity increases as it accelerates at the beginning of the journey. It remains the same in the middle of the journey (where there is no acceleration). It decreases as the object decelerates at the end of the journey.

To do this, we would also plot the slope of the velocity vs time graph. In this graph, the acceleration is constant in the three different stages of motion. As we noted earlier, the object is increasing speed and changing positions slowly in the beginning. The acceleration graph shows that the object was increasing at a positive constant acceleration during this time. In the middle, when the object was changing position at a constant velocity, the acceleration was 0. This is because the object is no longer changing its velocity and is moving at a constant rate. Towards the end of the motion, the object slows down. This is depicted as a negative value on the acceleration graph. Note that in this example, the motion of the object is still forward (positive), but since it is decelerating, the acceleration is negative.



Acceleration vs Time Graph: The object has positive acceleration as it speeds up at the beginning of the journey. It has no acceleration as it travels at constant velocity in the middle of the journey. Its acceleration is negative as it slows down at the end of the journey.





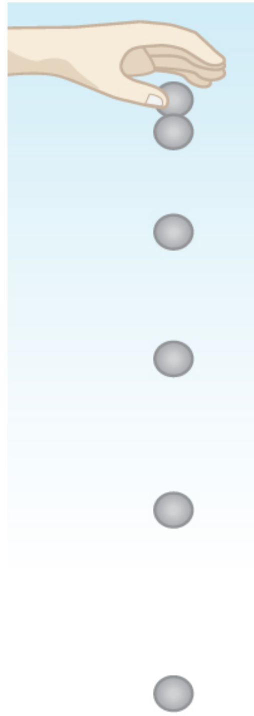
Graphing Motion: A brief introduction to particle diagrams and motion graphs.

Motion with Constant Acceleration

Constant acceleration occurs when an object’s velocity changes by an equal amount in every equal time period.

learning objectives

- Describe how constant acceleration affects the motion of an object



One-Dimensional Motion: When you drop an object, it falls vertically toward the center of the earth due to the constant acceleration of gravity.

An object experiencing constant acceleration has a velocity that increases or decreases by an equal amount for any constant period of time. Acceleration can be derived easily from basic kinematic principles. It is defined as the first time derivative of velocity (so the second derivative of position with respect to time):

$$a=\frac{\partial v}{\partial t}=\frac{\partial^2 x}{\partial t^2} \tag{5.2.3.1}$$

Assuming acceleration to be constant does not seriously limit the situations we can study and does not degrade the accuracy of our treatment, because in a great number of situations, acceleration *is* constant. When it is not, we can either consider it in separate parts of constant acceleration or use an average acceleration over a period of time.

The motion of falling objects is a simple, one-dimensional type of projectile motion in which there is no horizontal movement. For example, if you held a rock out and dropped it, the rock would fall only vertically downward toward the earth. If you were to throw the rock instead of just dropping it, it would follow a more projectile-like pattern, similar to the one a kicked ball follows.

Projectile motion is the motion of an object thrown or projected into the air and is subject only to the acceleration of gravity. The object thrown is called a projectile, and the object's path is called its trajectory. In two-dimensional projectile motion, there is both a vertical and a horizontal component.

Due to the algebraic properties of constant acceleration, there are kinematic equations that relate displacement, initial velocity, final velocity, acceleration, and time. A summary of these equations is given below.

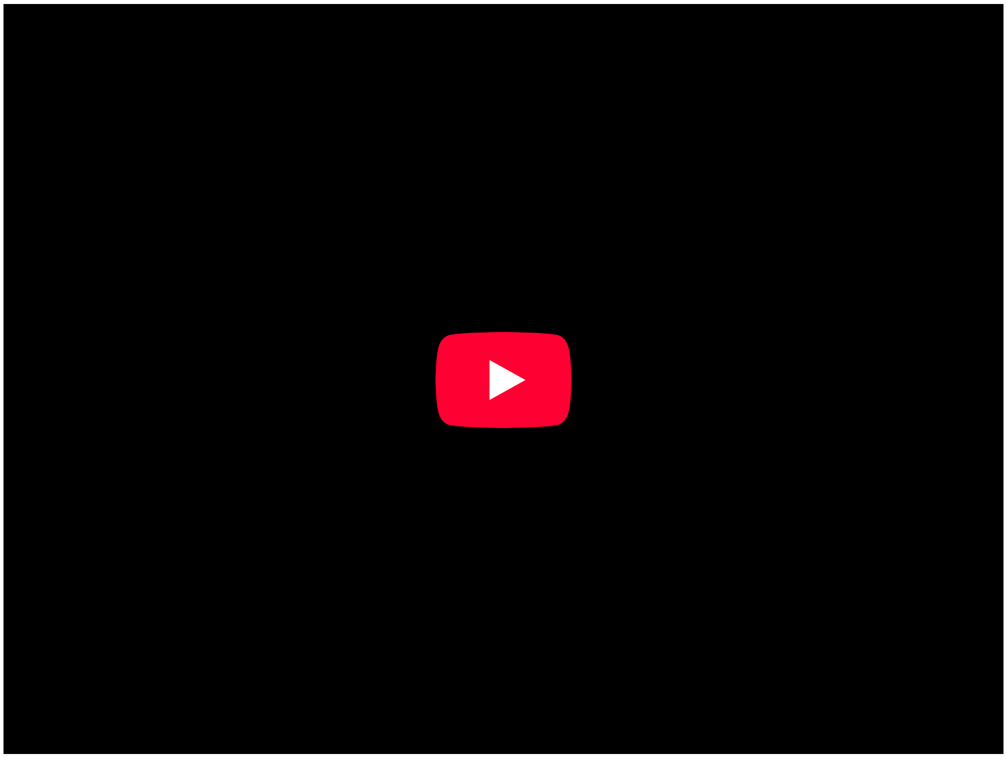
$$x=x_0+\bar{v}t \tag{5.2.3.2}$$

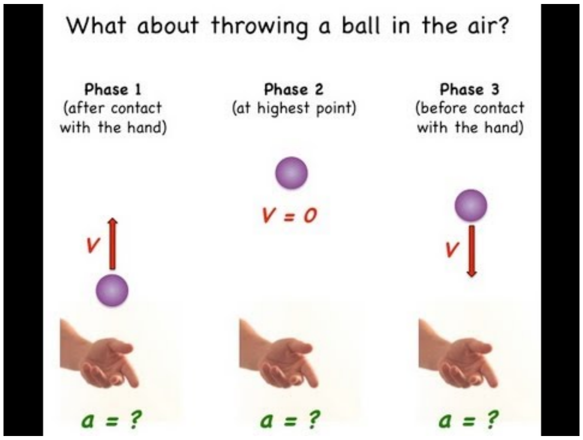
$$\bar{v}=\frac{v_0+v}{2} \tag{5.2.3.3}$$

$$v=v_0+at \tag{5.2.3.4}$$

$$x=x_0+v_0t+\frac{1}{2}at^2 \tag{5.2.3.5}$$

$$v^2=v_0^2+2a(x-x_0) \tag{5.2.3.6}$$





Constant Acceleration Explained with Vectors and Algebra: This video answers the question “what is acceleration? ".

Key Points

- Acceleration is the rate at which the velocity of a body changes with time.
- Acceleration is a vector that points in the same direction as the change in velocity, though it may not always be in the direction of motion.
- Because acceleration is velocity in m/s divided by time in s, we can derive a graph of acceleration from a graph of an object’s speed or position.
- Assuming acceleration to be constant does not seriously limit the situations we can study and does not degrade the accuracy of our treatment.
- Due to the algebraic properties of constant acceleration, there are kinematic equations that can be used to calculate displacement, velocity, acceleration, and time.
- Calculations with constant acceleration can be done in relation to one-dimensional motion as well as two-dimensional motion.

Key Terms

- **acceleration**: The amount by which a speed or velocity increases (and so a scalar quantity or a vector quantity).
- **velocity**: A vector quantity that denotes the rate of change of position with respect to time, or a speed with a directional component.
- **position**: A place or location.
- **kinematic**: of or relating to motion or kinematics

5.2.4: Problem-Solving for Basic Kinematics

Applications

There are four kinematic equations that describe the motion of objects without consideration of its causes.

learning objectives

- Choose which kinematics equation to use in problems in which the initial starting position is equal to zero

Kinematics is the branch of classical mechanics that describes the motion of points, bodies (objects), and systems of bodies (groups of objects) without consideration of the causes of motion. There are four kinematic equations when the initial starting position is the origin, and the acceleration is constant:

1. $\mathrm{v=v_0+at}$
2. $\mathrm{d=\frac{1}{2}(v_0+v)t}$ or alternatively $\mathrm{v_{average}=\frac{d}{t}}$
3. $\mathrm{d=v_0t+(\frac{at^2}{2})}$
4. $\mathrm{v^2=v_0^2+2ad}$

Notice that the four kinematic equations involve five kinematic variables: $\mathrm{d, v, v_0, a}$ and t. Each of these equations contains only four of the five variables and has a different one missing. This tells us that we need the values of three variables to obtain the value of the fourth and we need to choose the equation that contains the three known variables and one unknown variable for each specific situation.

Here the basic problem solving steps to use these equations:

Step one – Identify exactly what needs to be determined in the problem (identify the unknowns).

Step two – Find an equation or set of equations that can help you solve the problem.

Step three – Substitute the knowns along with their units into the appropriate equation, and obtain numerical solutions complete with units.

Step four – Check the answer to see if it is reasonable: Does it make sense?

Problem-solving skills are obviously essential to success in a quantitative course in physics. More importantly, the ability to apply broad physical principles, usually represented by equations, to specific situations is a very powerful form of knowledge. It is much more powerful than memorizing a list of facts. Analytical skills and problem-solving abilities can be applied to new situations, whereas a list of facts cannot be made long enough to contain every possible circumstance. Such analytical skills are useful both for solving problems in a physics class and for applying physics in everyday and professional life.

Motion Diagrams

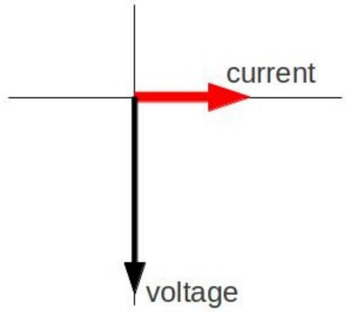
A motion diagram is a pictorial description of an object's motion and represents the position of an object at equally spaced time intervals.

learning objectives

- Construct a motion diagram

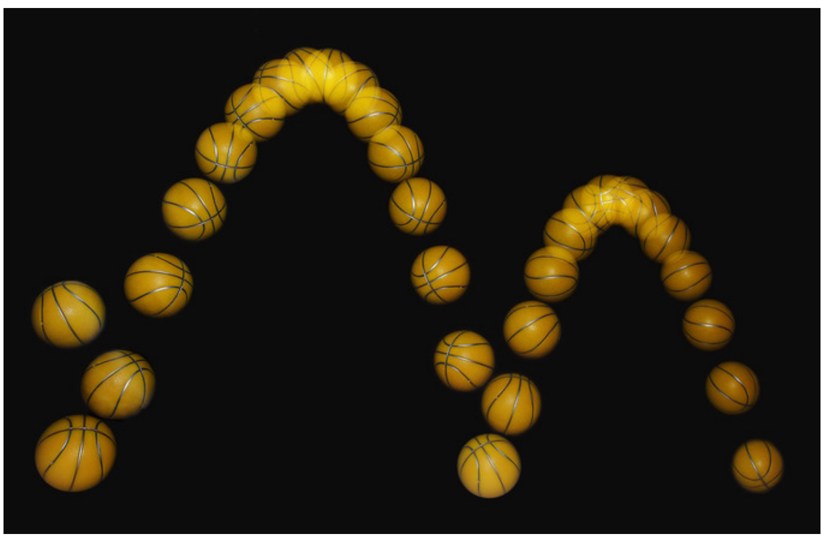
A motion diagram is a pictorial description of the motion of an object. It displays the object's location at various equally spaced times on the same diagram; shows an object's initial position and velocity; and presents several spots in the center of the diagram. These spots reveal whether or not the object has accelerated or decelerated. For simplicity, the object is represented by a simple shape, such as a filled circle, which contains information about an object's position at particular time instances. For this reason, a motion diagram is more information than a path diagram. It may also display the forces acting on the object at each time instance.

is a motion diagram of a simple trajectory. Imagine the object as a hockey puck sliding on ice. Notice that the puck covers the same distance per unit interval along the trajectory. We can conclude that the puck is moving at a constant velocity and, therefore, there is no acceleration or deceleration during the motion.



Puck Sliding on Ice: Motion diagram of a puck sliding on ice. The puck is moving at a constant velocity.

One major use of motion diagrams is the presentation of film through a series of frames taken by a camera; this is sometimes called stroboscopic technique (as seen in). Viewing an object on a motion diagram allows one to determine whether an object is speeding up or slowing down, or if it is at constant rest. As the frames are taken, we can assume that an object is at a constant rest if it occupies the same position over time. We can assume that an object is speeding up if there is a visible increase in the space between objects as time passes, and that it is slowing down if there is a visible decrease in the space between objects as time passes. The objects on the frame come very close together.



Bouncing Ball: A bouncing ball captured with a stroboscopic flash at 25 images per second.

Key Points

- The four kinematic equations involve five kinematic variables: d, v, v_0, a and t.
- Each equation contains only four of the five variables and has a different one missing.
- It is important to choose the equation that contains the three known variables and one unknown variable for each specific situation.
- Motion diagrams represent the motion of an object by displaying its location at various equally spaced times on the same diagram.
- Motion diagrams show an object's initial position and velocity and presents several spots in the center of the diagram. These spots reveal the object's state of motion.
- Motion diagrams contain information about an object's position at particular time instances and is therefore more informative than a path diagram.

Key Terms

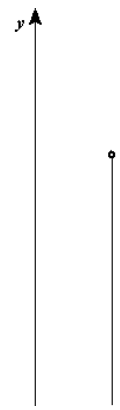
- **kinematics**: The branch of physics concerned with objects in motion.
- **stroboscopic**: Relating to an instrument used to make a cyclically moving object appear to be slow-moving, or stationary.
- **diagram**: A graph or chart.
- **motion**: A change of position with respect to time.

5.2.5: Free-Falling Objects

learning objectives

- Solve basic problems concerning free fall and distinguish it from other kinds of motion

The motion of falling objects is the simplest and most common example of motion with changing velocity. If a coin and a piece of paper are simultaneously dropped side by side, the paper takes much longer to hit the ground. However, if you crumple the paper into a compact ball and drop the items again, it will look like both the coin and the paper hit the floor simultaneously. This is because the amount of force acting on an object is a function of not only its mass, but also area. Free fall is the motion of a body where its weight is the only force acting on an object.



Free Fall: This clip shows an object in free fall.

Galileo also observed this phenomena and realized that it disagreed with the Aristotle principle that heavier items fall more quickly. Galileo then hypothesized that there is an upward force exerted by air in addition to the downward force of gravity. If air resistance and friction are negligible, then in a given location (because gravity changes with location), all objects fall toward the center of Earth with the *same constant acceleration, independent of their mass*, that constant acceleration is gravity. Air resistance opposes the motion of an object through the air, while friction opposes motion between objects and the medium through which they are traveling. The acceleration of free-falling objects is referred to as the acceleration due to gravity gg. As we said earlier, gravity varies depending on location and altitude on Earth (or any other planet), but the average acceleration due to gravity on Earth is 9.8 $\frac{m}{s^2}$. This value is also often expressed as a negative acceleration in mathematical calculations due to the downward direction of gravity.

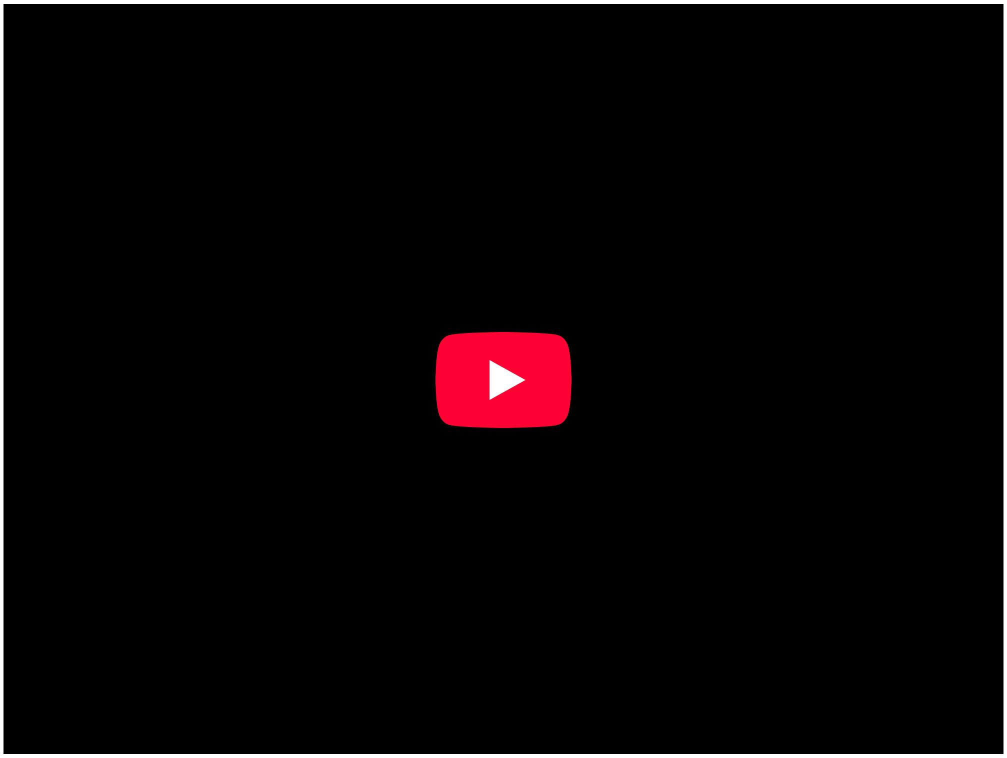
The best way to see the basic features of motion involving gravity is to start by considering straight up and down motion with no air resistance or friction. This means that if the object is dropped, we know the initial velocity is zero. Once the object is in motion, the object is in free-fall. Under these circumstances, the motion is one-dimensional and has constant acceleration, gg. The kinematic equations for objects experiencing free fall are:

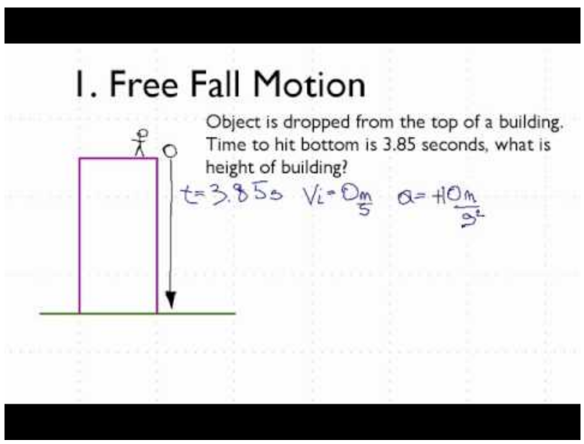
$$\mathrm{v} = \mathrm{v_0} - \mathrm{gt} \tag{5.2.5.1}$$

$$\mathrm{y} = \mathrm{y_0} + \mathrm{v_0 t} - \frac{1}{2}\mathrm{gt^2} \tag{5.2.5.2}$$

$$\mathrm{v^2} = \mathrm{v_0^2} - 2\mathrm{g}(\mathrm{y} - \mathrm{y_0}), \tag{5.2.5.3}$$

where $\mathrm{v = velocity, g = gravity, t = time,\ and\ y = vertical\ displacement.}$





Video $\mathbf{5.2.5.1}$ *: Free Fall Motion - Describes how to calculate the time for an object to fall if given the height and the height that an object fell if given the time to fall.*

Example $5.2.5.1$:

Some examples of objects that are in free fall include:

- A spacecraft in continuous orbit. The free fall would end once the propulsion devices turned on.
- An stone dropped down an empty well.
- An object, in projectile motion, on its descent.

Key Points

- The acceleration of free-falling objects is called the acceleration due to gravity, since objects are pulled towards the center of the earth.
- The acceleration due to gravity is constant on the surface of the Earth and has the value of 9.80 $\frac{\mathrm{m}}{\mathrm{s}^2}$.

Glossary

Acceleration

The amount by which a speed or velocity changes within a certain period of time (and so a scalar quantity or a vector quantity).

CHAPTER OVERVIEW

5.3: The Laws of Motion

Topic hierarchy

5.3.1: Introduction

Newton and His Laws

There are three laws of motion that describe the relationship between forces, mass, and acceleration.

learning objectives

- Apply three Newton's laws of motion to relate forces, mass, and acceleration

Newton's laws of motion describe the relationship between the forces acting on a body and its motion due to those forces. For example, if your car breaks down and you need to push it, you must exert a force with your hands on the car in order for it to move. The laws of motion will tell you how quickly the car will move from your pushing. There are three laws of motion:

First law: If an object experiences no net force, then its velocity is constant: the object is either at rest (if its velocity is zero), or it moves in a straight line with constant speed (if its velocity is nonzero). For example, if you don't push the car (no force), then it doesn't move.

Second law: The acceleration aa of a body is parallel and directly proportional to the net force F acting on the body, is in the direction of the net force, and is inversely proportional to the mass mm of the body:

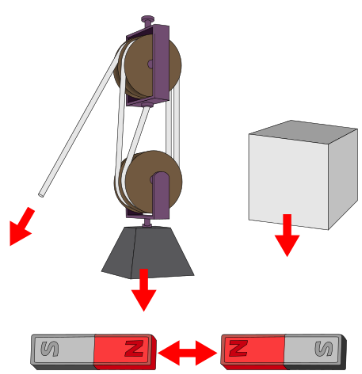
$$\mathrm{F}=\mathrm{m}\cdot\mathrm{a}\ \mathrm{or}\ \mathrm{a}=\frac{\mathrm{F}}{\mathrm{m}} \tag{5.3.1.1}$$

For example, if you push the car with a greater force it will accelerate more. But, if the car is more massive (mm is larger) then it won't accelerate as much from the same size force as a lighter car.

Third law: When a first body exerts a force F_1 on a second body, the second body simultaneously exerts a force $\mathrm{F}_2=-\mathrm{F}_1$ on the first body. This means that F_1 and F_2 are equal in magnitude and opposite in direction. For example, when you push a car, if it is exerting the same force on you that you are exerting on it, you might wonder why you don't move backwards? The answer is there are also forces from the ground on your feet pushing you forward. So, in fact, the car is pushing a force back on you that is of the same magnitude that you are using to push it forward.

In the figure below there are some practical examples illustrating the concept of force:

- Strain: by using a machine known as pulley you can easily raise or lower a massive body
- Gravitational Force: a massive body is attracted downward by the gravitational force practiced by the Earth
- Magnetic Force: two magnets repel each other when the same poles get closer



Examples of Force: Some situations in which forces are at play.

Key Points

- Acceleration of an object is proportional to the force on it.
- Force causes an object to move.
- Objects with more mass require more force to move.

Key Terms

- **force**: Any influence that causes an object to undergo a certain change, either concerning its movement, direction or geometrical construction.

5.3.2: Force and Mass

Force

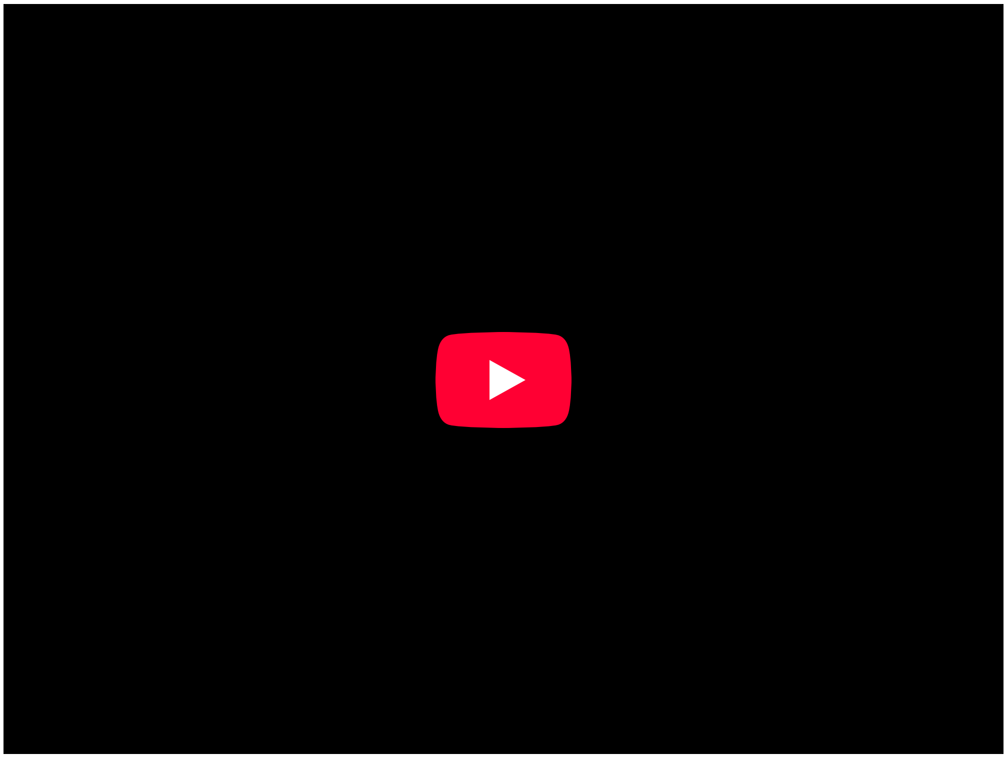
Force is any influence that causes an object to change, either concerning its movement, direction, or geometrical construction.

learning objectives

- Develop the relationship between mass and acceleration in determining force

Overview of Forces

In physics, a force is any influence that causes an object to undergo a certain change, either concerning its movement, direction, or geometrical construction. It is measured with the SI unit of Newtons. A force is that which can cause an object with mass to change its velocity, i.e., to accelerate, or which can cause a flexible object to deform. Force can also be described by intuitive concepts such as a push or pull. A force has both magnitude and direction, making it a vector quantity.



What is a force?: Describes what forces are and what they do.

Qualities of Force

The original form of Newton's second law states that the net force acting upon an object is equal to the rate at which its momentum changes. This law is further given to mean that the acceleration of an object is directly proportional to the net force acting on the object, is in the direction of the net force, and is inversely proportional to the mass of the object.

As we mentioned, force is a vector quantity. A vector is a one dimensional array with elements of both magnitude and direction. In a force vector, the mass, m, is the magnitude component and the acceleration, a, is the directional component. The equation for force is written:

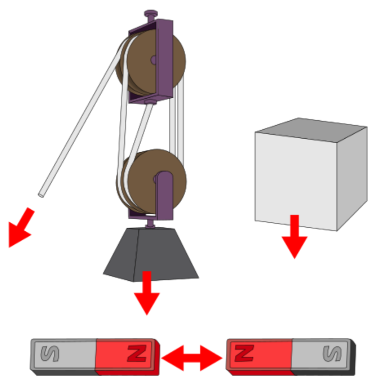
$$\mathrm{F} = \mathrm{m} \cdot \mathrm{a} \tag{5.3.2.1}$$

Related concepts to force include thrust, which increases the velocity of an object; drag, which decreases the velocity of an object; and torque which produces changes in rotational speed of an object. Forces which do not act uniformly on all parts of a body will also cause mechanical stresses, a technical term for influences which cause deformation of matter. While mechanical stress can remain embedded in a solid object, gradually deforming it, mechanical stress in a fluid determines changes in its pressure and volume.

Dynamics

Dynamics is the study of the forces that cause objects and systems to move. To understand this, we need a working definition of force. Our intuitive definition of force — that is, a push or a pull —is a good place to start. We know that a push or pull has both magnitude and direction (therefore, it is a vector quantity) and can vary considerably in each regard.

shows a few examples of the "push-pull" nature of force. The top left example is that of a pulley system. The force that someone would have to pull down on the cable would have to equal and exceed the force made by the mass the object and the effects of gravity on those object in order for the system to move up. The top right example shows that any object resting on a surface will still exert force on that surface. The bottom example is that of two magnets being attracted to each other due to magnetic force.



Examples of Force: Some situations in which forces are at play.

Mass

Mass is a physical property of matter that depends on size and shape of matter, and is expressed as kilograms by the SI system.

learning objectives

- Justify the significance of understanding mass in physics

What is Mass?

All elements have physical properties whose values can help describe an elements physical state. Changes to these properties can describe elemental transformations. Physical properties do not change the chemical nature of matter. The physical property we are covering in this atom is called mass.

Mass is defined as a quantitative measure of an object's resistance to acceleration. The terms mass and weight are often interchanged, however it is incorrect to do so. Weight is a different property of matter that, while related to mass, is not mass, but rather the amount of gravitational force acting on a given body of matter. Mass is an intrinsic property that never changes.

Units of Mass

In order to measure something, a standard value must be established to use in relation to the object of measurement. This relation is called a unit. The International System of Units (SI) measures mass in kilograms, or kg. There are other units of mass, including the following (only the first two are accepted by the SI system):

- t – Tonne; 1t = 1000kg
- u – atomic mass unit; 1u ~= 1.66×10^{-27}kg
- sl – slug
- lb – pound

Concepts Using Mass

- Weight – see
- Newtons Second Law – mass has a central role in determining the behavior of bodies. Newtons Second Law relates force f, exerted in a body of mass m, to the body's acceleration a: $\mathrm{F}=\mathrm{ma}$
- Momentum – mass relates a body's momentum, p, to its linear velocity, v: $\mathrm{p}=\mathrm{mv}$
- Kinetic Energy – mass relates kinetic energy, K to velocity, v: $\mathrm{K}=\frac{1}{2}\mathrm{m}|\mathrm{v}^2|$

Key Points

- Force is stated as a vector quantity, meaning it has elements of both magnitude and direction. Mass and acceleration respectively.
- In layman's terms, force is a push or pull that can be defined in terms of various standards.
- Dynamics is the study of the force that causes objects and systems to move or deform.
- External forces are any outside forces that act on a body, and internal forces are any force acting within a body.
- Mass is defined as a quantitative measure of an object's resistance to acceleration.
- According to Newton's second law of motion, if a body of fixed mass m is subjected to a single force F, its acceleration a is given by F/m.
- Mass is central in many concepts of physics, including:weight, momentum, acceleration, and kinetic energy.
- According to Newton's second law of motion, if a body of fixed mass m is subjected to a single force F, its acceleration a is given by F/m.

Key Terms

- **force**: A force is any influence that causes an object to undergo a certain change, either concerning its movement, direction or geometrical construction.
- **velocity**: A vector quantity that denotes the rate of change of position with respect to time, or a speed with a directional component.
- **vector**: A directed quantity, one with both magnitude and direction; the between two points.
- **mass**: The quantity of matter which a body contains, irrespective of its bulk or volume. It is one of four fundamental properties of matter. It is measured in kilograms in the SI system of measurement.

- File:Force examples.svg - Wikipedia, the free encyclopedia. **Provided by**: Wikipedia. **Located at**: **en.Wikipedia.org/w/index.php?title=File:Force_examples.svg&page=1**. **License**: *CC BY-SA: Attribution-ShareAlike*
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5.3.3: Newton’s Laws

The First Law: Inertia

Newton’s first law of motion describes inertia. According to this law, a body at rest tends to stay at rest, and a body in motion tends to stay in motion, unless acted on by a net external force.

learning objectives

- Define the First Law of Motion

History

Sir Isaac Newton was an English scientist who was interested in the motion of objects under various conditions. In 1687, he published a work called *Philosophiae Naturalis Principla Mathematica*, which described his three laws of motion. Newton used these laws to explain and explore the motion of physical objects and systems. These laws form the basis for mechanics. The laws describe the relationship between forces acting on a body and the motions experienced due to these forces. The three laws are as follows:

1. If an object experiences no net force, its velocity will remain constant. The object is either at rest and the velocity is zero or it moves in a straight line with a constant speed.
2. The acceleration of an object is parallel and directly proportional to the net force acting on the object, is in the direction of the net force, and is inversely proportional to the mass of the object.
3. When a first object exerts a force on a second object, the second object simultaneously exerts a force on the first object, meaning that the force of the first object and the force of the second object are equal in magnitude and opposite in direction.

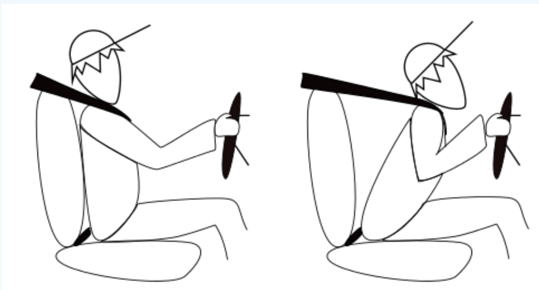
The First Law of Motion

You have most likely heard Newton’s first law of motion before. If you haven’t heard it in the form written above, you have probably heard that “a body in motion stays in motion, and a body at rest stays at rest.” This means that an object that is in motion will not change its velocity unless an unbalanced force acts upon it. This is called uniform motion. It is easier to explain this concept through examples.

Example $5.3.3.1$:

If you are ice skating, and you push yourself away from the side of the rink, according to Newton’s first law you will continue all the way to the other side of the rink. But, this won’t actually happen. Newton says that a body in motion will stay in motion until an outside force acts upon it. In this and most other real world cases, this outside force is friction. The friction between your ice skates and the ice is what causes you to slow down and eventually stop.

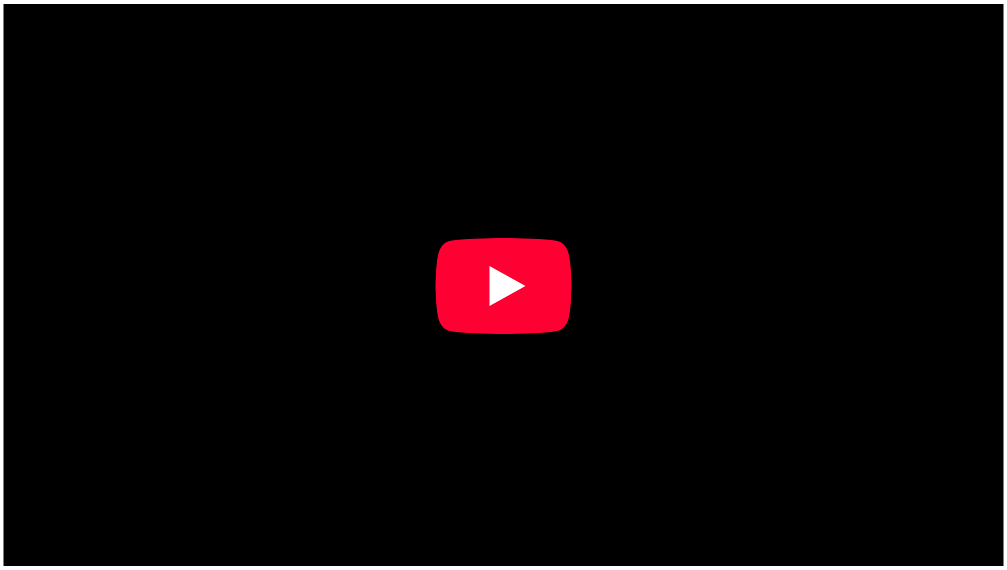
Let’s look at another situation. Refer to for this example. Why do we wear seat belts? Obviously, they’re there to protect us from injury in case of a car accident. If a car is traveling at 60 mph, the driver is also traveling at 60 mph. When the car suddenly stops, an external force is applied to the car that causes it to slow down. But there is no force acting on the driver, so the driver continues to travel at 60 mph. The seat belt is there to counteract this and act as that external force to slow the driver down along with the car, preventing them from being harmed.

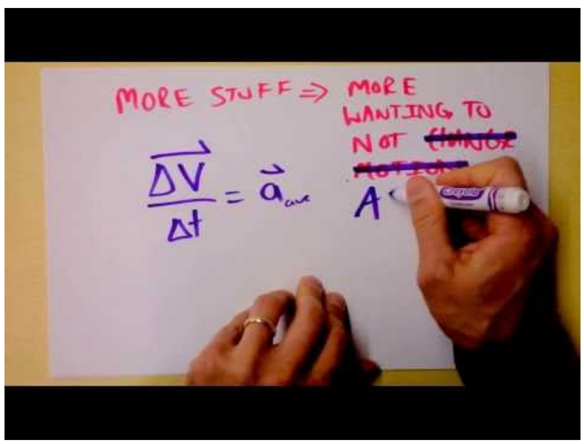


Newton’s First Law: Newton’s first law in effect on the driver of a car

Inertia

Sometimes this first law of motion is referred to as the law of inertia. Inertia is the property of a body to remain at rest or to remain in motion with constant velocity. Some objects have more inertia than others because the inertia of an object is equivalent to its mass. This is why it is more difficult to change the direction of a boulder than a baseball.





Doc Physics – Newton: Newton’s first law is hugely counterintuitive. You may have learned it in gradeschool, though. Let’s see it for the mind-blowing conclusion it really is.

The Second Law: Force and Acceleration

The second law states that the net force on an object is equal to the rate of change, or derivative, of its linear momentum.

learning objectives

- Define the Second Law of Motion

English scientist Sir Isaac Newton examined the motion of physical objects and systems under various conditions. In 1687, he published his three laws of motion in *Philosophiae Naturalis Principla Mathematica*. The laws form the basis for mechanics—they describe the relationship between forces acting on a body, and the motion experienced due to these forces. These three laws state:

1. If an object experiences no net force, its velocity will remain constant. The object is either at rest and the velocity is zero, or it moves in a straight line with a constant speed.
2. The acceleration of an object is parallel and directly proportional to the net force acting on the object, is in the direction of the net force and is inversely proportional to the mass of the object.

3. When a first object exerts a force on a second object, the second object simultaneously exerts a force on the first object, meaning that the force of the first object and the force of the second object are equal in magnitude and opposite in direction.

The first law of motion defines only the natural state of the motion of the body (i.e., when the net force is zero). It does not allow us to quantify the force and acceleration of a body. The acceleration is the rate of change in velocity; it is caused only by an external force acting on it. The second law of motion states that the net force on an object is equal to the rate of change of its linear momentum.

Linear Momentum

Linear momentum of an object is a vector quantity that has both magnitude and direction. It is the product of mass and velocity of a particle at a given time:

$$\mathrm{p=mv} \tag{5.3.3.1}$$

where, $\mathrm{p = momentum, m = mass,\ and\ v = velocity}$. From this equation, we see that objects with more mass will have more momentum.

The Second Law of Motion

Picture two balls of different mass, traveling in the same direction at the same velocity. If they both collide with a wall at the same time, the heavier ball will exert a larger force on the wall. This concept, illustrated below, explains Newton’s second law, which emphasizes the importance of force and motion, over velocity alone. It states: the net force on an object is equal to the rate of change of its linear momentum. From calculus we know that the rate of change is the same as a derivative. When we the linear momentum of an object we get:

Force and Mass: This animation demonstrates the connection between force and mass.

$$\mathrm{F} = \frac{\mathrm{dp}}{\mathrm{dt}} \tag{5.3.3.2}$$

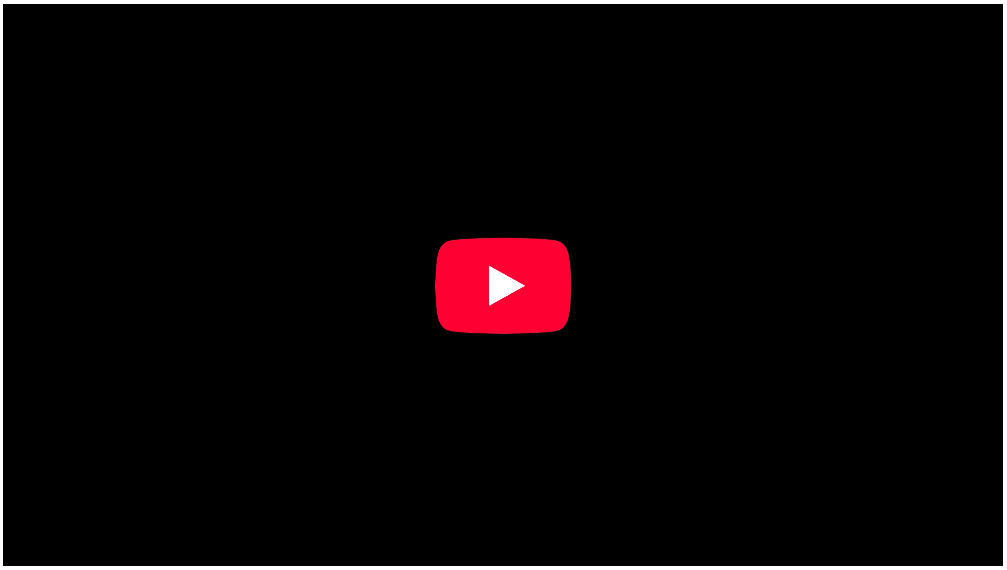
$$\mathrm{F} = \frac{\mathrm{d(m \cdot v)}}{\mathrm{dt}} \tag{5.3.3.3}$$

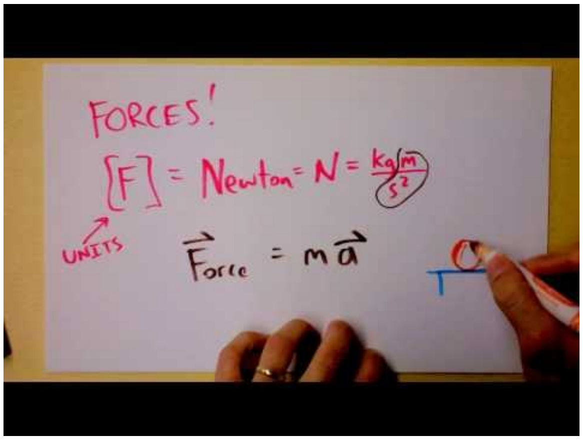
where, $\mathrm{F = Force\ and\ t = time}$. From this we can further simplify the equation:

$$\mathrm{F} = \mathrm{m}\frac{\mathrm{d(v)}}{\mathrm{dt}} \tag{5.3.3.4}$$

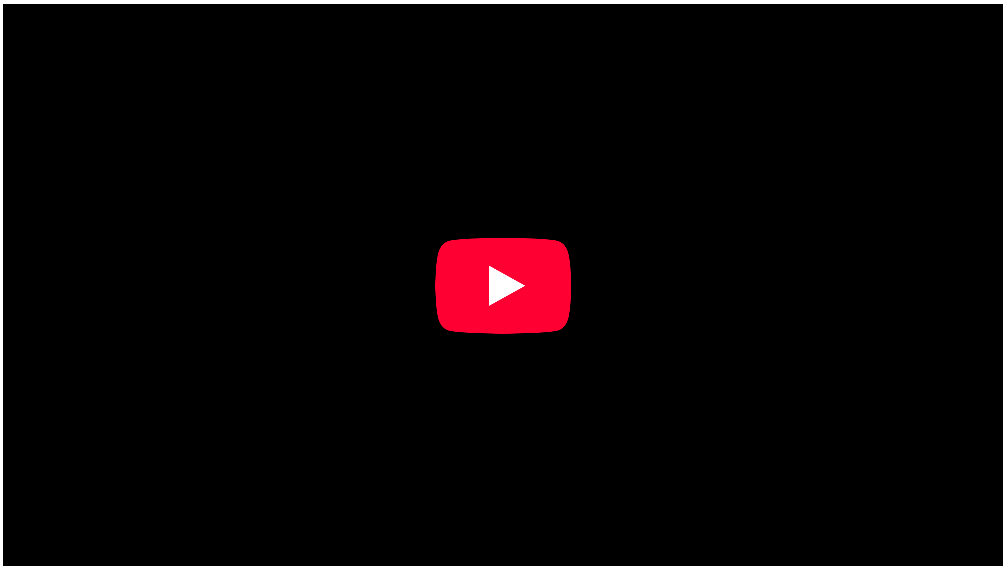
$$\mathrm{F} = \mathrm{m \cdot a} \tag{5.3.3.5}$$

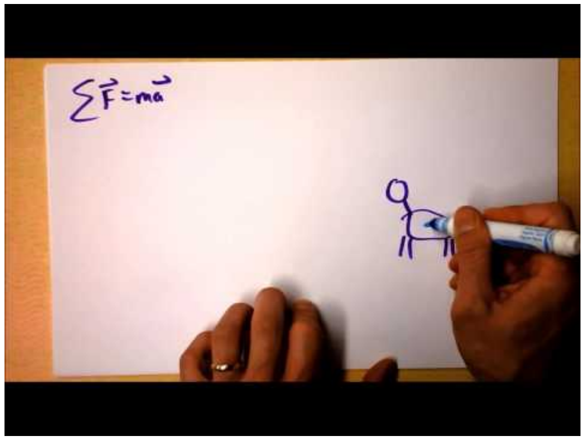
where, a=accelerationa=acceleration. As we stated earlier, acceleration is the rate of change of velocity, or velocity divided by time.





Newton’s Three Laws of Mechanics – Second Law – Part 1: Here we’ll see how many people can confuse your understanding of Newton’s 2nd law of motion through oversight, sloppy language, or cruel intentions.





Newton’s Three Laws of Mechanics – Second Law – Part Two: Equilibrium is investigated and Newton’s 1st law is seen as a special case of Newton’s 2nd law!

The Third Law: Symmetry in Forces

The third law of motion states that for every action, there is an equal and opposite reaction.

learning objectives

- Define the Third Law of Motion

Sir Isaac Newton was a scientist from England who was interested in the motion of objects under various conditions. In 1687, he published a work called *Philosophiae Naturalis Principla Mathematica*, which contained his three laws of motion. Newton used these laws to explain and explore the motion of physical objects and systems. These laws form the bases for mechanics. The laws describe the relationship between forces acting on a body, and the motion is an experience due to these forces. Newton’s three laws are:

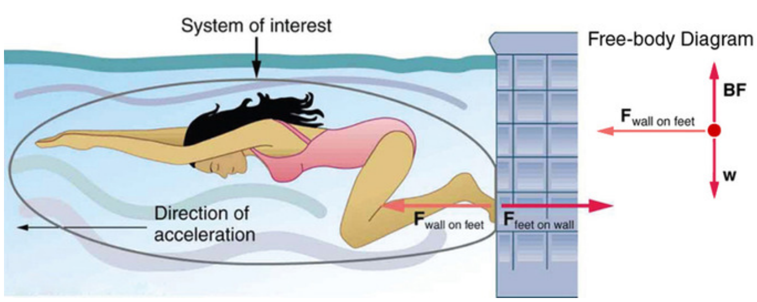
1. If an object experiences no net force, its velocity will remain constant. The object is either at rest and the velocity is zero or it moves in a straight line with a constant speed.
2. The acceleration of an object is parallel and directly proportional to the net force acting on the object, is in the direction of the net force and is inversely proportional to the mass of the object.
3. When a first object exerts a force on a second object, the second object simultaneously exerts a force on the first object, meaning that the force of the first object and the force of the second object are equal in magnitude and opposite in direction.

Newton’s Third Law of Motion

Newton’s third law basically states that for every action, there is an equal and opposite reaction. If object A exerts a force on object B, because of the law of symmetry, object B will exert a force on object A that is equal to the force acted on it:

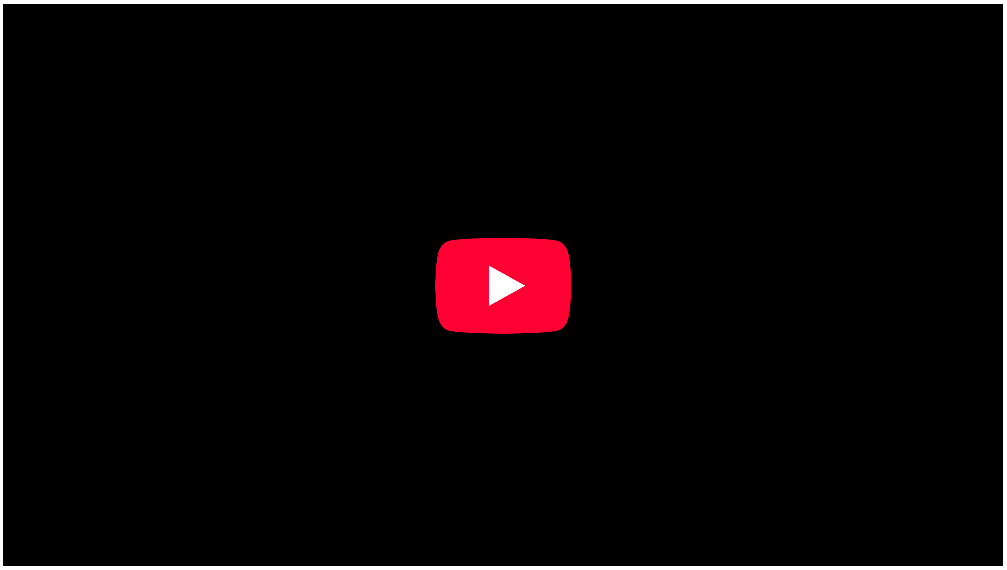
$$FA = -FB \tag{5.3.3.6}$$

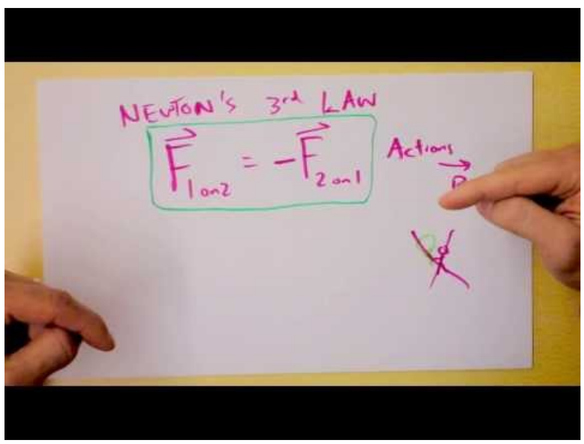
In this example, F_A is the action and F_B is the reaction. You have undoubtedly witnessed this law of motion. For example, take a swimmer who uses her feet to push off the wall in order to gain speed. The more force she exerts on the wall, the harder she pushes off. This is because the wall exerts the same force on her that she forces on it. She pushes the wall in the direction behind her, therefore the wall will exert a force on her that is in the direction in front of her and propel her forward.



Newton’s Third Law of Motion: When a swimmer pushes off the wall, the swimmer is using the third law of motion.

Take as another example, the concept of thrust. When a rocket launches into outer space, it expels gas backward at a high velocity. The rocket exerts a large backward force on the gas, and the gas exerts and equal and opposite reaction force forward on the rocket, causing it to launch. This force is called thrust. Thrust is used in cars and planes as well.





Newton’s Third Law: The most fundamental statement of basic physical reality is also the most often misunderstood. As your mom if she’s clear on Newton’s Third. Then ask her why things can move if every force has a paired opposite force all the time, forever.

Key Points

- Newton’s three laws of physics are the basis for mechanics.
- The first law states that a body at rest will stay at rest until a net external force acts upon it and that a body in motion will remain in motion at a constant velocity until acted on by a net external force.
- Net external force is the sum of all of the forcing acting on an object.
- Just because there are forces acting on an object doesn’t necessarily mean that there is a net external force; forces that are equal in magnitude but acting in opposite directions can cancel one another out.
- Friction is the force between an object in motion and the surface on which it moves. Friction is the external force that acts on objects and causes them to slow down when no other external force acts upon them.
- Inertia is the tendency of a body in motion to remain in motion. Inertia is dependent on mass, which is why it is harder to change the direction of a heavy body in motion than it is to change the direction of a lighter object in motion.
- Newton’s three laws of motion explain the relationship between forces acting on an object and the motion they experience due to these forces. These laws act as the basis for mechanics.

- The second law explains the relationship between force and motion, as opposed to velocity and motion. It uses the concept of linear momentum to do this.
- Linear momentum p, is the product of mass m, and velocity v : $p = mv$.
- The second law states that the net force is equal to the derivative, or rate of change of its linear momentum.
- By simplifying this relationship and remembering that acceleration is the rate of change of velocity, we can see that the second law of motion is where the relationship between force and acceleration comes from.
- If an object A exerts a force on object B, object B exerts an equal and opposite force on object A.
- Newton's third law can be seen in many everyday circumstances. When you walk, the force you use to push off the ground backwards makes you move forward.
- Thrust is an application of the third law of motion. A helicopter uses thrust to push the air under the propeller down, and therefore lift off the ground.

Key Terms

- **inertia**: The property of a body that resists any change to its uniform motion; equivalent to its mass.
- **friction**: A force that resists the relative motion or tendency to such motion of two bodies in contact.
- **uniform motion**: Motion at a constant velocity (with zero acceleration). Note that an object in motion will not change its velocity unless an unbalanced force acts upon it.
- **net force**: The combination of all the forces that act on an object.
- **momentum**: (of a body in motion) the product of its mass and velocity.
- **acceleration**: The amount by which a speed or velocity increases (and so a scalar quantity or a vector quantity).
- **symmetry**: Exact correspondence on either side of a dividing line, plane, center or axis.
- **thrust**: The force generated by propulsion, as in a jet engine.

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5.3.4: Other Examples of Forces

Weight

Weight is taken as the force on an object due to gravity, and is different than the mass of an object.

learning objectives

- Infer what factors other than gravity will contribute to the apparent weight of an object

In physics, it is important to differentiate the weight of an object from its mass. The mass of an object is an intrinsic quantity, independent of the location of the object. On the other hand, the weight of an object is an extrinsic quantity. It is considered as the force on an object due to gravity. Since gravitational acceleration changes depending on the location in the universe, weight does as well.

Mathematically, the weight of an object (W) can be found by multiplying its mass (m) by the acceleration due to gravity (g): $\mathrm{W} = \mathrm{M} \cdot \mathrm{g}$. The strength of gravity varies very little over the surface of the Earth. In fact, the greatest percent difference in the value of the acceleration due to gravity on Earth is 0.5%.

For most calculations involving the weight of an object on Earth, it is sufficient to assume that $\mathrm{g} = 9.8 \frac{\mathrm{m}}{\mathrm{s}^2}$.

The weight of an object has the same SI unit as force—the Newton ($1\mathrm{N} = 1\mathrm{kg} \cdot \frac{\mathrm{m}}{\mathrm{s}^2}$).

In US customary units, the weight of an object can be expressed in pounds. Keep in mind that in US units the pound is either a unit of force or of mass. If one must find the weight (as opposed to the mass) of an object in US units, it can be calculated in terms of pounds of force.

It is important to note that the apparent weight of an object (i.e., the weight of an object determined by a scale) will vary if forces other than gravity are acting upon the object. For example, if you weigh a given mass underwater you will find a different result than if you weigh that mass in air. In this case, the weight of the object varies due to the force of buoyancy. While the mass is in the water it displaces fluid, resulting in an upward force upon it. This upward force affects the net force that the mass exerts on the scale, and thus alters its "apparent" weight.

Spring Scale: A spring scale measures weight by finding the extent to which a spring is compressed.This is proportional to the force that a mass exerts on the scale due to its weight.

Normal Forces

The normal force comes about when an object contacts a surface; the resulting force is always perpendicular to the surface of contact.

learning objectives

- Evaluate Newton's Second and Third Laws in determining the normal force on an object

Overview

The normal force, $\mathrm{F_N}$, comes about when an object contacts a surface. According to Newton's third law, when one object exerts a force on a second object, the second object always exerts a force that is equal in magnitude and opposite in direction on the first object. This is the reason that the normal force exists.

A common situation in which a normal force exists is when a person stands on the ground. Because of Newton's third law, the ground exerts a force on the person that is equal in magnitude to the person's weight. In this simple case, the weight of the person and the opposing normal force are the only two forces considered on the person. The person remains still because the forces due to weight and the normal force create a net force of zero on the person.

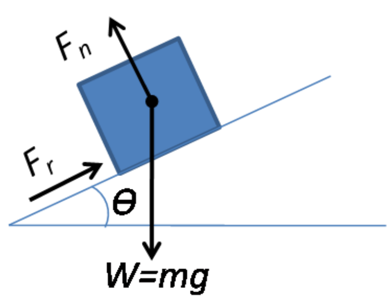
Forces on Inclined Planes

A more complex example of a situation in which a normal force exists is when a mass rests on an inclined plane. In this case, the normal force is not in the exact opposite direction as the force due to the weight of the mass. This is because the mass contacts the surface at an angle. By taking this angle into account, the magnitude of the normal force ($\mathrm{F_N}$) can be found from:

$$\mathrm{F_N} = \mathrm{mg}\cos(\theta), \tag{5.3.4.1}$$

where:

- m is the mass under consideration,
- g is the acceleration due to gravity,
- and θ is the angle between the inclined surface and the horizontal.



Inclined Plane: A mass rests on an inclined plane that is at an angle θ to the horizontal. The following forces act on the mass: the weight of the mass $(\mathrm{m} \cdot \mathrm{g})$,the force due to friction (Fr),and the normal force (Fn).

Another interesting example involving normal forces is when a person stands in an elevator. When the elevator goes up, the normal force is actually greater than the force due to gravity. In this situation there are only two forces acting on the person. The first is the force of gravity on the person, which does not change. The second is the normal force. By summing the forces and setting them equal to m⋅am⋅a (utilizing Newton's second law), we find:

$$\mathrm{F_N} - \mathrm{m} \cdot \mathrm{g} = \mathrm{m} \cdot \mathrm{a} \tag{5.3.4.2}$$

where:

- $\mathrm{F_N}$ is the normal force,
- $\mathrm{m} \cdot \mathrm{g}$ is the force due to gravity,
- m is the mass of the person,
- and a is the acceleration.

Since acceleration is positive, the normal force must actually be greater than the force due to gravity (the weight of the person).

Key Points

- Weight is taken to be the force on an object due to gravity.
- Weight and mass are not the same thing!
- The weight of a given mass will be different when the acceleration due to gravity is different.
- Apparent weight can change because of the effect of buoyancy.
- The strength of gravity is almost the same everywhere on the surface of the Earth.
- The normal force, F_N, comes about when an object contacts a surface.
- The normal force exists because for every force, there is always an equal and opposite force.
- The normal force is always perpendicular to the plane that the object contacts or rests on.

Key Terms

- **Gravitational acceleration**: Gravitational acceleration is the acceleration that an object undergoes due solely to gravity
- **perpendicular**: at or forming a right angle (to).
- **normal**: A line or vector that is perpendicular to another line, surface, or plane.

5.3.5: Problem-Solving

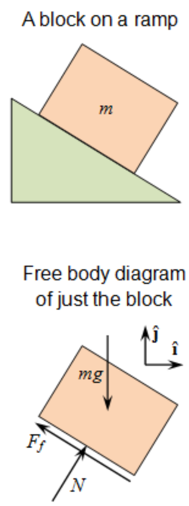
A General Approach

Basic problem-solving techniques can aid in the solution of problems involving motion (i.e., the laws of motion).

learning objectives

- Assess the laws of motion through practiced problem solving techniques

When dealing with the laws of motion, although knowledge of concepts and equations is important, understanding basic problem solving techniques can simplify the process of solving problems that may appear difficult. Your approach to problem solving can involve several key steps.



Free body diagram: An example of a drawing to help identify forces and directions.

First, gather all relevant information from the problem. Identify all quantities that are given (the *knowns*), then do the same for all quantities needed (the *unknowns*). Also, identify the physical principles involved (e.g., force, gravity, friction, etc.).

Next, a drawing may be helpful. Sometimes a drawing can even help determine the known and unknown quantities. It need not be a work of art, but it should be clear enough to illustrate proper dimension, (meaning one, two, or three dimensions). You can then use this drawing to determine which direction is positive and which is negative (making note of this on the drawing).

A next step is to use what is known to find the appropriate equation to find what is unknown. While it is easiest to find an equation that leaves only one unknown, sometimes this is not possible. In these situations, you can solve multiple equations to find the right answer. Remember that equations represent physical principles and relationships, so use the equations and drawings in tandem.

You may then substitute the knowns into the appropriate equations and find a numerical solution.

Check the answer to see if it is reasonable and makes sense. Your judgment will improve and fine tune as you solve more problems of this nature. This “judgement” step helps intuit the problem in terms of its conceptual meaning. If you can judge whether the answer is reasonable, you have a deeper understanding of physics than simply the mechanics of problem solving.

When solving problems, we tend to perform these steps in different order, as well as do several steps simultaneously. There is no rigid procedure that will work every time. Creativity and insight grow with experience. In time, the basics of problem solving can become relatively automatic.

Key Points

- Gathering all relevant information and identifying knowns and unknowns is an important first step.
- Always make a drawing to help identify directions of forces and to establish x, y, and z axes.
- Choose the correct equations, solve the problem, and check that the answer fits expectations numerically.

Key Terms

- **equation**: An assertion that two expressions are equal, expressed by writing the two expressions separated by an equal sign; from which one is to determine a particular quantity.

5.3.6: Vector Nature of Forces

Forces in Two Dimensions

Forces act in a particular direction and have sizes dependent upon how strong the push or pull is.

learning objectives

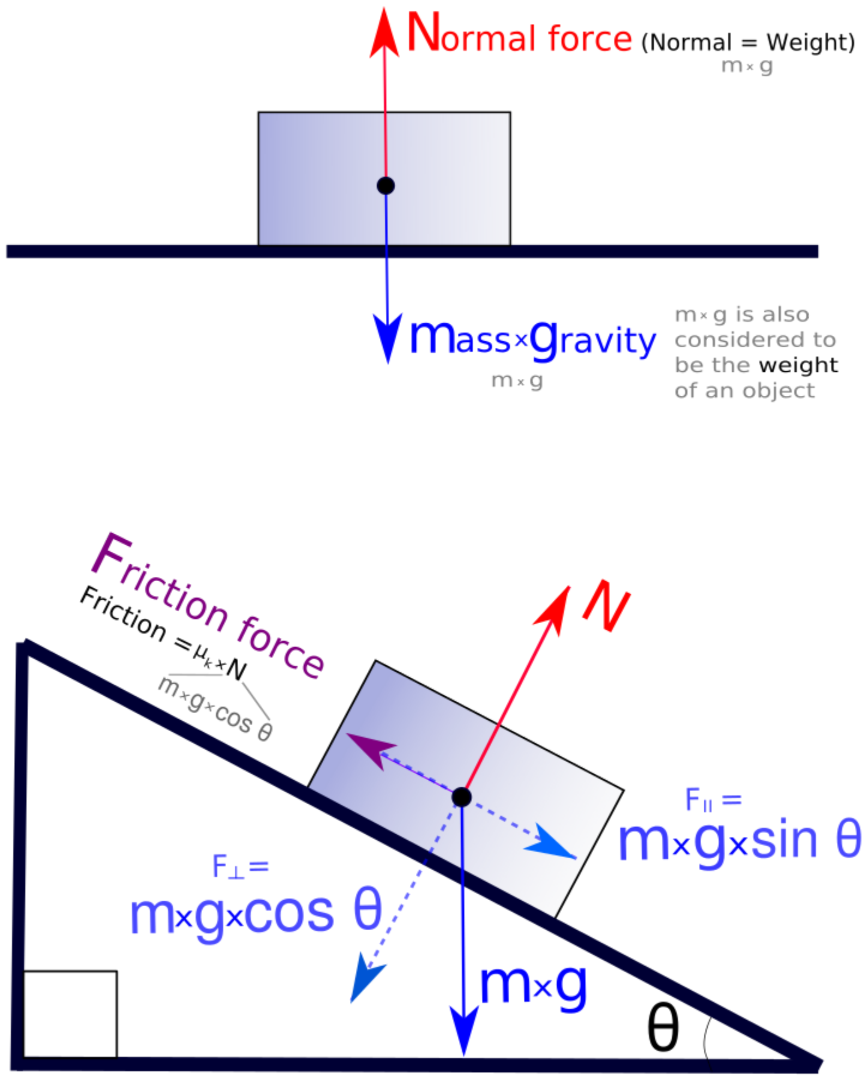
- Explain why forces are classified as “vector quantities”

Forces act in a particular direction and have sizes dependent upon how strong the push or pull is. Because of these characteristics, forces are classified as “vector quantities. ” This means that forces follow a different set of mathematical rules than physical quantities that do not have direction (denoted scalar quantities).

For example, when determining what happens when two forces act on the same object, it is necessary to know both the magnitude and the direction of both forces to calculate the result. If both of these pieces of information are not known for each force, the situation is ambiguous. For example, if you know that two people are pulling on the same rope with known magnitudes of force but you do not know which direction either person is pulling, it is impossible to determine what the acceleration of the rope will be. The two people could be pulling against each other as in tug of war or the two people could be pulling in the same direction. In this simple one-dimensional example, without knowing the direction of the forces it is impossible to decide whether the net force is the result of adding the two force magnitudes or subtracting one from the other. Associating forces with vectors avoids such problems.

When two forces act on a point particle, the resulting force or the resultant (also called the net force) can be determined by following the parallelogram rule of vector addition: the addition of two vectors represented by sides of a parallelogram gives an equivalent resultant vector which is equal in magnitude and direction to the transversal of the parallelogram. The magnitude of the resultant varies from the difference of the magnitudes of the two forces to their sum, depending on the angle between their lines of action.

Free-body diagrams can be used as a convenient way to keep track of forces acting on a system. Ideally, these diagrams are drawn with the angles and relative magnitudes of the force vectors preserved so that graphical vector addition can be done to determine the net force.



Forces as Vectors: Free-body diagrams of an object on a flat surface and an inclined plane. Forces are resolved and added together to determine their magnitudes and the net force.

Key Points

- When determining what happens when two forces act on the same object, it is necessary to know both the magnitude and the direction of both forces to calculate the result.
- When two forces act on a point particle, the resulting force or the resultant (also called the net force), can be determined by following the parallelogram rule of vector addition.
- Free-body diagrams can be used as a convenient way to keep track of forces acting on an object.

Key Terms

- **vector**: A directed quantity, one with both magnitude and direction; the between two points.
- **free-body diagram**: A free body diagram, also called a force diagram, is a pictorial representation often used by physicists and engineers to analyze the forces acting on a body of interest.
- **resultant**: A vector that is the vector sum of multiple vectors

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5.3.7: Further Applications of Newton's Laws

Applications of Newton's Laws

Net force affects the motion, postion and/or shape of objects (some important and commonly used forces are friction, drag and deformation).

learning objectives

- Explain the effect of forces on an object's motion and shape

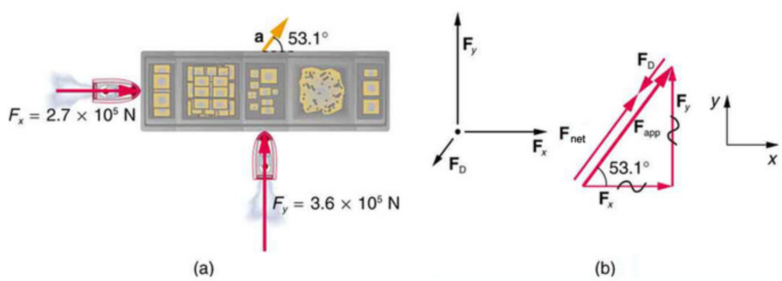
We know that a net force affects the motion, position and shape of an object. It is useful at this point to look at some particularly interesting and common forces that will provide further applications of Newton's laws of motion. Specifically, we will discuss the forces of friction, air or liquid drag, and deformation.

Friction

Friction is a force that resists movement between two surfaces sliding against each other. When surfaces in contact move relative to each other, the friction between the two surfaces converts kinetic energy into heat. This property can have a dramatic effect, as seen in the use of friction created by rubbing pieces of wood together to start a fire. Friction is not itself a fundamental force, but arises from fundamental electromagnetic forces between the charged particles constituting the two contacting surfaces.

Drag

Another interesting force in everyday life is the force of drag on an object when it is moving in a fluid (either gas or liquid). You feel this drag force when you move your hand through water, or through the wind. Like friction, the force of drag is a force that resists motion. As we will discuss in later units, the drag force is proportional to the velocity of the object moving through it. We see an illustrated example of drag force in.



Drag Force on a Barge: (a) A view from above of two tugboats pushing on a barge. (b) The free-body diagram for the ship contains only forces acting in the plane of the water. It omits the two vertical forces—the weight of the barge and the buoyant force of the water supporting it cancel and are not shown. Since the applied forces are perpendicular, the x– and y-axes are in the same direction as F_x and F_y. The problem quickly becomes a one-dimensional problem along the direction of F_{app}, since friction is in the direction opposite to F_{app}.

Deformation

We now move from consideration of forces that affect the motion of an object (such as friction and drag) to those that affect an object's shape. If a bulldozer pushes a car into a wall, the car will not move but it will noticeably change shape. The change in shape of an object due to the application of a force is a deformation. Even very small forces are known to cause some deformation. For small deformations, two important characteristics are observed. First, the object returns to its original shape when the force is removed (that is, the deformation is elastic for small deformations). Second, the size of the deformation is proportional to the force.

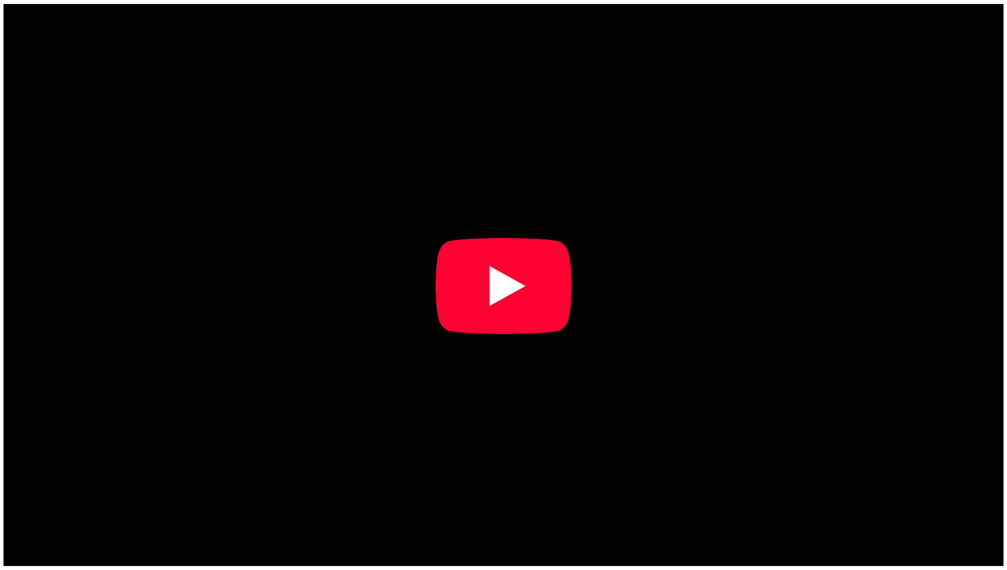
Friction: Kinetic

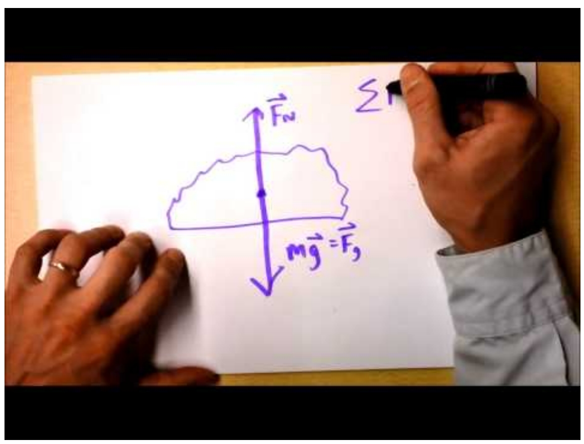
If two systems are in contact and moving relative to one another, then the friction between them is called kinetic friction.

learning objectives

- Explain the dynamics of energy for friction between two surfaces

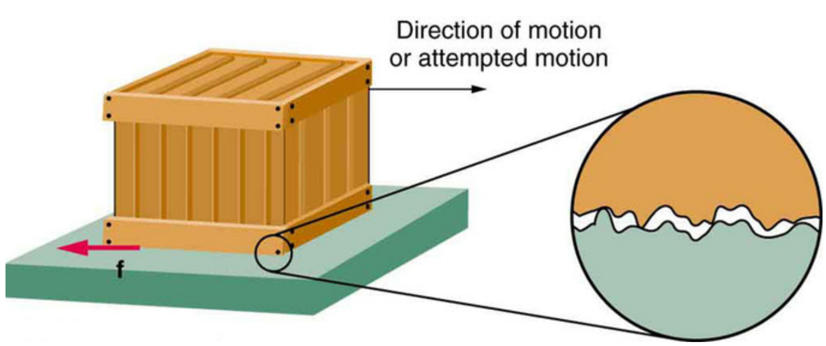
When surfaces in contact move relative to each other, the friction between the two surfaces converts kinetic energy into heat. This property can have dramatic consequences, as illustrated by the use of friction created by rubbing pieces of wood together to start a fire. Kinetic energy is converted to heat whenever motion with friction occurs, for example when a viscous fluid is stirred.





Kinetic Friction Introduction: Here, I'll explain the microscopic justification of friction and what we can know about it. The coefficient of friction, too!

Kinetic (or dynamic) friction occurs when two objects are moving relative to each other and rub together; a sled on the ground would be a good example of kinetic friction.



Friction: Frictional forces always oppose motion or attempted motion between objects in contact. Friction arises in part because of the roughness of the surfaces in contact, as seen in the expanded view. In order for the object to move, it must rise to where the peaks can skip along the bottom surface. Thus, a force is required just to set the object in motion. Some of the peaks will be broken off, also requiring a force to maintain motion. Much of the friction is actually due to attractive forces between molecules making up the two objects, so that even perfectly smooth surfaces are not friction-free. Such adhesive forces also depend on the substances the surfaces are made of, explaining, for example, why rubber-soled shoes slip less than those with leather soles.

The force of friction is what slows an object sliding over a surface. This force is what makes the brakes on cars work or causes resistance when you slide your hand across a surface. The force of friction can be represented by an equation: $\mathrm{F_{friction}} = \mu \mathrm{F_n}$. In this equation μ is something called the coefficient of friction. This is a unitless number that represents the strength of the friction of the object. A very “grippy” surface like rubber might have a high coefficient of friction, whereas a slippery surface like ice has a much lower coefficient. $\mathrm{F_n}$ is called the normal force and is the force of the surface pushing up on the object. In most cases on level ground, the normal force will be the equal and opposite of the object’s weight. In other words, it is the force that the surface must exert to keep the object from falling through.

The coefficient of kinetic friction is typically represented as μ_k and is usually less than the coefficient of static friction for the same materials.

Friction: Static

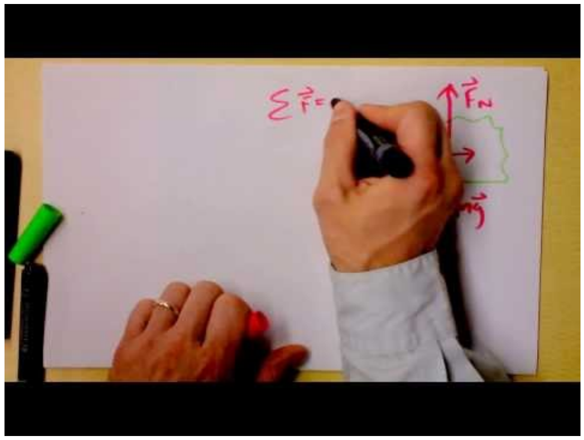
Static friction is a type of friction that occurs to resist motion when two objects are at rest against each other.

learning objectives

- Demonstrate the relationship of maximum force of static friction

Static Friction

Another type of frictional force is static friction, otherwise known as stiction. Like all friction, it acts to resist the motion of an object moving over a surface. Unlike kinetic friction, however, static friction acts to resist the start of motion.



Static Friction and some friction challenges: Here, I talk about sneaky ol' static friction.

Static friction is friction between two objects that are not moving relative to each other. This frictional force is what prevents a parked car from sliding down a hill, for example. Before an object at rest on a surface can move, it must overcome the force of static friction.

Static friction originates from multiple sources. For any given material on another material of the same composition, friction will be greater as the material surfaces become rougher (consider sandpaper) on the macroscopic level. Additionally, intermolecular forces can greatly influence friction when two materials are put into contact. When surface area is below the micrometer range, Van der Waals' forces, electrostatic interactions and hydrogen bonding can cause two materials to adhere to one another. A force is required to overcome these interactions and cause the surfaces to move across one another.

Like kinetic friction, the force of static friction is given by a coefficient multiplied by the normal force. The normal force is the force of the surface pushing up on the object, which is usually equal to the object's weight. The coefficient of static friction is usually greater than the coefficient of kinetic friction and is usually represented by μ_s.

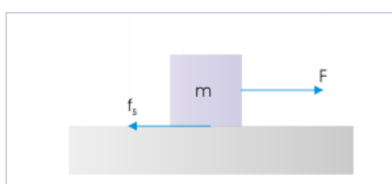
Putting these elements together gives the maximum force of static friction as:

$$\mathrm{F_s} = \mu_\mathrm{s}\mathrm{F_n} \tag{5.3.7.1}$$

In general, the force of static friction can be represented as:

$$\mathrm{F_s} \leq \mu_\mathrm{s}\mathrm{F_n} \tag{5.3.7.2}$$

As with all frictional forces, the force of friction can never exceed the force applied. Thus the force of static friction will vary between 0 and ${}_\mathrm{s}\mathrm{F_n}$ depending on the strength of the applied force. Any force smaller than ${}_\mathrm{s}\mathrm{F_n}$ attempting to slide one surface over the other is opposed by a frictional force of equal magnitude and opposite direction. Any force larger than that overcomes the force of static friction and causes sliding to occur. The instant sliding occurs, static friction is no longer applicable—the friction between the two surfaces is then called kinetic friction.



Static Friction: To move a block at rest on a surface, a force must be applied which is great enough to overcome the force of static friction.

Problem-Solving With Friction and Inclines

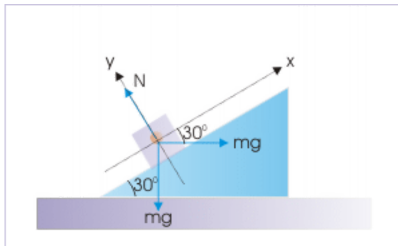
Combining motion on inclines with friction uses such concepts as equilibrium and contact force on an incline.

learning objectives

- Calculate the force of friction on an incline

Contact Force on an Incline

The incline plane has two contact or interface surfaces. One is the incline surface, where the block is placed and the other is the base of the incline, which is in contact with the surface underneath. The motion of the block, therefore, may depend on the motion of the incline itself.



Block and incline system: Forces on the block

When on an incline, calculating the force of friction is different than when the object is on a level surface. Recall that the force of friction depends on both the coefficient of friction and the normal force. $\mathrm{F_f} = \mu \mathrm{F_n}$ When on an incline with an angle θ, the normal force becomes $\mathrm{F_n} = \mathrm{mg}\cos(\theta)$

As always, the frictional force resists motion. If the block is being pushed up the incline the friction force points down the incline. If the block is being pulled down the incline, the friction force will hold the block up.

Equilibrium of Forces on an Incline

When not acted on by any other forces, only by gravity and friction, the frictional force will resist the tendency of the block to slide down the incline. If the frictional force is equal to the gravitational force the block will not slide down the incline. The block is said to be in equilibrium since the sum of the forces on it is 0.

Gravitational force down an incline is given by $\mathrm{mg}\sin(\theta)$.

Where θ is the angle the incline makes with the horizontal. For the block to be in equilibrium, the maximum force of friction $\mathrm{F_f} = \mu \mathrm{mg}\cos(\theta)$ must be greater than or equal to $\mathrm{F_G} = \mathrm{mg}\sin(\theta)$ FG=mgsin(θ). If the maximum frictional force is greater than the force of gravity, the sum of the forces is still 0. The force of friction can never exceed the other forces acting on it. The frictional forces only act to counter motion.

Drag

The drag force is the resistive force felt by objects moving through fluids and is proportional to the square of the object's speed.

learning objectives

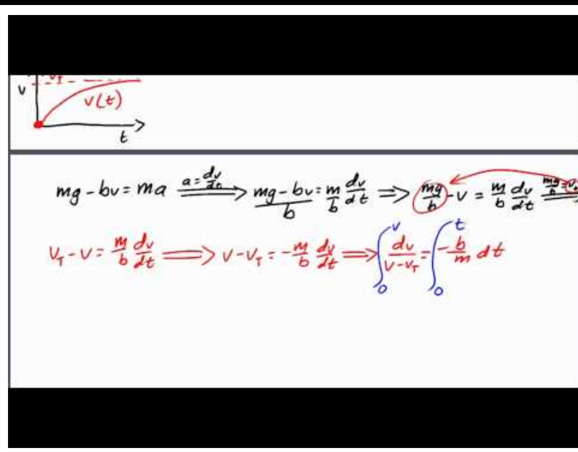
- Relate the magnitude of drag force to the speed of an object

Another interesting force in everyday life is the force of drag on an object when it is moving in a fluid (either a gas or a liquid). You feel the drag force when you move your hand through water. You might also feel it if you move your hand during a strong wind. The faster you move your hand, the harder it is to move. You feel a smaller drag force when you tilt your hand so only the side goes through the air—you have decreased the area of your hand that faces the direction of motion.

Like friction, the drag force always opposes the motion of an object. Unlike simple friction, the drag force is proportional to some function of the velocity of the object in that fluid. This functionality is complicated and depends upon the shape of the object, its size, its velocity, and the fluid it is in. Aerodynamic objects tend to have small surface areas and be designed to have low drag coefficients.

For most large objects such as bicyclists, cars, and baseballs not moving too slowly, the magnitude of the drag force $\mathrm{F_D}$ is found to be proportional to the square of the speed of the object. We can write this relationship mathematically as $\mathrm{FD} \propto \mathrm{v}^2$. When taking into account other factors, this relationship becomes $F_D = \frac{1}{\frac{2C}{\text{rhoAv}^2}}$, where C is known as the drag coefficient, a unit-less number that represents the aerodynamic properties of the object, A is the cross-sectional area of the object which is facing the direction of motion, and ρ is the density of the fluid the object is moving through.

Aerodynamic Shape: From racing cars to bobsled racers, aerodynamic shaping is crucial to achieving top speeds. Bobsleds are designed for speed. They are shaped like a bullet with tapered fins. (credit: U.S. Army, via Wikimedia Commons)



Retarding and Drag Forces: A brief look at retarding (drag) forces in physics, for students in introductory physics classes that use calculus. This video walks through a single scenario of an object experiencing a drag force where the drag force is proportional to the object's velocity.

Stress and Strain

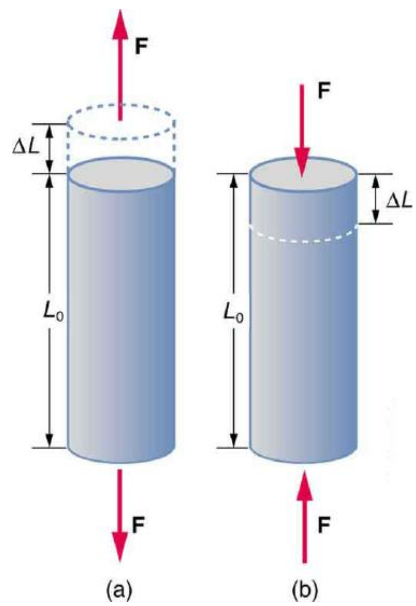
The ratio of force to area $\frac{F}{A}$ is called stress and the ratio of change in length to length $\frac{\Delta L}{L}$ is called the strain.

learning objectives

- Explain how forces affects the shape of an object

We now move from consideration of forces that affect the motion of an object (such as friction and drag) to those that affect an object's shape. If a bulldozer pushes a car into a wall, the car will not move past the wall but it will noticeably change shape. A change in shape due to the application of a force is a deformation. Even very small forces are known to cause some deformation. For small deformations, two important characteristics are observed. First, the object returns to its original shape when the force is removed—that is, the deformation is elastic for small deformations. Second, the size of the deformation is proportional to the force —that is, for small deformations, Hooke's law is obeyed. In equation form, Hooke's law is given by $F = k \cdot \Delta L$ where ΔL is the change in length and kk is a constant which depends on the material properties of the object.

Deformations come in several types: changes in length (tension and compression), sideways shear (stress), and changes in volume.



Tension/Compression: Tension: The rod is stretched a length ΔL when a force is applied parallel to its length. (b) Compression: The same rod is compressed by forces with the same magnitude in the opposite direction. For very small deformations and uniform materials, ΔL is approximately the same for the same magnitude of tension or compression. For larger deformations, the cross-sectional area changes as the rod is compressed or stretched.

The ratio of force to area $\frac{F}{A}$ is called stress and the ratio of change in length to length $\frac{\Delta L}{L}$ is called the strain.

Stress and strain are related to each other by a constant called Young's Modulus or the elastic modulus which varies depending on the material. Using Young's Modulus the relation between stress and strain is given by: $\text{stress} = Y \cdot \text{strain}$.

A material with a high elastic modulus is said to have high tensile strength. Such materials are very resistant to being stretched and require a large amount of force to deform a small amount.

Translational Equilibrium

An object is said to be in equilibrium when there is no external net force acting on it.

learning objectives

- Assess the role each type of equilibrium plays in mechanical devices

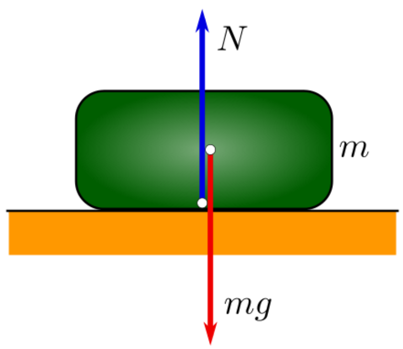
We are surrounded by great engineering architectures and mechanical devices, which are at rest in the frame of reference of Earth. A large part of engineering creations are static objects. Yet we also seek equilibrium of moving objects like that of floating ship, airplane cruising at high speed, and such other moving mechanical devices. In both cases – static or dynamic – net external forces and torques are zero.

A body is said to be in mechanical equilibrium when net external force is equal to zero and net external torque is also zero. Mathematically,

$$\Sigma \overrightarrow{\mathrm{F}}\mathrm{ext} = 0 \text{ and } \Sigma \overrightarrow{\tau}\mathrm{ext} = 0 \tag{5.3.7.3}$$

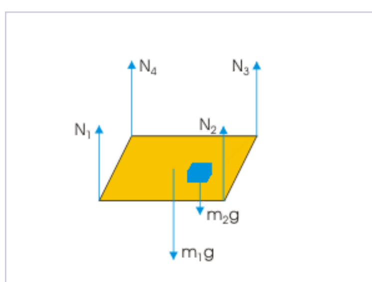
Since there is no net force on the object, the object does not accelerate. This implies two types of possible equilibrium. The first type, where all particles in the system are at rest and do have not velocity, is known as static equilibrium. In the second type, the object has a velocity, but since there are no net forces acting on it, the velocity remains constant. In the second case, the particle is said to be in dynamic equilibrium. Static or dynamic, these kinds of equilibrium can be categorized as translational equilibrium.

Examples of translational equilibrium are all around us. A book resting on a table is pushing down on the table with the force of its weight. The table, in turn, is pushing back on the book, keeping the book from falling through the table. Since neither the table nor the book are moving, this is an example of static equilibrium. The force of gravity on the book is perfectly counteracted by the force of the table pushing on it.



Forces Acting on an Object at Rest: A force diagram showing the forces acting on an object at rest on a surface. Notice that the amount of force that the table is pushing upward on the object (the N vector) is equal to the downward force of the object's weight (shown here as mg, as weight is equal to the object's mass multiplied by the acceleration due to gravity): because these forces are equal, the object is in a state of equilibrium (all the forces acting on it balance to zero).

An example of dynamic (or mechanical) equilibrium is an object sliding down a wedge. The force of gravity pulls the object down the wedge, but it is counteracted by the force of friction between the wedge and the object. If the force of friction is equal to the force of gravity, the object will proceed at a constant velocity.



Forces on a Table: These six forces are in equilibrium. The four forces of the table leg counteract the force of the table and the object pushing on them.

Connected Objects

Forces can be transferred from one object to another through connections.

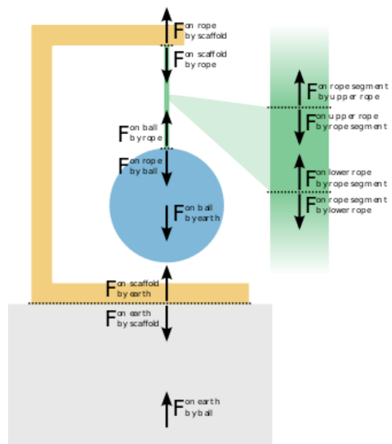
learning objectives

- Analyze the affect a rigid connection has on the movement of objects

The physics of connected objects is very similar to physics of simple objects. There are a variety of ways objects can be connected to each other, and a corresponding variety of mathematical ways to model such connections.

The simplest form of connection is a perfectly rigid connection. If two objects are connected by a perfectly rigid connector then they may be thought of as the same object. Perfectly rigid connectors cannot stretch nor deform, and transfer forces instantaneously from one side of the connection to the other. For example, given two blocks (both of mass 1 kg) connected by a perfectly rigid bar, if the first block is pulled with a force of 1 Newton, then both blocks will accelerate at the same time and the same acceleration. In this case the acceleration is $\frac{1}{2}\mathrm{m/s^2}$ —the same as if a force of mass 2 kg is exerted on one object. Thus it can be said that a perfectly rigid connection makes two objects into one large object. Of course, perfectly rigid connections do not exist in nature. Some deformation will always exist in any object as force travels along it. However, many materials are sufficiently rigid, so that using the perfectly rigid approximation is useful for simplicity’s sake.

One can think of the force transferring through the connection by means of the “tension” force. Tension is the pulling force exerted by a string, chain, or similar connector on another object. If two objects are connected by a string, a force exerted on one is balanced by a tension force in the string which pulls on the other. Of course, if the tension force is greater than the rope can withstand, the rope will break.



Tension Forces: The forces involved in supporting a ball by a rope. Tension is the force of the rope on the scaffold, the force of the rope on the ball, and the balanced forces acting on and produced by segments of the rope.

Circular Motion

An object in circular motion undergoes acceleration due to centripetal force in the direction of the center of rotation.

learning objectives

- Develop an understanding of uniform circular motion as an indicator for net external force

Uniform circular motion describes the motion of an object along a circle or a circular arc at constant speed. It is the basic form of rotational motion in the same way that uniform linear motion is the basic form of translational motion. However, the two types of motion are different with respect to the force required to maintain the motion.

Let us consider Newton’s first law of motion. It states that an object will maintain a constant velocity unless a net external force is applied. Therefore, uniform linear motion indicates the absence of a net external force. On the other hand, uniform circular motion requires that the velocity vector of an object constantly change direction. Since the velocity vector of the object is changing, an acceleration is occurring. Therefore, uniform *circular* motion indicates the *presence* of a net external force.

In uniform circular motion, the force is always perpendicular to the direction of the velocity. Since the direction of the velocity is continuously changing, the direction of the force must be as well.

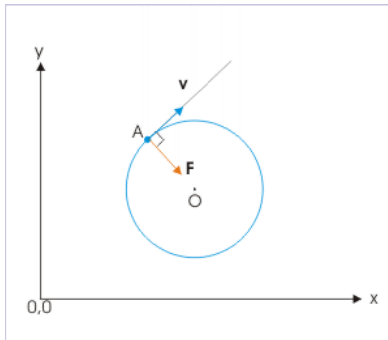
The direction of the velocity along the circular trajectory is tangential. The perpendicular direction to the circular trajectory is, therefore, the radial direction. Therefore, the force (and therefore the acceleration) in uniform direction motion is in the radial direction. For this reason, acceleration in uniform circular motion is recognized to “seek the center” — i.e., centripetal force.

The equation for the acceleration aa required to sustain uniform circular motion is:

$$a = \frac{v^2}{r} \tag{5.3.7.4}$$

where mm is the mass of the object, v is the velocity of the object, and r is the radius of the circle. Consequently, the net external force F_{net} required to sustain circular motion is:

$$F_{net} = \frac{m \cdot v^2}{r} \tag{5.3.7.5}$$



Uniform Circular Motion: In uniform circular motion, the centripetal force is perpendicular to the velocity. The centripetal force points toward the center of the circle, keeping the object on the circular track.

Key Points

- Friction is the force that resists relative motion between two surfaces sliding across each other. Friction converts kinetic energy into heat.
- Drag force is the force that resists motion of an object traveling through a fluid such as air or water. Drag force is proportional to the velocity of the object traveling.
- Deformation forces are forces caused by stretching or compressing a material. Some examples would be springs or elastics.
- Kinetic (or dynamic) friction occurs when two objects are moving relative to each other and rub together (like a sled on the ground).
- The force of friction can be represented by an equation $F_{friction} = \mu F_n$ where μ is the coefficient of friction and is a unitless number that represents the strength of the friction of the surface.
- Kinetic friction and static (stationary) friction use two different coefficients for the same material.
- Static friction is a force that acts to resist the start of motion. It is borne of macroscopic inconsistencies in the surfaces of materials in contact as well as intermolecular interactions between the materials, such as hydrogen bonding, Van der Waal’s interactions and electrostatic interactions.
- Static friction uses a different, usually higher, coefficient than kinetic friction does.
- The force of static friction is $F_{fs} = \mu_s F_n$. Where μ_s is the coefficient of static friction which varies by material and FnFn is the normal force.
- Motion on an incline is resisted by friction.
- The frictional force on an incline is dependent on the angle of the incline. $F_f = \mu mg \cos(\theta)$ is the maximum friction force on an incline.
- If the friction force is greater than or equal to the forces in the direction of motion, then the net force is 0 and the object is in equilibrium.
- Objects moving through a fluid feel a force which resists their motion. This force is known as the drag force.
- The drag force is proportional to the square of the velocity of the object relative to the fluid.
- The equation for drag is $F_D = \frac{1}{2} C_\rho A v^2$. C is a constant called the drag coefficient. ρρ is the density of the fluid. A is the surface area in the direction of motion.

- The ratio of force to area $\frac{F}{A}$ is called stress and the ratio of change in length to length $\frac{\Delta L}{L}$ is called the strain.
- Stress and strain are related to each other by a constant called Young's Modulus or the elastic modulus which varies depending on the material. Using Young's Modulus the relation between stress and strain is given by: $\text{stress} = Y \cdot \text{strain}$.
- A material with a high elastic modulus is said to have high tensile strength. Such materials are very resistant to being stretched and require a large amount of force to deform a small amount.
- When there is no external net force on an object, the object is said to be in equilibrium.
- When an object is in equilibrium, it does not accelerate. If it had a velocity, the velocity remains constant; if it was at rest, it remains at rest.
- An equilibrium in motion is known as dynamic equilibrium; an equilibrium at rest is a static equilibrium.
- If two objects are connected, a force on one has an effect on the other.
- Connections can often be approximated as completely rigid. In completely rigid cases, the connection does not deform and the force is transferred instantaneously.
- Tension is the force of a rope or cable or other connector on the object it is connected to. It is one way force is transferred between objects.
- An object that is undergoing circular motion has a velocity vector that is constantly changing direction.
- The force that is needed to maintain circular motion points toward the center of the circular path. It is therefore known as the centripetal force.
- The velocity of an object in circular motion is always tangent to the circle, and the centripetal force is always perpendicular to the velocity.

Key Terms

- **kinetic energy**: The energy possessed by an object because of its motion, equal to one half the mass of the body times the square of its velocity.
- **static**: Fixed in place; having no motion.
- **kinetic**: Of or relating to motion
- **friction**: A force that resists the relative motion or tendency to such motion of two bodies in contact.
- **incline**: A slope.
- **equilibrium**: The state of a body at rest or in uniform motion, the resultant of all forces on which is zero.
- **fluid**: Any substance which can flow with relative ease, tends to assume the shape of its container, and obeys Bernoulli's principle; a liquid, gas or plasma.
- **strain**: The amount by which a material deforms under stress or force, given as a ratio of the deformation to the initial dimension of the material and typically symbolized by ϵϵ is termed the engineering strain. The true strain is defined as the natural logarithm of the ratio of the final dimension to the initial dimension.
- **stress**: The internal distribution of force per unit area (pressure) within a body reacting to applied forces which causes strain or deformation and is typically symbolized by σ.
- **dynamic**: Changing; active; in motion.
- **torque**: A rotational or twisting effect of a force; (SI unit newton-meter or Nm; imperial unit foot-pound or ft-lb)
- **rigid**: Stiff, rather than flexible.
- **tangent**: a straight line touching a curve at a single point without crossing it at that point
- **perpendicular**: at or forming a right angle (to).

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CHAPTER OVERVIEW

5.4: Work and Energy

5.4.1: Introduction

Introduction to Work and Energy

Work is the energy associated with the action of a force.

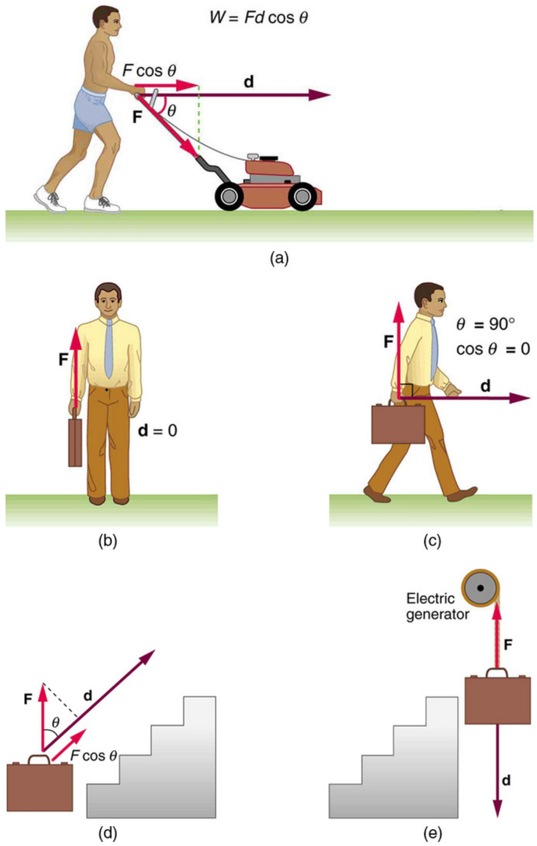
learning objectives

- Describe relationship between work, energy, and force

The work done on a system by a constant force is the product of the component of the force in the direction of motion times the distance through which the force acts. For one-way motion in one dimension, this is expressed in equation form as $\mathrm{W}=\mathrm{Fd}\cos\theta$, where W is work, F is the magnitude of the force on the system, d is the magnitude of the displacement of the system, and θ is the angle between the force vector F and the displacement vector d.

Work, Power, and Energy: Biology is useful.

Take this example of work in action from: (A) The work done by the force F on this lawn mower is $\mathrm{W}=\mathrm{Fd}\cos\theta$. Note that $\mathrm{W}=\mathrm{Fd}\cos\theta$ is the component of the force in the direction of motion. (B) A person holding a briefcase does no work on it, because there is no motion. No energy is transferred to or from the briefcase. (C) The person moving the briefcase horizontally at a constant speed does no work on it, and transfers no energy to it. (D) Work is done on the briefcase by carrying it up stairs at constant speed, because there is necessarily a component of force F in the direction of the motion. Energy is transferred to the briefcase and could, in turn, be used to do work. (E) When the briefcase is lowered, energy is transferred out of the briefcase and into an electric generator. Here the work done on the briefcase by the generator is negative, removing energy from the briefcase, because F and d are in opposite directions.



Examples of Work: This is how work in progress and energy co-exist and operate. Work is the energy associated with the action of a force.

In physics, a force is said to do work when it acts on a body so that there is a displacement of the point of application, however small, in the direction of the force. Thus a force does work when there is movement under the action of the force. The work done by a constant force of magnitude F on a point that moves a distance d in the direction of the force is the product:

$$\mathrm{W=Fd} \tag{5.4.1.1}$$

For example, if a force of 10 newton (F = 10 N) acts along point that travels two meters (d = 2 m), then it does the work W = (10 N)(2 m) = 20 N m = 20 J. This is approximately the work done lifting a 1 kg weight from the ground to over a person's head against the force of gravity. Notice that the work is doubled either by lifting twice the weight in the same distance or by lifting the same weight twice the distance.

Work is closely related to energy. The conservation of energy states that the change in total internal energy of a system equals the added heat minus the work performed by the system (see the first law of thermodynamics, and):

Baseball Pitcher: A baseball pitcher does work on a baseball by throwing the ball at some force, F, over some distance d, which for the average baseball field, is about 60 feet.

$$\delta E = \delta Q - \delta W \tag{5.4.1.2}$$

Also, from Newton's second law for rigid bodies, it can be shown that work on an object is equal to the change in kinetic energy of that object:

$$W = \Delta KE \tag{5.4.1.3}$$

The work of forces generated by a potential function is known as potential energy and the forces are said to be conservative. Therefore work on an object moving in a conservative force field is equal to minus the change of potential energy of the object:

$$W = -\Delta PE \tag{5.4.1.4}$$

This shows that work is the energy associated with the action of a force, and so has the physical dimensions and units of energy.

Key Points

- Work is the transfer of energy by a force acting on an object as it is displaced.
- The work done by a force is zero if the displacement is either zero or perpendicular to the force.
- The work done is positive if the force and displacement have the same direction, and the work done is negative if they have opposite direction.

Key Terms

- **energy**: A quantity that denotes the ability to do work and is measured in a unit dimensioned in mass × distance²/time² (ML^2/T^2) or the equivalent.

5.4.2: Work Done by a Constant Force

Force in the Direction of Displacement

The work done by a constant force is proportional to the force applied times the displacement of the object.

learning objectives

- Contrast displacement and distance in constant force situations

Work Done by a Constant Force

When a force acts on an object over a distance, it is said to have done work on the object. Physically, the work done on an object is the change in kinetic energy that that object experiences. We will rigorously prove both of these claims.

The term work was introduced in 1826 by the French mathematician Gaspard-Gustave Coriolis as “weight lifted through a height,” which is based on the use of early steam engines to lift buckets of water out of flooded ore mines. The SI unit of work is the newton-meter or joule (J).

Units

One way to validate if an expression is correct is to perform dimensional analysis. We have claimed that work is the change in kinetic energy of an object and that it is also equal to the force times the distance. The units of these two should agree. Kinetic energy – and all forms of energy – have units of joules (J). Likewise, force has units of newtons (N) and distance has units of meters (m). If the two statements are equivalent they should be equivalent to one another.

$$\mathrm{N \cdot m = kg \frac{m}{s^2} \cdot m = kg \frac{m^2}{s^2} = J} \tag{5.4.2.1}$$

Displacement versus Distance

Often times we will be asked to calculate the work done by a force on an object. As we have shown, this is proportional to the force and the distance which the object is displaced, not moved. We will investigate two examples of a box being moved to illustrate this.

Example Problems

Here are a few example problems:

(1.a) Consider a constant force of two newtons (F = 2 N) acting on a box of mass three kilograms (M = 3 kg). Calculate the work done on the box if the box is displaced 5 meters.

(1.b) Since the box is displaced 5 meters and the force is 2 N, we multiply the two quantities together. The object’s mass will dictate how fast it is accelerating under the force, and thus the time it takes to move the object from point a to point b. Regardless of how long it takes, the object will have the same displacement and thus the same work done on it.

(2.a) Consider the same box (M = 3 kg) being pushed by a constant force of four newtons (F = 4 N). It begins at rest and is pushed for five meters (d = 5m). Assuming a frictionless surface, calculate the velocity of the box at 5 meters.

(2.b) We now understand that the work is proportional to the change in kinetic energy, from this we can calculate the final velocity. What do we know so far? We know that the block begins at rest, so the initial kinetic energy must be zero. From this we algebraically isolate and solve for the final velocity.

$$\mathrm{Fd = \Delta KE = KE_f - 0 = \frac{1}{2} m v_f^2} \tag{5.4.2.2}$$

$$\mathrm{v_f = \sqrt{2 \frac{Fd}{m}} = \sqrt{2 \frac{4N \cdot 5m}{2kg}} = \sqrt{10} m/s} \tag{5.4.2.3}$$

We see that the final velocity of the block is approximately 3.15 m/s.

Force at an Angle to Displacement

A force does not have to, and rarely does, act on an object parallel to the direction of motion.

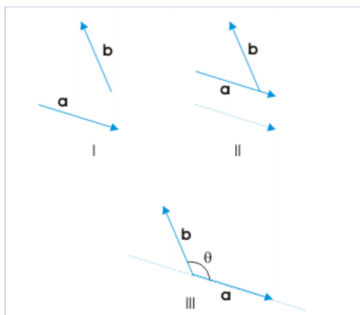
learning objectives

- Infer how to adjust one-dimensional motion for our three-dimensional world

The Fundamentals

Up until now, we have assumed that any force acting on an object has been parallel to the direction of motion. We have considered our motion to be one dimensional, only acting along the x or y axis. To best examine and understand how nature operators in our three-dimensional world, we will first discuss work in two dimensions in order to build our intuition.

A force does not have to, and rarely does, act on an object parallel to the direction of motion. In the past, we derived that $\mathrm{W=Fd}$; such that the work done on an object is the force acting on the object multiplied by the displacement. But this is not the whole story. This expression contains an assumed cosine term, which we do not consider for forces parallel to the direction of motion. “Why would we do such a thing? ” you may ask. We do this because the two are equivalent. If the angle of the force along the direction of motion is zero, such that the force is parallel to the direction of motion, then the cosine term equals one and does not change the expression. As we increase the force’s angle with respect to the direction of motion, less and less work is done along the direction that we are considering; and more and more work is being done in another, perpendicular, direction of motion. This process continues until we are perpendicular to our original direction of motion, such that the angle is 90, and the cosine term would equal zero; resulting in zero work being done along our original direction. Instead, we are doing work in another direction!



Angle: Recall that both the force and direction of motion are vectors. When the angle is 90 degrees, the cosine term goes to zero. When along the same direction, they equal one.

Let’s show this explicitly and then look at this phenomena in terms of a box moving along the x and y directions.

We have discussed that work is the integral of the force and the dot product respect to x. But in fact, dot product of force and a very small distance is equal to the two terms times cosine of the angle between the two. $\mathrm{F \times dx = Fd\cos(\theta)}$. Explicitly,

$$\int_{\mathrm{t_2}}^{\mathrm{t_1}} \mathrm{F \cdot dx} = \int_{\mathrm{t_2}}^{\mathrm{t_1}} \mathrm{Fd\cos\theta dx = Fd\cos\theta} \tag{5.4.2.4}$$

A Box Being Pushed

Consider a coordinate system such that we have x as the abscissa and y as the ordinate. More so, consider a box being pushed along the x direction. What happens in the following three scenarios?

- The box is being pushed parallel to the x direction?
- The box is being pushed at an angle of 45 degrees to the x direction?
- The box is being pushed at an angle of 60 degrees to the x direction?
- The box is being pushed at an angle of 90 degrees to the x direction?

In the first scenario, we know that all of the force is acting on the box along the x-direction, which means that work will only be done along the x-direction. More so, a vertical perspective the box is not moving – it is unchanged in the y direction. Since the force is acting parallel to the direction of motion, the angle is equal to zero and our total work is simply the force times the displacement in the x-direction.

In the second scenario, the box is being pushed at an angle of 45 degrees to the x-direction; and thus also a 45 degree angle to the y-direction. When evaluated, the cosine of 45 degrees is equal to $\frac{1}{\sqrt{2}}$, or approximately 0.71. This means is that 71% of the force is contributing to the work along the x-direction. The other 29% is acting along the y-direction.

In the third scenario, we know that the force is acting at a 60 degree angle to the x-direction; and thus also a 30 degree angle to the y-direction. When evaluated, cosine of 60 degrees is equal to 1/2. This means that the force is equally acting in the x and y-direction! The work done is linear with respect to both x and y.

In the last scenario, the box is being pushed at an angle perpendicular to the x direction. In other words, we are pushing the box in the y-direction! Thus, the box's position will be unchanged and experience no displacement along the x-axis. The work done in the x direction will be zero.

Key Points

- Understanding work is quintessential to understanding systems in terms of their energy, which is necessary for higher level physics.
- Work is equivalent to the change in kinetic energy of a system.
- Distance is not the same as displacement. If a box is moved 3 meters forward and then 4 meters to the left, the total displacement is 5 meters, not 7 meters.
- Work done on an object along a given direction of motion is equal to the force times the displacement times the cosine of the angle.
- No work is done along a direction of motion if the force is perpendicular.
- When considering force parallel to the direction of motion, we omit the cosine term because it equals 1 which does not change the expression.

Key Terms

- **work**: A measure of energy expended in moving an object; most commonly, force times displacement. No work is done if the object does not move.
- **dot product**: A scalar product.

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5.4.3: Work Done by a Variable Force

Work Done by a Variable Force

Integration is used to calculate the work done by a variable force.

learning objectives

- Describe approaches used to calculate work done by a variable force

Using Integration to Calculate the Work Done by Variable Forces

A force is said to do work when it acts on a body so that there is a displacement of the point of application in the direction of the force. Thus, a force does work when it results in movement.

The work done by a constant force of magnitude F on a point that moves a displacement Δx in the direction of the force is simply the product

$$W = F \cdot \Delta x \tag{5.4.3.1}$$

In the case of a variable force, integration is necessary to calculate the work done. For example, let's consider work done by a spring. According to the Hooke's law the restoring force (or spring force) of a perfectly elastic spring is proportional to its extension (or compression), but opposite to the direction of extension (or compression). So the spring force acting upon an object attached to a horizontal spring is given by:

$$Fs = -kx \tag{5.4.3.2}$$

that is proportional to its displacement (extension or compression) in the x direction from the spring's equilibrium position, but its direction is opposite to the x direction. For a variable force, one must add all the infinitesimally small contributions to the work done during infinitesimally small time intervals dt (or equivalently, in infinitely small length intervals $dx=v_x dt$). In other words, an integral must be evaluated:

$$W_s = \int_0^t Fs \cdot vdt = \int_0^t -kxv_x dt = \int_{x_o}^x -kxdx = -\frac{1}{2}k\Delta x^2 \tag{5.4.3.3}$$

This is the work done by a spring exerting a variable force on a mass moving from position x_o to x (from time 0 to time t). The work done is positive if the applied force is in the same direction as the direction of motion; so the work done by the object on spring from time 0 to time t, is:

$$W_a = \int_0^t Fa \cdot vdt = \int_0^t -F_s \cdot vdt = \frac{1}{2}k\Delta x^2 \tag{5.4.3.4}$$

in this relation F_a is the force acted upon spring by the object. F_a and F_s are in fact action- reaction pairs; and W_a is equal to the elastic potential energy stored in spring.

Using Integration to Calculate the Work Done by Constant Forces

The same integration approach can be also applied to the work done by a constant force. This suggests that *integrating* the product of force and distance is the general way of determining the work done by a force on a moving body.

Consider the situation of a gas sealed in a piston, the study of which is important in Thermodynamics. In this case, the Pressure (Pressure =Force/Area) is constant and can be taken out of the integral:

$$W = \int_a^b PdV = P\int_a^b dV = P\Delta V \tag{5.4.3.5}$$

Another example is the work done by gravity (a constant force) on a free-falling object (we assign the y-axis to vertical motion, in this case):

$$W = \int_{t_1}^{t_2} F \cdot vdt = \int_{t_1}^{t_2} mgv_y dt = mg\int_{y_1}^{y_2} dy = mg\Delta y \tag{5.4.3.6}$$

Notice that the result is *the same* as we would have obtained by simply evaluating the product of force and distance.

Units Used for Work

The SI unit of work is the joule (J), which is defined as the work done by a force of one newton moving an object through a distance of one meter.

Non-SI units of work include the erg, the foot-pound, the foot-pound, the kilowatt hour, the liter-atmosphere, and the horsepower-hour.

Key Points

- The work done by a constant force of magnitude F on a point that moves a displacement d in the direction of the force is the product: $W = Fd$.
- Integration approach can be used both to calculate work done by a variable force and work done by a constant force.
- The SI unit of work is the joule; non- SI units of work include the erg, the foot-pound, the foot-poundal, the kilowatt hour, the litre-atmosphere, and the horsepower-hour.

Key Terms

- **work**: A measure of energy expended in moving an object; most commonly, force times displacement. No work is done if the object does not move.
- **force**: A physical quantity that denotes ability to push, pull, twist or accelerate a body, which is measured in a unit dimensioned in mass × distance/time² (ML/T^2): SI: newton (N); CGS: dyne (dyn)

5.4.4: Work-Energy Theorem

Kinetic Energy and Work-Energy Theorem

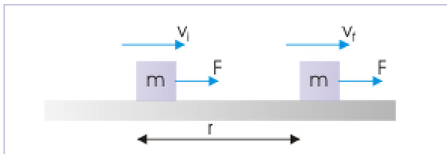
The work-energy theorem states that the work done by all forces acting on a particle equals the change in the particle's kinetic energy.

learning objectives

- Outline the derivation of the work-energy theorem

The Work-Energy Theorem

The principle of work and kinetic energy (also known as the work-energy theorem) states that the work done by the sum of all forces acting on a particle equals the change in the kinetic energy of the particle. This definition can be extended to rigid bodies by defining the work of the torque and rotational kinetic energy.



Kinetic Energy: A force does work on the block. The kinetic energy of the block increases as a result by the amount of work. This relationship is generalized in the work-energy theorem.

The work *W* done by the net force on a particle equals the change in the particle's kinetic energy K*E*:

$$\mathrm{W=\Delta KE=\frac{1}{2}mv_f^2-\frac{1}{2}mv_i^2} \tag{5.4.4.1}$$

where v_i and v_f are the speeds of the particle before and after the application of force, and *m* is the particle's mass.

Derivation

For the sake of simplicity, we will consider the case in which the resultant force *F* is constant in both magnitude and direction and is parallel to the velocity of the particle. The particle is moving with constant acceleration *a* along a straight line. The relationship between the net force and the acceleration is given by the equation *F* = *ma* (Newton's second law), and the particle's displacement *d*, can be determined from the equation:

$$\mathrm{v_f^2=v_i^2+2ad} \tag{5.4.4.2}$$

obtaining,

$$\mathrm{d=\frac{v_f^2-v_i^2}{2a}} \tag{5.4.4.3}$$

The work of the net force is calculated as the product of its magnitude (F=ma) and the particle's displacement. Substituting the above equations yields:

$$\mathrm{W=Fd=ma\frac{v_f^2-v_i^2}{2a}=\frac{1}{2}mv_f^2-\frac{1}{2}mv_i^2=KE_f-KE_i=\Delta KE} \tag{5.4.4.4}$$

Key Points

- The work *W* done by the net force on a particle equals the change in the particle's kinetic energy K*E*: $\mathrm{W=\Delta KE=\frac{1}{2}mv_f^2-\frac{1}{2}mv_i^2}$.
- The work-energy theorem can be derived from Newton's second law.
- Work transfers energy from one place to another or one form to another. In more general systems than the particle system mentioned here, work can change the potential energy of a mechanical device, the heat energy in a thermal system, or the electrical energy in an electrical device.

Key Terms

- **torque**: A rotational or twisting effect of a force; (SI unit newton-meter or Nm; imperial unit foot-pound or ft-lb)

5.4.5: Potential Energy and Conservation of Energy

Conservative and Nonconservative Forces

Conservative force—a force with the property that the work done in moving a particle between two points is independent of the path it takes.

learning objectives

- Describe properties of conservative and nonconservative forces

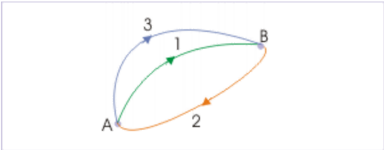
A conservative force is a force with the property that the work done in moving a particle between two points is independent of the path taken. Equivalently, if a particle travels in a closed loop, the net work done (the sum of the force acting along the path multiplied by the distance travelled) by a conservative force is zero.

A conservative force is dependent only on the position of the object. If a force is conservative, it is possible to assign a numerical value for the potential at any point. When an object moves from one location to another, the force changes the potential energy of the object by an amount that does not depend on the path taken. Gravity and spring forces are examples of conservative forces.

If a force is *not conservative*, then defining a scalar potential is not possible, because taking different paths would lead to conflicting potential differences between the start and end points. Nonconservative forces transfer energy *from* the object in motion (just like conservative force), but they do not transfer this energy back *to* the potential energy of the system to regain it during reverse motion. Instead, they transfer the energy from the system in an energy form which can not be used by the force to transfer it back to the object in motion. Friction is one such nonconservative force.

Path Independence of Conservative Force

Work done by the gravity in a closed path motion is zero. We can extend this observation to other conservative force systems as well. We imagine a closed path motion. We imagine this closed path motion be divided in two motions between points A and B as diagramed in Fig 1. Starting from point A to point B and then ending at point A via two work paths named 1 and 2 in the figure. The total work by the conservative force for the round trip is zero:



Motion Along Different Paths: Motion along different paths. For a conservative force, work done via different path is the same.

$$\mathrm{W}=\mathrm{W}_{\mathrm{AB1}}+\mathrm{W}_{\mathrm{BA2}}=0. \tag{5.4.5.1}$$

Let us now change the path for motion from A to B by another path, shown as path 3. Again, the total work by the conservative force for the round trip via new route is zero: $\mathrm{W}=\mathrm{W}_{\mathrm{AB3}}+\mathrm{W}_{\mathrm{BA2}}=0.$.

Comparing two equations, $\mathrm{W}_{\mathrm{AB1}}=\mathrm{W}_{\mathrm{AB3}}$. This is true for an arbitrary path. Therefore, work done for motion from A to B by conservative force along any paths are equal.

What is Potential Energy?

Potential energy is the energy difference between the energy of an object in a given position and its energy at a reference position.

learning objectives

- Relate the potential energy and the work

Potential energy is often associated with restoring forces such as a spring or the force of gravity. The action of stretching the spring or lifting the mass of an object is performed by an external force that works against the force field of the potential. This work is stored in the force field as potential energy. If the external force is removed the force field acts on the body to perform the work as it moves the body back to its initial position, reducing the stretch of the spring or causing the body to fall. The more formal definition is that potential energy is the energy difference between the energy of an object in a given position and its energy at a reference position.

Potential Energy in a Bow and Arrow: In the case of a bow and arrow, the energy is converted from the potential energy in the archer's arm to the potential energy in the bent limbs of the bow when the string is drawn back. When the string is released, the potential energy in the bow limbs is transferred back through the string to become kinetic energy in the arrow as it takes flight.

If the work for an applied force is independent of the path, then the work done by the force is evaluated at the start and end of the trajectory of the point of application. This means that there is a function $U(x)$, called a "potential," that can be evaluated at the two points $x(t = t_1)$ and $x(t_2)$ to obtain the work over any trajectory between these two points. It is tradition to define this function with a negative sign so that positive work is represented as a reduction in the potential:

$$W = \int_C F \cdot dx = \int_{x(t_1)}^{x(t_2)} F \cdot dx \tag{5.4.5.2}$$

$$= U(x(t_1)) - U(x(t_2)) = -\Delta U. \tag{5.4.5.3}$$

Examples of Potential Energy

There are various types of potential energy, each associated with a particular type of force. More specifically, every conservative force gives rise to potential energy. For example, the work of an elastic force is called elastic potential energy; work done by the gravitational force is called gravitational potential energy; and work done by the Coulomb force is called electric potential energy.

Gravity

Gravitational energy is the potential energy associated with gravitational force, as work is required to move objects against gravity.

learning objectives

- Generate an equation that can be used to express the gravitational potential energy near the earth

Gravitational energy is the potential energy associated with gravitational force (a conservative force), as work is required to elevate objects against Earth's gravity. The potential energy due to elevated positions is called gravitational potential energy, evidenced, for example, by water held in an elevated reservoir or behind a dam (as an example, shows Hoover Dam). If an object falls from one point to another point inside a gravitational field, the force of gravity will do positive work on the object, and the gravitational potential energy will decrease by the same amount.

Hoover Dam: Hoover dam uses the stored gravitational potential energy to generate electricity.

Potential Near Earth

Gravitational potential energy near the Earth can be expressed with respect to the height from the surface of the Earth. (The surface will be the zero point of the potential energy.) We can express the potential energy (gravitational potential energy) as:

$$\mathrm{PE} = \mathrm{mgh}, \tag{5.4.5.4}$$

where PE = potential energy measured in joules (J), m = mass of the object (measured in kg), and h = perpendicular height from the reference point (measured in m); g = gravitational acceleration ($9.8 m/s^2$). Near the surface of the Earth, g can be considered constant.

General Formula

However, over large variations in distance, the approximation that g is constant is no longer valid. Instead, we must use calculus and the general mathematical definition of work to determine gravitational potential energy. For the computation of the potential energy we can integrate the gravitational force, whose magnitude is given by Newton's law of gravitation (with respect to the distance r between the two bodies). Using that definition, the gravitational potential energy of a system of masses m and M at a distance r using gravitational constant G is:

$$\mathrm{U(r)} = \int_{\mathrm{r}} (\mathrm{G}\frac{\mathrm{mM}}{\mathrm{r}'^2})\mathrm{dr}' = -\mathrm{G}\frac{\mathrm{mM}}{\mathrm{r}} + \mathrm{K}, \tag{5.4.5.5}$$

where *K* is the constant of integration. Choosing the convention that *K*=0 makes calculations simpler, albeit at the cost of making *U* negative. For this choice, the potential at infinity is defined as 0.

Springs

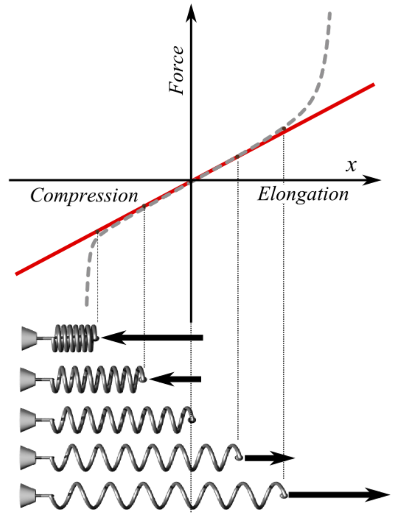
When a spring is stretched/compressed from its equilibrium position by x, its potential energy is give as $\mathrm{U} = \frac{1}{2}\mathrm{kx}^2$.

learning objectives

- Explain how potential energy is stored in springs

Spring force is conservative force, given by the Hooke's law: $\mathrm{F} = -\mathrm{kx}$, where k is spring constant, measured experimentally for a particular spring and x is the displacement. We would like to obtain an expression for the work done to the spring. From the conservation of mechanical energy (Check our Atom on "Conservation of Mechanical Energy), the work should be equal to the potential energy stored in spring. The displacement x is usually measured from the position of "neutral length" or "relaxed length"

– the length of spring corresponding to situation when spring is neither stretched nor compressed. We shall identify this position as the origin of coordinate reference (x=0).



Hooke's Law: Plot of applied force F vs. elongation X for a helical spring according to Hooke's law (solid line) and what the actual plot might look like (dashed line). Red is used extension, blue for compression. At bottom, schematic pictures of spring states corresponding to some points of the plot; the middle one is in the relaxed state (no force applied).

Let $\mathrm{x}=0$ and $\mathrm{x}=\mathrm{x}_{\mathrm{f}}(>0)$ be the initial and final positions of the block attached to the string. As the block slowly moves, we do work W on the spring: $\mathrm{W}=\int_{0}^{\mathrm{x}_{\mathrm{f}}}(\mathrm{kx})\mathrm{dx}=\frac{1}{2}\mathrm{kx}_{\mathrm{f}}^{2}$. When we stretch the spring. We have to apply force in the same direction as the displacement. (Technically, work is given as the inner product of the two vectors: force and displacement. $\mathrm{W}=\mathbf{F}\cdot\Delta\mathbf{x}$). Therefore, the overall sign in the integral is +, not -.

If the block is gently released from the stretched position (\mathrm{x = x_f}\), the stored potential energy in the spring will start to be converted to the kinetic energy of the block, and vice versa. Neglecting frictional forces, Mechanical energy conservation demands that, at any point during its motion,

$$\text{Total Energy} = \frac{1}{2}\mathrm{mv}^{2}+\frac{1}{2}\mathrm{kx}^{2} \tag{5.4.5.6}$$

$$= \frac{1}{2}\mathrm{kx}_{\mathrm{f}}^{2}=\text{constant}. \tag{5.4.5.7}$$

From the energy conservation, we can estimate that, by the time the block reaches x=0 position, its speed will be $\mathrm{v}(\mathrm{x}=0)=\sqrt{\frac{\mathrm{k}}{\mathrm{m}}}\mathrm{x}_{\mathrm{f}}$. The block will keep oscillating between x = -x_f and x_f.

Conservation of Mechanical Energy

Conservation of mechanical energy states that the mechanical energy of an isolated system remains constant without friction.

learning objectives

- Formulate the principle of the conservation of the mechanical energy

Conservation of mechanical energy states that the mechanical energy of an isolated system remains constant in time, as long as the system is free of all frictional forces. In any real situation, frictional forces and other non-conservative forces are always present, but in many cases their effects on the system are so small that the principle of conservation of mechanical energy can be used as a fair approximation. An example of a such a system is shown in. Though energy cannot be created nor destroyed in an isolated system, it can be internally converted to any other form of energy.

A Mechanical System: An example of a mechanical system: A satellite is orbiting the Earth only influenced by the conservative gravitational force and the mechanical energy is therefore conserved. This acceleration is represented by a green acceleration vector and the velocity is represented by a red velocity vector.

Derivation

Let us consider what form the work -energy theorem takes when only conservative forces are involved (leading us to the conservation of energy principle). The work-energy theorem states that the net work done by all forces acting on a system equals its change in kinetic energy (KE). In equation form, this is:

$$\mathrm{W_{net}=\frac{1}{2}mv^2-\frac{1}{2}mv_0^2=\Delta KE.} \tag{5.4.5.8}$$

If only conservative forces act, then $\mathrm{W_{net}=W_c}$, where $\mathrm{W_c}$ is the total work done by all conservative forces. Thus, $\mathrm{W_c=\Delta KE}$.

Now, if the conservative force, such as the gravitational force or a spring force, does work, the system loses potential energy (PE). That is, $\mathrm{W_c=-PE}$. Therefore,

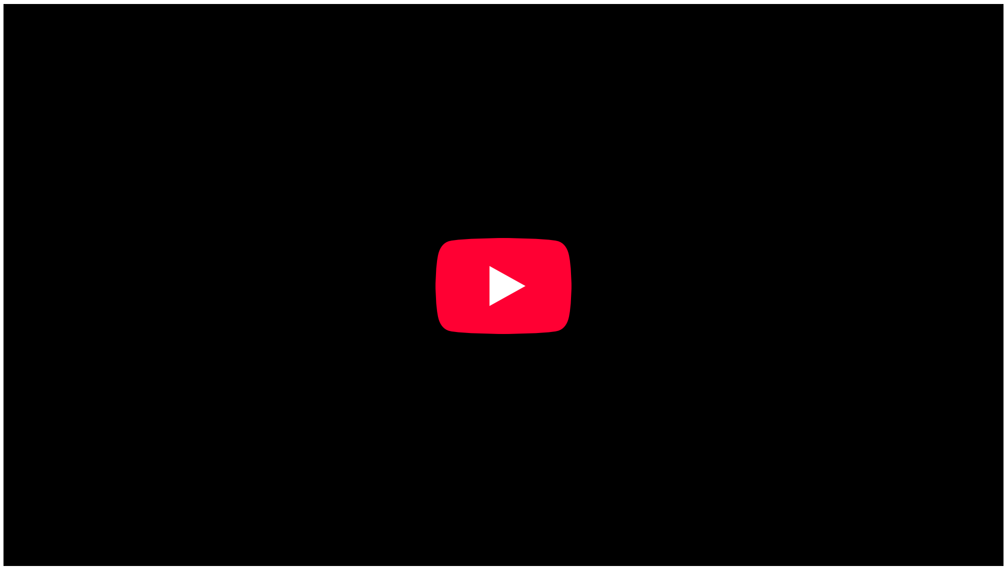
$$\mathrm{-\Delta PE=\Delta KE} \tag{5.4.5.9}$$

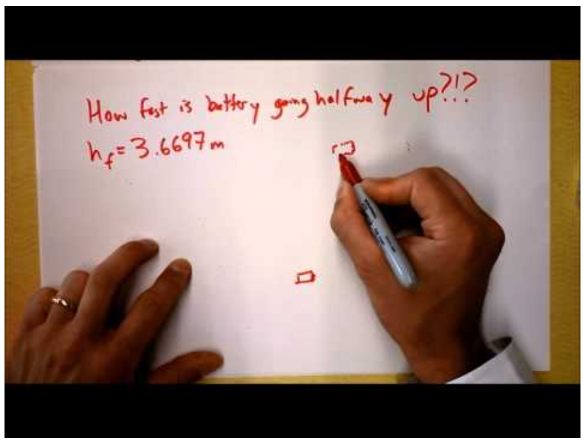
This equation means that the total kinetic and potential energy is constant for any process involving only conservative forces. That is,

$$\mathrm{KE+PE=const\ or\ KE_i+PE_i=KE_f+PE_f,} \tag{5.4.5.10}$$

where i and f denote initial and final values. This equation is a form of the work-energy theorem for conservative forces; it is known as the conservation of mechanical energy principle.

Remember that the law applies to the extent that all the forces are conservative, so that friction is negligible. The total kinetic plus potential energy of a system is defined to be its mechanical energy $\mathrm{(KE+PE)}$. In a system that experiences only conservative forces, there is a potential energy associated with each force, and the energy only changes form between KE and various types of PE (with the total energy remaining constant).





Conservation of Mechanical Energy: Worked example.

Problem Solving With the Conservation of Energy

To solve a conservation of energy problem determine the system of interest, apply law of conservation of energy, and solve for the unknown.

learning objectives

- Identify steps necessary to solve a conservation of energy problem

Problem-solving Strategy

You should follow a series of steps whenever you are problem solving:

Step One

Determine the system of interest and identify what information is given and what quantity is to be calculated. For example, let’s assume you have the problem with car on a roller coaster. You know that the cars of a roller coaster reach their maximum kinetic energy (KEKE) when at the bottom of their path. When they start rising, the kinetic energy begins to be converted to gravitational potential energy (PEgPEg). The sum of kinetic and potential energy in the system should remain constant, if losses to friction are ignored.

Determining Energy: The cars of a roller coaster reach their maximum kinetic energy when at the bottom of their path. When they start rising, the kinetic energy begins to be converted to gravitational potential energy. The sum of kinetic and potential energy in the system remains constant, ignoring losses to friction.

Step Two

Examine all the forces involved and determine whether you know or are given the potential energy from the work done by the forces. Then use step three or step four.

Step Three

If you know the potential energies (PE) for the forces that enter into the problem, then forces are all conservative, and you can apply conservation of mechanical energy simply in terms of potential and kinetic energy. The equation expressing conservation of energy is:

$$\mathrm{KE_i + PE_i = KE_f + PE_f}. \tag{5.4.5.11}$$

Step Four

If you know the potential energy for only some of the forces, then the conservation of energy law in its most general form must be used:

$$\mathrm{KE_i + PE_i + W_{nc} + OE_i = KE_f + PE_f + OE_f} \tag{5.4.5.12}$$

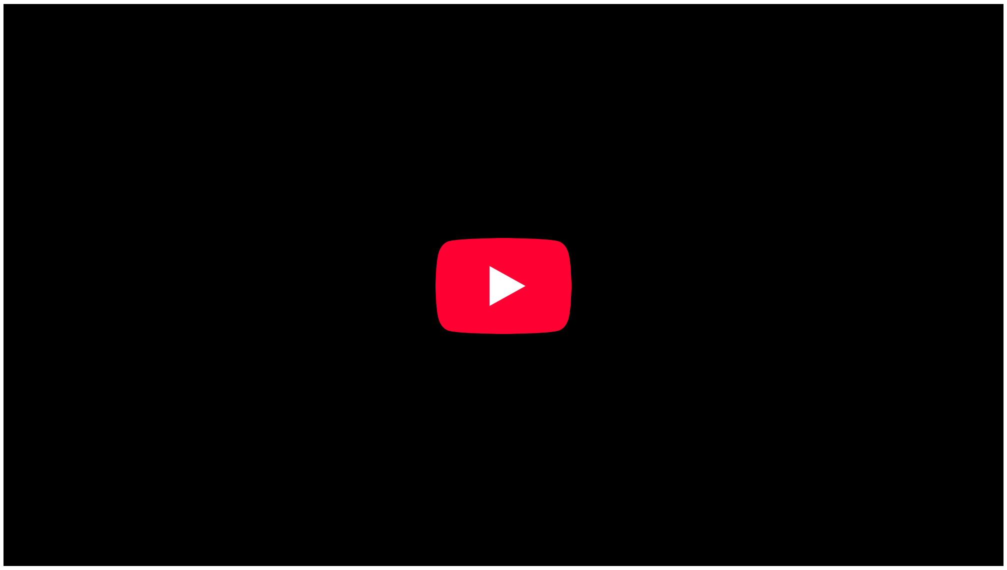
where $\mathrm{O_E}$ stand for all other energies, and $\mathrm{W_{nc}}$ stands for work done by non-conservative forces. In most problems, one or more of the terms is zero, simplifying its solution. Do *not* calculate $\mathrm{W_c}$, the work done by conservative forces; it is already incorporated in the PE terms.

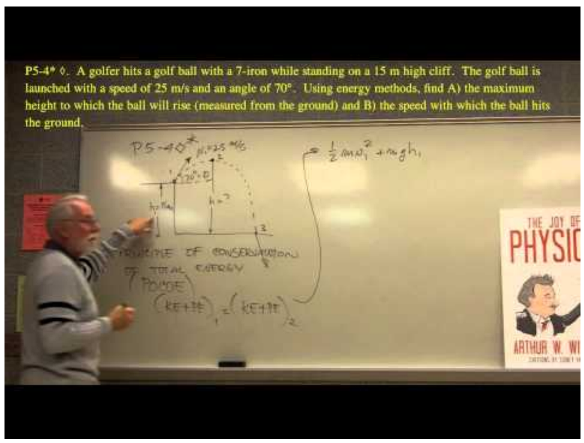
Step Five

You have already identified the types of work and energy involved (in step two). Before solving for the unknown, eliminate terms wherever possible to simplify the algebra. For example, choose height $\mathrm{h = 0}$ at either the initial or final point—this will allow to set PEg at zero. Then solve for the unknown in the customary manner.

Step Six

Check the answer to see if it is reasonable. Once you have solved a problem, reexamine the forms of work and energy to see if you have set up the conservation of energy equation correctly. For example, work done against friction should be negative, potential energy at the bottom of a hill should be less than that at the top, and so on.





Energy conservation: Part of a series of videos on physics problem-solving. The problems are taken from “The Joy of Physics. ” This one deals with energy conservation. The viewer is urged to pause the video at the problem statement and work the problem before watching the rest of the video.

Problem Solving with Dissipative Forces

In the presence of dissipative forces, total mechanical energy changes by exactly the amount of work done by nonconservative forces (W_c).

learning objectives

- Express the energy conservation relationship that can be applied to solve problems with dissipative forces

INTRODUCTION

We have seen a problem-solving strategy with the conservation of energy in the previous section. Here we will adopt the strategy for problems with dissipative forces. Since the work done by nonconservative (or dissipative) forces will irreversibly alter the energy of the system, the total mechanical energy (KE + PE) changes by exactly the amount of work done by nonconservative forces (W_c). Therefore, we obtain $\mathrm{KE_i + PE_i + W_{nc} = KE_f + PE_f}$, where KE and PE represent kinetic and potential energies respectively. Therefore, using the new energy conservation relationship, we can apply the same problem-solving strategy as with the case of conservative forces.

EXAMPLE

Consider the situation shown in, where a baseball player slides to a stop on level ground. Using energy considerations, calculate the distance the 65.0-kg baseball player slides, given that his initial speed is 6.00 m/s and the force of friction against him is a *constant 450 N.*

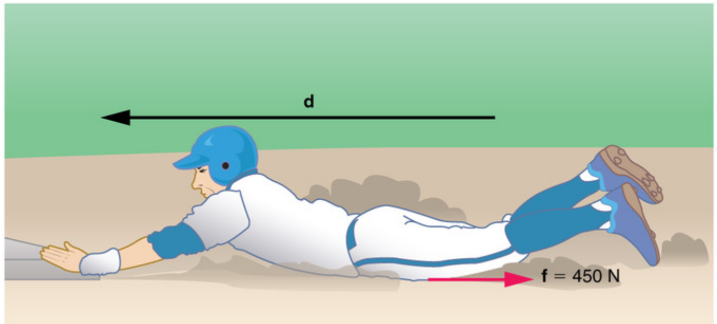


Fig 1: The baseball player slides to a stop in a distance d. In the process, friction removes the player's kinetic energy by doing an amount of work fd equal to the initial kinetic energy.

Strategy: Friction stops the player by converting his kinetic energy into other forms, including thermal energy. In terms of the work-energy theorem, the work done by friction (f), which is negative, is added to the initial kinetic energy to reduce it to zero. The work done by friction is negative, because f is in the opposite direction of the motion (that is, $\theta = 180^\circ$, and so $\cos\theta = -1$). Thus $W_{nc} = -fd$. The equation simplifies to $\frac{1}{2}mv_i^2 - fd = 0$.

Solution: Solving the previous equation for d and substituting known values yields, we get d = 2.60 m. The most important point of this example is that the amount of nonconservative work equals the change in mechanical energy.

Key Points

- If a particle travels in a closed loop, the net work done (the sum of the force acting along the path multiplied by the distance travelled) by a conservative force is zero.
- Conservative force is dependent only on the position of the object. If a force is conservative, it is possible to assign a numerical value for the potential at any point.
- Nonconservative force transfer the energy from the system in an energy form which can not be used by the force to transfer back to the object in motion.
- If the work for an applied force is independent of the path, then the work done by the force is evaluated at the start and end of the trajectory of the point of application. This means that there is a function U(x), called a " potential ".
- It is tradition to define the potential function with a negative sign so that positive work is represented as a reduction in the potential.
- Every conservative force gives rise to potential energy. Examples are elastic potential energy, gravitational potential energy, and electric potential energy.
- Gravitational potential energy near the earth can be expressed with respect to the height from the surface of the Earth as PE = mgh. g = gravitational acceleration ($9.8 m/s^2$). Near the surface of the Earth, g can be considered constant.
- Over large variations in distance, the approximation that g is constant is no longer valid and a general formula should be used for the potential. It is given as: $U(r) = \int_r (G\frac{mM}{r'^2})dr' = -G\frac{mM}{r} + K$.
- Choosing the convention that the constant of integration K=0 assumes that the potential at infinity is defined to be 0.
- The displacement of spring x is usually measured from the position of "neutral length " or "relaxed length". Often, it is most convenient to identify this position as the origin of coordinate reference (x=0).
- If the block is gently released from the stretched position ($x = x_f$), energy conservation tells us that $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kx_f^2 = \text{constant}$.
- If the block is released from the stretched position ($x = x_f$), by the time the block reaches x=0 position, its speed will be $v(x=0) = \sqrt{\frac{k}{m}}x_f$. The block will keep oscillating between $x = -x_f$ and x_f.
- The conservation of mechanical energy can be written as "$KE + PE = \text{const}$ ".
- Though energy cannot be created nor destroyed in an isolated system, it can be internally converted to any other form of energy.
- In a system that experiences only conservative forces, there is a potential energy associated with each force, and the energy only changes form between KE and various types of PE, with the total energy remaining constant.
- If you know the potential energies for the forces that enter into the problem, then forces are all conservative, and you can apply conservation of mechanical energy simply in terms of potential and kinetic energy. The equation expressing conservation of

energy is: $\mathrm{KE_i + PE_i = KE_f + PE_f}$. .

- If you know the potential energy for only some of the forces, then the conservation of energy law in its most general form must be used: $\mathrm{KE_i + PE_i + W_{nc} + OE_i = KE_f + PE_f + OE_f}$, where OE stands for all other energies.
- Once you have solved a problem, always check the answer to see if it is reasonable.
- Using the new energy conservation relationship

$$\mathrm{KE_i + PE_i + W_{nc} = KE_f + PE_f} \tag{5.4.5.13}$$

, we can apply the same problem-solving strategy as with the case of conservative forces.

- The most important point is that the amount of nonconservative work equals the change in mechanical energy.
- The work done by nonconservative (or dissipative) forces will irreversibly dissipated in the system.

Key Terms

- **potential**: A curve describing the situation where the difference in the potential energies of an object in two different positions depends only on those positions.
- **Coulomb force**: the electrostatic force between two charges, as described by Coulomb's law
- **potential**: A curve describing the situation where the difference in the potential energies of an object in two different positions depends only on those positions.
- **conservative force**: A force with the property that the work done in moving a particle between two points is independent of the path taken.
- **Hooke's law**: the principle that the stress applied to a solid is directly proportional to the strain produced. This law describes the behavior of springs and solids stressed within their elastic limit.
- **conservation**: A particular measurable property of an isolated physical system does not change as the system evolves.
- **isolated system**: A system that does not interact with its surroundings, that is, its total energy and mass stay constant.
- **frictional force**: Frictional force is the force resisting the relative motion of solid surfaces, fluid layers, and material elements sliding against each other.
- **kinetic energy**: The energy possessed by an object because of its motion, equal to one half the mass of the body times the square of its velocity.
- **potential energy**: The energy an object has because of its position (in a gravitational or electric field) or its condition (as a stretched or compressed spring, as a chemical reactant, or by having rest mass)
- **conservative force**: A force with the property that the work done in moving a particle between two points is independent of the path taken.
- **dissipative force**: A force resulting in dissipation, a process in which energy (internal, bulk flow kinetic, or system potential) is transformed from some initial form to some irreversible final form.

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5.4.6: Power

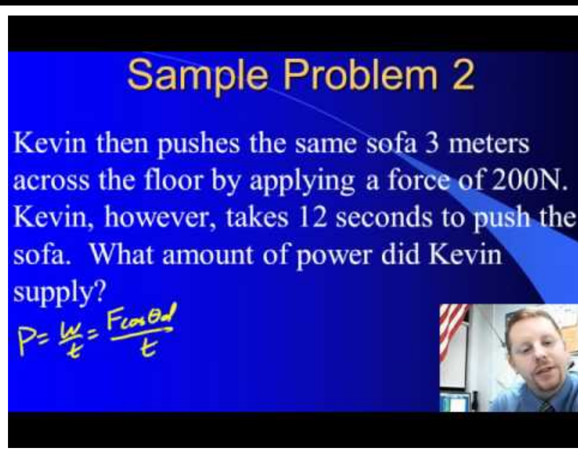
What is Power?

In physics, power is the rate of doing work—the amount of energy consumed per unit time.

learning objectives

- Relate power to the transfer, use, and transformation of different types of energy

In physics, power is the rate of doing work. It is the amount of energy consumed per unit time. The unit of power is the joule per second (J/s), known as the watt (in honor of James Watt, the eighteenth-century developer of the steam engine). For example, the rate at which a lightbulb transforms electrical energy into heat and light is measured in watts (W)—the more wattage, the more power, or equivalently the more electrical energy is used per unit time.



Power: A brief overview of power in an algebra-based physics course.

Energy transfer can be used to do work, so power is also the rate at which this work is performed. The same amount of work is done when carrying a load up a flight of stairs whether the person carrying it walks or runs, but more power is expended during the running because the work is done in a shorter amount of time. The output power of an electric motor is the product of the torque the motor generates and the angular velocity of its output shaft. The power expended to move a vehicle is the product of the traction force of the wheels and the velocity of the vehicle.

Examples of power are limited only by the imagination, because there are as many types as there are forms of work and energy. Sunlight reaching Earth's surface carries a maximum power of about 1.3 kilowatts per square meter (kW/m^2). A tiny fraction of this is retained by Earth over the long term. Our consumption rate of fossil fuels is far greater than the rate at which they are stored, so it is inevitable that they will be depleted. Power implies that energy is transferred, perhaps changing form. It is never possible to change one form completely into another without losing some of it as thermal energy. For example, a 60-W incandescent bulb converts only 5 W of electrical power to light, with 55 W dissipating into thermal energy. Furthermore, the typical electric power plant converts only 35 to 40 percent of its fuel into electricity. The remainder becomes a huge amount of thermal energy that must be dispersed as heat transfer, as rapidly as it is created. A coal-fired power plant may produce 1,000 megawatts; 1 megawatt (MW) is 106 W of electric power. But the power plant consumes chemical energy at a rate of about 2,500 MW, creating heat transfer to the surroundings at a rate of 1,500 MW.

Coal-fired Power Plant: Tremendous amounts of electric power are generated by coal-fired power plants such as this one in China, but an even larger amount of power goes into heat transfer to the surroundings. The large cooling towers here are needed to transfer heat as rapidly as it is produced. The transfer of heat is not unique to coal plants but is an unavoidable consequence of generating electric power from any fuel—nuclear, coal, oil, natural gas, or the like.

Humans: Work, Energy, and Power

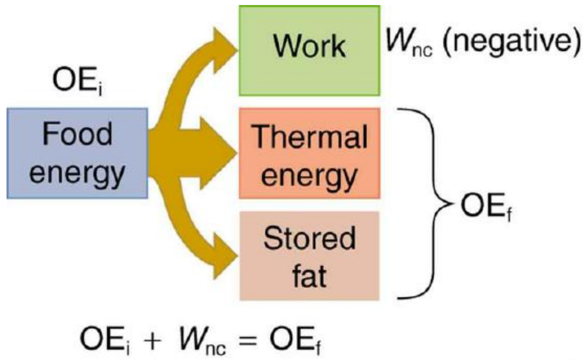
The human body converts energy stored in food into work, thermal energy, and/or chemical energy that is stored in fatty tissue.

learning objectives

- Identify what factors play a role in basal metabolic rate (BMR)

Humans: Work, Energy, and Power

Our own bodies, like all living organisms, are energy conversion machines. Conservation of energy implies that the chemical energy stored in food is converted into work, thermal energy, or stored as chemical energy in fatty tissue, as shown in. Energy consumed by humans is converted to work, thermal energy, and stored fat. By far the largest fraction goes to thermal energy, although the fraction varies depending on the type of physical activity. The fraction going into each form depends both on how much we eat and on our level of physical activity. If we eat more than is needed to do work and stay warm, the remainder goes into body fat.



Energy Conversion in Humans: Energy consumed by humans is converted to work, thermal energy, and stored fat. By far the largest fraction goes to thermal energy, although the fraction varies depending on the type of physical activity.

Functions that Require Energy

All bodily functions, from thinking to lifting weights, require energy. The many small muscle actions accompanying all quiet activity, from sleeping to head scratching, ultimately become thermal energy, as do less visible muscle actions by the heart, lungs, and digestive tract. Shivering, in fact, is an involuntary response to low body temperature that pits muscles against one another to produce thermal energy in the body (and do no work). The kidneys and liver consume a surprising amount of energy, but the biggest surprise of all it that a full 25% of all energy consumed by the body is used to maintain electrical potentials in all living cells. (Nerve cells use this electrical potential in nerve impulses.) This bioelectrical energy ultimately becomes mostly thermal energy, but some is utilized to power chemical processes such as in the kidneys and liver, and in fat production.

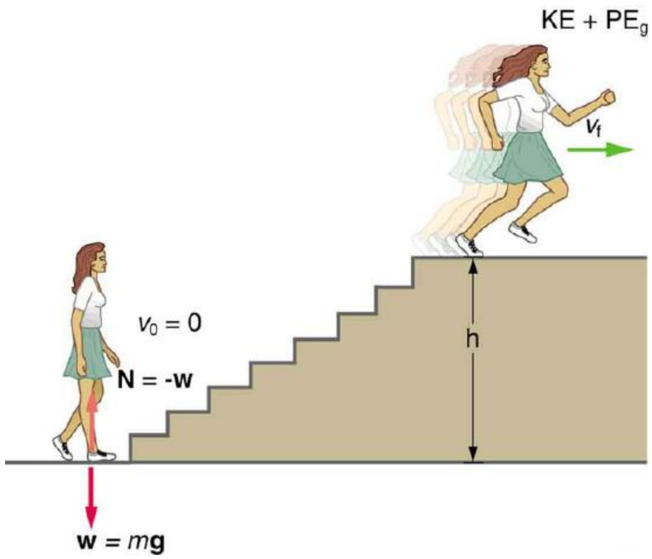
Basal Metabolic Rate

The rate at which the body uses food energy to sustain life and to do different activities is called the metabolic rate. The total energy conversion rate of a person at rest is called the basal metabolic rate (BMR) and is divided among various systems in the body. The largest fraction goes to the liver and spleen, with the brain coming next. Of course, during vigorous exercise, the energy consumption of the skeletal muscles and heart increase markedly. About 75% of the calories burned in a day go into these basic functions. The BMR is a function of age, gender, total body weight, and amount of muscle mass (which burns more calories than body fat). Athletes have a greater BMR due to this last factor.

Useful Work

Work done by a person is sometimes called useful work, which is work done on the outside world, such as lifting weights. Useful work requires a force exerted through a distance on the outside world, and so it excludes internal work, such as that done by the heart when pumping blood. Useful work does include that done in climbing stairs or accelerating to a full run, because these are accomplished by exerting forces on the outside world. Forces exerted by the body are nonconservative, so that they can change the mechanical energy (KE+PE) of the system worked upon, and this is often the goal.

For example, what is the power output for a 60.0-kg woman who runs up a 3.00 m high flight of stairs in 3.50 s, starting from rest but having a final speed of 2.00 m/s?.



Woman Running Up Stairs: When this woman runs upstairs starting from rest, she converts the chemical energy originally from food into kinetic energy and gravitational potential energy. Her power output depends on how fast she does this.

Her power output depends on how fast she does this. The work going into mechanical energy is W= KE + PE. At the bottom of the stairs, we take both KE and PE_g as initially zero; thus,

$$W = KE_f + PE_g = \frac{1}{2}mv_f^2 + mgh \tag{5.4.6.1}$$

where h is the vertical height of the stairs. Because all terms are given, we can calculate W and then divide it by time to get power. Substituting the expression for W into the definition of power given in the previous equation, $P = \frac{W}{t}$ yields

$$P = \frac{W}{t} = \frac{\frac{1}{2}mv_f^2 + mgh}{t} \tag{5.4.6.2}$$

Entering known values yields

$$P = \frac{0.5(60.0kg)(2.00m/s)^2 + (60.0kg)(9.80m/s^2)(3.00m)}{(3.50s)} = \frac{120J + 1764J}{3.50s} = 538W \tag{5.4.6.3}$$

The woman does 1764 J of work to move up the stairs compared with only 120 J to increase her kinetic energy; thus, most of her power output is required for climbing rather than accelerating.

Energy consumption is directly proportional to oxygen consumption because the digestive process is basically one of oxidizing food. We can measure the energy people use during various activities by measuring their oxygen use. Approximately 20 kJ of energy are produced for each liter of oxygen consumed, independent of the type of food.

Key Points

- Power implies that energy is transferred, perhaps changing form.
- Energy transfer can be used to do work, so power is also the rate at which this work is performed.
- The unit of power is the joule per second (J/s), known as the watt.
- The rate at which the body uses food energy to sustain life and to do different activities is called the metabolic rate, and the corresponding rate when at rest is called the basal metabolic rate (BMR).
- The energy included in the basal metabolic rate is divided among various systems in the body, with the largest fraction going to the liver and spleen, and the brain coming next.
- About 75% of food calories are used to sustain basic body functions included in the basal metabolic rate.
- Work done by a person is sometimes called useful work, which is work done on the outside world, such as lifting weights.
- The energy consumption of people during various activities can be determined by measuring their oxygen use, because the digestive process is basically one of oxidizing food.

Key Terms

- **power**: A measure of the rate of doing work or transferring energy.
- **watt**: In the International System of Units, the derived unit of power; the power of a system in which one joule of energy is transferred per second.
- **basal metabolic rate**: The amount of energy expended while at rest in a neutrally temperate environment, in the post-absorptive state.

5.4.7: CASE STUDY- World Energy Use

World Energy Use

The most prominent sources of energy used in the world are non-renewable (i.e., unsustainable).

learning objectives

- Explain why renewable energy sources must be found and utilized

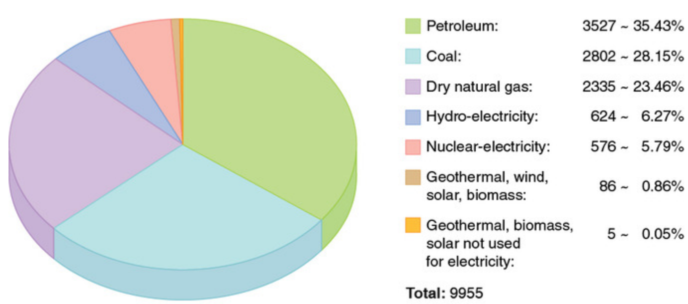
Energy Use

World energy consumption is the total amount of energy used by all humans on the planet (measured on a per-year basis). This measurement is the sum of all energy sources (and purposes) in use. Who measures this? Several organizations publish this data, including the International Energy Agency (IEA), the US Energy Information Administration (EIA), and the European Environment Agency. This data is useful because evaluating this information to discover trends might yield energy issues not currently being addressed, thereby encouraging the search for solutions. The IEA established a goal of limiting global warming to 2 degrees Celsius, but this goal is becoming more difficult to reach each year that the necessary action is not taken. In global energy use, fossil fuels make up a substantial portion. In 2011 they received over $500 billion in subsidies—six times more than that received by renewable energy sources.

Implementing new practices that will utilize different, renewable energy sources is important because having access to energy is important—it maintains our quality of life. Fossil fuels, however, are not sustainable at the rate they are currently used. About 40% of the world's energy comes from oil, but oil prices are dependent on uncertain factors (such as availability, politics, and world events). The United States alone uses 24% of the world's oil per year, yet it makes up only 4.5% of the world's population! In 2008, total worldwide energy consumption was 474 exajoules (474×10^{18} J=132,000 TWh)—equivalent to an average power usage of 15 terawatts (1.504×10^{13} W). Potential renewable energy sources include: solar energy at 1600 EJ (444,000 TWh), wind power at 600 EJ (167,000 TWh), geothermal energy at 500 EJ (139,000 TWh), biomass at 250 EJ (70,000 TWh), hydropower at 50 EJ (14,000 TWh) and ocean energy at 1 EJ (280 TWh).

Types of Energy

shows a pie chart of world energy usage by category—both renewable and nonrenewable sources. Renewable energy comes from sources with an unlimited supply. This includes energy from water, wind, the sun, and biomass. In the US, only 10% of energy comes from renewable sources (mostly hydroelectric energy). Nonrenewable sources makes up 85% of worldwide energy usage—from sources that eventually will be depleted, such as oil, natural gases and coal.



World Energy Use: This chart shows that the primary worldwide energy sources nonrenewable. If new practices are not put in place now, this model will not be sustainable.

Energy Needs

In the last 50 years, the global energy demand has tripled due to the number of developing countries and innovations in technology. It is projected to triple again over the next 30 years. In Europe, many in such developing areas recognize that the need for renewable energy sources, as the present course of energy usage cannot be sustained indefinitely. While renewable energy development makes up a only small percentage of the field, strides are being made in natural energy, particularly wind energy.

For example, by the year 2020 Germany plans to meet 10% of their total energy usage and 20% of its electricity usage with renewable resources. While some countries are making improvements in this field, coal usage is still a huge problem. In China, two thirds of the energy used each year is from commercial coal energy. India imports 50% of its oil, and 70% of its electricity is produced from coal, which is highly polluting.

Key Points

- The energy consumption increases with the increasing number of developing areas. In order for this development to continue, while maintaining quality of life, new and renewable energy sources must be found and utilized.
- Renewable energy comes from sources that will never deplete, no matter how much is used. An example of this is wind energy, which had been growing in popularity in countries like India and Germany.
- Nonrenewable energy makes up 85% the energy used on earth—the most popular form of energy being oil.

Key Terms

- **fossil fuel**: Any fuel derived from hydrocarbon deposits such as coal, petroleum, natural gas and, to some extent, peat; these fuels are irreplaceable, and their burning generates the greenhouse gas carbon dioxide.
- **renewable energy**: Energy that can be replenished at the same rate as it is used.

5.4.8: Further Topics

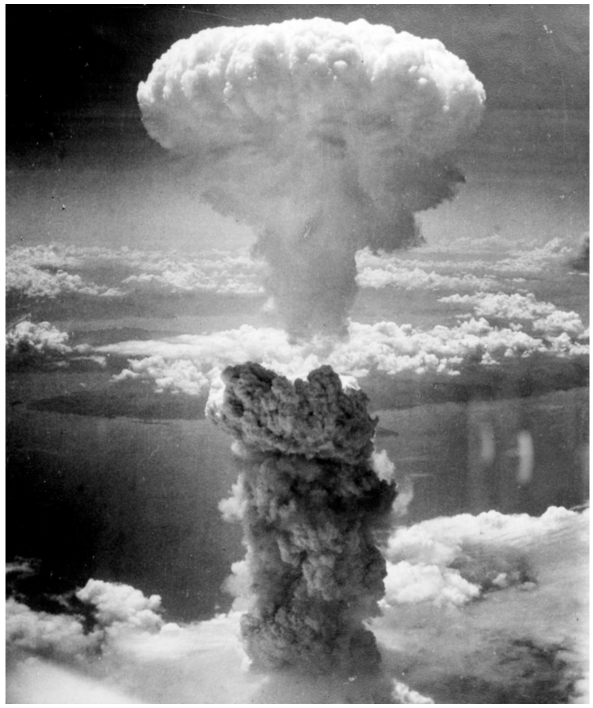
learning objectives

- Compare the different forms of energy interrelate to one another

Other Forms of Energy

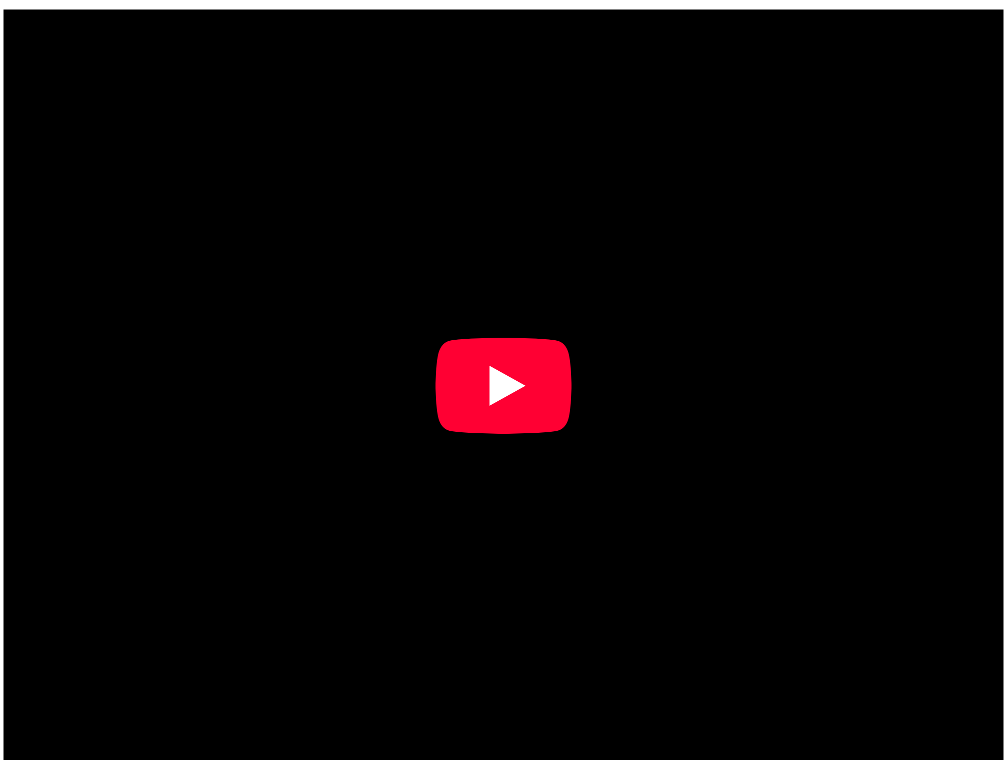
Thermal, chemical, electric, radiant, nuclear, magnetic, elastic, sound, mechanical, luminous, and mass are forms that energy can exist in. Energy can come in a variety of forms. These forms include:

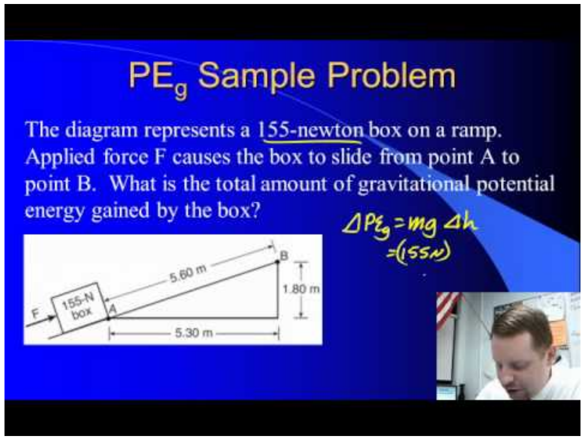
- Thermal Energy: This is energy associated with the microscopic random motion of particles in the media under consideration. An example of something that stores thermal energy is warm bath water.
- Chemical Energy: This is energy due to the way that atoms are arranged in molecules and various other collections of matter. An example of something that stores chemical energy is food. When your body digests and metabolizes food it utilizes its chemical energy.
- Electric Energy: This is energy that is from electrical potential energy, a result of Coulombic forces. Electrical potential energy is associated with the way that point charges in a system are arranged. An example of something that stores electric energy is a capacitor. A capacitor collects positive charge on one plate and negative charge on the other plate. Energy is thus stored in the resulting electrostatic field.
- Radiant Energy: This is any kind of electromagnetic radiation (see key term). An example of an electromagnetic wave is light.
- Nuclear Energy: This type of energy is liberated during the nuclear reactions of fusion and fission. Examples of things that utilize nuclear energy include nuclear power plants and nuclear weapons.
- Magnetic Energy: Technically magnetic energy is electric energy; the two are related by Maxwell's equations. An example of something that stores magnetic energy is a superconducting magnet used in an MRI.
- Elastic Energy: This is potential mechanical energy that is stored in the configuration of a material or physical system as work is performed to distort its volume or shape. An example of something that stores elastic energy is a stretched rubber band.
- Sound Energy: This is energy that is associated with the vibration or disturbance of matter. An example of something that creates sound energy is your voice box (larynx).
- Mechanical Energy: This is energy that is associated with the motion and position of an object. It is the sum of all of the kinetic and potential energy that the object has. An example of something that utilizes mechanical energy is a pendulum.
- Luminous Energy: This is energy that can be seen because it is visible light. An example of luminous energy is light from a flashlight.
- Mass: Can be converted to energy via: $\mathrm{E=mc^2}$. For example, mass is converted into energy when a nuclear bomb explodes.



Atomic bomb explosion: The mushroom cloud of the atomic bombing of Nagasaki, Japan

In each of the aforementioned forms, energy exists as either kinetic energy, potential energy, or a combination of both. It is important to note that the above list is not necessarily complete as we may discover new forms of energy in the future such as “dark energy. ” Also, each of the forms that energy can take on (as listed above) are not necessarily mutually exclusive. For example, luminous energy is radiant energy.





Types of Energy: A brief overview of energy, kinetic energy, gravitational potential energy, and the work-energy theorem for algebra-based physics students.

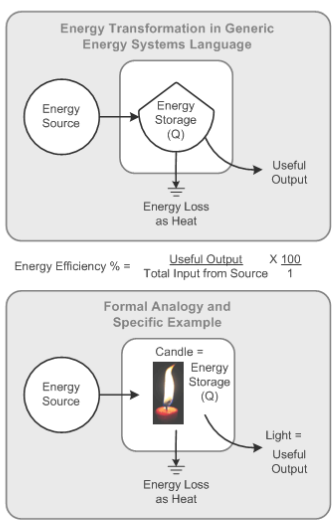
Energy Transformations

Energy transformation occurs when energy is changed from one form to another, and is a consequence of the first law of thermodynamics.

learning objectives

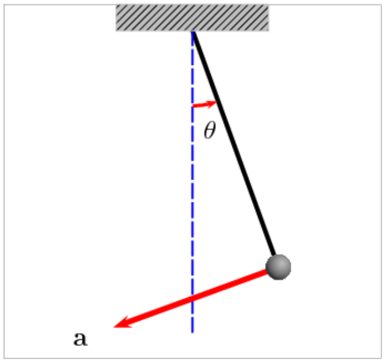
- Summarize the consequence of the first law of thermodynamics on the total energy of a system

Energy transformation occurs when energy is changed from one form to another. It is a consequence of the first law of thermodynamics that the total energy of a given system can only be changed when energy is added or subtracted from the system. Often it appears that energy has been lost from a system when it simply has been transformed. For example, an internal combustion engine converts the potential chemical energy in gasoline and oxygen into heat energy. This heat energy is then converted to kinetic energy, which is then used to propel the vehicle that is utilizing the engine. The technical term for a device that converts energy from one form to another is a *transducer*.



Energy Transformation: These figures illustrate the concepts of energy loss and useful energy output.

When analyzing energy transformations, it is important to consider the efficiency of conversion. The efficiency of conversions describes the ratio between the useful output and input of an energy conversion machine. Some energy transformations can occur with an efficiency of essentially 100%. For example, imagine a pendulum in a vacuum. As illustrated in, when the pendulum's mass reaches its maximum height, all if its energy exists in the form of potential energy. However, when the pendulum is at its lowest point, all of its energy exists in the form of kinetic energy.



Pendulum: This animation shows the velocity and acceleration vectors for a pendulum. One may note that at the maximum height of the pendulum's mass, the velocity is zero. This corresponds to zero kinetic energy and thus all of the energy of the pendulum is in the form of potential energy. When the pendulum's mass is at its lowest point, all of its energy is in the form of kinetic energy and we see its velocity vector has a maximum magnitude here.

Other energy transformations occur with a much lower efficiency of conversion. For example, the theoretical limit of the energy efficiency of a wind turbine (converting the kinetic energy of the wind to mechanical energy) is 59%. The process of photosynthesis is able to transform the light energy of the sun into chemical energy that can be used by a plant with an efficiency of conversion of a mere 6%.

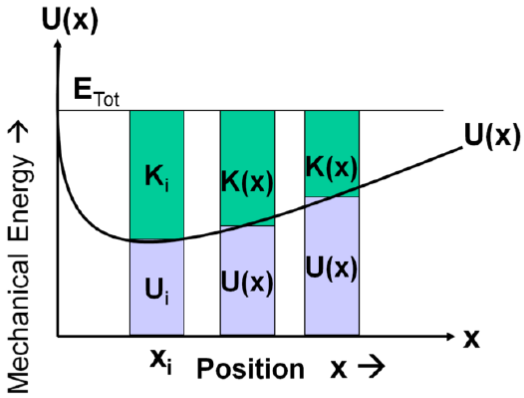
Potential Energy Curves and Equipotentials

A potential energy curve plots potential energy as a function of position; equipotential lines trace lines of equal potential energy.

learning objectives

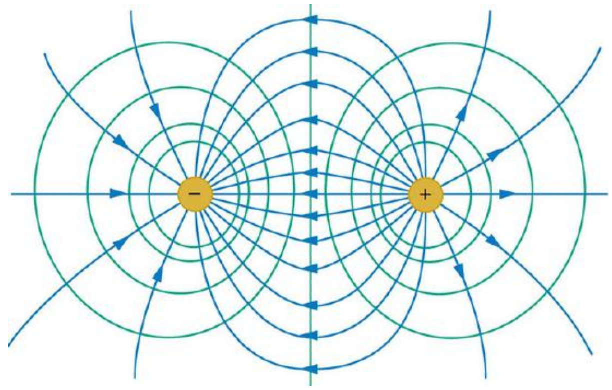
- Derive the potential of a point charge

A potential energy curve plots the potential energy of an object as a function of that object's position. For example, see. The system under consideration is a closed system, so the total energy of the system remains constant. This means that the kinetic and potential energy always have to sum to be the same value. We observe that the potential energy increases as the kinetic energy decreases and vice versa. The utility of a potential energy curve is that we can quickly determine the potential energy of the object in question at a given position.



Potential Energy Curve: This figure illustrates the potential energy of a particle as a function of position. The kinetic energy is also shown and is abbreviated K.

Equipotential lines trace lines of equal potential energy. In, if you were to draw a straight horizontal line through the center, that would be an equipotential line. In and, if you travel along an equipotential line, the electric potential will be constant.



Equipotential Lines for Two Equal and Opposite Point Charges: Electric field (blue) and equipotential lines (green) for two equal and opposite charges

Let us examine the physical explanation for the equipotentials lines in. The equation for the potential of a point charge is $\mathrm{V}=\frac{\mathrm{kQ}}{\mathrm{r}}$, where *V* is the potential, *k* is a constant with a value of $8.99 \cdot 10^9$ N m^2/C^2, *Q* is the magnitude of the point charge, and *r*is the distance from the charge. So, every point that is the same distance from the point charge will have the same electric potential energy. Therefore, if we draw a circle around the point charge, every point on the circle will have the same potential energy.

Work (*W*) is a measure of the change in potential energy $(\Delta \mathrm{PE}): \mathrm{W}=-$\DeltaPE. Since the potential energy does not change along an equipotential line, you do not need to do any work to move along one. However, you *do* need to do work to move from one equipotential line to another. Recall that work is zero if force is perpendicular to motion; in the figures shown above, the forces resulting from the electric field are in the same direction as the electric field itself. So we note that each of the equipotential lines must be perpendicular to the electric field at every point.

Key Points

- Thermal, chemical, electric, radiant, nuclear, magnetic, elastic, sound, mechanical, luminous, and mass are forms that energy can exist in.
- Energy exists as either kinetic energy, potential energy, or a combination of both.
- We may discover new forms of energy (like “dark energy”) in the future.
- The total energy of a given system can only be changed when energy is added or subtracted from the system.
- Often it appears that energy has been lost from a system when it simply has been transformed.
- The efficiency of conversions describes the ratio between the useful output of an energy conversion machine and the input.
- A potential energy curve plots the potential energy of an object as a function of its position.
- Equipotential lines trace lines of equal potential energy.
- You do not need to do any work to move along an equipotential line.

Key Terms

- **fusion**: A nuclear reaction in which nuclei combine to form more massive nuclei with the concomitant release of energy.
- **electromagnetic radiation**: radiation (quantized as photons) consisting of oscillating electric and magnetic fields oriented perpendicularly to each other, moving through space
- **fission**: The process of splitting the nucleus of an atom into smaller particles; nuclear fission.
- **pendulum**: A body suspended from a fixed support so that it swings freely back and forth under the influence of gravity; it is commonly used to regulate various devices such as clocks.
- **first law of thermodynamics**: A version of the law of conservation of energy, specialized for thermodynamical systems. It is usually formulated by stating that the change in the internal energy of a closed system is equal to the amount of heat supplied to the system, minus the amount of work done by the system on its surroundings.
- **potential energy**: The energy an object has because of its position (in a gravitational or electric field) or its condition (as a stretched or compressed spring, as a chemical reactant, or by having rest mass)
- **kinetic energy**: The energy possessed by an object because of its motion, equal to one half the mass of the body times the square of its velocity.

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CHAPTER OVERVIEW

5.5: Static Equilibrium, Elasticity, and Torque

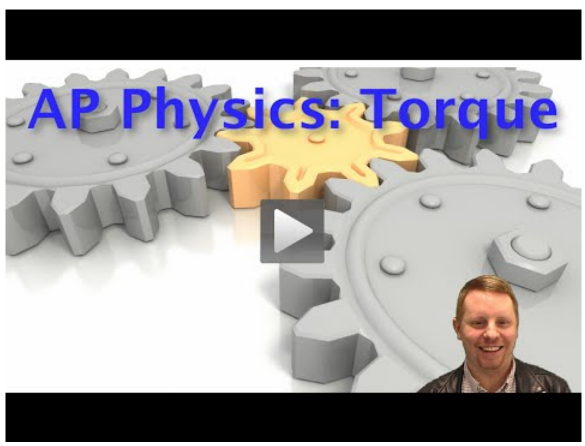
Topic hierarchy

5.5.1: Introduction

learning objectives

- Describe the effect of the torque on an object

Torque about a point is a concept that denotes the tendency of force to turn or rotate an object in motion. This tendency is measured in general about a point, and is termed as *moment of force*. The torque in angular motion corresponds to force in translation. It is the “cause” whose effect is either angular acceleration or angular deceleration of a particle in general motion. Quantitatively, it is defined as a vector given by:



Torque: A brief introduction to torque for students studying rotational motion in algebra-based physics courses such as AP Physics 1 and Honors Physics.

$$mathrmT = r \times F \tag{5.5.1.1}$$

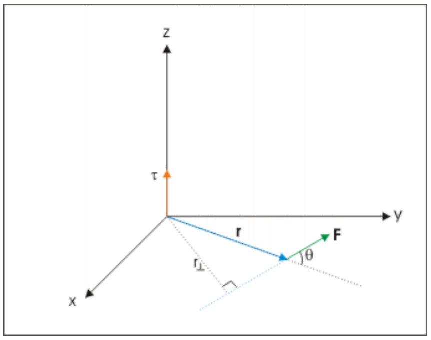
Rotation is a special case of angular motion. In the case of rotation, torque is defined with respect to an axis such that vector “r” is constrained as perpendicular to the axis of rotation. In other words, the plane of motion is perpendicular to the axis of rotation. Clearly, the torque in rotation corresponds to force in translation.

Torque is the cross product of force cross length of the moment arm; it is involved whenever there is a rotating object. Torque can also be expressed in terms of the angular acceleration of the object.

The determination of torque’s direction is relatively easier than that of angular velocity. The reason for this is simple: the torque itself is equal to vector product of two vectors, unlike angular velocity which is one of the two operands of the vector product.

Clearly, if we know the directions of two operands here, the direction of torque can easily be interpreted.

Since torque depends on both the force and the distance from the axis of rotation, the SI units of torque are newton-meters.



Torque: Torque in terms of moment arm.

Key Points

- Torque is found by multiplying the applied force by the distance to the axis of rotation, called the moment arm.
- Torque is to rotation as force is to motion.
- The unit of torque is the newton-meter.

Key Terms

- **vector**: A directed quantity, one with both magnitude and direction; the between two points.
- **angular velocity**: A vector quantity describing an object in circular motion; its magnitude is equal to the speed of the particle and the direction is perpendicular to the plane of its circular motion.
- **angular motion**: The motion of a body about a fixed point or fixed axis (as of a planet or pendulum). It is equal to the angle passed over at the point or axis by a line drawn to the body.

5.5.2: Conditions for Equilibrium

learning objectives

- Identify the first condition of equilibrium

First Condition of Equilibrium

For an object to be in equilibrium, it must be experiencing no acceleration. This means that both the net force and the net torque on the object must be zero. Here we will discuss the first condition, that of zero net force.

In the form of an equation, this first condition is:

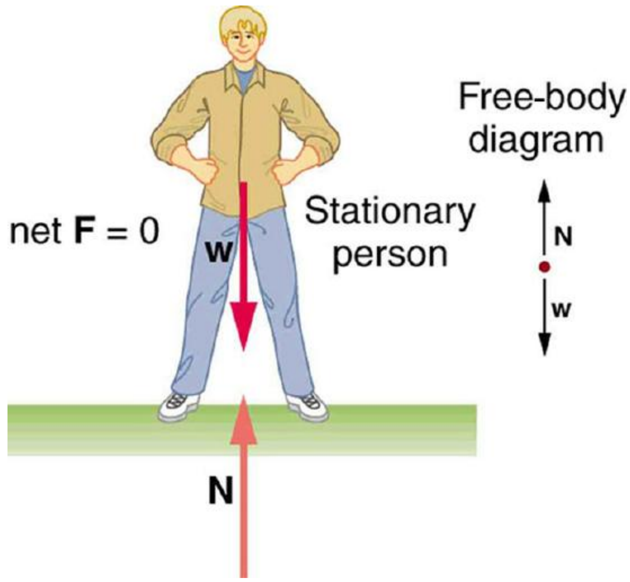
$F_{net} = 0.$

In order to achieve this conditon, the forces acting along *each* axis of motion must sum to zero. For example, the net external forces along the typical *x*– and *y*-axes are zero. This is written as

$\text{net } F_x = 0 \text{ and net } F_y = 0.$

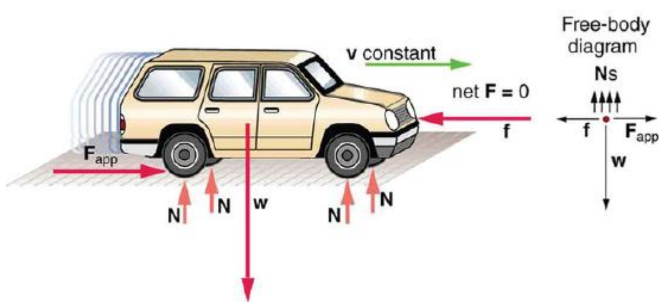
The condition $F_{net} = 0$ must be true for both static equilibrium, where the object's velocity is zero, and dynamic equilibrium, where the object is moving at a constant velocity.

Below, the motionless person is in static equilibrium. The forces acting on him add up to zero. Both forces are vertical in this case.



Person in Static Equilibrium: This motionless person is in static equilibrium.

Below, the car is in dynamic equilibrium because it is moving at constant velocity. There are horizontal and vertical forces, but the net external force in any direction is zero. The applied force between the tires and the road is balanced by air friction, and the weight of the car is supported by the normal forces, here shown to be equal for all four tires.



A Car in Dynamic Equilibrium: This car is in dynamic equilibrium because it is moving at constant velocity. The forces in all directions are balanced.

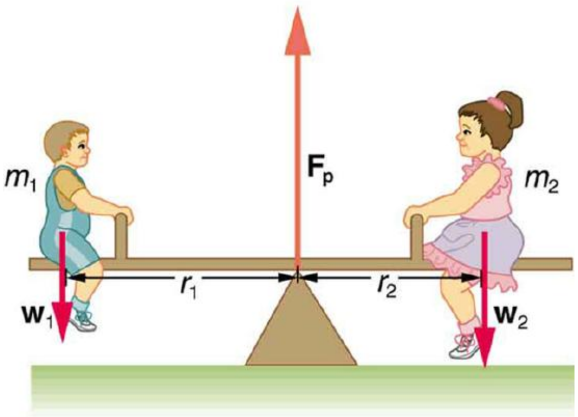
Second Condition

The second condition of static equilibrium says that the net torque acting on the object must be zero.

learning objectives

- Identify the second condition of static equilibrium

A child's seesaw, shown in, is an example of static equilibrium. An object in static equilibrium is one that has no acceleration in any direction. While there might be motion, such motion is constant.



Two children on a seesaw: The system is in static equilibrium, showing no acceleration in any direction.

If a given object is in static equilibrium, both the net force and the net torque on the object must be zero. Let's break this down:

Net Force Must Be Zero

The net force acting on the object must be zero. Therefore all forces balance in each direction. For example, a car moving along a highway at a constant speed is in equilibrium, as it is not accelerating in any forward or vertical direction. Mathematically, this is stated as $\mathrm{F_{net}} = \mathrm{ma} = 0$.

Net Torque Must Be Zero

The second condition necessary to achieve equilibrium involves avoiding accelerated rotation (maintaining a constant angular velocity). A rotating body or system can be in equilibrium if its rate of rotation is constant and remains unchanged by the forces acting on it.

To understand what factors affect rotation, let us think about what happens when you open an ordinary door by rotating it on its hinges.The magnitude, direction, and point of application of the force are incorporated into the definition of the physical quantity called torque—the rotational equivalent of a force. It is a measure of the effectiveness of a force in changing or accelerating a rotation (changing the angular velocity over a period of time).

In equation form, the magnitude of torque is defined to be $\tau = \mathrm{rF}\sin\theta$ where τ (the Greek letter tau) is the symbol for torque, r is the distance from the pivot point to the point where the force is applied, F is the magnitude of the force, and θ is the angle between the force and the vector directed from the point of application to the pivot point.

Two-Component Forces

In equilibrium, the net force and torque in any particular direction equal zero.

learning objectives

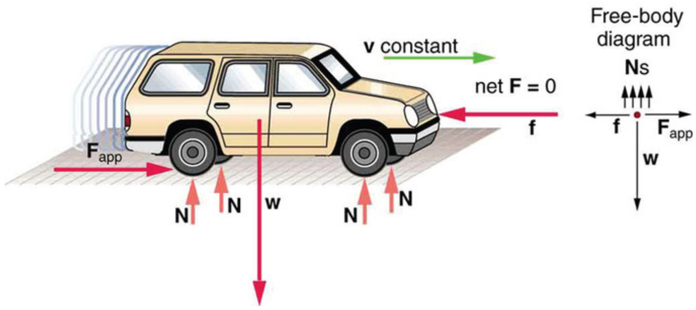
- Calculate the net force and the net torque for an object in equilibrium

An object with constant velocity has zero acceleration. A motionless object still has constant (zero) velocity, so motionless objects also have zero acceleration. Newton's second law states that:

$$\sum \mathrm{F} = \mathrm{ma} \tag{5.5.2.1}$$

so objects with constant velocity also have zero net external force. This means that all the forces acting on the object are balanced — that is to say, they are in equilibrium.

This rule also applies to motion in a specific direction. Consider an object moving along the *x*-axis. If no net force is applied to the object along the *x*-axis, it will continue to move along the *x*-axis at a constant velocity, with no acceleration.



Car Moving at Constant Velocity: A moving car for which the net x and y force components are zero

We can easily extend this rule to the *y*-axis. In any system, unless the applied forces cancel each other out (i.e., the resultant force is zero), there will be acceleration in the direction of the resultant force. In static systems, in which motion does not occur, the sum of the forces in all directions always equals zero. This concept can be represented mathematically with the following equations:

$$\sum \mathrm{F_x} = \mathrm{ma_x} = 0 \tag{5.5.2.2}$$

$$\sum \mathrm{F_y} = \mathrm{ma_y} = 0 \tag{5.5.2.3}$$

This rule also applies to rotational motion. If the resultant moment about a particular axis is zero, the object will have no rotational acceleration about the axis. If the object is not spinning, it will not start to spin. If the object is spinning, it will continue to spin at the same constant angular velocity. Again, we can extend this to moments about the *y*-axis as well. We can represent this rule mathematically with the following equations:

$$\sum \tau_{\mathrm{x}} = \mathrm{I}\alpha_{\mathrm{x}} = 0 \tag{5.5.2.4}$$

$$\sum \tau_{\mathrm{y}} = \mathrm{I}\alpha_{\mathrm{y}} = 0 \tag{5.5.2.5}$$

Key Points

- There are two conditions that must be met for an object to be in equilibrium.
- The first condition is that the net force on the object must be zero for the object to be in equilibrium.
- If net force is zero, then net force along any direction is zero.
- The second condition necessary to achieve equilibrium involves avoiding accelerated rotation.
- A rotating body or system can be in equilibrium if its rate of rotation is constant and remains unchanged by the forces acting on it.
- The magnitude of torque about a axis of rotation is defined to be $\tau = \mathrm{rF}\sin\theta$.
- In equilibrium, the net force in all directions is zero.
- If the net moment of inertia about an axis is zero, the object will have no rotational acceleration about the axis.
- In each direction, the net force takes the form: $\sum \mathrm{F} = \mathrm{ma} = 0$ and the net torque take the form: $\sum \tau = \mathrm{I}\alpha = 0$ where the sum represents the vector sum of all forces and torques acting.

Key Terms

- **force**: A physical quantity that denotes ability to push, pull, twist or accelerate a body which is measured in a unit dimensioned in mass × distance/time² (ML/T^2): SI: newton (N); CGS: dyne (dyn)
- **torque**: A rotational or twisting effect of a force; (SI unit newton-meter or Nm; imperial unit foot-pound or ft-lb)
- **translation**: Motion of a body on a linear path, without deformation or rotation, i.e. such that every part of the body moves at the same speed and in the same direction; also (in physics), the linear motion of a body considered independently of its rotation.
- **equilibrium**: The state of a body at rest or in uniform motion, the resultant of all forces on which is zero.

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5.5.3: Stability

learning objectives

- Explain the relationship between how center of mass is defined and static equilibrium

For an object to be in static equilibrium, we expect it to stay in the same state indefinitely. If it starts accelerating away from its current position, it would hardly be in equilibrium. To quantify equilibrium for a single object, there are two conditions:

1. The net external force on the object is zero: $\sum_i F_i = F_{net} = 0$
2. The net external torque, regardless of choice of origin, is also zero: $\sum_i r_i \times F_i = \sum_i \tau_i = \tau_{net} = 0$

Those two conditions hold regardless of whether the object we are talking about is a single point particle, a rigid body, or a collection of discrete particles. Being in equilibrium means that we expect no changes to the linear momentum or the angular momentum. Note that this does not mean that the system is not moving or rotating; instead it simply means that its movement will not change as time goes on.

In a special case when the external forces are governed by some potential (e.g. gravitational potential) we can gain insight into the nature of the equilibrium. From the definition of a potential we know that $F_{ext} = -\frac{dU(x)}{dx}|_{x_0}$. When the first derivative is zero, we can take the second derivative to find whether the equilibrium is stable or unstable. Explicitly, if the potential is concave-up at x_0, $\frac{d^2U(x)}{dx^2}|_{x_0} > 0$, then the system is stable; conversely, if the potential is concave-down, then the equilibrium is unstable. If the second derivative is zero or does not exist, then the equilibrium is neutral—neither stable nor unstable.

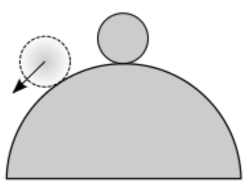
Mathematically, we can view this as a Taylor series expansion on the force slightly away from equilibrium,

$$F(x_0+\delta x) = F(x_0) + \frac{dF(x)}{dx}\,|_{x_0}\,\delta x F(x0+\delta x) = -\frac{dU(x)}{dx}\,|_{x_0} + (-\frac{d^2U(x)}{dx^2}\,|_{x_0})\delta x \quad ,$$

and when it is initially at equilibrium,

$$F(x_0) = 0 F(x_0) = 0 F(x_0+\delta x) = -\frac{dU(x)}{dx}\,|_{x_0} + (-\frac{d^2U(x)}{dx^2}\,|_{x_0})\delta x U(y) = mgy \quad .$$

If the ball is at the top of the hill (where the potential is concave-down) it is possible for it to be perfectly balanced, and therefore at equilibrium. But if it gets pushed just slightly to the side, then it will roll down the hill with increasing speed, and the equilibrium is unstable.



Unstable Equilibrium: A ball on top of a hill can initially be balanced, but if it moves slightly left or right, it gets pushed further and further away from the initial equilibrium position. This is an example of unstable equilibrium.

Our notion of "balance" comes directly from the formulation of equilibrium. For something to be "balanced" means that the net external forces are zero. For example, a coin could balance standing up on a table. Initially the coin will feel no net external force or torque; it is in equilibrium. But if pushed slightly to the side, it will become "off-balance," experiencing both a force and a torque causing it to fall to the table. It might have been initially "balanced" and at equilibrium, but it was an unstable equilibrium, prone to being disturbed. But why all this talk of external forces, with no mention of internal forces? The reason is that all the internal forces must sum to zero. This follows directly from Newton's Third Law, $F_{12} = -F_{21}$. Every time we consider a force from particle 1 on particle 2 inside of a system, we know that it will later be cancelled out by the corresponding force from particle 2 on particle 1. We could include those forces in the sum, but it is unnecessary and internal forces are often more complicated than internal forces.

This differentiation between internal and external forces is a powerful one. It also implies that you can trace the motion of the system as a whole (ignoring motion inside the system) through the net external force acting on a center of mass. A center of mass acts as if it has the entire mass of the system, located at one point, and only feels external forces. Its position is defined as the weighted average of all the particles in the system: $\frac{R=\sum_i m_i r_i}{\sum_i m_i}$ or if we have a continuous density of mass, $\rho(r)$, then we can integrate: $R = \frac{\int V\rho(r) r dV}{\int V\rho(r) dV}$. The power of the center of mass is that it hides all the details of what is happening internally. We do not always want to lose the information of what is happening internally, but it is a useful tool to remember, when dealing with a number of complicated interactions.

Key Points

- Equilibrium is defined by no net forces or torques.
- Stability of an equilibrium can be determined by the second derivative of the potential.
- Defining a center of mass allows a simple way to study the behavior of a system or object as a whole.
- Stable equilibrium requires a restoring force. This restoring force can be derived by a Taylor expansion of the force, F(x).

Key Terms

- **stable equilibrium**: The response [of a system in static equilibrium] to a small perturbation is forces that tend to restore the equilibrium.
- **center of mass**: The center of mass (COM) is the unique point at the center of a distribution of mass in space that has the property that the weighted position vectors relative to this point sum to zero.
- **static equilibrium**: the physical state in which all components of a system are at rest and the net force is equal to zero throughout the system

5.5.4: Solving Statics Problems

learning objectives

- Formulate and apply six steps to solve static problems

Statics is the study of forces in equilibrium. Recall that Newton's second law states:

$$\sum \mathrm{F} = \mathrm{ma} \tag{5.5.4.1}$$

Therefore, for all objects moving at constant velocity (including a velocity of 0 — stationary objects), the net external force is zero. There are forces acting, but they are balanced — that is to say, they are "in equilibrium. "

When solving equilibrium problems, it might help to use the following steps:

1. First, ensure that the problem you're solving is in fact a static problem—i.e., that no acceleration (including angular acceleration) is involved Remember: $\sum \mathrm{F} = \mathrm{ma} = 0$ for these situations. If rotational motion is involved, the condition $\sum \tau = \mathrm{I}\alpha = 0$ must also be satisfied, where is torque, is the moment of inertia, and is the angular acceleration.
2. Choose a pivot point. Often this is obvious because the problem involves a hinge or a fixed point. If the choice is not obvious, pick the pivot point as the location at which you have the most unknowns. This simplifies things because forces at the pivot point create no torque because of the cross product: $\tau = \mathrm{rF}$
3. Write an equation for the sum of torques, and then write equations for the sums of forces in the *x* and *y* directions. Set these sums equal to 0. Be careful with your signs.
4. Solve for your unknowns.
5. Insert numbers to find the final answer.
6. Check if the solution is reasonable by examining the magnitude, direction, and units of the answer. The importance of this last step cannot be overstated, although in unfamiliar applications, it can be more difficult to judge reasonableness. However, these judgments become progressively easier with experience.

Key Points

- First, ensure that the problem you're solving is in fact a static problem—i.e., that no acceleration (including angular acceleration) is involved.
- Choose a pivot point — use the location at which you have the most unknowns.
- Write equations for the sums of torques and forces in the *x* and *y* directions.
- Solve the equations for your unknowns algebraically, and insert numbers to find final answers.

Key Terms

- **torque**: A rotational or twisting effect of a force; (SI unit newton-meter or Nm; imperial unit foot-pound or ft-lb)
- **moment of inertia**: A measure of a body's resistance to a change in its angular rotation velocity

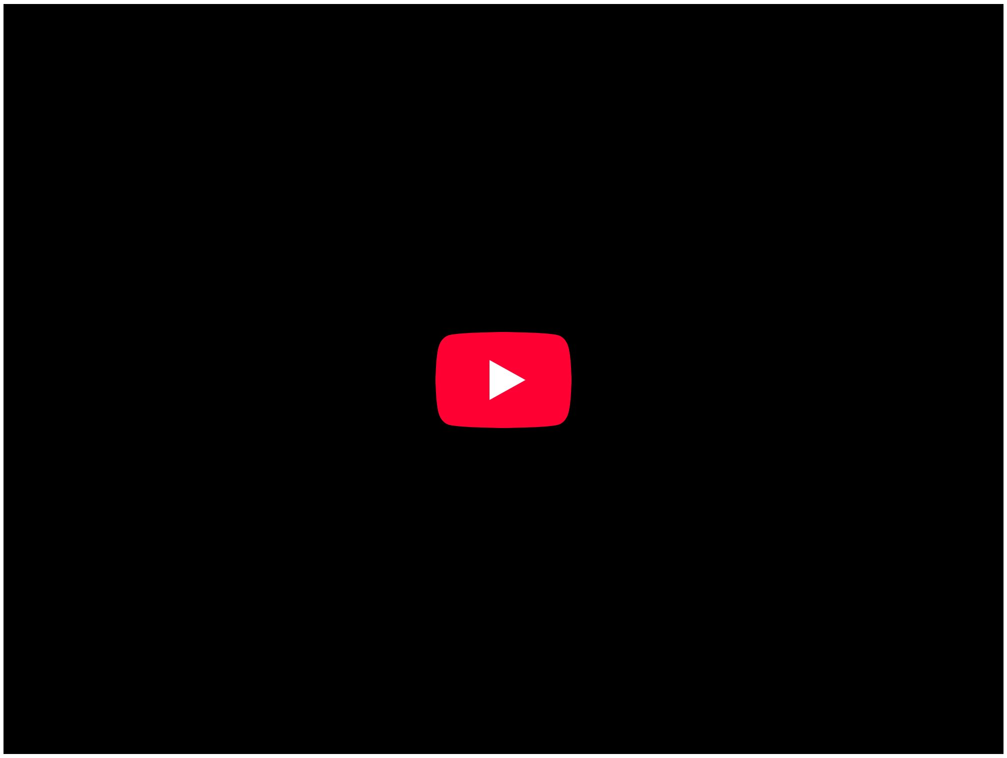
5.5.5: Applications of Statics

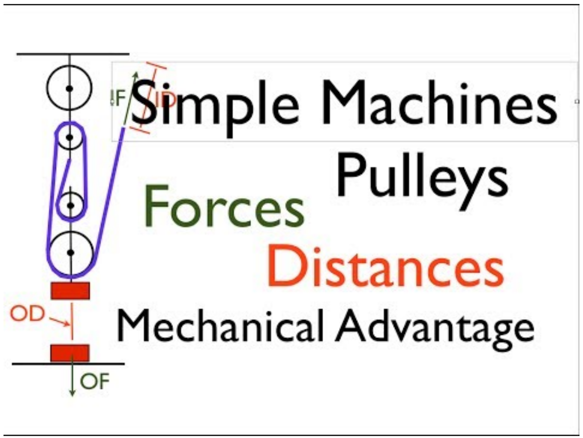
learning objectives

- Develop an understanding of how a machine applies force to work against a load force

Simple Machines

A simple machine is a device that changes the direction or magnitude of a force. They can be described as the simplest mechanisms that use mechanical advantage (or leverage) to multiply force. Usually, the term “simple machine” is referring to one of the six classical simple machines, defined by Renaissance scientists.





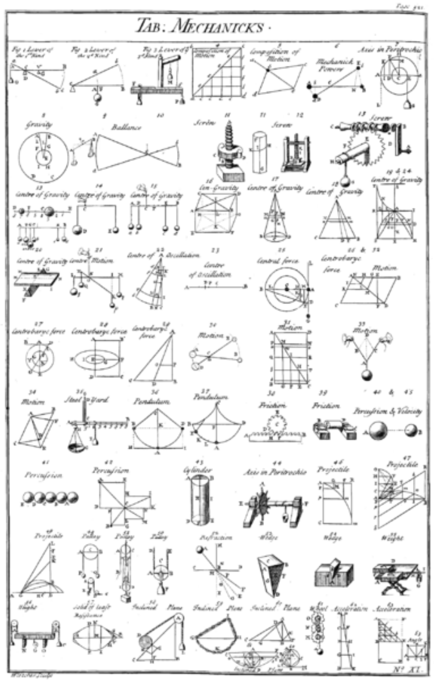
Simple Machines, Pulleys; Forces, Distances and MA: Describes the following terms as they relate to simple machine; input force, output force, input distance, output distance, mechanical advantage.

Simple machines are devices used to multiply or augment a force that we apply—often at the expense of a distance through which we apply the force. Some common examples include:

- Lever

- Wheel and Axle
- Pulley
- Inclined Plane
- Wedge
- Screw

When a device with a specific movement, called a mechanism, is joined with others to form a machine, these machines can be broken down into elementary movements. For example, a bicycle is a mechanism made up of wheels, levers, and pulleys.

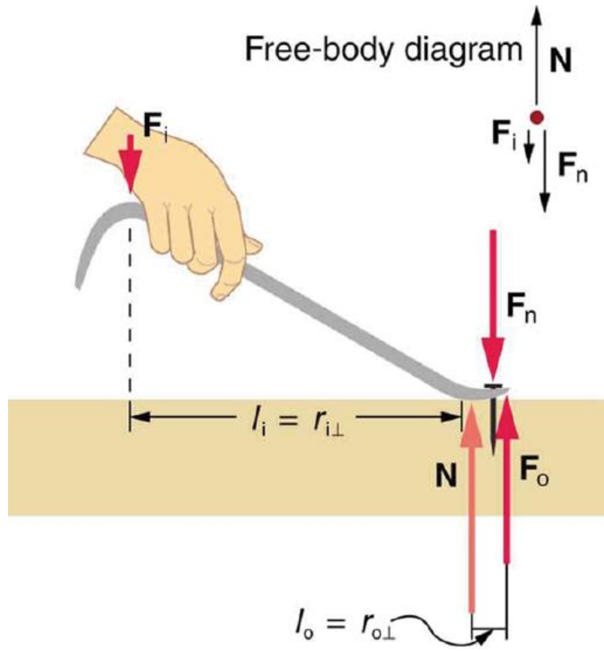


Simple Machines: Table of simple mechanisms, from Chambers' Cyclopedia, 1728. [1] Simple machines provide a "vocabulary" for understanding more complex machines.

Mechanics

A simple machine has an applied force that works against a load force. If there are no frictional losses, the work done on the load is equal to the work done by the applied force. This allows an increase in the output force at the cost of a proportional decrease in distance moved by the load. The ratio of the output force to the input force is the mechanical advantage of the machine. If the machine does not absorb energy, its mechanical advantage can be calculated from the machine's geometry. For instance, the mechanical advantage of a lever is equal to the ratio of its lever arms.

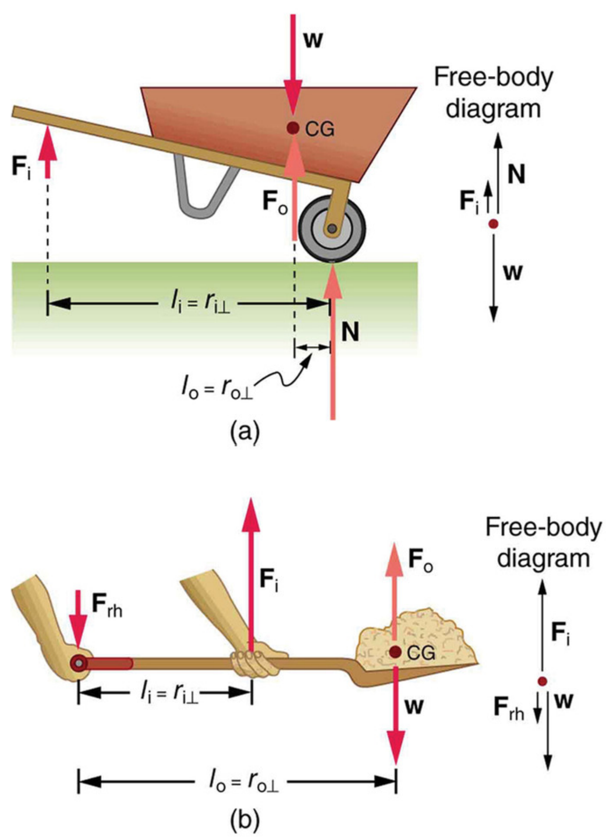
Simple machines which do not experience frictional losses are called ideal machines. For these ideal machines, the power in (rate of energy input) in equal to the power out (rate of energy output): Pin=PoutPin=Pout.



Lever: The amount of force produced by a machine can not be greater than the amount of force put into it.

Further Examples

Wheelbarrows and shovels are also examples of simple machines (these utilize levers). They use only three forces: the input force, output force, and force on the pivot. In the case of wheelbarrows, the output force is between the pivot (wheel's axle) and the input force. In the shovel, the input force is between the pivot and the load.



Examples of Simple Machines: Both of these machines use the concept of levers.

Arches and Domes

Arches and domes are structures that exhibit structural strength and can span large areas with no intermediate supports.

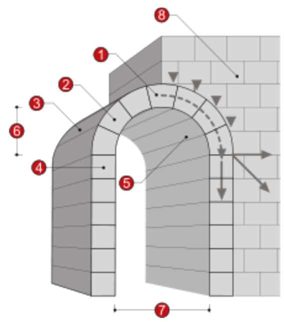
learning objectives

- Explain how an arch exhibits structural strength and how a dome can span a large area without intermediate supports

Arches and domes are structures that exhibit structural strength and can span large areas with no intermediate supports. In this atom, we will discuss the history and physics behind arches and domes.

Arches

An arch is a structure that spans a space, and supports structure and weight above it. Arches have been being built from as long ago as the second millennium, but were not used for a variety of structures until the Romans took advantage of their capabilities. Arches are a pure compression form. They span large areas by resolving forces into compressive stresses and eliminating tensile stresses (referred to as arch action). As the forces in an arch are carried toward the ground, the arch will push outward at the base (called thrust). As the height of the arch decreases, the outward thrust increases. To prevent the arch from collapsing, the thrust needs to be restrained, either with internal ties or external bracing. This external bracing is often called an abutment, as shown in.



Arches: A masonry arch1. Keystone 2. Voussoir 3. Extrados 4. Impost 5. Intrados 6. Rise 7. Clear span 8. Abutment

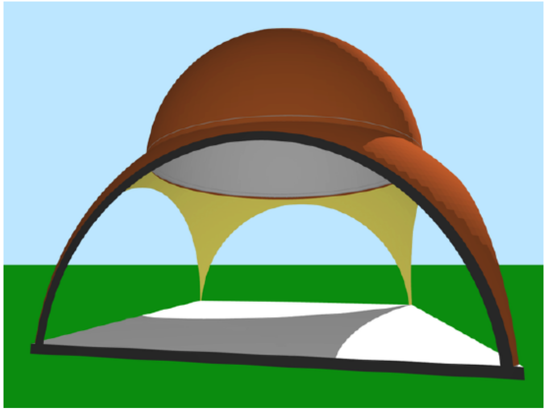
The most common true arch configurations are the fixed arch, the two-hinged arch and the three-hinged arch. The fixed arch is most often used in reinforced concrete bridge and tunnel construction, where the spans are short. Because it is subject to additional internal stress caused by thermal expansion and contraction, this type of arch is considered to be statically indeterminate. The two-hinged arch is most often used to bridge long spans. This type of arch has pinned connections at the base. Unlike the fixed arch, the pinned base is able to rotate, allowing the structure to move freely and compensate for the thermal expansion and contraction caused by changes in outdoor temperature. Because the structure is pinned between the two base connections, which can result in additional stresses, the two-hinged arch is also statically indeterminate, although not to the degree of the fixed arch.

Domes

A dome is an element of architecture that resembles the hollow upper half of a sphere. Dome structures made of various materials (from mud to stone, wood, brick, concrete, metal, glass and plastic) and have a long architectural lineage extending into prehistory.

A dome is basically an arch that has been rotated around its central vertical axis. Domes have the same properties and capabilities of arches, they can span large areas without intermediate supports and have a great deal of structural strength. When the base of a dome is not the same shape as its supporting walls, for example when a circular dome is on a square structure, techniques are employed to transition between the two. Pendentives are triangular sections of a sphere used to transition from the flat surfaces of supporting walls to the round base of a dome.

Domes can be divided into two kinds, simple and compound. Simple domes use pendentives that are part of the same sphere as the dome itself. Compound domes are part of the structure of a large sphere below that of the dome itself, forming a circular base, as shown in.



Compound Dome: A compound dome (red) with pendentives (yellow) from a sphere of greater radius than the dome.

Muscles and Joints

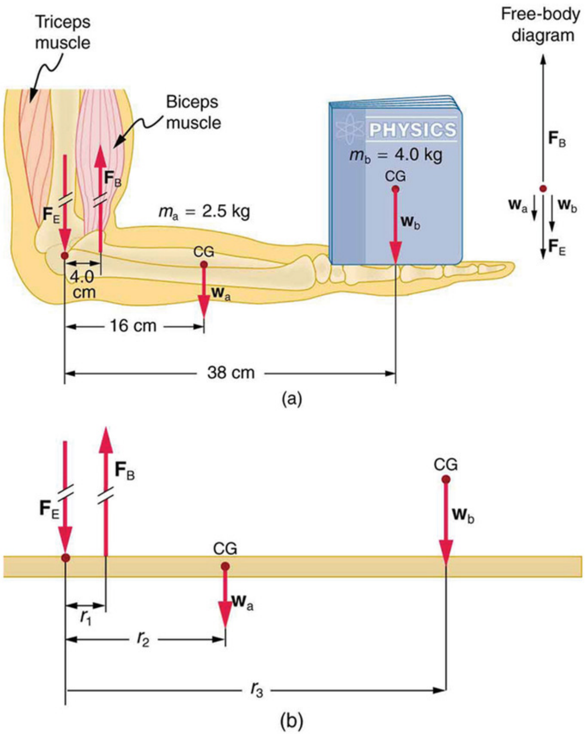
Most skeletal muscles and joints exert much larger forces within the body than the limbs will apply to the outside world.

learning objectives

- Explain the forces exerted by muscles

Muscles and Joints

Muscles and joints involve very interesting applications of statics. Muscles can only contract, so they occur in pairs. In the arm, the biceps muscle is a flexor: it closes the limb. The triceps muscle is an extensor that opens the limb. This configuration is typical of skeletal muscles, bones, and joints in humans and other vertebrates. Most skeletal muscles exert much larger forces within the body than the limbs will apply to the outside world. The reason is clear, since most muscles are attached to bones via tendons close to joints, causing these systems to have mechanical advantages much less than one. Viewing them as simple machines, the input force is much greater than the output force, as seen in.



The Forearm of a Person Holding a Book: (a.) The biceps exert a force FB to support the weight of the forearm and the book. The triceps are assumed to be relaxed. (b.) An approximately equivalent mechanical system with the pivot at the elbow joint

Very large forces are also created in the joints. Because muscles can contract but not expand beyond their resting length, joints and muscles often exert forces that act in opposite directions, and thus subtract. Forces in muscles and joints are largest when their load is far from the joint. For example, in racquet sports like tennis, the constant extension of the arm during game play creates large forces. The mass times the lever arm of a tennis racquet is an important factor, and many players use the heaviest racquet they can handle. It is no wonder that joint deterioration and damage to the tendons in the elbow, such as 'tennis elbow,' can result from repetitive motion, undue torques, and possible poor racquet selection in such sports.

Various tried techniques for holding and using a racquet, bat, or stick can not only increase sporting prowess but can minimize fatigue and long-term damage to the body. Training coaches and physical therapists use the knowledge of the relationships between forces and torques in the treatment of muscles and joints. In physical therapy, an exercise routine can apply a particular force and torque, which can revive muscles and joints in time. Some exercises should be performed under water, thus requiring the exertion of more force and further strengthening muscles.

Key Points

- The six classifications of simple machines were established by renaissance scientists; they are as follows: lever, wheel and axle, pulley, inclined plane, wedge and screw.
- Simple machines can be joined with other devices to create a more complicated machine. These building blocks are used to explain how machines work.
- The force output by a simple machine can exceed the force that was put into the machine.
- Arches span large areas by resolving forces into compressive stresses and eliminating tensile stresses.
- The most common true arch configurations are the fixed arch, the two-hinged arch, and the three-hinged arch.
- A dome is basically an arch that has been rotated around its central vertical axis.
- Domes are basically arches that have been rotated on their vertical axis, and have the same capabilities and properties of arches.
- Domes can be divided into two kinds, simple and compound.
- It is helpful to view muscles as a simple machines and draw them as free body diagrams.
- In muscles, the input force is often much greater than the output force.
- Very large forces are also created in the joints. Because muscles can contract but not expand beyond their resting length, joints and muscles often exert forces that act in opposite directions, and thus subtract.

Key Terms

- **machine**: A mechanical or electrical device that performs or assists in the performance of human tasks, whether physical or computational, laborious or for entertainment.
- **leverage**: A force amplified by means of a lever rotating around a pivot.
- **mechanical advantage**: In a simple machine, the ratio of the output force to the input force.
- **compressive stress**: Stress on materials that leads to a smaller volume.
- **tensile stress**: Stress state leading to expansion; that is, the length of a material tends to increase in the tensile direction while the volume remains constant.
- **pendentive**: The concave triangular sections of vaulting that provide the transition between a dome and the square base on which it is set and transfer the weight of the dome.
- **muscle**: A contractile form of tissue which animals use to effect movement.
- **joint**: Any part of the body where two bones join, in most cases allowing that part of the body to be bent or straightened.

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5.5.6: Elasticity, Stress, Strain, and Fracture

learning objectives

- Identify properties of elastic objects

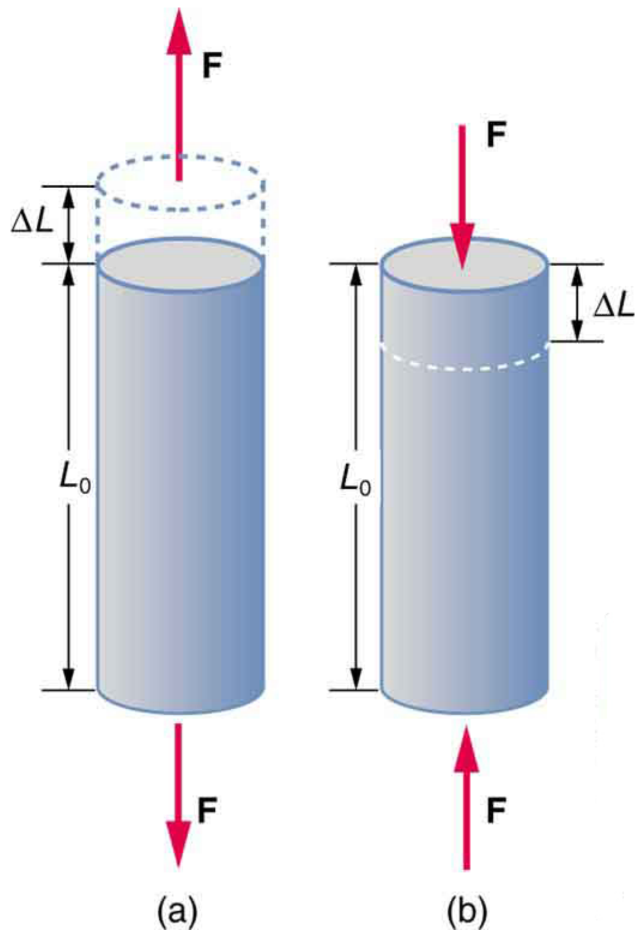
We now move from consideration of forces that affect the motion of an object (such as friction and drag) to those that affect an object's shape. If a bulldozer pushes a car into a wall, the car will not move once it hits the wall, but it will noticeably change shape. A change in shape due to the application of a force is a deformation. Even very small forces are known to cause some deformation. For small deformations, two important characteristics are observed. First, the object returns to its original shape when the force is removed—that is, the deformation is elastic for small deformations. Second, the size of the deformation is proportional to the force—that is, for small deformations, Hooke's law is obeyed. In equation form, Hooke's law is given by $\mathrm{F}=\mathrm{k}\Delta\mathrm{L}$, where $\Delta\mathrm{L}$ is the change in length.

Elasticity is a measure of how difficult it is to stretch an object. In other words it is a measure of how small kk is. Very elastic materials like rubber have small kk and thus will stretch a lot with only a small force.

Stress is a measure of the force put on the object over the area.

Strain is the change in length divided by the original length of the object.

Experiments have shown that the change in length (ΔL) depends on only a few variables. As already noted, ΔL is proportional to the force F and depends on the substance from which the object is made. Additionally, the change in length is proportional to the original length L0 and inversely proportional to the cross-sectional area of the wire or rod. For example, a long guitar string will stretch more than a short one, and a thick string will stretch less than a thin one.



Tension/Compression: Tension: The rod is stretched a length ΔL when a force is applied parallel to its length. (b) Compression: The same rod is compressed by forces with the same magnitude in the opposite direction. For very small deformations and uniform materials, ΔL is approximately the same for the same magnitude of tension or compression. For larger deformations, the cross-sectional area changes as the rod is compressed or stretched.

Fracture

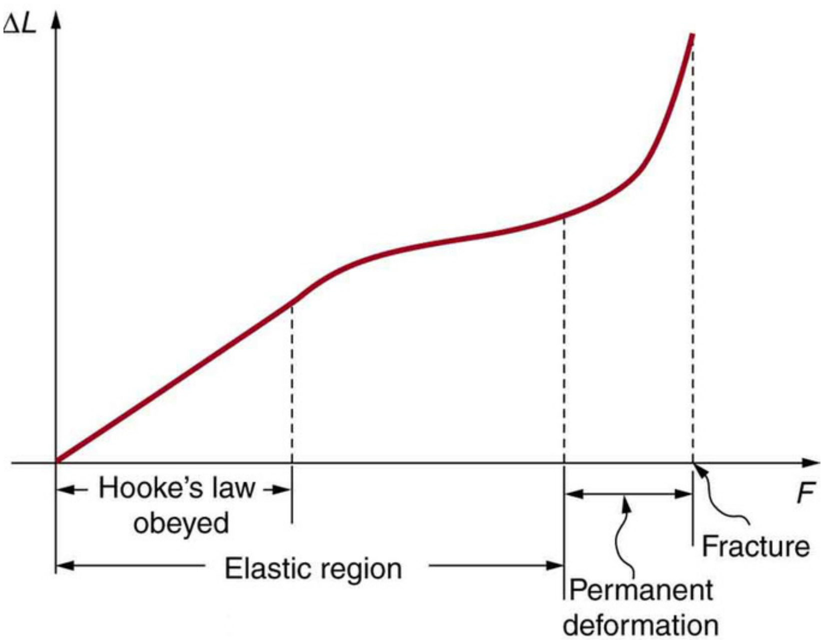
Fracture is caused by a strain placed on an object such that it deforms beyond its elastic limit and breaks.

learning objectives

- Relate fracture with the elastic limit of a material

Materials cannot stretch forever. When a strain is applied to a material it deforms elastically proportional to the force applied. However, after it has deformed a certain amount, the object can no longer take the strain and will break or fracture. The zone in which it bends under strain is called the elastic region. In that region the object will bend and then return to its original shape when the force is abated. Past that point, if more strain is added, the object may permanently deform and eventually fracture.

Fracture strength, also known as breaking strength, is the stress at which a specimen fails via fracture. This is usually determined for a given specimen by a tensile test, which charts the stress-strain curve. The final recorded point is the fracture strength.



Fracture: This is a graph of deformation ΔL versus applied force F. The straight segment is the linear region where Hooke's law is obeyed. The slope of the straight region is 1k. For larger forces, the graph is curved but the deformation is still elastic—L will return to zero if the force is removed. Still greater forces permanently deform the object until it finally fractures. The shape of the curve near fracture depends on several factors, including how the force F is applied. Note that in this graph the slope increases just before fracture, indicating that a small increase in F is producing a large increase in L near the fracture.

Bones, on the whole, do not fracture due to tension or compression. Rather they generally fracture due to sideways impact or bending, resulting in the bone shearing or snapping. The behavior of bones under tension and compression is important because it determines the load the bones can carry. Bones are classified as weight-bearing structures such as columns in buildings and trees. Weight-bearing structures have special features; columns in building have steel-reinforcing rods while trees and bones are fibrous. The bones in different parts of the body serve different structural functions and are prone to different stresses. Thus, the bone in the top of the femur is arranged in thin sheets separated by marrow while, in other places, the bones can be cylindrical and filled with marrow or just solid. Overweight people have a tendency toward bone damage due to sustained compressions in bone joints and tendons.

Key Points

- Elasticity is a measure of the deformation of an object when a force is applied. Objects that are very elastic like rubber have high elasticity and stretch easily.
- Stress is force over area.
- Strain is change in length over original length.
- Most objects behave elastically for small strains and return to their original shape after being bent.
- If the strain on an object is greater than the elastic limit of the object, it will permanently deform or eventually fracture.
- Fracture strength is a measure of the force needed to break an object.

Key Items

- **deformation**: A transformation; change of shape.
- **strain**: The amount by which a material deforms under stress or force, given as a ratio of the deformation to the initial dimension of the material and typically symbolized by ε is termed the engineering strain. The true strain is defined as the natural logarithm of the ratio of the final dimension to the initial dimension.
- **elastic**: Capable of stretching; particularly, capable of stretching so as to return to an original shape or size when force is released.

5.5.7: The Center of Gravity

learning objectives

- Describe how the center of mass of an oddly shaped object is found

Center of Gravity

When people think of objects, they think of them as singular particles of matter. In fact, every object is made up of millions of particles, all of which behave differently when moved. When people observe a stick being thrown in the air, it seems as though the entire object is moving at the same trajectory and velocity, but each particle is being subjected to a different motion in space and acceleration, depending on its place. The different parts of the body have different motions. shows the motion of a stick in the air: it seems to rotate around a single point. Three-dimensional bodies have a property called the center of mass, or center of gravity. This center of mass's main characteristic is that it appears to carry the whole mass of the body.

Center of Gravity: Although the center of mass is in the midpoint of the stick, all of the particles are moving as well.

The center of mass does not actually carry all the mass, despite appearances. Given a hollow sphere, the center is the center of mass, even though it does not actually have anything in it. As seen in, it looks as if the external forces of gravity appear to be working only on the center of mass, but each particle is being pushed or pulled by gravity. The center of mass is much easier to use when discussing bodies, because no one has to analyze each individual particle.

Mathematical Expression: The mathematical relation of center of gravity is read as: 'the position of the center of mass and weighted average of the position of the particles. '

Specifically: 'the total mass x the position of the center of mass= ∑ the mass of the individual particle x the position of the particle. ' The center of mass is a geometric point in three-dimensional volume. When using the definition above, it yields the following equation for center of mass:

$$\mathrm{r_{COM}} = \frac{\Sigma \mathrm{m_i r_i}}{\mathrm{M}} \tag{5.5.7.1}$$

where r is the reference axis x, y, or z; *m* is individual mass;$\mathrm{r_i}$ is the individual position; and M *M* is the total mass.

When taking the center of mass of an oddly shaped object, it is helpful to break it down into smaller sections whose mass and properties are easier to evaluate, and then add the products of the individual masses and positions and divide by the total mass.

Center of Mass: This child's toy uses the principles of 'center of mass' to stay balanced on a finger.

Key Points

- The center of mass 's main characteristic is that it appears to carry the whole mass of the body.
- The total mass x the position of the center of mass= ∑ mass of the individual particle x the position of the particle.
- The center of mass is a geometric point in three-dimensional volume. By using the definition above, the following equation for center of mass can be derived: $\mathrm{r_{COM}} = \Sigma \frac{\mathrm{m_i r_i}}{\mathrm{M}}$.

Key Terms

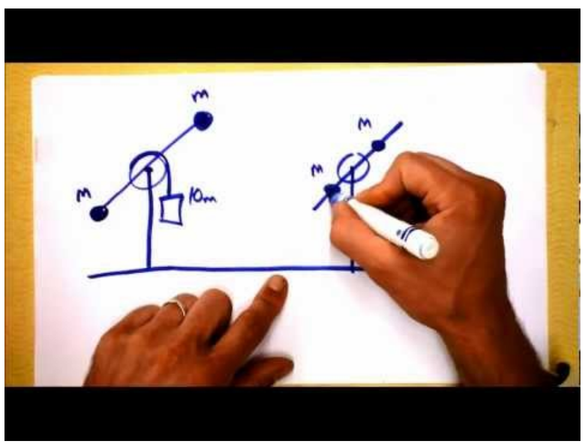
- **center of mass**: The center of mass (COM) is the unique point at the center of a distribution of mass in space that has the property that the weighted position vectors relative to this point sum to zero.

5.5.8: Torque and Angular Acceleration

learning objectives

- Express the relationship between the torque and the angular acceleration in a form of equation

Torque and angular acceleration are related by the following formula where is the objects moment of inertia and αα is the angular acceleration.



Torque, Angular Acceleration, and the Role of the Church in the French Revolution: Why do things change their angular velocity? Soon, you'll know.

Just like Newton's Second Law, which is force is equal to the mass times the acceleration, torque obeys a similar law. If you replace torque with force and rotational inertia with mass and angular acceleration with linear acceleration, you get Newton's Second Law back out. In fact, this equation is Newton's second law applied to a system of particles in rotation about a given axis. It makes no assumptions about constant rotational velocity.

The net torque about an axis of rotation is equal to the product of the rotational inertia about that axis and the angular acceleration, as shown in Figure 1.

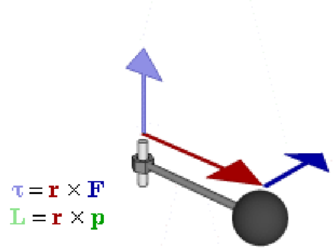


Figure 1: Relationship between force (F), torque (τ), momentum (p), and angular momentum (L) vectors in a rotating system

Similar to Newton's Second Law, angular motion also obeys Newton's First Law. If no outside forces act on an object, an object in motion remains in motion and an object at rest remains at rest. With rotating objects, we can say that unless an outside torque is applied, a rotating object will stay rotating and an object at rest will not begin rotating.

If a turntable were spinning counter clockwise (when viewed from the top), and you applied your fingers to opposite sides the turntable would begin to slow its spinning. From a translational viewpoint, at least, there would be no net force applied to the turntable. The force that points to one side would be cancelled by the force that points to the other. The forces of the two fingers would cancel. Therefore, the turntable would be in translational equilibrium. Despite that, the rotational velocity would be decreased meaning that the acceleration would no longer be zero. From this we might conclude that just because a rotating object is in translational equilibrium, it is not necessarily in rotational equilibrium.

Key Points

- When a torque is applied to an object it begins to rotate with an acceleration inversely proportional to its moment of inertia.
- This relation can be thought of as Newton's Second Law for rotation. The moment of inertia is the rotational mass and the torque is rotational force.
- Angular motion obeys Newton's First Law. If no outside forces act on an object, an object in motion remains in motion and an object at rest remains at rest.

Key Terms

- **angular acceleration**: The rate of change of angular velocity, often represented by α.
- **torque**: A rotational or twisting effect of a force; (SI unit newton-meter or Nm; imperial unit foot-pound or ft-lb)
- **rotational inertia**: The tendency of a rotating object to remain rotating unless a torque is applied to it.

CHAPTER OVERVIEW

5.6: Fluids

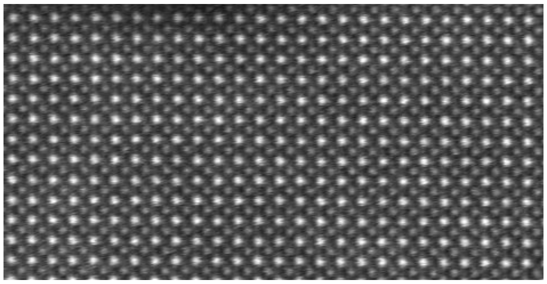
5.6.1: Introduction

learning objectives

- Assess the distinguishing characteristics of the four states of matter

There are a number of properties that can be observed in a material that identify what state the matter is in — solid, liquid, gas, or plasma.

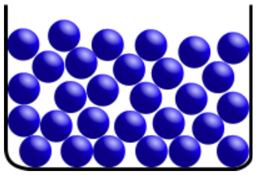
Solids



Solid: Solids are in a state of matter that maintains a fixed volume and shape.

A solid is in a state of matter that maintains a fixed volume and shape. A solid's particles fit closely together. The forces between the particles are so strong that the particles can not move freely; they can only vibrate. This causes a solid to be a stable, non-compressible shape with definite volume.

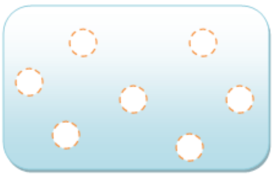
Liquids



Liquid: Liquids maintain a fixed volume, but their shape will mold to the shape of the container they are being held in.

A liquid maintains a fixed volume, but its shape will mold to the shape of the container it is being held in. In, you can see that even though the liquid's shape is determined by the container, it has a free surface that is not controlled by the container. The particles are close together but not as close as in solids; they are still able to move around, which causes the liquid to flow. Liquids usually have a higher volume than their solid counterparts, except for certain molecules, such as H_2O (water).

Gases

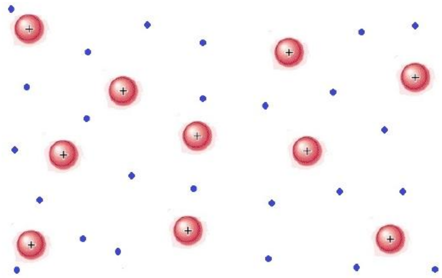


Gas: The particles are much farther from each other, usually a farther length than the size of the particles, and move a lot

shows a gas, whose particles move around a lot and are much farther apart from each other, usually farther apart than the diameter of the particles themselves. The gas behaves like a liquid; the particles are moving but are still attracted to each other, so they still flow. Unlike a solid or a liquid, the gas will try to fill whatever container it is in, adapting its volume accordingly.

Plasma

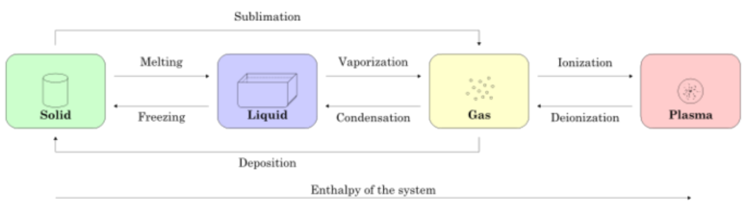
Plasma is a gas that has been ionized. That is to say, sufficient energy has been supplied to the gas such that the electrons have enough energy to escape their atoms or molecules. Plasma contains ions and electrons that can move around freely. Matter in the plasma state has variable volume and shape. Plasma is the most common form of visible matter in the universe. Lightning, sparks, neon lights, and the sun are all examples of matter in the plasma state.



Plasma: Matter in the plasma state has variable volume and shape, but as well as neutral atoms, it contains a significant number of ions and electrons, both of which can move around freely.

Phase Transitions

shows the different states of matter and how they can change from one to another as a function of enthalpy and pressure and temperature changes. A solid can change to a liquid with an enthalpy increase. The process of a liquid going to a solid is known as melting. A liquid can change into a gas when it hits its boiling point or can even enter a plasma state if the enthalpy is increased enough. When the enthalpy is lowered, a liquid can transform into a solid through freezing. Sometimes, a solid can skip freezing and go directly to a gaseous state; this process is called sublimation.



States of Matter: This figure illustrates the relationship between the enthalpy of a system and the state of matter that the system is in.

What is a Fluid?

A fluid is a substance that continually deforms (flows) under an applied shear stress.

learning objectives

- Explain the properties of substances under an applied shear stress

A fluid is a substance that continually deforms (flows) under an applied shear stress. Fluids are a subset of the states of matter and include three of the four states—liquids, gases, and plasma (shown in).

Four Fundamental States of Matter: Four fundamental states of matter: 1) top left corner corresponds to solid; 2) top right corner corresponds to liquid; 3) bottom left corner corresponds to gas; 4) bottom right corner corresponds to plasma.

Liquids form a free surface (i.e., a surface not created by the container) while gases do not. The distinction between solids and fluid is not entirely obvious. The distinction is made by evaluating the viscosity of the substance. Silly Putty can be considered to behave like a solid or a fluid, depending on the time period over which it is observed. It is best described as a viscoelastic fluid.

Fluids display properties such as:

- a) not resisting deformation or resisting it only lightly (viscosity), and
- b) the ability to flow (also described as the ability to take on the shape of the container).

This also means that all fluids have the property of fluidity. These properties are typically a function of their inability to support a shear stress in static equilibrium.

Solids can be subjected to shear stresses, and normal stresses—both compressive and tensile. In contrast, ideal fluids can only be subjected to normal, compressive stress (called pressure). Real fluids display viscosity and so are capable of being subjected to low levels of shear stress.

Although the term *fluid* includes both the liquid and gas phases, it is also commonly used as a synonym for *liquid*, with no implication that gas could also be present. For example, "brake fluid" is hydraulic oil and will not perform its required function if there is gas in it. This colloquial usage of the term is also common in the fields of medicine and nutrition (e.g., "take plenty of fluids").

Key Points

- Solids are non-compressible and have constant volume and constant shape.
- Liquids are non-compressible and have constant volume but can change shape. A liquid's shape is dictated by the shape of the container it is in.
- Gases do not have a constant volume or shape; they not only take the shape of the container they are in, they try to fill the entire container.
- Matter in the plasma state has variable volume and shape. Plasma contains ions and electrons, both of which can move around freely.
- Fluids are a subset of the states of matter and include liquids, gases, and plasma.
- Fluids display properties such as: not resisting deformation or resisting it only lightly (viscosity), and the ability to flow (also described as the ability to take on the shape of the container).
- Ideal fluids can only be subjected to normal, compressive stress which is called pressure. Real fluids display viscosity and thus are capable of being subjected to low levels of shear stress.

Key Items

- **plasma**: a state of matter consisting of partially ionized gas
- **enthalpy**: the total amount of energy in a system, including both the internal energy and the energy needed to displace its environment
- **sublimation**: the transition of a substance from the solid phase directly to the vapor state such that it does not pass through the intermediate, liquid phase
- **fluidity**: A measure of the extent to which something is fluid. The reciprocal of its viscosity.
- **viscosity**: A quantity expressing the magnitude of internal friction in a fluid, as measured by the force per unit area resisting uniform flow.
- **shear stress**: The component of stress that causes parallel layers of a material to move relative to each other in their own planes.

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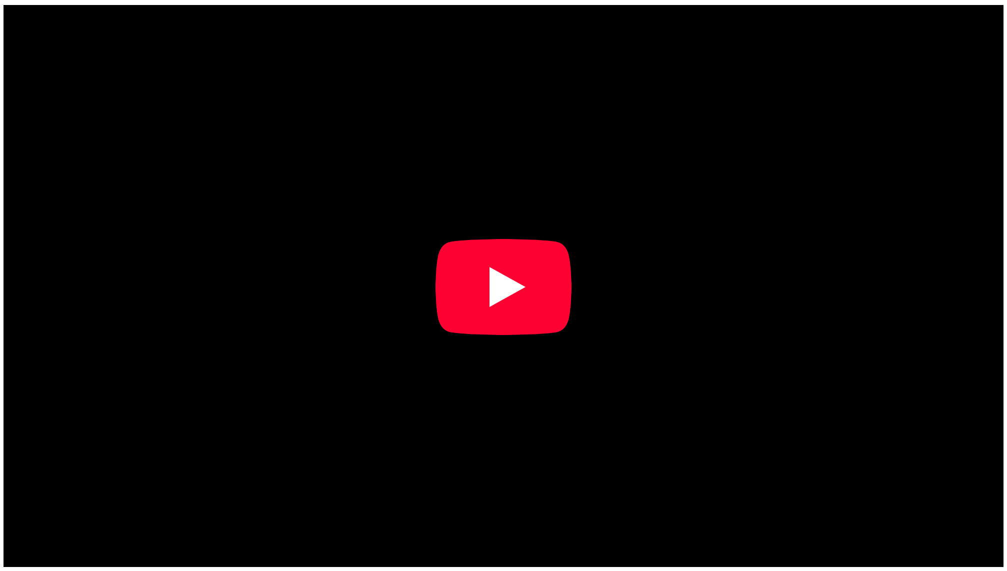
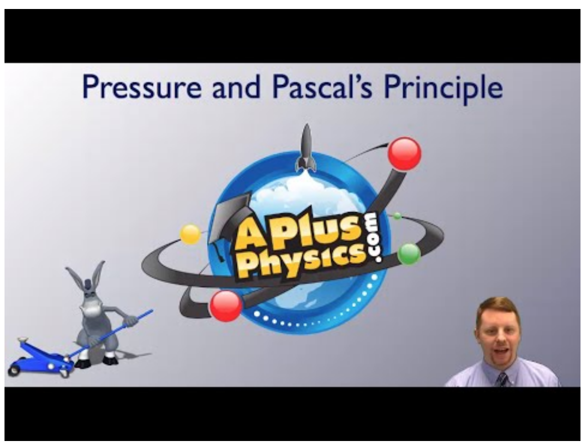
5.6.2: Density and Pressure

learning objectives

- Identify factors that determine the pressure exerted by the gas

Pressure is an important physical quantity—it plays an essential role in topics ranging from thermodynamics to solid and fluid mechanics. As a scalar physical quantity (having magnitude but no direction), pressure is defined as the force per unit area applied perpendicular to the surface to which it is applied. Pressure can be expressed in a number of units depending on the context of use.





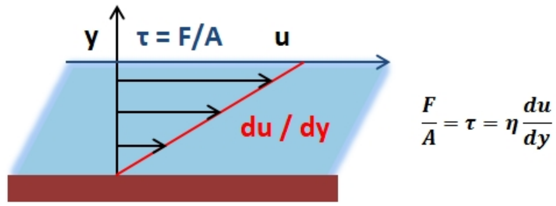
Pressure and Pascal's Principle: A brief introduction to pressure and Pascal's Principle, including hydraulics.

Units, Equations and Representations

In SI units, the unit of pressure is the Pascal (Pa), which is equal to a Newton / meter^2 (N/m^2). Other important units of pressure include the pound per square inch (psi) and the standard atmosphere (atm). The elementary mathematical expression for pressure is given by:

$$\text{pressure} = \frac{\text{Force}}{\text{Area}} = \frac{\text{F}}{\text{A}} \tag{5.6.2.1}$$

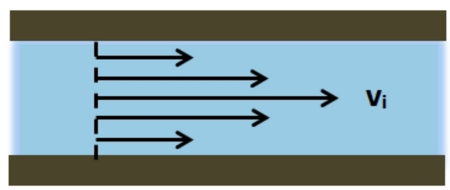
where p is pressure, F is the force acting perpendicular to the surface to which this force is applied, and A is the area of the surface. Any object that possesses weight, whether at rest or not, exerts a pressure upon the surface with which it is in contact. The magnitude of the pressure exerted by an object on a given surface is equal to its weight acting in the direction perpendicular to that surface, divided by the total surface area of contact between the object and the surface. shows the graphical representations and corresponding mathematical expressions for the case in which a force acts perpendicular to the surface of contact, as well as the case in which a force acts at angle θ relative to the surface.



Representation of Pressure: This image shows the graphical representations and corresponding mathematical expressions for the case in which a force acts perpendicular to the surface of contact, as well as the case in which a force acts at angle θ relative to the surface.

Pressure as a Function of Surface Area

Since pressure depends only on the force acting perpendicular to the surface upon which it is applied, only the force component perpendicular to the surface contributes to the pressure exerted by that force on that surface. Pressure can be increased by either increasing the force or by decreasing the area or can oppositely be decreased by either decreasing the force or increasing the area. illustrates this concept. A rectangular block weighing 1000 N is first placed horizontally. It has an area of contact (with the surface upon which it is resting) of 0.1 m^2, thus exerting a pressure of 1,000 Pa on that surface. That same block in a different configuration (also in Figure 2), in which the block is placed vertically, has an area of contact with the surface upon which it is resting of 0.01 m^2, thus exerting a pressure of 10,000 Pa—10 times larger than the first configuration due to a decrease in the surface area by a factor of 10.



Pressure as a Function of Surface Area: Pressure can be increased by either increasing the force or by decreasing the area or can oppositely be decreased by either decreasing the force or increasing the area.

A good illustration of this is the reason a sharp knife is far more effective for cutting than a blunt knife. The same force applied by a sharp knife with a smaller area of contact will exert a much greater pressure than a blunt knife having a considerably larger area of contact. Similarly, a person standing on one leg on a trampoline causes a greater displacement of the trampoline than that same person standing on the same trampoline using two legs—not because the individual exerts a larger force when standing on one leg, but because the area upon which this force is exerted is decreased, thus increasing the pressure on the trampoline. Alternatively, an object having a weight larger than another object of the same dimensionality and area of contact with a given surface will exert a greater pressure on that surface due to an increase in force. Finally, when considering a given force of constant magnitude acting on a constant area of a given surface, the pressure exerted by that force on that surface will be greater the larger the angle of that force as it acts upon the surface, reaching a maximum when that force acts perpendicular to the surface.

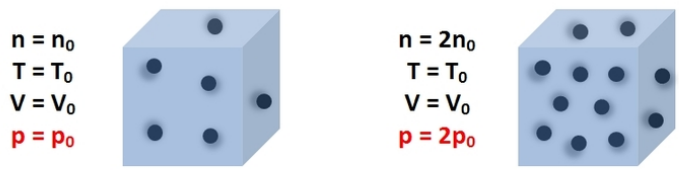
Liquids and Gases: Fluids

Just as a solid exerts a pressure on a surface upon which it is in contact, liquids and gases likewise exert pressures on surfaces and objects upon which they are in contact with. The pressure exerted by an ideal gas on a closed container in which it is confined is best analyzed on a molecular level. Gas molecules in a gas container move in a random manner throughout the volume of the container, exerting a force on the container walls upon collision. Taking the overall average force of all the collisions of the gas molecules confined within the container over a unit time allows for a proper measurement of the effective force of the gas molecules on the container walls. Given that the container acts as a confining surface for this net force, the gas molecules exert a pressure on the container. For such an ideal gas confined within a rigid container, the pressure exerted by the gas molecules can be calculated using the ideal gas law:

$$\mathrm{p} = \frac{\mathrm{nRT}}{\mathrm{V}} \tag{5.6.2.2}$$

where n is the number of gas molecules, R is the ideal gas constant (R = 8.314 J mol^{-1} K^{-1}), T is the temperature of the gas, and V is the volume of the container.

The pressure exerted by the gas can be increased by: increasing the number of collisions of gas molecules per unit time by increasing the number of gas molecules; increasing the kinetic energy of the gas by increasing the temperature; or decreasing the volume of the container. offers a representation of the ideal gas law, as well as the effect of varying the equation parameters on the gas pressure. Another common type of pressure is that exerted by a static liquid or hydrostatic pressure. Hydrostatic pressure is most easily addressed by treating the liquid as a continuous distribution of matter, and may be considered a measure of energy per unit volume or energy density. We will further discuss hydrostatic pressure in other sections.



$$p = \frac{nRT}{V}$$

n	n_0	$2n_0$	n_0	n_0
T	T_0	T_0	T_0	T_0
V	V_0	V_0	V_0	$2V_0$
p	p_0	$2p_0$	$2p_0$	$½p_0$

Pressure of an Ideal Gas: This image is a representation of the ideal gas law, as well as the effect of varying the equation parameters on the gas pressure.

Variation of Pressure With Depth

Pressure within static fluids depends on the properties of the fluid, the acceleration due to gravity, and the depth within the fluid.

learning obectives

- Identify factors that determine the pressure exerted by static liquids and gases

Pressure is defined in simplest terms as force per unit area. However, when dealing with pressures exerted by gases and liquids, it is most convenient to approach pressure as a measure of energy per unit volume by means of the definition of work (W = F·d). The derivation of pressure as a measure of energy per unit volume from its definition as force per unit area is given in. Since, for gases and liquids, the force acting on a system contributing to pressure does not act on a specific point or particular surface, but rather as a distribution of force, analyzing pressure as a measure of energy per unit volume is more appropriate. For liquids and gases at rest, the pressure of the liquid or gas at any point within the medium is called the hydrostatic pressure. At any such point within a medium, the pressure is the same in all directions, as if the pressure was not the same in all directions, the fluid, whether it is a gas or liquid, would not be static. Note that the following discussion and expressions pertain only to incompressible fluids at static equilibrium.

$$\Delta p = \frac{8\eta Q \Delta x}{\pi r^4}$$

Energy per Unit Volume: This equation is the derivation of pressure as a measure of energy per unit volume from its definition as force per unit area.

The pressure exerted by a static liquid depends only on the depth, density of the liquid, and the acceleration due to gravity. gives the expression for pressure as a function of depth within an incompressible, static liquid as well as the derivation of this equation from the definition of pressure as a measure of energy per unit volume (ρ is the density of the gas, g is the acceleration due to gravity, and h is the depth within the liquid). For any given liquid with constant density throughout, pressure increases with increasing depth. For example, a person under water at a depth of h_1 will experience half the pressure as a person under water at a depth of $h_2 = 2h_1$. For many liquids, the density can be assumed to be nearly constant throughout the volume of the liquid and, for virtually all practical applications, so can the acceleration due to gravity (g = 9.81 m/s^2). As a result, pressure within a liquid is therefore a function of depth only, with the pressure increasing at a linear rate with respect to increasing depth. In practical applications involving calculation of pressure as a function of depth, an important distinction must be made as to whether the absolute or relative pressure within a liquid is desired. Equation 2 by itself gives the pressure exerted by a liquid relative to atmospheric pressure, yet if the absolute pressure is desired, the atmospheric pressure must then be added to the pressure exerted by the liquid alone.

$$R_e = \frac{V}{I} \quad \rightarrow \quad \begin{cases} R_h \rightarrow R_e \\ \Delta p \rightarrow V \\ I \rightarrow Q \end{cases} \quad \rightarrow \quad \Delta p = \frac{8\eta Q \Delta x}{\pi r^4} \quad \rightarrow \quad R_h = \frac{\Delta p}{Q} = \frac{8\eta \Delta x}{\pi r^4}$$

Pressure as Energy per Unit Volume: This equation gives the expression for pressure as a function of depth within an incompressible, static liquid as well as the derivation of this equation from the definition of pressure as a measure of energy per unit volume (ρ is the density of the gas, g is the acceleration due to gravity, and h is the depth within the liquid).

When analyzing pressure within gases, a slightly different approach must be taken as, by the nature of gases, the force contributing to pressure arises from the average number of gas molecules occupying a certain point within the gas per unit time. Thus the force contributing to the pressure of a gas within the medium is not a continuous distribution as for liquids and the barometric equation given in must be utilized to determine the pressure exerted by the gas at a certain depth (or height) within the gas (p_0 is the pressure at h = 0, M is the mass of a single molecule of gas, g is the acceleration due to gravity, k is the Boltzmann constant, T is the temperature of the gas, and h is the height or depth within the gas). Equation 3 assumes that the gas is incompressible and that the pressure is hydrostatic.

$$p = p_0 e^{-\frac{Mgh}{kT}}$$

Pressure within a gas: The force contributing to the pressure of a gas within the medium is not a continuous distribution as for liquids and the barometric equation given in this figure must be utilized to determine the pressure exerted by the gas at a certain depth (or height) within the gas (p0 is the pressure at h = 0, M is the mass of a single molecule of gas, g is the acceleration due to gravity, k is the Boltzmann constant, T is the temperature of the gas, and h is the height or depth within the gas)

Static Equilibrium

Any region or point, or any static object within a static fluid is in static equilibrium where all forces and torques are equal to zero.

learning obectives

- Identify required conditions for a fluid to be in rest

Static equilibrium is a particular state of a physical system. It is qualitatively described by an object at rest and by the sum of all forces, with the sum of all torques acting on that object being equal to zero. Static objects are in static equilibrium, with the net force and net torque acting on that object being equal to zero; otherwise there would be a driving mechanism for that object to undergo movement in space. The analysis and study of objects in static equilibrium and the forces and torques acting on them is called statics—a subtopic of mechanics. Statics is particularly important in the design of static and load bearing structures. As it pertains to fluidics, static equilibrium concerns the forces acting on a static object within a fluid medium.

Fluids

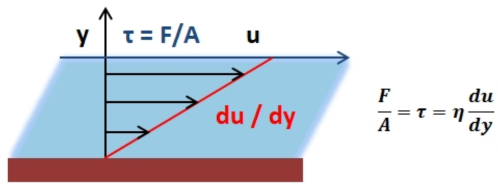
For a fluid at rest, the conditions for static equilibrium must be met at any point within the fluid medium. Therefore, the sum of the forces and torques at any point within the static liquid or gas must be zero. Similarly, the sum of the forces and torques of an object at rest within a static fluid medium must also be zero. In considering a stationary object within a liquid medium at rest, the forces acting at any point in time and at any point in space within the medium must be analyzed. For a stationary object within a static liquid, there are no torques acting on the object so the sum of the torques for such a system is immediately zero; it need not concern analysis since the torque condition for equilibrium is fulfilled.

Density

At any point in space within a static fluid, the sum of the acting forces must be zero; otherwise the condition for static equilibrium would not be met. In analyzing such a simple system, consider a rectangular region within the fluid medium with density ρ_L (same density as the fluid medium), width w, length l, and height h, as shown in. Next, the forces acting on this region within the medium are taken into account. First, the region has a force of gravity acting downwards (its weight) equal to its density object, times its volume of the object, times the acceleration due to gravity. The downward force acting on this region due to the fluid above the region is equal to the pressure times the area of contact. Similarly, there is an upward force acting on this region due to the fluid below the region equal to the pressure times the area of contact. For static equilibrium to be achieved, the sum of these forces must be zero, as shown in. Thus for any region within a fluid, in order to achieve static equilibrium, the pressure from the fluid below the region must be greater than the pressure from the fluid above by the weight of the region. This force which counteracts the weight of a region or object within a static fluid is called the buoyant force (or buoyancy).

image

Static Equilibrium of a Region Within a Fluid: This figure shows the equations for static equilibrium of a region within a fluid.



Region Within a Static Fluid: This figure is a free body diagram of a region within a static fluid.

In the case on an object at stationary equilibrium within a static fluid, the sum of the forces acting on that object must be zero. As previously discussed, there are two downward acting forces, one being the weight of the object and the other being the force exerted by the pressure from the fluid above the object. At the same time, there is an upwards force exerted by the pressure from the fluid below the object, which includes the buoyant force. shows how the calculation of the forces acting on a stationary object within a static fluid would change from those presented in if an object having a density ρ_S different from that of the fluid medium is surrounded by the fluid. The appearance of a buoyant force in static fluids is due to the fact that pressure within the fluid changes as depth changes. The analysis presented above can furthermore be extended to much more complicated systems involving complex objects and diverse materials.

Pascal’s Principle

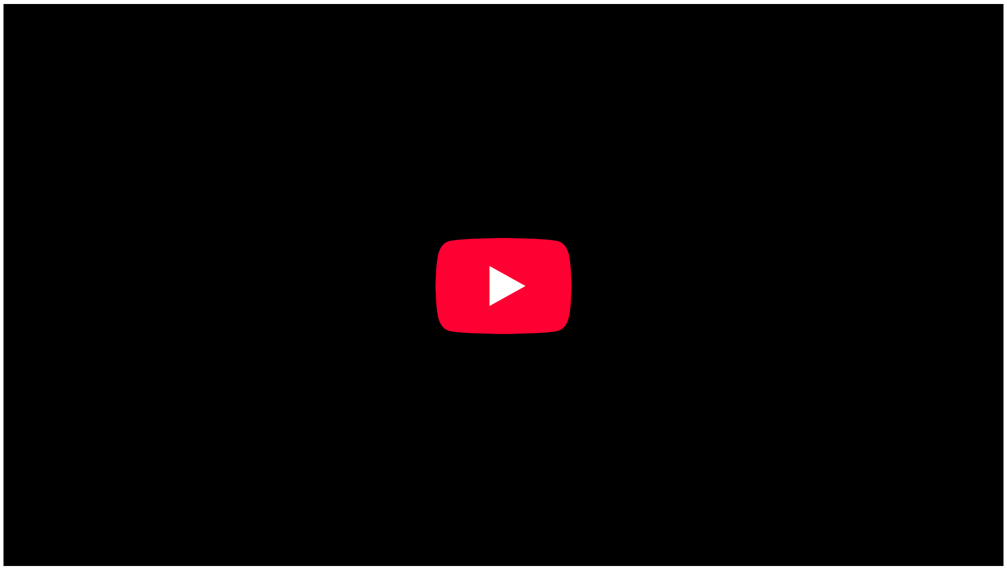
Pascal’s Principle states that pressure is transmitted and undiminished in a closed static fluid.

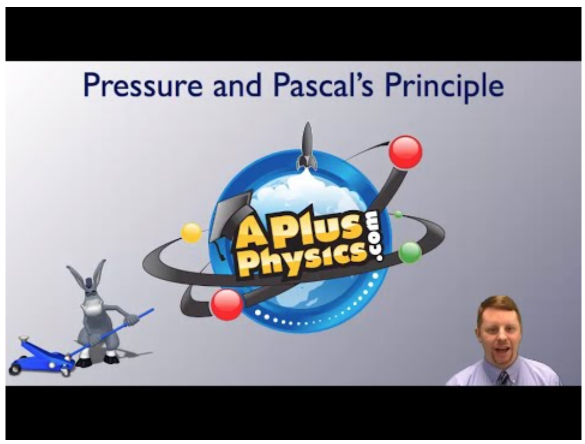
learning obectives

- Apply Pascal’s Principle to describe pressure behavior in static fluids

Pascal’s Principle

Pascal’s Principle (or Pascal’s Law) applies to static fluids and takes advantage of the height dependency of pressure in static fluids. Named after French mathematician Blaise Pascal, who established this important relationship, Pascal’s Principle can be used to exploit pressure of a static liquid as a measure of energy per unit volume to perform work in applications such as hydraulic presses. Qualitatively, Pascal’s Principle states that pressure is transmitted undiminished in an enclosed static liquid. Quantitatively, Pascal’s Law is derived from the expression for determining the pressure at a given height (or depth) within a fluid and is defined by Pascal’s Principle:





Pressure and Pascal's Principle: A brief introduction to pressure and Pascal's Principle, including hydraulics.

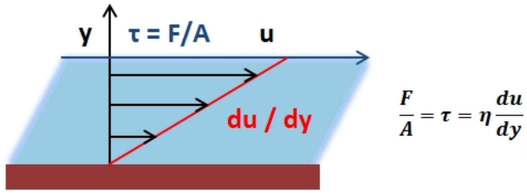
$$p_2 = p_1 + \Delta p, \Delta p = \rho g \Delta h \tag{5.6.2.3}$$

where p_1 is the external applied pressure, ρ is the density of the fluid, Δh is the difference in height of the static liquid, and g is the acceleration due to gravity. Pascal's Law explicitly determines the pressure difference between two different heights (or depths) within a static liquid. As, by Pascal's Law, a change in pressure is linearly proportional to a change in height within an incompressible, static liquid of constant density, doubling the height between the two points of reference will double the change of pressure, while halving the height between the two points will half the change in pressure.

Enclosed Static Liquids

While Pascal's Principle applies to any static fluid, it is most useful in terms of applications when considering systems involving rigid wall closed column configurations containing homogeneous fluids of constant density. By exploiting the fact that pressure is transmitted undiminished in an enclosed static liquid, such as in this type of system, static liquids can be used to transform small amounts of force into large amounts of force for many applications such as hydraulic presses.

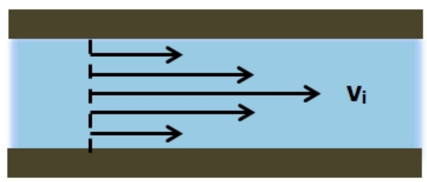
As an example, referring to, a downwards force of 10 N is applied to a bottle filled with a static liquid of constant density ρ at the spout of cross-sectional area of 5 cm^2, yielding an applied pressure of 2 N/cm^2. The cross-sectional area of the bottle changes with height so that at the bottom of the bottle the cross-sectional area is 500 cm^2. As a result of Pascal's Law, the pressure change (pressure applied to the static liquid) is transmitted undiminished in the static liquid so that the applied pressure is 2 N/m^2 at the bottom of the bottle as well. Furthermore, the hydrostatic pressure due to the difference in height of the liquid is given by Equation 1 and yields the total pressure at the bottom surface of the bottle. Since the cross-sectional area at the bottom of the bottle is 100 times larger than at the top, the force contributing to the pressure at the bottom of the bottle is 1000 N plus the force from the weight of the static fluid in the bottle. This example shows how, through Pascal's Principle, the force exerted by a static fluid in a closed system can be multiplied by changing the height and the surface area of contact.



Pressure Applied to a Hydrostatic Fluid: A downwards force of 10 N is applied to a bottle filled with a static liquid of constant density ρ at the spout of cross-sectional area of 5 cm2, yielding an applied pressure of 2 N/cm2.

Pressure Transmitted Throughout an Entire Fluid

As stated by Pascal's Principle, the pressure applied to a static fluid in a closed container is transmitted throughout the entire fluid. Taking advantage of this phenomenon, hydraulic presses are able to exert a large amount of force requiring a much smaller amount of input force. This gives two different types of hydraulic press configurations, the first in which there is no difference in height of the static liquid and the second in which there is a difference in height Δh of the static liquid. In the first configuration, a force F_1 is applied to a static liquid of density ρ across a surface area of contact A_1, yielding an input pressure of P_2. On the other side of the press configuration, the fluid exerts an output pressure P_1 across a surface area of contact A_2, where $A_2 > A_1$. By Pascal's Principle, $P_1 = P_2$, yielding a force exerted by the static fluid of F_2, where $F_2 > F_1$. Depending on the applied pressure and geometry of the hydraulic press, the magnitude of F_2 can be changed. In the second configuration, the geometry of the system is the same, except that the height of the fluid on the output end is a height Δh less than the height of the fluid at the input end. The difference in height of the fluid between the input and the output ends contributes to the total force exerted by the fluid. For a hydraulic press, the force multiplication factor is the ratio of the output to the input contact areas.



Hydraulic Press Diagrams: Two different types of hydraulic press configurations, the first in which there is no difference in height of the static liquid and the second in which there is a difference in height Δh of the static liquid.

Gauge Pressure and Atmospheric Pressure

Pressure is often measured as gauge pressure, which is defined as the absolute pressure minus the atmospheric pressure.

learning obectives

- Explain the relationship among absolute pressure, gauge pressure, and atmospheric pressure

Atmospheric Pressure

An important distinction must be made as to the type of pressure quantity being used when dealing with pressure measurements and calculations. Atmospheric pressure is the magnitude of pressure in a system due to the atmosphere, such as the pressure exerted by air molecules (a static fluid) on the surface of the earth at a given elevation. In most measurements and calculations, the atmospheric pressure is considered to be constant at 1 atm or 101,325 Pa, which is the atmospheric pressure under standard conditions at sea level.

Atmospheric pressure is due to the force of the molecules in the atmosphere and is a case of hydrostatic pressure. Depending on the altitude relative to sea level, the actual atmospheric pressure will be less at higher altitudes and more at lower altitudes as the weight of air molecules in the immediate atmosphere changes, thus changing the effective atmospheric pressure. Atmospheric pressure is a measure of absolute pressure and can be affected by the temperature and air composition of the atmosphere but can generally be accurately approximated to be around standard atmospheric pressure of 101,325 Pa. Within the majority of earth's atmosphere, pressure varies with height according to. In this equation p_0 is the pressure at sea level (101,325 Pa), g is the acceleration due to gravity, M is the mass of a single molecule of air, R is the universal gas constant, T_0 is the standard temperature at sea level, and h is the height relative to sea level.

$$\Delta p = \frac{8\eta Q\Delta x}{\pi r^4}$$

Pressure and Height: Atmospheric pressure depends on altitude or height.

Gauge Pressure

For most applications, particularly those involving pressure measurements, it is more practical to use gauge pressure than absolute pressure as a unit of measurement. Gauge pressure is a relative pressure measurement which measures pressure relative to atmospheric pressure and is defined as the absolute pressure minus the atmospheric pressure. Most pressure measuring equipment give the pressure of a system in terms of gauge pressure as opposed to absolute pressure. For example, tire pressure and blood pressure are gauge pressures by convention, while atmospheric pressures, deep vacuum pressures, and altimeter pressures must be absolute.

For most working fluids where a fluid exists in a closed system, gauge pressure measurement prevails. Pressure instruments connected to the system will indicate pressures relative to the current atmospheric pressure. The situation changes when extreme vacuum pressures are measured; absolute pressures are typically used instead.

To find the absolute pressure of a system, the atmospheric pressure must then be added to the gauge pressure. While gauge pressure is very useful in practical pressure measurements, most calculations involving pressure, such as the ideal gas law, require pressure values in terms of absolute pressures and thus require gauge pressures to be converted to absolute pressures.

Measurements: Gauge Pressure and the Barometer

Barometers are devices used for measuring atmospheric and gauge pressure indirectly through the use of hydrostatic fluids.

learning obectives

- Compare design and operation of aneroid and hydrostatic based barometers

Gauge Pressure

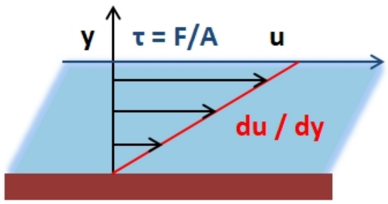
In practice, pressure is most often measured in terms of gauge pressure. Gauge pressure is the pressure of a system above atmospheric pressure. Since atmospheric pressure is mostly constant with little variation near sea level, where most practical pressure measurements are taken, it is assumed to be approximately 101,325 Pa. Modern pressure measuring devices sometimes have incorporated mechanisms to account for changes in atmospheric pressure due to elevation changes. Gauge pressure is much more convenient than absolute pressure for practical measurements and is widely used as an established measure of pressure. However, it is important to determine whether it is necessary to use absolute (gauge plus atmospheric) pressure for calculations, as is often the case for most calculations, such as those involving the ideal gas law. Pressure measurements have been accurately taken since the mid-1600s with the invention of the traditional barometer. Barometers are devices used to measure pressure and were initially used to measure atmospheric pressure.

Hydrostatic Based Barometers

Early barometers were used to measure atmospheric pressure through the use of hydrostatic fluids. Hydrostatic based barometers consist of columnar devices usually made from glass and filled with a static liquid of consistent density. The columnar section is sealed, holds a vacuum, and is partially filled with the liquid while the base section is open to the atmosphere and makes an interface with the surrounding environment. As the atmospheric pressure changes, the pressure exerted by the atmosphere on the fluid reservoir exposed to the atmosphere at the base changes, increasing as the atmospheric pressure increases and decreasing as the atmospheric pressure decreases. This change in pressure causes the height of the fluid in the columnar structure to change, increasing in height as the atmosphere exerts greater pressure on the liquid in the reservoir base and decreasing as the atmosphere exerts lower pressure on the liquid in the reservoir base. The height of the liquid within the glass column then gives a measure of the atmospheric pressure. Pressure, as determined by hydrostatic barometers, is often measured by determining the height of the liquid in the barometer column, thus the torr as a unit of pressure, but can be used to determine pressure in SI units. Hydrostatic based barometers most commonly use water or mercury as the static liquid. While the use of water is much less hazardous than mercury, mercury is often a better choice for fabricating accurate hydrostatic barometers. The density of mercury is much higher than that of water, thus allowing for higher accuracy of measurements and the ability to fabricate more compact hydrostatic barometers. In theory, a hydrostatic barometer can be placed in a closed system to measure the absolute pressure and the gauge pressure of the system by subtracting the atmospheric pressure.

Aneroid Barometer

Another type of barometer is the aneroid barometer, which consists of a small, flexible sealed metal box called an aneroid cell. The aneroid cell is made from beryllium-copper alloy and is partially evacuated. A stiff spring prevents the aneroid cell from collapsing. Small changes in external air pressure cause the cell to expand or contract. This expansion and contraction is amplified by mechanical mechanisms to give a pressure reading. Such pressure measuring devices are more practical than hydrostatic barometers for measuring system pressures. Many modern pressure measuring devices are pre-engineered to output gauge pressure measurements. While the aneroid barometer is the underlying mechanism behind many modern pressure measuring devices, pressure can also be measured using more advanced measuring mechanisms.



$$\frac{F}{A} = \tau = \eta \frac{du}{dy}$$

Hydrostatic Column Barometer: The concept of determining pressure using the fluid height in a hydrostatic column barometer

$$\Delta p = \frac{8\eta Q \Delta x}{\pi r^4}$$

Variation of Pressure with Height: The density of the liquid is p, g is the acceleration due to gravity, and h is the height of the fluid in the barometer column.

Pressure in the Body

Pressure plays an essential role in a number of critical bodily functions including respiration and blood circulation.

learning obectives

- Explain role played by pressure in the circulatory and respiratory systems

The Role of Pressure in the Circulatory System

Pressure plays an essential role in various critical bodily systems that are necessary for survival. One such critical bodily system which relies on pressure for functionality is the circulatory system, which is an example of a closed fluid system under pressure. The circulatory system is responsible for transporting oxygen and essential nutrients to all organs within the body as well as removing waste materials from these organs. Blood can be regarded as a viscous liquid contained within the circulatory system that travels throughout this closed system as a result of pressure and pressure differences within the circulatory system.

As the volume of blood within the circulatory system is confined to the veins, arteries, and capillaries there is a pressure within this closed system. Furthermore, through a complicated system of veins, arteries, and capillaries of varying diameter as well as valves and the heart acting as a continuous pump, pressure differences arise within the circulatory system that result in the potential for blood to circulate throughout the circulatory system, thus carrying out essential bodily functions for survival.

Pressure within the circulatory system is referred to as blood pressure, and is a primary and crucial vital sign which can be used to diagnose or indicate a number of medical conditions. Blood pressure varies throughout the body as well as from one individual to another and depends on a number of factors such as heart rate, blood volume, resistance of the circulatory system (veins, arteries, and capillaries), and the viscosity of blood. Any medical conditions affecting any of these factors will have an effect on blood pressure and the overall health of the circulatory system.

$$R_e = \frac{V}{I} \quad \rightarrow \quad \begin{cases} R_h \rightarrow R_e \\ \Delta p \rightarrow V \\ I \rightarrow Q \end{cases} \quad \rightarrow \quad \Delta p = \frac{8\eta Q \Delta x}{\pi r^4} \quad \rightarrow \quad R_h = \frac{\Delta p}{Q} = \frac{8\eta \Delta x}{\pi r^4}$$

Approximation for Mean Arterial Pressure: In practice, the mean arterial pressure (MAP) can be approximated from easily obtainable blood pressure measurements.

The mean arterial pressure (MAP) is the average pressure over a cardiac cycle and is determined by, where CO is the cardiac outputs, SVR is the systemic vascular resistance, and CVP is the central venous pressure (CVP). In practice, the mean arterial pressure (MAP) can be approximated from easily obtainable blood pressure measurements in, where P_{sys} is the measured systolic pressure and P_{dias} is the measured diastolic pressure. One particularly common and dangerous circulatory system condition is partial blockage of blood vessels due to a number of factors, such as plaque build-up from high cholesterol, which results in a reduction of the effective blood vessel cross-sectional diameter and a corresponding reduction in blood flow rate and thus an increase in blood pressure to restore normal blood flow according to Poiseuille's Law.

$$\Delta p = \frac{8\eta Q \Delta x}{\pi r^4}$$

Equation for Mean Arterial Pressure: The mean arterial pressure (MAP) is the average pressure over a cardiac cycle and is determined this equation, where CO is the cardiac outputs, SVR is the systemic vascular resistance, and CVP is the central venous pressure (CVP).

The Role of Pressure in the Respiratory System

Pressure also plays an essential role in the respiratory system, as it is responsible for the breathing mechanism. Pressure differences between the lungs and the atmosphere create a potential for air to enter the lungs, resulting in inhalation. The mechanism resulting in inhalation is due to lowering of the diaphragm, which increases the volume of the thoracic cavity surrounding the lungs, thus lowering its pressure as determined by the ideal gas law. The reduction in pressure of the thoracic cavity, which normally has a negative gauge pressure, thus keeping the lungs inflated, pulls air into the lungs, inflating the alveoli and resulting in oxygen transport needed for respiration. As the diaphragm restores and moves upwards, pressure within the thoracic cavity increases, resulting in exhalation. The cycle repeats itself, resulting in the respiration which as discussed is mechanically due to pressure changes. Without pressure in the body, and the corresponding potential that it has for dynamic bodily processes, essential functions such as blood circulation and respiration would not be possible.

Key Points

- Pressure is a scalar quantity defined as force per unit area. Pressure only concerns the force component perpendicular to the surface upon which it acts, thus if the force acts at an angle, the force component along the direction perpendicular to the surface must be used to calculate pressure.
- The pressure exerted on a surface by an object increases as the weight of the object increases or the surface area of contact decreases. Alternatively the pressure exerted decreases as the weight of the object decreases or the surface area of contact increases.
- Pressure exerted by ideal gases in confined containers is due to the average number of collisions of gas molecules with the container walls per unit time. As such, pressure depends on the amount of gas (in number of molecules), its temperature, and the volume of the container.
- Hydrostatic pressure refers to the pressure exerted by a fluid (gas or liquid) at any point in space within that fluid, assuming that the fluid is incompressible and at rest.
- Pressure within a liquid depends only on the density of the liquid, the acceleration due to gravity, and the depth within the liquid. The pressure exerted by such a static liquid increases linearly with increasing depth.
- Pressure within a gas depends on the temperature of the gas, the mass of a single molecule of the gas, the acceleration due to gravity, and the height (or depth) within the gas.
- Hydrostatic balance is the term used for a region or stationary object within a static fluid which is at static equilibrium, and for which the sum of all forces and sum of all torques is equal to zero.
- A region or static object within a stationary fluid experiences downward forces due to the weight of the region or object, and the pressure exerted from the fluid above the region or object, as well as an upward force due to the pressure exerted from the fluid

below the region or object.
- For a region or static object within a static fluid, the downward force due to the weight of the region or object is counteracted by the upward buoyant force, which is equal to the weight of the fluid displaced by the region or object.
- Pascal's Principle is used to quantitatively relate the pressure at two points in an incompressible, static fluid. It states that pressure is transmitted, undiminished, in a closed static fluid.
- The total pressure at any point within an incompressible, static fluid is equal to the sum of the applied pressure at any point in that fluid and the hydrostatic pressure change due to a difference in height within that fluid.
- Through the application of Pascal's Principle, a static liquid can be utilized to generate a large output force using a much smaller input force, yielding important devices such as hydraulic presses.
- Atmospheric pressure is a measure of absolute pressure and is due to the weight of the air molecules above a certain height relative to sea level, increasing with decreasing altitude and decreasing with increasing altitude.
- Gauge pressure is the additional pressure in a system relative to atmospheric pressure. It is a convenient pressure measurement for most practical applications.
- While gauge pressure is more convenient for practical measurements, absolute pressure is necessary for most pressure calculations, thus the atmospheric pressure must be added to the gauge pressure for calculations.
- Gauge pressure is the pressure of a system above atmospheric pressure, which must be converted to absolute pressure for most calculations.
- The barometer is a device which uses hydrostatic fluids to directly determine atmospheric pressure and may be used to indirectly measure the gauge pressure of systems.
- The hydrostatic column barometer uses a liquid like water or mercury for functionality, while the aneroid barometer uses an evacuated flexible metal cell.
- Pressure, along with the potential for work arising from differences in pressure, plays an essential role in the functionality of several critical bodily functions and systems necessary for survival.
- The circulatory system relies on pressure differences for circulating blood, along with oxygen, necessary nutrients, and waste products throughout the body.
- Respiration is made possible as a result of pressure differences between the thoracic cavity, the lungs, and the environment and is largely regulated by movement of the diaphragm.

Key Terms

- **ideal gas**: Theoretical gas characterized by random motion whose individual molecules do not interact with one another and are chemically inert.
- **kinetic energy**: The energy associated with a moving particle or object having a certain mass.
- **incompressible**: Unable to be compressed or condensed.
- **static equilibrium**: the physical state in which all components of a system are at rest and the net force is equal to zero throughout the system
- **Buoyancy**: The power of supporting a body so that it floats; upward pressure exerted by the fluid in which a body is immersed.
- **torque**: Something that produces or tends to produce torsion or rotation; the moment of a force or system of forces tending to cause rotation.
- **equilibrium**: A state of rest or balance due to the equal action of opposing forces.
- **hydraulic press**: Device that uses a hydraulic cylinder (closed static fluid) to generate a compressive force.
- **Gauge Pressure**: The pressure of a system above atmospheric pressure.
- **Torr**: A unit of pressure equal to one millimeter of mercury (760 torr = 101,325 Pa).
- **Aneroid Barometer**: A device for measuring pressure, often specially calibrated for use as an altimeter, consisting of a box or chamber partially exhausted of air, having an elastic top and a pointer to indicate the degree of compression of the top caused by the external air.
- **Thoracic Cavity**: A hollow place or space, or a potential space, within the body or one of its organs.
- **Poiseuille's Law**: The law that the velocity of a liquid flowing through a capillary is directly proportional to the pressure of the liquid and the fourth power of the radius of the capillary and is inversely proportional to the viscosity of the liquid and the length of the capillary.
- **Alveoli**: Small air sacs or cavities in the lung that give the tissue a honeycomb appearance and expand its surface area for the exchange of oxygen and carbon dioxide.

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5.6.3: Archimedes' Principle

learning objectives

- Calculate the direction of the buoyancy force

When you rise from soaking in a warm bath, your arms may feel strangely heavy. This effect is due to the loss of the buoyant support of the water. What creates this buoyant force ? Why is it that some things float and others do not? Do objects that sink get any support at all from the fluid? Is your body buoyed by the atmosphere, or are only helium balloons affected?

Buoyant Force: Cause and Calculation

We find the answers to the above questions in the fact that in any given fluid, pressure increases with depth. When an object is immersed in a fluid, the upward force on the bottom of an object is greater than the downward force on the top of the object. The result is a net upward force (a buoyant force) on any object in any fluid. If the buoyant force is greater than the object's weight, the object will rise to the surface and float. If the buoyant force is less than the object's weight, the object will sink. If the buoyant force equals the object's weight, the object will remain suspended at that depth. The buoyant force is always present in a fluid, whether an object floats, sinks or remains suspended.

The buoyant force is a result of pressure exerted by the fluid. The fluid pushes on all sides of an immersed object, but as pressure increases with depth, the push is stronger on the bottom surface of the object than in the top (as seen in).

You can calculate the buoyant force on an object by adding up the forces exerted on all of an object's sides. For example, consider the object shown in.

The top surface has area A and is at depth h_1; the pressure at that depth is:

$$P_1 = h_1 \rho g, \tag{5.6.3.1}$$

where ρρ is the density of the fluid and $g \approx 9.81 \frac{m}{s^2}$ is the gravitational acceleration. The magnitude of the force on the top surface is:

$$F_1 = P_1 A = h_1 \rho g A. \tag{5.6.3.2}$$

This force points downwards. Similarly, the force on the bottom surface is:

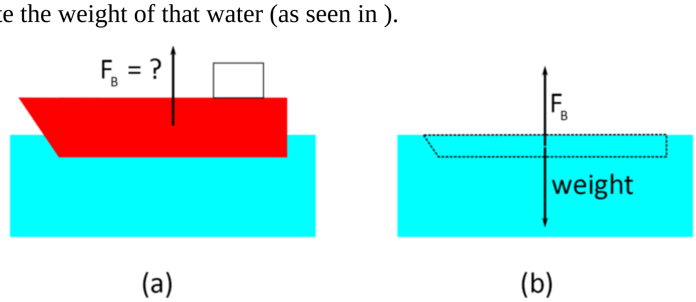
$$F_2 = P_2 A = h_2 \rho g A \tag{5.6.3.3}$$

and points upwards. Because it is cylindrical, the net force on the object's sides is zero—the forces on different parts of the surface oppose each other and cancel exactly. Thus, the net upward force on the cylinder due to the fluid is:

$$F_B = F_2 - F_1 = \rho g A (h_2 - h_1) \tag{5.6.3.4}$$

The Archimedes Principle

Although calculating the buoyant force in this way is always possible it is often very difficult. A simpler method follows from the Archimedes principle, which states that the buoyant force exerted on a body immersed in a fluid is equal to the weight of the fluid the body displaces. In other words, to calculate the buoyant force on an object we assume that the submersed part of the object is made of water and then calculate the weight of that water (as seen in).



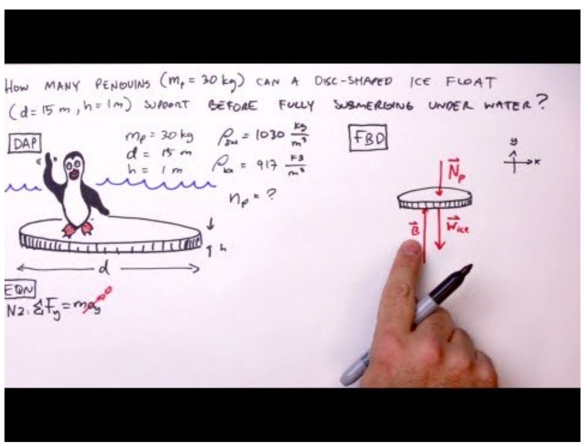
Archimedes principle: The buoyant force on the ship (a) is equal to the weight of the water displaced by the ship—shown as the dashed region in (b).

The principle can be stated as a formula:

$$F_B = w_{fl} \tag{5.6.3.5}$$

The reasoning behind the Archimedes principle is that the buoyancy force on an object depends on the pressure exerted by the fluid on its submerged surface. Imagine that we replace the submerged part of the object with the fluid in which it is contained, as in (b). The buoyancy force on this amount of fluid must be the same as on the original object (the ship). However, we also know that the buoyancy force on the fluid must be equal to its weight, as the fluid does not sink in itself. Therefore, the buoyancy force on the original object is equal to the weight of the "displaced fluid" (in this case, the water inside the dashed region (b)).

The Archimedes principle is valid for any fluid—not only liquids (such as water) but also gases (such as air). We will explore this further as we discuss applications of the principle in subsequent sections.



Archimedes' Principle – Simple Example: We use Archimedes' Principle to determine the number of penguins an ice float can dryly support.

Complete Submersion

The buoyancy force on a completely submerged object of volume is $F_B = V\rho g$.

learning objectivies

- Identify factors determining the buoyancy force on a completely submerged object

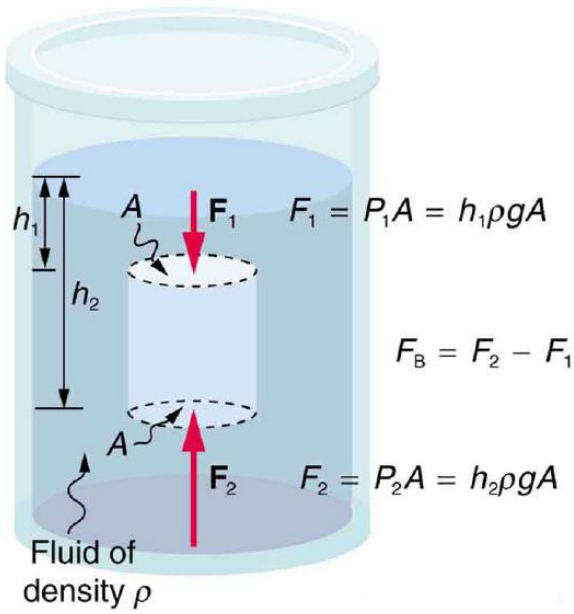
The Archimedes principle is easiest to understand and apply in the case of entirely submersed objects. In this section we discuss a few relevant examples. In general, the buoyancy force on a completely submerged object is given by the formula:

$$F_B = V\rho g, \tag{5.6.3.6}$$

where V is the volume of the object, ρ is the density of the fluid, and g is gravitational acceleration. This follows immediately from the Archimedes' principle, and the fact that the object is completely submerged (and so the volume of the fluid displaced is just the volume of the object).

Cylinder

In the previous section, we calculated the buoyancy force on a cylinder (shown in) by considering the force exerted on each of the cylinder's sides. Now, we'll calculate this force using Archimedes' principle. The buoyancy force on the cylinder is equal to the weight of the displaced fluid. This weight is equal to the mass of the displaced fluid multiplied by the gravitational acceleration:



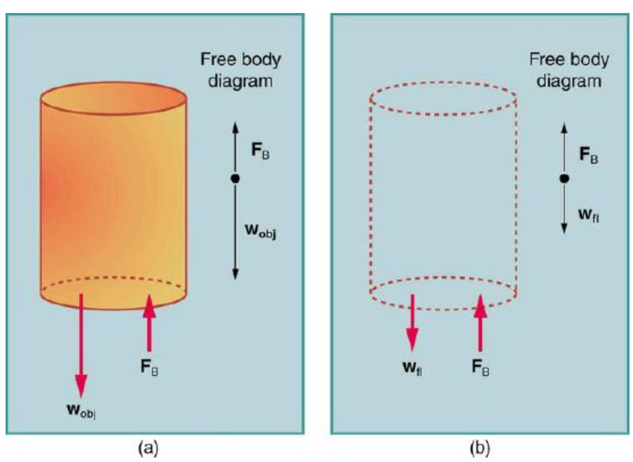
Buoyant force: The fluid pushes on all sides of a submerged object. However, because pressure increases with depth, the upward push on the bottom surface (F2) is greater than the downward push on the top surface (F1). Therefore, the net buoyant force is always upwards.

$$F_B = w_{fl} = m_{fl}g \tag{5.6.3.7}$$

The mass of the displaced fluid is equal to its volume multiplied by its density:

$$m_{fl} = V_{fl}\rho \tag{5.6.3.8}$$

However (*and this is the crucial point*), the cylinder is entirely submerged, so the volume of the displaced fluid is just the volume of the cylinder (see), and:



Archimedes principle: The volume of the fluid displaced (b) is the same as the volume of the original cylinder (a).

$$m_{fl} = V_{fl}\rho = V_{cylinder}\rho. \tag{5.6.3.9}$$

The volume of a cylinder is the area of its base multiplied by its height, or in our case:

$$V_{cylinder} = A(h_2 - h_1). \tag{5.6.3.10}$$

Therefore, the buoyancy force on the cylinder is:

$$F_B = m_{fl}g = V_{cylinder}\rho g = (h_1 - h_2)\rho g A. \tag{5.6.3.11}$$

This is the same result obtained in the previous section by considering the force due to the pressure exerted by the fluid.

Helium Airship

Consider the USS Macon, a helium-filled airship (shown in). Its envelope (the "balloon") contained 184,059.5 cubic meters of helium. Ignoring the small volume of the gondola, what was the buoyancy force on this airship? If the airship weighed 108,000 kg, how much cargo could it carry? Assume the density of air is 1.225 kg per meter cubed. The buoyancy force on an airship is due to the air in which it is immersed. Although we don't know the exact shape of the airship, we know its volume and the density of the air, and thus we can calculate the buoyancy force:

Helium airship: The USS Macon, a 1930s helium-filled airship.

$$F_B = V_{\rho g} = 184,059.5\ \mathrm{kg} \times 1.225 \frac{\mathrm{kg}}{\mathrm{m}^3} \times 9.81 \frac{\mathrm{m}}{\mathrm{s}^2} \approx 2.212 \times 10^6\ \mathrm{N} \tag{5.6.3.12}$$

To find the cargo capacity of the airship, we subtract the weight of the airship from the buoyancy force:

$$F_{cargo} = F_B - mg = 2.21 \times 10^6\ \mathrm{N} - 1.08 \times 10^5\ \mathrm{kg} \times 9.81 \frac{\mathrm{m}}{\mathrm{s}^2} = 1.15 \times 10^6\ \mathrm{N} \tag{5.6.3.13}$$

The mass the airship can carry is:

$$m_{cargo} = \frac{F_{cargo}}{g} = 1.2 \times 10^5\ \mathrm{kg} = 120\ \mathrm{tons}. \tag{5.6.3.14}$$

Flotation

An object floats if the buoyancy force exerted on it by the fluid balances its weight.

learning objectives

- Express the relationship between the buoyancy force and the weight for a floating object

Why do some objects float, but others don't? If you put a metal coin into a glass of water it will sink. But most ships are built of metal, and they float. So how is this possible?

Condition for Flotation

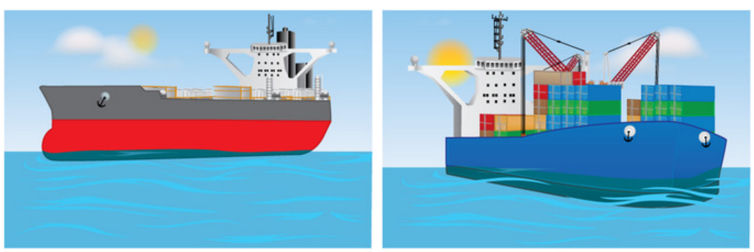
An object will float if the buoyancy force exerted on it by the fluid balances its weight, i.e. if FB=mgFB=mg.

But the Archimedes principle states that the buoyant force is the weight of the fluid displaced. So, for a floating object on a liquid, the weight of the displaced liquid is the weight of the object. Thus, only in the special case of floating does the buoyant force acting on an object equal the object's weight. Consider a one-ton block of solid iron. As iron is nearly eight times denser than water, it displaces only 1/8 ton of water when submerged, which is not enough to keep it afloat. Suppose the same iron block is reshaped into a bowl. It still weighs one ton, but when it is put in water, it displaces a greater volume of water than when it was a block. The deeper the iron bowl is immersed, the more water it displaces, and the greater the buoyant force acting on it. When the buoyant force equals one ton, it will sink no further.

When any boat displaces a weight of water equal to its own weight, it floats. This is often called the "principle of flotation" where a floating object displaces a weight of fluid equal to its own weight. Every ship, submarine, and dirigible must be designed to displace a weight of fluid equal to its own weight. A 10,000-ton ship must be built wide enough to displace 10,000 tons of water before it sinks too deep in the water. The same is true for vessels in air (as air is a fluid): A dirigible that weighs 100 tons displaces at least 100 tons of air; if it displaces more, it rises; if it displaces less, it falls. If the dirigible displaces exactly its weight, it hovers at a constant altitude.

Flotation and Density

Density plays a crucial role in Archimedes' principle. The average density of an object is what ultimately determines whether it floats. If its average density is less than that of the surrounding fluid, it will float. This is because the fluid, having a higher density, contains more mass and thus more weight in the same volume. The buoyant force, which equals the weight of the fluid displaced, is thus greater than the weight of the object. Likewise, an object denser than the fluid will sink. The extent to which a floating object is submerged depends on how the object's density is related to that of the fluid. For example, an unloaded ship has a lower density, and less of it is submerged compared with the same ship loaded with cargo. We can derive a quantitative expression for the fraction submerged by considering density. The fraction submerged is the ratio of the volume submerged to the volume of the object, or



(a) (b)

Density and Submersion: An unloaded ship (a) floats higher in the water than a loaded ship (b).

$$\text{fraction submerged} = \frac{V_{sub}}{V_{obj}} = \frac{V_{fl}}{V_{obj}} \tag{5.6.3.15}$$

The volume submerged equals the volume of fluid displaced, which we call V_{fl}. Now we can obtain the relationship between the densities by substituting $\rho = mV$ into the expression. This gives

$$\text{fraction submerged} = \frac{m_{fl}/\rho_{fl}}{m_{obj}/\bar{\rho}_{obj}} \tag{5.6.3.16}$$

where $\bar{\rho}_{obj}$ is the average density of the object and ρflρflis the density of the fluid. Since the object floats, its mass and that of the displaced fluid are equal, and so they cancel from the equation, leaving

$$\text{fraction submerged} = \frac{\bar{\rho}_{obj}}{\rho_{fl}}. \tag{5.6.3.17}$$

There are a couple things to note about this expression:

1. Note that it mentions the average density of the object. This can be much less than the density of the material the object is made of. For instance, a steel ship is actually mostly filled with air (think of the corridors, cargo holds, etc.), so its average density is

between that of air and steel. To be more precise, the average density is defined as the total mass of an object divided by its total volume: $\bar{\rho} = \frac{m}{V}$.
2. This formula makes sense only if the density of the object is smaller than the density of the fluid. Otherwise, the fraction submerged becomes greater than one—a sign that the object does not float at all, but it sinks!

Key Points

- The buoyancy force is caused by the pressure exerted by the fluid in which an object is immersed.
- The buoyancy force always points upwards because the pressure of a fluid increases with depth.
- You can calculate the buoyancy force either directly by computing the force exerted on each of the object's surfaces, or indirectly by finding the weight of the displaced fluid.
- If an object is completely submerged, the volume of the fluid displaced is equal to the volume of the object.
- The buoyancy force on hot-air balloons, dirigibles and other objects can be calculated by assuming that they are entirely submerged in air.
- The buoyancy force does not depend on the shape of the object, only on its volume.
- The buoyancy force experienced by an object depends on its shape.
- The fraction of an object's volume that's submerged is given by the ratio of its average density to that of the fluid: $\frac{\bar{\rho}_{obj}}{\rho_{fl}}$.
- An object floats if the buoyancy force exerted on it by the fluid balances its weight.

Key Terms

- **buoyant force**: An upward force exerted by a fluid that opposes the weight of an immersed object.
- **Archimedes principle**: The buoyant force exerted on a body immersed in a fluid is equal to the weight of the fluid the body displaces.

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5.6.4: Cohesion and Adhesion

learning objectives

- Explain the phenomena of surface tension and capillary action

Attractive forces between molecules of the same type are called cohesive forces. Liquids can, for example, be kept in open containers because cohesive forces hold the molecules together. Attractive forces between molecules of different types are called adhesive forces. Such forces cause liquid drops to cling to window panes, for example. In this section we examine effects of cohesive and adhesive forces in liquids.

Surface Tension

Surface tension is a contractive tendency of the surface of a liquid that allows it to resist an external force. It is shown, for example, in the floating of some objects on the surface of water, even though they are denser than water, and in the ability of some insects (e.g., water striders) to run on water's surface. This property is caused by cohesion of similar molecules and is responsible for many of the behaviors of liquids.

The cohesive forces among liquid molecules are responsible for the phenomenon of surface tension, as shown in. In the bulk of the liquid, each molecule is pulled equally in every direction by neighboring liquid molecules, resulting in a net force of zero. The molecules at the surface do not have other molecules on all sides of them and therefore are pulled inwards. This creates some internal pressure and forces liquid surfaces to contract to the minimal area.

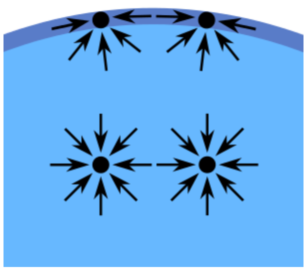


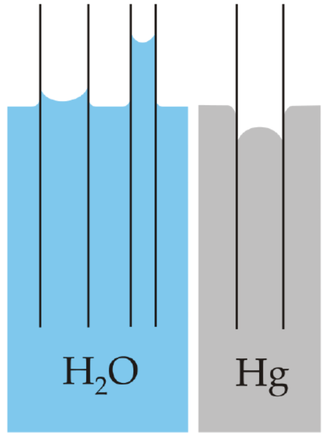
Diagram of Surface-Tension Forces: Diagram of the forces on molecules of a liquid

Surface tension has the unit of force per unit length, or of energy per unit area. The two units are equivalent. However, when we refer to energy per unit of area, we use the term surface energy, which is more general in that it applies to solids as well as liquids.

Capillary Action

Capillary action, or capillarity, is the ability of a liquid to flow in narrow spaces without the assistance of, and in opposition to, external forces like gravity. The effect can be seen in the drawing-up of liquids between the hairs of a paint-brush, in a thin tube, in porous materials such as paper, in some non-porous materials such as liquified carbon fiber, and in a cell. It occurs because of intermolecular attractive forces between the liquid and solid surrounding surfaces. If the diameter of the tube is sufficiently small, then the combination of surface tension (which is caused by cohesion within the liquid) and adhesive forces between the liquid and the container act to lift the liquid.

With some pairs of materials, such as mercury and glass (see), the intermolecular forces within the liquid exceed those between the solid and the liquid, so a convex meniscus forms, and capillary action works in reverse.



Capillarity: Capillary action of water compared to mercury, in each case with respect to glass

Key Points

- Attractive forces between molecules of the same type are called cohesive forces.
- Attractive forces between molecules of different types are called adhesive forces.
- Surface tension is a contractive tendency of the surface of a liquid that allows it to resist an external force.
- Capillary action, or capillarity, is the ability of a liquid to flow in narrow spaces without the assistance of, and in opposition to, external forces such as gravity.

Key Term

- **Pressure**: the amount of force that is applied over a given area divided by the size of that area
- **intermolecular**: from one molecule to another; between molecules

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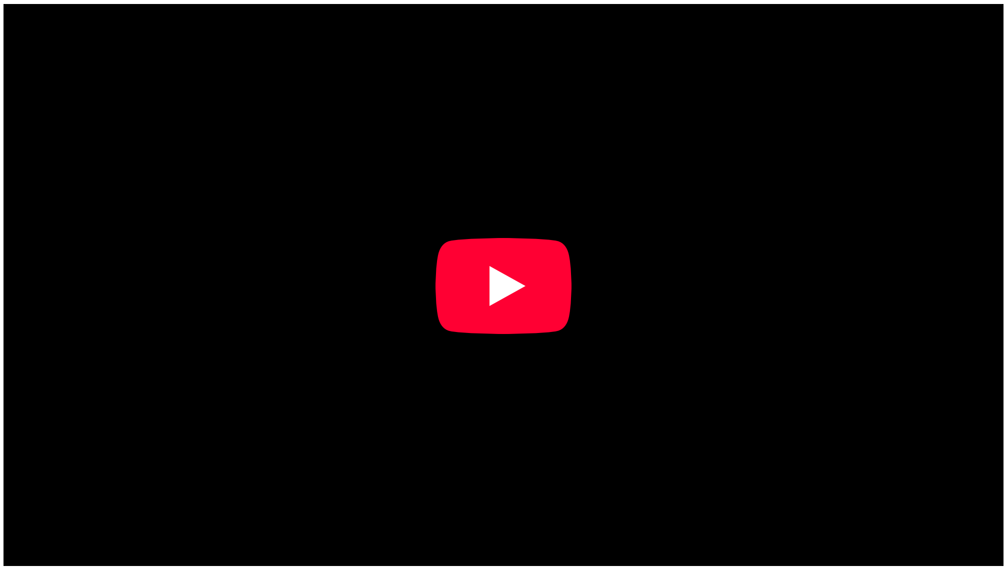
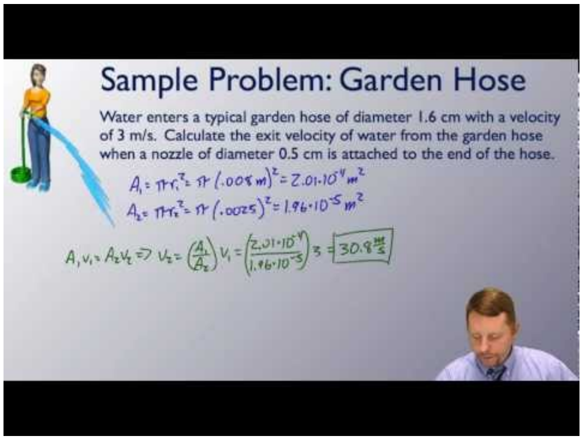
5.6.5: Fluids in Motion

learning objectives

- Determine the flow rate based on velocity and area or elapsed time and justify the use of continuity in expressing properties of a fluid and its motion

The flow rate of a fluid is the volume of fluid which passes through a surface in a given unit of time. It is usually represented by the symbol Q.





Continuity Equation for Fluids: A brief introduction to the Continuity Equation for Fluids.

Flow Rate

Volumetric flow rate is defined as

$$Q = v \times a, \tag{5.6.5.1}$$

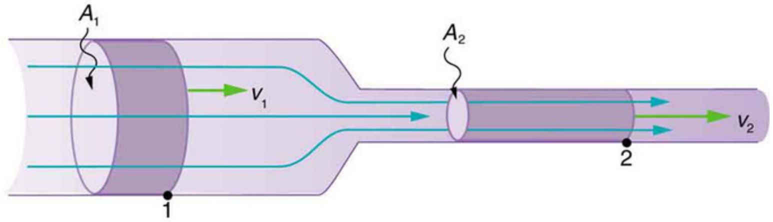
where Q is the flow rate, v is the velocity of the fluid, and a is the area of the cross section of the space the fluid is moving through. Volumetric flow rate can also be found with

$$Q = \frac{V}{t} \tag{5.6.5.2}$$

where Q is the flow rate, V is the Volume of fluid, and t is elapsed time.

Continuity

The equation of continuity works under the assumption that the flow in will equal the flow out. This can be useful to solve for many properties of the fluid and its motion:



Flow in = Flow out: Using the known properties of a fluid in one condition, we can use the continuity equation to solve for the properties of the same fluid under other conditions.

$$Q_1 = Q_2 \tag{5.6.5.3}$$

This can be expressed in many ways, for example: $A_1 v_1 = A_2 v_2$. The equation of continuity applies to any incompressible fluid. Since the fluid cannot be compressed, the amount of fluid which flows into a surface must equal the amount flowing out of the surface.

Applying the Continuity Equation

You can observe the continuity equation's effect in a garden hose. The water flows through the hose and when it reaches the narrower nozzle, the velocity of the water increases. Speed increases when cross-sectional area decreases, and speed decreases when cross-sectional area increases. This is a consequence of the continuity equation. If the flow Q is held constant, when the area A decreases, the velocity v must increase proportionally. For example, if the nozzle of the hose is half the area of the hose, the velocity must double to maintain the continuous flow.

Key Points

- Flow rate can be expressed in either terms of cross sectional area and velocity, or volume and time.
- Because liquids are incompressible, the rate of flow into an area must equal the rate of flow out of an area. This is known as the equation of continuity.
- The equation of continuity can show how much the speed of a liquid increases if it is forced to flow through a smaller area. For example, if the area of a pipe is halved, the velocity of the fluid will double.
- Although gases often behave as fluids, they are not incompressible the way liquids are and so the continuity equation does not apply.

Key Terms

- **incompressible**: Unable to be compressed or condensed.
- **continuity**: Lack of interruption or disconnection; the quality of being continuous in space or time.

5.6.6: Deformation of Solids

Learning objectives

- Explain how length of an object is determined

Length

In geometric measurements, length is the longest dimension of an object. In other contexts “length” is the measured dimension of an object. For example: it is possible to cut a length of a wire which is shorter than wire thickness. Length may be distinguished from height, which is vertical extent, and width or breadth, which are the distance from side to side, measuring across the object at right angles to the length.

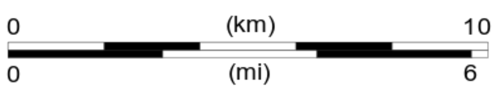
Length is a measure of one dimension, whereas area is a measure of two dimensions (length squared) and volume is a measure of three dimensions (length cubed). In most systems of measurement, the unit of length is a fundamental unit, from which other units are defined.

After Albert Einstein’s Special Relativity Theory, length can no longer be thought of being constant in all reference frames. Thus, a ruler that is one meter long in one frame of reference will not be one meter long in a reference frame that is travelling at a velocity relative to the first frame. This means that the length of an object is variable depending on the observer.

Units

One of the oldest units of length measurement used in the ancient world was the ‘cubit,’ which was the length of the arm from the tip of the finger to the elbow. This could then be subdivided into shorter units like the foot, hand (which at 4 inches is still used today for expressing the height of horses) or finger, or added together to make longer units like the stride. The cubit could vary considerably due to the different sizes of people.

In the physical sciences and engineering, when one speaks of “units of length”, the word “length” is synonymous with “distance”. There are several units that are used to measure length. Units of length may be based on lengths of human body parts, the distance traveled in a number of paces, the distance between landmarks or places on the Earth, or arbitrarily on the length of some fixed object.In the International System of Units (SI), the basic unit of length is the meter and is now defined in terms of the speed of light. The centimeter and the kilometer, derived from the meter, are also commonly used units. In U.S. customary units, English or Imperial system of units, commonly used units of length are the inch, the foot, the yard, and the mile. Units used to denote distances in the vastness of space, as in astronomy, are much longer than those typically used on Earth and include the astronomical unit, the light-year, and the parsec.



Length: The metric length of one kilometre is equivalent to the imperial measurement of 0.62137 miles.

Shape

The shape of an object is a description of space that the object takes up; the shape can change if the object is deformed.

learning objectives

- Describe effects of deformations, rotations, and magnifications

Shape

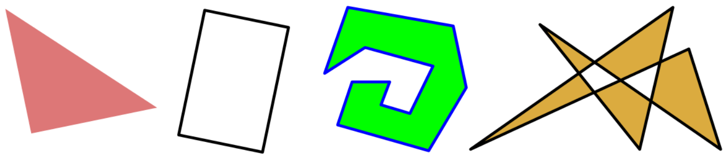
The shape of an object located in some space is a geometrical description of the part of that space occupied by the object, as determined by its external boundary – abstracting from location and orientation in space, size, and other properties such as color, content, and material composition

Simple and Complex Shapes

Simple shapes can be described by basic geometry objects such as a set of two or more points, a line, a curve, a plane, a plane figure (e.g. square or circle), or a solid figure (e.g. cube or sphere). Most shapes occurring in the physical world are complex. Some, such as plant structures and coastlines, may be so arbitrary as to defy traditional mathematical description – in which case they may be analyzed by differential geometry, or as fractals.

In geometry, two subsets of a Euclidean space have the same shape if one can be transformed to the other by a combination of translations, rotations (together also called rigid transformations), and uniform scalings. In other words, the shape of a set of points is all the geometrical information that is invariant to translations, rotations, and size changes. Having the same shape is an equivalence relation, and accordingly a precise mathematical definition of the notion of shape can be given as being an equivalence class of subsets of a Euclidean space having the same shape.

Shapes of physical objects are equal if the subsets of space these objects occupy satisfy the definition above. In particular, the shape does not depend on the size and placement in space of the object.



Shapes: Examples of shapes.

Volume

Volume is a measure of the three-dimensional space an object occupies, *usually* taken in terms of length, width and height.

learning objectives

- Explain how is volume measured geometrically

Volume is the quantity of three-dimensional space contained by a closed boundary; it is the space that a substance (solid, liquid, gas or plasma) or shape occupies or contains. Volume is often quantified numerically using an SI derived unit, the cubic meter. However, for liquids the unit of volume used is known as the liter (equivalent to 0.001 cubic meters).

Measuring Volume: A measuring cup can be used to measure volumes of liquids. This cup measures volume in units of cups, fluid ounces and millilitres.

Volume is measured geometrically by multiplying an object's three dimensions—usually taken as length, width and height. Some common volumes are taken as follows:

- The volume of a cube: length times width times height.
- The volume of a cylinder: the cross-sectional area times the height of the cylinder.
- The volume of a sphere: 4/3 times the radius cubed times pi.

The volume of a solid can be determined by the volume of liquid it displaces when submerged.

The volume of a container is generally understood as the capacity of the container, meaning the amount of fluid (gas or liquid) the container can hold, rather than the amount of space the container itself displaces. Measuring cups, as seen in, work by taking a known cross sectional area of a cup and multiplying that by a variable height. Since liquid will always cover the cross section (if there is enough liquid), adding more liquid will increase the height inside the container.

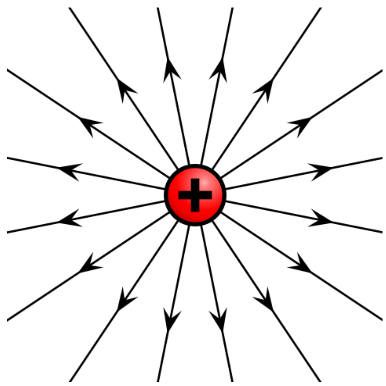
Liquids take the shape of their container, filling up the minimum height needed. Gases, on the other hand, take up the maximum amount of volume possible. Thus a measuring cup can accurately measure the volume of a liquid, whereas a gas will always fill the entire container, more or less uniformly, no matter how little gas there is.

Stress and Strain

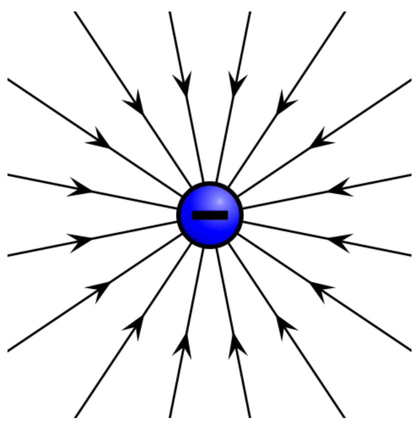
A point charge creates an electric field that can be calculated using Coulomb's Law.

The electric field of a point charge is, like any electric field, a vector field that represents the effect that the point charge has on other charges around it. The effect is felt as a force and when charged particles are not in motion this force is known as the electrostatic force. The electrostatic force is, much like gravity, a force that acts at a distance. Therefore, we rationalize this action at a distance by saying that charges create fields around them that have effects on other charges.

Given a point charge, or a particle of infinitesimal size that contains a certain charge, electric field lines emanate radially in all directions. If the charge is positive, field lines point radially away from it; if the charge is negative, field lines point radially towards it.



Electric field of positive point charge: The electric field of a positively charged particle points radially away from the charge.

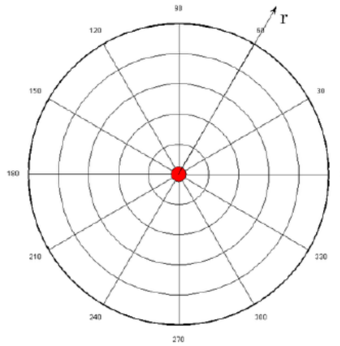


Electric field of negative point charge: The electric field of a negatively charged particle points radially toward the particle.

The reason for these directions can be seen in the derivation of the electric field of a point charge. Let's first take a look at the definition of electric field of a point particle:

$$\overrightarrow{\mathrm{E}} = \frac{1}{4\pi\epsilon_{\mathrm{o}}}\frac{\mathrm{q}}{\mathrm{r}^{2}}\hat{\mathrm{r}} = \mathrm{k}\frac{\mathrm{q}}{\mathrm{r}^{2}}\hat{\mathrm{r}}. \tag{5.6.6.1}$$

In the above equation, q represents the charge of the particle creating the electric field and the constant k is a result of simply lumping the constants together. This charge is either positive or negative. If the charge is positive, as shown above, the electric field will be pointing in a positive radial direction from the charge q (away from the charge) and the following text explains why. The above equation is defined in radial coordinates which can be seen in.



Radial Coordinate System: The electric field of a point charge is defined in radial coordinates. The positive r direction points away from the origin, and the negative r direction points toward the origin. The electric field of a point charge is symmetric with respect to the θ direction.

If we now place another positive charge, Q (called the test charge), at some radial distance, R, away from the original particle, the test charge will feel a force given by

$$\overrightarrow{\mathrm{F}}=\mathrm{Q}\overrightarrow{\mathrm{E}}=\mathrm{Q}\frac{1}{4\pi\epsilon_{\mathrm{o}}}\frac{\mathrm{q}}{\mathrm{R}^2}\hat{\mathrm{r}} \tag{5.6.6.2}$$

The thing to keep in mind is that the force above is acting on the test charge Q, in the positive radial direction as defined by the original charge q. This means that because the charges are both positive and will repel one another, the force on the test charge points away from the original charge.

If the test charge were negative, the force felt on that charge would be

$$\overrightarrow{\mathrm{F}}=\mathrm{Q}\overrightarrow{\mathrm{E}}=-\mathrm{Q}\frac{1}{4\pi\epsilon_{\mathrm{o}}}\frac{\mathrm{q}}{\mathrm{R}^2}\hat{\mathrm{r}} \tag{5.6.6.3}$$

Notice that this points in the negative $\hat{\mathrm{r}}$ direction, which is toward the original charge. This makes sense because opposite charges attract and the force on the test charge will tend to push it toward the original positive charge creating the field. The above mathematical description of the electric field of a point charge is known as Coulomb ‘s Law.

Key Points

- Length is typically a measure of the longest dimension of an object.
- The deformation of an object is typically a change in length.
- The SI unit of length is the meter.
- The shape of an object is a representation of the space taken up by the object.
- Deformations can change the shape of an object.
- Objects that have the same shape can be transformed into each other by rotation or magnification.
- Volume is often quantified numerically using an SI derived unit, the cubic meter. However, for liquids the unit of volume used is known as the liter (equivalent to 0.001 cubic meters).
- This can also be understood as the amount of fluid a submerged object displaces.
- Volume can be measured for geometrically regular objects by simple formulas. However, more complicated objects are easier to measure with fluid displacement.
- The electric field is a vector field around a charged particle that represents the force that other charged particles would feel when placed near the particle creating the electric field.
- Given a point charge, or a particle of infinitesimal size, that contains a certain charge, electric field lines emanate from equally in all radial directions.
- If the point charge is positive, field lines point away from it; if the charge is negative, field lines point towards it.

Key Terms

- **dimension**: A measure of spatial extent in a particular direction, such as height, width or breadth, or depth.
- **special relativity**: A theory that (neglecting the effects of gravity) reconciles the principle of relativity with the observation that the speed of light is constant in all frames of reference.
- **plane**: A level or flat surface.
- **Euclidean**: Adhering to the principles of traditional geometry, in which parallel lines are equidistant.
- **cross section**: A section formed by a plane cutting through an object, usually at right angles to an axis.
- **dimension**: A measure of spatial extent in a particular direction, such as height, width or breadth, or depth.
- **coulomb's law**: the mathematical equation calculating the electrostatic force vector between two charged particles
- **vector field**: a construction in which each point in a Euclidean space is associated with a vector; a function whose range is a vector space

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CHAPTER OVERVIEW

5.7: Fluid Dynamics and Its Applications

5.7.1: Overview

learning objectives

- Interpret the circulatory system in terms of your knowledge of fluid dynamics

We have discussed many situations in which fluids are static, though there are many situations where fluids flow. For example, a column of smoke rises from a camp fire, water streams from a fire hose, blood courses through your veins. Why does rising smoke curl and twist? How does a nozzle increase the speed of water emerging from a hose? How does the body regulate blood flow? Fluid dynamics, the physics of fluids in motion, allows us to answer these and many other questions.

Application in the Circulatory System

For example, consider the circulatory system—a connected series of tubes with fluid flowing through them. The heart is the driver of the circulatory system, generating cardiac output (CO) by rhythmically contracting and relaxing. This creates changes in regional pressures and (combined with a complex valvular system in the heart and the veins) ensures that the blood moves around the circulatory system in one direction. The "beating" of the heart generates pulsatile blood flow, conducted into the arteries across the micro-circulation and then back via the venous system to the heart.

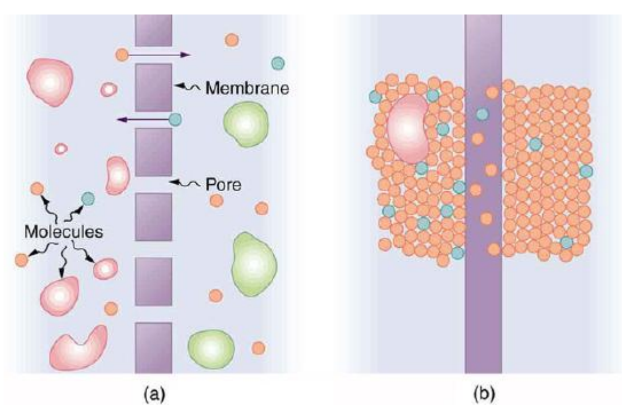
The aorta, the main artery, leaves the left side of the heart and proceeds to divide into smaller and smaller arteries that first become arterioles and eventually become capillaries, through which oxygen transfer occurs. The capillaries connect to venules, into which the deoxygenated blood passes from the cells back into the blood. The blood then travels back through the network of veins to the right heart. The micro-circulation (arterioles, capillaries and venules) constitutes most of the area of the vascular system and is the site of the transfer of O_2 into the cells.

The venous system returns the de-oxygenated blood to the right heart where it is pumped into the lungs to become oxygenated. This is also where CO_2 and other gaseous wastes are exchanged and expelled during breathing. Blood then returns to the left side of the heart where it begins the process again. The heart, vessels and lungs are all actively involved in maintaining healthy cells and organs, and all influence the fluid dynamics of the blood.

Fluids and Diffusion

Now consider how nutrients are transported through a human body. Diffusion is the movement of substances due to random thermal molecular motion. Fluids can even diffuse through solids (such as fumes or odors entering ice cubes). Diffusion is the dominant mechanism by which the exchange of nutrients and waste products occurs between the blood and tissue, and between air and blood in the lungs. In the evolutionary process, as organisms became larger they needed quicker methods of transportation than net diffusion, due to the larger distances involved in the transport. This factor lead to the development of circulatory systems. Less sophisticated, single-celled organisms still rely totally on diffusion for the removal of waste products and the uptake of nutrients.

Another important form of fluid movement is osmosis—the transport of water through a semipermeable membrane (shown in) from a region of high concentration to a region of low concentration. It is driven by the imbalance in water concentration. Similarly, dialysis is the transport of any other molecule through a semipermeable membrane due to its concentration difference. Both osmosis and dialysis are used by the kidneys to cleanse the blood, and the medical application of dialysis through machinery is important in the treatment of individuals with failing kidney function.



A Semipermeable Membrane: A semipermeable membrane with small pores that allow only small molecules to pass through.

Flow Rate and Velocity

Flow velocity and volumetric flow rates are important quantities in fluid dynamics used to quantify motion of a fluid and are interrelated.

learning objectives

- Assess the significance of studying volumetric flow in addition to flow velocity

Fluid dynamics is the study of fluids in motion and corresponding phenomena. A fluid in motion has a velocity, just as a solid object in motion has a velocity. Like the velocity of a solid, the velocity of a fluid is the rate of change of position per unit of time. In mathematical terms, the velocity of a fluid is the derivative of the position vector of the fluid with respect to time, and is therefore itself a vector quantity. The flow velocity vector is a function of position, and if the velocity of the fluid is not constant then it is also a function of time. Equation 1 shows the mathematical expression for the velocity of a fluid in motion. As a vector quantity, fluid velocity must have at least one non-zero directional component and may have up to three non-zero directional components. The velocity vector has non-zero components in any orthogonal direction along which motion of the fluid occurs.

$$\Delta p = \frac{8\eta Q \Delta x}{\pi r^4}$$

Flow Velocity: Mathematical Expression for Flow Velocity

Turbulent Flow vs. Laminar Flow

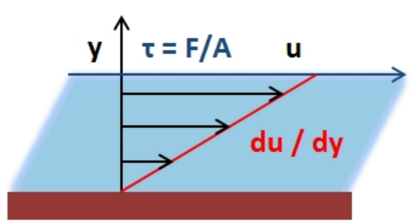
Fluid velocity can be affected by the pressure of the fluid, the viscosity of the fluid, and the cross-sectional area of the container in which the fluid is travelling. These factors affect fluid velocity depending on the nature of the fluid flow—particularly whether the flow is turbulent or laminar in nature. In the case of turbulent flow, the flow velocity is complex in nature and thus hard to predict; it must be analyzed on a system per system basis. In the case of Laminar flow, however, fluid flow is much simpler and flow velocity can be accurately calculated using Poiseuille's Law. In SI units, fluid flow velocity is expressed in terms of meters per seconds. The magnitude of the fluid flow velocity is the fluid flow speed. Fluid flow velocity effectively describes everything about the motion of a fluid.

Volumetric Flow

In addition to flow velocity, volumetric flow rate is an important quantity in fluid dynamics analysis. Volumetric flow is defined as the volume of fluid that passes through a given surface per unit time. Qualitatively, Figure 1 shows the notion of volumetric flow rate regarding a cross-sectional surface of area A. Mathematically, volumetric flow rate is the derivative of the volume of fluid that passes through a given surface with respect to time; in SI units this is expressed as meterscubed per second. Volumetric flow rate is related to the flow velocity vector as the surface integral with respect to the surface in question. If the surface area in question is a flat, plane cross-section, the surface integral reduces as shown in Equation 2, where A is the surface area of the surface in question and v is the flow velocity of the fluid.

$$R_e = \frac{V}{I} \quad \rightarrow \quad \begin{cases} R_h \rightarrow R_e \\ \Delta p \rightarrow V \\ I \rightarrow Q \end{cases} \quad \rightarrow \quad \Delta p = \frac{8\eta Q \Delta x}{\pi r^4} \quad \rightarrow \quad R_h = \frac{\Delta p}{Q} = \frac{8\eta \Delta x}{\pi r^4}$$

Volumetric Flow Rate: Volumetric Flow Rate Surface Integral and its simplification



$$\frac{F}{A} = \tau = \eta \frac{du}{dy}$$

Flow Velocity – Volumetric Flow Rate Relation: This figure shows the relation between flow velocity and volumetric flow rate.

Moreover, only the flow velocity component parallel to the surface normal of the surface in question, or alternatively the flow velocity component perpendicular to the surface in question contributes to the volumetric flow rate. Figure 1 and Equation 2 illustrate decomposition of the flow velocity vector, making an angle θ with respect to the normal of the surface plane in order to calculate volumetric flow rate through that surface. Thus, volumetric flow rate for a given fluid velocity and cross-sectional surface area increases as θ decreases, and is maximized when $\theta = 0$. Volumetric flow rate is an important scalar quantity in fluid dynamics and is used widely in fluid flow measurements. Volumetric flow rate can be converted to mass flow rate if the density of the fluid is known. Flow of fluids through a closed system is often analyzed as a hydraulic circuit analogous to electron flow in an electronic circuit where: 1) the volumetric fluid flow is analogous to the electric current, 2) pressure is analogous to the voltage, and 3) fluid velocity is analogous to current density.

Key Points

- There are many fluids in biology and understanding their behavior in motion is crucial to effective medicine.
- The heart pumps a fluid, blood, throughout a series of tubes in the body.
- Circulation may be understood through a study of fluid dynamics.
- Diffusion is the dominant mechanism by which the exchange of nutrients and waste products occur between the blood and tissue, and between air and blood in the lungs.
- Both osmosis and dialysis are used by the kidneys to cleanse the blood, and the medical application of dialysis through machinery is important in the treatment of individuals with failing kidney function.
- Flow velocity is a vector quantity used to describe the motion of a fluid. It can be easily determined for laminar flow but complex to determine for turbulent flow.
- Volumetric flow rate is the volume of a liquid that passes through a given surface per unit time. It is found from the flow velocity and the surface area of the surface through which the fluid passes.
- Fluid flow through a closed hydraulic system is analyzed much like electron flow through an electronic circuit —where volumetric flow rate is analogous to current, flow velocity is analogous to current density, and pressure is analogous to voltage (electrical potential).

Key Terms

- **vascular**: Of, pertaining to, or containing vessels that conduct or circulate fluids (such as blood, lymph, or sap) through the body of an animal or plant.
- **osmosis**: The net movement of solvent molecules from a region of high solvent potential to a region of lower solvent potential through a partially permeable membrane.
- **dialysis**: A method of separating molecules or particles of different sizes by differential diffusion through a semipermeable membrane.
- **Laminar Flow**: Non-turbulent motion of a fluid in which parallel layers have different velocities relative to each other.
- **Turbulent Flow**: The motion of a fluid having local velocities and pressures that fluctuate randomly.

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5.7.2: Flow in Tubes

learning objectives

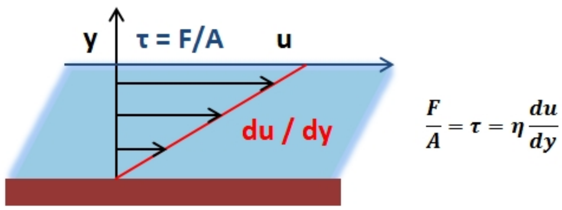
- Contrast turbulent and laminar flow in constant velocity

Virtually all moving fluids exhibit viscosity, which is a measure of the resistance of a fluid to flow. Viscosity is a basic property necessary for the analysis of fluid flow.

Measure of Fluid Friction

It describes a fluid's internal resistance to movement and can be thought of as a measure of fluid friction. The greater the viscosity, the 'thicker' the fluid and the more the fluid will resist movement.

Mathematically, viscosity is a proportionality constant relating an applied shear stress to the resulting shear velocity and is given, along with a representative diagram, (see). As shown, when a force is applied to a fluid, creating a shear stress, the fluid will undergo a certain displacement. The viscosity of the fluid is then its inherent resistance to undergo this displacement.



Representation of Viscosity: A proportionality constant relating an applied shear stress to the resulting shear velocity.

Different fluids exhibit different viscous behavior yet, in this analysis, only Newtonian fluids (fluids with constant velocity independent of applied shear stress) will be considered. Viscosity in fluids generally decreases with increasing temperature. The study of the viscous nature of fluids is called *rheology*.

In analyzing the properties of moving fluids, it is necessary to determine the nature of flow of the fluid. This is generally split into two categories, laminar and turbulent flow.

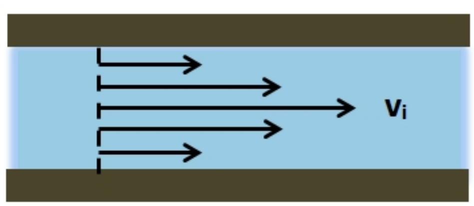
Turbulent Flow

Turbulent flow is characterized by irregular flow of a fluid in which there are both inconsistent flow patterns and velocity variations throughout the volume of the fluid in motion. Analysis of turbulent flow can be very complex and often requires advanced mathematical analysis to simulate flow in systems on a near case-by-case basis.

It occurs when the Reynolds number is above a certain critical threshold while mixed turbulent–laminar flow occurs within a range of Reynolds number below this threshold value. At the lower limit of this mixed turbulent–laminar flow Reynolds number region there is another critical threshold value, below which only laminar flow is possible.

Laminar Flow

Laminar flow consists of a regular-flow pattern with constant-flow velocity throughout the fluid volume and is much easier to analyze than turbulent flow.



Relative Magnitudes of Velocity Vectors: Laminar fluid flow in a circular pipe at the same direction.

Laminar flow is often encountered in common hydraulic systems, such as where fluid flow is through an enclosed, rigid pipe; the fluid is incompressible, has constant viscosity, and the Reynolds number is below this lower critical threshold value. It is characterized by the flow of a fluid in parallel layers, in which there is no disruption or interaction between the different layers, and in which each layer flows at a different velocity along the same direction. The variation in velocity between adjacent parallel layers is due to the viscosity of the fluid and resulting shear forces.

This figure (see) gives a representation of the relative magnitudes of the velocity vectors of each of these layers for laminar fluid flow through a circular pipe, in a direction parallel to the pipe axis.

$$\Delta p = \frac{8\eta Q \Delta x}{\pi r^4}$$

Poiseuille's Equation: Can be used to determine the pressure drop of a constant viscosity fluid exhibiting laminar flow through a rigid pipe.

Considering laminar flow of a constant density, incompressible fluid such as for a Newtonian fluid traveling in a pipe, with a Reynolds number below the upper limit level for fully laminar flow, the pressure difference between two points along the pipe can be found from the volumetric flow rate, or vice versa. For such a system with a pipe radius of r, fluid viscosity η, distance between the two points along the pipe $\Delta x = x_2 - x_1$, and the volumetric flow rate Q, of the fluid, the pressure difference between the two points along the pipe Δp is given by Poiseuille's equation (see).

This equation is valid for laminar flow of incompressible fluids only, and may be used to determine a number of properties in the hydraulic system, if the others are known or can be measured. In practice, Poiseuille's equation holds for most systems involving laminar flow of a fluid, except at regions where features disrupting laminar flow, such as at the ends of a pipe, are present.

Poiseuille's equation as given in this example (see) is analogous to Ohm 's equation for determining the resistance in an electronic circuit and is of great practical use in hydraulic-circuit analysis.

$$R_e = \frac{V}{I} \quad \rightarrow \quad \begin{cases} R_h \rightarrow R_e \\ \Delta p \rightarrow V \\ I \rightarrow Q \end{cases} \quad \rightarrow \quad \Delta p = \frac{8\eta Q \Delta x}{\pi r^4} \quad \rightarrow \quad R_h = \frac{\Delta p}{Q} = \frac{8\eta \Delta x}{\pi r^4}$$

Poiseuille's Equation: Analogous to Ohm's Law Analogy

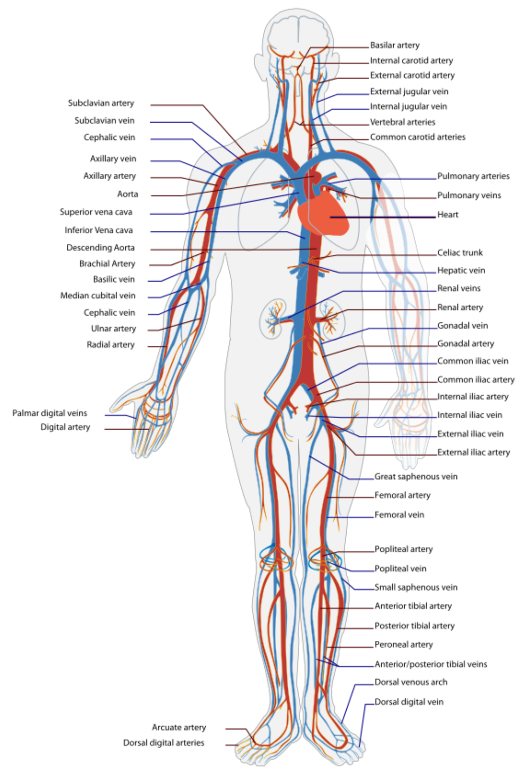
Blood Flow

Blood flow is the continuous running of blood through the cardiovascular system, which consists of the vessels and the heart.

learning objectives

- Outline how normal plasma behaves in a mammalian cardiovascular system

Blood flow is the continuous running of blood through vessels in the cardiovascular system (the mammalian cardiovascular system is shown in). Blood is the viscous fluid composed of plasma and cells. The composition of the blood includes plasma, red blood cells, white blood cells and platelets. In microcirculation, the properties of the blood cells have an important influence on flow.



An illustrative overview of the mammalian cardiovascular system: Keep in mind that both circular paths are working simultaneously and not in a sequential manner as the numbering in the illustration might suggest. Both the ventricles are working together in harmony; as tiny amounts of blood are moving in the pulmonary circuit, the remainder of the blood moves through the systemic circuit.

The cardiovascular system, which consists of blood vessels and the heart, helps to distribute nutrients, O_2, and other products of metabolism. The blood moves in the blood vessels, while the heart serves as the pump for the blood. The vessel walls of the heart are elastic and movable, therefore causing the blood and the wall to exert forces on each other and in turn influencing their respective motion.

The major quantity of interest in describing the motion of blood particles is velocity—the rate of change of the position of an object with time:

$$\mathrm{v}=\frac{\Delta \mathrm{x}}{\Delta \mathrm{t}} \tag{5.7.2.1}$$

Blood velocities in arteries are higher during systole than during diastole. One parameter to quantify this difference is pulsatility index (PI), which is equal to the difference between the peak systolic velocity and the minimum diastolic velocity divided by the mean velocity during the cardiac cycle.

Another important parameter is the acceleration—the rate of change of velocity: $\mathrm{a}=\frac{\Delta \mathrm{v}}{\Delta \mathrm{t}}$

Normal plasma behaves like a Newtonian fluid at rates of shear. Typical values for the viscosity of normal human plasma at 37°C is $1.2\mathrm{Nsm}^{-2}$. The viscosity of normal plasma varies with temperature in the same way as does that of its solvent, water. (a 5°C increase of temperature in the physiological range reduces plasma viscosity by about 10%).

The osmotic pressure of the plasma affects the mechanics of the circulation in several ways. An alteration of the osmotic pressure difference across the membrane of a blood cell causes a shift of water and a change in cell volume. The change, both in shape and flexibility, affects the mechanical properties of whole blood. Therefore, a change in plasma osmotic pressure alters the hematocrit (the volume concentration of red cells in the whole blood) by redistributing water between the intravascular and extravascular spaces. This in turn affects the mechanics of the whole blood.

Key Points

- Viscosity is the resistance of a fluid to flow. Virtually all fluids have viscosity which generally changes as a function of temperature; although different types of fluids exhibit different types of fluid–shear velocity dependencies.
- Laminar flow of a fluid is characterized by its flow in parallel layers in which there is no disruption or interaction between the different layers, and in which each layer flows at a different velocity along the same direction.
- Poiseuille's equation pertains to moving incompressible fluids exhibiting laminar flow. It relates the difference in pressure at different spatial points to volumetric flow rate for fluids in motion in certain cases, such as in the flow of fluid through a rigid pipe.
- The major quantity of interest in describing the motion of blood particles is the velocity – the rate of change of the position of an object with time: $\mathrm{v}=\frac{\Delta \mathrm{x}}{\Delta \mathrm{t}}$.
- Blood velocities in arteries are higher during systole than during diastole.
- The mechanics of the circulation depends on osmotic pressure of plasma.

Key Terms

- **viscosity**: The property of a fluid that resists the force which tends to cause it to flow.
- **shear stress**: The external force acting on an object or surface parallel to the slope or plane in which it lies; the stress tending to produce shear.
- **Reynolds Number**: A dimensionless number, $\frac{\mathrm{v}\rho\mathrm{l}}{\eta}$, where v is the fluid velocity, ρ the density, η the viscosity and l a dimension of the system. The value of the number indicates the type of fluid flow.
- **systole**: The rhythmic contraction of the heart, by which blood is driven through the arteries.
- **vessel**: A tube or canal that carries fluid in an animal or plant.
- **diastole**: The phase or process of relaxation and dilation of the heart chambers, between contractions, during which they fill with blood; an instance of the process.

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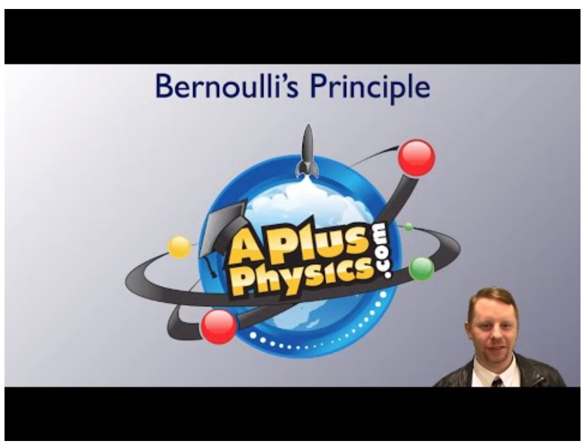
5.7.3: Bernoulli’s Equation

learning objectives

- Adapt Bernoulli’s equation for flows that are either unsteady or compressible

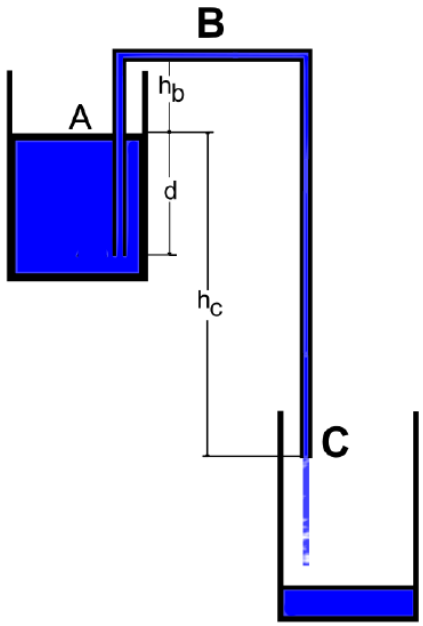
Application of Bernoulli’s Equation

The relationship between pressure and velocity in ideal fluids is described quantitatively by Bernoulli’s equation, named after its discoverer, the Swiss scientist Daniel Bernoulli (1700–1782). Bernoulli’s equation states that for an incompressible and inviscid fluid, the total mechanical energy of the fluid is constant. (An inviscid fluid is assumed to be an ideal fluid with no viscosity.)



***Bernoulli’s Principle**: A brief introduction to Bernoulli’s Principle for students studying fluids.*

The total mechanical energy of a fluid exists in two forms: potential and kinetic. The kinetic energy of the fluid is stored in static pressure, psps, and dynamic pressure, 12ρV212ρV2, where \rho is the fluid density in (SI unit: kg/m^3) and V is the fluid velocity (SI unit: m/s). The SI unit of static pressure and dynamic pressure is the pascal.



***Syphoning**: Syphoning fluid between two reservoirs. The flow rate out can be determined by drawing a streamline from point (A) to point (C).*

Static pressure is simply the pressure at a given point in the fluid, dynamic pressure is the kinetic energy per unit volume of a fluid particle. Thus, a fluid will not have dynamic pressure unless it is moving. Therefore, if there is no change in potential energy along a streamline, Bernoulli's equation implies that the total energy along that streamline is constant and is a balance between static and dynamic pressure. Mathematically, the previous statement implies:

$$\mathrm{p_s} + \frac{1}{2}\rho \mathrm{V}^2 = \mathrm{constant} \tag{5.7.3.1}$$

along a streamline. If changes there are significant changes in height or if the fluid density is high, the change in potential energy should not be ignored and can be accounted for with,

$$\Delta \mathrm{PE} = \rho \mathrm{g} \Delta \mathrm{h}. \tag{5.7.3.2}$$

This simply adds another term to the above version of the Bernoulli equation and results in

$$\mathrm{p_s} + \frac{1}{2}\rho \mathrm{V}^2 + \rho \mathrm{g} \Delta \mathrm{h} = \mathrm{constant}. \tag{5.7.3.3}$$

Deriving Bernoulli's Equation

The Bernoulli equation can be derived by integrating Newton's 2nd law along a streamline with gravitational and pressure forces as the only forces acting on a fluid element. Given that any energy exchanges result from conservative forces, the total energy along a streamline is constant and is simply swapped between potential and kinetic.

Applying Bernoulli's Equation

Bernoulli's equation can be applied when syphoning fluid between two reservoirs. Another useful application of the Bernoulli equation is in the derivation of Torricelli's law for flow out of a sharp edged hole in a reservoir. A streamline can be drawn from the top of the reservoir, where the total energy is known, to the exit point where the static pressure and potential energy are known but the dynamic pressure (flow velocity out) is not.

Adapting Bernoulli's Equation

The Bernoulli equation can be adapted to flows that are both unsteady and compressible. However, the assumption of inviscid flow remains in both the unsteady and compressible versions of the equation. Compressibility effects depend on the speed of the flow relative to the speed of sound in the fluid. This is determined by the dimensionless quantity known as the Mach number. The Mach number represents the ratio of the speed of an object moving through a medium to the speed of sound in the medium.

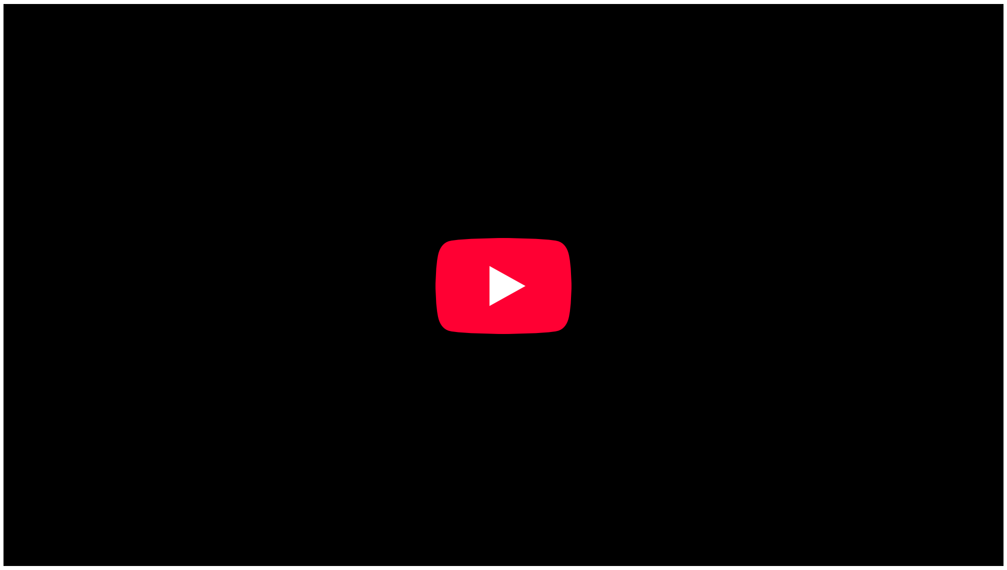
Torricelli's Law

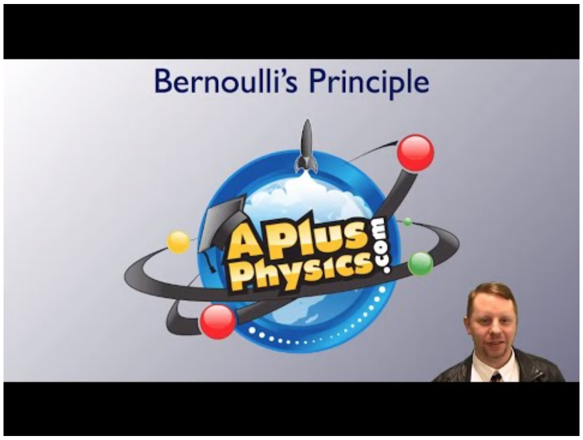
Torricelli's law is theorem about the relation between the exit velocity of a fluid from a hole in a reservoir to the height of fluid above the hole.

learning objectives

- Infer the exit velocity through examining the Bernoulli equation

Torricelli's law is theorem in fluid dynamics about the relation between the exit velocity of a fluid from a sharp-edged hole in a reservoir to the height of the fluid above that exit hole. This relationship applies for an "ideal" fluid (inviscid and incompressible) and results from an exchange of potential energy,





Torricelli's Principle: A brief introduction to Torricelli's Principle for students studying fluids.

mghmgh, for kinetic energy,

$\frac{1}{2}\rho v^2$, at the exit.

This relationship can be derived by applying the Bernoulli equation between the top of the reservoir and the exit hole. Applying Bernoulli between the top of a reservoir and an exit hole at a height h below the top of the reservoir results in,

$$PE \leftrightarrow KE$$

***Exchange of Energy**: Potential energy at the top of the reservoir becomes kinetic energy at the exit.*

$$p_t + \frac{1}{2}\rho v_t^2 + \rho g h_t = p_e + \frac{1}{2}\rho v_e^2 + \rho g h_e \tag{5.7.3.4}$$

where subscript t implies evaluation at the top of the reservoir and subscript e implies evaluation at the exit. If we assume both the top of reservoir and the exit are open to the atmosphere, the zero for potential energy is at the exit hole, and the fluid velocity at the top of the reservoir is essentially zero (large reservoir, small hole), we arrive at

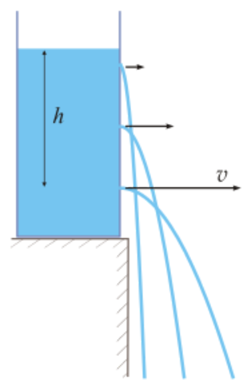
$$\rho g h_t = \frac{1}{2}\rho v_e^2 \tag{5.7.3.5}$$

This can be solved for the exit velocity, resulting in,

$$v_e = \sqrt{2 g h_t} \tag{5.7.3.6}$$

where again h_t is the height difference between the top of the reservoir and the exit hole. Due to the assumption of an ideal fluid, all forces acting on the fluid are conservative and thus there is an exchange between potential and kinetic energy. The result is that the velocity acquired by the fluid is the same that a body would acquire when simply dropped from the height h_t.

A simple experiment to test Torricelli’s law involves filling a soda bottle with water and puncturing the bottom with a small hole (about 1 cm in diameter). As the height in the reservoir decreases, the exit velocity will decrease as well. The exit velocity can be increased by capping the top of the reservoir and pressurizing it.



Toricelli’s Law: The exit velocity depends on the height of the fluid above the exit hole.

$$\mu = 0$$

$$\nabla \cdot \vec{u} = 0$$

Ideal Fluid: Applies to an ideal fluid (inviscid, incompressible)

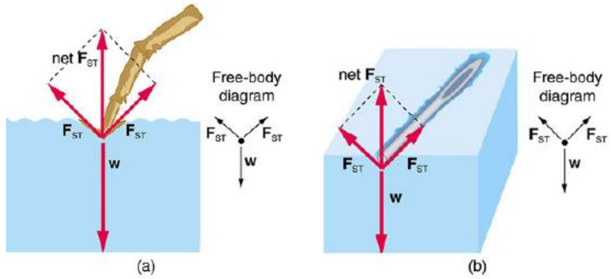
Surface Tension

The tendency of the surface of a liquid to resist a force and behave like a membrane and is a result of cohesion between liquid molecules.

learning objectives

- Summarize the cause for different surface tensions at a liquid’s surface

Surface tension is the tendency of a liquid surface to resist forces applied to it. This effect is a result of cohesion of the molecules of the liquid causing the surface of the liquid to contract to the smallest area possible. This effect is visible in nature with water strider insects that are able to walk on water. Also, a paper clip or pin can be supported by the surface tension at a water air interface.

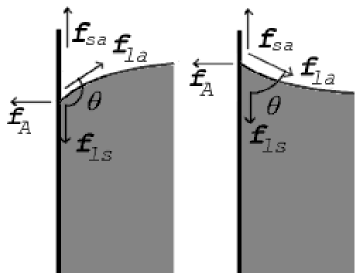


Surface Tension FBD*: Force diagrams showing the direction of forces for water supporting a water strider (insect) foot and a pin. In both cases, the vertical component of the surface tension is enough to support the weight of the object.*

In the bulk of the liquid, the molecules are pulled equally in all directions. The molecules at the surface feel a greater attractive force toward the bulk material than the interface material.

The surface of a liquid is an interface between another fluid, a solid body, or both. Therefore, the surface tension will be a property of the interface rather than simply the liquid. Adhesion describes the attractive force between molecules of different types. The surface of a liquid in a container is an interface between the liquid, the air, and the container. Where the surfaces meet, forces must be in equilibrium. This results in a contact angle at the interface. The contact angle is measured in the liquid and depends on the relative strength of cohesive forces in the liquid and adhesive forces between the liquid and interface materials. If liquid molecules are strongly attracted to the molecules of the solid surface (adhesive forces > cohesive forces), the drop will tend to spread out and the contact angle will be close to zero degrees. If the cohesive forces are greater than the adhesive forces, the resulting contact angles will be large and will form a more circular drop.

Water Droplet on Leaf*: When a water droplet forms on a leaf, the cohesive forces between the water molecules are greater than the adhesive forces between the water and leaf surface. The leaf is a hydrophobic surface.*



Contact Angle*: The contact angle is the angle, measured in the fluid, that results when a liquid-gas interface, meets a solid surface.*

When the liquid is water, a surface where the contact angle is small is said to be hydrophilic. Large contact angles are present on hydrophobic surfaces. The contact angle determines the wettability of the surface.

Key Points

- The simplest form of Bernoulli's equation (steady and incompressible flow) states that the sum of mechanical energy, potential energy and kinetic energy, along a streamline is constant. Therefore, any increase in one form results in a decrease in the other.
- Bernoulli's equation considers only pressure and gravitational forces acting on the fluid particles. Therefore, if there is no change in height along a streamline, Bernoulli's equation becomes a balance between static pressure and velocity.
- The steady-state, incompressible Bernoulli equation, can be derived by integrating Newton's 2nd law along a streamline.
- Torricelli's law applies to an inviscid, incompressible fluid ("ideal" fluid).
- You can ascertain results from applying the Bernoulli equation between the top of the reservoir and the exit hole.
- The relationship arises from an exchange of potential energy at the top of the reservoir to kinetic energy at the exit.
- The final kinetic energy is equivalent to what a solid body would acquire when falling from height h.
- Surface tension is a result of cohesion between the molecules of the liquid. The molecules at the surface of the liquid feel an attractive force pulling them toward the bulk of the liquid more than the solid or fluid at the interface.
- When a liquid-solid-gas interface is encountered, the contact angle represents a measure of the relative strength of adhesive and cohesive forces.
- The contact angle determines the wettability of a surface.

Key Terms

- **viscosity**: A quantity expressing the magnitude of internal friction in a fluid, as measured by the force per unit area resisting uniform flow.
- **Ideal Fluid**: An inviscid and incompressible fluid
- **incompressible**: Unable to be compressed or condensed.
- **inviscid**: A fluid with zero viscosity (internal friction). In reality viscosity is always present. However, it is often very small compared with other forces (e.g. gravity, pressure) and for common fluids (water and air) the fluid can be approximated as having zero viscosity.
- **cohesion**: Various intermolecular forces that hold solids and liquids together.
- **wettability**: The ability of a solid surface to reduce the surface tension of a liquid in contact with it such that it spreads over the surface and wets it.
- **adhesion**: The ability of a substance to stick to an unlike substance.

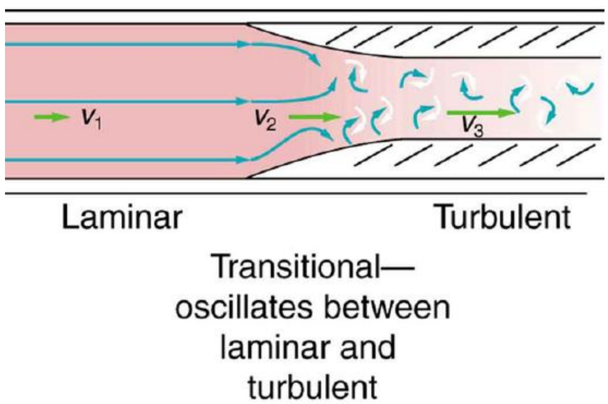
5.7.4: Other Applications

learning objectives

- Predict if flow will be laminar or turbulent

It is possible to predict if flow will be laminar or turbulent. At low velocity, flow in a very smooth tube or around a smooth, streamlined object will be laminar. At high velocity, even the flow in a smooth tube or around a smooth object will experience turbulence. However, between low and high velocity, flow is more difficult to predict. In fact, at intermediate velocities, flow may oscillate back and forth indefinitely between laminar and turbulent.

An occlusion (narrowing) of an artery, such as shown in, is likely to cause turbulence because of the irregularity of the blockage, as well as the complexity of blood as a fluid. Turbulence in the circulatory system (such as aneurysms, or ballooning of arteries) is noisy and can sometimes be detected with a stethoscope (such as when measuring diastolic pressure in the upper arm's partially collapsed brachial artery). These turbulent sounds, at the onset of blood flow when the cuff pressure becomes sufficiently small, are called *Korotkoff* sounds. Heart murmurs, consistent with their name, are sounds produced by turbulent flow around damaged and insufficiently closed heart valves. Another method of detecting this type of turbulence is ultrasound, used as a medical indicator in a process analogous to Doppler-shift radar (used to detect storms).



Turbulent Flow in an Artery: Flow is laminar in the large part of this blood vessel and turbulent in the part narrowed by plaque, where velocity is high. In the transition region, the flow can oscillate chaotically between laminar and turbulent flow.

Turbulence manifests in other areas, with varying causes. During an airplane flight, for example, the turbulence experienced is due to the mixing of warm and cold air in the atmosphere, causing the airplane to shake. The mixing currents in oceans creates a similar effect.

The phenomenon of turbulent air flow must be accounted for in many applications. For example, race cars are unable to follow each other around fast corners because the leading car creates turbulent air flow in its wake (this can lead to under-steering).

Industrial equipment, such as pipes, ducts, and heat exchangers are often designed to induce the flow regime of interest (laminar or turbulent). When flow is turbulent, particles exhibit additional transverse motion. This enhances the rate of energy and momentum exchange between them, increasing the heat transfer. Turbulent flow is thus desirable in applications where a relatively cool fluid is mixed with a warmer fluid to reduce the temperature of the warmer fluid.

It is imperative to take into account turbulent flow when designing certain structures, such as a bridge support, as shown in. In the late summer and fall, when river flow is slow, water flows smoothly around the support legs. In the spring, when the flow is faster, the flow may start off laminar but it is quickly separated from the leg and becomes turbulent. The bridge supports must be designed so that they can withstand the turbulent flow of the water in the spring.

Longtown Bridge: Turbulent flow is visible around the bridge supports of the Longtown bridge.

Motionof an Object in a Viscous Field

Objects moving in a viscous fluid feel a resistive force proportional to the viscosity of the fluid.

learning objectives

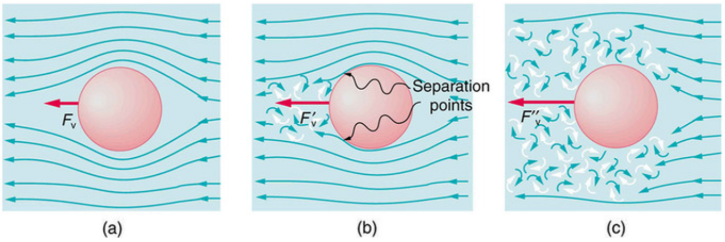
- Assess the relationship of the parameters to one another in determining the inertia of an object moving in fluid

Overview

A moving object in a viscous fluid is equivalent to a stationary object in a flowing fluid stream. (For example, when you ride a bicycle at 10 m/s in still air, you feel the air in your face exactly as if you were stationary in a 10-m/s wind.) Flow of the stationary fluid around a moving object may be laminar, turbulent, or a combination of the two. Just as with flow in tubes, it is possible to predict when a moving object creates turbulence. We use another form of the Reynolds number N′R, defined for an object moving in a fluid to be

$$\mathrm{N‘R} = \frac{\rho \mathrm{vL}}{\eta} \tag{5.7.4.1}$$

where L is a characteristic length of the object (a sphere's diameter, for example), the fluid density, its viscosity, and v the object's speed in the fluid. If N′R is less than about 1, flow around the object can be laminar, particularly if the object has a smooth shape. The transition to turbulent flow occurs for N′R between 1 and about 10, depending on surface roughness and so on. Depending on the surface, there can be a *turbulent wake* behind the object with some laminar flow over its surface. For an N′R between 10 and 10^6, the flow may be either laminar or turbulent and may oscillate between the two. For N′R greater than about 10^6, the flow is entirely turbulent, even at the surface of the object. (See.) Laminar flow occurs mostly when the objects in the fluid are small, such as raindrops, pollen, and blood cells in plasma.



Motion of an object in a viscous fluid.: (a) Motion of this sphere to the right is equivalent to fluid flow to the left. Here the flow is laminar with N′R less than 1. There is a force, called viscous drag FV, to the left on the ball due to the fluid's viscosity. (b) At a higher speed, the flow becomes partially turbulent, creating a wake starting where the flow lines separate from the surface. Pressure in the wake is less than in front of the sphere, because fluid speed is less, creating a net force to the left F′V that is significantly greater than for laminar flow. Here N′R is greater than 10. (c) At much higher speeds, where N′R is greater than 10^6, flow becomes turbulent everywhere on the surface and behind the sphere. Drag increases dramatically.

Viscous Drag

One of the consequences of viscosity is a resistance force called viscous drag FVFV that is exerted on a moving object. This force typically depends on the object's speed (in contrast with simple friction). Experiments have shown that for laminar flow (N′R less than about one) viscous drag is proportional to speed, whereas for N′R between about 10 and 106, viscous drag is proportional to speed squared. (This relationship is a strong dependence and is pertinent to bicycle racing, where even a small headwind causes significantly increased drag on the racer. Cyclists take turns being the leader in the pack for this reason.) For N′R greater than 106, drag increases dramatically and behaves with greater complexity. For laminar flow around a sphere, FVFV is proportional to fluid viscosity, the object's characteristic size L, and its speed v. All of which makes sense—the more viscous the fluid and the larger the object, the more drag we expect. Recall Stoke's law FS=6πrηvFS=6πrηv. For the special case of a small sphere of radius R, moving slowly in a fluid of viscosity, the drag force FSFS is given by

FS=6πRηvFS=6πRηv.

Molecular Transport Phenomena

Molecular transport phenomena are ways in which molecules are transported from one region to another. These include diffusion and osmosis.

learning objectives

- Predict the role diffusion plays in blood transport throughout the body

Diffusion

Atoms and molecules are in constant motion at any temperature. In fluids they move about randomly even in the absence of macroscopic flow.

Diffusion is the movement of substances due to random thermal molecular motion. Fluids, like fish fumes or odors entering ice cubes, can even diffuse through solids. Diffusion is a slow process over macroscopic distances. The densities of common materials are great enough that molecules cannot travel very far before having a collision that can scatter them in any direction, including straight backward. More massive molecules diffuse more slowly.

Another interesting point is that the diffusion rate for oxygen in air is much greater than for oxygen in water. In water, an oxygen molecule makes many more collisions in its random walk and is slowed considerably. In water, an oxygen molecule moves only about 40μm in 1 s. (Each molecule actually collides about 1010 times per second!). Finally, note that diffusion constants increase with temperature, because average molecular speed increases with temperature. This is because the average kinetic energy of molecules, $1/2mv^2$, is proportional to absolute temperature. Because diffusion is typically very slow, its most important effects occur over small distances. For example, the cornea of the eye gets most of its oxygen by diffusion through the thin tear layer covering it.

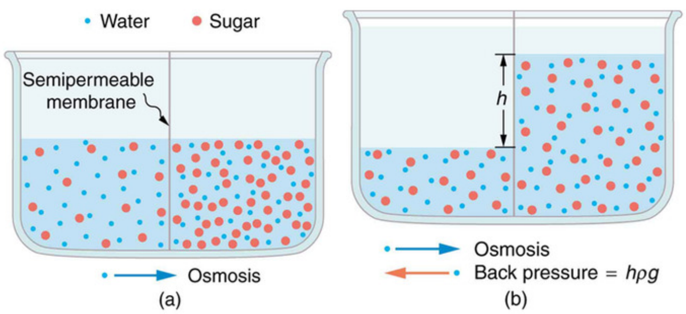
If you very carefully place a drop of food coloring in a still glass of water, it will slowly diffuse into the colorless surroundings until its concentration is the same everywhere. This type of diffusion is called free diffusion, because there are no barriers inhibiting it. Let us examine its direction and rate. Molecular motion is random in direction, and so simple chance dictates that more molecules will move out of a region of high concentration than into it. The net rate of diffusion is higher initially than after the process is partially completed. The rate of diffusion is proportional to the concentration difference. Many more molecules will leave a region of high concentration than will enter it from a region of low concentration. In fact, if the concentrations were the same, there would be no net movement. The rate of diffusion is also proportional to the diffusion constant D, which is determined experimentally. Many of the factors that affect the rate are hidden in the diffusion constant D. For example, temperature and cohesive and adhesive forces all affect values of D. Diffusion is the dominant mechanism by which the exchange of nutrients and waste products occur between the blood and tissue, and between air and blood in the lungs. In the evolutionary process, as organisms became larger, they needed quicker methods of transportation than net diffusion, because of the larger distances involved in the transport, leading to the development of circulatory systems. Less sophisticated, single-celled organisms still rely totally on diffusion for the removal of waste products and the uptake of nutrients.

Food Coloring: Food coloring spreading on a thin water film.

Osmosis and Dialysis – Diffusion Across Different Membranes

Some of the most interesting examples of diffusion occur through barriers that affect the rates of diffusion. For example, when you soak a swollen ankle in Epsom salt, water diffuses through your skin. Many substances regularly move through cell membranes; oxygen moves in, carbon dioxide moves out, nutrients go in, and wastes go out, for example. Because membranes are thin structures (typically 6.5×10^{-9} to 10×10^{-9} m across) diffusion rates through them can be high.

Diffusion through membranes is an important method of transport. Membranes are generally selectively permeable, or semipermeable. In other types of membranes, the molecules may actually dissolve in the membrane or react with molecules in the membrane while moving across. Membrane function, in fact, is the subject of much current research, involving not only physiology but also chemistry and physics. Osmosis is driven by the imbalance in water concentration. For example, water is more concentrated in your body than in Epsom salt. When you soak a swollen ankle in Epsom salt, the water moves out of your body into the lower-concentration region in the salt. Similarly, dialysis is the transport of any other molecule through a semipermeable membrane due to its concentration difference. Both osmosis and dialysis are used by the kidneys to cleanse the blood.



Diffusion: (a) Two sugar-water solutions of different concentrations, separated by a semipermeable membrane that passes water but not sugar. Osmosis will be to the right, since water is less concentrated there. (b) The fluid level rises until the back pressure ρgh equals the relative osmotic pressure; then, the net transfer of water is zero.

Pumps and the Heart

The heart pumps blood through the body by contracting and relaxing, increasing and decreasing the pressure.

learning objectives

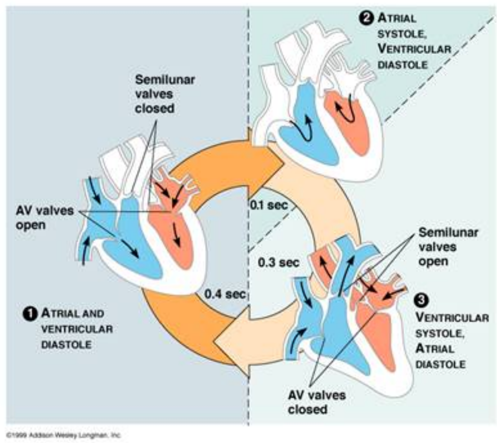
- Contrast systole and diastole in cardiovascular circulation

The Heart and its Parts

The heart is made up of four chambers. Two atria at the top of the heart receive blood and two ventricles at the bottom of the heart pump blood out of the heart. The septum divides the left and right side of the heart, while the valves of the heart ensure that blood only flows in one direction.They include the tricuspid valve-found between the right atrium and the right ventricle-and the mitral valve-found between the left atrium and the left ventricle. The list of heart valves also includes the semi-lunar valves, which are located at the bottom of the aorta and pulmonary artery. Strong tendinous chords attached to valves prevent them from turning inside out when they close.

The human heart will undergo over 3 billion contraction cycles during a normal lifetime. A complete cardiac cycle is one round of the heart pumping blood and consists of two parts: systole (contraction of the heart muscle) and diastole (relaxation of the heart muscle). During the cycle, the top half of the heart works as one unit, while the bottom half of the heart works as one unit.

The heart beat can be heard as a sound that the valves make when they close. The ‘lub’ sound is made when the atrio ventricular valves close and the ‘dub’ sound is made when the semi lunar valves close. Blood pressure is produced by the left ventricle contractions.The rhythm of ventricle diastole, often just referred to as diastole, causes the pulse, which can be felt by holding two fingers to the side of the throat.



Cardiac Cycle: The heart pumps blood through the body.

Key Points

- For low velocity, flow in a smooth tube will be laminar.
- At higher velocities or if there are obstructions, the flow turns turbulent.
- Turbulent flow is very chaotic, with rapid variations in velocity and pressure.
- Viscous fluids exert a resitive force on objects attempting to move through them.
- This resistive force is called viscous drag and is proportional to the viscosity of the fluid and the motion of the object.
- An object moving in a fluid can be thought of as a stationary object in a moving fluid.
- Diffusion is the movement of molecules due to random thermal motion.
- Osmosis is the movement of molecules due to different concentrations. Molecules will move from regions of high concentrations to lower concentrations.
- These transport phenomena can take place through membranes if the pressure is great enough.

Key Terms

- **turbulent**: Being in, or causing, disturbance or unrest.
- **streamlined**: Designed to offer little resistance to the flow of fluid, especially by having sleek, graceful lines.
- **laminar**: Of fluid motion, smooth and regular, flowing as though in different layers.
- **viscosity**: A quantity expressing the magnitude of internal friction in a fluid, as measured by the force per unit area resisting uniform flow.
- **turbulence**: Disturbance in a gas or fluid, characterized by evidence of internal motion or unrest.
- **diffusion**: the intermingling of the molecules of a fluid due to random thermal agitation
- **ventricle**: One of two lower chambers of the heart.
- **contraction**: A reversible reduction in size.
- **atrium**: An upper chamber of the heart that receives blood from the veins and forces it into a ventricle. In higher vertebrates, the right atrium receives blood from the superior vena cava and inferior vena cava, and the left atrium receives blood from the left and right pulmonary veins.

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CHAPTER OVERVIEW

5.8: Waves and Vibrations

Topic hierarchy

5.8.1: Introduction

learning objectives

- Contrast mechanical and electromagnetic waves

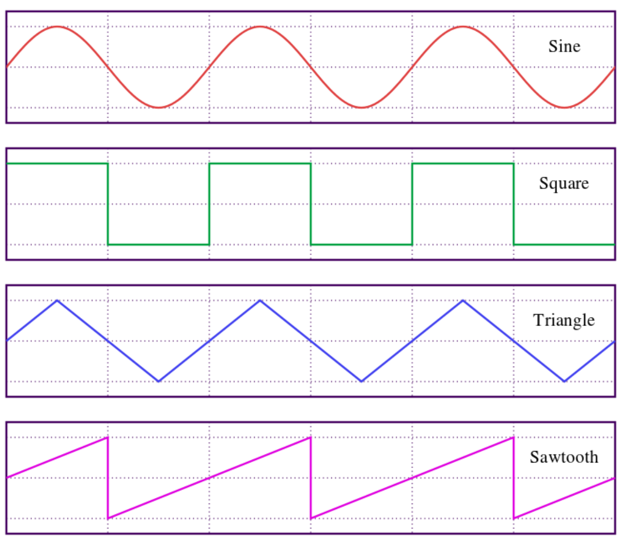
Overview

A wave is an oscillation that travels through space, accompanied by a transfer of energy. Wave motion transfers energy from one point to another, often with no permanent displacement of the particles of the medium —that is, with little or no associated mass transport. They consist, instead, of oscillations or vibrations around almost fixed locations. There are two main types of waves. Mechanical waves propagate through a medium, and the substance of this medium is deformed. The deformation reverses itself owing to restoring forces resulting from its deformation.

The second main type of wave, electromagnetic waves, do not require a medium (although they may still propagate through a medium). Instead, they consist of periodic oscillations in electrical and magnetic fields generated by charged particles, and can therefore travel through a vacuum.

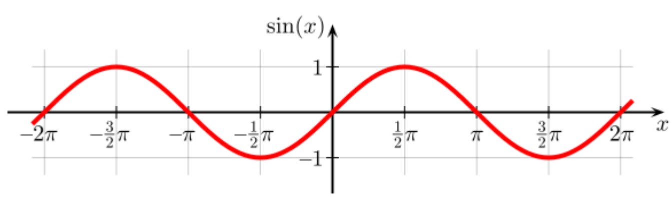
Wave Equation

The shape of a wave can take the form of any function that repeats itself over some characteristic spatial scale λλ, the wavelength (see). More generally, waveforms are scalar functions uu which satisfy the wave equation, $\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u$. This equation simply states that the acceleration of the waveform (Left: second derivative with respect to time) is proportional to the Laplacian (Right: second spatial derivative) of the same waveform. The constant of proportionality, c^2, is the square of the propagation speed of the wave.



Common waveforms: A sample of several common, simple waveforms. A waveform is a function that repeats in space.

Sine Wave



Plot of Sine: The sine function graphed on the Cartesian plane. In this graph, the angle *x* is given in radians (π = 180°).

Consider one of the most common waveforms, the sinusoid. A general form of a sinusoidal wave is $y(x, t) = A\sin(kx - \omega t + \phi)$, where A is the amplitude of the wave, ω is the wave's angular frequency, k is the wavenumber, and ϕ is the phase of the sine wave given in radians. This waveform gives the displacement position ("y") of a particle in a medium from its equilibrium as a function of both position "x" and time "t".

By taking derivatives, it is evident that the wave equation given above holds for $c = \frac{\omega}{k}$, which is also called the phase speed of the wave. To find the velocity of a particle in the medium at x and t, we take the temporal derivative of the waveform to get $\partial y(x, t)\partial t = -A\omega\cos(kx - \omega t + \phi)$. Likewise, to find the acceleration of the displaced particle in the medium at x and t, we take the second derivative to get $\frac{\partial^2 y(x,t)}{\partial t^2} = -A\omega^2\sin(kx - \omega t + \phi)$. Note the phase relationship among the trigonometric functions in $y(x, t), y'(x, t), y"(x, t)$. When the particle displacement is maximum or minimum, the velocity is 0. When the displacement is 0, particle velocity is either maximum or minimum. Similarly, the particle acceleration is maximum (or minimum) when the particle displacement is minimum (or maximum), respectively.

Arbitrary Wave

We looked closely into the sinusoidal wave. But how about waves that has a general form? One important aspect of the wave equation is its linearity: the wave equation is linear in *u* and it is left unaltered by translations in space and time. Since a wave with an arbitrary shape can be represented by a sum of many sinusoidal waves (this is called Fourier analysis), we can generate a great variety of solutions of the wave equation by translating and summing sine waves that we just looked closely into.

Key Points

- The wave equation requires that the second time derivative of the waveform be proportional to its second spatial derivative.
- Waveforms describe the shape of physical waves, and can take the form of any function that repeats in space.
- One of the most common waveforms in physics is the sinusoid. Since any arbitray waveform can be generated by adding a set of sine waves, the physics governing a wave of an arbitrary shape can be described by using its sine wave components.

Key Terms

- **waveform**: The shape of a physical wave, such as sound or electromagnetic radiation. The shape can be any function that repeats in space.
- **Fourier analysis**: The study of the way general functions may be represented or approximated by sums of simpler trigonometric functions.

5.8.2: Hooke’s Law

learning objectives

- Generate the mathematical expression of Hooke’s law

In mechanics (physics), Hooke’s law is an approximation of the response of elastic (i.e., springlike) bodies. It states: the extension of a spring is in direct proportion with the load applied to it. For instance, the spring is pulled downwards with either no load, F_p, or twice F_p.

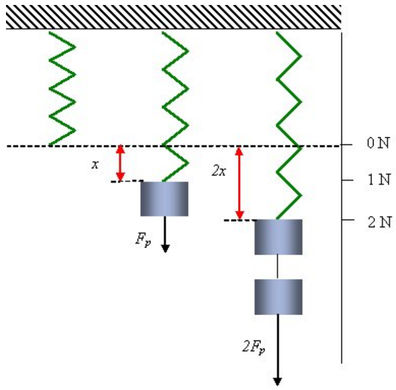
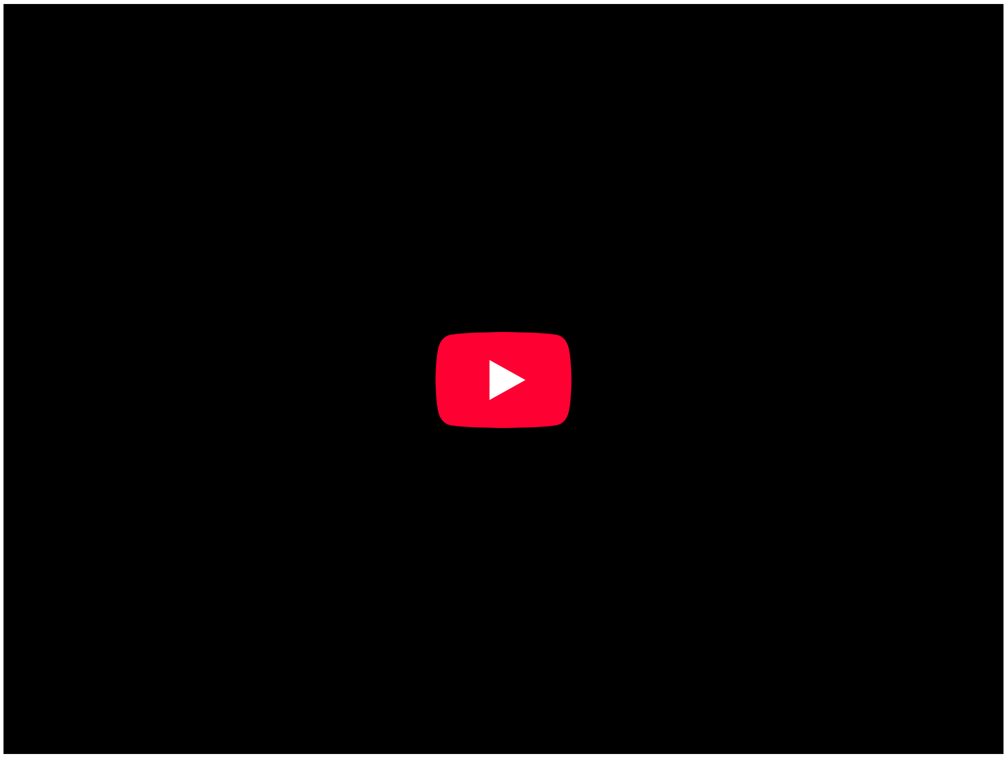
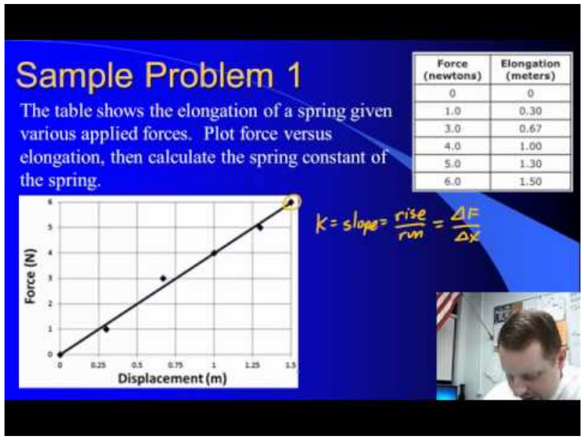


Diagram of Hooke’s Law: The extension of the spring is linearly proportional to the force.





Springs and Hooke's Law: A brief overview of springs, Hooke's Law, and elastic potential energy for algebra-based physics students.

Many materials obey this law of elasticity as long as the load does not exceed the material's elastic limit. Materials for which Hooke's law is a useful approximation are known as linear-elastic or "Hookean" materials. Hookean materials are broadly defined and include springs as well as muscular layers of the heart. In simple terms, Hooke's law says that stress is directly proportional to strain. Mathematically, Hooke's law is stated as:

$$\mathrm{F} = -\mathrm{kx} \tag{5.8.2.1}$$

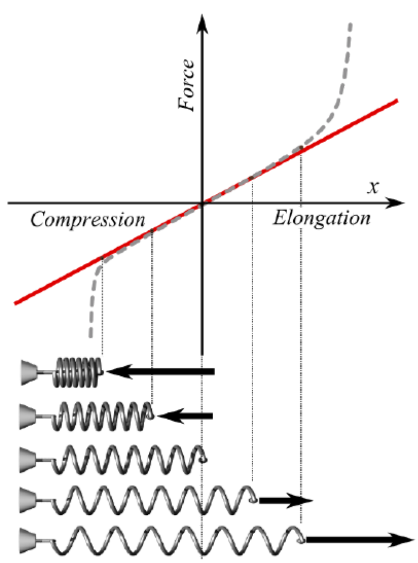
where:

- x is the displacement of the spring's end from its equilibrium position (a distance, in SI units: meters);
- F is the restoring force exerted by the spring on that end (in SI units: N or kg·m/s^2); and
- k is a constant called the rate or spring constant (in SI units: N/m or kg/s^2). When this holds, the behavior is said to be linear. If shown on a graph, the line should show a direct variation.

It's possible for multiple springs to act on the same point. In such a case, Hooke's law can still be applied. As with any other set of forces, the forces of many springs can be combined into one resultant force.

When Hooke's law holds, the behavior is linear; if shown on a graph, the line depicting force as a function of displacement should show a direct variation. There is a negative sign on the right hand side of the equation because the restoring force always acts in the opposite direction of the displacement (for example, when a spring is stretched to the left, it pulls back to the right).

Hooke's law is named after the 17th century British physicist Robert Hooke, and was first stated in 1660 as a Latin anagram, whose solution Hooke published in 1678 as *Ut tensio, sic vis*, meaning, "As the extension, so the force."



Hooke's Law: The red line in this graph illustrates how force, F, varies with position according to Hooke's law. The slope of this line corresponds to the spring constant k. The dotted line shows what the actual (experimental) plot of force might look like. The pictures of spring states at the bottom of the graph correspond to some points of the plot; the middle one is in the relaxed state (no force applied).

Elastic Potential Energy

If a force results in only deformation, with no thermal, sound, or kinetic energy, the work done is stored as elastic potential energy.

learning objectives

- Express elastic energy stored in a spring in a mathematical form

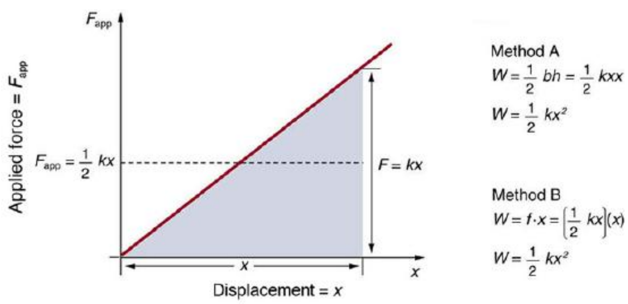
Elastic Potential Energy

In order to produce a deformation, work must be done. That is, a force must be exerted through a distance, whether you pluck a guitar string or compress a car spring. If the only result is deformation and no work goes into thermal, sound, or kinetic energy, then all the work is initially stored in the deformed object as some form of potential energy. Elastic energy is the potential mechanical energy stored in the configuration of a material or physical system when work is performed to distort its volume or shape. For example, the potential energy PE_{el} stored in a spring is

$$\mathrm{PE_{el}} = \frac{1}{2}\mathrm{kx^2} \tag{5.8.2.2}$$

where k is the elastic constant and x is the displacement.

It is possible to calculate the work done in deforming a system in order to find the energy stored. This work is performed by an applied force F_{app}. The applied force is exactly opposite to the restoring force (action-reaction), and so $\mathrm{F_{app}} = \mathrm{kx}$. A graph shows the applied force versus deformation x for a system that can be described by Hooke's law. Work done on the system is force multiplied by distance, which equals the area under the curve, or $\frac{1}{2}\mathrm{kx^2}$ (Method A in the figure). Another way to determine the work is to note that the force increases linearly from 0 to kx, so that the average force is $\frac{1}{2}\mathrm{kx}$, the distance moved is x, and thus



Applied force versus deformation: A graph of applied force versus distance for the deformation of a system that can be described by Hooke's law is displayed. The work done on the system equals the area under the graph or the area of the triangle, which is half its base multiplied by its height, or $\mathrm{W} = \frac{1}{2}\mathrm{kx^2}$.

$\mathrm{W} = \mathrm{F_{app}d} = (\frac{1}{2}\mathrm{kx})(\mathrm{x}) = \frac{1}{2}\mathrm{kx^2}$ (Method B in the figure).

Elastic energy of or within a substance is static energy of configuration. It corresponds to energy stored principally by changing the inter-atomic distances between nuclei. Thermal energy is the randomized distribution of kinetic energy within the material, resulting in statistical fluctuations of the material about the equilibrium configuration. There is some interaction, however. For example, for some solid objects, twisting, bending, and other distortions may generate thermal energy, causing the material's temperature to rise. This energy can also produce macroscopic vibrations sufficiently lacking in randomization to lead to oscillations that are merely the exchange between (elastic) potential energy within the object and the kinetic energy of motion of the object as a whole.

Key Points

- Mathematically, Hooke's Law can be written as $\mathrm{F} = -\mathrm{kx}$.
- Many materials obey this law as long as the load does not exceed the material's elastic limit.

- The rate or spring constant, k, relates the force to the extension in SI units: N/m or kg/s2.
- In order to produce a deformation, work must be done.
- The potential energy stored in a spring is given by $\mathrm{PE_{el}} = \frac{1}{2}\mathrm{kx}^2$, where k is the spring constant and x is the displacement.
- Deformation can also be converted into thermal energy or cause an object to begin oscillating.

Key Terms

- **elasticity**: The property by virtue of which a material deformed under the load can regain its original dimensions when unloaded
- **deformation**: A transformation; change of shape.
- **kinetic energy**: The energy possessed by an object because of its motion, equal to one half the mass of the body times the square of its velocity.
- **oscillating**: Moving in a repeated back-and-forth motion.

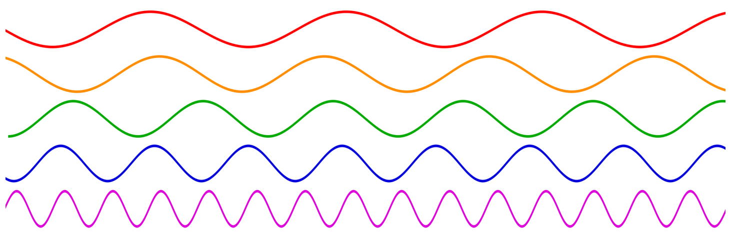
5.8.3: Periodic Motion

learning objectives

- Practice converting between frequency and period

Period and Frequency

The usual physics terminology for motion that repeats itself over and over is *periodic motion*, and the time required for one repetition is called the *period*, often expressed as the letter *T*. (The symbol *P* is not used because of the possible confusion with momentum.) One complete repetition of the motion is called a cycle. The frequency is defined as the number of cycles per unit time. Frequency is usually denoted by a Latin letter *f* or by a Greek letter *v* (nu). Note that period and frequency are reciprocals of each other.



Sinusoidal Waves of Varying Frequencies: Sinusoidal waves of various frequencies; the bottom waves have higher frequencies than those above. The horizontal axis represents time.

$$\mathrm{f}=\frac{1}{\mathrm{T}} \tag{5.8.3.1}$$

For example, if a newborn baby's heart beats at a frequency of 120 times a minute, its period (the interval between beats) is half a second. If you calibrate your intuition so that you expect *large frequencies* to be paired with *short periods*, and vice versa, you may avoid some embarrassing mistakes on physics exams.

Units

Locomotive Wheels: The locomotive's wheels spin at a frequency of f cycles per second, which can also be described as ω radians per second. The mechanical linkages allow the linear vibration of the steam engine's pistons, at frequency f, to drive the wheels.

In SI units, the unit of frequency is the *hertz* (Hz), named after the German physicist Heinrich Hertz: *1 Hz* indicates that an event repeats once per second. A traditional unit of measure used with rotating mechanical devices is revolutions per minute, abbreviated *RPM*. 60 RPM equals one hertz (i.e., one revolution per second, or a period of one second). The SI unit for period is the second.

Angular Frequency

Often periodic motion is best expressed in terms of angular frequency, represented by the Greek letter ω (omega). Angular frequency refers to the angular displacement per unit time (e.g., in rotation) or the rate of change of the phase of a sinusoidal waveform (e.g., in oscillations and waves), or as the rate of change of the argument of the sine function.

$$\mathrm{y(t)}=\sin(\theta(\mathrm{t}))=\sin(\omega \mathrm{t})=\sin(2\pi \mathrm{ft}) \tag{5.8.3.2}$$

$$\omega=2\pi \mathrm{f} \tag{5.8.3.3}$$

Angular frequency is often represented in units of radians per second (recall there are 2π radians in a circle).

Period of a Mass on a Spring

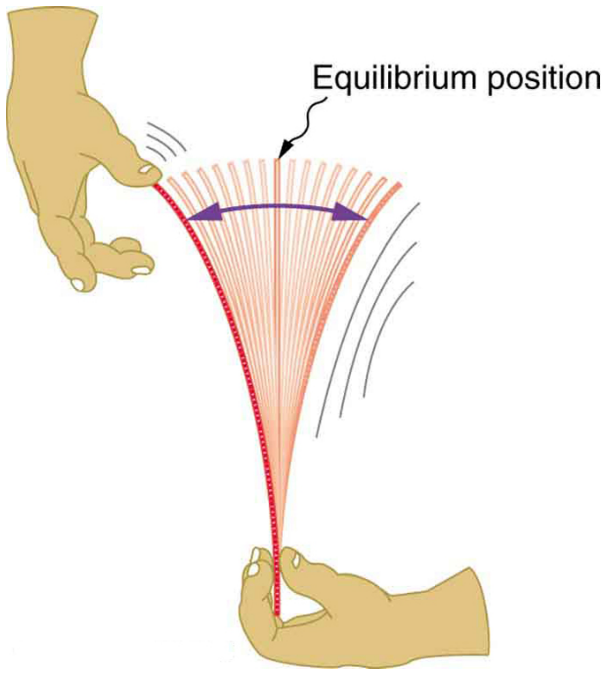
The period of a mass m on a spring of spring constant k can be calculated as $T = 2\pi\sqrt{\frac{m}{k}}$.

learning objectives

- Identify parameters necessary to calculate the period and frequency of an oscillating mass on the end of an ideal spring

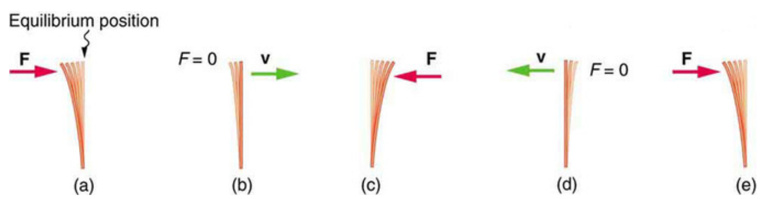
Understanding the Restoring Force

Newton's first law implies that an object oscillating back and forth is experiencing forces. Without force, the object would move in a straight line at a constant speed rather than oscillate. It is important to understand how the force on the object depends on the object's position. If an object is vibrating to the right and left, then it must have a leftward force on it when it is on the right side, and a rightward force when it is on the left side. In one dimension, we can represent the direction of the force using a positive or negative sign, and since the force changes from positive to negative there must be a point in the middle where the force is zero. This is the equilibrium point, where the object would stay at rest if it was released at rest. It is common convention to define the origin of our coordinate system so that *x* equals zero at equilibrium.



Oscillating Ruler: When displaced from its vertical equilibrium position, this plastic ruler oscillates back and forth because of the restoring force opposing displacement. When the ruler is on the left, there is a force to the right, and vice versa.

Consider, for example, plucking a plastic ruler shown in the first figure. The deformation of the ruler creates a force in the opposite direction, known as a *restoring force*. Once released, the restoring force causes the ruler to move back toward its stable equilibrium position, where the net force on it is zero. However, by the time the ruler gets there, it gains momentum and continues to move to the right, producing the opposite deformation. It is then forced to the left, back through equilibrium, and the process is repeated until dissipative forces (e.g., friction) dampen the motion. These forces remove mechanical energy from the system, gradually reducing the motion until the ruler comes to rest.



Restoring force, momentum, and equilibrium: (a) The plastic ruler has been released, and the restoring force is returning the ruler to its equilibrium position. (b) The net force is zero at the equilibrium position, but the ruler has momentum and continues to move to the right. (c) The restoring force is in the opposite direction. It stops the ruler and moves it back toward equilibrium again. (d) Now the ruler has momentum to the left. (e) In the absence of damping (caused by frictional forces), the ruler reaches its original position. From there, the motion will repeat itself.

Hooke's Law

The simplest oscillations occur when the restoring force is directly proportional to displacement. The name that was given to this relationship between force and displacement is Hooke's law:

$$\mathrm{F} = \mathrm{kx} \tag{5.8.3.4}$$

Here, F is the restoring force, x is the displacement from equilibrium or deformation, and k is a constant related to the difficulty in deforming the system (often called the spring constant or force constant). Remember that the minus sign indicates the restoring force is in the direction opposite to the displacement. The force constant k is related to the rigidity (or stiffness) of a system—the larger the force constant, the greater the restoring force, and the stiffer the system. The units of k are newtons per meter (N/m). For example, k is directly related to Young's modulus when we stretch a string. A typical physics laboratory exercise is to measure restoring forces created by springs, determine if they follow Hooke's law, and calculate their force constants if they do.

Mass on a Spring

A common example of an objecting oscillating back and forth according to a restoring force directly proportional to the displacement from equilibrium (i.e., following Hooke's Law) is the case of a mass on the end of an ideal spring, where "ideal" means that no messy real-world variables interfere with the imagined outcome.

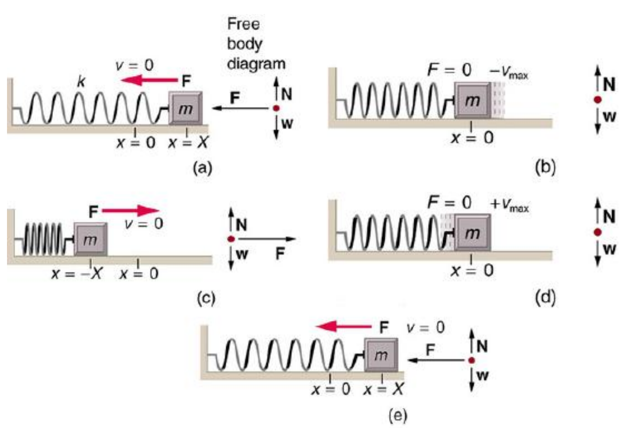
The motion of a mass on a spring can be described as Simple Harmonic Motion (SHM), the name given to oscillatory motion for a system where the net force can be described by Hooke's law. We can now determine how to calculate the period and frequency of an oscillating mass on the end of an ideal spring. The period T can be calculated knowing only the mass, m, and the force constant, k:

$$\mathrm{T} = 2\pi\sqrt{\frac{\mathrm{m}}{\mathrm{k}}} \tag{5.8.3.5}$$

When dealing with $\mathrm{f} = \frac{1}{\mathrm{T}}$, the frequency is given by:

$$\mathrm{f} = \frac{1}{2\pi}\sqrt{\frac{\mathrm{k}}{\mathrm{m}}} \tag{5.8.3.6}$$

We can understand the dependence of these equations on m and k intuitively. If one were to increase the mass on an oscillating spring system with a given k, the increased mass will provide more inertia, causing the acceleration due to the restoring force F to decrease (recall Newton's Second Law: $\mathrm{F} = \mathrm{ma}$). This will lengthen the oscillation period and decrease the frequency. In contrast, increasing the force constant k will increase the restoring force according to Hooke's Law, in turn causing the acceleration at each displacement point to also increase. This reduces the period and increases the frequency. The maximum displacement from equilibrium is known as the *amplitude X*.



Motion of a mass on an ideal spring: An object attached to a spring sliding on a frictionless surface is an uncomplicated simple harmonic oscillator. When displaced from equilibrium, the object performs simple harmonic motion that has an amplitude X and a period T. The object's maximum speed occurs as it passes through equilibrium. The stiffer the spring is, the smaller the period T. The greater the mass of the object is, the greater the period T. (a) The mass has achieved its greatest displacement X to the right and now the restoring force to the left is at its maximum magnitude. (b) The restoring force has moved the mass back to its equilibrium point and is now equal to zero, but the leftward velocity is at its maximum. (c) The mass's momentum has carried it to its maximum displacement to the right. The restoring force is now to the right, equal in magnitude and opposite in direction compared to (a). (d) The equilibrium point is reach again, this time with momentum to the right. (e) The cycle repeats.

Simple Harmonic Motion

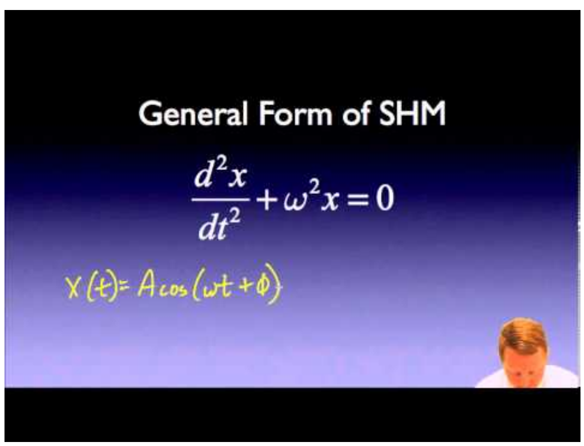
Simple harmonic motion is a type of periodic motion where the restoring force is directly proportional to the displacement.

learning objectives

- Relate the restoring force and the displacement during the simple harmonic motion

Simple Harmonic Motion

Simple harmonic motion is a type of periodic motion where the restoring force is directly proportional to the displacement (i.e., it follows Hooke's Law). It can serve as a mathematical model of a variety of motions, such as the oscillation of a spring. In addition, other phenomena can be approximated by simple harmonic motion, such as the motion of a simple pendulum, or molecular vibration.



Simple Harmonic Motion: A brief introduction to simple harmonic motion for calculus-based physics students.

Simple harmonic motion is typified by the motion of a mass on a spring when it is subject to the linear elastic restoring force given by Hooke's Law. A system that follows simple harmonic motion is known as a *simple harmonic oscillator.*

Dynamics of Simple Harmonic Oscillation

For one-dimensional simple harmonic motion, the equation of motion (which is a second-order linear ordinary differential equation with constant coefficients) can be obtained by means of Newton's second law and Hooke's law.

$$F_{net} = m\frac{d^2x}{dt^2} = -kx, \tag{5.8.3.7}$$

where *m* is the mass of the oscillating body, *x* is its displacement from the equilibrium position, and *k* is the spring constant. Therefore:

$$\frac{d^2x}{dt^2} = -(\frac{k}{m})x. \tag{5.8.3.8}$$

Solving the differential equation above, a solution which is a sinusoidal function is obtained.

$$x(t) = c_1\cos(\omega t) + c_2\sin(\omega t) = A\cos(\omega t - \varphi), \tag{5.8.3.9}$$

where

$$\omega = \sqrt{\frac{k}{m}}, \tag{5.8.3.10}$$

$$A = \sqrt{c_1^2 + c_2^2}, \tag{5.8.3.11}$$

$$\tan\varphi = (\frac{c_2}{c_1}). \tag{5.8.3.12}$$

In the solution, c_1 and c_2 are two constants determined by the initial conditions, and the origin is set to be the equilibrium position. Each of these constants carries a physical meaning of the motion: A is the amplitude (maximum displacement from the equilibrium position), $\omega = 2\pi f$ is the angular frequency, and φ is the phase.

We can use differential calculus and find the velocity and acceleration as a function of time:

$$v(t) = \frac{dx}{dt} = -A\omega\sin(\omega t - \varphi) \tag{5.8.3.13}$$

$$a(t) = \frac{d^2x}{dt^2} = -A\omega^2\cos(\omega t - \varphi). \tag{5.8.3.14}$$

Acceleration can also be expressed as a function of displacement:

$$a(t) = -\omega^2 x. \tag{5.8.3.15}$$

Then since $\omega = 2\pi f$,

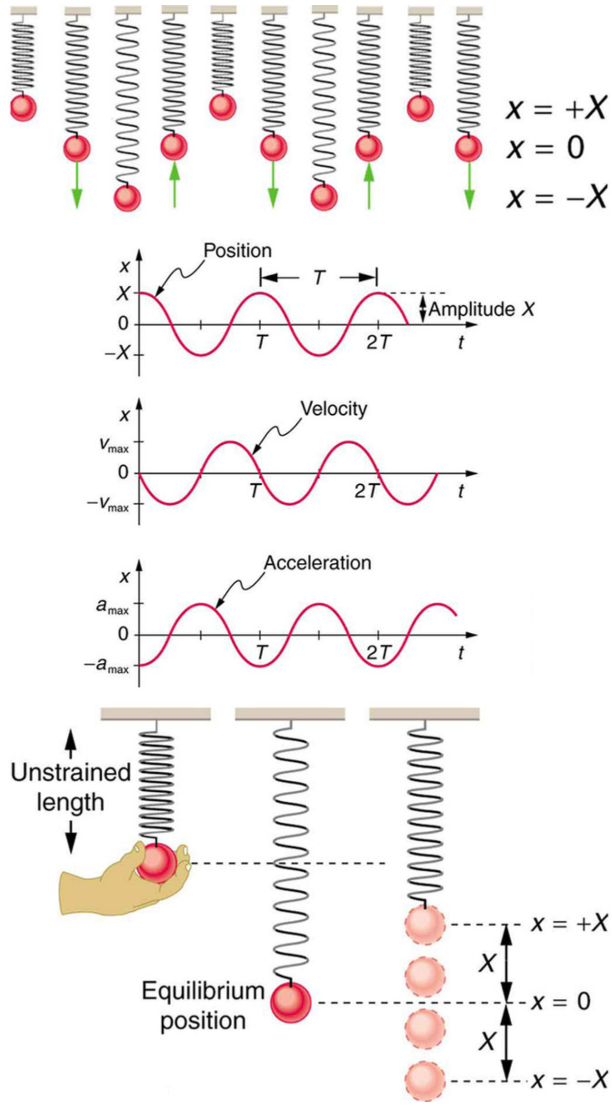
$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}. \tag{5.8.3.16}$$

Recalling that $T = \frac{1}{f}$,

$$T = 2\pi\sqrt{\frac{m}{k}}. \tag{5.8.3.17}$$

Using Newton's Second Law, Hooke's Law, and some differential Calculus, we were able to derive the period and frequency of the mass oscillating on a spring that we encountered in the last section! Note that the period and frequency are completely independent of the amplitude.

The below figure shows the simple harmonic motion of an object on a spring and presents graphs of x(t),v(t), and a(t) versus time. You should learn to create mental connections between the above equations, the different positions of the object on a spring in the cartoon, and the associated positions in the graphs of x(t), v(t), and a(t).



Visualizing Simple Harmonic Motion: Graphs of x(t),v(t), and a(t) versus t for the motion of an object on a spring. The net force on the object can be described by Hooke's law, and so the object undergoes simple harmonic motion. Note that the initial position has the vertical displacement at its maximum value X; v is initially zero and then negative as the object moves down; and the initial acceleration is negative, back toward the equilibrium position and becoming zero at that point.

Simple Harmonic Motion and Uniform Circular Motion

Simple harmonic motion is produced by the projection of uniform circular motion onto one of the axes in the x-y plane.

learning objectives

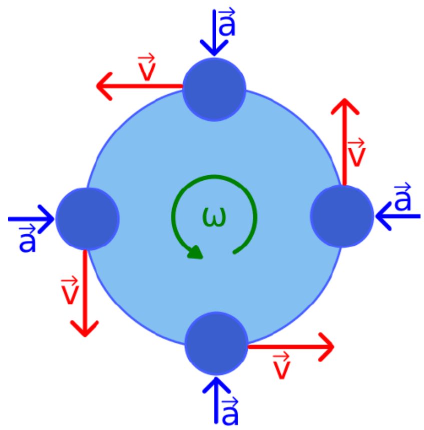
- Describe relationship between the simple harmonic motion and uniform circular motion

Uniform Circular Motion

Uniform circular motion describes the motion of a body traversing a circular path at constant speed. The distance of the body from the center of the circle remains constant at all times. Though the body's speed is constant, its velocity is not constant: velocity (a vector quantity) depends on both the body's speed and its direction of travel. Since the body is constantly changing direction as it travels around the circle, the velocity is changing also. This varying velocity indicates the presence of an acceleration called the centripetal acceleration. Centripetal acceleration is of constant magnitude and directed at all times towards the center of the circle. This acceleration is, in turn, produced by a centripetal force —a force in constant magnitude, and directed towards the center.

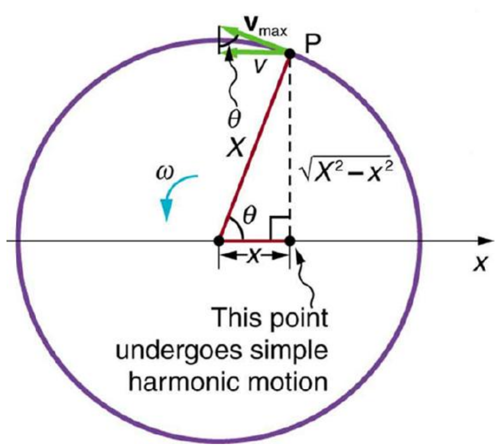
Velocity

The above figure illustrates velocity and acceleration vectors for uniform motion at four different points in the orbit. Since velocity *v* is tangent to the circular path, no two velocities point in the same direction. Although the object has a constant speed, its direction is always changing. This change in velocity is due to an acceleration, *a*, whose magnitude is (like that of the velocity) held constant, but whose direction also is always changing. The acceleration points radially inwards (centripetally) and is perpendicular to the velocity. This acceleration is known as centripetal acceleration.



Uniform Circular Motion (at Four Different Point in the Orbit): Velocity v and acceleration a in uniform circular motion at angular rate ω; the speed is constant, but the velocity is always tangent to the orbit; the acceleration has constant magnitude, but always points toward the center of rotation

Displacement around a circular path is often given in terms of an angle θ. This angle is the angle between a straight line drawn from the center of the circle to the objects starting position on the edge and a straight line drawn from the objects ending position on the edge to center of the circle. See for a visual representation of the angle where the point p started on the x- axis and moved to its present position. The angle θ describes how far it moved.



Projection of Uniform Circular Motion: A point P moving on a circular path with a constant angular velocity ω is undergoing uniform circular motion. Its projection on the x-axis undergoes simple harmonic motion. Also shown is the velocity of this point around the circle, v−max, and its projection, which is v. Note that these velocities form a similar triangle to the displacement triangle.

For a path around a circle of radius *r*, when an angle θ(measured in radians) is swept out, the distance traveled on the edge of the circle is $s = r\theta$. You can prove this yourself by remembering that the circumference of a circle is 2πr, so if the object traveled around the whole circle (one circumference) it will have gone through an angle of 2π radians and traveled a distance of 2πr. Therefore, the speed of travel around the orbit is:

$$v = r\frac{d\theta}{dt} = r\omega, \tag{5.8.3.18}$$

where the angular rate of rotation is ω. (Note that $\omega = \frac{v}{r}$.) Thus,*v* is a constant, and the velocity vector *v* also rotates with constant magnitude *v*, at the same angular rate ω.

Acceleration

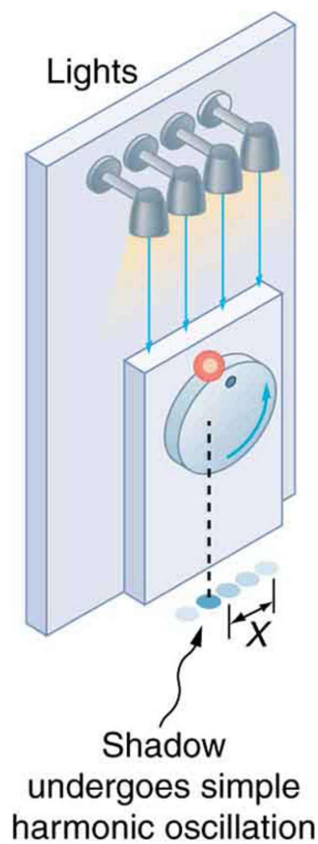
The acceleration in uniform circular motion is always directed inward and is given by:

$$a = v\frac{d\theta}{dt} = v\omega = \frac{v^2}{r}. \tag{5.8.3.19}$$

This acceleration acts to change the direction of *v*, but not the speed.

Simple Harmonic Motion from Uniform Circular Motion

There is an easy way to produce simple harmonic motion by using uniform circular motion. The figure below demonstrates one way of using this method. A ball is attached to a uniformly rotating vertical turntable, and its shadow is projected onto the floor as shown. The shadow undergoes simple harmonic motion.



Shadow of a Ball Undergoing Simple Harmonic Motion: The shadow of a ball rotating at constant angular velocity ω on a turntable goes back and forth in precise simple harmonic motion.

The next figure shows the basic relationship between uniform circular motion and simple harmonic motion. The point *P* travels around the circle at constant angular velocity ω. The point *P* is analogous to the ball on a turntable in the figure above. The projection of the position of *P* onto a fixed axis undergoes simple harmonic motion and is analogous to the shadow of the object. At a point in time assumed in the figure, the projection has position *x* and moves to the left with velocity *v*. The velocity of the point *P* around the circle equals $|v_{max}|$. The projection of $|v_{max}|$ on the x-axis is the velocity *v* of the simple harmonic motion along the x-axis.

To see that the projection undergoes simple harmonic motion, note that its position *x* is given by:

$$x = X\cos\theta, \tag{5.8.3.20}$$

where $\theta = \omega t$, ω is the constant angular velocity, and X is the radius of the circular path. Thus,

$$x = X\cos \omega t. \tag{5.8.3.21}$$

The angular velocity ω is in radians per unit time; in this case 2π radians is the time for one revolution T. That is, $\omega = \frac{2\pi}{T}$. Substituting this expression for ω, we see that the position x is given by:

$$x(t) = \cos(\frac{2\pi t}{T}) = \cos(2\pi f t). \tag{5.8.3.22}$$

Note: This equation should look familiar from our earlier discussion of simple harmonic motion.

The Simple Pendulum

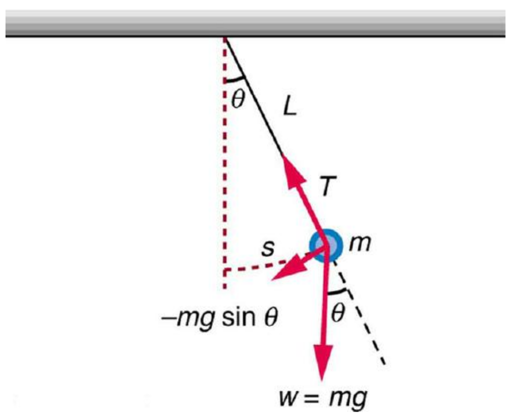
A simple pendulum acts like a harmonic oscillator with a period dependent only on L and g for sufficiently small amplitudes.

learning objectives

- Identify parameters that affect the period of a simple pendulum

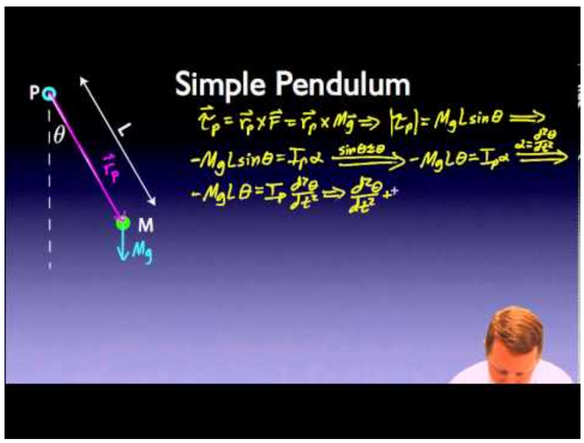
The Simple Pendulum

A pendulum is a weight suspended from a pivot so that it can swing freely. When a pendulum is displaced sideways from its resting equilibrium position, it is subject to a restoring force; after it reaches its highest point in its swing, gravity will accelerate it back toward the equilibrium position. When released, the restoring force combined with the pendulum's mass causes it to oscillate about the equilibrium position, swinging back and forth.



Simple Pendulum: A simple pendulum has a small-diameter bob and a string that has a very small mass but is strong enough not to stretch appreciably. The linear displacement from equilibrium is s, the length of the arc. Also shown are the forces on the bob, which result in a net force of −mgsinθ toward the equilibrium position—that is, a restoring force.

For small displacements, a pendulum is a simple harmonic oscillator. A simple pendulum is defined to have an object that has a small mass, also known as the pendulum bob, which is suspended from a wire or string of negligible mass, such as shown in the illustrating figure. Exploring the simple pendulum a bit further, we can discover the conditions under which it performs simple harmonic motion, and we can derive an interesting expression for its period.



Pendulums: A brief introduction to pendulums (both ideal and physical) for calculus-based physics students from the standpoint of simple harmonic motion.

We begin by defining the displacement to be the arc length s. We see from the figure that the net force on the bob is tangent to the arc and equals $-mg\sin\theta$. (The weight mg has components $mg\cos\theta$ along the string and $mg\sin\theta$ tangent to the arc.) Tension in the string exactly cancels the component $mg\cos\theta$ parallel to the string. This leaves a net restoring force drawing the pendulum back toward the equilibrium position at $\theta = 0$.

Now, if we can show that the restoring force is directly proportional to the displacement, then we have a simple harmonic oscillator. In trying to determine if we have a simple harmonic oscillator, we should note that for small angles (less than about 15º), $\sin\theta\approx\theta$ ($\sin\theta$ and θ differ by about 1% or less at smaller angles). Thus, for angles less than about 15º, the restoring force F is

$$\mathrm{F} \approx -\mathrm{mg}\theta. \tag{5.8.3.23}$$

The displacement s is directly proportional to θ. When θ is expressed in radians, the arc length in a circle is related to its radius (L in this instance) by:

s=Lθs=Lθ

so that

$$\theta = \mathrm{sL}. \tag{5.8.3.24}$$

For small angles, then, the expression for the restoring force is:

$$\mathrm{F} \approx \frac{\mathrm{mgL}}{\mathrm{s}}. \tag{5.8.3.25}$$

This expression is of the form of Hooke's Law:

$$\mathrm{F} \approx-\mathrm{kx} \tag{5.8.3.26}$$

where the force constant is given by $k=mg/L$ and the displacement is given by $x=s$. For angles less than about 15º, the restoring force is directly proportional to the displacement, and the simple pendulum is a simple harmonic oscillator.

Using this equation, we can find the period of a pendulum for amplitudes less than about 15º. For the simple pendulum:

$$\mathrm{T}=2 \pi \sqrt{\frac{\mathrm{m}}{\mathrm{k}}}=2 \pi \sqrt{\frac{\mathrm{m}}{\frac{\mathrm{mg}}{\mathrm{L}}}}. \tag{5.8.3.27}$$

Thus,

$$\mathrm{T}=2 \pi \sqrt{\frac{\mathrm{L}}{\mathrm{g}}} \tag{5.8.3.28}$$

or the period of a simple pendulum. This result is interesting because of its simplicity. The only things that affect the period of a simple pendulum are its length and the acceleration due to gravity. The period is completely independent of other factors, such as mass. Even simple pendulum clocks can be finely adjusted and accurate. Note the dependence of T on g. If the length of a pendulum is precisely known, it can actually be used to measure the acceleration due to gravity. If θ is less than about 15º, the period T for a pendulum is nearly independent of amplitude, as with simple harmonic oscillators. In this case, the motion of a pendulum as a function of time can be modeled as:

$$\theta(\mathrm{t})=\theta_{\mathrm{o}} \cos (\frac{2 \pi \mathrm{t}}{\mathrm{T}}) \tag{5.8.3.29}$$

For amplitudes larger than 15º, the period increases gradually with amplitude so it is longer than given by the simple equation for T above. For example, at an amplitude of θ_0 = 23° it is 1% larger. The period increases asymptotically (to infinity) as $\theta_0$approaches 180°, because the value θ_0 = 180° is an unstable equilibrium point for the pendulum.

The Physical Pendulum

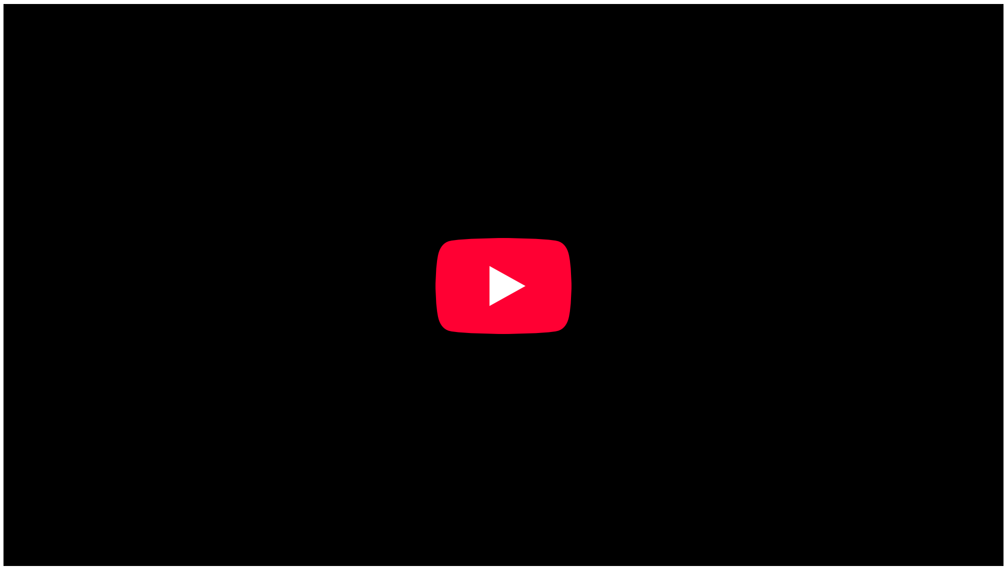
The period of a physical pendulum depends upon its moment of inertia about its pivot point and the distance from its center of mass.

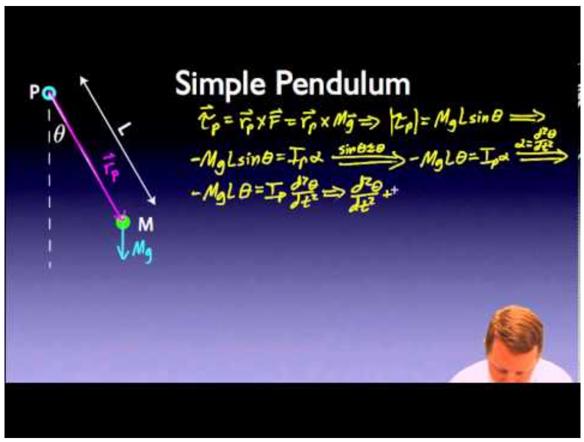
learning objectives

- Identify parameters that affect the period of a physical pendulum

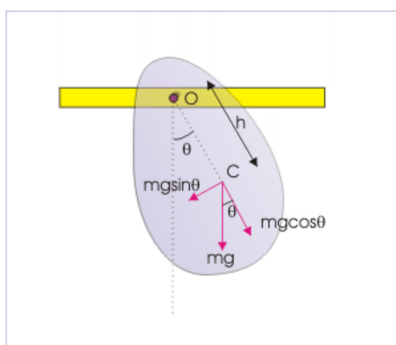
The Physical Pendulum

Recall that a simple pendulum consists of a mass suspended from a massless string or rod on a frictionless pivot. In that case, we are able to neglect any effect from the string or rod itself. In contrast, a *physical pendulum* (sometimes called a compound pendulum) may be suspended by a rod that is not massless or, more generally, may be an arbitrarily-shaped, rigid body swinging by a pivot (see). In this case, the pendulum's period depends on its moment of inertia around the pivot point.





Pendulums – Physical Pendulum: A brief introduction to pendulums (both ideal and physical) for calculus-based physics students from the standpoint of simple harmonic motion.



A Physical Pendulum: An example showing how forces act through center of mass. We can calculate the period of this pendulum by determining the moment of inertia of the object around the pivot point.

Gravity acts through the center of mass of the rigid body. Hence, the length of the pendulum used in equations is equal to the linear distance between the pivot and the center of mass (h).

The equation of torque gives:

$$\tau = \mathrm{I}\alpha, \tag{5.8.3.30}$$

where α is the angular acceleration, τ is the torque, and I is the moment of inertia.

The torque is generated by gravity so:

$$\tau = \mathrm{mgh} \sin\theta, \tag{5.8.3.31}$$

where *h* is the distance from the center of mass to the pivot point and θ is the angle from the vertical.

Hence, under the small-angle approximation sin\theta \approx \theta,

$$\alpha \approx -\frac{\mathrm{mgh}\theta}{\mathrm{I}}. \tag{5.8.3.32}$$

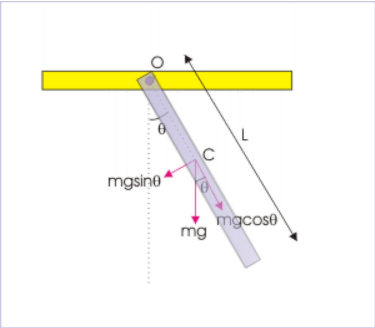
This is of the same form as the conventional simple pendulum and this gives a period of:

$$\mathrm{T} = 2\pi\sqrt{\frac{\mathrm{I}}{\mathrm{mgh}}}. \tag{5.8.3.33}$$

And a frequency of:

\[\mathrm{f=\dfrac{1}{T}=\dfrac{1}{2π}\sqrt{\dfrac{mgh}{I}}.}\]

In case we know the moment of inertia of the rigid body, we can evaluate the above expression of the period for the physical pendulum. For illustration, let us consider a uniform rigid rod, pivoted from a frame as shown (see). Clearly, the center of mass is at a distance *L/2* from the point of suspension:



Uniform Rigid Rod: A rigid rod with uniform mass distribution hangs from a pivot point. This is another example of a physical pendulum.

$$\mathrm{h} = \frac{\mathrm{L}}{2}. \tag{5.8.3.34}$$

The moment of inertia of the rigid rod about its center is:

$$\mathrm{I_c} = \frac{\mathrm{mL^2}}{12}. \tag{5.8.3.35}$$

However, we need to evaluate the moment of inertia about the pivot point, not the center of mass, so we apply the parallel axis theorem:

$$\mathrm{I_o} = \mathrm{I_c} + \mathrm{mh^2} = \frac{\mathrm{mL^2}}{12} + \mathrm{m}\left(\frac{\mathrm{L}}{2}\right)^2 = \frac{\mathrm{mL^2}}{3}. \tag{5.8.3.36}$$

Plugging this result into the equation for period, we have:

$$\mathrm{T} = 2\pi\sqrt{\frac{\mathrm{I}}{\mathrm{mgh}}} = 2\pi\sqrt{\frac{\mathrm{2mL^2}}{\mathrm{3mgL}}} = 2\pi\sqrt{\frac{\mathrm{2L}}{\mathrm{3g}}}. \tag{5.8.3.37}$$

The important thing to note about this relation is that the period is still independent of the mass of the rigid body. However, it is not independent of the *mass distribution* of the rigid body. A change in shape, size, or mass distribution will change the moment of inertia. This, in turn, will change the period.

As with a simple pendulum, a physical pendulum can be used to measure *g*.

Energy in a Simple Harmonic Oscillator

The total energy in a simple harmonic oscillator is the constant sum of the potential and kinetic energies.

learning objectives

- Explain why the total energy of the harmonic oscillator is constant

Energy in a Simple Harmonic Oscillator

To study the energy of a simple harmonic oscillator, we first consider all the forms of energy it can have. Recall that the potential energy (*PE)*, stored in a spring that follows Hooke's Law is:

$$\mathrm{PE}=\frac{1}{2}\mathrm{kx}^2, \tag{5.8.3.38}$$

where PE is the potential energy, *k* is the spring constant, and *x* is the magnitude of the displacement or deformation. Because a simple harmonic oscillator has no *dissipative forces* , the other important form of energy is kinetic energy (*KE)*. Conservation of energy for these two forms is:

$$\mathrm{KE}+\mathrm{PE}=\text{constant}, \tag{5.8.3.39}$$

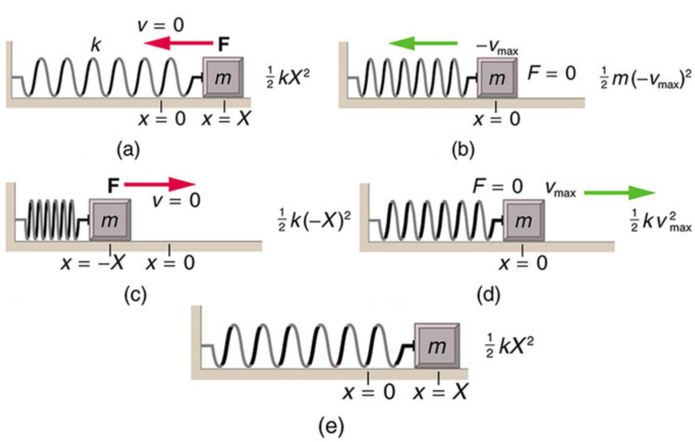
which can be written as:

$$\frac{1}{2}\mathrm{mv}^2+\frac{1}{2}\mathrm{kx}^2=\text{constant}. \tag{5.8.3.40}$$

This statement of conservation of energy is valid for *all* simple harmonic oscillators, including ones where the gravitational force plays a role. For example, for a simple pendulum we replace the velocity with $v=L\omega$, the spring constant with $k=mg/L$, and the displacement term with $x=L\theta$. Thus:

$$\frac{1}{2}\mathrm{mL}^2\omega^2+\frac{1}{2}\mathrm{mgL}\theta^2=\text{constant}. \tag{5.8.3.41}$$

In the case of undamped, simple harmonic motion, the energy oscillates back and forth between kinetic and potential, going completely from one to the other as the system oscillates. So for the simple example of an object on a frictionless surface attached to a spring, as shown again (see), the motion starts with all of the energy stored in the spring. As the object starts to move, the elastic potential energy is converted to kinetic energy, becoming entirely kinetic energy at the equilibrium position. It is then converted back into *elastic potential energy* by the spring, the velocity becomes zero when the kinetic energy is completely converted, and so on. This concept provides extra insight here and in later applications of simple harmonic motion, such as alternating current circuits.



Energy in a Simple Harmonic Oscillator: The transformation of energy in simple harmonic motion is illustrated for an object attached to a spring on a frictionless surface. (a) The mass has achieved maximum displacement from equilibrium. All energy is potential energy. (b) As the mass passes through the equilibrium point with maximum speed all energy in the system is in kinetic energy. (c) Once again, all energy is in the potential form, stored in the compression of the spring (in the first panel the energy was stored in the extension of the spring). (d) Passing through equilibrium again all energy is kinetic. (e) The mass has completed an entire cycle.

The conservation of energy principle can be used to derive an expression for velocity v. If we start our simple harmonic motion with zero velocity and maximum displacement (x=X), then the total energy is:

$$\mathrm{E}=\frac{1}{2}\mathrm{kX}^{2}. \tag{5.8.3.42}$$

This total energy is constant and is shifted back and forth between kinetic energy and potential energy, at most times being shared by each. The conservation of energy for this system in equation form is thus:

$$\frac{1}{2}\mathrm{mv}^{2}+\frac{1}{2}\mathrm{kx}^{2}=\frac{1}{2}\mathrm{kX}^{2}. \tag{5.8.3.43}$$

Solving this equation for v yields:

$$\mathrm{v}=\pm\sqrt{\frac{\mathrm{k}}{\mathrm{m}}(\mathrm{X}^{2}-\mathrm{x}^{2})}. \tag{5.8.3.44}$$

Manipulating this expression algebraically gives:

$$\mathrm{v}=\pm\sqrt{\frac{\mathrm{k}}{\mathrm{m}}}\mathrm{X}\sqrt{1-\frac{\mathrm{x}^{2}}{\mathrm{X}^{2}}}, \tag{5.8.3.45}$$

and so:

$$\mathrm{v}=\pm\mathrm{v}_{\max}\sqrt{1-\frac{\mathrm{x}^{2}}{\mathrm{X}^{2}}}, \tag{5.8.3.46}$$

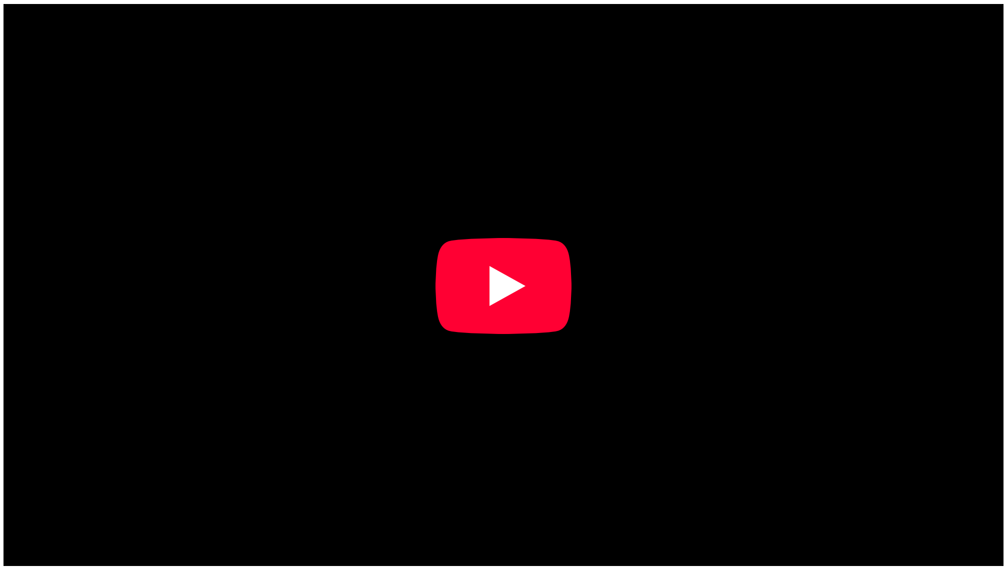
where:

$$\mathrm{v}_{\max}=\sqrt{\frac{\mathrm{k}}{\mathrm{m}}}\mathrm{X}. \tag{5.8.3.47}$$

From this expression, we see that the velocity is a maximum (v_{max}) at x=0. Notice that the maximum velocity depends on three factors. It is directly proportional to amplitude. As you might guess, the greater the maximum displacement, the greater the maximum velocity. It is also greater for stiffer systems because they exert greater force for the same displacement. This observation is seen in the expression for v_{max}; it is proportional to the square root of the force constant k. Finally, the maximum velocity is smaller for objects that have larger masses, because the maximum velocity is inversely proportional to the square root of m. For a given force, objects that have large masses accelerate more slowly.

A similar calculation for the simple pendulum produces a similar result, namely:

$$\omega_{\max}=\sqrt{\frac{\mathrm{g}}{\mathrm{L}}}\theta_{\max}. \tag{5.8.3.48}$$



Experience with a simple harmonic oscillator: A known mass is hung from a spring of known spring constant and allowed to oscillate. The time for one oscillation (period) is measured. This value is compared to a predicted value, based on the mass and spring constant.

Sinusoidal Nature of Simple Harmonic Motion

The solutions to the equations of motion of simple harmonic oscillators are always sinusoidal, i.e., sines and cosines.

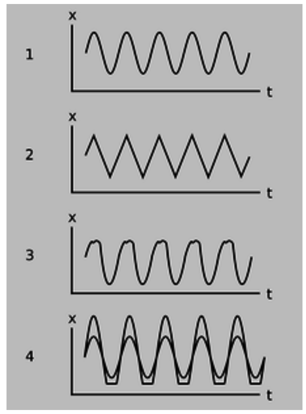
learning objectives

- Review factors responsible for the sinusoidal behavior of uniform circular motion

Sinusoidal Nature of Simple Harmonic Motion

Why are sine waves so common?

If the mass -on-a-spring system discussed in previous sections were to be constructed and its motion were measured accurately, its *x*–*t* graph would be a near-perfect sine-wave shape, as shown in. It is called a “sine wave” or “sinusoidal” even if it is a cosine, or a sine or cosine shifted by some arbitrary horizontal amount. It may not be surprising that it is a wiggle of this general sort, but why is it a specific mathematically perfect shape? Why is it not a sawtooth shape, like in (2); or some other shape, like in (3)? It is notable that a vast number of apparently unrelated vibrating systems show the same mathematical feature. A tuning fork, a sapling pulled to one side and released, a car bouncing on its shock absorbers, all these systems will exhibit sine-wave motion under one condition: the amplitude of the motion must be small.



Sinusoidal and Non-Sinusoidal Vibrations: Only the top graph is sinusoidal. The others vary with constant amplitude and period, but do no describe simple harmonic motion.

Hooke's Law and Sine Wave Generation

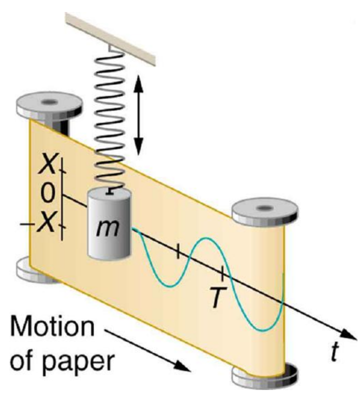
The key to understanding how an object vibrates is to know how the force on the object depends on the object's position. If a system follows Hooke's Law, the restoring force is proportional to the displacement. As touched on in previous sections, there exists a second order differential equation that relates acceleration and displacement.

$$\mathrm{F_{net}} = \mathrm{m}\frac{\mathrm{d^2x}}{\mathrm{dt^2}} = -\mathrm{kx}. \tag{5.8.3.49}$$

When this general equation is solved for the position, velocity and acceleration as a function of time:

- $\mathrm{x(t)} = \mathrm{A}\cos(\omega \mathrm{t} - \varphi)$
- $\mathrm{v(t)} = \frac{\mathrm{dx}}{\mathrm{dt}} = -\mathrm{A}\omega\sin(\omega \mathrm{t} - \varphi)$
- $\mathrm{a(t)} = \frac{\mathrm{d^2x}}{\mathrm{dt^2}} = -\mathrm{A}\omega^2\cos(\omega \mathrm{t} - \varphi)$

These are all sinusoidal solutions. Consider a mass on a spring that has a small pen inside running across a moving strip of paper as it bounces, recording its movements.

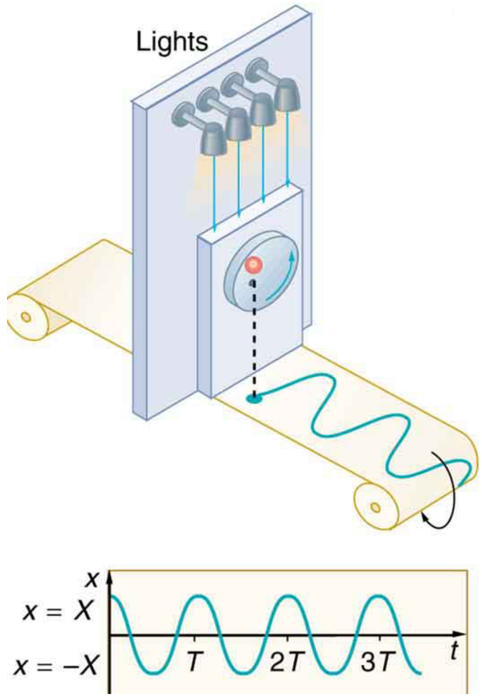


Mass on Spring Producing Sine Wave: The vertical position of an object bouncing on a spring is recorded on a strip of moving paper, leaving a sine wave.

The above equations can be rewritten in a form applicable to the variables for the mass on spring system in the figure.

- $\mathrm{x(t)} = \mathrm{X}\cos(\frac{2\pi \mathrm{t}}{\mathrm{T}})$
- $\mathrm{v(t)} = -\mathrm{v_{max}}\sin(\frac{2\pi \mathrm{t}}{\mathrm{T}})$
- $\mathrm{a(t)} = -\frac{\mathrm{kX}}{\mathrm{m}}\cos(\frac{2\pi \mathrm{t}}{\mathrm{T}})$

Recall that the projection of uniform circular motion can be described in terms of a simple harmonic oscillator. Uniform circular motion is therefore also sinusoidal, as you can see from.



Sinusoidal Nature of Uniform Circular Motion: The position of the projection of uniform circular motion performs simple harmonic motion, as this wavelike graph of x versus t indicates.

Instantaneous Energy of Simple Harmonic Motion

The equations discussed for the components of the total energy of simple harmonic oscillators may be combined with the sinusoidal solutions for *x(t)*, *v(t)*, and *a(t)* to model the changes in kinetic and potential energy in simple harmonic motion.

The kinetic energy *K* of the system at time t is:

$$\mathrm{K(t)} = \frac{1}{2}\mathrm{mv}^2(\mathrm{t}) = \frac{1}{2}\mathrm{m}\omega^2\mathrm{A}^2\sin^2(\omega\mathrm{t}-\varphi) = \frac{1}{2}\mathrm{kA}^2\sin^2(\omega\mathrm{t}-\varphi). \tag{5.8.3.50}$$

The potential energy *U* is:

$$\mathrm{U(t)} = \frac{1}{2}\mathrm{kx}^2(\mathrm{t}) == \frac{1}{2}\mathrm{kA}^2\cos^2(\omega\mathrm{t}-\varphi). \tag{5.8.3.51}$$

Summing *K(t)* and *U(t)* produces the total mechanical energy seen before:

$$\mathrm{E} = \mathrm{K}+\mathrm{U} = \frac{1}{2}\mathrm{kA}^2. \tag{5.8.3.52}$$

Key Points

- Motion that repeats itself regularly is called periodic motion. One complete repetition of the motion is called a cycle. The duration of each cycle is the period.
- The frequency refers to the number of cycles completed in an interval of time. It is the reciprocal of the period and can be calculated with the equation f=1/T.
- Some motion is best characterized by the angular frequency (ω). The angular frequency refers to the angular displacement per unit time and is calculated from the frequency with the equation $\omega = 2\pi\mathrm{f}$.
- If an object is vibrating to the right and left, then it must have a leftward force on it when it is on the right side, and a rightward force when it is on the left side.
- The restoring force causes an oscillating object to move back toward its stable equilibrium position, where the net force on it is zero.
- The simplest oscillations occur when the restoring force is directly proportional to displacement. In this case the force can be calculated as $\mathrm{F} = -\mathrm{kx}$, where F is the restoring force, k is the force constant, and x is the displacement.

- The motion of a mass on a spring can be described as *Simple Harmonic Motion* (SHM): oscillatory motion that follows Hooke's Law.
- The period of a mass on a spring is given by the equation $T=2\pi\sqrt{\frac{m}{k}}$
- Simple harmonic motion is often modeled with the example of a mass on a spring, where the restoring force obey's Hooke's Law and is directly proportional to the displacement of an object from its equilibrium position.
- Any system that obeys simple harmonic motion is known as a simple harmonic oscillator.
- The equation of motion that describes simple harmonic motion can be obtained by combining Newton's Second Law and Hooke's Law into a second-order linear ordinary differential equation: $F_{net}=m\frac{d^2x}{dt^2}=-kx.$
- Uniform circular motion describes the movement of an object traveling a circular path with constant speed. The one-dimensional projection of this motion can be described as simple harmonic motion.
- In uniform circular motion, the velocity vector v is always tangent to the circular path and constant in magnitude. The acceleration is constant in magnitude and points to the center of the circular path, perpendicular to the velocity vector at every instant.
- If an object moves with angular velocity ω around a circle of radius r centered at the origin of the x-y plane, then its motion along each coordinate is simple harmonic motion with amplitude r and angular frequency ω.
- A simple pendulum is defined as an object that has a small mass, also known as the pendulum bob, which is suspended from a wire or string of negligible mass.
- When displaced, a pendulum will oscillate around its equilibrium point due to momentum in balance with the restoring force of gravity.
- When the swings (amplitudes) are small, less than about 15º, the pendulum acts as a simple harmonic oscillator with period $T=2\pi\sqrt{\frac{L}{g}}$, where L is the length of the string and g is the acceleration due to gravity.
- A physical pendulum is the generalized case of the simple pendulum. It consists of any rigid body that oscillates about a pivot point.
- For small amplitudes, the period of a physical pendulum only depends on the moment of inertia of the body around the pivot point and the distance from the pivot to the body's center of mass. It is calculated as:$T=2\pi\sqrt{\frac{I}{mgh}}$.
- The period is still independent of the total mass of the rigid body. However, it is not independent of the mass distribution of the rigid body. A change in shape, size, or mass distribution will change the moment of inertia and thus, the period.
- The sum of the kinetic and potential energies in a simple harmonic oscillator is a constant, i.e., $KE+PE=\text{constant}$. The energy oscillates back and forth between kinetic and potential, going completely from one to the other as the system oscillates.
- In a spring system, the conservation equation is written as: $\frac{1}{2}mv^2+\frac{1}{2}kx^2=\text{constant}=\frac{1}{2}kX^2$, where X is the maximum displacement.
- The maximum velocity depends on three factors: amplitude, the stiffness factor, and mass: $v_{max}=\sqrt{\frac{k}{m}}X$.
- For simple harmonic oscillators, the equation of motion is always a second order differential equation that relates the acceleration and the displacement. The relevant variables are x, the displacement, and k, the spring constant.
- Solving the differential equation above always produces solutions that are sinusoidal in nature. For example, $x(t), v(t), a(t), K(t),$ and $U(t)$ all have sinusoidal solutions for simple harmonic motion.
- Uniform circular motion is also sinusoidal because the projection of this motion behaves like a simple harmonic oscillator.

Key Terms

- **period**: The duration of one cycle in a repeating event.
- **angular frequency**: The angular displacement per unit time.
- **frequency**: The quotient of the number of times n a periodic phenomenon occurs over the time t in which it occurs: f = n / t.
- **Restoring force**: A variable force that gives rise to an equilibrium in a physical system. If the system is perturbed away from the equilibrium, the restoring force will tend to bring the system back toward equilibrium. The restoring force is a function only of position of the mass or particle. It is always directed back toward the equilibrium position of the system
- **amplitude**: The maximum absolute value of some quantity that varies.
- **simple harmonic oscillator**: A device that implements Hooke's law, such as a mass that is attached to a spring, with the other end of the spring being connected to a rigid support, such as a wall.
- **oscillator**: A pattern that returns to its original state, in the same orientation and position, after a finite number of generations.

- **centripetal acceleration**: Acceleration that makes a body follow a curved path: it is always perpendicular to the velocity of a body and directed towards the center of curvature of the path.
- **uniform circular motion**: Movement around a circular path with constant speed.
- **simple pendulum**: A hypothetical pendulum consisting of a weight suspended by a weightless string.
- **physical pendulum**: A pendulum where the rod or string is not massless, and may have extended size; that is, an arbitrarily-shaped, rigid body swinging by a pivot. In this case, the pendulum's period depends on its moment of inertia around the pivot point.
- **mass distribution**: Describes the spatial distribution, and defines the center, of mass in an object.
- **elastic potential energy**: The energy stored in a deformable object, such as a spring.
- **dissipative forces**: Forces that cause energy to be lost in a system undergoing motion.
- **sinusoidal**: In the form of a wave, especially one whose amplitude varies in proportion to the sine of some variable (such as time).

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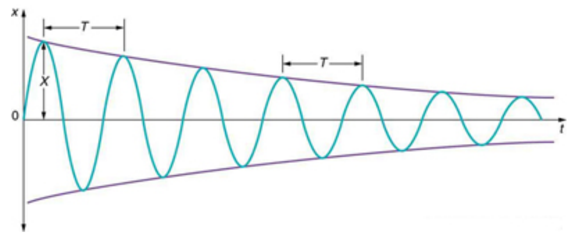
5.8.4: Damped and Driven Oscillations

learning objectives

- Describe the time evolution of the motion of the damped harmonic oscillator

The Physical Situation

The simple harmonic oscillator describes many physical systems throughout the world, but early studies of physics usually only consider ideal situations that do not involve friction. In the real world, however, frictional forces – such as air resistance – will slow, or dampen, the motion of an object. Sometimes, these dampening forces are strong enough to return an object to equilibrium over time.



Damped Harmonic Motion: Illustrating the position against time of our object moving in simple harmonic motion. We see that for small damping, the amplitude of our motion slowly decreases over time.

The simplest and most commonly seen case occurs when the frictional force is proportional to an object's velocity. Note that other cases exist which may lead to nonlinear equations which go beyond the scope of this example.

Consider an object of mass *m* attached to a spring of constant *k*. Let the damping force be proportional to the mass' velocity by a proportionality constant, *b,* called the *vicious damping coefficient.* We can describe this situation using Newton's second law, which leads to a second order, linear, homogeneous, ordinary differential equation. We simply add a term describing the damping force to our already familiar equation describing a simple harmonic oscillator to describe the general case of damped harmonic motion.

$$\mathrm{F_{net}} = \mathrm{m}\frac{\mathrm{d^2x}}{\mathrm{dt^2}} + \mathrm{b}\frac{\mathrm{dx}}{\mathrm{dt}} + \mathrm{kx} = 0 \tag{5.8.4.1}$$

$$= \frac{\mathrm{d^2x}}{\mathrm{dt^2}} + \frac{\mathrm{b}}{\mathrm{m}}\frac{\mathrm{dx}}{\mathrm{dt}} + \frac{\mathrm{k}}{\mathrm{m}}\mathrm{x} = 0 \tag{5.8.4.2}$$

$$= \frac{\mathrm{d^2x}}{\mathrm{dt^2}} + \gamma\frac{\mathrm{dx}}{\mathrm{dt}} + \omega_0^2\mathrm{x} = 0 \tag{5.8.4.3}$$

$$\omega_0^2 = \frac{\mathrm{k}}{\mathrm{m}}, \gamma = \frac{\mathrm{b}}{\mathrm{m}} \tag{5.8.4.4}$$

This notation uses $\frac{\mathrm{d^2x}}{\mathrm{dt^2}}$, the acceleration of our object, $\frac{\mathrm{dx}}{\mathrm{dt}}$, the velocity of our object, ω_0, undamped angular frequency of oscillation, and ɣ, which we can call the damping ratio.

Solving the Differential Equation; Interpreting Results

We solve this differential equation for our equation of motion of the system, x(t). We assume a solution in the form of an exponential, where a is a constant value which we will solve for.

$$\mathrm{x(t)} = \mathrm{e^{at}} \tag{5.8.4.5}$$

Plugging this into the differential equation we find that there are three results for a, which will dictate the motion of our system. We can solve for a by using the quadratic equation.

$$\mathrm{F_{net}} = \mathrm{a^2x} + \gamma\mathrm{ax} + \omega_0^2\mathrm{x} = 0 \tag{5.8.4.6}$$

$$= \mathrm{a^2} + \gamma\mathrm{a} + \omega_0^2 = 0 \tag{5.8.4.7}$$

$$\mathrm{a} = \frac{\gamma \pm \sqrt{\gamma^2 - 4\omega_0^2}}{2} \tag{5.8.4.8}$$

The physical situation has three possible results depending on the value of a, which depends on the value of what is under our radical. This expression can be positive, negative, or equal to zero which will result in overdamping, underdamping, and critical damping, respectively.

$\gamma^2 > 4\omega_0^2$ is the *Over Damped* case. In this case, the system returns to equilibrium by exponentially decaying towards zero. The system will not pass the equilibrium position more than once.

$\gamma^2 < 4\omega_0^2$ is the *Under Damped* case. In this case, the system oscillates as it slowly returns to equilibrium and the amplitude decreases over time. Figure 1 depicts an underdamped case.

$\gamma^2 = 4\omega_0^2$ is the*Critically Damped* case. In this case, the system returns to equilibrium very quickly without oscillating and without passing the equilibrium position at all.

Driven Oscillations and Resonance

Driven harmonic oscillators are damped oscillators further affected by an externally applied force.

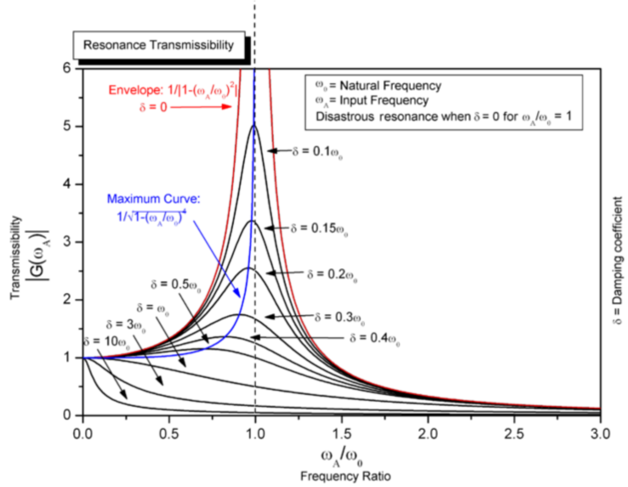
learning objectives

- Describe a driven harmonic oscillator as a type of damped oscillator

In classical mechanics, a harmonic oscillator is a system that, when displaced from its equilibrium position, experiences a restoring force, F, proportional to the displacement, $\mathrm{xF} \rightarrow = -\mathrm{k}\overrightarrow{\mathrm{x}}$ where k is a positive constant. If a frictional force (damping) proportional to the velocity is also present, the harmonic oscillator is described as a damped oscillator.

Driven harmonic oscillators are damped oscillators further affected by an externally applied force F(t). Newton's second law takes the form $\mathrm{F(t)} - \mathrm{kx} - \mathrm{c}\frac{\mathrm{dx}}{\mathrm{dt}} = \mathrm{m}\frac{\mathrm{d^2x}}{\mathrm{dt^2}}$. It is usually rewritten into the form $\frac{\mathrm{d^2x}}{\mathrm{dt^2}} + 2\zeta\omega_0\frac{\mathrm{dx}}{\mathrm{dt}} + \omega_0^2\mathrm{x} = \frac{\mathrm{F(t)}}{\mathrm{m}}$. This equation can be solved exactly for any driving force, using the solutions z(t) which satisfy the unforced equation: $\frac{\mathrm{d^2z}}{\mathrm{dt^2}} + 2\zeta\omega_0\frac{\mathrm{dz}}{\mathrm{dt}} + \omega_0^2\mathrm{z} = 0$, and which can be expressed as damped sinusoidal oscillations$\mathrm{z(t)} = \mathrm{Ae}^{-\zeta\omega_0\mathrm{t}}\sin\left(\sqrt{1-\zeta^2}\,\omega_0\mathrm{t} + \varphi\right)$ in the case where $\zeta \leq 1$. The amplitude A and phase φ determine the behavior needed to match the initial conditions. In the case $\zeta < 1$ and a unit step input with $\mathrm{x}(0) = 0$ the solution is: $\mathrm{x(t)} = 1 - \mathrm{e}^{-\zeta\omega_0\mathrm{t}}\frac{\sin\left(\sqrt{1-\zeta^2}\omega_0\mathrm{t}+\varphi\right)}{\sin(\varphi)}$ with phase φ given by $\cos\varphi = \zeta$. The time an oscillator needs to adapt to changed external conditions is of the order \(\mathrm{τ =\frac{1}{(ζ_0)}}\). In physics, the adaptation is called relaxation, and τ is called the relaxation time.

In the case of a sinusoidal driving force: $\frac{\mathrm{d^2x}}{\mathrm{dt^2}} + 2\zeta\omega_0\frac{\mathrm{dx}}{\mathrm{dt}} + \omega_0^2\mathrm{x} = \frac{1}{\mathrm{m}}\mathrm{F}_0\sin(\omega\mathrm{t})$, where F_0 is the driving amplitude and ωω is the driving frequency for a sinusoidal driving mechanism. This type of system appears in AC driven RLC circuits (resistor-inductor-capacitor) and driven spring systems having internal mechanical resistance or external air resistance. The general solution is a sum of a transient solution that depends on initial conditions, and a steady state that is independent of initial conditions and depends only on the driving amplitude F_0, driving frequency ωω, undamped angular frequency ω_0, and the damping ratio ζ. For a particular driving frequency called the resonance, or resonant frequency $\omega_\mathrm{r} = \omega_0\sqrt{1-2\zeta^2}$, the amplitude (for a given F_0) is maximum. This resonance effect only occurs when $\zeta < 1/\sqrt{2}$, i.e. for significantly underdamped systems. For strongly underdamped systems the value of the amplitude can become quite large near the resonance frequency (see).



Resonance: Steady state variation of amplitude with frequency and damping of a driven simple harmonic oscillator.

Key Points

- To describe a damped harmonic oscillator, add a velocity dependent term, bx, where b is the *vicious dampingcoefficient*.
- Solve the differential equation for the equation of motion, x(t).
- Depending on the values of the damping coefficient and undamped angular frequency, the results will be one of three cases: an under damped system, an over damped system, or a critically damped system.
- Newton's second law takes the form $\mathrm{F(t)-kx-c\frac{dx}{dt}=m\frac{d^2x}{dt^2}}$ for driven harmonic oscillators.
- The resonance effect occurs only in the underdamped systems.
- For strongly underdamped systems the value of the amplitude can become quite large near the resonance frequency.

Key Terms

- **Under Damped**: "The condition in which damping of an oscillator causes it to return to equilibrium with the amplitude gradually decreasing to zero; system returns to equilibrium faster but overshoots and crosses the equilibrium position one or more times. "
- **Critically Damped**: "The condition in which the damping of an oscillator causes it to return as quickly as possible to its equilibrium position without oscillating back and forth about this position. "
- **Over Damped**: "The condition in which damping of an oscillator causes it to return to equilibrium without oscillating; oscillator moves more slowly toward equilibrium than in the critically damped system. "
- **oscillator**: A pattern that returns to its original state, in the same orientation and position, after a finite number of generations.
- **equilibrium**: The state of a body at rest or in uniform motion, the resultant of all forces on which is zero.
- **force**: A physical quantity that denotes ability to push, pull, twist or accelerate a body which is measured in a unit dimensioned in mass × distance/time² (ML/T²): SI: newton (N); CGS: dyne (dyn)

- Driven oscillations. **Provided by**: Wikipedia. **Located at**: **en.Wikipedia.org/wiki/Driven_oscillations%23Driven_harmonic_oscillators**. **License**: ***CC BY-SA: Attribution-ShareAlike***
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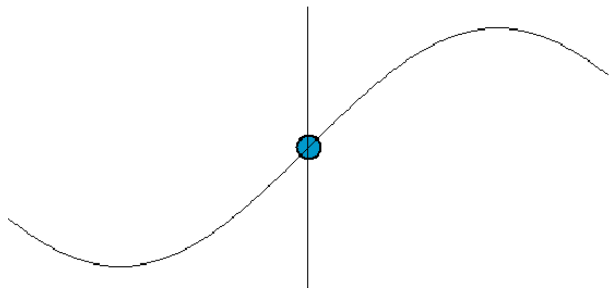
5.8.5: Waves

learning objectives

- Describe process of energy and mass transfer during wave motion

Vibrations and waves are extremely important phenomena in physics. In nature, oscillations are found everywhere. From the jiggling of atoms to the large oscillations of sea waves, we find examples of vibrations in almost every physical system. In physics a wave can be thought of as a disturbance or oscillation that travels through space-time, accompanied by a transfer of energy. Wave motion transfers energy from one point to another, often with no permanent displacement of the particles of the medium —that is, with little or no associated mass transport. They consist, instead, of oscillations or vibrations around almost fixed locations.

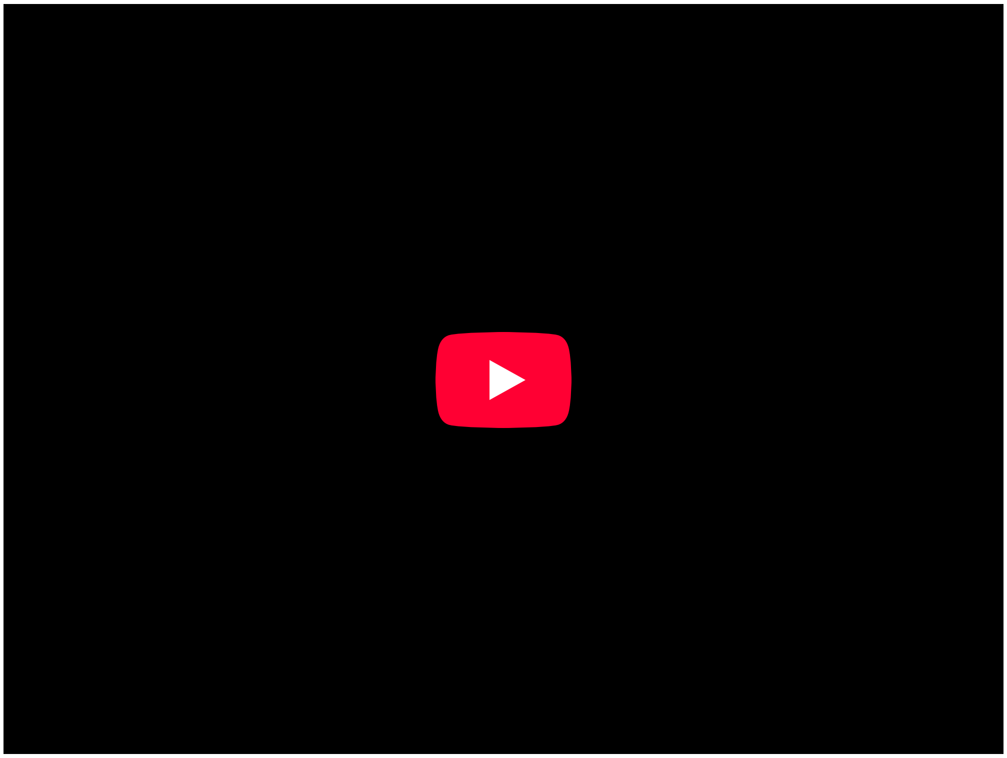
The emphasis of the last point highlights an important misconception of waves. Waves transfer energy not mass. An easy way to see this is to imagine a floating ball a few yards out to sea. As the waves propagate (i.e., travel) towards the shore, the ball will not come towards the shore. It may come to shore eventually due to the tides, current or wind, but the waves themselves will not carry the ball with them. A wave only moves mass perpendicular to the direction of propagation—in this case up and down, as illustrated in the figure below:

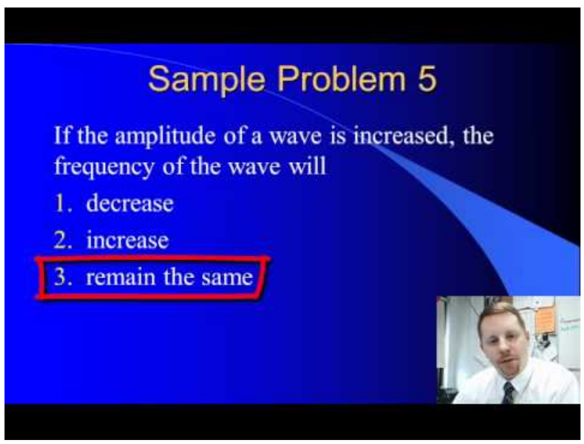


Wave motion: The point along the axis is analogous to the floating ball at sea. We notice that while it moves up and down it does not move in the direction of the wave’s propagation.

A wave can be transverse or longitudinal depending on the direction of its oscillation. Transverse waves occur when a disturbance causes oscillations perpendicular (at right angles) to the propagation (the direction of energy transfer). Longitudinal waves occur when the oscillations are parallel to the direction of propagation. While mechanical waves can be both transverse and longitudinal, all electromagnetic waves are transverse. Sound, for example, is a longitudinal wave.

The description of waves is closely related to their physical origin for each specific instance of a wave process. For example, acoustics is distinguished from optics in that sound waves are related to a mechanical rather than an electromagnetic (light) wave transfer caused by vibration. Therefore, concepts such as mass, momentum, inertia or elasticity become crucial in describing acoustic (as distinct from optic) wave processes. This difference in origin introduces certain wave characteristics particular to the properties of the medium involved. In this chapter we will closely examine the difference between longitudinal and transverse waves along with some of the properties they possess. We will also learn how waves are fundamental in describing motion of many applicable physical systems.





The Wave Equation: A brief introduction to the wave equation, discussing wave velocity, frequency, wavelength, and period.

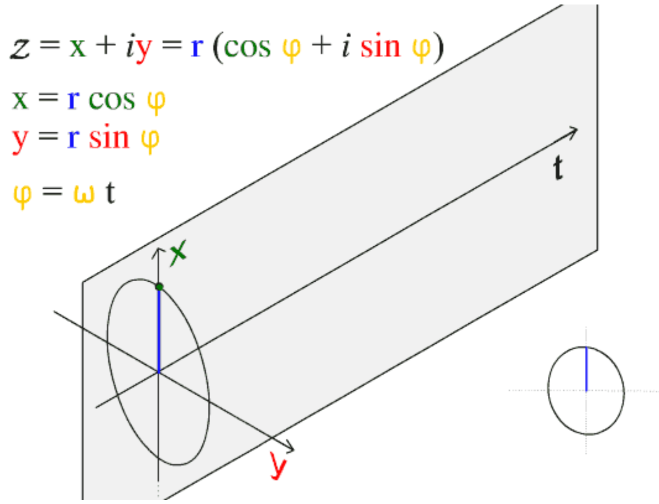
Transverse Waves

Transverse waves propagate through media with a speed →vwv→w orthogonally to the direction of energy transfer.

> **learning objectives**
>
> - Describe properties of the transverse wave

A transverse wave is a moving wave that consists of oscillations occurring perpendicular (or right angled) to the direction of energy transfer. If a transverse wave is moving in the positive *x*-direction, its oscillations are in up and down directions that lie in the *y*–*z* plane. Light is an example of a transverse wave. For transverse waves in matter, the displacement of the medium is perpendicular to the direction of propagation of the wave. A ripple on a pond and a wave on a string are easily visualized transverse waves.

Transverse waves are waves that are oscillating perpendicularly to the direction of propagation. If you anchor one end of a ribbon or string and hold the other end in your hand, you can create transverse waves by moving your hand up and down. Notice though, that you can also launch waves by moving your hand side-to-side. This is an important point. There are two independent directions in which wave motion can occur. In this case, these are the *y* and *z* directions mentioned above. depicts the motion of a transverse wave. Here we observe that the wave is moving in *t* and oscillating in the *x*-*y* plane. A wave can be thought as comprising many particles (as seen in the figure) which oscillate up and down. In the figure we observe this motion to be in *x*-*y* plane (denoted by the red line in the figure). As time passes the oscillations are separated by units of time. The result of this separation is the sine curve we expect when we plot position versus time.

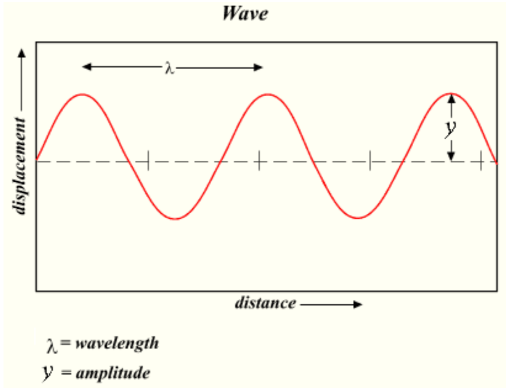


Sine Wave: The direction of propagation of this wave is along the t axis.

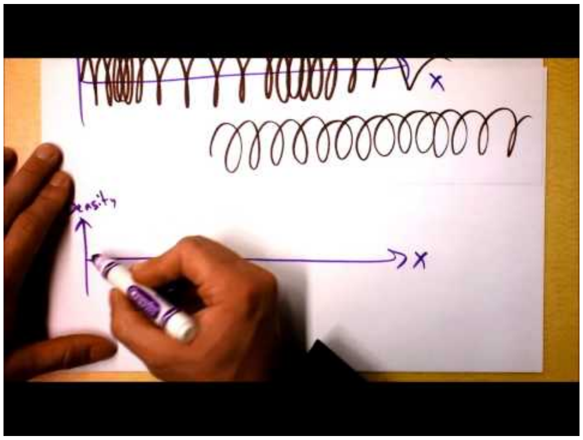
When a wave travels through a medium–i.e., air, water, etc., or the standard reference medium (vacuum)–it does so at a given speed: this is called the speed of propagation. The speed at which the wave propagates is denoted and can be found using the following formula:

$$v = f\lambda \tag{5.8.5.1}$$

where *v* is the speed of the wave, *f* is the frequency , and is the wavelength. The wavelength spans crest to crest while the amplitude is 1/2 the total distance from crest to trough. Transverse waves have their applications in many areas of physics. Examples of transverse waves include seismic S (secondary) waves, and the motion of the electric (E) and magnetic (M) fields in an electromagnetic plane waves, which both oscillate perpendicularly to each other as well as to the direction of energy transfer. Therefore an electromagnetic wave consists of two transverse waves, visible light being an example of an electromagnetic wave.



Wavelength and Amplitude: The wavelength is the distance between adjacent crests. The amplitude is the 1/2 the distance from crest to trough.



Two Types of Waves: Longitudinal vs. Transverse: Even ocean waves!

Longitudinal Waves

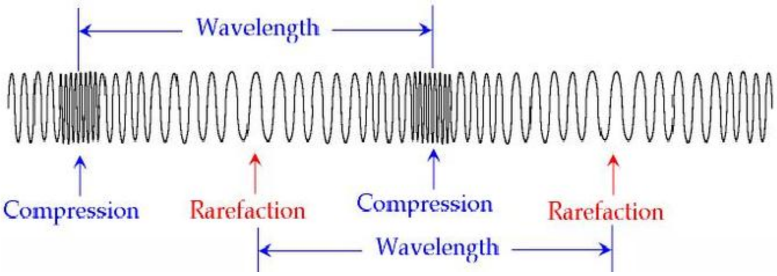
Longitudinal waves, sometimes called compression waves, oscillate in the direction of propagation.

learning objectives

- Give properties and provide examples of the longitudinal wave

Longitudinal Waves

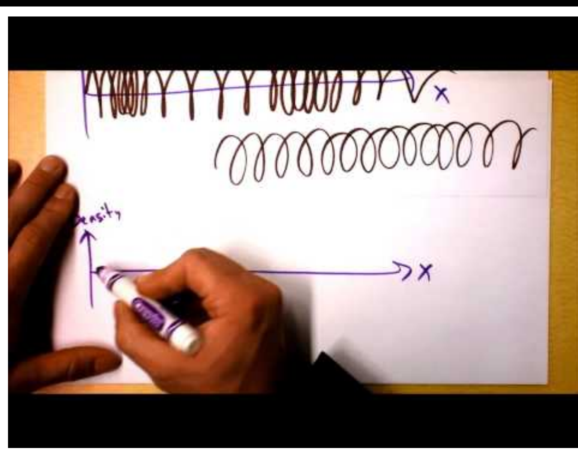
Longitudinal waves have the same direction of vibration as their direction of travel. This means that the movement of the medium is in the same direction as the motion of the wave. Some longitudinal waves are also called compressional waves or compression waves. An easy experiment for observing longitudinal waves involves taking a Slinky and holding both ends. After compressing and releasing one end of the Slinky (while still holding onto the end), a pulse of more concentrated coils will travel to the end of the Slinky.



Longitudinal Waves: A compressed Slinky is an example of a longitudinal wave. The wve propagates in the same direction of oscillation.

Like transverse waves, longitudinal waves do not displace mass. The difference is that each particle which makes up the medium through which a longitudinal wave propagates oscillates along the axis of propagation. In the example of the Slinky, each coil will oscillate at a point but will not travel the length of the Slinky. It is important to remember that energy, in this case in the form of a pulse, is being transmitted and not the displaced mass.

Longitudinal waves can sometimes also be conceptualized as pressure waves. The most common pressure wave is the sound wave. Sound waves are created by the compression of a medium, usually air. Longitudinal sound waves are waves of alternating pressure deviations from the equilibrium pressure, causing local regions of compression and rarefaction. Matter in the medium is periodically displaced by a sound wave, and thus oscillates. When people make a sound, whether it is through speaking or hitting something, they are compressing the air particles to some significant amount. By doing so, they create transverse waves. When people hear sounds, their ears are sensitive to the pressure differences and interpret the waves as different tones.



Two Types of Waves: Longitudinal vs. Transverse: Even ocean waves!

Water Waves

Water waves can be commonly observed in daily life, and comprise both transverse and longitudinal wave motion.

learning objectives

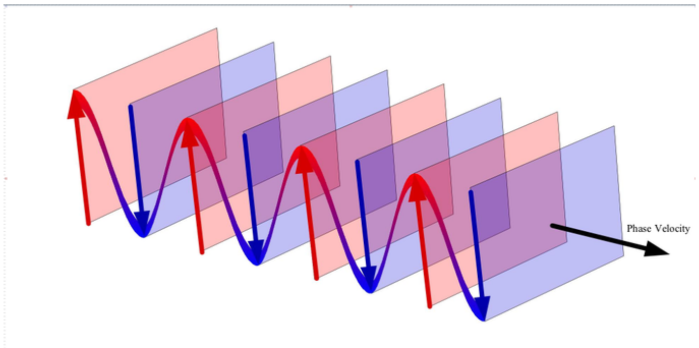
- Describe particle movement in water waves

Water waves, which can be commonly observed in our daily lives, are of specific interest to physicists. Describing detailed fluid dynamics in water waves is beyond the scope of introductory physics courses. Although we often observe water wave propagating in 2D, in this atom we will limit our discussion to 1D propagation.

Water waves: Surface waves in water

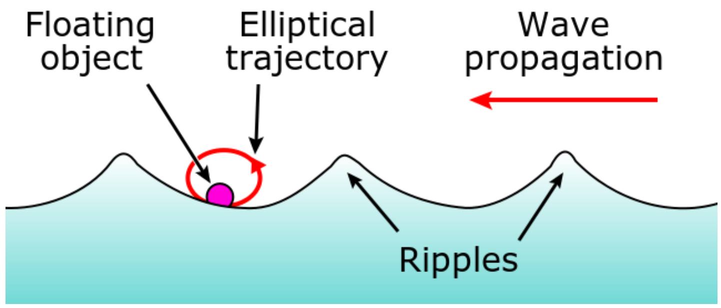
The uniqueness of water waves is found in the observation that they comprise both transverse and longitudinal wave motion. As a result, the particles composing the wave move in clockwise circular motion, as seen in. Oscillatory motion is highest at the surface and diminishes exponentially with depth. Waves are generated by wind passing over the surface of the sea. As long as the waves propagate slower than the wind speed just above the waves, there is an energy transfer from the wind to the waves. Both air pressure differences between the upwind and the lee side of a wave crest, as well as friction on the water surface by the wind (making the water to go into the shear stress), contribute to the growth of the waves.

In the case of monochromatic linear plane waves in deep water, particles near the surface move in circular paths, creating a combination of longitudinal (back and forth) and transverse (up and down) wave motions. When waves propagate in shallow water (where the depth is less than half the wavelength), the particle trajectories are compressed into ellipses. As the wave amplitude (height) increases, the particle paths no longer form closed orbits; rather, after the passage of each crest, particles are displaced slightly from their previous positions, a phenomenon known as Stokes drift.



Plane wave: We see a wave propagating in the direction of the phase velocity. The wave can be thought to be made up of planes orthogonal to the direction of the phase velocity.

Since water waves transport energy, attempts to generate power from them have been made by utilizing the physical motion of such waves. Although larger waves are more powerful, wave power is also determined by wave speed, wavelength, and water density. Deep water corresponds with a water depth larger than half the wavelength, as is a common case in the sea and ocean. In deep water, longer-period waves propagate faster and transport their energy faster. The deep-water group velocity is half the phase velocity. In shallow water for wavelengths larger than about twenty times the water depth (as often found near the coast), the group velocity is equal to the phase velocity. These methods have proven viable in some cases but do not provide a fully sustainable form of renewable energy to date.



Water waves: The motion water waves causes particles to follow clockwise circular motion. This is a result of the wave having both transverse and longitudinal properties.

Wavelength, Freqency in Relation to Speed

Waves are defined by its frequency, wavelength, and amplitude among others. They also have two kinds of velocity: phase and group velocity.

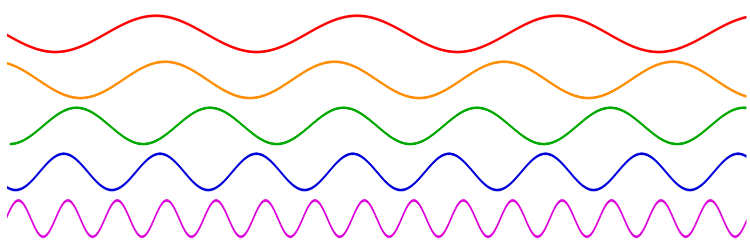
learning objectives

- Identify major characteristic properties of waves

Characteristics of Waves

Waves have certain characteristic properties which are observable at first notice. The first property to note is the amplitude. The amplitude is half of the distance measured from crest to trough. We also observe the wavelength, which is the spatial period of the wave (e.g. from crest to crest or trough to trough). We denote the wavelength by the Greek letter λλ.

The frequency of a wave is the number of cycles per unit time — one can think of it as the number of crests which pass a fixed point per unit time. Mathematically, we make the observation that,



Frequencies of different sine waves.: The red wave has a low frequency sine there is very little repetition of cycles. Conversely we say that the purple wave has a high frequency. Note that time increases along the horizontal.

$$\mathrm{f}=\frac{1}{\mathrm{T}} \tag{5.8.5.2}$$

where T is the period of oscillation. Frequency and wavelength can also be related-* with respects to a “speed” of a wave. In fact,

\[\mathrm{v=fλ}]

where v is called the wave speed, or more commonly,the phase velocity, the rate at which the phase of the wave propagates in space. This is the velocity at which the phase of any one frequency component of the wave travels. For such a component, any given phase of the wave (for example, the crest) will appear to travel at the phase velocity.

Finally, the group velocity of a wave is the velocity with which the overall shape of the waves’ amplitudes — known as the modulation or envelope of the wave — propagates through space. In, one may see that the overall shape (or “envelope”) propagates to the right, while the phase velocity is negative.

Fig 2: This shows a wave with the group velocity and phase velocity going in different directions. (The group velocity is positive and the phase velocity is negative.)

Energy Transportation

Waves transfer energy which can be used to do work.

learning objectives

- Relate direction of energy and wave transportation

Energy transportion is essential to waves. It is a common misconception that waves move mass. Waves carry energy along an axis defined to be the direction of propagation. One easy example is to imagine that you are standing in the surf and you are hit by a significantly large wave, and once you are hit you are displaced (unless you hold firmly to your ground!). In this sense the wave has done work (it applied a force over a distance). Since work is done over time, the energy carried by a wave can be used to generate power.

Water Wave: Waves that are more massive or have a greater velocity transport more energy.

Similarly we find that electromagnetic waves carry energy. Electromagnetic radiation (EMR) carries energy—sometimes called radiant energy—through space continuously away from the source (this is not true of the near-field part of the EM field). Electromagnetic waves can be imagined as a self-propagating transverse oscillating wave of electric and magnetic fields. EMR also carries both momentum and angular momentum. These properties may all be imparted to matter with which it interacts (through work). EMR is produced from other types of energy when created, and it is converted to other types of energy when it is destroyed. The photon is the quantum of the electromagnetic interaction, and is the basic “unit” or constituent of all forms of EMR. The quantum nature of light becomes more apparent at high frequencies (or high photon energy). Such photons behave more like particles than lower-frequency photons do.

Electromagnetic Wave: Electromagnetic waves can be imagined as a self-propagating transverse oscillating wave of electric and magnetic fields. This 3D diagram shows a plane linearly polarized wave propagating from left to right.

In general, there is a relation of waves which states that the velocity (v) of a wave is proportional to the frequency (f) times the wavelength (λ):

$$v = f\lambda \tag{5.8.5.3}$$

We also know that classical momentum pp is given by p=mvp=mv which relates to force via Newton's second law: $F = \frac{dp}{dt}$

EM waves with higher frequencies carry more energy. This is a direct result of the equations above. Since $v \propto f$ we find that higher frequencies imply greater velocity. If velocity is increased then we have greater momentum which implies a greater force (it gets a little bit tricky when we talk about particles moving close to the speed of light, but this observation holds in the classical sense). Since energy is the ability of an object to do work, we find that for $W = Fd$ a greater force correlates to more energy transfer. Again, this is an easy phenomenon to experience empirically; just stand in front of a faster wave and feel the difference!

Key Points

- A wave can be thought of as a disturbance or oscillation that travels through space-time, accompanied by a transfer of energy.
- The direction a wave propagates is perpendicular to the direction it oscillates for transverse waves.
- A wave does not move mass in the direction of propagation; it transfers energy.
- Transverse waves oscillate in the z-y plane but travel along the x axis.
- A transverse wave has a speed of propagation given by the equation $v = f\lambda$.
- The direction of energy transfer is perpendicular to the motion of the wave.
- While longitudinal waves oscillate in the direction of propagation, they do not displace mass since the oscillations are small and involve an equilibrium position.
- The longitudinal 'waves' can be conceptualized as pulses that transfer energy along the axis of propagation.
- Longitudinal waves can be conceptualized as pressure waves characterized by compression and rarefaction.
- The particles which make up a water wave move in circular paths.
- If the waves move slower than the wind above them, energy is transfered from the wind to the waves.
- The oscillations are greatest on the surface of the wave and become weaker deeper in the fluid.
- The wavelength is the spatial period of the wave.
- The frequency of a wave refers to the number of cycles per unit time and is not to be confused with angular frequency.
- The phase velocity can be expressed as the product of wavelength and frequency.
- Waves which are more massive transfer more energy.
- Waves with greater velocities transfer more energy.
- Energy of a wave is transported in the direction of the waves transportation.

Key Terms

- **medium**: The material or empty space through which signals, waves or forces pass.
- **direction of propagation**: The axis along which the wave travels.
- **wave**: A moving disturbance in the energy level of a field.
- **wavelength**: The length of a single cycle of a wave, as measured by the distance between one peak or trough of a wave and the next; it is often designated in physics as λ, and corresponds to the velocity of the wave divided by its frequency.
- **trough**: A long, narrow depression between waves or ridges.
- **speed of propagation**: The speed at which a wave moves through a medium.
- **crest**: The ridge or top of a wave.
- **transverse wave**: Any wave in which the direction of disturbance is perpendicular to the direction of travel.
- **rarefaction**: a reduction in the density of a material, especially that of a fluid
- **Longitudinal**: Running in the direction of the long axis of a body.
- **compression**: to increase in density; the act of compressing, or the state of being compressed; compaction
- **phase velocity**: The velocity of propagation of a pure sine wave of infinite extent and infinitesimal amplitude.
- **group velocity**: The propagation velocity of the envelope of a modulated travelling wave, which is considered as the propagation velocity of information or energy contained in it.

- **plane wave**: A constant-frequency wave whose wavefronts (surfaces of constant phase) are infinite parallel planes of constant peak-to-peak amplitude normal to the phase velocity vector.
- **wave speed**: The absolute value of the velocity at which the phase of any one frequency component of the wave travels.
- **wavelength**: The length of a single cycle of a wave, as measured by the distance between one peak or trough of a wave and the next; it is often designated in physics as λ, and corresponds to the velocity of the wave divided by its frequency.
- **frequency**: The quotient of the number of times n a periodic phenomenon occurs over the time t in which it occurs: $f = \frac{n}{t}$.
- **energy**: A quantity that denotes the ability to do work and is measured in a unit dimensioned in mass × distance^2/time^2 (ML^2/T^2) or the equivalent.
- **power**: A measure of the rate of doing work or transferring energy.
- **work**: A measure of energy expended in moving an object; most commonly, force times displacement. No work is done if the object does not move.

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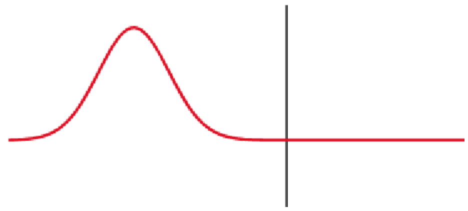
5.8.6: Wave Behavior and Interaction

learning objectives

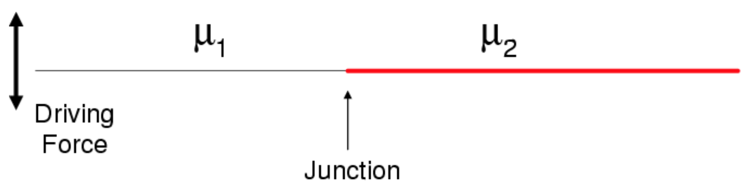
- Distinguish transmission and reflection phenomena

When the medium through which a wave travels suddenly changes, the wave often experiences partial transmission and partial refection at the interface. Reflection is a wave phenomenon that changes the direction of a wavefront at an interface between two different media so that the wavefront returns into the medium from which it originated. Transmission permits the passage of wave, with some or none of the incident wave being absorbed. Reflection and transmission often occur at the same time.



Partial Transmittance and Partial Reflectance: A wave experiences partial transmittance and partial reflectance when the medium through which it travels suddenly changes.

Consider a long string made by connecting two sub-strings with different density μ_1, μ_2. When the string is driven by an external force, partial reflection and transmission occur as in the figure above. For the incoming, reflected, and transmitted waves, we can try a solution of the following forms:



Two Strings With Different Density: Two strings with different density are connected and driven by an external driving force.

$$y_{inc} = A\cos(k_1 x - \omega t) \tag{5.8.6.1}$$
$$y_{ref} = B\cos(k_1 x + \omega t) \tag{5.8.6.2}$$
$$y_{trans} = C\cos(k_2 x - \omega t) \tag{5.8.6.3}$$

k_1 and k_2 are determined by the speed of the wave in each medium. We choose our coordinates such that the junction of two sub-strings is located at x=0. In choosing a trial solution for the waves, we assumed that the incident and transmitted waves travel to the right, while the reflected waves travel to the left. (This is why the '+' sign is chosen before ωtωt in the reflected wave. On the left side of the junction, we have

$$y_l = y_{inc} + y_{ref} = A\cos(k_1 x - \omega t). \tag{5.8.6.4}$$

On the right side, we have

$$y_r = y_{trans} = C\cos(k_2 x - \omega t) \tag{5.8.6.5}$$

We will impose additional restriction on the waves by applying "boundary conditions" at x=0. At the boundary x=0, the wave must be continuous and there should be no kinks in it. Thus we must have

$$y_l(x=0,t) = y_r(x=0,t) \tag{5.8.6.6}$$
$$\frac{\partial y_l(x,t)}{\partial x})_{x=0} = (\frac{\partial y_r(x,t)}{\partial x})_{x=0} \tag{5.8.6.7}$$

From the first equation, we get $A+B=C$. From the second equation, we get $A-B=(\frac{k_2}{k_1})C$.

Thus, we get the following result.

$$A = \frac{1}{2}(1 + \frac{k_2}{k_1})C \tag{5.8.6.8}$$

$$B = \frac{1}{2}(1 - \frac{k_2}{k_1})C \tag{5.8.6.9}$$

We can define the transmission (t) and reflection (r) coefficients as

$$t = \frac{C}{A} = \frac{2k_1}{k_1 + k_2}, r = \frac{B}{A} = \frac{k_1 - k_2}{k_1 + k_2}. \tag{5.8.6.10}$$

Superposition and Interference

A wave may have a complicated shape that can result from superposition and interference of several waves.

learning objectives

- Distinguish destructive and constructive interference and identify conditions that are required for the superposition of waves

Most waves do not look very simple. They look are often more complex than the simple water waves often considered in textbooks. Simple waves may be created by a simple harmonic oscillation, and thus have a sinusoidal shape. Complex waves are more interesting, even beautiful, but they look formidable. Most waves appear complex because they result from several simple waves adding together. Luckily, the rules for adding waves are quite simple.

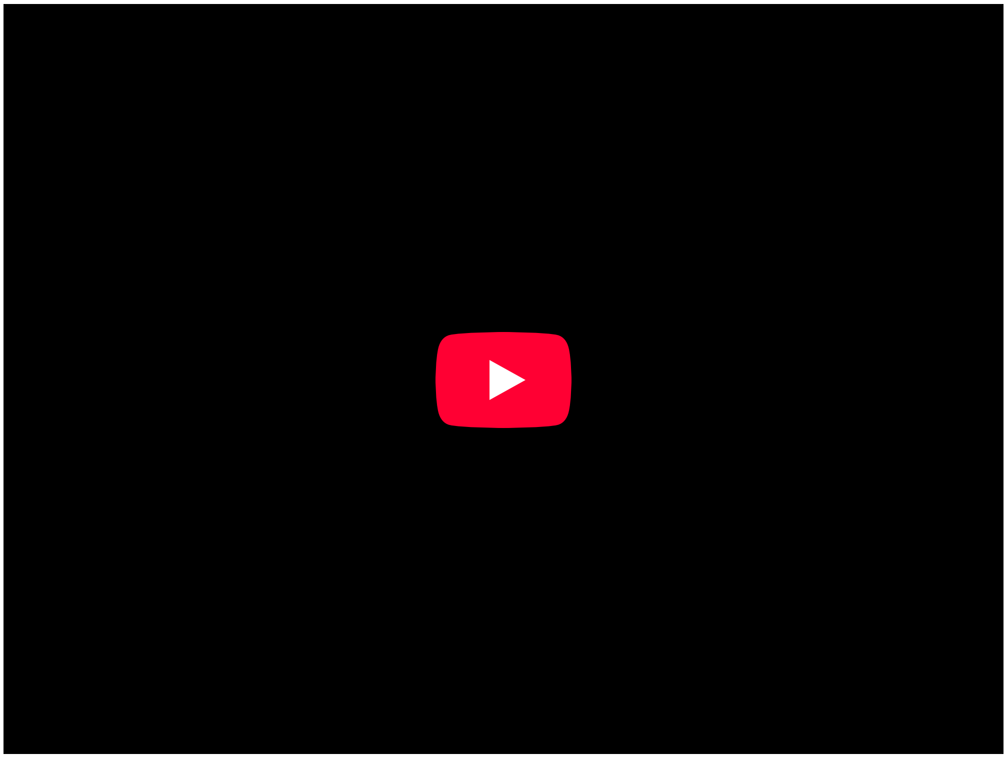
Complex Pattern of Waves: These waves result from the superposition of several waves from different sources, producing a complex pattern.

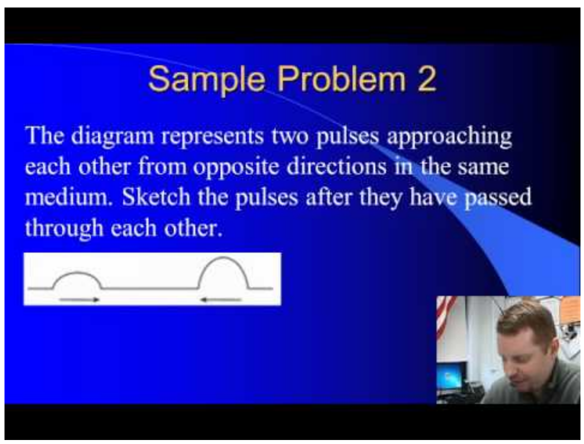
Superposition

When two or more waves arrive at the same point, they superimpose themselves on one another. More specifically, the disturbances of waves are superimposed when they come together—a phenomenon called superposition. Each disturbance corresponds to a force, and forces add. If the disturbances are along the same line, then the resulting wave is a simple addition of the disturbances of the individual waves—that is, their amplitudes add.

Interference

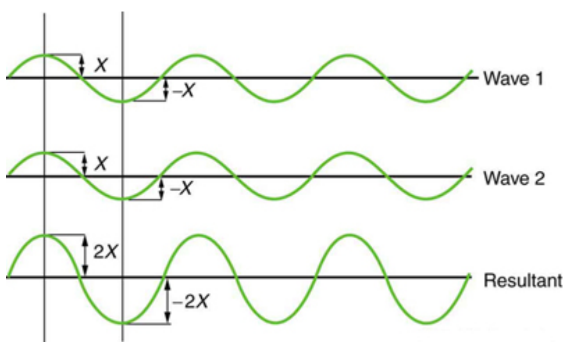
As a result of superposition of waves, interference can be observed. Interference is an effect caused by two or more waves.





Wave Interference: A brief introduction to constructive and destructive wave interference and the principle of superposition.

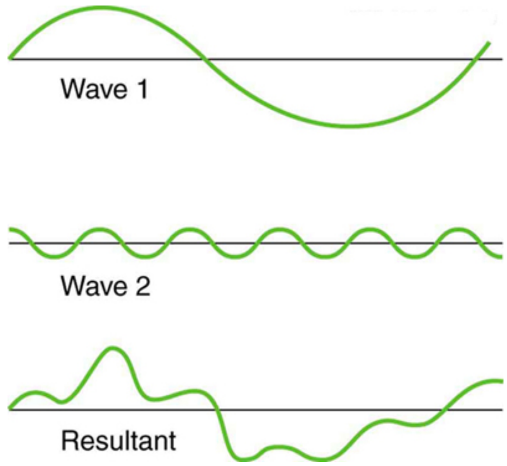
When two identical waves arrive at the same point exactly in phase the crests of the two waves are precisely aligned, as are the troughs. This superposition produces pure constructive interference. Because the disturbances add, constructive interference may produce a wave that has twice the amplitude of the individual waves, but has the same wavelength.



Constructive Interference: Pure constructive interference of two identical waves produces one with twice the amplitude, but the same wavelength.

If two identical waves that arrive exactly out of phase—that is, precisely aligned crest to trough—they may produce pure destructive interference. Because the disturbances are in the opposite direction for this superposition, the resulting amplitude may be zero for destructive interference, and the waves completely cancel.

While pure constructive and pure destructive interference do occur, they require precisely aligned identical waves. The superposition of most waves produces a combination of constructive and destructive interference and can vary from place to place and time to time. Here again, the disturbances add and subtract, producing a more complicated looking wave.



Superposition of Non-Identical Waves: Superposition of non-identical waves exhibits both constructive and destructive interference.

Standing Waves and Resonance

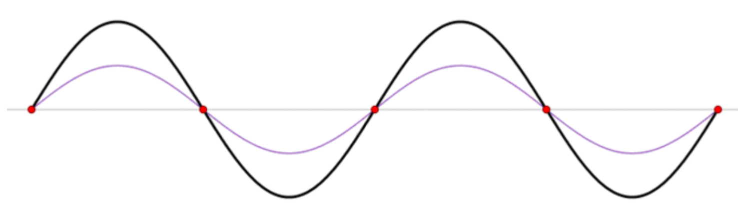
A standing wave is one in which two waves superimpose to produce a wave that varies in amplitude but does not propagate.

learning objectives

- Describe properties of a standing wave

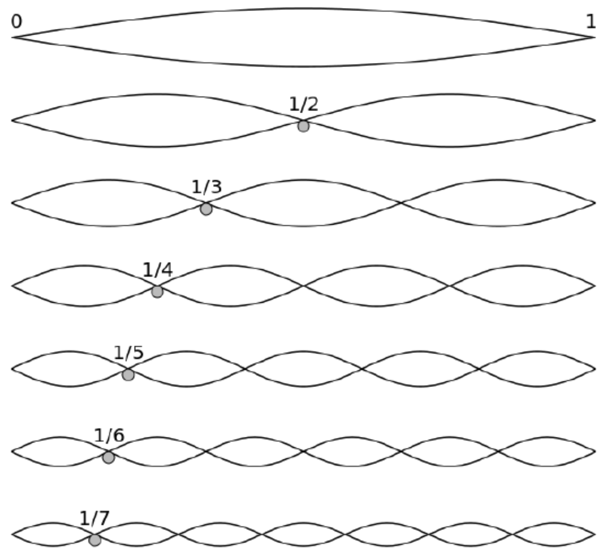
Standing Wave

Sometimes waves do not seem to move, but rather they just vibrate in place. These waves are formed by the superposition of two or more moving waves for two identical waves moving in opposite directions. The waves move through each other with their disturbances adding as they go by. If the two waves have the same amplitude and wavelength then they alternate between constructive and destructive interference. The resultant looks like a wave standing in place and, thus, is called a standing wave.



Standing Wave: A standing wave (black) depicted as the sum of two propagating waves traveling in opposite directions (red and blue).

Standing waves are found on the strings of musical instruments and are due to reflections of waves from the ends of the string. shows seven standing waves that can be created on a string that is fixed at both ends. Nodes are the points where the string does not move; more generally, nodes are where the wave disturbance is zero in a standing wave. The fixed ends of strings must be nodes, too, because the string cannot move there. The word antinode is used to denote the location of maximum amplitude in standing waves. Standing waves on strings have a frequency that is related to the propagation speed v_w of the disturbance on the string. The wavelength λ is determined by the distance between the points where the string is fixed in place.

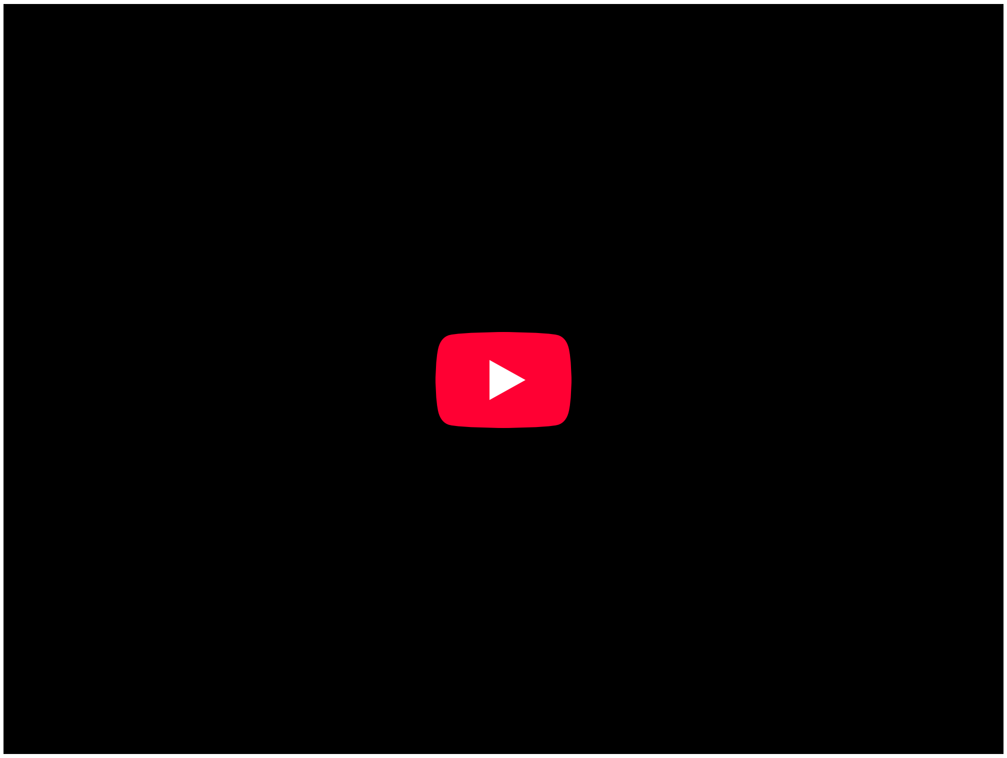


Standing Waves: Standing waves in a string, the fundamental mode and the first six overtones.

The lowest frequency, called the fundamental frequency, is thus for the longest wavelength, twice the length of the string. The overtones or harmonics are multiples of the fundamental frequency. shows the fundamental mode along with six overtones.

Resonance

A closer look at earthquakes provides evidence for conditions appropriate for resonance: standing waves, and constructive and destructive interference. A building may be vibrated for several seconds with a driving frequency matching that of the natural frequency of the vibration of the building—producing a resonance resulting in one building collapsing while neighboring buildings do not. Often buildings of a certain height are devastated while other taller buildings remain intact. The building height matches the condition for setting up a standing wave for that particular height. As the earthquake waves travel along the surface of Earth and reflect off denser rocks, constructive interference occurs at certain points. Often areas closer to the epicenter are not damaged while areas farther away are damaged.



Resonance: A brief overview of resonance, targeted toward introductory physics students.

Harmonic Wave Functions

When vibrations in the string are simple harmonic motion, waves are described by harmonic wave functions.

learning objectives

- Express relationship between the wave number and the wavelength, and frequency and period, of the harmonic wave function

In this Atom we shall consider wave motion resulting from harmonic vibrations and discuss harmonic transverse wave in the context of a string. We assume there is no loss of energy during transmission of wave along the string. This can be approximated when the string is light and taught. In such condition, if we oscillate the free end in harmonic manner, then the vibrations in the string are simple harmonic motion (SHM), perpendicular to the direction of wave motion. The amplitude of wave form remains intact through its passage along the string.

We know that a traveling wave function representing motion in x-direction has the form:

$$\mathrm{y(x,t) = Af(ax-bt).} \tag{5.8.6.11}$$

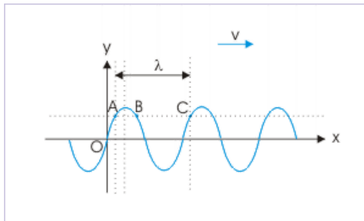
(Read our Atom on "Mathematical Representation of a Traveling Wave. ") For the case of harmonic vibration, we represent harmonic wave motion in terms of either harmonic sine or cosine function:

$$y(x, t) = A\sin(kx - \omega t). \tag{5.8.6.12}$$

Harmonic Oscillatory Properties

Each particle (or a small segment of string) vibrates in SHM. The particle attains the greatest speed at the mean position and reduces to zero at extreme positions. On the other hand, acceleration of the particle is greatest at extreme positions and zero at the mean position. The vibration of particle is represented by a harmonic sine or cosine function. For x=0:

$$y(x = 0, t) = A\sin(\omega t) = A\sin(\omega t). \tag{5.8.6.13}$$



Harmonic Waves: Harmonic waves are described by sinusoidal functions. The wavelength is equal to linear distance between repetitions of transverse disturbance or phase.

Clearly, the displacement in y-direction is described by the bounded sine or cosine function. The important point here is to realize that oscillatory attributes (like time period, angular and linear frequency) of wave motion is same as that of vibration of a particle in transverse direction.

We know that time period in SHM is equal to time taken by the particle to complete one oscillation. It means that displacement of the particle from the mean position at a given position such as x=0 has same value after time period "T" for:

$\omega T = 2\pi$. Therefore, $\omega = \frac{2\pi}{T}$.

Similarly, displacement of the particle from the mean position at a given time such as t=0 has same value by change the position by ", where $k\lambda = 2\pi$. k is called wavenumber.

We can determine speed of the wave by noting that wave travels a linear distance " in one period (T). Thus, speed of wave is given by:

$$v = \frac{\lambda}{T} = \frac{\omega}{k}. \tag{5.8.6.14}$$

Refraction

Refraction is a surface phenomenon that occurs as the change in direction of a wave due to a change in its medium.

learning objectives

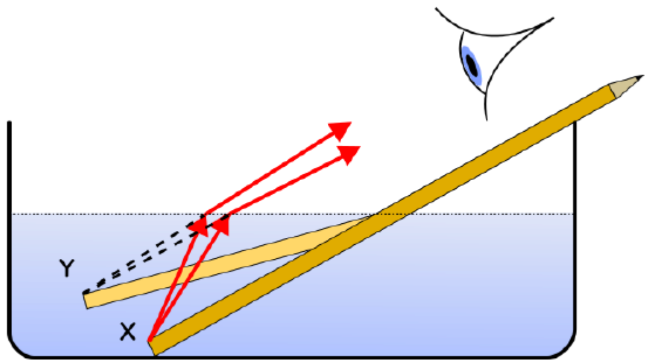
- Formulate the law of conservation of energy and momentum as it is applied to refraction

Refraction is the change in direction of a wave due to a change in its medium. Essentially, it is a surface phenomenon—mainly in governance to the law of conservation of energy and momentum. Due to change of medium, the phase velocity of the wave is changed but its frequency remains constant (most commonly observed when a wave passes from one medium to another at any angle other than 90° or 0°). Refraction of light is the most commonly observed phenomenon, but any type of wave can refract when it interacts with a medium (e.g., when sound waves pass from one medium into another or when water waves move into water of a different depth). Refraction is described by Snell's law, which states that for a given pair of media and a wave with a single frequency, the ratio of the sines of the angle of incidence θ1 and angle of refraction θ2 is equivalent to the ratio of phase velocities (v_1/v_2) in the two media, or equivalently, to the opposite ratio of the indices of refraction (n_2/n_1):

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1}. \tag{5.8.6.15}$$

In optics, refraction is a phenomenon that often occurs when waves travel from a medium with a given refractive index to a medium with another at an oblique angle. For example, a light ray will refract as it enters and leaves glass, assuming there is a change in refractive index. A ray traveling along the normal (perpendicular to the boundary) will change speed, but not direction. Refraction still occurs in this case. Understanding of refraction led to the invention of lenses and the refracting telescope.

Refraction can be seen when looking into a bowl of water, as illustrated in. Air has a refractive index of about 1.0003, and water has a refractive index of about 1.33. If a person looks at a straight object, such as a pencil or straw, placed partially in the water at a slant, the object appears to bend at the water's surface. This is due to the bending of light rays as they move from the water to the air. Once the rays reach the eye, the eye traces them back as straight lines (lines of sight). The lines of sight (shown as dashed lines) intersect at a higher position than where the actual rays originated (causing the pencil to appear higher and the water to appear shallower than they actually are).



Refraction in Water: An object (in this case a pencil) partially immersed in water looks bent due to refraction: the light waves from X change direction and so seem to originate at Y. (More accurately, for any angle of view, Y should be vertically above X, and the pencil should appear shorter, not longer as shown.)

Diffraction

Diffraction refers to various phenomena such as the bending of waves around obstacles and the spreading out of waves past small openings.

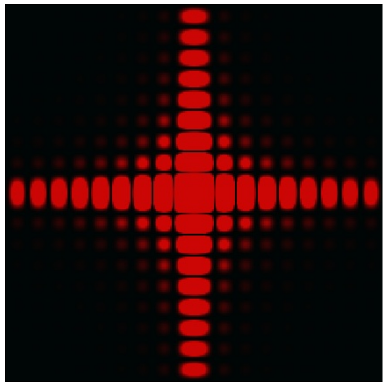
learning objectives

- Describe the phenomenon of diffraction, according to classical physics

Diffraction refers to various phenomena which occur when a wave encounters an obstacle. In classical physics, the diffraction phenomenon is described as the apparent bending of waves around small obstacles and the spreading out of waves past small openings. Similar effects occur when a light wave travels through a medium with a varying refractive index, or a sound wave travels through one with varying acoustic impedance.

Diffraction occurs with all waves, including sound waves, water waves, and electromagnetic waves such as visible light, X-rays and radio waves. As physical objects have wave-like properties (at the atomic level), diffraction also occurs with matter and can be studied according to the principles of quantum mechanics.

Diffraction effects are generally most pronounced for waves whose wavelengths are roughly similar to the dimensions of the diffracting objects. If the obstructing object provides multiple, closely-spaced openings, a complex pattern of varying intensity can result. This is due to the superposition, or interference, of different parts of a wave that travel to the observer by different paths. A good example would be diffraction gratings.



Intensity Pattern: Intensity pattern formed on a screen by diffraction from a square aperture.

The effects of diffraction are often seen in everyday life. The most striking examples of diffraction are those involving light. For example, the closely spaced tracks on a CD or DVD act as a diffraction grating to form the familiar rainbow pattern seen when looking at a disk. This principle can be extended to engineer a grating with a structure such that it will produce any diffraction pattern desired, like the hologram on a credit card. Diffraction in the atmosphere by small particles can cause a bright ring to be visible around a bright light source like the sun or the moon. A shadow of a solid object, using light from a compact source, shows small fringes near its edges. The speckle pattern which is observed when laser light falls on an optically rough surface is also a diffraction phenomenon. All these effects are a consequence of the fact that light propagates as a wave.

Mathematical Represenation of a Traveling Wave

The most general solution of the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ is given as $u(x,t) = f(x+ct) + g(x-ct)$, where f and g are arbitrary functions.

learning objectives

- Formulate solution of the wave equation for a traveling wave

In general, one dimensional waves satisfy the 1D wave equation:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}. \tag{5.8.6.16}$$

For example, a sinusoidal form

$$u(x,t) = A\sin(kx - \omega t) \tag{5.8.6.17}$$

is a solution of the wave equation for $c = \frac{\omega}{k}$. In this atom, we will obtain a general mathematical form of a traveling wave.

Solving the Wave Equation

First, we notice that any function u(x,t) satisfying

$$\frac{\partial u}{\partial t} = \pm c \frac{\partial u}{\partial x} (\text{Eq. } 1) \tag{5.8.6.18}$$

is a solution to the wave equation. To show this, note that

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\partial u}{\partial t}\right) = \pm c \frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}\right) \tag{5.8.6.19}$$

$$= \pm c \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial t}\right) = c^2 \frac{\partial^2 u}{\partial x^2}. \tag{5.8.6.20}$$

In the middle, we used the equation 1 along with the fact that partial derivatives are interchangeable.

To solve the equation 1, let's introduce new variables: $\phi = x - ct, \psi = x + ct.$ From the chain rules,

$$\frac{\partial}{\partial t} = \frac{\partial \phi}{\partial t}\frac{\partial}{\partial \phi} + \frac{\partial \psi}{\partial t}\frac{\partial}{\partial \psi} = -c\frac{\partial}{\partial \phi} + c\frac{\partial}{\partial \psi}. \tag{5.8.6.21}$$

$$\frac{\partial}{\partial x} = \frac{\partial \phi}{\partial x}\frac{\partial}{\partial \phi} + \frac{\partial \psi}{\partial x}\frac{\partial}{\partial \psi} = \frac{\partial}{\partial \phi} + \frac{\partial}{\partial \psi}. \tag{5.8.6.22}$$

With the change of variables, the equation 1 becomes $\frac{\partial u_+}{\partial \phi} = 0$ for the equation with the "+" sign and $\frac{\partial u_-}{\partial \psi} = 0$ for the "-" sign. Therefore, we see that

$$u_+(\phi, \psi) = f(\psi), u_-(\phi, \psi) = g(\phi), \tag{5.8.6.23}$$

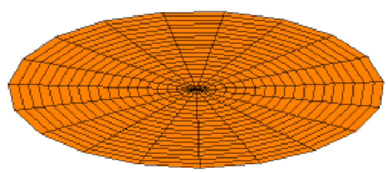
where f and g are arbitrary functions. Converting back to the original variables of x and t, we conclude that the solution of the original wave equation is

$$u(x, t) = f(x + ct) + g(x - ct). \tag{5.8.6.24}$$

f(x+ct) represents a left-going traveling wave, while g(x-ct) represents a right-going traveling wave. In other words, solutions of the 1D wave equation are sums of a left traveling function f and a right traveling function g. "Traveling" means that the shape of these individual arbitrary functions with respect to x stays constant, however the functions are translated left and right with time at the speed c. This solution was derived by Jean le Rond d'Alembert.

Boundary Condition

Any function that contains "x+ct" or "x-ct" can be a solution of the wave equation. The wave function is further determined by taking additional information, usually given as boundary conditions and some others. For example, in the case of a string in a guitar, we know that the wave has zero amplitude at both ends: u(x=0)=u(x=L)=0. Also, the shape of the function at an instance can be provided to determine the function.



Wave Equation in Two Dimensions: A solution of the wave equation in two dimensions with a zero-displacement boundary condition along the entire outer edge.

Energy, Intensity, Frequency, and Amplitude

The energy in a wave is proportional to its amplitude squared and the intensity of a wave is defined as power per unit area.

learning objectives

- Describe relationship between the energy and the amplitude, and energy and intensity, of a wave

All waves carry energy. This is seen in practical applications (e.g., in medicine), as well as effects in nature. Some examples of are:

- ultrasound used for deep-heat treatment of muscle strains
- a laser beam to burn away malignant tissue
- water waves that erode beaches
- earthquakes that topple cities

The amount of energy in a wave is related to its amplitude. Large-amplitude earthquakes produce large ground displacements, as seen in. Loud sounds have higher pressure amplitudes and come from larger-amplitude source vibrations than soft sounds. Large ocean breakers erode the shore more than small ones. More quantitatively, a wave is a displacement that is resisted by a restoring force. The larger the displacement x, the larger the force F=-kx needed to create it. Because work W is related to force multiplied by distance (Fx) and energy is put into the wave by the work done to create it, the energy in a wave is related to amplitude. In fact, a wave's energy is directly proportional to its amplitude squared because:

Earthquake Destruction: The destructive effect of an earthquake is palpable evidence of the energy carried in these waves. The Richter scale rating of earthquakes is related to both their amplitude and the energy they carry.

$$W = \int F(x)dx = \frac{1}{2}kx^2. \tag{5.8.6.25}$$

The energy effects of a wave depend on time as well as amplitude. For example, the longer deep-heat ultrasound is applied, the more energy it transfers. Therefore, power is more appropriate than energy to describe the “intensity” of a wave. Waves can also be concentrated or spread out. Sunlight, for example, can be focused to burn wood. Earthquakes “spread out” so they do less damage the farther they spread from their source. In both cases, changing the area the waves cover has important effects. All these pertinent factors are included in the definition of intensity I as power (P) per unit area:

$$I = \frac{P}{A}, \tag{5.8.6.26}$$

where P is the power carried by the wave through area A.

Energy vs. Frequency

In classic wave theory, energy of a wave doesn’t depend on the frequency of the wave. However, this is not the case in the microscopic world, as shown in experiments on photoelectric effects (see our Atom on “Photoelectric Effect”). As Einstein postulated to explain photoelectric effects, a quantum of light (photon) carries a specific amount of energy proportional to the frequency of light. Although you can increase the number of photons by increasing the intensity of a beam, the energy of individual photons in the beam is determined by the frequency of the beam.

Key Points

- Reflection is a wave phenomenon that changes the direction of a wavefront at an interface between two different media so that the wavefront returns into the medium from which it originated.
- At the boundary, a wave must be continuous and there should be no kinks in it.
- By imposing boundary conditions, we can solve wave equation and get the form of the waves. Reflection and transmission coefficients are defined as ratio of reflected/transmitted amplitudes and the incoming amplitude.
- The disturbances of waves are superimposed when they come together—a phenomenon called superposition.
- As a result of superposition of waves, inteference can be observed. Interference is an effect caused by two or more waves. Waves can interfere constructively or destructively.
- The superposition of most waves produces a combination of constructive and destructive interference and can vary from place to place and time to time.
- If two waves with the same amplitude and wavelength travel in opposite directions they alternate between constructive and destructive interference. The resultant looks like a wave standing in place and, thus, is called a standing wave.
- Nodes are points of no motion in standing waves. An antinode is the location of maximum amplitude of a standing wave.
- During an earthquake, buildings with a certain height may collapse more easily. This occurs when the building height matches the condition for setting up a standing wave for that particular height.
- We represent harmonic wave motion in terms of either harmonic sine or cosine function: $y(x,t) = A\sin(kx - \omega t)$.
- k and ω in the harmonic wave functinon are related to wavelength and period as follows: $k = \frac{2\pi}{\lambda}, \omega = \frac{2\pi}{T}$.
- The speed of a harmonic wave is given by ω/k.
- Refraction is mainly in governance to the law of conservation of energy and momentum. Due to change of medium, the phase velocity of the wave is changed but its frequency remains constant.

- Refraction is described by Snell's law, which states that for a given pair of media and a wave with a single frequency, $\frac{\sin\theta_1}{\sin\theta_2}=\frac{v_1}{v_2}=\frac{n_2}{n_1}$.
- An object partially immersed in water looks bent due to refraction.
- Diffraction is a wave phenomenon. It occurs with all waves, including sound waves, water waves, and electromagnetic waves such as visible light, X-rays and radio waves.
- Diffraction effects are generally most pronounced for waves whose wavelengths are roughly similar to the dimensions of the diffracting objects.
- The effects of diffraction are often seen in everyday life. All these effects are a consequence of the fact that light propagates as a wave.
- Any function u(x,t) satisfying $\frac{\partial u}{\partial t}=\pm c\frac{\partial u}{\partial x}$ is a solution the wave equation. To solve this new equation, we introduced new variables $\phi=x-ct, \psi=x+ct$.
- The solutions of the 1D wave equation are sums of a left traveling function and a right traveling function.
- The wave function is further determined by taking additional information, usually given as boundary conditions and some others.
- The energy effects of a wave depend on the amplitude and duration (time) of the wave. Waves can also be concentrated or spread out. Considering all these factors, intensity is defined as power per unit area.
- In the classical wave theory, energy of a wave doesn't depend on the frequency of the wave. However, the energy of individual photons in a beam is determined by the frequency of the beam.
- Wave's energy is directly proportional to its amplitude squared.

Key Terms

- **boundary condition**: A set of restraints at the boundaries, used to solve a differential equation.
- **superposition**: The summing of two or more field contributions occupying the same space.
- **interference**: An effect caused by the superposition of two systems of waves, such as a distortion on a broadcast signal due to atmospheric or other effects.
- **constructive interference**: Occurs when waves interfere with each other crest to crest and the waves are exactly in phase with each other.
- **destructive interference**: Occurs when waves interfere with each other crest to trough (peak to valley) and are exactly out of phase with each other.
- **resonance**: The increase in the amplitude of an oscillation of a system under the influence of a periodic force whose frequency is close to that of the system's natural frequency.
- **simple harmonic motion**: (SHM) — Oscillating motion (as of a pendulum) in which the acceleration of the oscillator has an equal magnitude but opposite direction to the displacement of it from the equilibrium position.
- **Snell's law**: A formula used to describe the relationship between the angles of incidence and refraction.
- **refractive index**: The ratio of the speed of light in air or vacuum to that in another medium.
- **wave equation**: An important second-order linear partial differential equation for the description of waves such as sound waves, light waves, and water waves.
- **Restoring force**: If the system is perturbed away from the equilibrium, the restoring force will tend to bring the system back toward equilibrium. The restoring force is a function only of position of the mass or particle. It is always directed back toward the equilibrium position of the system.An example is the action of a spring. An idealized spring exerts a force that is proportional to the amount of deformation of the spring from its equilibrium length, exerted in a direction to oppose the deformation. Pulling the spring to a greater length causes it to exert a force that brings the spring back toward its equilibrium length. The amount of force can be determined by multiplying the spring constant of the spring by the amount of stretch.
- **ultrasound**: Sound with a frequency greater than the upper limit of human hearing; approximately 20 kilohertz.
- **photoelectric effects**: In photoelectric effects, electrons are emitted from matter (metals and non-metallic solids, liquids or gases) as a consequence of their absorption of energy from electromagnetic radiation.

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5.8.7: Waves on Strings

learning objectives

- Calculate the speed of a wave on a string

When studying waves, it is helpful to use a string to observe the physical properties of waves visually. Imagine you are holding one end of a string, and the other end is secured and the string is pulled tight. Now, if you were to flick the string either up and down. The wave that occurs due to this motion is called a transverse wave. A transverse wave is defined as a wave where the movement of the particles of the medium is perpendicular to the direction of the propagation of the wave. Figure 1 shows this in a diagram. In this case, the medium through which the waves propagate is the rope. The wave traveled from one end to the other, while the rope moved up and down.

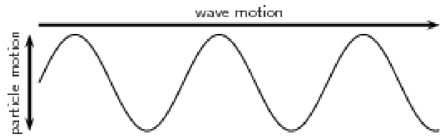


Figure 1: In transverse waves, the media the wave is traveling in moves perpendicular to the direction of the wave.

Wave Properties

Transverse waves have what are called peaks and troughs. The peak is the crest, or top point of the wave and the trough is the valley or bottom point of the wave. Refer to Figure 2 for a visual representation of these terms.The amplitude is the maximum displacement of a particle from its equilibrium position.Wavelength, usually denoted with a lambda (λ) and measured in meters, is the distance from either one peak to the next peak, or one trough to the next trough.Period, usually denoted as T and measured in seconds, is the time it takes for two successive peaks, or one wavelength, to pass through a fixed point.Frequency, *f*, is the number of wavelengths that pass through a given point in 1 second. Frequency is measured by taking the reciprocal of a period: $\mathrm{f}=\frac{1}{\mathrm{T}}$

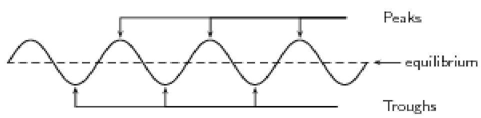


Figure 2: Peaks are the top most points of the waves and troughs are the bottom, or valleys of the waves.

Speed of a Wave on a String

Velocity is found by dividing the distance traveled by the time it took to travel that distance. In waves, this is found by dividing the wavelength by the period: v=λTv=λTWe can take the inverse proportionality to period and frequency and apply it to this situation:

$$\mathrm{v}=\frac{\lambda}{\mathrm{T}} \tag{5.8.7.1}$$

$$\mathrm{v}=\lambda \frac{1}{\mathrm{T}} \tag{5.8.7.2}$$

$$\mathrm{v}=\lambda \mathrm{f} \tag{5.8.7.3}$$

Speed of a Wave on a Vibrating String

Another example of waves on strings are of the waves on vibrating strings, such as in musical instruments. Pianos and guitars both use vibrating strings to produce music. In these cases, the frequency is what characterizes the pitch and therefore the note. The speed of a wave on this kind of string is proportional to the square root of the tension in the string and inversely proportional to the square root of the linear density of the string:$\mathrm{v}=\sqrt{\frac{\mathrm{T}}{\mu}}$

Reflections

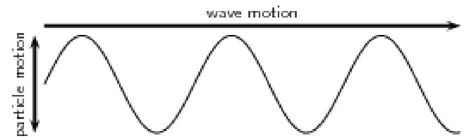
When transverse waves in strings meet one end, they are reflected, and when the incident wave meets the reflected wave, interference occurs.

learning objectives

- Explain when a standing wave occurs

Overview

Imagine you are holding one end of a string, and the other end is secured and the string is pulled tight. Now, if you were to flick the string either up and down. The wave that occurs due to this motion is called a transverse wave. A transverse wave is defined as a wave where the movement of the particles of the medium is perpendicular to the direction of the propagation of the wave. shows this in a diagram. In this case, the medium through which the waves propagate is the rope. The wave traveled from one end to the other, while the rope moved up and down.



Transverse Wave: Diagram of a transverse wave. The wave motion moves perpendicular to the medium it is traveling in.

Properties of Waves

- Transverse waves have what are called peaks and troughs. The peak is the crest, or top point of the wave and the trough is the valley or bottom point of the wave.
- The amplitude is the maximum displacement of a particle from its equilibrium position.
- Wavelength, usually denoted with a lambda (λ) and measured in meters, is the distance from either one peak to the next peak, or one trough to the next trough.
- Period, usually denoted as T and measured in seconds, is the time it takes for two successive peaks, or one wavelength, to pass through a fixed point.
- Frequency, f, is the number of wavelengths that pass through a given point in 1 second. Frequency is measured by taking the reciprocal of a period: $f = \frac{1}{T}$
- Transverse waves can occur while being fixed at the end point or while being free at the end point.

Reflections of Transverse Waves

The way in which a transverse wave reflects depends on whether or not it is fixed at both ends. First we will look at waves that are fixed at both ends:

shows an image of a transverse wave that is reflected from a fixed end. When a transverse wave meets a fixed end, the wave is reflected, but inverted. This swaps the peaks with the troughs and the troughs with the peaks.

Transverse Wave With a Fixed End Point: A transverse wave that is fixed at the end point. The reflected wave is inverted.

is an image of a transverse wave on a string that meets a free end. The wave is reflected, but unlike a transverse wave with a fixed end, it is not inverted.

Transverse Wave With a Free End: When a transverse wave meets a free end, it is reflected.

Standing Waves

When either of the two scenarios of wave reflection occurs, the incident wave meets the reflected wave. These waves move past each other in opposite directions, causing interference. When these two waves have the same frequency, the product of this is called the standing waves. Standing waves appear to be standing still, hence the name. To understand how standing waves occur, we can analyze them further: When the incident wave and reflected wave first meet, both waves have an amplitude is zero. As the waves continue to move past each other, they continue to interfere with each other either constructively of destructively.

As you may remember from previous atoms, when waves are completely in phase and interfere with each other constructively, they are amplified, and when they are completely out of phase and interfere destructively they cancel out. As the waves continue to move past each other, and are reflected from the opposite end, they continue to interfere both ways, and a standing wave is produced.

Every point in the medium containing a standing wave oscillates up and down and the amplitude of the oscillations depends on the location of the point. When we observe standing waves on strings, it looks like the wave is not moving and standing still. The principle of standing waves is the basis of resonance and how many musical instruments get their sound. The points in a standing wave that appear to remain flat and do not move are called nodes. The points which reach the maximum oscillation height are called antinodes.

Key Points

- The type of wave that occurs in a string is called a transverse wave. In a transverse wave, the wave direction is perpendicular the the direction that the string oscillates in.
- The period of a wave is indirectly proportional to the frequency of the wave: $\mathrm{T}=\frac{1}{\mathrm{f}}$.
- The speed of a wave is proportional to the wavelength and indirectly proportional to the period of the wave: $\mathrm{v}=\frac{\lambda}{\mathrm{T}}$.
- This equation can be simplified by using the relationship between frequency and period:$\mathrm{v}=\lambda \mathrm{f}$.
- When a transverse wave on a string is fixed at the end point, the reflected wave is inverted from the incident wave. When a transverse wave on a string is free at the end point, the reflected wave is not inverted from the incident wave.
- A standing wave occurs when an incident wave meets a reflected wave on a string.
- The points in a standing wave that appear to remain flat and do not move are called nodes. The points which reach the maximum oscillation height are called antinodes.
- Every point in the medium containing a standing wave oscillates up and down and the amplitude of the oscillations depends on the location of the point.
- A standing wave has some points that remain flat due to destructive interference. These are called antinodes.
- The points on a standing wave that have reached maximum oscillation do so from constructive interference, and are called nodes.

Key Terms

- **transverse wave**: Any wave in which the direction of disturbance is perpendicular to the direction of travel.
- **oscillate**: To swing back and forth, especially if with a regular rhythm.
- **amplitude**: The maximum absolute value of some quantity that varies.
- **standing wave**: A wave form which occurs in a limited, fixed medium in such a way that the reflected wave coincides with the produced wave. A common example is the vibration of the strings on a musical stringed instrument.
- **transverse wave**: Any wave in which the direction of disturbance is perpendicular to the direction of travel.

CHAPTER OVERVIEW

5.9: Sound

Topic hierarchy

5.9.1: Introduction

learning objectives

- Explain how humans can characterize sound

Before delving too far into the physics of calculating sound waves (studying things like the Doppler Effect, for example), lets address some basics of sound. Sound is a wave—a longitudinal wave of pressure that travels through compressible medias (i.e., solid, liquid, gaseous, or made of plasma). There is no sound in a vacuum; by definition, a vacuum is a space free of any particles or matter. Thus there in a vacuum, there is no media through which sound waves can travel. Following are some characteristics of sound:

- Sound travels in longitudinal waves. When drawn these are also called sinusoidal waves, a visual example of which is shown in (we will cover this in more detail in a different section).
- Sound waves have frequency; that is, the pitch of sounds goes up or down.
- The amplitude of a sound determines its volume (loudness).
- Tone is a measure of the quality of a sound wave.
- Sound travels faster in a hot meduim, or in a solid. It also travels faster at sea level (where air pressure is higher).
- Sound intensity is the energy transmitted over a certain area. Intensity is a measure of the sound's frequency.
- Ultrasound uses sound waves with high frequencies to see things normally hard to detect, like tumors. Animals, like bats and dolphins, use ultrasound (echolocation) to navigate and locate things. Ships also use a similar technique (known as SONAR) to locate things underwater. (This point will be discussed further in a more advanced Atom.)

Sound Perception

Every sound wave has properties that define its frequency, such as wavelength, amplitude and intensity. Calculating these properties is outside the scope of this atom and will be addressed later. For now, it is important to know the basics of sound. As with light waves, sound frequencies have a range. Each living creature has a different level of sound perception. For example, consider the following examples of sound ranges (in Hz, Hertz):

- Humans 20 – 20,000 Hz
- Dogs 50 – 45,000 Hz
- Bats 20 – 120,000 Hz

By this comparison, humans have a relatively low sound perception.

Sound Speed

Breaking the Sound Barrier: This familiar image is of a plane that is moving faster than the speed of sound.

As mentioned previously, the speed at which sound travels depends on the media through which the sound is traveling. It is much faster in a solid than in a liquid or gas. The general formula for calculating the speed of sound is given as:

$$\mathrm{c}=\sqrt{\frac{\mathrm{K}}{\rho}}, \tag{5.9.1.1}$$

where K is the coefficient of stiffness of the material (also called the Bulk modulus) and p is the density of the material. We will examine this further in another section. Generally, the expression 'faster than the speed of sound' refers to 344 m/s. is an image demonstrating a plane moving faster than the speed of sound. This general measurement is taken at sea level—at a temperature of 21 degrees Celsius under normal atmospheric conditions.

Frequency of Sound Waves

Frequency is the number of occurrences of a repeating event per unit of time. The perception of frequency is called pitch.

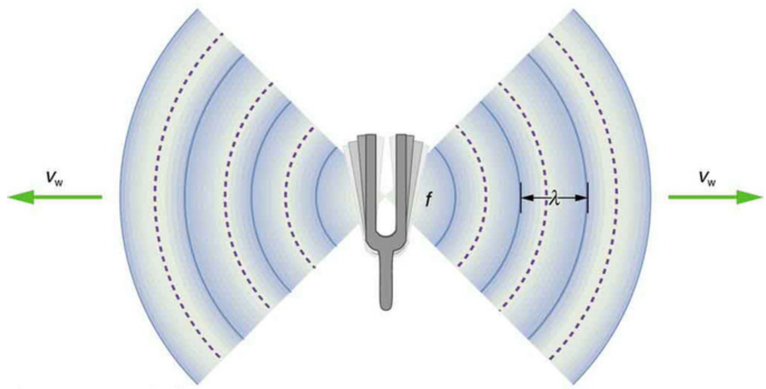
learning objectives

- Relate frequency with the wavelength and speed of sound

Sound waves, like all other waves, have a property called frequency. Frequency is the number of occurrences of a repeating event per unit of time. The perception of frequency is called pitch.

Frequency is dependent on wavelength and the speed of sound. It is calculated with the following equation: $f = \frac{v_s}{\lambda}$

This figure,, shows how the frequency is connected to wavelength.



Frequency: A sound wave emanates from a source vibrating at a frequency f, propagates at v, and has a wavelength λ.

Alternatively, you can use the frequency and the wavelength to find the speed of sound in a specific medium. Remember that sound travels at different speeds in different mediums; sound moves fastest through a solid. The following equation is used to find the specific speed of sound, and is often easier to use than the standard speed of sound equation: *vs=f∗λv_s=f*\lambda*

A period is the duration of one cycle of a repeating event, and is the reciprocal or inverse of the frequency. The following animation shows different frequencies and their periods, from lowest to highest.

Frequency Animation: Three flashing lights, from lowest frequency (top) to highest frequency (bottom). *f* is the frequency in hertz (*Hz*); or the number of cycles per second. *T* is the period in seconds (*s*); or the number of seconds per cycle. *T* and *f* are reciprocals.

Hertz

The SI unit of frequency is called a Hertz, denoted Hz. A hertz is defined as the number of cycles per second. For example, 100 Hz signifies 100 cycles per second.

Different species can hear different frequency ranges. Humans can only hear from 20 Hz to 20,000 Hz, while dogs can hear up to 60,000 Hz. Bats can hear the highest ranges, up to 120,000 Hz. Bats use this super hearing, or ultrasound, to locate objects and prey. By bouncing sound waves off of another object and hearing how long it takes for the sound to echo back to them, they are able to approximate the distance between themselves and the object. This is called echolocation.

Sound Production: Vibrating String and Air Columns

Sound can be produced by many different devices. A vibrating string or air column can both create music and have unique properties.

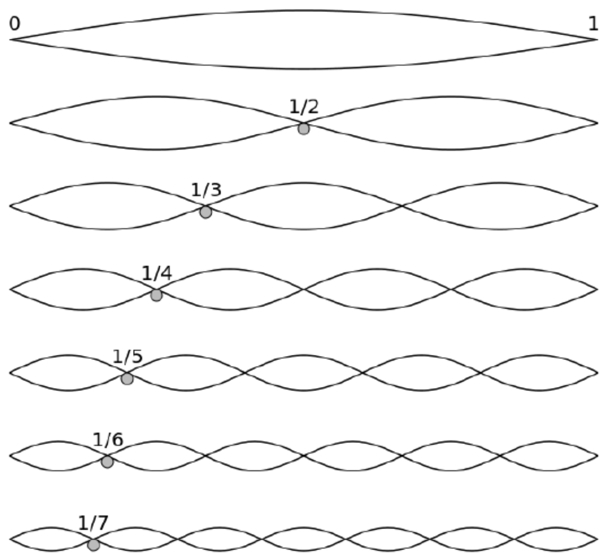
learning objectives

- Calculate the frequency of the sound wave produced by the string and a column of air

Sound can be produced in many ways, by many different instruments and devices. A vibrating string or air column can both make interesting music and it is good to understand the physics behind both.

Vibrating Strings

There are many instruments that produce sound based on strings. Guitars, cellos, pianos and many other examples. These sounds are produced by standing waves in the strings. These waves and their frequencies are constant, and therefor the sound and pitch produced by them is constant. This figure shows a visual of a standing wave in a string: The speed of the wave is proportional to the root of the string tension, and inversely proportional to the root of the string density, shown by the following equation $v=\sqrt{\frac{T}{\mu}}$ Pitch, and the way the sound is perceived depends on the frequency of the sound wave. Using the wave velocity, you can find the frequency using this equation: $f=\frac{v}{2L}$, where L is the length of the string.

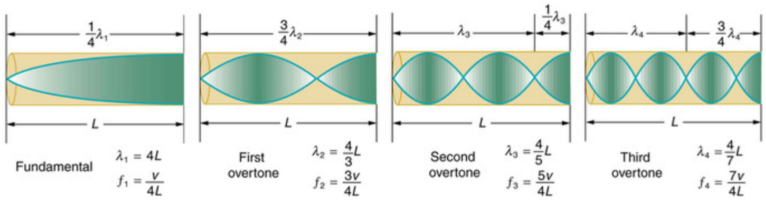


Standing Waves in a String: Vibration, standing waves in a string. The fundamental and the first 6 overtones form a harmonic series.

Air Columns

Many instruments make music by causing a vibration to a column of air in a resonator. These are usually called wind instruments. Examples of these are flutes, recorders, and saxophones. They are either open at both ends, or closed at one end and open at the other. The pitch of the sound is a function of the frequency. There are many factors that go into finding the frequency. Lets start with the tubes that are closed at one end and open at the other.

Closed Air Tubes:The maximum displacement of the air occurs at the open end of the tube, and is called the antinode. The air movement is constrained at the closed end, and there is no displacement, and this is called the node. The distance from the node to the antinode is 1/4 the length of one wavelength, and equal to the length of the tube, as shown in this equation:$\lambda = 4\mathrm{L}$This can also be seen in this figure: The frequency is equal to the speed of sound in the air divided by the wavelength, or:$\mathrm{f} = \frac{\mathrm{v_w}}{\lambda} = \frac{\mathrm{v_w}}{4\mathrm{L}}$, where $\mathrm{v_w}$ is the speed of sound in the air, which we learned how to find in a previous atom.



An Air column in a tube closed at one end: The fundamental and three lowest overtones for a tube closed at one end. All have maximum air displacements at the open end and none at the closed end.

Open Air tubes:Air tubes can also be open at both ends. They are very similar to the ones we talked about above, but there is an antinode at both ends, since they are both open, as shown in this figure: Since there is an antinode at both ends, we can see that the length of a wavelength is found by this equation: $\lambda = 2\mathrm{L}$ And the frequency can be found in the following equation:$\mathrm{f} = \frac{\mathrm{v_w}}{\lambda} = \frac{\mathrm{v_w}}{2\mathrm{L}}$

Quality of Sound

Sound quality is an assessment of accuracy or enjoyability of how a sound is perceived.

Skills to Develop

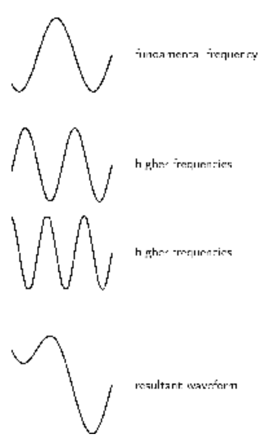
- Identify factors that influence the sound quality of a reproduction of music

Sound quality is an assessment of accuracy or enjoyability of how a sound is perceived. It can be measured objectively or subjectively. What is the difference? Objective measurement is taken when tools are used to gauge accuracy. Subjective measurement is more of an opinion. When human listeners hear a sound and compare it to another sound they have heard, and decide which one they enjoy more, this is a subjective measurement of sound quality.

Music Sound Quality

The quality of sound of live music is referred to as its tone. What makes this tone? Why does the same note being played on a piano sound different from that note played on a flute, or a guitar? When you hear a note, you mostly hear the fundamental frequency. But there are other harmonics present. You hear these, but they are much, much fainter than the fundamental, or main frequency. These are called *overtones*.

Each instrument has a different mix of these overtones, and this is why the sounds of similar notes sound different. The quality of this sound depends on these overtones to not take over what you are hearing. The overtones and fundamental frequency combine to make a unique frequency that you hear, which is illustrated in.



Sound Frequencies: The quality of a tone depends on its mixture of harmonics.

Reproduced Sound Quality

The sound quality of a reproduction of music depends on many factors. These include:

- Recording equipment
- Processing and mastering
- Reproduction equipment
- Listening environment

When you listen to a recording of music, you are hearing not only the music itself, but also any interference that may have been mistakenly recorded as well. The simplest form of digitally-stored audio is PCM.

As samples are placed closer together in time, higher frequencies can be reproduced. According to the sampling theorem, any signal with bandwidth *B* can be perfectly described by more than *2B* samples per second. Audio must be sampled at above 40kHz: 44.1kHz for CD recordings and 48kHz for DVD recordings.

The amount of space required to store PCM depends on the number of bits per sample, the number of samples per second, and the number of channels. For CD audio, this is 44,100 samples per second, 16 bits per sample, and 2 channels for stereo audio leading to 1,411,200 bits per second.

Speed of Sound

The speed of sound is is the distance traveled in a unit of time by a sound wave through an elastic medium, and is usually given as 344 m/s.

Skills to Develop

- Calculate the speed of sound from the properties of the media

Speed of Soud

The speed of sound is is the distance traveled in a unit of time by a sound wave through an elastic medium. This medium can be a solid, liquid, gas or even plasma. The speed of sound is dependent on the properties of the media the sound is travelling through. The general value given for the speed of sound is the speed of a sound wave in air, at sea level, at normal atmospheric pressure; that number is 344 m/s. However, this number is not constant. Sound travels faster in a solid than in a liquid, and faster in a liquid that in a gas.

Types of Sound Waves: Compression and Shear

There are two different kinds of sound waves: *compression waves* and *shear waves*. Compression waves can travel through any media, but shear waves can only travel through solids. The speed of a compression wave is determined by the media's compression capacity, shear modulus, and density, while the speed of the shear wave is only determined by the shear modulus and density. The shear modulus is a measurement of the elasticity or rigidity of a material. Calculating this is outside of the scope of this atom, but there are tables which tell you its value for materials.

Calculating the Speed of Sound

The speed of sound is usually denoted by cc, and a general equation can be used to calculate it. This is called the Newton-Laplace equation:c=√Kρc=KρK is the coefficient of stiffness, and p is the density of the media. From this equation, it is easy to see that the speed of sound will increase with stiffness and decrease with density. This is a very general equation, there are more specific derivations, for example:

The speed of sound in air at sea level is given by the following equation: $\mathrm{c_{air}} = 331\,\frac{\mathrm{m}}{\mathrm{s}} \times \sqrt{\frac{\mathrm{T}}{273\mathrm{K}}}$ T is the temperature in Kelvin.

Mach Number

You may have heard the term *Mach number* in relation to speed of space craft or jets before. This is a ratio of an object's speed in relation to the speed of sound. The Mach number is given by the following, dimensionless equation: $\mathrm{M} = \frac{\mathrm{v}}{\mathrm{a}}$ – Mach numberv – Velocity of object a – Speed of sound in medium. If something is travelling at the speed of sound, that would make the equation equal to 1, and can be denoted as Mach 1. shows a jet that is travelling at the speed of sound or faster. The *vapor cone* is made just before it reaches the speed of sound and is caused by a sudden drop in air pressure.

Faster than the Speed of Sound: This is a jet that is just about to break the sound barrier.

Key Points

- Sound travels in longitudinal sinusoidal waves.
- Humans can characterize sound by frequency, amplitude and tone.
- The speed at which sound travels depends on the media through which the sound is traveling. It can be calculated using: $c = \sqrt{\frac{K}{\rho}}$, where K is the coefficient of stiffness of the material (also called the Bulk modulus) and p is the density of the material.
- Frequency is dependent on wavelength and the speed of sound. It is calculated with the following equation: $f = \frac{v_s}{\lambda}$
- A period is the duration of one cycle of a repeating event, and is the reciprocal or inverse of the frequency.
- The SI unit of frequency is called a Hertz, denoted Hz. A hertz is defined as the number of cycles per second. For example, 100 Hz signifies 100 cycles per second.
- The frequency, and therefore the pitch of a string instrument depends on the velocity of the sound wave and the length of the string. The frequency is found by: $f = \frac{\sqrt{\frac{T}{\mu}}}{2L}$.
- Sound in a tube of air that is closed at one end is found by the following equation: $f = \frac{v_w}{\lambda} = \frac{v_w}{4L}$.
- Sound in a tube of air that is open at one end is found by the following equation: $f = \frac{v_w}{\lambda} = \frac{v_w}{2L}$.
- Objective measurement is taken when tools are used to gauge accuracy. Subjective measurement is more of an opinion. When human listeners hear a sound and compare it to another sound they have heard, and decide which one they enjoy more, this is a subjective measurement of sound quality.
- The quality of sound of live music is referred to as its tone. When you hear a note, you mostly hear the fundamental frequency. But there are other harmonics present. You hear these, but they are much, much fainter than the fundamental, or main frequency. These are called overtones.
- The sound quality of a reproduction of music depends on many factors. These include: recording equipment, processing and mastering,reproduction equipment, and even listening environment.
- Sound can travel through any compressible material. These media can be solid, liquid, gas, or even plasma.
- The speed of sound is dependent on the properties of the material it travels through. It will travel faster through a solid than a liquid, and faster through a liquid than a gas.
- The general number given for the speed of sound is calculated at sea level, in air, at normal atmospheric pressure. That value is 344 m/s.

Key Terms

- **frequency**: The quotient of the number of times n a periodic phenomenon occurs over the time t in which it occurs: $f = \frac{n}{t}$.
- **Hertz**: Measurement of sound frequency.
- **media**: General term for different types of materials.
- **period**: The duration of one cycle in a repeating event.
- **node**: Point on a wave where there is no displacement.
- **antinode**: A region of maximum amplitude situated between adjacent nodes of a vibrating body, such as a string
- **Subjective measurement**: Based on a comparison to a previous experience, opinion.
- **Objective measurement**: Taken by tools to gauge accuracy.
- **elasticity**: The property by virtue of which a material deformed under the load can regain its original dimensions when unloaded
- **kelvin**: in the International System of Units, the base unit of thermodynamic temperature; 1/273.16 of the thermodynamic temperature of the triple point of water; symbolized as K

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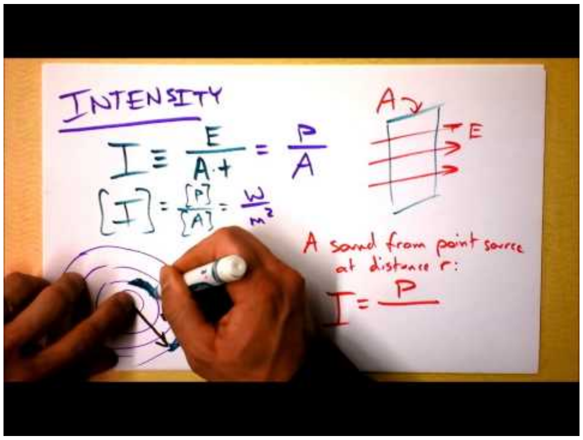
5.9.2: Sound Intensity and Level

learning objectives

- Calculate sound intensity from power

Overview of Intensity

Sound Intensity is the power per unit area carried by a wave. Power is the rate that energy is transferred by a wave.

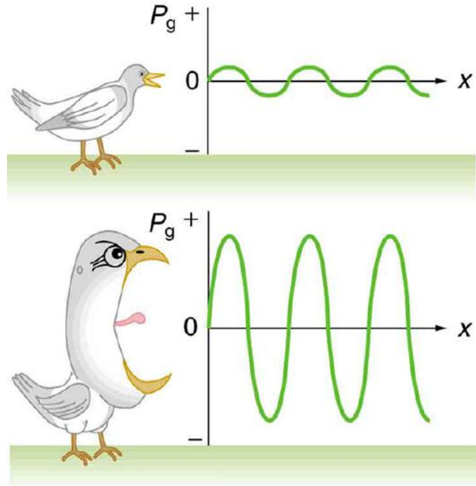


Sound Intensity and Decibels

The equation used to calculate this intensity, I, is:$II = \frac{P}{A}$ Where P is the power going through the area, A. The SI unit for intensity is watts per meter squared or,Wm2Wm2. This is the general intensity formula, but lets look at it from a sound perspective.

Sound Intensity

Sound intensity can be found from the following equation: $I = \frac{\Delta p^2}{2\rho v_w}$. Δp – change in pressure, or amplitudeρ – density of the material the sound is traveling throughv_w – speed of observed sound.Now we have a way to calculate the sound intensity, so lets talk about observed intensity. The pressure variation, amplitude, is proportional to the intensity, So it is safe to say that the larger your sound wave oscillation, the more intense your sound will be. This figure shows this concept.



Sound Intensity: Graphs of the gauge pressures in two sound waves of different intensities. The more intense sound is produced by a source that has larger-amplitude oscillations and has greater pressure maxima and minima. Because pressures are higher in the greater-intensity sound, it can exert larger forces on the objects it encounters

Although the units for sound intensity are technically watts per meter squared, it is much more common for it to be referred to as decibels, dB. A decibel is a ratio of the observed amplitude, or intensity level to a reference, which is 0 dB. The equation for this is: \(\mathrm{β=10 \log _{10} \frac{I}{I_0}*β*}\) – decibel levelI – Observed intensityI_0– Reference intensity.For more on decibels, please refer to the Decibel Atom.

For a reference point on intensity levels, below are a list of a few different intensities:

- 0 dB, I = $1×10^{-12}$ –> Threshold of human hearing
- 10 dB, I = $1×10^{-11}$ –> Rustle of leaves
- 60 dB, I = $1×10^{-6}$ –> Normal conversation
- 100 dB, I = $1×10^{-2}$ –> Loud siren
- 160 dB, I = $1×10^{4}$–> You just burst your eardrums

Human Perception of Sound

The study of human perception of sound is called psychoacoustics.

Skills to Develop

- Explain how frequency is perceived by humans

The study of the human perception of sound is called psychoacoustics. Many factors go into hearing, including wave properties, sensory and brain processes. First, the wave has to be made, and it has a specific wavelength and frequency. Then the sound wave reaches the human ear, and is processed through many areas. Finally, the sound wave makes it through the ear and to the human brain, where even more action happens. You might think that when something makes a noise that you hear it instantaneously but, in reality, it goes through many steps first.

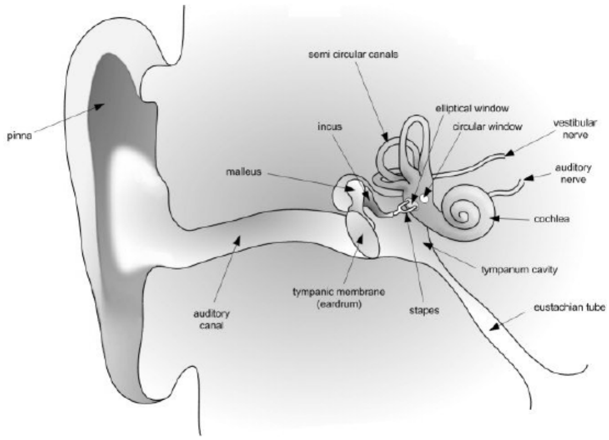
Wave Properties

We are not going to go into too much detail about the wave's physical properties, since it is out of the scope of this atom, but remember:

- Frequency is perceived by humans as pitch;
- The sound intensity is the amplitude;
- Humans can only hear a specific range of sound, usually from 20 Hz to 20,000 Hz;
- The factors that go into a sound are its intensity, frequency and overtones (which are like interference, or background noises).

The Human Ear

The human ear is made up of three main sections, as shown in:



The Human Ear: A detailed diagram of the human ear.

1. The outer ear
2. The middle ear
3. The inner ear

We are going to start where a sound wave would start and follow it on its journey from outside your ear all the way to your brain. When you look at someone’s ear, you are really only seeing the pinna, the outer most portion of the ear. It collects and focuses the sound wave. The wave then goes through your ear canal to the eardrum. The sound waves cause the eardrum to vibrate. Then we are in the middle ear, which has three very, very small bones: the malleus, incus and stapes. These can also be referred to as the hammer, anvil and stirrup, respectively. These three bones transmit the signal to the elliptical window. This is the beginning of the inner ear. The sound waves are then transmitted from the elliptical window through the inner ear’s semicircular canals, the cochlea, and the audio nerve, which is filled with fluid. This fluid is what allows the body to detect movements and maintain balance. Your cochlea is shaped like a snail, and is full of teeny tiny hairs. These hairs vibrate differently depending on the frequencies. These vibrations release electrical impulses to the auditory nerve and are then sent to your brain, where they are understood as sound. So while this seems to happen very quickly, sound waves have to travel a long way before you ever hear anything!

Decibels

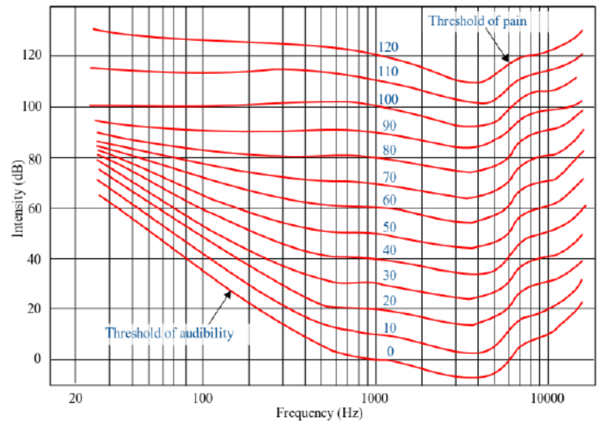
The decibel is a logarithmic unit used to quantify sound levels, by comparing a physical quantity to a reference level.

Skills to Develop

- Identify how decibel is quantified in acoustics

The decibel, dB, is commonly used to quantify sound levels, although it is not a unit of sound, but a unit of pressure. The decibel is a logarithmic unit that indicates the ratio if a physical quantity to a reference level. It is one tenth of a Bel, which was named after the inventor of the telephone, Alexander Graham Bell. The word decibell comes from the prefix, deci, that is 1/10 of the word it precedes For more information on how to convert units, refer to the unit conversion atom. Although the decibel can be used to talk about a number of different subjects, in this atom we are going to cover its use in acoustics and sound level.

In acoustics, the decibel is quantified relative to a reference which has been set at a sound pressure level of 20 micropascals, and is called a 0 dB. This reference level is a typical threshold of human hearing perception. The following equation is used to calculate the sound pressure level, or amplitude: $\mathrm{amplitude[dB]} = 20 \log_{10} \frac{\mathrm{s}}{\mathrm{s_o}}$ $\mathrm{s_o}$ is the reference pressure which is 20 micropascals or 0 dB, and s is the observed sound pressure.The human ear has a standard sound threshold of 120 dB, which expressed logarithmically is around 10^{12}. This is a standard threshold, but it also depends on frequency. Loudness is a measure of sound intensity taking frequency into account, and is called a A-weighted decibel, dB(A), or a phon. This figure shows The Fletcher Munson Chart, which demonstrates the different sound frequencies and decibels that the human ear perceives as the same.



The Fletcher Munson Chart: The Fletcher-Munson equal-loudness contours. Phons are labelled in blue

Key Points

- Sound intensity can be found from the following equation: $\mathrm{I}=\frac{\Delta \mathrm{p}^2}{2 \rho \mathrm{v_w}}$. Δ p – change in pressure, or amplitude ρ – density of the material the sound is traveling through v_w– speed of observed sound.
- The larger your sound wave oscillation, the more intense your sound will be.
- Although the units for sound intensity are technically watts per meter squared, it is much more common for it to be referred to as decibels, dB.
- Frequency is perceived by humans as pitch. The sound intensity is what humans can hear, and is generally only a specific range of sound, usually from 20 Hz to 20,000 Hz. The factors that go into a sound are its intensity, frequency and overtones (which are like interference or background noises).
- Your ear is made up of three major sections: the inner, middle and outer ear.
- Your cochlea, which is in your inner ear, not only transmits sound waves to your brain, but also contains a liquid that helps humans maintain their balance.
- In acoustics, the decibel is quantified relative to a reference which has been set at a sound pressure level of 20 micropascals, and is called a 0 dB.
- The following equation is used to calculate the sound pressure level, or amplitude: $\mathrm{amplitude}[\mathrm{dB}]=20\log_{10}\frac{\mathrm{s}}{\mathrm{s_o}}$. s_o is the reference pressure which is 20 micropascals or 0 dB, and s is the observed sound pressure.
- The human ear has a standard sound threshold of 120 dB, which expressed logarithmically is around 1012. This is a standard threshold, but it also depends on frequency. Loudness is a measure of sound intensity taking frequency into account, and is called a A-weighted decibel, dB(A), or a phon.

Key Terms

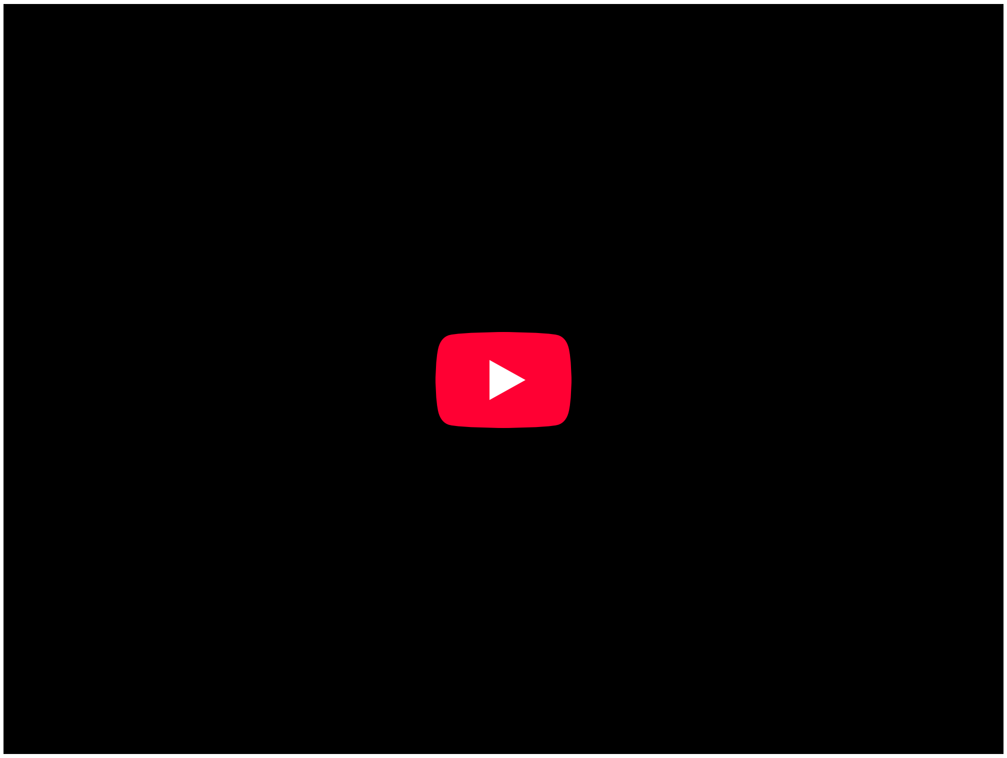
- **decibel**: A common measure of sound intensity that is one tenth of a bel on the logarithmic intensity scale. It is defined as $\mathrm{dB}=10\log_{10}(\mathrm{P_1}/\mathrm{P_2})$, where $\mathrm{P_1}$ and $\mathrm{P_2}$ are the relative powers of the sound.
- **amplitude**: The maximum absolute value of some quantity that varies.
- **eardrum**: A thin membrane that separates the outer ear from the middle ear and transmits sound from the air to the malleus.
- **cochlea**: The complex, spirally coiled, tapered cavity of the inner ear in which sound vibrations are converted into nerve impulses.
- **phon**: A unit of apparent loudness, equal in number to the intensity in decibels of a 1,000-hertz tone judged to be as loud as the sound being measured.

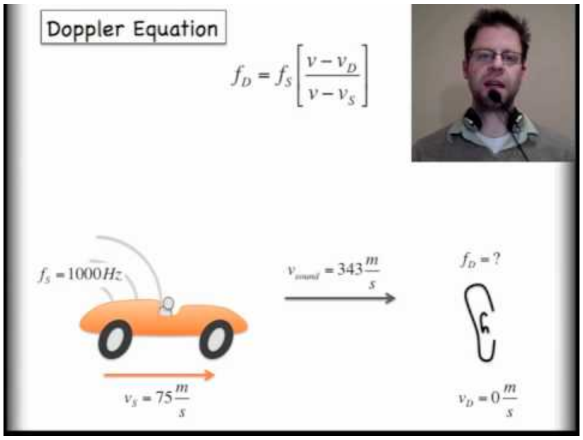
5.9.3: Doppler Effect and Sonic Booms

learning objectives

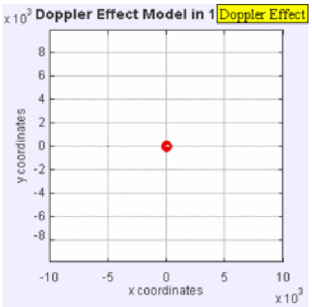
- Identify parameters required to calculate the frequency perceived by the observer moving towards the sound source

In this atom, we are going to cover the Doppler effect, but specifically when the observer is the one in motion.





Sound and the Doppler Effect: This video introduces sound waves. The first video describes the basics of sound while the second video looks at the Doppler Effect.



The Doppler Effect: The same sound source is radiating sound waves at a constant frequency in the same medium. However, now the sound source is moving to the right with a speed υs = 0.7 c (Mach 0.7). The wave-fronts are produced with the same frequency as before. However, since the source is moving, the centre of each new wavefront is now slightly displaced to the right. As a result, the wave-fronts begin to bunch up on the right side (in front of) and spread further apart on the left side (behind) of the source.

When the observer moves toward an sound source, each successive wave is encountered sooner than the previous wave. Thus, it will take just a little less time for the observer to hear the next one. Since the time between waves is reduced, the frequency is increased. Similarly if the observer is moving away from the sound source, the frequency, and therefor pitch, is decreased. While the frequency will change whether the observer or sound source is moving, it is easier to show with the sound source as the one moving. This figure demonstrated the sound source moving:

Unless the observer is moving directly towards the sound source, this angle needs to be taken into account when calculating the newly perceived frequency. Before we can start this calculation, we must know:

- The original sound wave frequency, f_0
- The velocity of the observer, v_r
- The speed of sound in the air, or medium, c
- The angle of the line of sight from the observer to the sound source, θθ

Although the sound waves are being emitted from the sound source at a uniform frequency, the observer is perceiving them differently. The equation for the perceived wave frequency is as follows: $\mathrm{f}=\frac{\mathrm{c} \pm \mathrm{v}_{\mathrm{r}, \text { new }}}{\mathrm{c}} \times \mathrm{f}_0$ And the equation for v_{r}, is: $\mathrm{v}_{\mathrm{r}, \text { new }}=\mathrm{v}_{\mathrm{r}} \cos \theta$ If the observer is moving towards the sound source, you are going to use a plus sign in front of the observers velocity. If the observer is moving away from sound source, you are going to use a negative sign in front of the observers velocity.

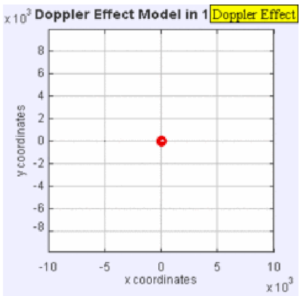
Moving Source

The Doppler effect is the apparent change in frequency of a wave when the observer and the source of the wave move relative to each other.

learning objectives

- Identify parameters required to calculate the perceived frequency of the sound source moving towards an observer

The Doppler Effect —When the Sound Source is in Motion



The Doppler Effect: The same sound source is radiating sound waves at a constant frequency in the same medium. However, now the sound source is moving to the right with a speed υs = 0.7 c (Mach 0.7). The wave-fronts are produced with the same frequency as before. However, since the source is moving, the centre of each new wavefront is now slightly displaced to the right. As a result, the wave-fronts begin to bunch up on the right side (in front of) and spread further apart on the left side (behind) of the source.

When the sound source moves toward an observer, each successive wave is emitted closer to the observer than the previous wave and takes just a little less time to reach the observer than the previous one. Since the time between waves is reduced, the frequency is increased. Similarly, if the sound source is moving away from the observer, the frequency (and therefore pitch) is decreased. While the frequency will change whether the observer or sound source is moving, the effect is more easily demonstrated by the sound source. This Doppler Effect is illustrated in.

Unless the observer is moving directly towards the sound source, this angle must be considered when calculating the newly perceived frequency. Before attempting this calculation, we must know:

- The original sound wave frequency, f_0
- The velocity of the observer, v_r
- The speed of sound in the air, or medium, c
- The angle of the line of sight from the observer to the sound source, θ

Although the sound waves are being emitted from the sound source at a uniform frequency, the observer is perceiving them differently. The equation for the perceived wave frequency is as follows: $f = \frac{c}{c \pm v_{s,new}} * f_0$ And the equation for v_s, is:

$$v_{s,new} = v_s \cos\theta \tag{5.9.3.1}$$

If the sound source is moving towards the observer, a plus sign is used in front of the sound source's velocity. If the sound source is moving away from the observer, then a negative sign is used in front of the sound source's velocity.

General Case

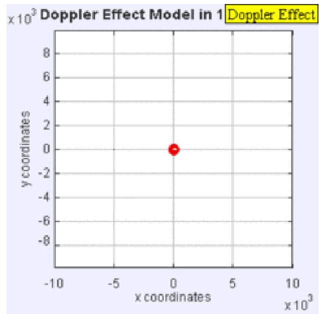
The Doppler effect is the apparent change in frequency of a wave when the observer and the source of the wave move relative to each other.

learning objectives

- Compare change in the frequency due to the doppler effect when the object in motion moves towards or away from the other object

Have you ever wondered why as a firetruck speeds by, the sound of the siren seems to change? Or where the very distinguishable sound at a race track comes from as the cars whisk by you? Well believe it or not, this is science! The name of this is called the Doppler effect, named for the scientist who discovered it. His name was Christian Doppler, and he discovered it around 1840 in Prague. He preformed the experiment in two segments. In the first experiment, he had a band play music while aboard a moving train, and would have observers listen to the music in a stationary location while the train went by. In the second experiment, he had a group of observers sit aboard a moving train, while the band played in a stationary location. In both cases, observers noticed the phenomenon. The change in sound perception can be explained through relativity.

When the sound source moves toward an observer, each successive wave is emitted closer to the observer than the previous wave. Thus, it will take just a little less time to reach the observer than the previous one. Since the time between waves is reduced, the frequency is increased. Similarly if the sound source is moving away from the observer, the frequency, and therefor pitch, is decreased. If the observer is moving away from the sound source, the frequency will be lowered, and if the observer moves closer to the sound source, the frequency is increased. While the frequency will change whether the observer or sound source is moving, it is easier to show with the sound source as the one moving. This figure demonstrated the sound source moving:



The Doppler Effect: The same sound source is radiating sound waves at a constant frequency in the same medium. However, now the sound source is moving to the right with a speed υs = 0.7 c (Mach 0.7). The wave-fronts are produced with the same frequency as before. However, since the source is moving, the centre of each new wavefront is now slightly displaced to the right. As a result, the wave-fronts begin to bunch up on the right side (in front of) and spread further apart on the left side (behind) of the source.

In classical physics, where the speeds of source and the receiver relative to the medium are lower than the velocity of waves in the medium, the relationship between observed frequency *(f)* and emitted frequency (f_o) is given by: $\mathrm{f} = \frac{(\mathrm{c+vr})}{(\mathrm{c+vs})} \times \mathrm{f}_0$ where c – velocity of the sound waves in the medium, v_r– velocity of the observer or reciever, v_s – velocity of the sound source, and f_0 – original frequency of the sound waves.

The above formula assumes that the source is either directly approaching or receding from the observer. If the source approaches the observer at an angle (but still with a constant velocity), the observed frequency that is first heard is higher than the object's emitted frequency. Thereafter, there is a monotonic decrease in the observed frequency as it gets closer to the observer, through equality when it is closest to the observer, and a continued monotonic decrease as it recedes from the observer. When the observer is very close to the path of the object, the transition from high to low frequency is very abrupt. When the observer is far from the path of the object, the transition from high to low frequency is gradual.

If the speeds and are small compared to the speed of the wave, the relationship between observed frequency and emitted frequency is approximately

Observed frequency: $\mathrm{f} = (1 + \frac{\Delta \mathrm{v}}{\mathrm{c}})\mathrm{f}_0$

Change in frequency: $\Delta \mathrm{f} = \frac{\Delta \mathrm{v}}{\mathrm{c}}\mathrm{f}_0$

where $\Delta \mathrm{v}$ is the velocity of the receiver relative to the source: it is positive when the source and the receiver are moving towards each other.

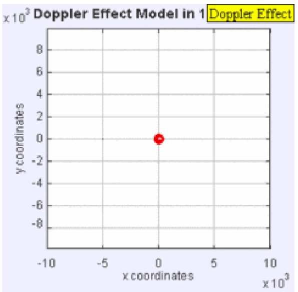
Sonic Booms

A sonic boom is the sound associated with the shock waves created by an object traveling through the air faster than the speed of sound.

> **learning objectives**
>
> - Identify conditions that lead to a sonic boom and discuss its properties

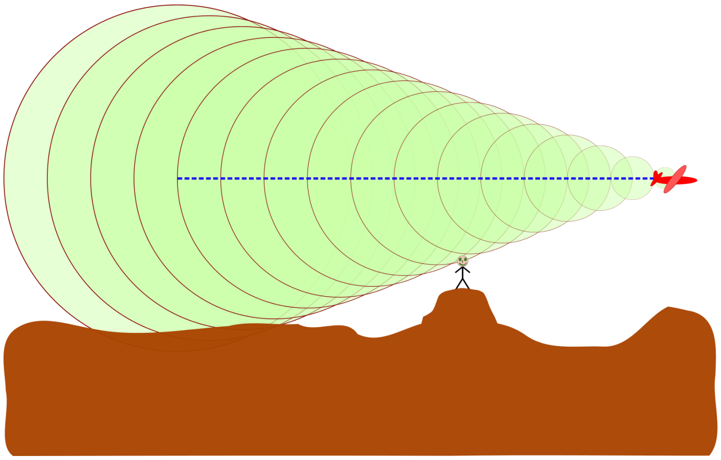
Sonic Booms

A sonic boom is a sound associated with the shock waves created by an object traveling through the air faster than the speed of sound. It can be viewed as a Doppler effect on steroids; sonic booms generate an enormous amount of energy and sound like explosions. The first man-made object to ever create this phenomenon was a bullwhip. The 'crack' of the whip is a result of this sonic boom. This version of a Doppler effect is demonstrated by.



Sonic Boom Gif: The sound source has now broken through the sound speed barrier, and is traveling at 1.4 times the speed of sound, (Mach 1.4). Since the source is moving faster (with a speed) than the sound waves it creates, it actually leads the advancing wavefront. The sound source will pass by a stationary observer (with a speed) before the observer actually hears the sound it creates.

When the sound source passes through the air, it creates a series of pressure waves. These waves are travelling at the speed of sound, and as the speed of the sound source increases, the waves, not being able to get out of each other's way, are forced together. They eventually merge into a single shock wave traveling at the speed of sound. This is a critical speed, known as Mach. The shock waves radiate out from the sound source, and create a "Mach cone'. The half-angle, αα, can be found using the equation



Sonic Boom: A sonic boom produced by an aircraft moving at M=2.92, calculated from the cone angle of 20 degrees. An observer hears the boom when the shock wave, on the edges of the cone, crosses his or her location

$$\sin \alpha = \frac{\mathrm{v_s}}{\mathrm{v_r}}. \tag{5.9.3.2}$$

From previous atoms, we know that $\frac{\mathrm{v_s}}{\mathrm{v_r}}$ is the sound source's Mach number.

At the front of the sound source, there is a sudden rise in pressure, while at the end of the source there is a decreasing pressure. This 'overpressure profile' is known as an N-wave. There is a big boom when there is a sudden change in pressure, and since the pressure changes twice, this is a double boom.

Key Points

- When the object in motion moves towards the other, the frequency is increased because the time between successive sound waves is shortened. Therefore the pitch is higher.
- When the object in motion moves away from the other, the frequency is decreased because the time between successive sound waves is lengthened. Therefor the pitch is lowered.
- Unless the objects are in each other's direct path, you need to account for the angle they are at relative to each other. The following equation needs to be substituted for the 'movers' velocity. The angle used needs to be the angle from the line of sight of the observer to the sound source. vradial=v∗cosθvradial=v∗cosθ.
- When the object in motion moves towards the other, the frequency is increased because the time between successive sound waves is shortened (therefore the pitch is higher).

- When the object in motion moves away from the other, the frequency is decreased because the time between successive sound waves is lengthened (therefore the pitch is lowered).
- Unless the objects are in each other's direct path, you need to account for their angle relative to each other. The following equation must be substituted for the 'movers' velocity. The angle used must be the angle from the line of sight of the observer to the sound source.
- When the object in motion moves towards the other, the frequency is increased because the time between successive sound waves is shortened. Therefore the pitch is higher.
- When the object in motion moves away from the other, the frequency is decreased because the time between successive sound waves is lengthened. Therefor the pitch is lowered.
- If the speeds and are small compared to the speed of the wave, the relationship between observed frequency and emitted frequency is approximately: $\mathrm{f} = (1 + \frac{\Delta \mathrm{v}}{\mathrm{c}})\mathrm{f}_0$ where $\Delta \mathrm{v}$ is the velocity of the receiver relative to the source: it is positive when the source and the receiver are moving towards each other.
- A sonic boom happens when a sound source passes an observer at a speed either very close to, or faster than the speed of sound.
- Sonic booms generate an enormous amount of energy, and sound like explosions.
- The half angle of the cone of sound waves produced by the sonic boom can be found by taking the inverse sine of the object's Mach number.

Key Terms

- **Doppler's effect**: Apparent change in frequency of a wave when the observer and the source of the wave move relative to each other.
- **classical physics**: All aspects of physics developed before the rise of quantum mechanics.
- **frequency**: The quotient of the number of times n a periodic phenomenon occurs over the time t in which it occurs: $\mathrm{f} = \mathrm{n}/\mathrm{t}$.
- **Mach number**: The ratio of the velocity of a body to that of sound in the surrounding medium.
- **sonic boom**: The audible effect of a shock wave in the air, especially one caused by an aircraft flying faster than the speed of sound

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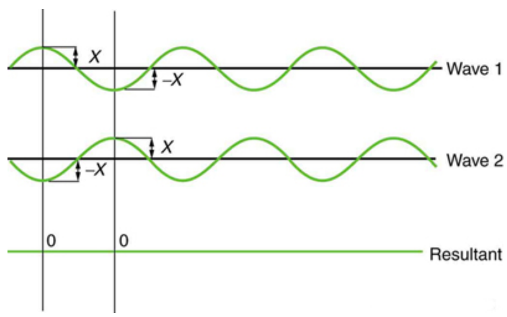
5.9.4: Interactions with Sound Waves

learning objectives

- Identify conditions required for the superposition of two waves

Waves are most commonly described by variations in some parameter through space and time—height in a water wave, pressure in a sound wave, or the electromagnetic field in a light wave. The value of this parameter is called the amplitude of the wave; the wave itself is a function specifying the amplitude at each point.

When two or more waves arrive at the same point, they superimpose themselves onto one another. More specifically, the disturbances of waves are superimposed when they come together (a phenomenon called superposition). Each disturbance corresponds to a force, or amplitude (and the forces add). If the disturbances are along the same line, the resulting wave is a simple addition of the disturbances of the individual waves. That is, their amplitudes add.

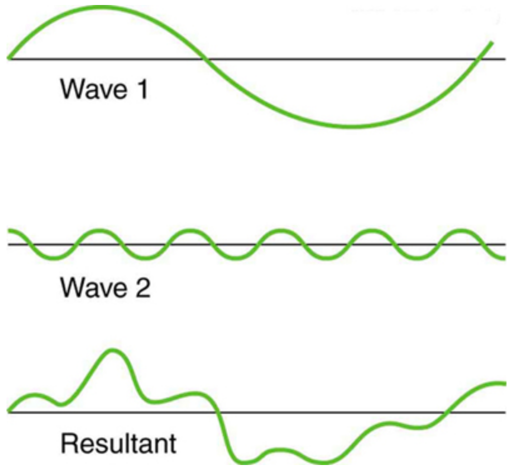


Superposition: Superposition is when two waves add together. In this figure, the two waves add together and cancel out leaving no wave. This is destructive interference.

Superposition of waves leads to what is known as interference, which manifests in two types: constructive and destructive. Constructive interference occurs when two waves add together in superposition, creating a wave with cumulatively higher amplitude, as shown in. In destructive interference, the two waves add together but cancel out (like adding a positive and negative number). Destructive interference is shown in.

While pure constructive and pure destructive interference do occur, they require precisely aligned identical waves. The superposition of most waves produces a combination of constructive and destructive interference, and can vary from place to place and time to time. Sound from a stereo, for example, can be loud in one spot but quiet in another. Varying loudness means the sound waves add partially constructively and partially destructively at different locations. A stereo has at least two speakers creating sound waves, and waves can reflect from walls. All these waves superimpose. An example of sounds that vary over time from constructive to destructive is found in the combined whine of airplane jets heard by a stationary passenger. The combined sound can fluctuate up and down in volume as the sound from the two engines varies in time from constructive to destructive.

These examples are of waves that are similar. illustrates that when non-identical waves superimpose, the outcome is a mixture of constructive and destructive interference.



Superposition of Non-Identical Waves: Superposition of non-identical waves exhibits both constructive and destructive interference.

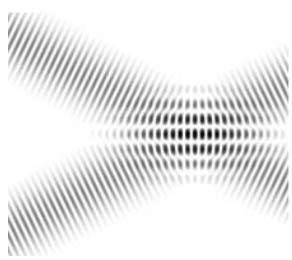
Interference

Interference occurs when multiple waves interact with each other, and is a change in amplitude caused by several waves meeting.

> **learning objectives**
>
> - Contrast constructive and destructive interference

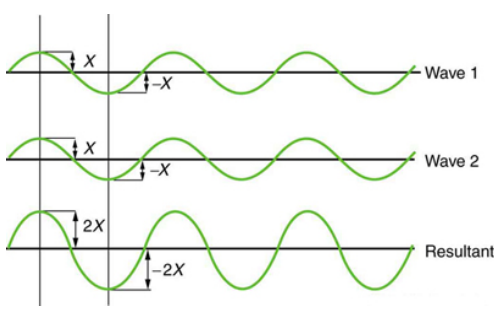
Unlike solid objects, two waves can share a point in space. In physics, interference is a phenomenon in which two waves (passing through the same point) superimpose to form a resultant wave of greater or lower amplitude. Interference usually refers to the interaction of waves that are correlated or coherent with each other (i.e, "interfere" with each other), either because they come from the same source or because they have the same or nearly the same frequency.

The effects of interference can be observed with all types of waves, for example, light, radio, acoustic and surface water waves. The idea that interference is caused by superposition means that when two waves meet their two amplitudes (their maximum absolute value) combine together.



Interference: Two overlapping waves exhibit interference.

Interference can be constructive or destructive. In constructive interference, the two amplitudes of the waves add together and result in a higher displacement than would have been the case if there were only one wave. An example of constructive interference may be seen in.



Constructive Interference: Pure constructive interference of two identical waves produces one with twice the amplitude, but the same wavelength.

Destructive interference is when two waves add together and the result is a smaller displacement than would have been the case. An example of destructive interference can be seen in. When the waves have opposite amplitudes at the point they meet they can destructively interfere, resulting in no amplitude at that point. For example, this is how noise cancelling headphones work. By playing a sound with the opposite amplitude as the incoming sound, the two sound waves destructively interfere and this cancel each other out.

Beats

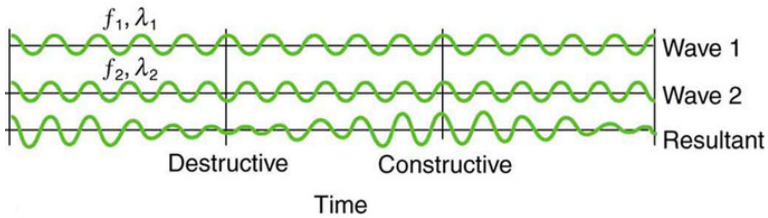
The superposition of two waves of similar but not identical frequencies produces a pulsing known as a beat.

learning objectives

- Identify superposition conditions that lead to beat

Striking two adjacent keys on a piano produces a warbling combination (usually considered unpleasant to the ear). The culprit is the superposition of two waves of similar but not identical frequencies. When two waves of similar frequency arrive at the same point and superimpose, they alternately constructively and destructively interfere. This alternating is known as a beat because it produces an unpleasant pulsing sound.

Another example is often noticeable in a taxiing jet aircraft (particularly the two-engine variety). The loudness of the combined sound of the engines increases and decreases. This varying loudness occurs because the sound waves have similar but not identical frequencies. The discordant warbling of the piano and the fluctuating loudness of the jet engine noise are both due to alternately constructive and destructive interference as the two waves go in and out of phase. illustrates this phenomenon graphically.



Beat Frequency: Beats are produced by the superposition of two waves of slightly different frequencies but identical amplitudes.The waves alternate in time between constructive interference and destructive interference, giving the resulting wave a time-varying amplitude.

The wave resulting from the superposition of two similar-frequency waves has a frequency that is the average of the two. This wave fluctuates in amplitude, or beats, with a frequency called the beat frequency. We can determine the beat frequency mathematically by adding two waves together.

One can also measure the beat frequency directly. When you hear a beat coming from two discordant sounds (say, two notes on a piano) you can count the number of beats per second. The number of beats per second, or the beat frequency, shows the difference in frequency between the two notes. Musicians often use this phenomena to ensure that two notes are in tune (if they are in tune then there are no beats).

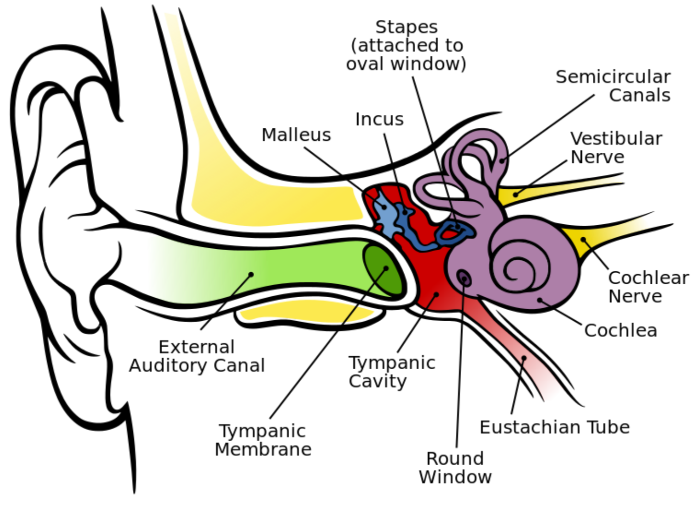
The Ear

The ear is the sensory organ that picks up sound waves from the air and turns them into nerve impulses that can be sent to the brain.

learning objectives

- Describe how sound waves are collected and transformed into nerve impulses

Sound waves are vibrations in the air. The ear is the sensory organ that picks up sound waves from the surrounding air and turns them into nerve impulses, which are then sent to the brain. The sound waves carry a lot of information — language, music, and noise — all mixed together. The task of the ear is to turn the signals in these waves of bouncing air molecules into electrical nerve signals while keeping as much of the information in the signal as possible. (It's the brain's job to then sort the signals and make sense of them.) It's not easy to turn one kind of signal into another without losing information, but the ear is well designed for the task.



Anatomy of the Human Ear: Anatomy of the human ear; the length of the auditory canal is exaggerated for viewing purposes

Air surrounds the head and fills the ear canal and middle ear. Therefore, when the outer part of the ear collects sound and the middle ear amplifies this sound pressure, these processes occur in the medium of air. However, the hollow channels of the inner ear (which is embedded in the temporal bone, the densest bone of the body) are filled with liquid. So as the sound travels into the inner ear, it passes from the medium of air into a liquid medium. These inner-ear channels contain a sensory epithelium that is studded with hair cells. The microscopic "hairs" are structural protein filaments that project out into the fluid. The hair cells release a chemical neurotransmitter when stimulated. Sound waves moving through fluid push the filaments; if the filaments bend over enough, the hair cells fire chemical signals. In this way sound waves are transformed into nerve impulses. The nerve impulses travel from the left and right ears through the eighth cranial nerve to both sides of the brain stem and up to the part of the cerebral cortex dedicated to sound (auditory cortex, located in the temporal lobe).

Applications: Ultrasound, Sonar, and Medical Imaging

Sound waves reflect off different materials differently (when the reflections are collected, they can provide information and images).

learning objectives

- Discuss application of sound waves in medicine and navigation

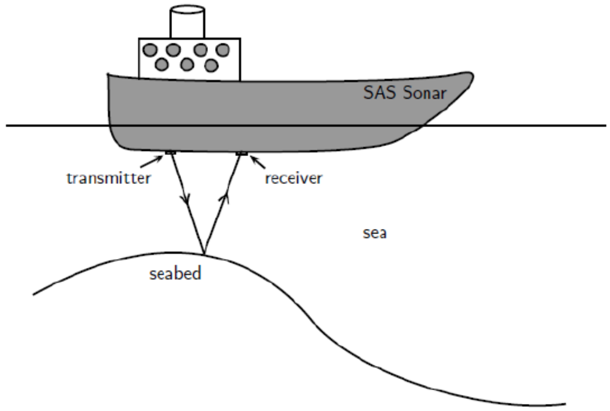
The controlled use of sound waves has many applications in science. Controlled reflection of such waves allows images to be received.

Ultrasound

Ultrasound is sound with a frequency higher than 20 kHz. This is above the human range of hearing. The most common use of ultrasound, creating images, has industrial and medical applications. The use of ultrasound to create images is based on the reflection and transmission of a wave at a boundary. When an ultrasound wave travels inside an object that is made up of different materials (such as the human body), each time it encounters a boundary (e.g., between bone and muscle, or muscle and fat), part of the wave is reflected and part of it is transmitted. The reflected rays are detected and used to construct an image of the object.

Sonar

illustrates how a ship on the ocean utilizes the reflecting properties of sound waves to determine the depth of the ocean. A sound wave is transmitted and bounces off the seabed. Because the speed of sound is known and the time lapse between sending and receiving the sound can be measured, the distance from the ship to the bottom of the ocean can be determined. This technique is called *sonar* (originally an acronym for SOund Navigation And Ranging).



Sonar: Ships on the ocean make use of the reflecting properties of sound waves to determine the depth of the ocean. A sound wave is transmitted and bounces off the seabed. Because the speed of sound is known and the time lapse between sending and receiving the sound can be measured, the distance from the ship to the bottom of the ocean can be determined.

Just as ships on the ocean, certain animals, like dolphins and bats, make use of sounds waves (sonar) to navigate or find their way. Ultrasound waves are sent out then reflected off the objects around the animal. Bats or dolphins then use the reflected sounds to form a "picture" of their surroundings (this is known as echolocation).

Key Points

- When two waves occupy the same point, superposition occurs. Superposition results in adding the two waves together.
- Constructive interference is when two waves superimpose and the resulting wave has a higher amplitude than the previous waves.
- Destructive interference is when two waves superimpose and cancel each other out, leading to a lower amplitude.
- Most wave superpositions involve a mixture of constructive and destructive interference since the waves are not perfectly identical.
- Interference is a phenomenon of wave interactions. When two waves meet at a point, they interfere with each other.
- There are two types of interference, constructive and destructive.
- In constructive interference, the amplitudes of the two waves add together resulting in a higher wave at the point they meet.
- In destructive interference, the two waves cancel out resulting in a lower amplitude at the point they meet.
- When two waves of similar frequencies interfere, the result is a beat frequency.
- A beat frequency is a pulsing sound that goes up and down in loudness.
- As the two waves go in and out of phase, the varying constructive and destructive interference makes the wave grow and shrink in amplitude. For sound waves this produces a beating sound.
- The task of the ear is to turn the signals in the waves of bouncing air molecules into electrical nerve signals while keeping as much of the information in the signals as possible.
- Sound is collected in the outer part of the ear; sound pressure is amplified through the middle part of the ear and is passed from the medium of air into a liquid medium.
- That sound pressure is amplified through the middle portion of the ear and passed from the medium of air into a liquid medium.
- Sound waves moving through the fluid in the inner ear stimulate hair cells, making them release chemical neurotransmitters. In this way sound waves are transformed into nerve impulses.
- When waves encounter a boundary between two materials, part of the wave is reflected and part is transmitted.
- By using high frequency sound waves, doctors can create images of parts of the body normally not visible.
- By transmitting sound waves and measuring the time between the transmission and receiving the reflection, ships can use sound waves to navigate. This is called sonar.

Key Terms

- **superimpose**: To place an object over another object.
- **displacement**: A vector quantity that denotes distance with a directional component.
- **amplitude**: The maximum absolute value of some quantity that varies.
- **coherent**: Of waves having the same direction, wavelength and phase, as light in a laser.

- **frequency**: The quotient of the number of times n a periodic phenomenon occurs over the time tt in which it occurs: \(\mathrm{f=n/t\).
- **interfere**: (of waves) To be correlated with each other when overlapped or superposed.
- **superposition**: The summing of two or more field contributions occupying the same space.
- **epithelium**: a membranous tissue composed of one or more layers of cells that forms the covering of most internal and external surfaces of the body and its organs (internally, the lining of vessels and other small cavities; externally, the skin)
- **nerve impulse**: the signal transmitted along a nerve fiber, either in response to a stimulus (such as touch, pain, or heat), or as an instruction (such as causing a muscle to contract)
- **neurotransmitter**: any substance, such as acetylcholine or dopamine, responsible for sending nerve signals across a synapse between two neurons

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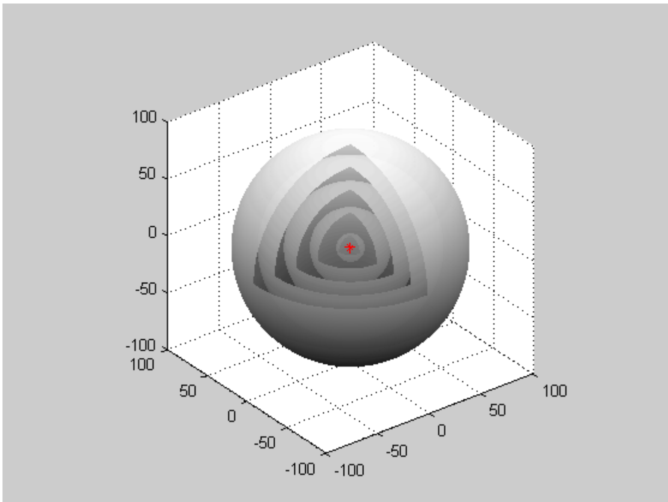
5.9.5: Further Topics

learning objectives

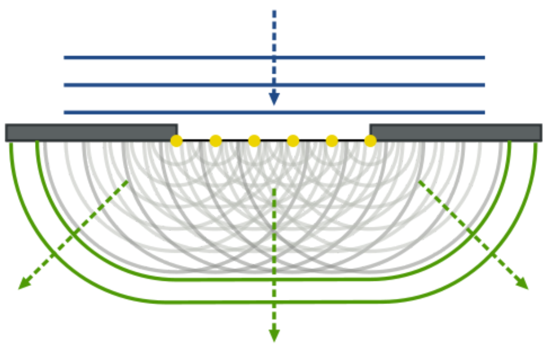
- Identify sources of spherical and plane wave patterns

Spherical Waves

Christiaan Huygens was a Dutch scientist who developed a useful technique for determining how and where waves propagate. In 1678, he proposed that every point that a luminous disturbance touches becomes itself a source of a spherical wave; the sum of these secondary waves determines the form of the wave at any subsequent time. The Huygen-Fresnel Principle shows that as the waves interact with each other, they interfere either constructively or destructively. Constructive interference occurs when waves are completely in phase with each other and amplifies the waves. Destructive interference occurs when waves are exactly out of phase with either other, and if waves are perfectly out of phase with each other, the wave will be canceled out completely. Since the waves all come from one point source, the waves happen in a spherical pattern. All the waves come from a single point source and are spherical.



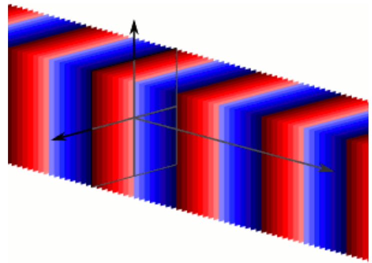
Spherical Wave: *When waves are produced from a point source, they are spherical waves.*



Huygen-Fresnel Principle: The Huygen-Fresnel Principle uses the law of refraction. Each point on the wave produces waves which interfere with each other either constructively or destructively.

Plane Waves

A plane wave is a constant-frequency wave whose wavefronts (surfaces of constant phase) are infinite parallel planes of constant peak-to-peak amplitude normal to the phase velocity vector. It is not possible in practice to have a true plane wave; only a plane wave of infinite extent will propagate as a plane wave. However, many waves are approximately plane waves in a localized region of space. For example, a localized source such as an antenna produces a field that is approximately a plane wave far from the antenna in its far-field region. Similarly, if the length scales are much longer than the wave's wavelength, as is often the case for light in the field of optics, one can treat the waves as light rays which correspond locally to plane waves.



Plane Wave*: Plane waves are an infinite number of wavefronts normal to the direction of the propogation.*

Standing Waves on a String

Standing wave occurs due to the interference when transverse waves in strings are reflected and the incident and reflected waves meet.

learning objectives

- Identify when a standing wave occurs

A standing wave is a wave that appears stationary, meaning it remains in a constant position. In a string, a standing wave is a type of transverse wave—where the movement of the particles of the medium is perpendicular to the direction of the propagation of the wave. A standing wave can occur when two identical waves moving in different directions along the string interfere.

There are two scenarios of waves in strings: the string is fixed at both ends, or the string is fixed at one end and free at the other. A transverse wave will move along the string until it reaches the other end. It is then reflected from that end and starts to move back towards the original direction; at this point interference occurs. shows a transverse wave that is reflected from a fixed end. When a transverse wave meets a fixed end, the wave is reflected, but inverted. This swaps the peaks with the troughs and the troughs with the peaks. diagrams a transverse wave on a string that meets a free end. The wave is reflected, but unlike a transverse wave with a fixed end, it is not inverted.

Free End Reflection: The wave is reflected, but unlike a transverse wave with a fixed end, it is not inverted.

Fixed End Reflection: When a transverse wave meets a fixed end, the wave is reflected, but inverted.

Standing Waves

When either of the two scenarios of wave reflection occurs, the incident wave meets the reflected wave. These waves move past each other in opposite directions, causing interference. When these two waves have the same frequency, the product of this is called the standing waves. Standing waves appear to be standing still, hence the name. illustrates a very slow moving standing wave. (One application of the principle of standing waves is in music with the concept of resonance—and how many musical instruments, like guitars and pianos, get their sound.) Let us now examine how standing waves occur.

Standing Wave on a String: This is what a standing wave would look like if you were to slow it down. The wave is caused by an incident wave on a string being reflected and then traveling back in the direction it came from. The two waves then meet and interfere with each other causing this phenomenon.

Constructive vs. Destructive Interference

When the incident wave and reflected wave first meet, both waves have an amplitude is zero. As the waves continue to move past each other they continue to interfere with each other, either constructively of destructively. As discussed in previous atoms, when waves are completely in phase and interfere with each other constructively they are amplified, and when they are completely out of phase and interfere destructively they cancel out. As the waves continue to move past each other, and are reflected from the opposite end, they continue to interfere both ways; a standing wave is produced. Every point in the medium containing a standing wave oscillates up and down, and the amplitude of the oscillations depends on the location of the point. When we observe a standing wave on strings, it appears the wave is not moving but standing still. In summary:

- The points which reach the maximum oscillation height are called antinodes, and are results of complete constructive interference.
- The points in a standing wave that appear to remain flat and do not move are called nodes. These are due to complete destructive interference.

Standing Waves in Air Columns

Standing waves in air columns is the physical phenomenon that gives wind instruments their resonance and, therefore, sound.

learning objectives

- Identify the type of a standing wave in an air column

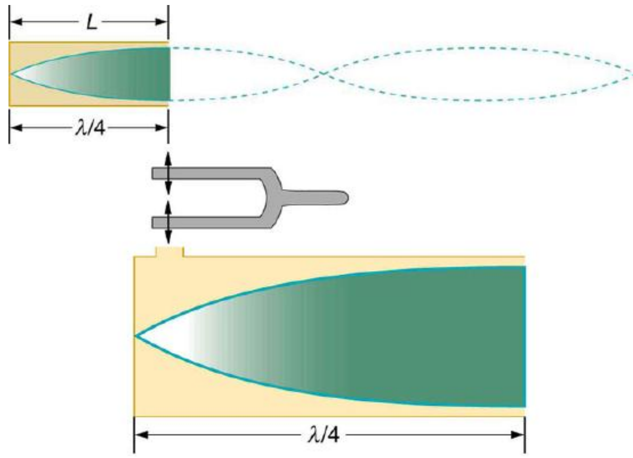
Standing Waves

A standing wave is a wave that appears to be stationary, meaning it remains in a constant position. In an air column, a standing wave can form as either a longitudinal or transverse wave. A transverse wave, you may remember, is a wave where the movement of the particles of the medium is perpendicular to the direction of the propagation of the wave. A longitudinal wave, on the other hand, is parallel to the direction of propagation. A standing wave can occur when two identical waves moving in different directions interfere.

Air Columns

When a standing wave is formed in a tube, the standing wave has a maximum air displacement at the open end called an antinode. Here, the motion is unconstrained. At the closed end, there is no displacement; this is called a node, and the air is halted. The distance from a node to antinode is 1/4 of a wavelength, and is equal to the length of the tube.

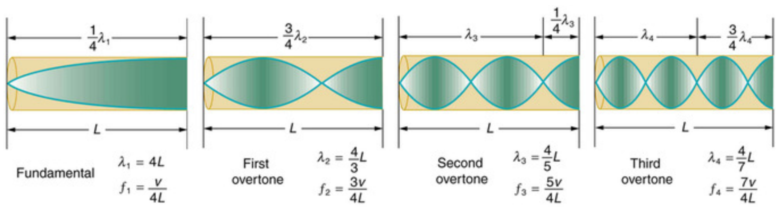
shows that this resonance can be produced by a vibration introduced at or near the closed end of the tube as well. This is considered to be a natural vibration of the air column independently of how it is induced. Given that maximum air displacements are possible at the open end and none at the closed end, there are other, shorter wavelengths that can resonate in the tube. For example, a standing wave can have three-fourths of its wavelength in the tube, or $L=(3/4)\lambda$. This can be rendered as follows:



Standing Wave in Air Column: The same standing wave is created in the tube by a vibration introduced near its closed end.

$$\lambda' = \frac{4\mathrm{L}}{3} \tag{5.9.5.1}$$

Continuing this process reveals a whole series of shorter-wavelength and higher-frequency sounds that resonate in the tube. We use specific terms for the resonances in any system. The lowest resonant frequency is called the fundamental, while all higher resonant frequencies are called overtones. All resonant frequencies are integral multiples of the fundamental, and they are collectively called harmonics. The fundamental is the first harmonic, the first overtone is the second harmonic, and so on. shows hows the fundamental and the first three overtones (the first four harmonics) in a tube closed at one end.



The First Four Harmonics: The fundamental and three lowest overtones for a tube closed at one end. All have maximum air displacements at the open end and none at the closed end.

Now let us look for a pattern in the resonant frequencies for a simple tube that is closed at one end. The fundamental has =4L, and frequency is related to wavelength and the speed of sound as given by the following:

$$\mathbf{v}_{\mathrm{w}} = \mathrm{f}\lambda \tag{5.9.5.2}$$

Solving for f in this equation gives a more helpful form:

$$\mathrm{f} = \frac{\mathrm{v}_{\mathrm{w}}}{\lambda} = \frac{\mathbf{v}_{\mathrm{w}}}{4\mathrm{L}} \tag{5.9.5.3}$$

Here, f is frequency, v_w is speed of sound in air, λ is wavelength, and L is the length of the air column. The first overtone has = 4L/3. From this, we can deduce the following:

$$\mathrm{f}' = 3\frac{v_{\mathrm{w}}}{4\mathrm{L}} = 3\mathrm{f} \tag{5.9.5.4}$$

Because f′ = 3f, we call the first overtone the third harmonic. Continuing this process, we see a pattern that can be generalized in a single expression. The resonant frequencies of a tube closed at one end are:

$$\mathrm{f}_{\mathrm{n}} = \mathrm{n}\frac{\mathrm{vw}}{4\mathrm{L}}, \mathrm{n} = 1, 3, 5 \tag{5.9.5.5}$$

Here, f1 is the fundamental, f3 is the first overtone, and so on. It is interesting that the resonant frequencies depend on the speed of sound and, hence, on temperature. This dependence poses a noticeable problem for organs in old unheated cathedrals, and it is also the reason why musicians commonly bring their wind instruments to room temperature before playing them.

Example 5.9.5.1:

The fundamental and overtones can be present simultaneously in a variety of combinations. For example, middle C on a trumpet has a sound distinctively different from middle C on a clarinet, both instruments being modified versions of a tube closed at one end. The fundamental frequency is the same (and usually the most intense), but the overtones and their mix of intensities are different and subject to shading by the musician. This mix is what gives various musical instruments (and human voices) their distinctive characteristics, whether they have air columns, strings, sounding boxes, or drumheads. In fact, much of our speech is determined by shaping the cavity formed by the throat and mouth and positioning the tongue to adjust the fundamental and combination of overtones. Simple resonant cavities can be made to resonate with the sound of the vowels, for example.

Forced Vibrations and Resonance

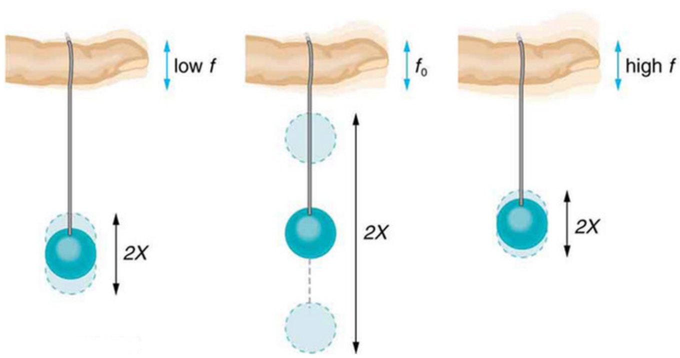
The phenomenon of driving a system with a frequency equal to its natural frequency is called resonance.

learning objectives

- Explain the relationship between the resonance curve and damping

Forced Vibration and Resonance

Many people have played with toys involving an object supported by an elastic band: something like the paddle ball suspended from a finger in. Say a person drives the paddle ball by moving his or her finger up and down at a certain frequency. In this example, he or she is causing a forced oscillation (or vibration). At first the finger is held steady, and the ball bounces up and down with a small amount of damping. If the finger is moved up and down slowly, the ball will follow along without bouncing much on its own.

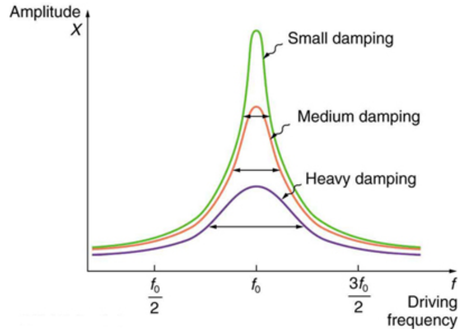


Paddle Balls and Frequencies: The paddle ball on its rubber band moves in response to the finger supporting it. If the finger moves with the natural frequency f0 of the ball on the rubber band, then a resonance is achieved, and the amplitude of the ball's oscillations increases dramatically. At higher and lower driving frequencies, energy is transferred to the ball less efficiently, and it responds with lower-amplitude oscillations.

As the frequency at which the finger is moved up and down increases, the ball will respond by oscillating with increasing amplitude. After driving the ball at its natural frequency, the ball's oscillations increase in amplitude with each oscillation for as long as it is driven. The phenomenon of driving a system with a frequency equal to its natural frequency is called resonance. As the driving frequency gets progressively higher than the resonant or natural frequency, the amplitude of the oscillations becomes smaller until the oscillations nearly disappear and the finger simply moves up and down with little effect on the ball.

Effects of Damping

In real life, most oscillators have damping present in the system. It is interesting that the widths of the resonance curves shown in depend on damping: the less the damping, the narrower the resonance. For a driven oscillator to resonate at a very specific frequency, there needs to be as little damping as possible, as is the case for piano strings and many other musical instruments. Conversely, for small-amplitude oscillations, such as in a car's suspension system, there needs to be heavy damping. Heavy damping reduces the amplitude, but the tradeoff is that the system responds at more frequencies.



Damping: The amplitude of a harmonic oscillator is a function of the frequency of the driving force. The curves represent the same oscillator with the same natural frequency but with different amounts of damping. Resonance occurs when the driving frequency equals the natural frequency, and the greatest response is for the least amount of damping. The narrowest response is also for the least amount of damping.

Example 5.9.5.2:

These features of driven harmonic oscillators apply to a huge variety of systems. When tuning a radio, for example, people are adjusting the resonant frequency of the radio circuit so that it only oscillates to the desired station's broadcast (or driving) frequency. The more selective the radio is in discriminating between stations, the smaller its damping. A child on a swing is driven by a parent at the swing's natural frequency to achieve maximum amplitude. In all of these cases, the efficiency of energy transfer from the driving force into the oscillator is best at resonance. On gravel roads that are corrugated, if people travel at the 'wrong' speed, the bumps are very noticeable. At other speeds, it is difficult to feel the bumps at all. shows a photograph of a famous example (the Tacoma Narrows Bridge) of the destructive effects of a driven harmonic oscillation. Unfortunately, heavy winds happened to drive the bridge at its natural frequency, leading to the collapse.

Collapse of the Tacoma Narrows Bridge: In 1940, the Tacoma Narrows Bridge in Washington state collapsed. Heavy cross winds drove the bridge into oscillations at its resonant frequency. The damping decreased when support cables broke loose and started to slip over the towers, allowing increasingly greater amplitudes until the structure failed. (credit: PRI's Studio 360, via Flickr)

Key Points

- The waves either interfere with each other constructively or destructively, which will either amplify or minimize the wave, respectively.
- Spherical waves are emitted from a single point source in a spherical shape.
- A plane wave is a constant-frequency wave whose wavefronts (surfaces of constant phase) are infinite parallel planes of constant peak-to-peak amplitude normal to the phase velocity vector.
- Although it is not possible in practice to have a true plane wave, many waves approximate plane wave behavior.
- The reflected wave is inverted from the incident wave when a transverse wave on a string is fixed at the end point. The reflected wave is not inverted from the incident wave when a transverse wave on a string is free at the end point.
- A standing wave occurs when an incident wave meets a reflected wave on a string.
- A standing wave contains nodes (points that remain flat due to the destructive interference) and antinodes (points with maximum oscillation due to the constructive interference).

- Every point in the string oscillates up and down and the amplitude of the oscillations depends on the location of the point.
- A standing wave has some points that remain flat due to destructive interference. These are called antinodes.
- The points on a standing wave that have reached maximum oscillation do so from constructive interference, and are called nodes.
- A standing wave in an air column is a transverse wave.
- A node occurs at the closed end of an air tube where there is no wave displacement.
- An antinode occurs at the open end of the air tube where the maximum displacement occurs.
- The resonant frequencies of a tube closed at one end are as follows: $\mathrm{f_n = n\frac{vw}{4L}}$, n=1,3,5.
- Any oscillator has a natural frequency. Driven at the natural frequency, oscillations increase in amplitude with each oscillation.
- The widths of the resonance curves depend on damping: less damping corresponds to a narrower resonance. With less damping, the resonance peak becomes more pronounced.
- The relationship between resonance and damping applies to a huge variety of systems. Damping is often reduced or enhanced to induce the desired response of an oscillator.

Key Terms

- **wavefront**: An imaginary surface passing through points of a medium oscillating in phase.
- **destructive interference**: Occurs when waves interfere with each other crest to trough (peak to valley) and are exactly out of phase with each other.
- **constructive interference**: Occurs when waves interfere with each other crest to crest and the waves are exactly in phase with each other.
- **transverse wave**: Any wave in which the direction of disturbance is perpendicular to the direction of travel.
- **natural frequency**: The frequency at which a system vibrates on its own. For a spring (spring constant k) with an object of mass m attached, the natural frequency is given as $\mathrm{f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}}$.
- **damping**: The reduction in the magnitude of oscillations by the dissipation of energy

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